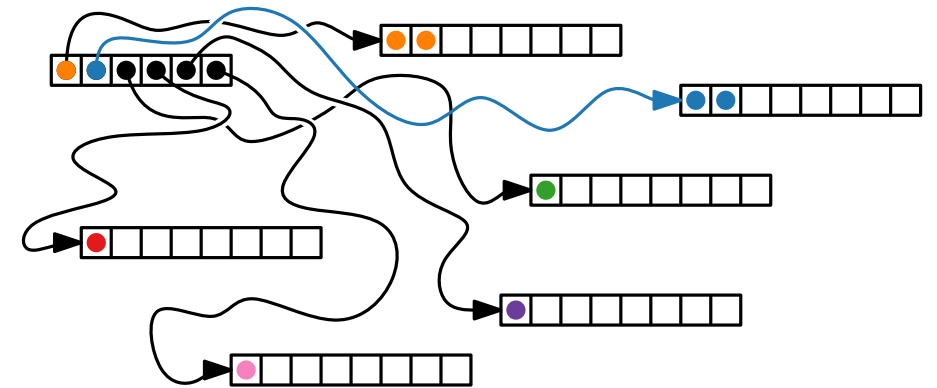
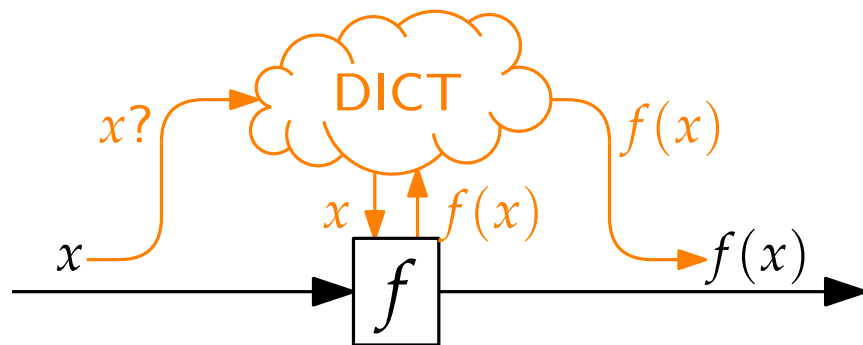


Advanced Algorithms

Algorithms in Practice

Computers are Fast · IntroSort · Knapsack DP · Swisstable



Theory & Practice

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- What if?**
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 - Two algorithms have similar g (e.g. n and $n \log(n)$) but for one c is much higher?

*“In theory, there is no difference between
theory and practice;*

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but, in practice, there is.”*

Do We Need More Detail?

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[Knuth 1998]

	Step	Operations	Time
	$N3$	CMPA, JG, JE	$3.5u$
Either	$\left\{ \begin{array}{l} N4 \\ N5 \end{array} \right.$	$\left\{ \begin{array}{l} \text{STA, INC} \\ \text{INC, LDA, CMPA, JGE} \end{array} \right.$	$\left\{ \begin{array}{l} 3u \\ 6u \end{array} \right.$
Or	$\left\{ \begin{array}{l} N8 \\ N9 \end{array} \right.$	$\left\{ \begin{array}{l} \text{STX, INC} \\ \text{DEC, LDX, CMPX, JGE} \end{array} \right.$	$\left\{ \begin{array}{l} 3u \\ 6u \end{array} \right.$

Thus about $12.5u$ is spent on each record in each pass, and the total running time will be asymptotically $12.5N \lg N$, for both the average case and the worst case. This is slower than quicksort's average time, and it may not be enough better than heapsort to justify taking twice as much memory space, since the asymptotic running time of Program 5.2.3H is never more than $18N \lg N$.

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What about? ■ Caching?

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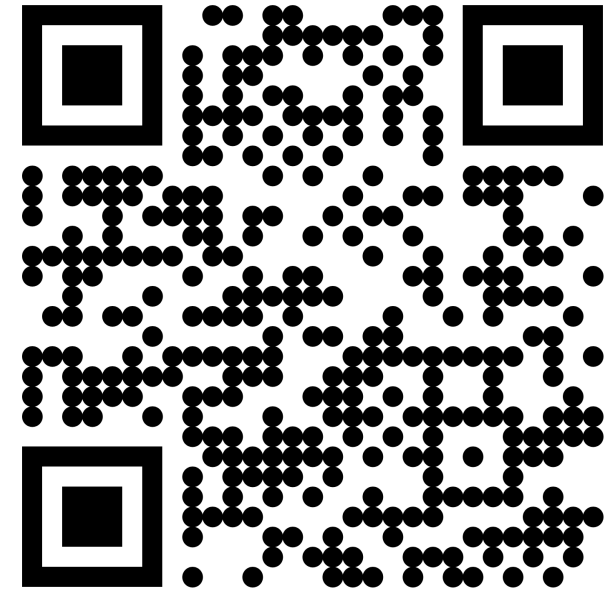
Computers are Fast

Event	Latency
1 CPU cycle	0.3 ns
Level 1 cache access	0.9 ns
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How to Sort a Million 32-bit Integers

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Well-known comparison-based sorting algorithms

	best-case	average	worst-case	in-place	stable
Insertion sort	$\mathcal{O}(n)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$	✓	✓
Heap sort	$\mathcal{O}(n \log(n))$	$\mathcal{O}(n \log(n))$	$\mathcal{O}(n \log(n))$	✓	✗
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Quicksort With an Emergency Exit

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Idea. If for any partition the recursion reaches a depth of $a \log(n)$ (for a given parameter a) stop and sort this section with heap sort.

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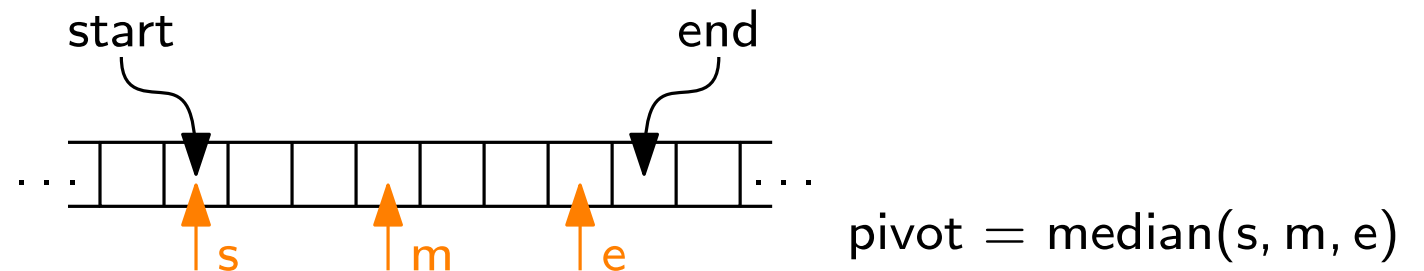
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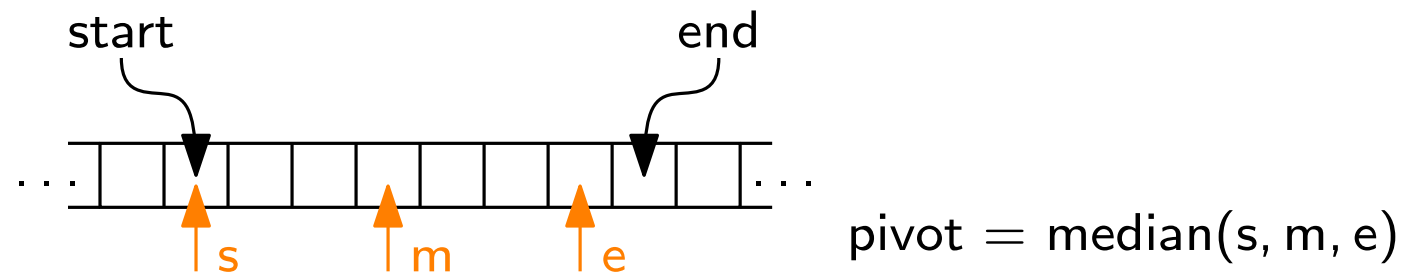
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$a = 2$

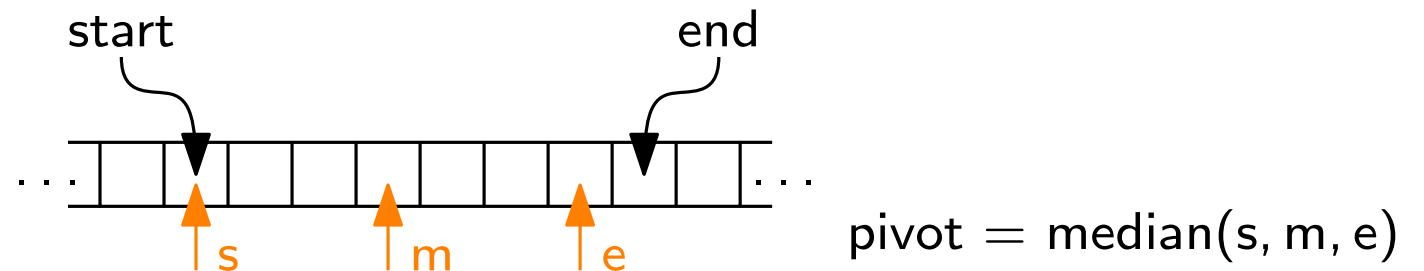
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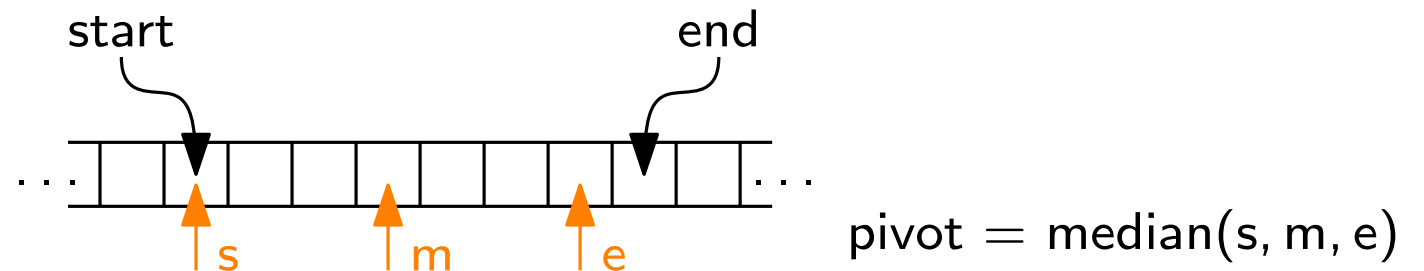
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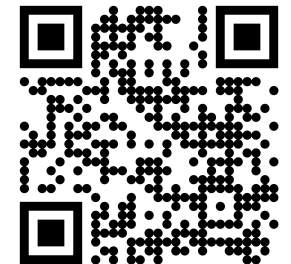
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see (and hear) it!



Advanced Algorithms

Algorithms in Practice

Implementing the Knapsack Dynamic Program

KNAPSACK

Input: Finite set U , for each $u \in U$ a size $s(u) \in \mathbb{Z}^+$ and a value $v(u) \in \mathbb{Z}^+$, and positive integers B and K .

Task: Is there a subset $U' \subseteq U$ such that $\sum_{u \in U'} s(u) \leq B$ and such that $\sum_{u \in U'} v(u) \geq K$?

[Garey, Johnson 1979; Karp 1972]

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Optimization Problem: Maximize K

Example:

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-------	-----

\$ 12	△ 6
-------	-----

\$ 4	△ 1
------	-----

\$ 6	△ 2
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	$\sum_{u \in U'} s(u)$	$\sum_{u \in U'} v(u)$
Greedy	6	12

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Value/Size-Ratio

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Greedy	6	12
Value/Size-Ratio	3	10

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OPT		

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Value/Size-Ratio	3	10
OPT	6	16

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Greedy	6	12
Value/Size-Ratio	3	10
OPT	6	16

KNAPSACK is (weakly) NP-complete but can be solved in pseudo-polynomial time by dynamic programming.

A Dynamic Program for KNAPSACK

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Pick an arbitrary order on the items U .

$f(i, s) = \max$ **value** with size limit s using only items $1 \dots i$.

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Recursion:

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$

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Runtime?

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Runtime:

$$\mathcal{O}(|U| \cdot B)$$

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Result:

$$f(|U|, B)$$

Runtime:

$$\mathcal{O}(|U| \cdot B) \longleftarrow \text{“pseudo-polynomial”}$$

Implement it

```
case class Item(weight: Int, value: Int)
```


Implement it

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case class Item(weight: Int, value: Int)

def solve(items: IndexedSeq[Item], weight: Int) =
  def go(i: Int, weight: Int): Int =
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def solve(items: IndexedSeq[Item], weight: Int) =
  def go(i: Int, weight: Int): Int =
    val item = items(i)
    if i == 0 then
      if item.weight <= weight then item.value else 0
    else if item.weight <= weight then
      go(i - 1, weight) max
        (go(i - 1, weight - item.weight) + item.value)
    else go(i - 1, weight)
```

Implement it

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        (go(i - 1, weight - item.weight) + item.value)
    else go(i - 1, weight)

  go(items.size - 1, weight)
```

Expectation Management

[Skiena 2012]

n	$f(n)$	$\lg n$	n	$n \lg n$	n^2	2^n	$n!$
10		0.003 μs	0.01 μs	0.033 μs	0.1 μs	1 μs	3.63 ms
20		0.004 μs	0.02 μs	0.086 μs	0.4 μs	1 ms	77.1 years
30		0.005 μs	0.03 μs	0.147 μs	0.9 μs	1 sec	8.4×10^{15} yrs
40		0.005 μs	0.04 μs	0.213 μs	1.6 μs	18.3 min	
50		0.006 μs	0.05 μs	0.282 μs	2.5 μs	13 days	
100		0.007 μs	0.1 μs	0.644 μs	10 μs	4×10^{13} yrs	
1,000		0.010 μs	1.00 μs	9.966 μs	1 ms		
10,000		0.013 μs	10 μs	130 μs	100 ms		
100,000		0.017 μs	0.10 ms	1.67 ms	10 sec		
1,000,000		0.020 μs	1 ms	19.93 ms	16.7 min		
10,000,000		0.023 μs	0.01 sec	0.23 sec	1.16 days		
100,000,000		0.027 μs	0.10 sec	2.66 sec	115.7 days		
1,000,000,000		0.030 μs	1 sec	29.90 sec	31.7 years		

Figure 2.4: Growth rates of common functions measured in nanoseconds

Running Times

$$|U| = 30$$

$$B = 10|U|$$

Running Times

$$|U| = 30$$

Scala naïve 1.15 s

$$B = 10|U|$$

Running Times

	$ U = 30$	100
Scala naïve	1.15 s	

$B = 10|U|$

Running Times

	$ U = 30$	$ U = 100$
Scala naïve	1.15 s	$\approx 2\,700 \times$ age of the universe

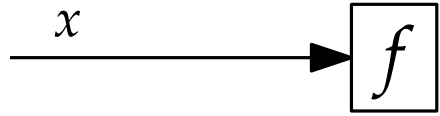
$$B = 10|U|$$

Memoization

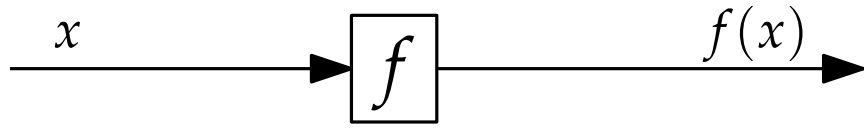
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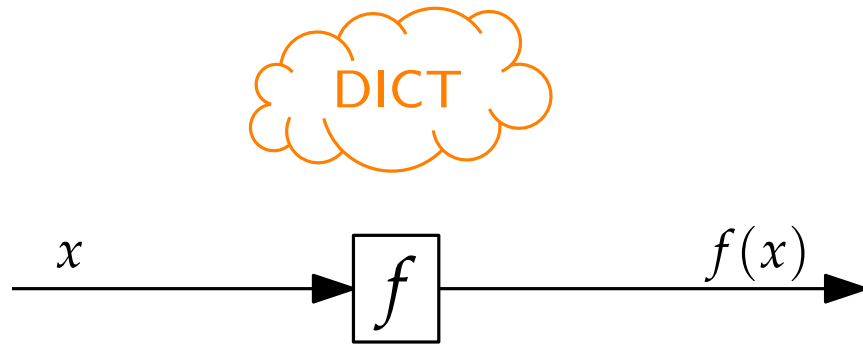
Memoization



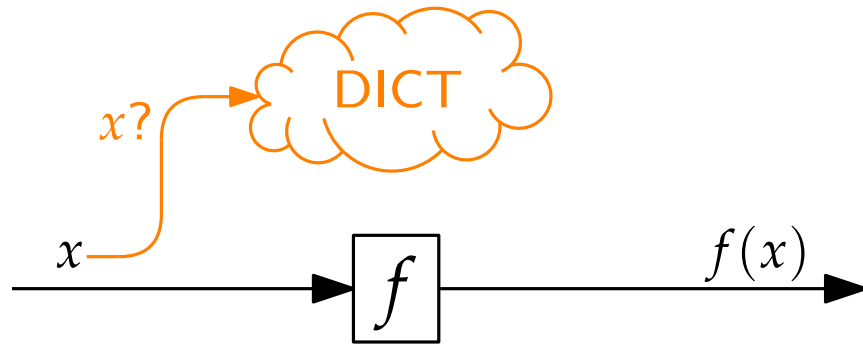
Memoization



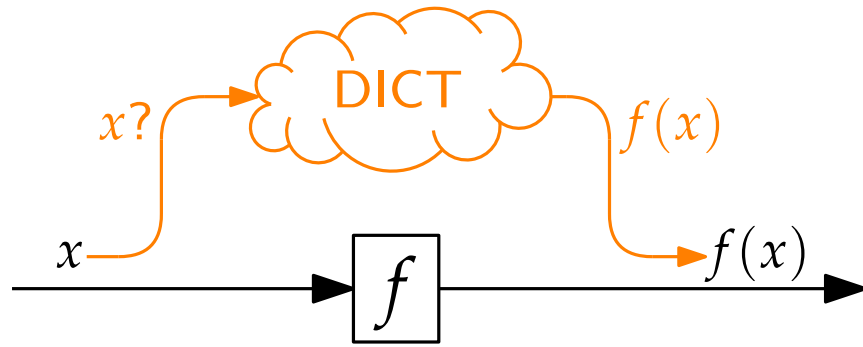
Memoization



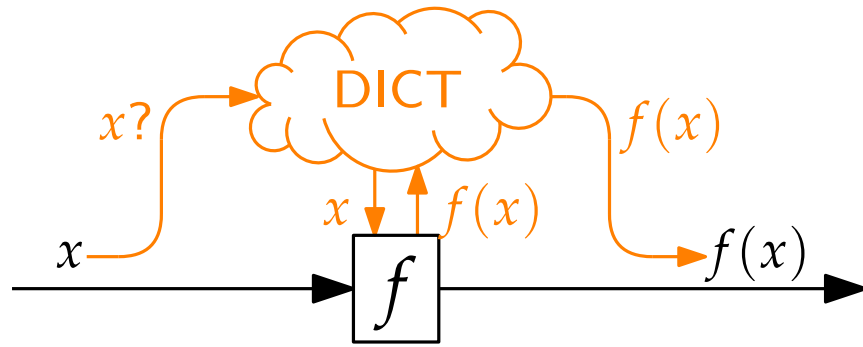
Memoization



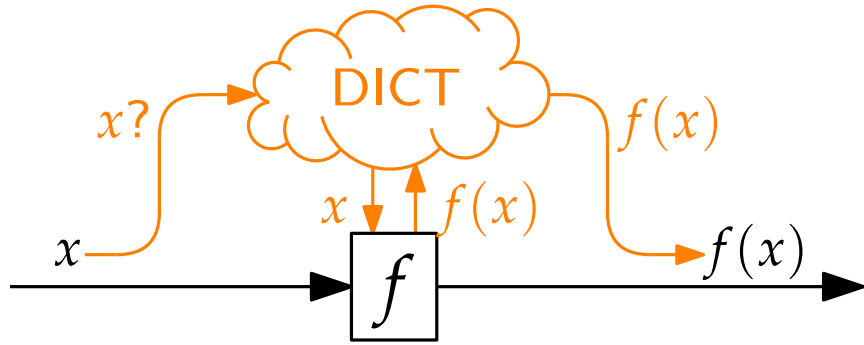
Memoization



Memoization

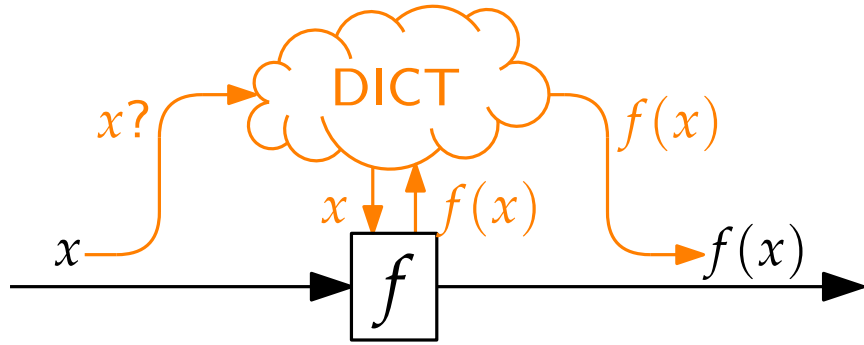


Memoization



```
def memoized[K, V](f: K => V): K => V =  
  val dict = mutable.Map.empty[K, V]  
  (key: K) => dict.getOrElseUpdate(key, f(key))
```

Memoization



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def memoized[K, V](f: K => V): K => V =  
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```

```
lazy val go: ((Int, Int)) => Int =  
  memoized((args: (Int, Int)) =>  
    val (i, weight) = args  
    // fill in the body of the old go  
  )
```

Running Times

	$ U = 30$	100
Scala naïve	1.15 s	

$B = 10|U|$

Running Times

	$ U = 30$	100
--	------------	-------

Scala naïve	1.15 s	
-------------	--------	--

Scala memoized	230 ms	
----------------	--------	--

$B = 10|U|$

Running Times

	$ U = 30$	$ U = 100$
--	------------	-------------

Scala naïve	1.15 s	
-------------	--------	--

Scala memoized	230 ms	267 ms
----------------	--------	--------

$B = 10|U|$

Running Times

$ U =$	30	100	1K	$B = 10 U $
Scala naïve	1.15 s			
Scala memoized	230 ms	267 ms		

Running Times

$ U =$	30	100	1K	$B = 10 U $
Scala naïve	1.15 s			
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Running Times

```
Exception in thread "main" java.lang.StackOverflowError
  at scala.runtime.BoxesRunTime.equalsNumNum(BoxesRunTime.java:148)
  at scala.runtime.BoxesRunTime.equalsNumObject(BoxesRunTime.java:138)
  at scala.runtime.BoxesRunTime.equals2(BoxesRunTime.java:127)
  at scala.runtime.BoxesRunTime.equals(BoxesRunTime.java:119)
  at scala.Tuple2.equals(Tuple2.scala:24)
  at scala.runtime.BoxesRunTime.equals2(BoxesRunTime.java:133)
  at scala.runtime.BoxesRunTime.equals(BoxesRunTime.java:119)
  at scala.collection.mutable.HashMap$Node.findNode(HashMap.scala:621)
  at scala.collection.mutable.HashMap.getOrElseUpdate(HashMap.scala:449)
  at y.Memoized$.momoized$$anonfun$1(Y.scala:64)
  at y.Memoized$.go$lzyINIT1$1$$anonfun$1(Y.scala:72)
  at y.Memoized$.momoized$$anonfun$1$$anonfun$1(Y.scala:64)
  at scala.collection.mutable.HashMap.getOrElseUpdate(HashMap.scala:454)
  at y.Memoized$.momoized$$anonfun$1(Y.scala:64)
  ...
```


Running Times

$ U =$	30	100	1K
Scala naïve	1.15 s		
Scala memoized	230 ms	267 ms	⚡ so
Scala TCO			

$$B = 10|U|$$

Covered in this talk:



Running Times

$ U =$	30	100	1K
Scala naïve	1.15 s		
Scala memoized	230 ms	267 ms	⚡ so
Scala TCO		273 ms	

$$B = 10|U|$$

Covered in this talk:



Running Times

$ U =$	30	100	1K
Scala naïve	1.15 s		
Scala memoized	230 ms	267 ms	⚡ so
Scala TCO		273 ms	2.30 s

$$B = 10|U|$$

Covered in this talk:



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s		

Covered in this talk:



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	...	

Covered in this talk:



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	...	

```
Exception in thread "main" java.lang.OutOfMemoryError: Java heap s
  at java.base/java.lang.Integer.valueOf(Integer.java:1077)
  at scala.runtime.BoxesRunTime.boxToInteger(BoxesRunTime.java:6
  at y.TCO$.go$3(Y.scala:112)
  at y.TCO$.solve(Y.scala:121)
  at y.TCO$.runTco(Y.scala:82)
  at y.runTco.main(Y.scala:80)
```

Covered in this talk:



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	

Covered in this talk:



Reducing the Memory Footprint

Idea:

Reducing the Memory Footprint

- Idea:**
- Use Iteration instead of Recursion
 - Use Arrays instead of a Dictionary

Reducing the Memory Footprint

```
def solve(items: Seq[Item], weight: Int) =  
    val dict = Array.fill(items.size, weight + 1)(0)
```

Reducing the Memory Footprint

```
def solve(items: Seq[Item], weight: Int) =  
  val dict = Array.fill(items.size, weight + 1)(0)  
  val head = items.head  
  for w <- head.weight to weight do dict(0)(w) = head.value
```

Reducing the Memory Footprint

```
def solve(items: Seq[Item], weight: Int) =  
  val dict = Array.fill(items.size, weight + 1)(0)  
  val head = items.head  
  for w <- head.weight to weight do dict(0)(w) = head.value  
  for  
    (item, i) <- items.zipWithIndex.tail  
    w <- 0 to weight  
  do  
    val dontTake = dict(i - 1)(w)  
    if w >= item.weight then  
      val take = dict(i - 1)(w - item.weight) + item.value  
      dict(i)(w) = dontTake max take  
    else dict(i)(w) = dontTake
```

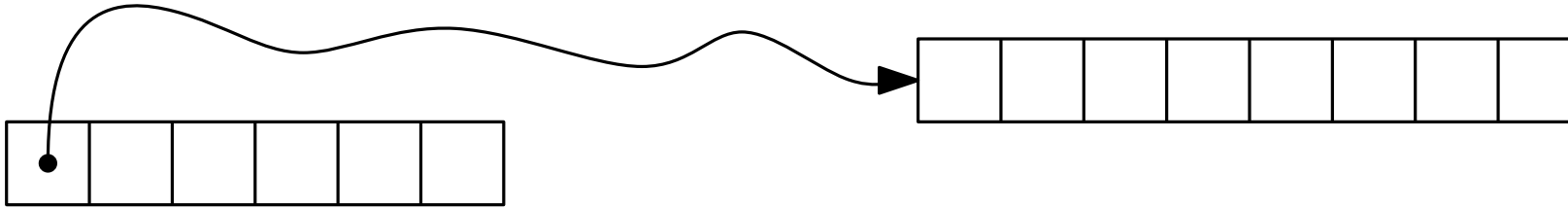
Reducing the Memory Footprint

```
def solve(items: Seq[Item], weight: Int) =  
  val dict = Array.fill(items.size, weight + 1)(0)  
  val head = items.head  
  for w <- head.weight to weight do dict(0)(w) = head.value  
  for  
    (item, i) <- items.zipWithIndex.tail  
    w <- 0 to weight  
  do  
    val dontTake = dict(i - 1)(w)  
    if w >= item.weight then  
      val take = dict(i - 1)(w - item.weight) + item.value  
      dict(i)(w) = dontTake max take  
    else dict(i)(w) = dontTake  
  dict(items.size - 1)(weight)
```

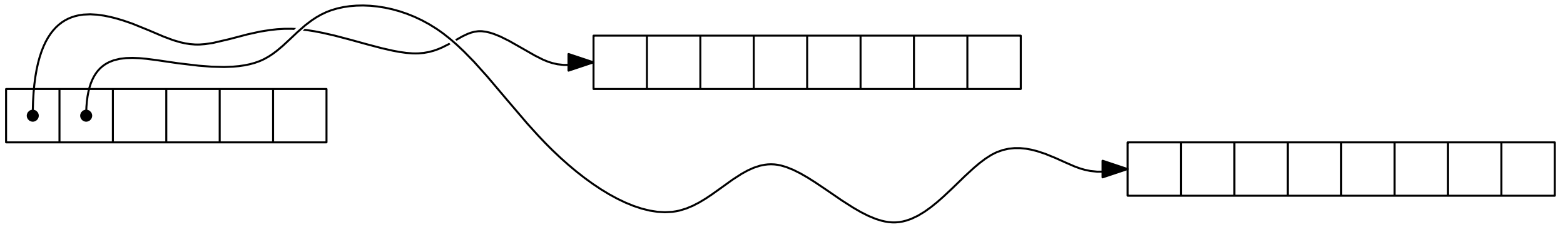
Arrays and Memory Locality



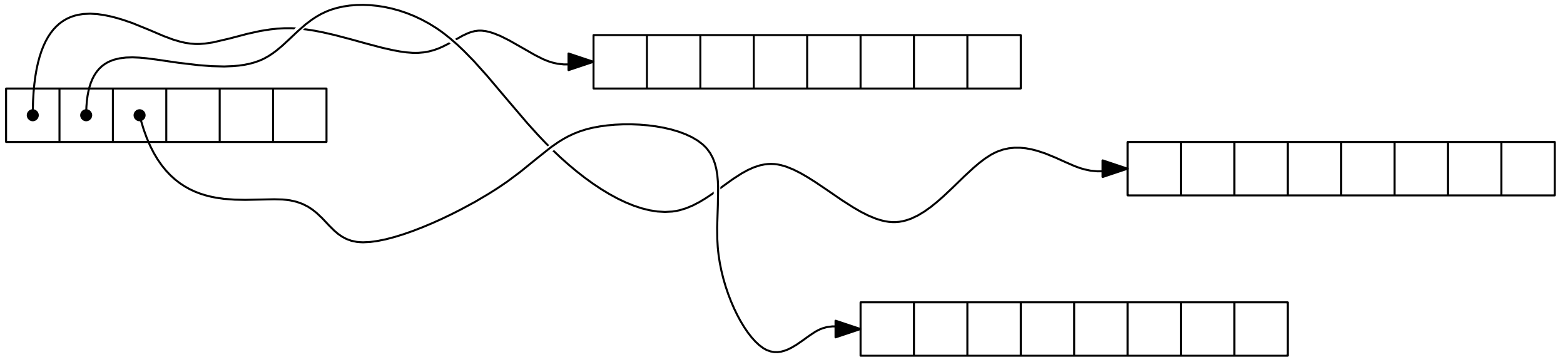
Arrays and Memory Locality



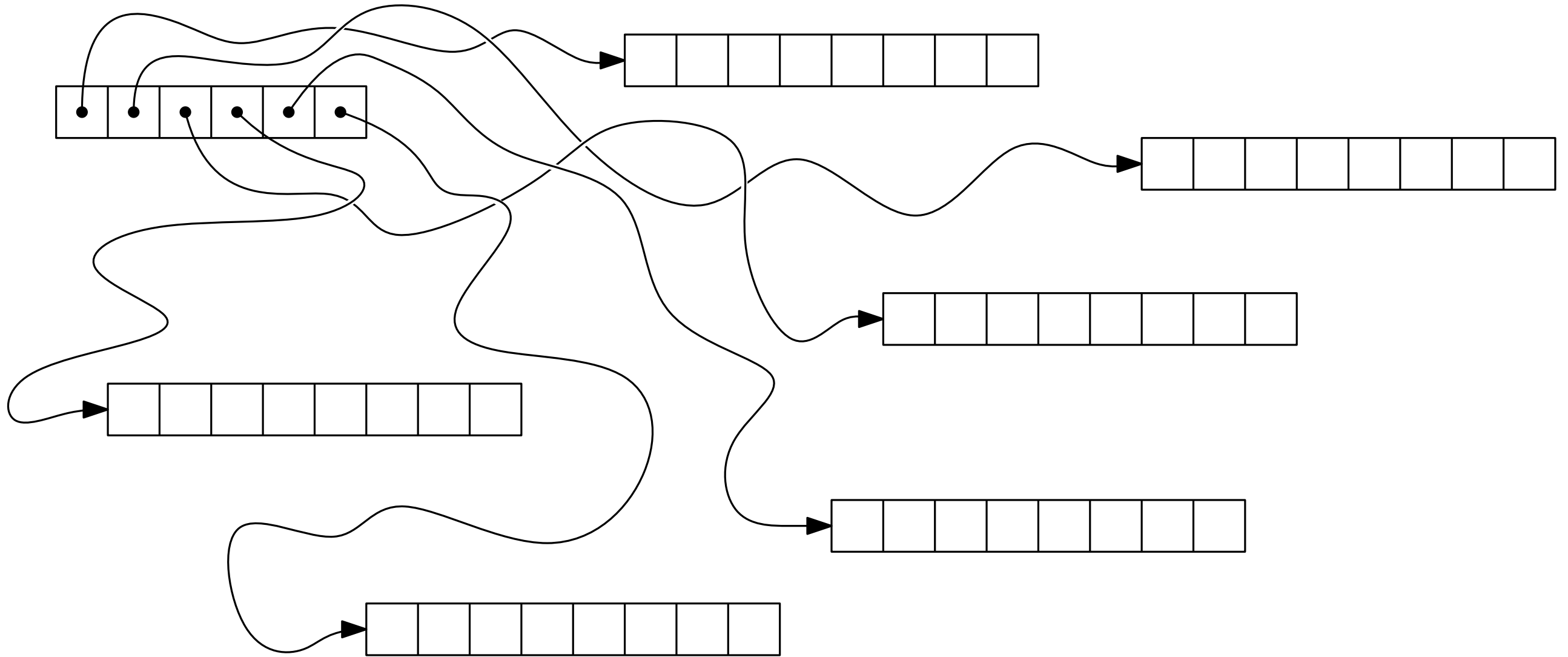
Arrays and Memory Locality



Arrays and Memory Locality

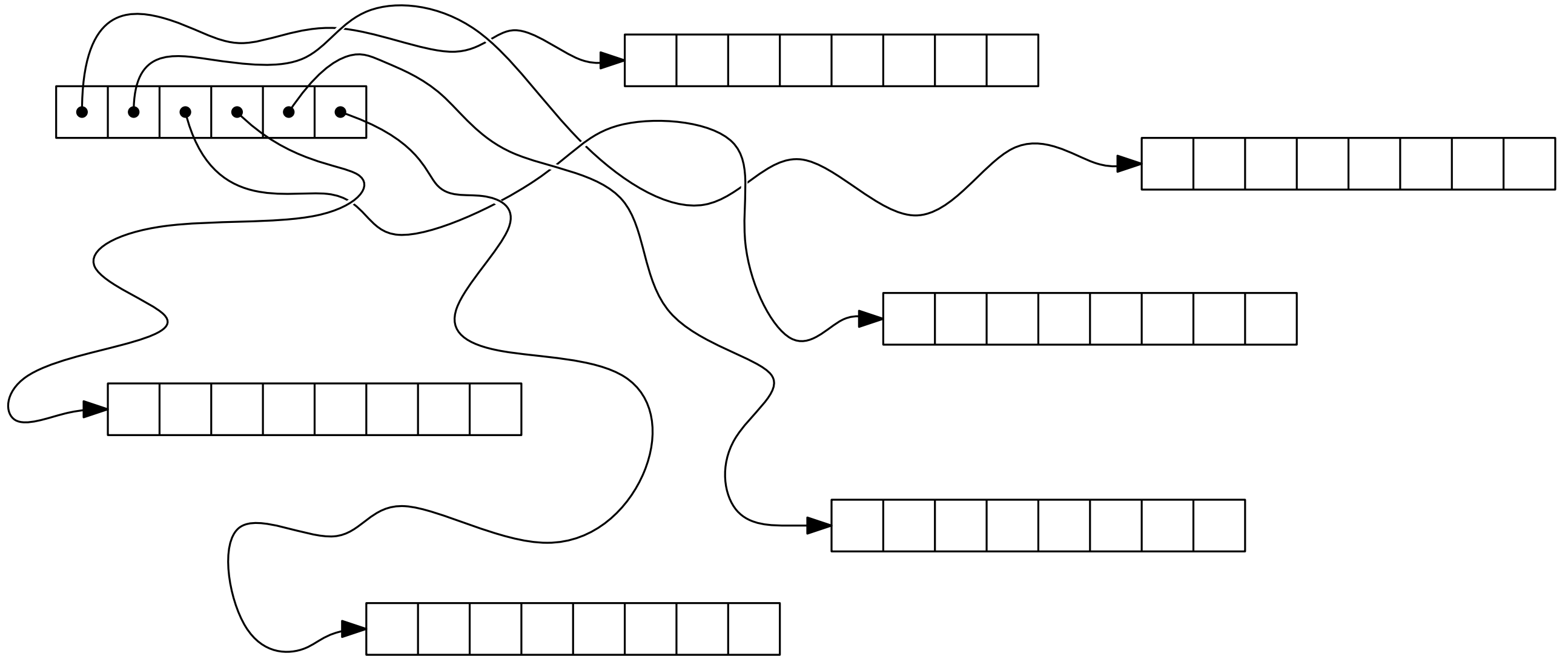


Arrays and Memory Locality



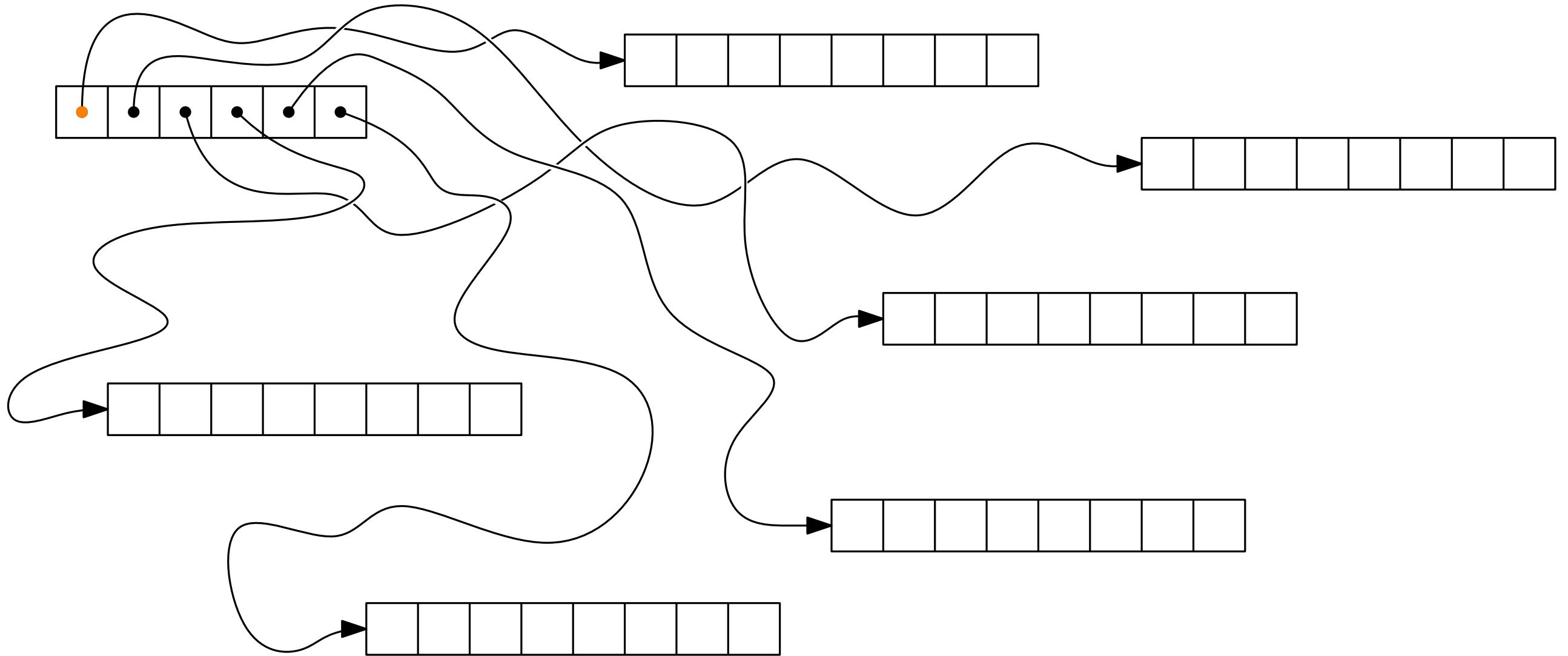
Arrays and Memory Locality

table(outer)(inner)

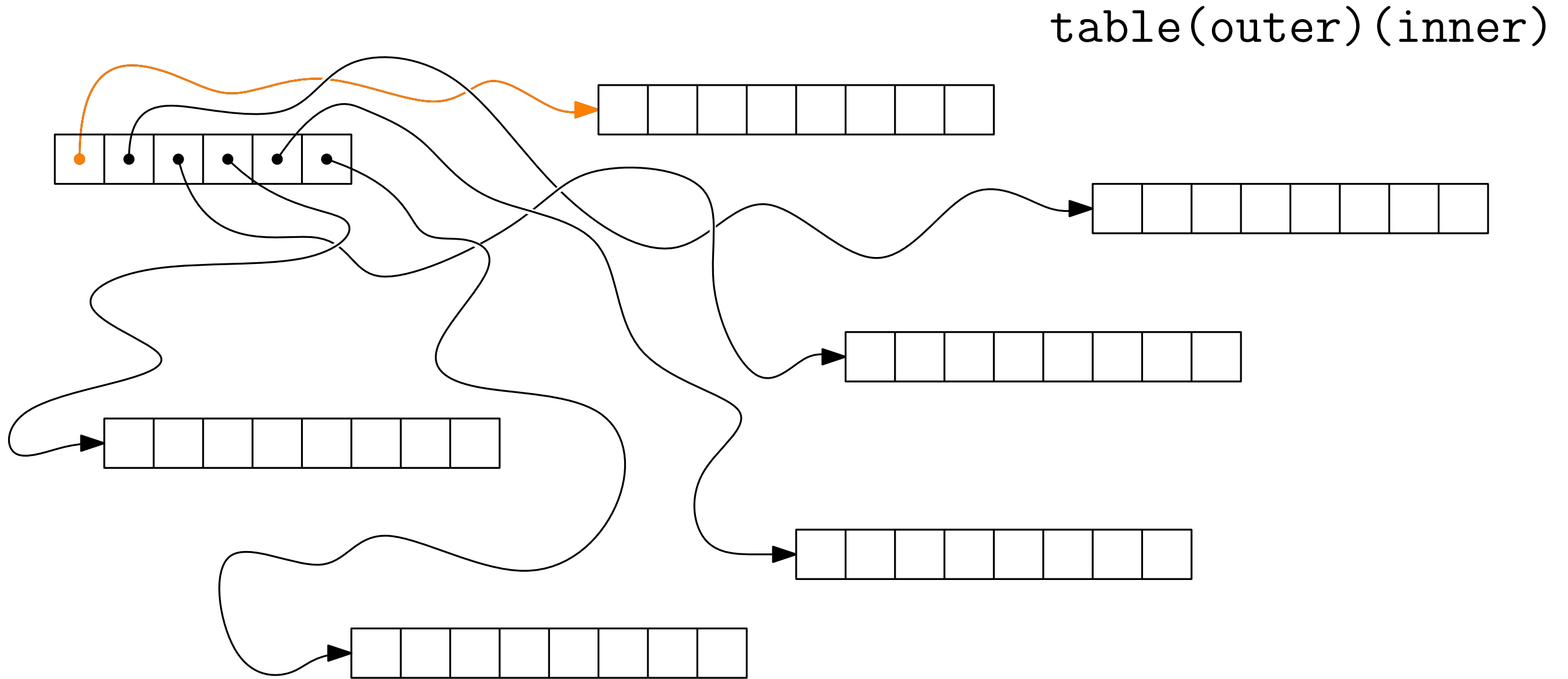


Arrays and Memory Locality

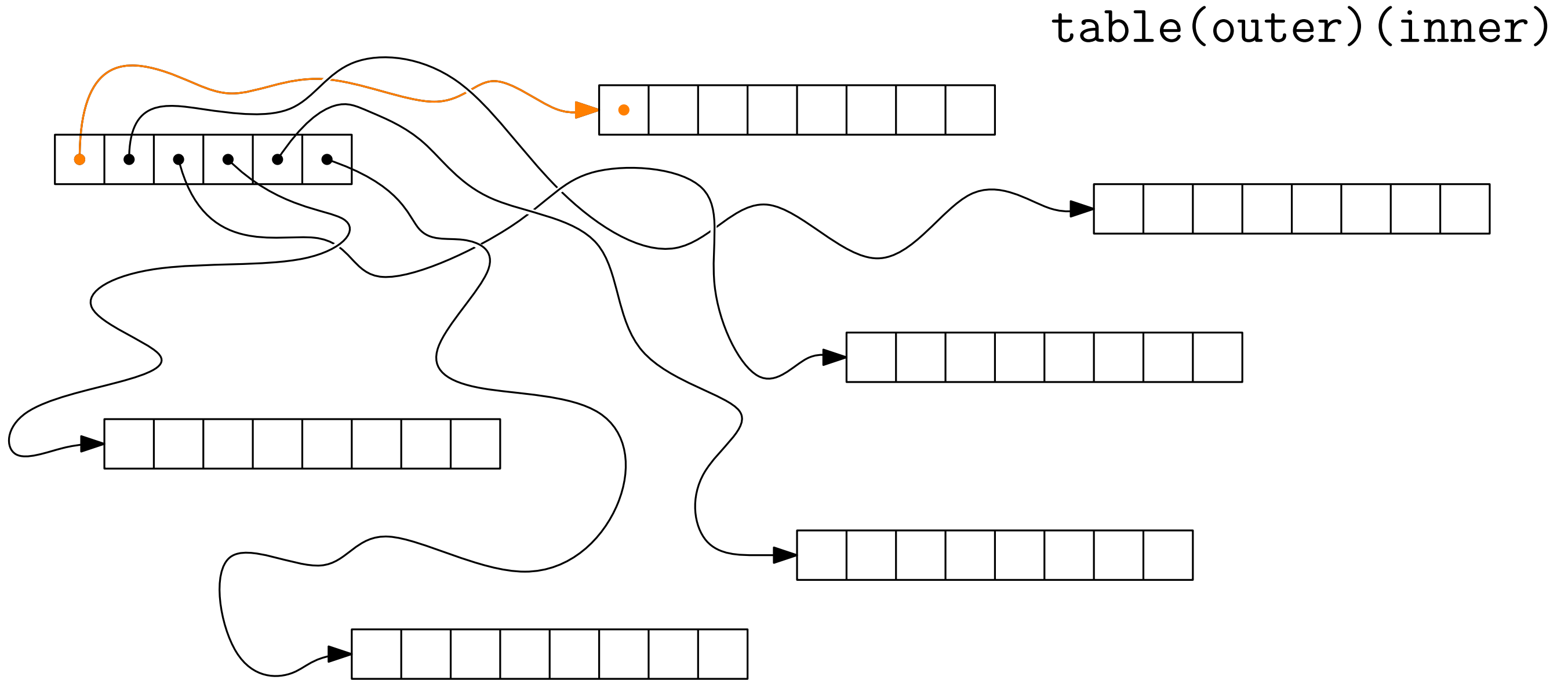
table(outer)(inner)



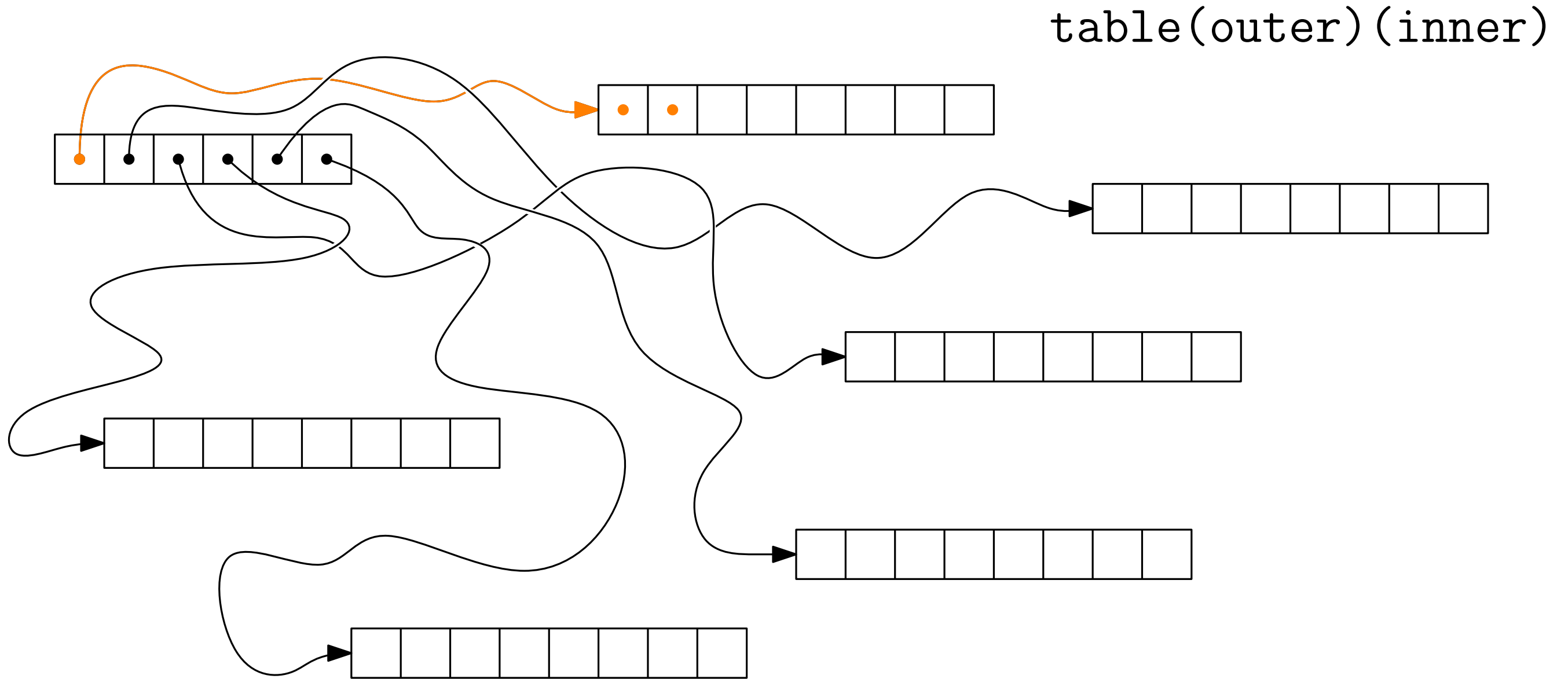
Arrays and Memory Locality



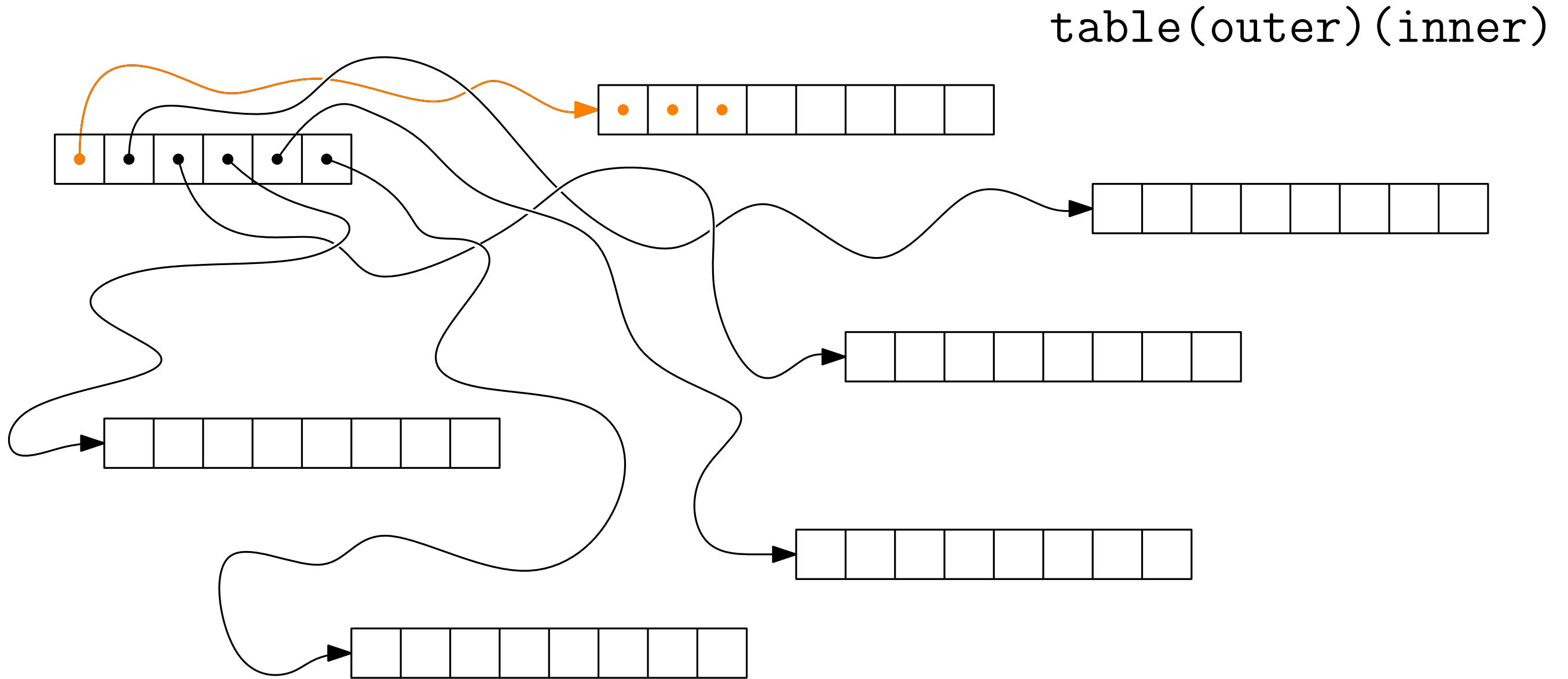
Arrays and Memory Locality



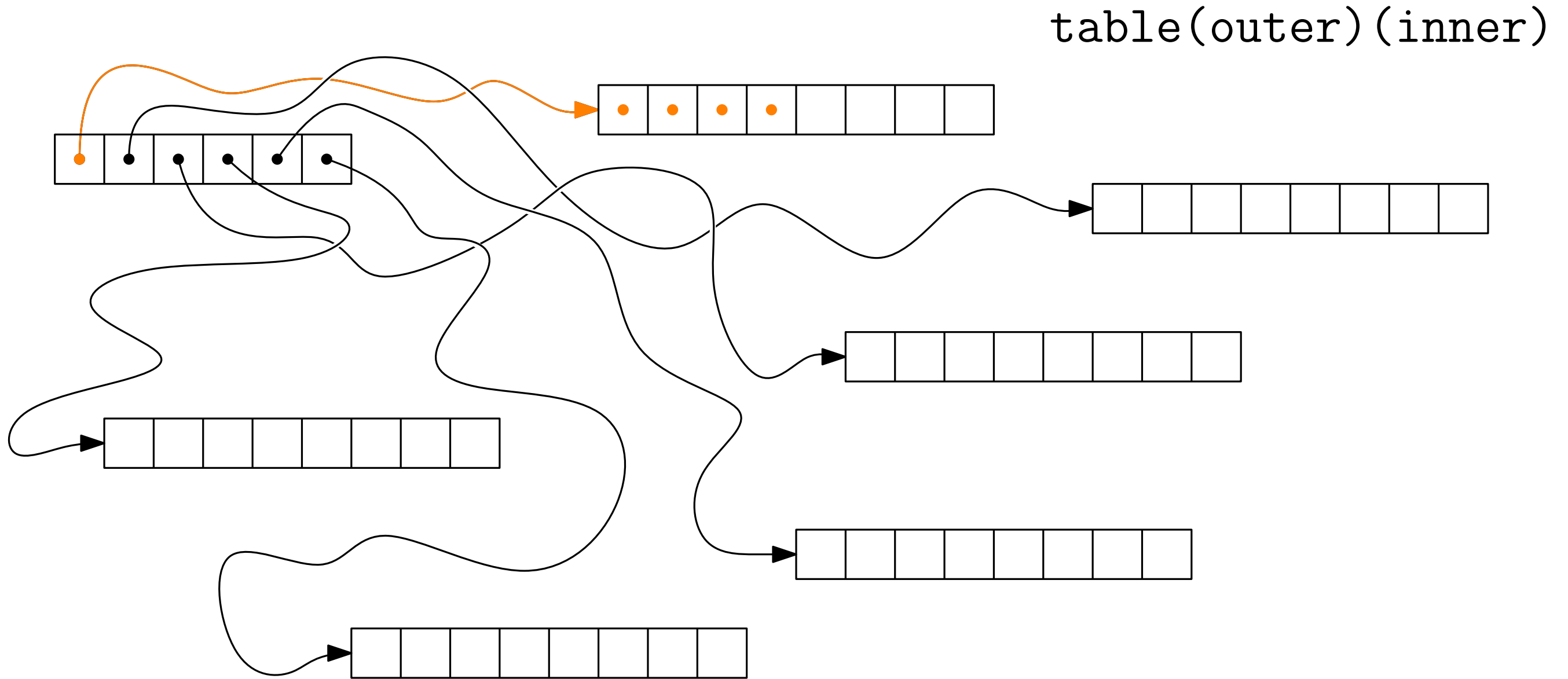
Arrays and Memory Locality



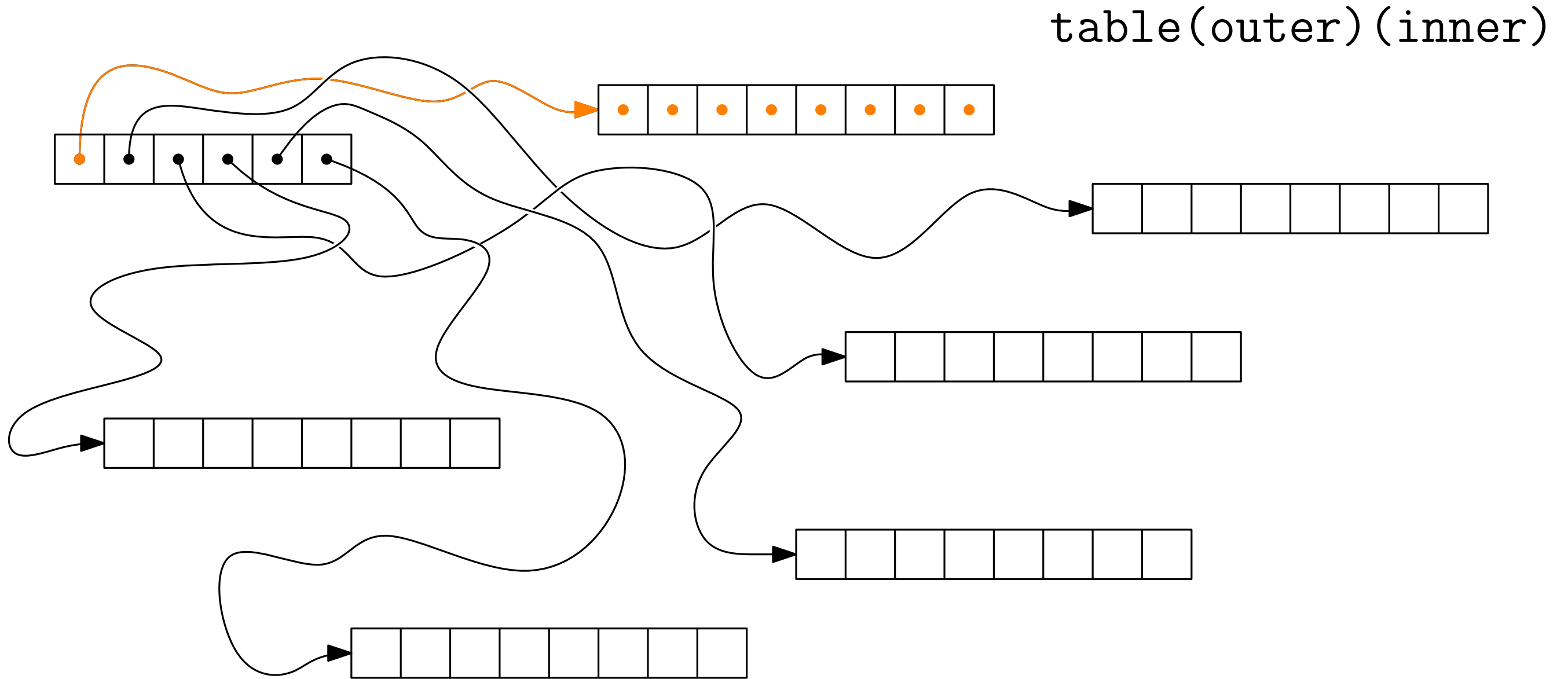
Arrays and Memory Locality



Arrays and Memory Locality

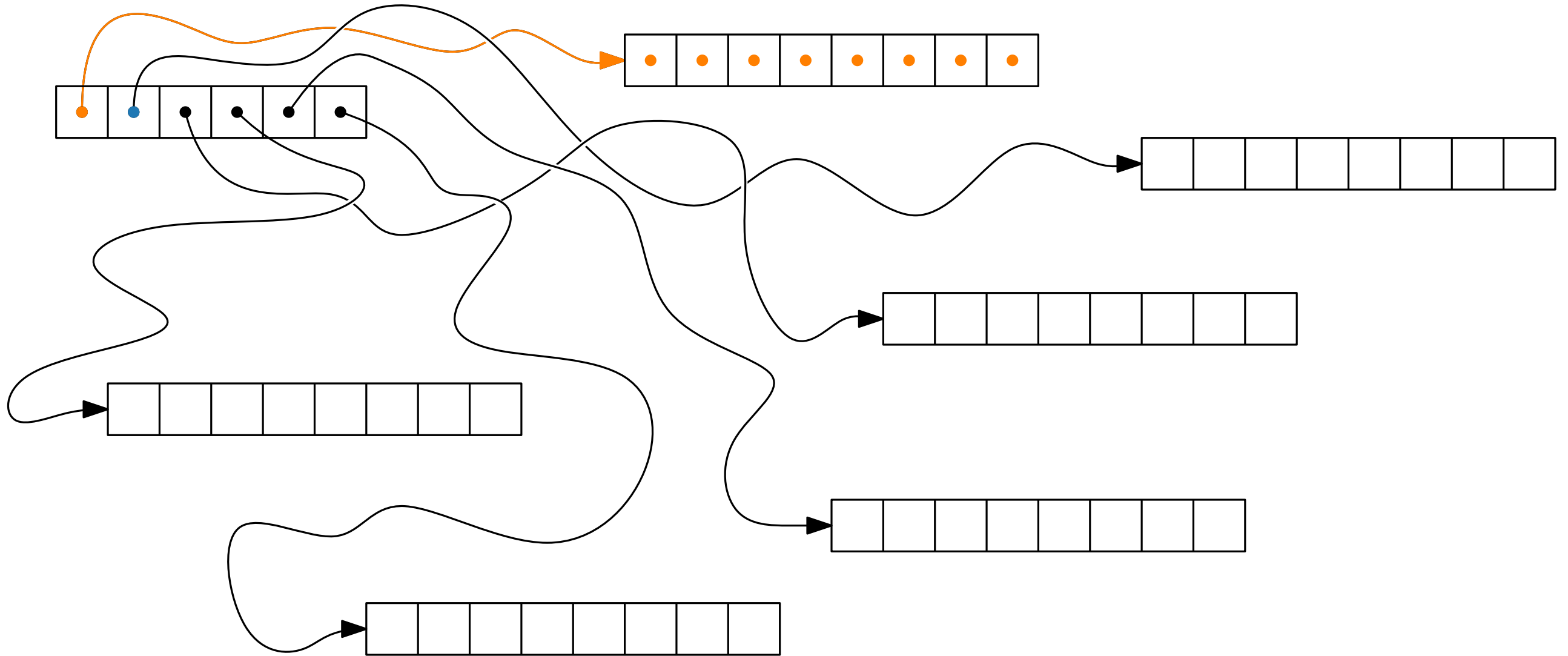


Arrays and Memory Locality

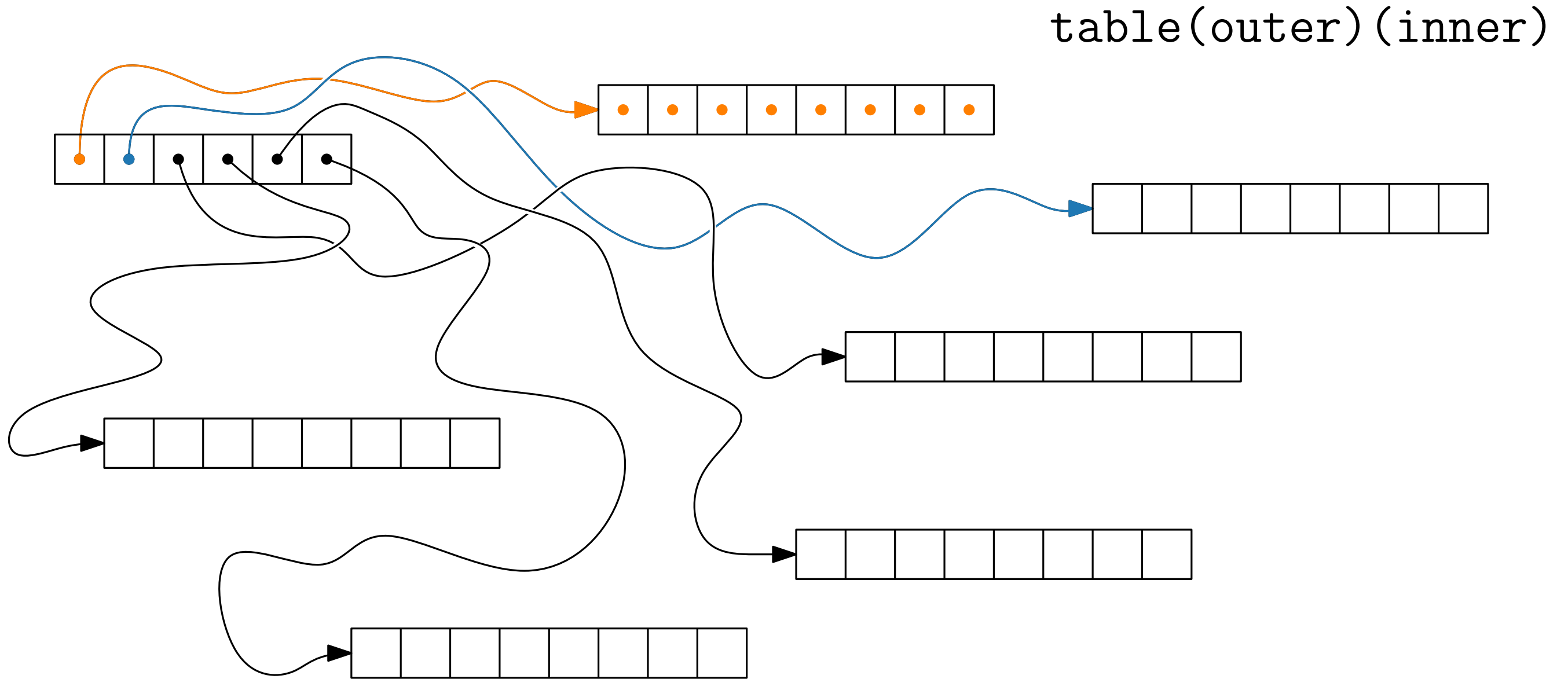


Arrays and Memory Locality

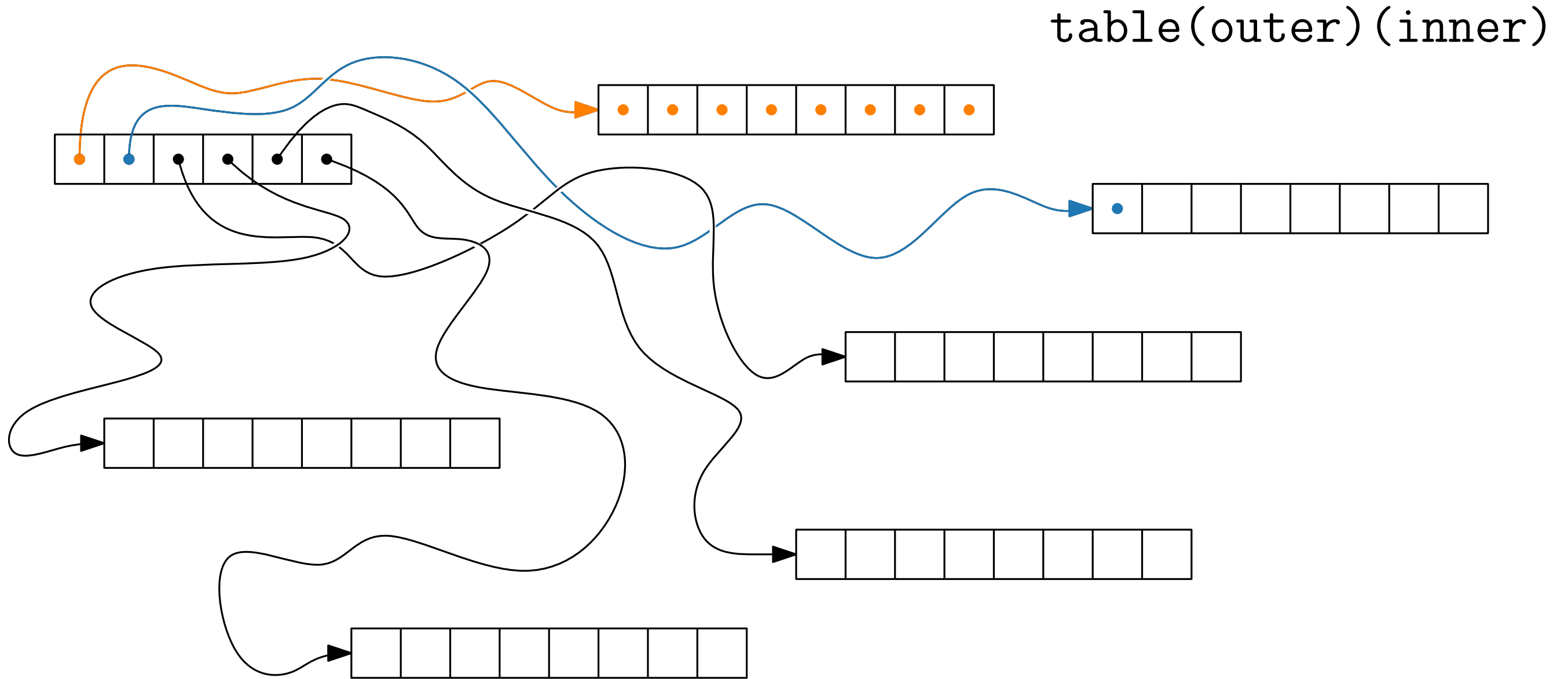
table(outer)(inner)



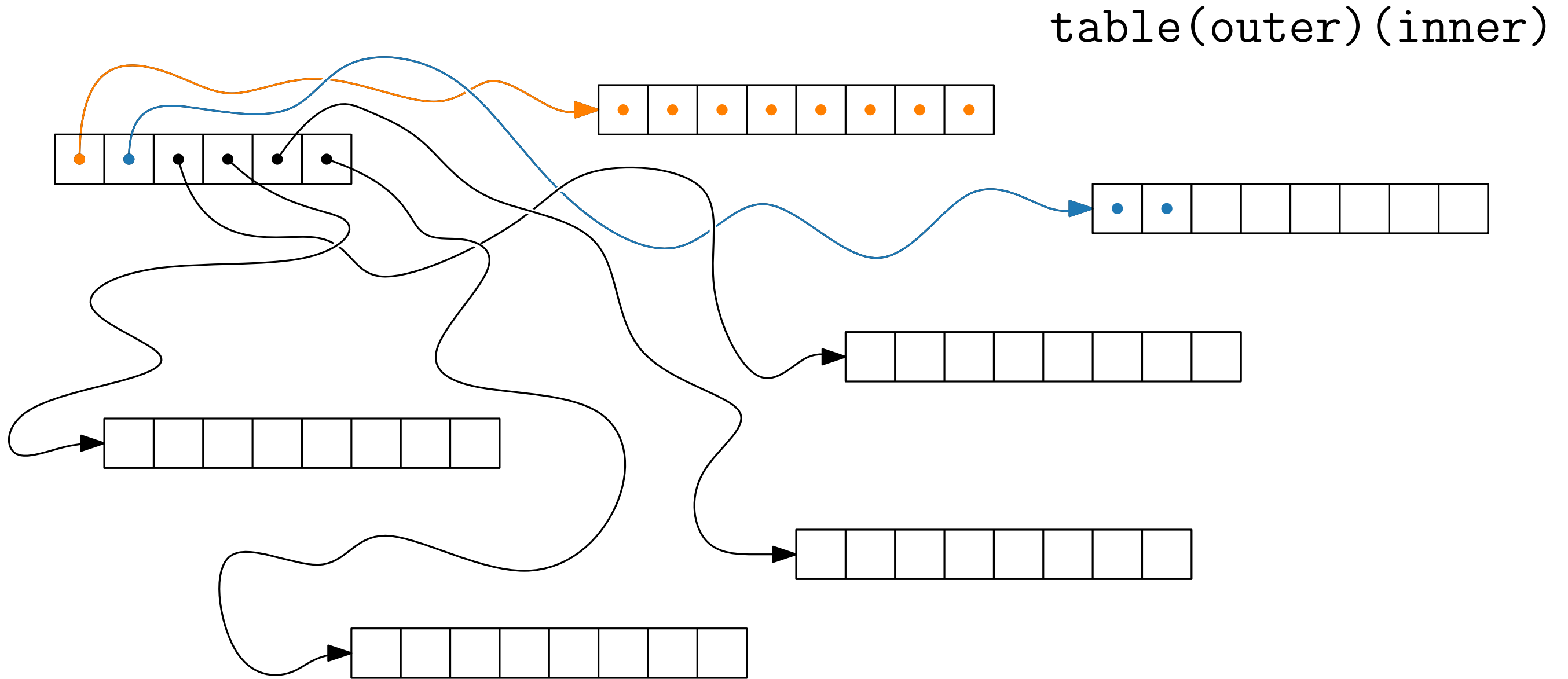
Arrays and Memory Locality



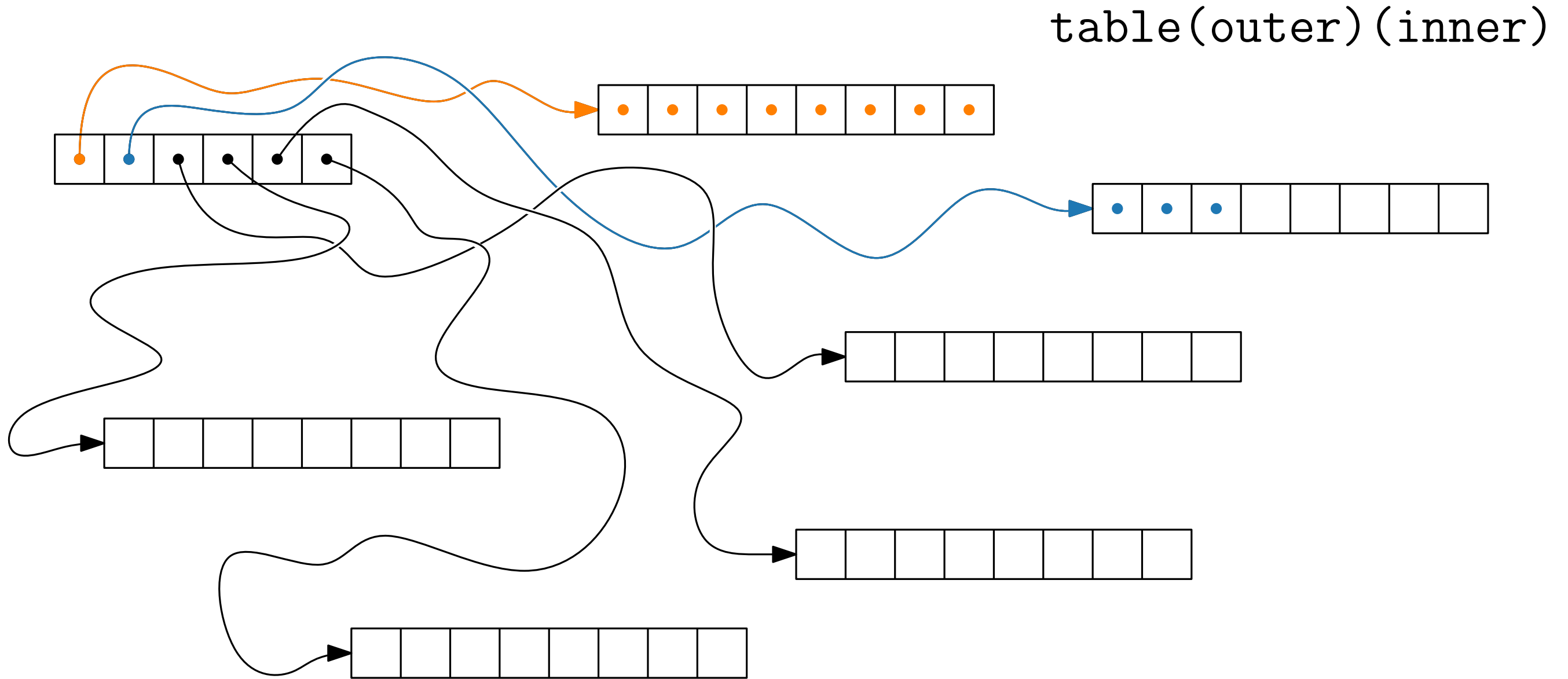
Arrays and Memory Locality



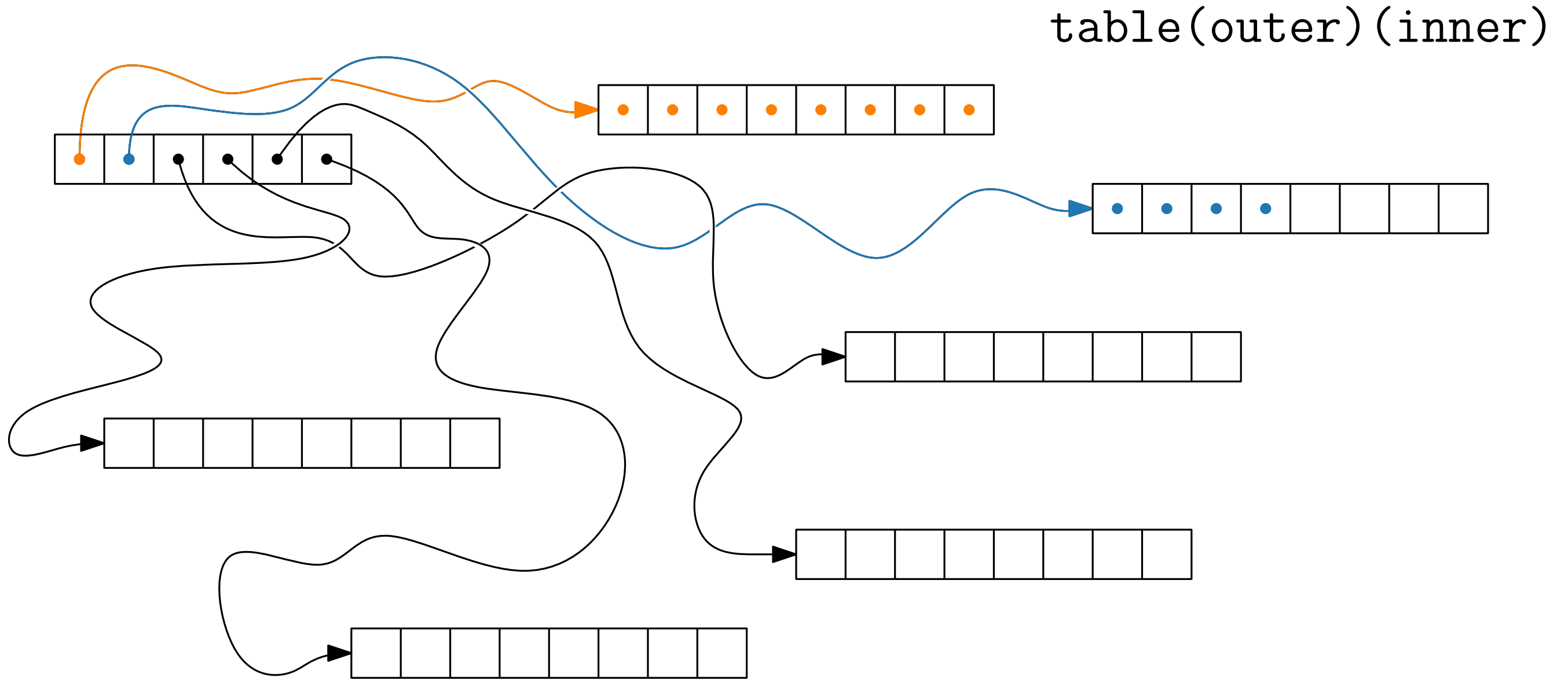
Arrays and Memory Locality



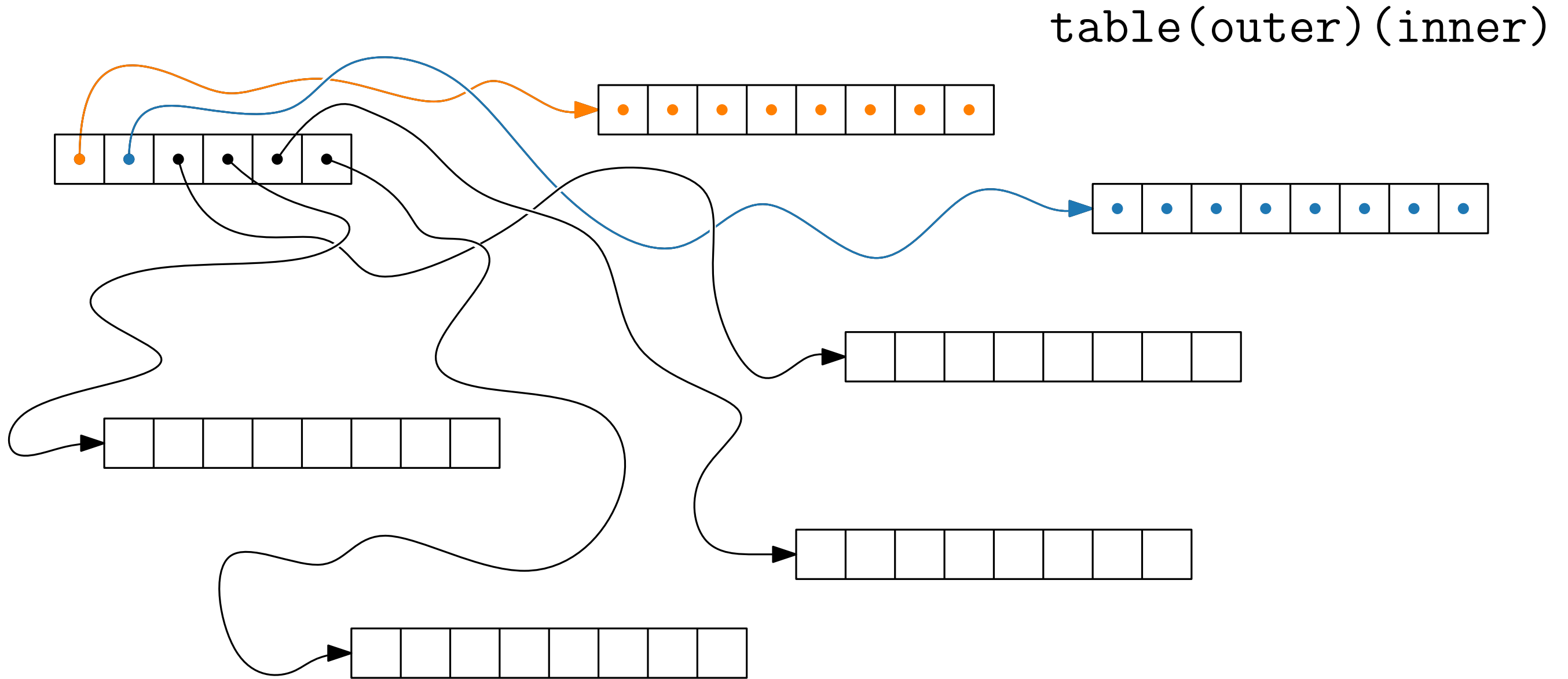
Arrays and Memory Locality



Arrays and Memory Locality

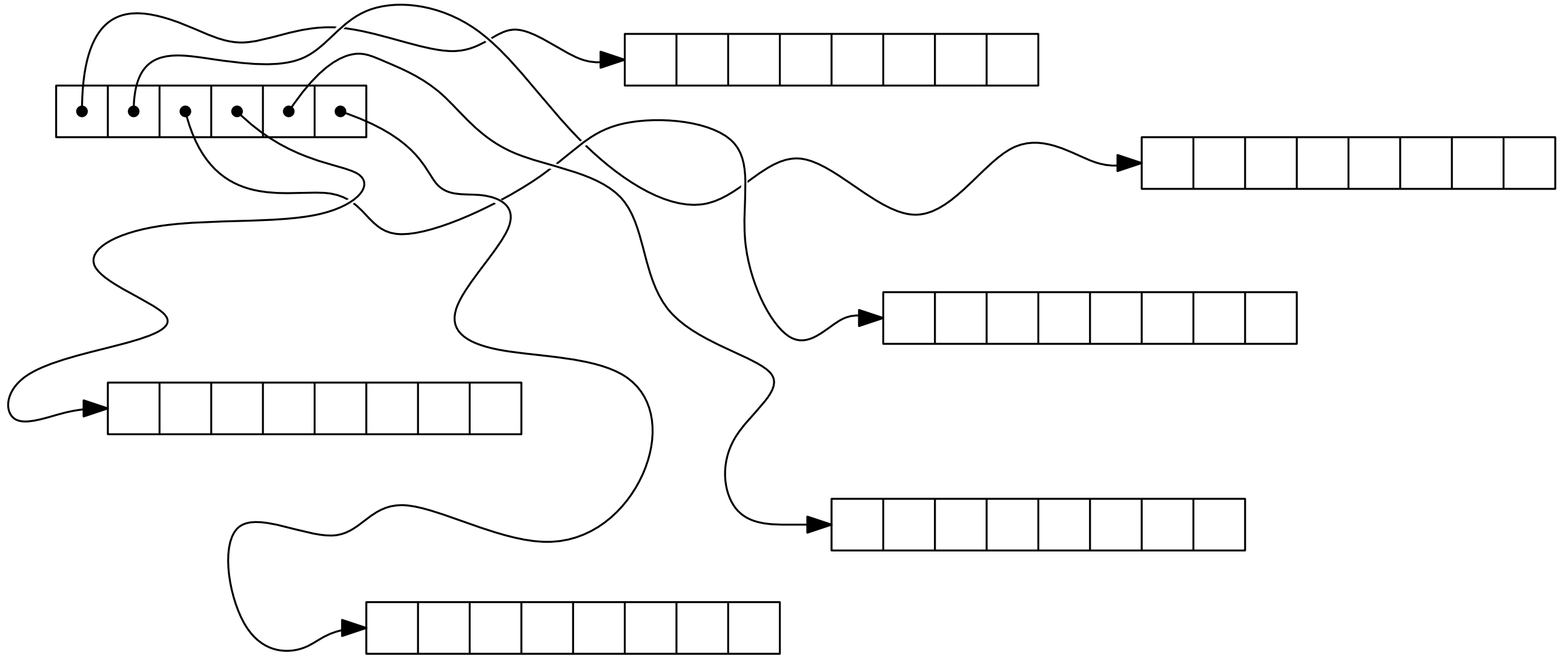


Arrays and Memory Locality



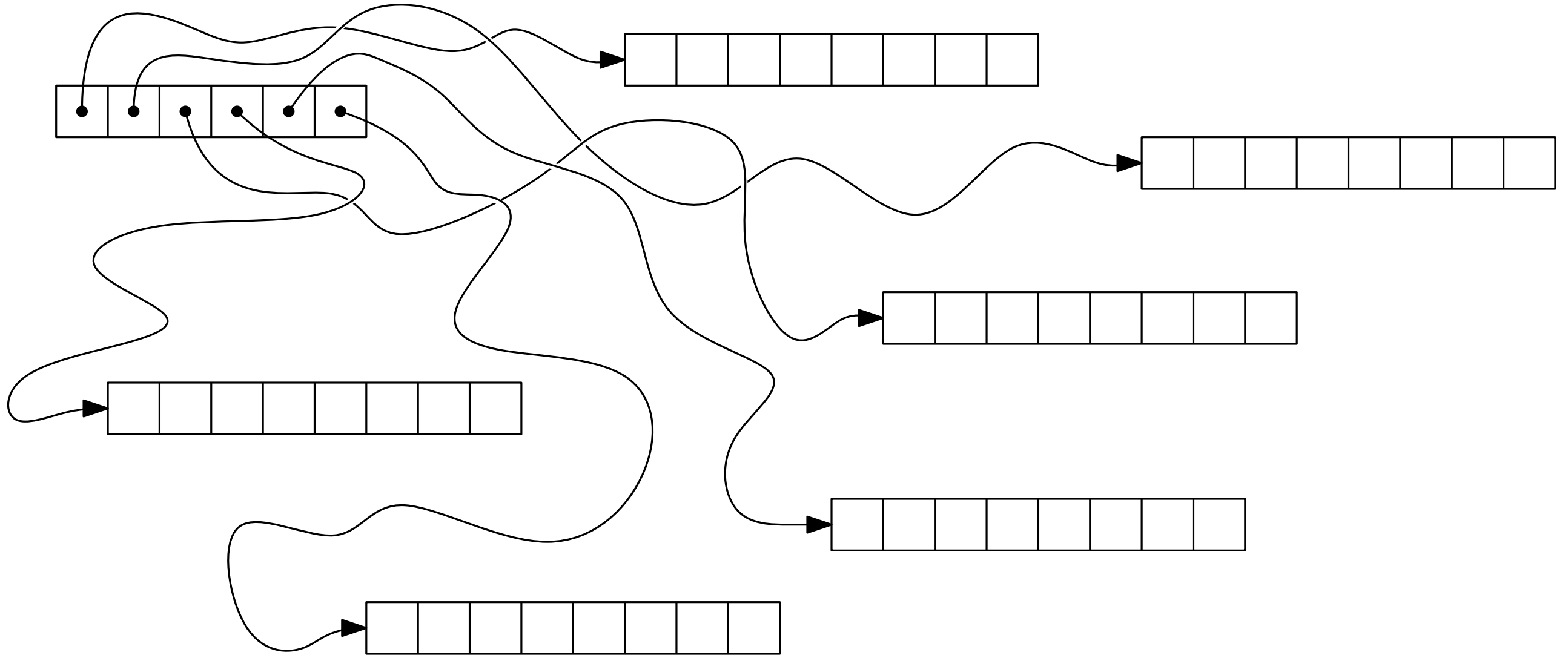
Arrays and Memory Locality

table(outer)(inner)



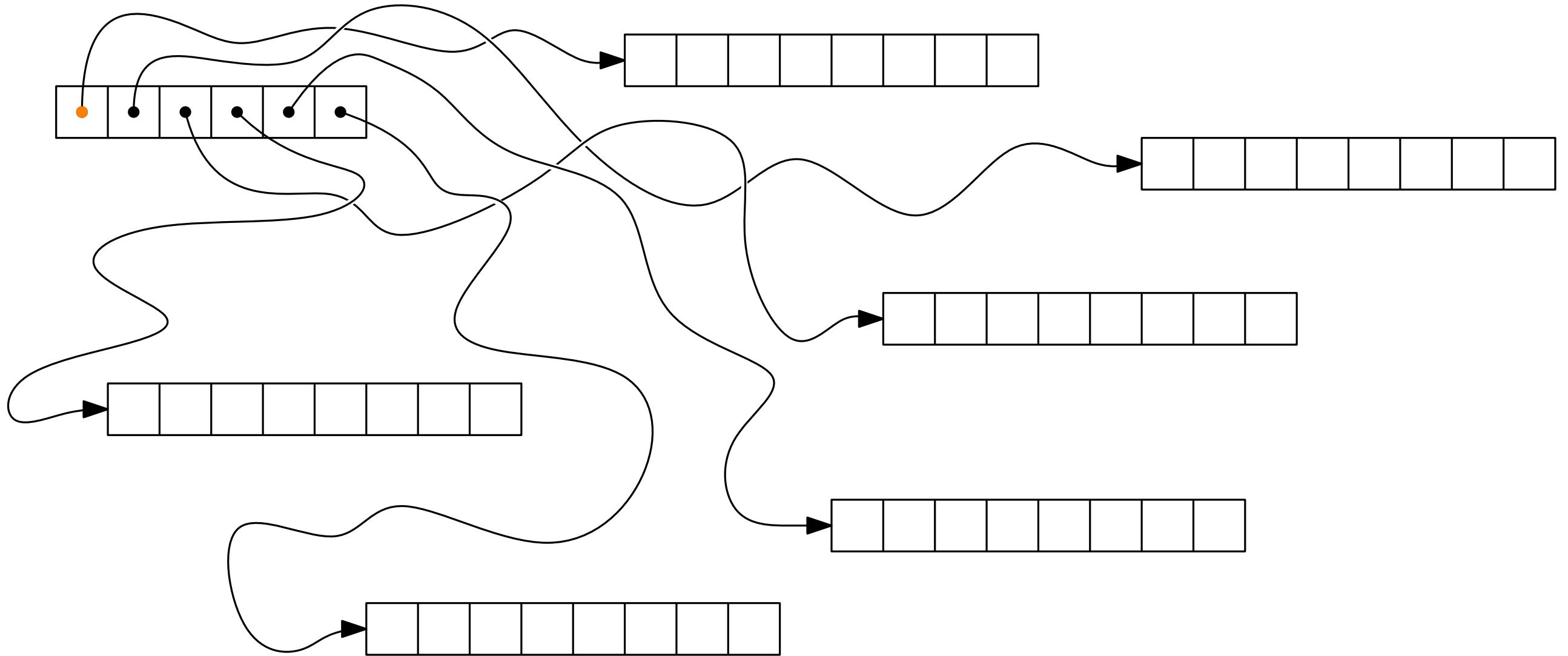
Arrays and Memory Locality

table(inner)(outer)



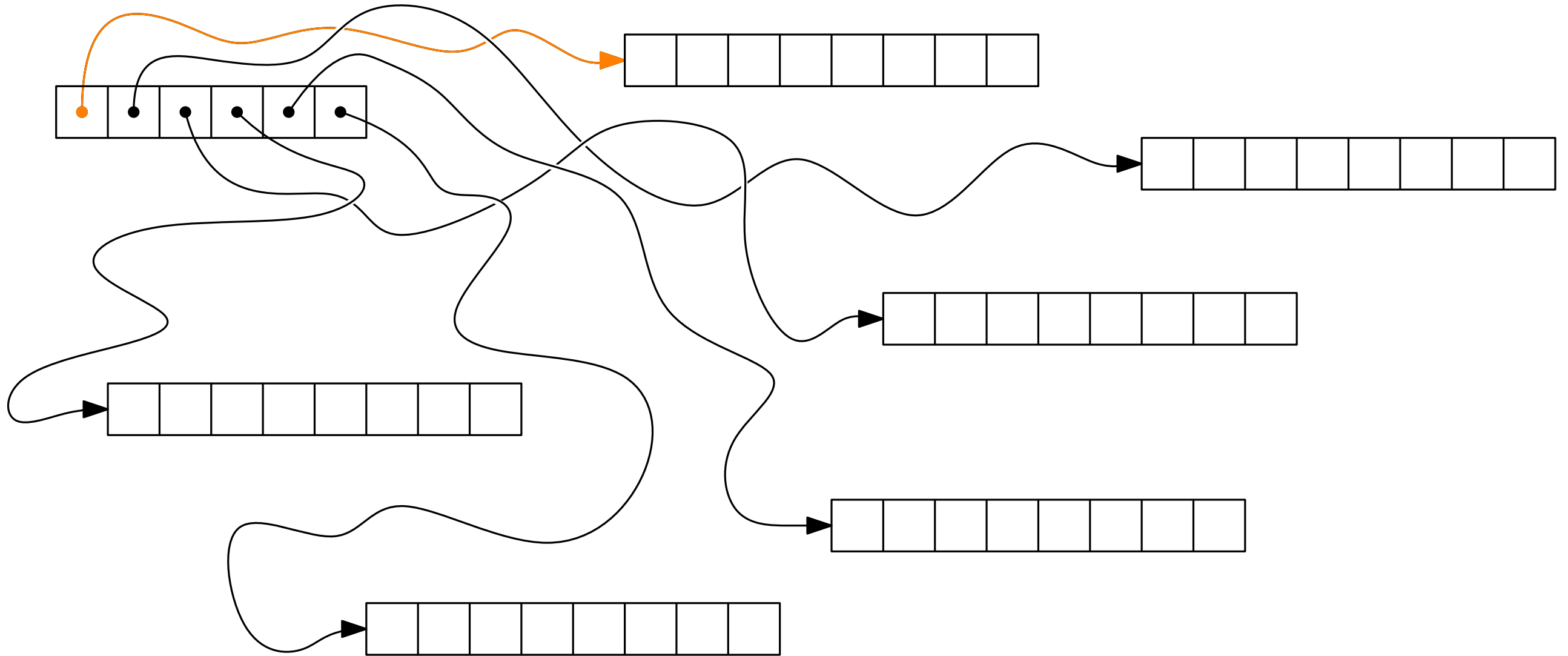
Arrays and Memory Locality

table(inner)(outer)



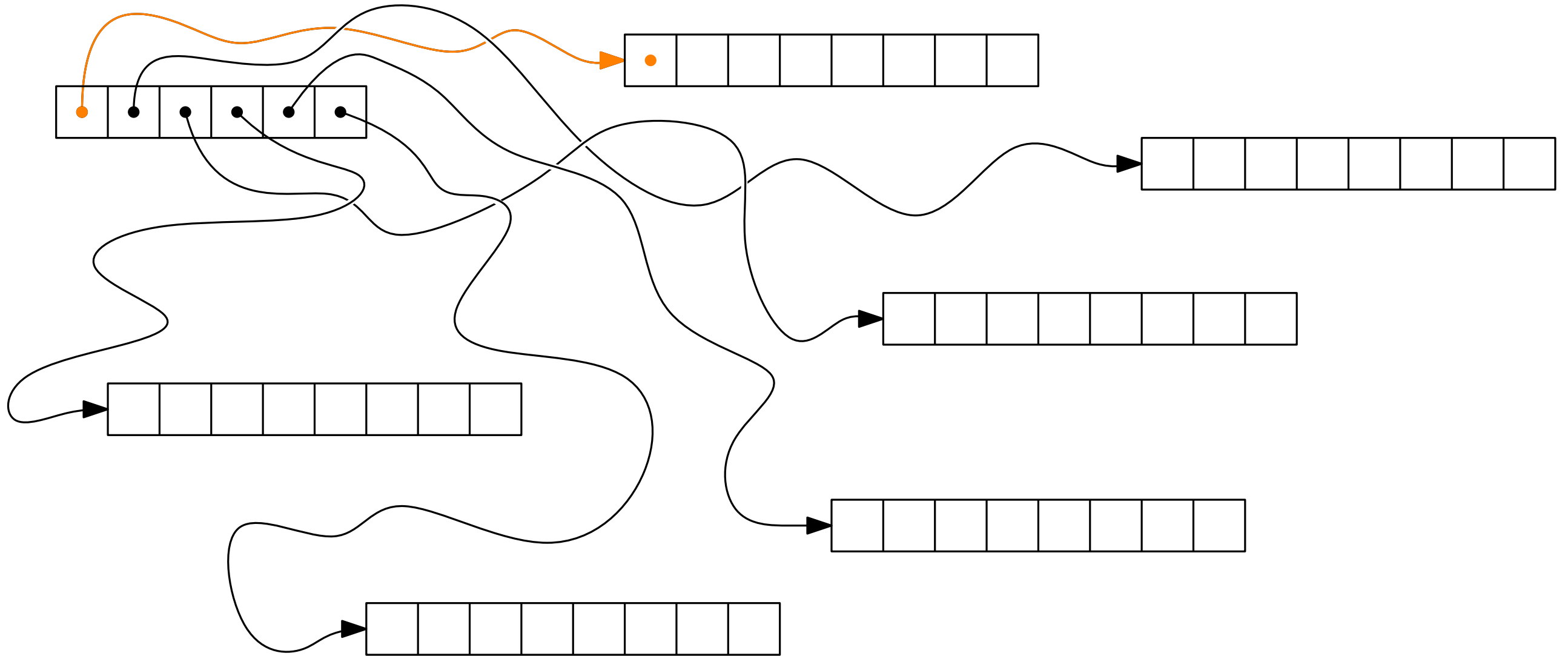
Arrays and Memory Locality

table(inner)(outer)

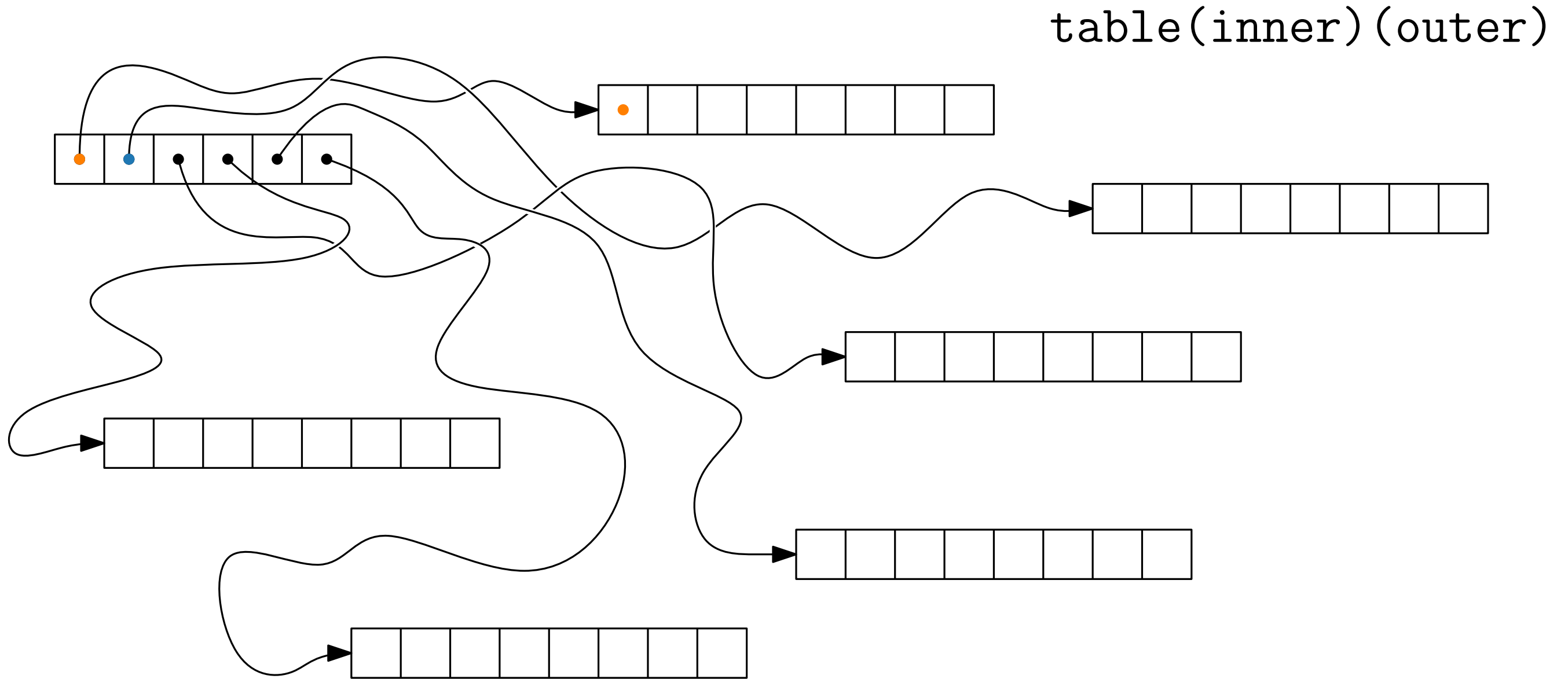


Arrays and Memory Locality

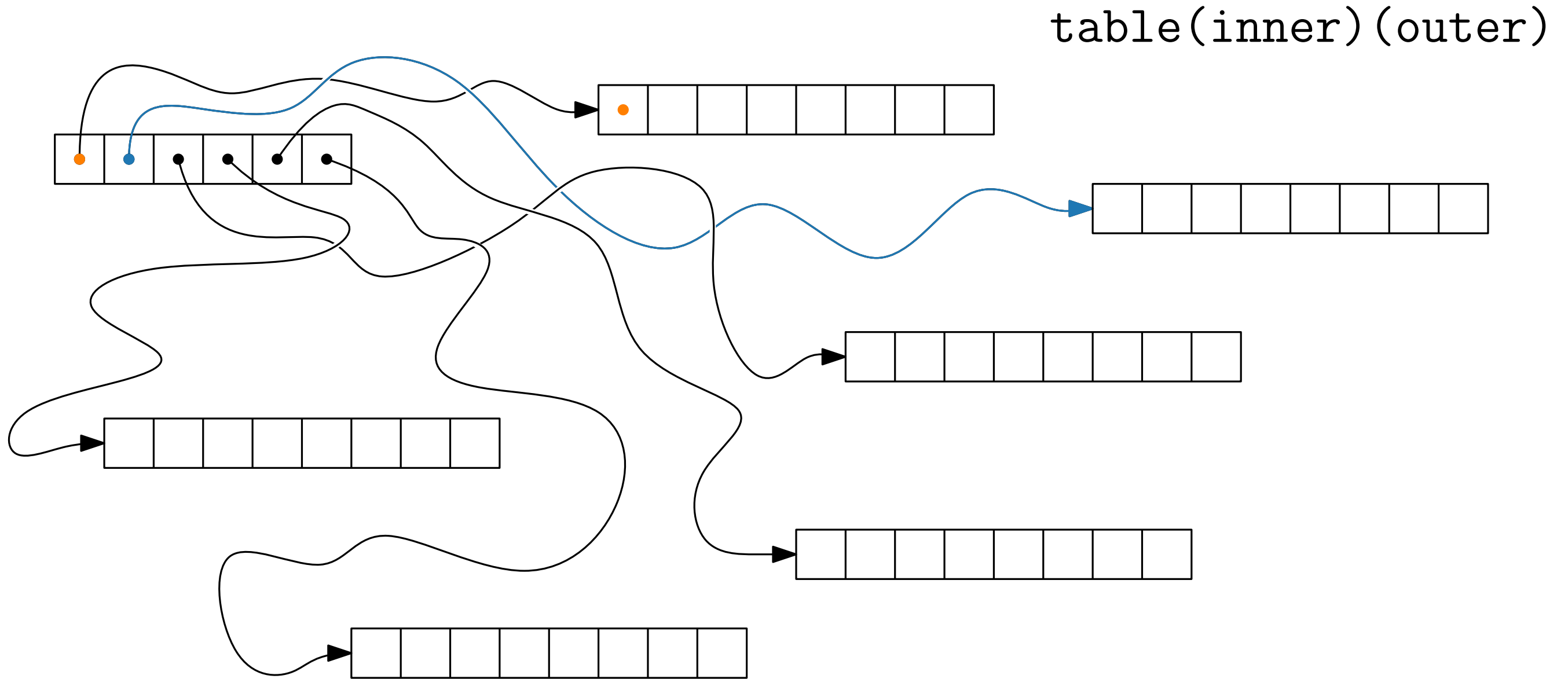
table(inner)(outer)



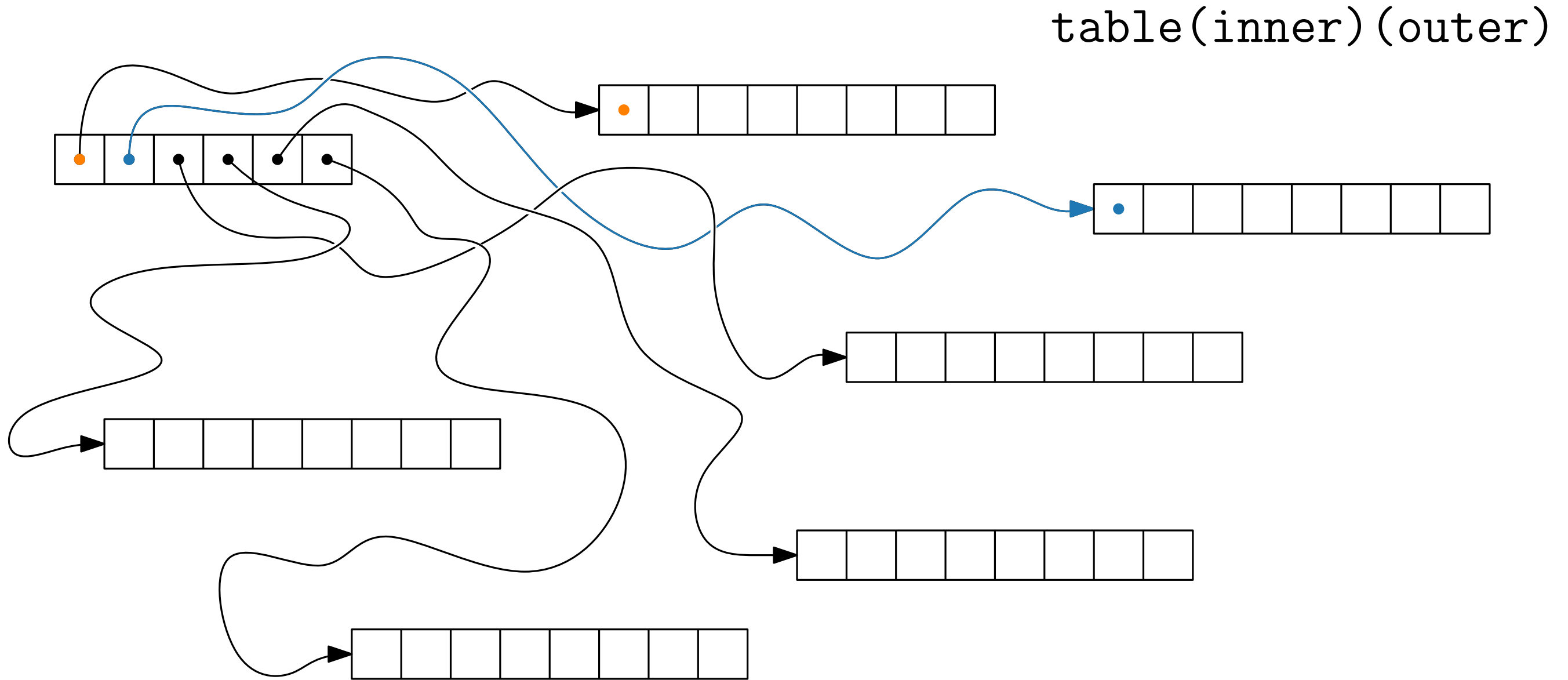
Arrays and Memory Locality



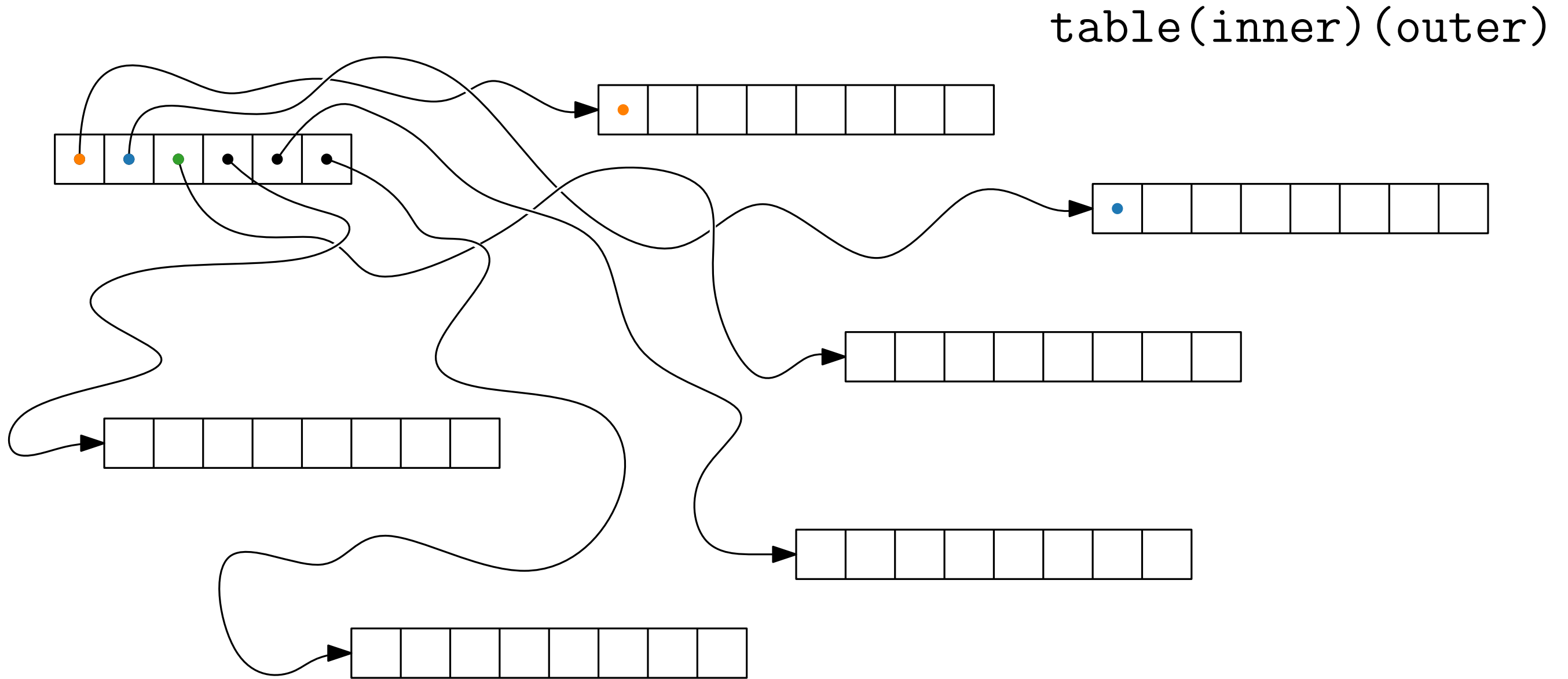
Arrays and Memory Locality



Arrays and Memory Locality

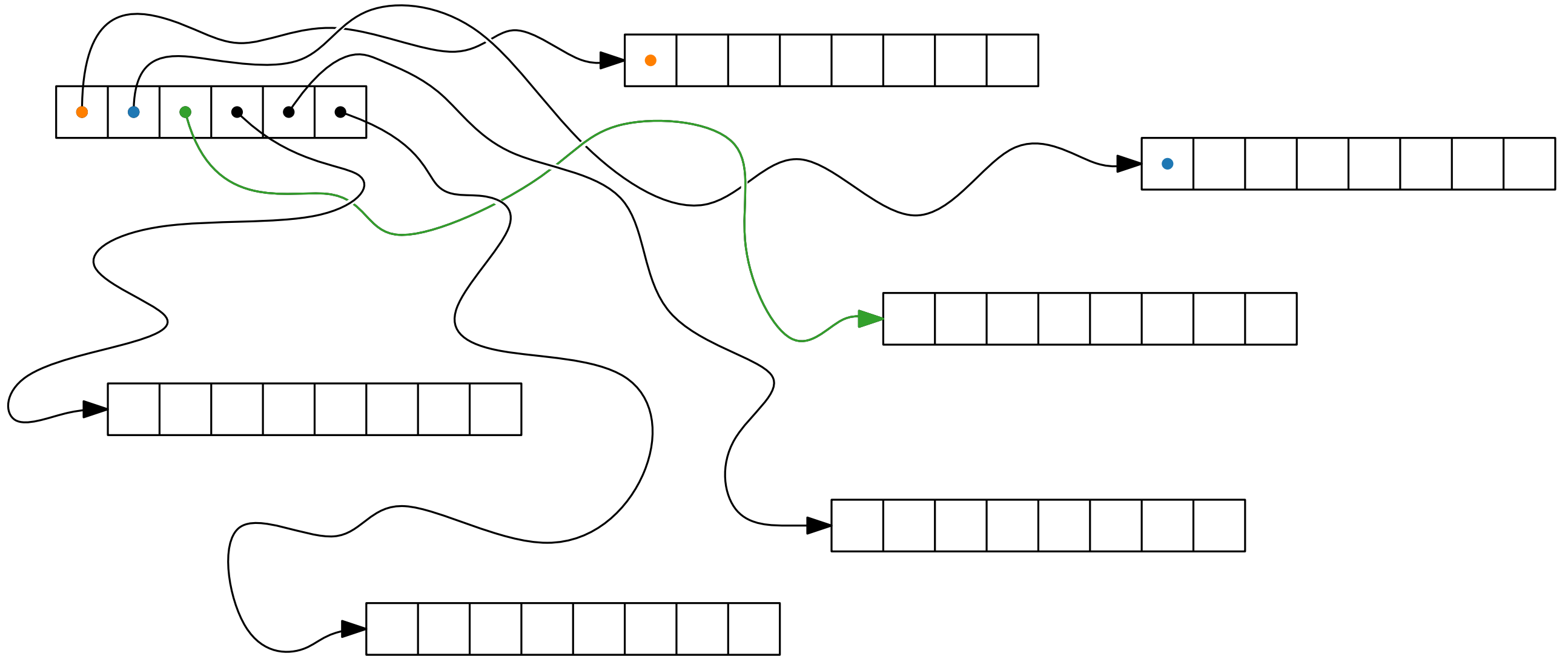


Arrays and Memory Locality

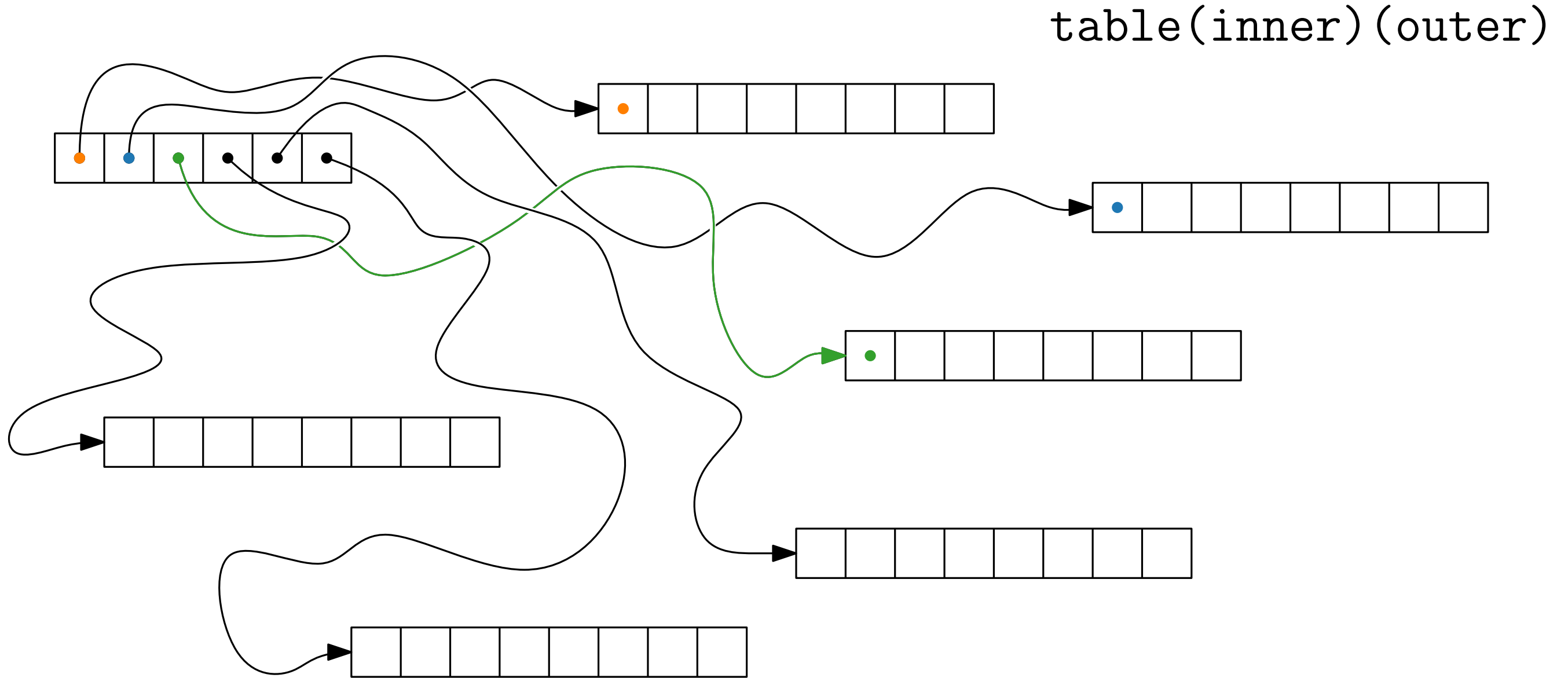


Arrays and Memory Locality

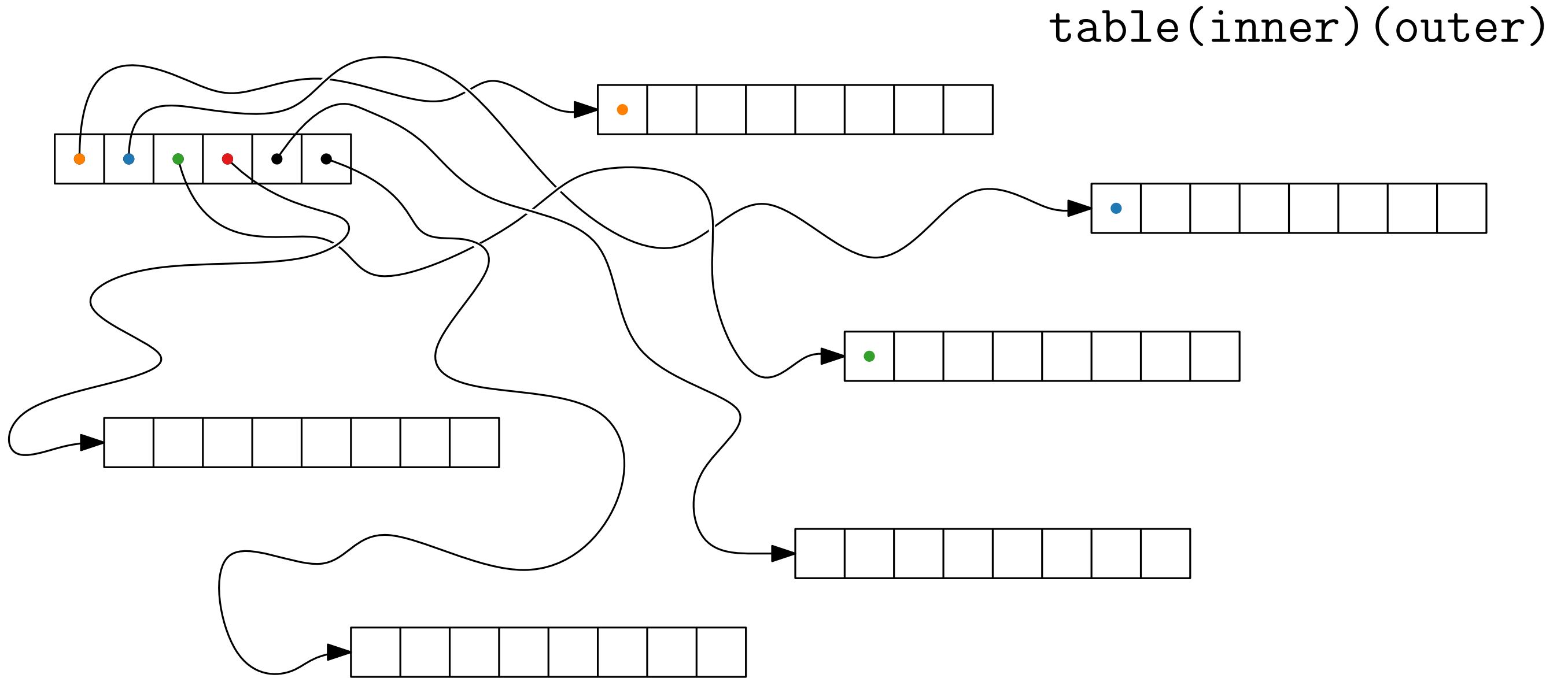
table(inner)(outer)



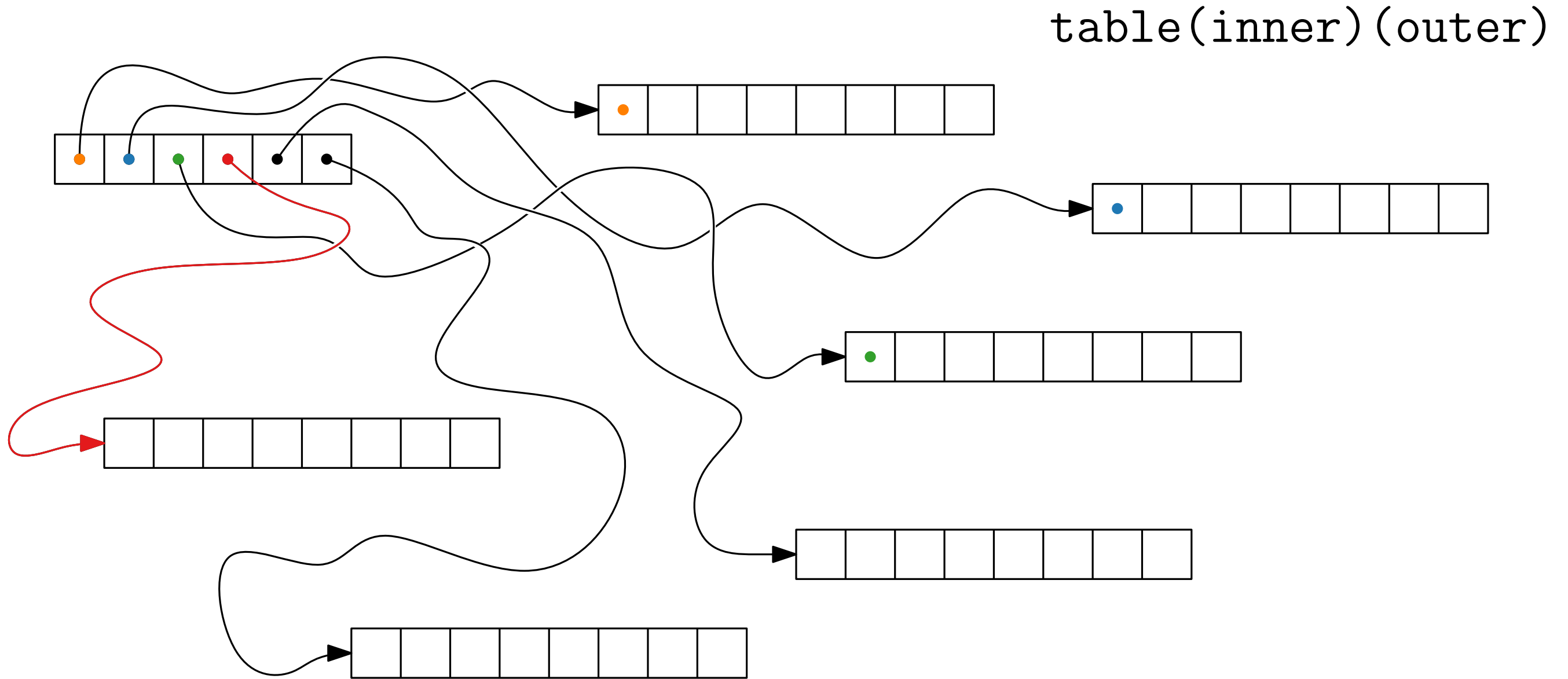
Arrays and Memory Locality



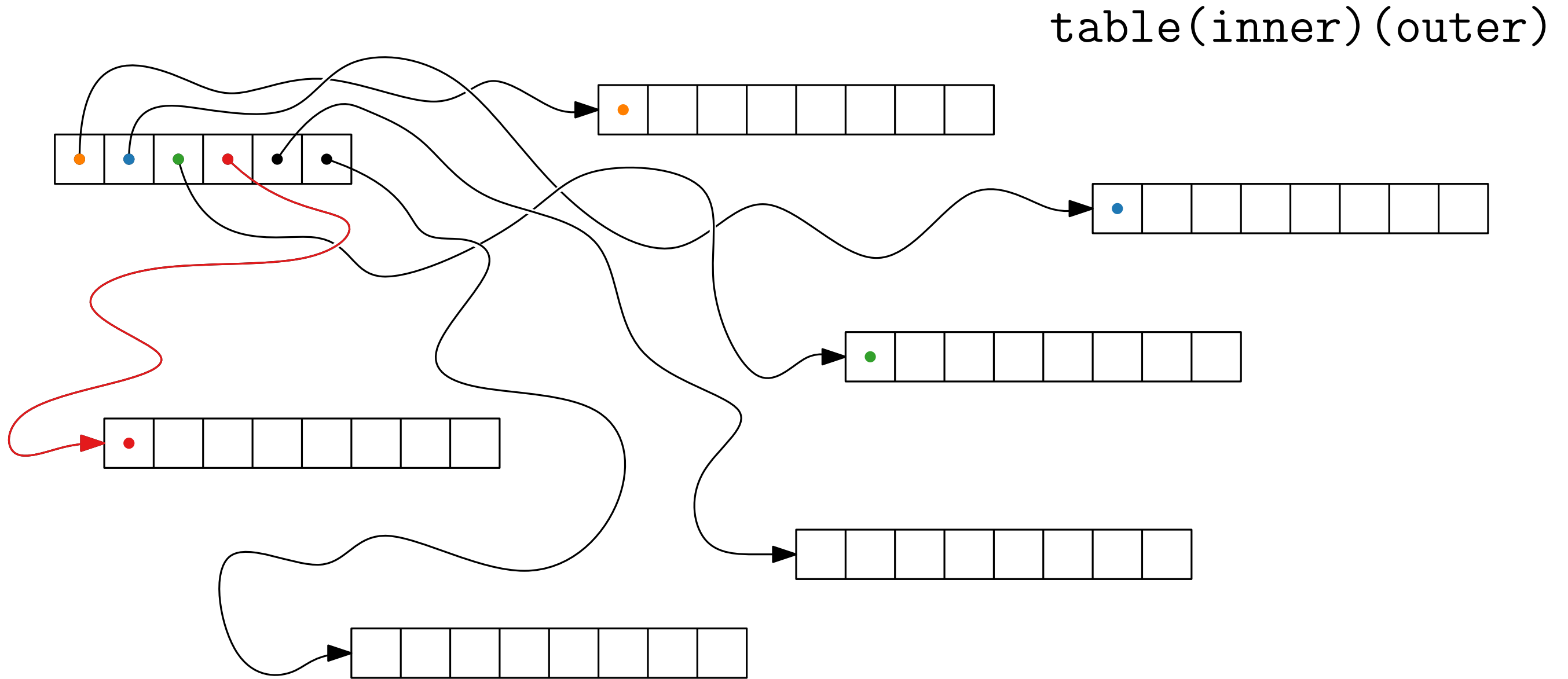
Arrays and Memory Locality



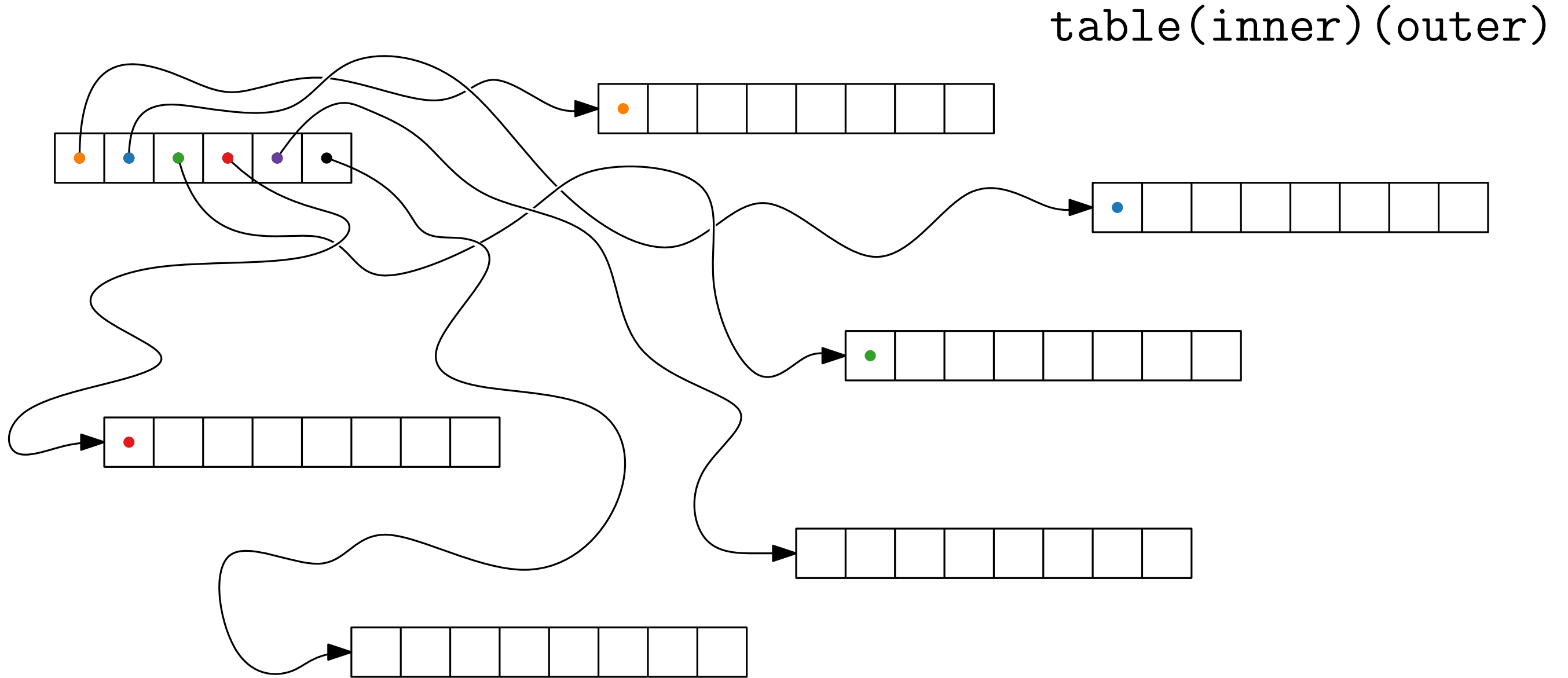
Arrays and Memory Locality



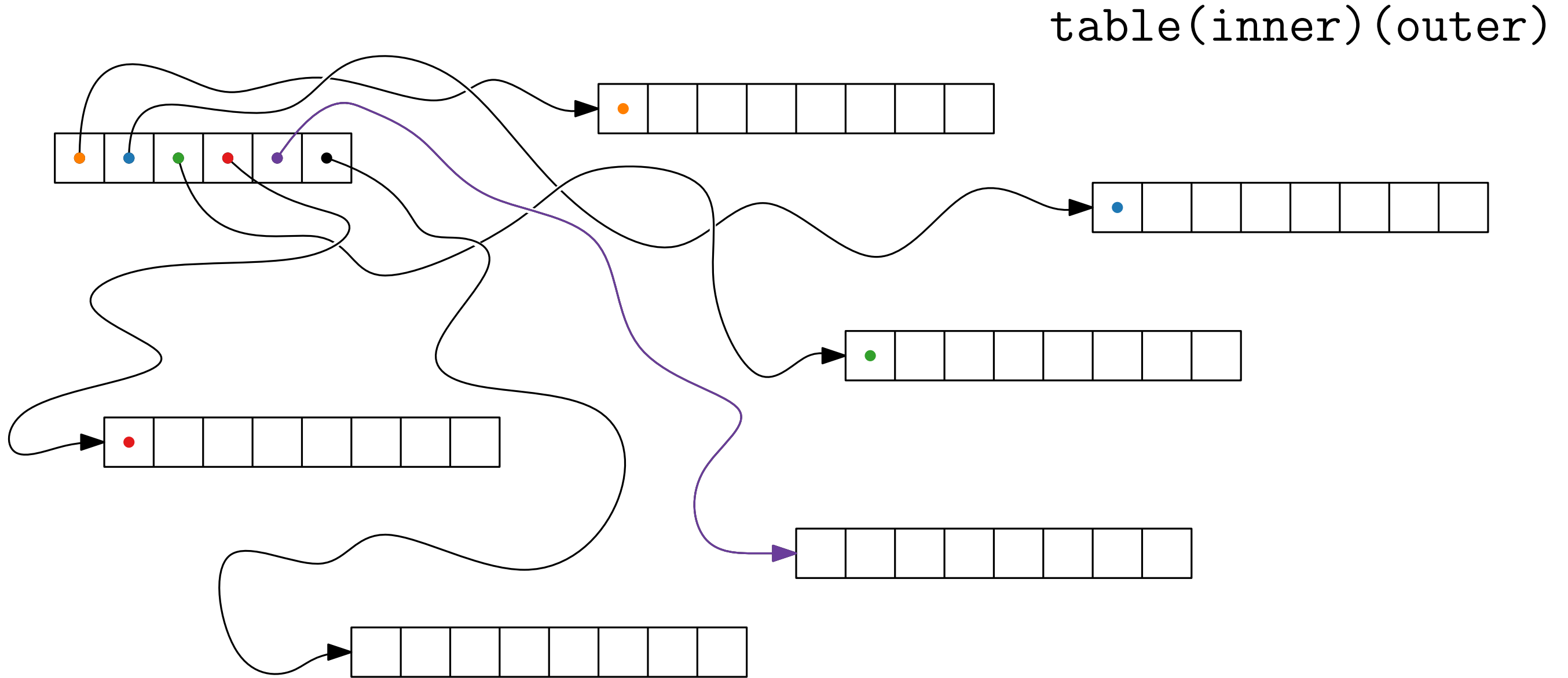
Arrays and Memory Locality



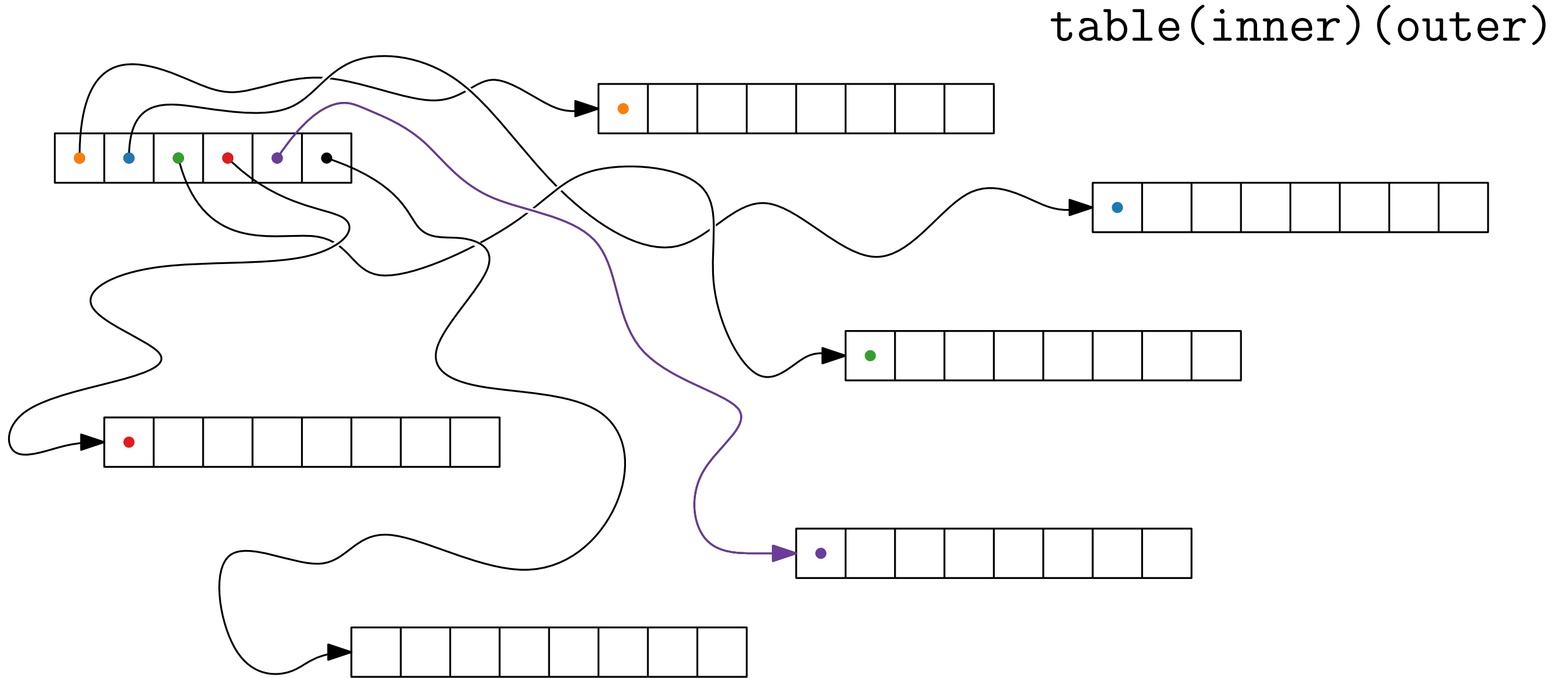
Arrays and Memory Locality



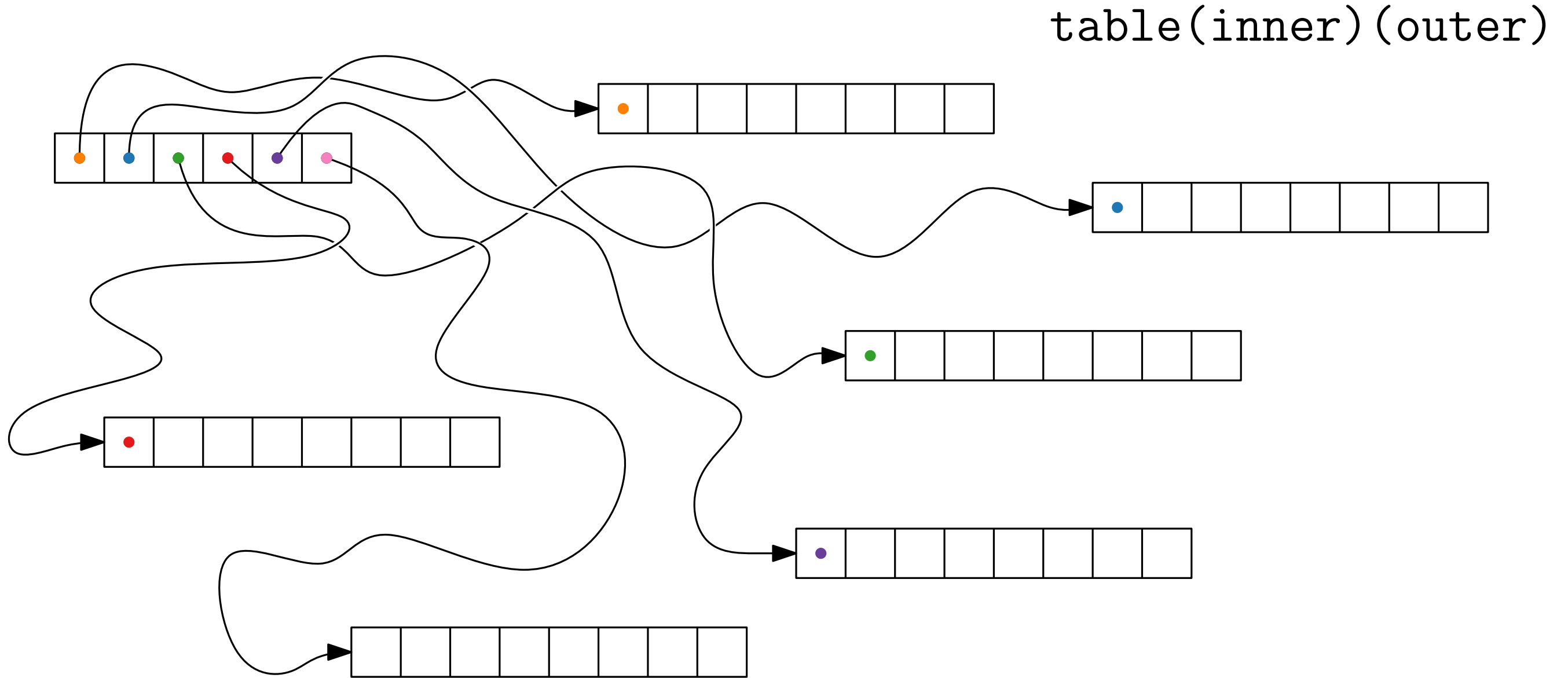
Arrays and Memory Locality



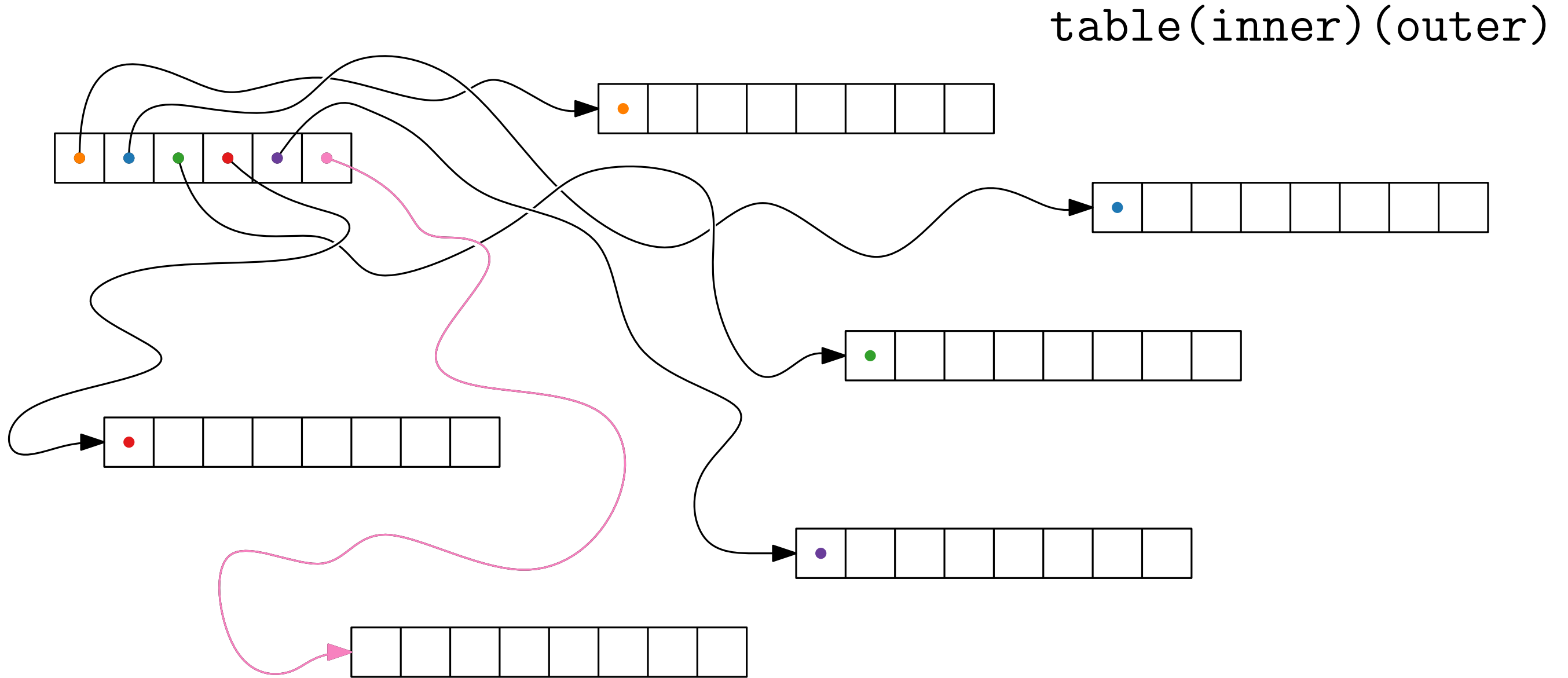
Arrays and Memory Locality



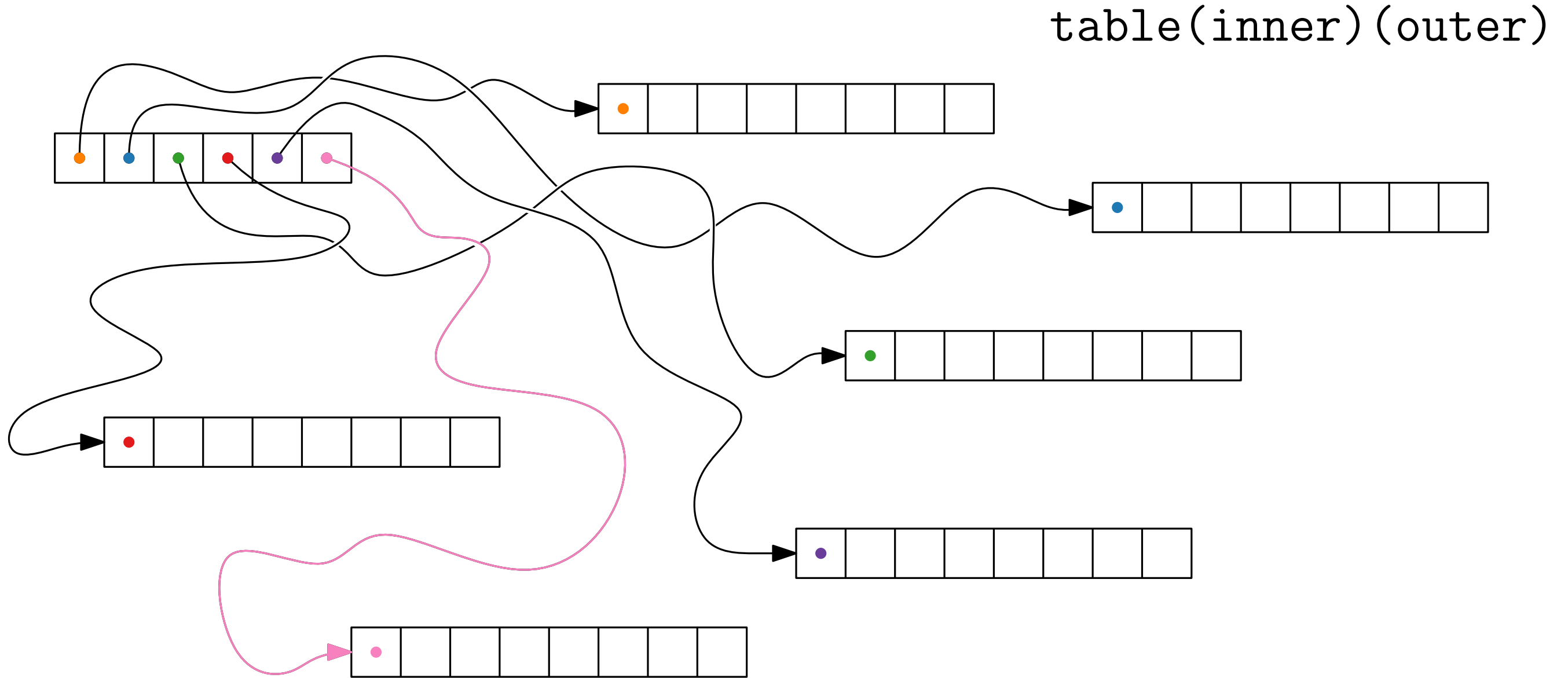
Arrays and Memory Locality



Arrays and Memory Locality

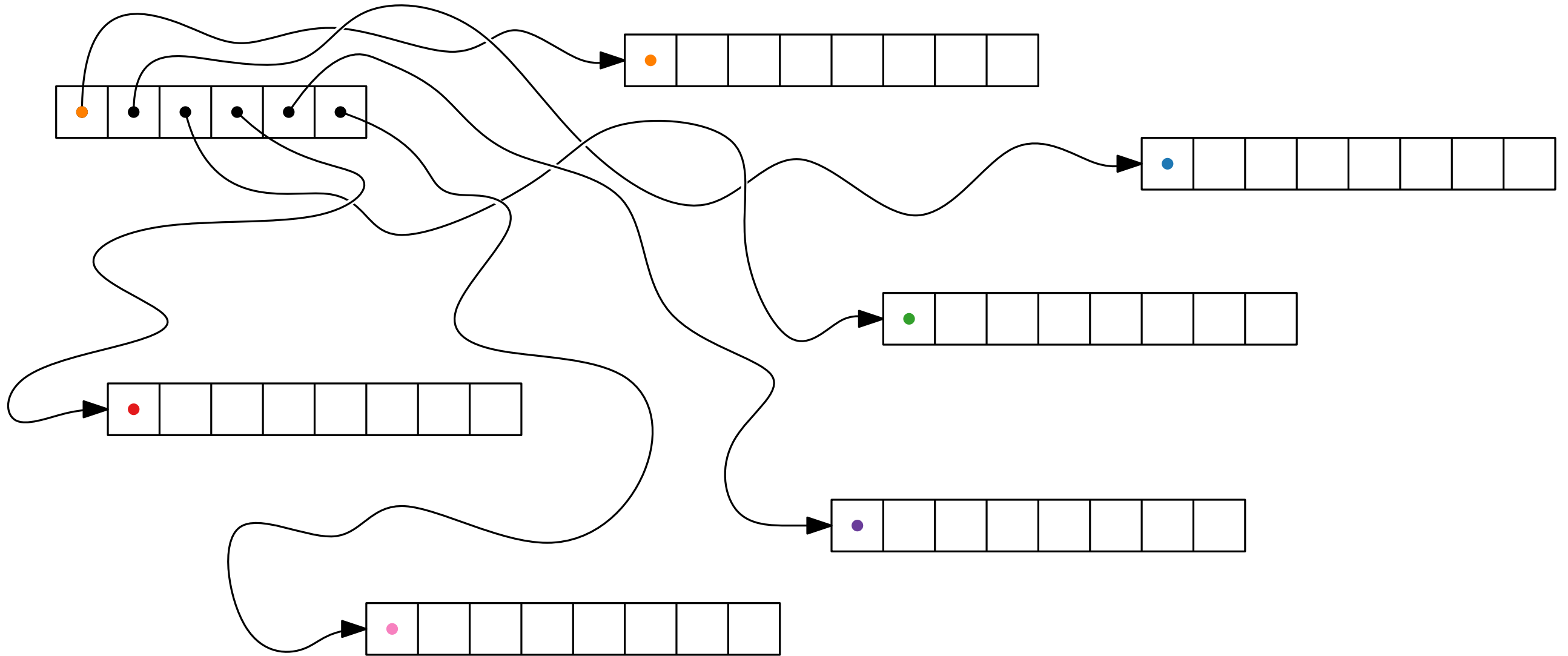


Arrays and Memory Locality

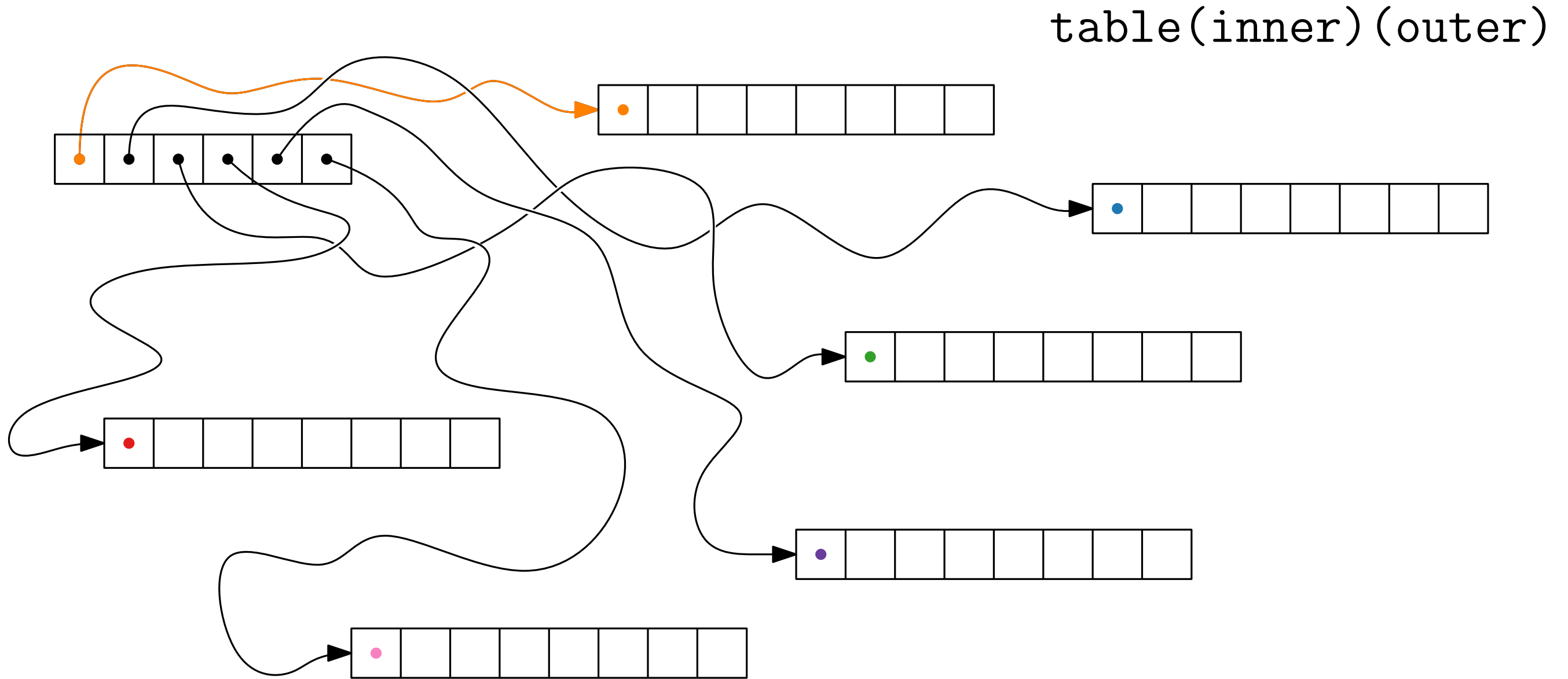


Arrays and Memory Locality

table(inner)(outer)

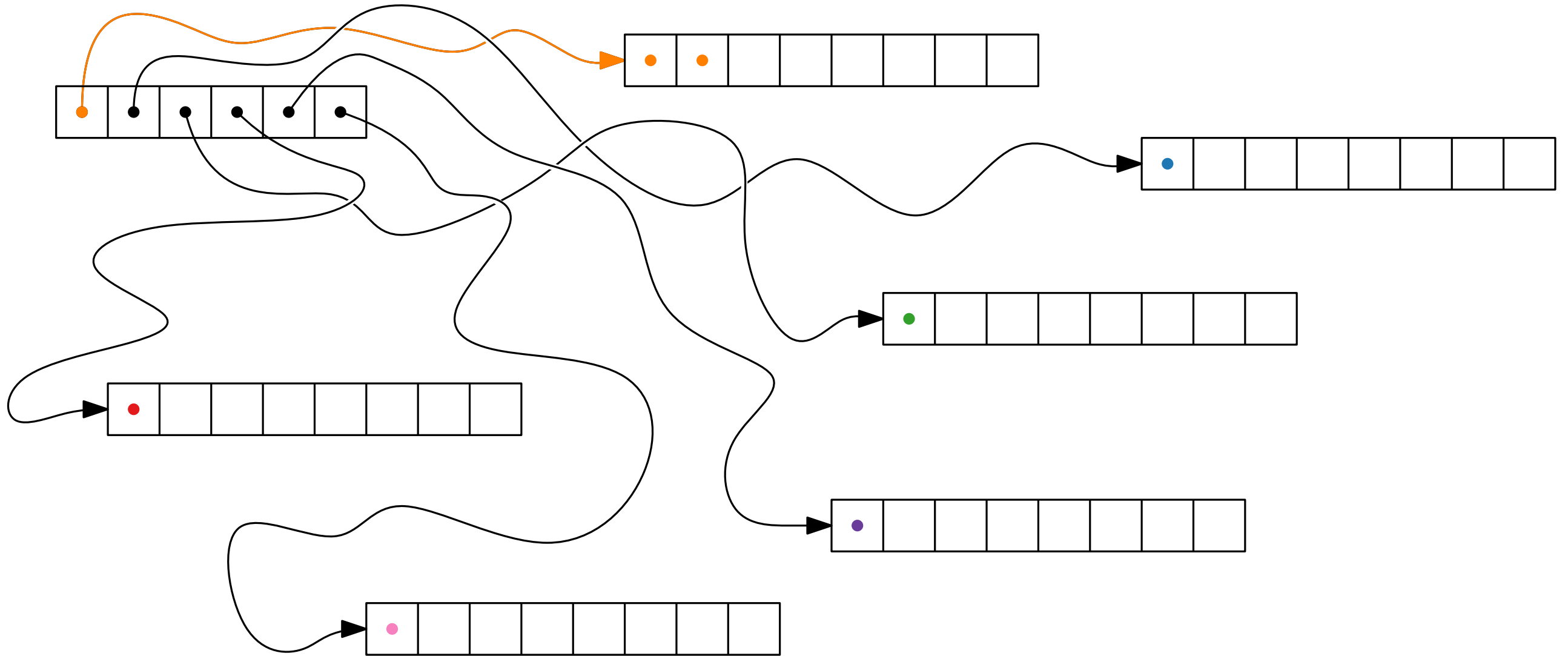


Arrays and Memory Locality

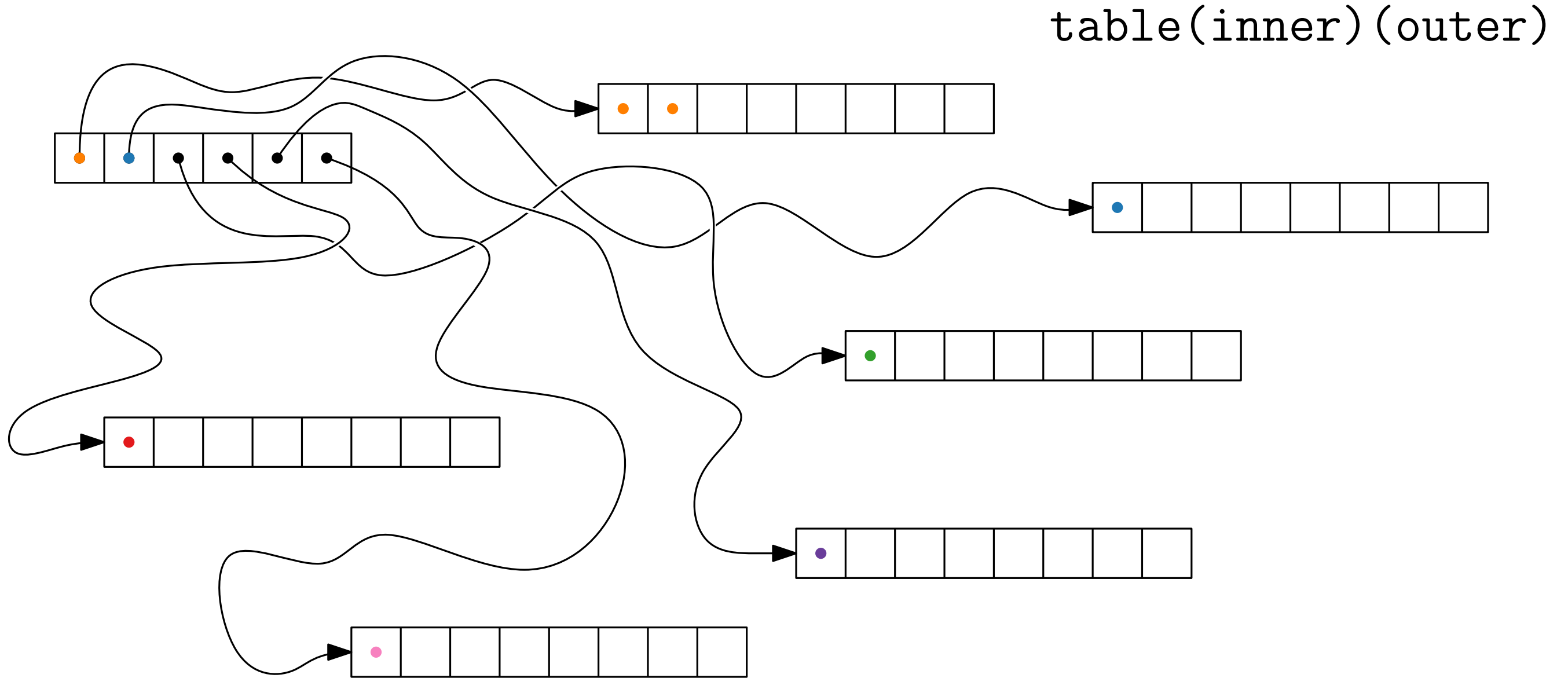


Arrays and Memory Locality

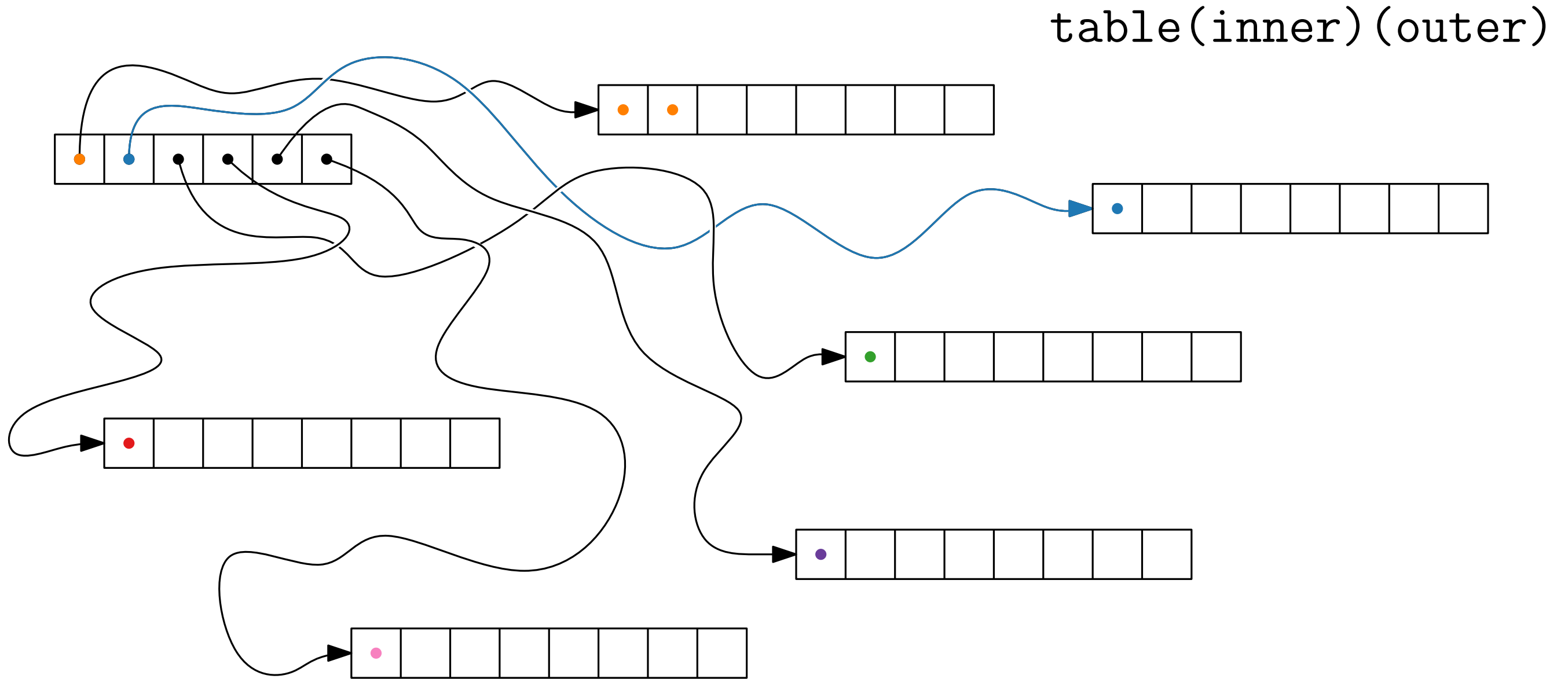
table(inner)(outer)



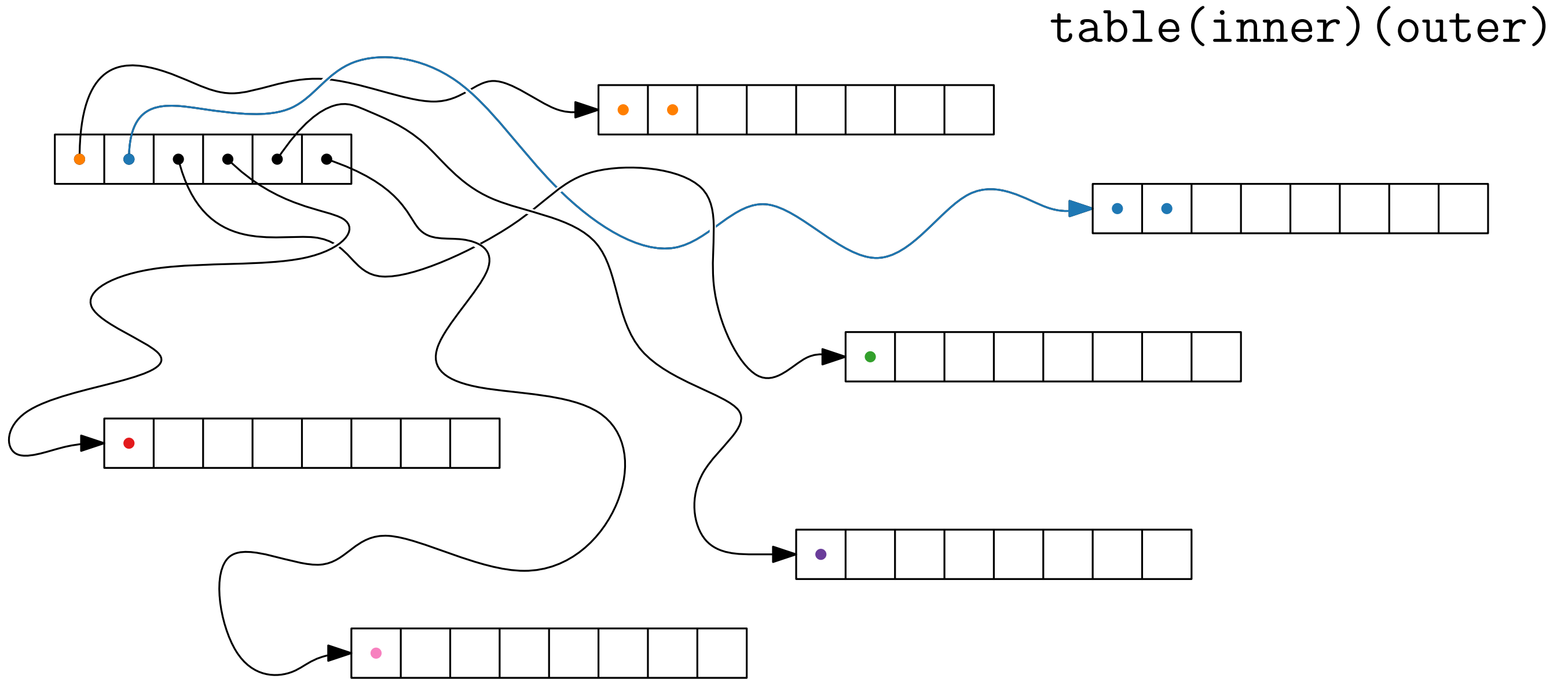
Arrays and Memory Locality



Arrays and Memory Locality



Arrays and Memory Locality



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays					

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays			458 ms		

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays			458 ms	27.5 s	

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s-s(i)) \end{cases} \quad \text{if } s \geq s(i)$$

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$

A horizontal array of 16 empty square cells. The first cell is labeled "prev" to its left. The last cell is labeled " $i - 1$ " to its left, with the text in orange.

[illegible]

$$f(i, s) = \max \begin{cases} f(i-1, s) & \leftarrow \text{don't take} \\ v(i) + f(i-1, s-s(i)) & \text{if } s \geq s(i) \end{cases}$$

prev

[illegible]

$$f(i, s) = \max \begin{cases} f(i-1, s) & \leftarrow \text{don't take} \\ v(i) + f(i-1, s-s(i)) & \text{if } s \geq s(i) \end{cases}$$

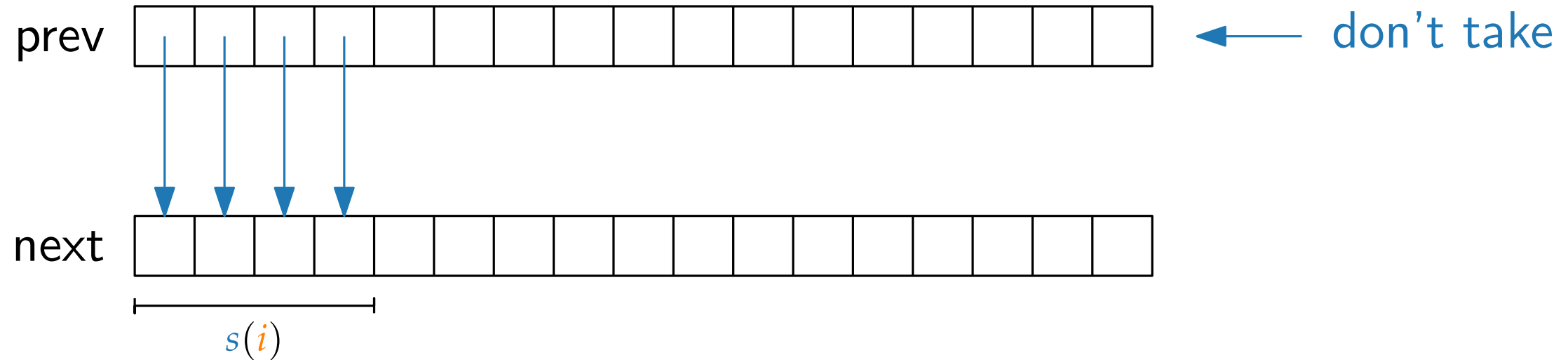
prev

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

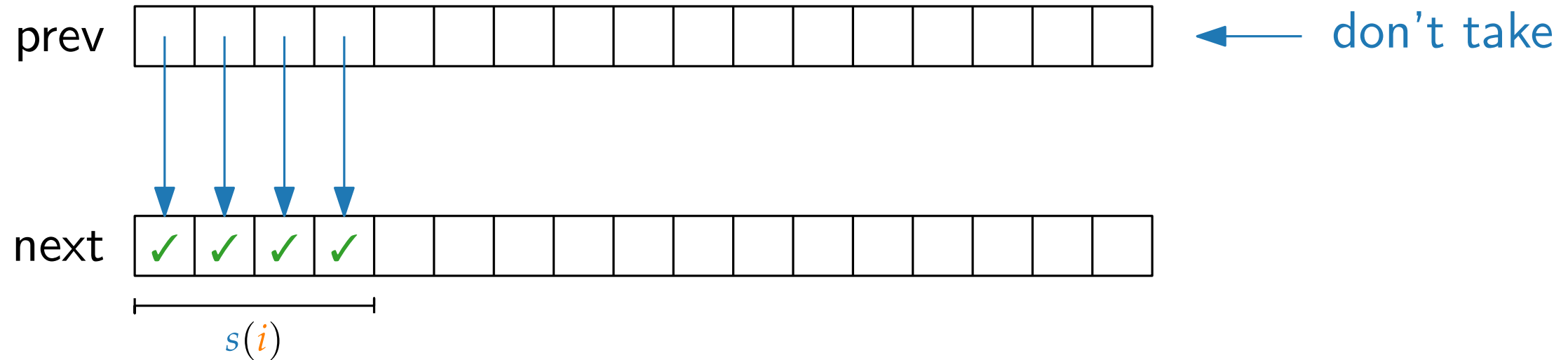
← don't take

next

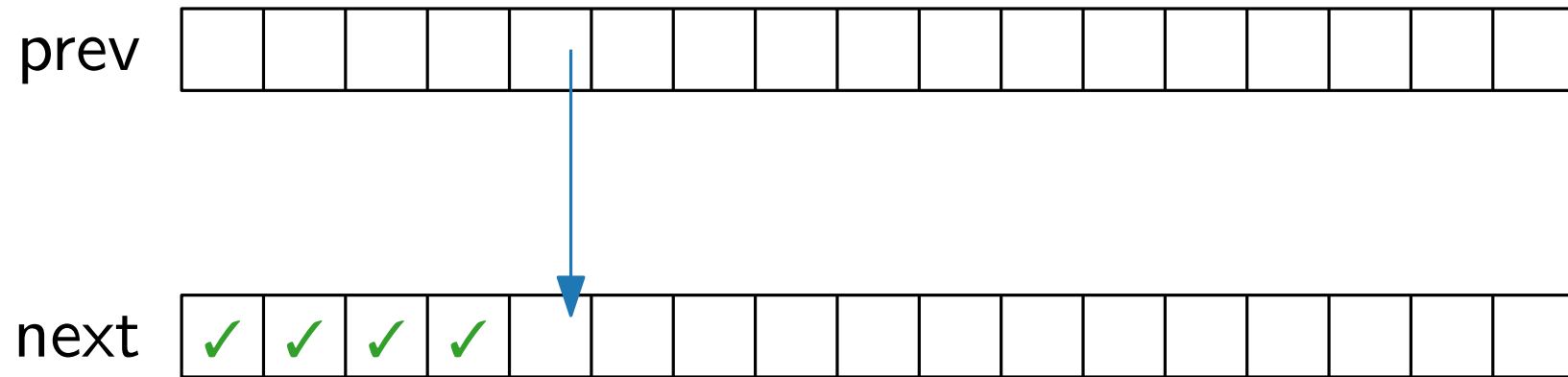
$$f(i, s) = \max \begin{cases} f(i-1, s) & \leftarrow \text{don't take} \\ v(i) + f(i-1, s - s(i)) & \text{if } s \geq s(i) \end{cases}$$



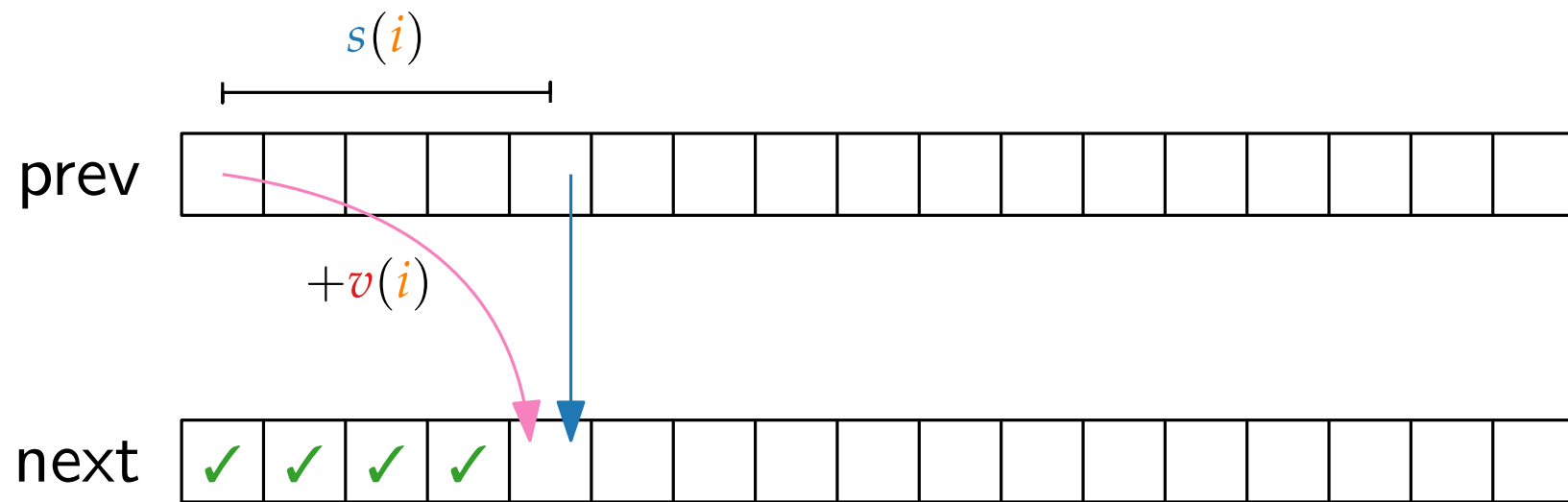
$$f(i, s) = \max \begin{cases} f(i-1, s) & \leftarrow \text{don't take} \\ v(i) + f(i-1, s - s(i)) & \text{if } s \geq s(i) \end{cases}$$



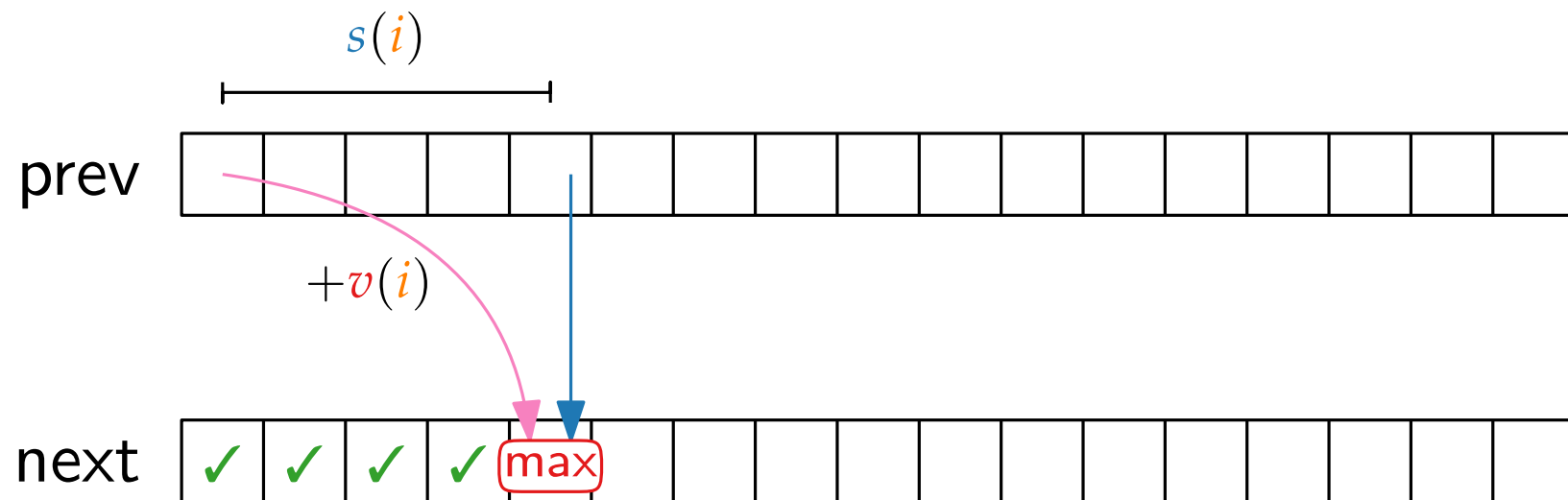
$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$



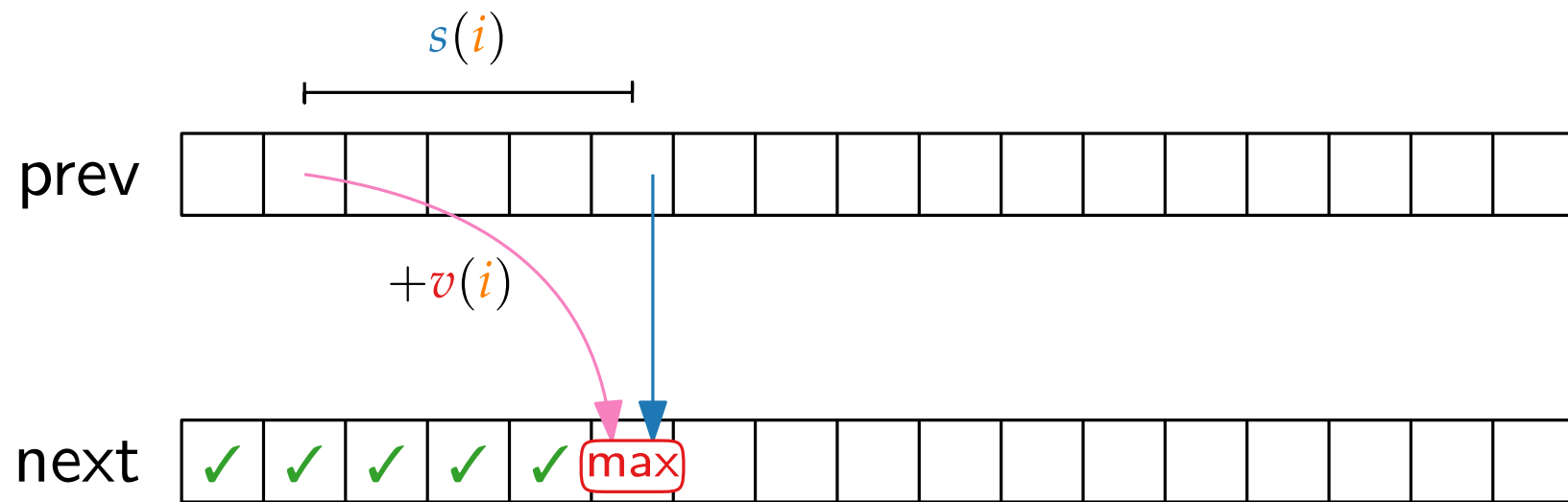
$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$



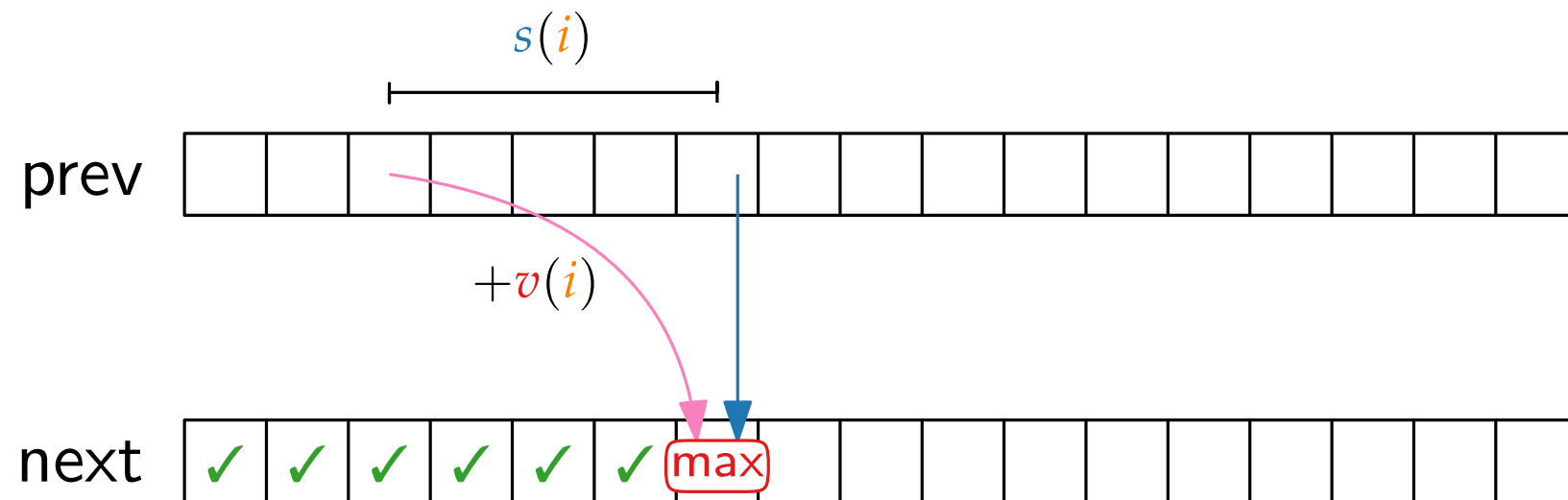
$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$



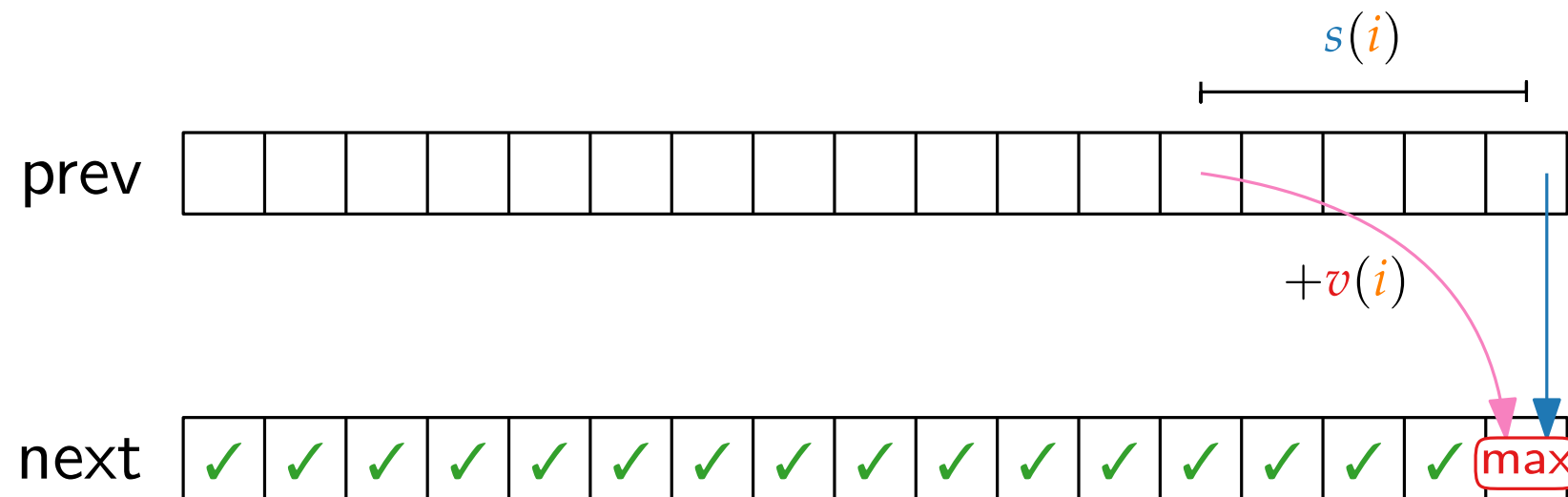
$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$



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Hint:

Use optimized procedures provided by your operating system!

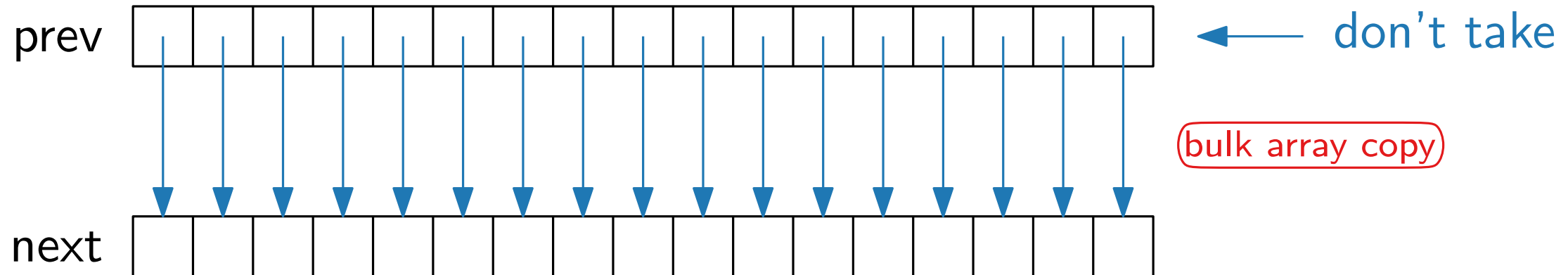
prev

[illegible]

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s-s(i)) \end{cases} \quad \text{if}$$

Hint:

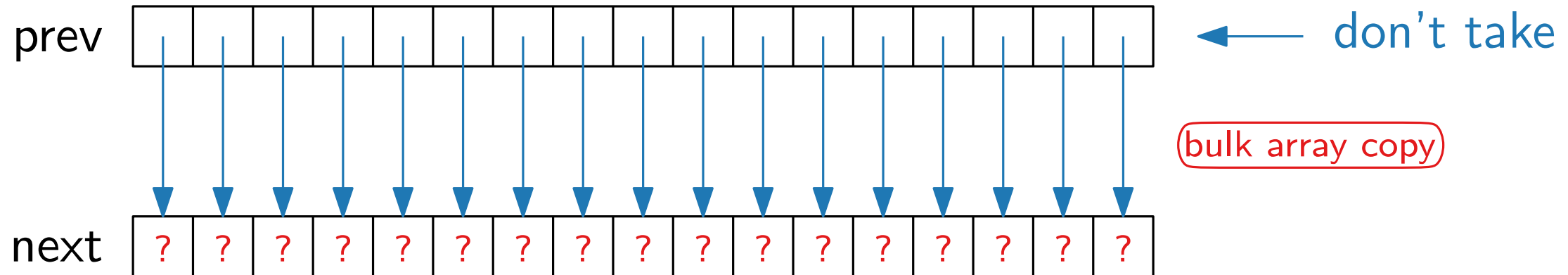
Use optimized procedures provided by your operating system!



$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s-s(i)) \end{cases} \quad \text{if}$$

Hint:

Use optimized procedures provided by your operating system!



$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if } s \geq s(i)$$

Hint:

Use optimized procedures provided by your operating system!

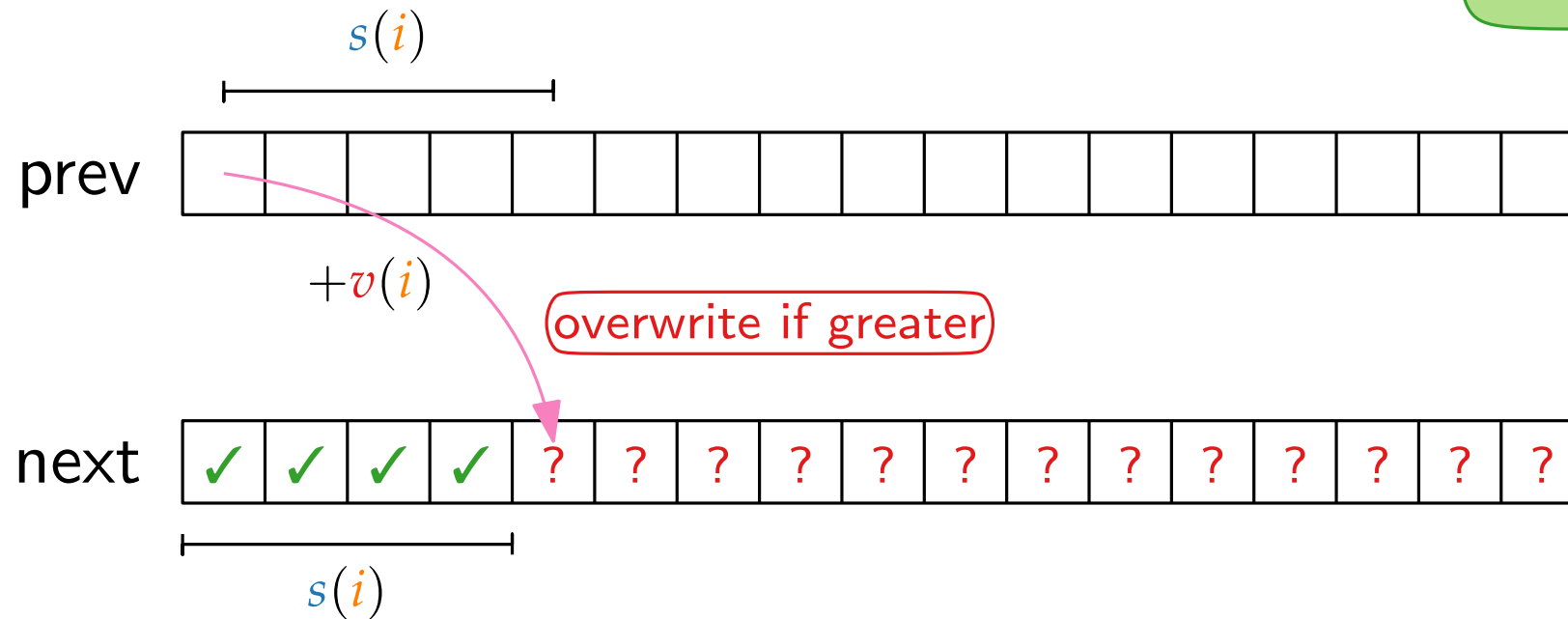
prev

[illegible]

$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if}$$

Hint:

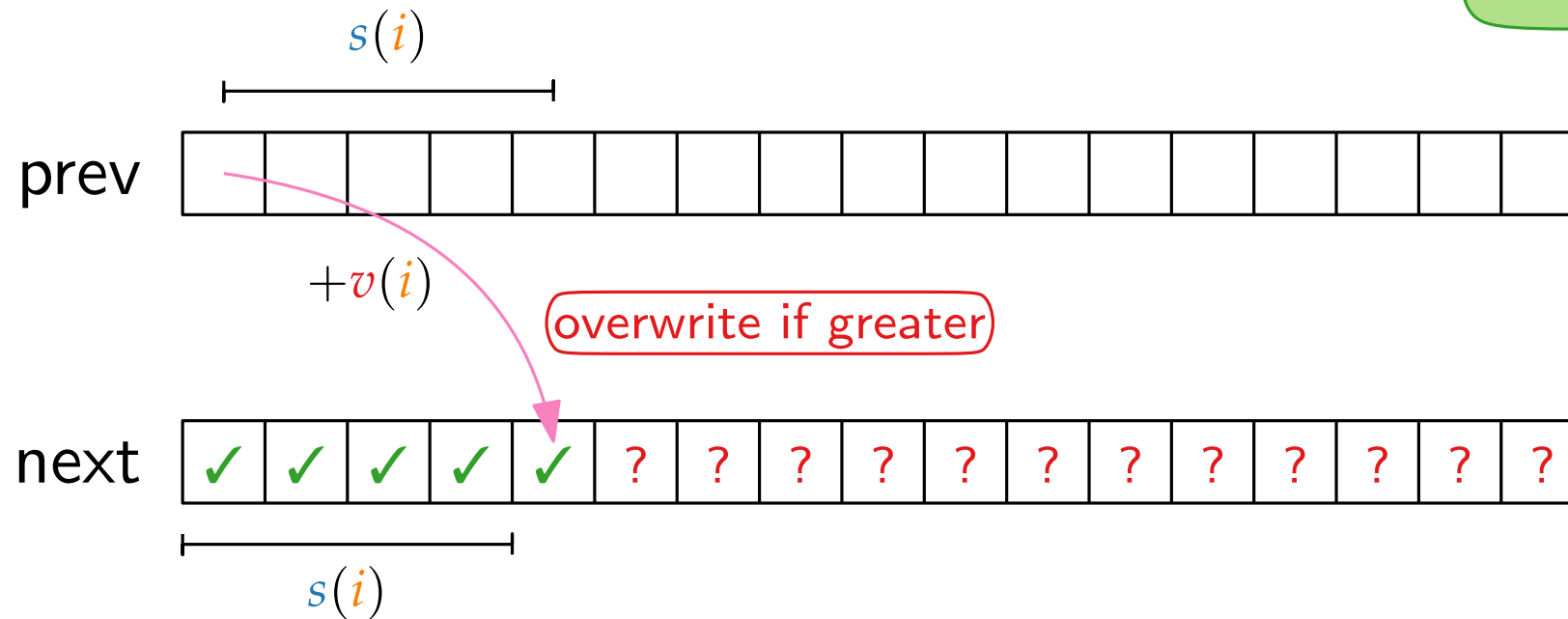
Use optimized procedures provided by your operating system!



$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \quad \text{if}$$

Hint:

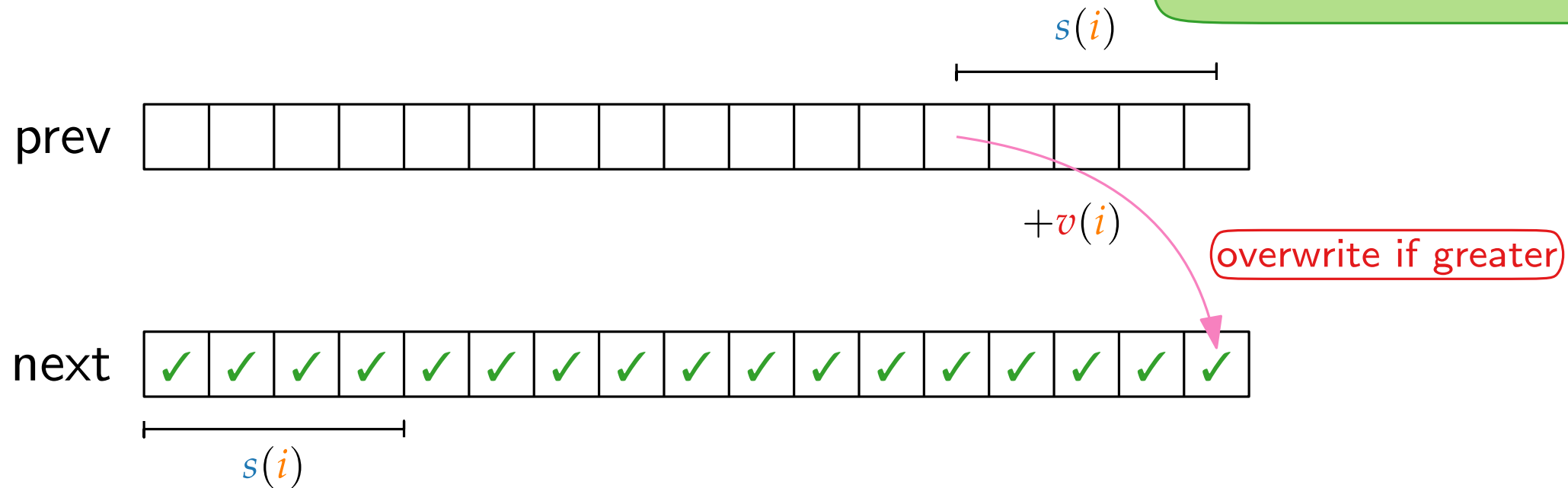
Use optimized procedures provided by your operating system!



$$f(i, s) = \max \begin{cases} f(i-1, s) \\ v(i) + f(i-1, s - s(i)) \end{cases} \text{ if } s \geq s(i)$$

Hint:

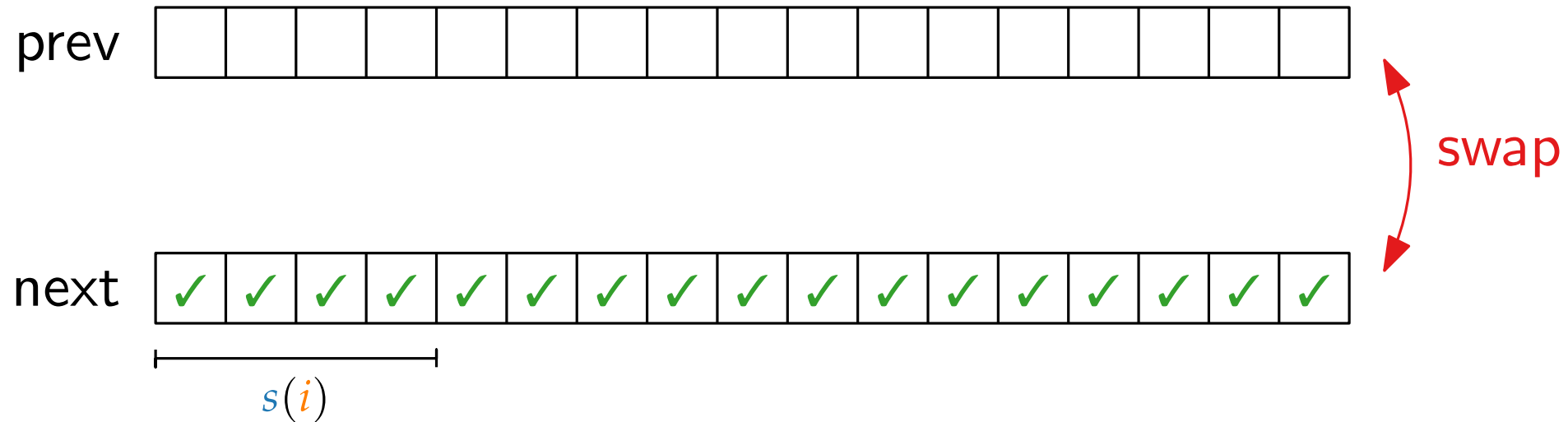
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Hint:

Use optimized procedures provided by your operating system!



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays			458 ms	27.5 s	
Scala bulk copy					

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Try other programming languages?

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$ U =$	30	100	1K	10K	$B = 10 U $
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Try other programming languages? **Sure!**

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Running Times

	$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve		1.15 s				
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Scala Arrays				458 ms	27.5 s	
Scala bulk copy					437 ms	
Java					636 ms	

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
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Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays			458 ms	27.5 s	
Scala bulk copy				437 ms	
Java				636 ms	

we can fix this!



Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
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Java				636 ms	
node.js					

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
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Java				636 ms	
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Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
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Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3					

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
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Java				636 ms	
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Python3				96.8 s	

Running Times

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Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3					

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Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	

Running Times

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Scala bulk copy				437 ms	
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Rust (safe)					

Running Times

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Scala Arrays			458 ms	27.5 s	
Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	
Rust (safe)				300 ms	

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
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Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	
Rust (safe)				300 ms	
C++ (by tvd)					

Running Times

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Scala Arrays			458 ms	27.5 s	
Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	
Rust (safe)				300 ms	
C++ (by tvd)				212 ms	

Running Times

$ U =$	30	100	1K	10K	$B = 10 U $
Scala naïve	1.15 s				
Scala memoized	230 ms	267 ms	⚡ so		
Scala TCO		273 ms	2.30 s	⚡ oom	
Scala Arrays			458 ms	27.5 s	
Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	
Rust (safe)				300 ms	
C++ (by tvd)				212 ms	
Rust with AVX2					

Running Times

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Scala Arrays			458 ms	27.5 s	
Scala bulk copy				437 ms	
Java				636 ms	
node.js				957 ms	
Python3				96.8 s	
Pypy3				2.48 s	
Rust (safe)				300 ms	
C++ (by tvd)				212 ms	
Rust with AVX2				43 ms	

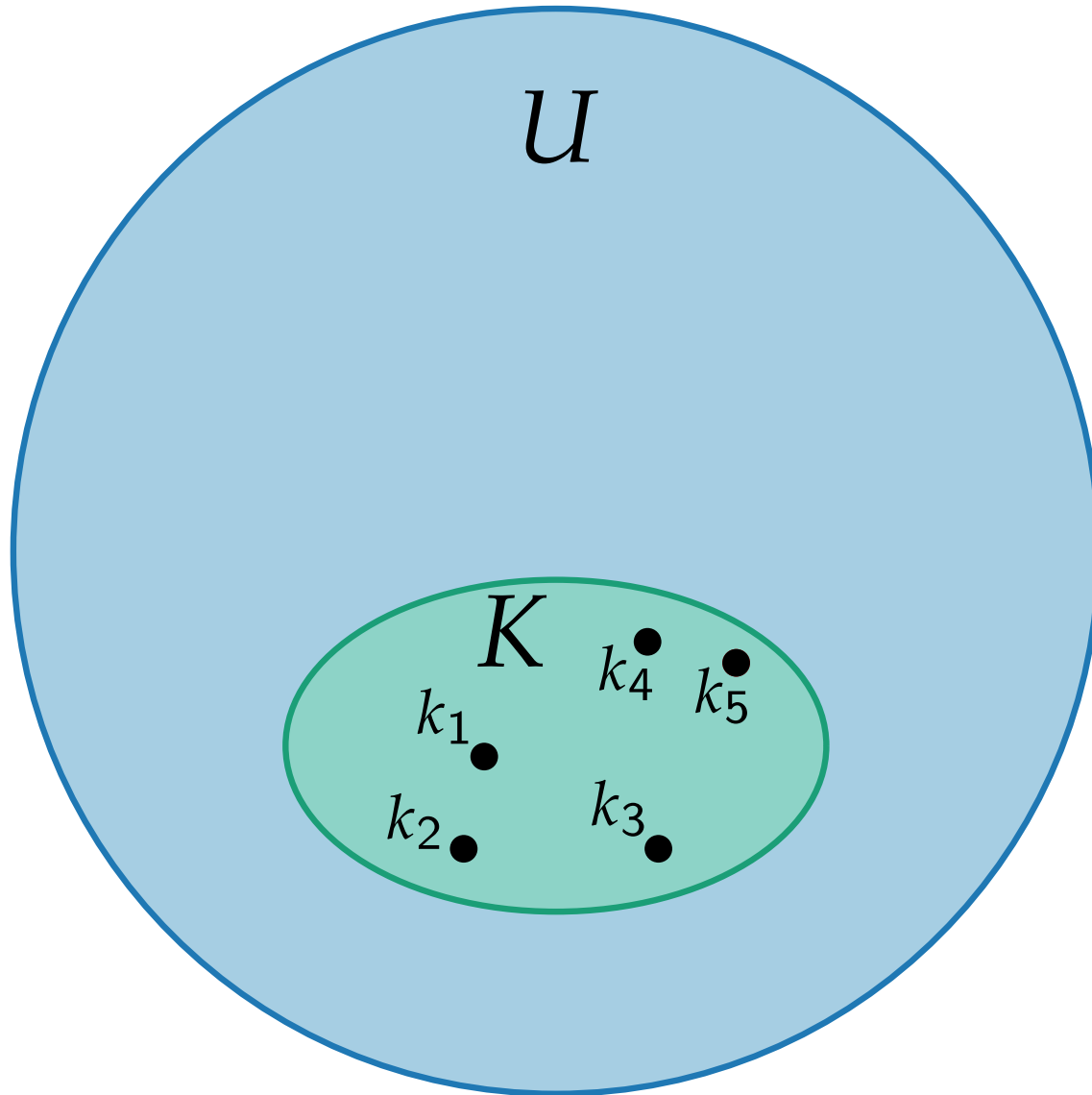
Advanced Algorithms

Algorithms in Practice

Hash Maps: A Look Under the Hood

Recap: Hash Tables

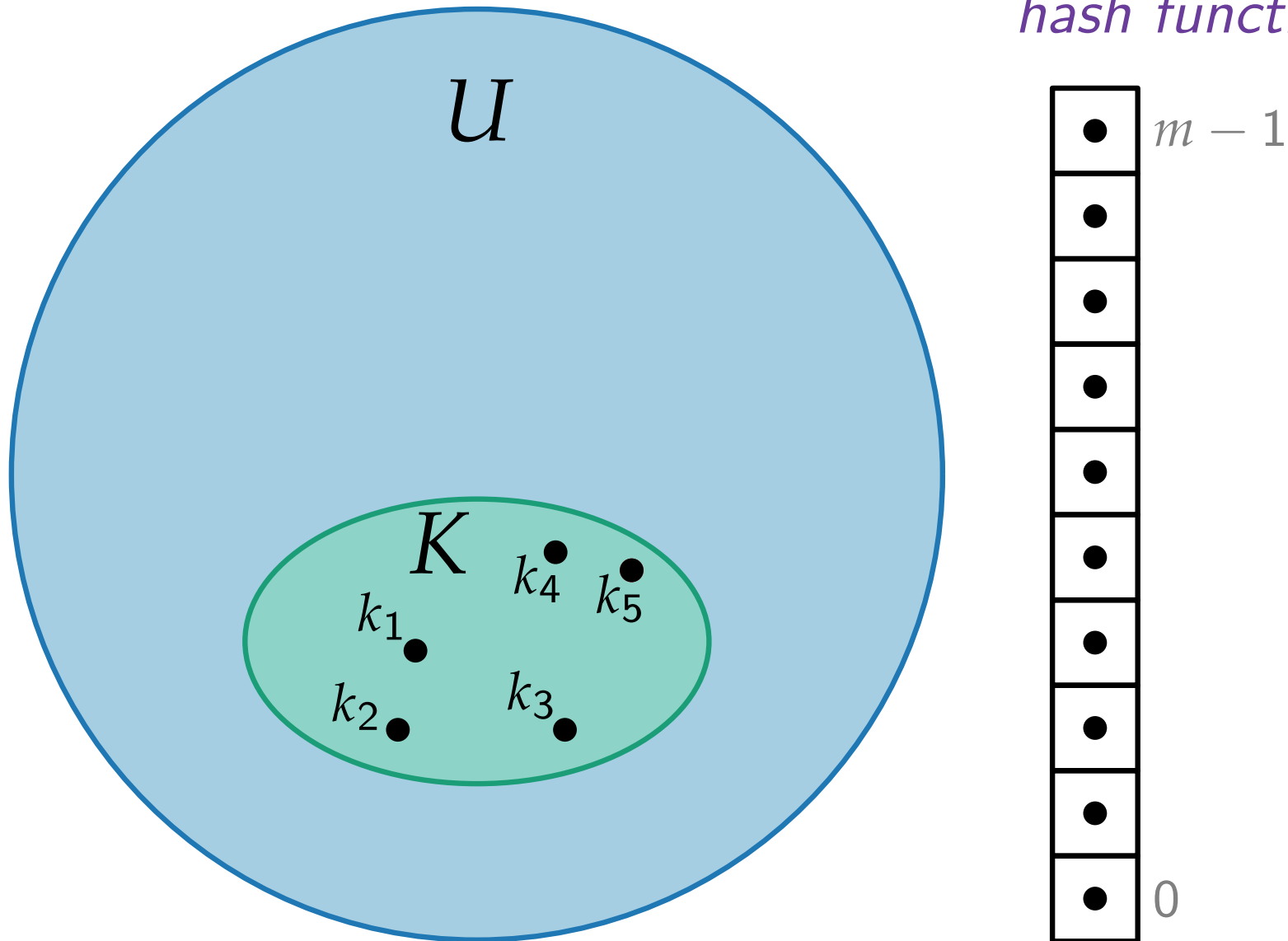
Assumption: large universe U (that means $|U| \gg |K|$)



Recap: Hash Tables

Assumption: large universe U (that means $|U| \gg |K|$)

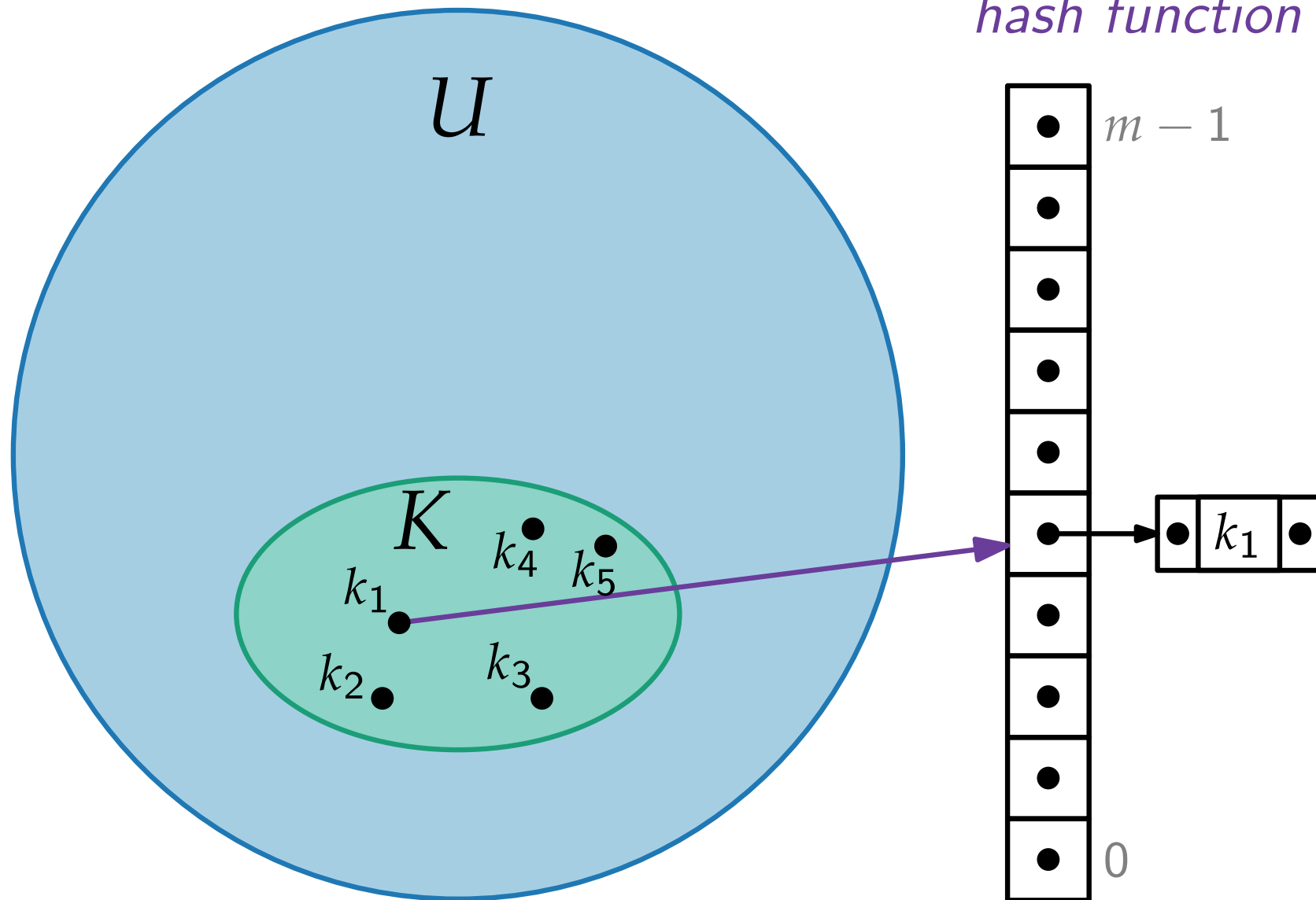
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



Recap: Hash Tables

Assumption: large universe U (that means $|U| \gg |K|$)

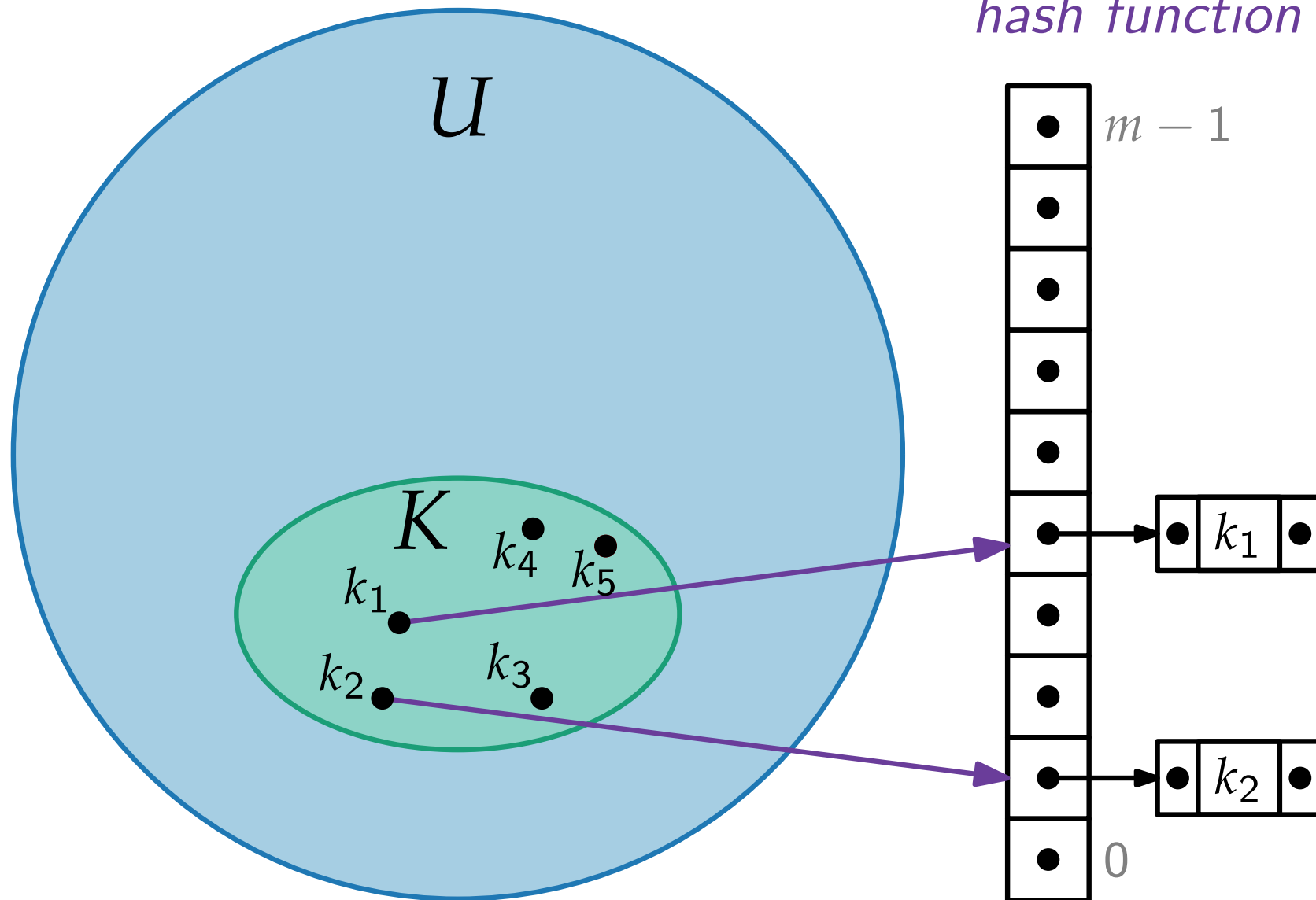
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



Recap: Hash Tables

Assumption: large universe U (that means $|U| \gg |K|$)

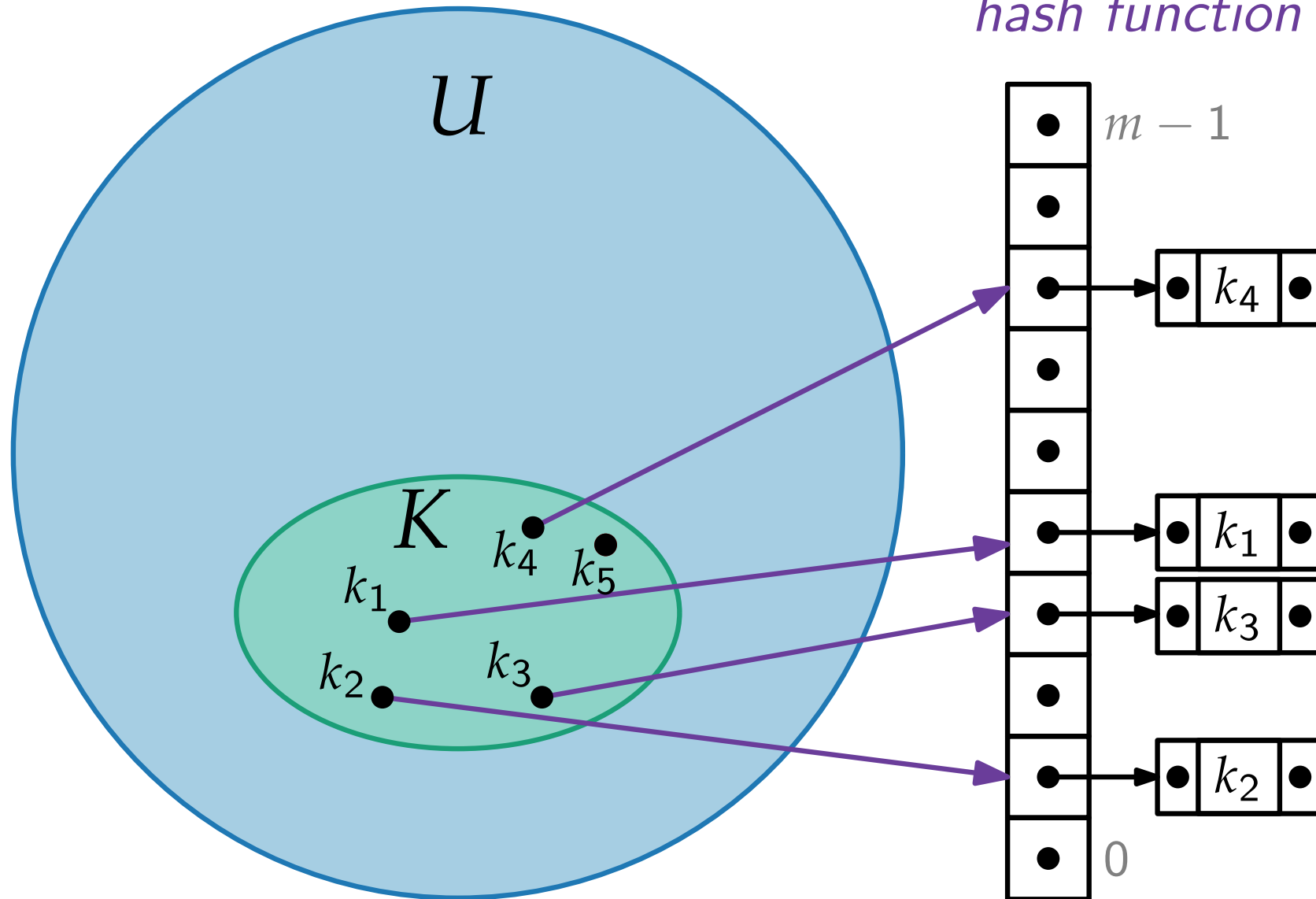
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



Recap: Hash Tables

Assumption: large universe U (that means $|U| \gg |K|$)

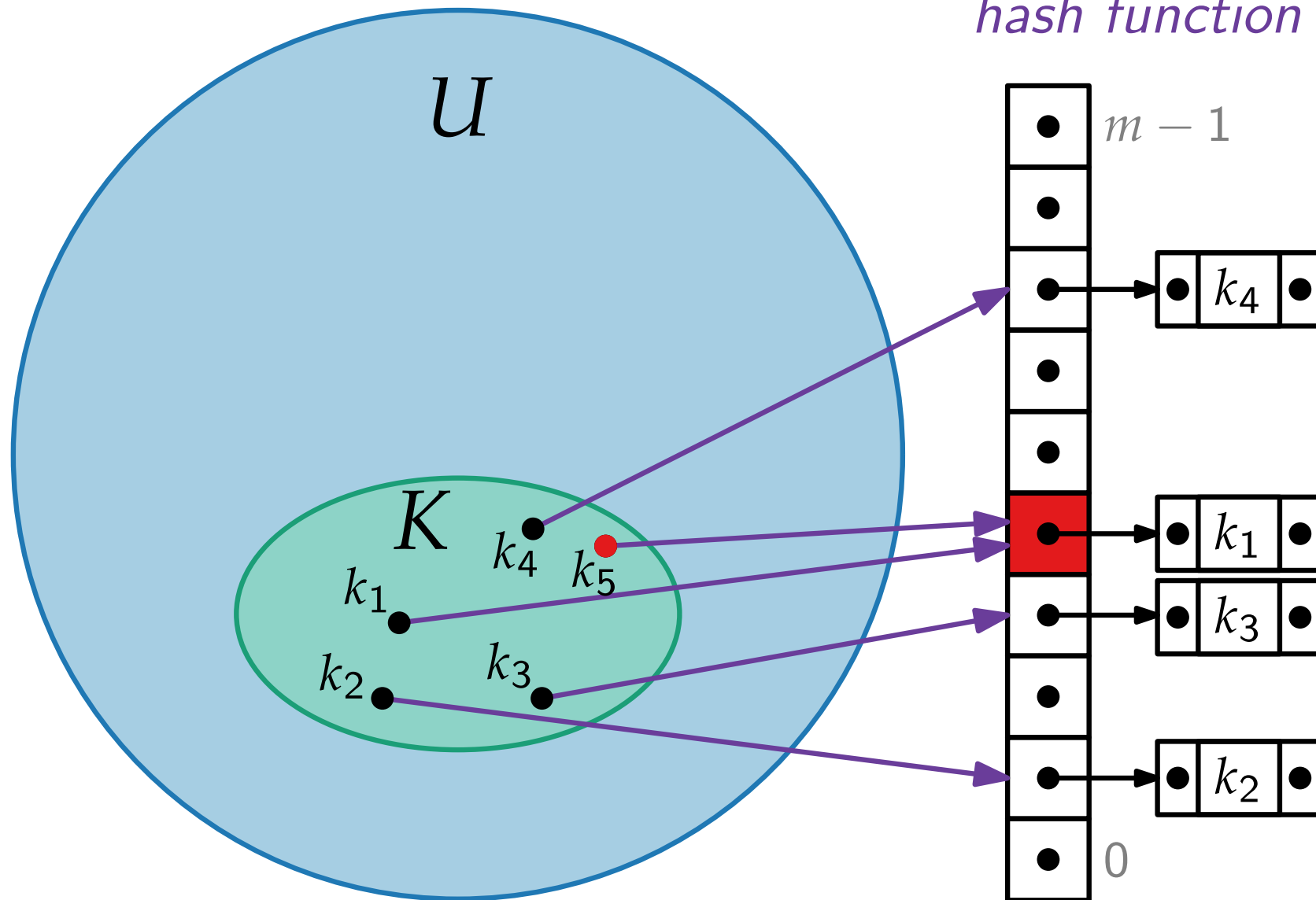
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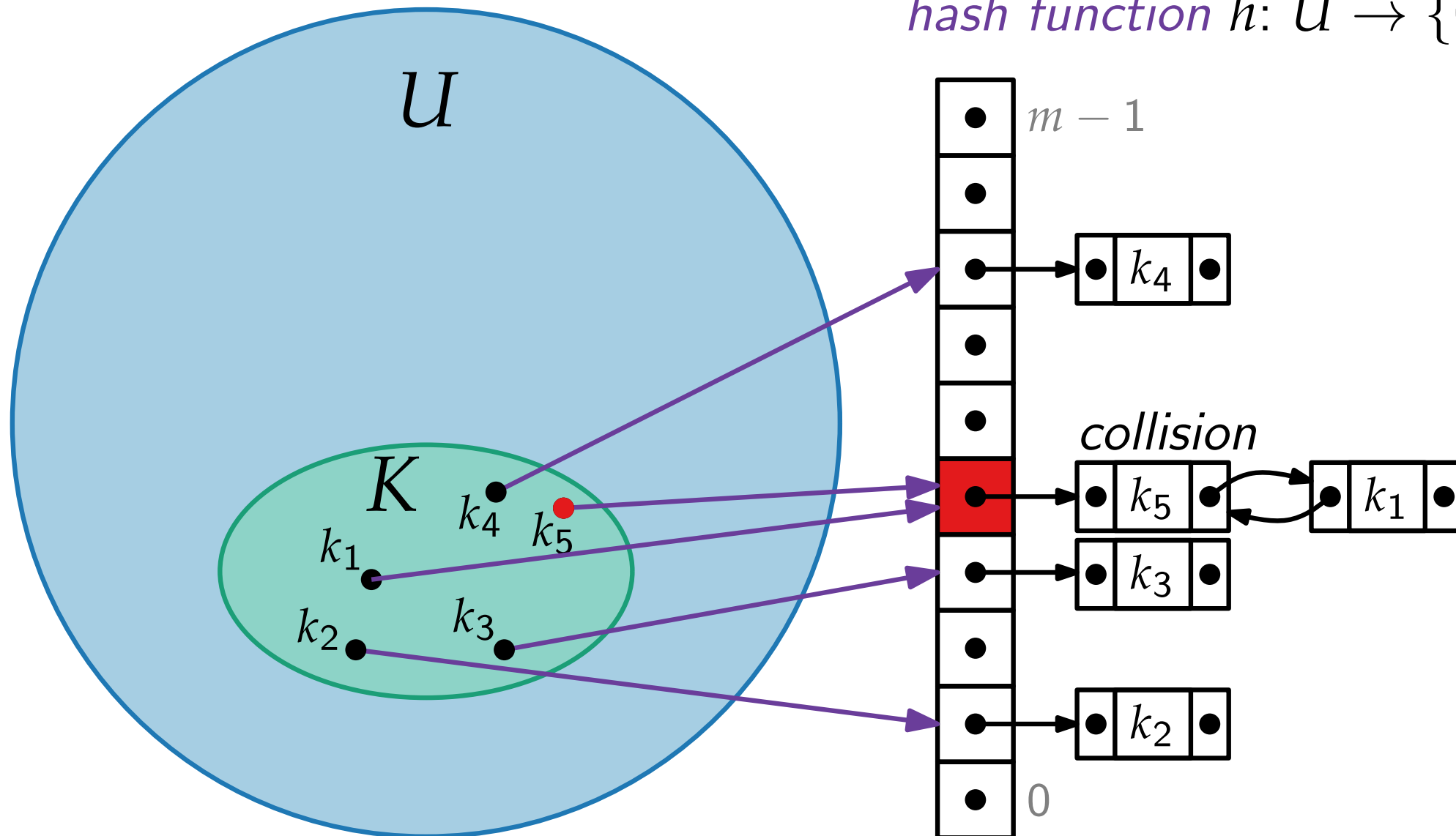
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



Recap: Hash Tables

Assumption: large universe U (that means $|U| \gg |K|$)

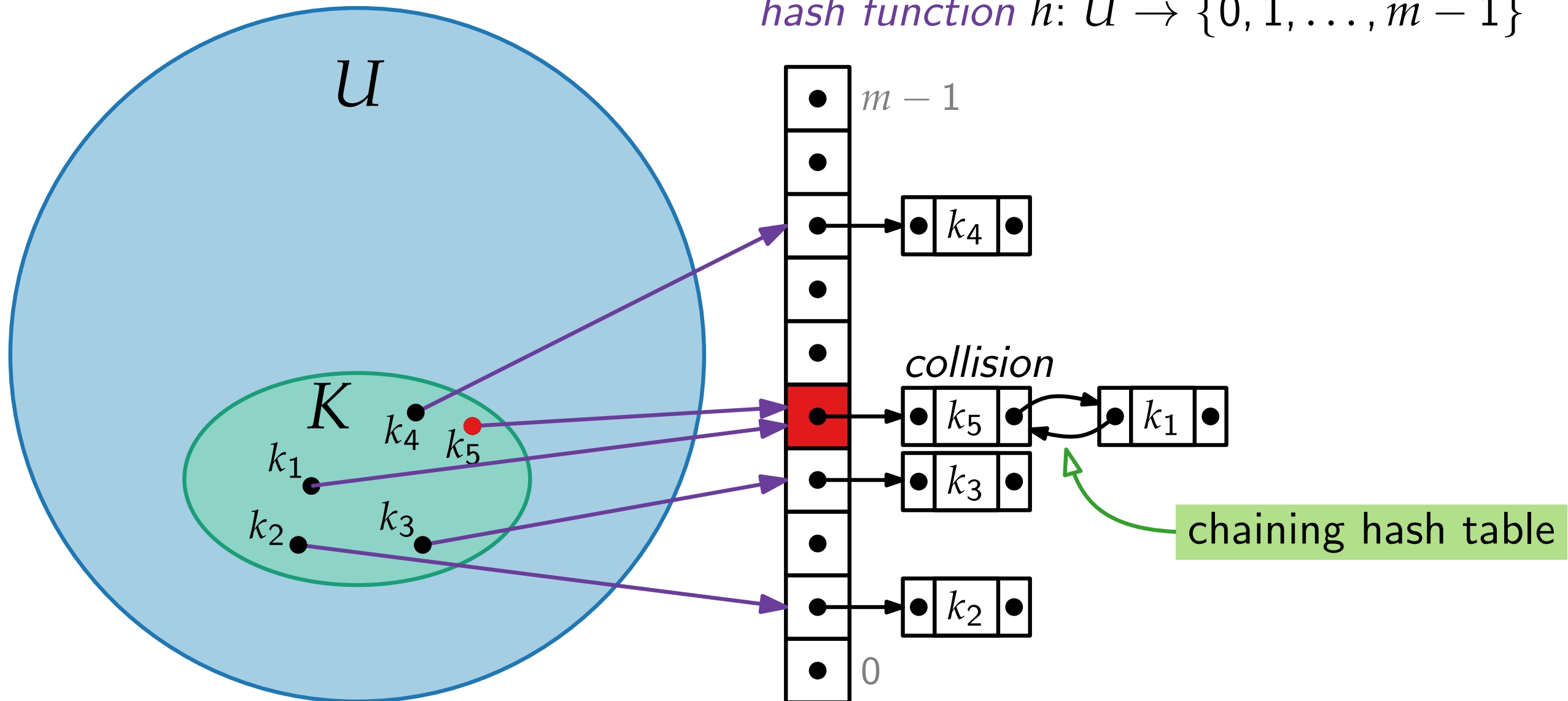
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



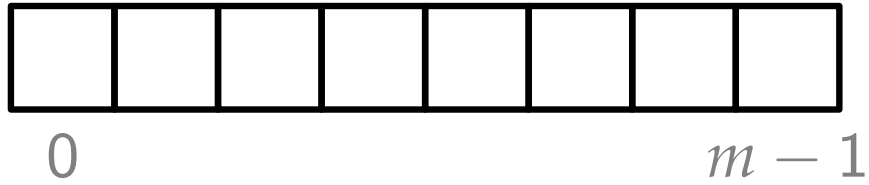
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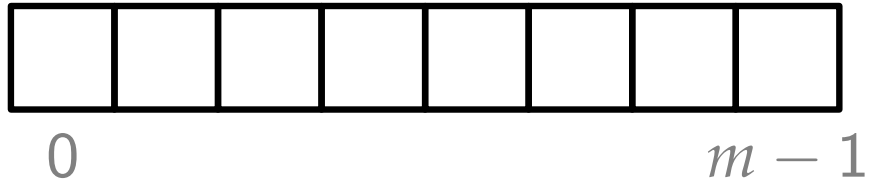
hash function $h: U \rightarrow \{0, 1, \dots, m-1\}$



Probing Hash Table



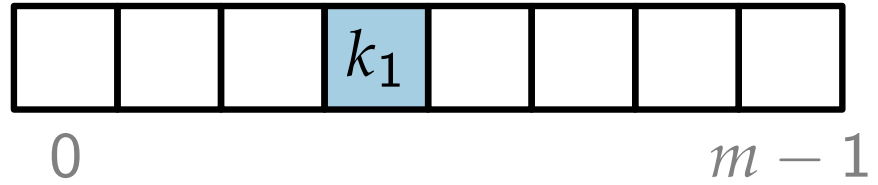
Probing Hash Table



insert(k_1)

$$h(k_1) = 3$$

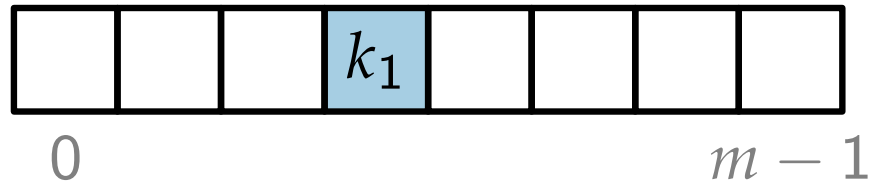
Probing Hash Table



insert(k_1)

$$h(k_1) = 3$$

Probing Hash Table



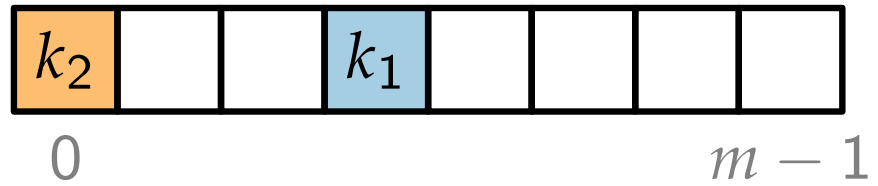
insert(k_1)

insert(k_2)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

Probing Hash Table



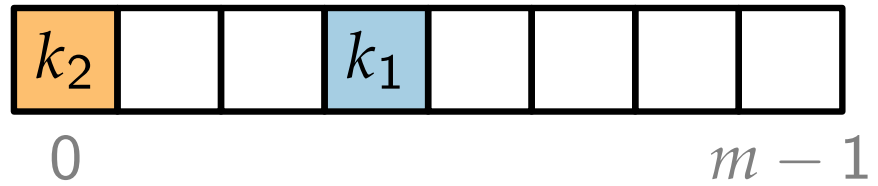
insert(k_1)

insert(k_2)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

Probing Hash Table



insert(k_1)

insert(k_2)

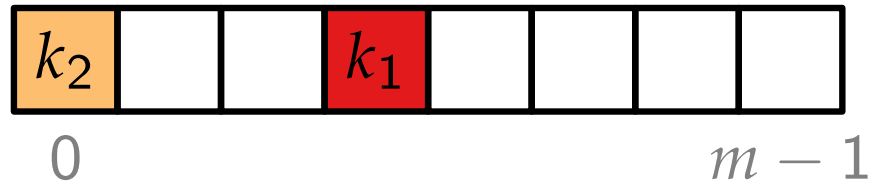
insert(k_3)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

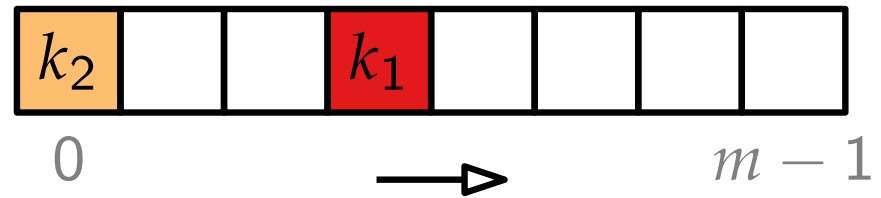
insert(k_3)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

Probing Hash Table



insert(k_1)

“linear probing”

insert(k_2)

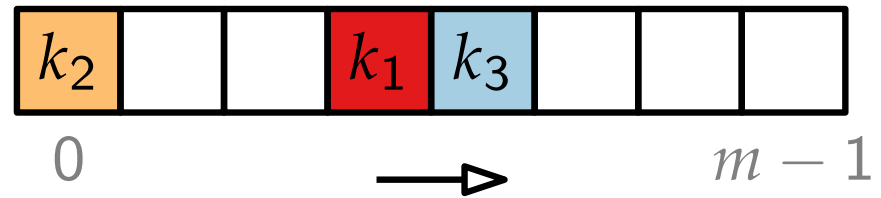
insert(k_3)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

Probing Hash Table



insert(k_1)

“linear probing”

insert(k_2)

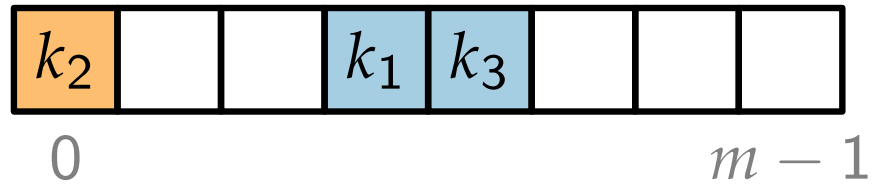
insert(k_3)

$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

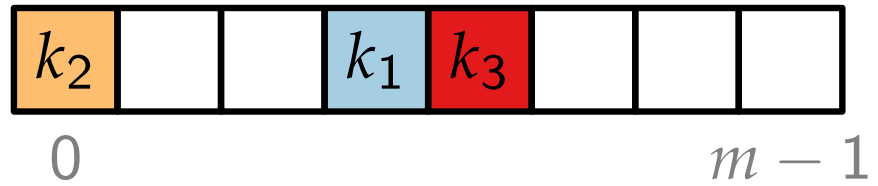
$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

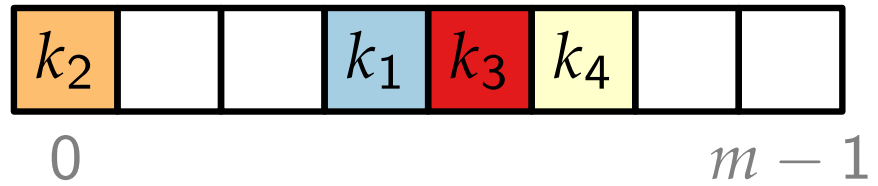
$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

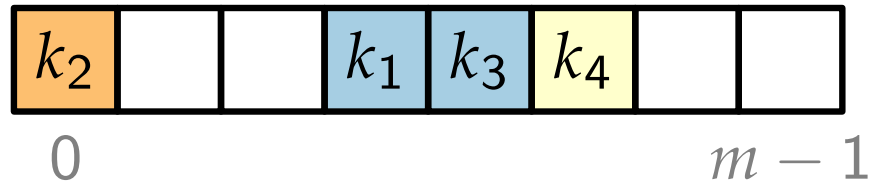
$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

$$h(k_1) = 3$$

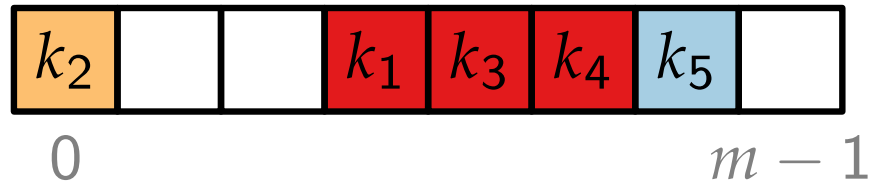
$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

$$h(k_5) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

$$h(k_1) = 3$$

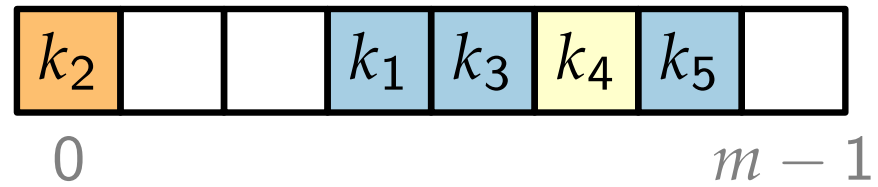
$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

$$h(k_5) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

delete(k_3)

$$h(k_1) = 3$$

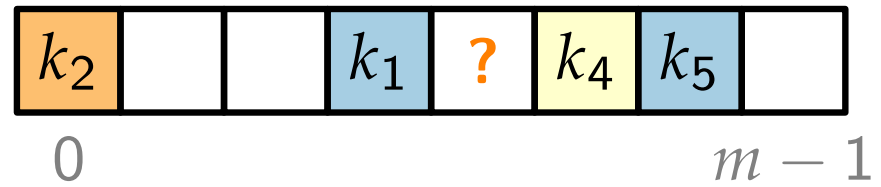
$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

$$h(k_5) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

delete(k_3)

$$h(k_1) = 3$$

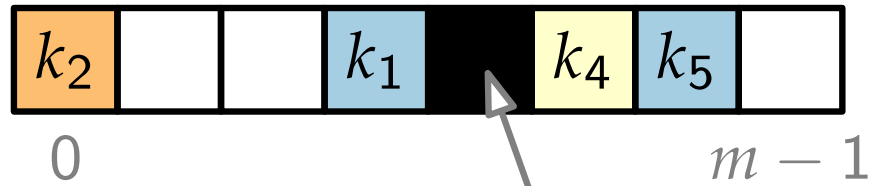
$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

$$h(k_5) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

delete(k_3)



$$h(k_1) = 3$$

$$h(k_2) = 0$$

$$h(k_3) = 3$$

$$h(k_4) = 4$$

$$h(k_5) = 3$$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

delete(k_3)

including tombstones!

load factor: $\alpha = \frac{|K|}{m}$

$h(k_1) = 3$

$h(k_2) = 0$

$h(k_3) = 3$

$h(k_4) = 4$

$h(k_5) = 3$

Probing Hash Table



insert(k_1)

insert(k_2)

insert(k_3)

insert(k_4)

insert(k_5)

delete(k_3)

including tombstones!

load factor: $\alpha = \frac{|K|}{m}$

$\alpha > \alpha_{\max}$? Resize & Rebuild!

$h(k_1) = 3$

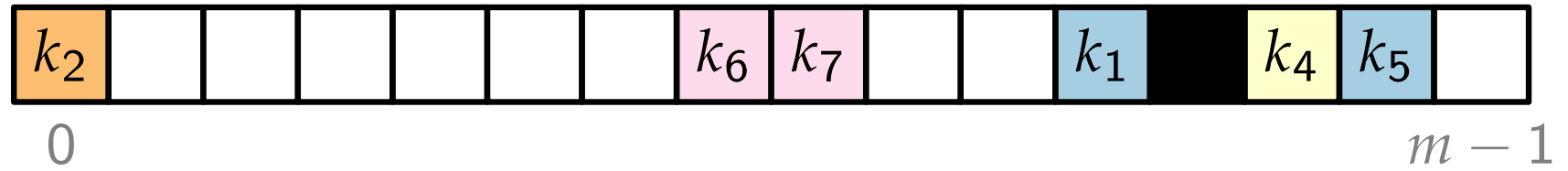
$h(k_2) = 0$

$h(k_3) = 3$

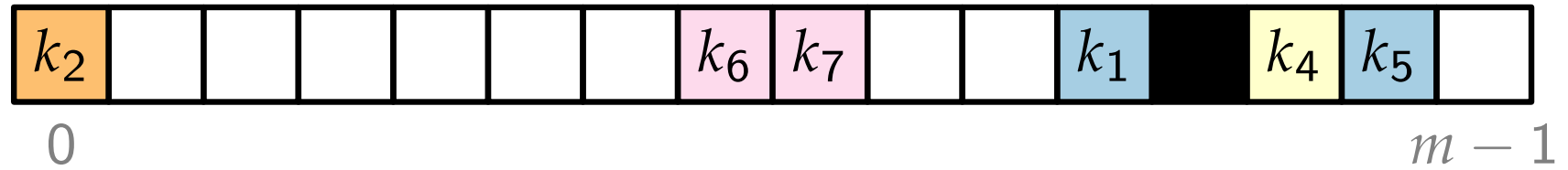
$h(k_4) = 4$

$h(k_5) = 3$

Swisstable



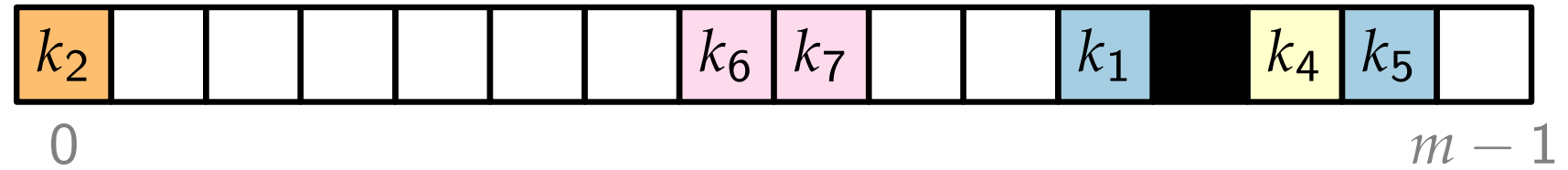
Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Swisstable

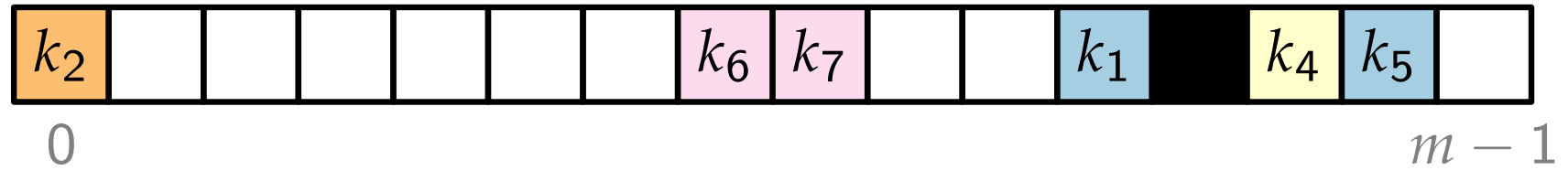


Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

$$h : \mathcal{U} \rightarrow \text{u64}$$

Swisstable



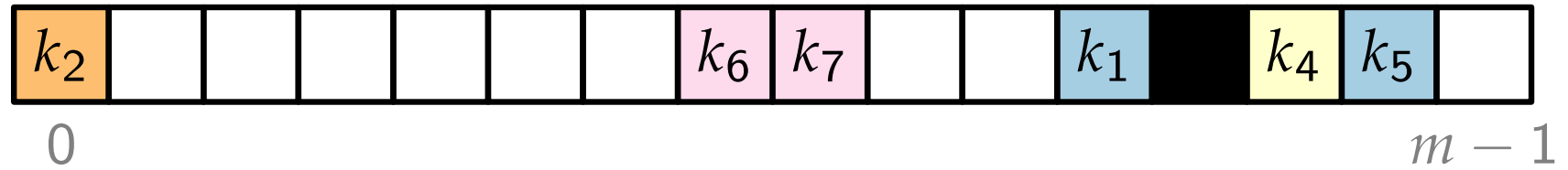
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

$$h : U \rightarrow \text{u64}$$

$$H_1(k) = h(k) \bmod m$$

Swisstable



Ingredients:

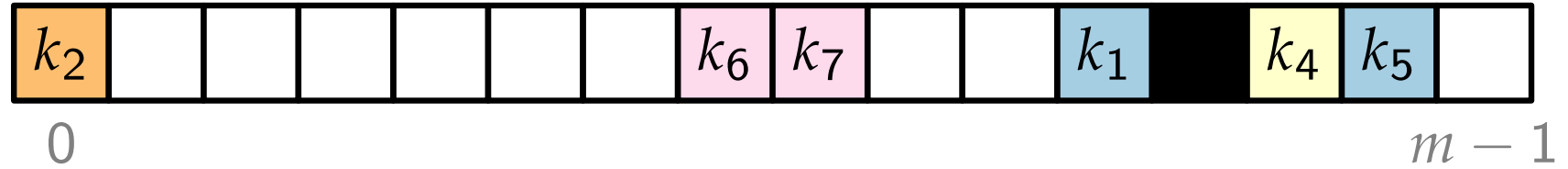
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

$$h : U \rightarrow \text{u64}$$

$$H_1(k) = h(k) \bmod m$$

$$H_2(k) = h(k) \gg 57 \quad \text{"first 7 bits"}$$

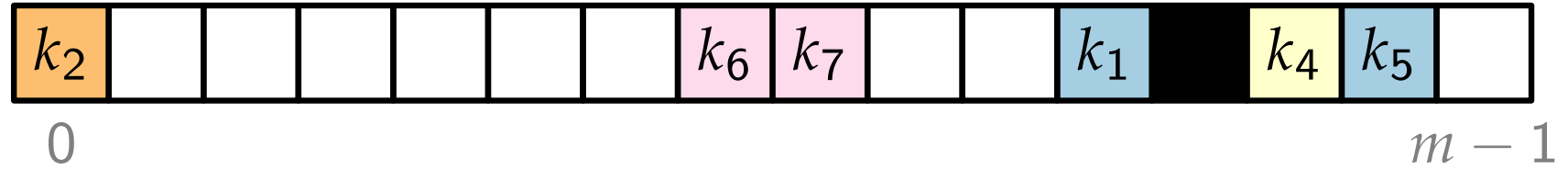
Swisstable



Ingredients:

- derive two hash functions
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- SIMD instructions

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

m bytes of

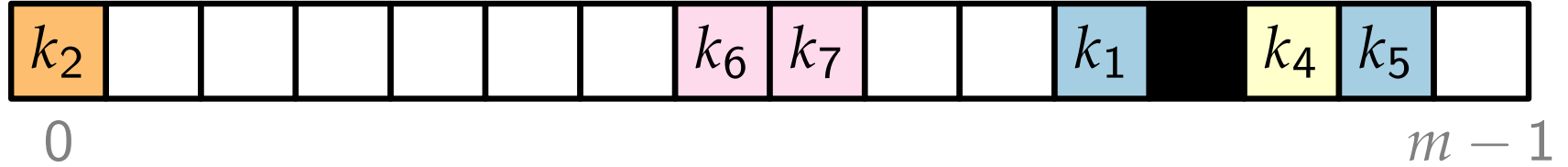
$0 \times \text{FF}$ = empty

0×80 = deleted



$0 \times 00 \dots 0 \times 7\text{F}$ = occupied

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

 m bytes of

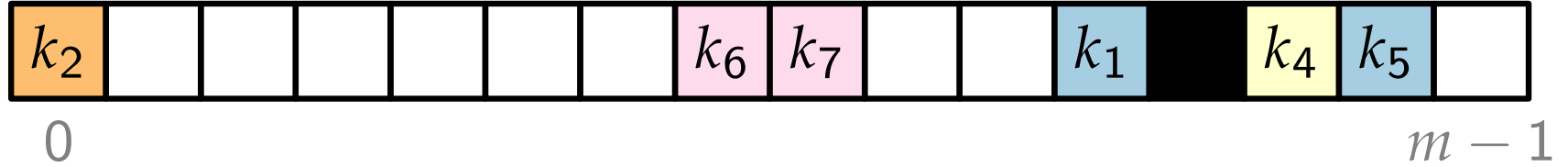
$0 \times \text{FF}$ = empty

0x80 = deleted 

$0 \times 00 \dots 0 \times 7F = \text{occupied}$

use H_2 for the occupied bytes

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

 m bytes of

$0 \times \text{FF}$ = empty

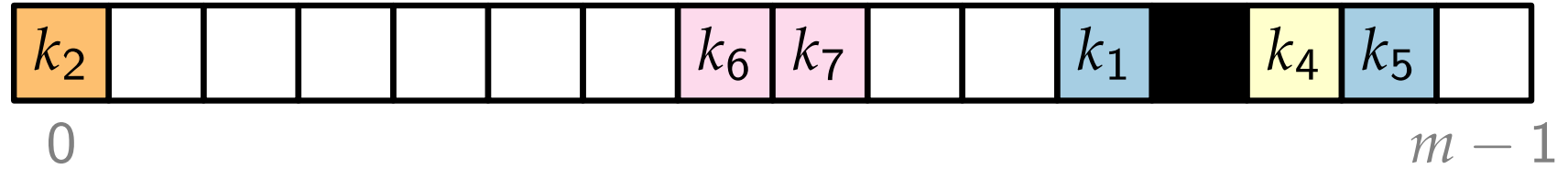
0x80 = deleted 

$0 \times 00 \dots 0 \times 7F = \text{occupied}$

use H_2 for the occupied bytes

+ reprise of the first 16 bytes

Swisstable



Ingredients:

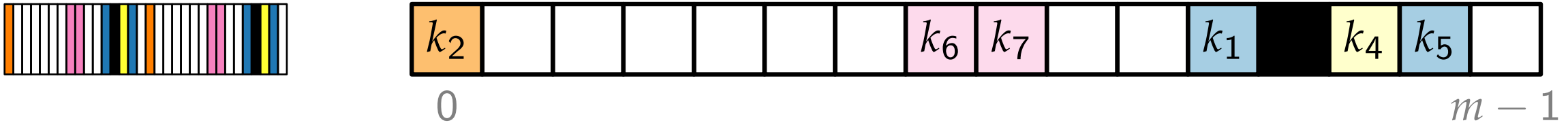
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

`_mm_set1_epi8`

`_mm_cmpeq_epi8`

`_mm_movemask_epi8`

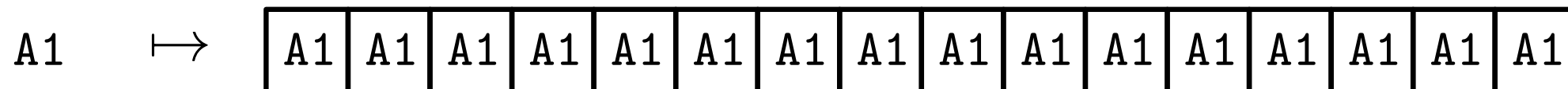
Swisstable



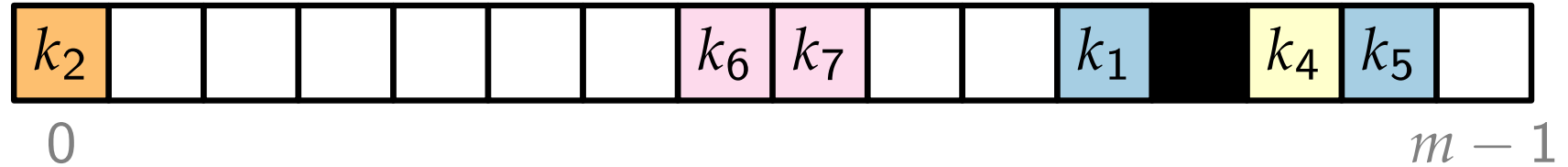
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

_mm_set1_epi8



Swisstable



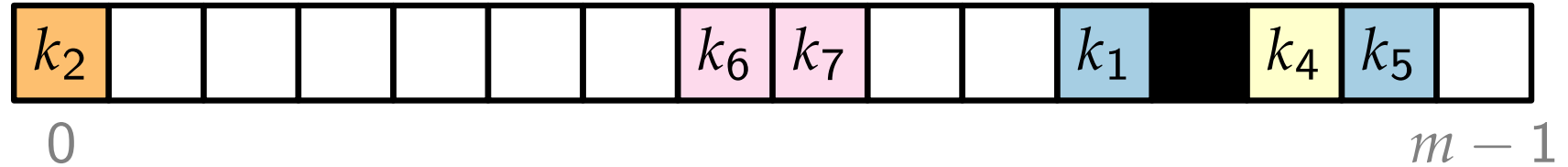
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

`_mm_cmpeq_epi8`

A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1	A1
05	7B	A1	82	CF	40	6A	37	42	A1	9B	58	E2	FA	36	00

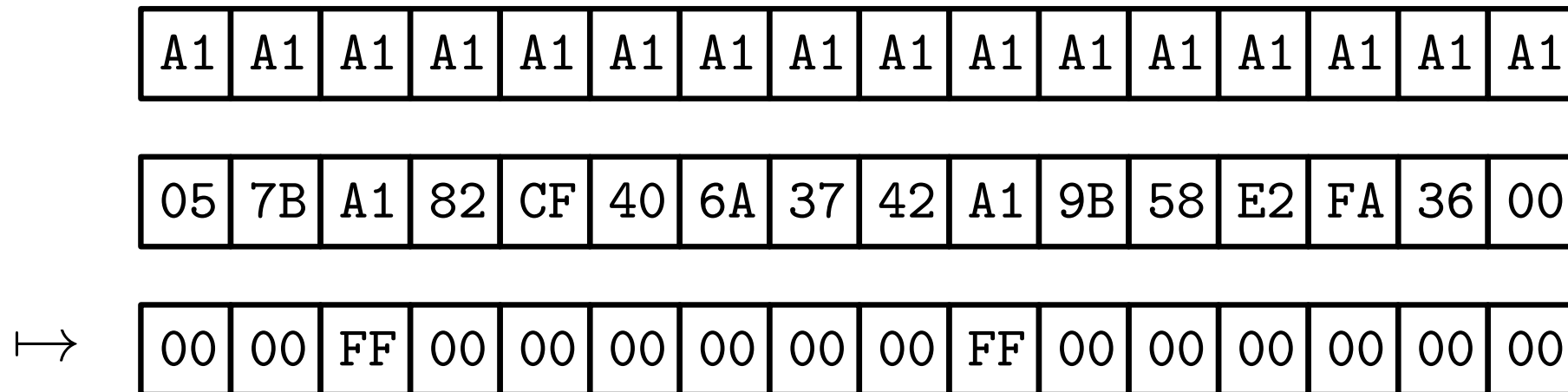
Swisstable



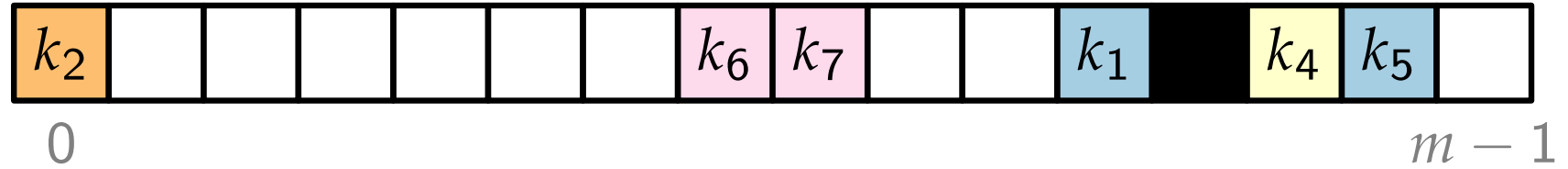
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

`_mm_cmpeq_epi8`



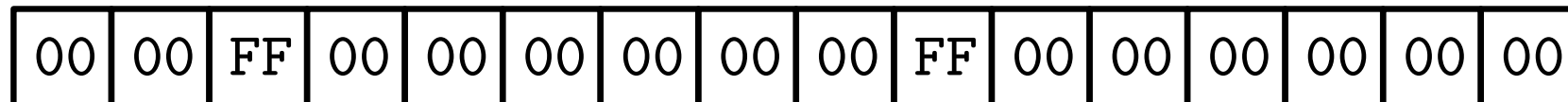
Swisstable



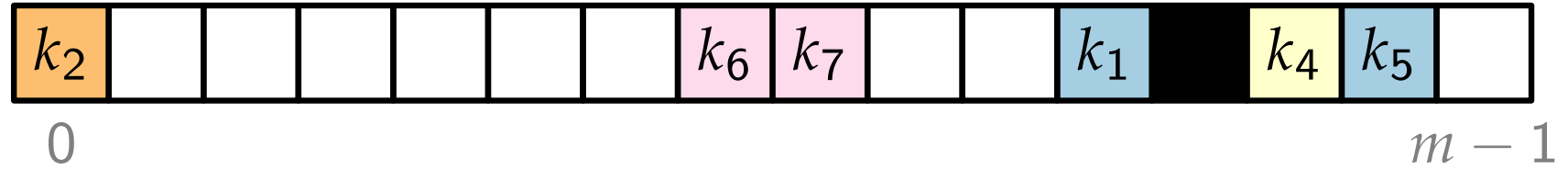
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

`_mm_movemask_epi8`



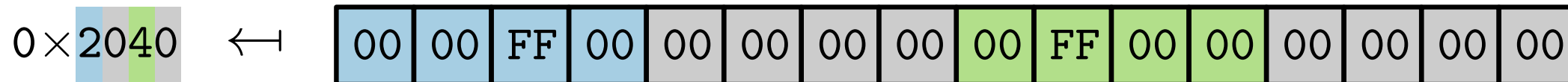
Swisstable



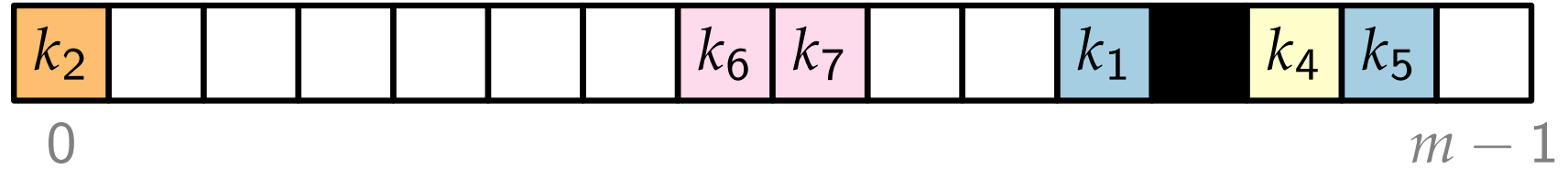
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

`_mm_movemask_epi8`



Swisstable



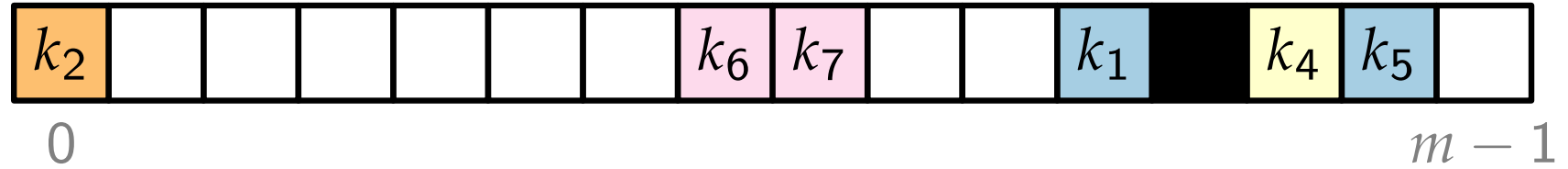
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

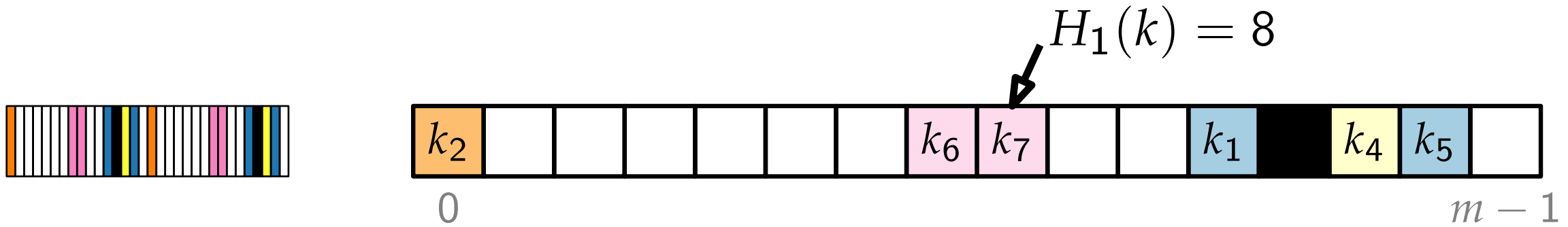
Steps:

1. start at bucket $H_1(k)$

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

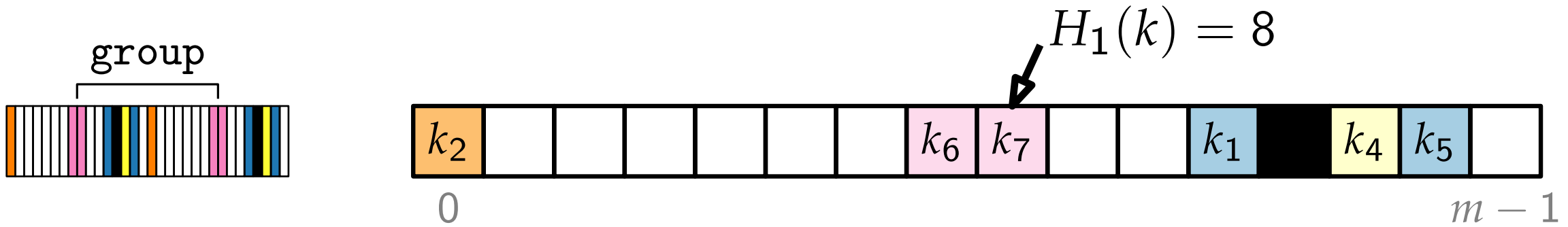
Steps:

1. start at bucket $H_1(k)$

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

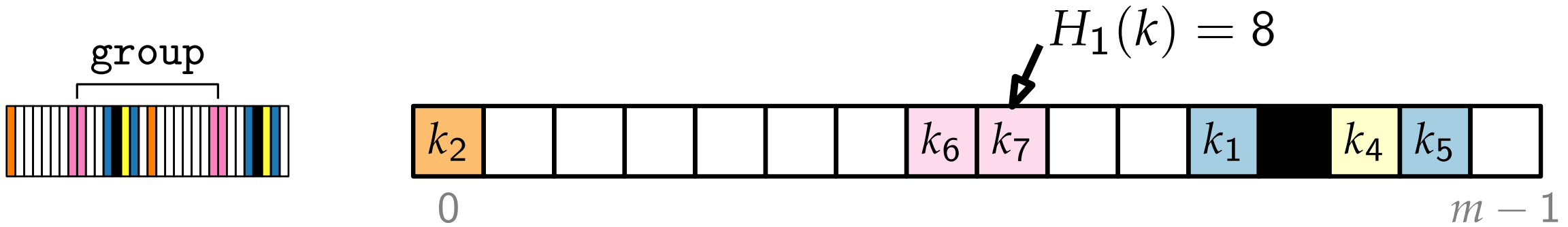
Steps:

1. start at bucket $H_1(k)$
2. search group for $H_2(k)$

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

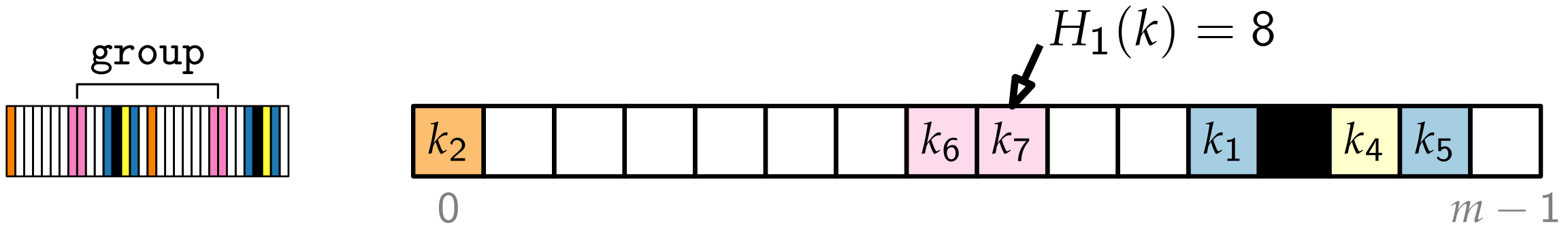
Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Steps:

1. start at bucket $H_1(k)$
2. search group for $H_2(k)$
3. for each match, check keys, return true if found

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

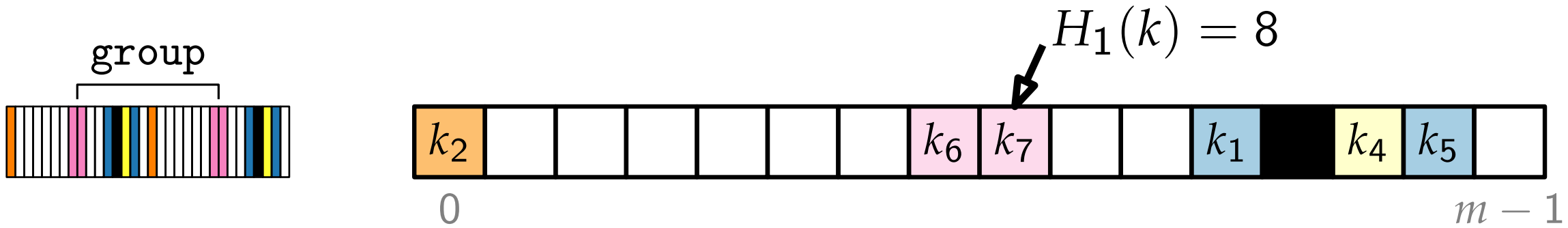
Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Steps:

1. start at bucket $H_1(k)$
2. search group for $H_2(k)$
3. for each match, check keys, return true if found
4. search group for an empty bucket

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

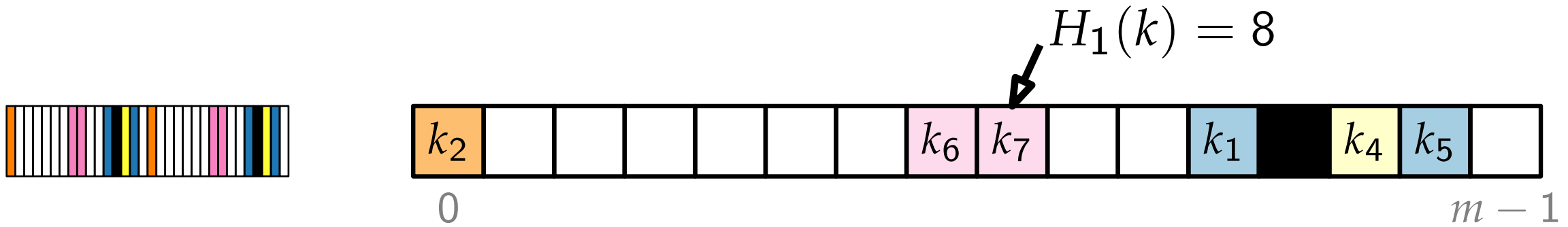
Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Steps:

1. start at bucket $H_1(k)$
2. search group for $H_2(k)$
3. for each match, check keys, return true if found
4. search group for an empty bucket
5. return false if found
6. otherwise, check the next group

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

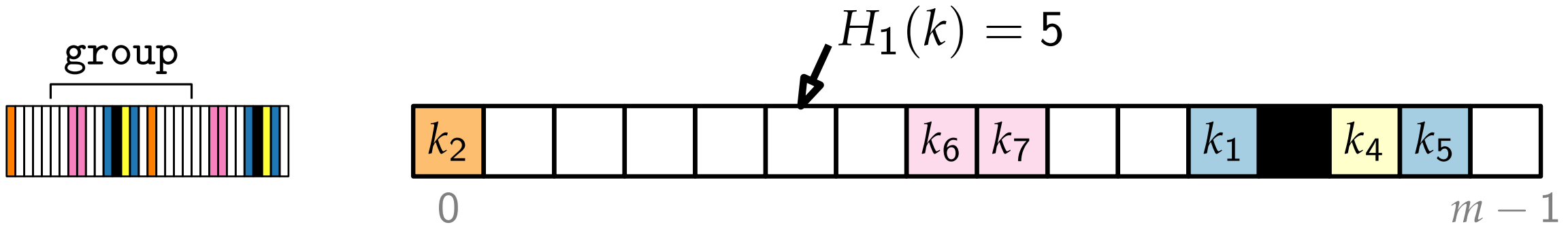
Case 1: k is in the table

find k , then replace it

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

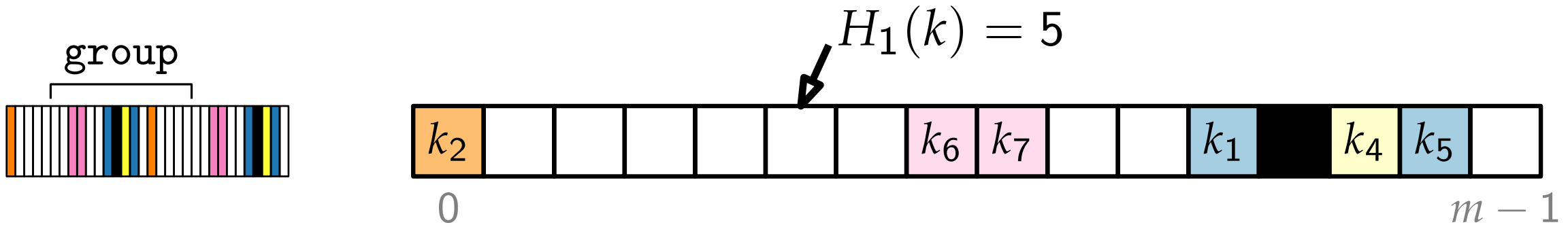
Case 1: k is in the table

find k , then replace it

Case 2: k is not in the table

1. start at bucket $H_1(k)$
2. search group for empty or deleted

Swisstable



Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

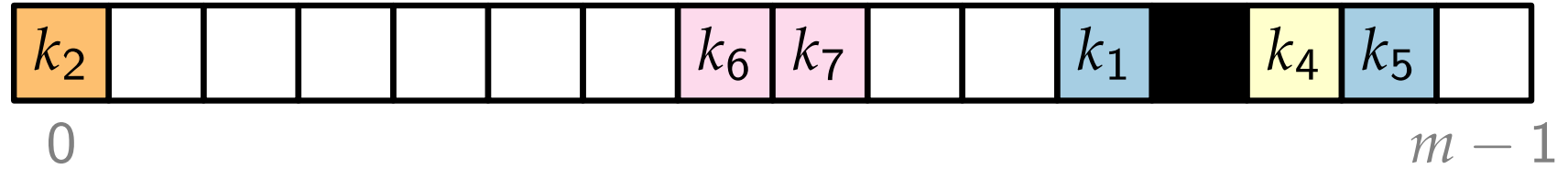
Case 1: k is in the table

find k , then replace it

Case 2: k is not in the table

1. start at bucket $H_1(k)$
2. search group for empty or deleted
3. if found, insert k
4. otherwise, check the next group

Swisstable



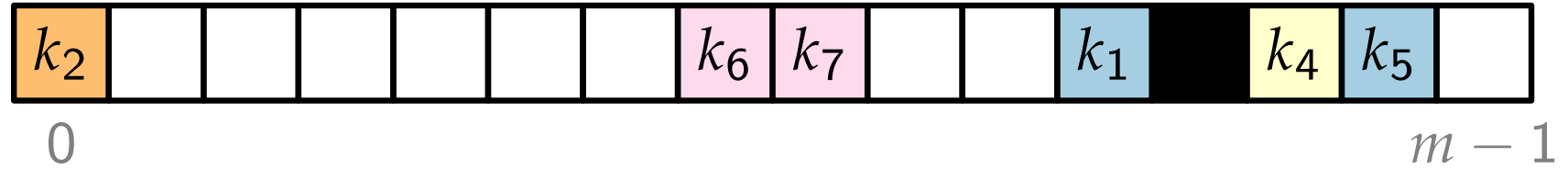
Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$

Swisstable



Ingredients:

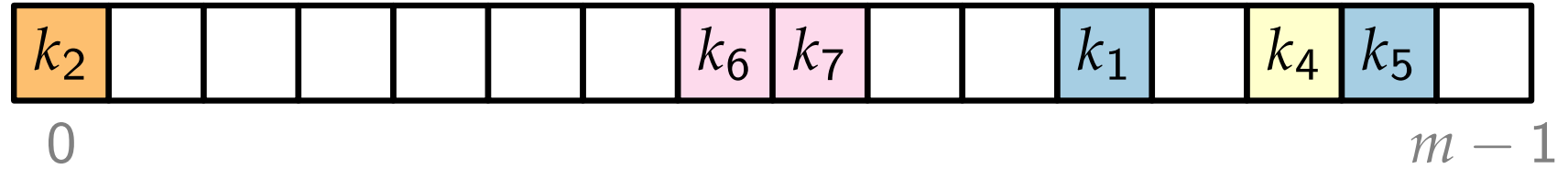
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$



Swisstable



Ingredients:

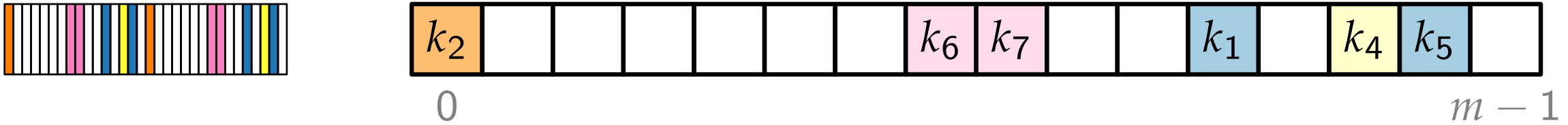
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- `find( $k$ : Key)`
- `insert( $k$ : Key)`
- `delete( $k$ : Key)`



Swisstable



Ingredients:

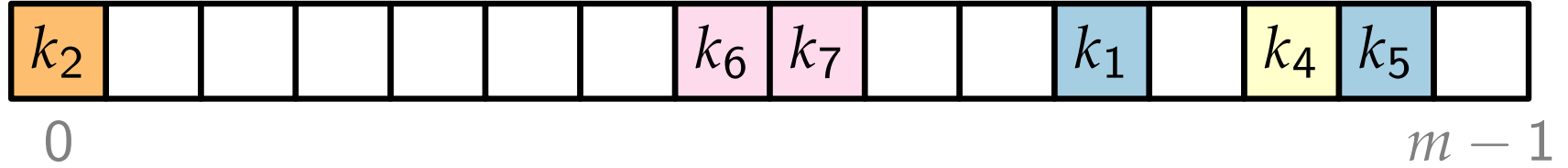
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions



Operations:

- `find(k: Key)`
- `insert(k: Key)`
- `delete(k: Key)`

Swisstable

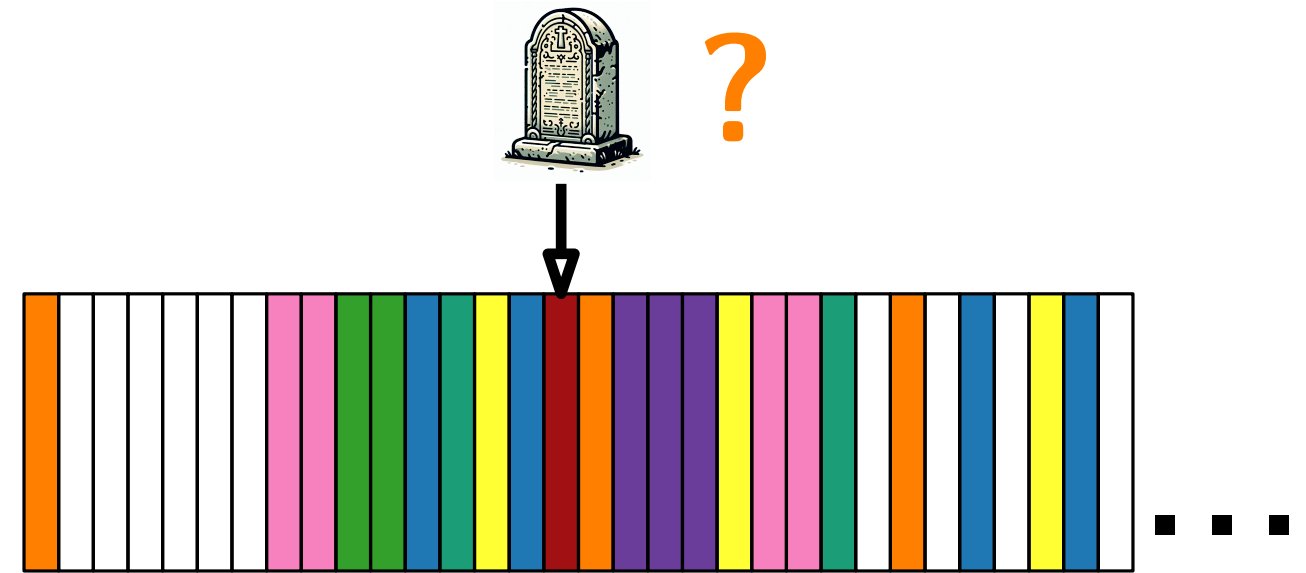


Ingredients:

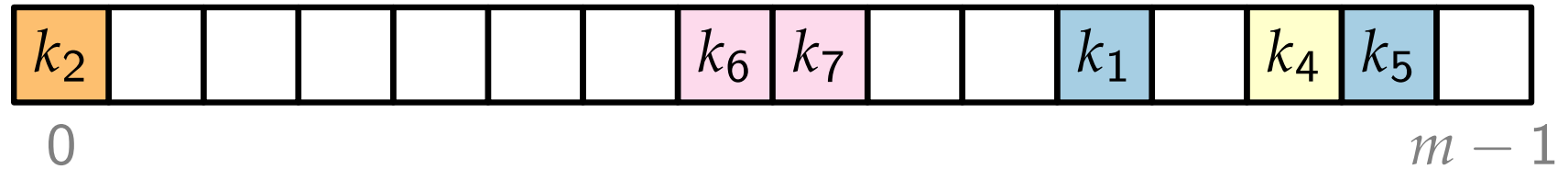
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- find(k : Key)
- insert(k : Key)
- delete(k : Key)



Swisstable

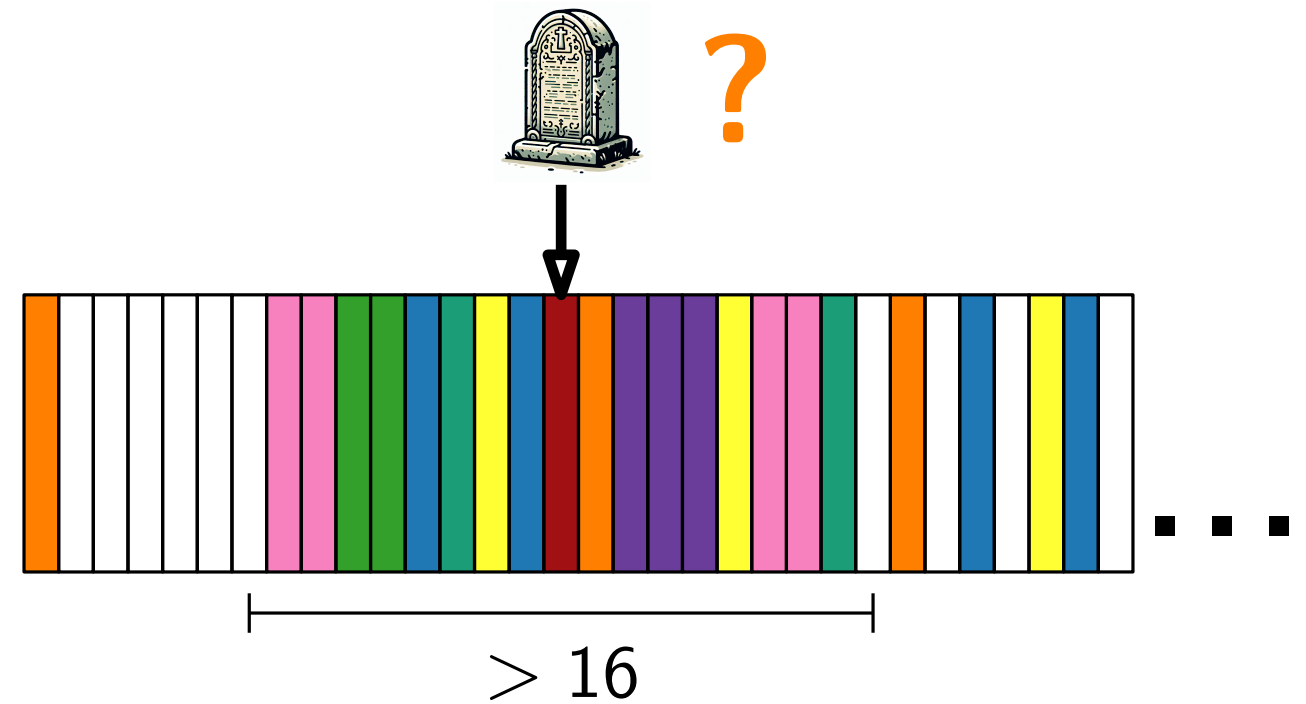


Ingredients:

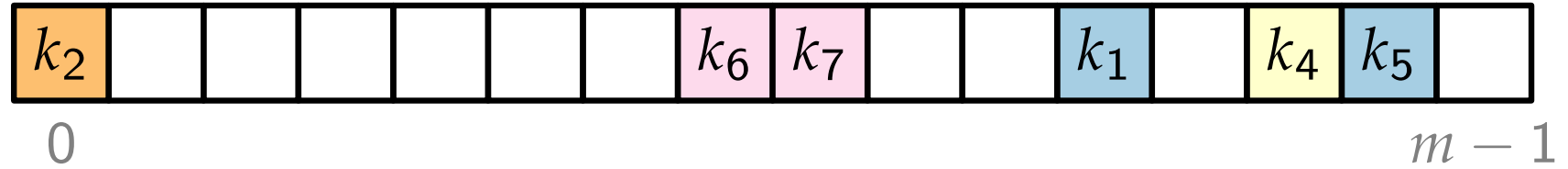
- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

- find(k : Key)
- insert(k : Key)
- delete(k : Key)



Swisstable

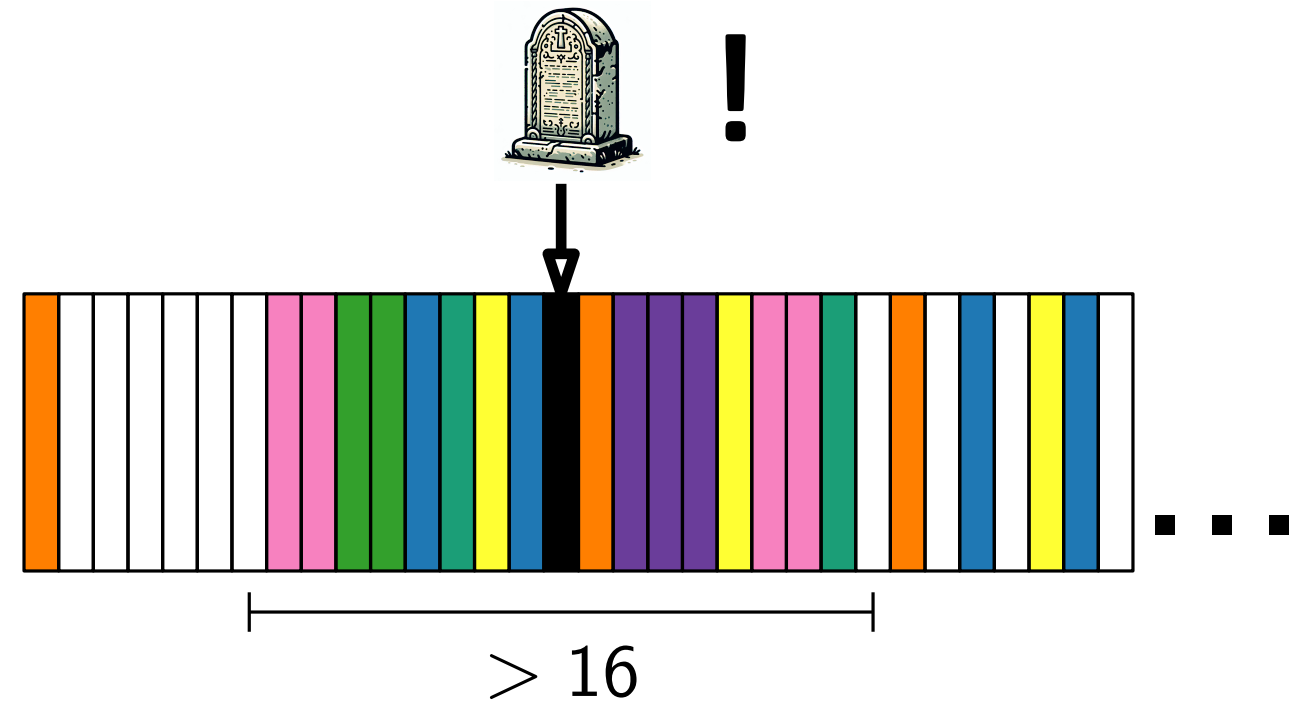


Ingredients:

- derive two hash functions
- use $m + 16$ bytes of extra metadata
- SIMD instructions

Operations:

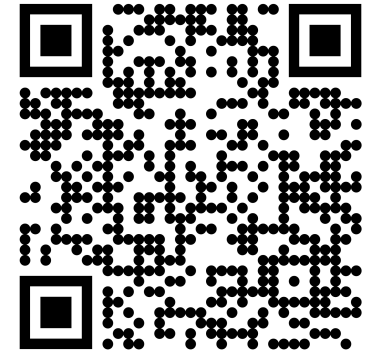
- $\text{find}(k: \text{Key})$
- $\text{insert}(k: \text{Key})$
- $\text{delete}(k: \text{Key})$



Further Reading

Slides based on:

- “Designing a Fast, Efficient, Cache-friendly Hash Table, Step by Step” by Matt Kulukundis



Further Reading

Slides based on:

- “Designing a Fast, Efficient, Cache-friendly Hash Table, Step by Step” by Matt Kulukundis
- “Swisstable, a Quick and Dirty Description” by Aria Beingessner

