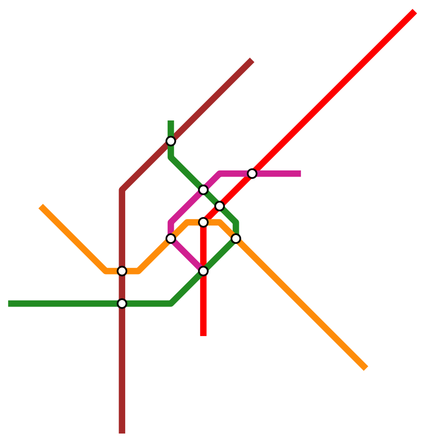
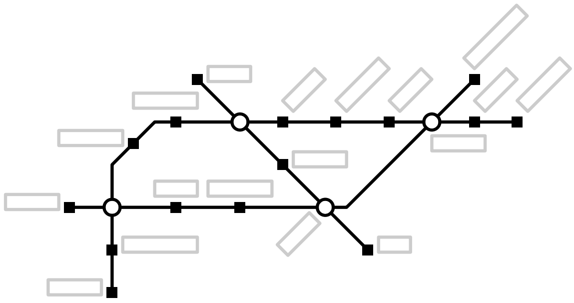
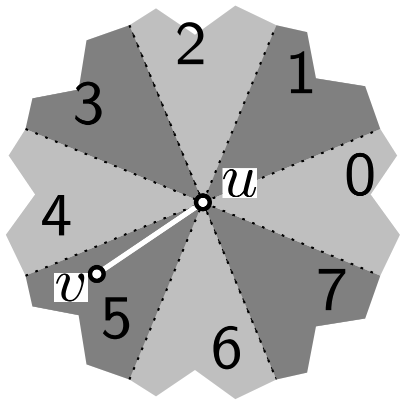
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Visualization of Graphs

Lecture 13:
Octilinear Graph Drawing
Metro Map Layout



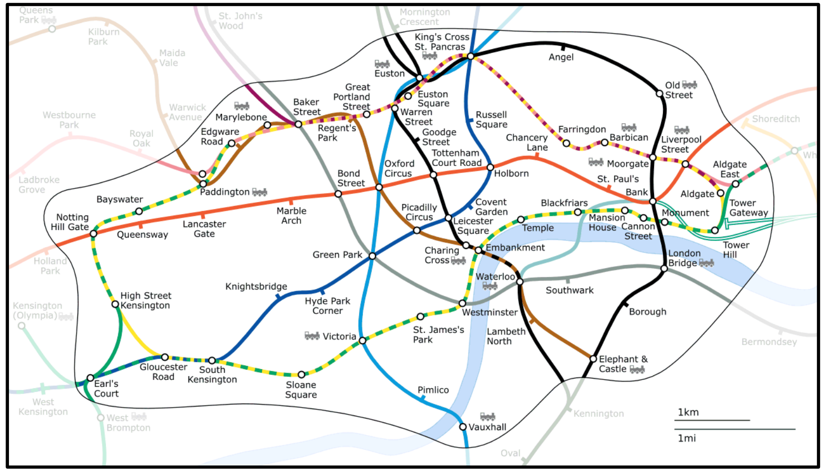


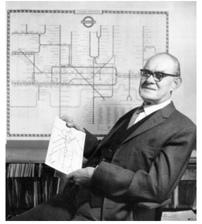


Johannes Zink

Summer semester 2024

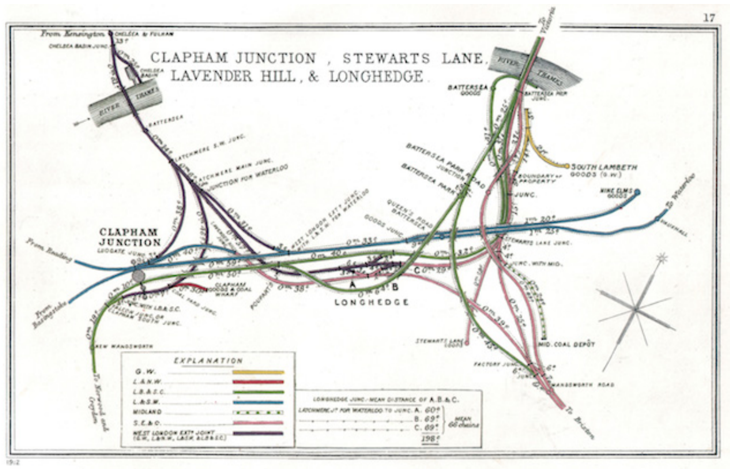
What is a Schematic Metro Map?





- map/diagram that shows stations connected by metro lines
- focus on topology rather than geography
- goal: easy-to-use visual navigation aid for passengers
  - “*How do I quickly get from A to B?*”
  - “*Where do I need to change trains?*”
- distorts scale and geometry
- metro map design still a largely manual process
- optimizing network layout computationally challenging

Subtasks in Metro Map Layout



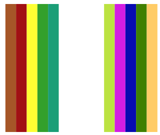
Input:
- geographically embedded railway network G
- set $\mathcal{L}$ of metro lines serving G

Output:
- **optimal** metro map layout (whatever it means)

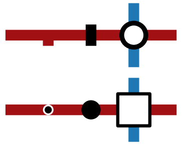
Divide the task into several subtasks:

- rendering and design choices
- network layout
- station labeling
- metro line routing

colors fonts symbols

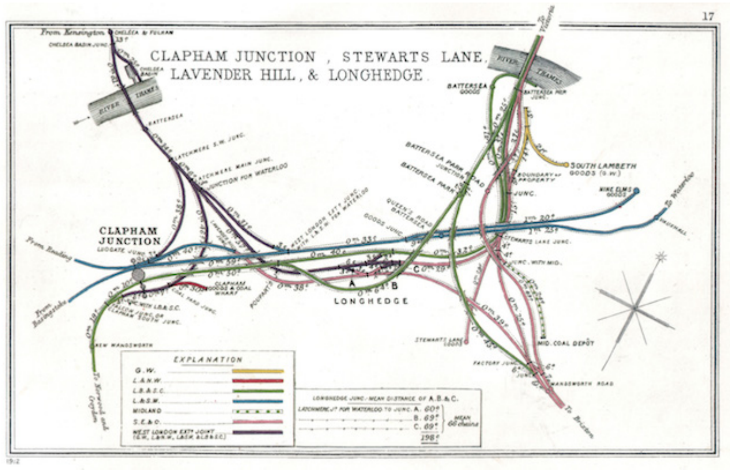


Paddington
PADDINGTON
Paddington



$\rightarrow$ very salient,
but not a computational problem

Subtasks in Metro Map Layout



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→

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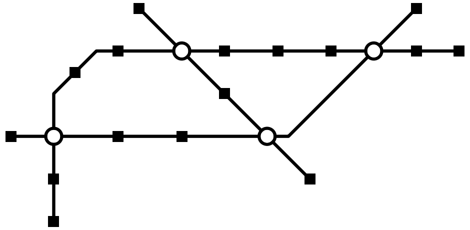
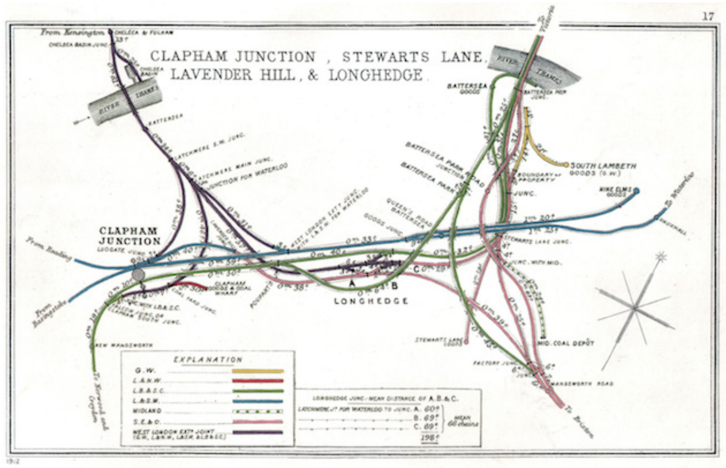


determine geometry of network layout

Subtasks in Metro Map Layout



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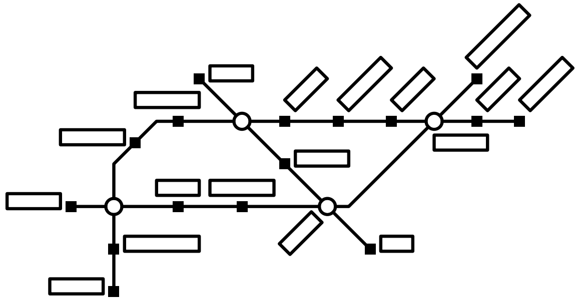
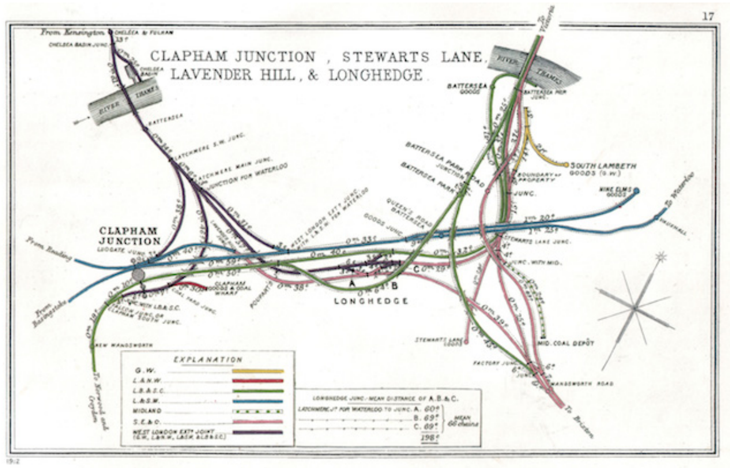


determine positions of station names

Subtasks in Metro Map Layout



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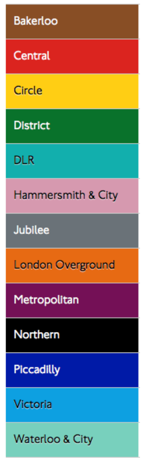

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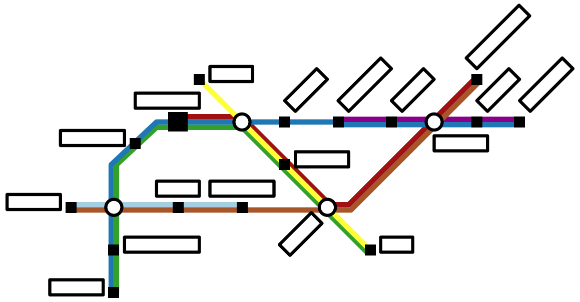
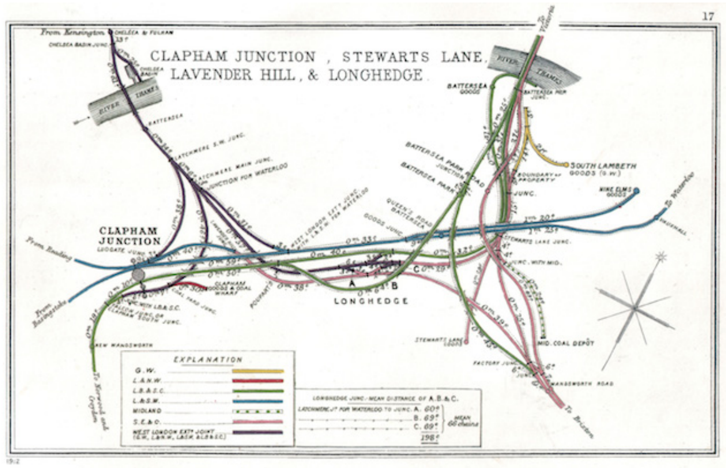


determine line routing and ordering of bundles

Subtasks in Metro Map Layout



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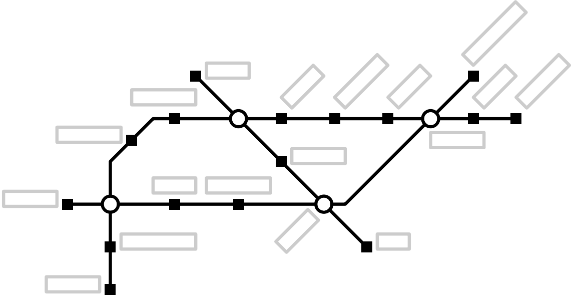
Divide the task into several subtasks:

- rendering and design choices
- network layout
- station labeling
- metro line routing

focus today



Formalizing the Network Layout Problem

Given
- graph $G = (V, E)$ geometrically embedded in $\mathbb{R}^2$
- vertex set V (stations)
- edge set E (rail links)
- set of paths $\mathcal{L}$ (metro lines in G)

Find a **schematic** layout of $(G, \mathcal{L})$ that
- satisfies a set of layout constraints
- optimizes a set of quality criteria

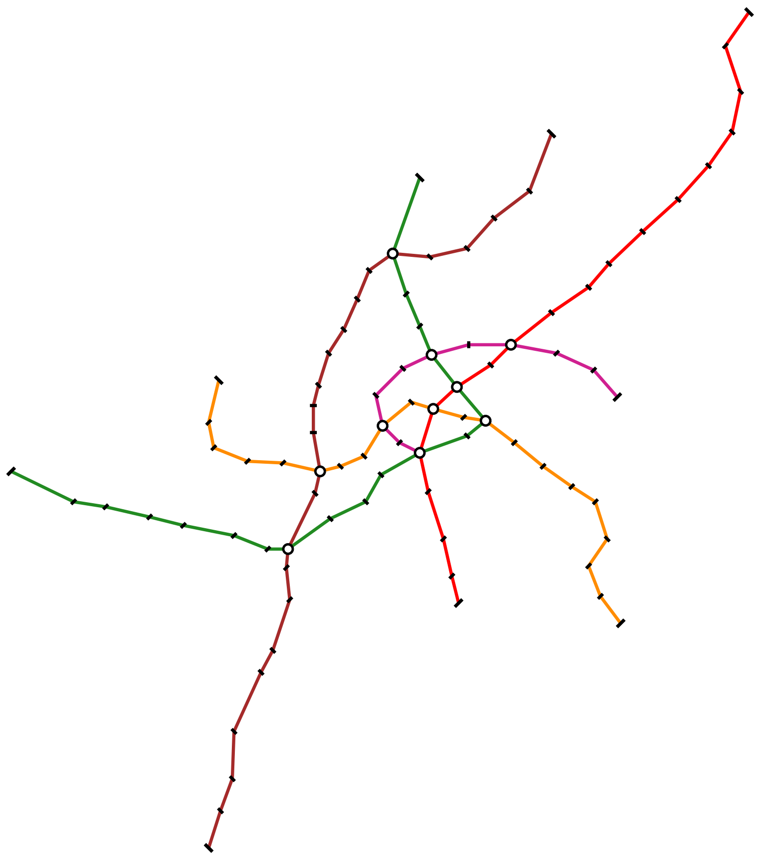
But what are the constraints and quality criteria?

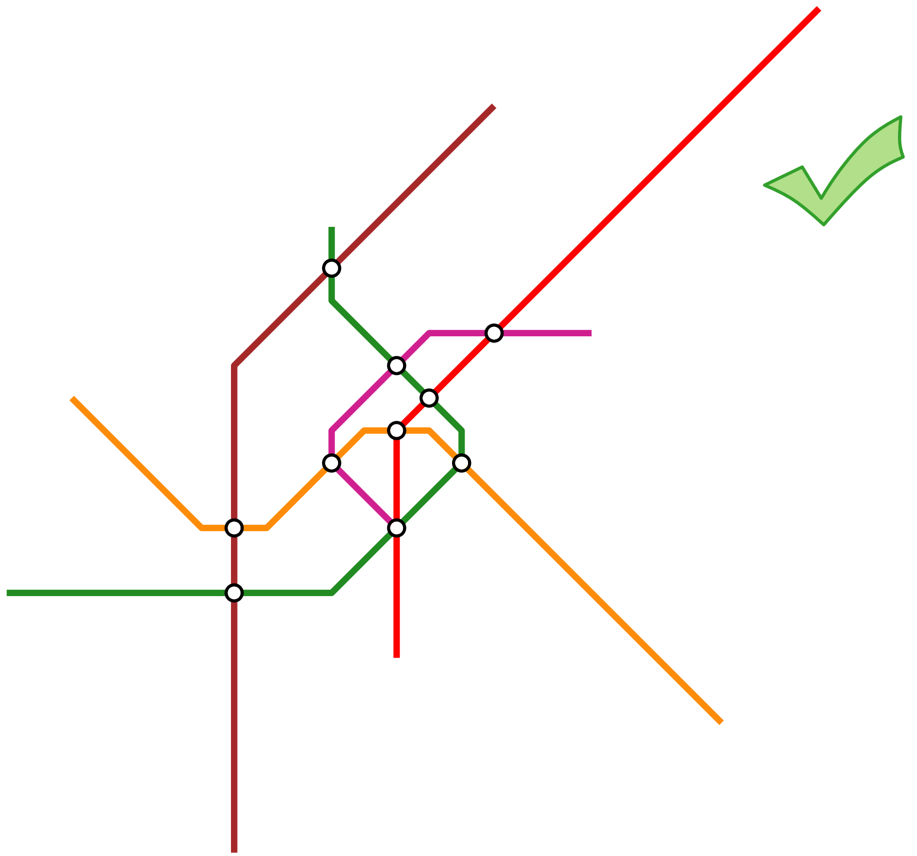
$\rightarrow$ extract common principles of existing, manually designed metro maps

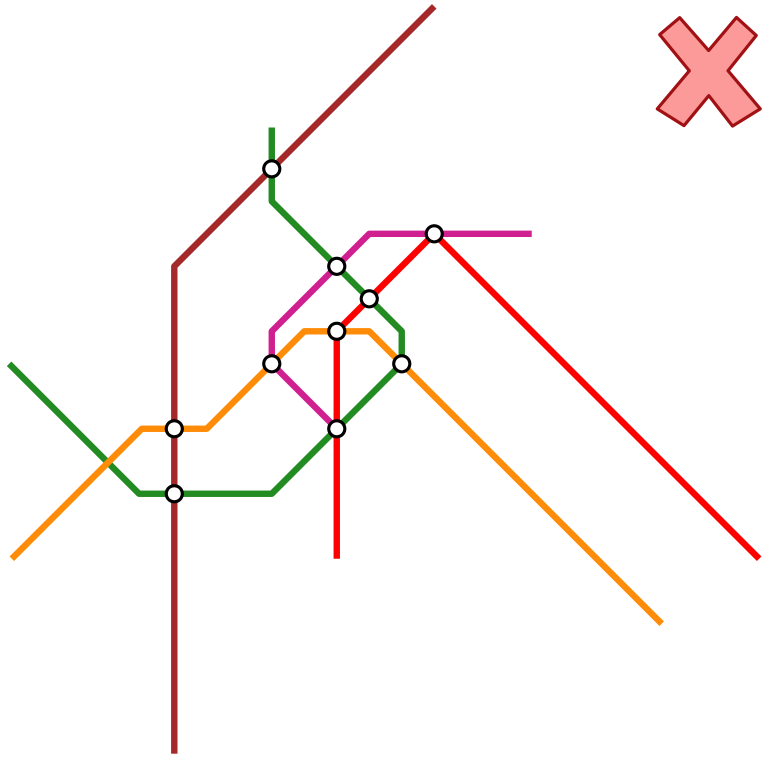
Design Rules

(R1) Do not change the network topology.

- no new crossings
- no changes in circular vertex orders





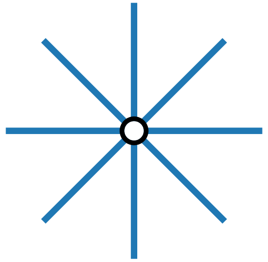


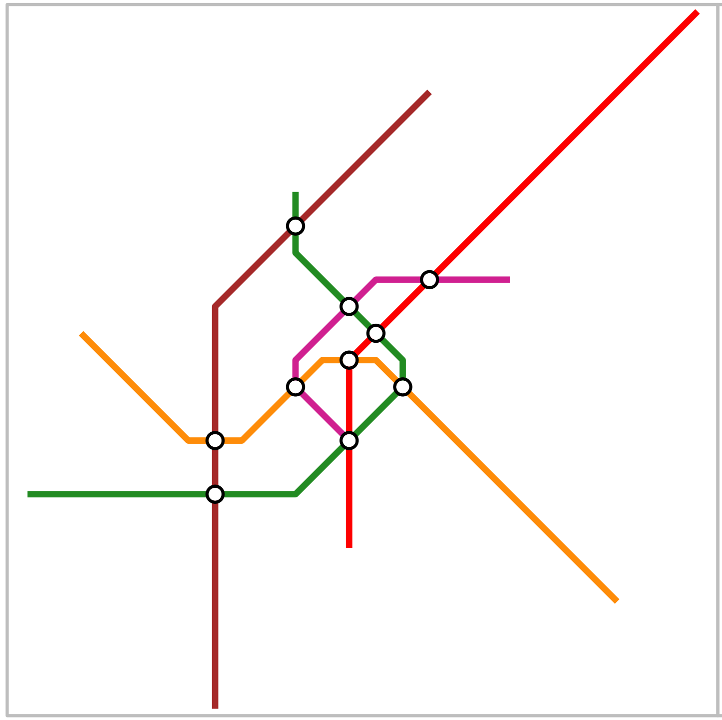
Design Rules

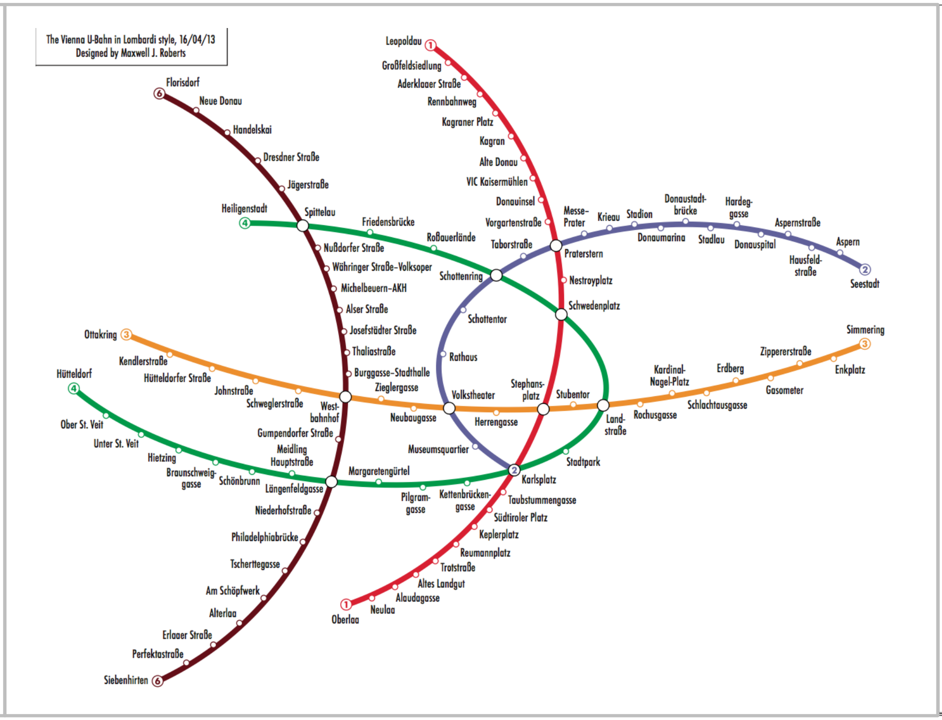
(R1) Do not change the network topology.

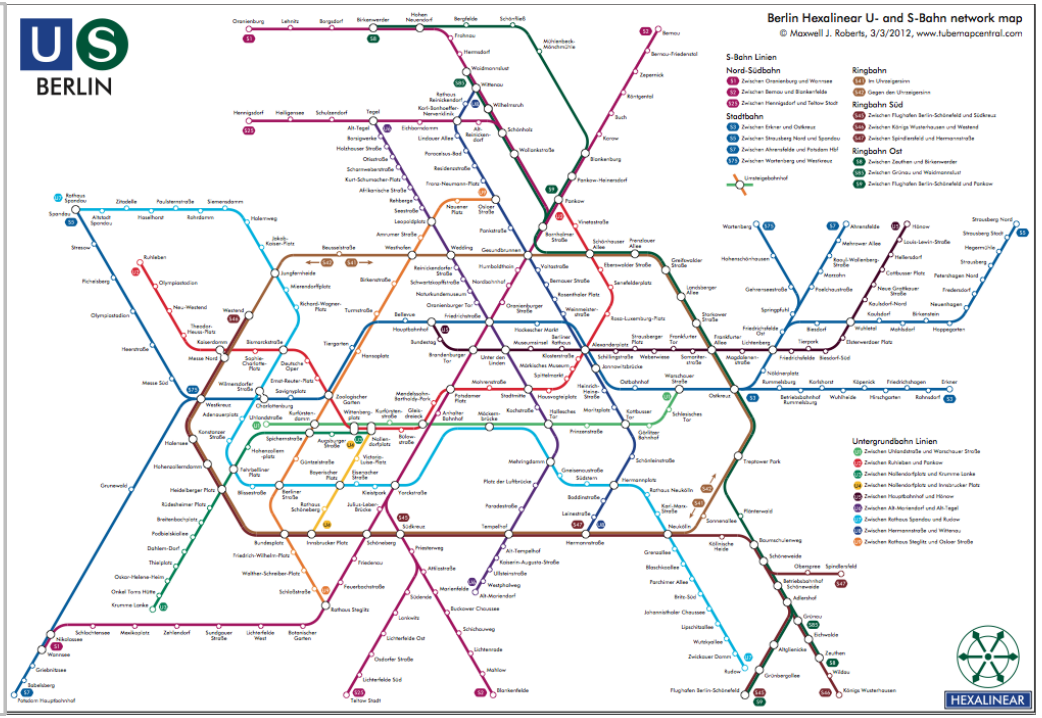
(R2) Restrict edge orientations.

- mostly octilinear (octolinear) orientation systems
- curvilinear and other alternative orientation systems also exist









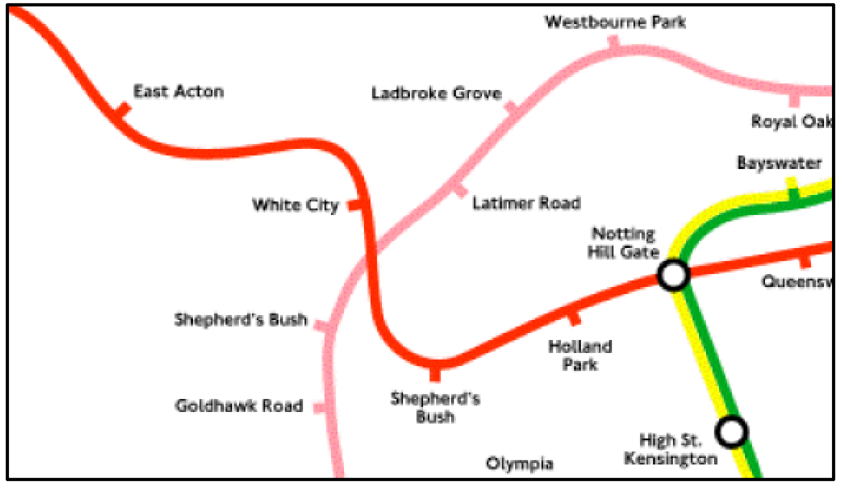
Design Rules

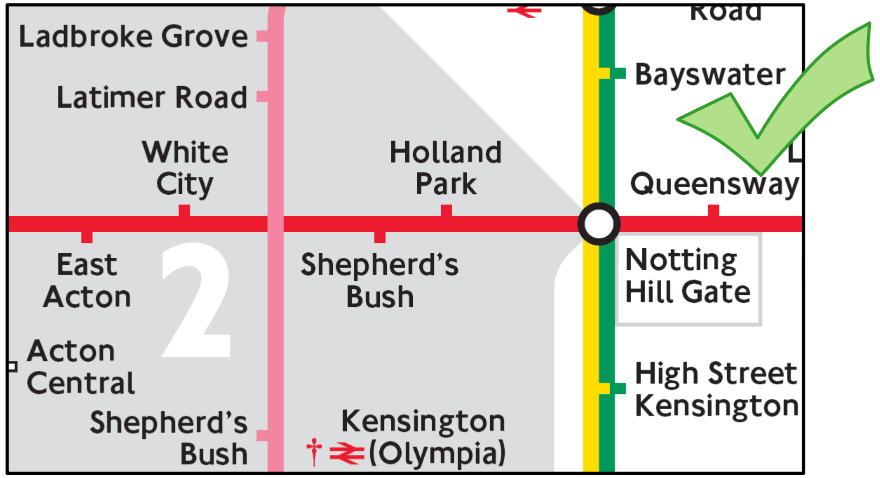
(R1) Do not change the network topology.

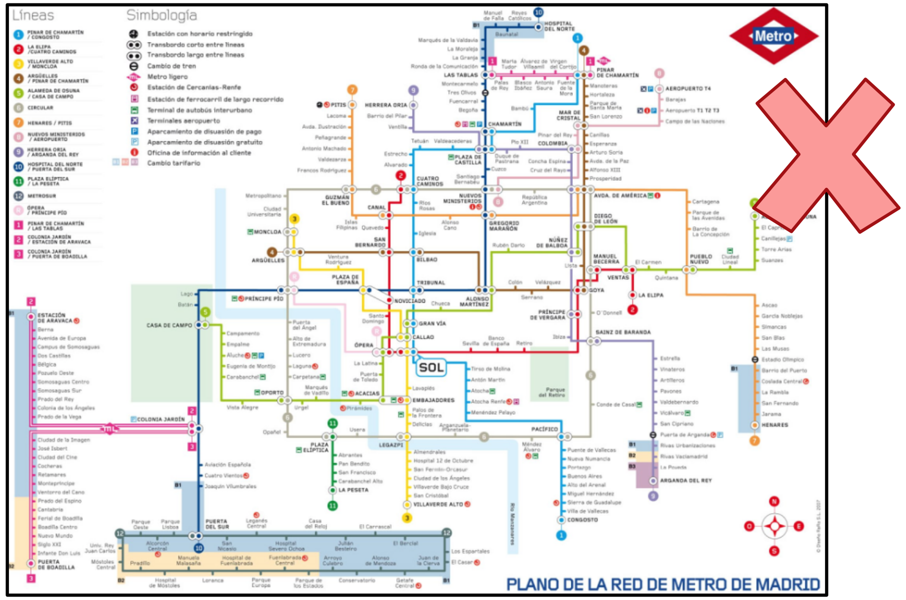
(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

- avoid bends
- prefer obtuse bend angles
- for curves: prefer uniform curvature, few inflection points







Design Rules

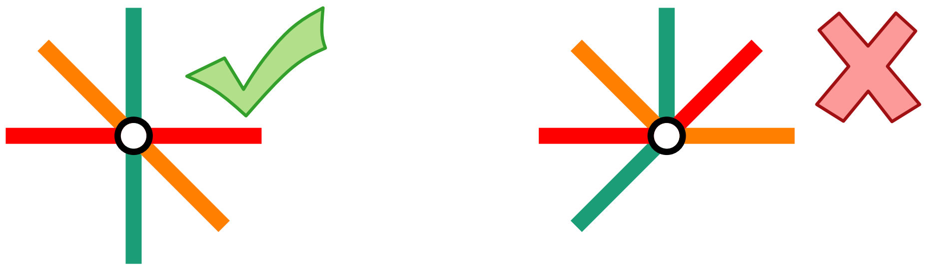
(R1) Do not change the network topology.

(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

- avoids visual ambiguities in complex stations



Design Rules

(R1) Do not change the network topology.

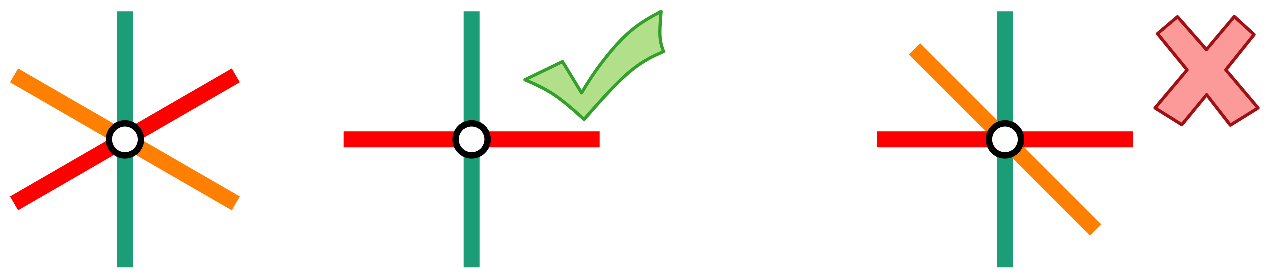
(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

(R5) Use large angular resolution in stations.

- distributes edges evenly for balanced appearance



Design Rules

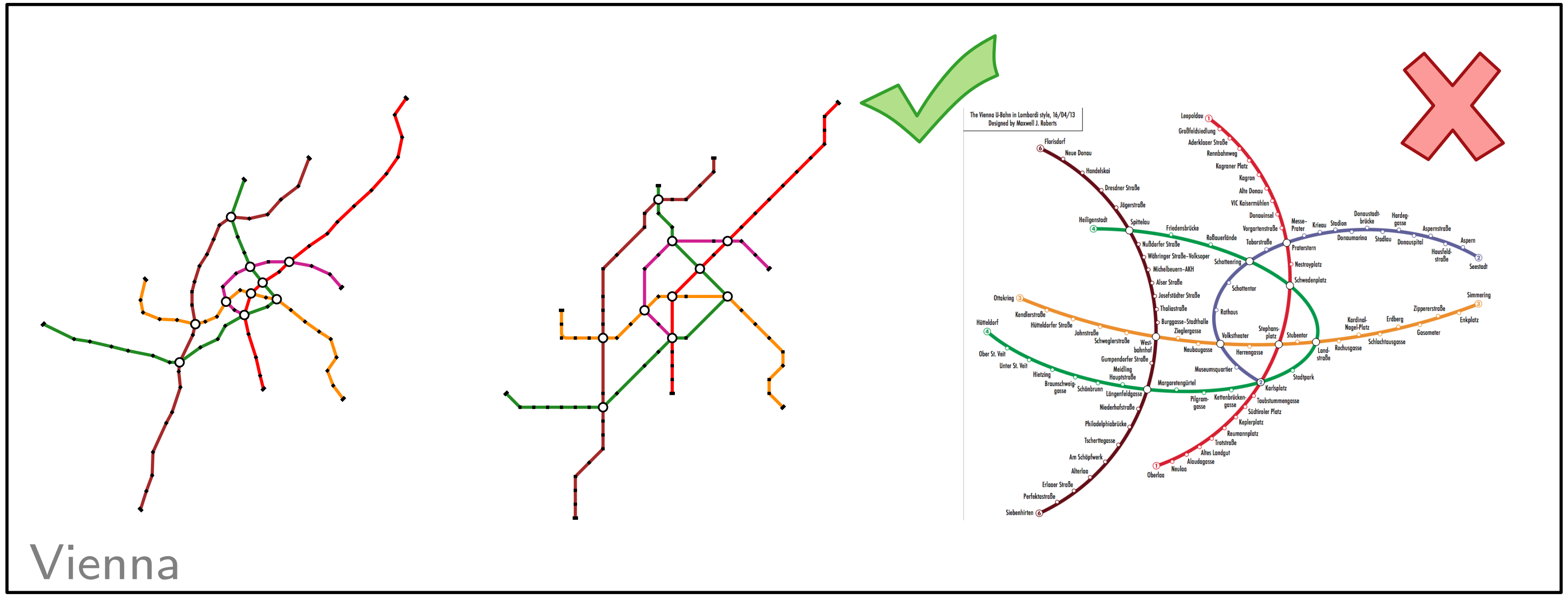
(R1)

(R2)

(R3)

(R4)

(R5)



(R6) Minimize geometric distortion and displacement.

- maintains resemblance to geography
- preserves user's mental map
- applicable locally or globally

topographicity

Design Rules

(R1) Do not change the network topology.

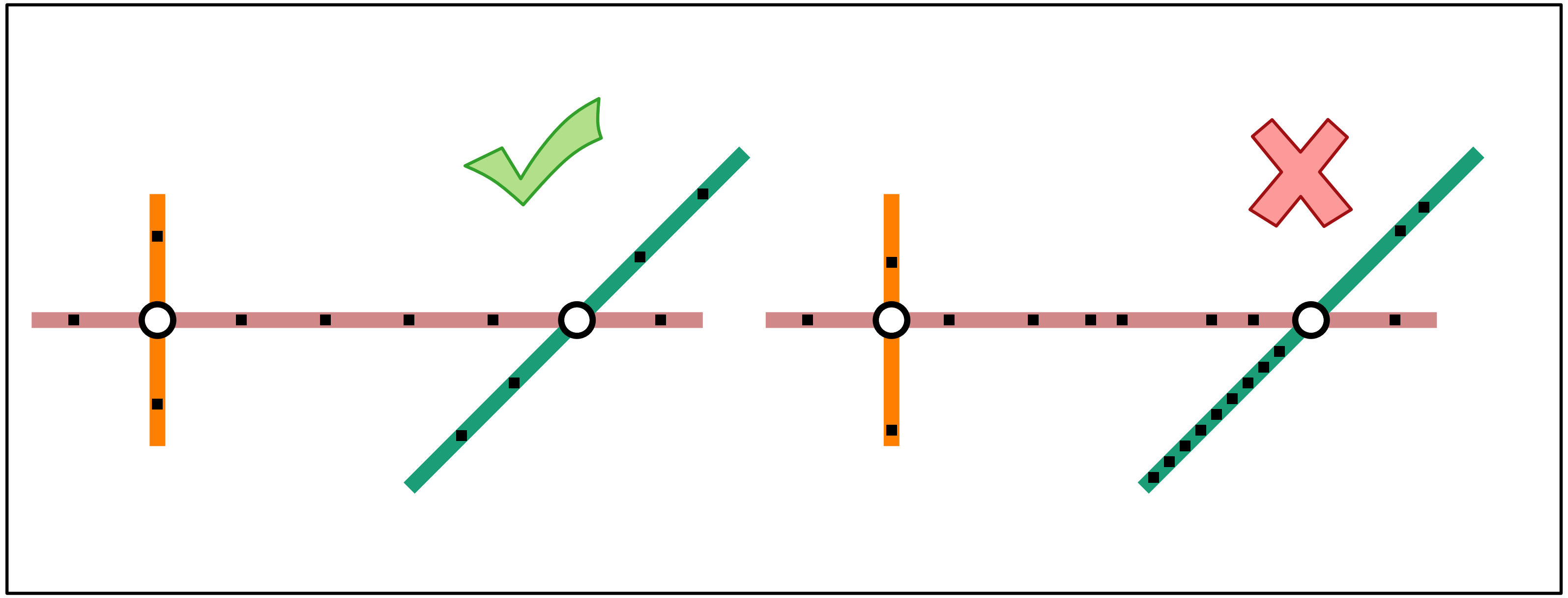
(R2)

(R3)

(R4)

(R5)

(R6)



(R7) Use uniform edge lengths.

- geographic distances less important
- network hop-distances more important
- balanced appearance

Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

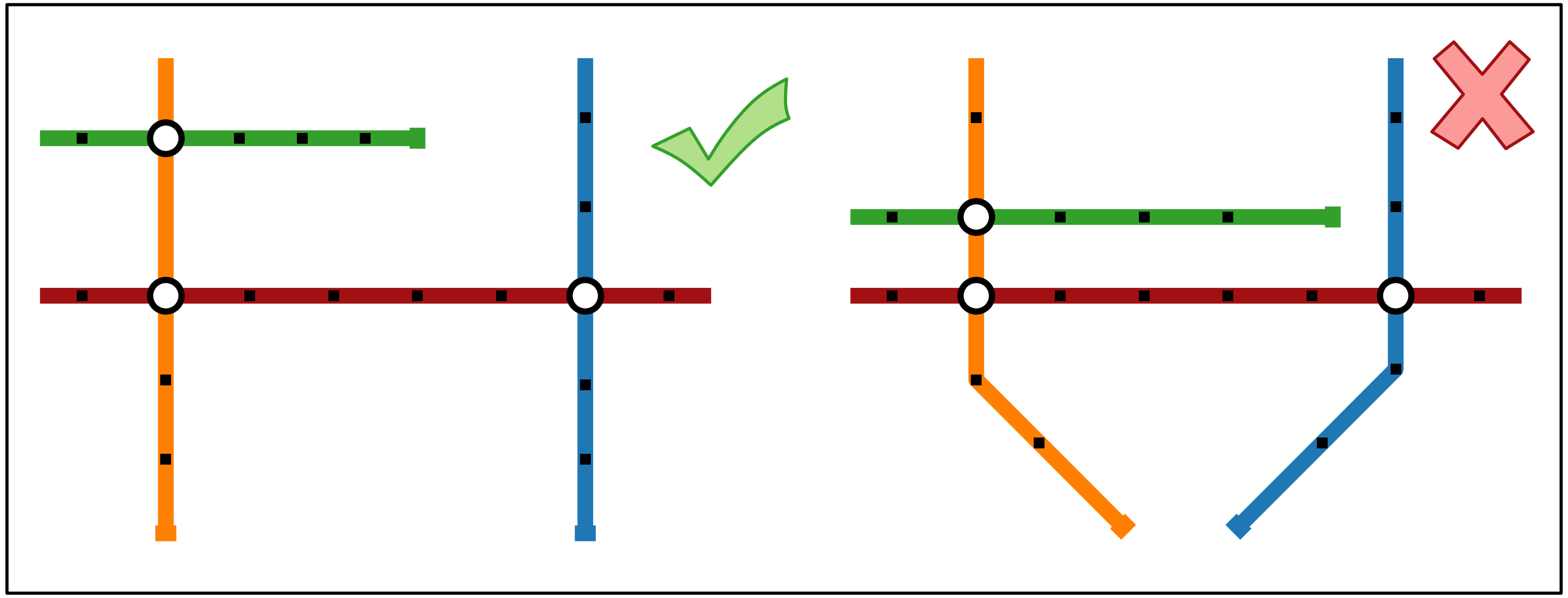
(R3)

(R4)

(R5)

(R6)

(R7)



(R8) Keep unrelated features apart.

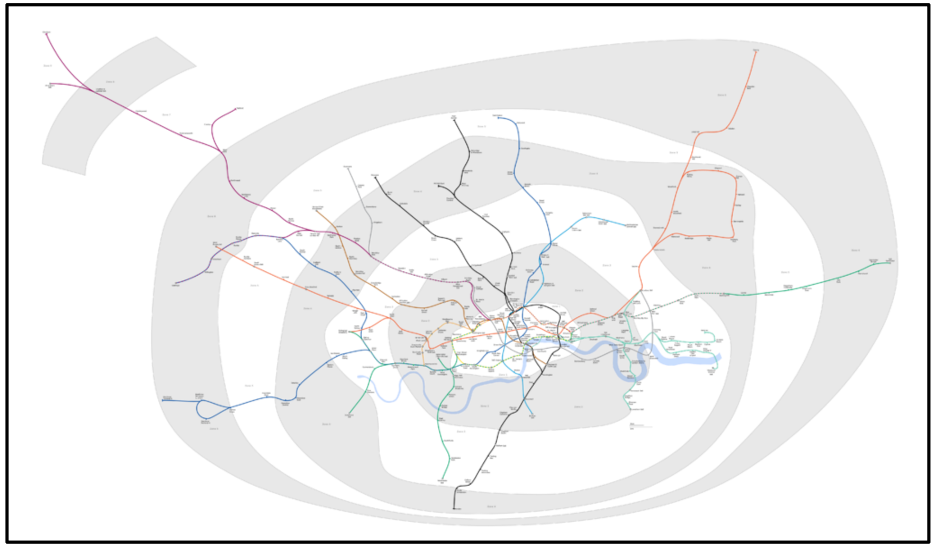
- guarantees minimum clearance between features
- avoids ambiguities

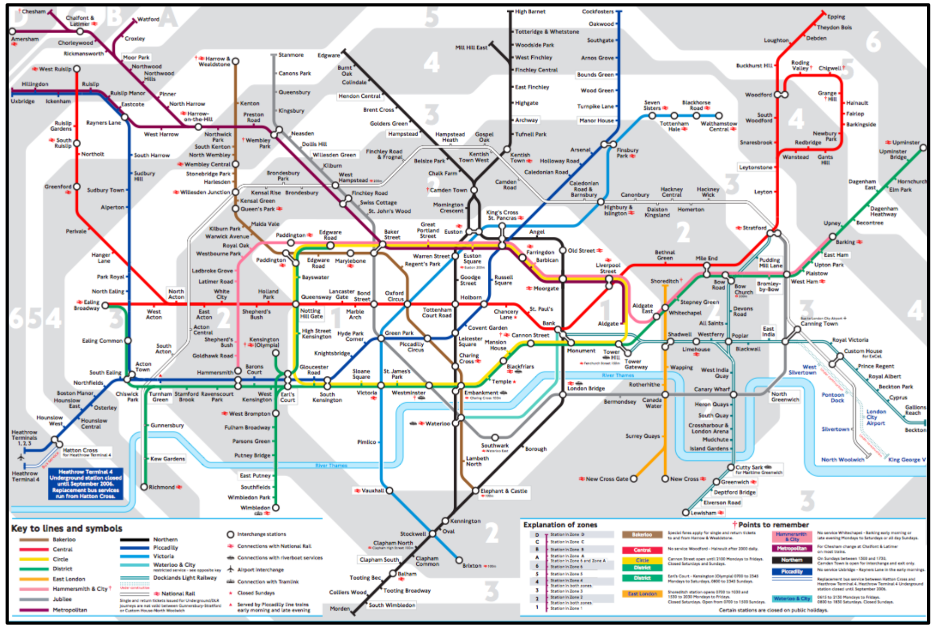
Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

(R3)





(R4)

(R5)

(R6)

(R7)

(R8)

(R9) Avoid large empty spaces.

- balances local feature density
- possibly fill gaps with legends

Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

(R5) Use large angular resolution in stations.

(R6) Minimize geometric distortion and displacement.

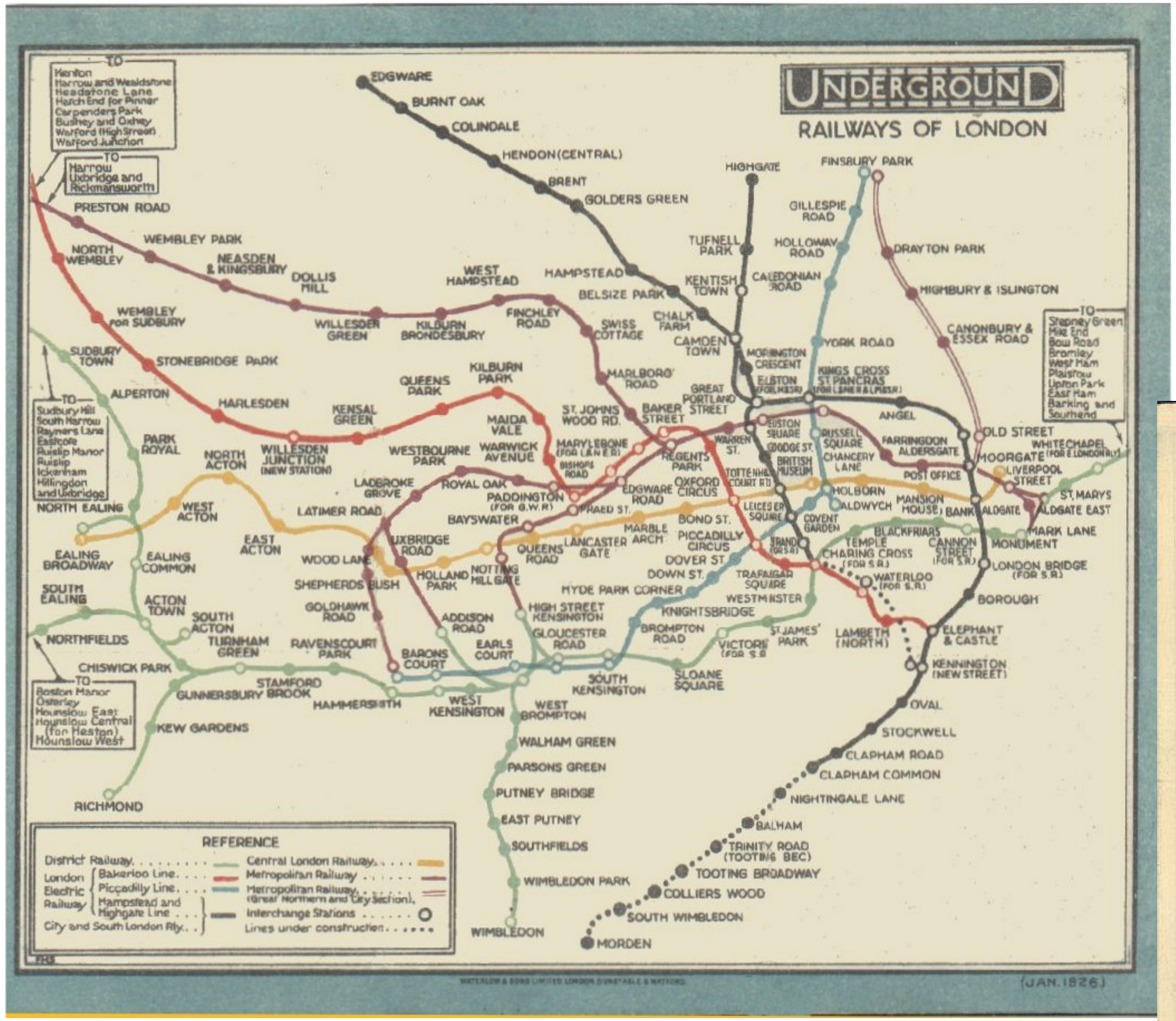
(R7) Use uniform edge lengths.

(R8) Keep unrelated features apart.

(R9) Avoid large empty spaces.

$\rightarrow$ **rules are potentially conflicting and need priorities**

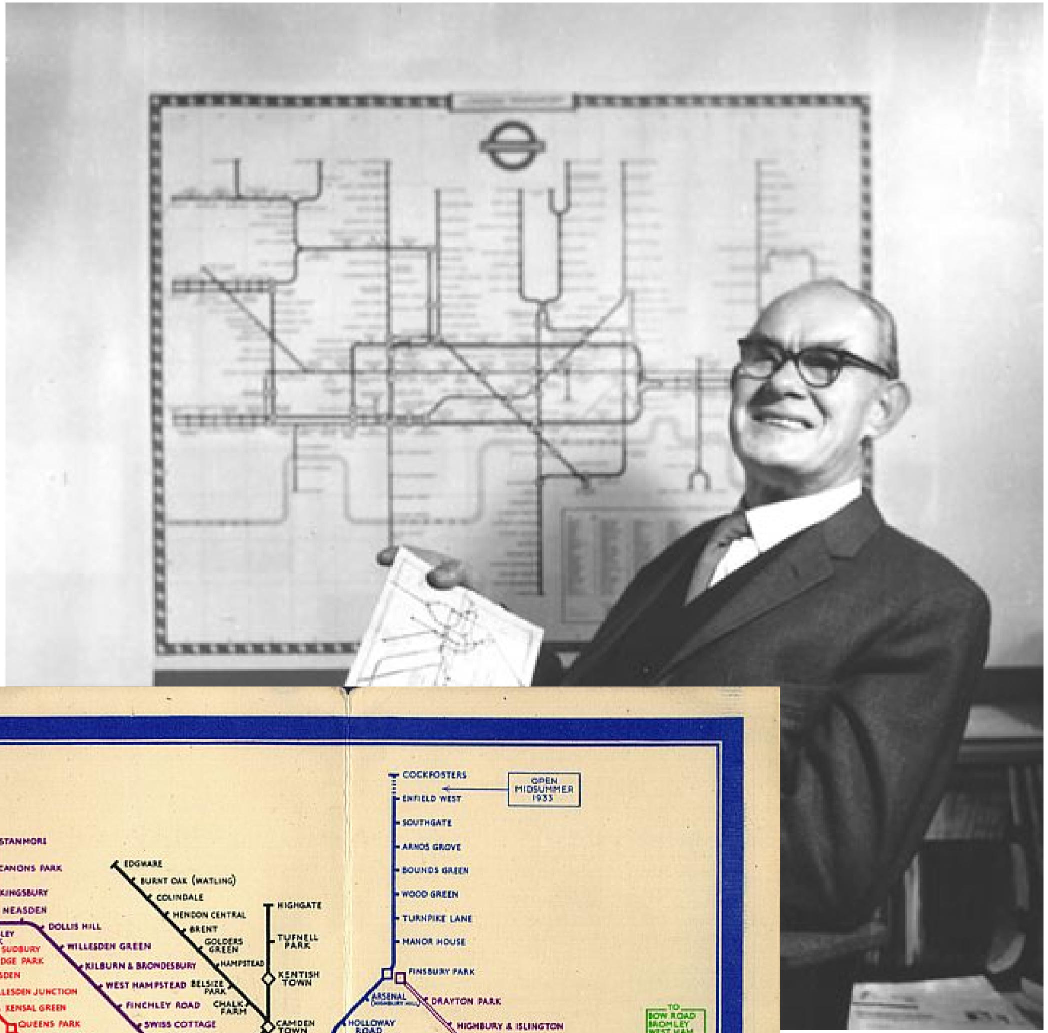
A Bit of History



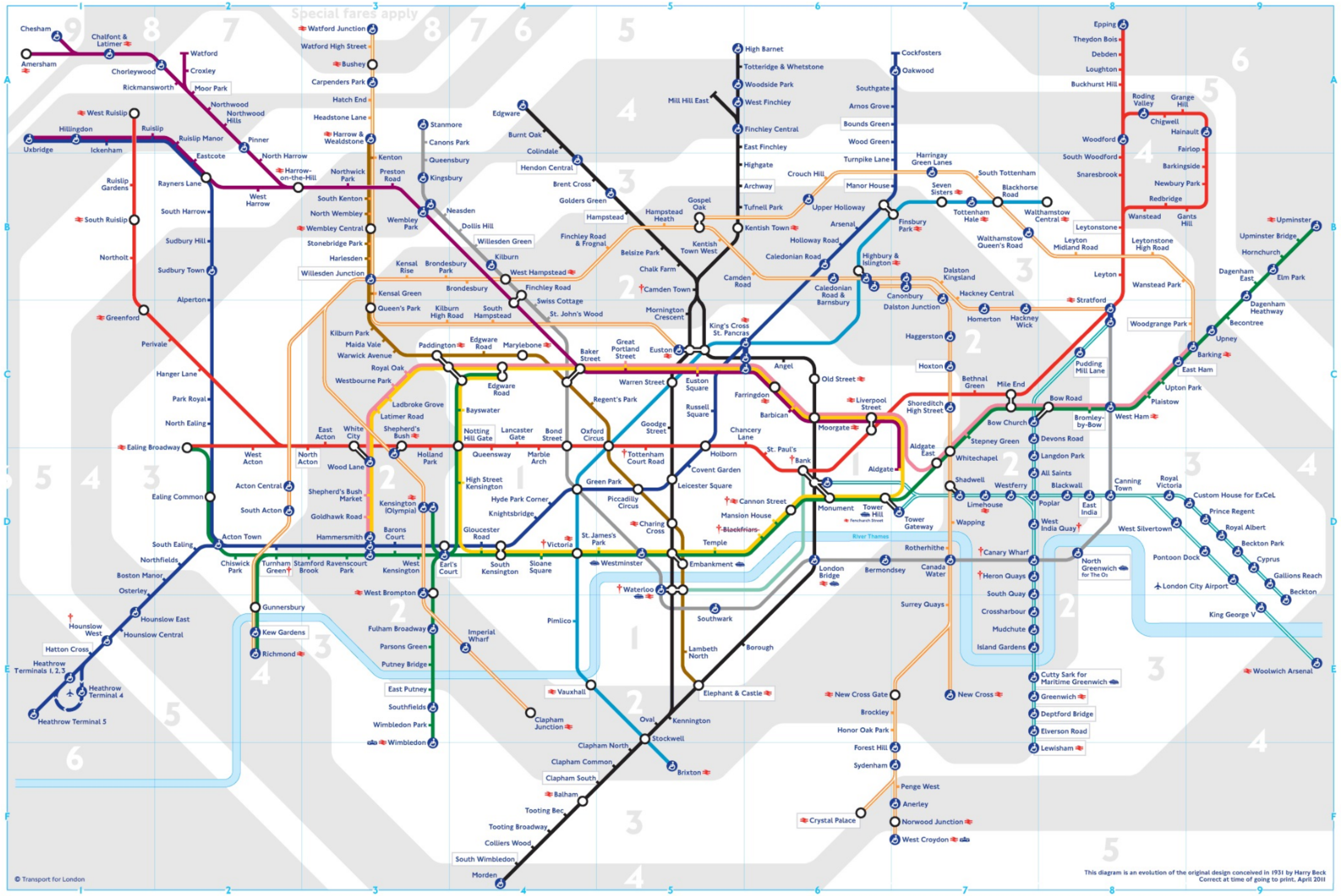
London 1927 (Fred H. Stingemore)

(c) Mike Yashworth

Henry Beck
London 1933



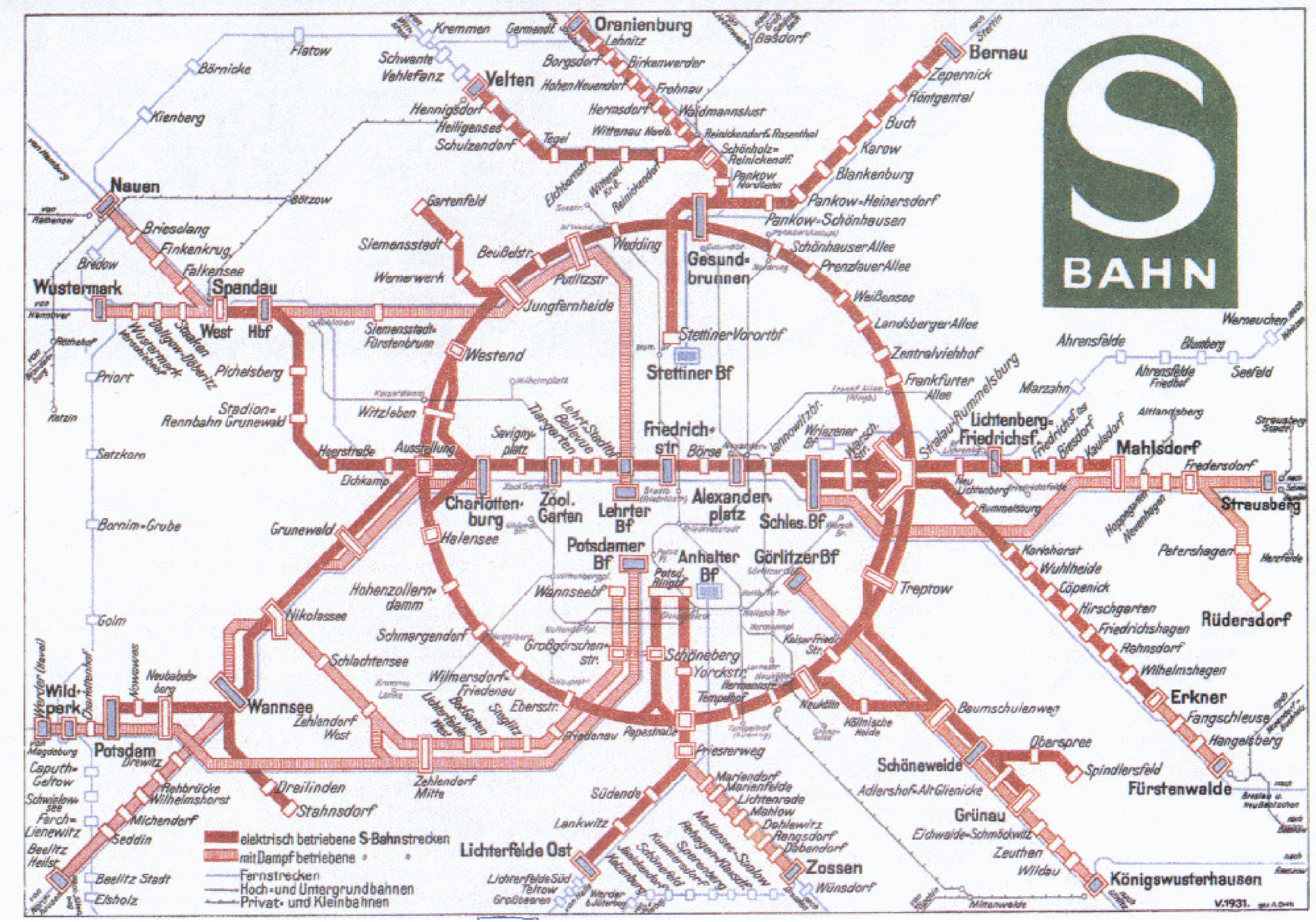
A Bit of History



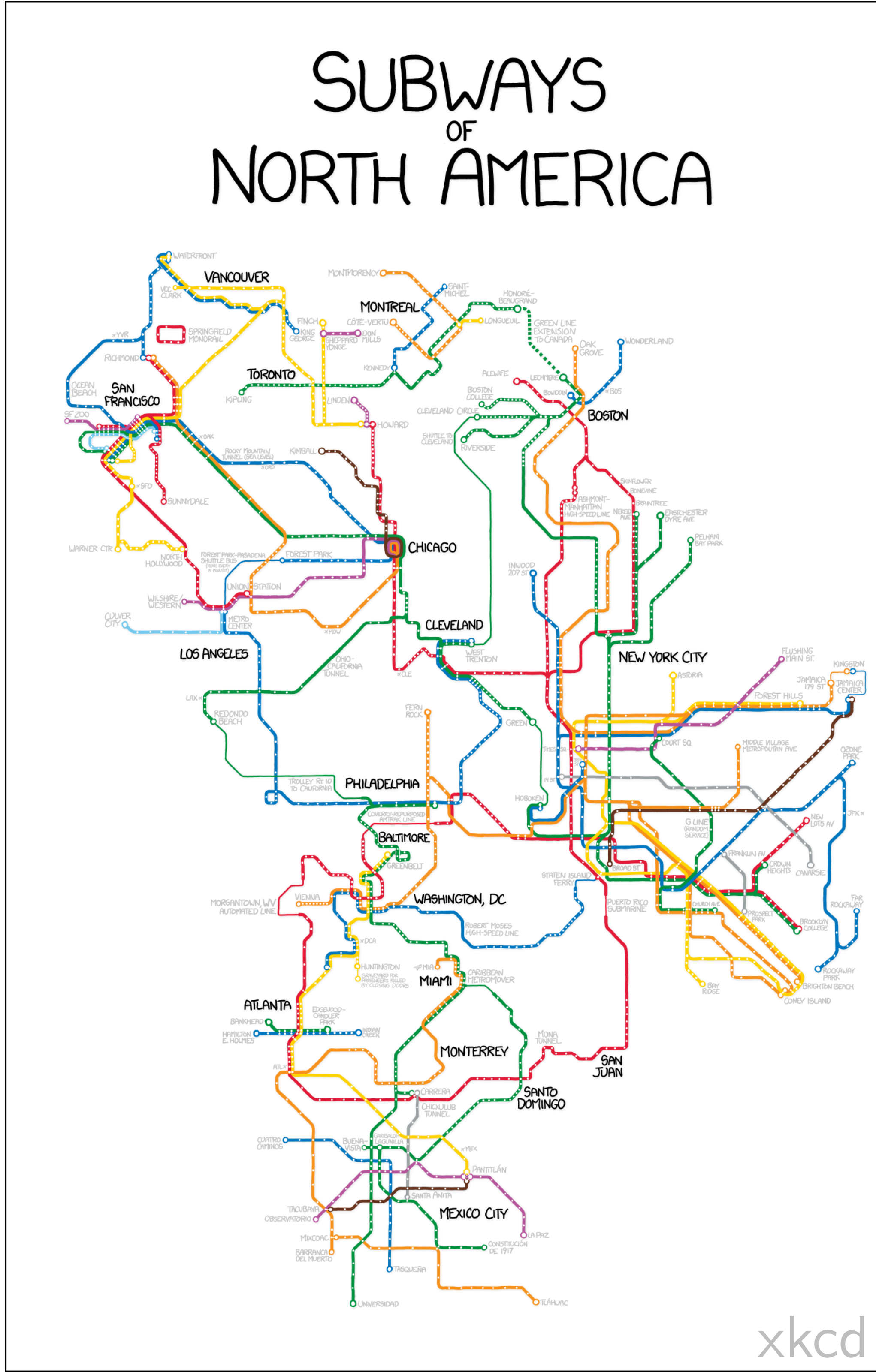
Tube Map of London
voted Design Icon 2006
(2nd after Concorde)

A Bit of History

Berlin 1931
(redrawn by Maxwell Roberts)



A Bit of History

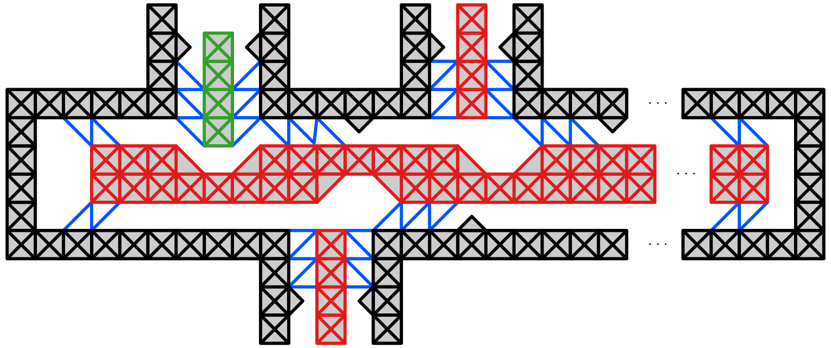


Complexity

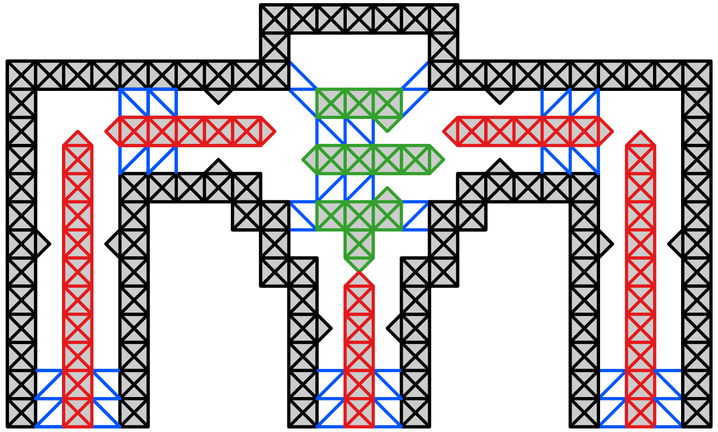
Theorem 1. [Nöllenburg 2005]
For an embedded graph with vertex degrees at most 8, **bend minimization** (R3) is NP-hard if **preserving topology** (R1) and **octilinearity** (R2) are required.

Proof sketch.

Reduction from Boolean satisfiability problem PLANAR-3SAT using rigid "mechanical" gadgets.



variable gadget (false)



clause gadget (false, false, false)

Remark.

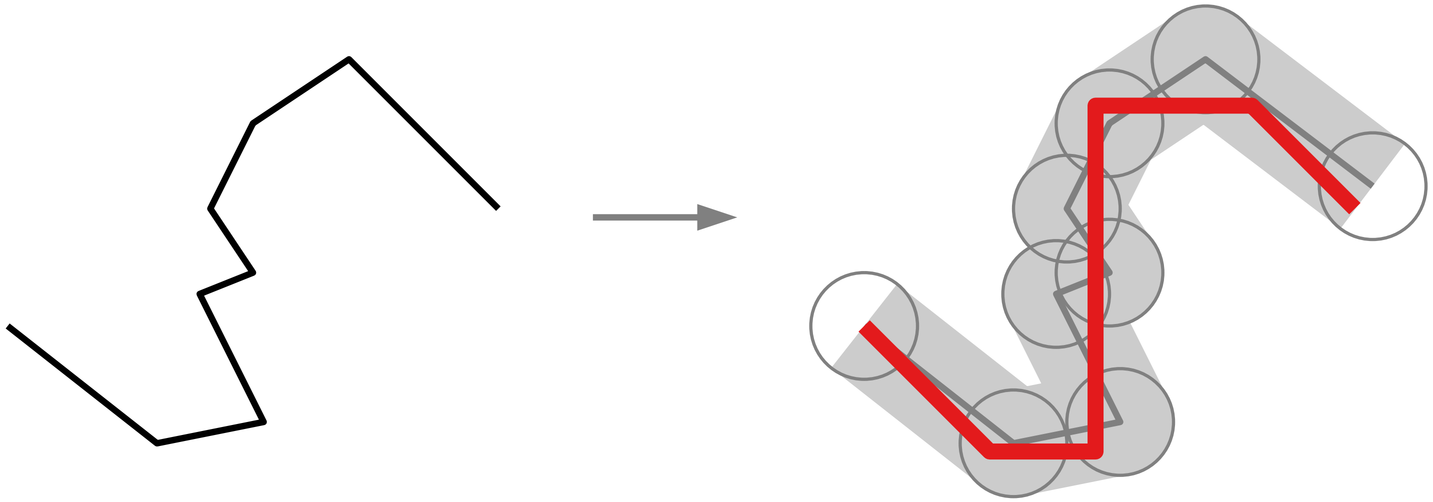
- no efficient exact algorithms to expect
- same problem without diagonals (rectilinear) is efficiently solvable [Tamassia '87] ($\rightarrow$ lecture 6)

Path-Based Schematization

Goal. Solve restricted problem, where G is a path (or polyline).

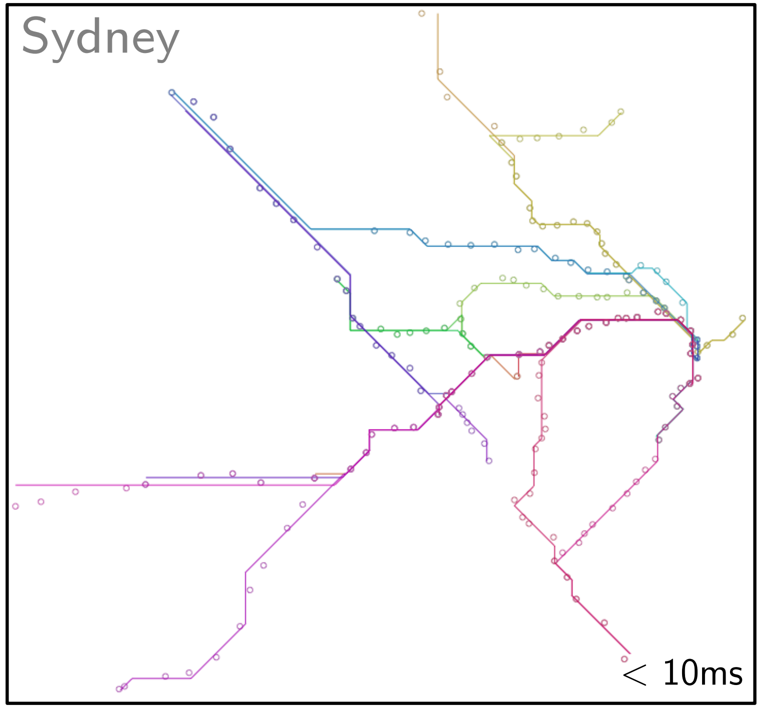
Constraints.
- $\mathcal{C}$-oriented edges (e.g. octilinear) **(R2)**
- bounded displacement **(R6)**

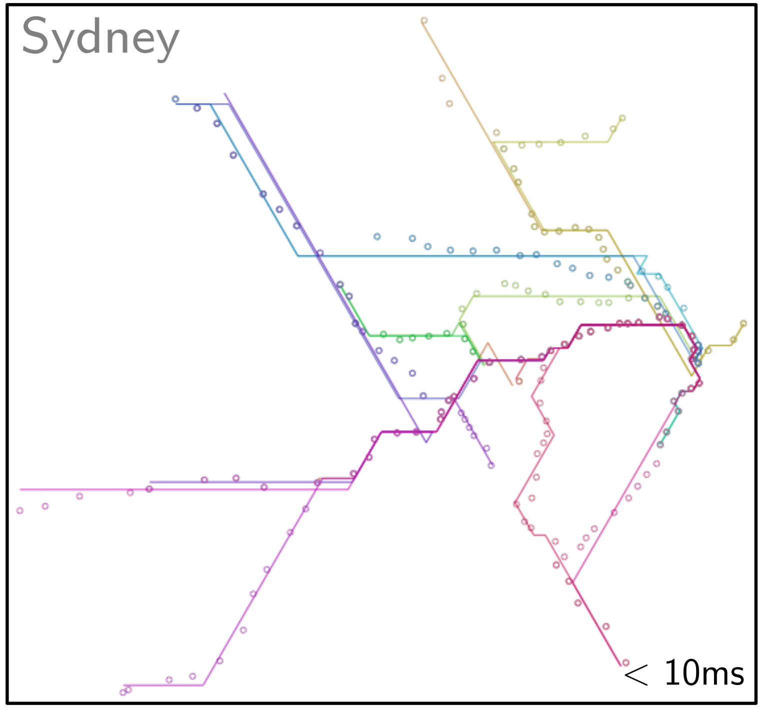
Criteria.
- minimize number of links **(R3)**



Path-based schematization – example

Theorem 2. [Dwyer, Hurst, Merrick 2008]
For a path P of length n and an orientation set $\mathcal{C}$, a $\mathcal{C}$-oriented schematized path can heuristically be fitted to the vertices in $O(|\mathcal{C}|n)$ time (or $O(|\mathcal{C}|n \log n)$) using least-squares regression.





$\mathcal{C}$-oriented Route Sketches

Theorem 3. [Delling, Gemsa, Nöllenburg, Pajor 2010]
Given a monotone path P and a set $\mathcal{C}$ of admissible edge slopes, we can compute, in $O(n^2)$+solve(LP) time, a $\mathcal{C}$-oriented schematization of P that

- preserves the orthogonal order of P,
- has minimum slope deviation,
- has minimum total length.

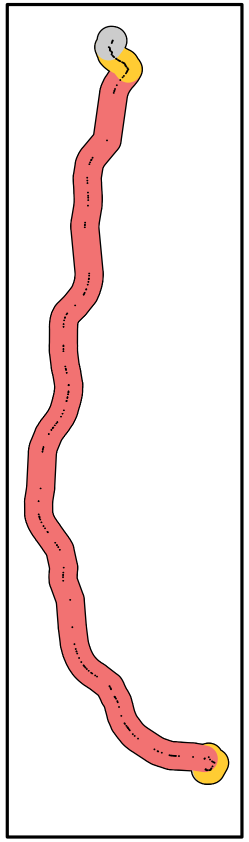
relative north-south-east-west relationships of all vertices

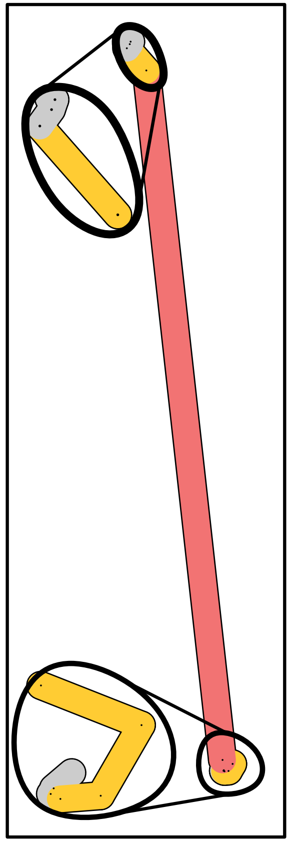
Proof.
- dynamic programming for slope assignment
- LP for length assignment

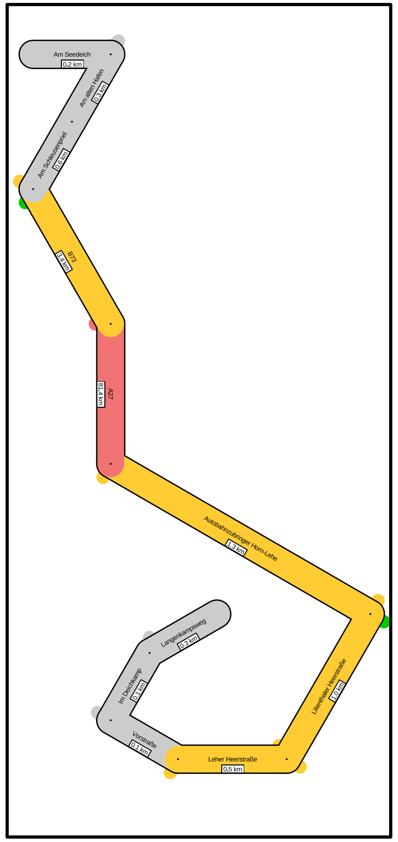
Theorem 4. [Brandes & Pampel 2009, Gemsa et al. 2011]
The d-regular (non-monotone) route sketch problem is NP-hard for any $d \geq 1$, where $\mathcal{C} = \{i \cdot 90°/d \mid i \in \mathbb{Z}\}$.

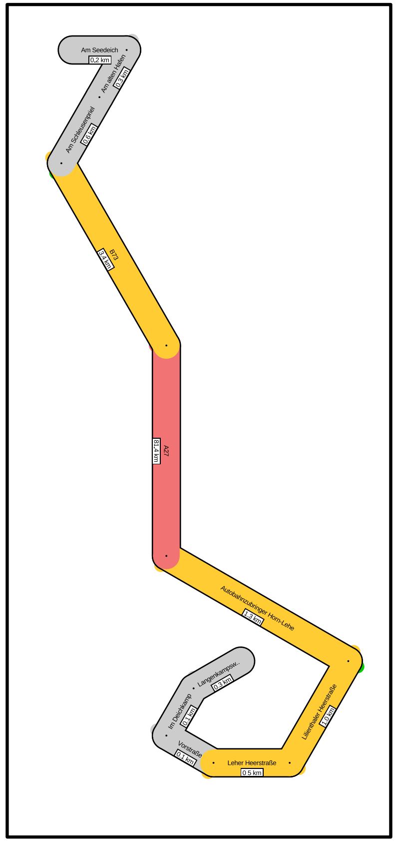
$\mathcal{C}$-oriented Route Sketches

Example. Bremen to Cuxhaven [Gemsa, Nöllenburg, Pajor, Rutter 2011]


input


simplified


optimized by MIP


including length order constraint

Path-Based Schematization – Discussion

Pros.

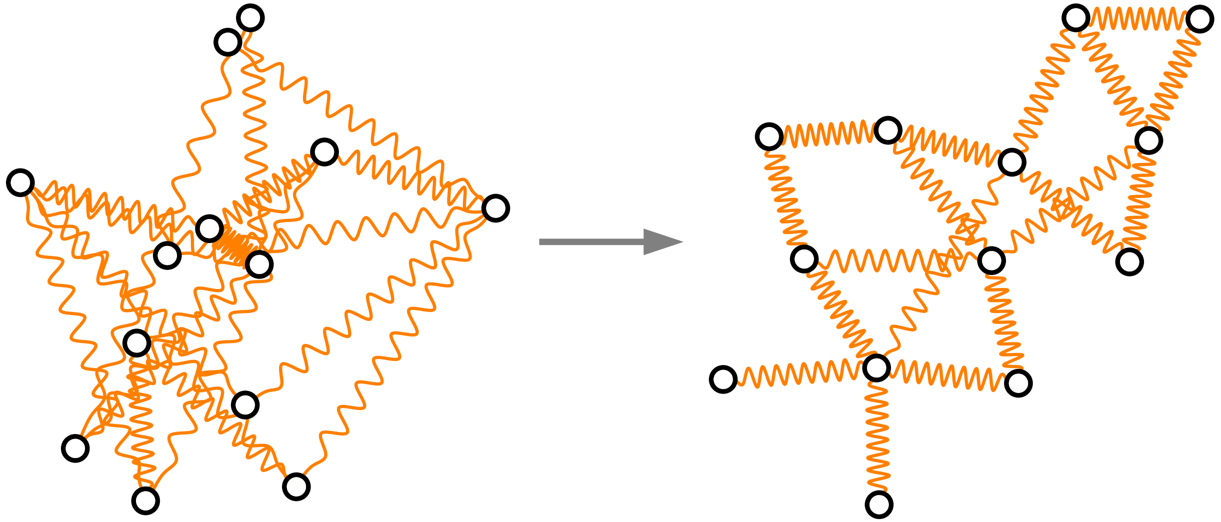
- polynomial running times
- $\mathcal{C}$-orientation and bounded displacement guaranteed
- bend minimization
- extends to metro networks:
  - decompose metro network into paths
  - schematize individual paths
  - glue schematized paths together at interchanges

Cons.

- no guarantee on network topology **(R1)**
- distortion/displacement too limited for metro maps

Force-Based Schematization

Idea. Apply well-known force-based graph drawing approach ($\rightarrow$ lecture 2)



Recall.

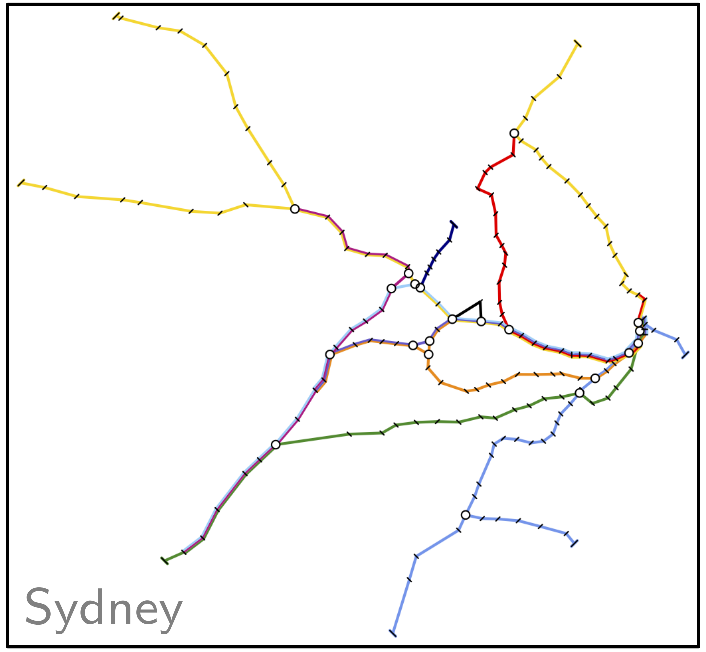
- vertices are charged particles repelling each other
- edges are springs pulling edges into target length
- iteratively calculate and apply forces until system stabilizes

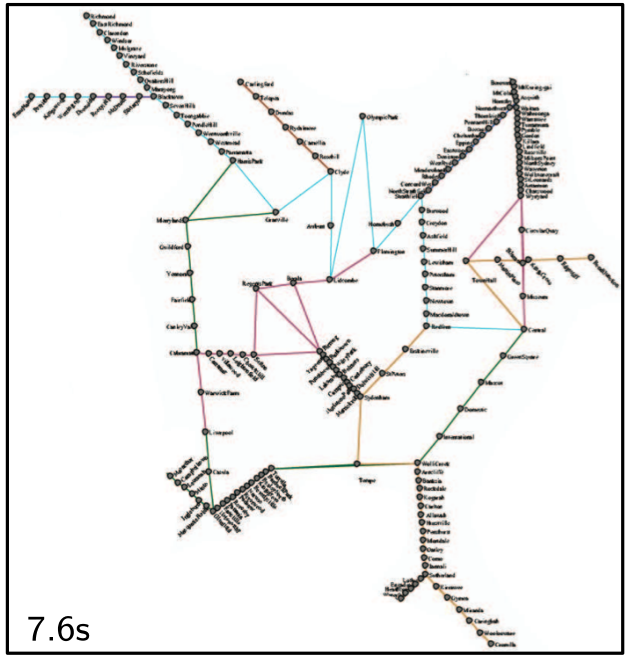
$\rightarrow$ define additional forces to model
subset of metro map design rules

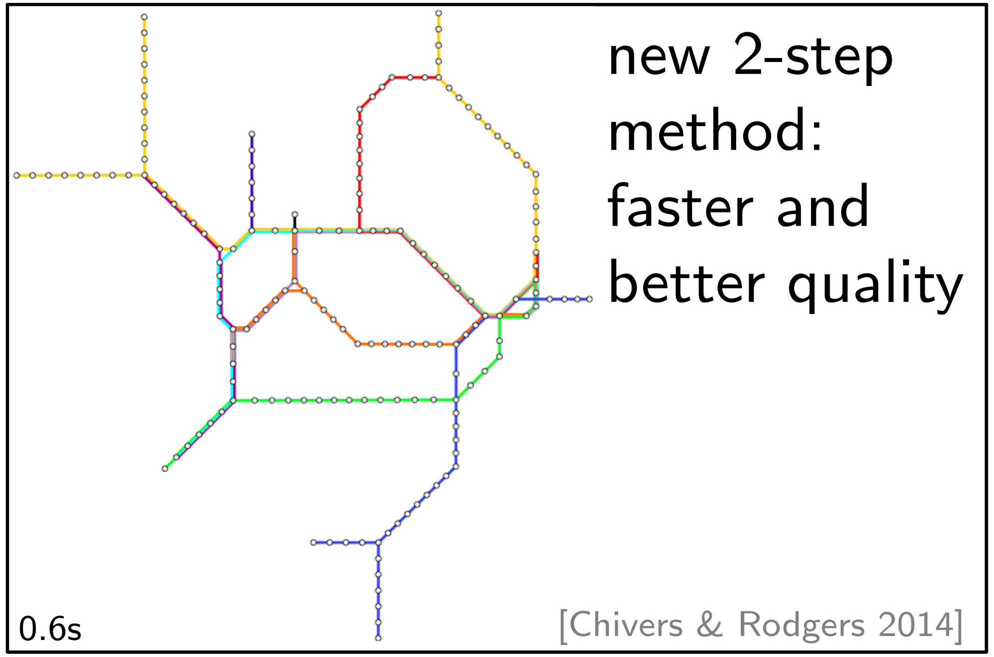
Force-Based Schematization – Octilinear

[Hong, Merrick, do Nascimento 2006]

- Contract deg-2 vertices into weighted edges: **(R3)** (Draw metro lines simple and monotone.)
- Define octilinear magnetic field attracting edges: **(R2)** (Restrict edge orientations.)
- Apply only topology-preserving vertex moves: **(R1)** (Do not change the network topology.)
- Spring lengths model uniform edge lengths: **(R7)** (Use uniform edge lengths.)
- Vertex repulsion models feature separation: **(R8)** (Keep unrelated features apart.)
- station labels are placed in an independent second step.



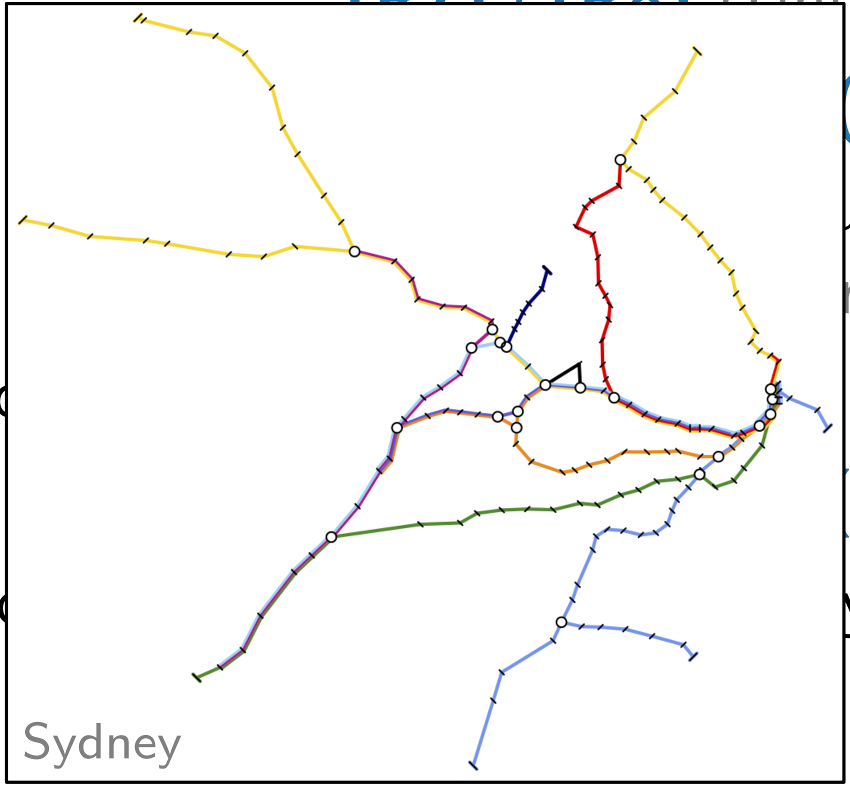




Force-Based Schematization – Bézier

[Fink, Haverkort, Nöllenburg, Roberts, Schuhmann, Wolff 2012]

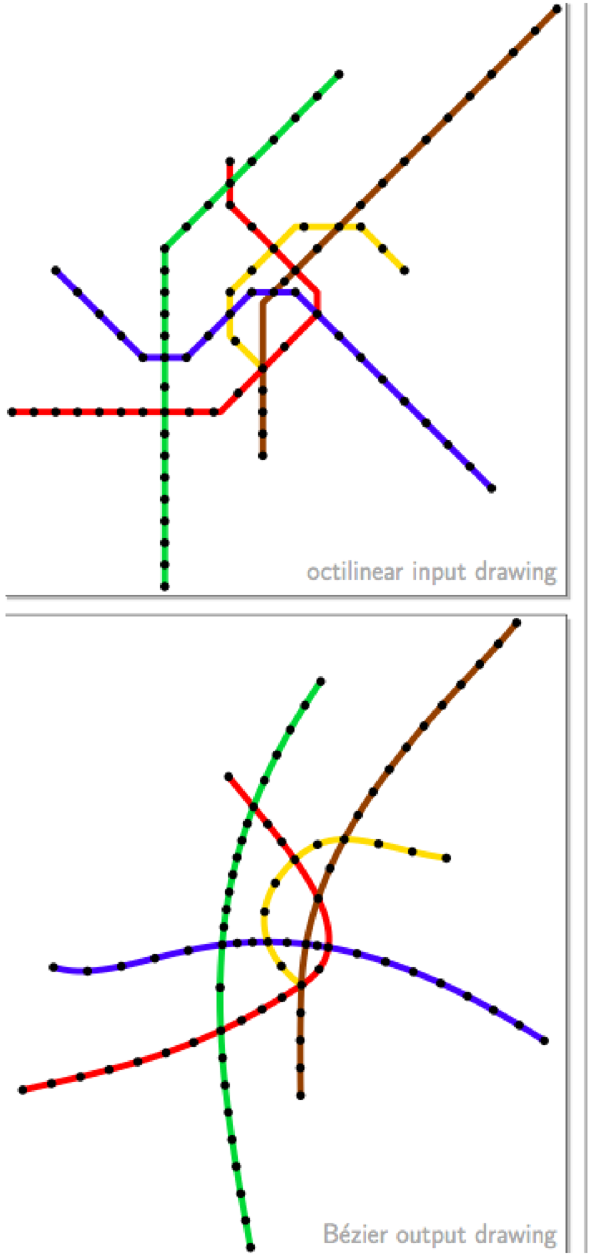
- convert (octilinear) input layout into Bézier curves
  $\rightarrow$ vertices and control points, both subject to forces
- metro line subcurves share tangents in common vertex
  (R4) (Let lines pass straight through interchanges.)
- (standard) attractive and repulsive forces
  (R7) + (R8) (Uniform edge lengths. + Keep unrelated features apart.)
- (weak) … **(R6)** (M… & displacement.)
- forces i… urvature
  … raw met… one as possible.)
- merge c…)
- forces i… **(R5)** (U… stations.)
- apply t… y **(R1)** … topology.)

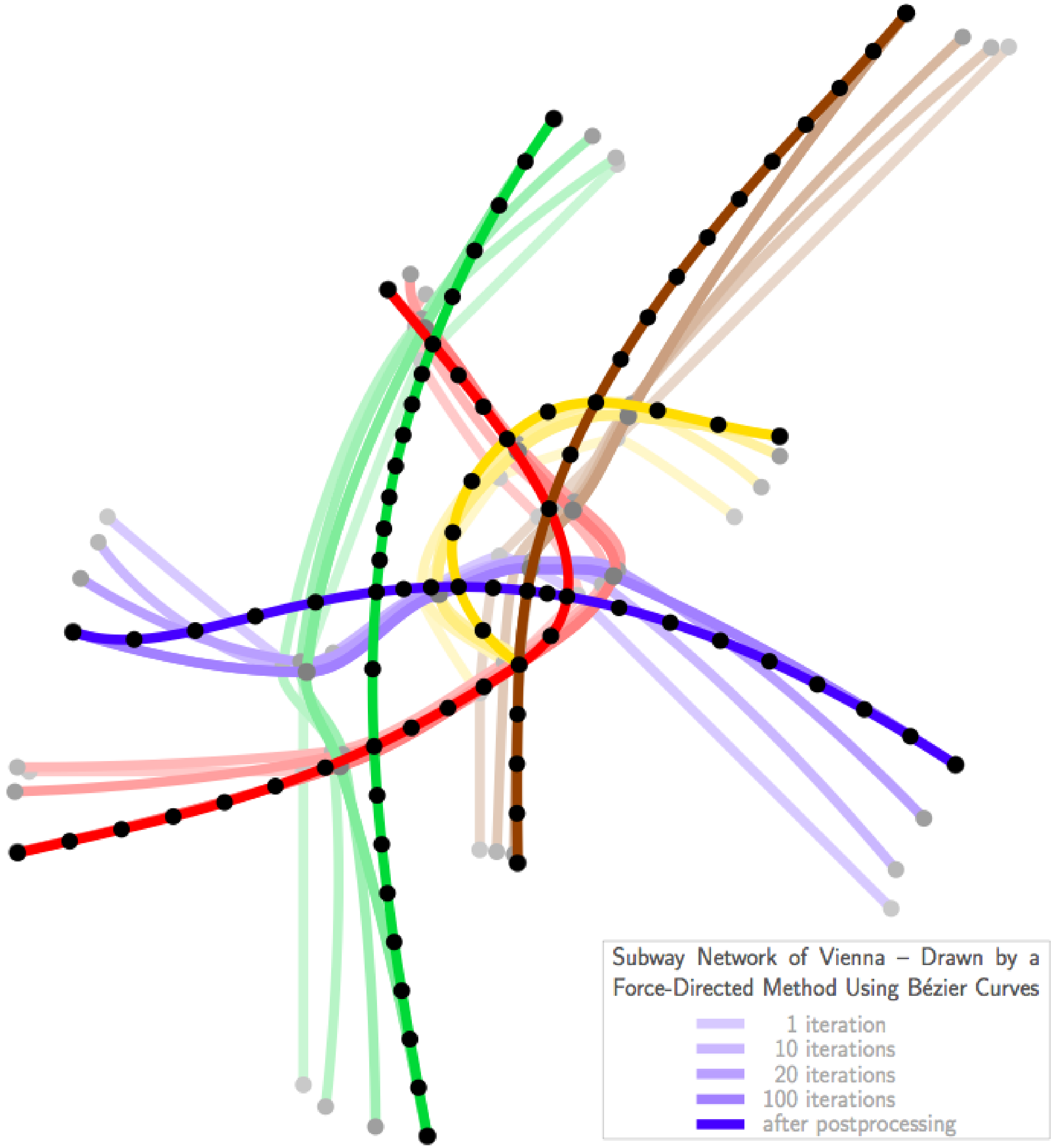


Force-Based Schematization – Bézier

[Fink, Haverkort, Nöllenburg, Roberts, Schuhmann, Wolff 2012]

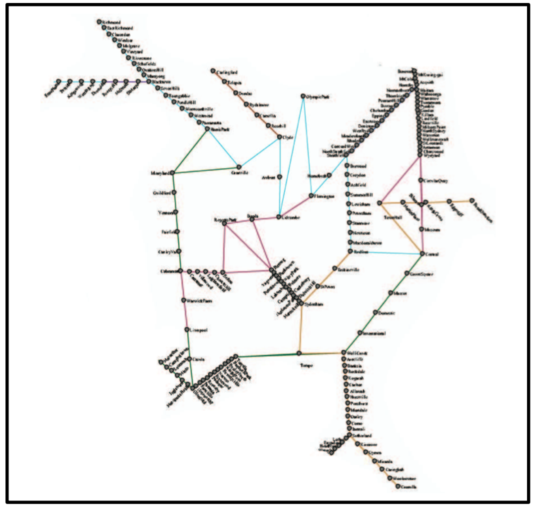
Vienna:





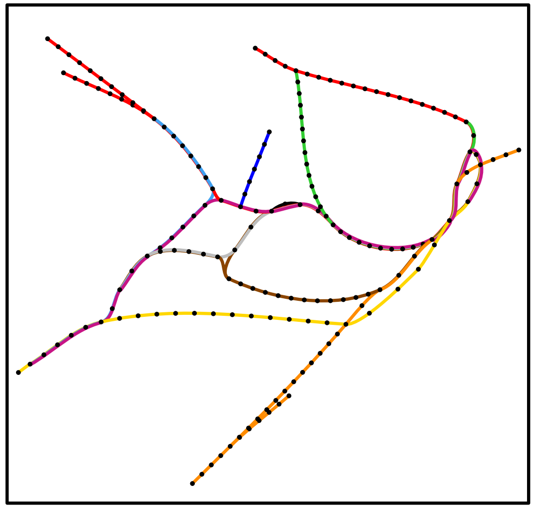
Force-Based Schematization – Discussion

Octilinear.



- guarantees topology **(R1)**
- slower than path-based algorithms, but still fast enough
- no strict enforcement of octilinearity **(R2)**
- quite unbalanced edge lengths **(R7)**
- bends in interchanges **(R4)**
- no distortion restriction **(R6)**

Bézier.

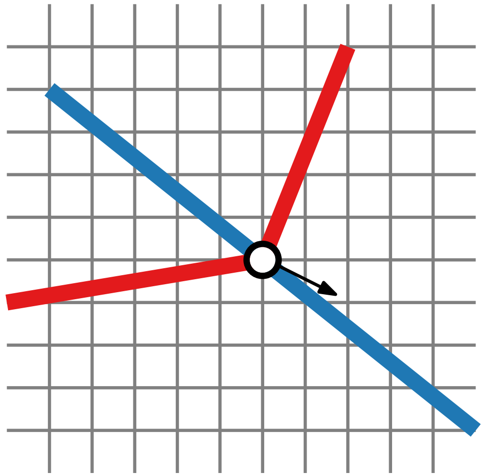


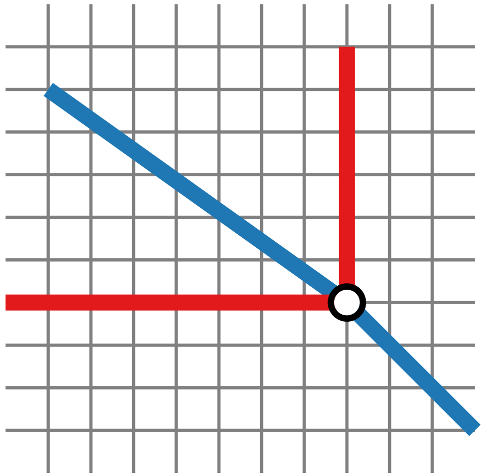
- guarantees topology **(R1)**
- first curvilinear metro map algorithm
- takes almost all design rules into account
- sometimes small angles and distances **(R5)+(R8)**
- works well on small and medium instances, but difficulties with more complex networks

Local Schematization

Idea.

- define a (multi-criteria) layout quality function
- improve layout quality step-by-step by locally moving vertices to better nearby (grid) positions
- applicable optimization techniques: hill climbing, simulated annealing, ant colony optimization, . . .





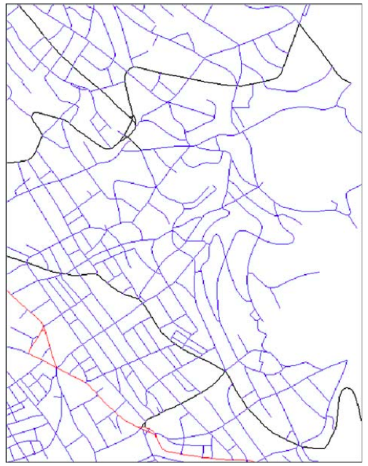
Local Schematization

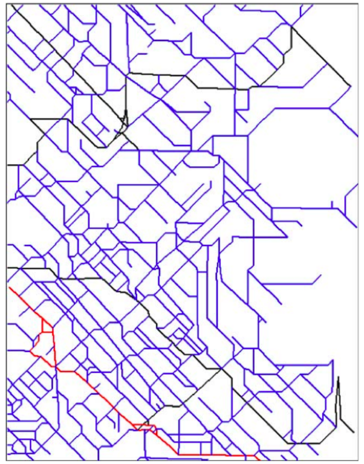
Idea.

- define a (multi-criteria) layout quality function
- improve layout quality step-by-step by locally moving vertices to better nearby (grid) positions
- applicable optimization techniques: hill climbing, simulated annealing, ant colony optimization, . . .

[Avelar & Müller 2000]

- calculate best vertex position in each criterion (octilinearity **(R2)**, min. separation **(R8)**)
- move vertex to average of positions without violating topology **(R1)**


Zürich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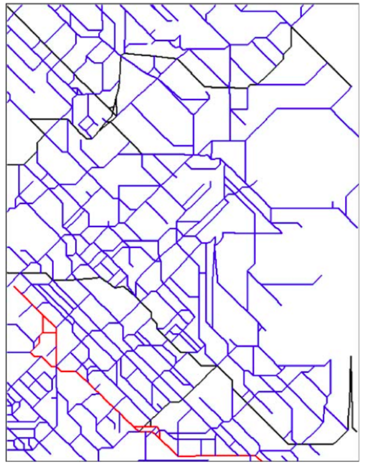
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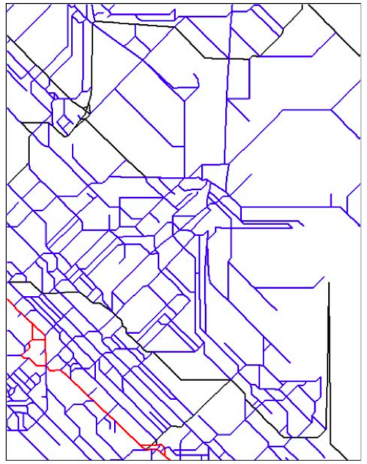
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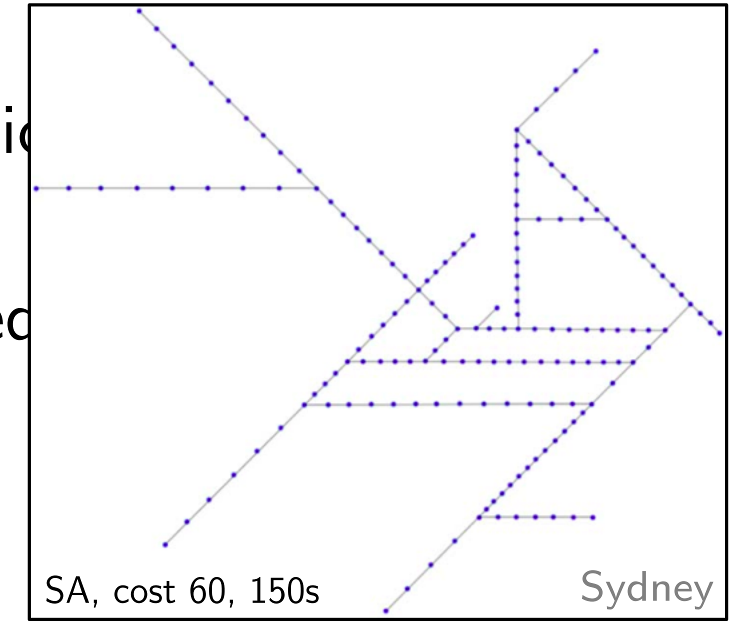
Local Schematization

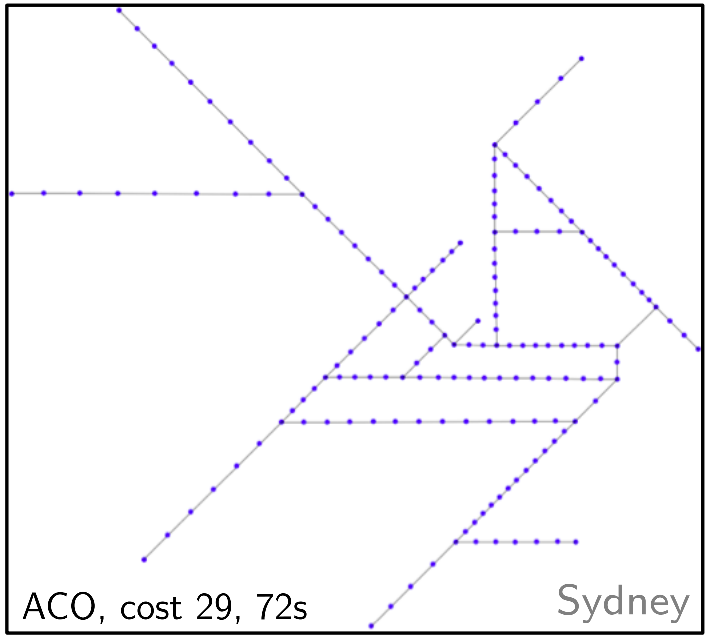
Idea.

- define a (multi-criteria) layout quality function
- improve layout quality step-by-step by locally moving vertic to better nearby (grid) positions
- applicable optimization techniques: hill climbing, simulated annealing, ant colony optimization, . . .

[Ware, Taylor, Anand, Thomas 2006, Ware & Richards 2013]

- weighted multicriteria function
- contract degree-2 vertices prior to optimization
- implemented more design rules (topology **(R1)**, octilinearity **(R2)**, displacement **(R6)**, edge lengths **(R7)**, separation **(R8)**)
- simulated annealing (2006) and ant colony optimization (2013)



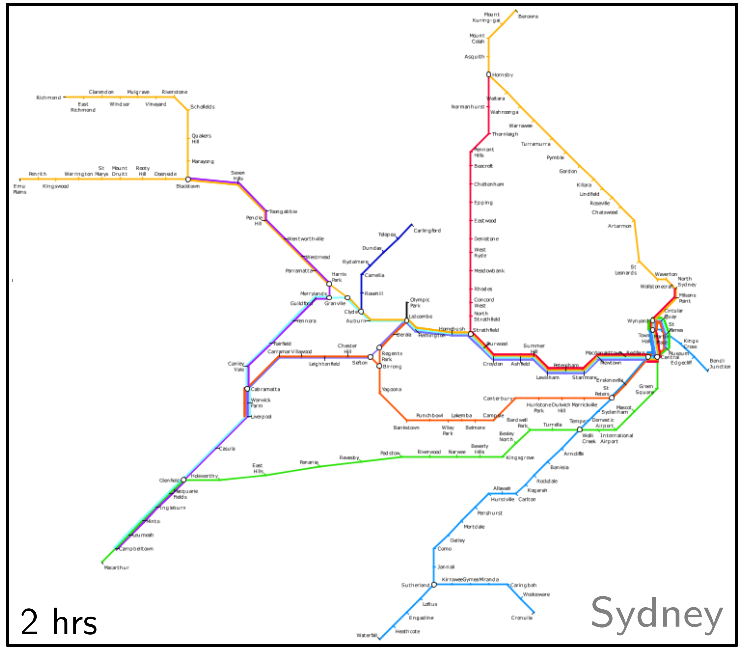


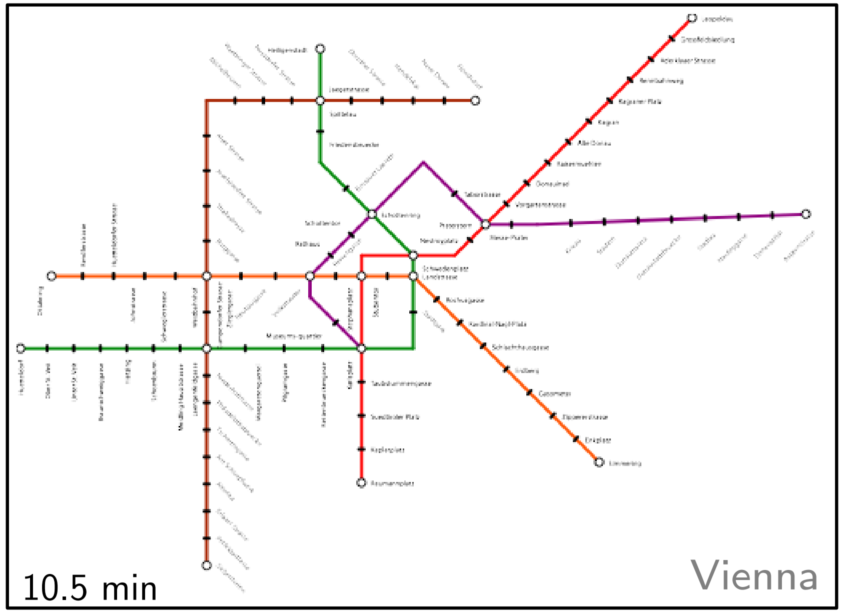
Local Schematization

[Stott, Rodgers, Burkhard, Meier, Smis 2011]

Idea.

- design rules as before
- additionally include metro map specific criteria (bend minimization **(R3)**, interchange straightness **(R4)**, angular resolution **(R5)**, relative positions **(R6)**)
- integrate alternating label placement rounds
- some ad-hoc fixes for local minima situations





Local Schematization – Discussion

Pros.

- flexible framework, easy to integrate new criteria
- recent methods improved visual layout quality
- integration of layout and labeling

Cons.

- optimization of criteria, but no guarantees (except topology)
- susceptible to local minima
- long running times

Mixed-Integer Programming

[Nöllenburg & Wolff 2011]

- find exact optimum solution using combinatorial optimization
- split design rules into **hard** and **soft** constraints
- model constraints as linear (in)equalities and linear objective function
  $\rightarrow$ mixed-integer programming **MIP**
- integrate overlap-free station labeling in same model
- can use sophisticated optimization tools as black box solvers
  (e.g., CPLEX, Gurobi)

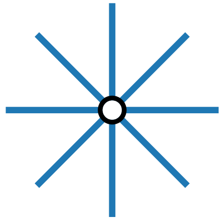
Hard Constraints

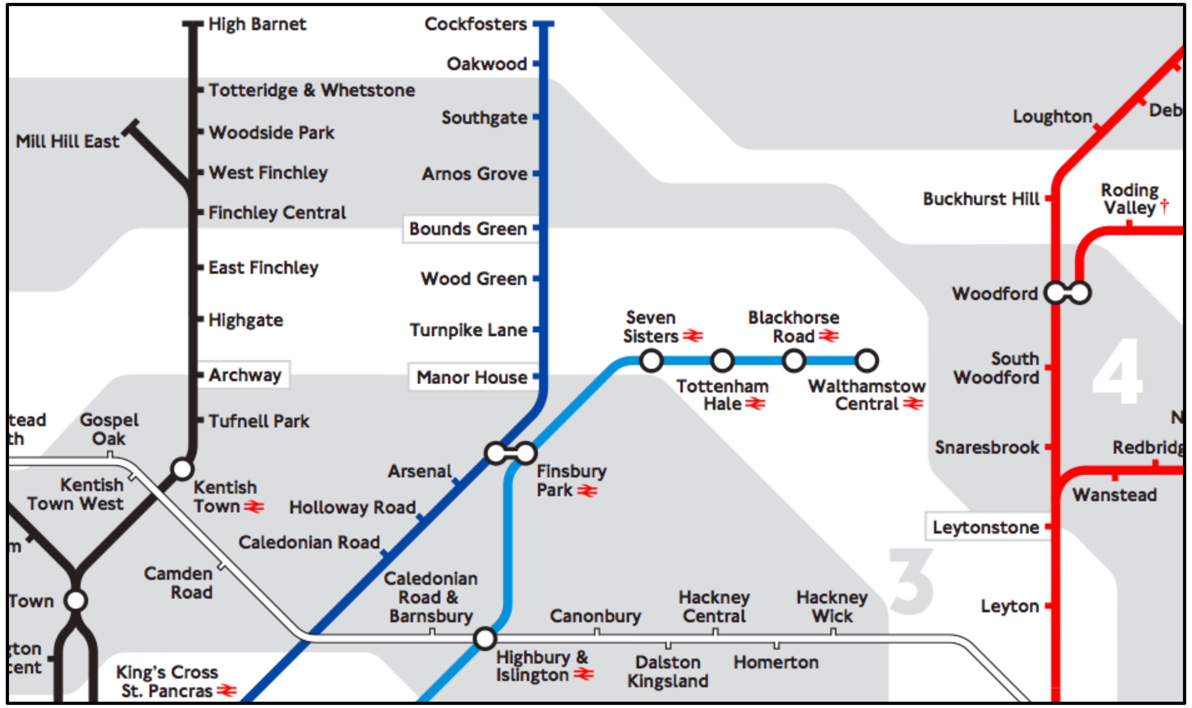
(R1) preserve embedding/**topology** and **planarity**

(R2) draw all edges **octilinearly**

(R7) enforce **minimum edge length** ℓ_{min}

(R8) enforce **minimum feature separation** d_{min}



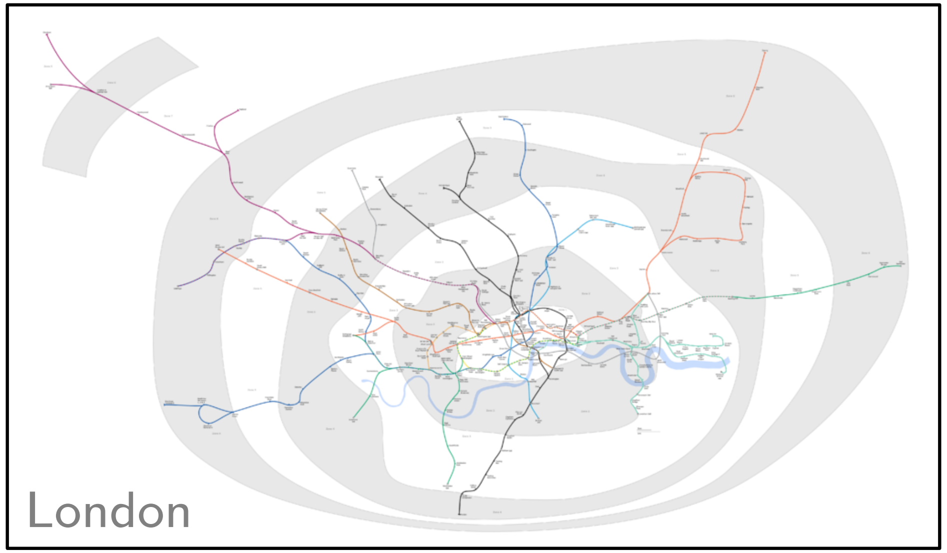


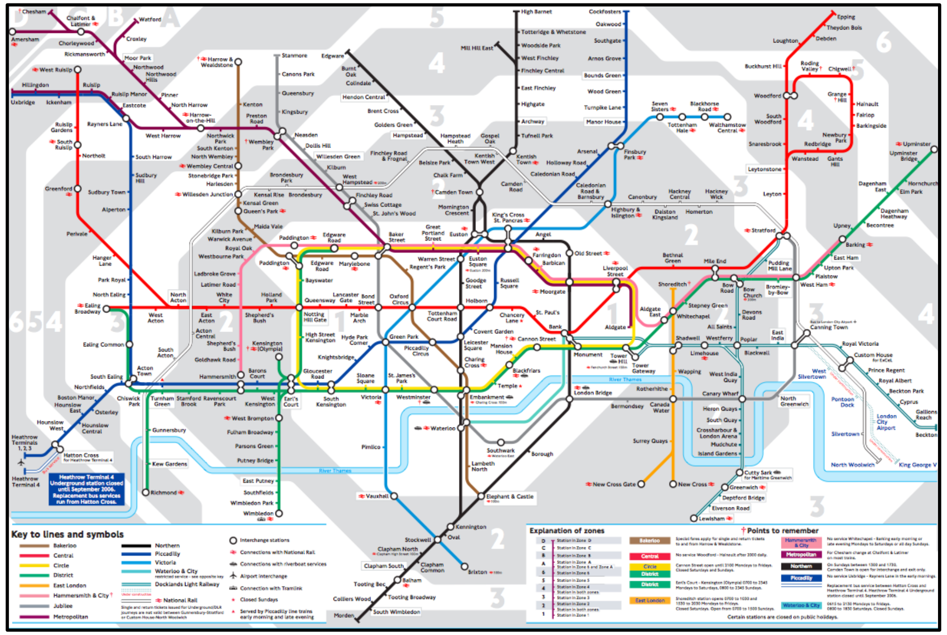
Soft Constraints

(R3+R4) draw lines in $\mathcal{L}$ with **few bends**

(R6) minimize geometric **distortion**

(R7) minimize total **edge length**

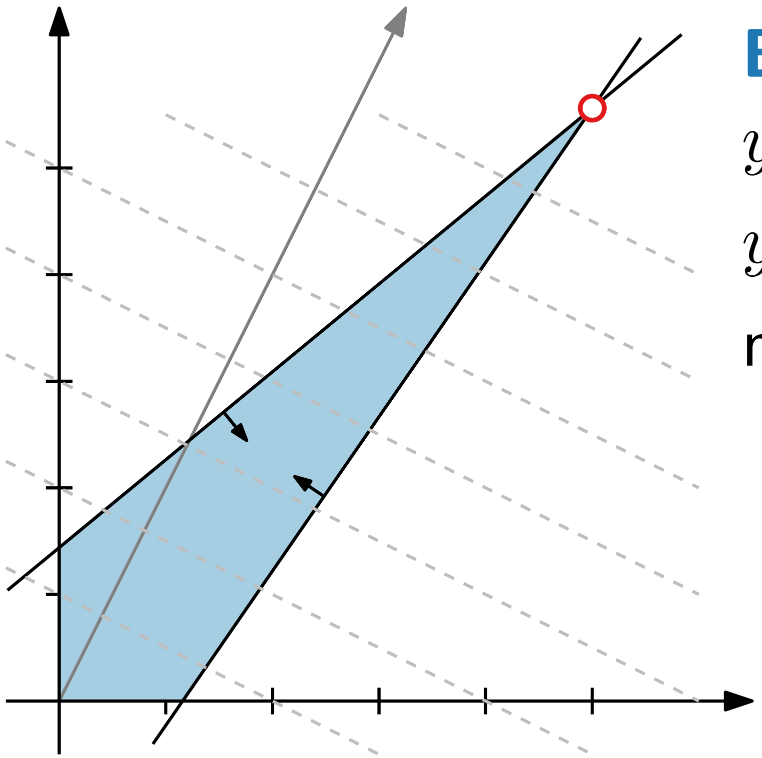




Linear Programming

Linear Programming (LP) is an efficient optimization method for

- linear constraints
- linear objective function
- real-valued variables



Example.

$y \leq 0.9x + 1.5$

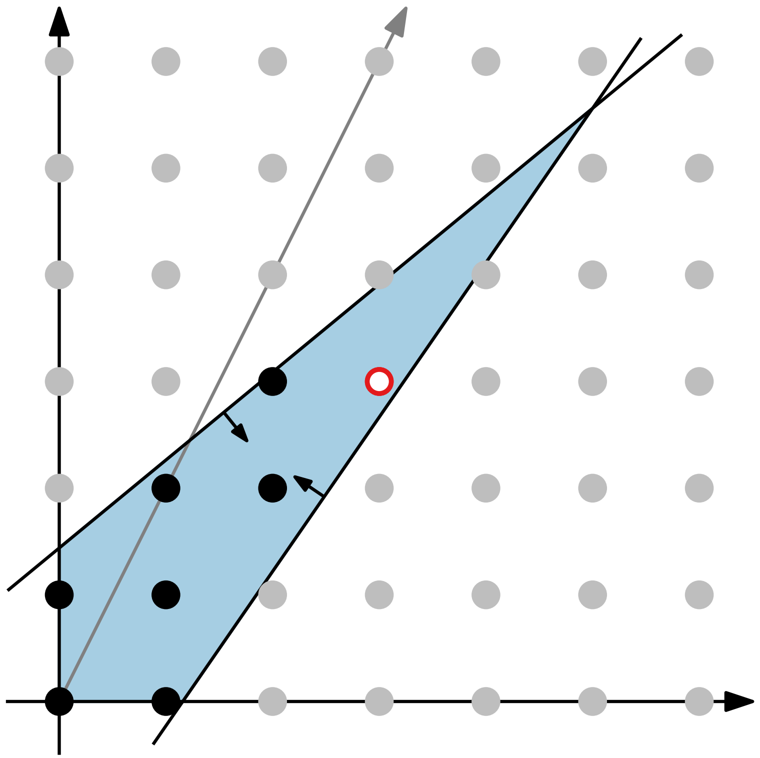
$y \geq 1.4x - 1.3$

maximize $x + 2y$

Linear Programming

Linear Programming (LP) is an efficient optimization method for

- linear constraints
- linear objective function
- real-valued variables

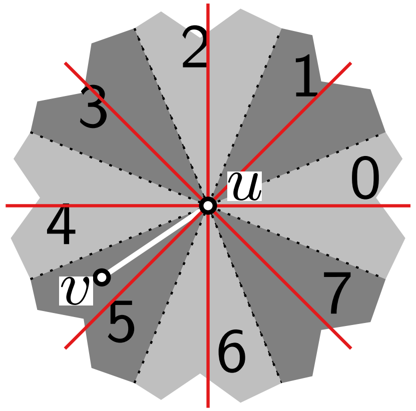


Mixed Integer Programming (MIP)

- in addition binary and integer variables
- NP-hard optimization problem
- still method of choice for many practical optimization tasks

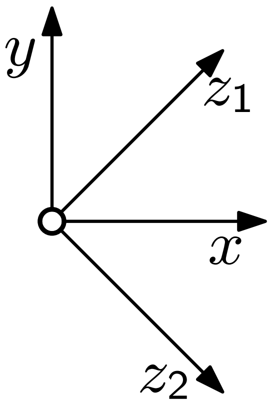
> **Theorem 5.**
> Metro map layout can be modeled as MIP such that
> - hard constraints $\rightarrow$ linear constraints
> - soft constraints $\rightarrow$ linear objective function

Sectors and Coordinates



Sectors.

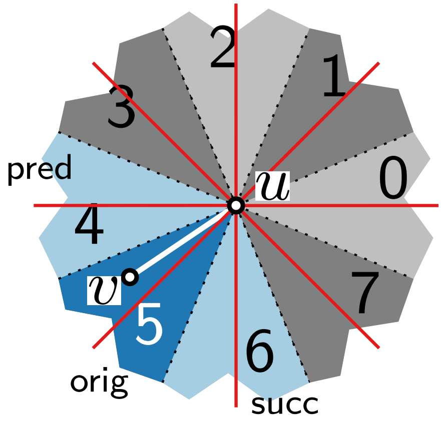
- for each vertex u partition the plane into eight sectors numbered 0–7
  here: $\text{sec}(u, v) = 5$ in the input
- number octilinear edge directions accordingly
  here, e.g., $\text{dir}(u, v) = 5$ in the solution (output)



Coordinates.

- assign (redundant) z_1- and z_2-coordinates to each vertex v
  - $z_1(v) = \frac{1}{2} \cdot (x(v) + y(v))$
  - $z_2(v) = \frac{1}{2} \cdot (x(v) - y(v))$

Octilinearity and Relative Positions



Goal.
Draw the edge uv

- octilinearly **(R2)**
- with minimum length $\ell = \ell_{uv}$ **(R7)**
- restricted to the three best directions **(R6)**

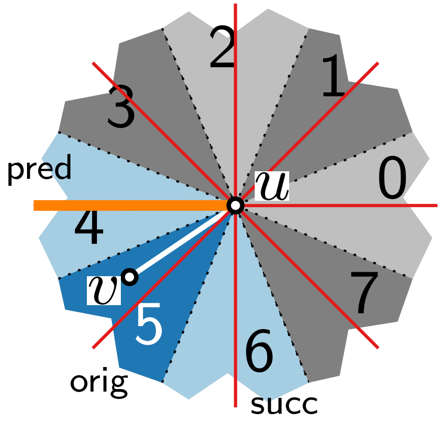
How to model this using linear constraints in a MIP?

Introduce **binary variables**

$$\alpha_{\mathsf{pred}}(u, v) + \alpha_{\mathsf{orig}}(u, v) + \alpha_{\mathsf{succ}}(u, v) = 1$$

to select exactly one of the three sectors.

Octilinearity and Relative Positions



very large constant (how large, actually?)

Predecessor sector.

$$\begin{aligned} y(u) - y(v) &\leq M(1 - \alpha_{\mathsf{pred}}(u,v)) \\ -y(u) + y(v) &\leq M(1 - \alpha_{\mathsf{pred}}(u,v)) \\ x(u) - x(v) &\geq -M(1 - \alpha_{\mathsf{pred}}(u,v)) + \ell \end{aligned}$$

How does this work?

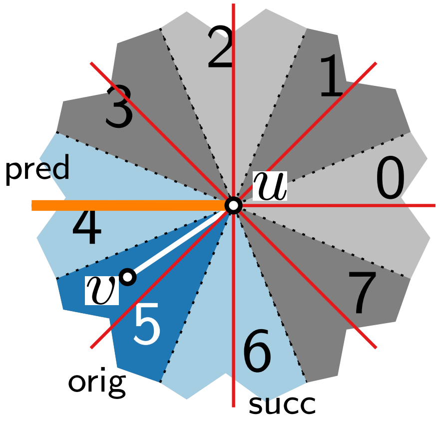
Case 1: $\alpha_{\mathsf{pred}}(u,v) = 1$

$$\left.\begin{aligned} y(u) - y(v) &\leq 0 \\ -y(u) + y(v) &\leq 0 \end{aligned}\right\} \Rightarrow y(u) = y(v)$$
$$x(u) - x(v) \geq \ell$$

Case 2: $\alpha_{\mathsf{pred}}(u,v) = 0$

$$\left.\begin{aligned} y(u) - y(v) &\leq M \\ -y(u) + y(v) &\leq M \\ x(u) - x(v) &\geq -M + \ell \end{aligned}\right\} \text{all arbitrary}$$

Octilinearity and Relative Positions



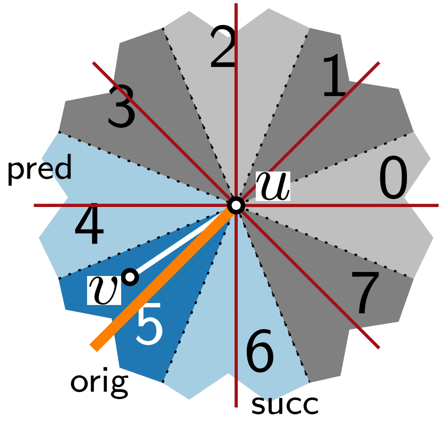
very large constant

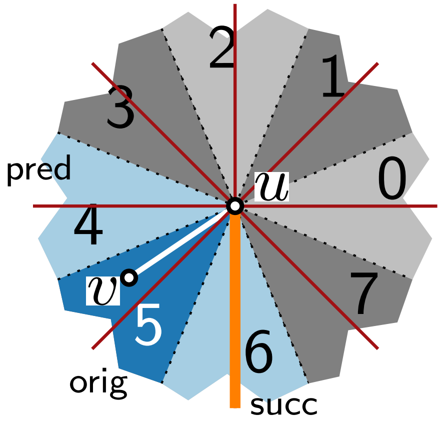
Predecessor sector.

$$\begin{aligned} y(u) - y(v) &\leq & M(1 - \alpha_{\mathsf{pred}}(u,v)) \\ -y(u) + y(v) &\leq & M(1 - \alpha_{\mathsf{pred}}(u,v)) \\ x(u) - x(v) &\geq & -M(1 - \alpha_{\mathsf{pred}}(u,v)) + \ell \end{aligned}$$

Original sector.

$$\begin{aligned} z_2(u) - z_2(v) &\leq & M(1 - \alpha_{\mathsf{orig}}(u,v)) \\ -z_2(u) + z_2(v) &\leq & M(1 - \alpha_{\mathsf{orig}}(u,v)) \\ z_1(u) - z_1(v) &\geq & -M(1 - \alpha_{\mathsf{orig}}(u,v)) + \ell \end{aligned}$$





Successor sector.

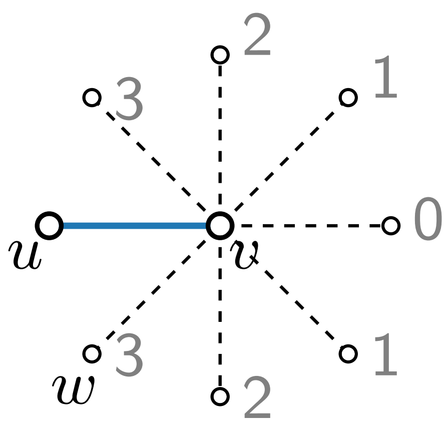
$$\begin{aligned} x(u) - x(v) &\leq & M(1 - \alpha_{\mathsf{succ}}(u,v)) \\ -x(u) + x(v) &\leq & M(1 - \alpha_{\mathsf{succ}}(u,v)) \\ y(u) - y(v) &\geq & -M(1 - \alpha_{\mathsf{succ}}(u,v)) + \ell \end{aligned}$$

Objective Function

- models the three soft constraints
- weighted sum of individual cost functions

  minimize $\lambda_{\text{bends}}\,\text{cost}_{\text{bends}} + \lambda_{\text{length}}\,\text{cost}_{\text{length}} + \lambda_{\text{dist}}\text{cost}_{\text{dist}}$

Example: line bends (R3/R4)



Edges uv and vw on a line $L \in \mathcal{L}$

- draw as straight as possible
- increasing cost bend(u, v, w) for increasing acuteness of $\angle(\overline{uv}, \overline{vw})$

$$\text{cost}_{\text{bends}} = \sum_{L \in \mathcal{L}} \sum_{uv, vw \in L} \text{bend}(u, v, w)$$

To assign bend(u, v, w) correctly, we need to define some linear constraints based on the direction variables dir(u, v) and dir(v, w).

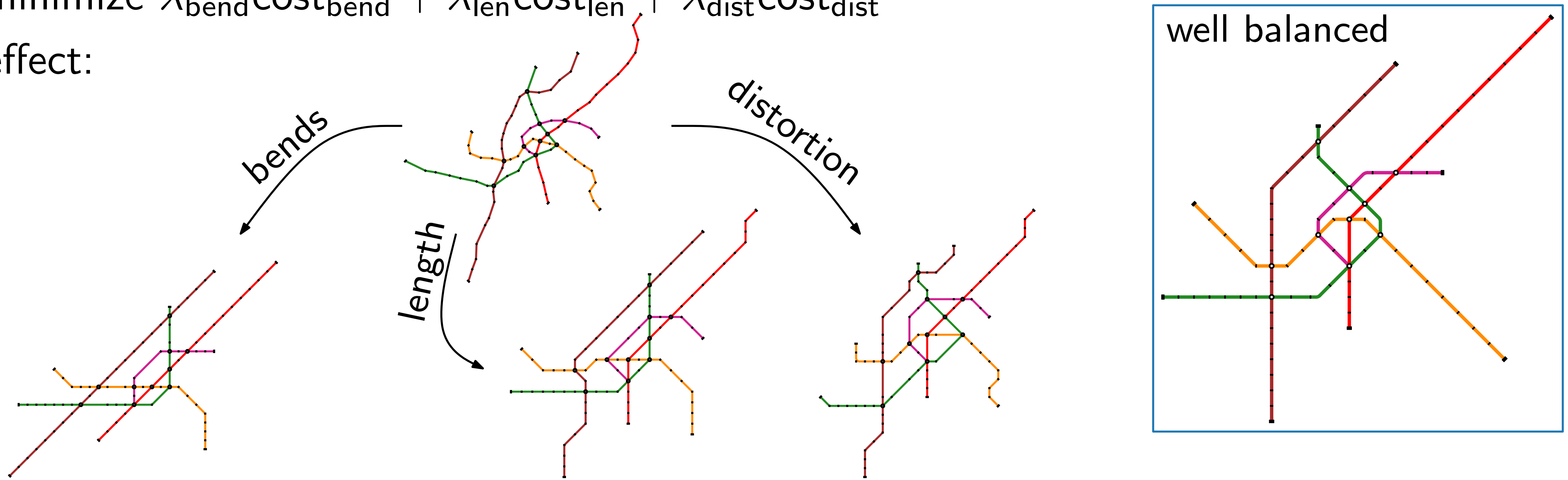
Overview MIP model

Constraints.

- linearization of all hard constraints
- $O(n^2)$ variables and constraints (due to planarity)

Objective function.

- weighted sum of the three soft constraints
- minimize $\lambda_{\text{bend}}\text{cost}_{\text{bend}} + \lambda_{\text{len}}\text{cost}_{\text{len}} + \lambda_{\text{dist}}\text{cost}_{\text{dist}}$
- effect:

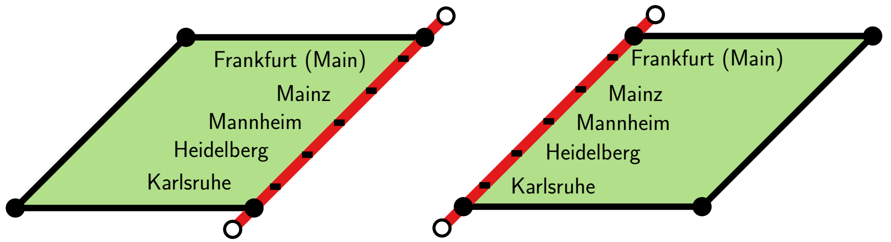


Station Labeling

- unlabeled map mostly useless
- labels need space
- labels may not overlap each other
- graph labeling problem is NP-hard [Kakoulis & Tollis 2001]

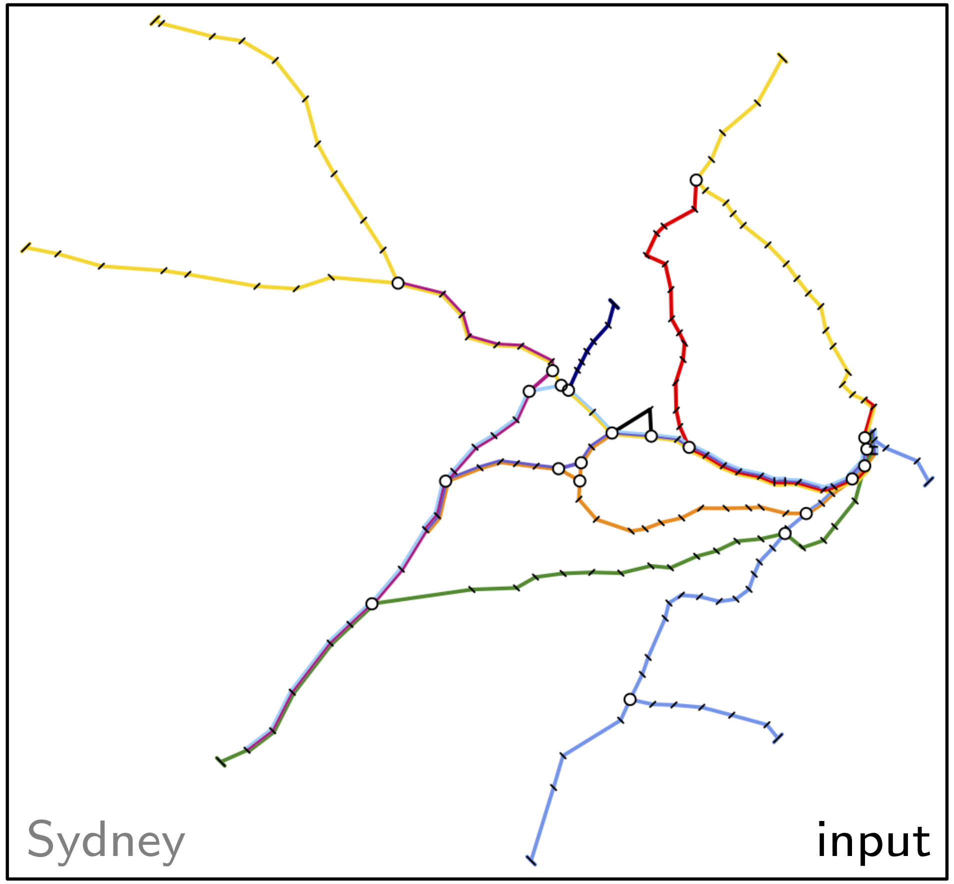
Paderborn

→ combine layout & labeling for optimal results!



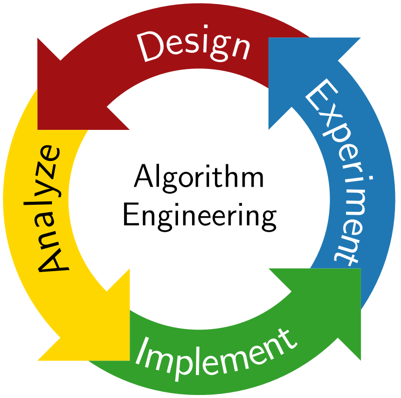
- parallelogram as special metro line
- switching sides allowed

Example: Sydney

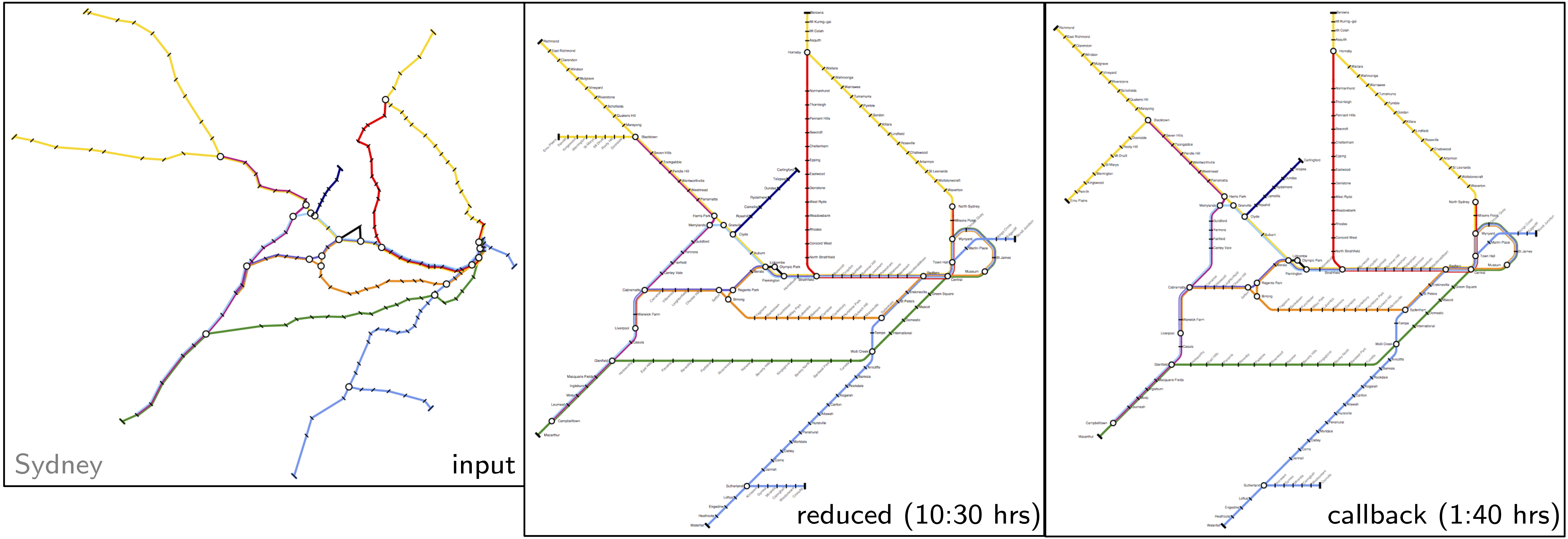


Input	$\lvert V \rvert$	$\lvert E \rvert$	faces	$\lvert \mathcal{L} \rvert$
full	174	183	11	10
reduced	88	97		
labeled	242	270	30	

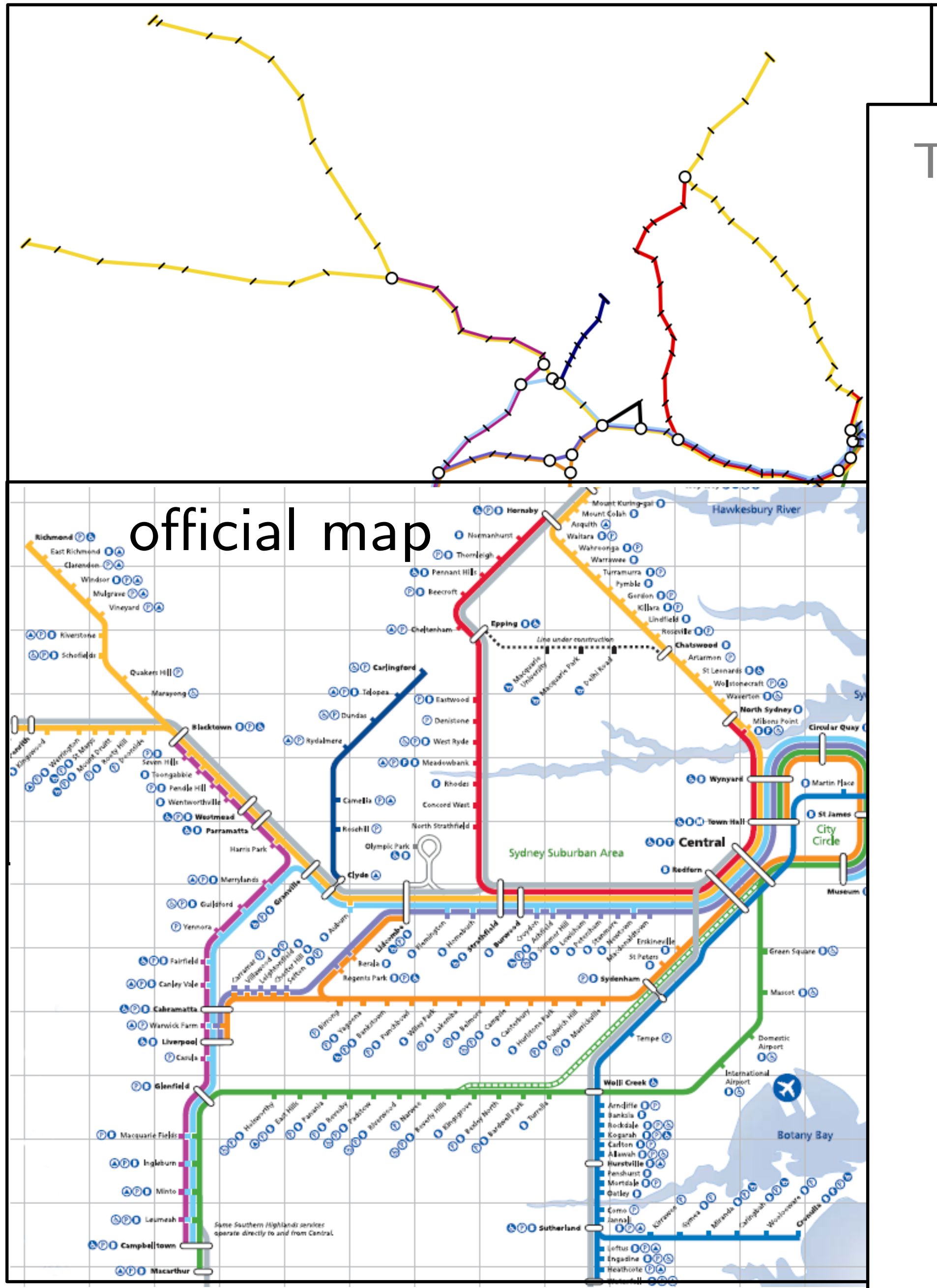
MIP	constraints	variables
full	1,191,406	290,137
callback	21,988	92,681

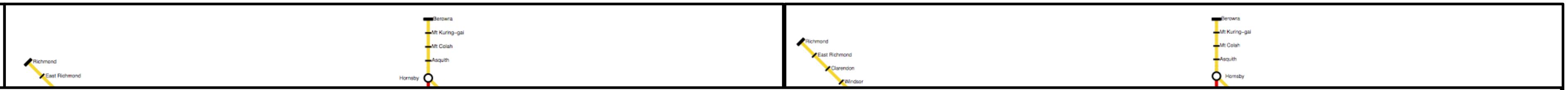


Example: Sydney

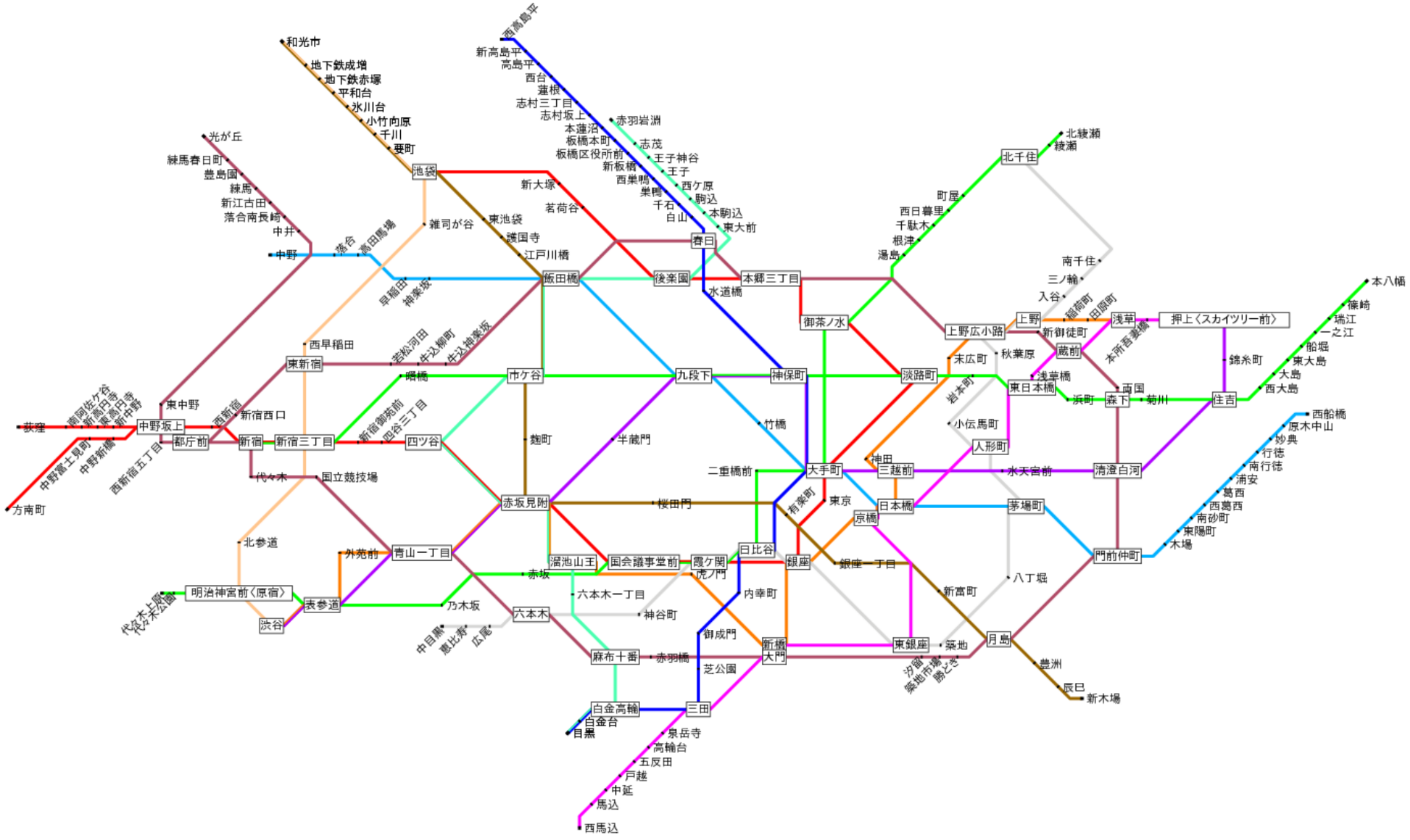


Example: Sydney & Tokyo





Tokyo



- multi-phase MIP model [Onda, Moriguchi, Imai 2018]

5 hrs

Mixed-Integer Programming – Discussion

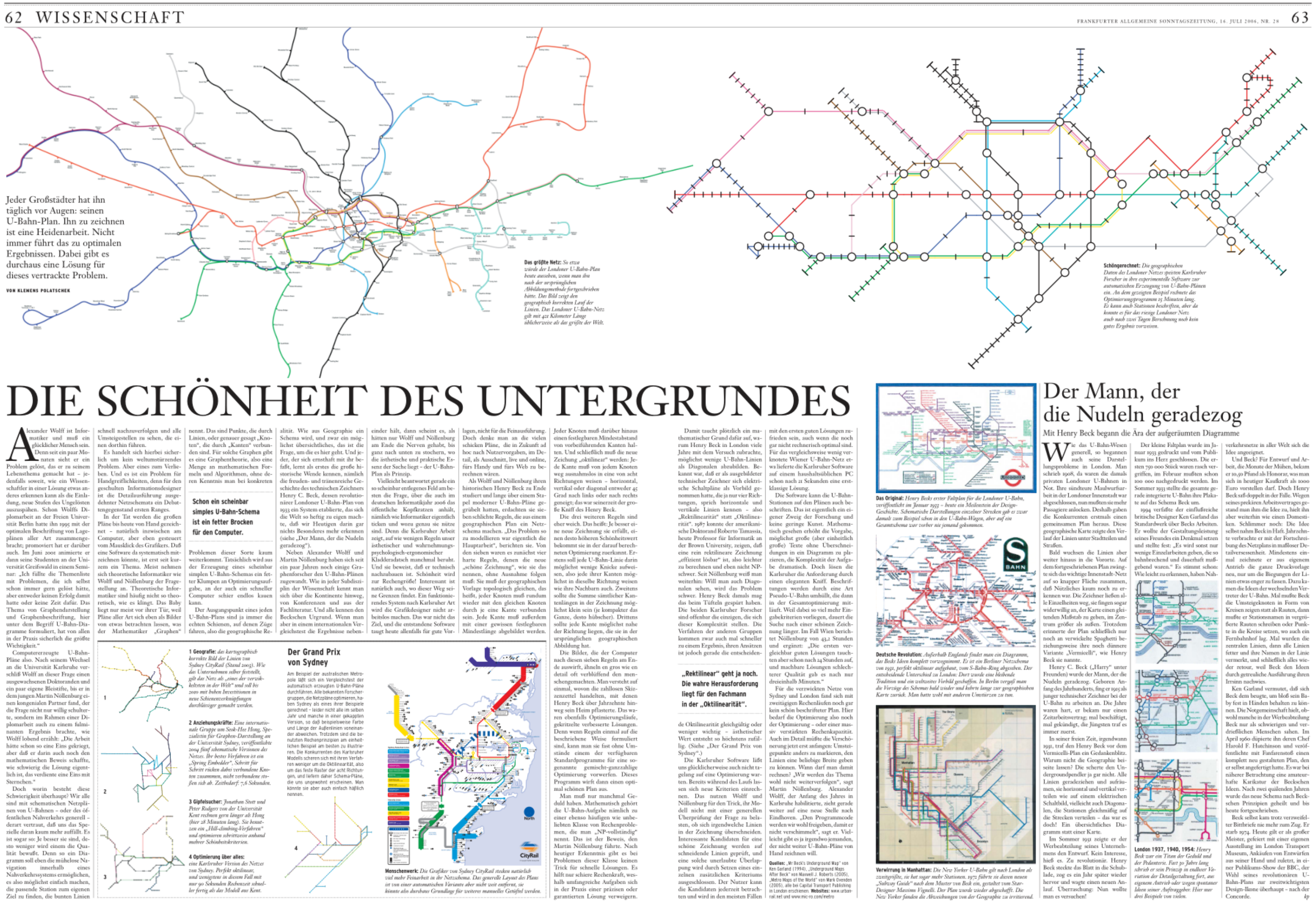
Pros.

- flexible framework, but integration and linearization of new criteria requires some effort
- high layout and labeling quality
- theoretical guarantees
- can integrate user constraints dynamically

Cons.

- long, sometimes unpredictable running times
- for large labeled networks no proof of optimality
- solutions only as good as the model specification

Mixed-Integer Programming – Discussion



Least-Squares Schematization

[Wang & Chi 2011]

Idea.
- model layout problem as minimization of a set of (squared) energy terms
- variables for vertex positions and edge slopes
- use iterative numerical optimization method

- 3-step approach
  - compute topologically correct non-octilinear layout optimizing angular resolution **(R5)**, uniform edge lengths **(R7)**, displacement **(R6)**
  - octilinearly discretize edge orientations **(R2)** by extra energy term
  - optimize label placement by energy minimization in fixed layout
- described for a focus route, but generalizes to entire maps

Discussion.
- very fast method for good quality layouts
- no guarantee on constraints unless final energy is zero

Summary

- variety of layout methods evolved over the last 15–20 years
- many shared design rules
- trade-off between speed and quality, but quite reasonable maps can be computed in a matter of seconds to minutes
- many approaches are customizable and open to new criteria
- current trend: beyond-octilinear metro maps

Why automated maps?

- base layouts for graphic designers (semi-automated process)
- large quantities of individual or special-purpose maps

Challenges

- global quality criteria like symmetry, harmony, coherence, balance
- take thick edge bundles and large vertices into account

Literature

- [Nöllenburg '14] A Survey on Automated Metro Map Layout Methods
- [Dwyer, Hurst, Merrick '08] A fast and simple heuristic for metro map path simplification
- [Delling, Gemsa, Nöllenburg, Pajor '10] Path Schematization for Route Sketches
- [Brandes & Pampel '09] On the Hardness of Orthogonal-Order Preserving Graph Drawing
- [Gemsa, Nöllenburg, Pajor, Rutter '11] On d-Regular Schematization of Embedded Paths
- [Hong, Merrick, do Nascimento '06] Automatic visualisation of metro maps
- [Chivers & Rodgers '14] Octilinear Force-Directed Layout with Mental MapPreservation for Schematic Diagrams
- [Fink, Haverkort, Nöllenburg, Roberts, Schuhmann, Wolff '12] Drawing metro maps using Bézier curves
- [Avelar & Müller '00] Generating topologically correct schematicmaps
- [Ware, Taylor, Anand, Thomas '06] Automated production of schematic maps for mobile application
- [Ware & Richards '13] An ant colony system algorithm for automatically schematizing transport network data set
- [Stott, Rodgers, Burkhard, Meier, Smis '11] Automatic metro map layout using multicriteria optimization
- [Nöllenburg & Wolff '11] Drawing and labeling high-quality metro maps by mixed-integer programming
- [Onda, Moriguchi, Imai '18] Automatic Drawing for Tokyo Metro Map
- [Wang & Chi '11] Focus+context metro maps