The Decimal System

The decimal system was developed by the interaction of mathematicians from China and from India and is commonly used nowadays all over the world. First of all, we will recall what this system looks like. Any natural number n can be represented as:

$$n = \sum_{j=0}^{m} a_j * 10^j, \ where \ a_j \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

An interesting fact is that even though we have sixty as a base in the sexagesimalsystem (and not ten as in the decimal system), we count up to the number 59 in the decimal system, and only then do we begin bundling in blocks of sixty.

Now we will have a closer look at the history of the decimal system. The Chinese used a base-10-system from the earliest times on, "but the forms of the numbers and the mode of representation changed over the years" [Katz, 2009, p. 197]. In the Shang dynasty (around 18th century until 11th century B.C.), they had different symbols for the numbers 1 to 9. Moreover, they also had special symbols for each of the powers of 10, so their system did not look quite like ours nowadays. There were records found from the fourth century BC "of a physical system of representing numbers by counting rods, small bamboo rods about 10 cm long. These were manipulated on a counting board in which rods were arranged in vertical columns standing for the various powers of 10", [Katz, 2009, p. 198]. So, the more recent Chinese system now looked more similar to ours. A picture of what such a board and addition with it looked like can be seen in Figure 1. Furthermore, one could also use it to handle subtractions, multiplications, divisions and even root extractions.

It should be noted that the Chinese also made use of negative numbers from around 100 B.C. on (in the times of the Han dynasty), way earlier than the Europeans. One of the first people from Europe to use negative numbers was Leonardo of Pisa (better known as Fibonacci) in the 13th century, cf. [Wußing, 2003, p. 5].

Figure 1: Calculating with a Chinese counting board, the lines were represented by rods, [Wußing, 2008, p. 53]

Along the trade routes, the decimal system found its way to India, where it went through further development. In India, there were already symbols for the first nine numbers that had their origins in the Brahmi system of writing. Those symbols can be seen in the following graphic. Some of those symbols already looked pretty similar to our modern ones, for example the 7.

Einer	Ziffern	1	2	3	4	5	6	7	8	9
Zehner	Verzifferung	10	20	30	40	50	60	70	80	90
Hunderter und Tausender	Stellenschrift	100	200	500	1000	4000	70000			

Figure 2: The Brahmi symbols of writing, [Wußing, 2008, p. 98].

The Indians took over the place value system with the base of 10, but they used their own symbols (replacing the rods) and dropped their old system where they had symbols for every number, even the ones that are higher than 9. That was around the year 600 (cf. [Katz, 2009, p. 233]). From this time on, the decimal system did not undergo drastic changes anymore, except for the introduction of the number zero by the Indians (see Chapter *The number Zero*). Merchants brought the system from India to the Islamic countries (in Bagdad, it was around the 8th century) and even back to China. From Arabia thereafter, the decimal system made its triumphal march into the European countries.

References

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