

Approximation Algorithms

Lecture 2:

SETCOVER and SHORTESTSUPERSTRING

Part I:

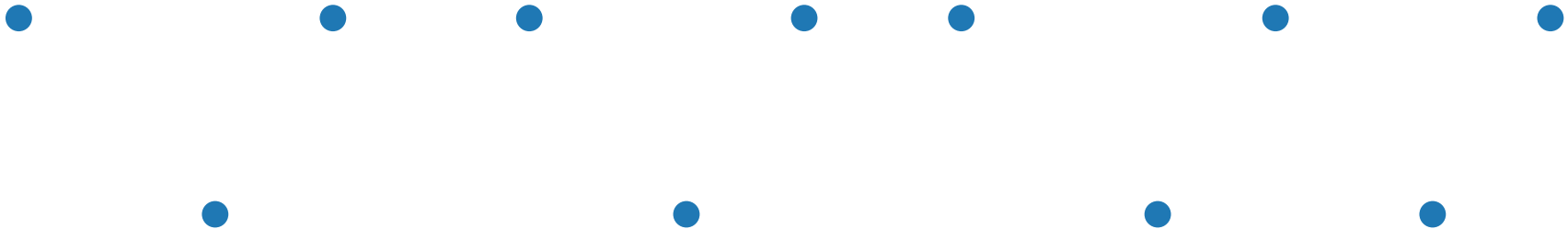
SETCOVER

SETCOVER (card.)

Let U be some **ground set** (universe),

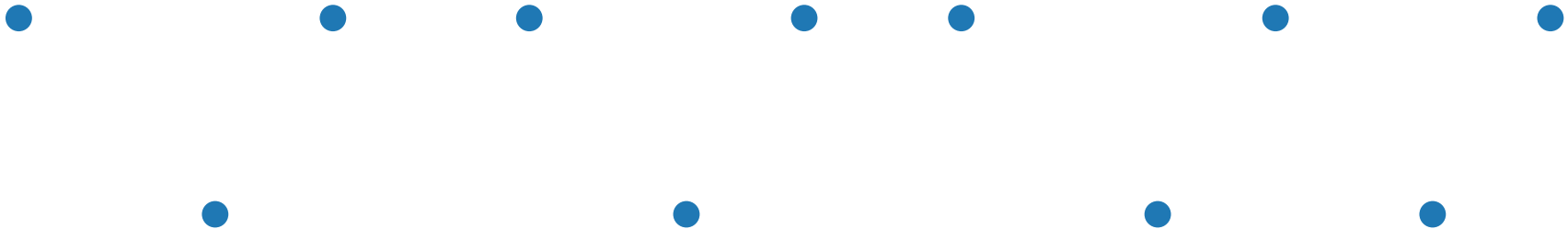
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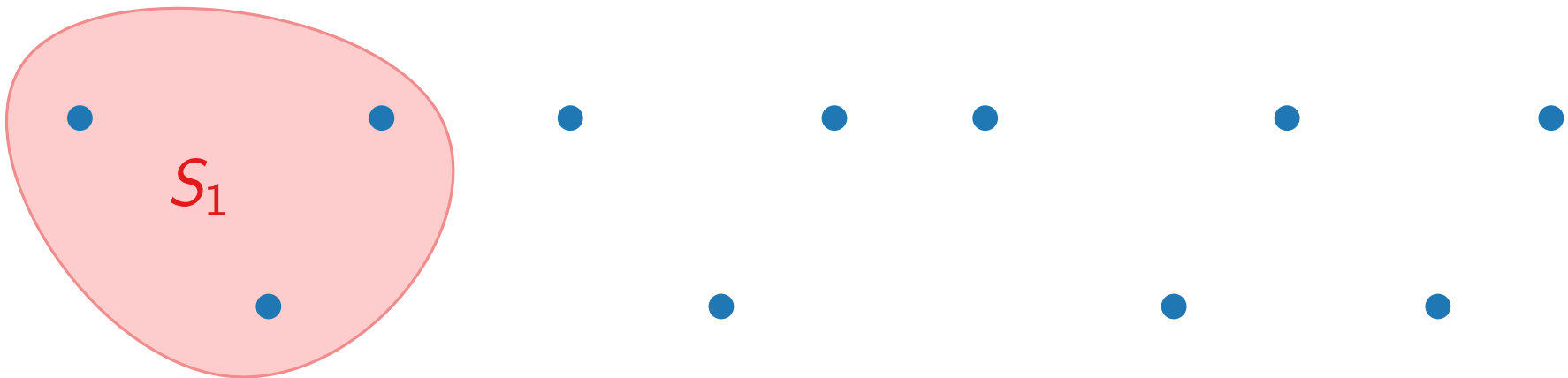
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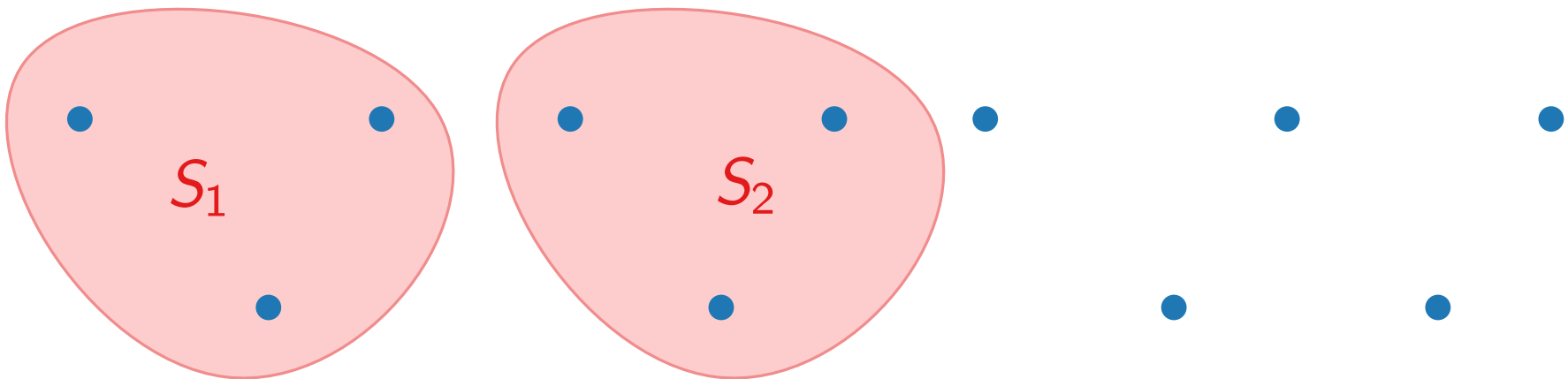
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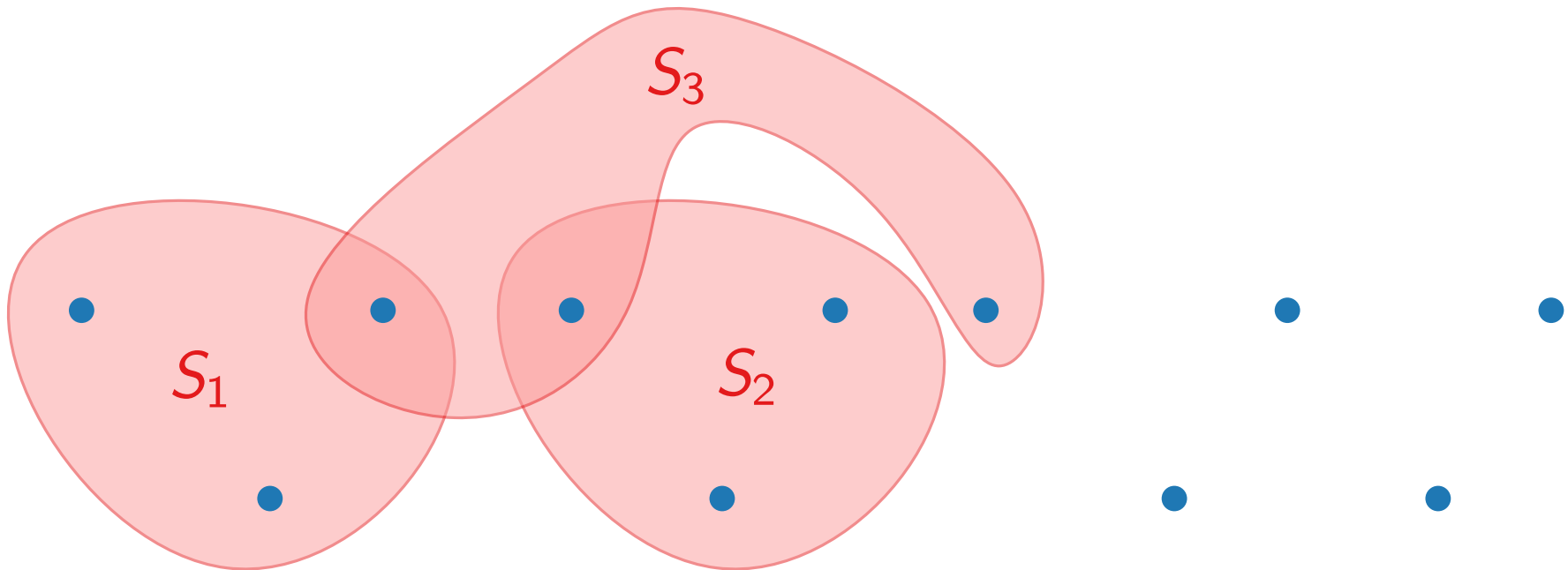
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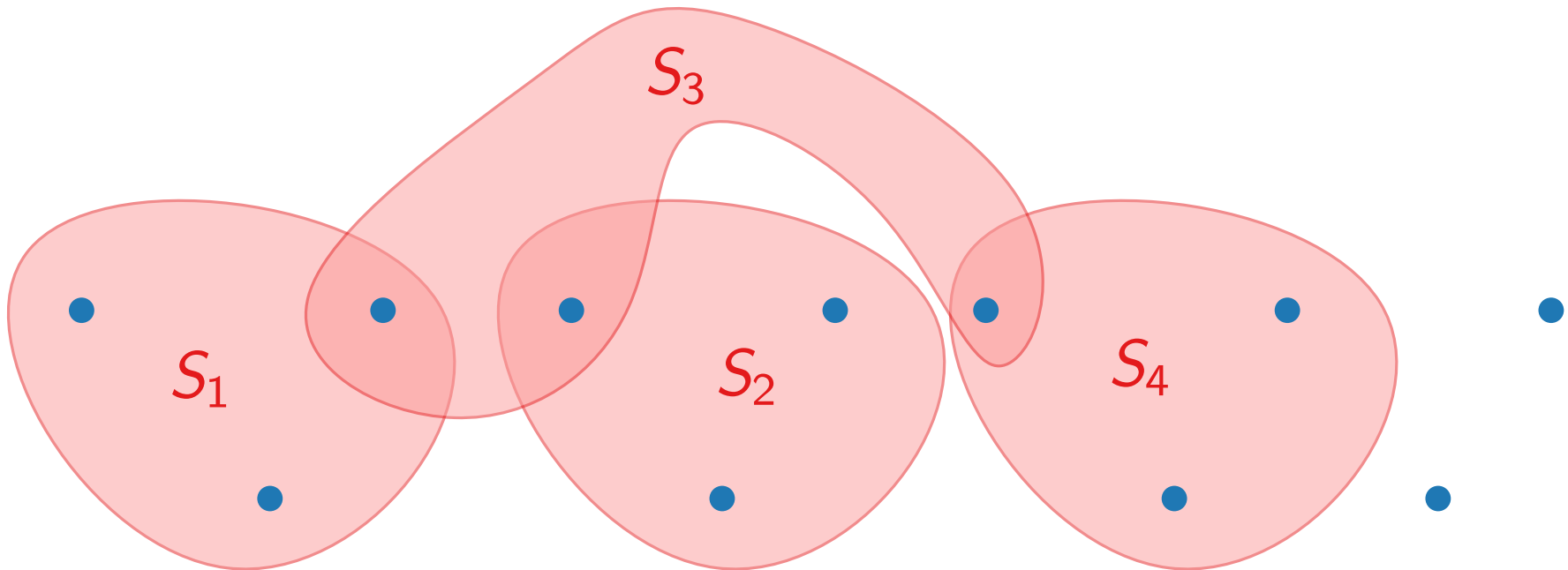
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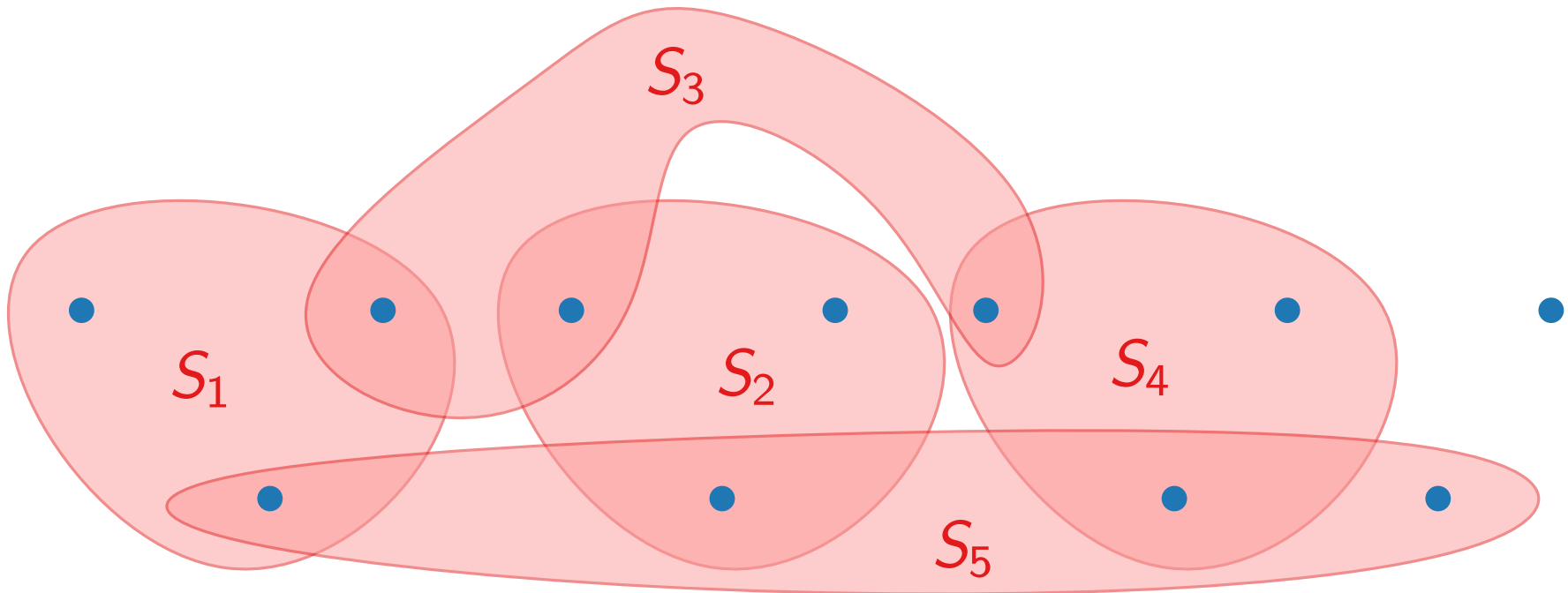


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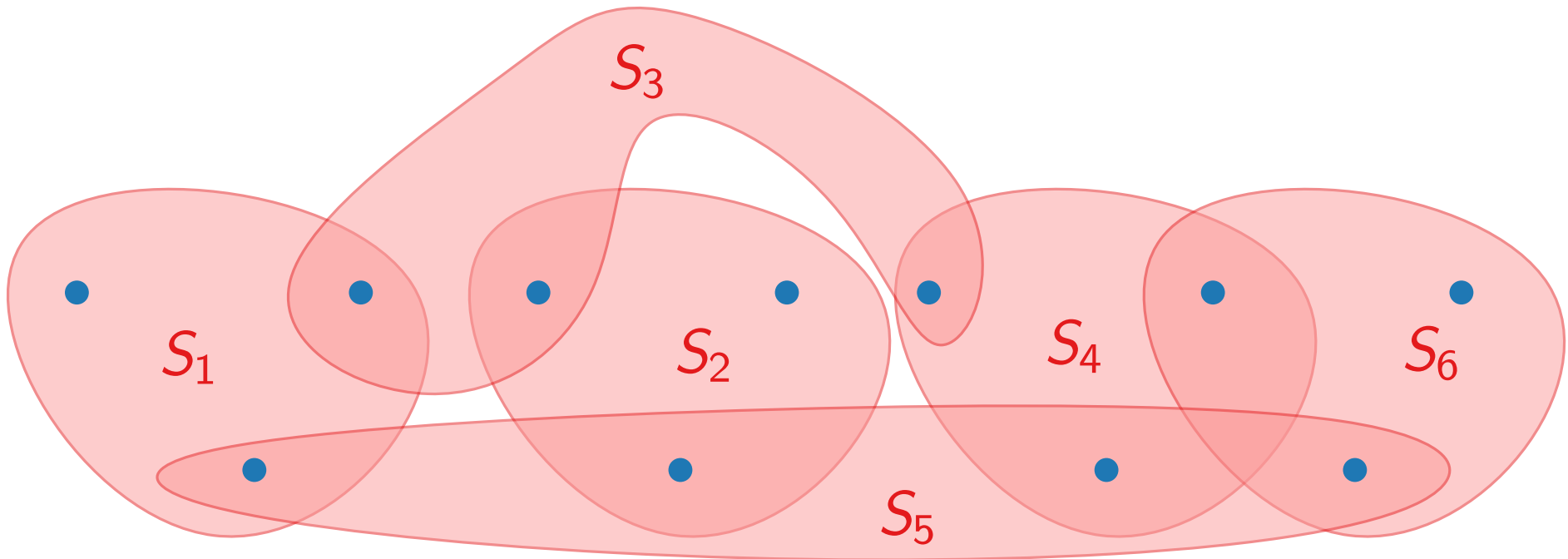


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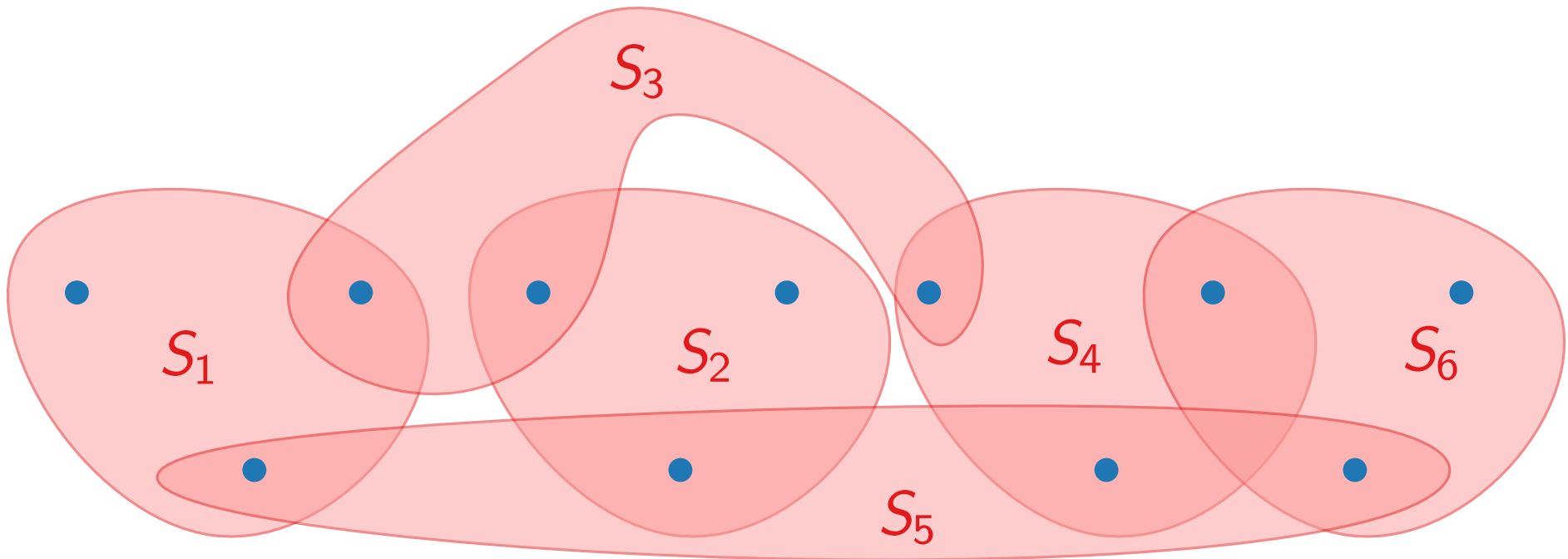
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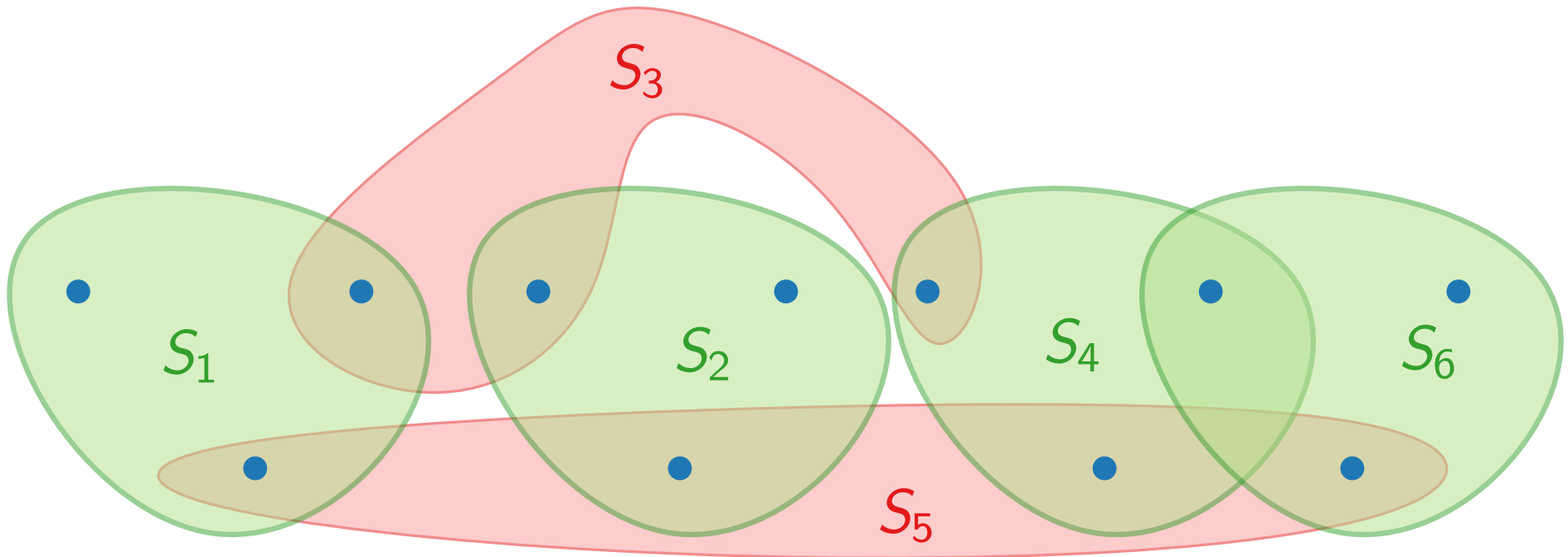
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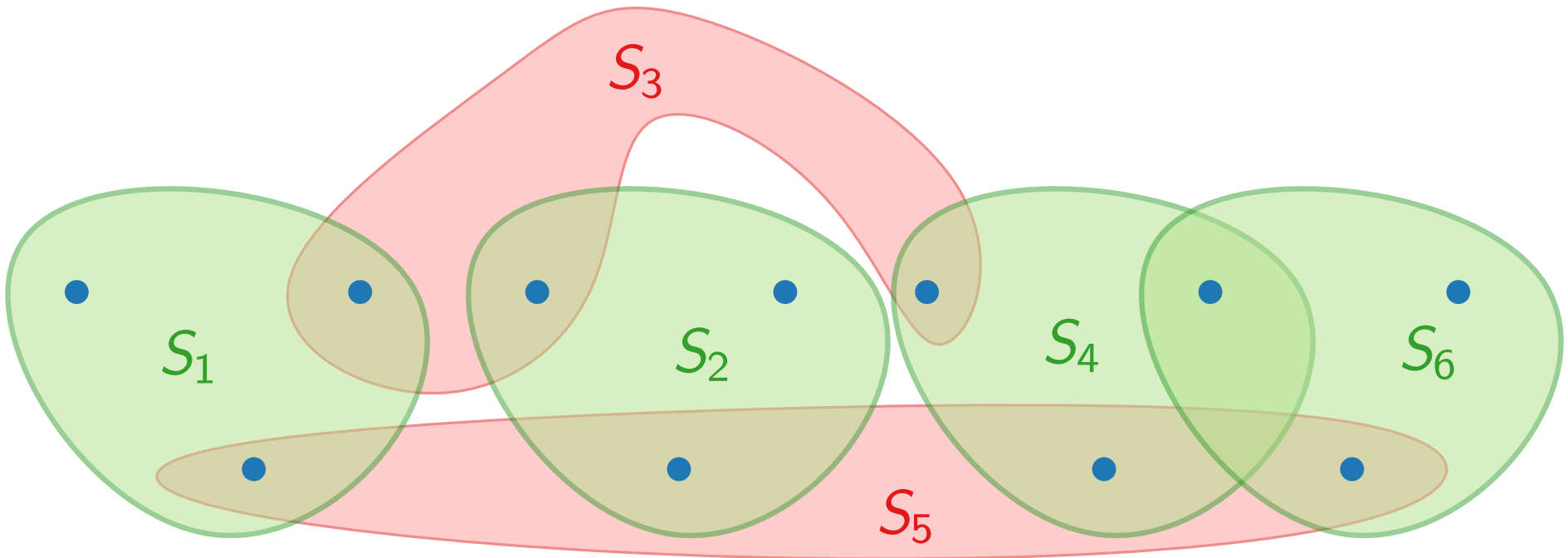
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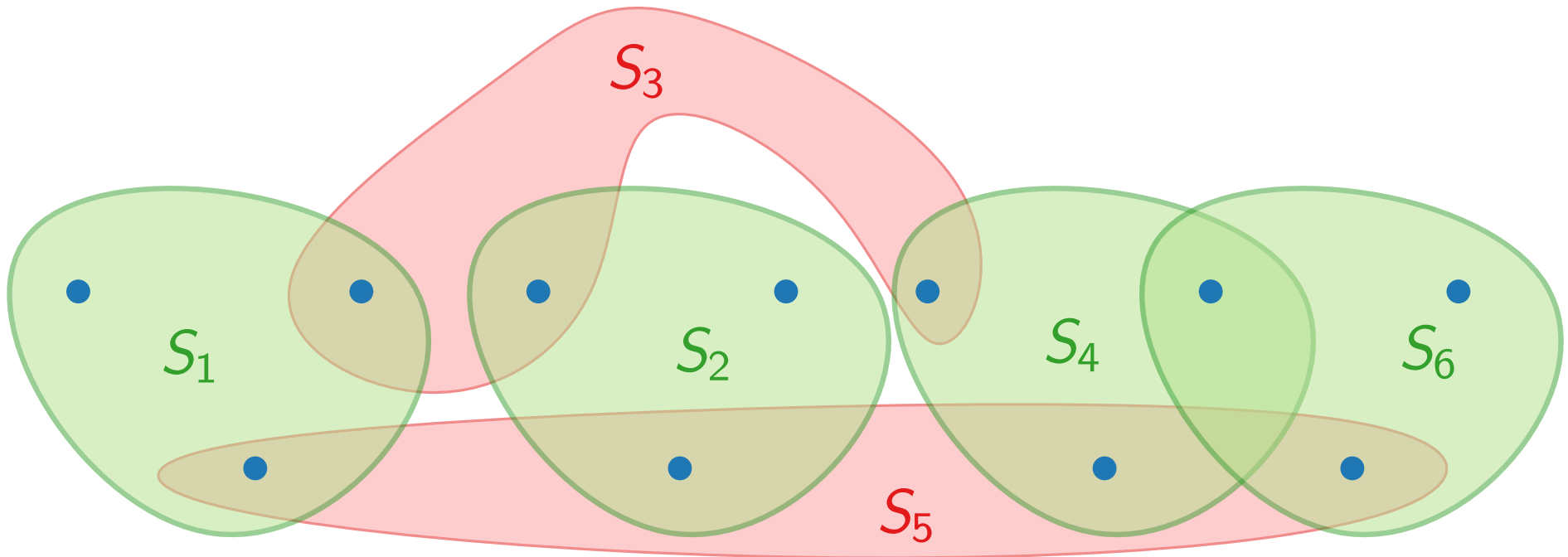


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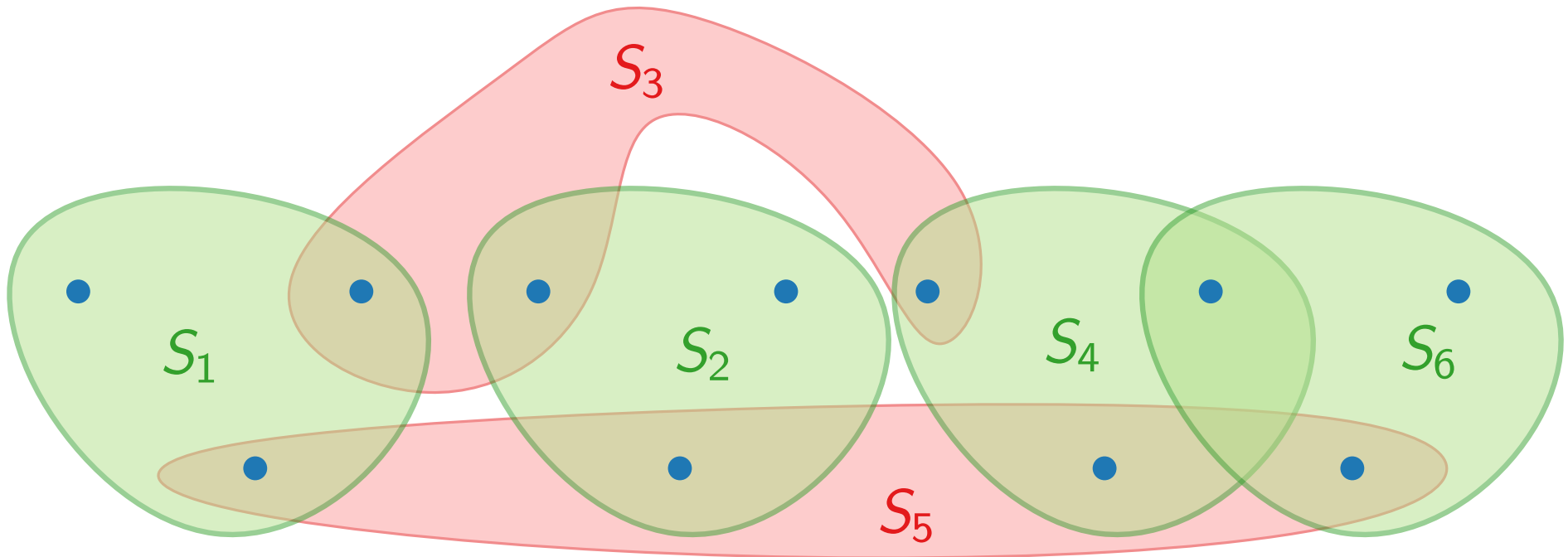


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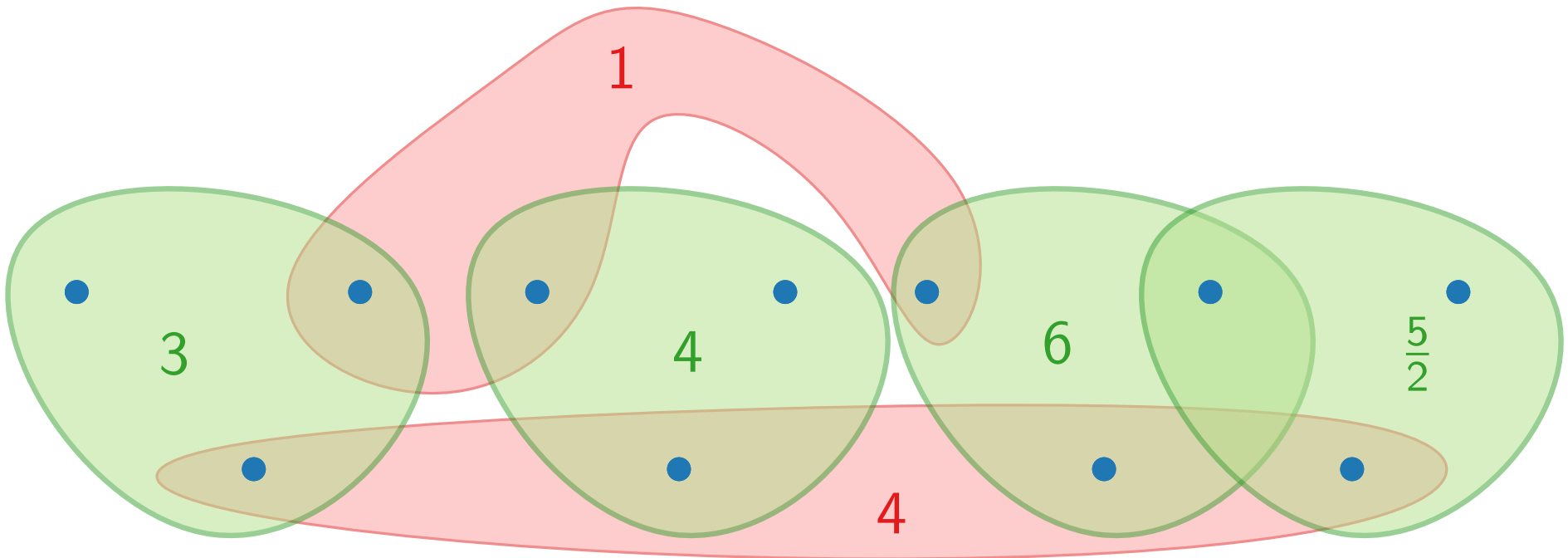


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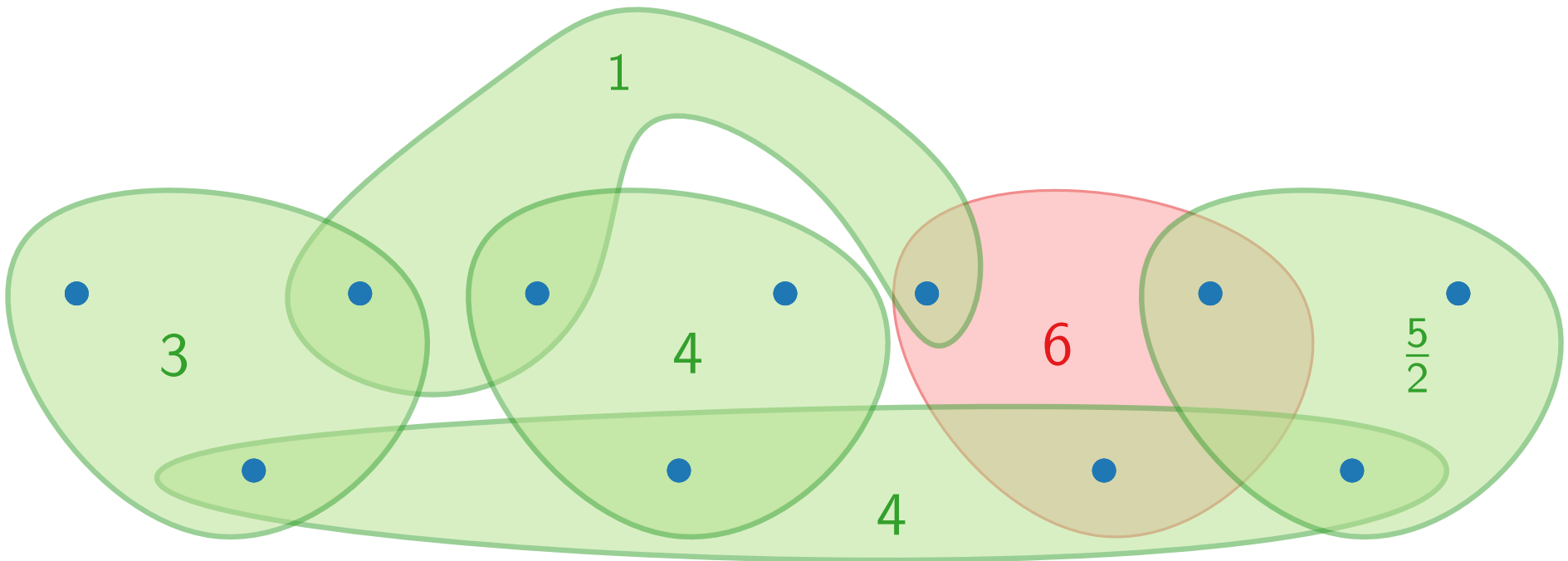


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Approximation Algorithms

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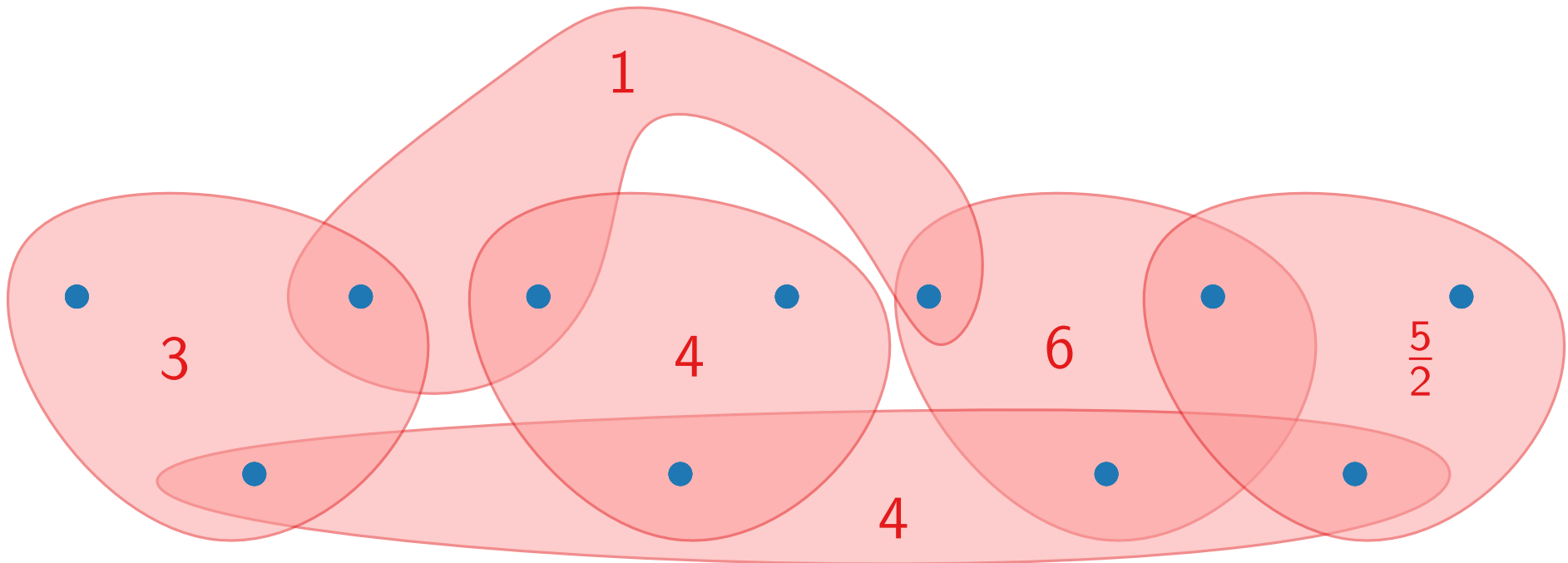
SETCOVER and SHORTESTSUPERSTRING

Part II:

Greedy for SETCOVER

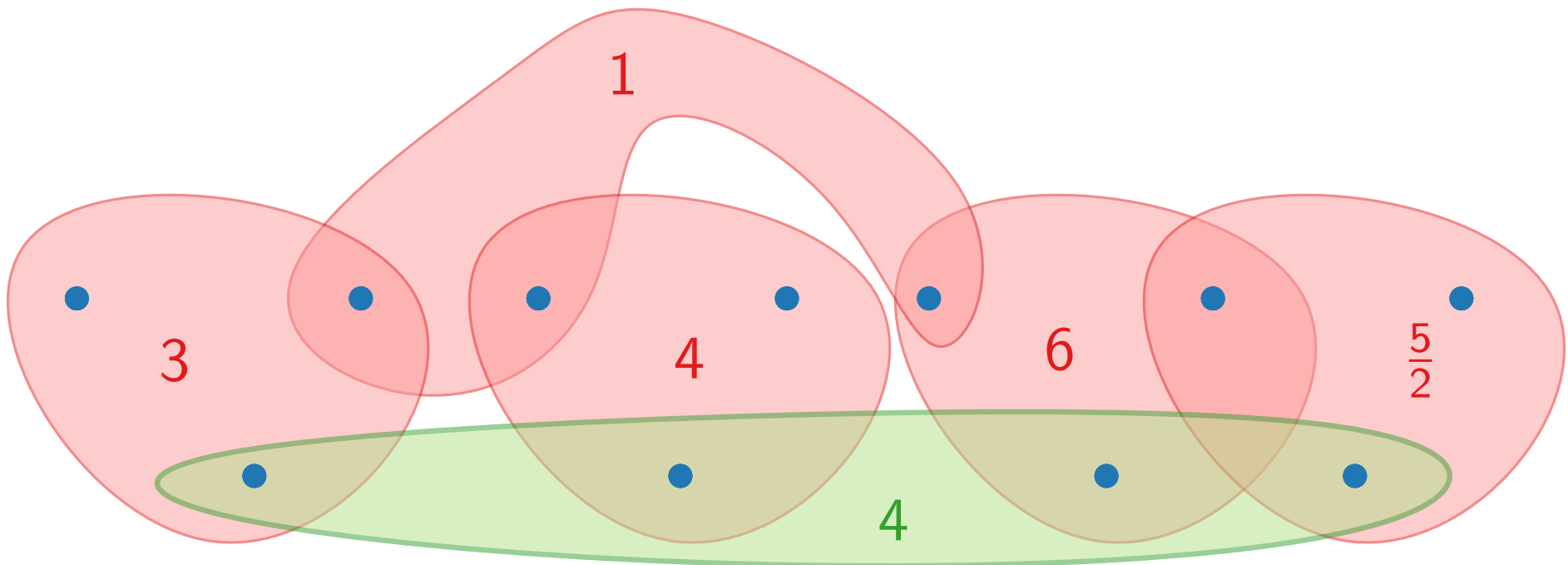
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What is the real cost of picking a set?



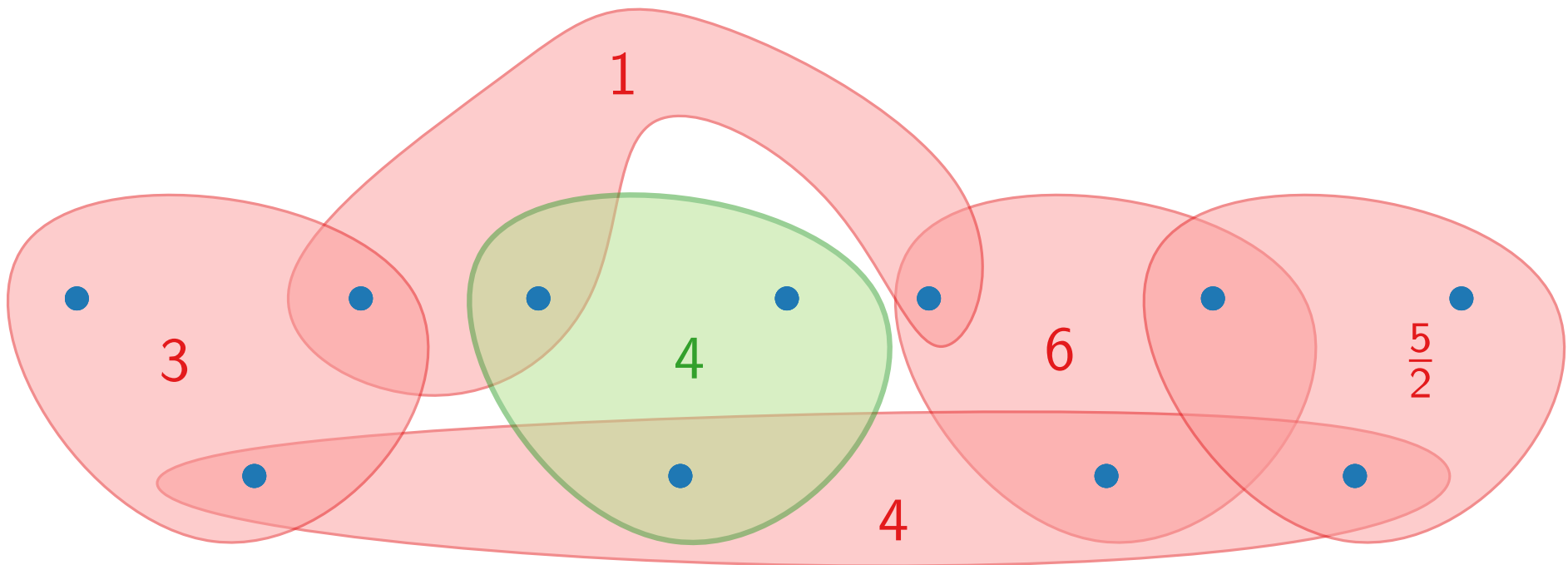
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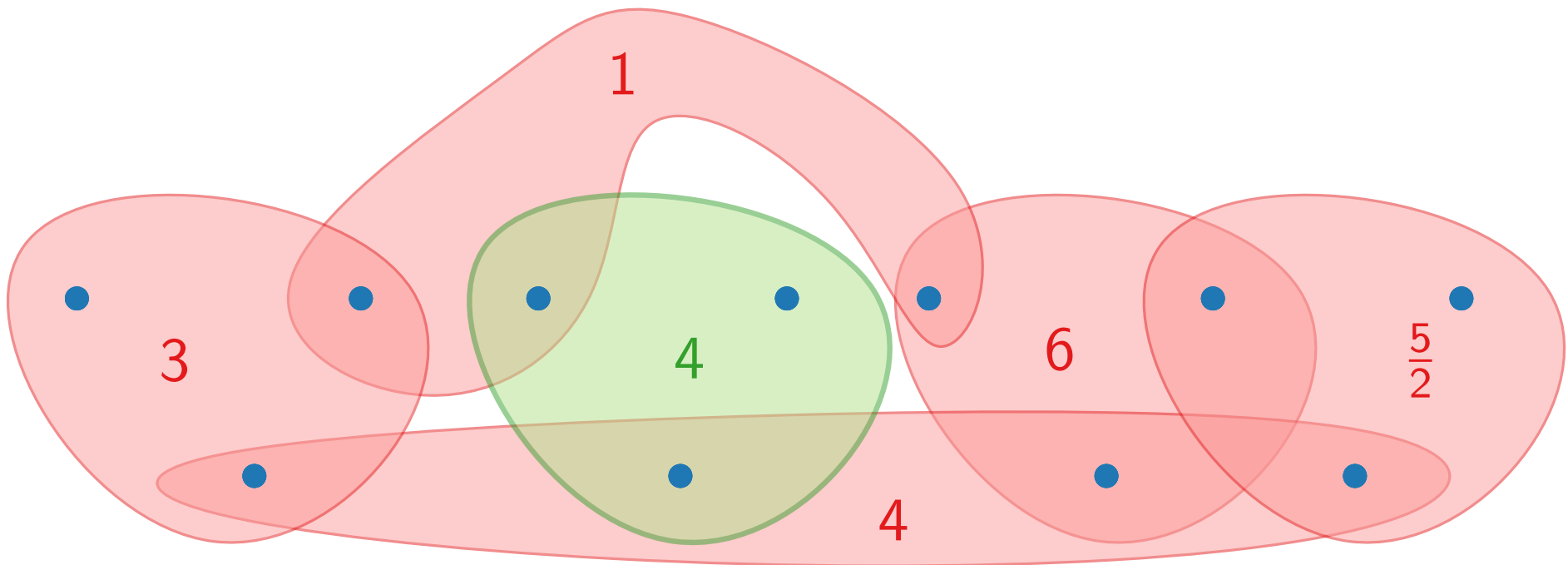
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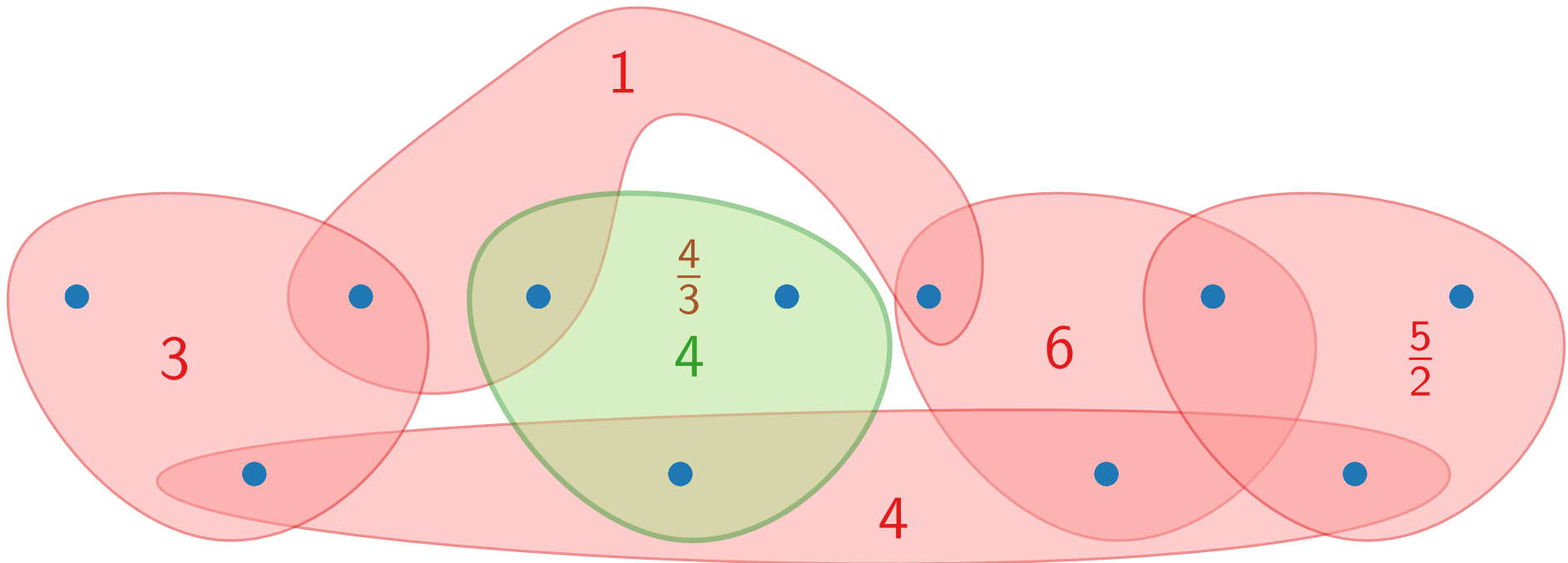
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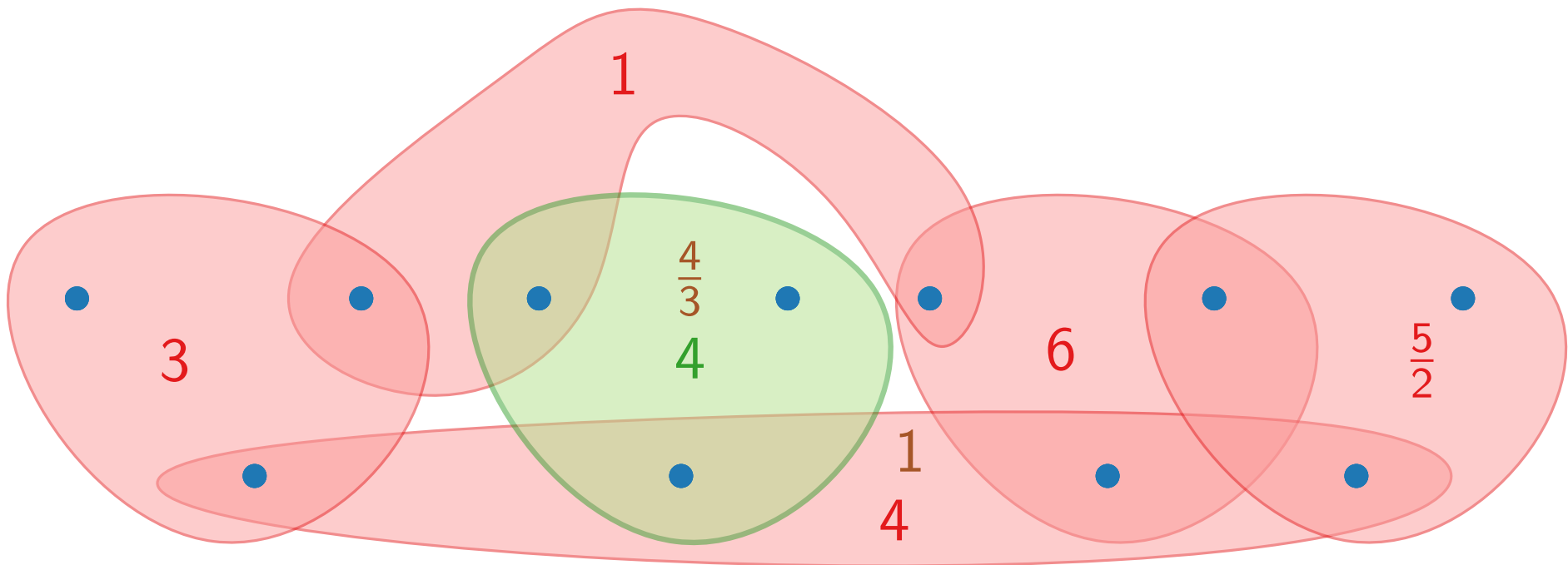
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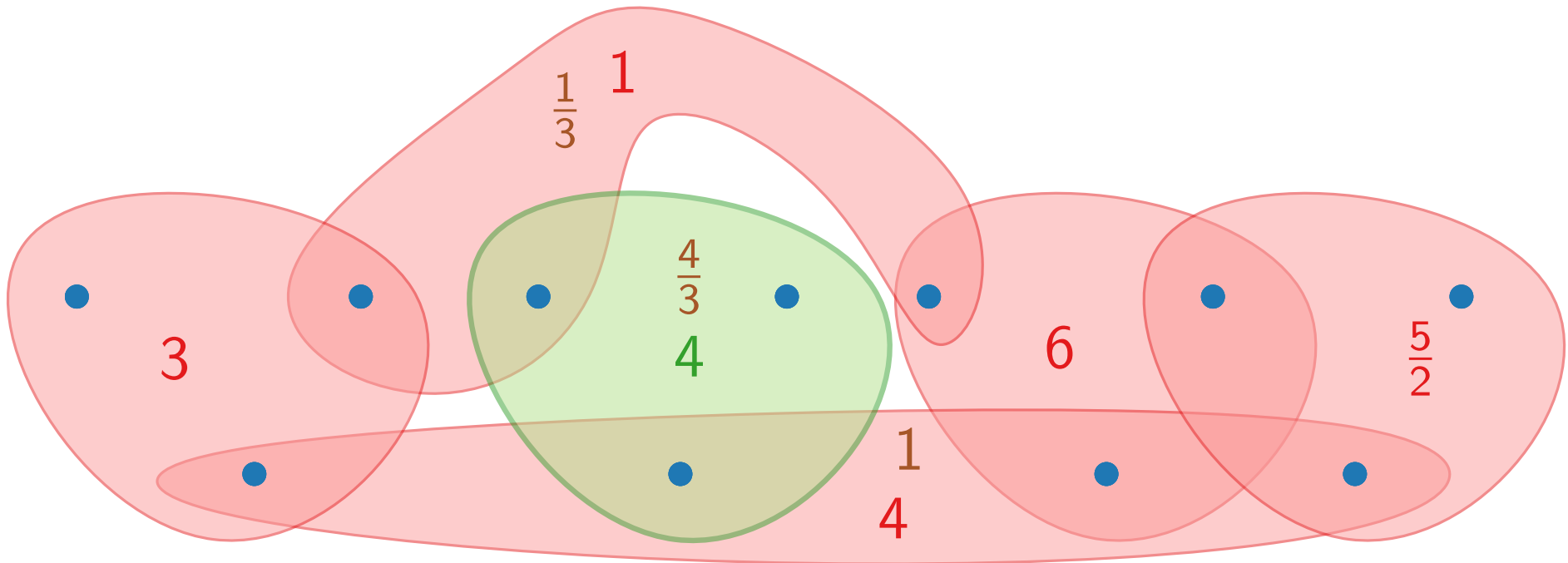
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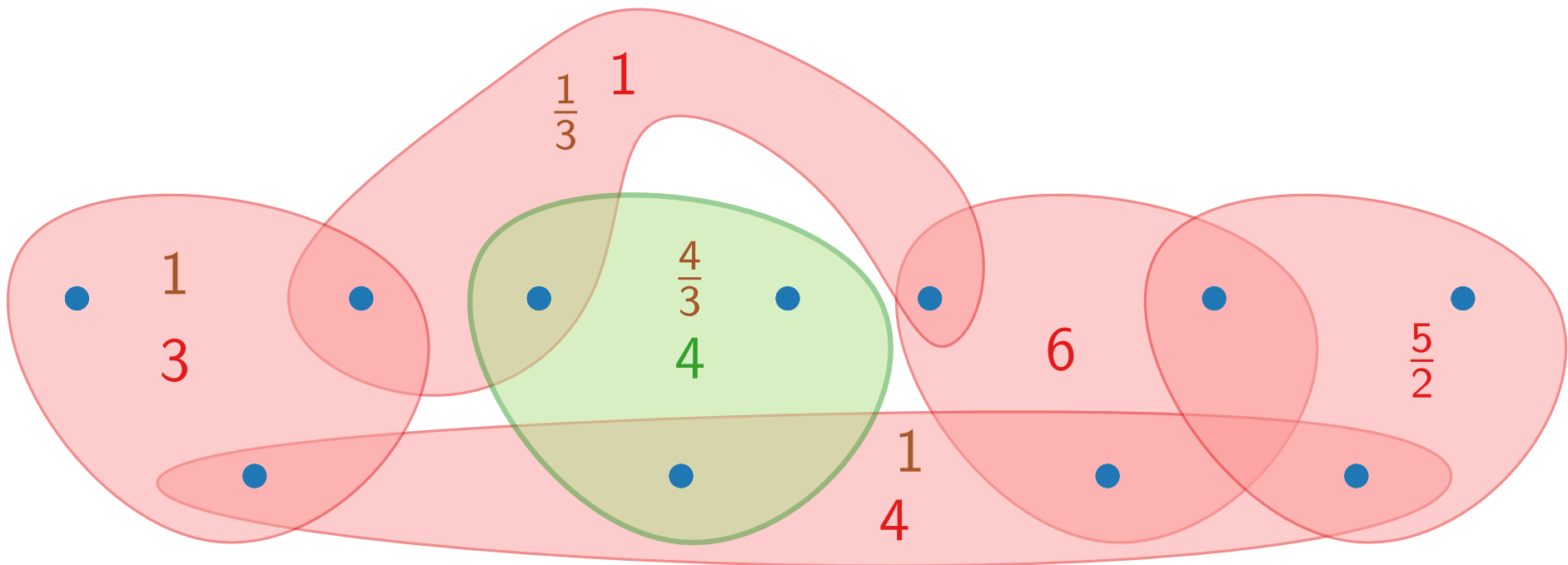
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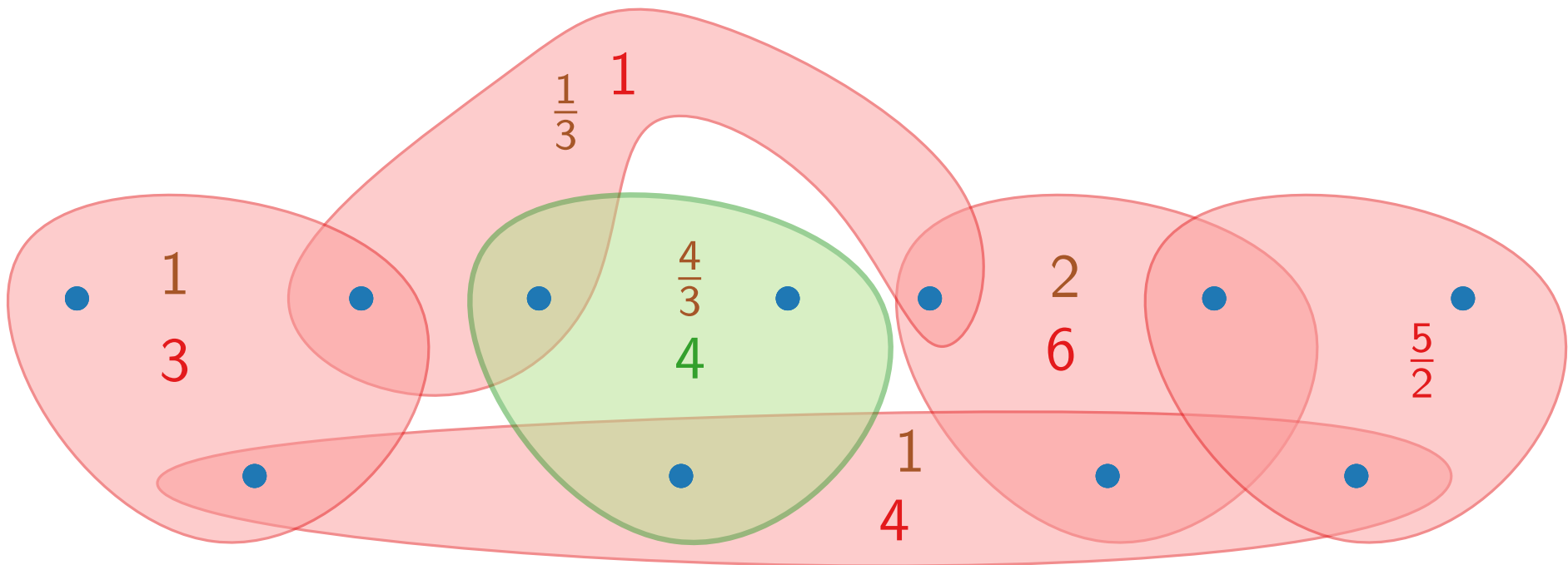
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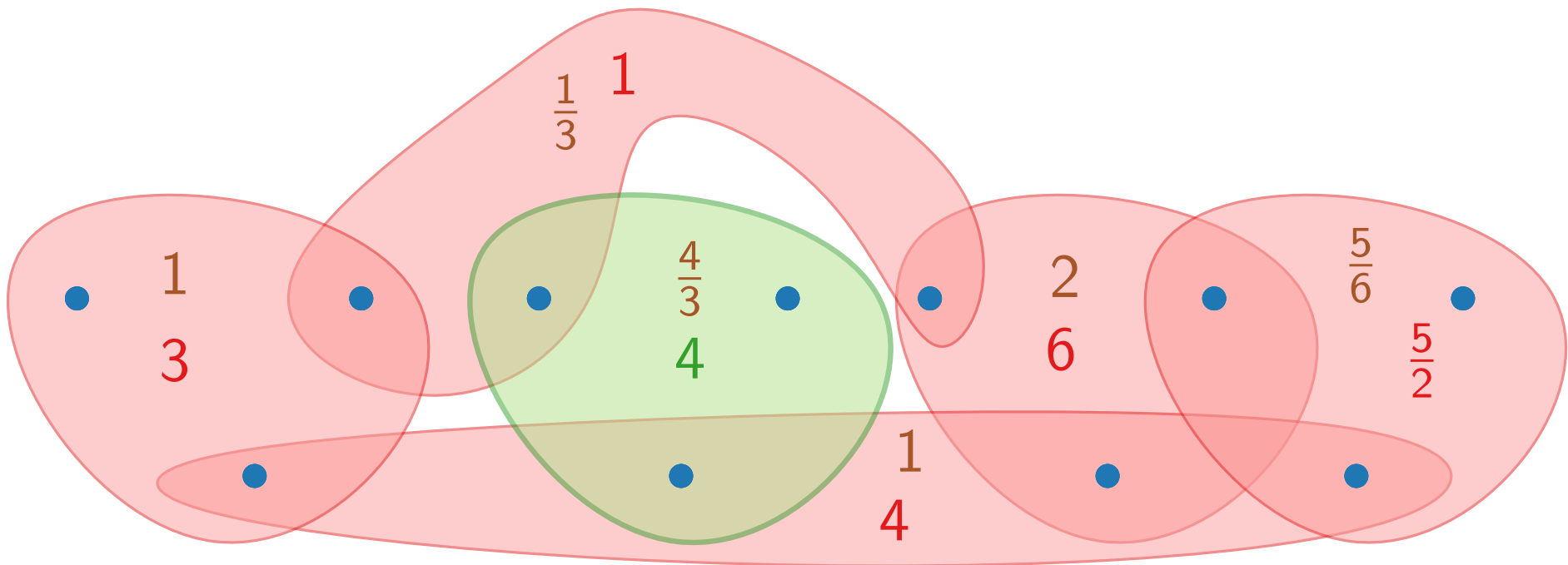
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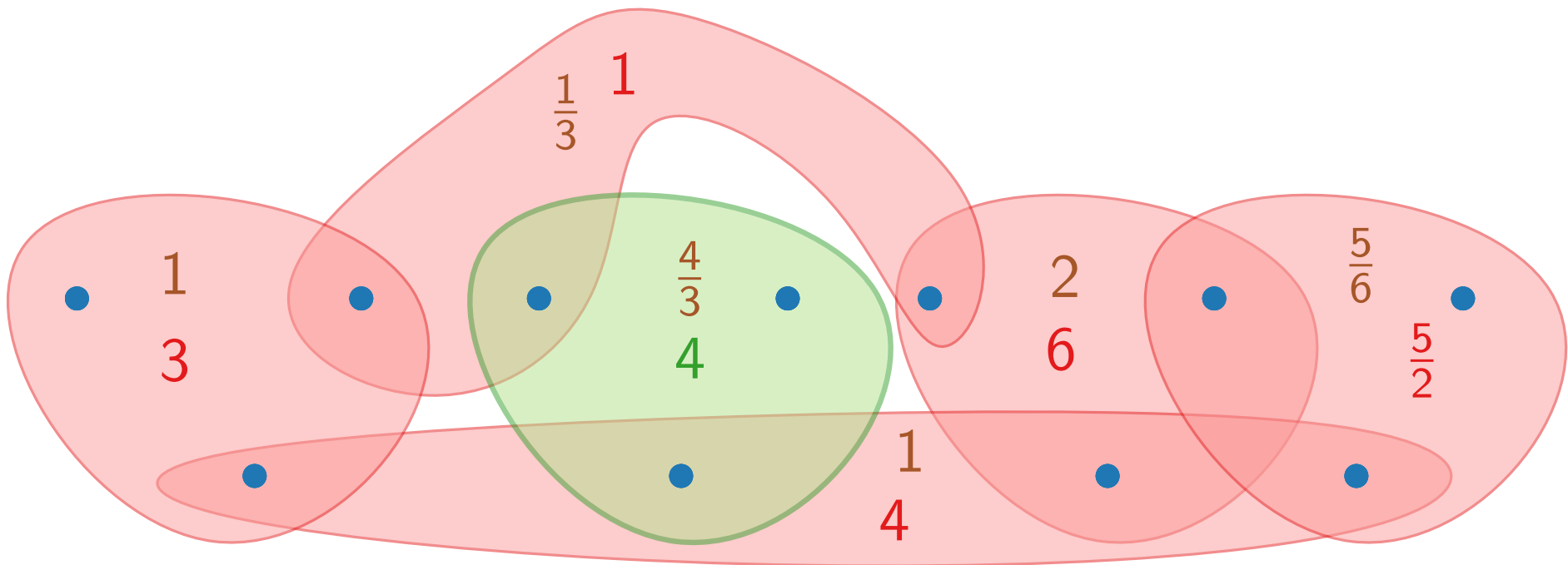


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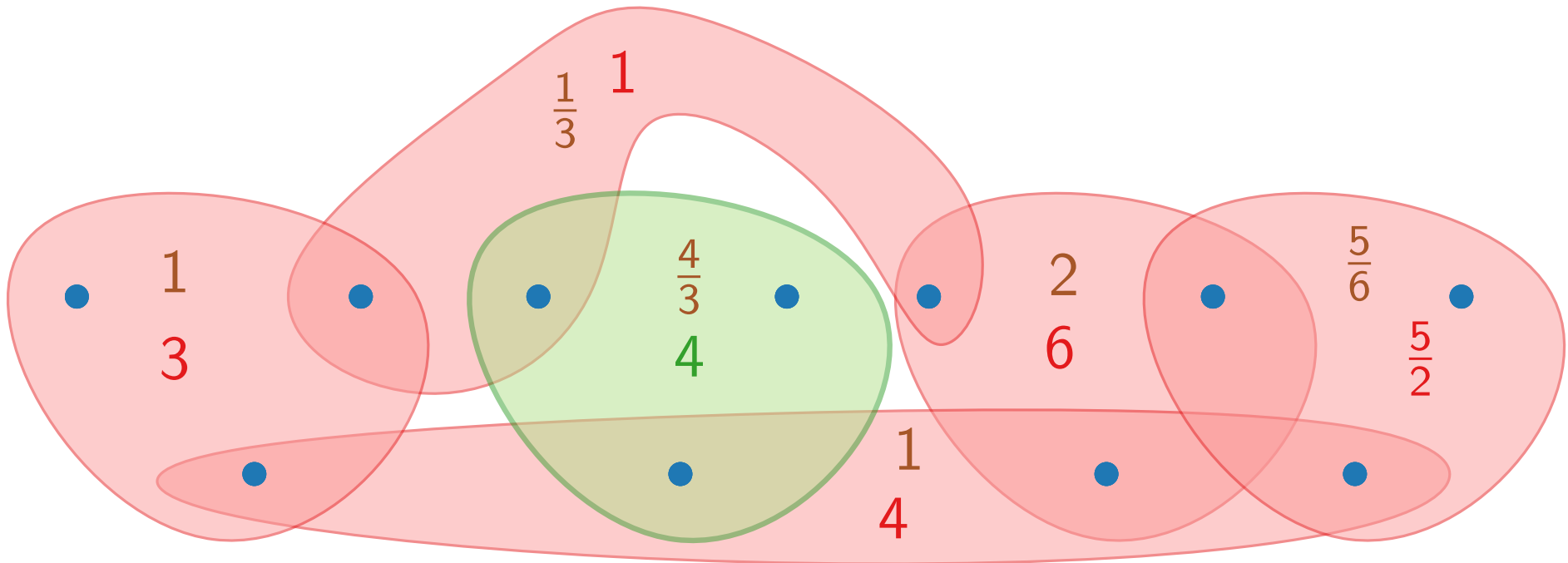
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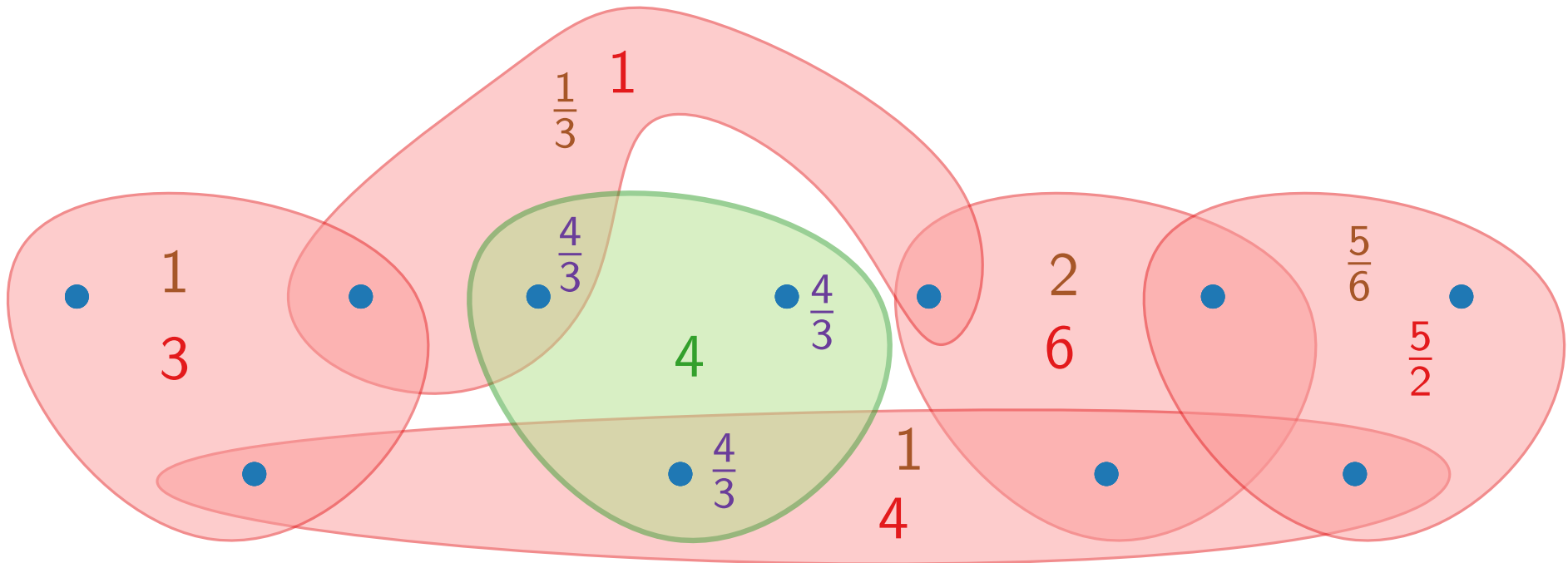
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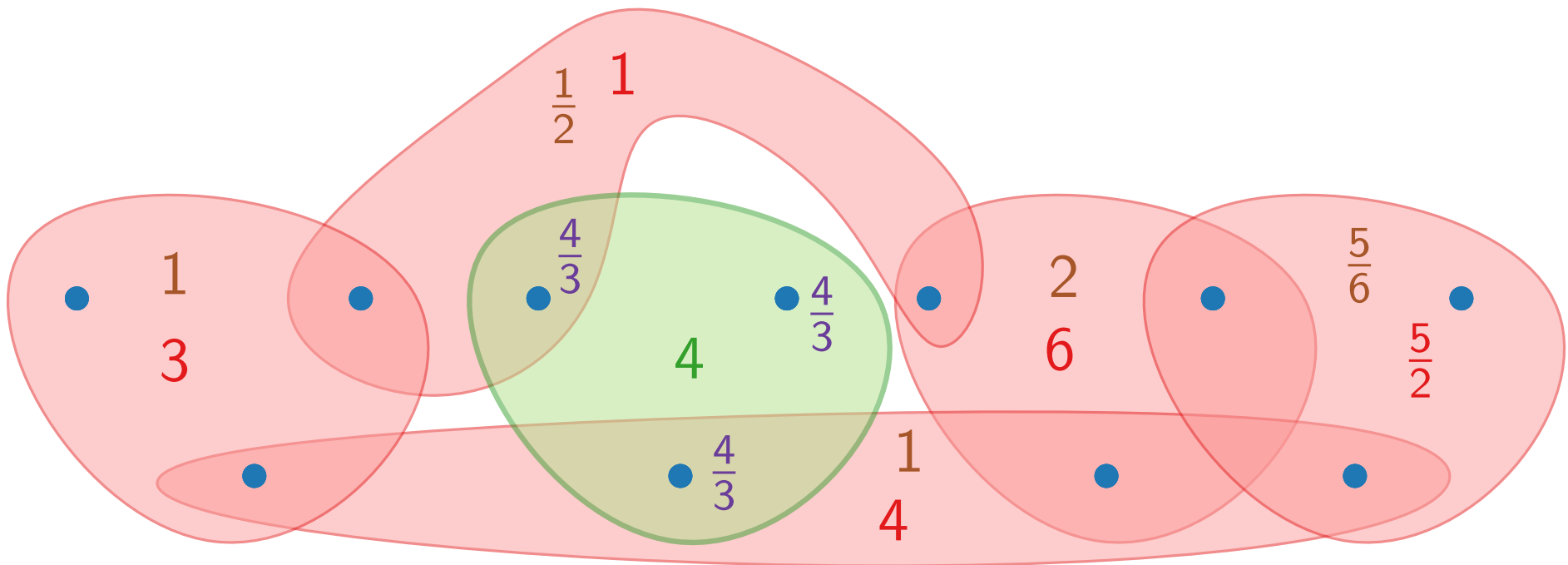
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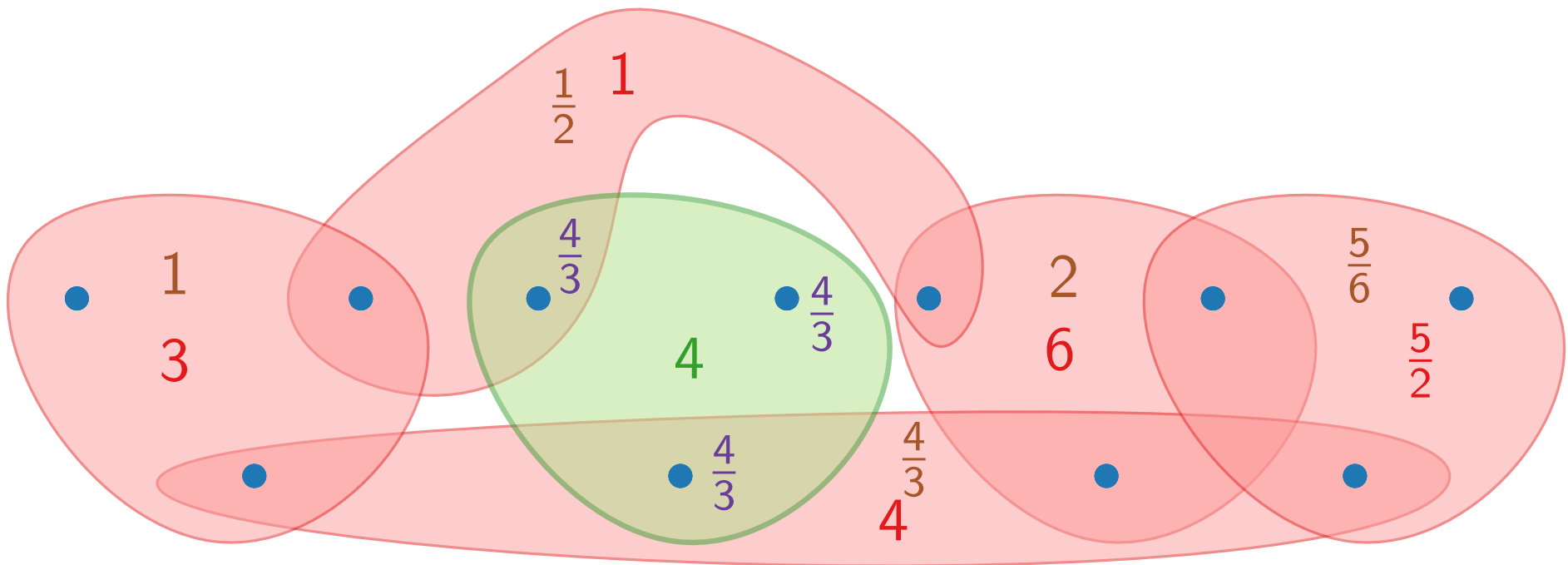
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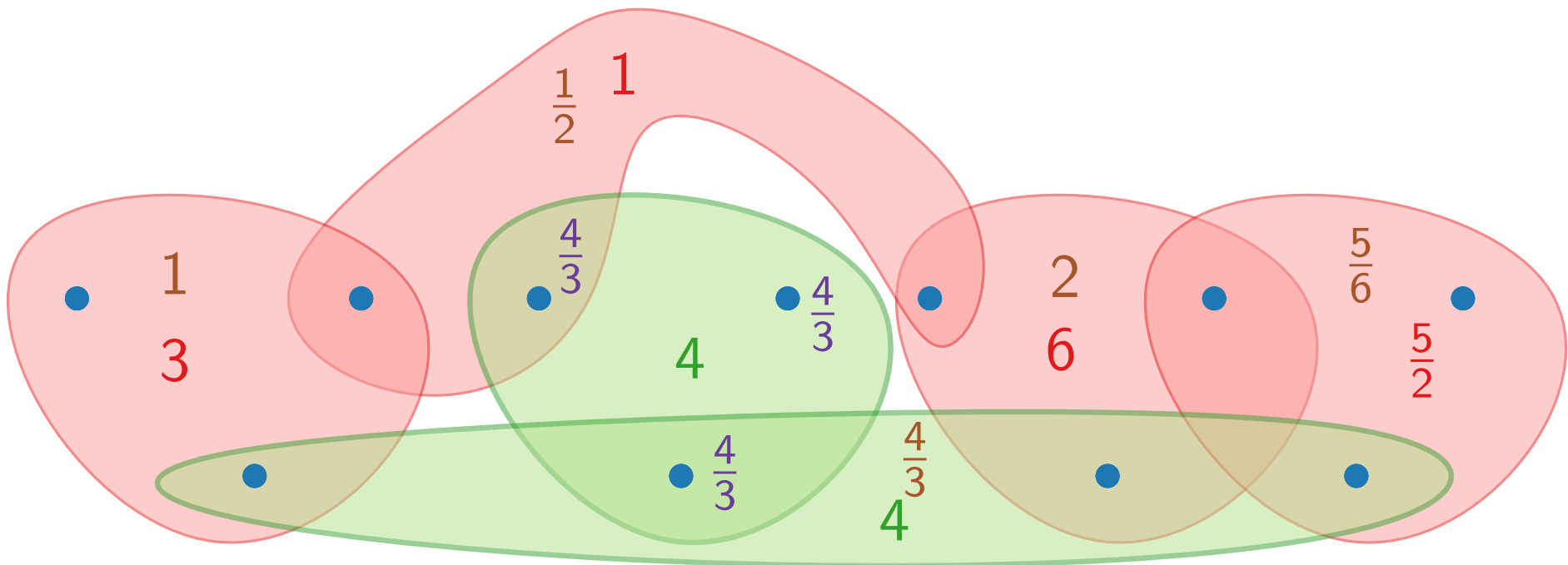
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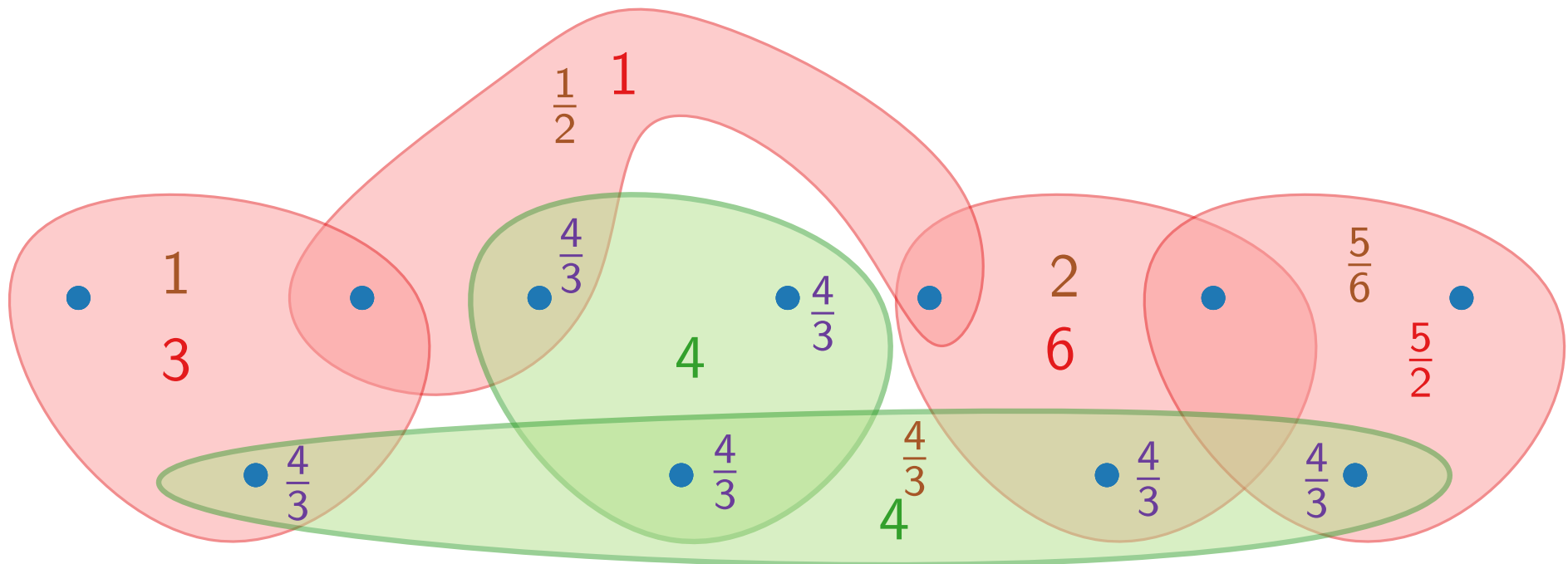
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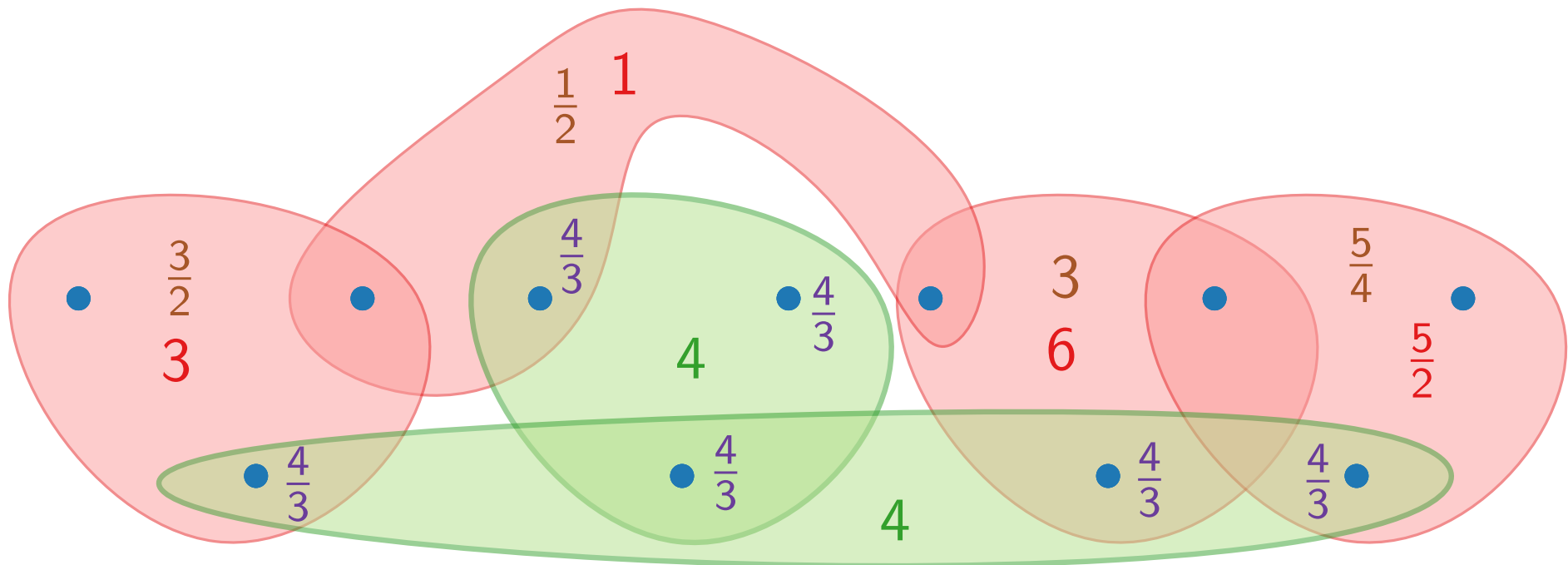
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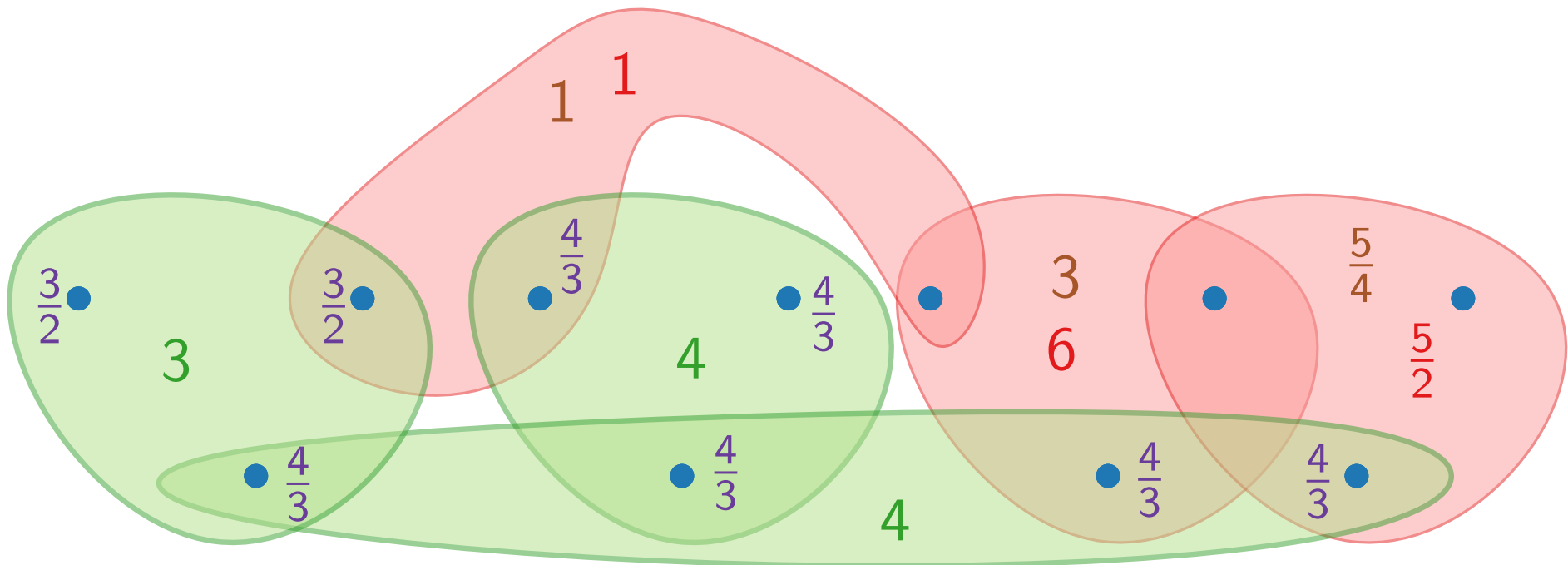
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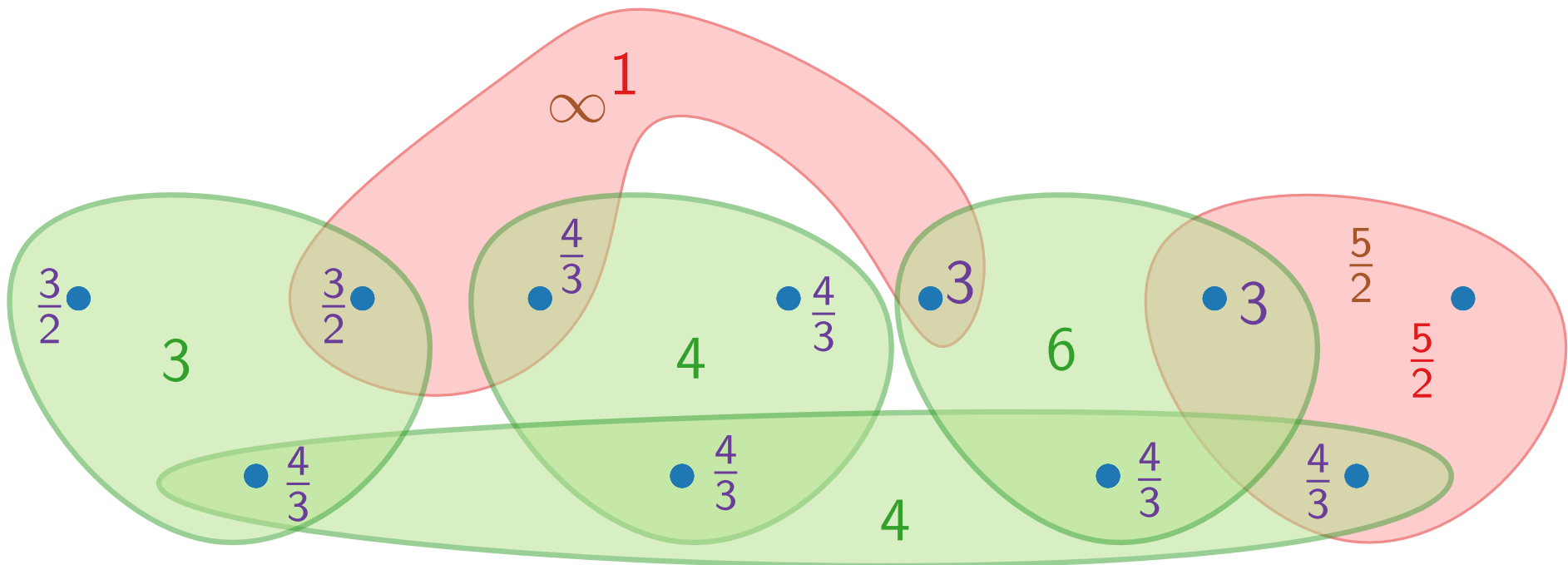
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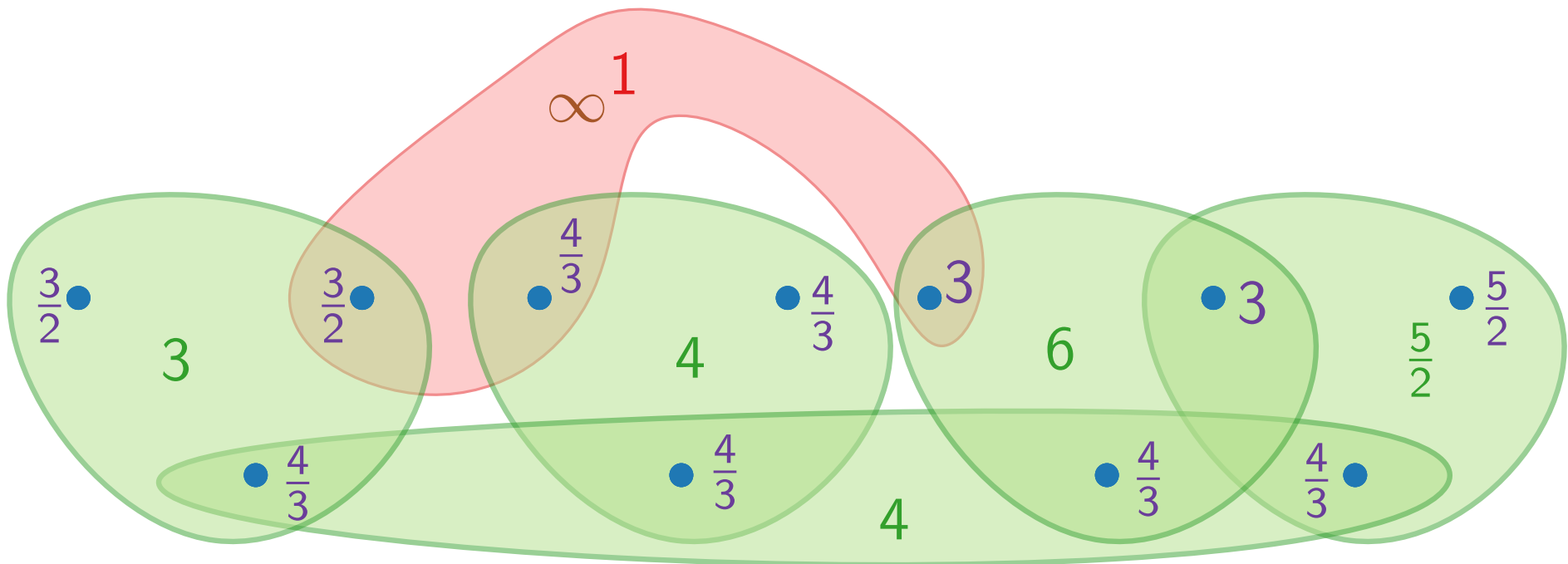
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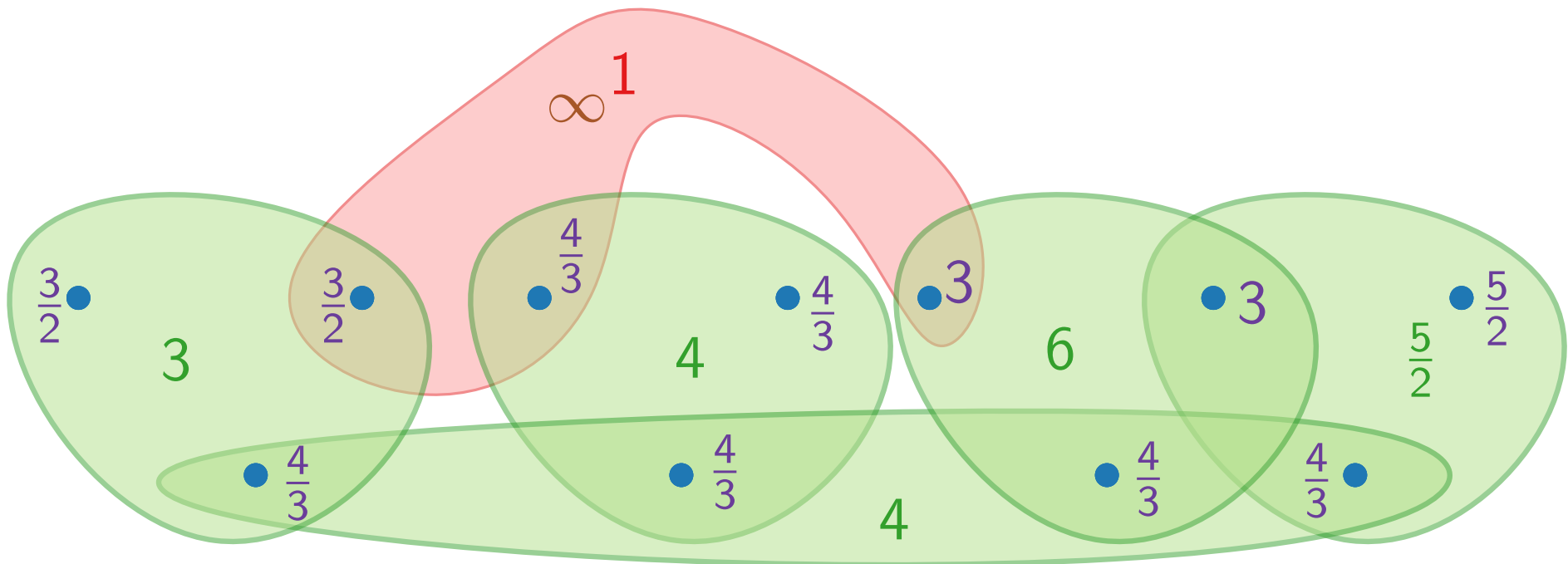
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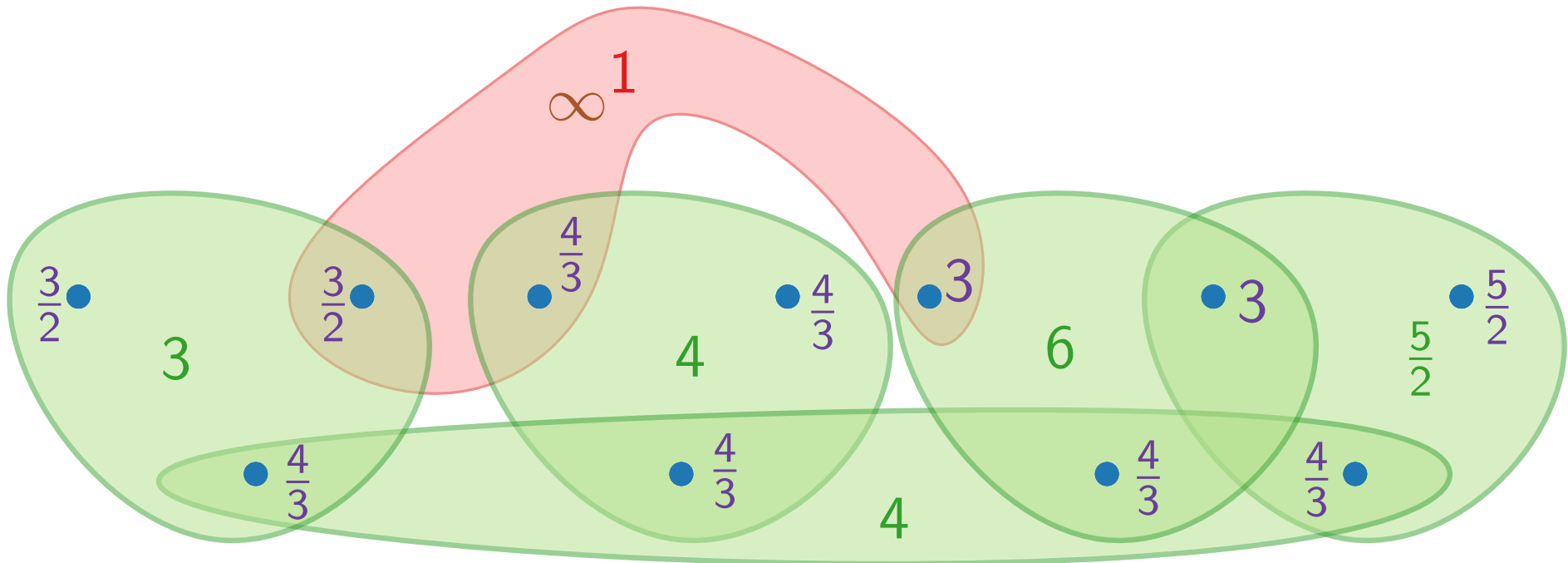
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$$\text{total cost: } \sum_{u \in U} \text{price}(u)$$

Greedy: Always choose the set with minimum **per-element cost**.



Greedy for SETCOVER

GreedySetCover(U , S , c)

$C \leftarrow \emptyset$

$S' \leftarrow \emptyset$

return S'

// Cover of U

Greedy for SETCOVER

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$C \leftarrow \emptyset$

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while $C \neq U$ **do**

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GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

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while $C \neq U$ **do**

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$S' \leftarrow S' \cup \{S\}$

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Approximation Algorithms

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SETCOVER and SHORTESTSUPERSTRING

Part III: Analysis

Analysis

Theorem. GreedySetCover is a factor- $\mathcal{H}_k$ approximation algorithm for SETCOVER, where k is the cardinality of the largest set in $\mathcal{S}$ and

$$\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k} \rightarrow 0.5 + \ln k.$$

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Lemma. Let $S \in \mathcal{S}$, and let $u_1, \dots, u_\ell$ be the elements of S in the order in which they are covered (“bought”) by GreedySetCover. Then, for every $j \in \{1, \dots, \ell\}$: $\text{price}(u_j) \leq$

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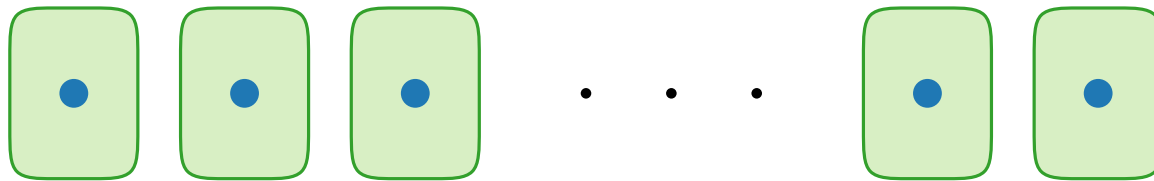
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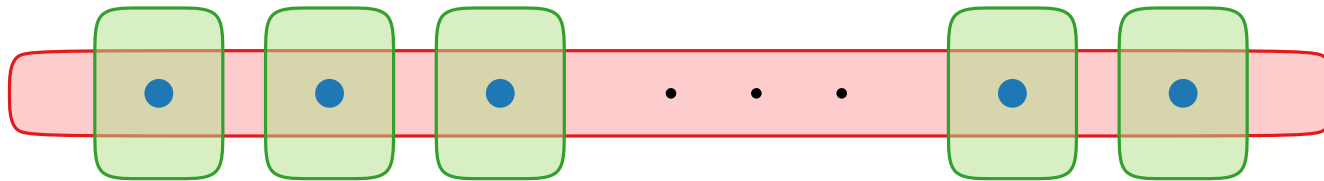
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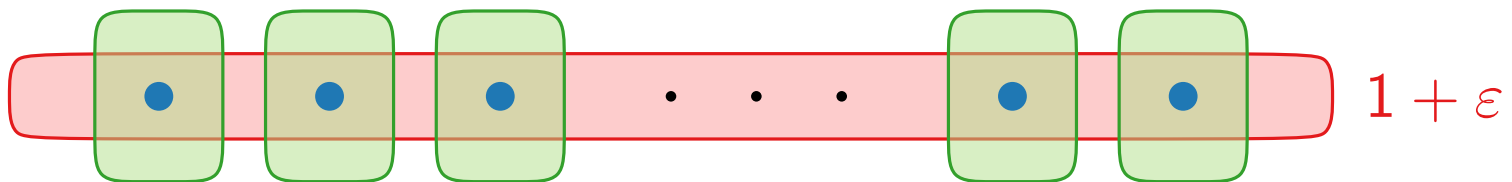
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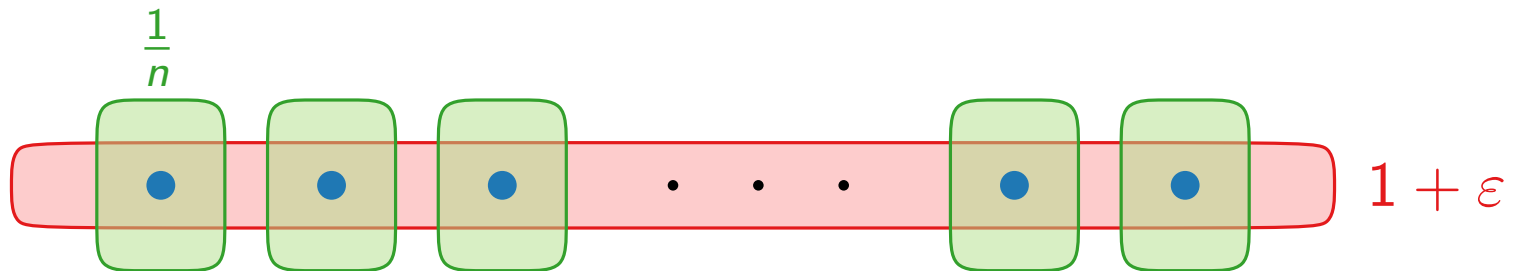
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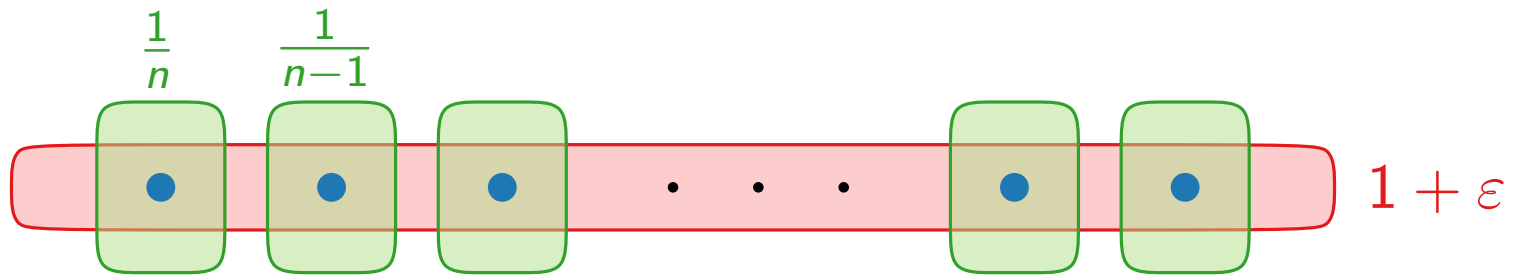
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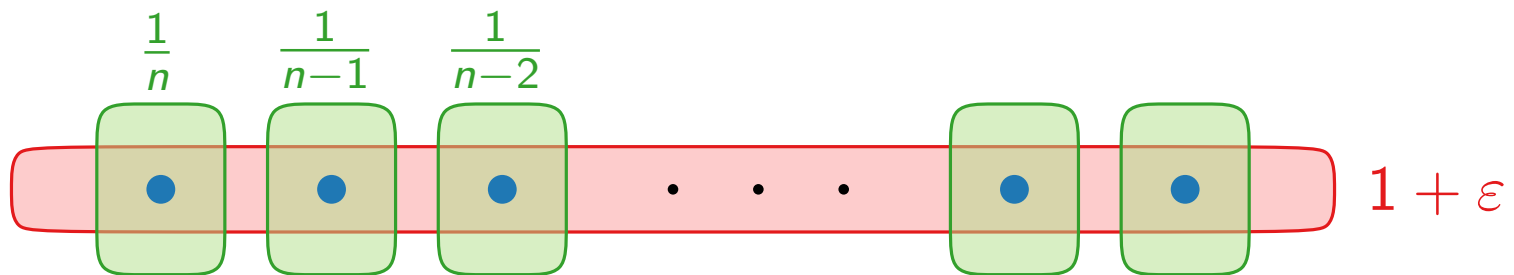
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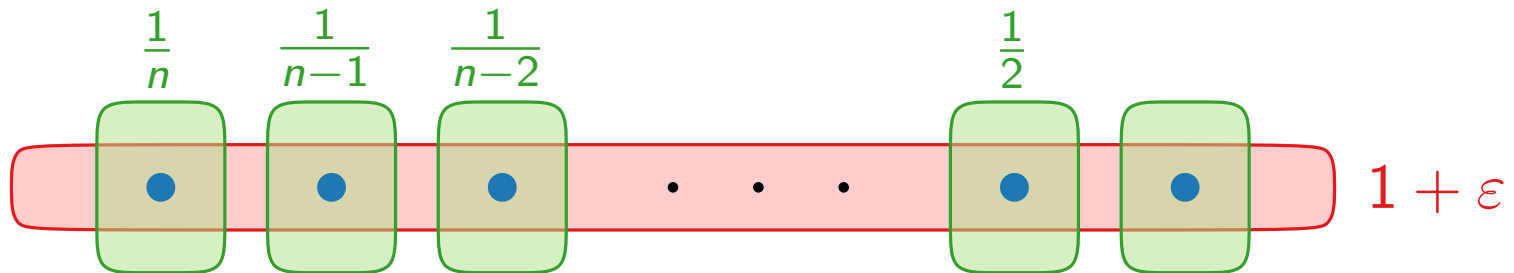
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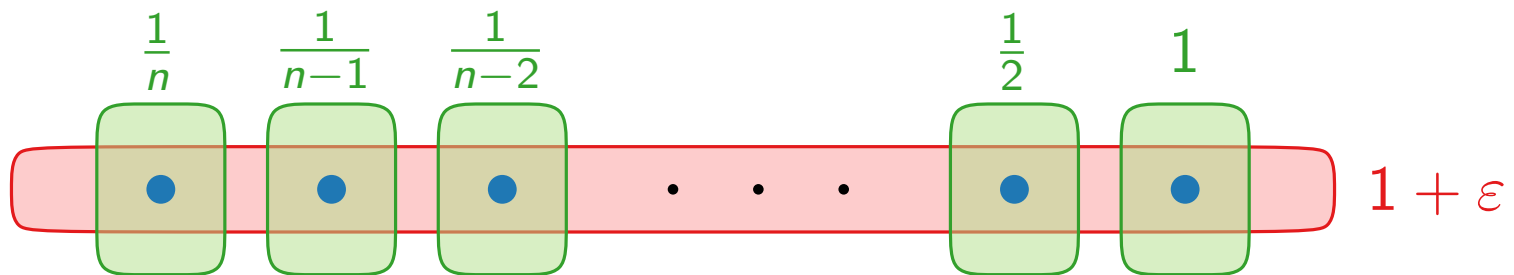
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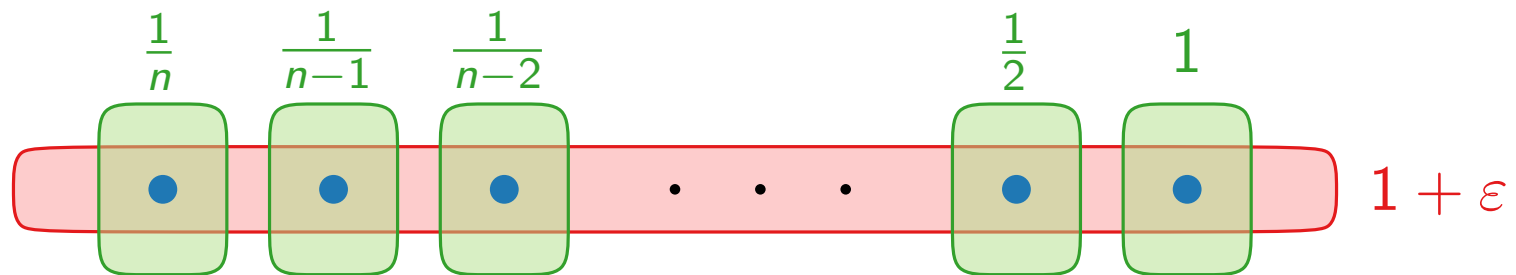
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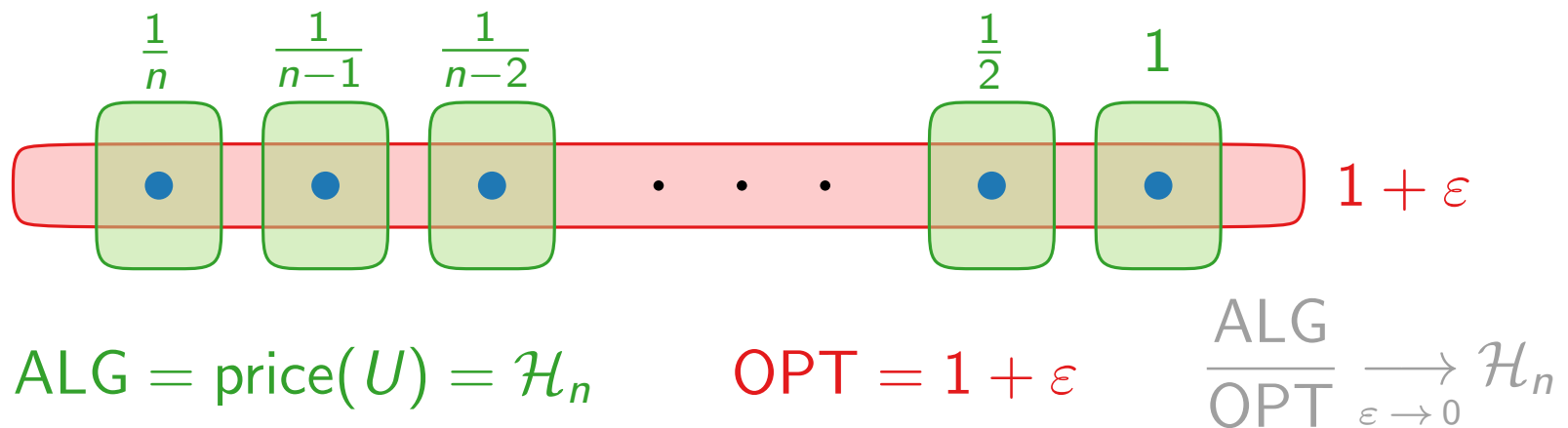
$$\text{ALG} = \text{price}(U) = \mathcal{H}_n$$

$$\text{OPT} = 1 + \varepsilon$$

$$\frac{\text{ALG}}{\text{OPT}} \xrightarrow{\varepsilon \rightarrow 0} \mathcal{H}_n$$

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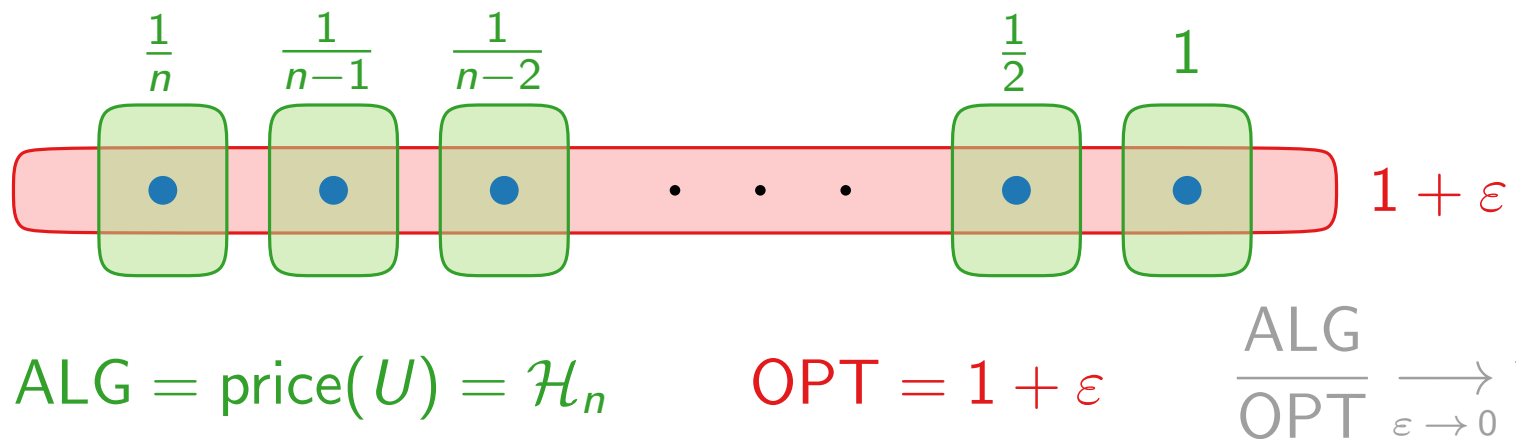
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No – for any $\varepsilon > 0$, it is NP-hard to approximate SETCOVER with factor $(1 - \varepsilon) \cdot \ln n$.
 [Feige, JACM 1998]
 [Dinur, Steurer, STOC 2014]

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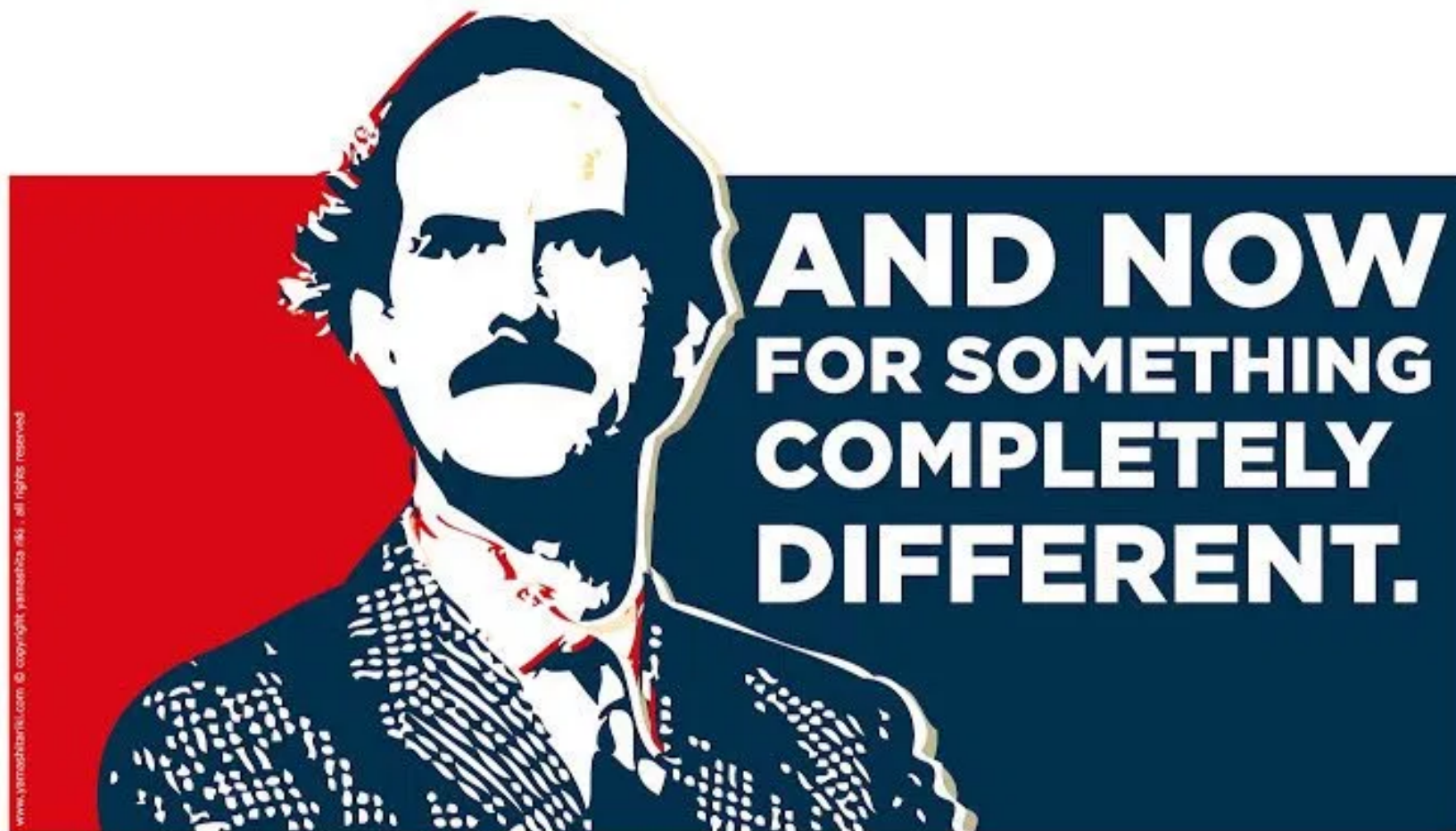
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No – for any $\epsilon > 0$, there is a family $\mathcal{S}$ with factor $(1 + \epsilon) \ln k$.



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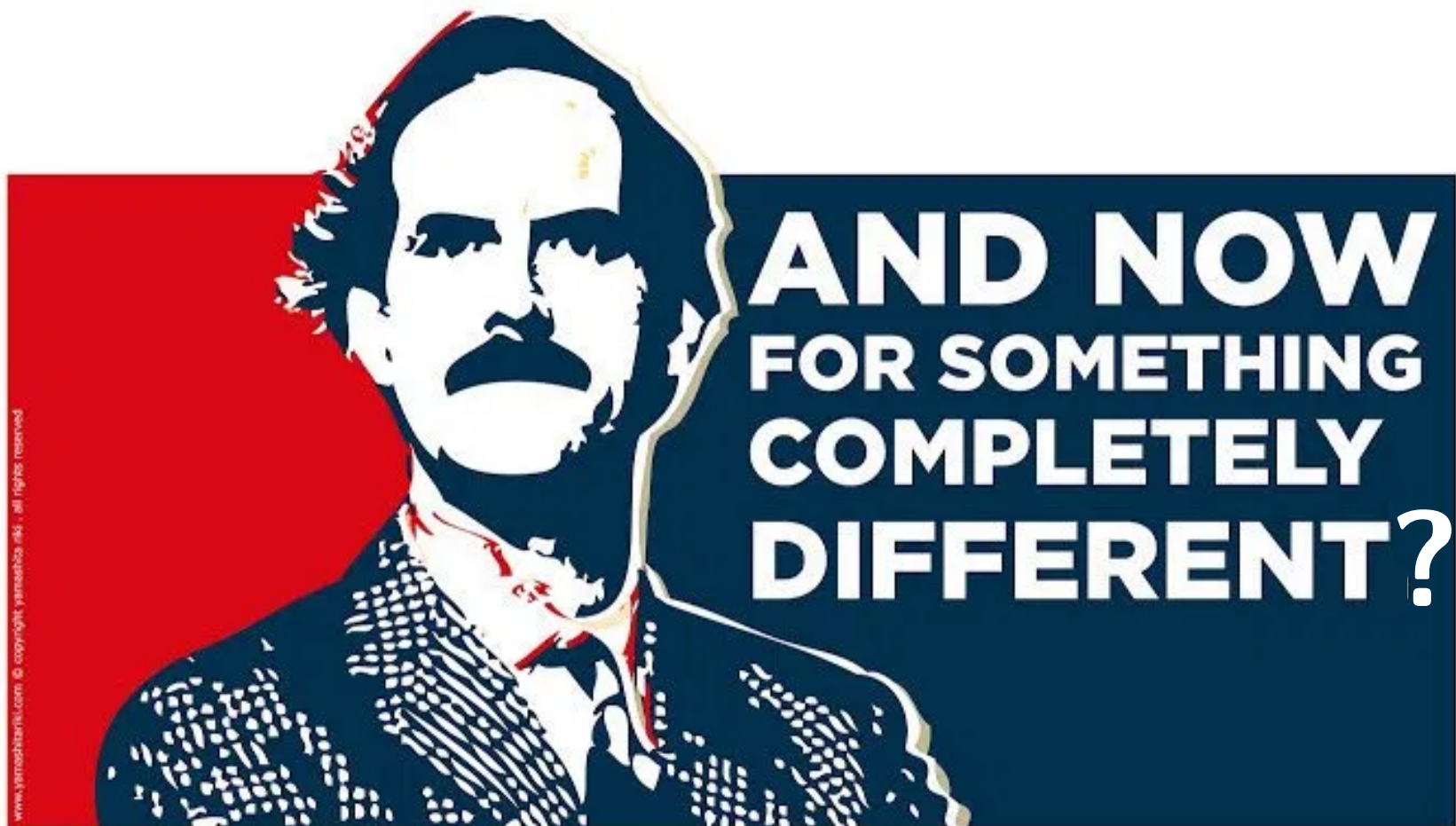
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Approximation Algorithms

Lecture 2:

SETCOVER and SHORTESTSUPERSTRING

Part IV:

SHORTESTSUPERSTRING

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W.l.o.g.: No string s_i is a substring of any other string s_j .

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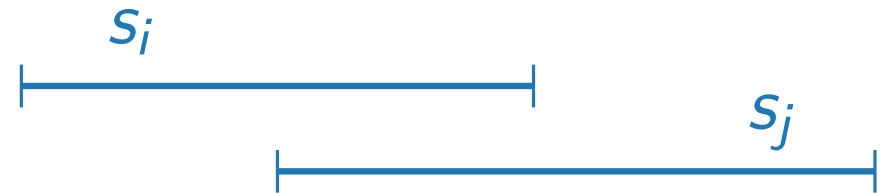


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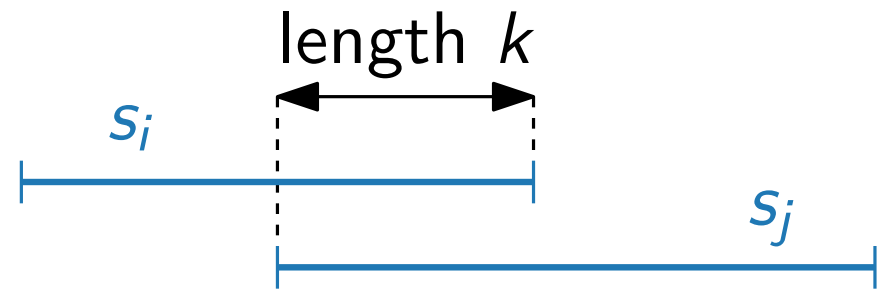


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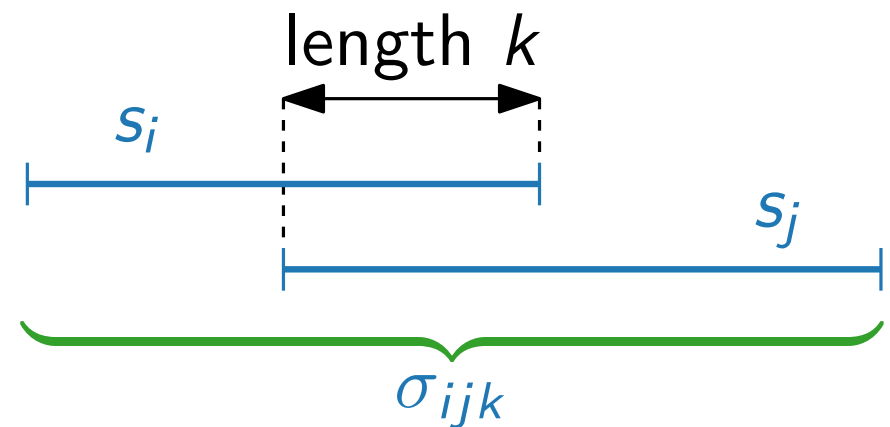


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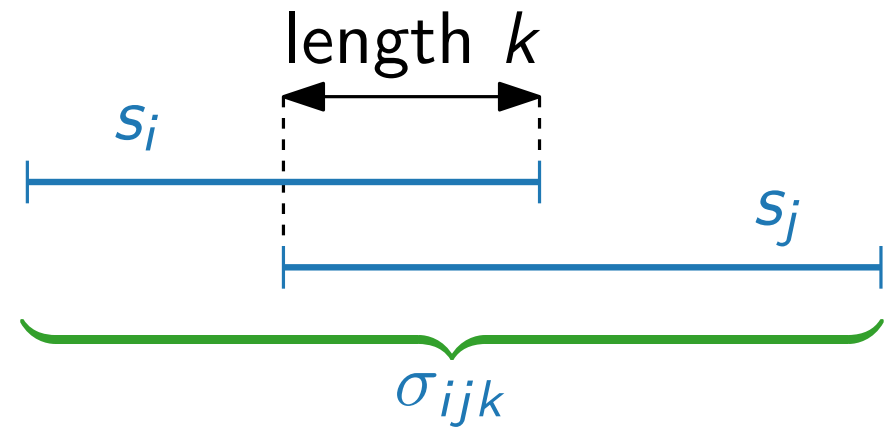
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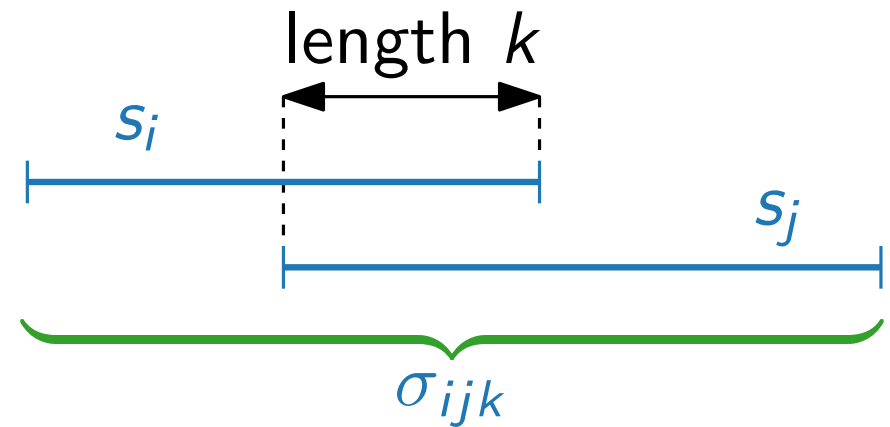
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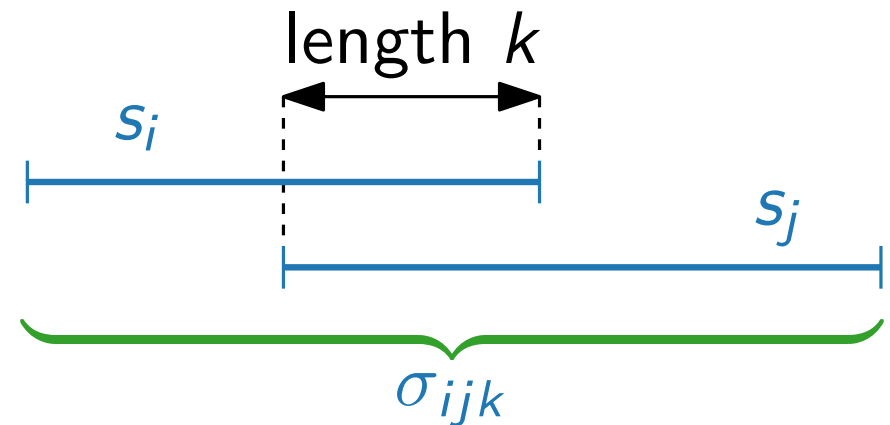
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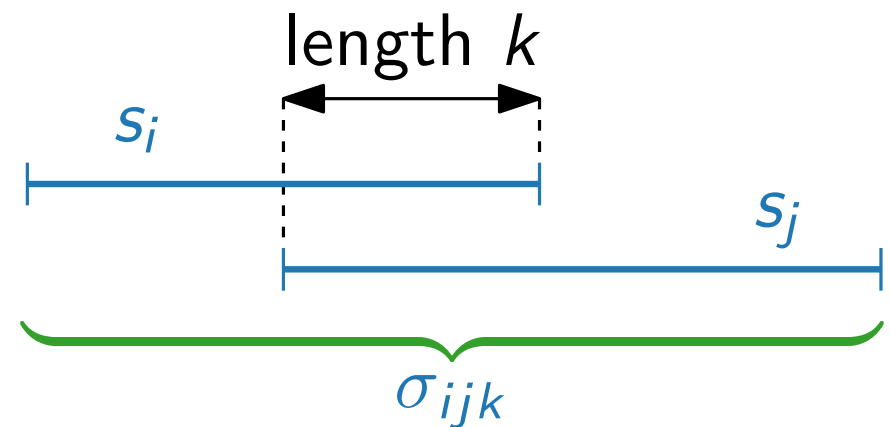
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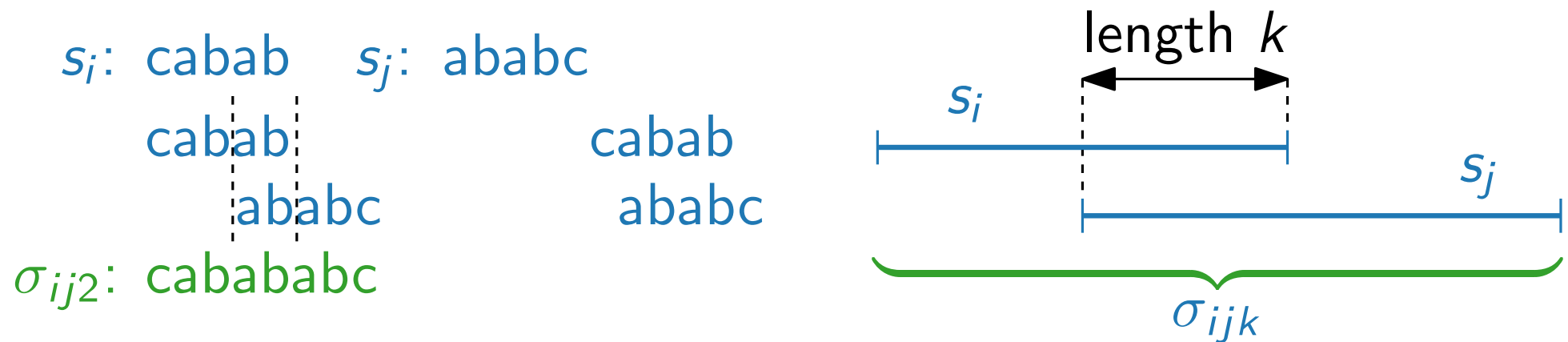


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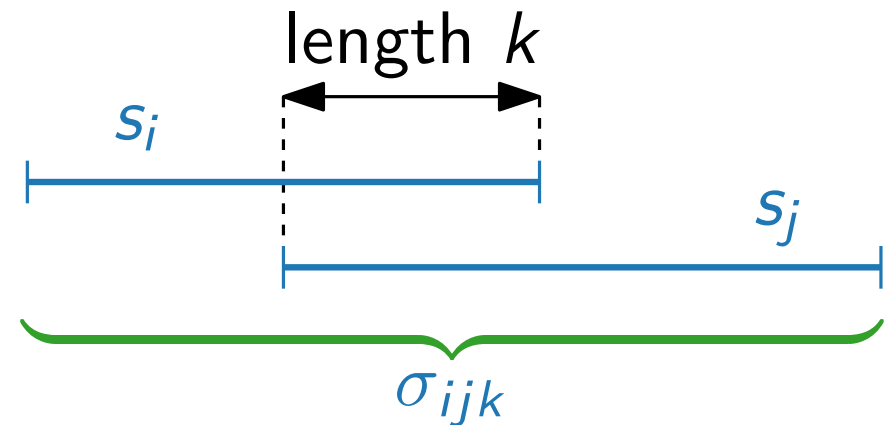
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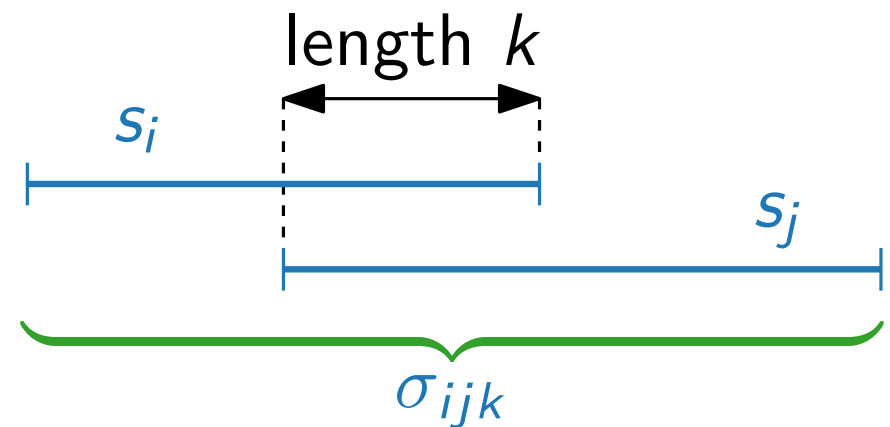
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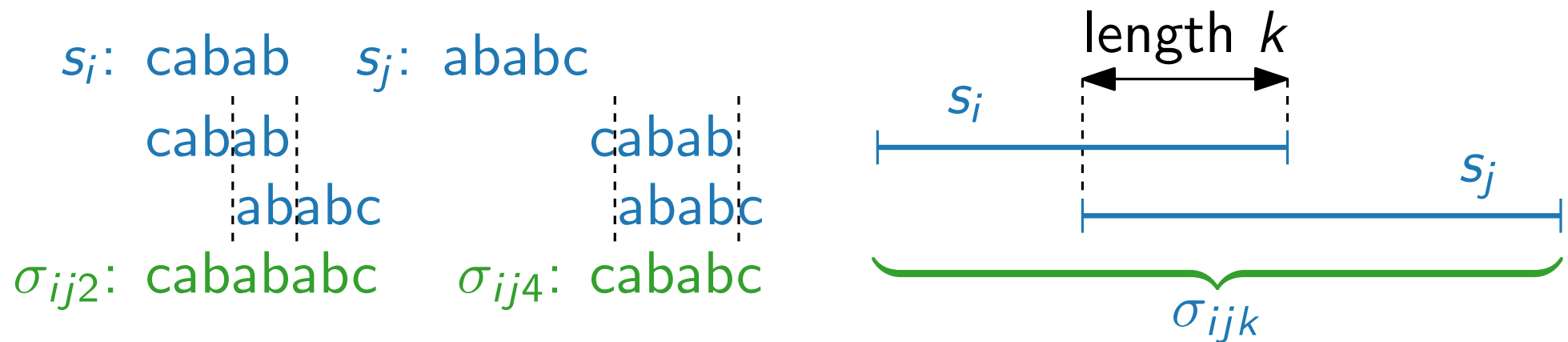
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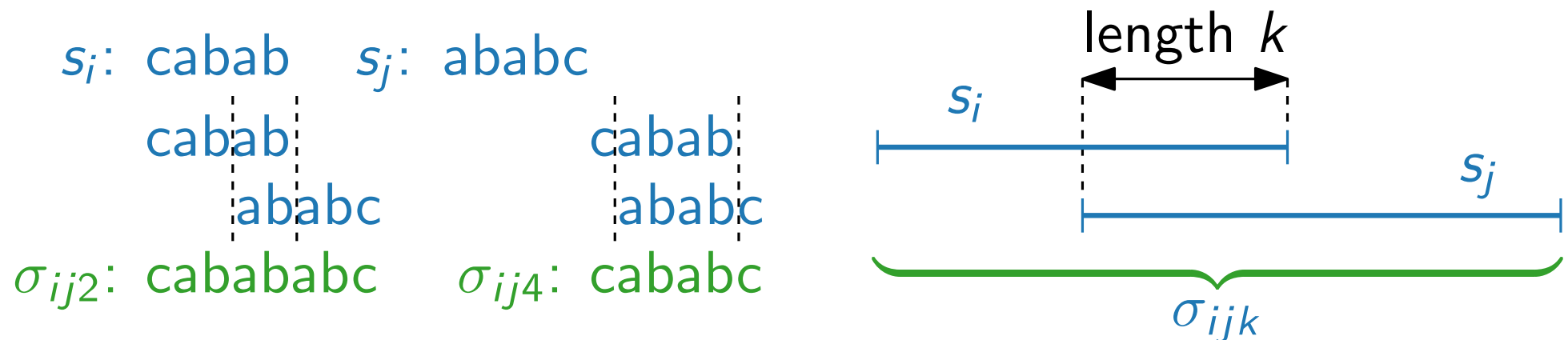
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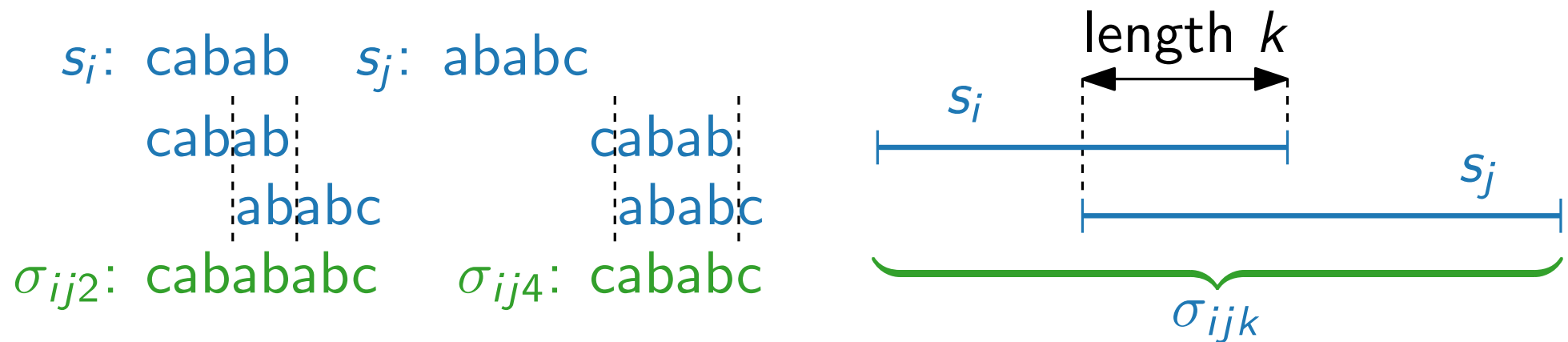
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Approximation Algorithms

Lecture 2:

SETCOVER and SHORTESTSUPERSTRING

Part V:

Solving SHORTESTSUPERSTRING via SETCOVER

Relating SSS and SETCOVER

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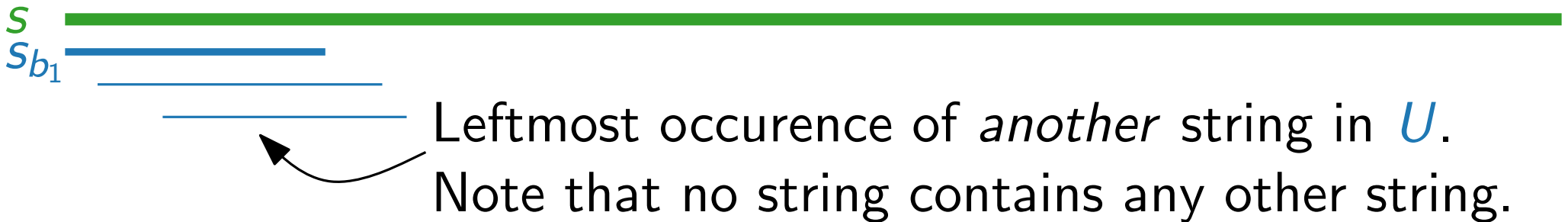
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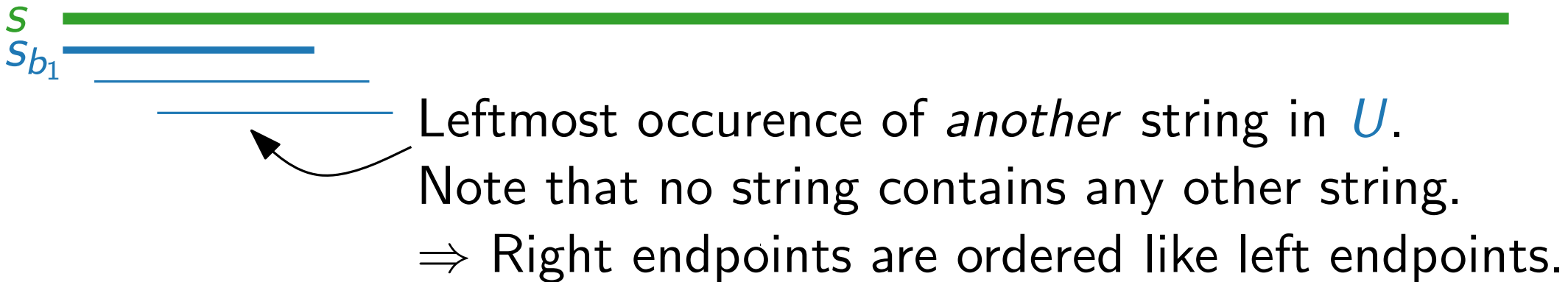
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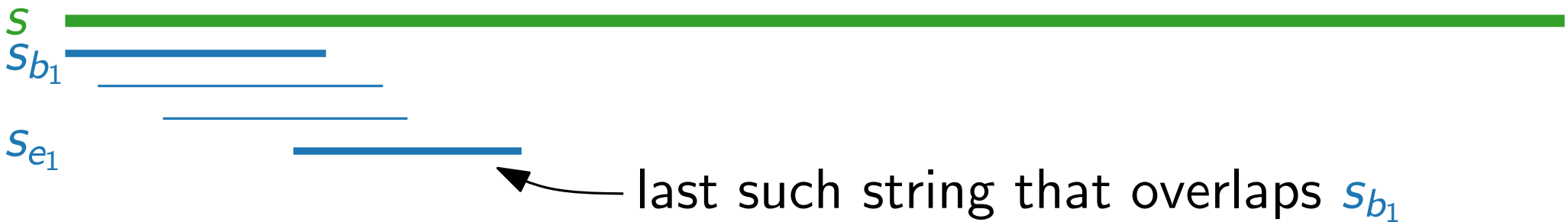
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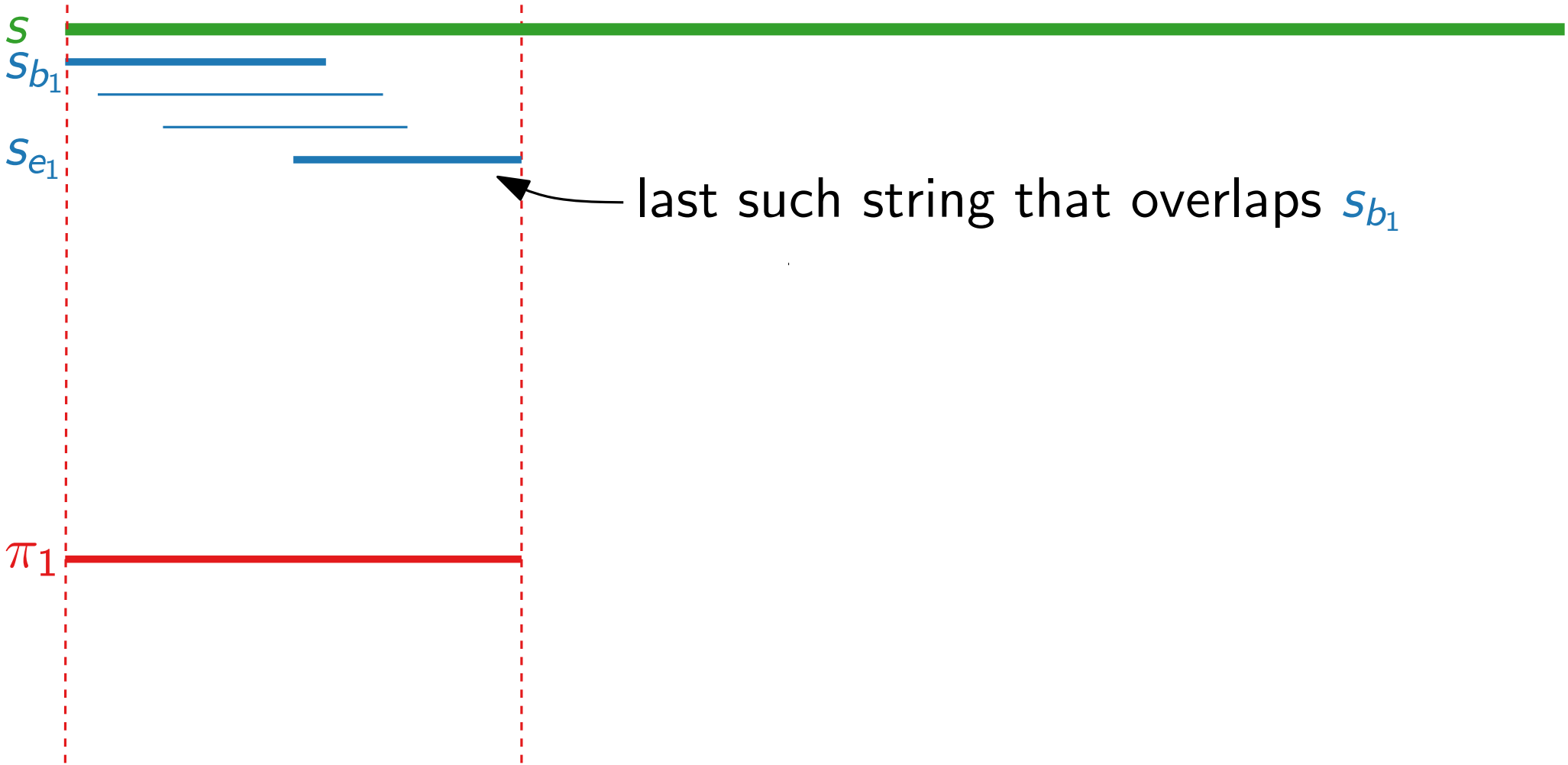
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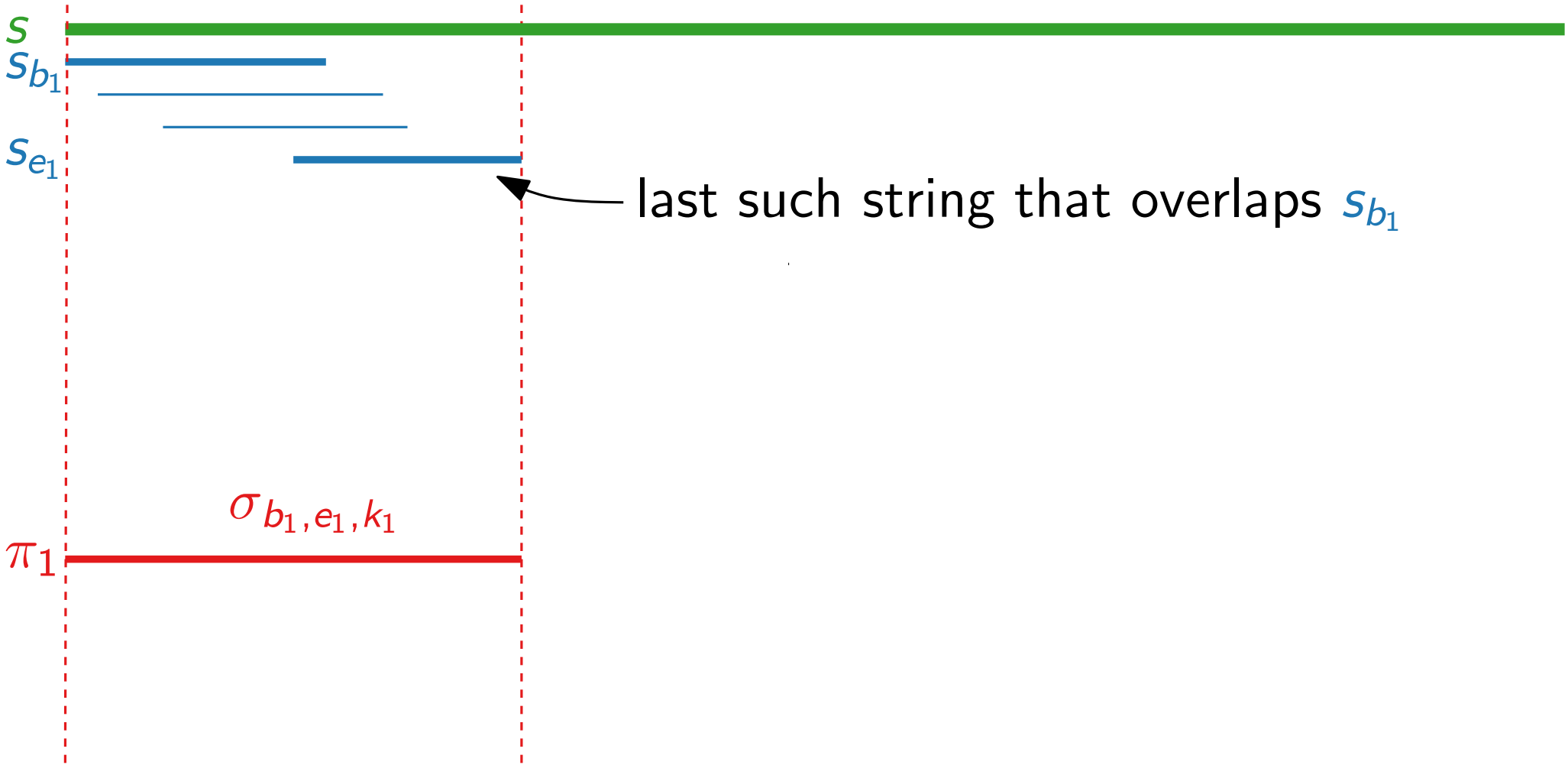
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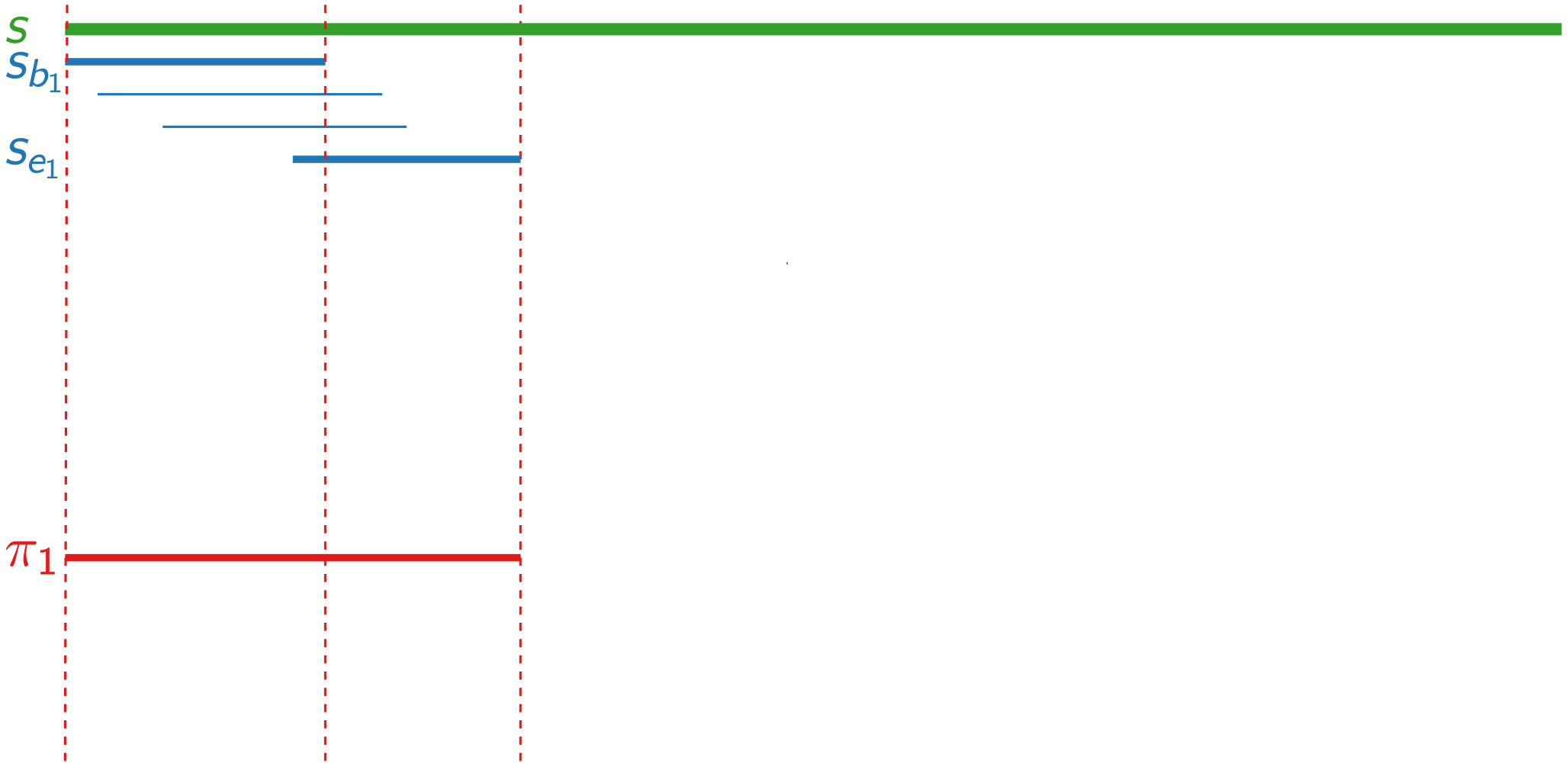
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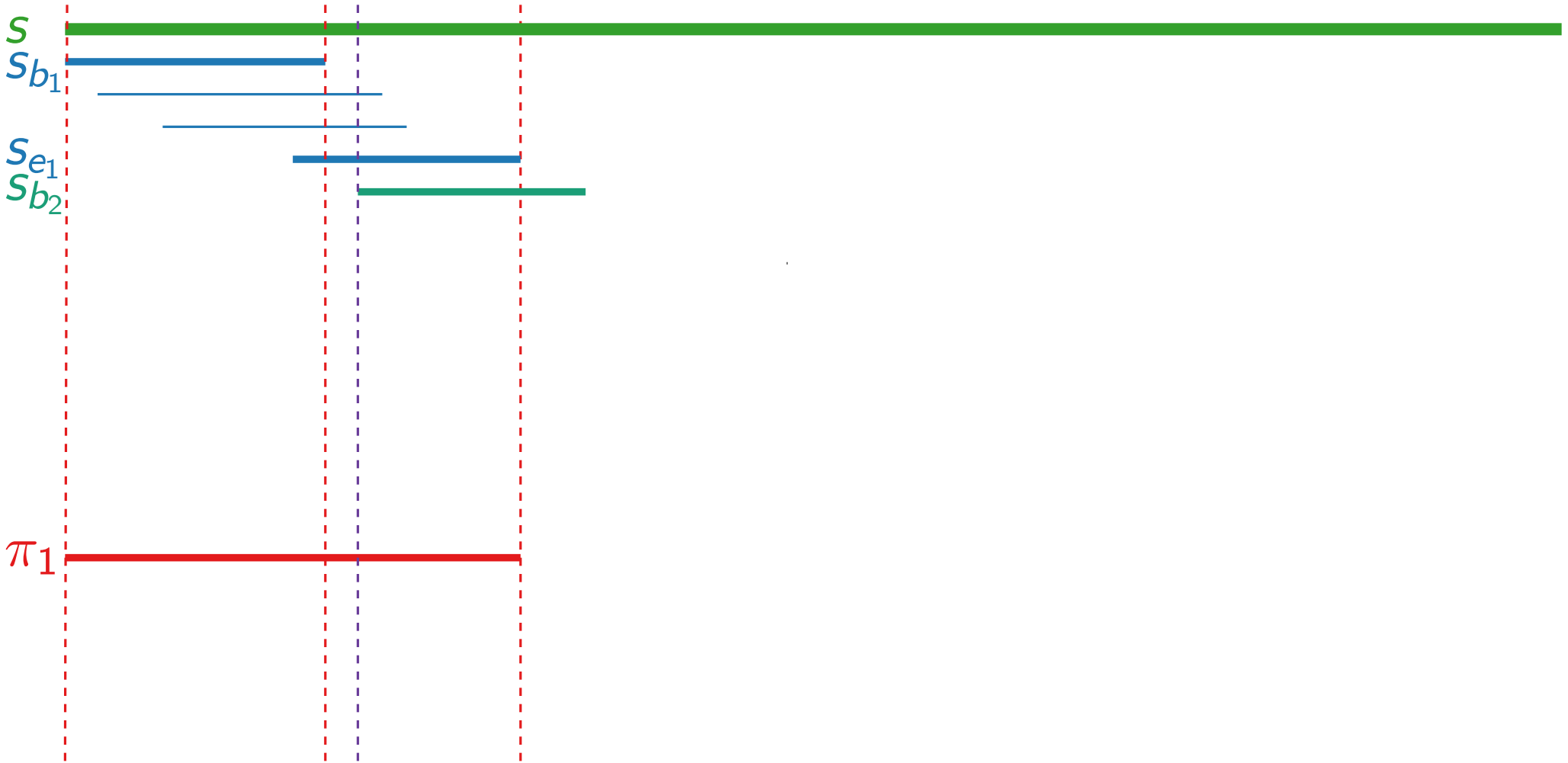
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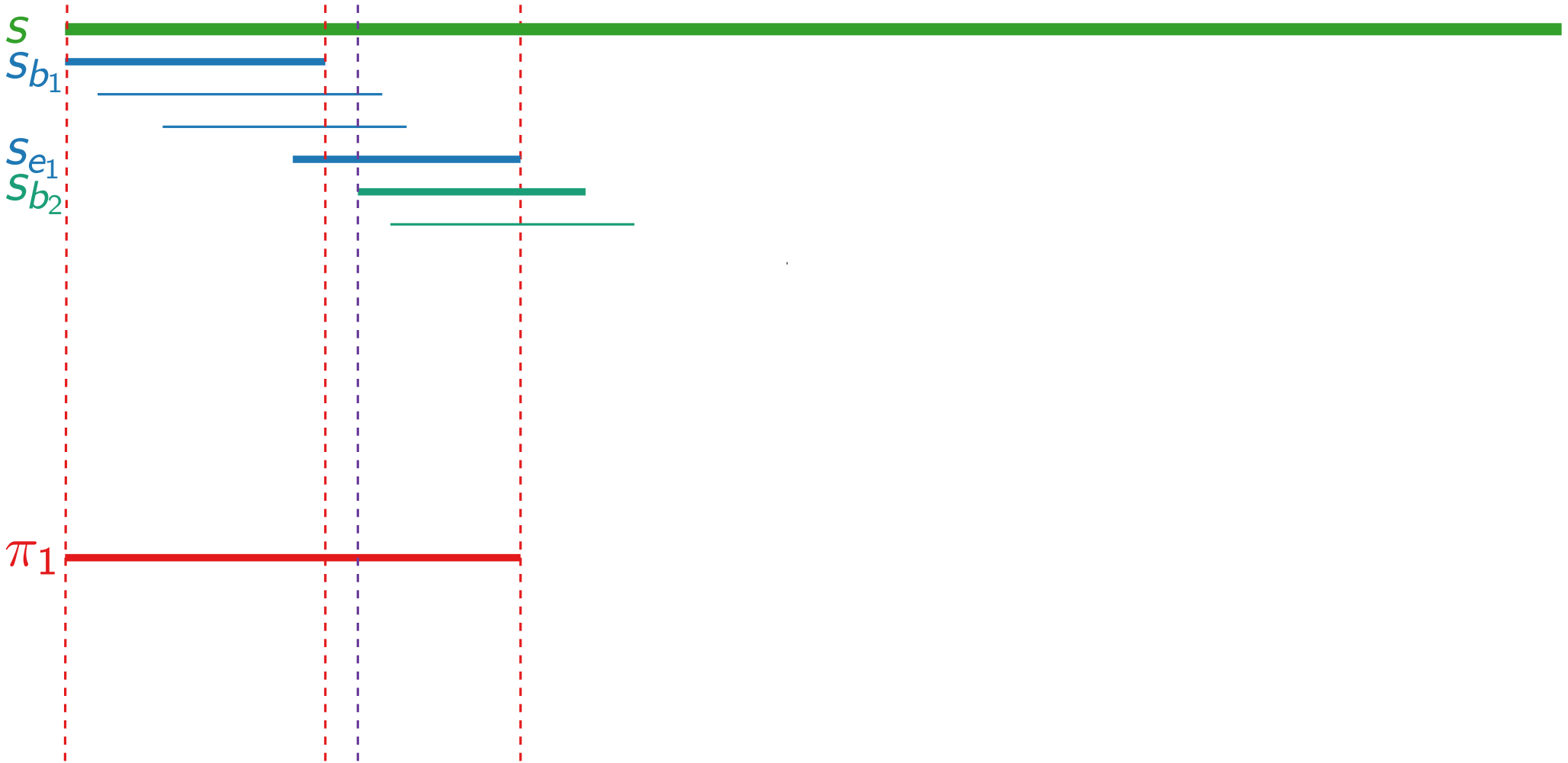
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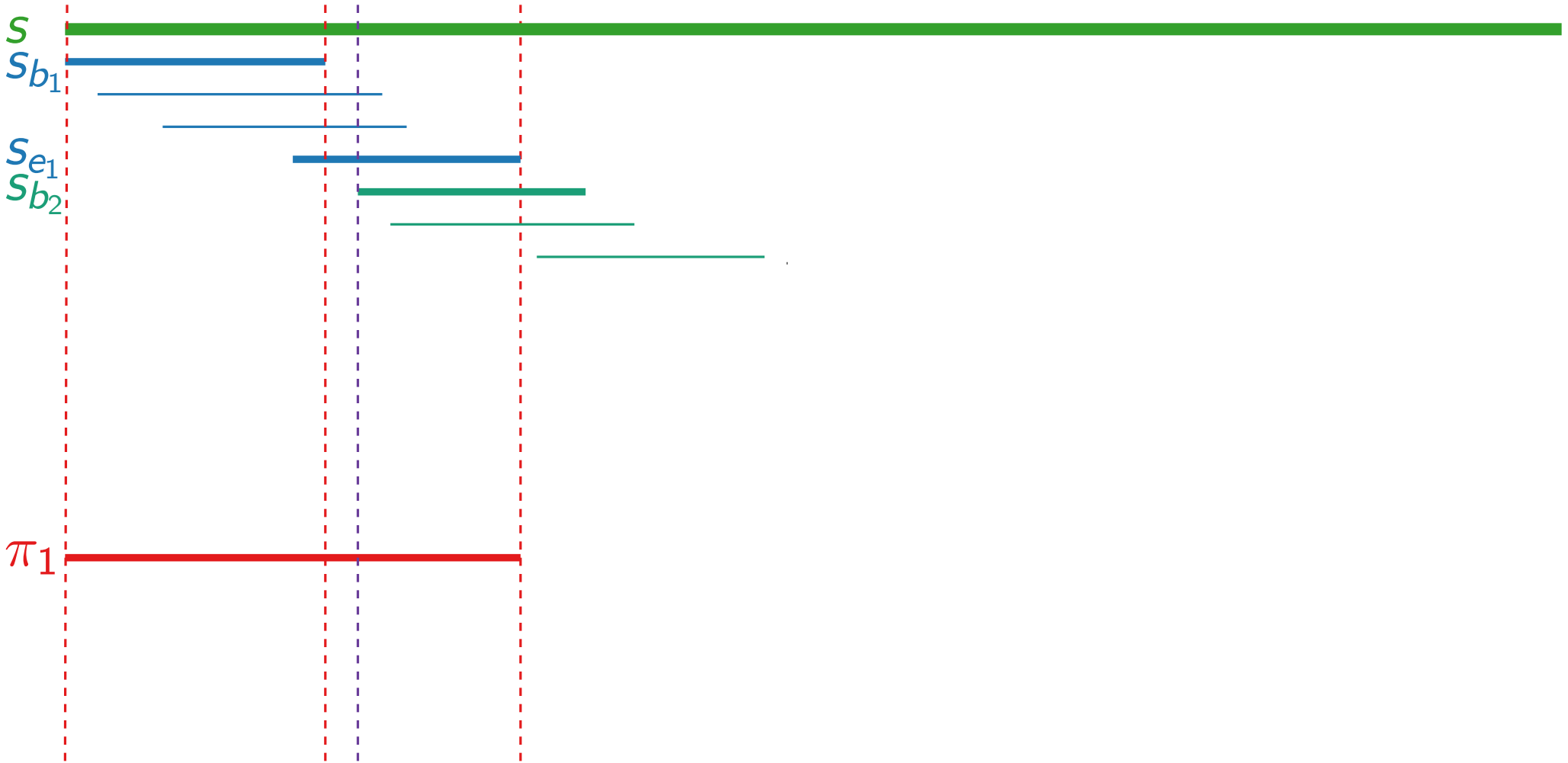
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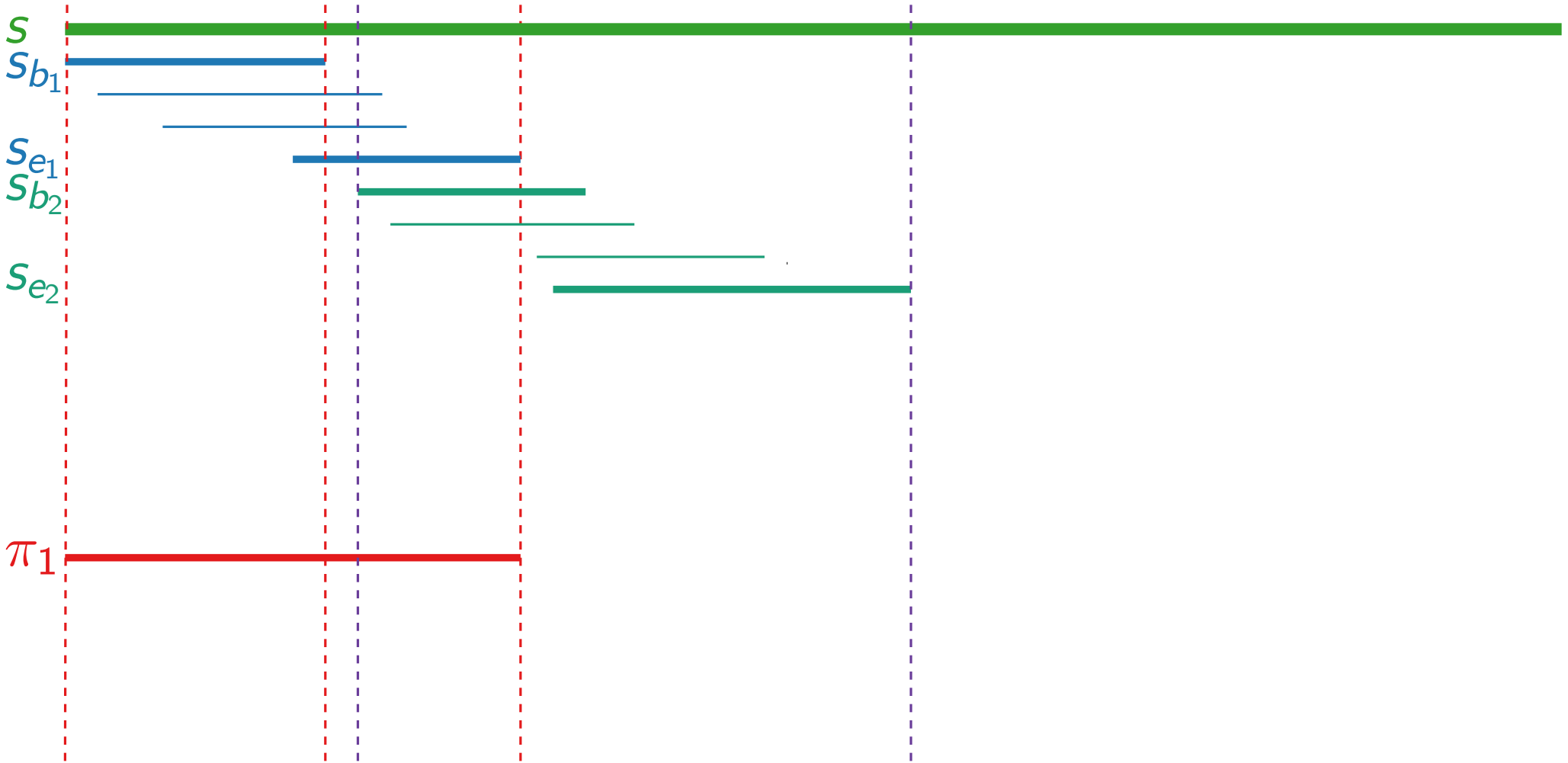
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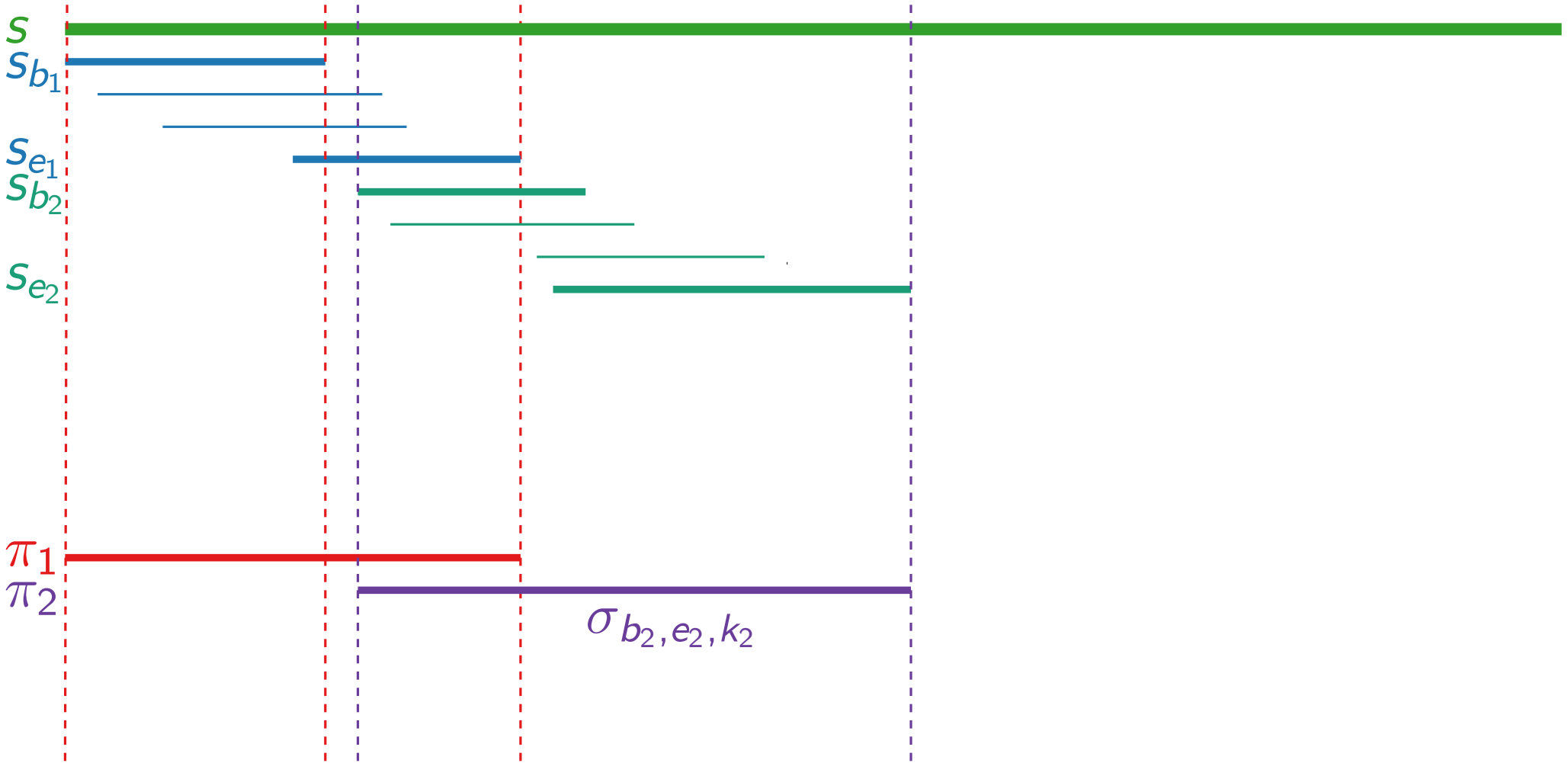
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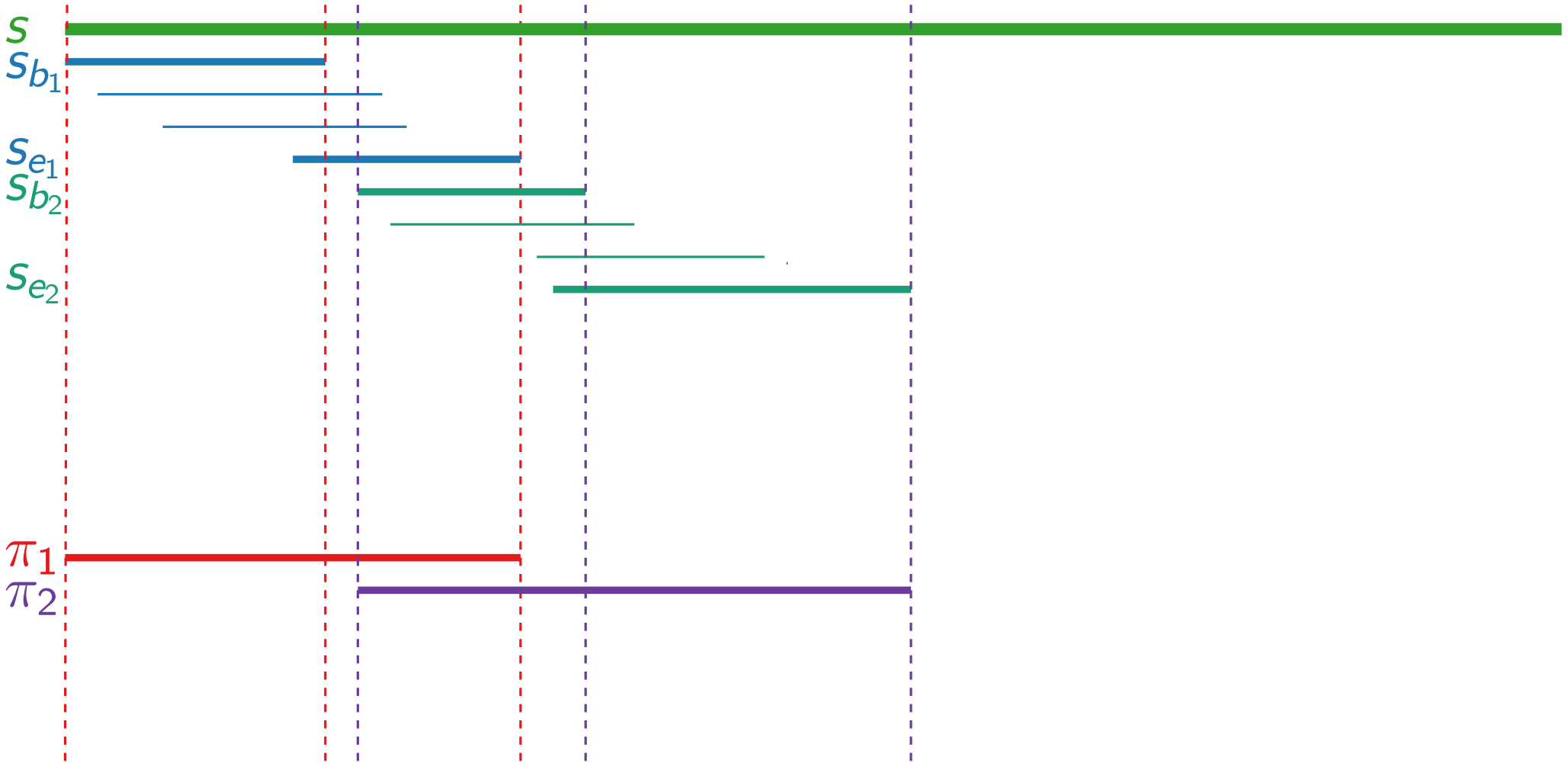
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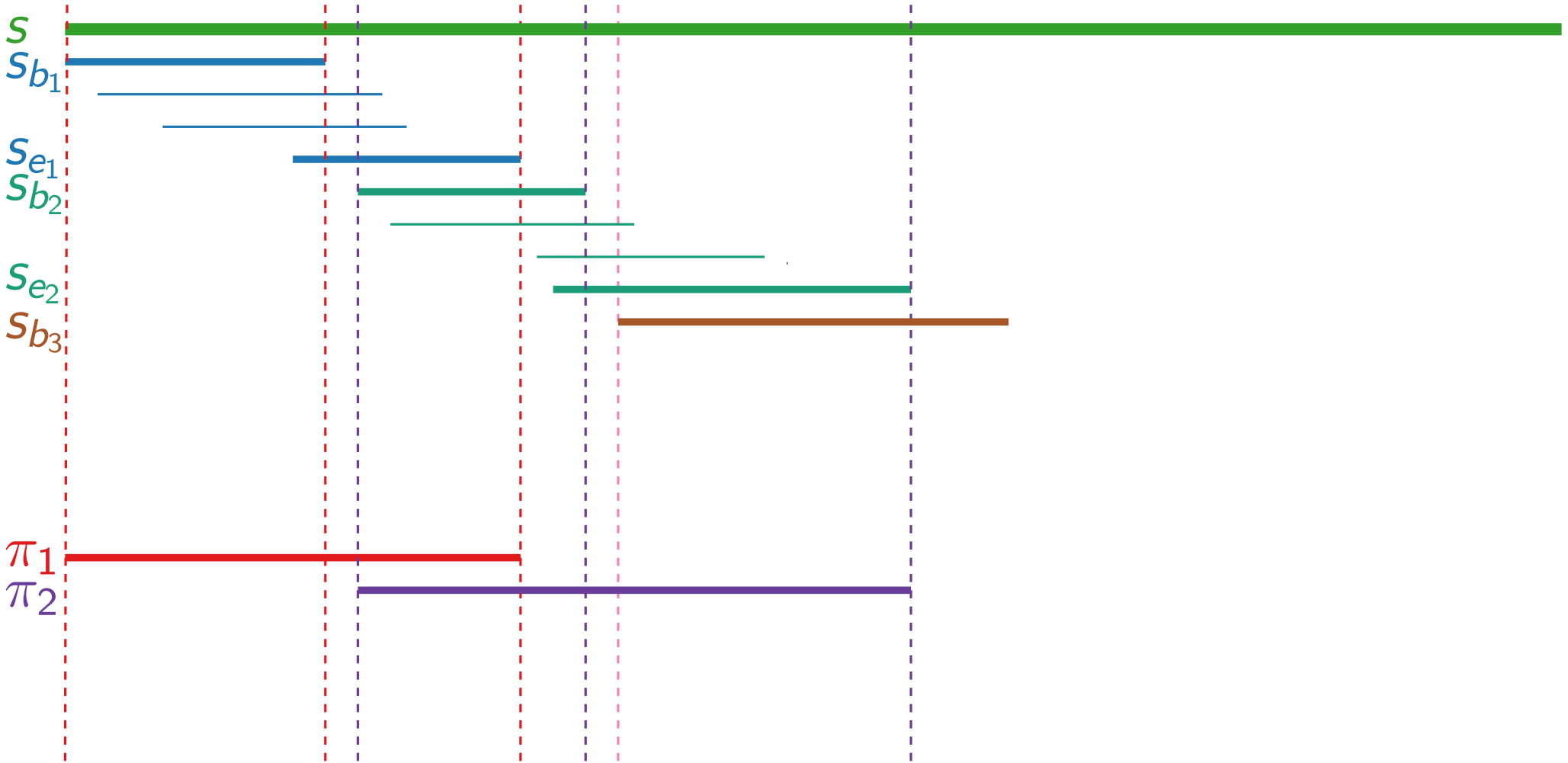
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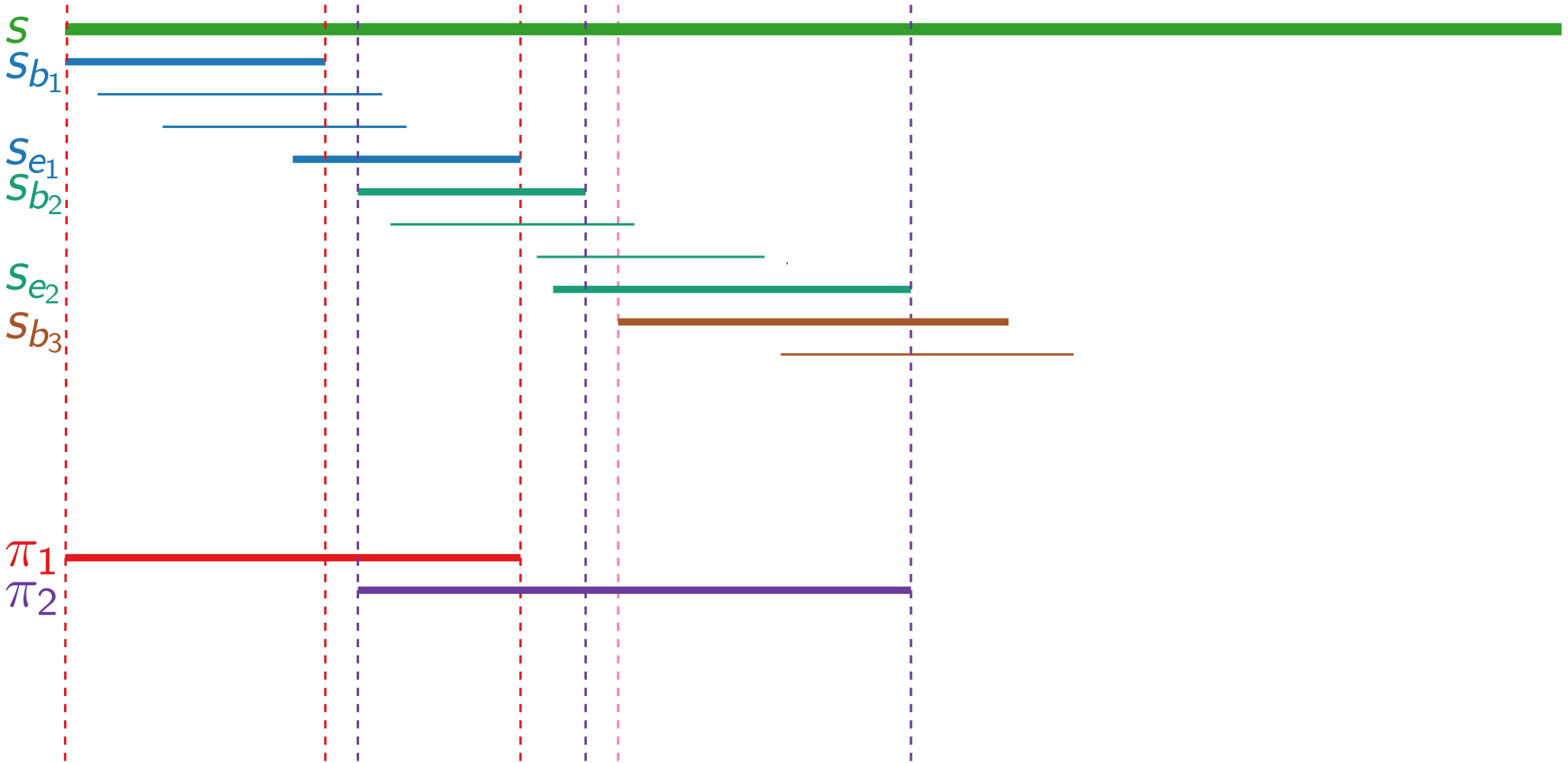
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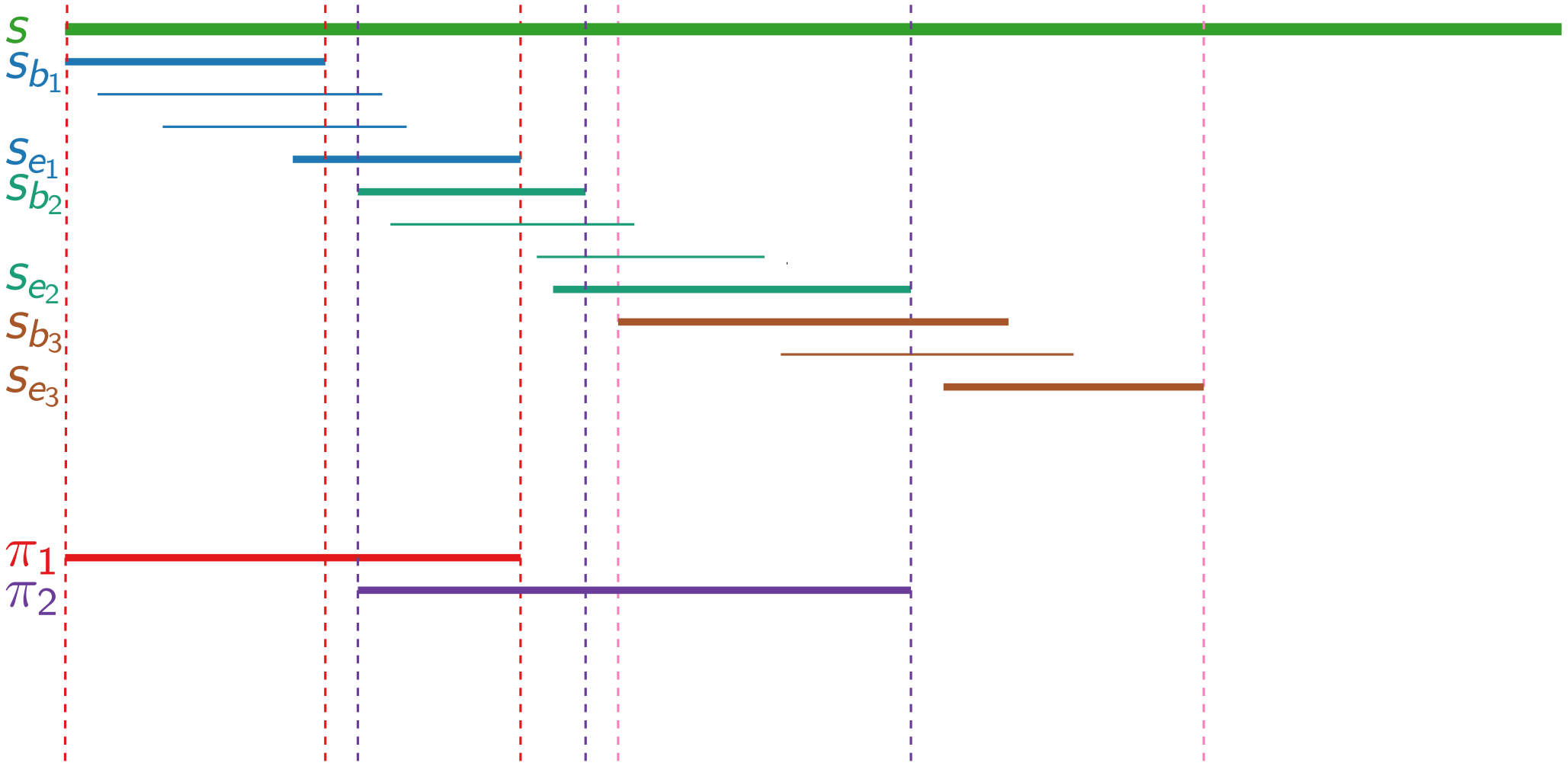
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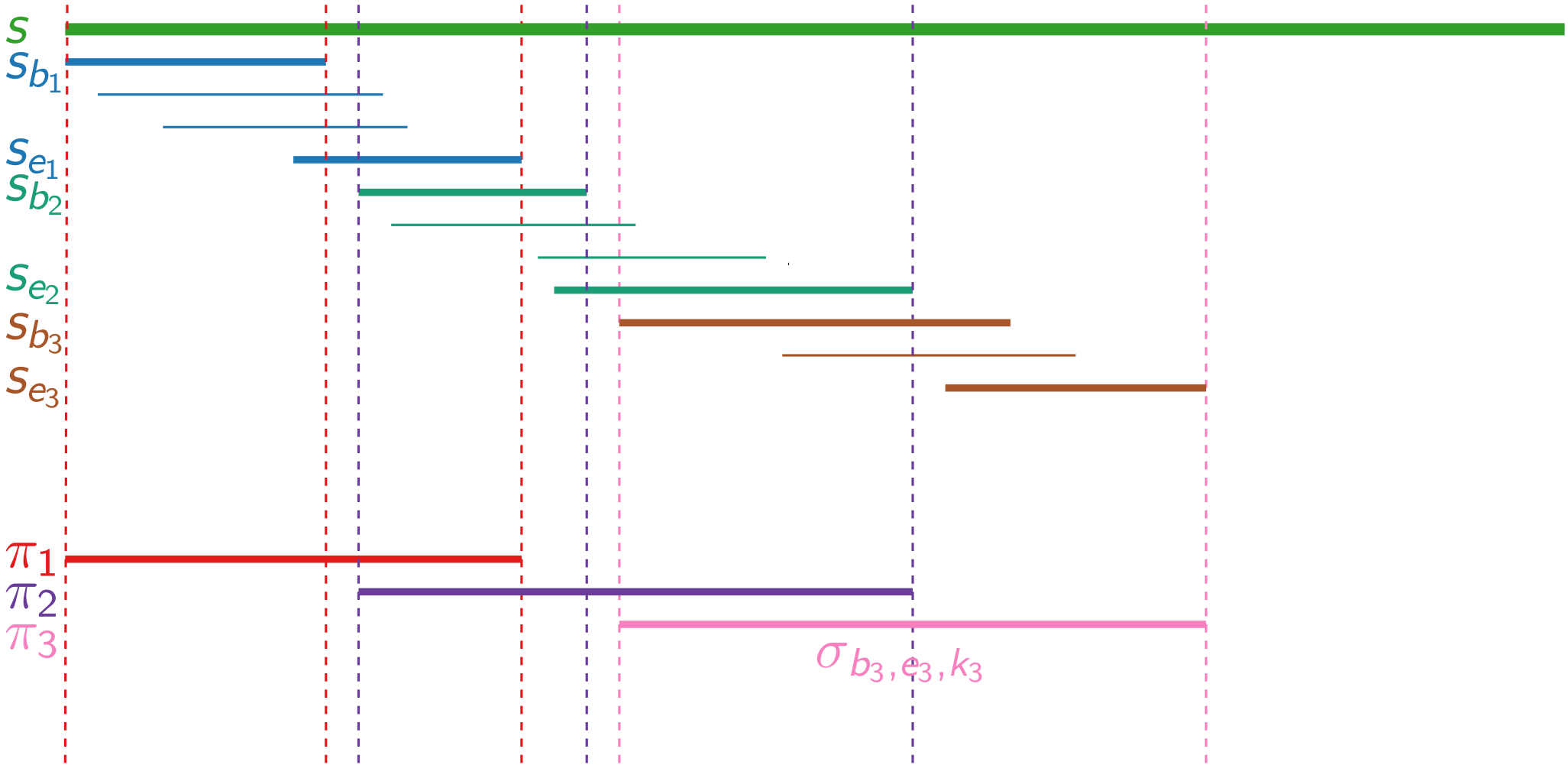
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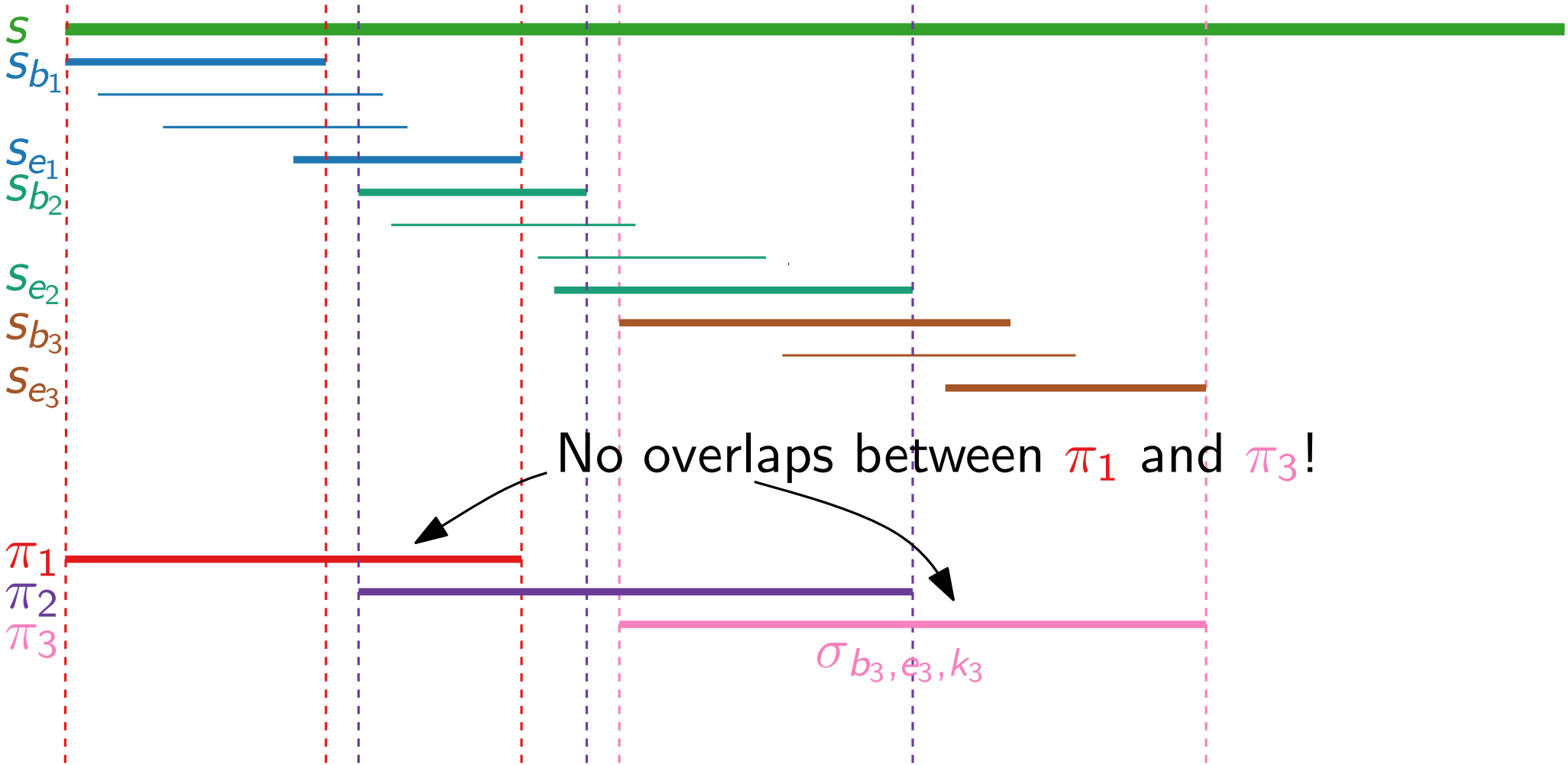
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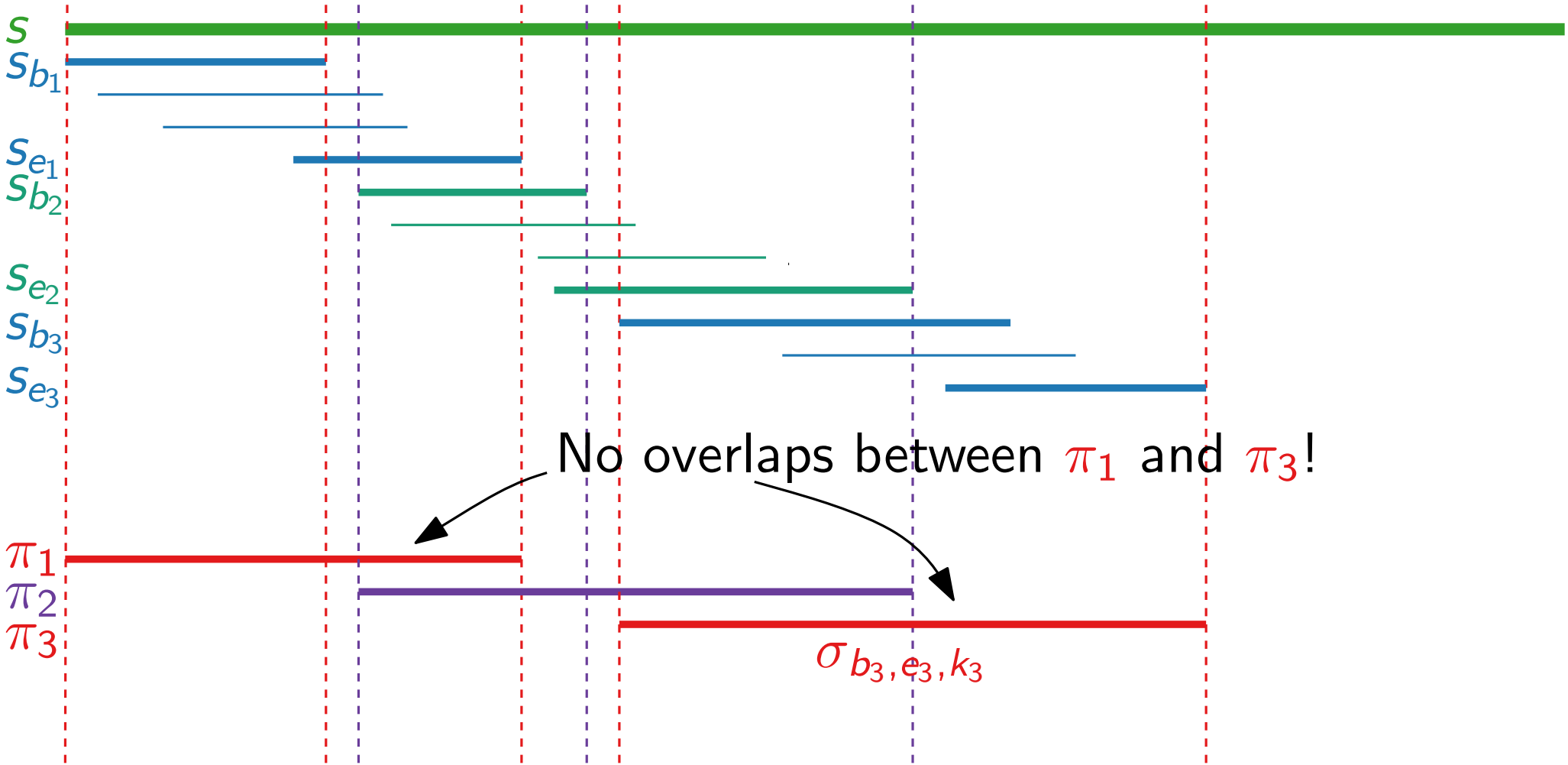
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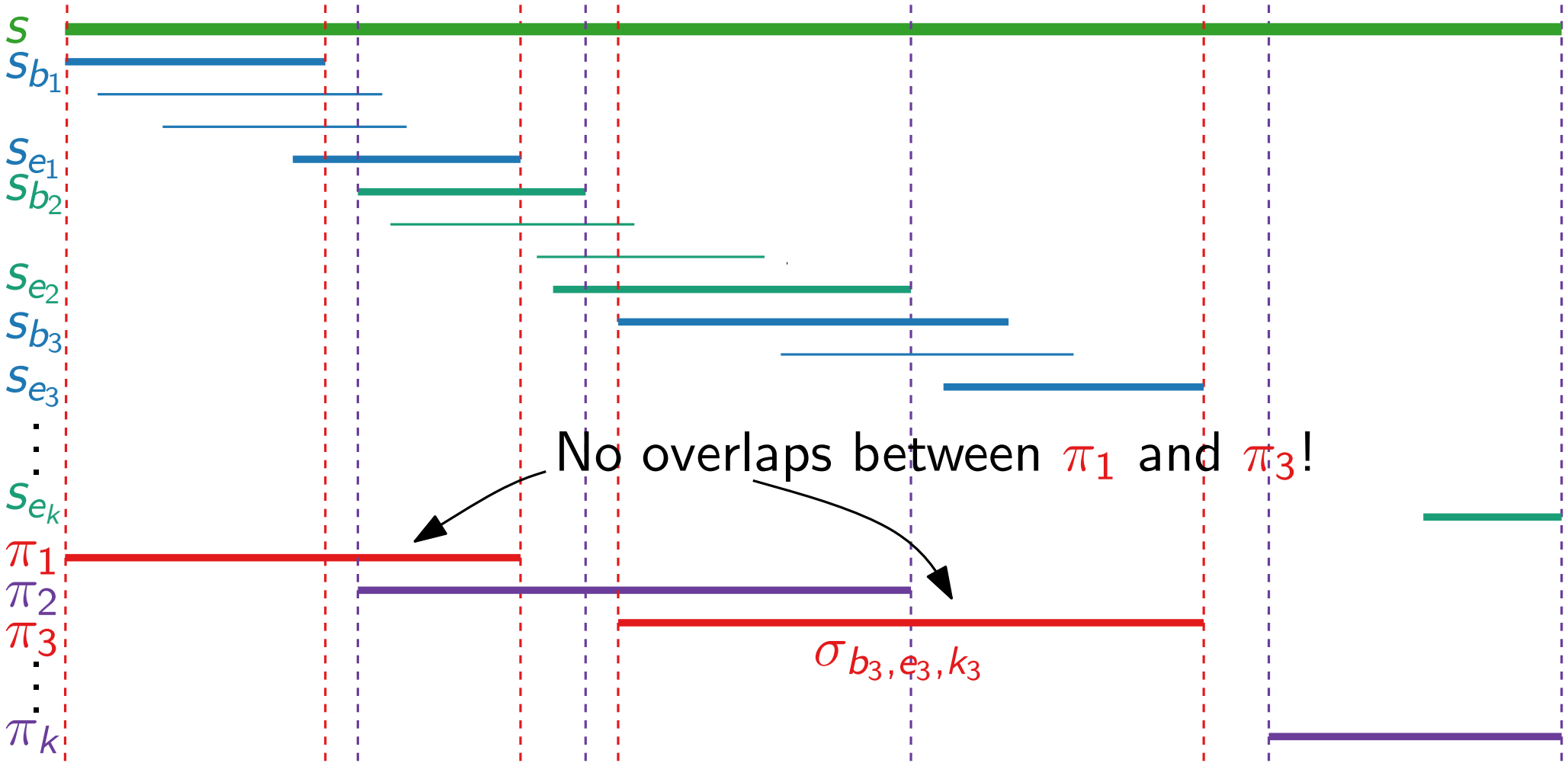
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Theorem. This algorithm is a factor- $2\mathcal{H}_n$ approximation algorithm for SHORTESTSUPERSTRING.

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- `SHORTESTSUPERSTRING` cannot be approximated within factor $\frac{333}{332} \approx 1.003$ (unless $P = NP$).
[Karpinski & Schmied: CATS 2013]