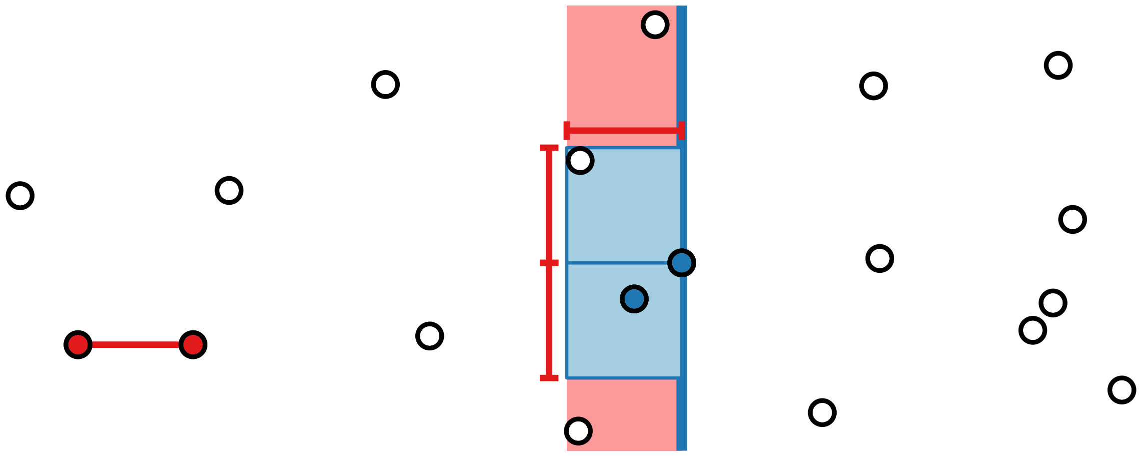
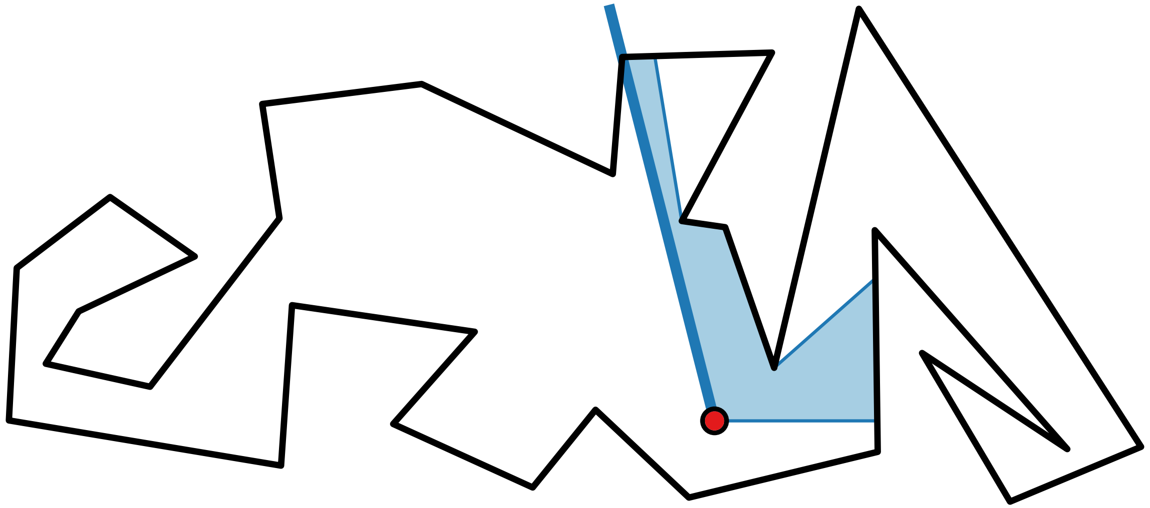
Advanced Algorithms

Computational Geometry
Sweep-Line Algorithms

Johannes Zink · WS23/24





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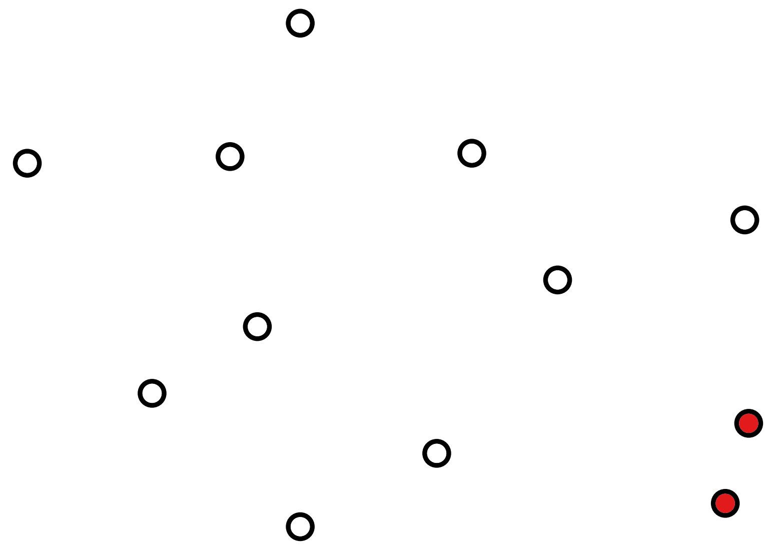
Some problems:

- Closest Pair



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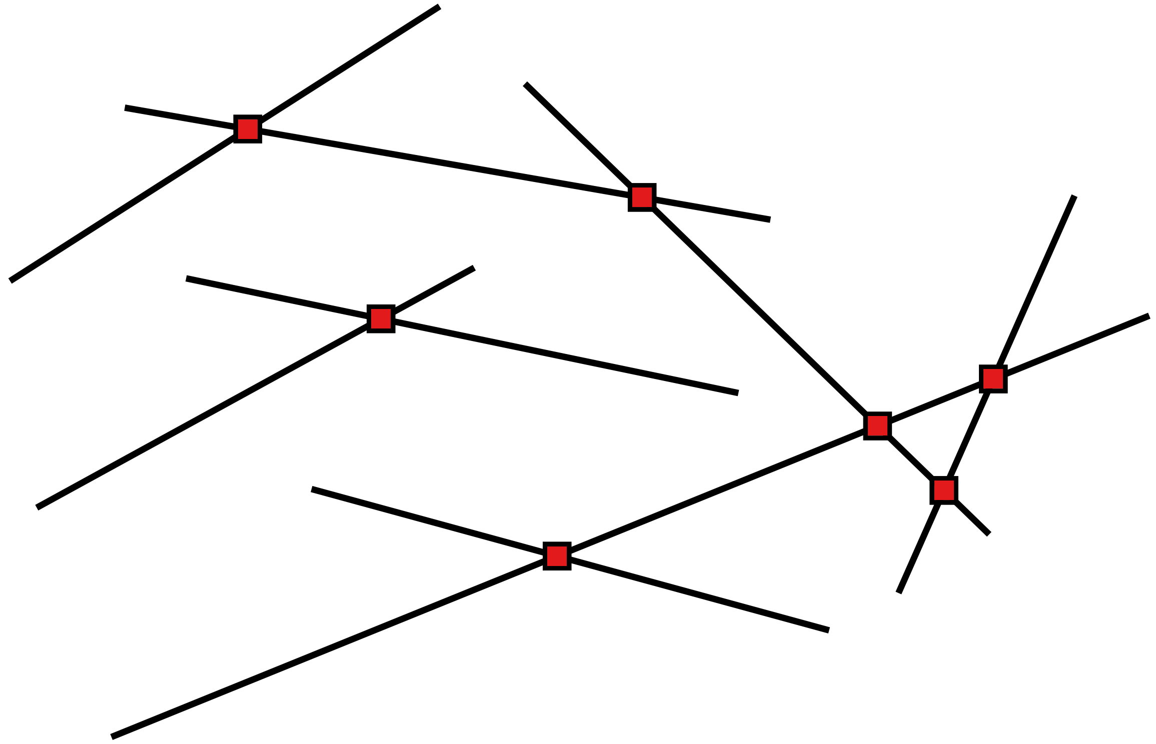

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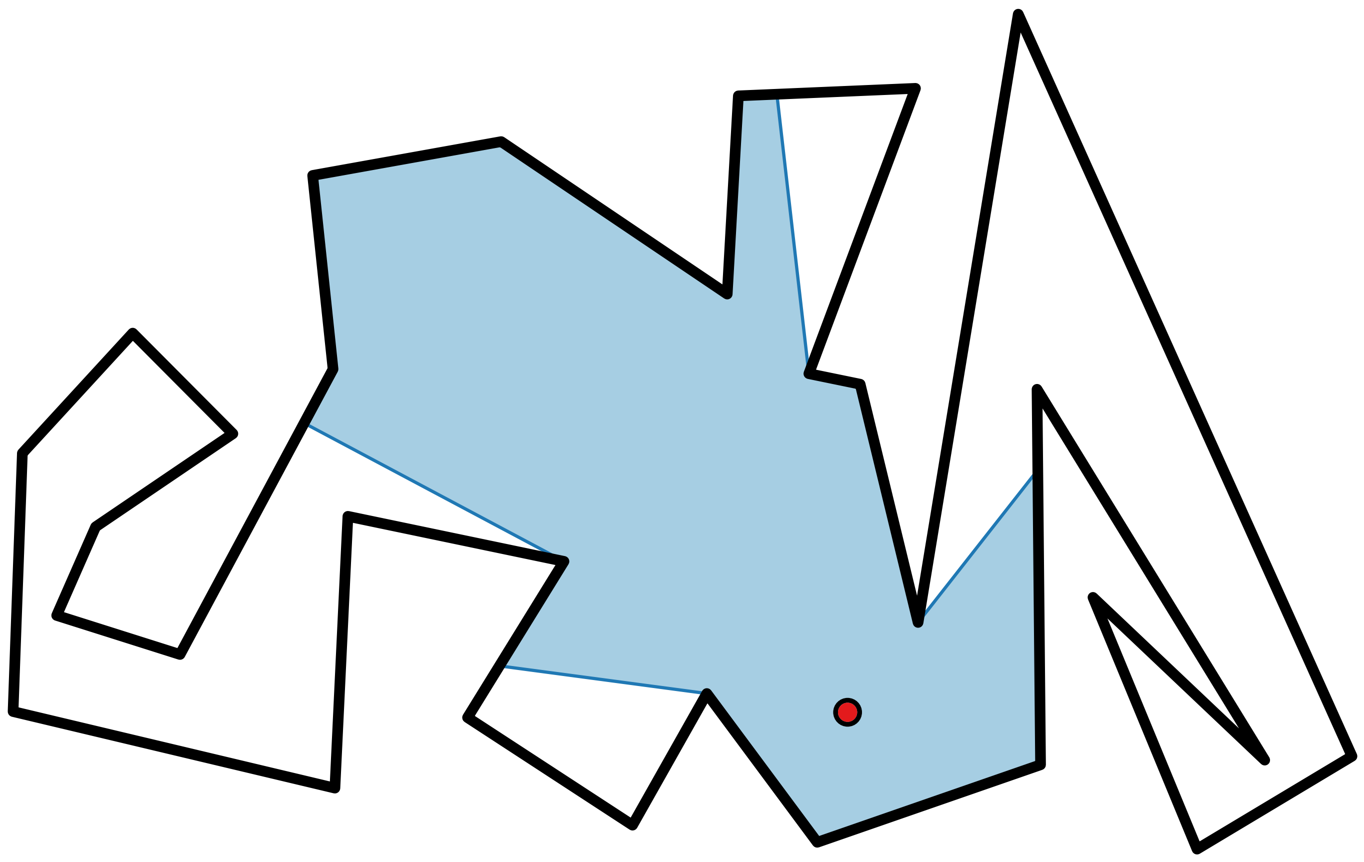

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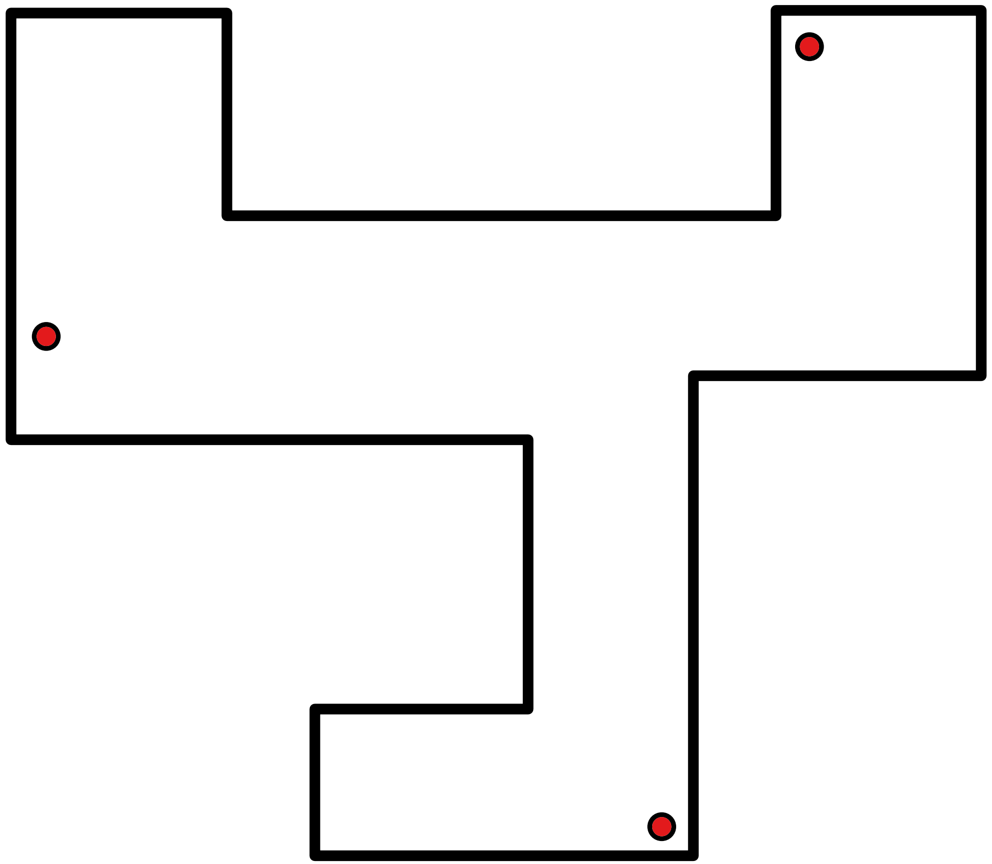

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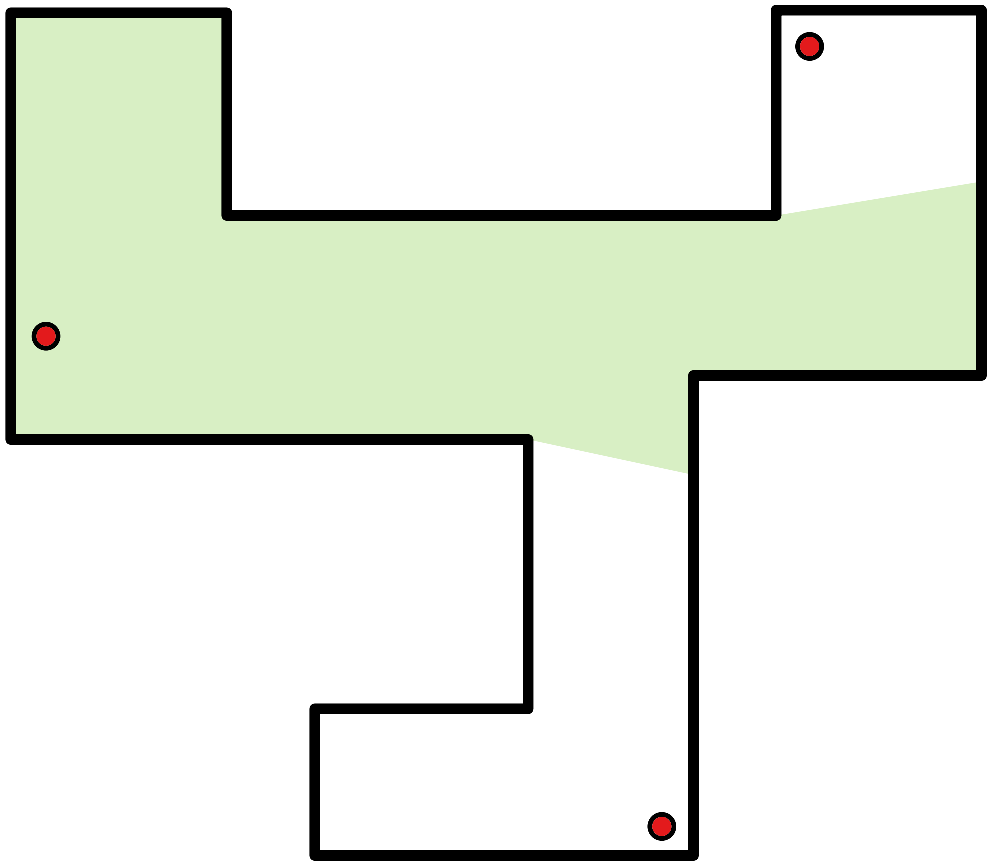

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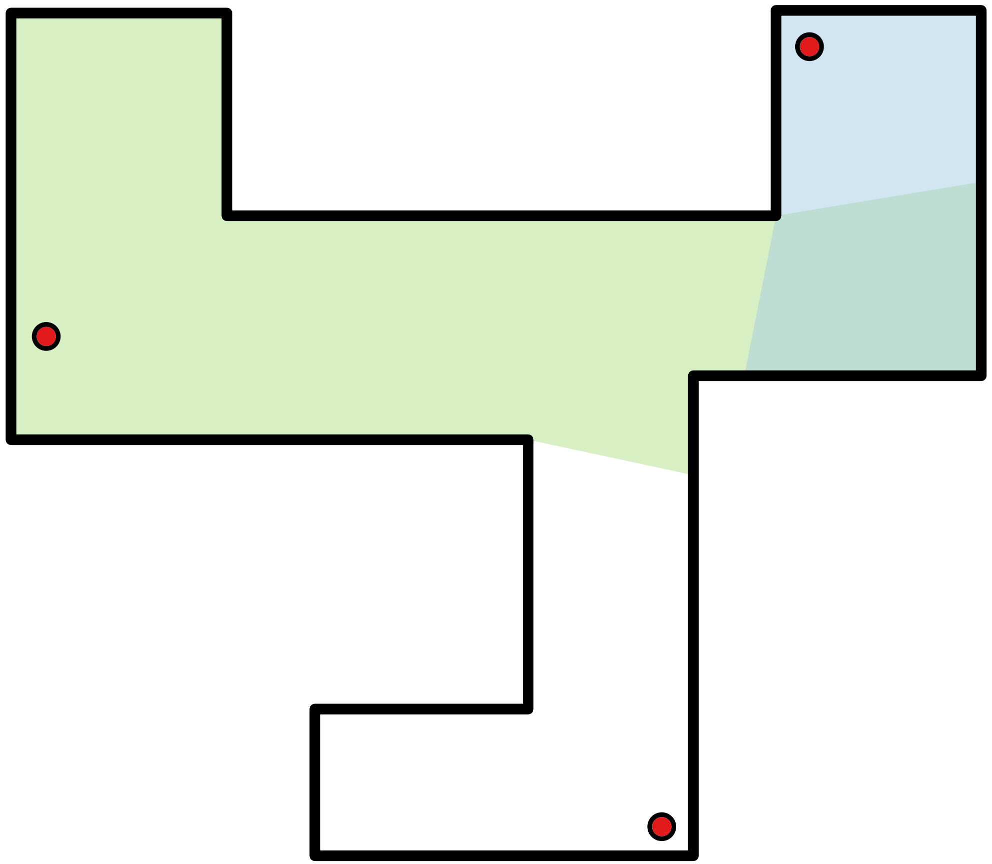

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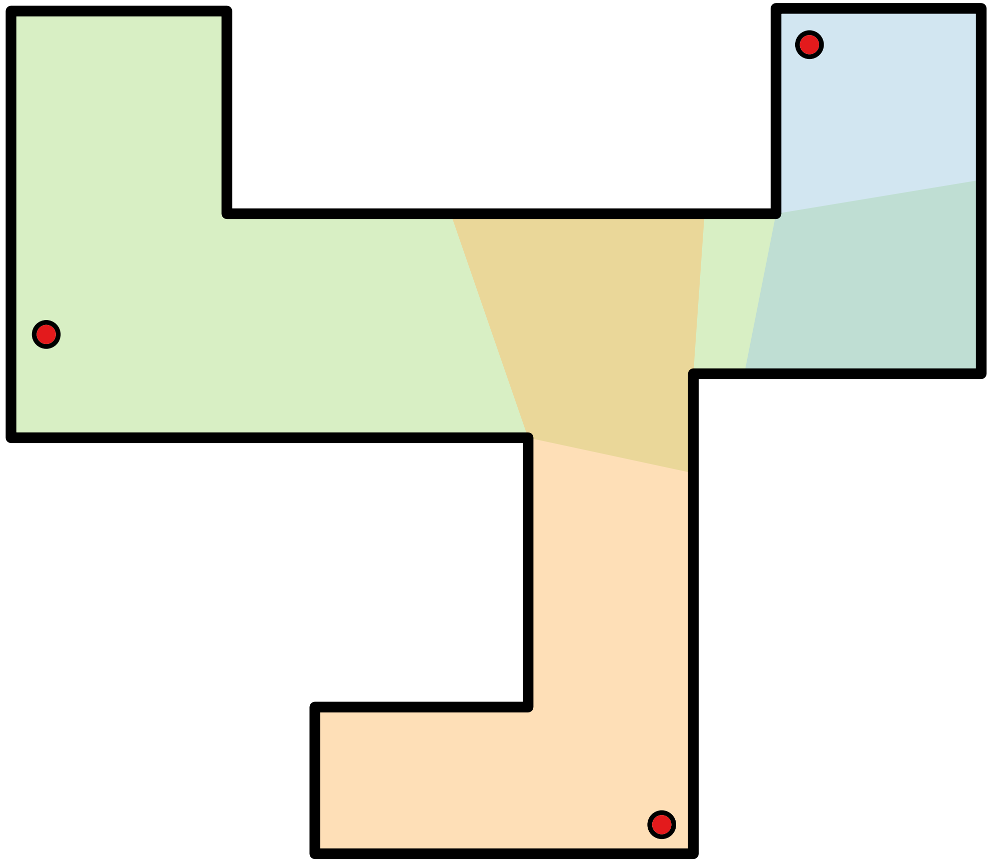

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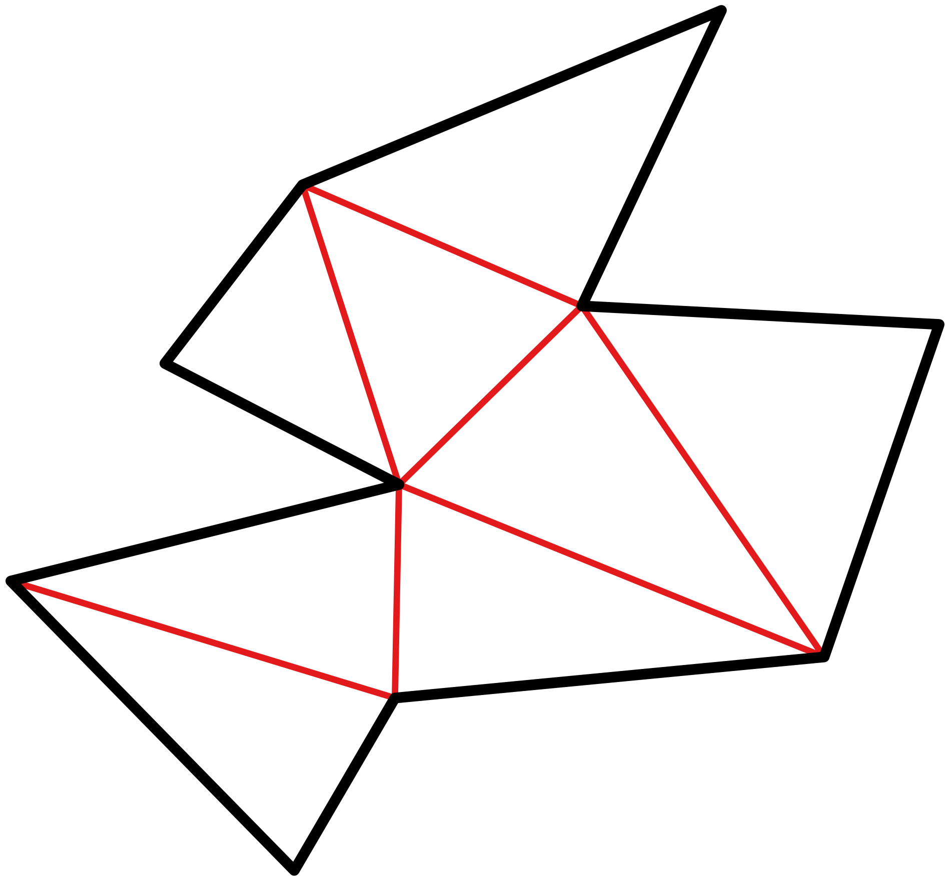

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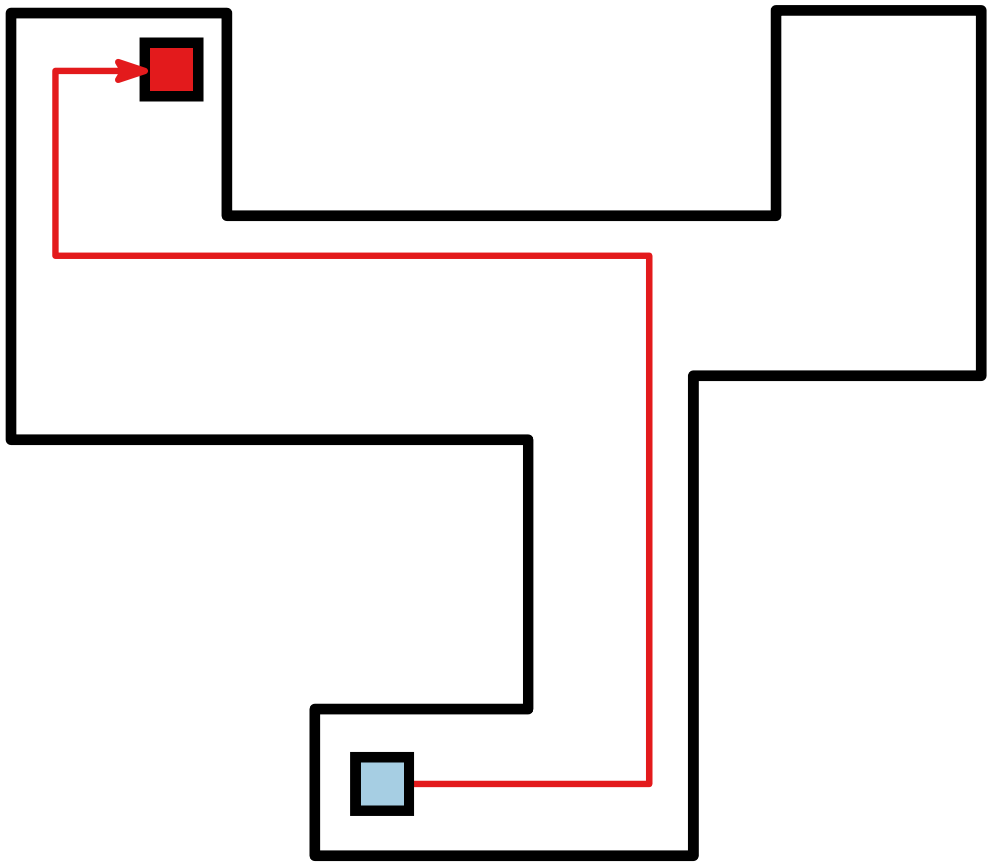

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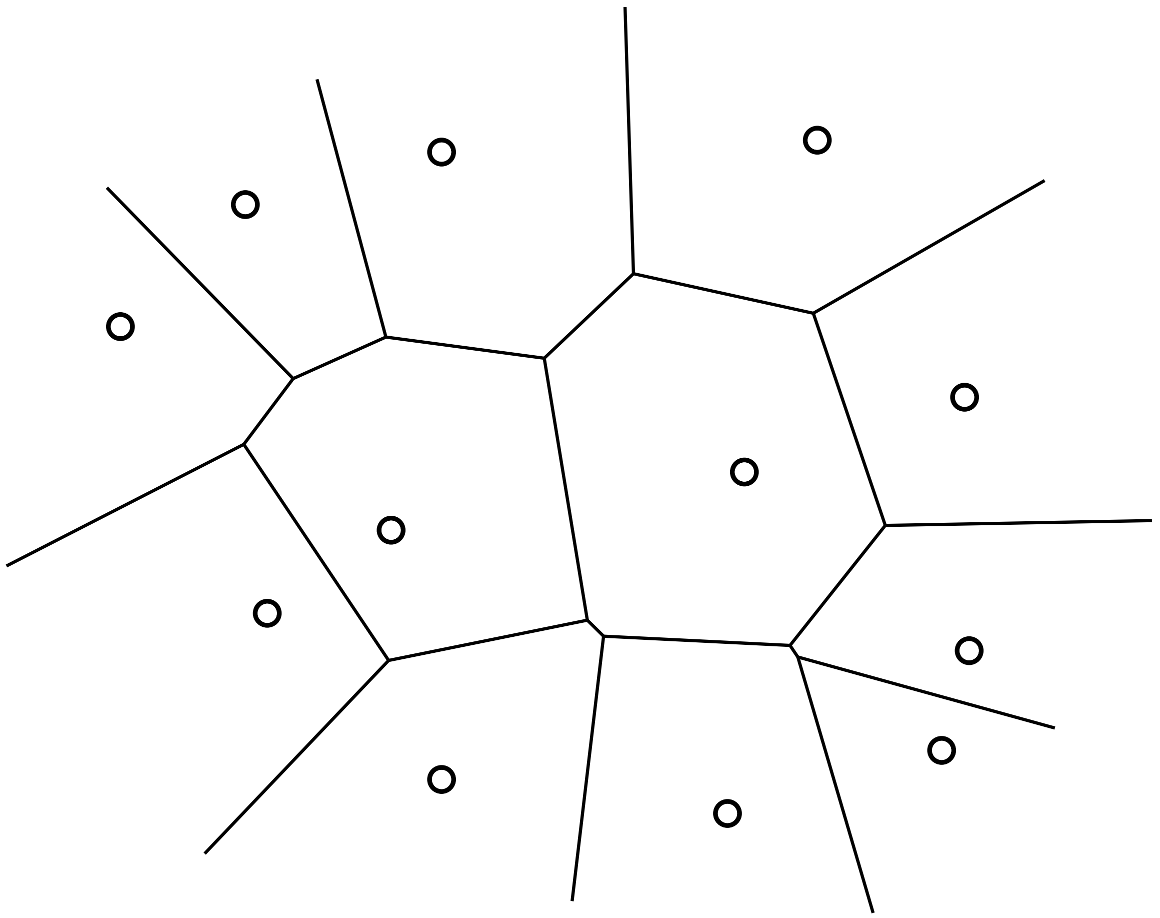
- Finding the closest post office



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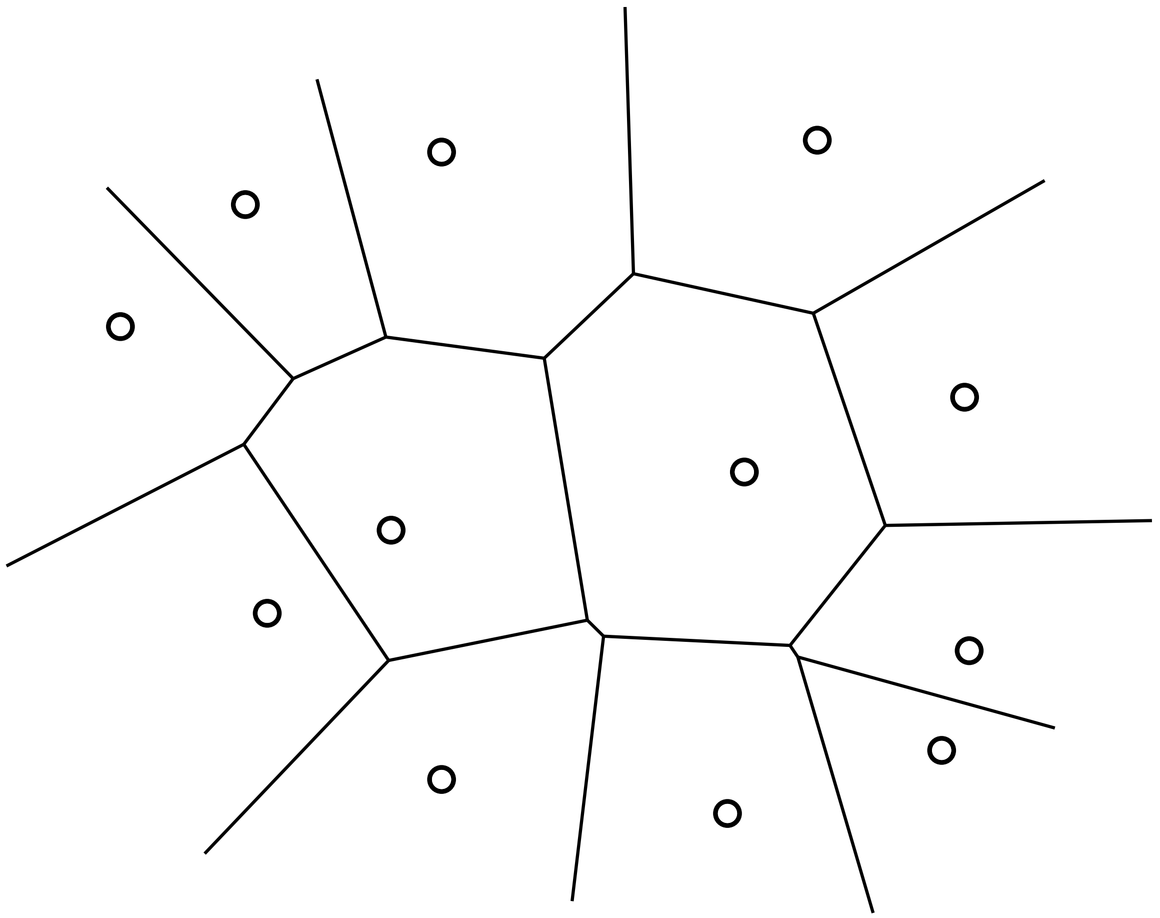
- Finding the closest post office
- and many more.

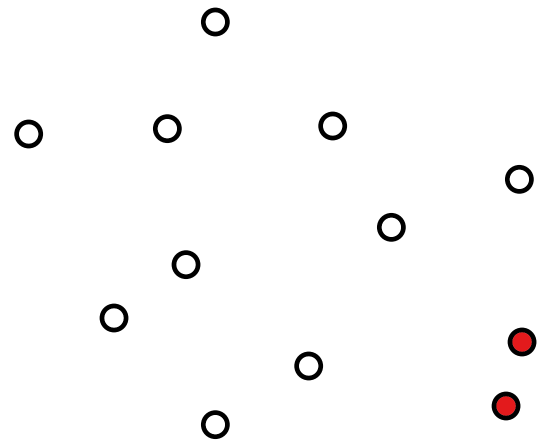


We offer an entire course on computational geometry in the winter term!

Closest Pair

Given: (multi-)set of points $P \subseteq \mathbb{R}^2$.

Task: Find a pair of distinct elements $p_a, p_b \in P$ such that the Euclidean distance $||p_a - p_b||$ is minimum.



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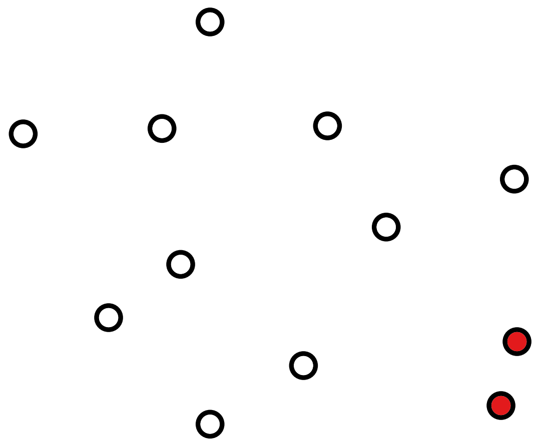

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Deterministic algorithms:

Brute-force $\mathcal{O}(n^2)$

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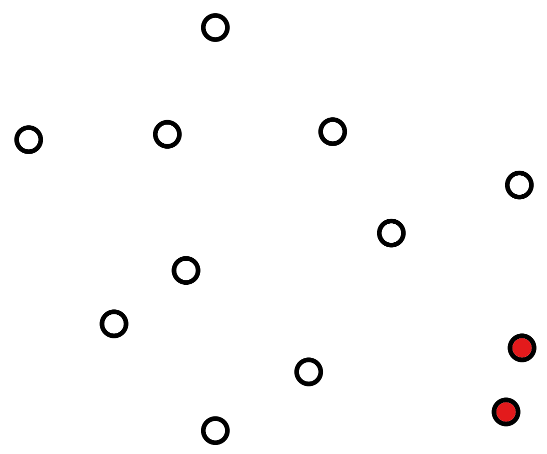

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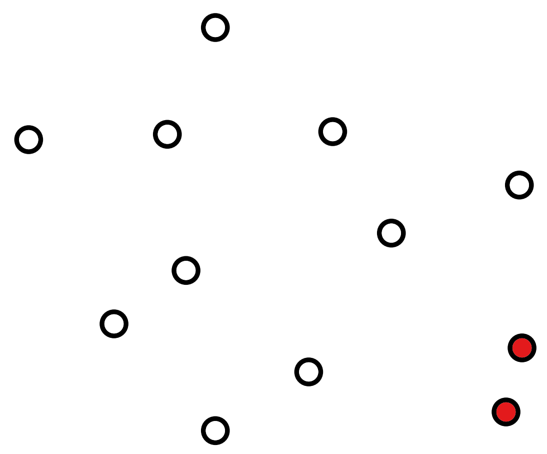

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Randomized algorithm:

Randomized incremental construction	$\mathcal{O}(n)$	(expected runtime)

later in this course!

A Randomized Incremental Algorithm for Closest Pair

Define $P_i = \{p_1, p_2, \ldots, p_i\}$ and let δ_i be the dis… …osest pair in P_i.

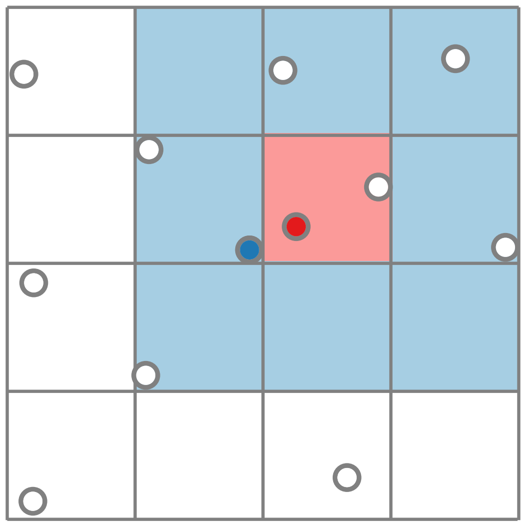
Idea: $\delta_2 = ||p_1, p_2||$. Compute $\delta_3, \delta_4, \ldots, \delta$… …g the points iteratively.

Suppose we have already determin…

Consider a square grid with c… …$\delta_{i-1} \times \delta_{i-1}$.

Add the point p_i. If δ_i… …n p_i must be part of each closest pa…

Moreover, p… …n the cell of p_i or one of the adjacen…



(simple) exercise

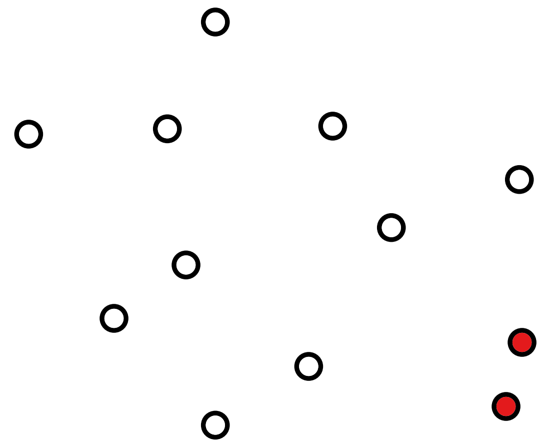
Ea… …e cells contains at most $\mathcal{O}(1)$ points of P_{i-1} ($\Leftarrow$ packing argument).

The co…inates of the cell of p_i can be determined in $\mathcal{O}(1)$ time assuming the floor function can be computed in $\mathcal{O}(1)$ time.

$\Rightarrow$ The test $\delta_i < \delta_{i-1}$ can be performed in $\mathcal{O}(1)$ time assuming P_{i-1} is stored in a suitable dictionary for the nonempty cells (implementable via dynamic perfect hashing).

Upcoming lecture on randomized algorithms

Closest Pair



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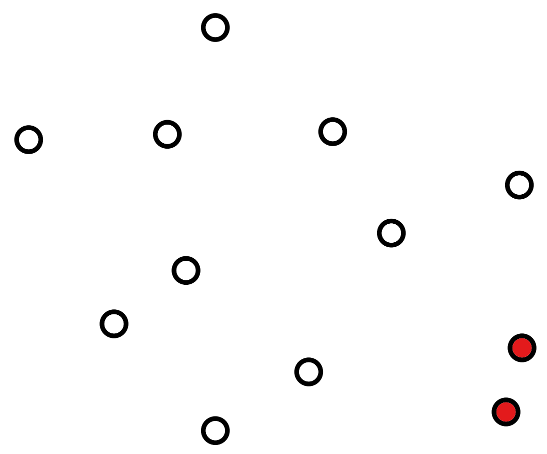

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Deterministic algorithms:

Brute-force	$\mathcal{O}(n^2)$	
Divide and conquer (recall from ADS)	$\mathcal{O}(n \log n)$	(optimal)
Sweep line	$\mathcal{O}(n \log n)$	(optimal) **now!**

Randomized algorithm:

Randomized incremental construction	$\mathcal{O}(n)$	(expected runtime)

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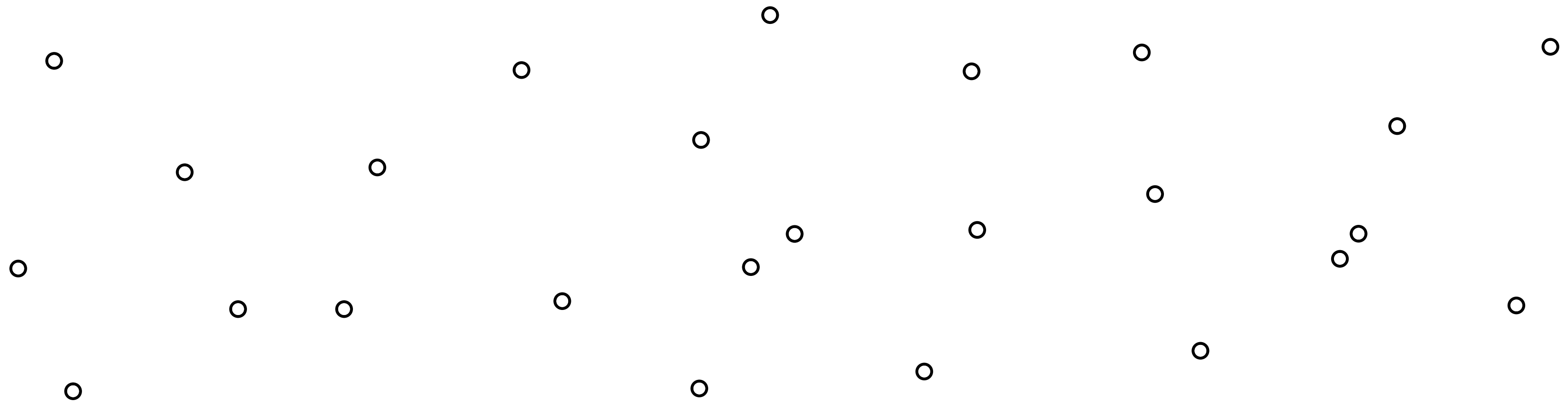
A Sweep Line Approach for Closest Pair

Assumption: The points in P have pairwise distinct x-coordinates.

Idea: Sweep the plane from left to right with a vertical line ℓ (the **sweep line**).



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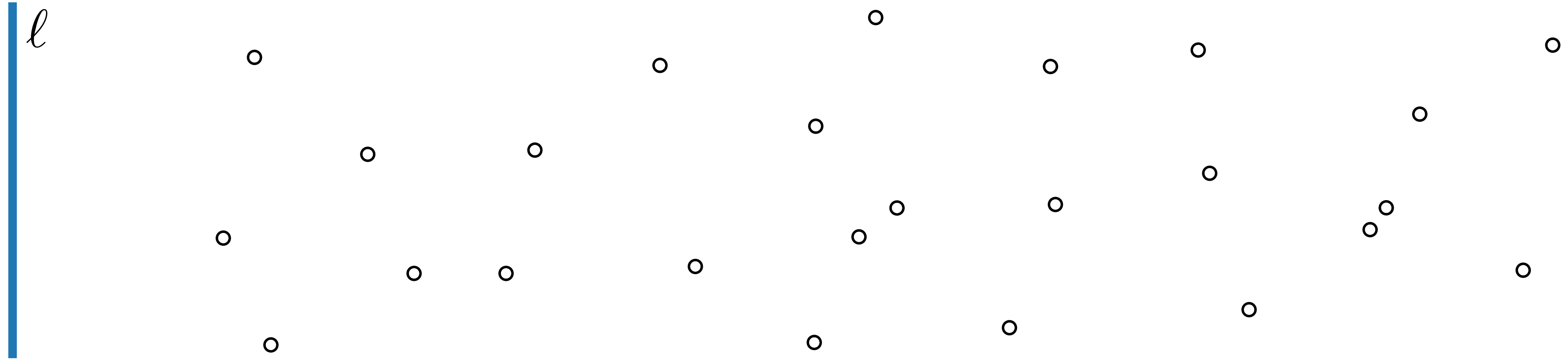

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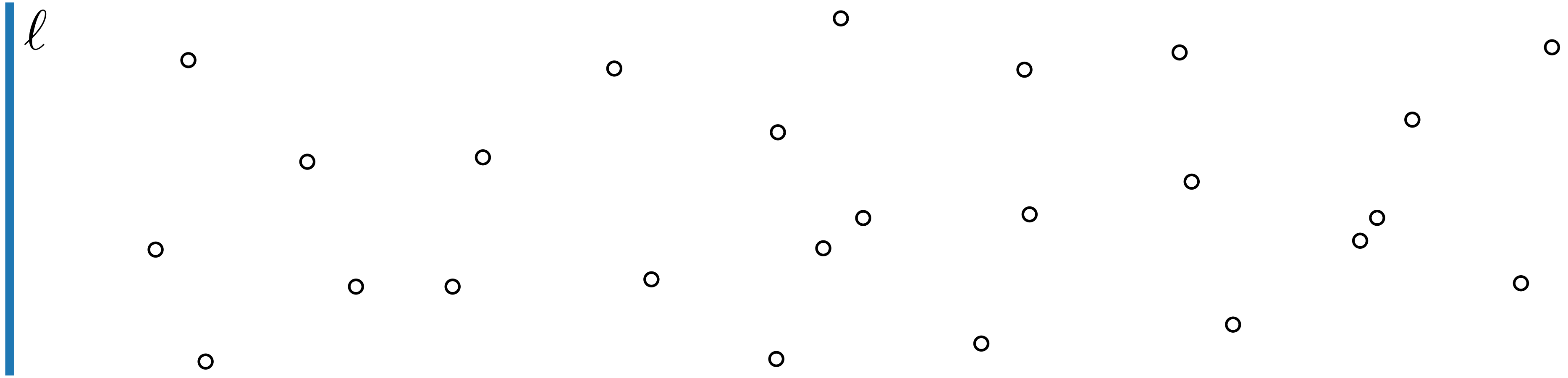

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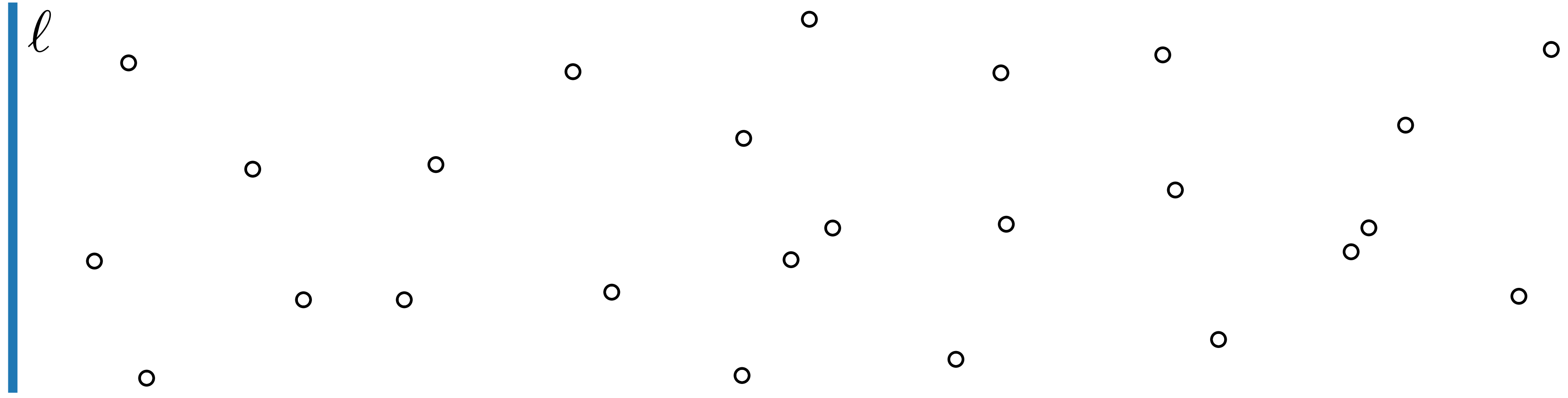

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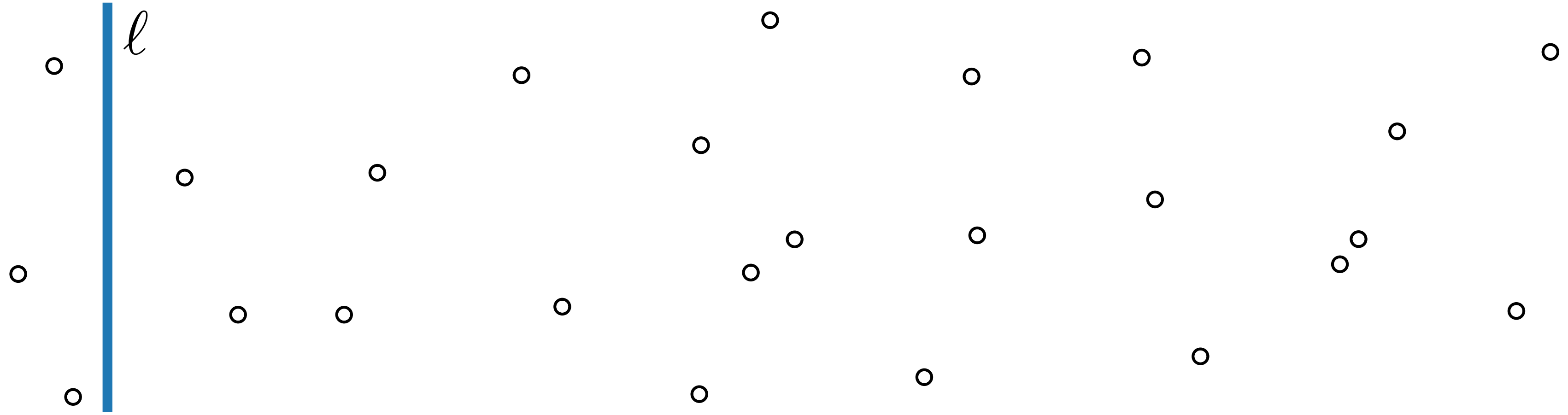

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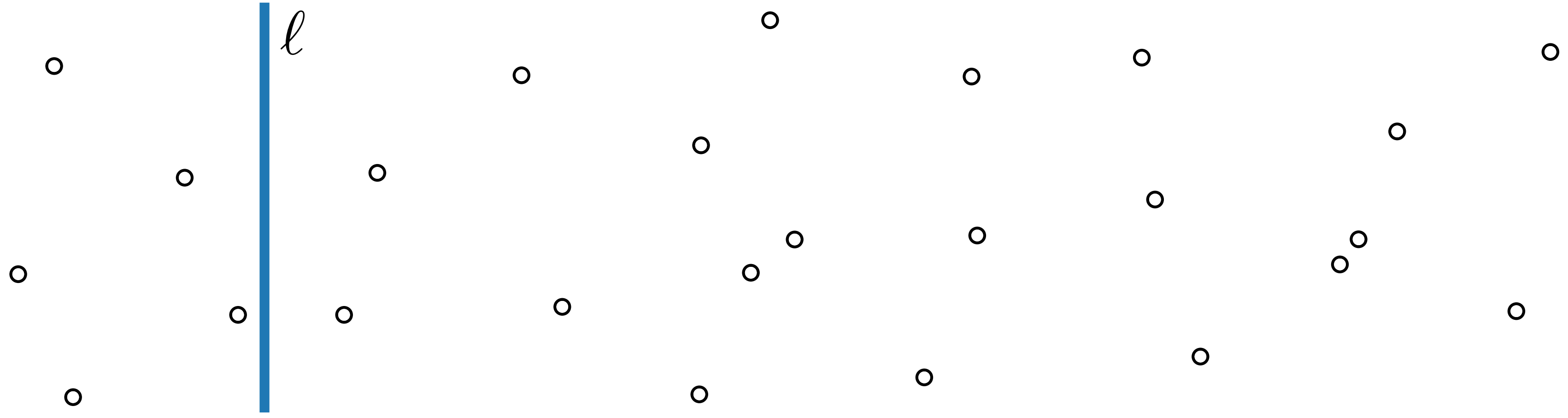

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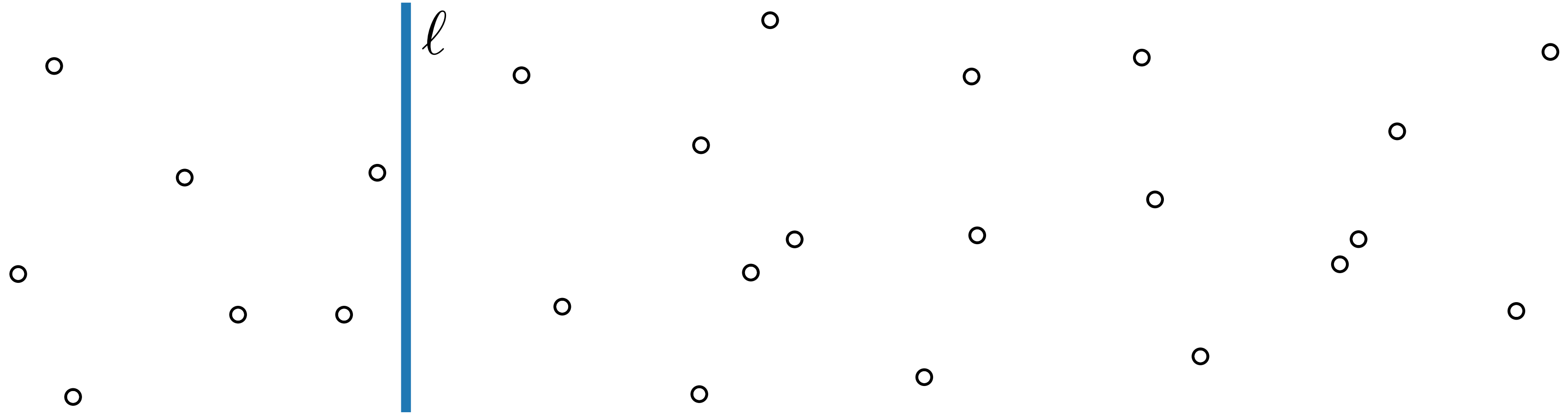

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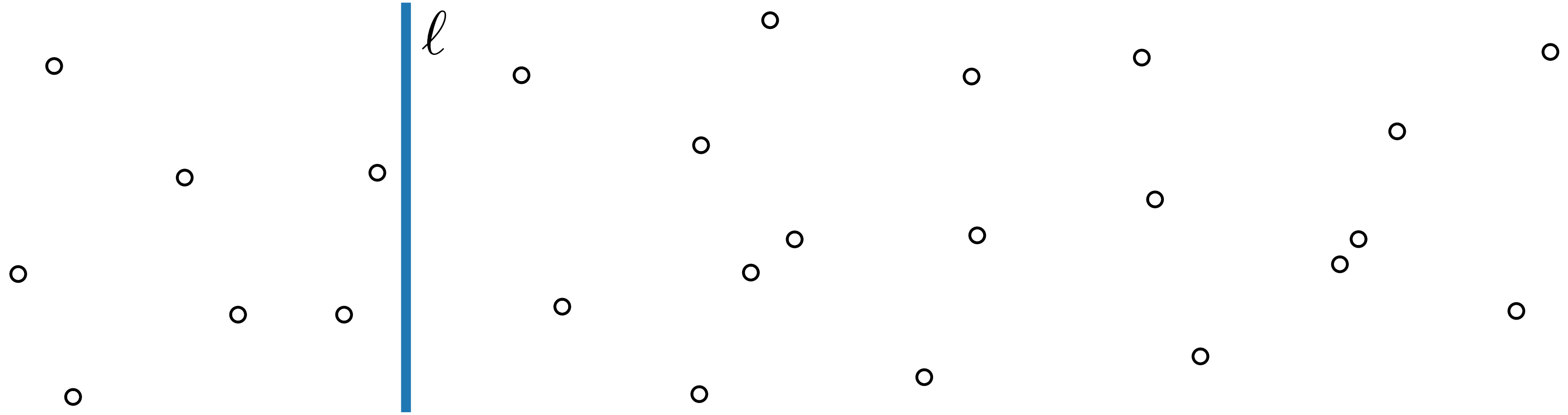

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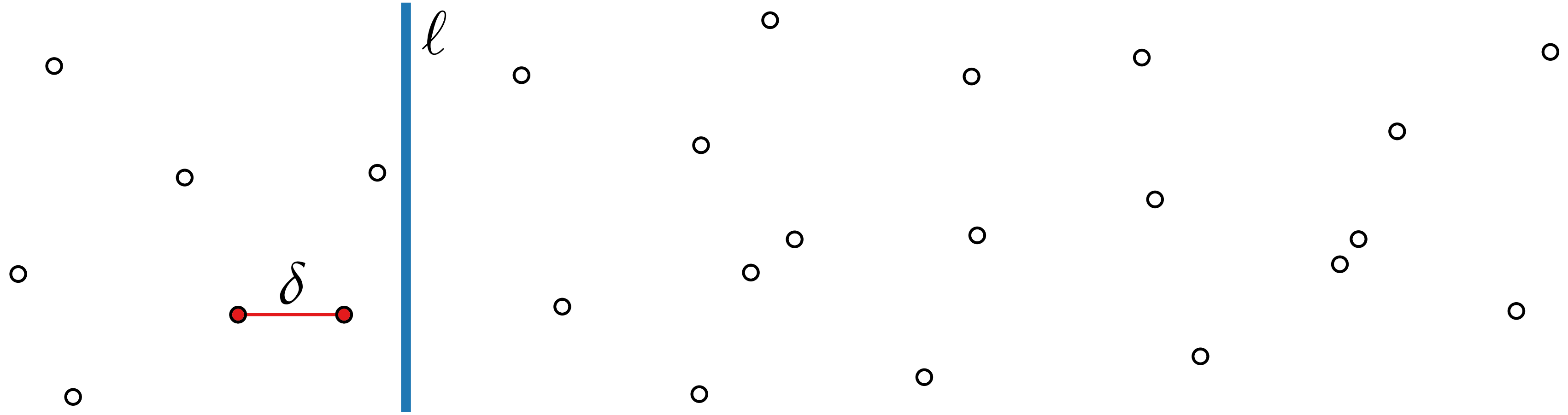

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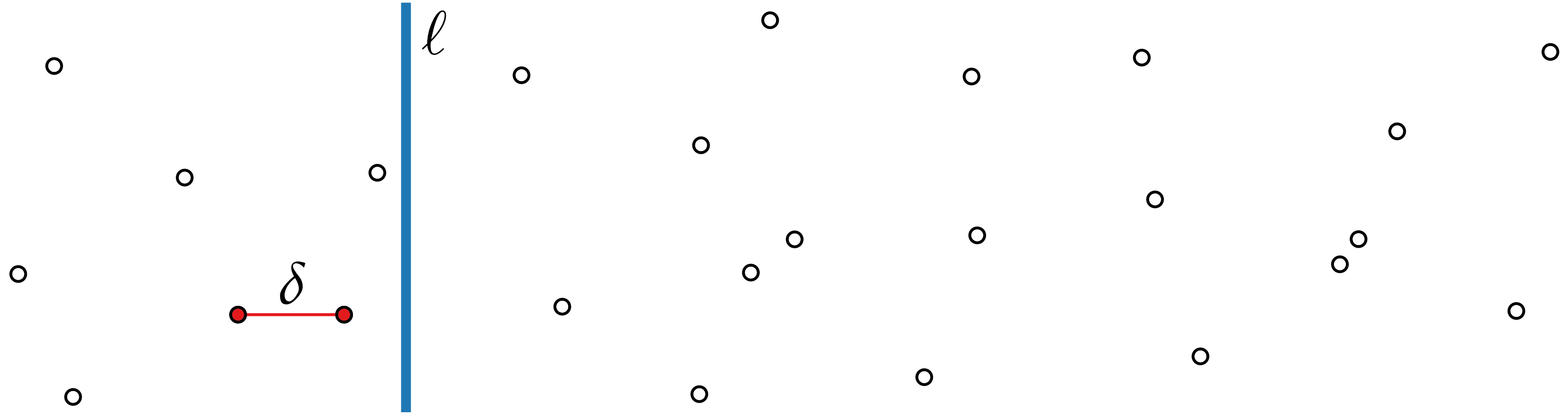

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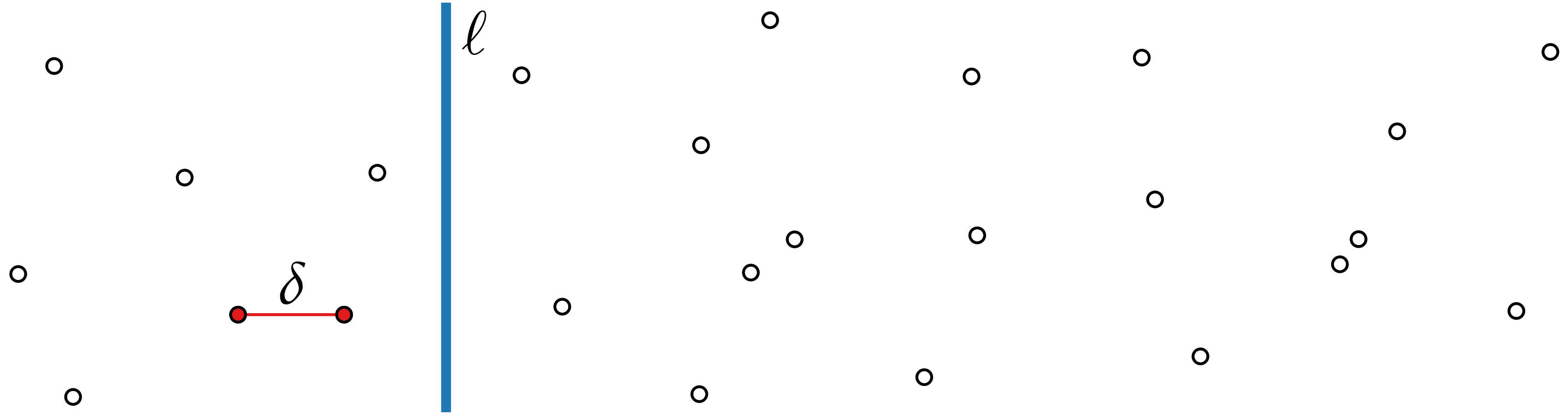

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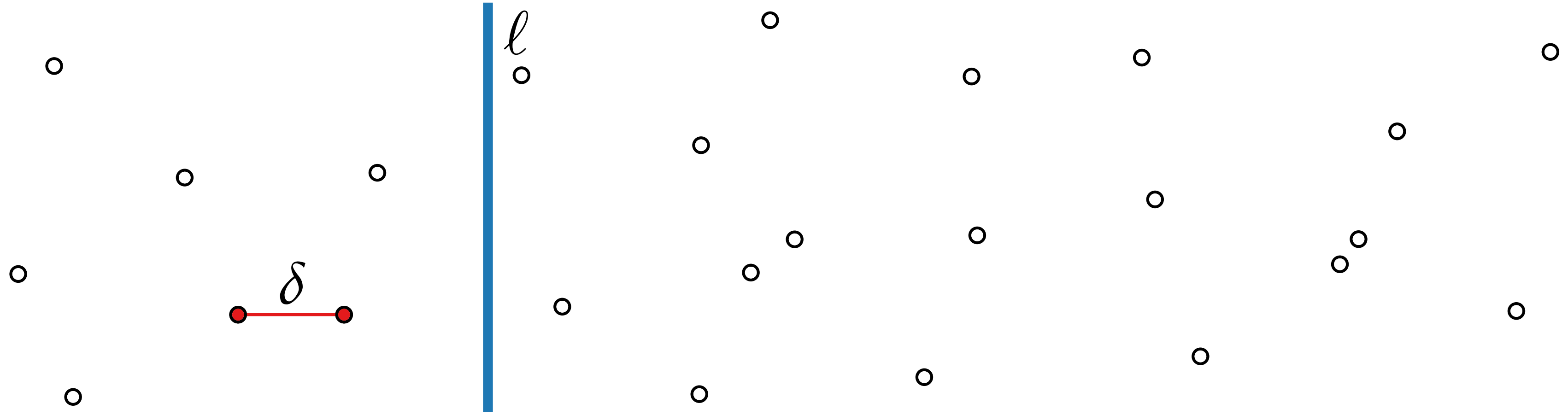

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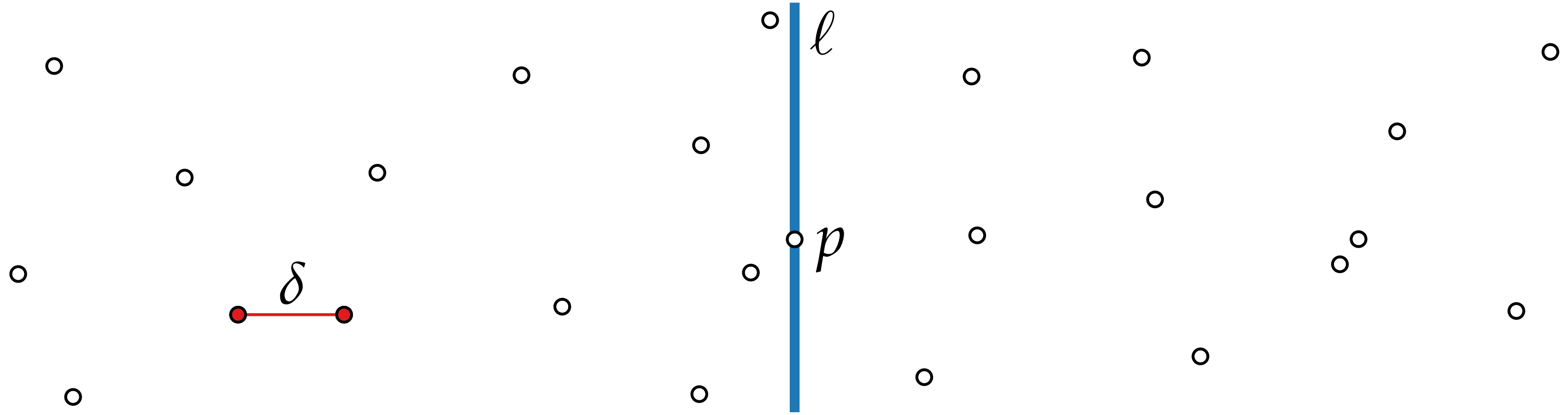

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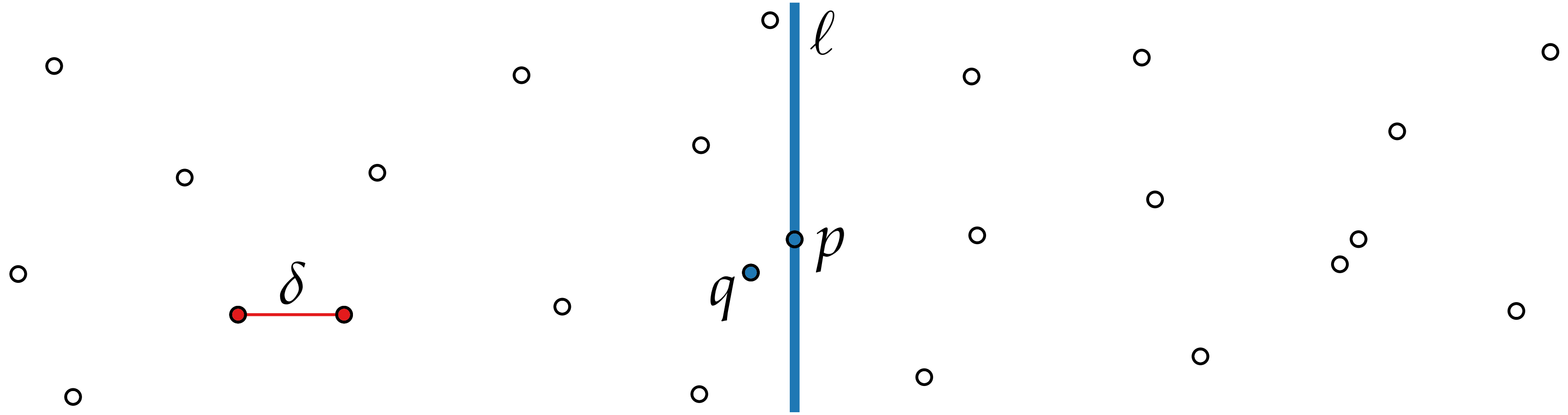

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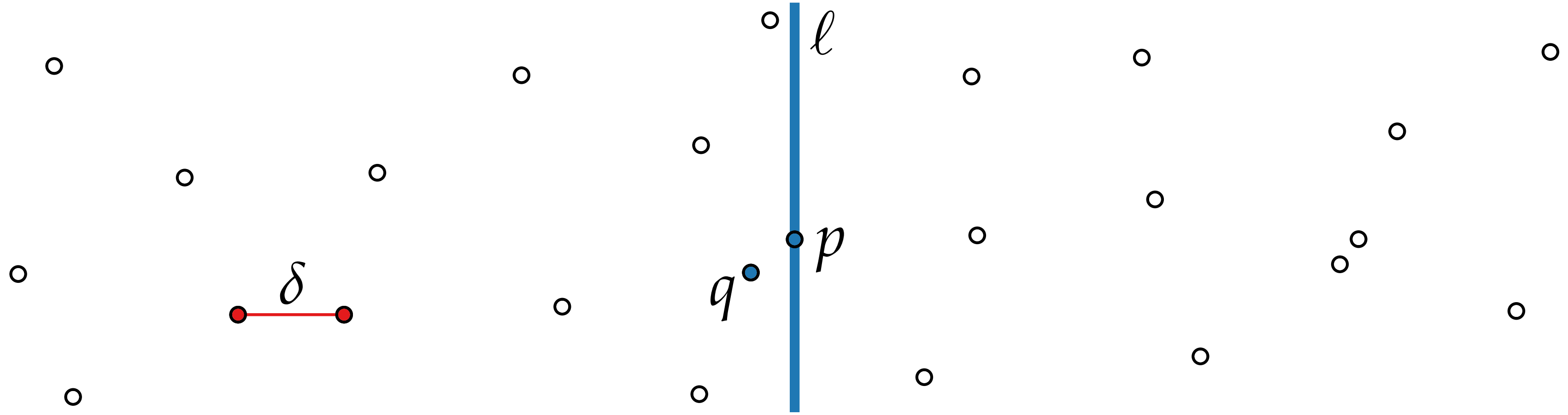

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What do we know about the location of q?

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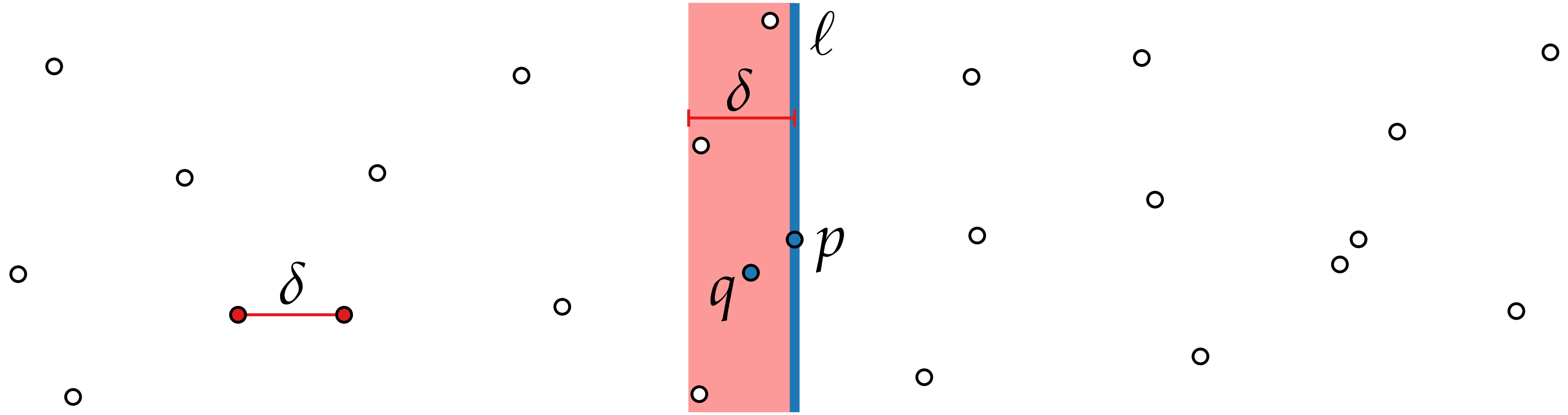

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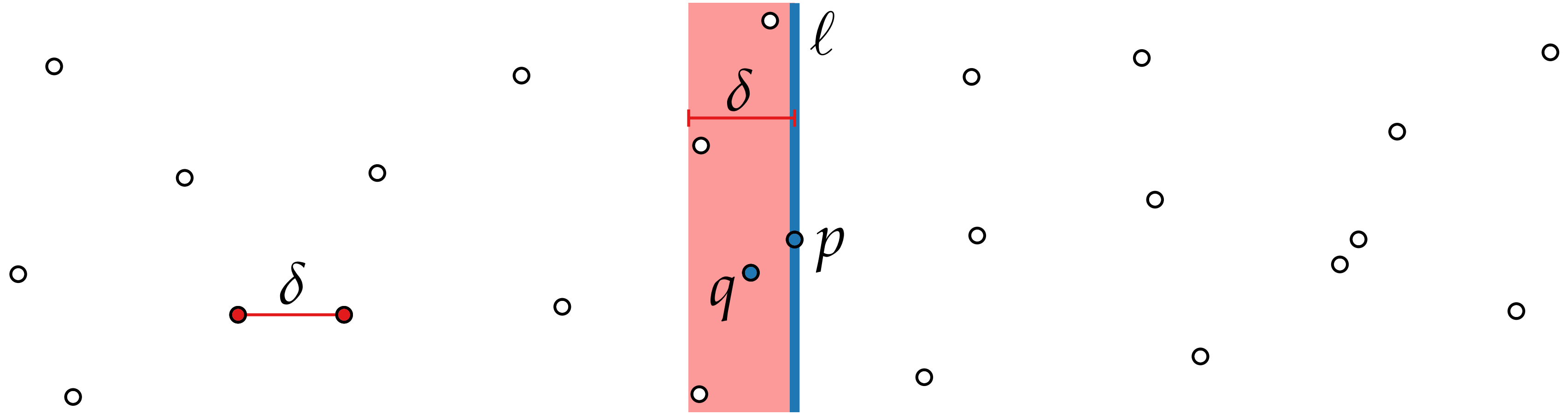

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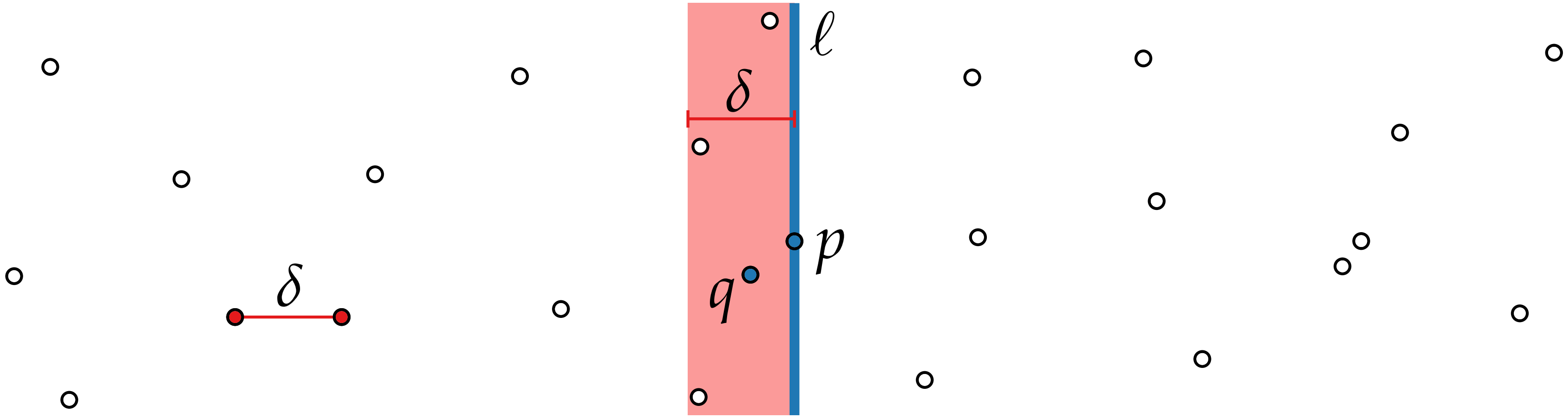

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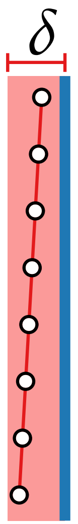

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All of them!

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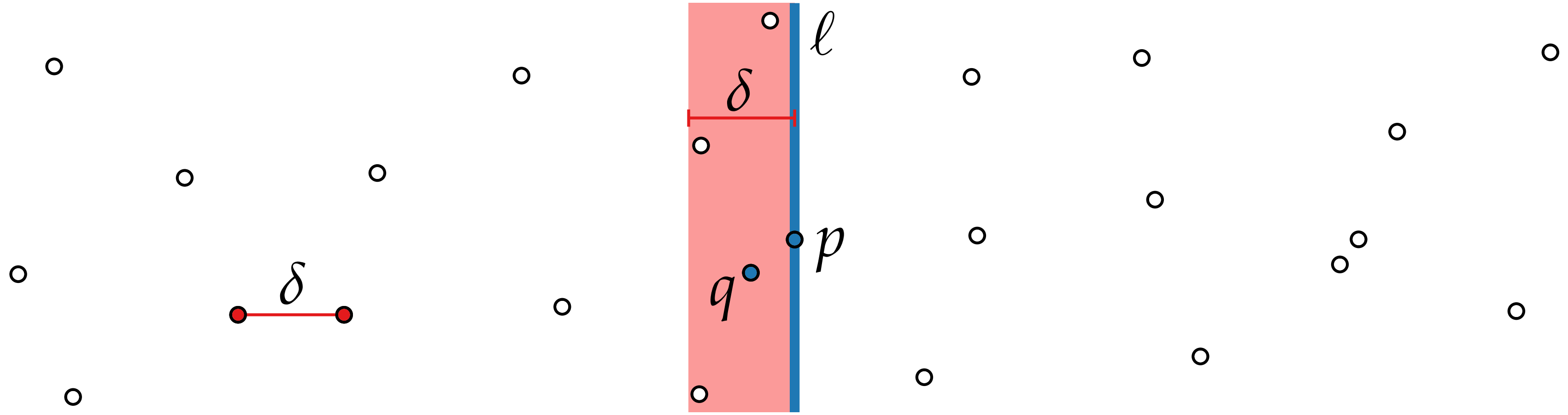

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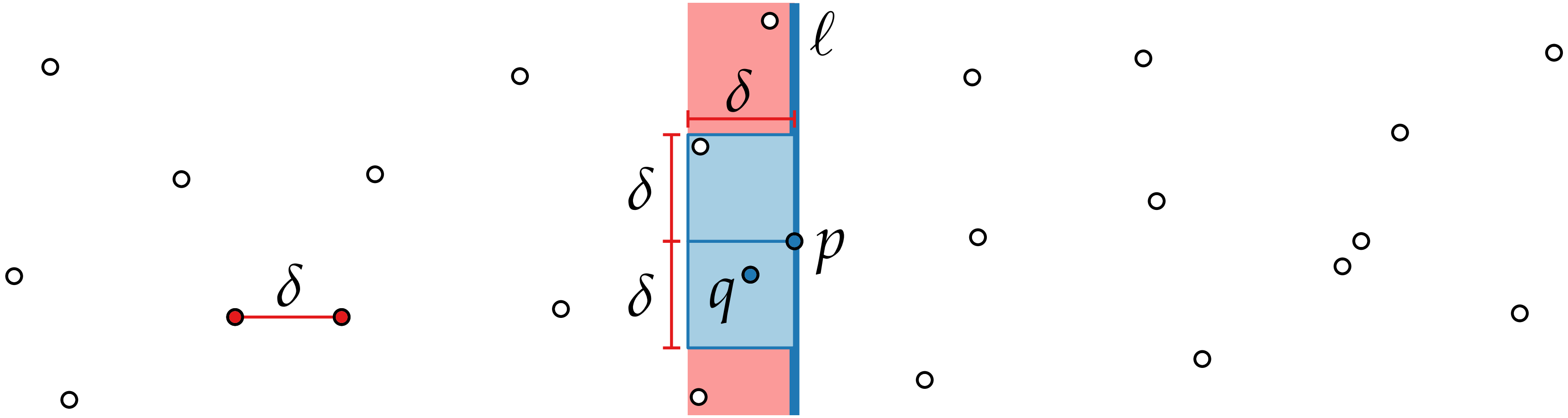


Invariant: a closest pair of the points to the left of ℓ and its distance δ is already known.

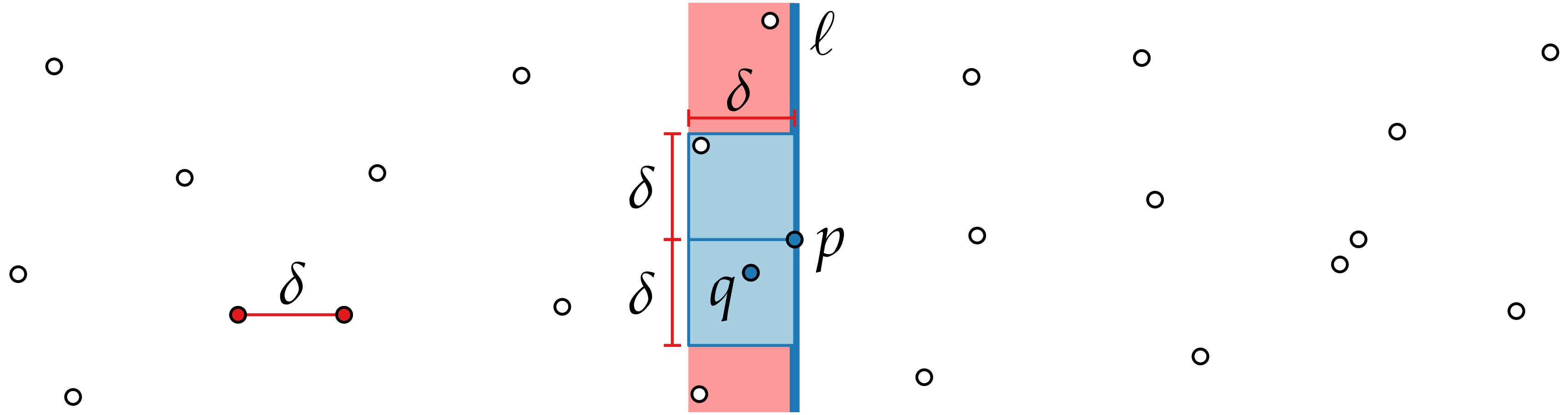
Observations:

- This partial solution can only change when ℓ sweeps a point p of P.
- Each new closest pair consists of p and a point q with distance $< \delta$ to ℓ.
- q needs to be located in a $\delta \times 2\delta$ rectangle R to the left of p.

A Sweep Line Approach for Closest Pair

Assumption: The points in P have pairwise distinct x-coordinates.

Idea: Sweep the plane from left to right with a vertical line ℓ (the **sweep line**).



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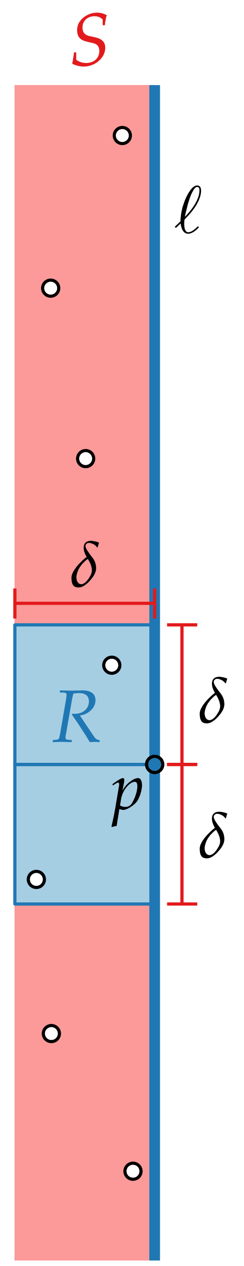
- This partial solution can only change when ℓ sweeps a point p of P.
- Each new closest pair consists of p and a point q with distance $< \delta$ to ℓ.
- q needs to be located in a $\delta \times 2\delta$ rectangle R to the left of p.
- R contains $\mathcal{O}(1)$ points of $P \setminus \{p\}$ since their pairwise distance is $\geq \delta$. (packing argument)

Computing the Points in R Efficiently

Let S denote the vertical slab of width δ to the left of ℓ.



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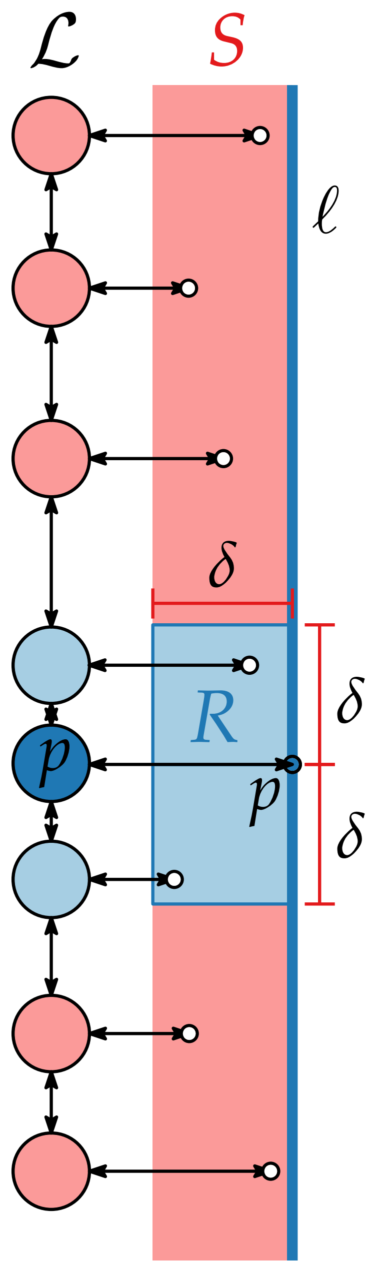
Assume that the points $P \cap S$ are stored in a **linked list** $\mathcal{L}$ sorted according to their y-coordinates.



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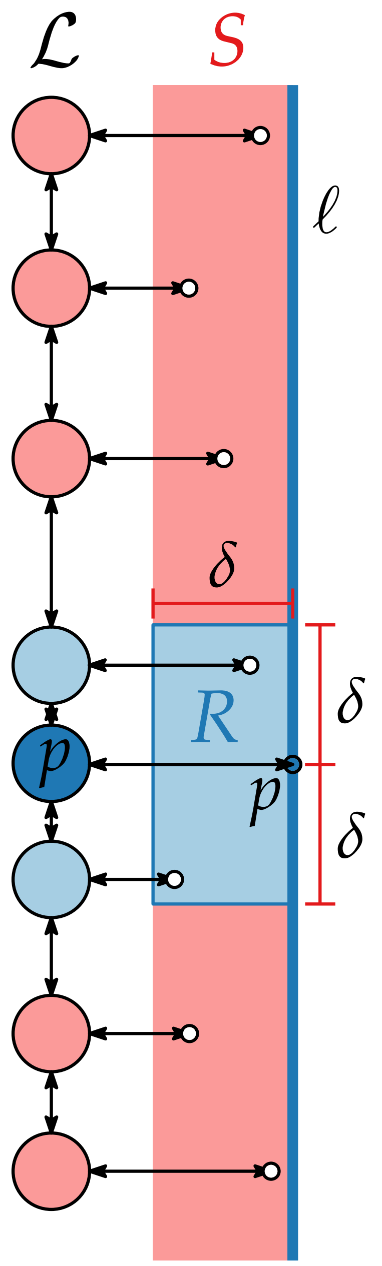

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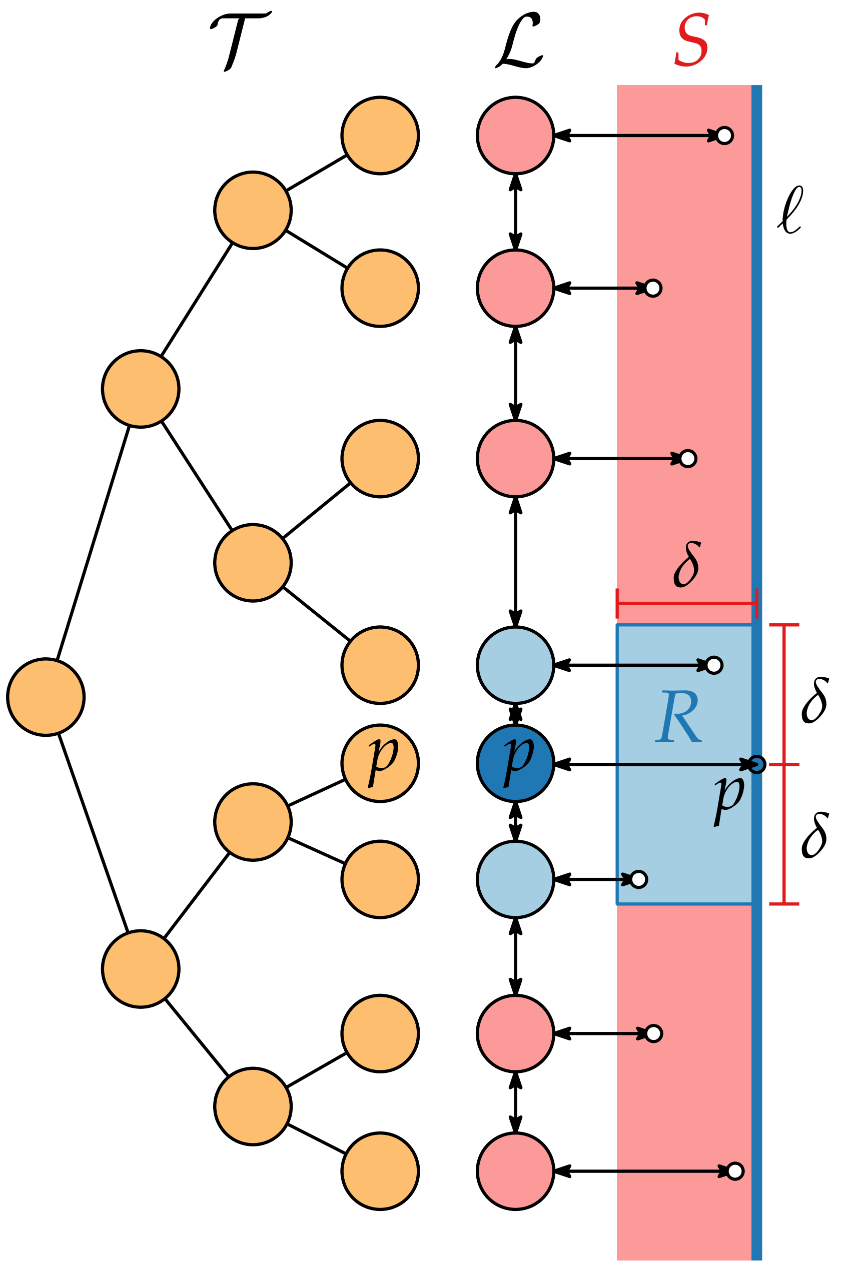
To ensure that $\mathcal{L}$ can be updated efficiently, we additionally store the points $P \cap S$ in a **balanced binary search tree** $\mathcal{T}$ using the y-coordinates as keys.



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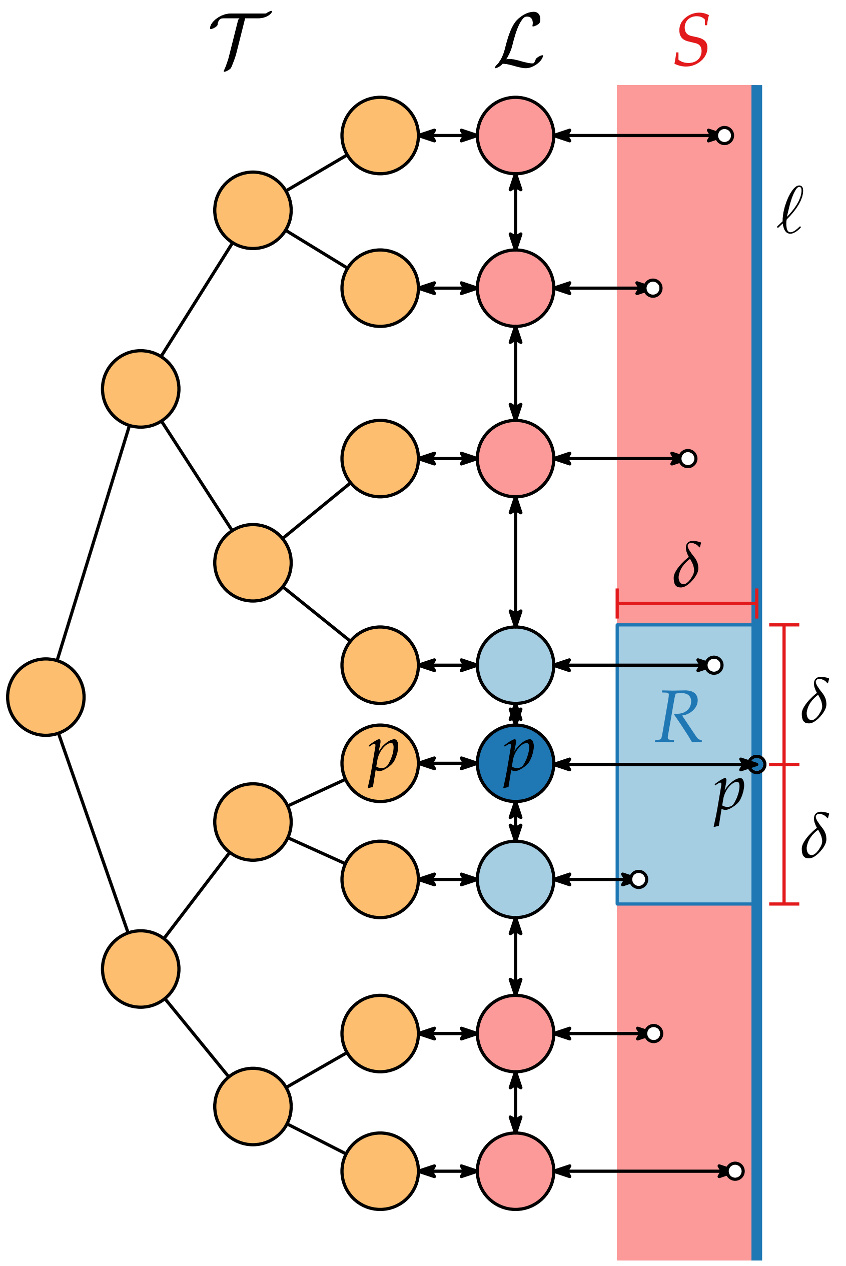

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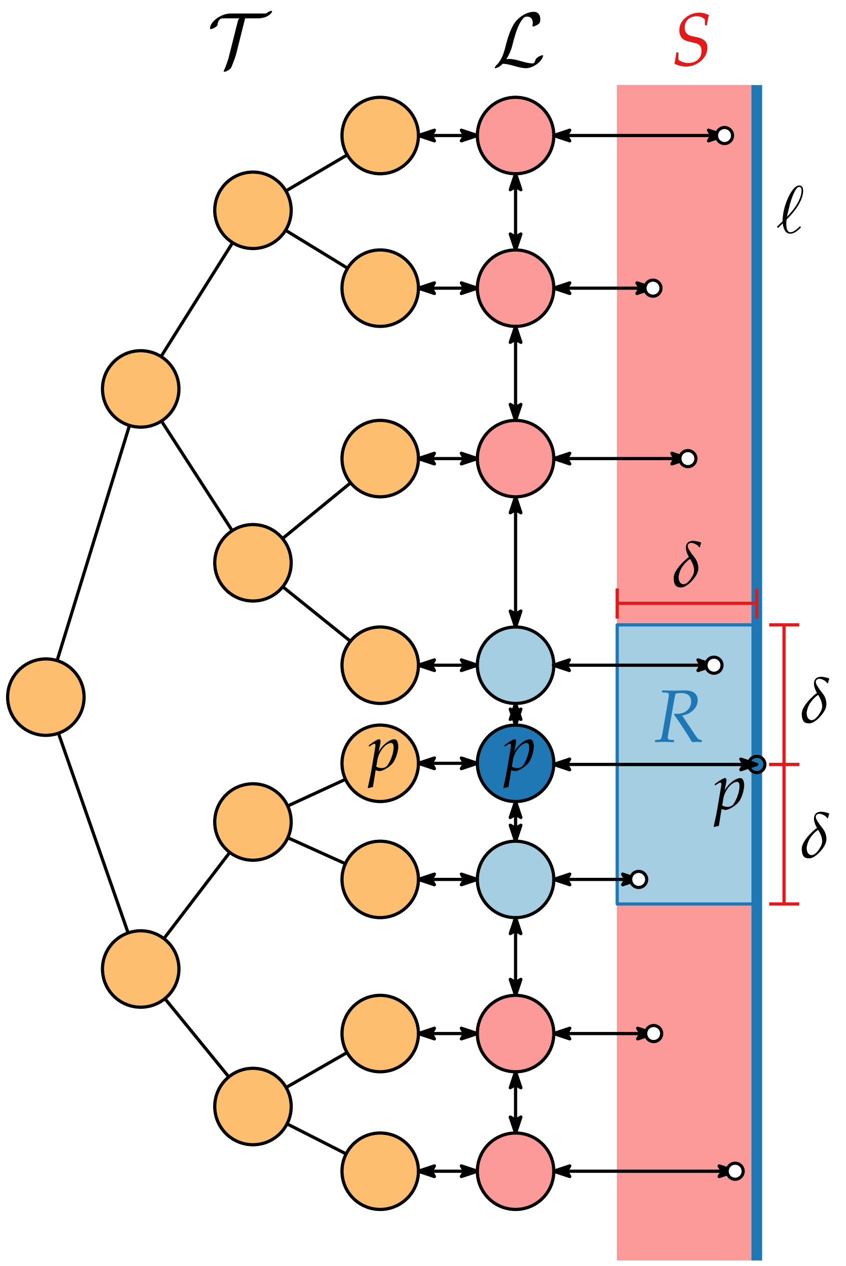

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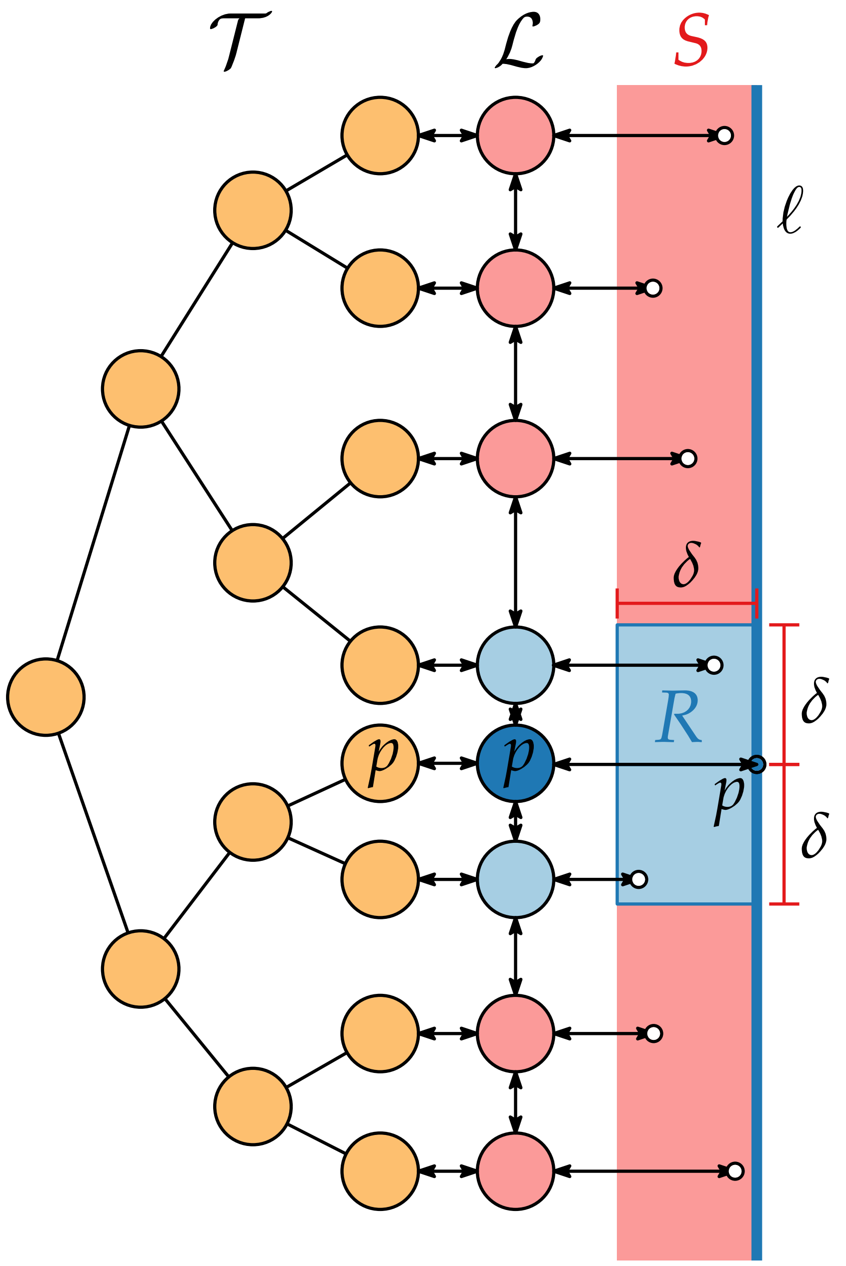

Invariant 2: when we reach a point p, $\mathcal{T}$ and $\mathcal{L}$ contain exactly the points in $P \cap S$.

Algorithm

$p_1, p_2, \ldots, p_n =$ points of P sorted according to their x-coordinates
$P_{\min} =$ nil // current closest pair
$\delta = \infty$ // distance of current closest pair
$k = 1$ // index of the left-most point in $\mathcal{L}$ and $\mathcal{T}$
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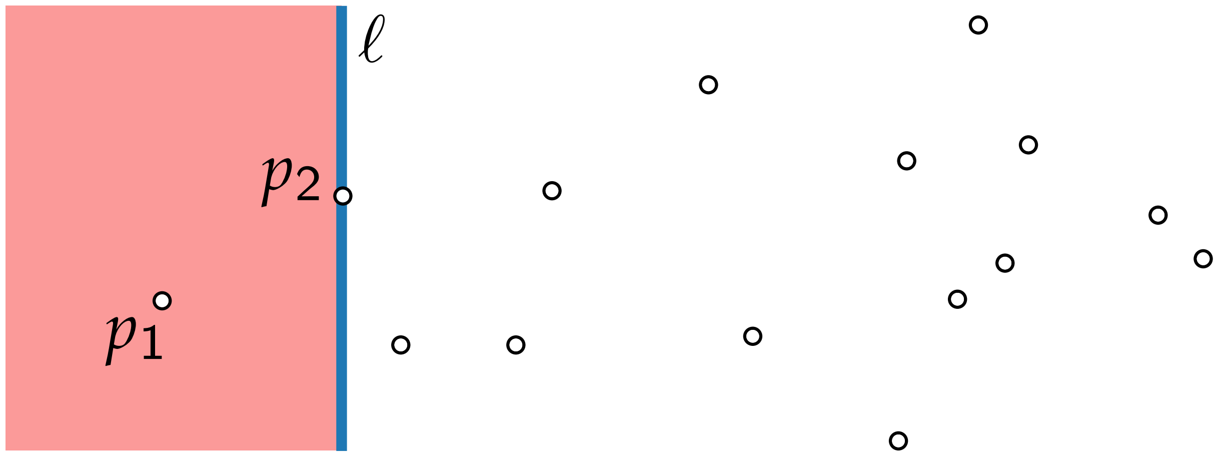

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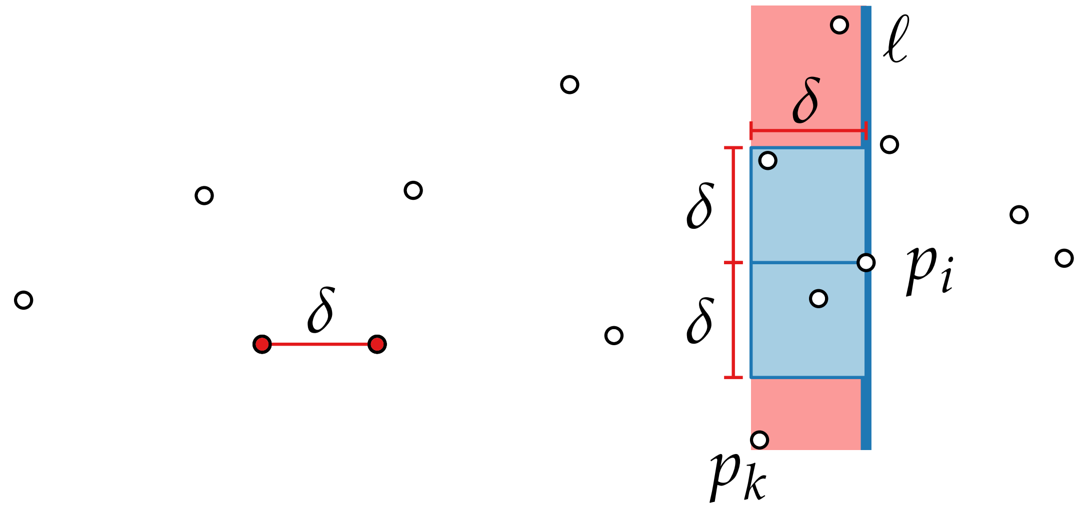

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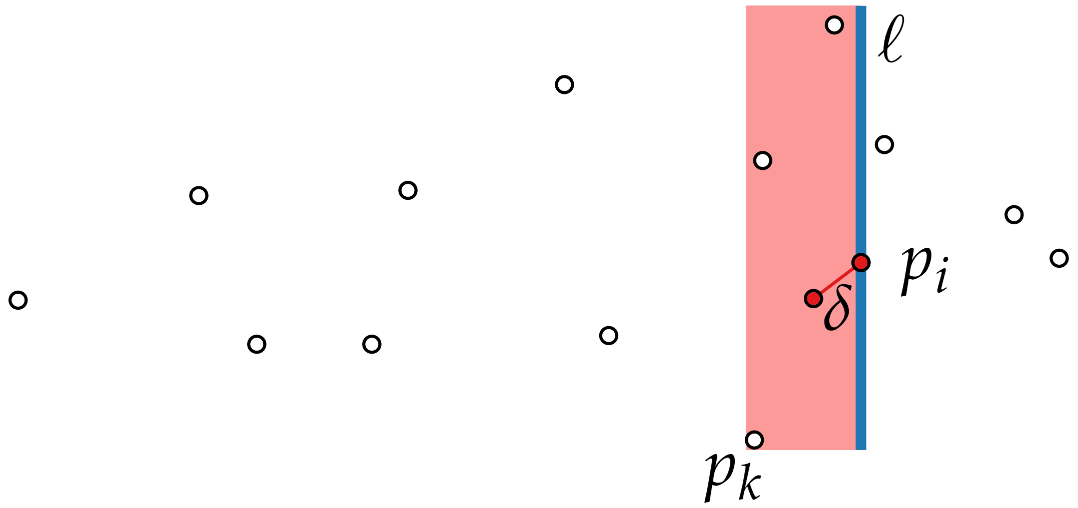

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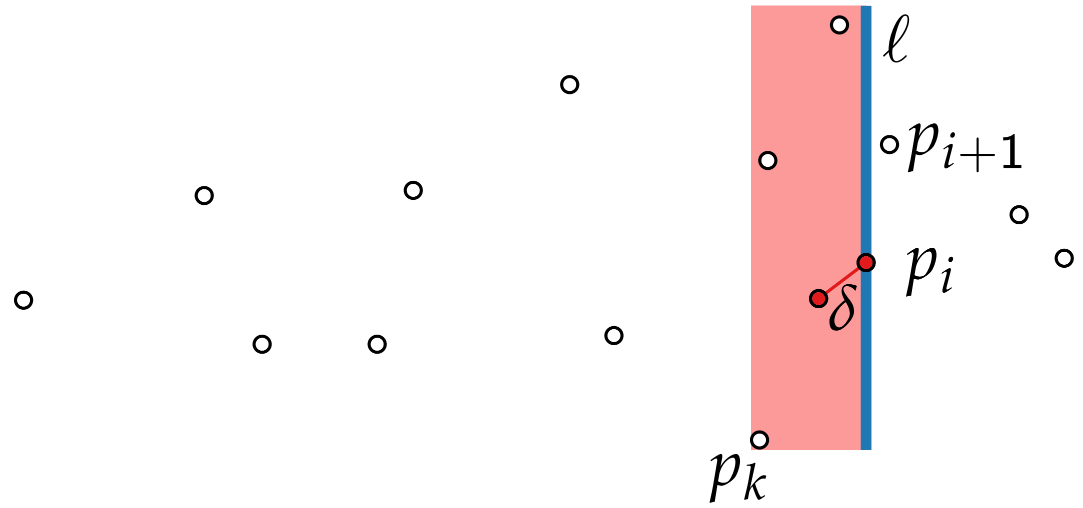

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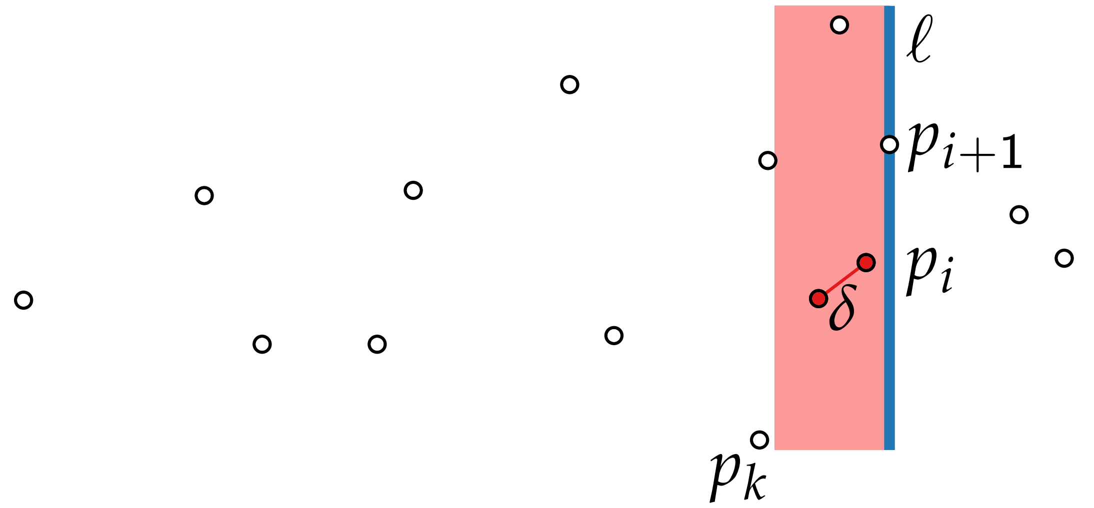

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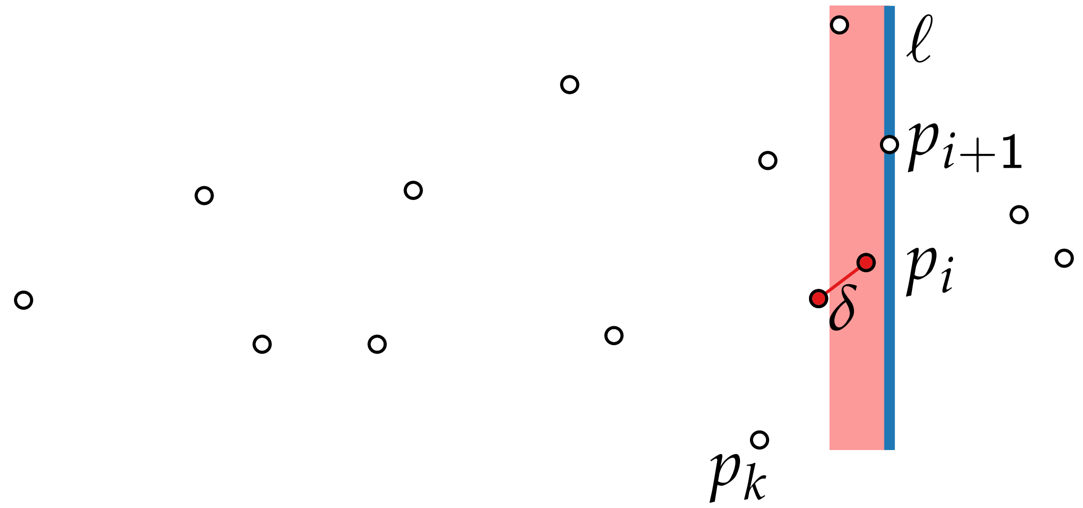

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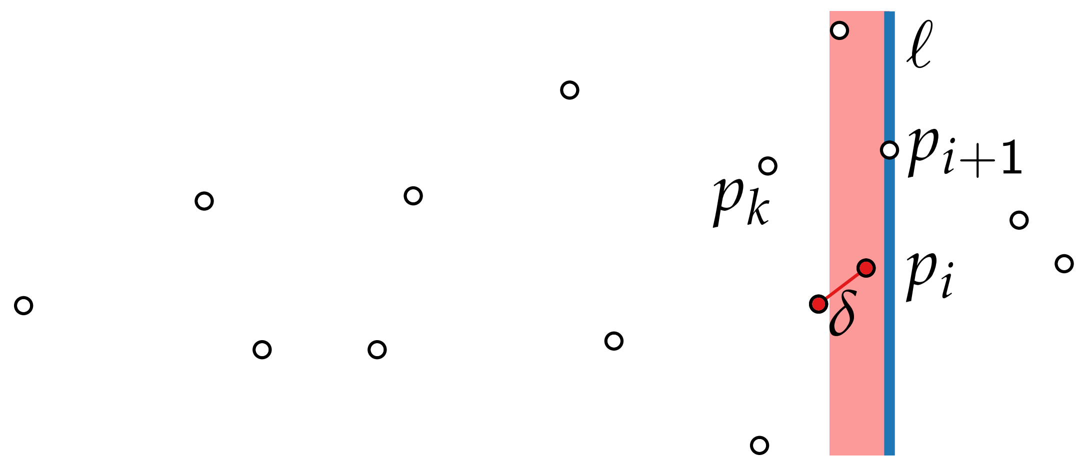

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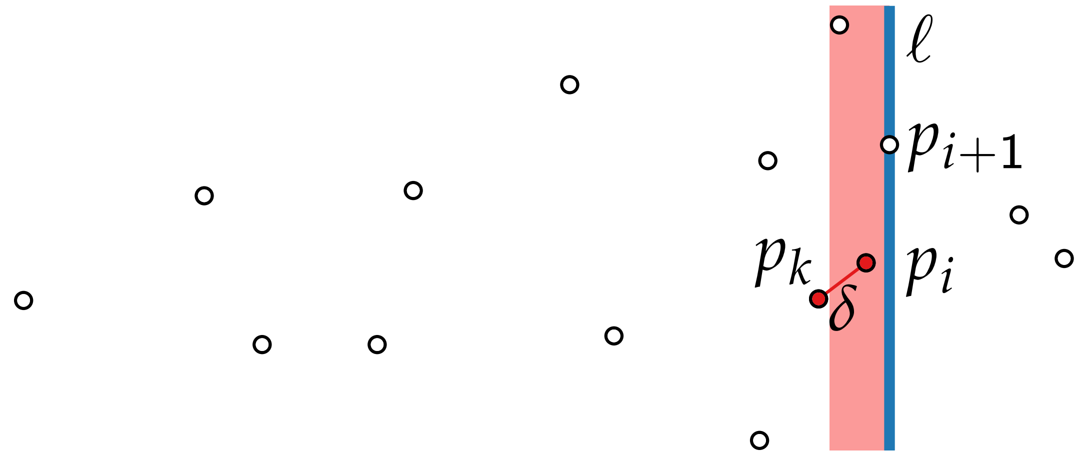

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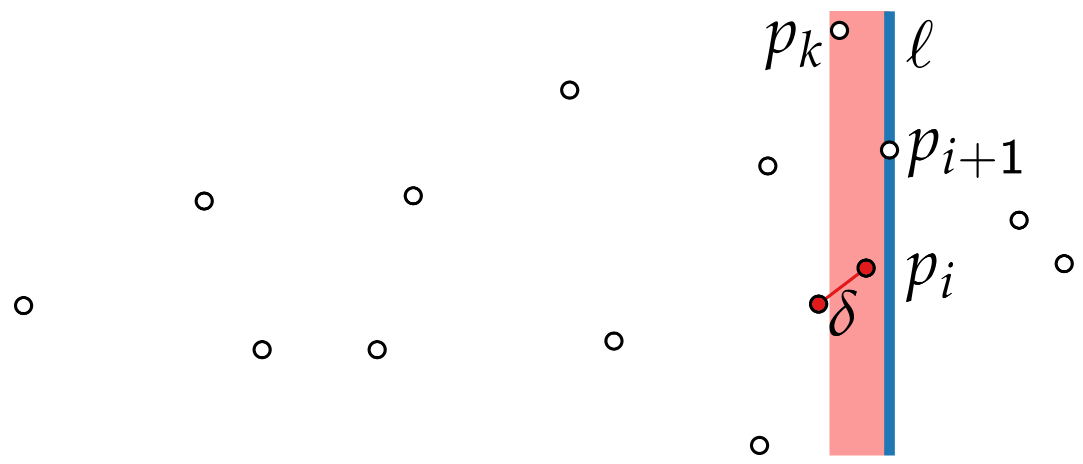

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$p_1, p_2, \ldots, p_n$ = points of P sorted according to their x-coordinates $\mathcal{O}(n \log n)$

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initialize $\mathcal{L}$ and $\mathcal{T}$ with p_1

```
for i = 2, 3, ..., n do
    insert p_i into L and T                                  O(log n)
    for p_j ∈ [y(p_i) − δ, y(p_i) + δ] \ {p_i} do            O(1)
        if ||p_j − p_i|| < δ do                                         O(1)
            P_min = {p_j, p_i};  δ = ||p_j − p_i||
    while x(p_k) < x(p_{i+1}) − δ do                         O(n) in total
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$\Rightarrow$ **Total runtime:** $\mathcal{O}(n \log n)$

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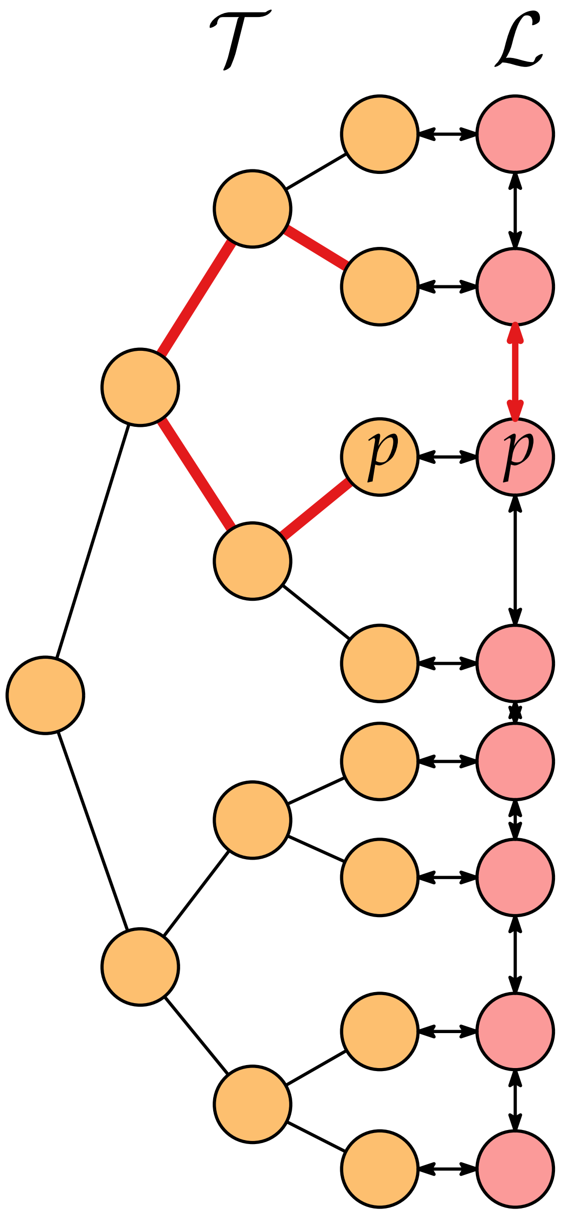

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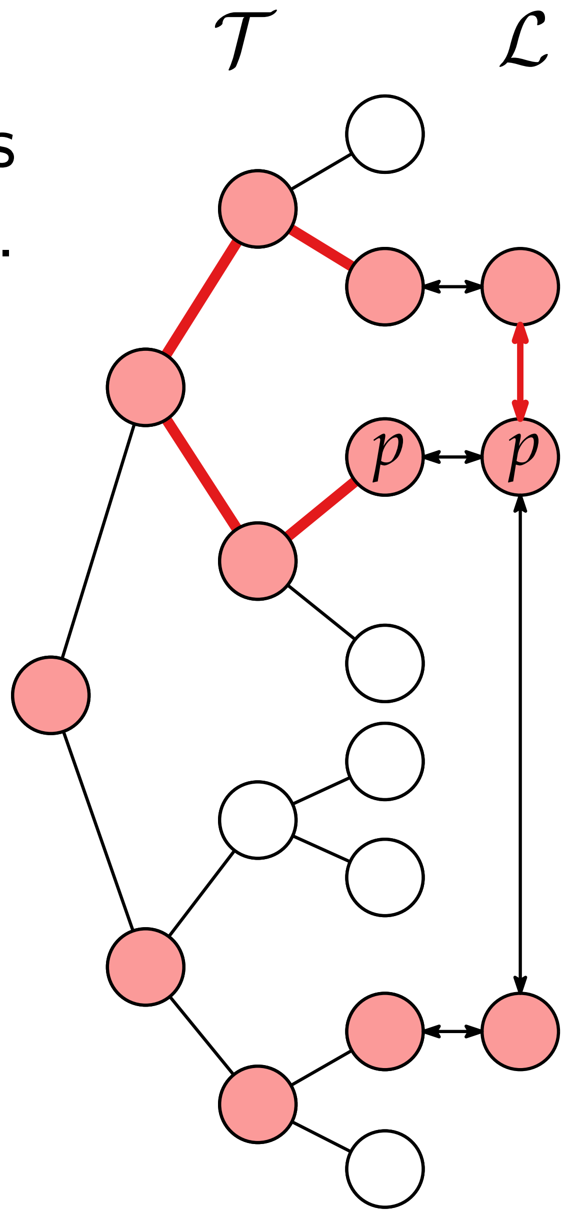

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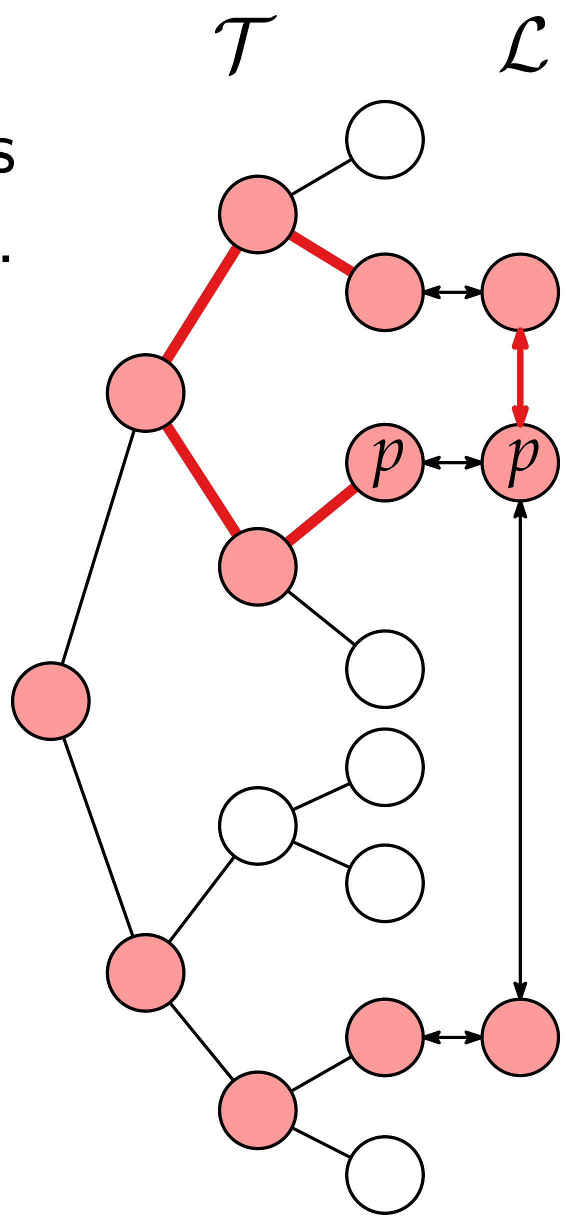
- We assumed that the points in P have pairwise distinct x-coordinates. This situation can be established by rotating P or tilting ℓ slightly.
  Simply, visit the points in lexicographical order!



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Main idea: Sweep the plane with a line ℓ while maintaining two invariants:

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The sweep line paradigm is **powerful** and leads to **simple** algorithms for many problems:
computing Voronoi diagrams, crossings in an arrangement of line segments,
intersection/union of two polygons, decompositions of polygons,
certain triangulations, visibility polygons, ...

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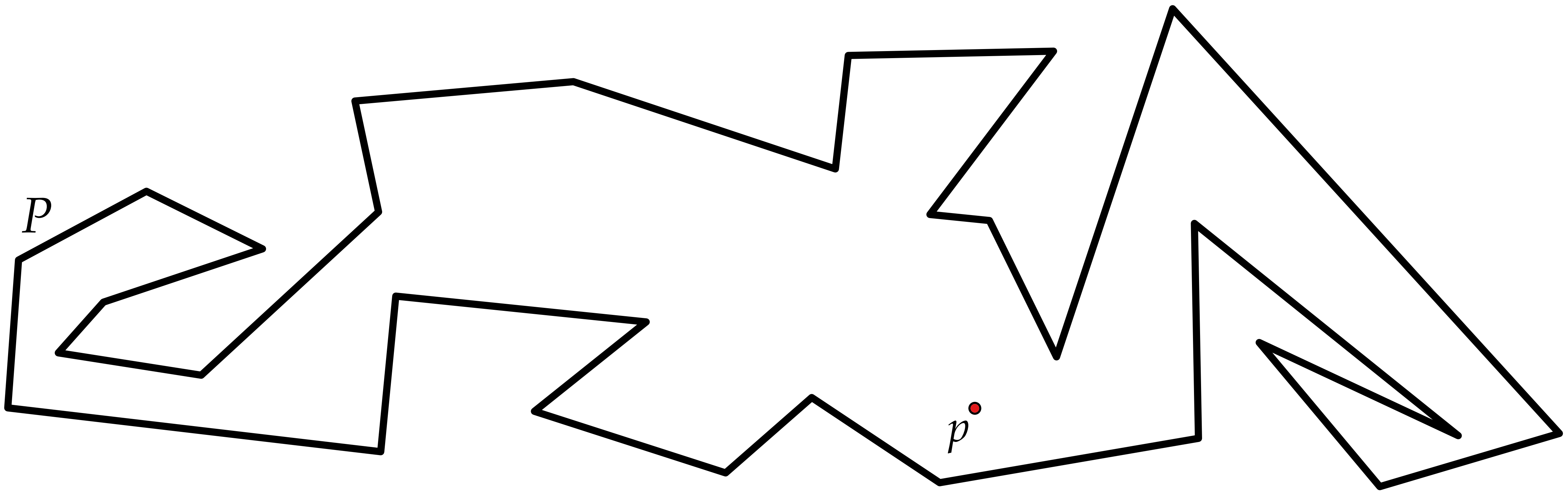

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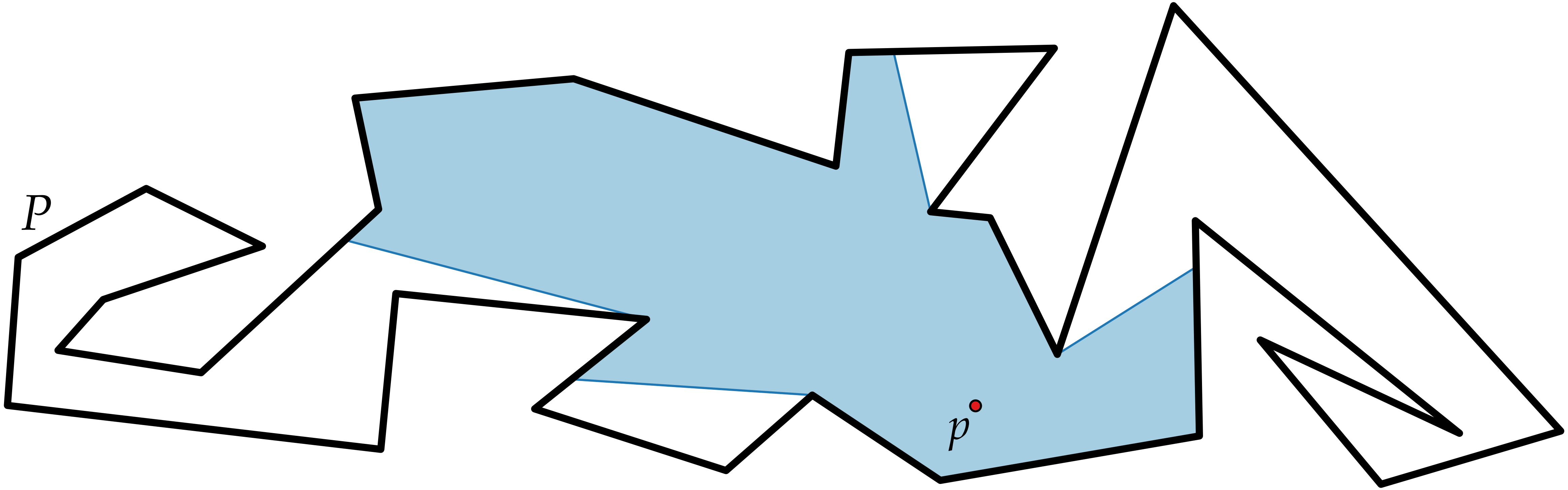

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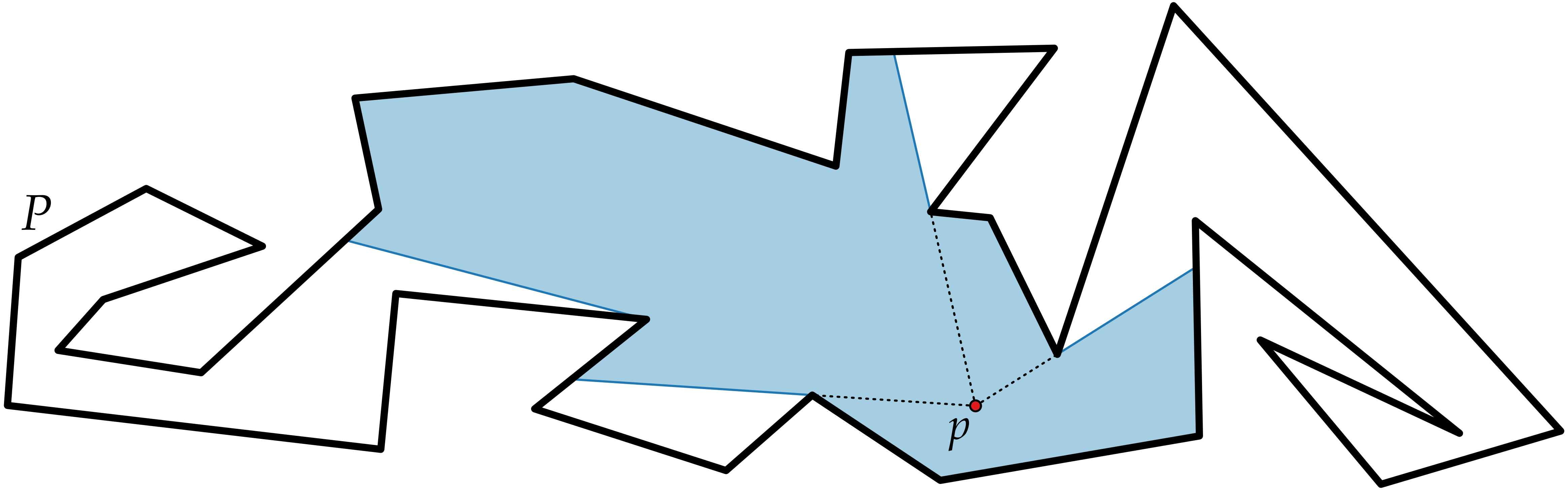

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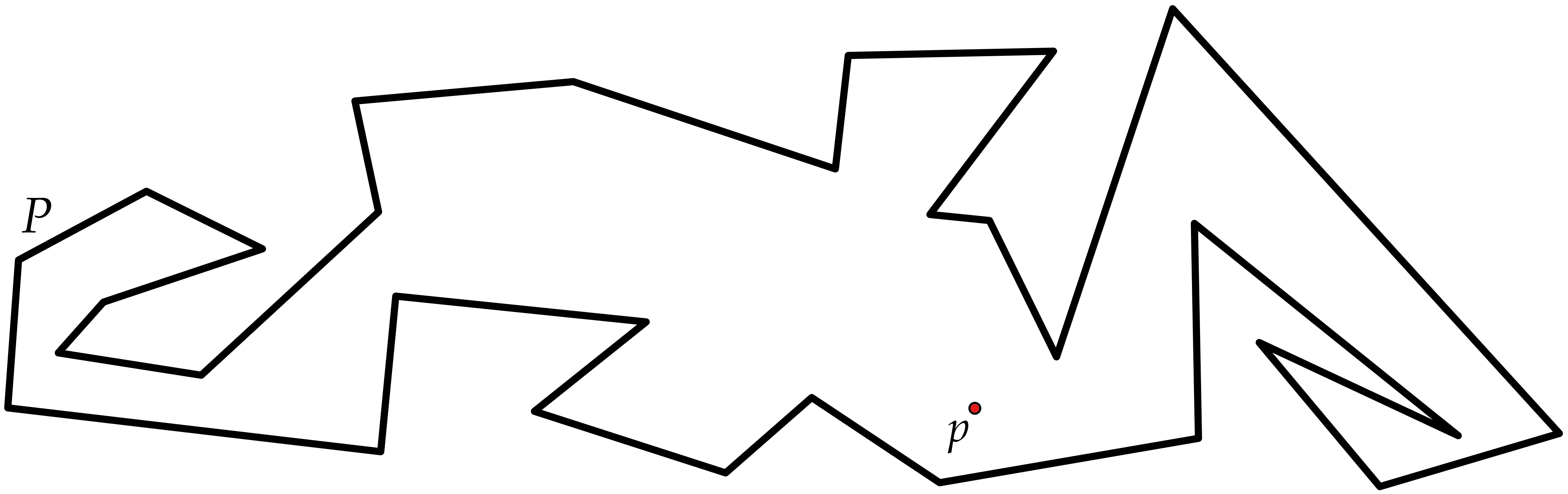

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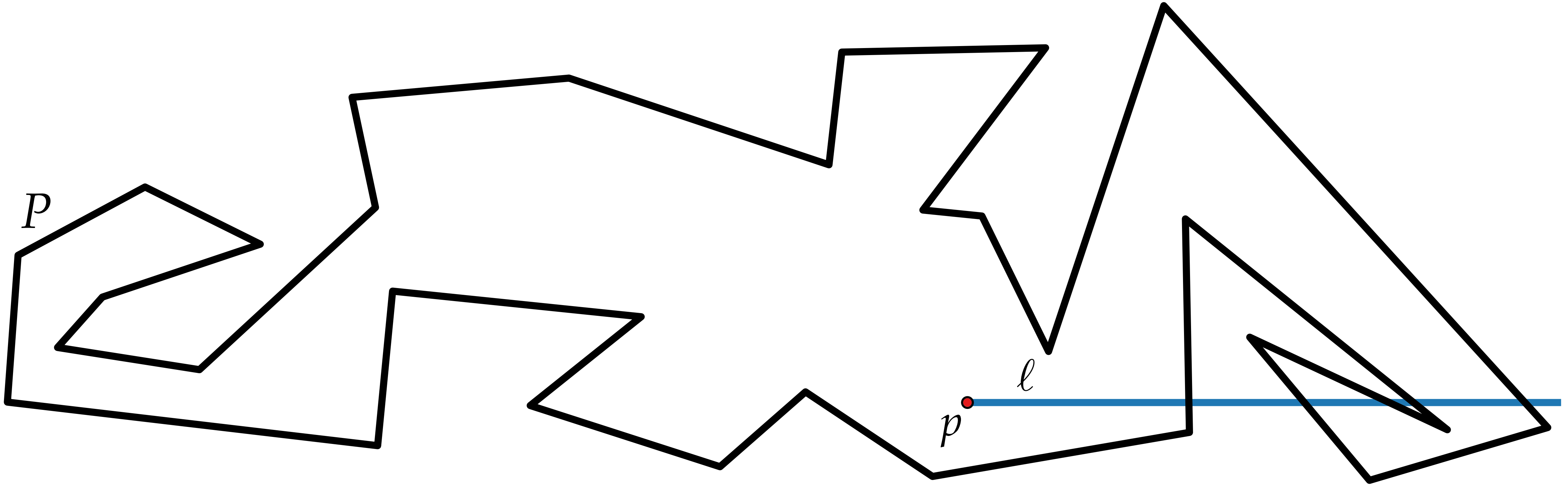

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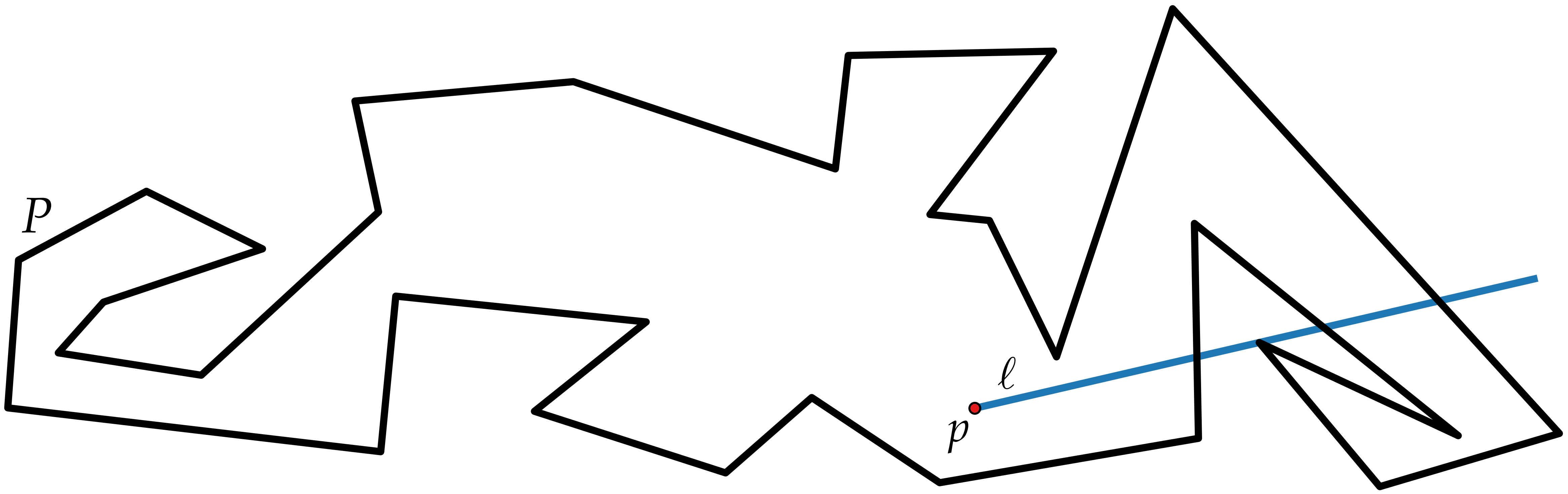

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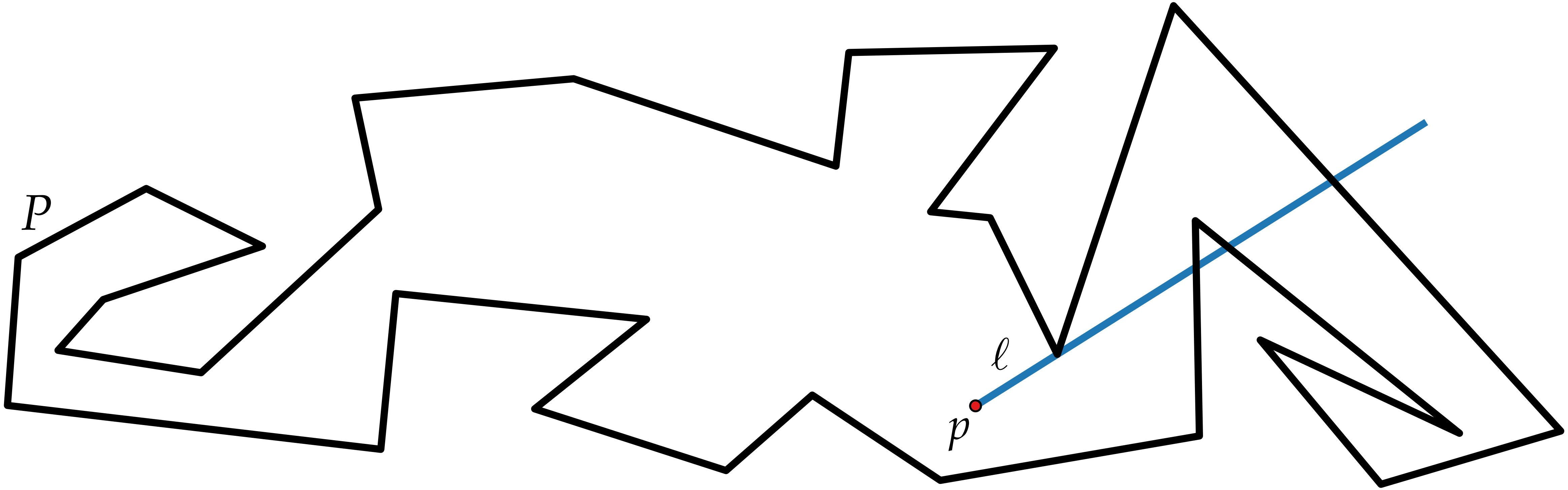

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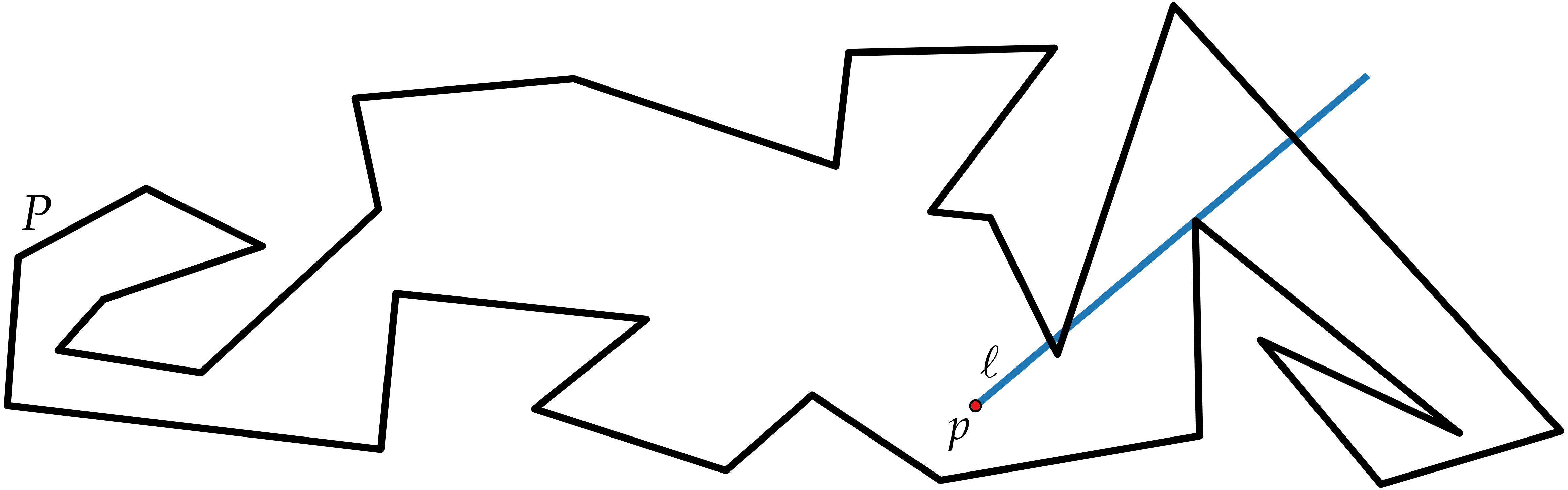

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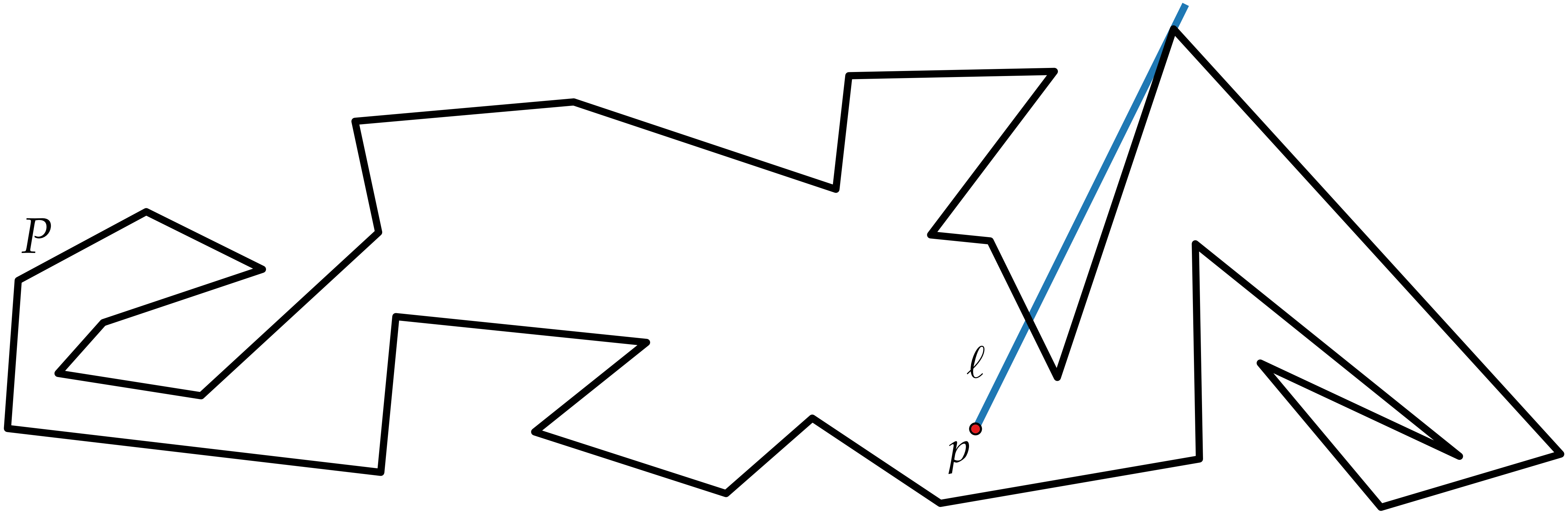

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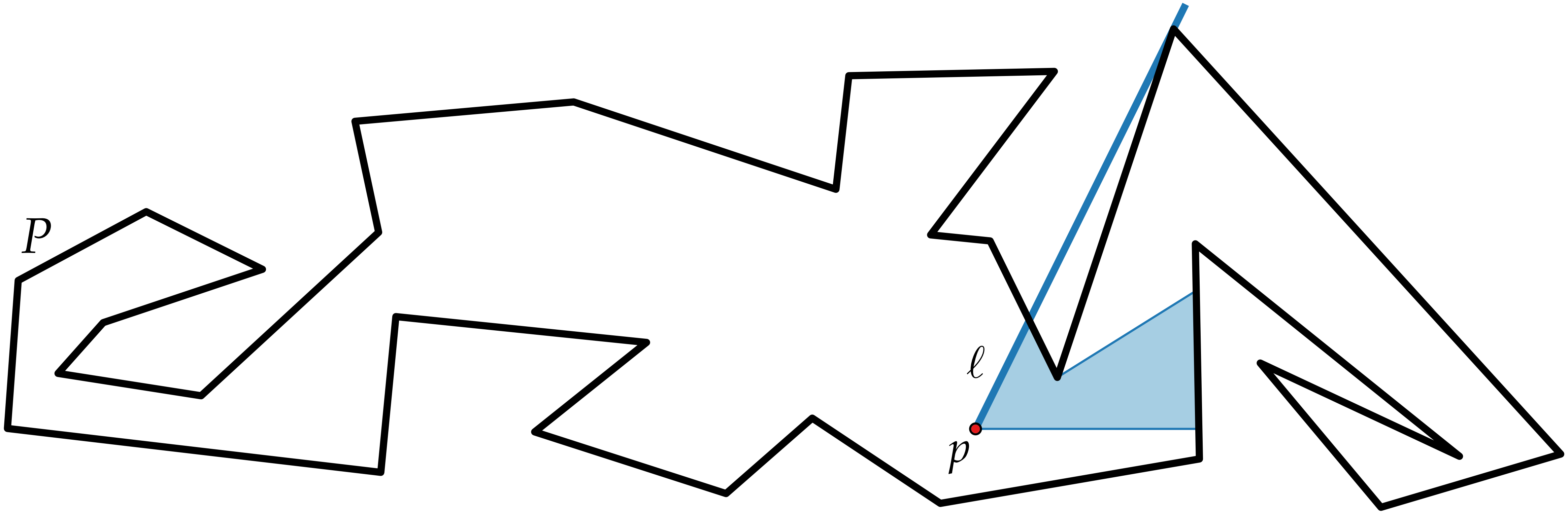

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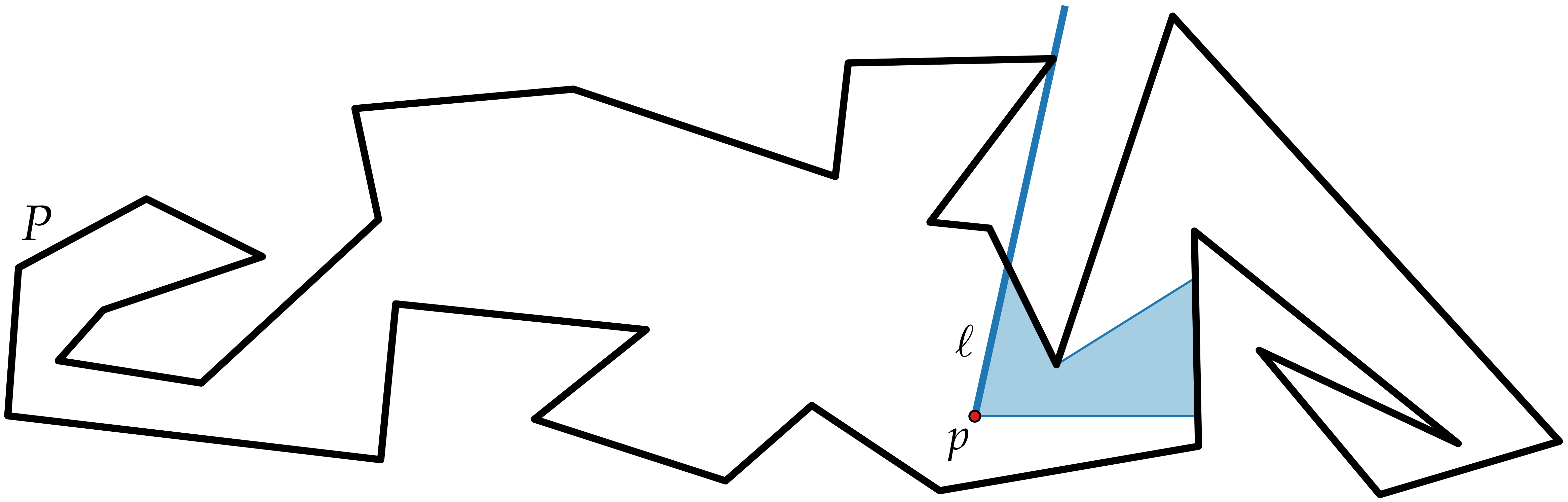

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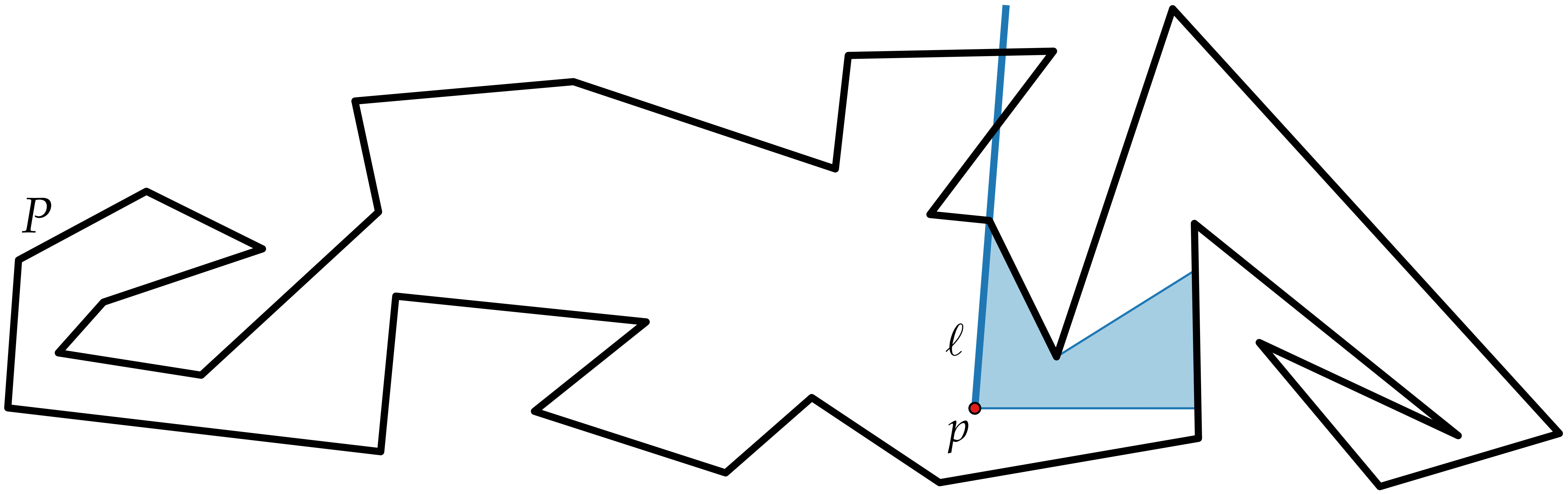

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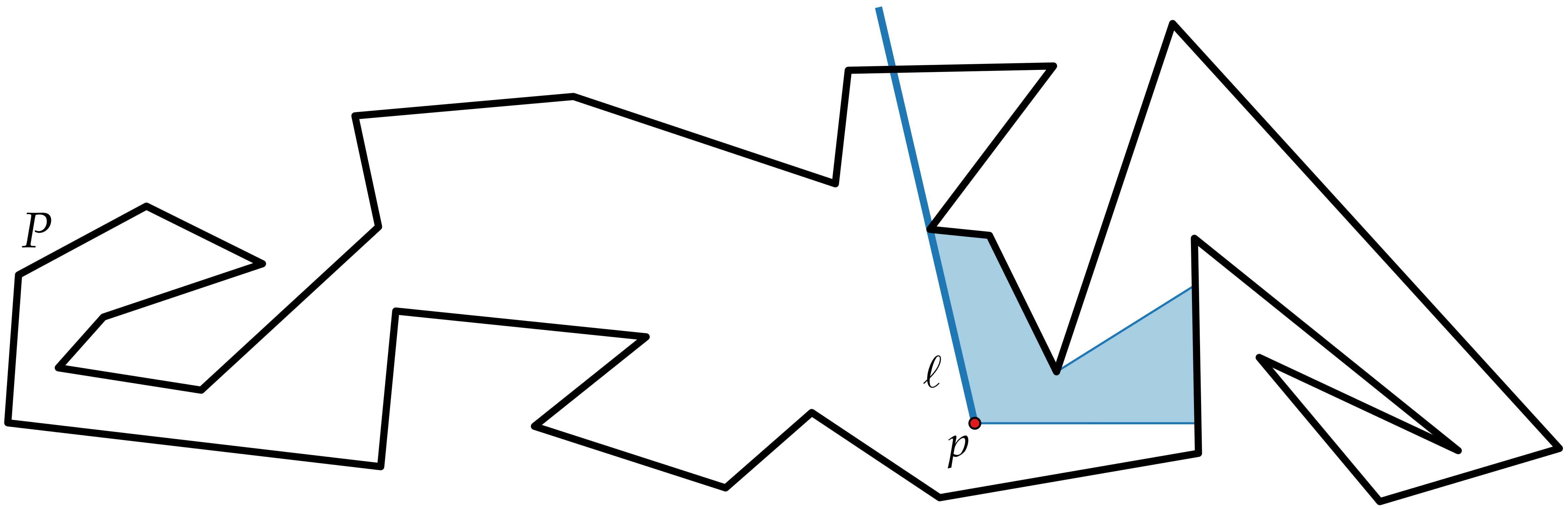

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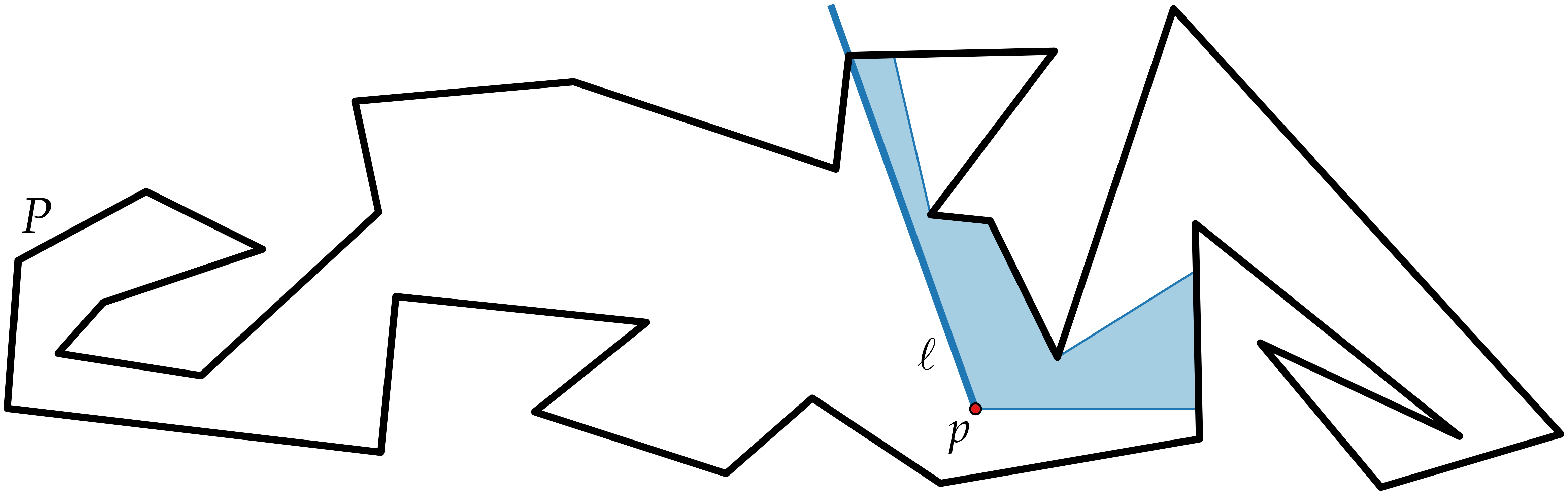

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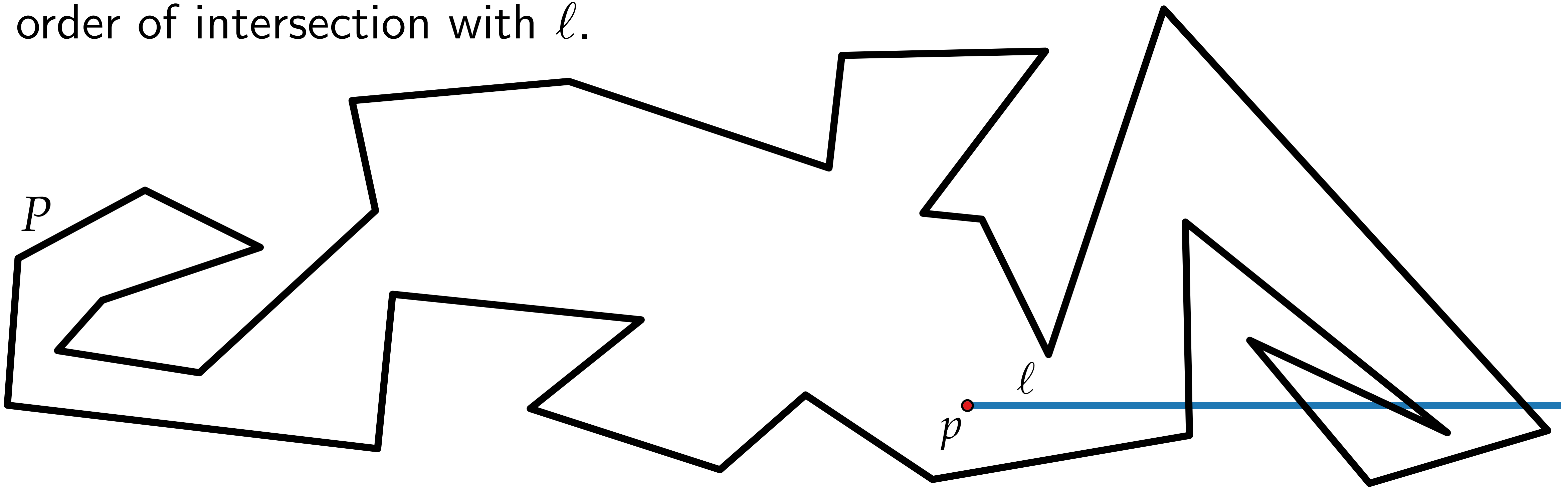

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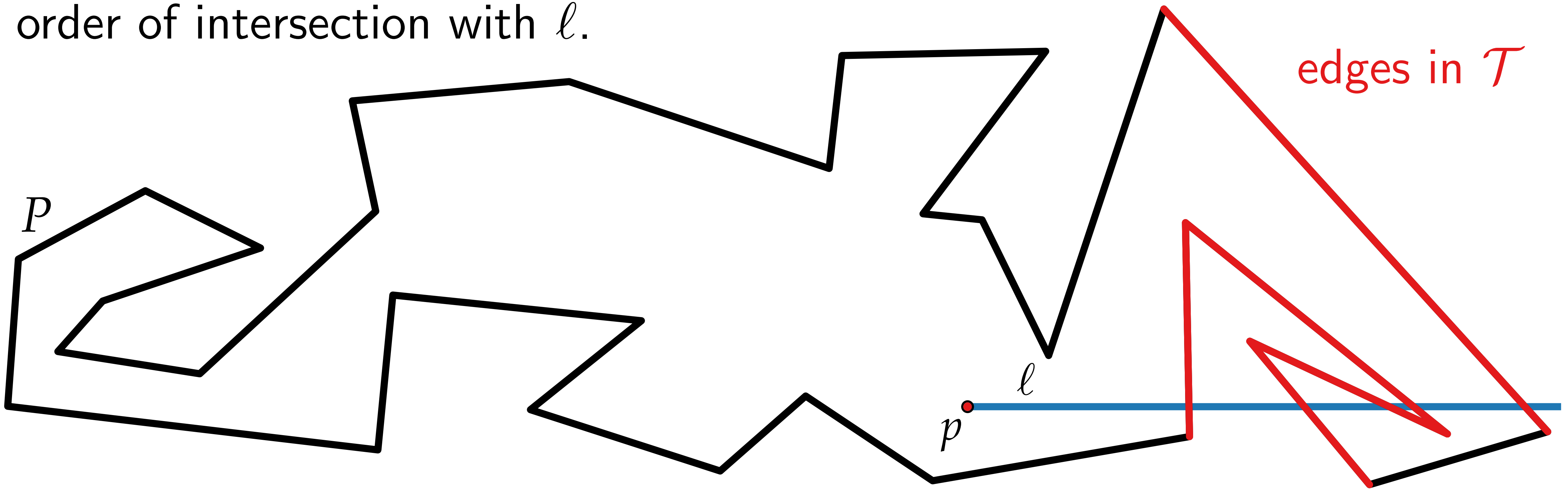

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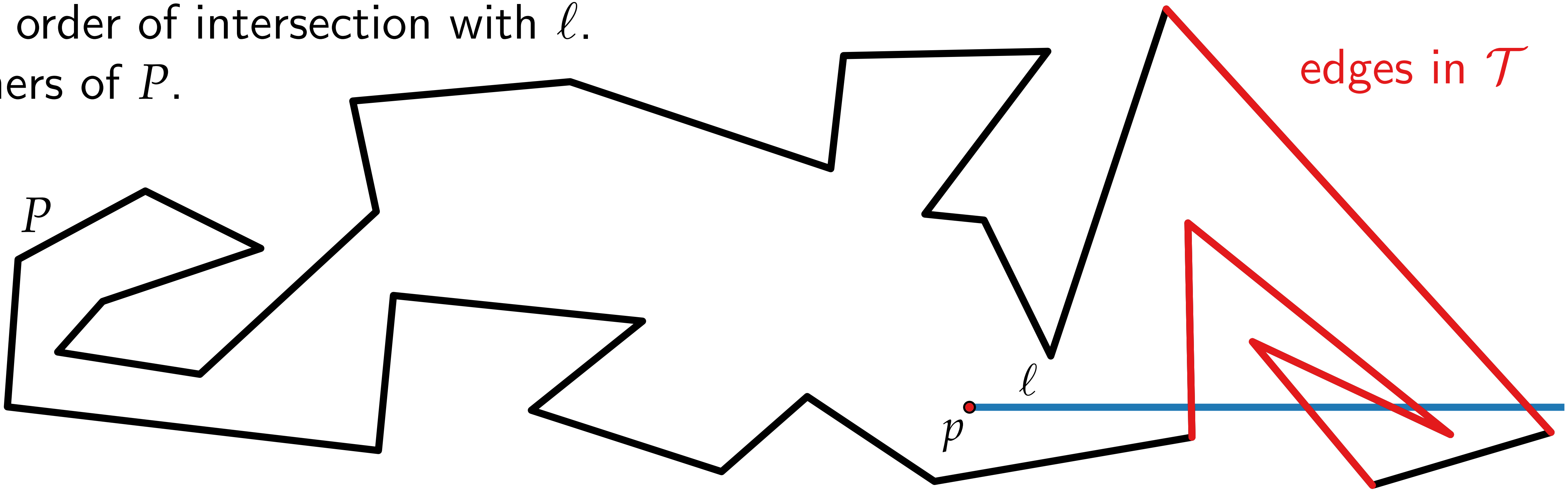

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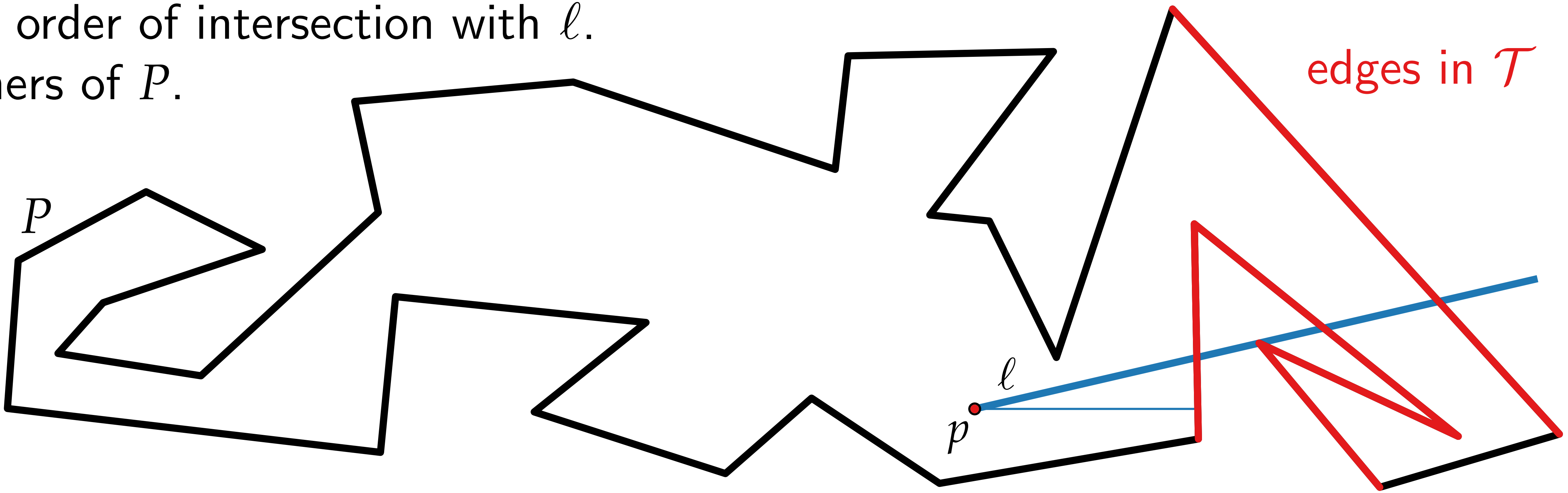

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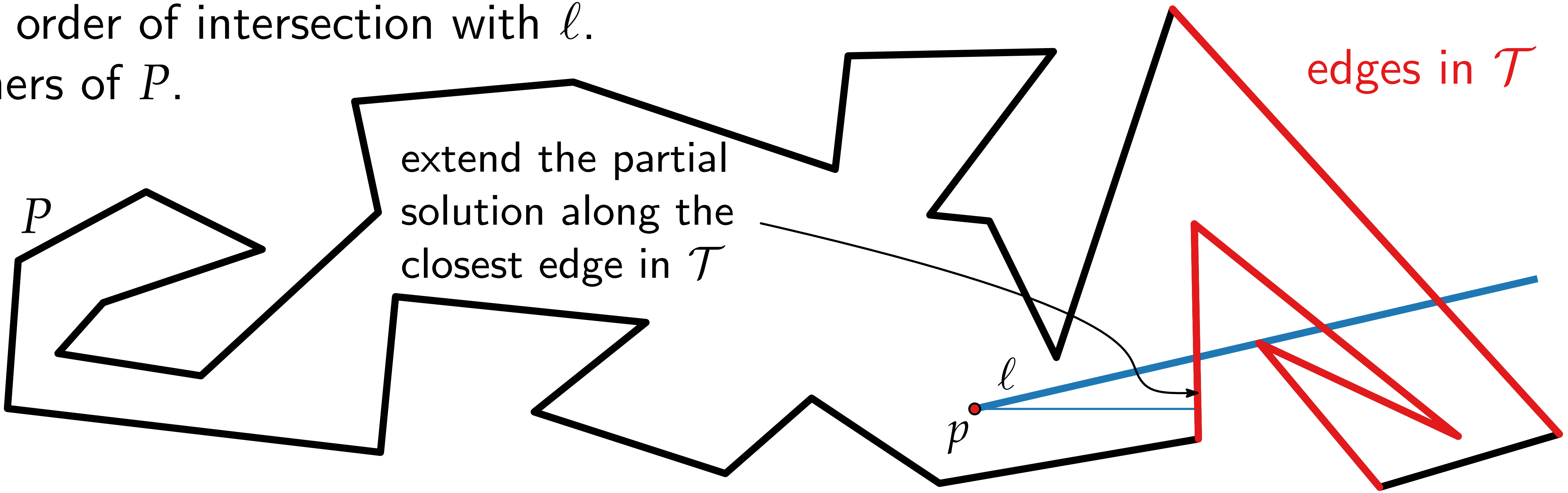

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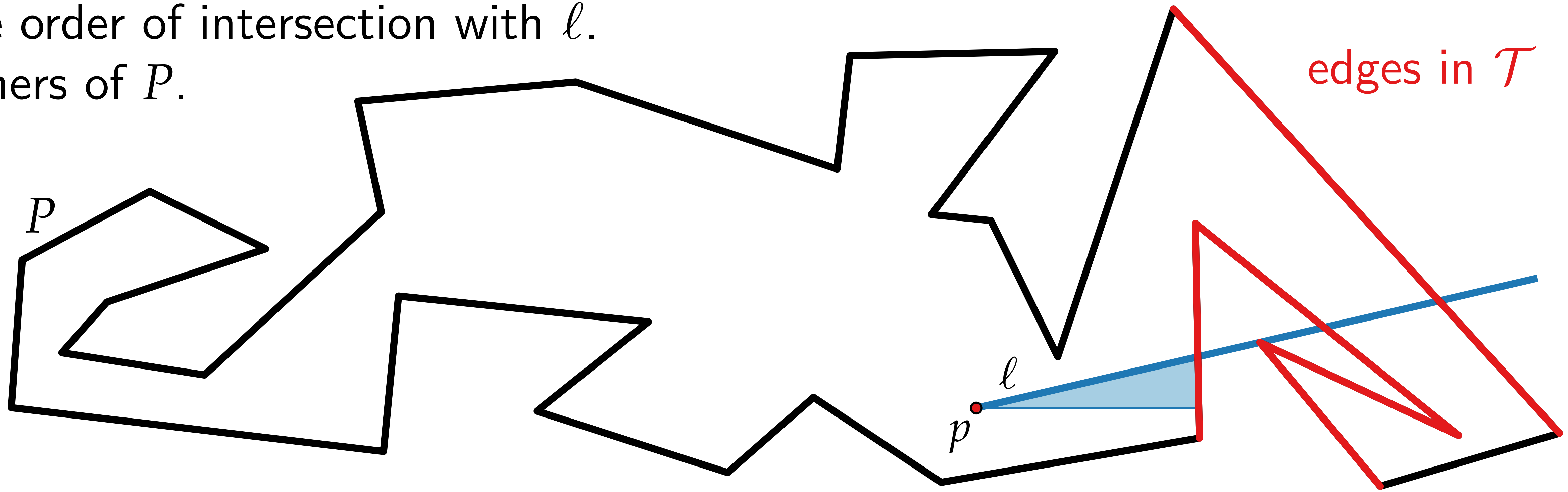

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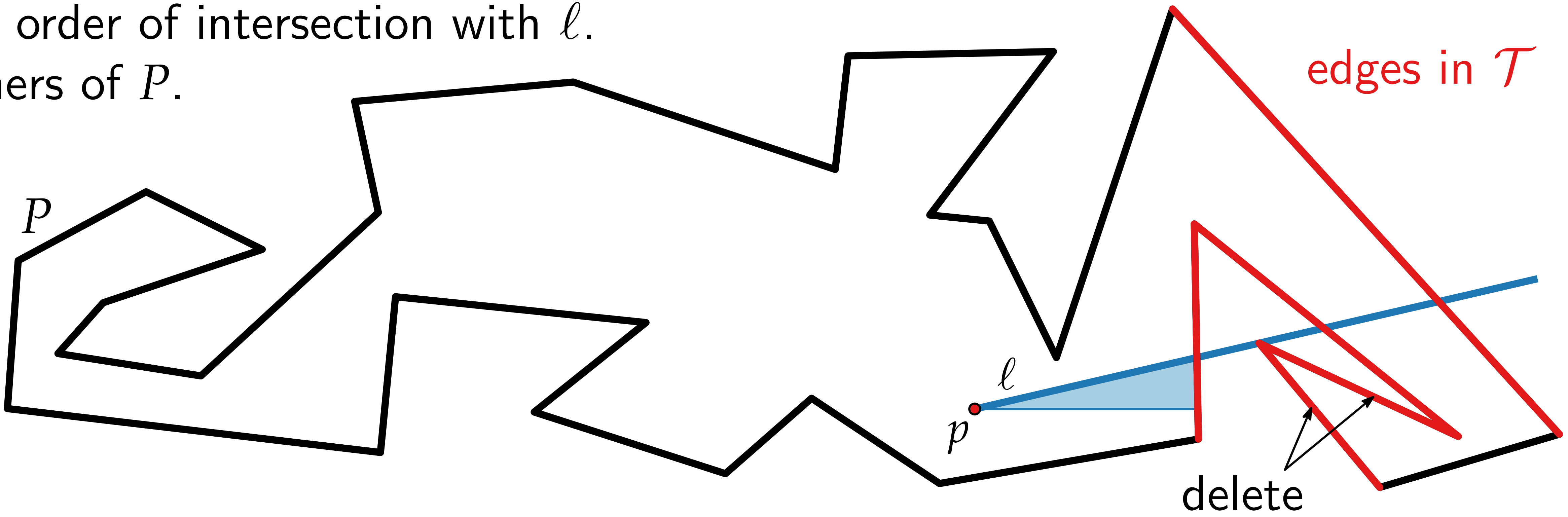

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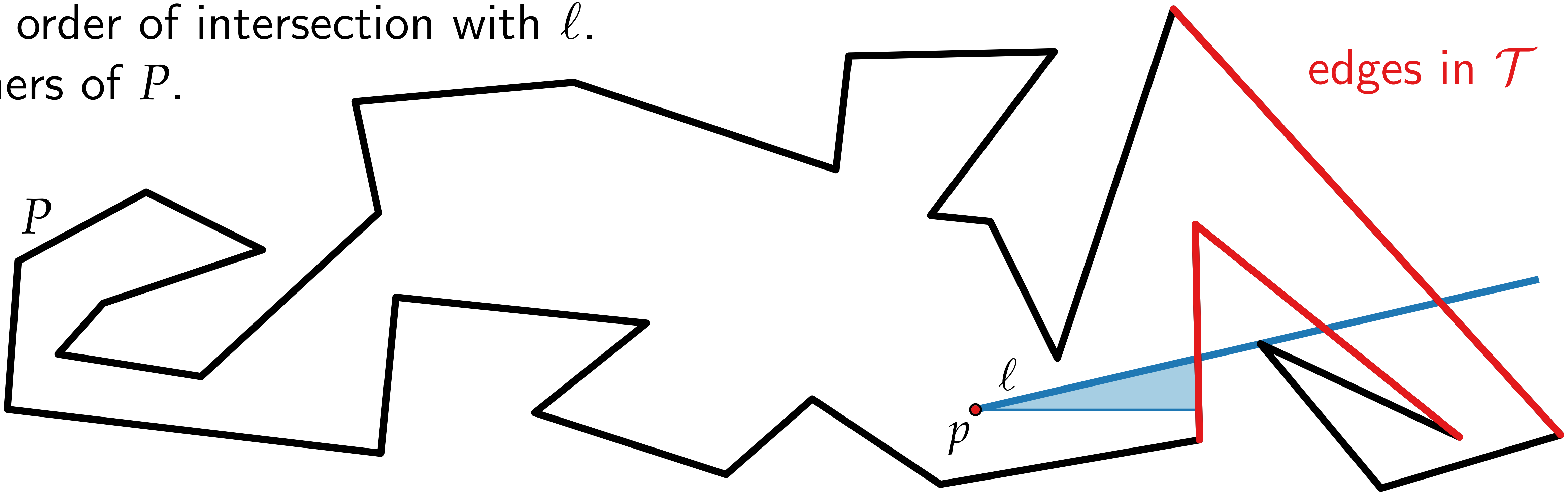

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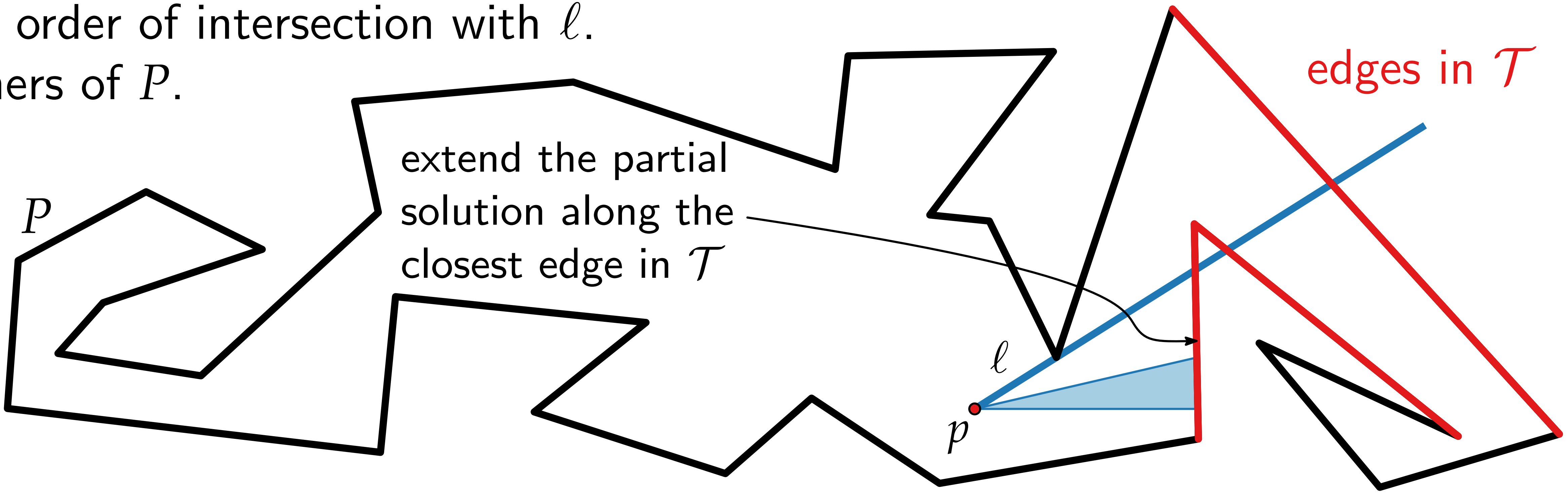

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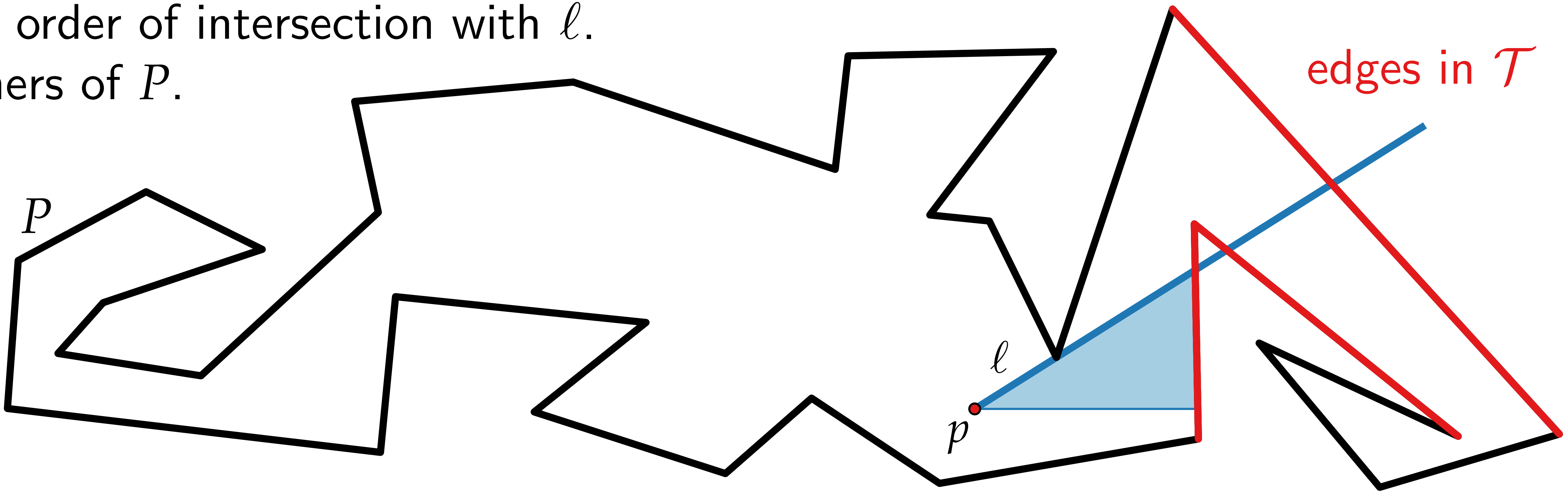

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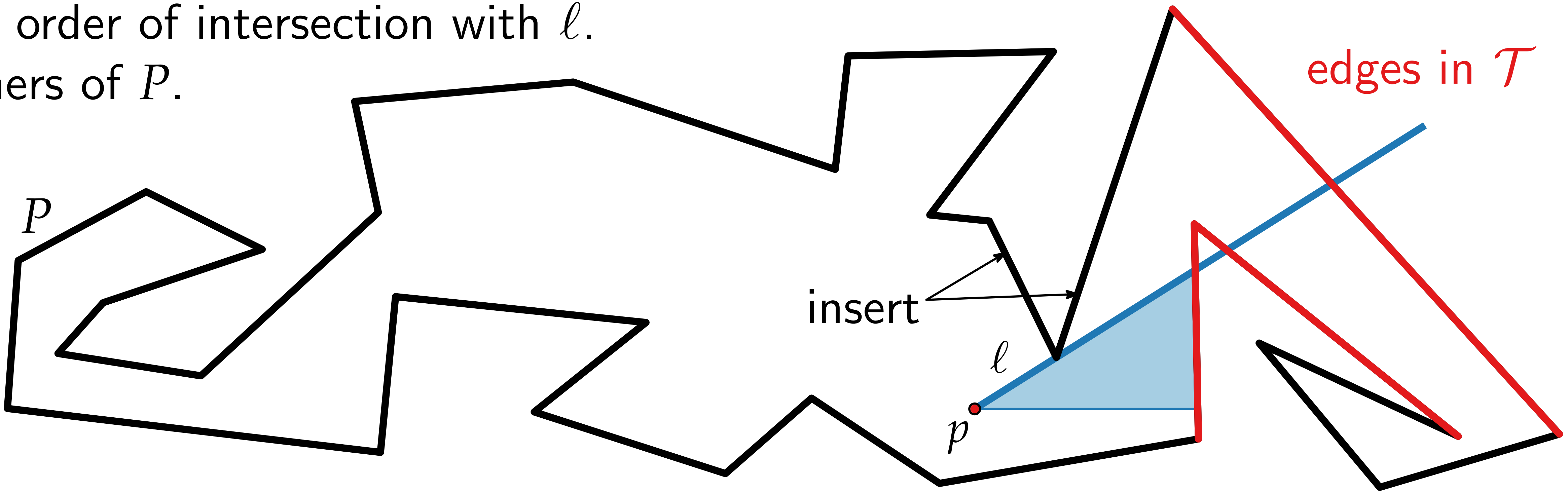

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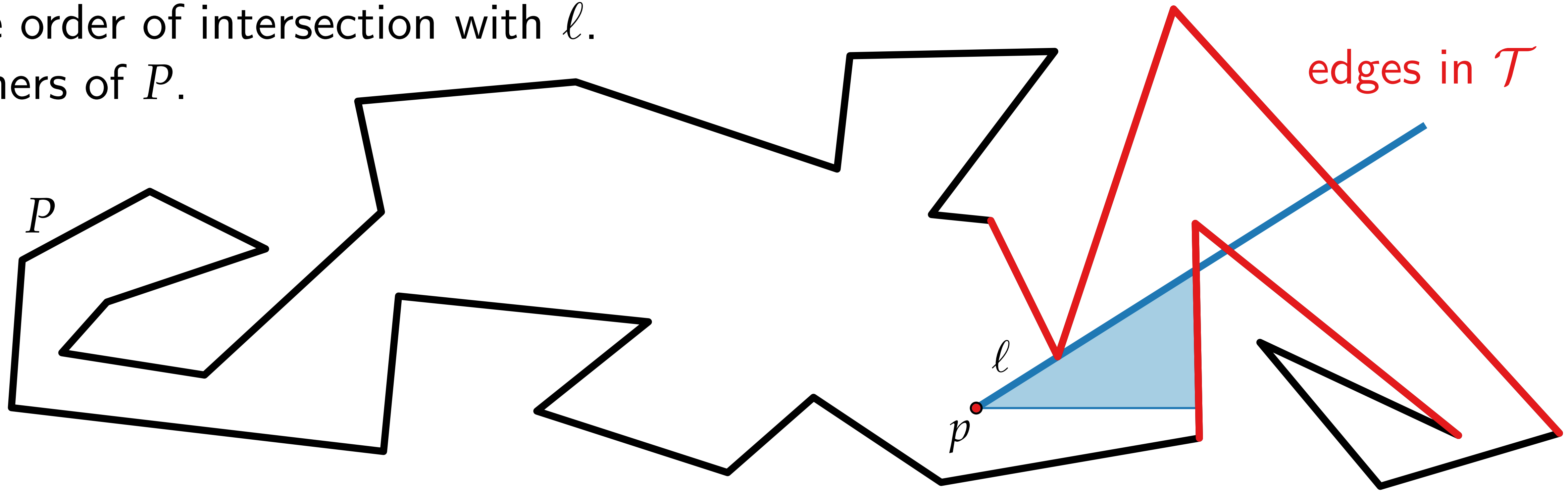
Task: Compute the visibility polygon of p with respect to P.

Idea: Sweep a **ray** ℓ radially around p.

Sweep line status: Edges of P intersected by ℓ are stored in a balanced binary search tree $\mathcal{T}$ in the order of intersection with ℓ.

Events: Corners of P.



Outlook: Computing Visibility Polygons

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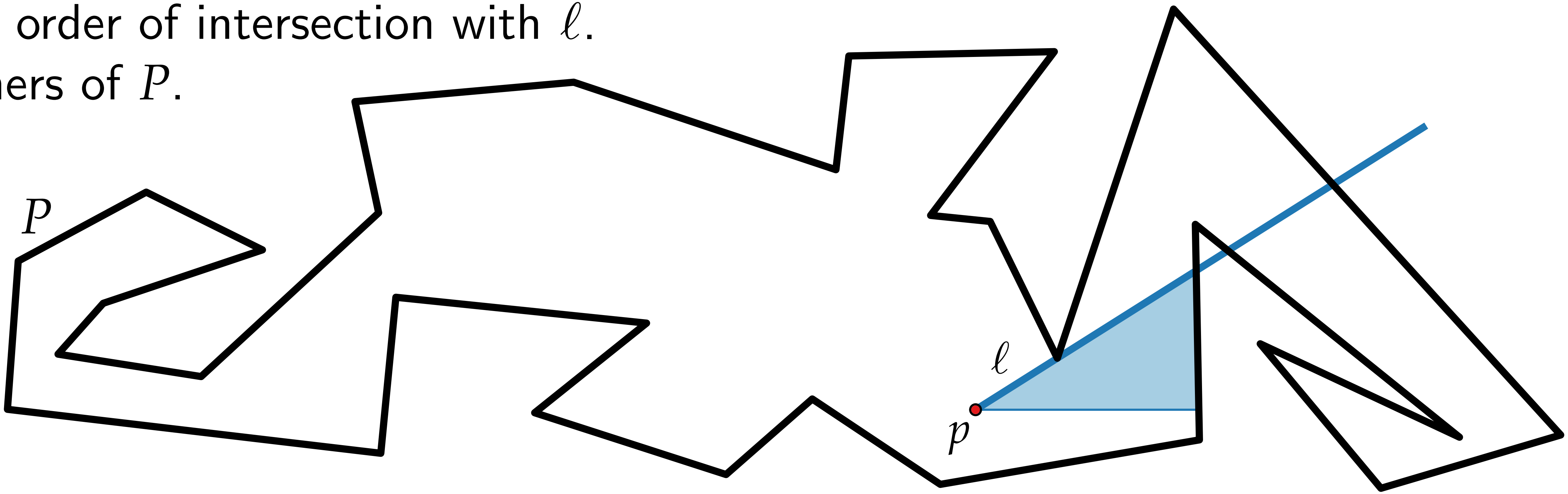

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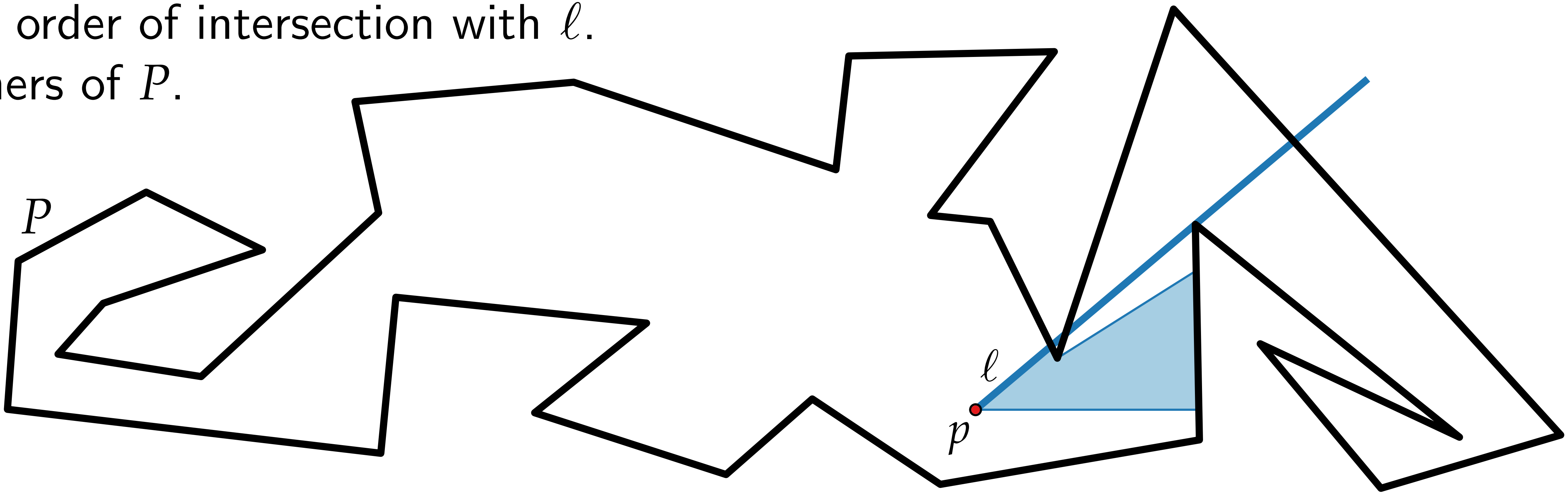

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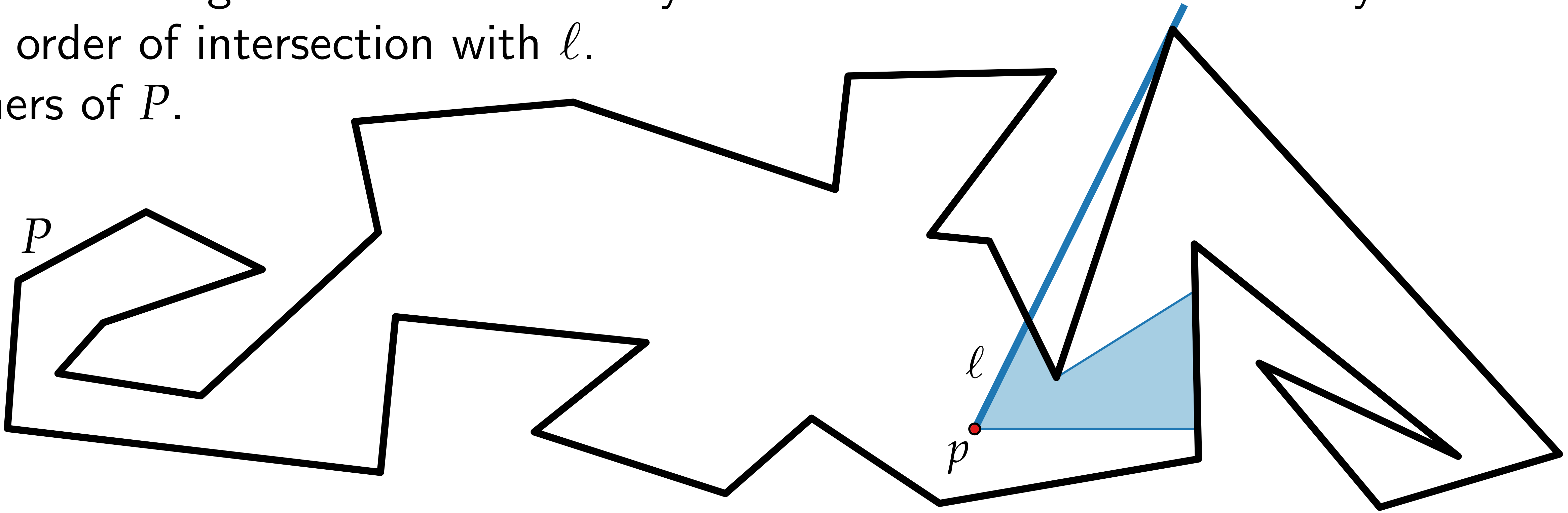

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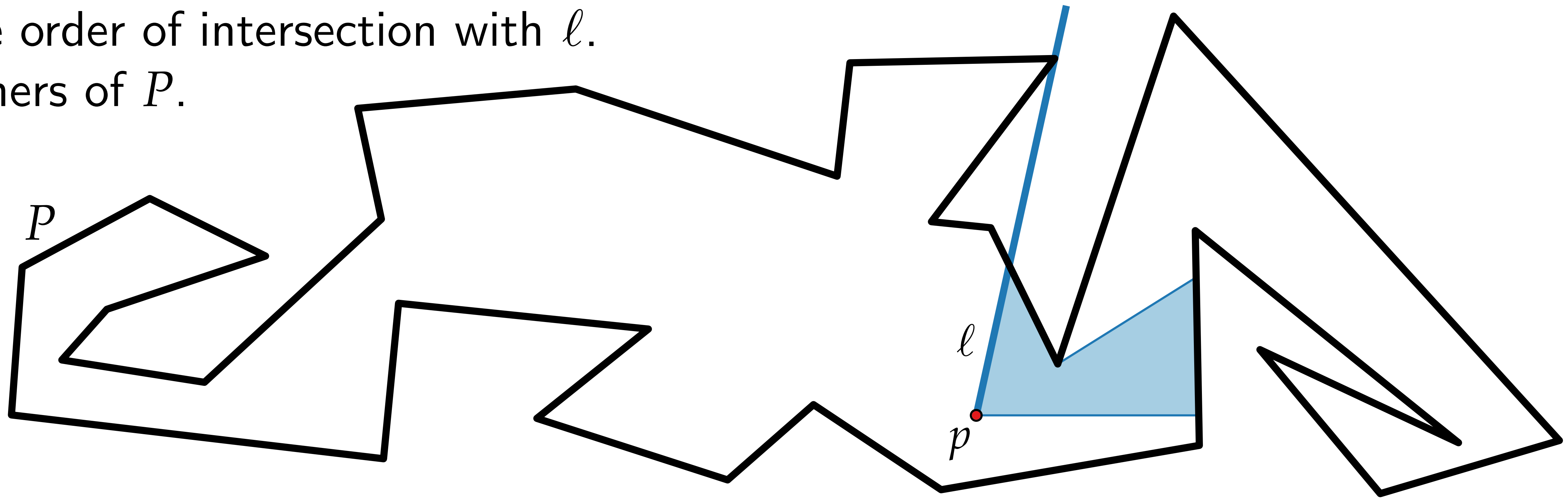

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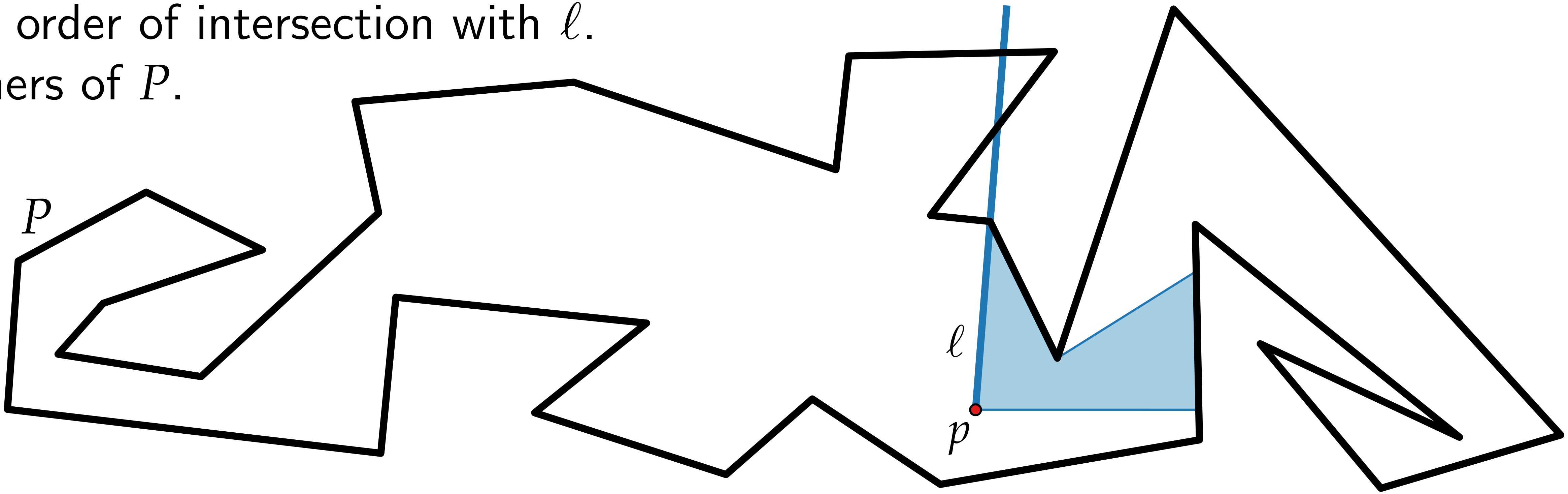

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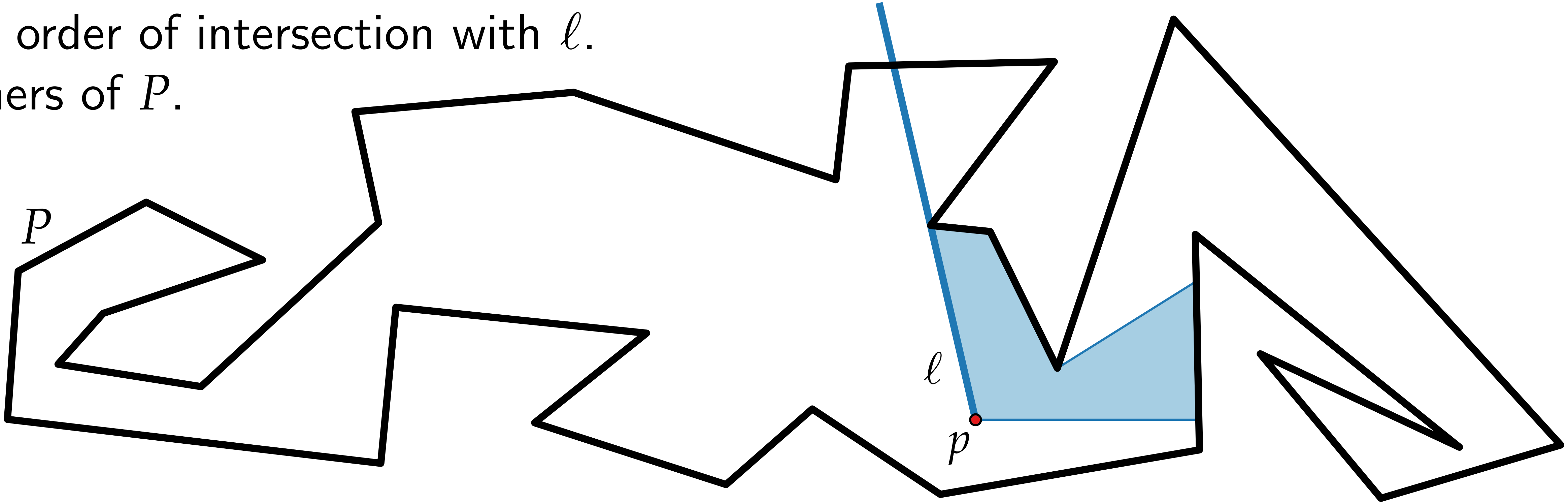

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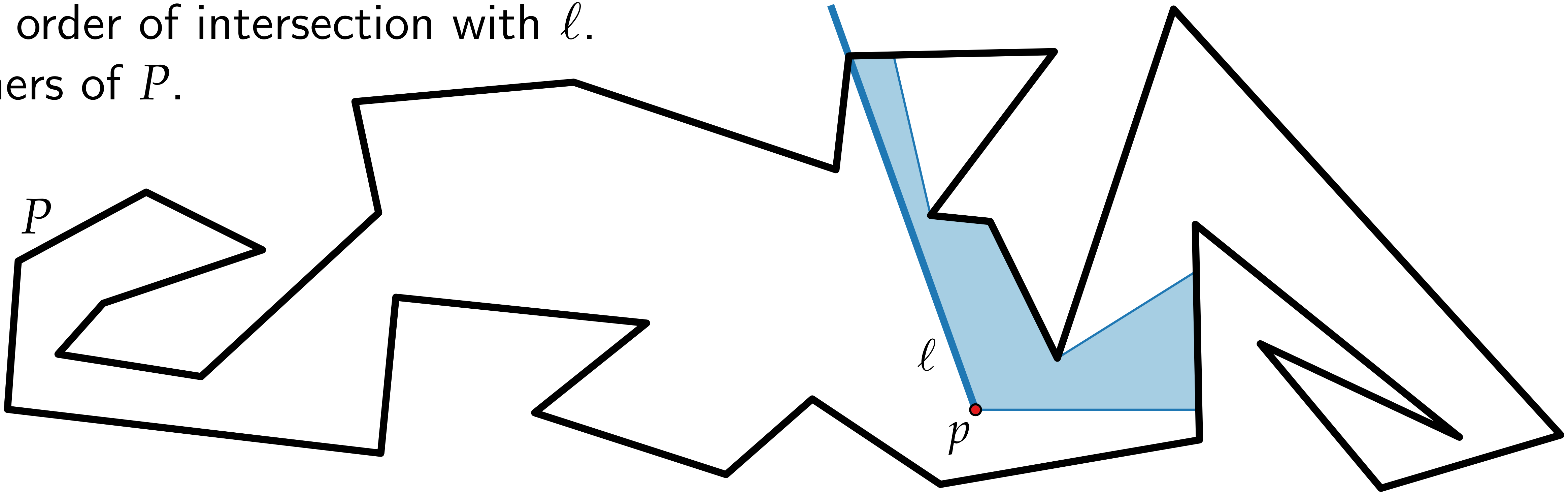

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Total runtime: $\mathcal{O}(n \log n)$

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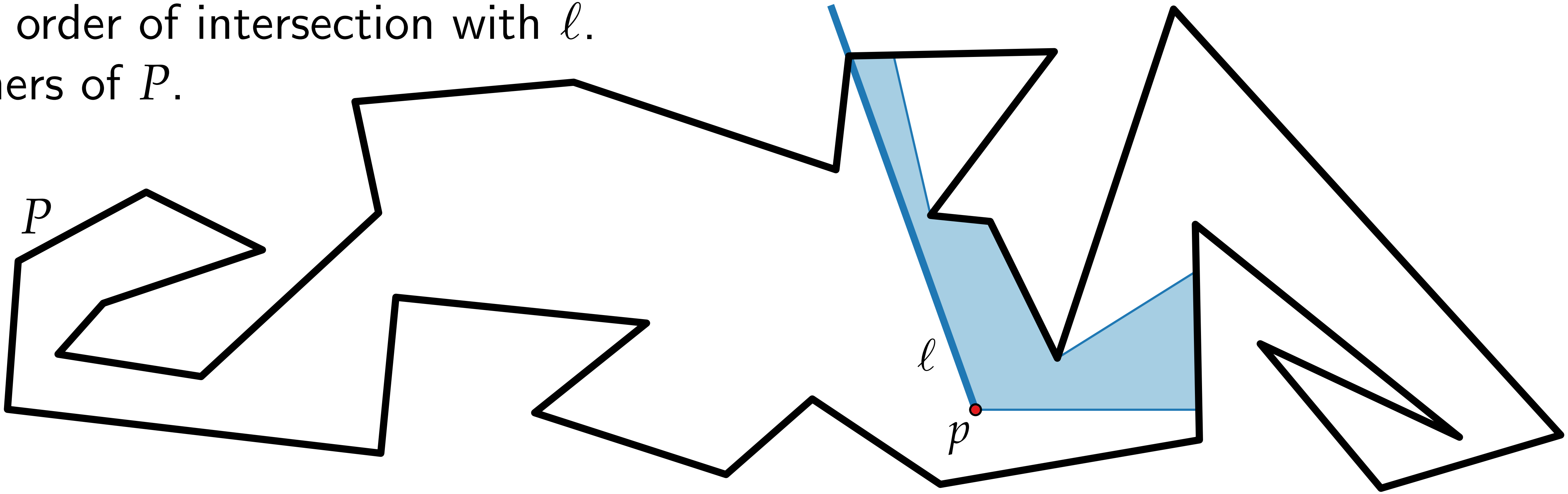

Literature

Rolf Klein. Algorithmische Geometrie: Grundlagen, Methoden, Anwendungen. Springer Verlag 2005.