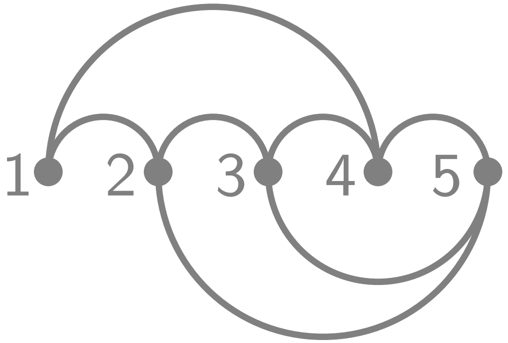
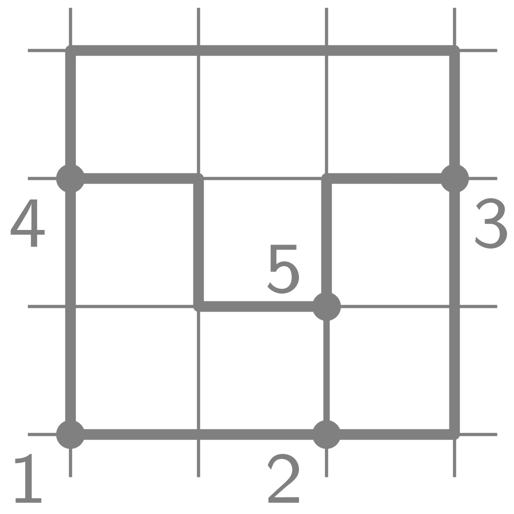
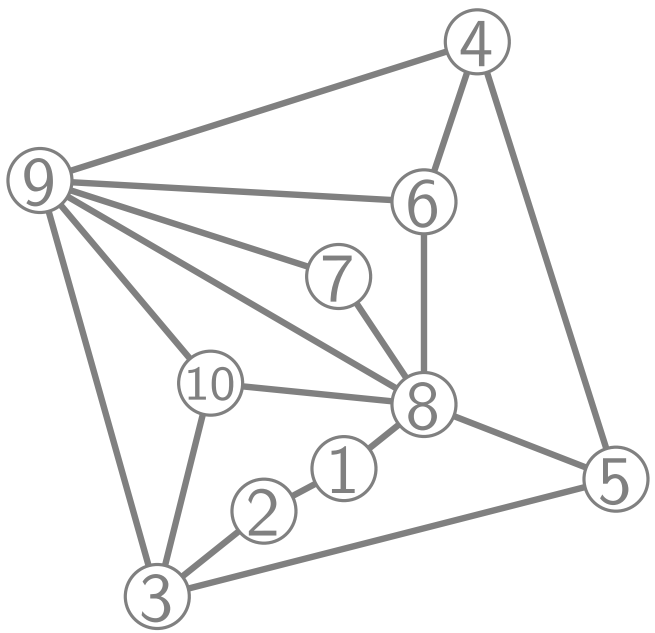
Visualization of Graphs

Lecture 1a: The Graph Visualization Problem





Johannes Zink



Summer semester 2024

Organizational

Lectures:

- Johannes Zink (M4, room 01.007, johannes.zink@uni-wuerzburg.de)

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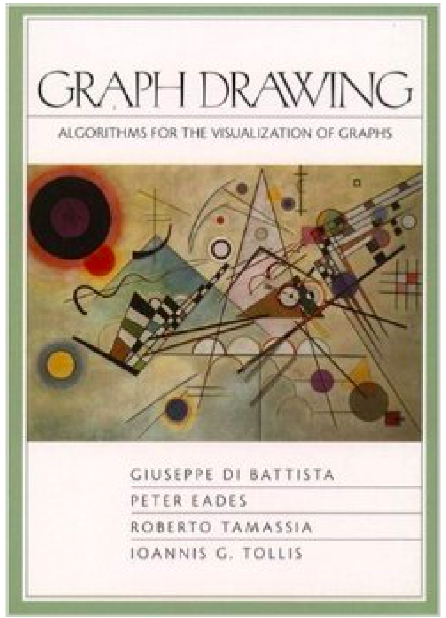

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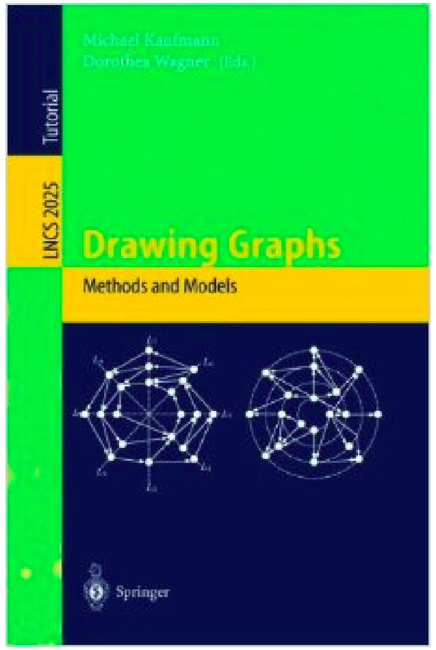

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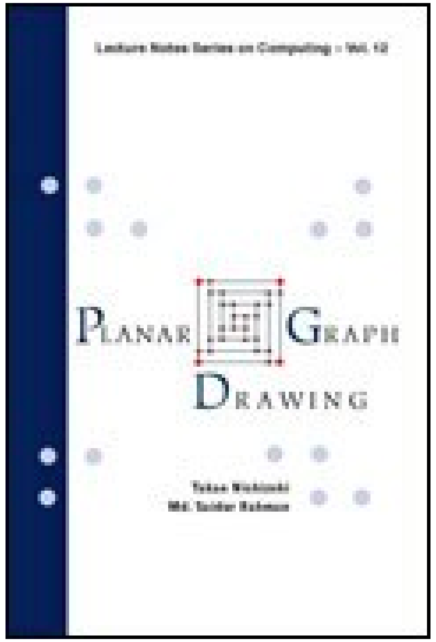

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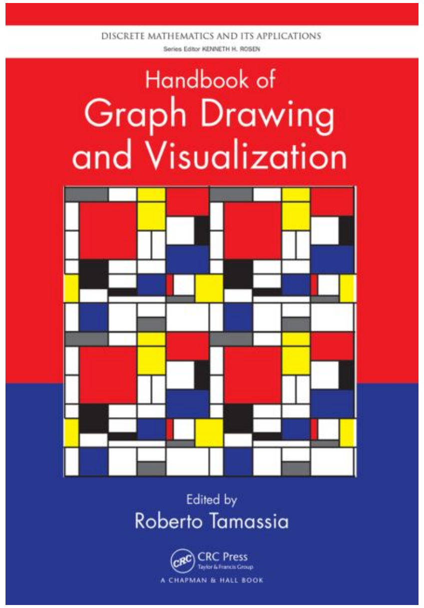

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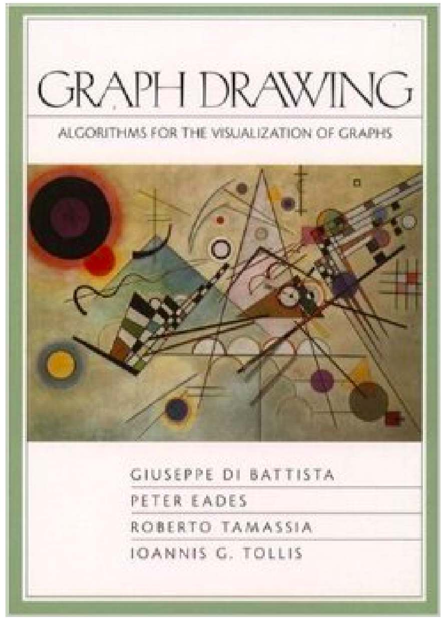

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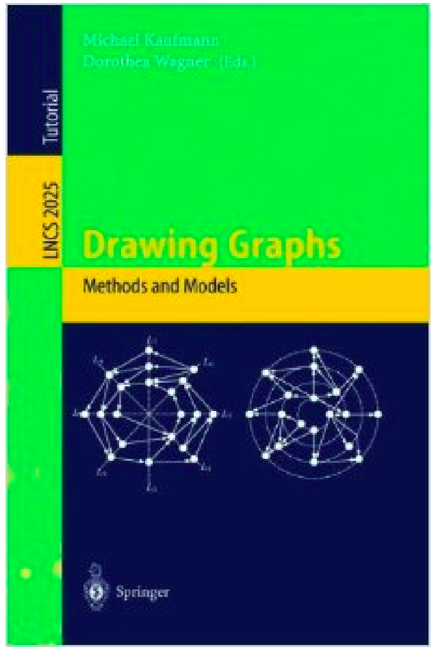

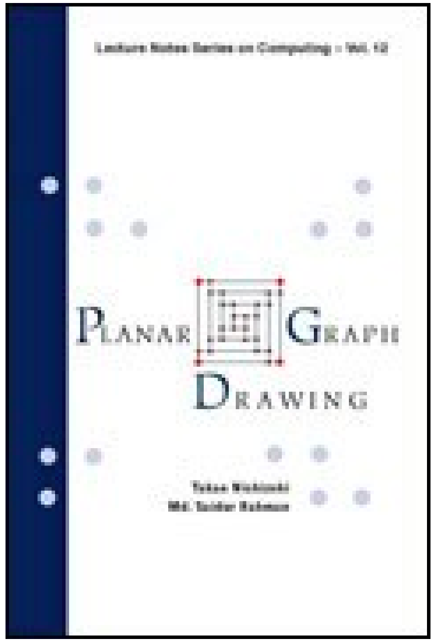

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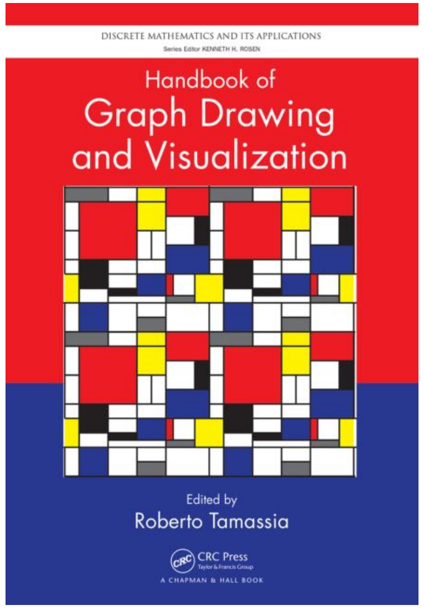

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- proofs

What Is This Course About?

Topics

- Drawing Trees and Series-Parallel Graphs
- Force-Based Drawing Algorithms and Tutte Embedding
- Straight-Line Drawings of Planar Graphs
- Upwards Planar Drawings
- Orthogonal Grid Drawings
- Contact Representations
- Hierarchical Layouts of Directed Graphs
- Visibility Representations
- The Crossing Lemma
- Linear Layouts
- Beyond Planarity
- Octilinear Drawings for Metro Maps

Graphs and Their Representations

What is a graph?

- graph G
- vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$
- edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$,
  where each edge is a pair of vertices.

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- Set notation

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- Adjacency list

v_1:	v_2, v_8	v_6:	v_4, v_8, v_9
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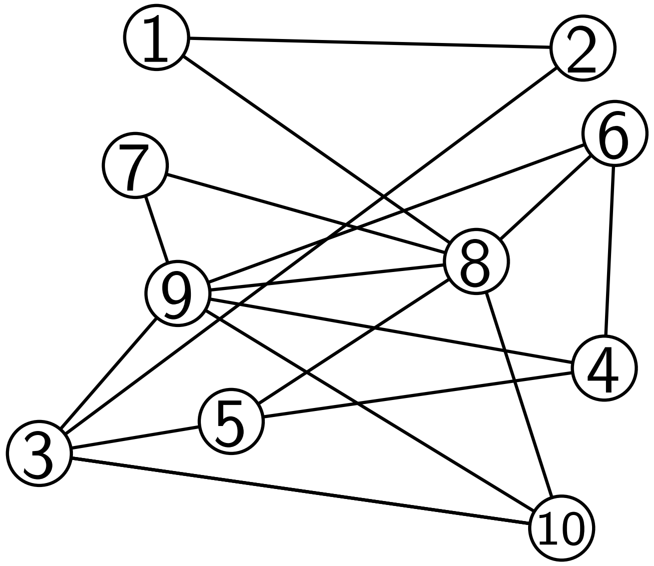
- Drawing



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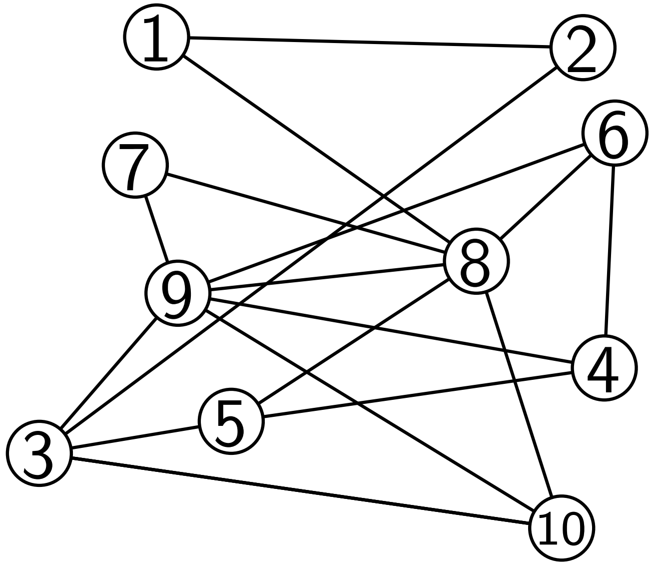

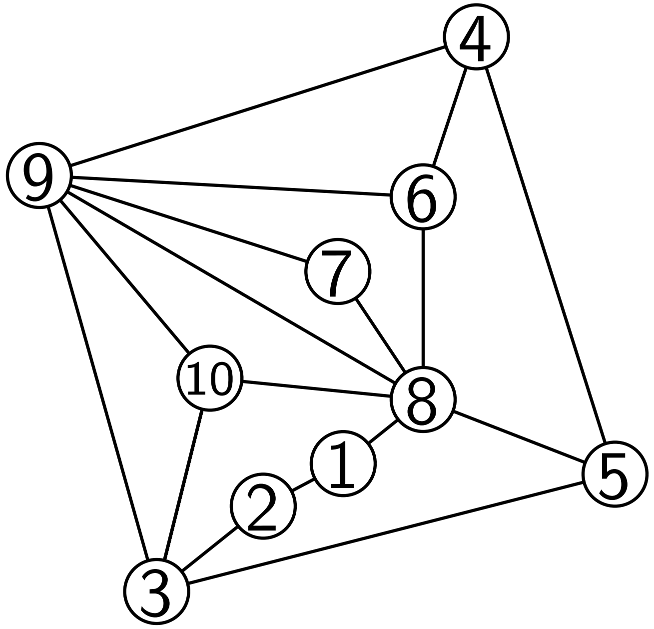

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- Road networks
- Power grids
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Abstract networks

- Social networks
- Communication networks
- Phylogenetic networks
- Metabolic networks
- Class/Object Relation Digraphs (UML)
- ...

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We need algorithms that draw graphs automatically to make networks more accessible to humans.

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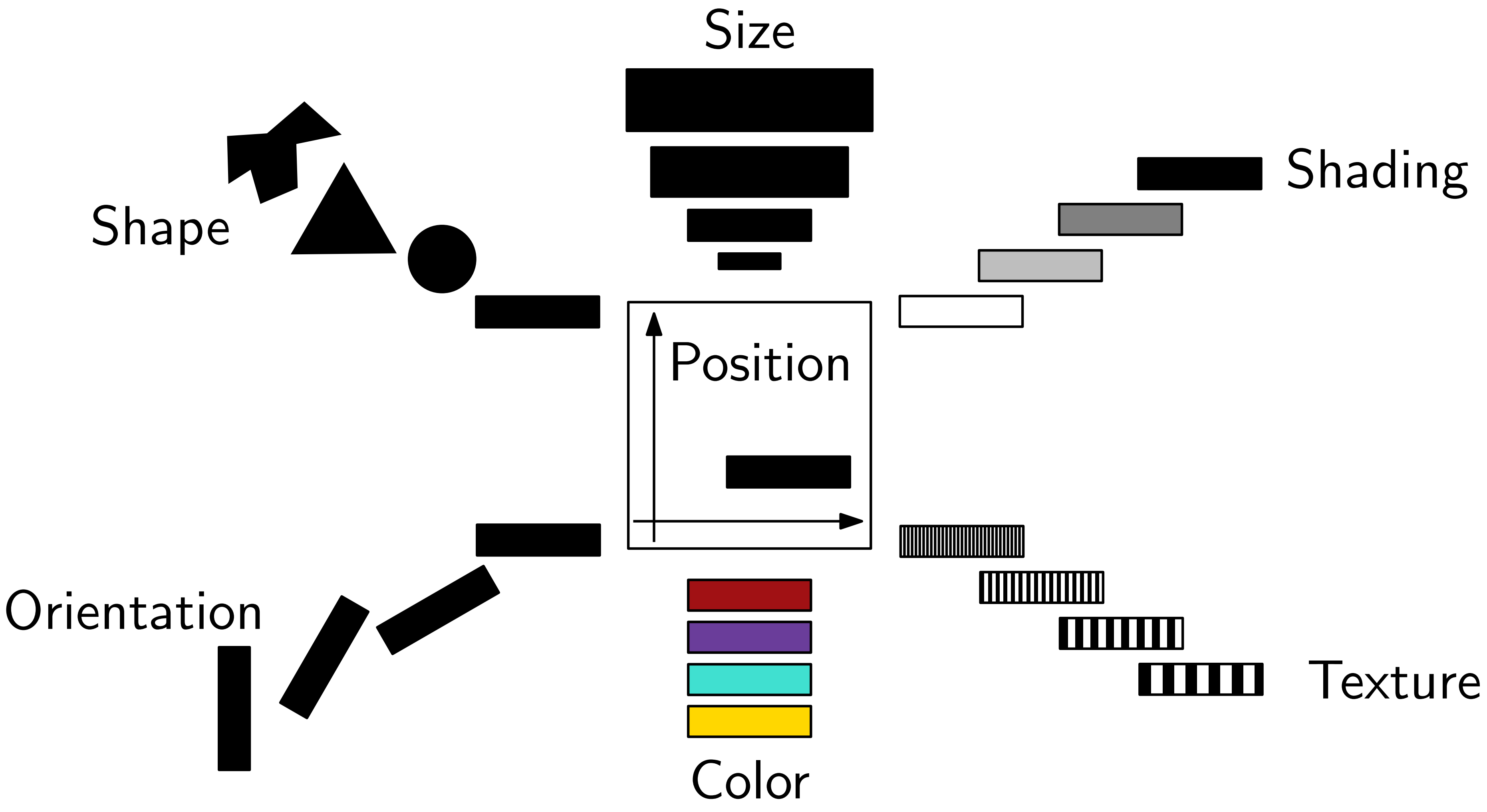

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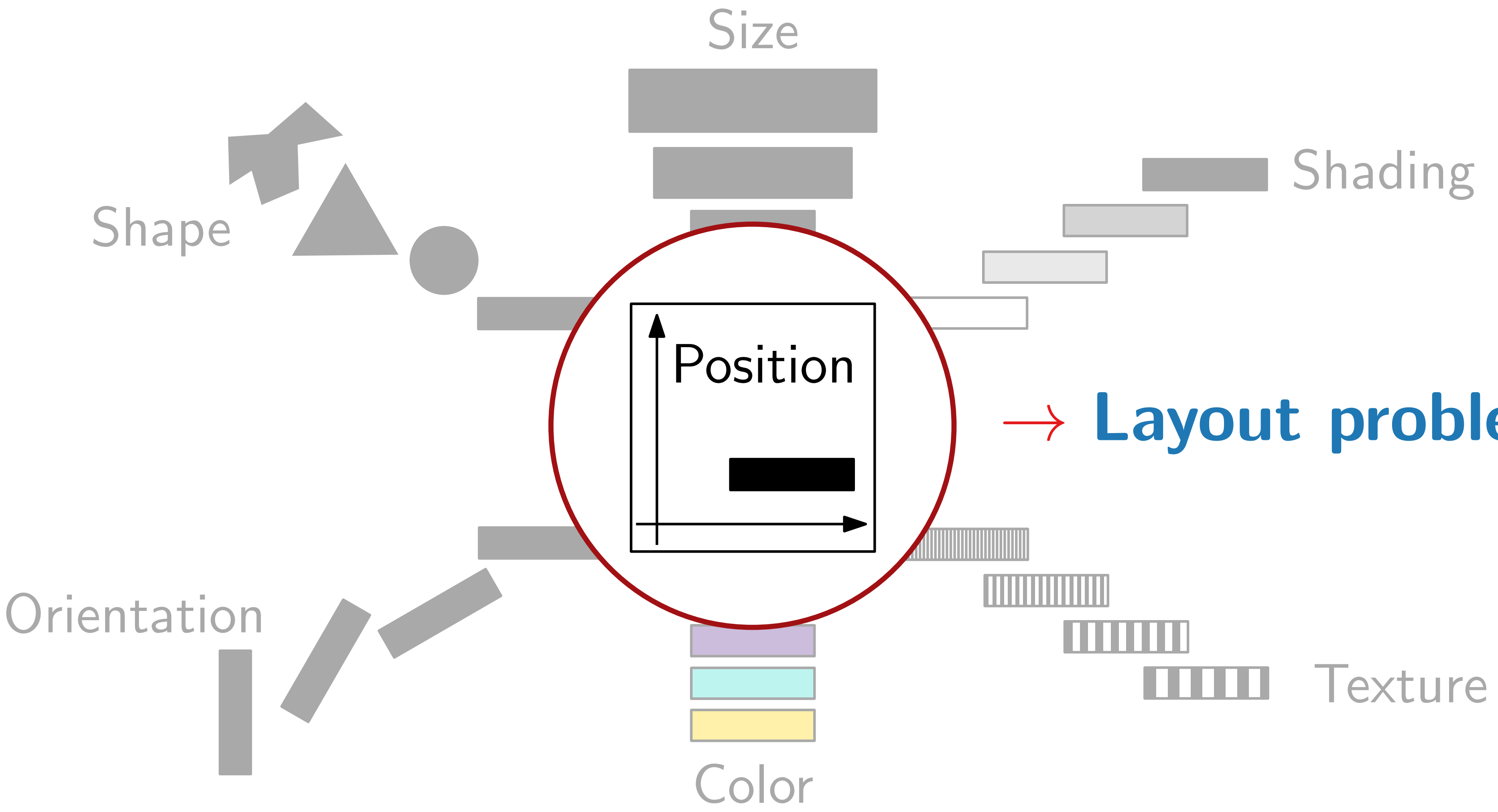

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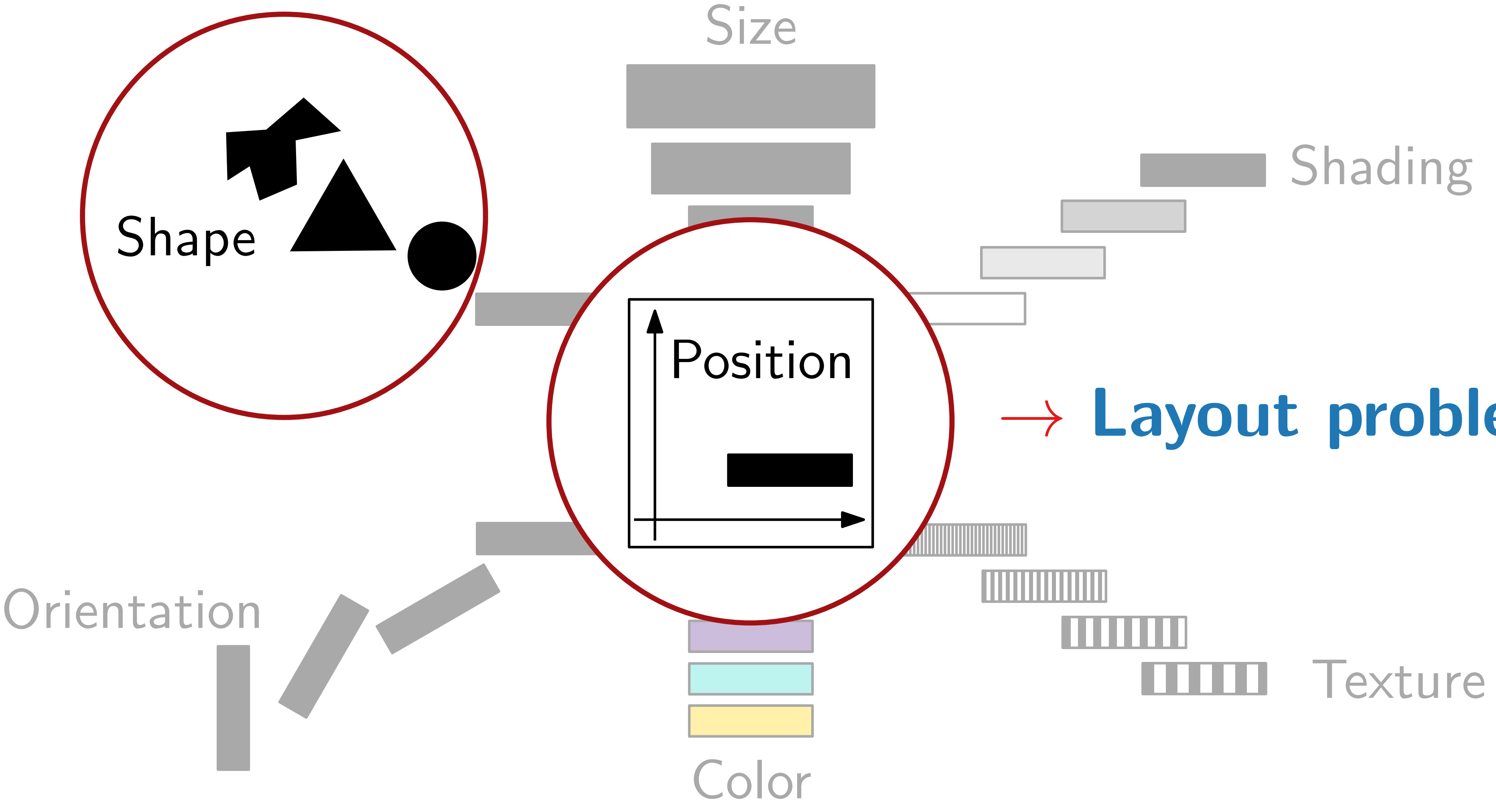

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The Layout Problem

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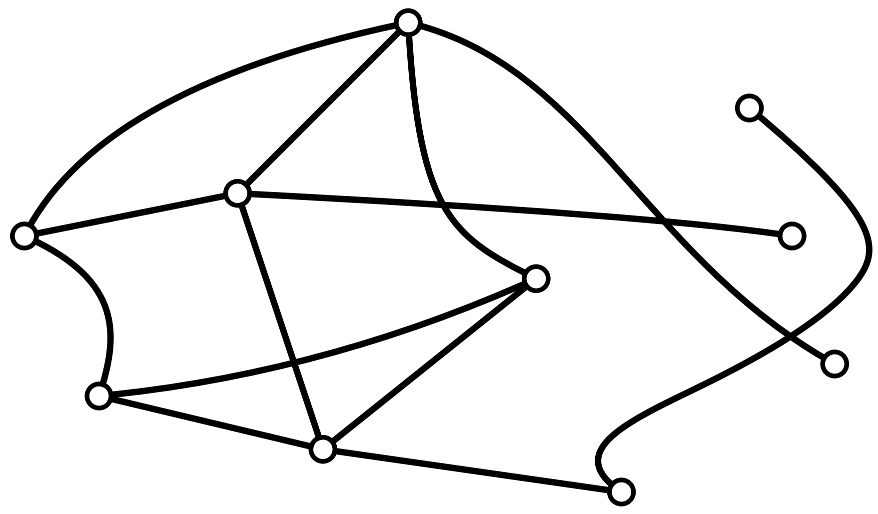

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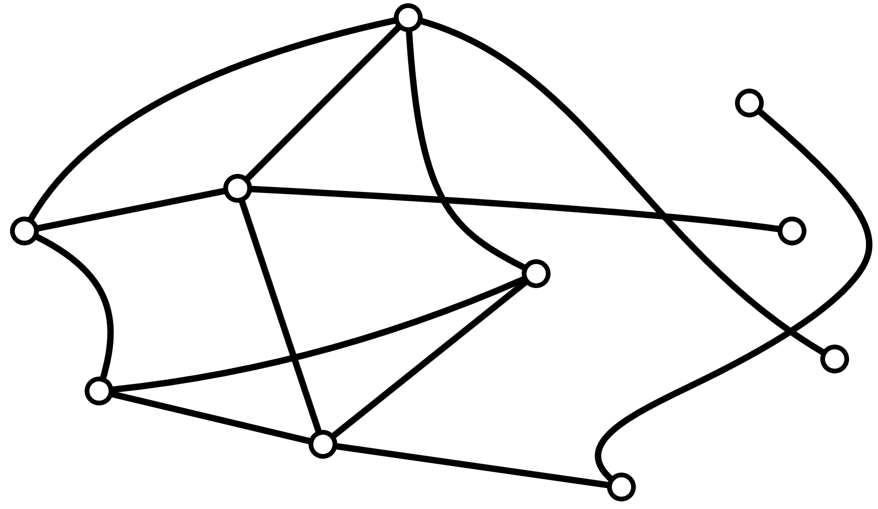
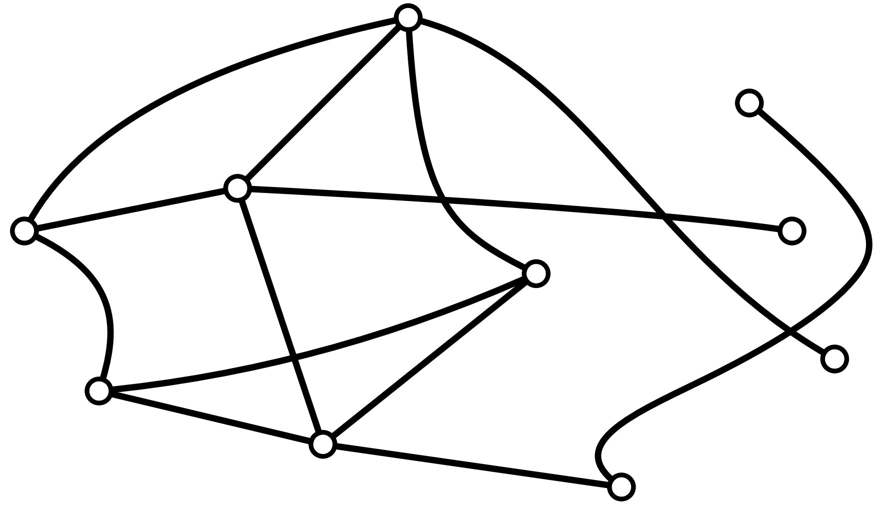


Graph Visualization Problem

in: graph G

out:

The Layout Problem



- *Here* restricted to the **standard representation**, so-called node–link diagrams.

Graph Visualization Problem

in: graph G

out: nice drawing Γ of G

- $\Gamma \colon V(G) \to \mathbb{R}^2$, vertex $v \mapsto$ point $\Gamma(v)$
- $\Gamma \colon E(G) \to$ simple, open curves in $\mathbb{R}^2$
  $\{u, v\} \mapsto \Gamma(\{u, v\})$ with endpoints $\Gamma(u)$ and $\Gamma(v)$

The Layout Problem?

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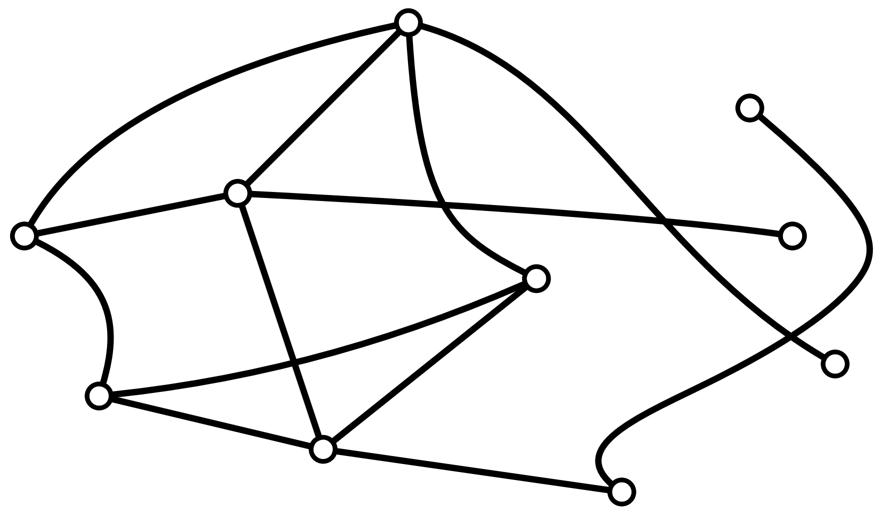

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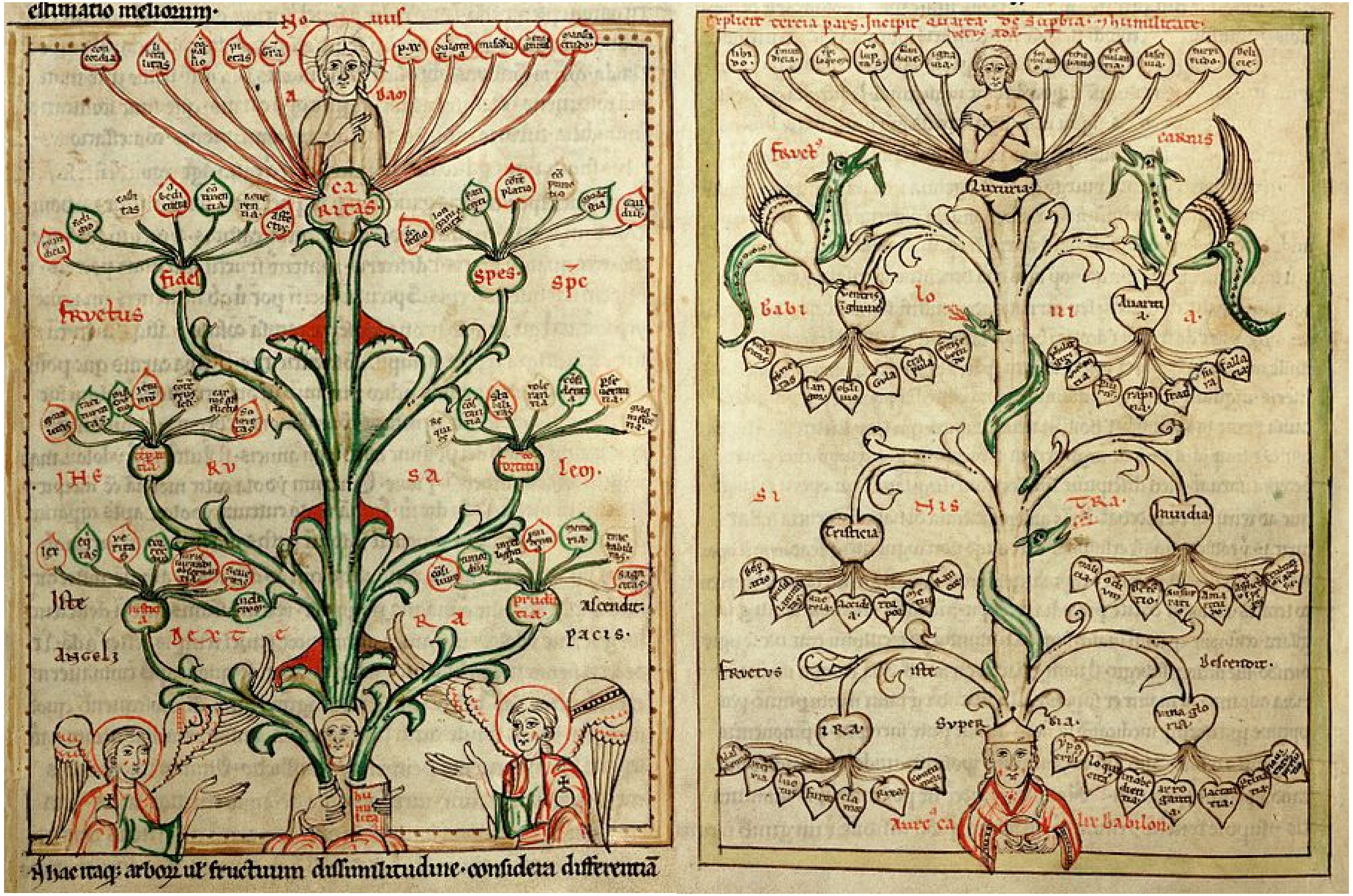
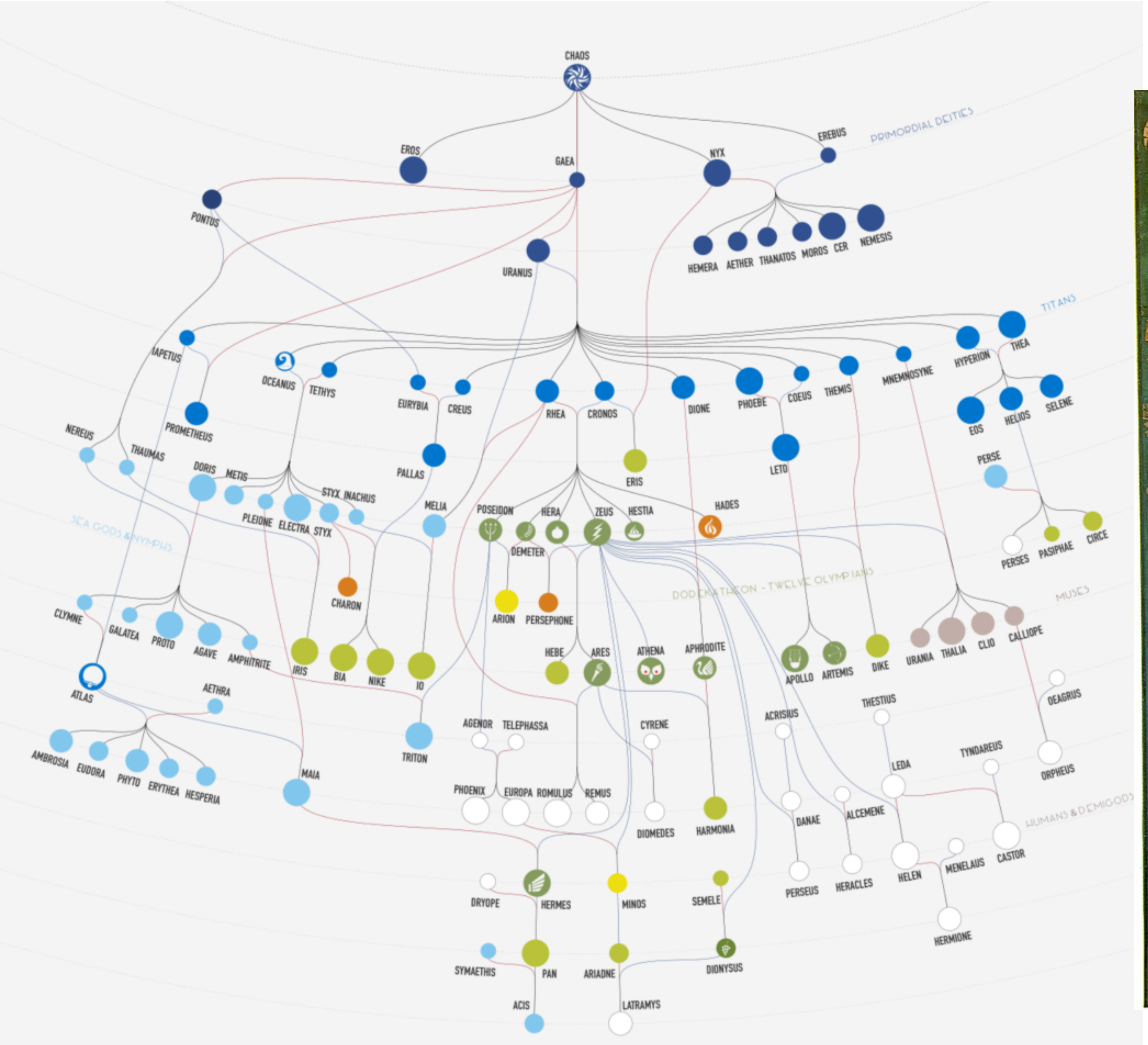
But what is a **nice** drawing?

Tree of Virtues and Tree of Vice

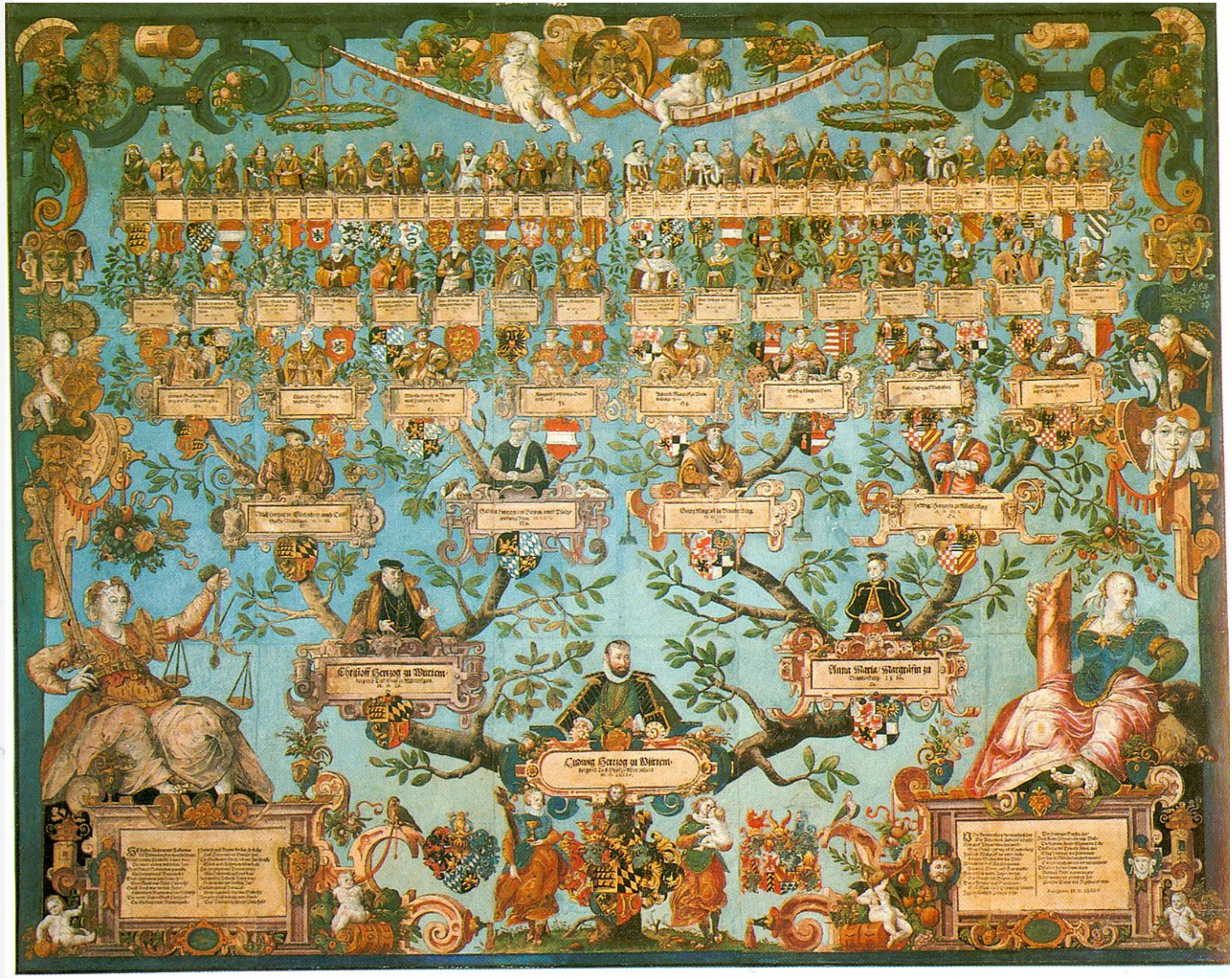
ca. 1200



Social Networks – Family Trees

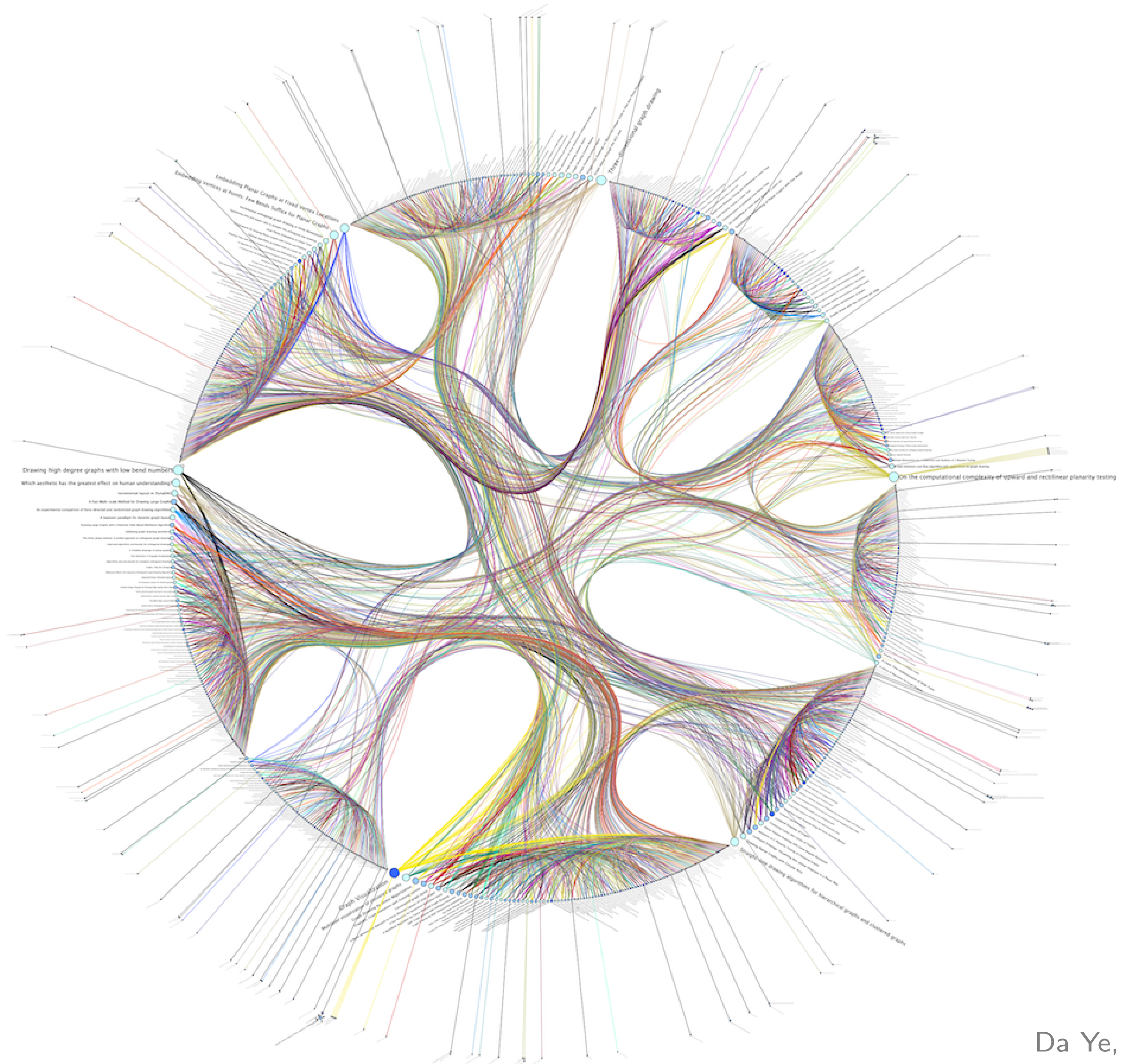


J. Klawitter, T. Mchedlidze, *Link:* go.uniwue.de/myth-poster



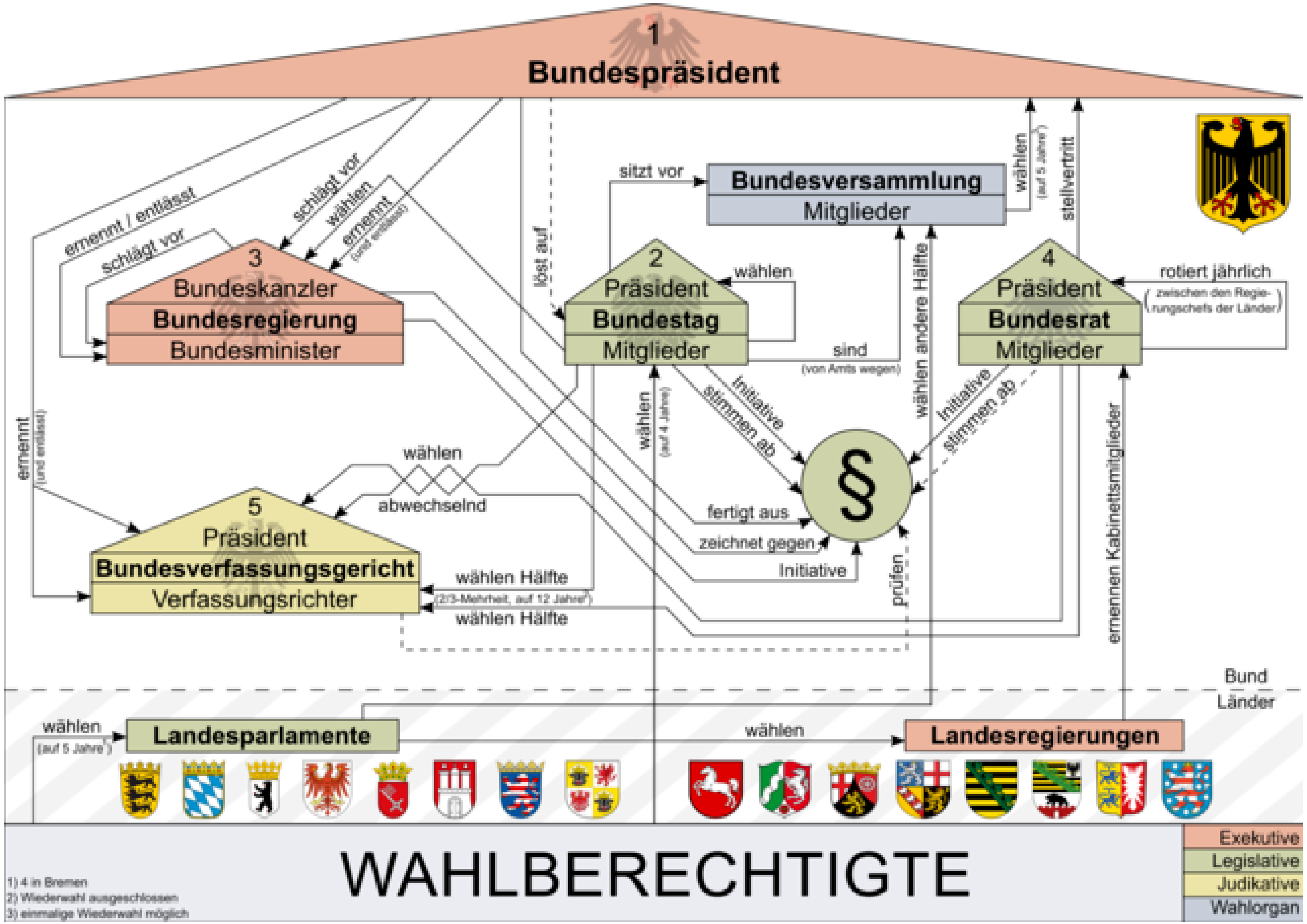
Ahnentafel Herzog Ludwig von Württemberg, 1585

Social Network – Citation Graph

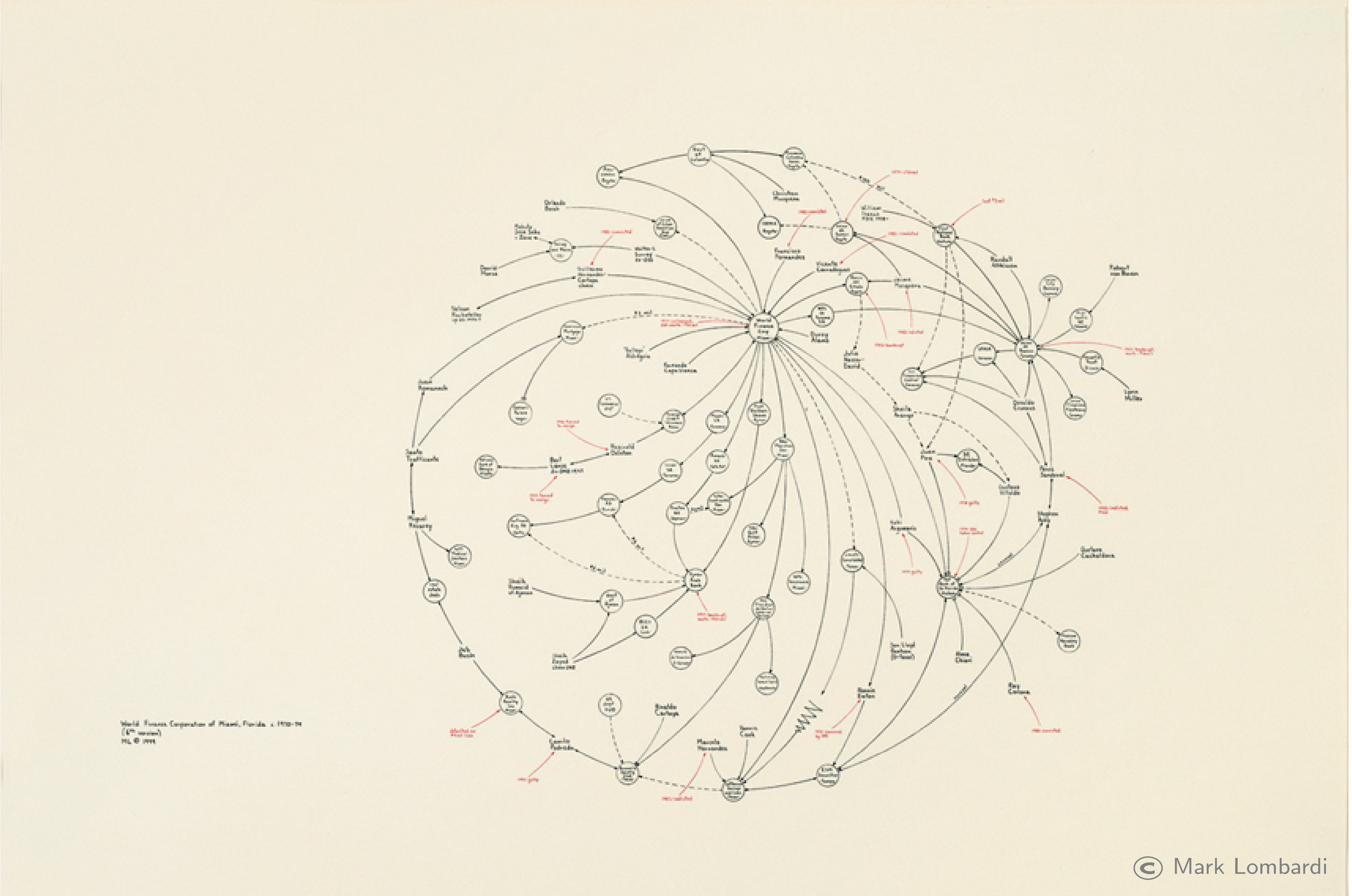


Da Ye, *Link:* https://go.uniwue.de/citation-graph

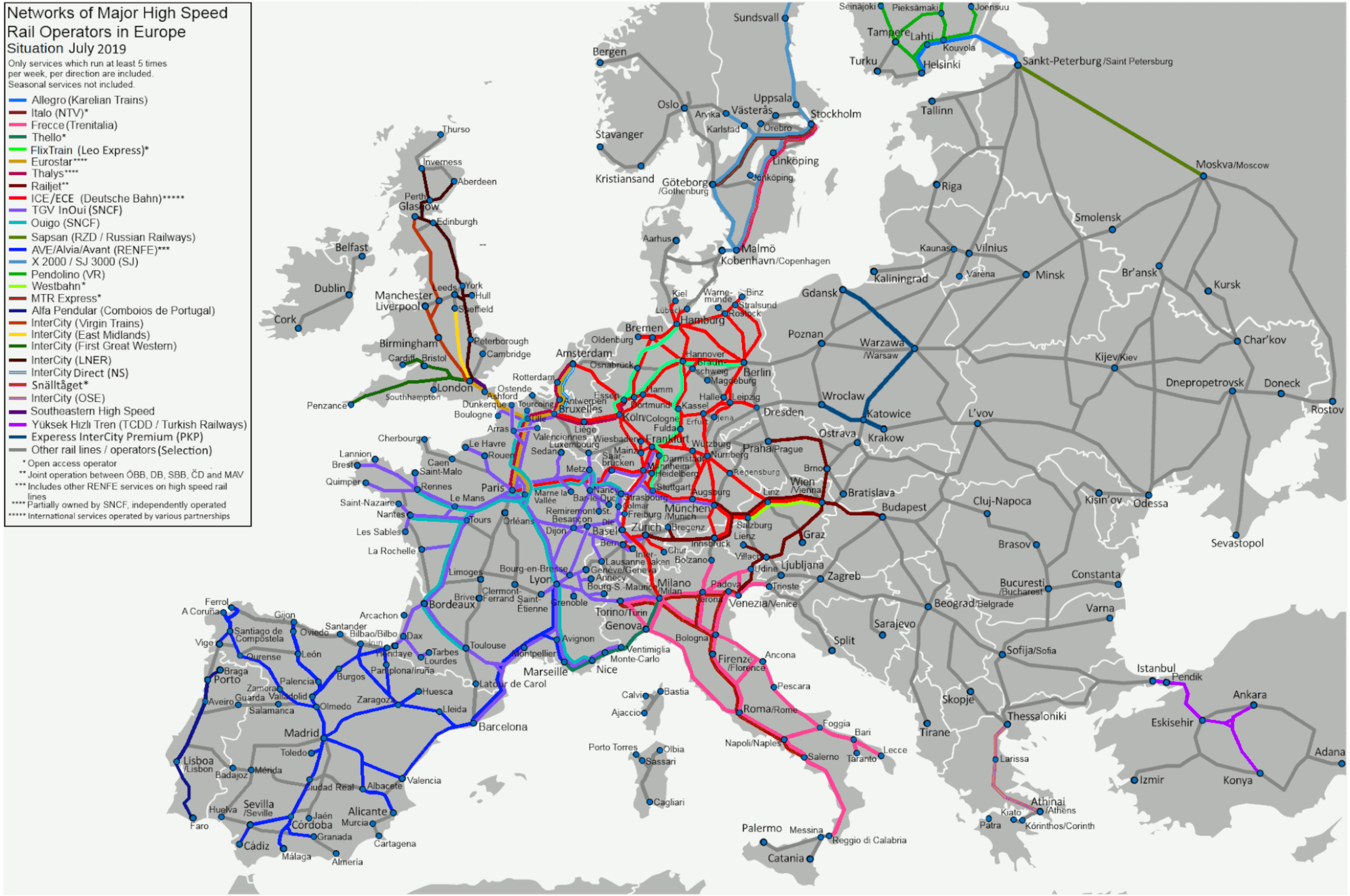
Social Network – Organizational Chart



Social Network – World Finance Corporation



Transportation Network – European High-Speed Railroads

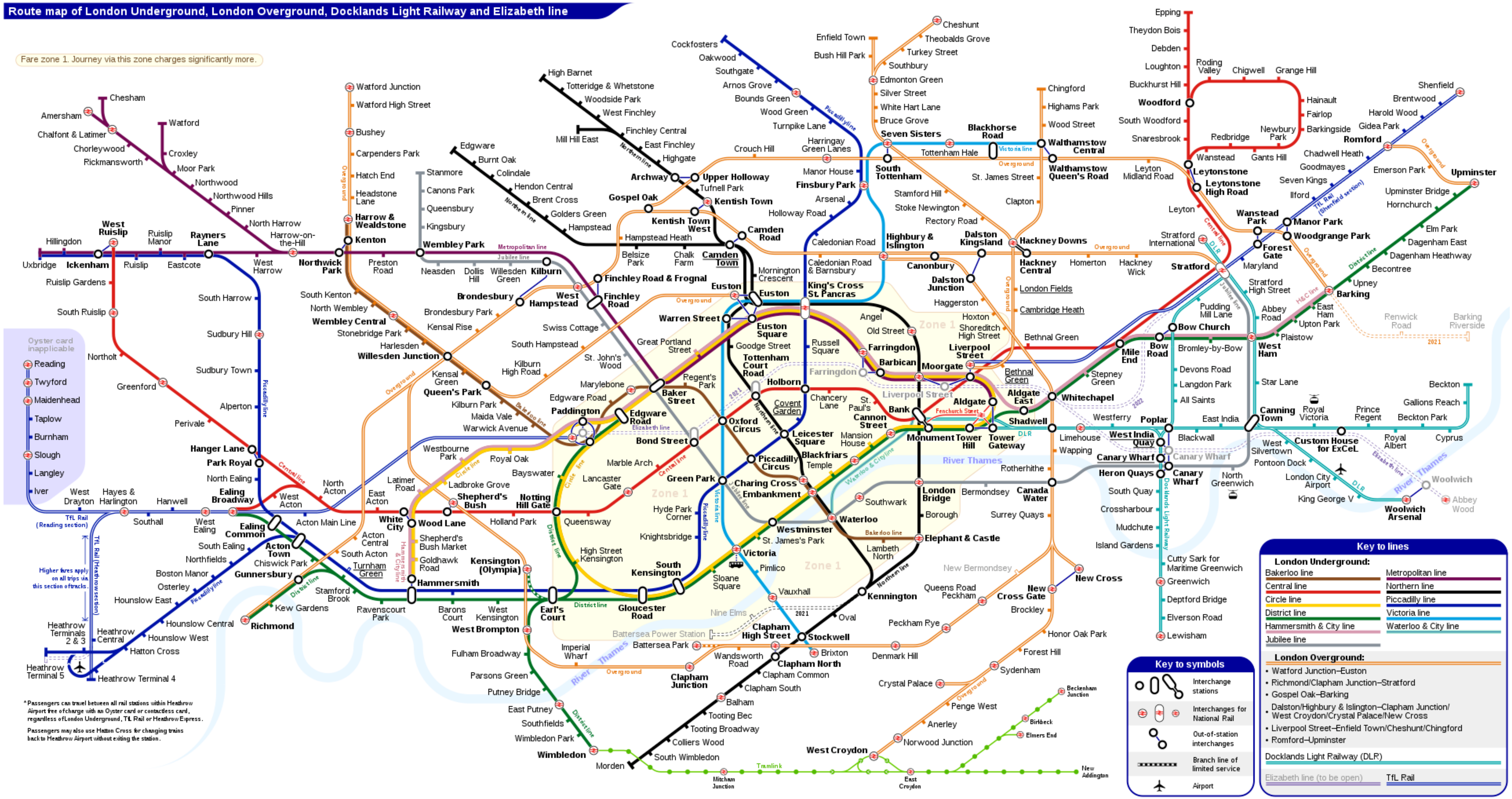


Source: Wiki Commons: Networks of Major High Speed Rail Operators in Europe - CC BY-SA 3.0

Transportation Network – London Underground

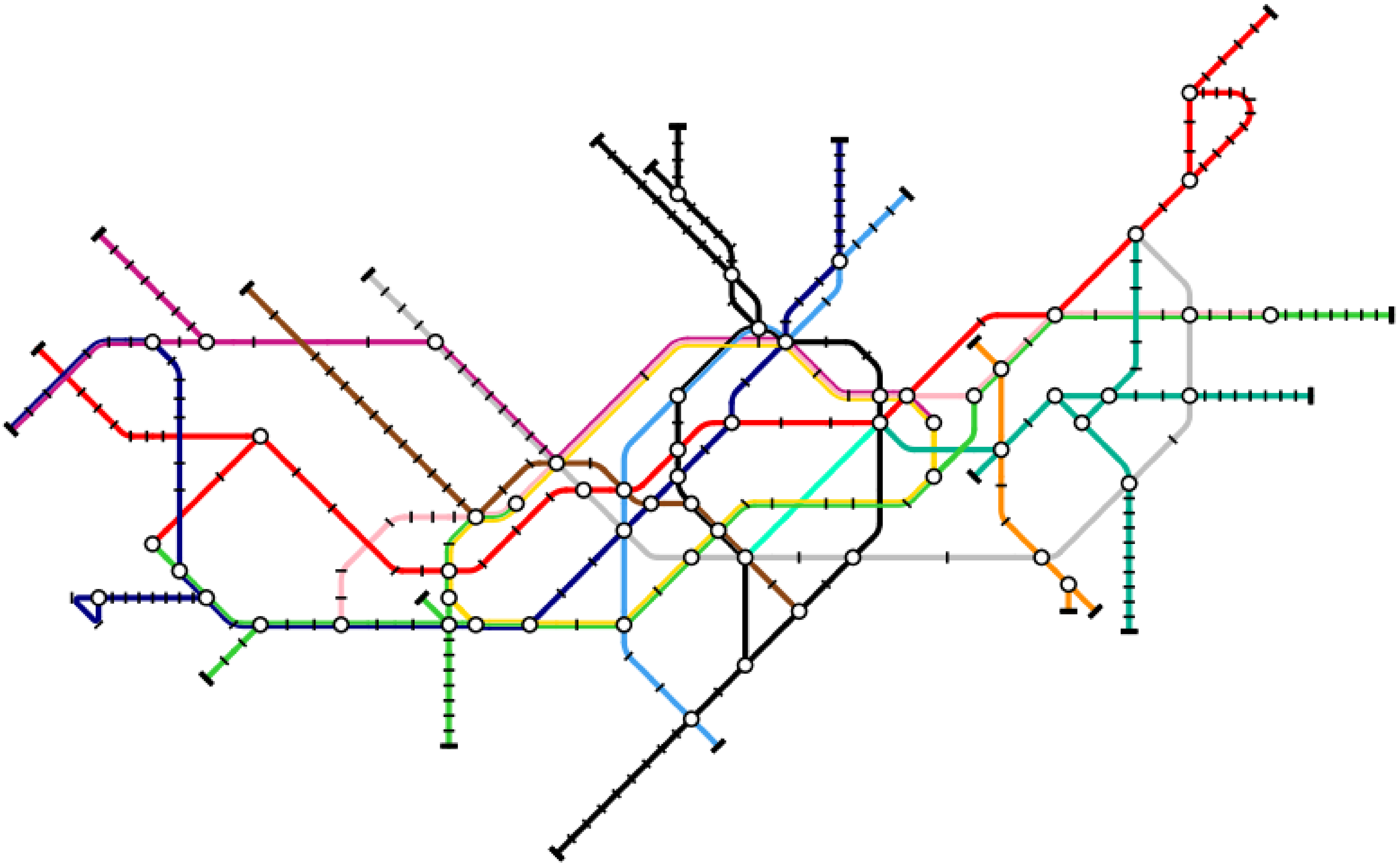
Source: Wiki Commons: London Underground full map - CC BY-SA 3.0

Transportation Network – London Underground



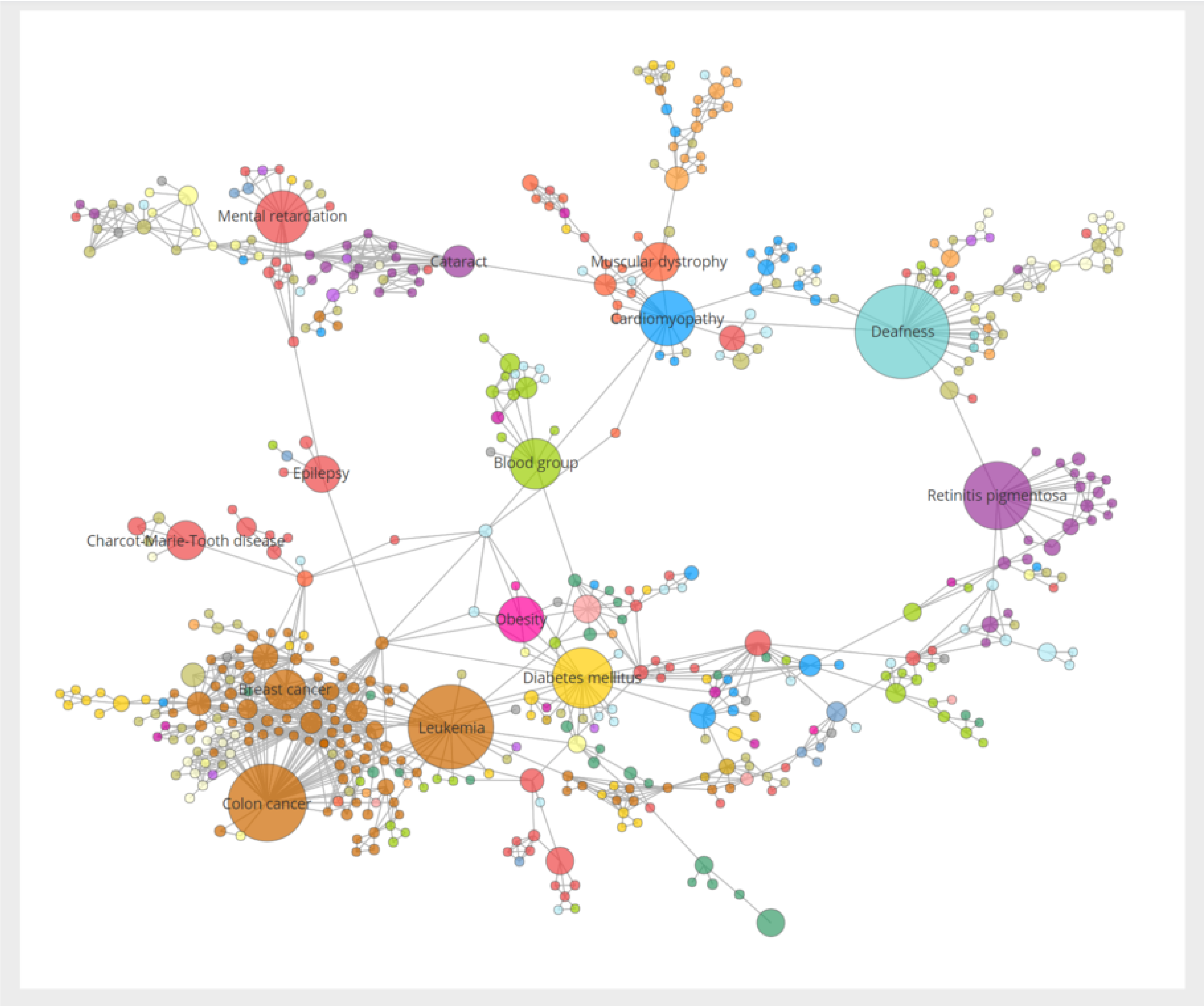
Source: Wiki Commons: London Underground Overground DLR Crossrail map - CC BY-SA 4.0

Transportation Network – London Underground



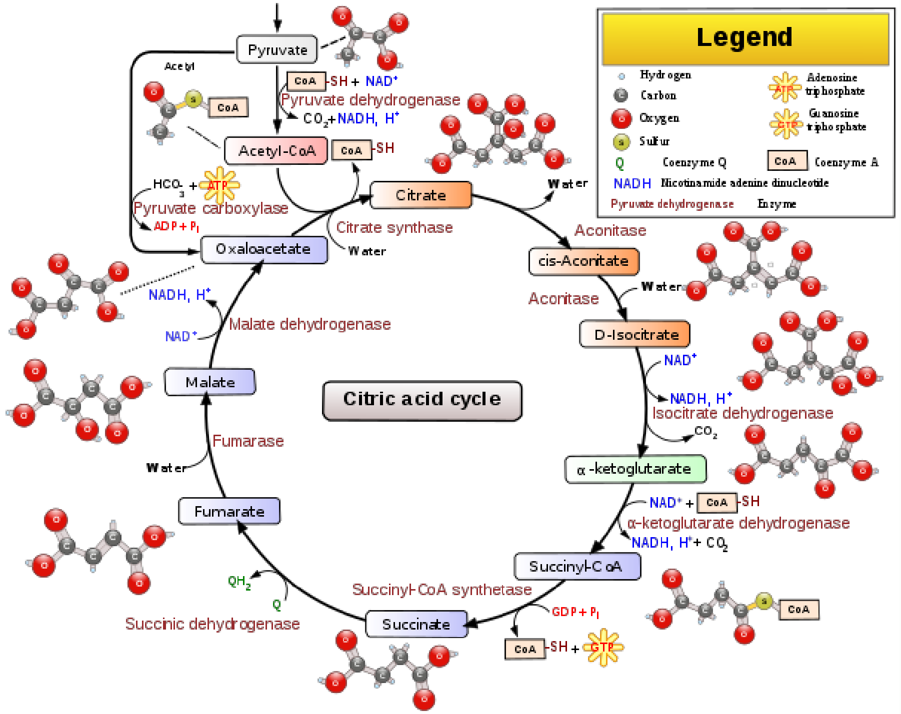
Drawing and Labeling High-Quality Metro Maps by Mixed-Integer Programming. Nöllenburg & Wolff. TVCG 2011, https://dx.doi.org/10.1109/TVCG.2010.81

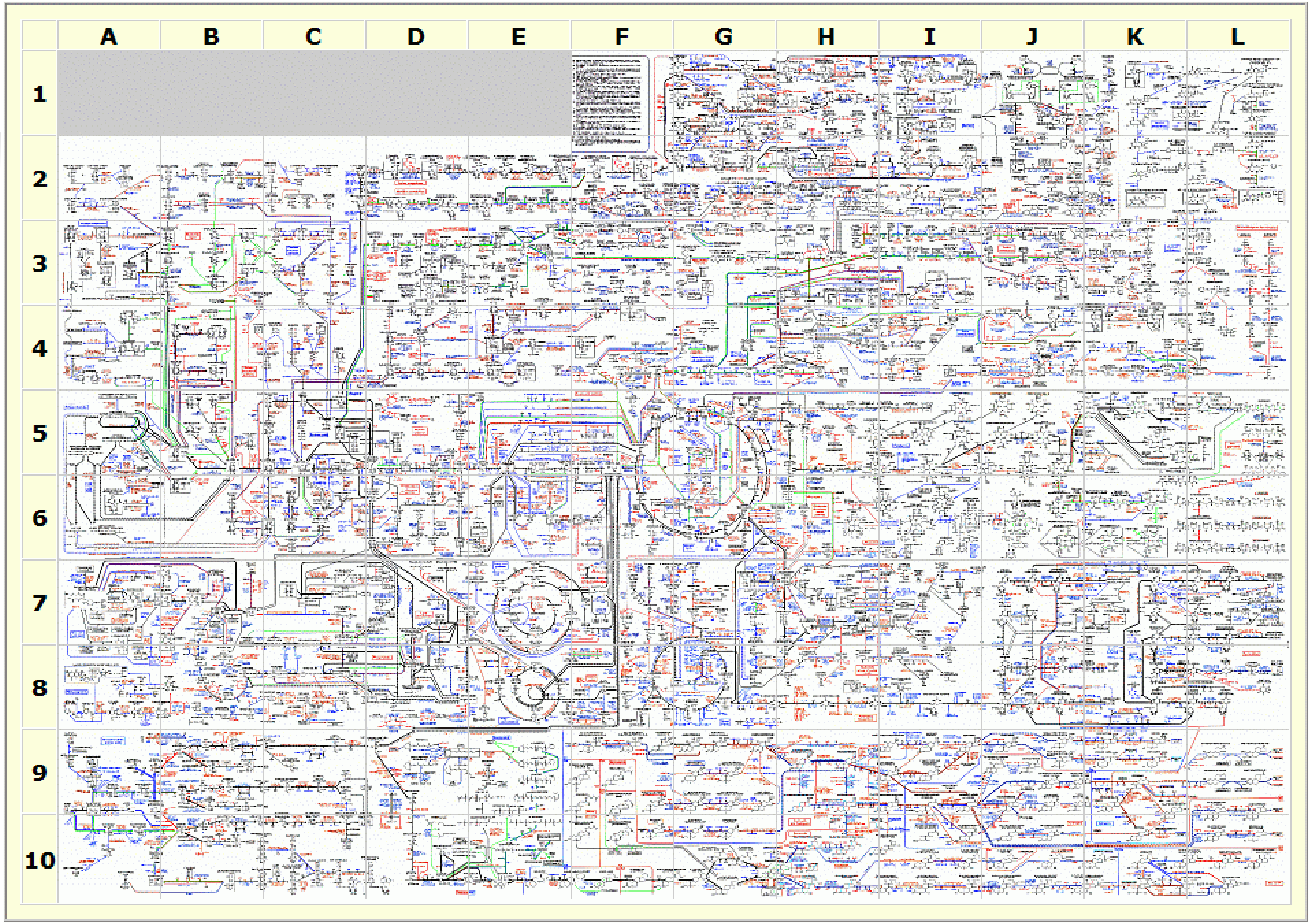
Bioinformatics – Disease Interaction



Source: Wiki Commons: Human disease network - CC BY-SA 4.0

Bioinformatics – Molecular Metabolic Network

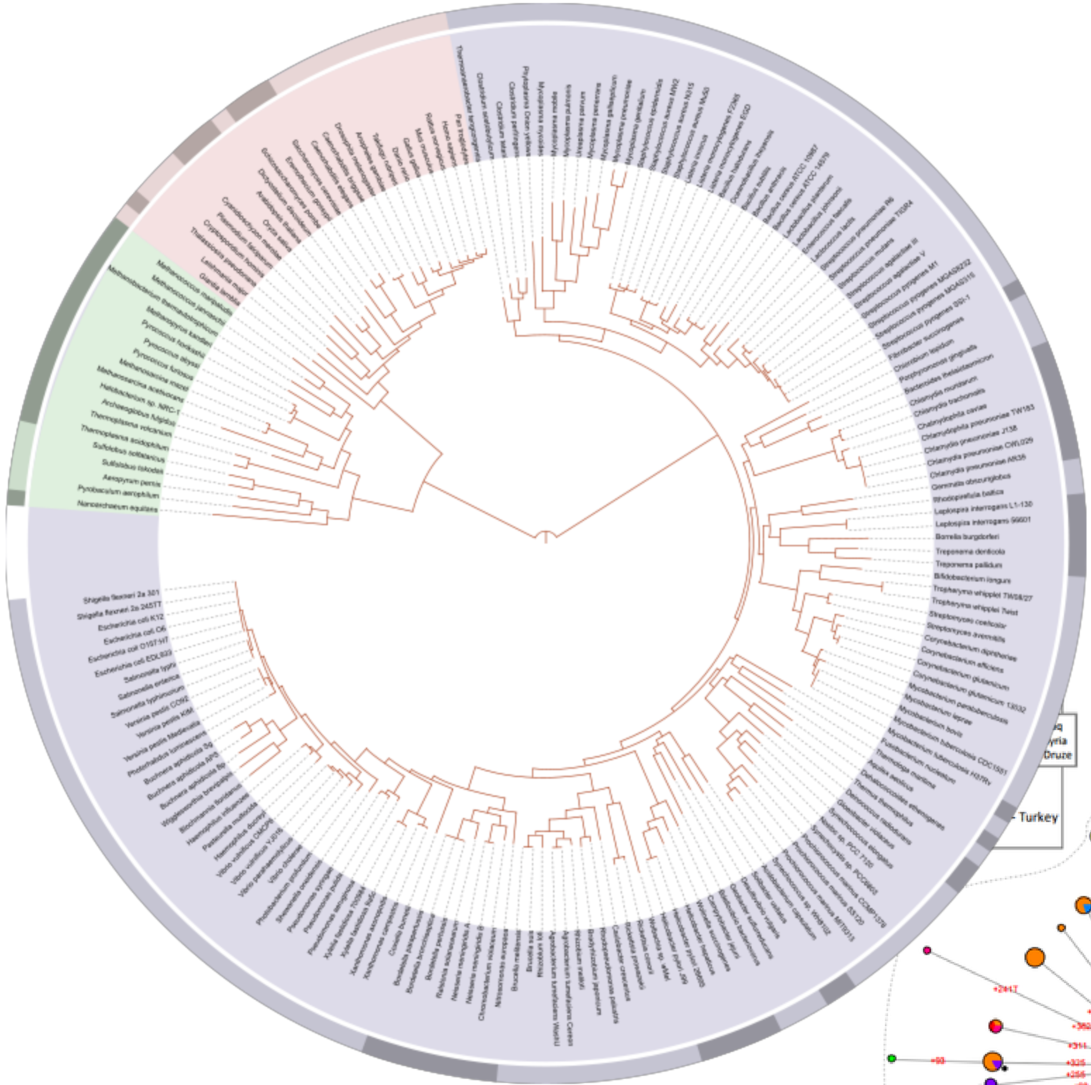


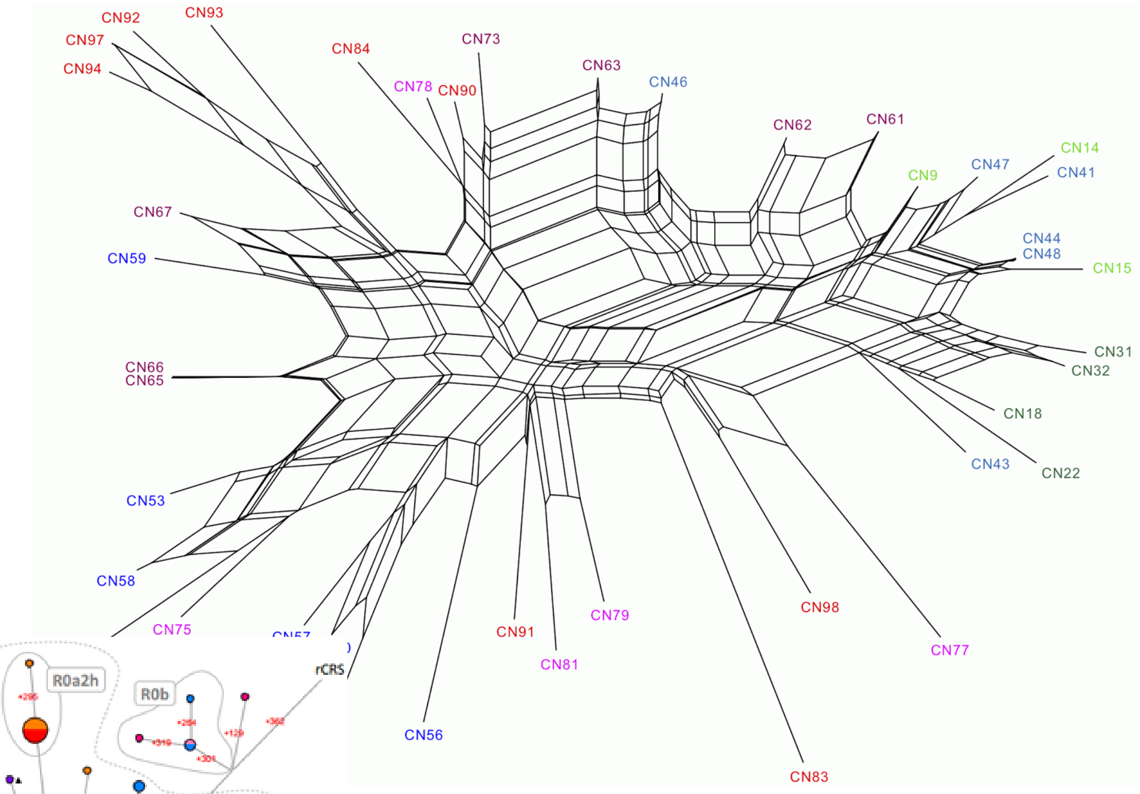


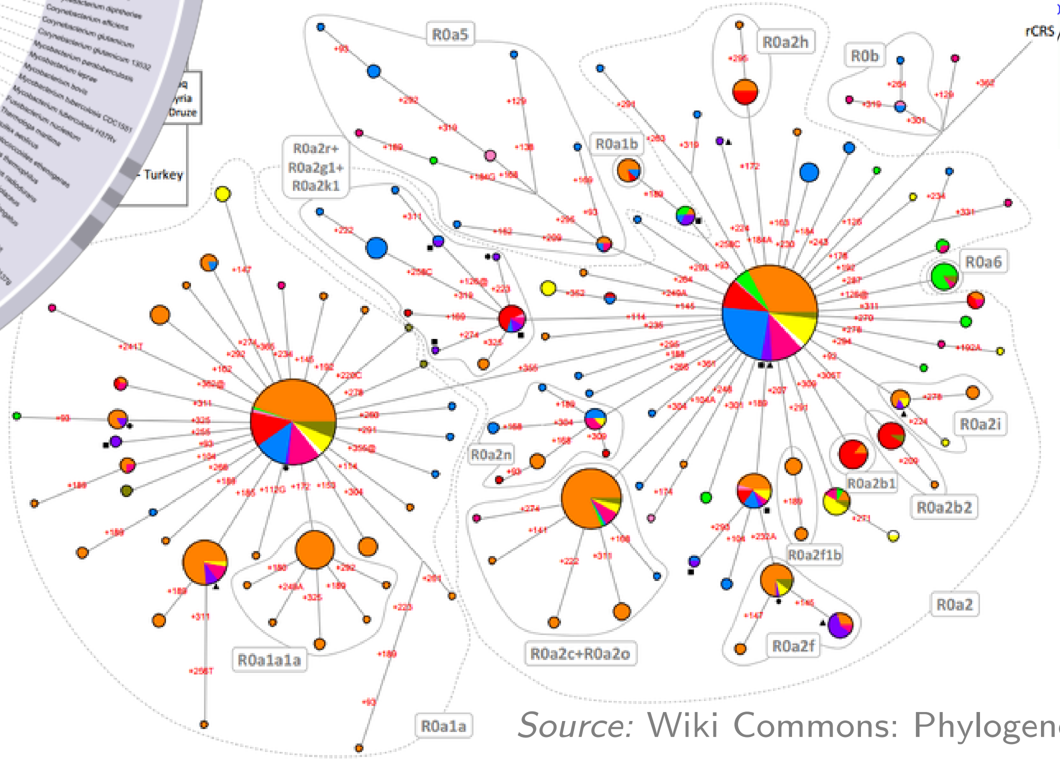
Source: Wiki Commons: Citric acid cycle with aconitate 2 - CC BY-SA 3.0

Source: Thiele et al., Nature Biotechnology 31, 419–425 (2013)

Bioinformatics – Phylogenetic Trees & Networks

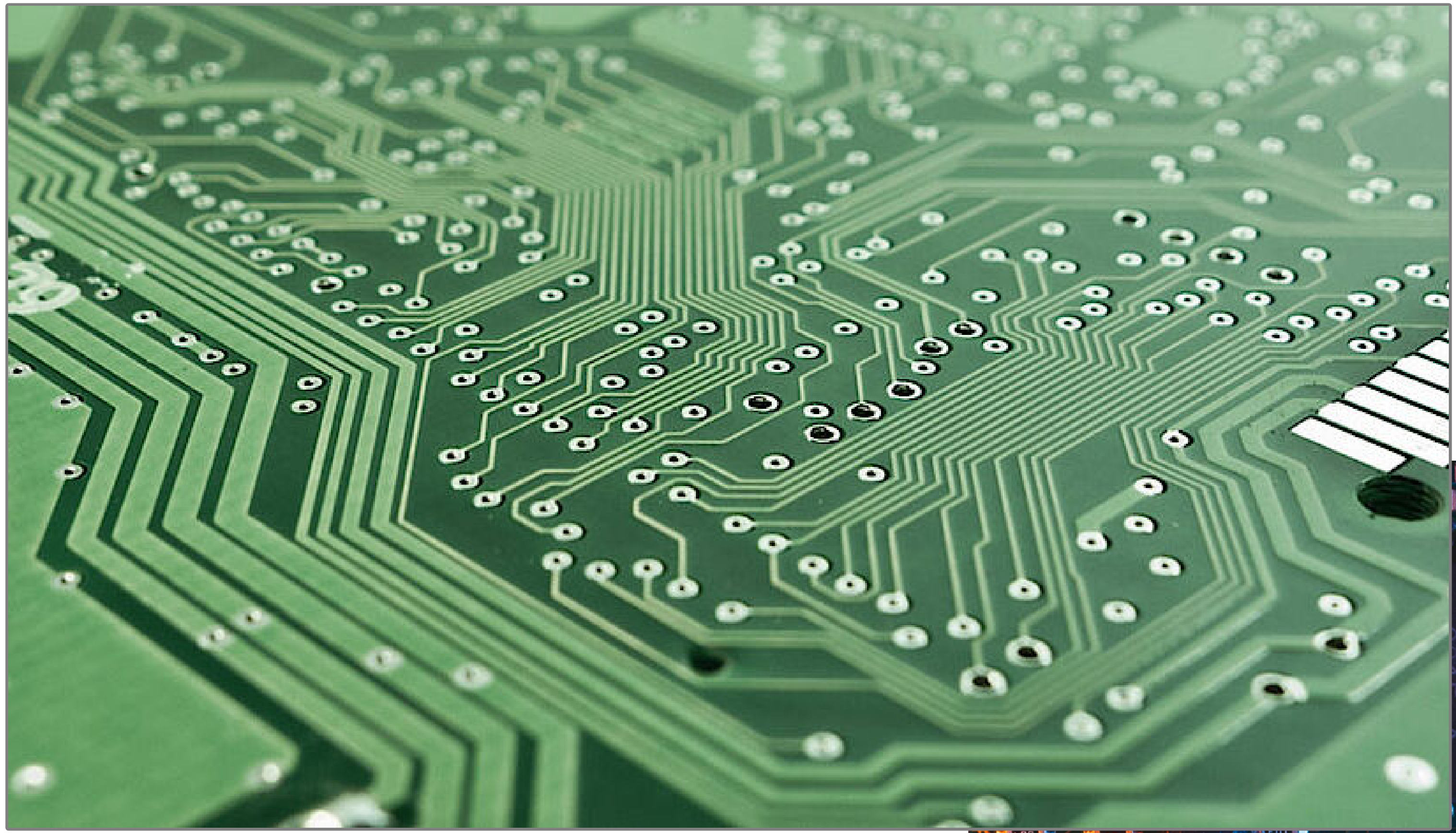




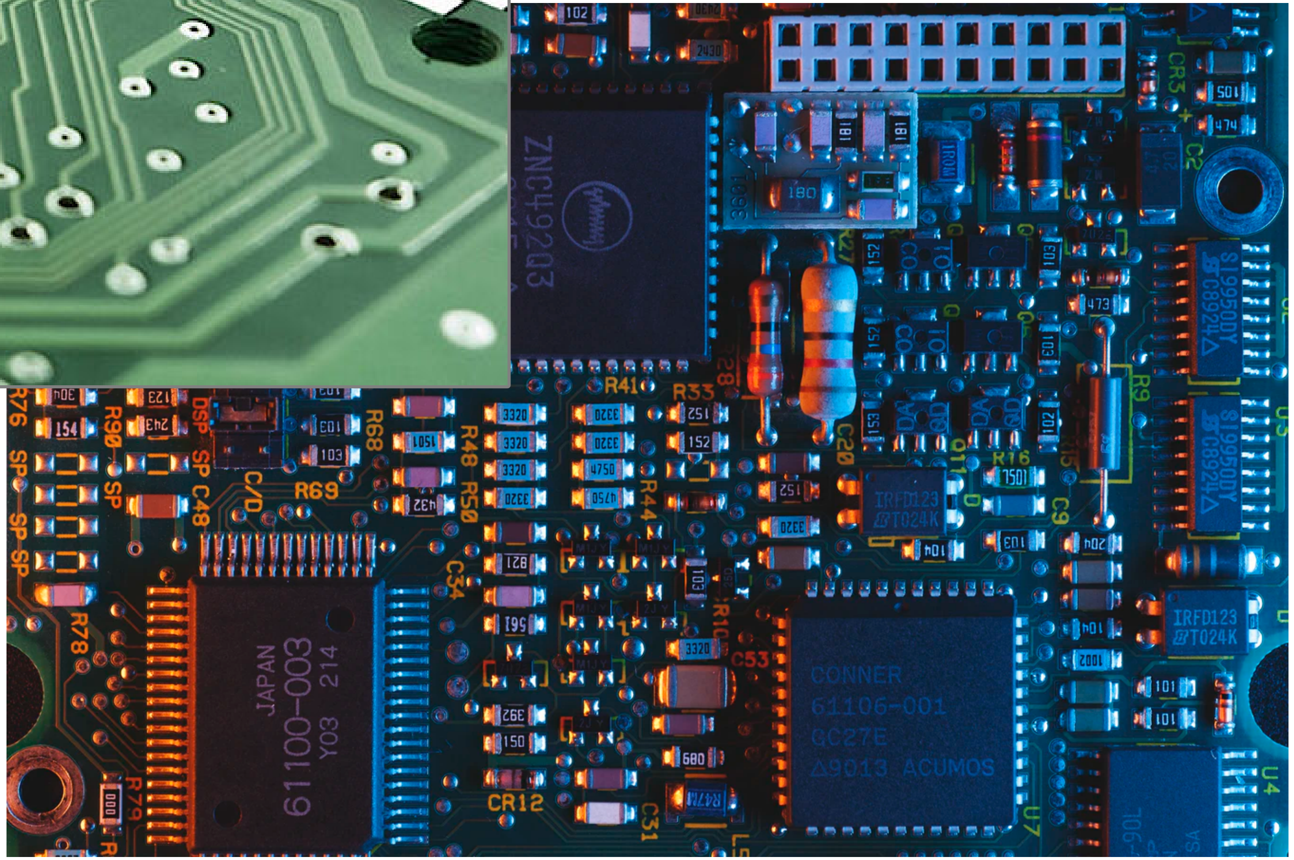


Source: Wiki Commons: Phylogenetic network of HVS-I variation - CC BY 4.0

Technical Network – Very Large-Scale Integration (VLSI)

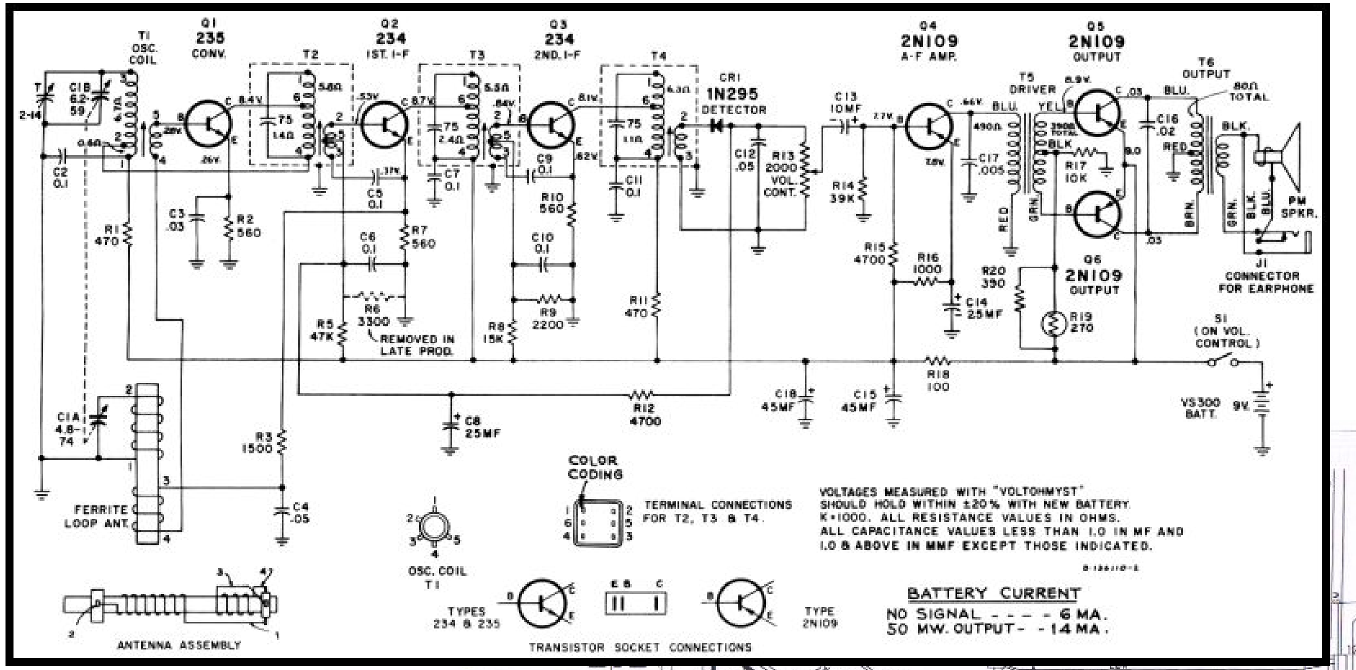


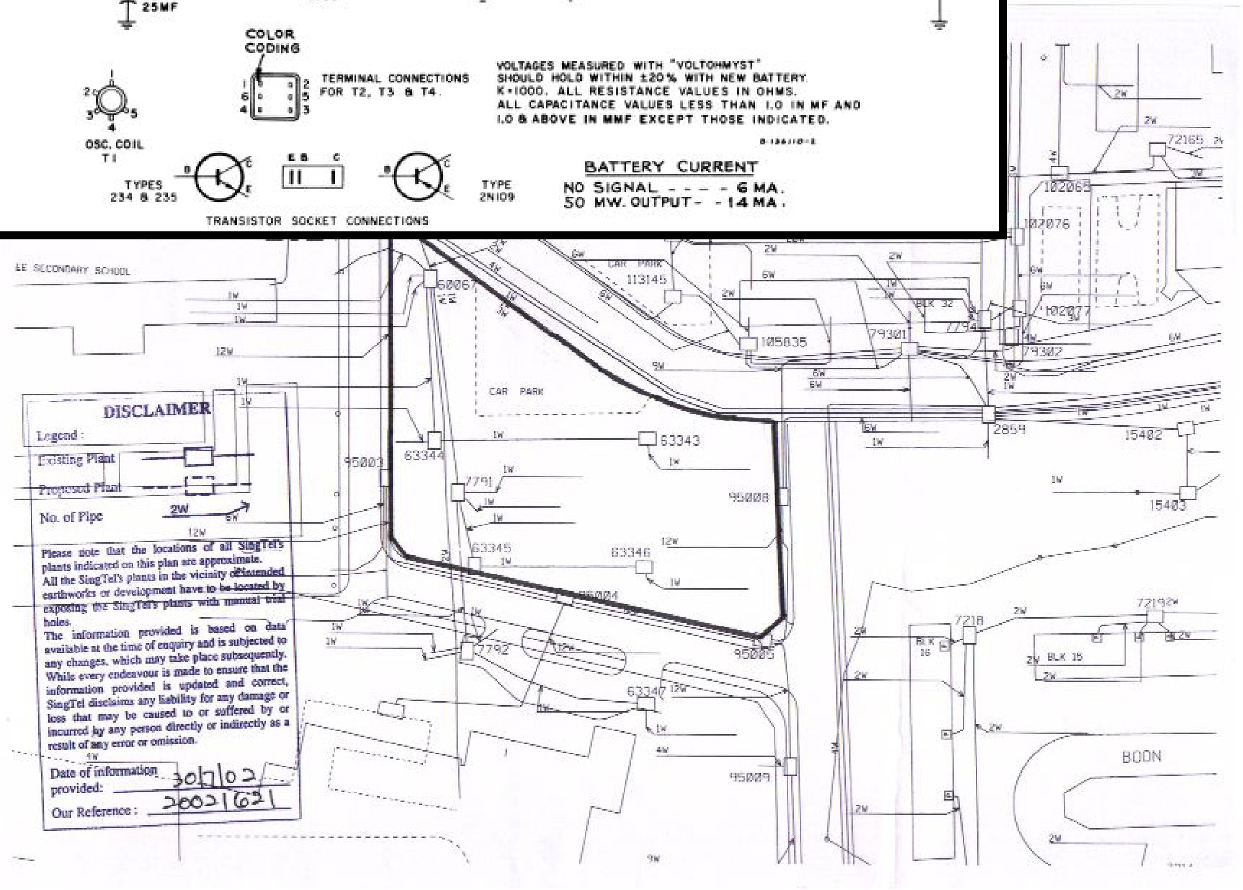
Source: Pixabay

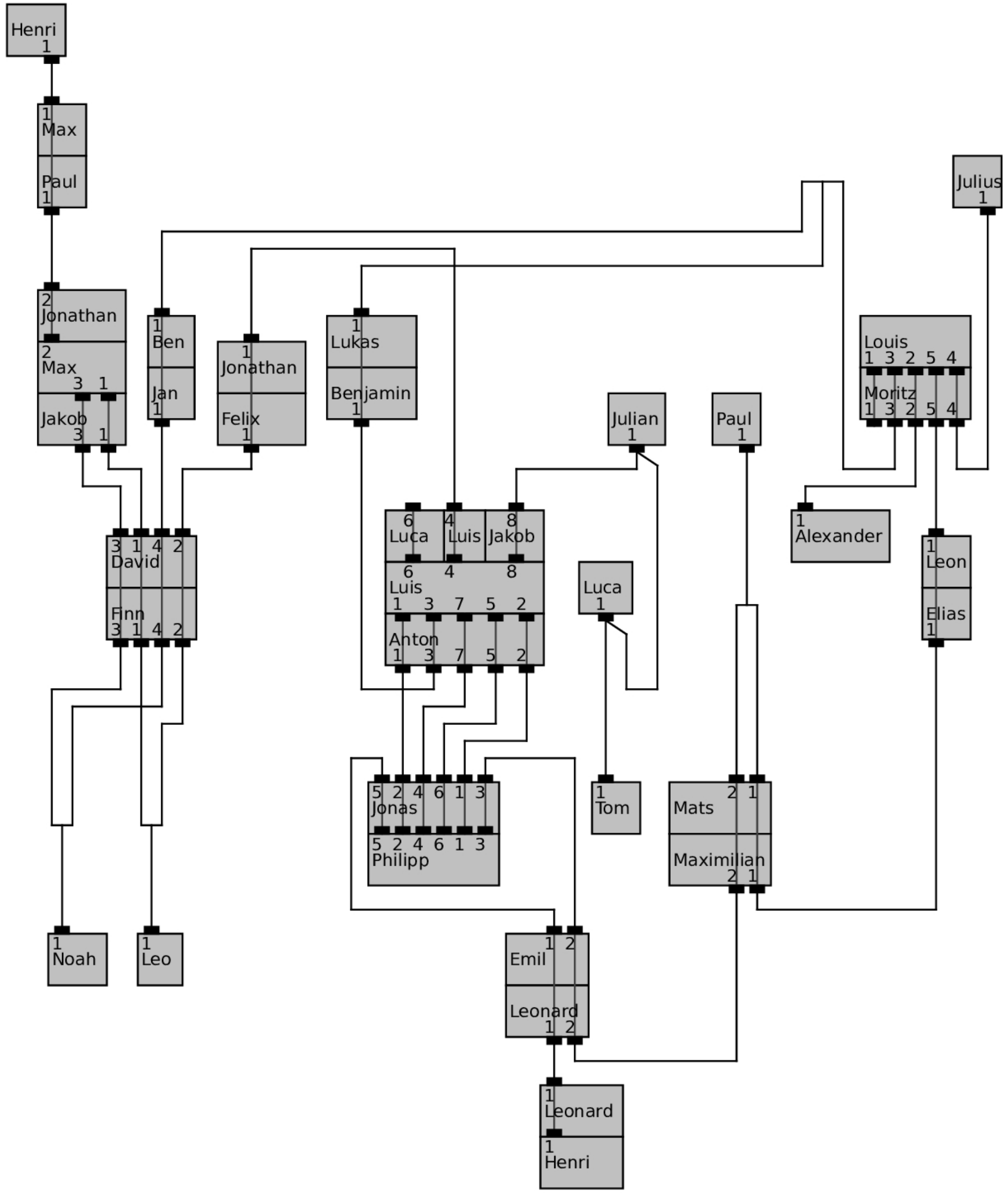


Source: Wiki Commons: Diopsis - CC BY-SA 3.0

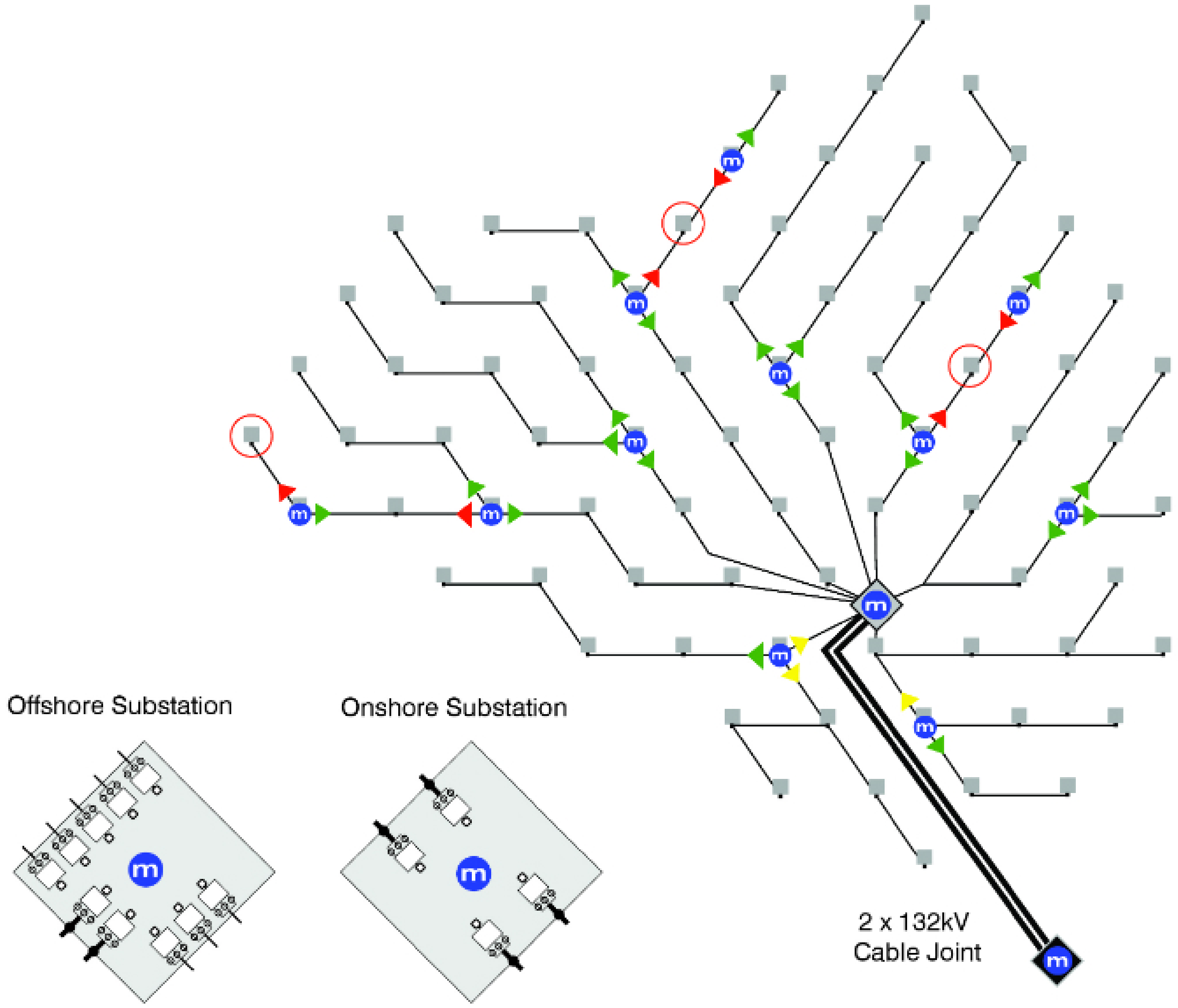
Technical Network – Transistor Diagram, Wiring Diagram





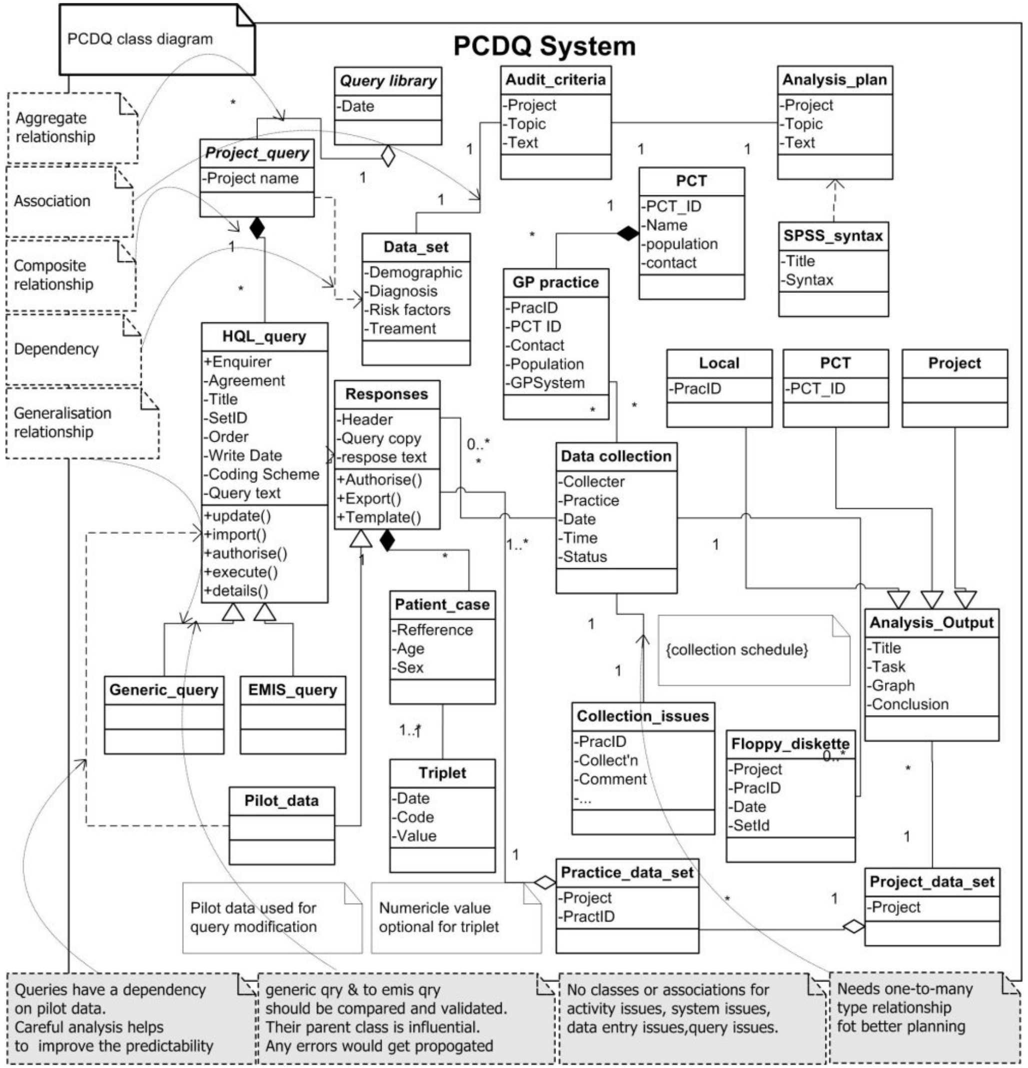


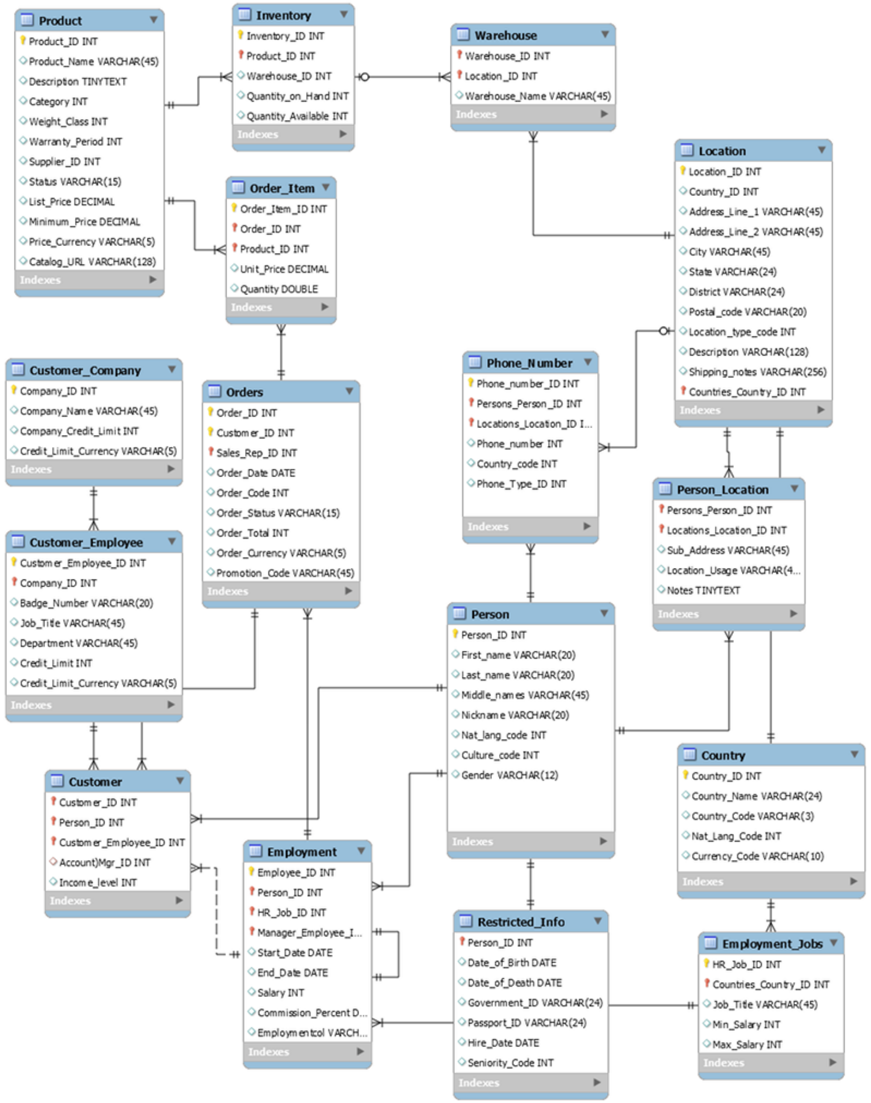
Technical Networks – Offshore Wind Farms



Source: Wiki Commons: Alpha Ventus Windmills -

Technical Network – UML Diagram

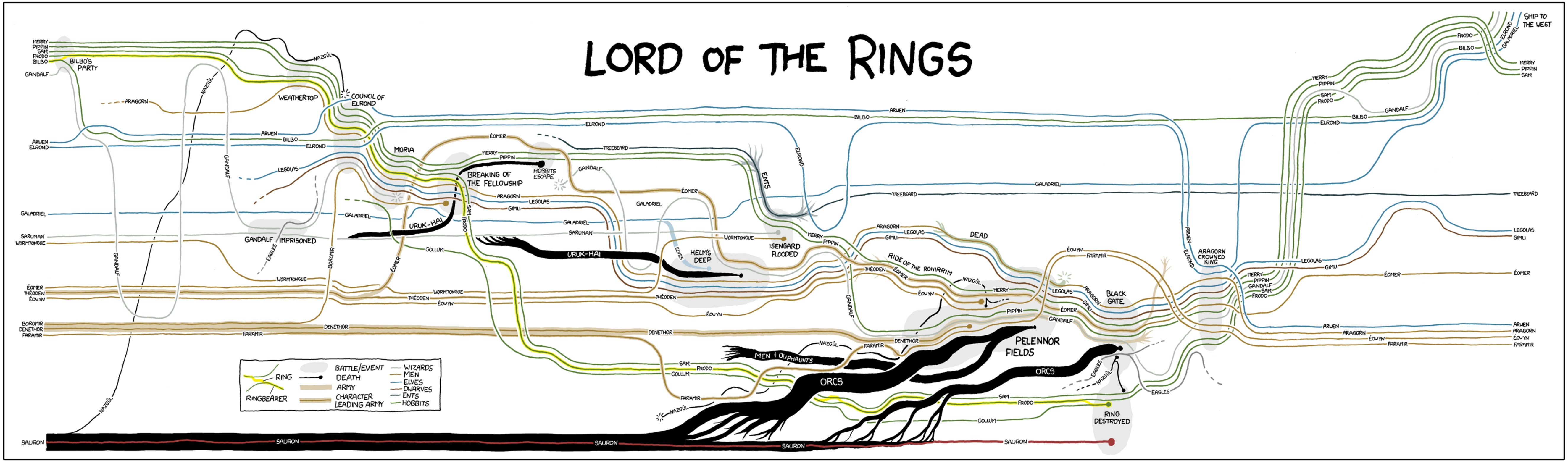




© AWS

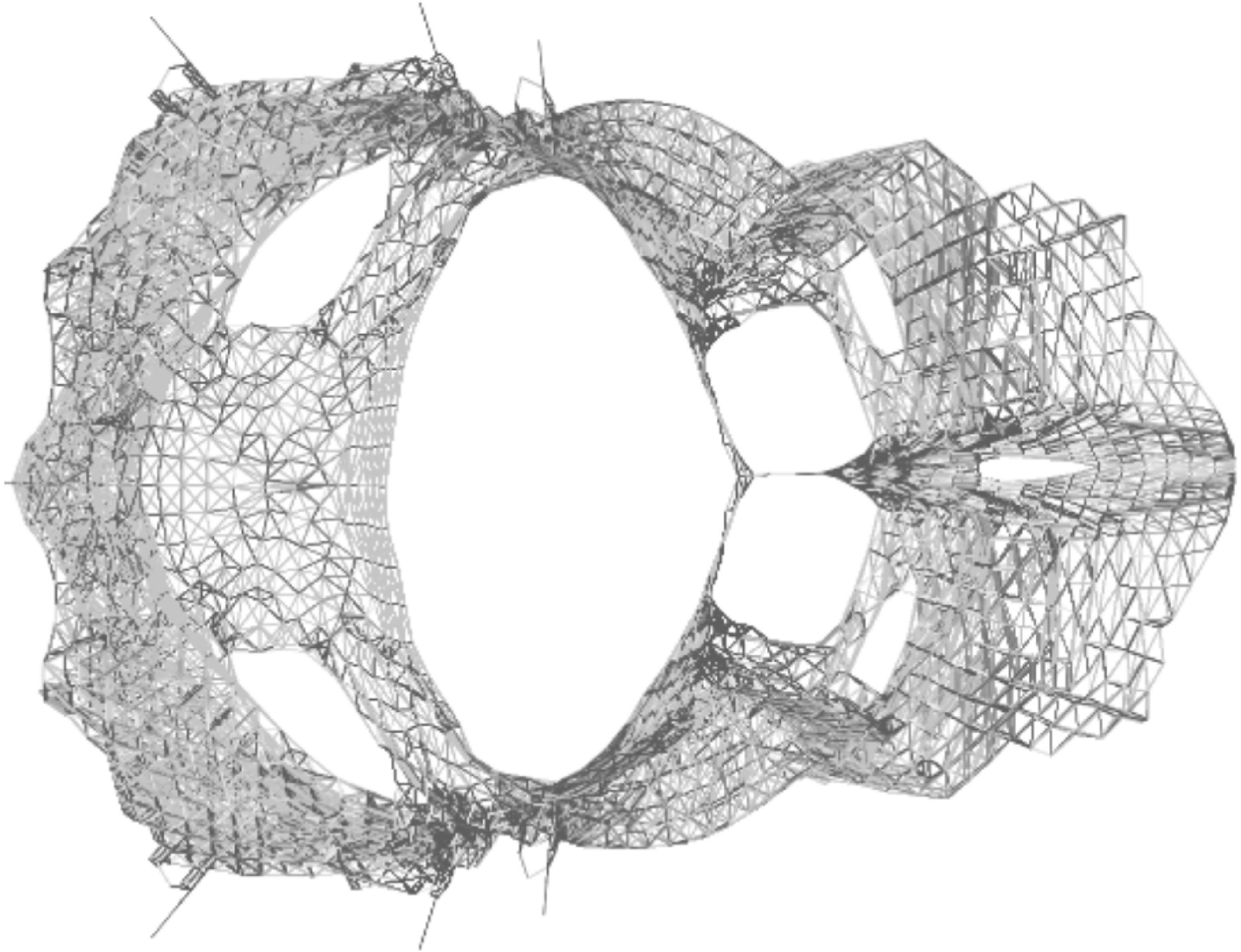
Temporal Graph Layout – Storylines

THESE CHARTS SHOW MOVIE CHARACTER INTERACTIONS.
THE HORIZONTAL AXIS IS TIME. THE VERTICAL GROUPING OF THE
LINES INDICATES WHICH CHARACTERS ARE TOGETHER AT A GIVEN TIME.

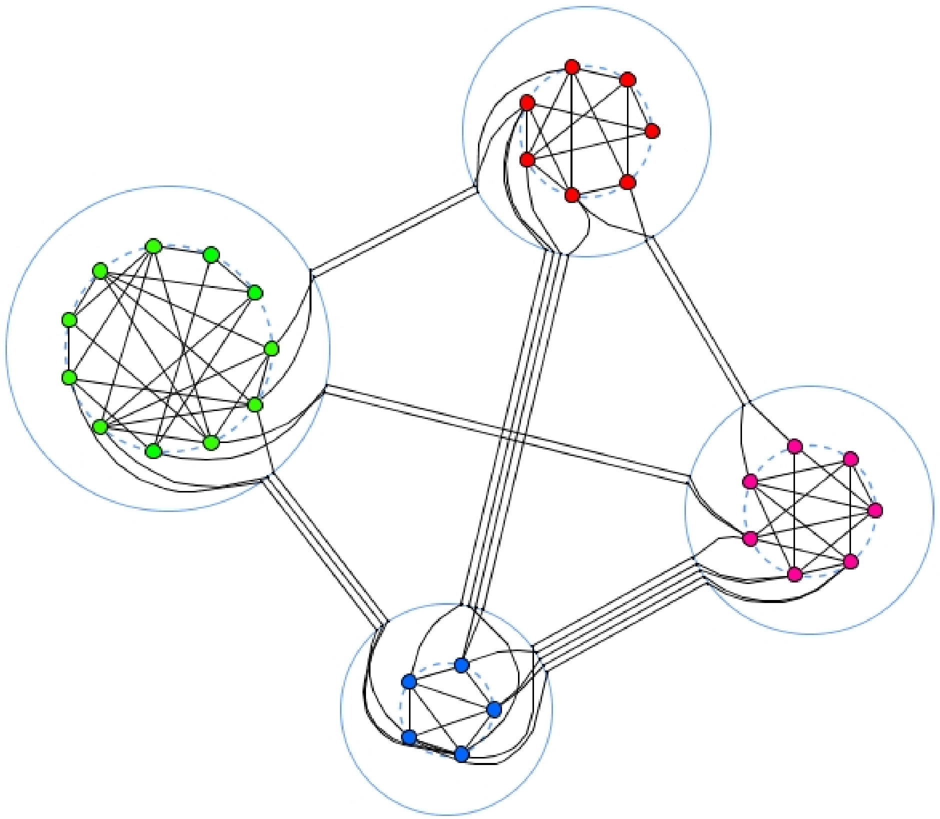


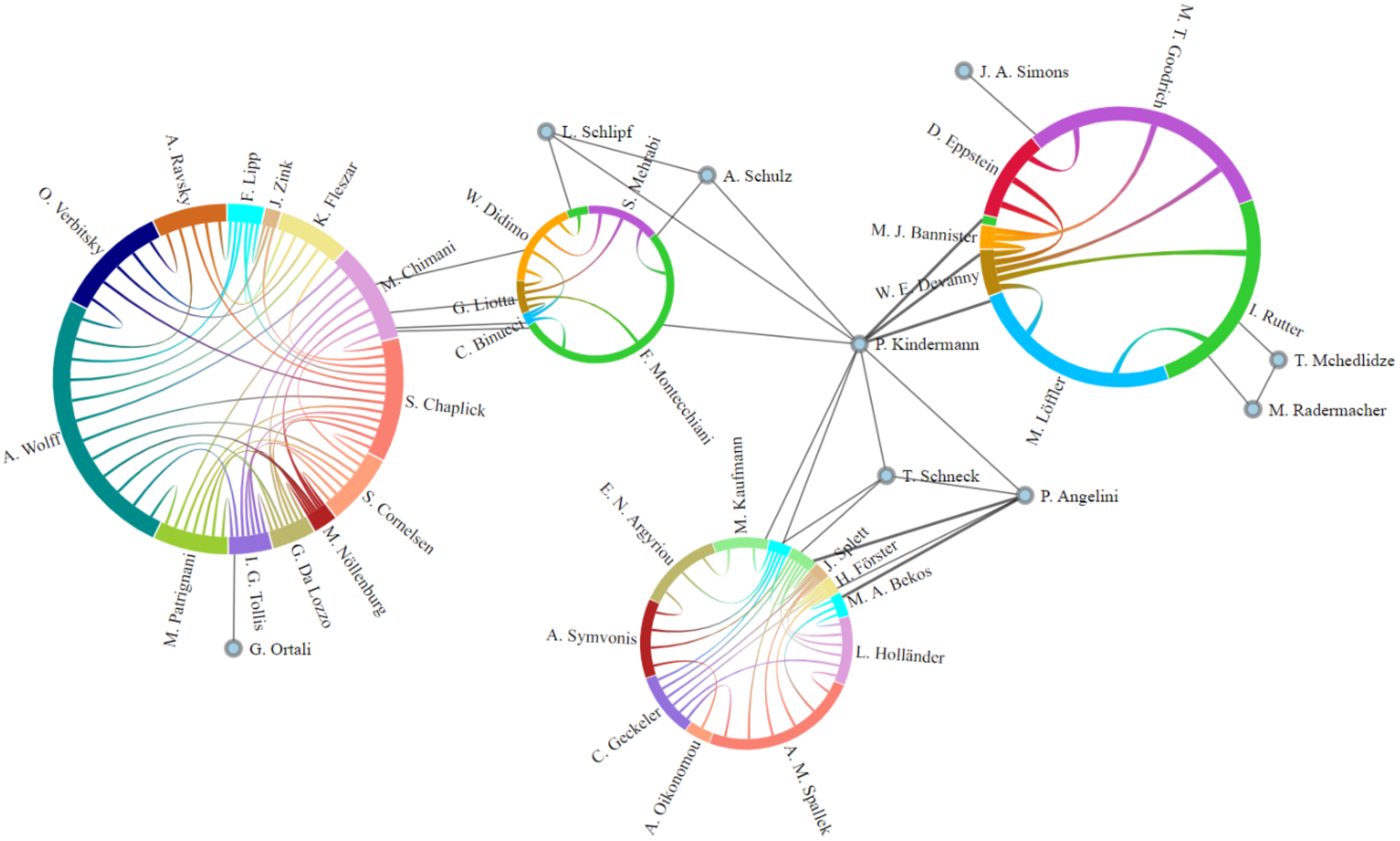
Source and more: xkcd Comic 657 – xkcd.com/657/

Large Graphs – Object Mesh



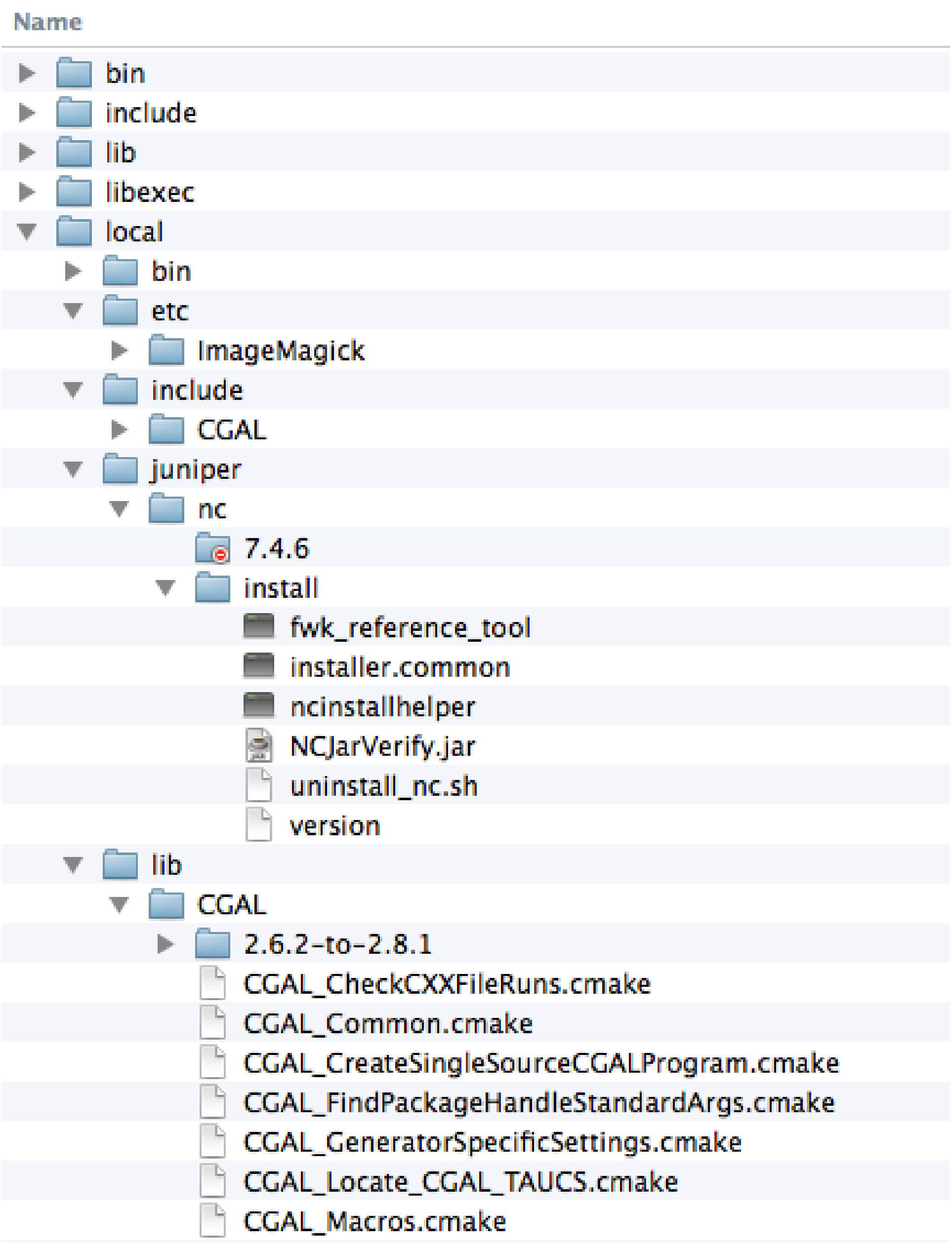
General Graphs – Micro–Macro Layout

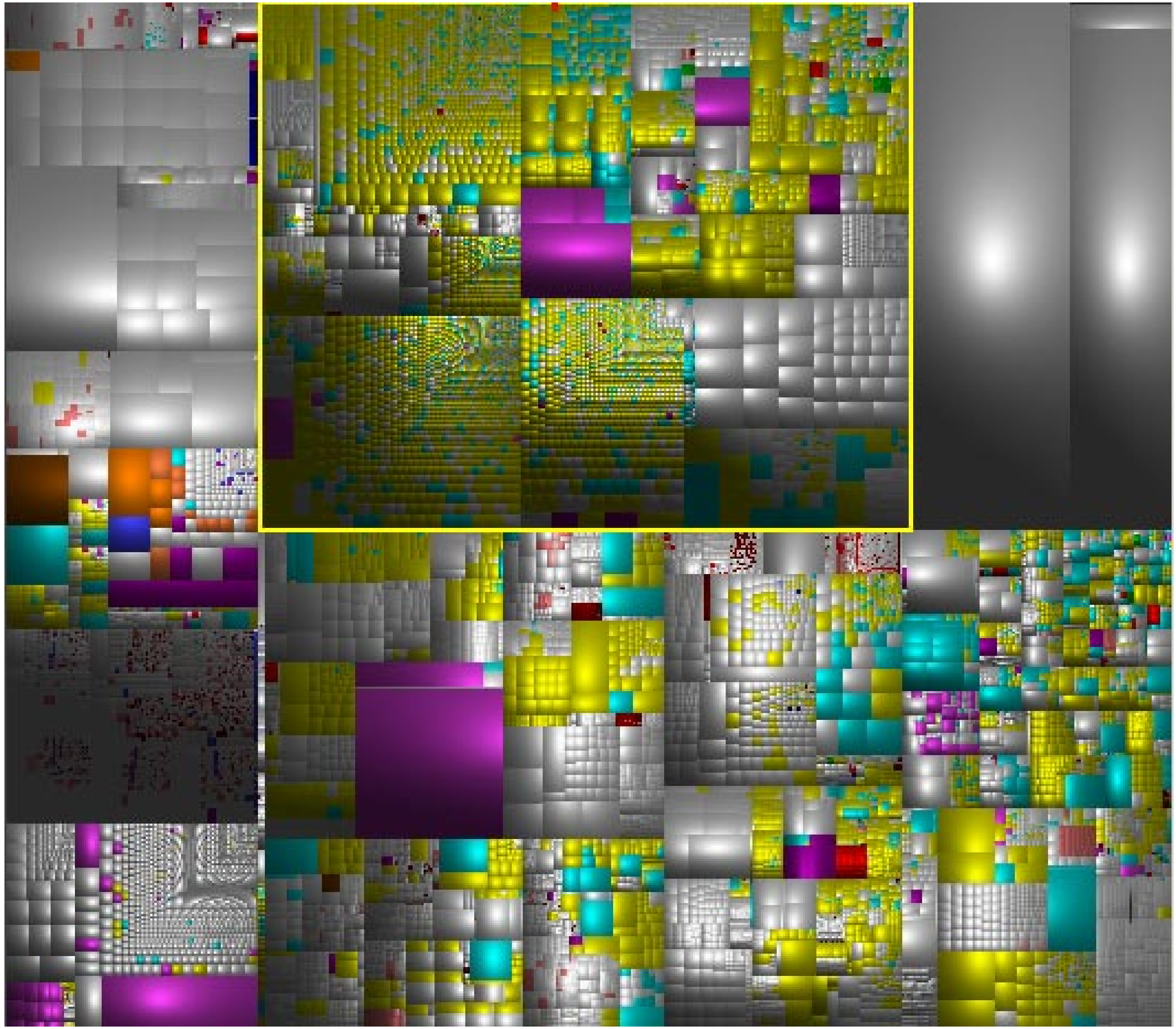




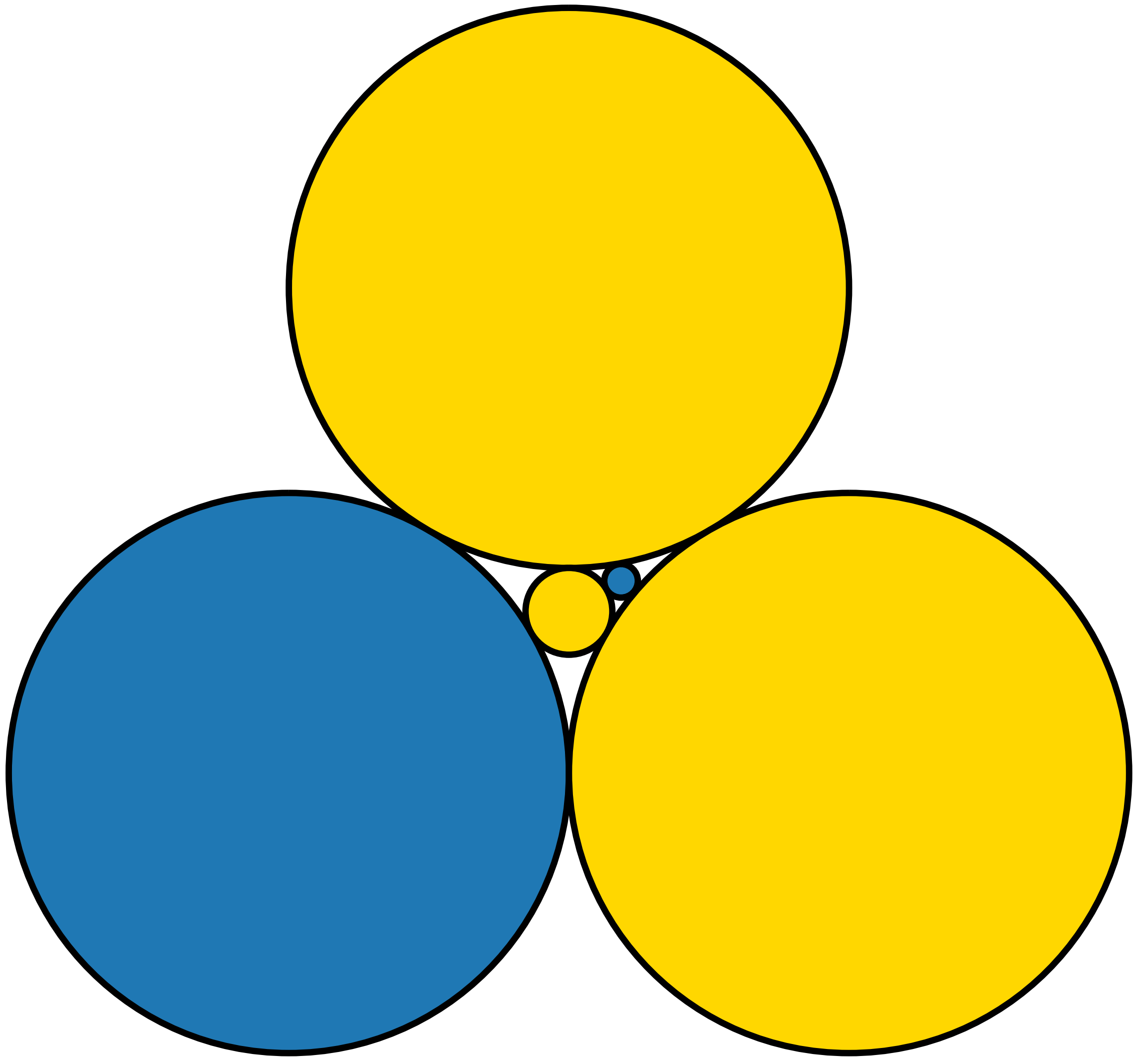
Source: Angori et al., ChordLink: A New Hybrid Visualization Model, GD'19 (2019)

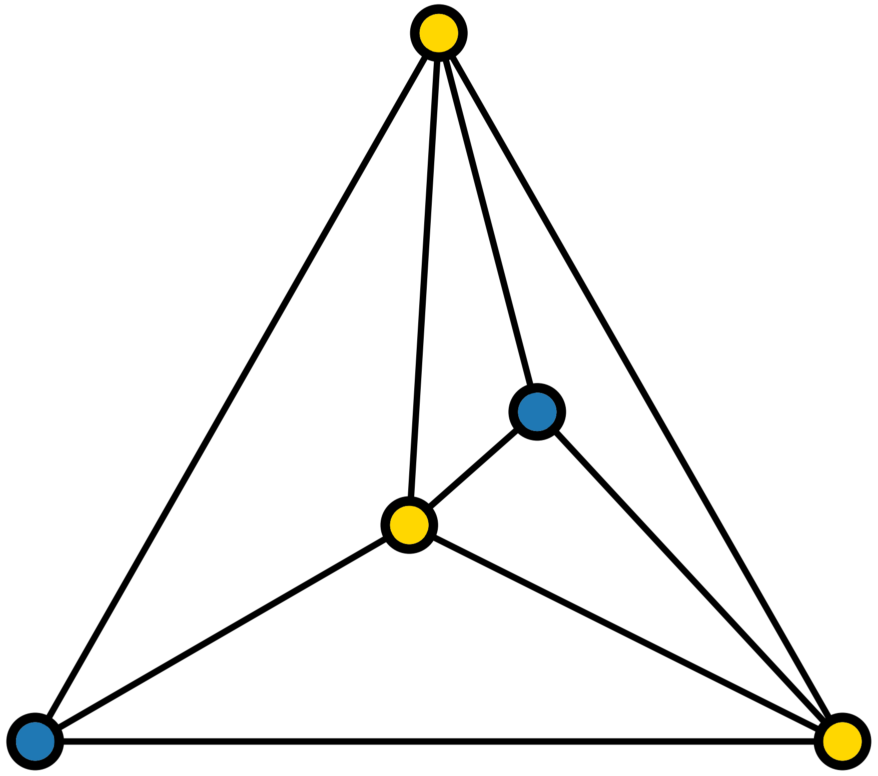
Alternative Representations – Treemap





Alternative Representations – Contact Graphs





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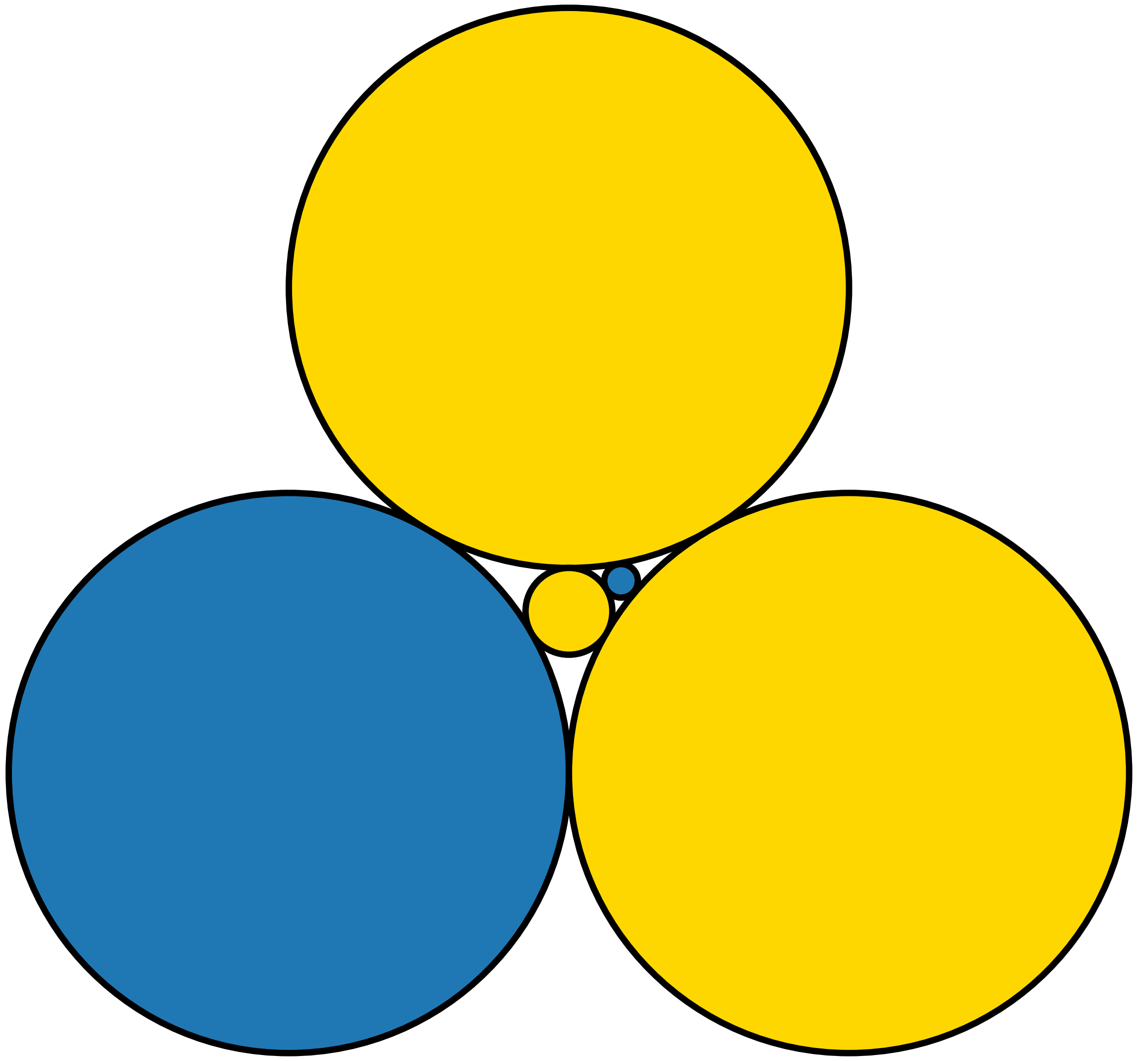
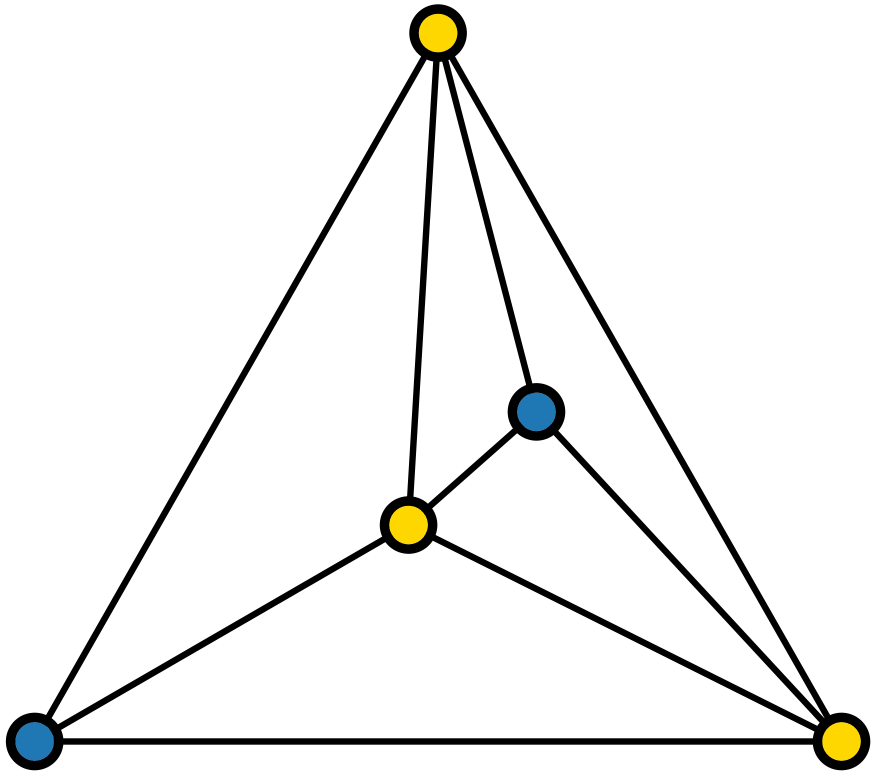


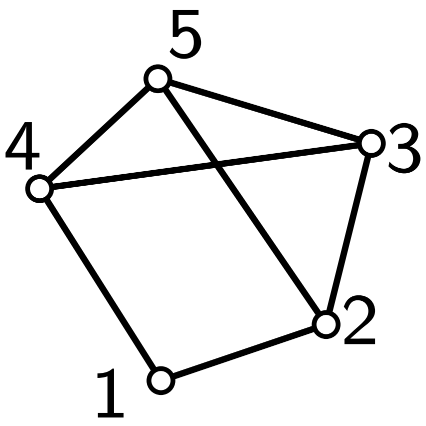


- For more examples see `visualcomplexity.com`

Requirements of a Graph Layout

1. Drawing conventions and requirements, e.g.,

Requirements of a Graph Layout



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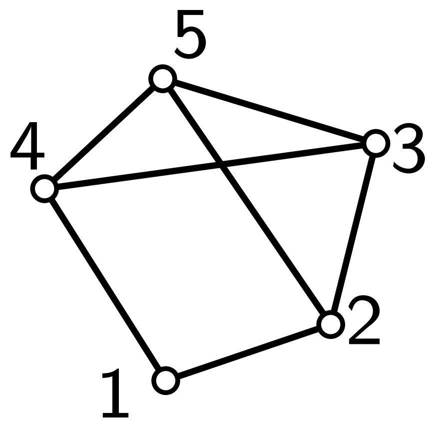

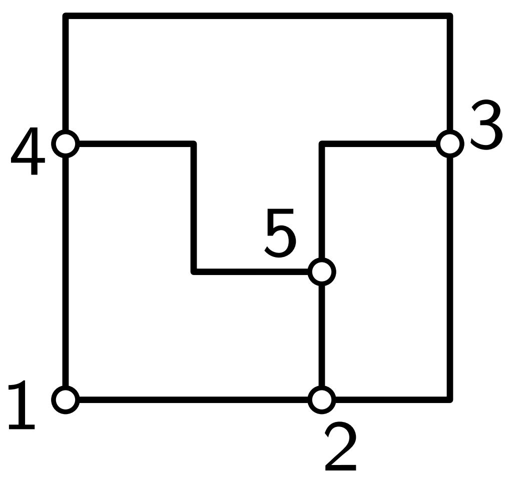

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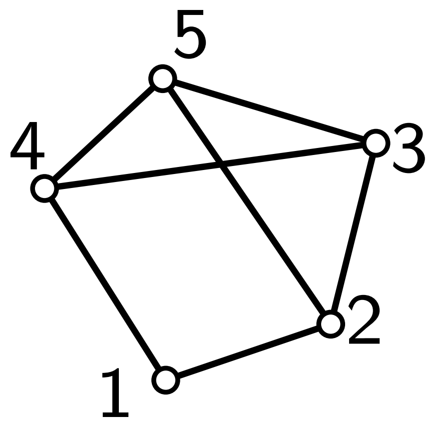

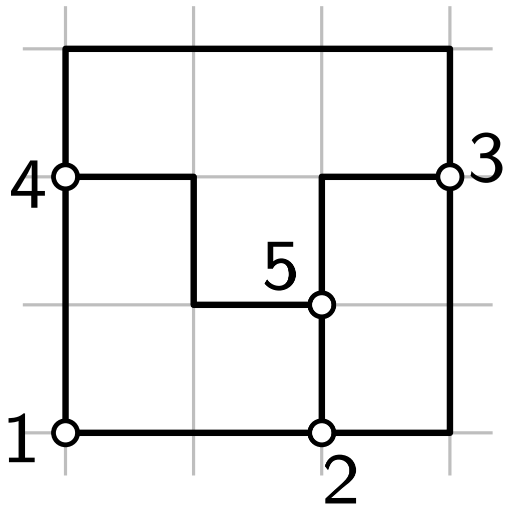

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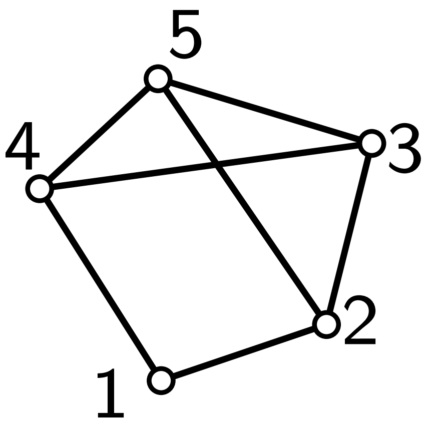

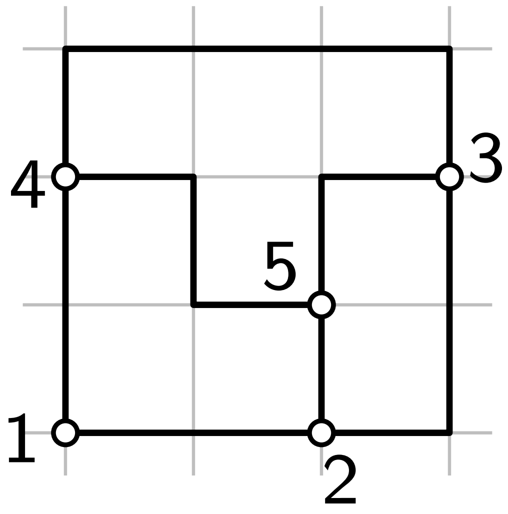

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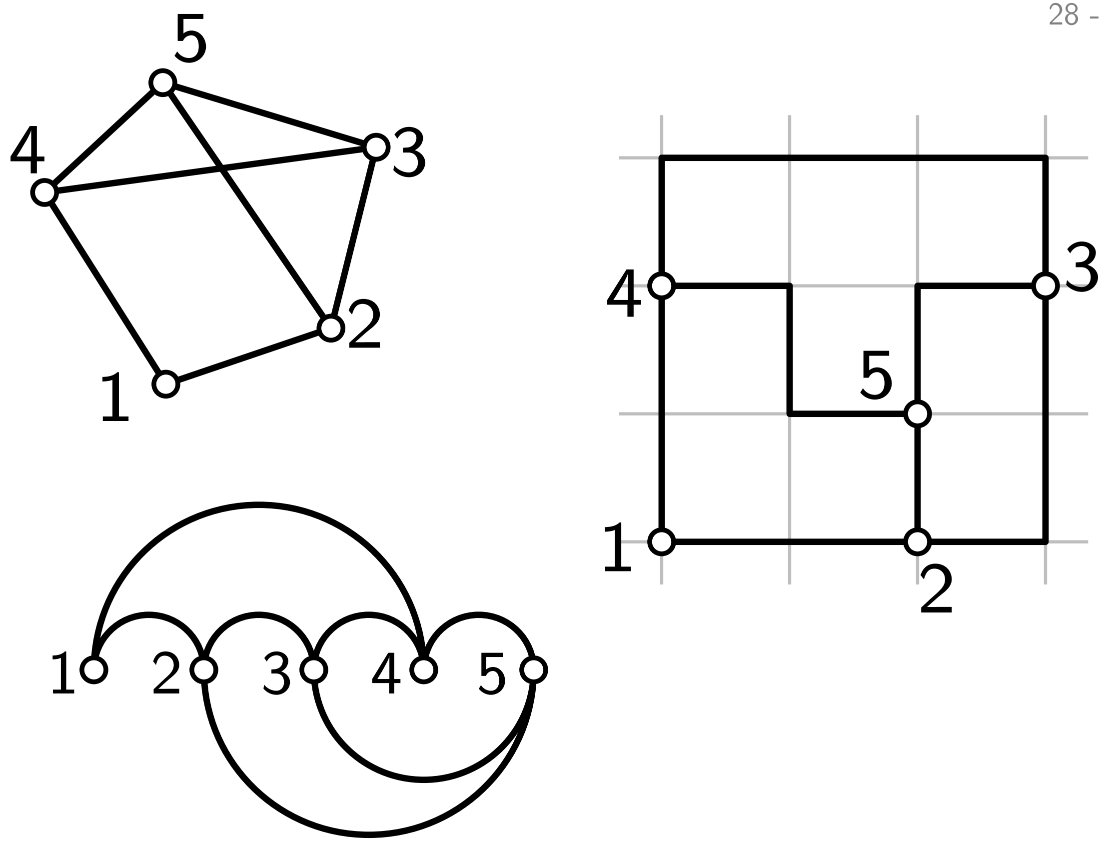

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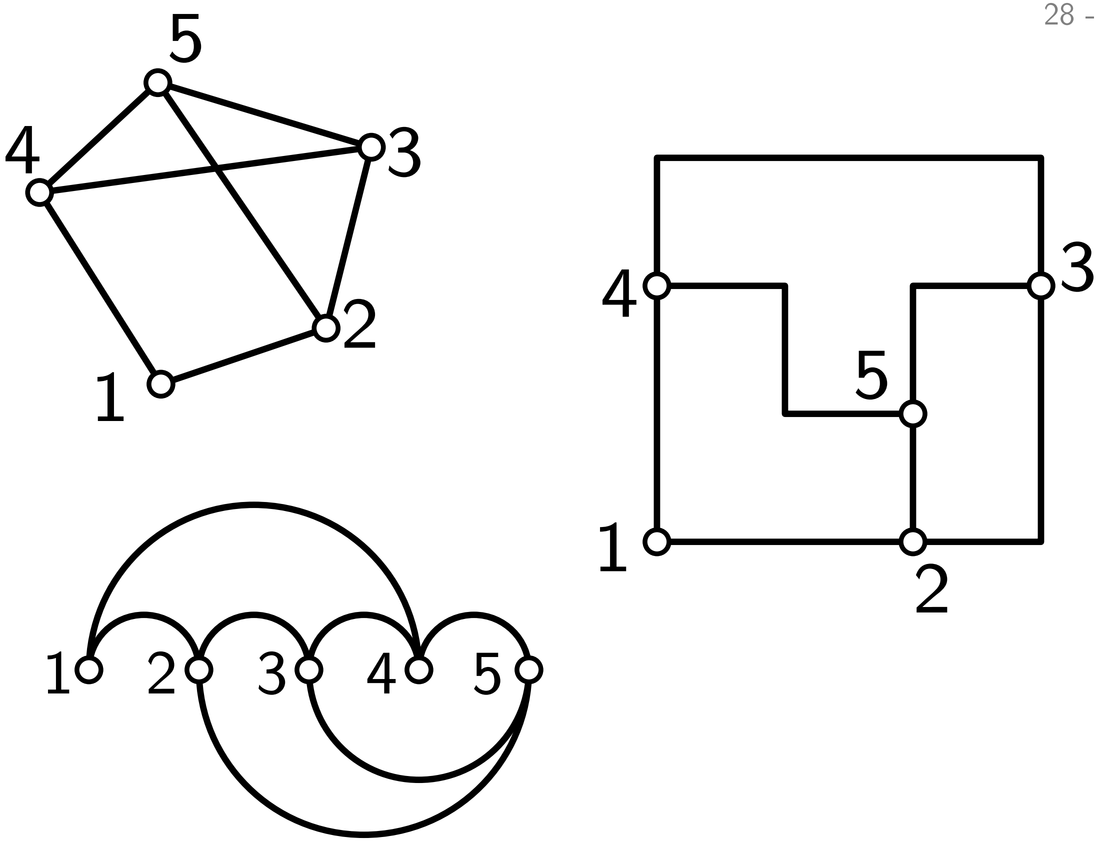

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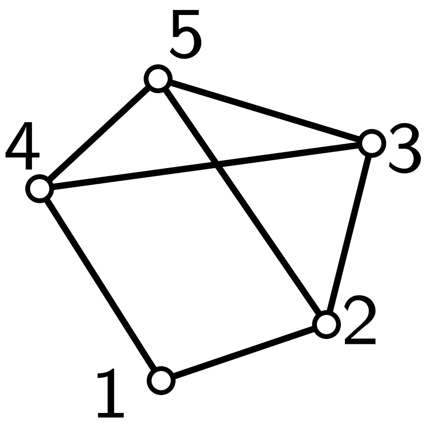

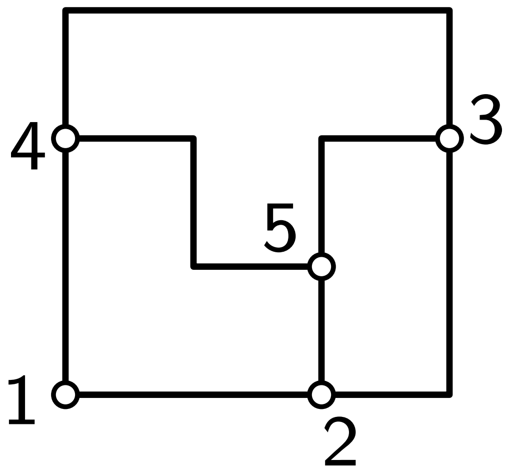

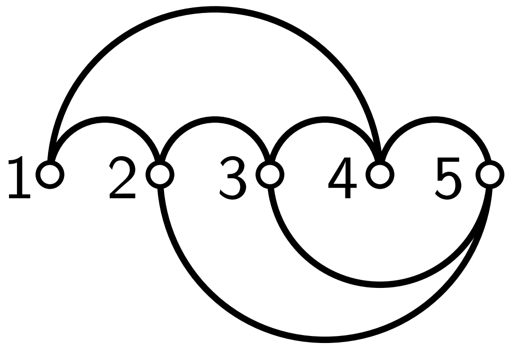

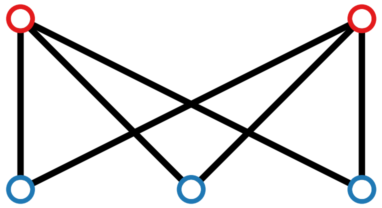

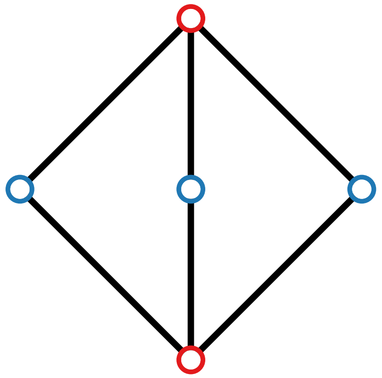

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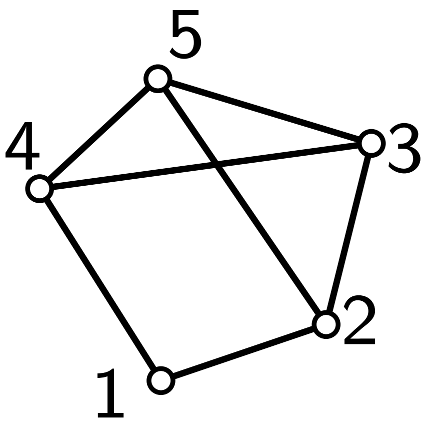

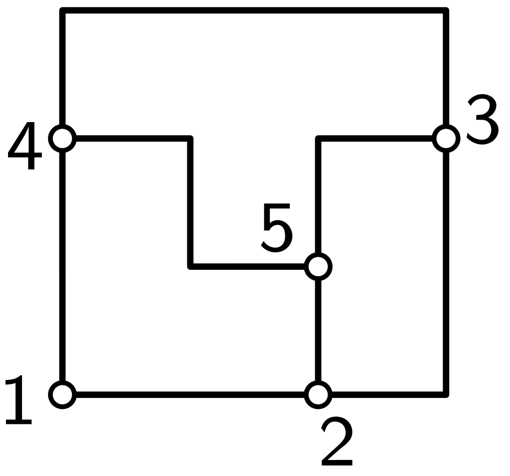

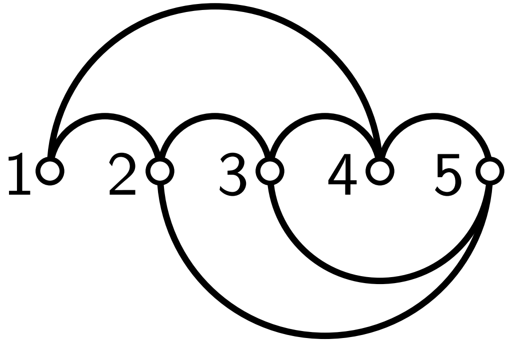

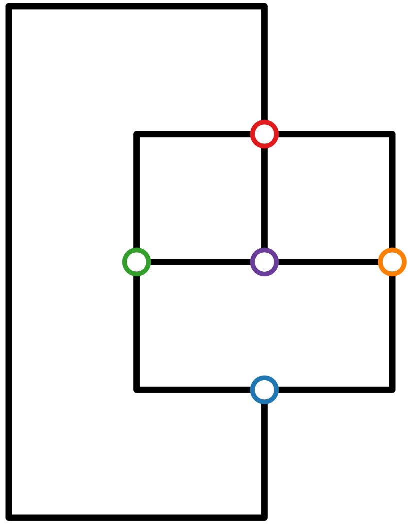

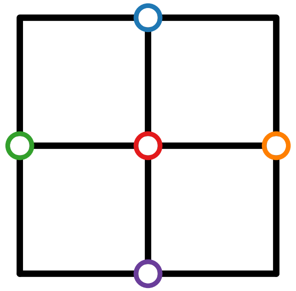

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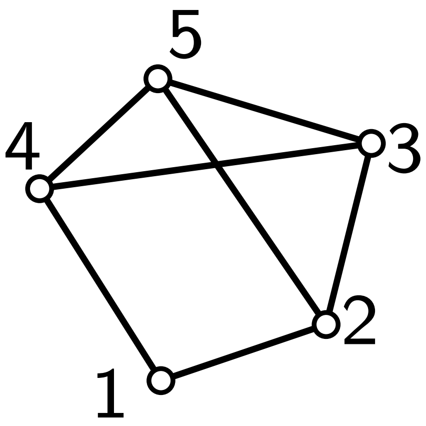

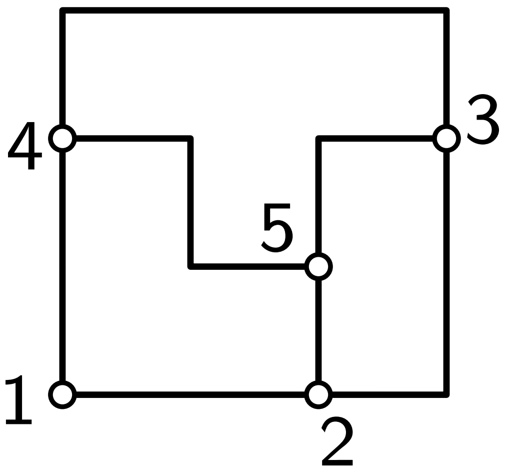

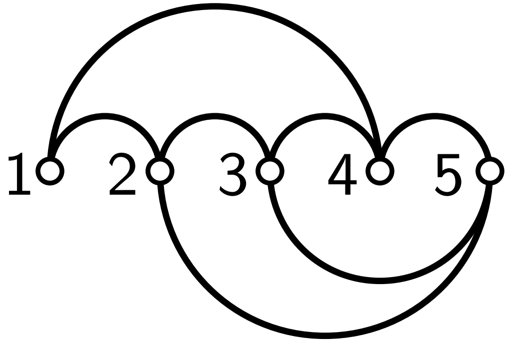

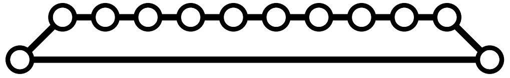

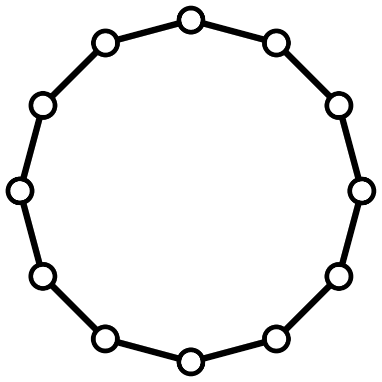

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   - without crossing
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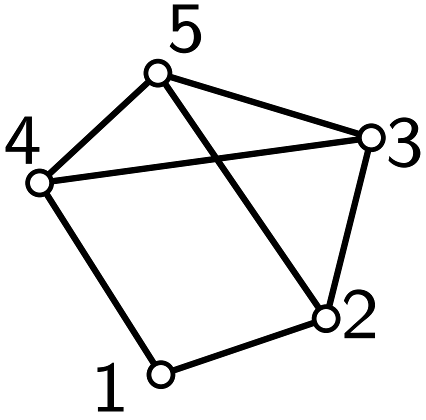
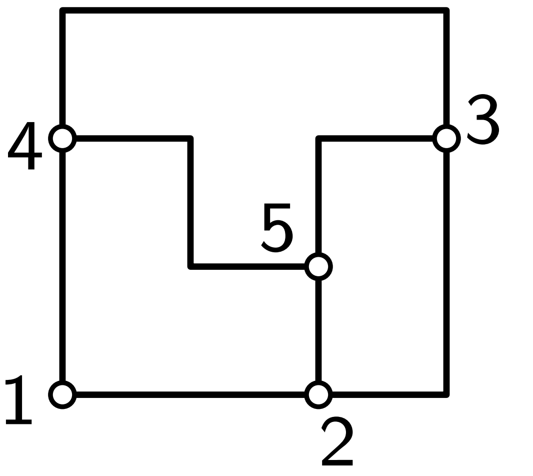
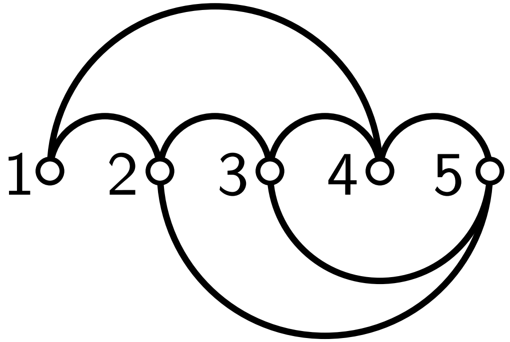
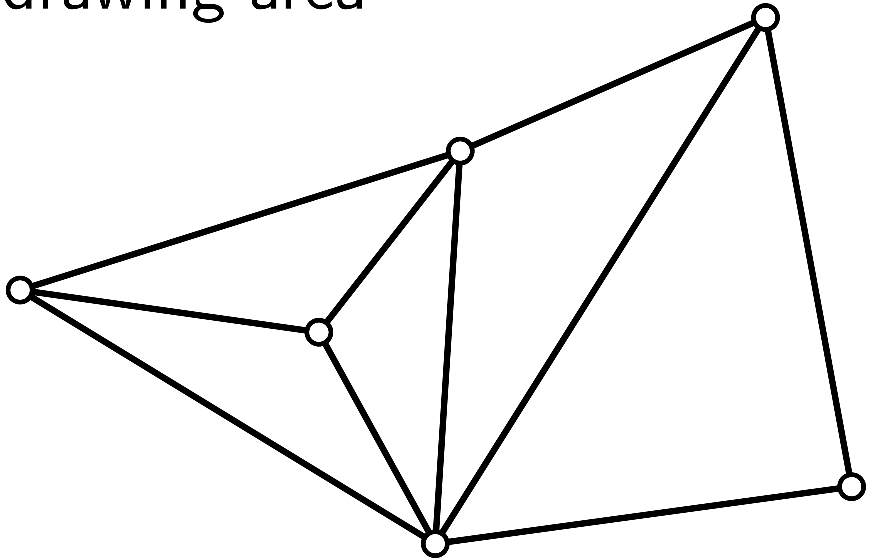
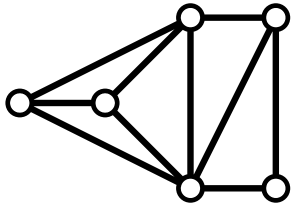
   - crossing/bend minimization
   - edge length uniformity
   - minimizing total edge length/drawing area











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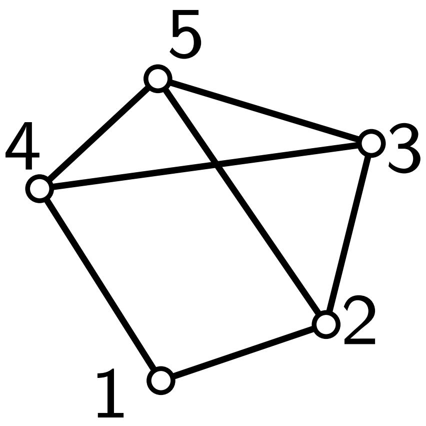

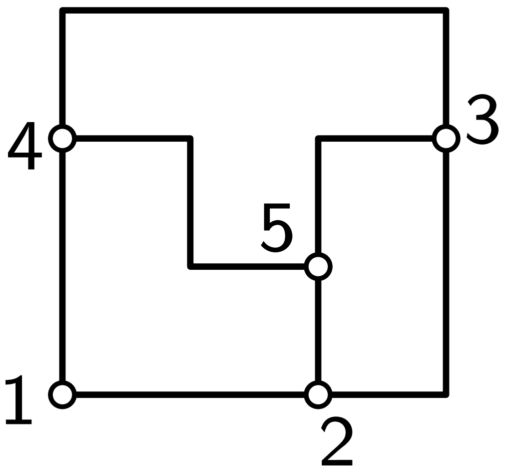

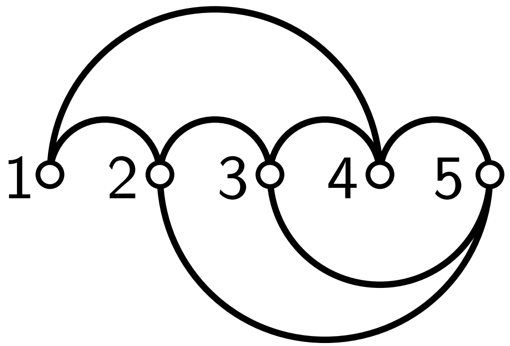

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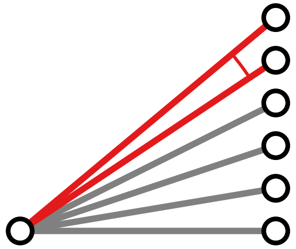

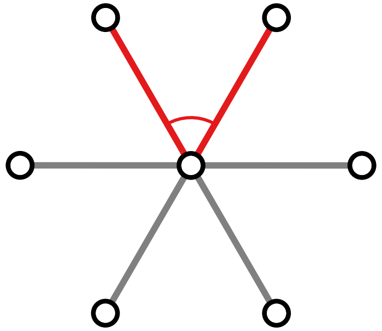

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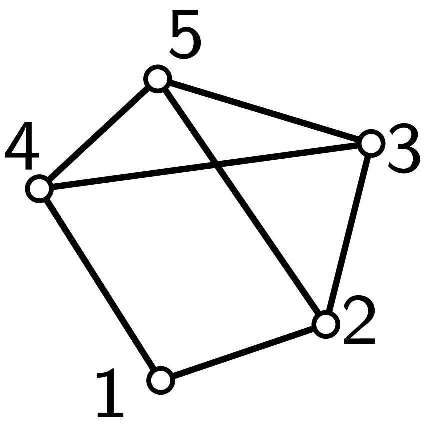

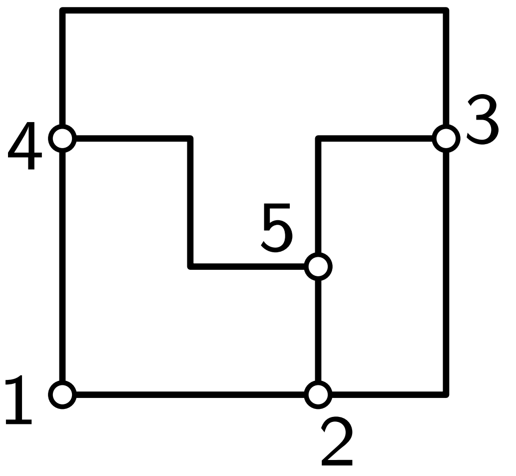

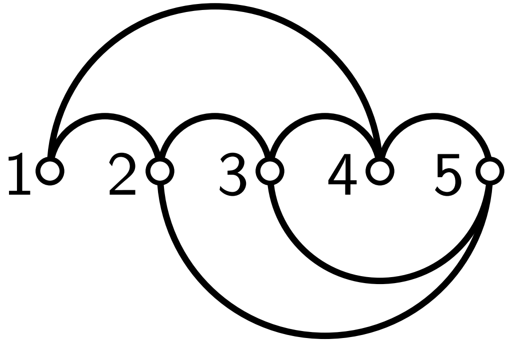

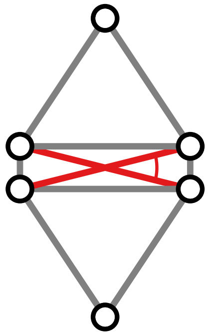

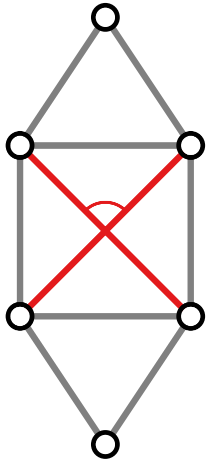

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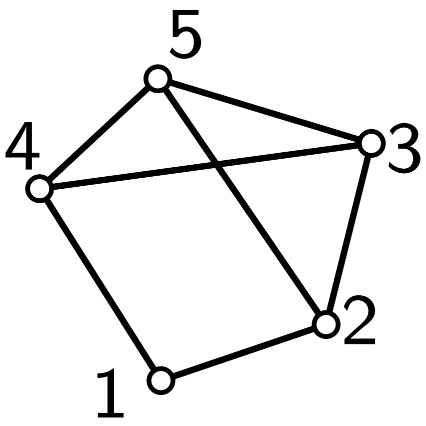

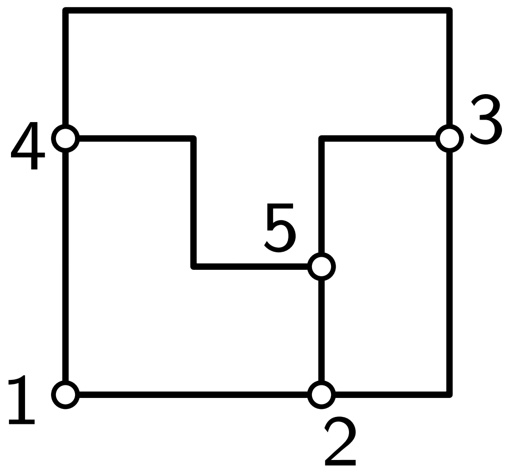

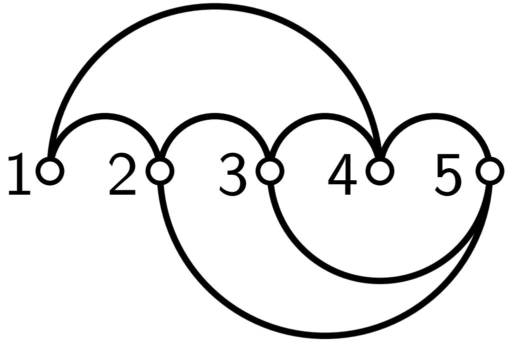

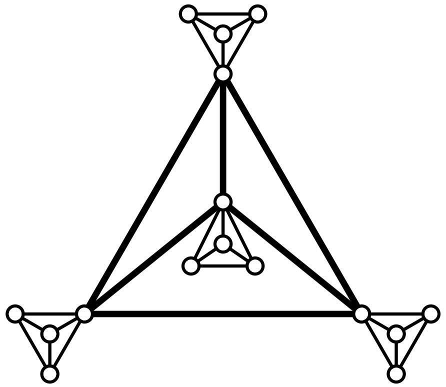

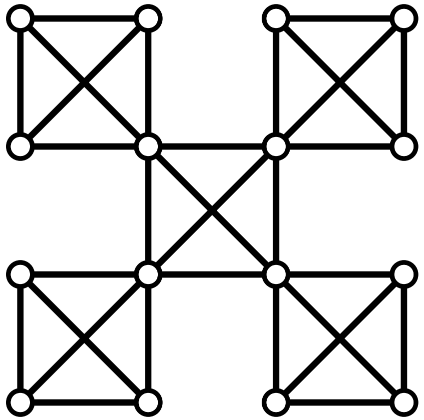

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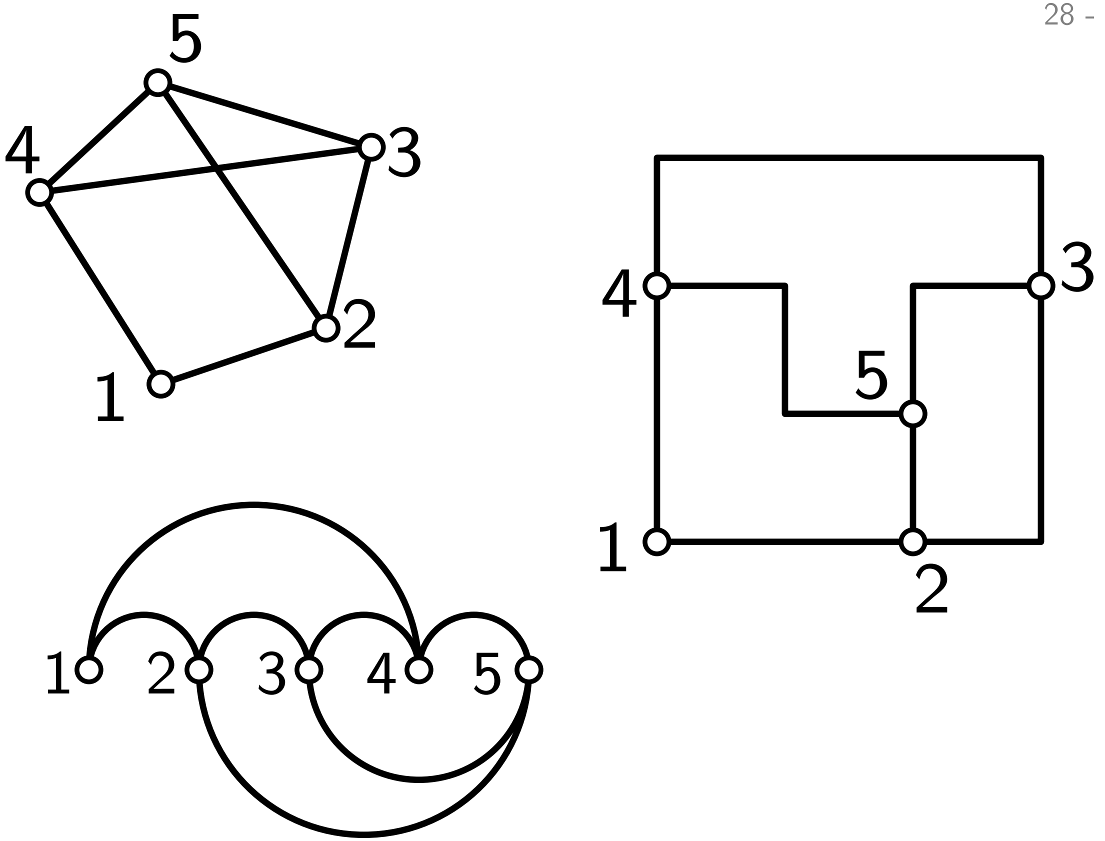

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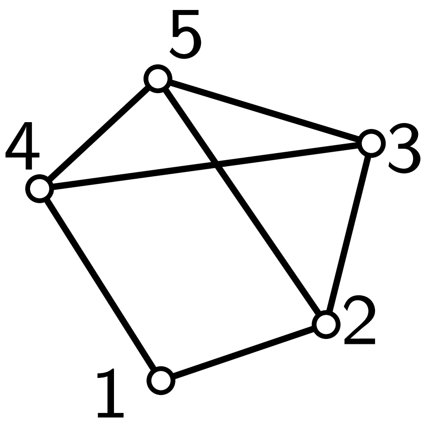

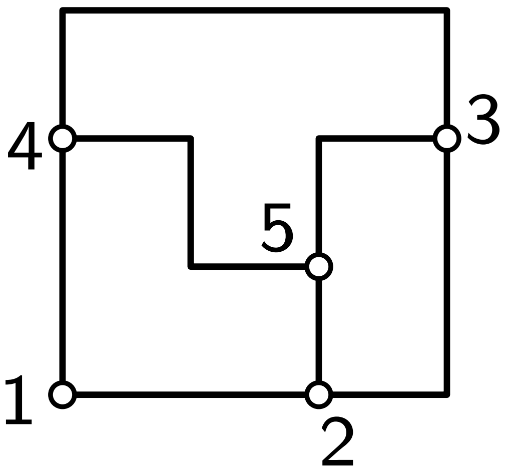

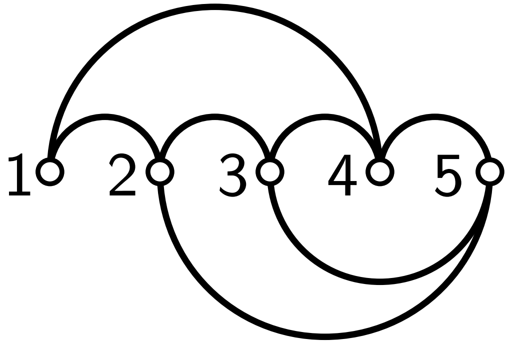

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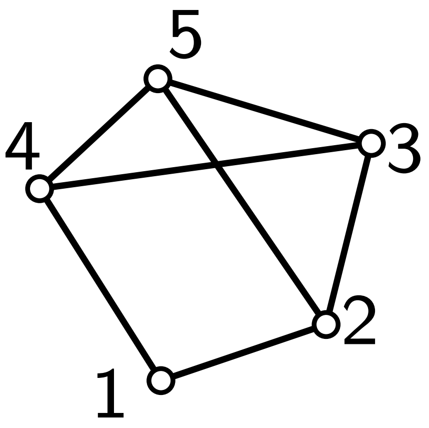

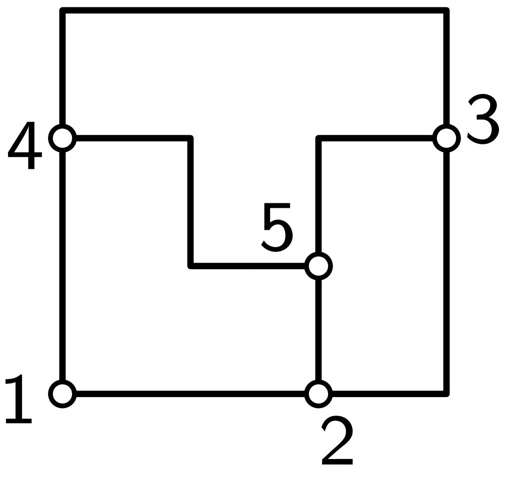

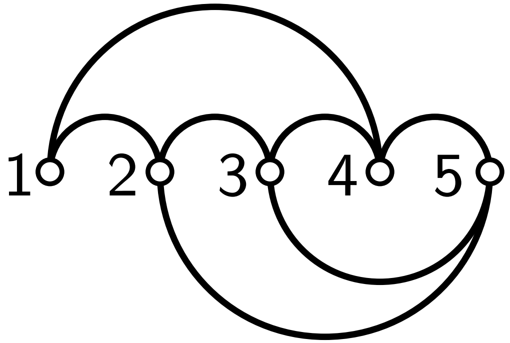

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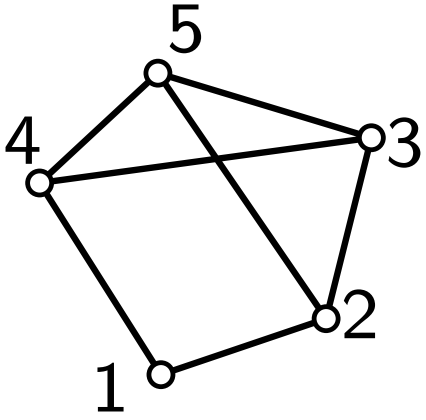

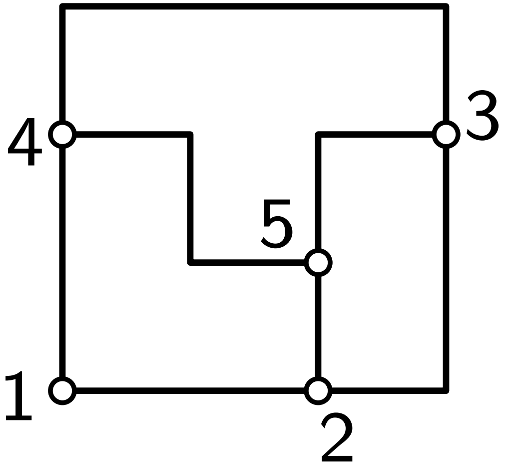

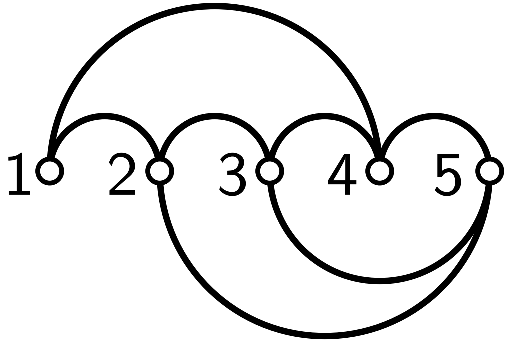

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   - restrictions on groups of vertices/edges (e.g., “clustered”).

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Graph Visualization Problem (more general)

in: Graph G

out: Drawing Γ of G such that

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- Interested in implementing a program for the live challenge? May be done as a Praktikum!