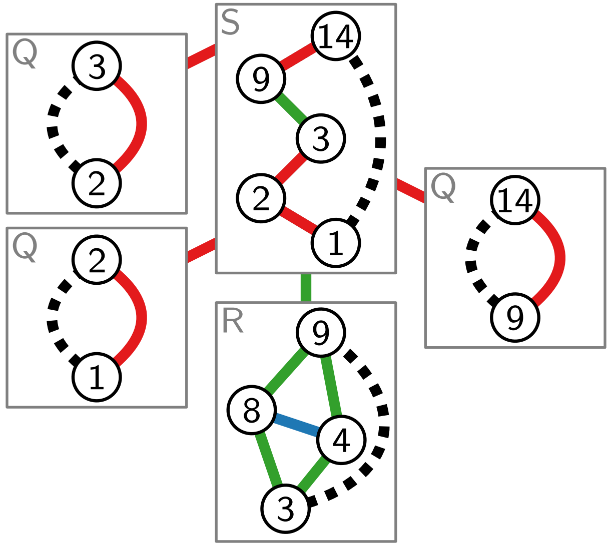
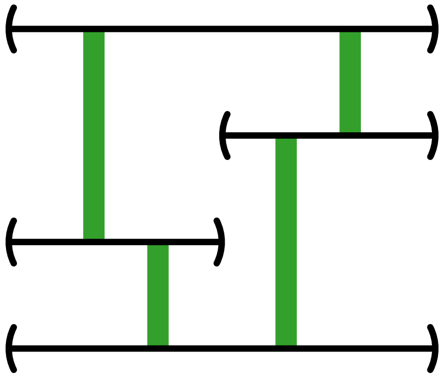
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension

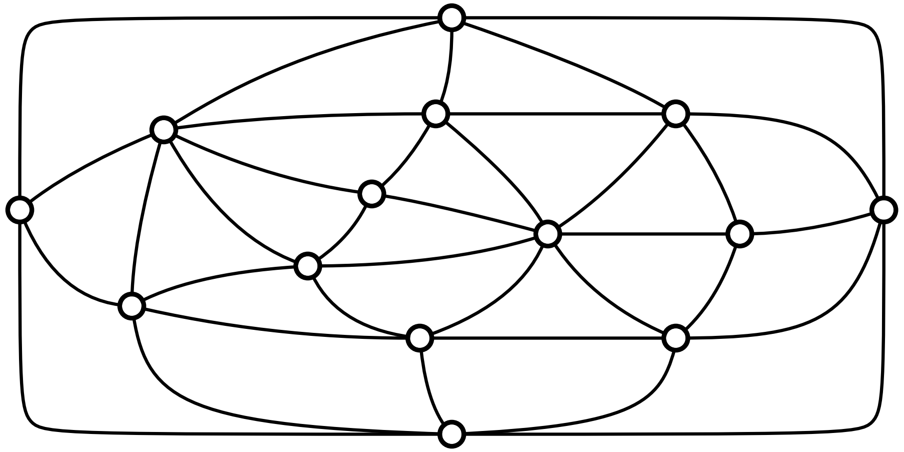


Johannes Zink



Partial Representation Extension Problem

Let $G = (V, E)$ be a graph.



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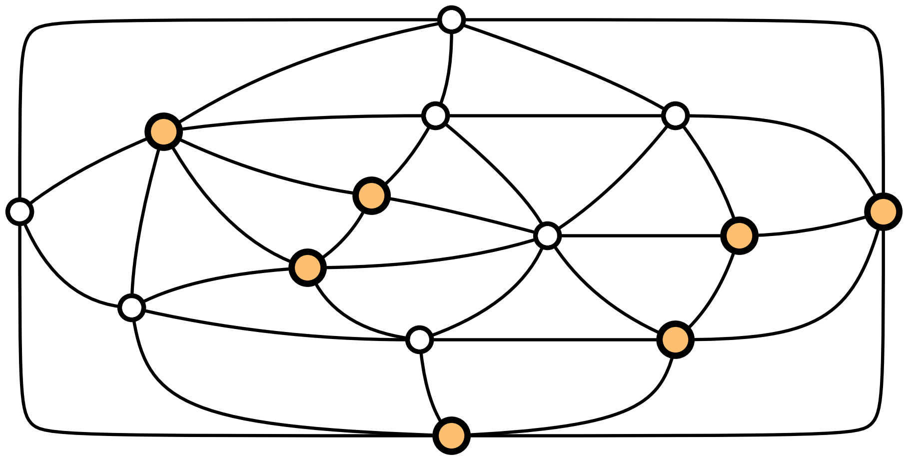
Let $V' \subseteq V$



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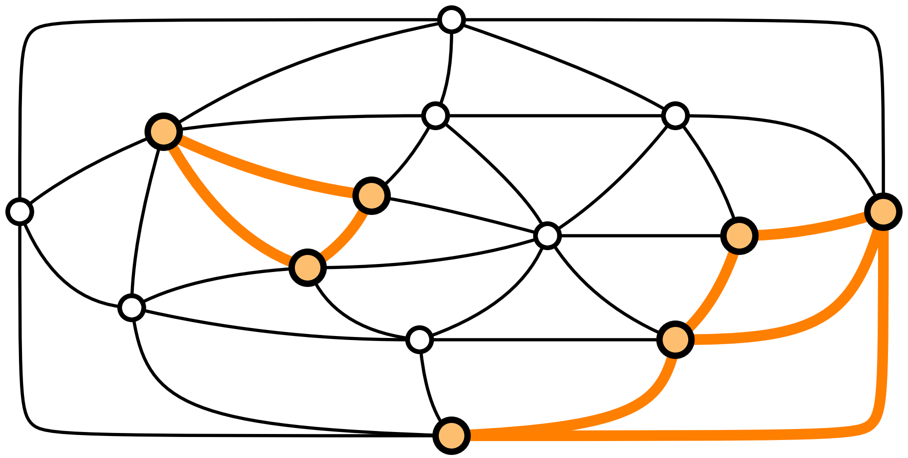
Let $G = (V, E)$ be a graph.

Let $V' \subseteq V$ and $H = G[V']$



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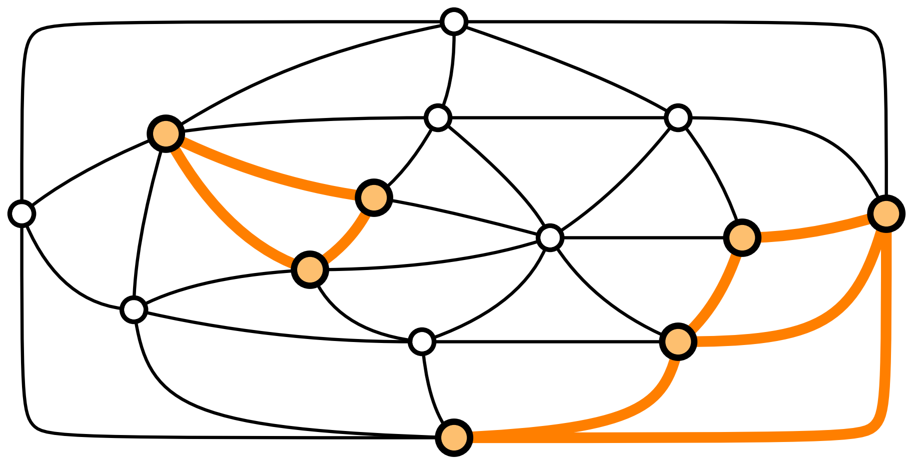

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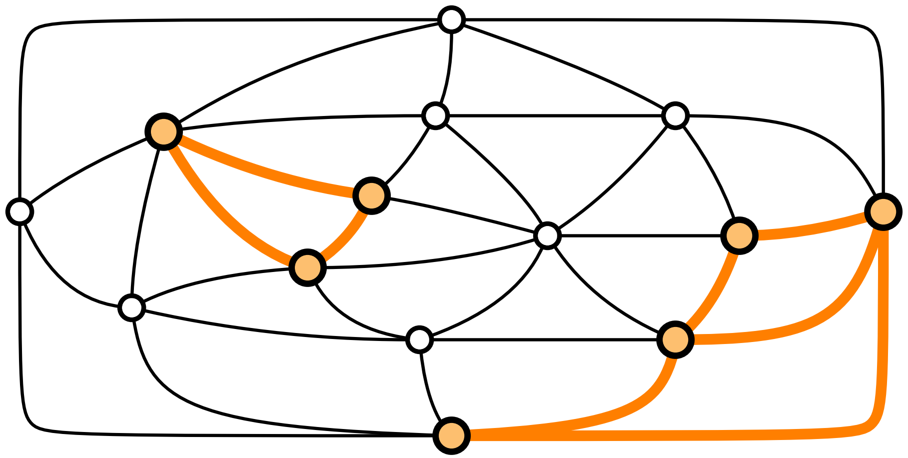

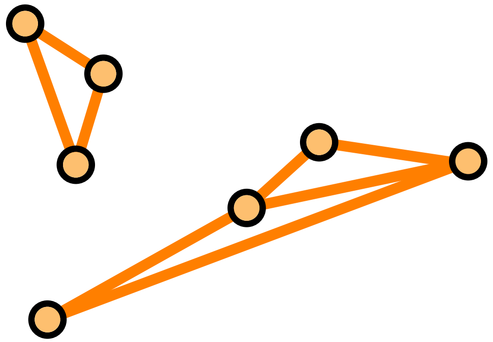

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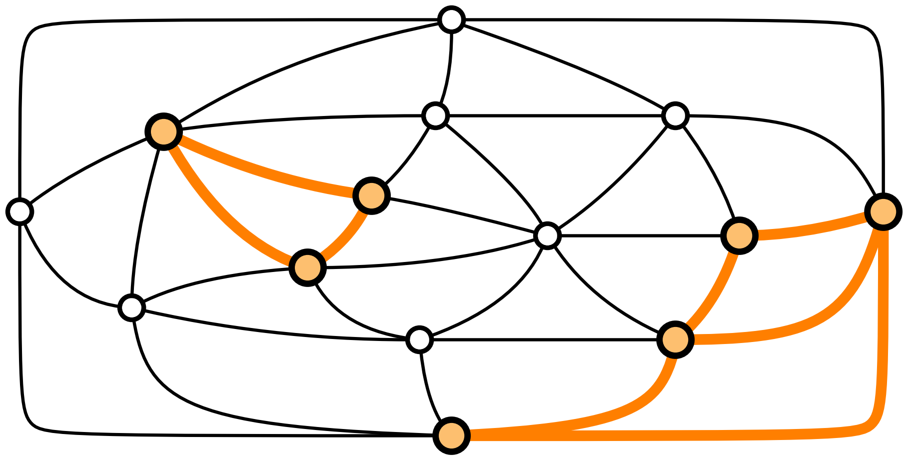

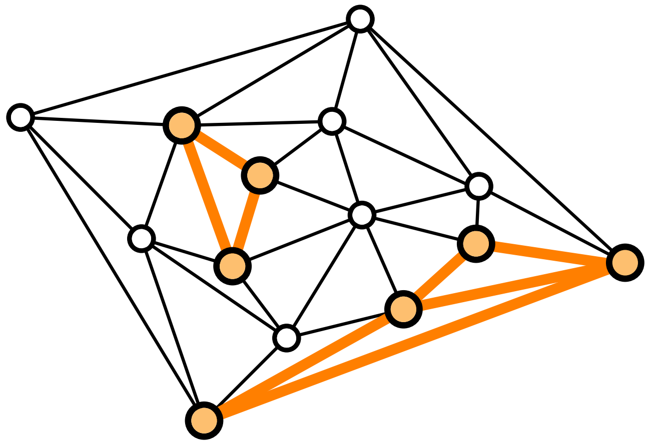

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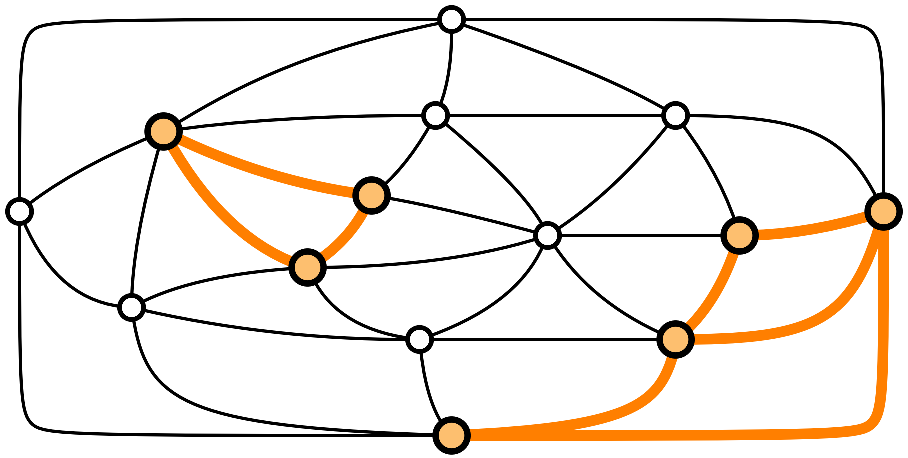

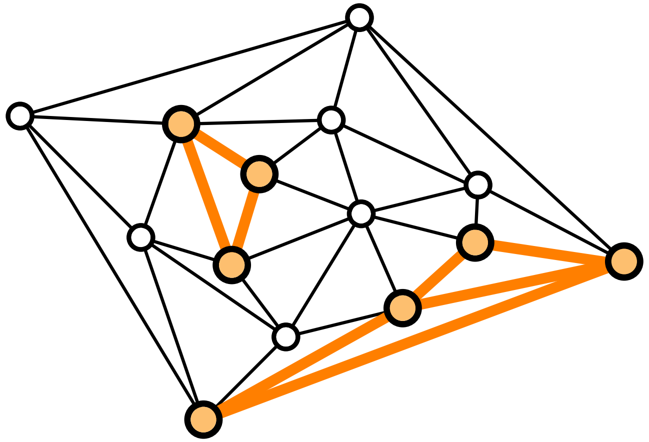

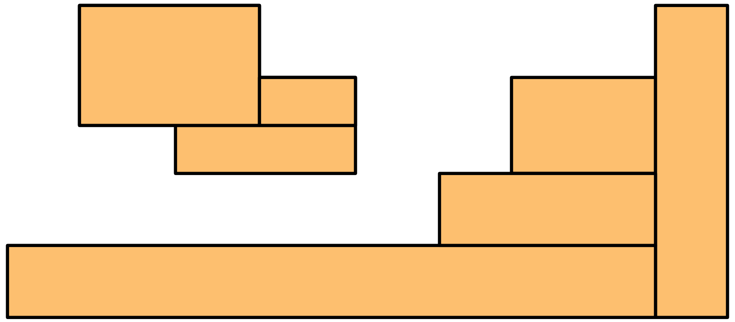

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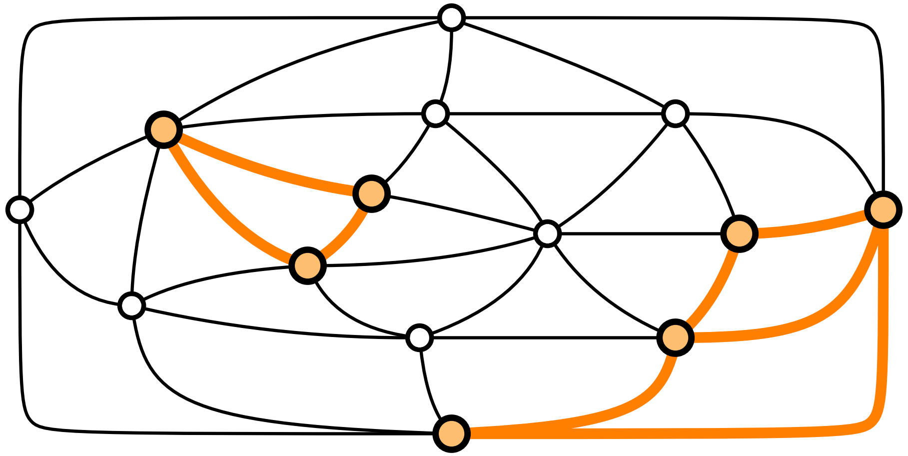

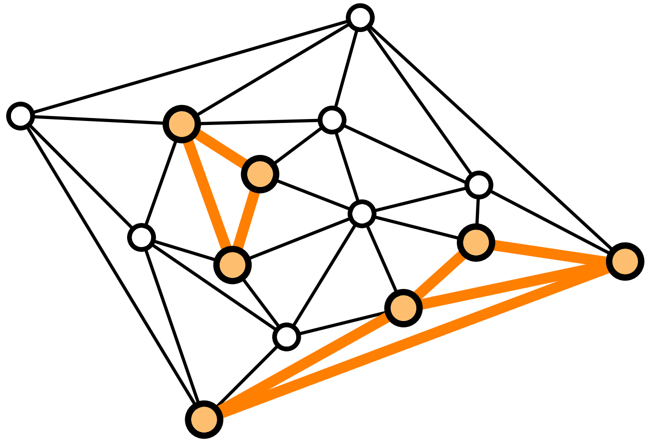

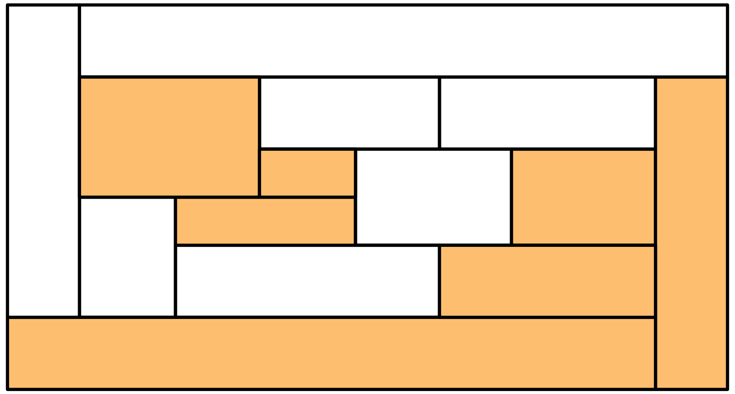

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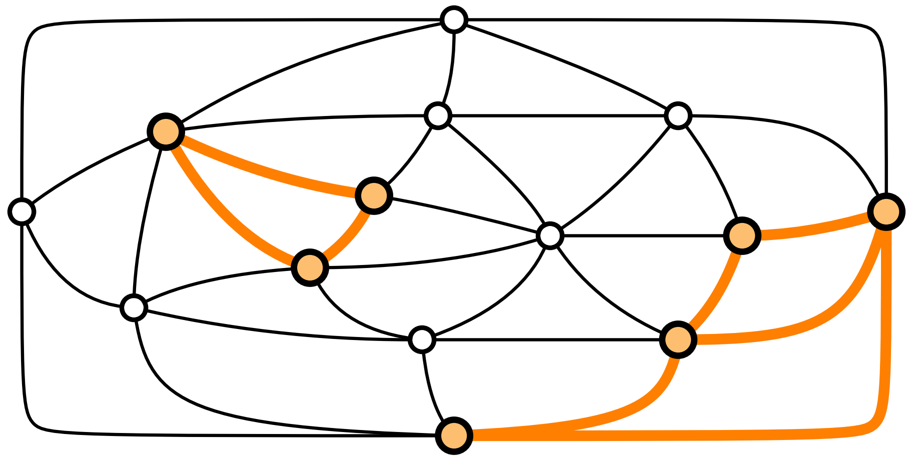

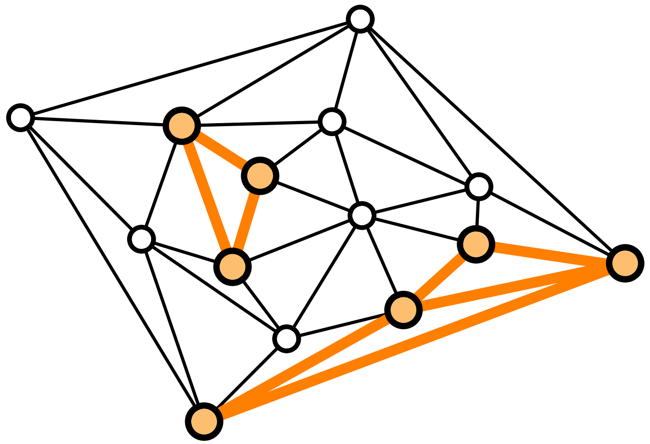

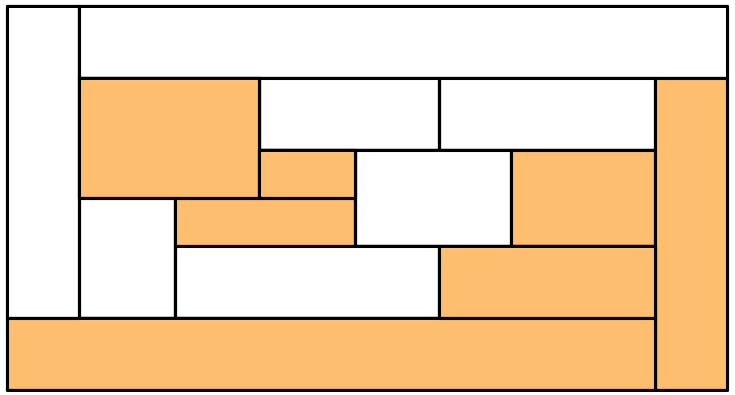

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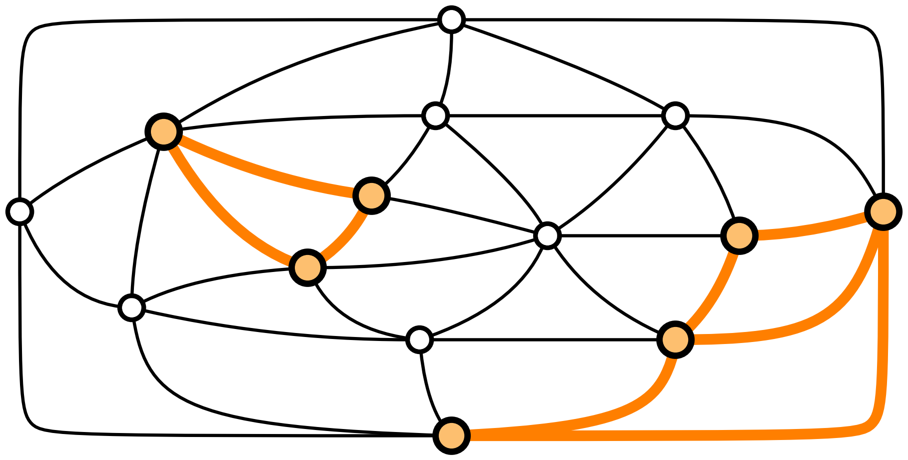

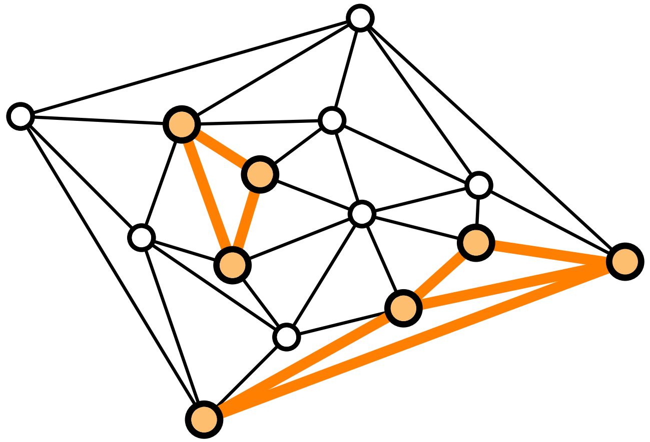

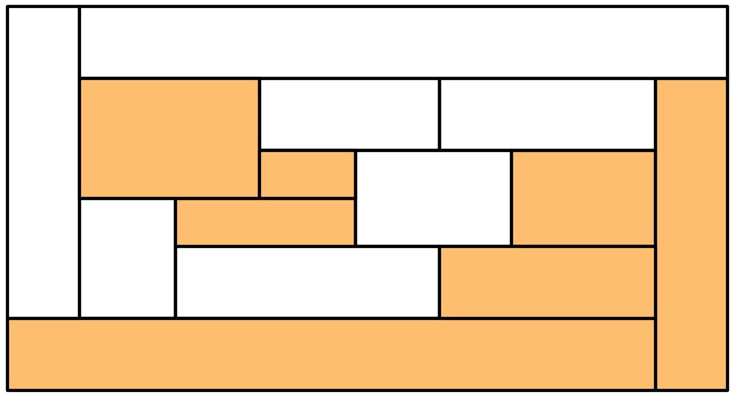

Polytime for:

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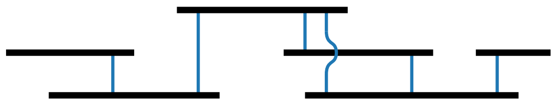

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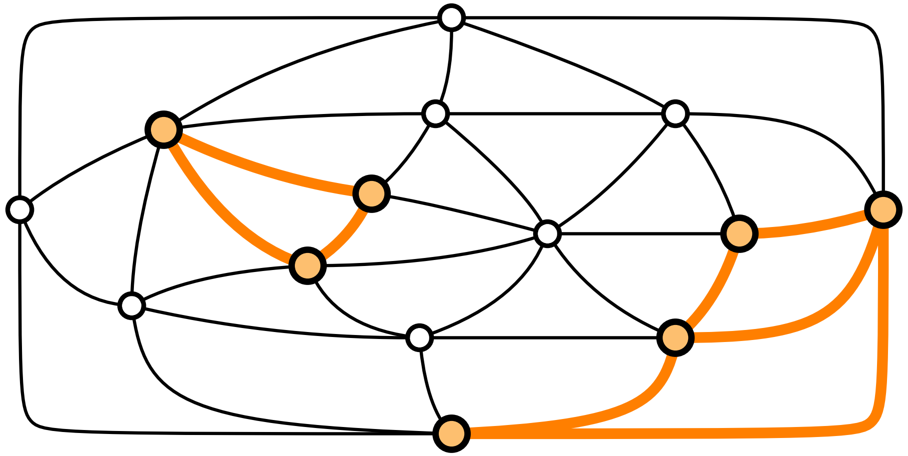

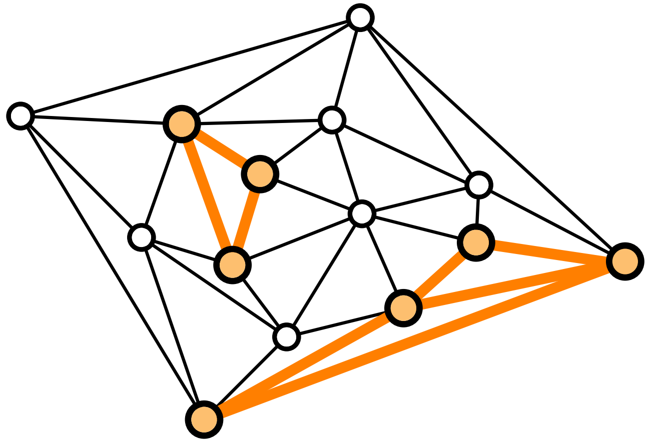

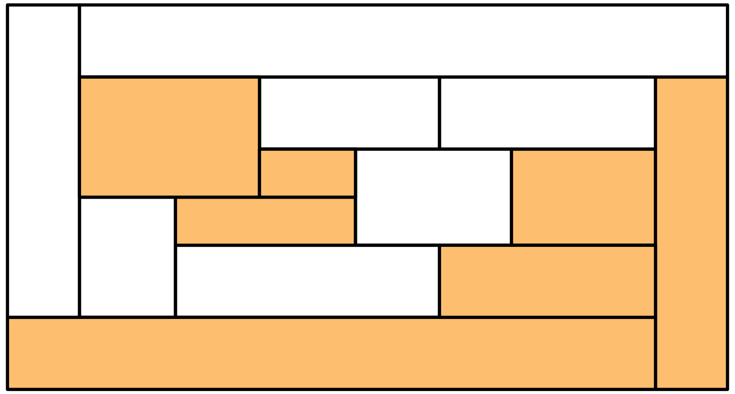

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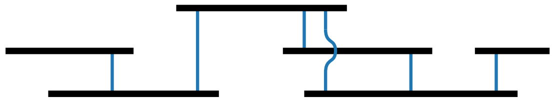

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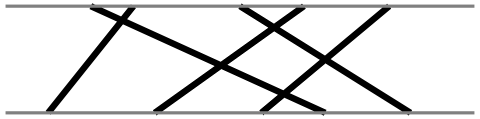

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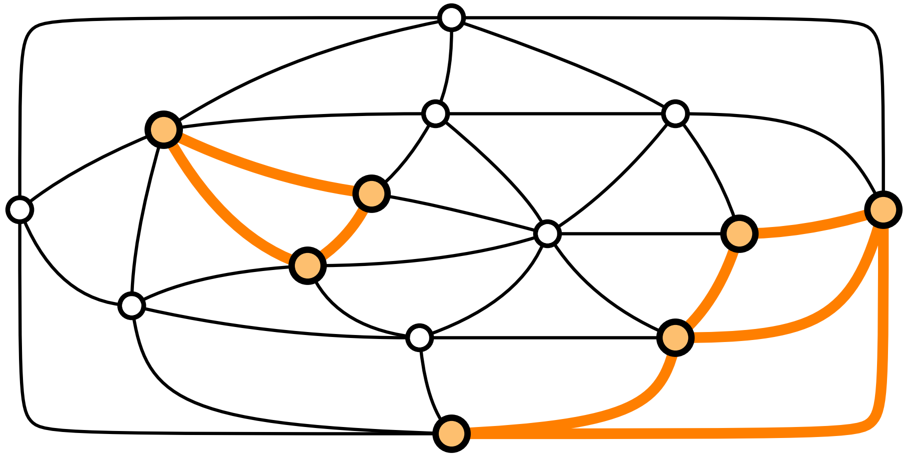

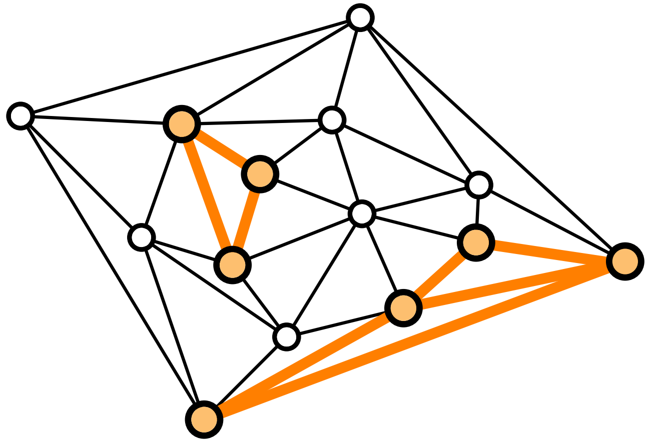

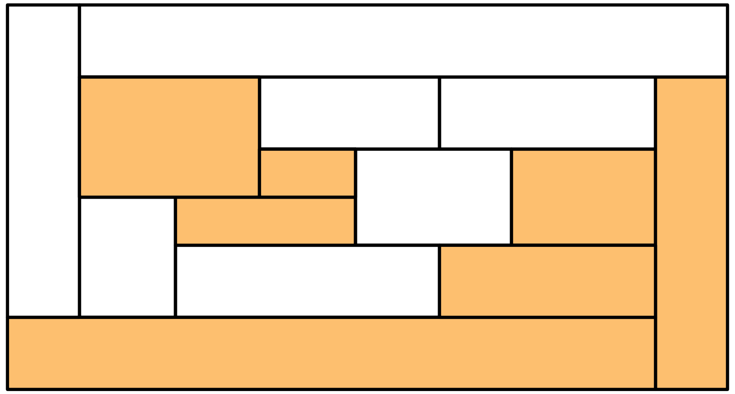

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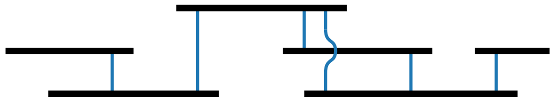

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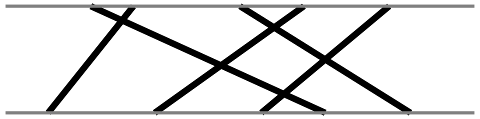

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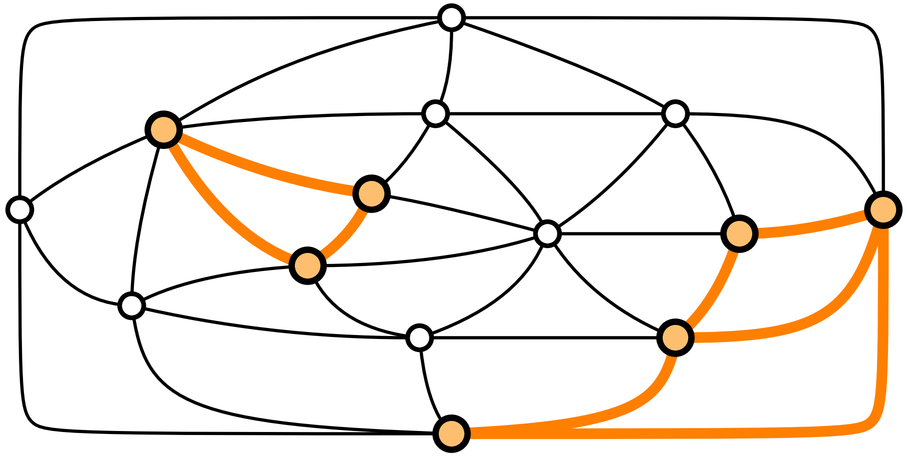

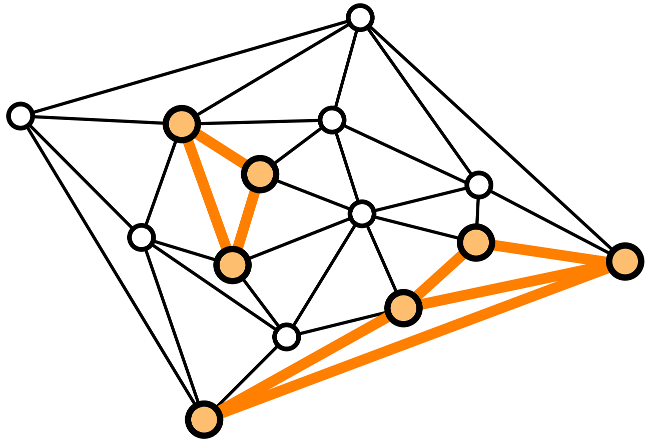

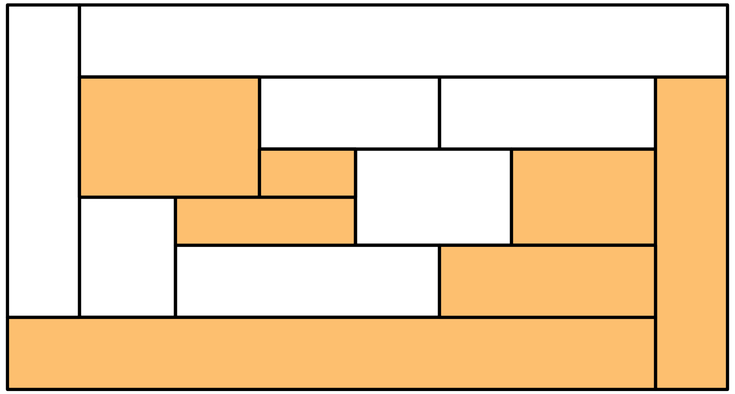

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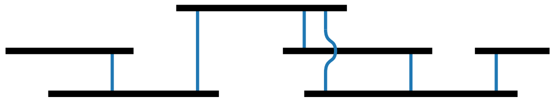

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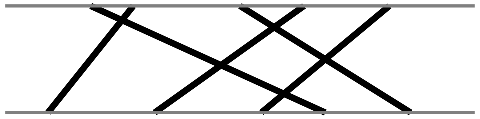

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NP-hard for:

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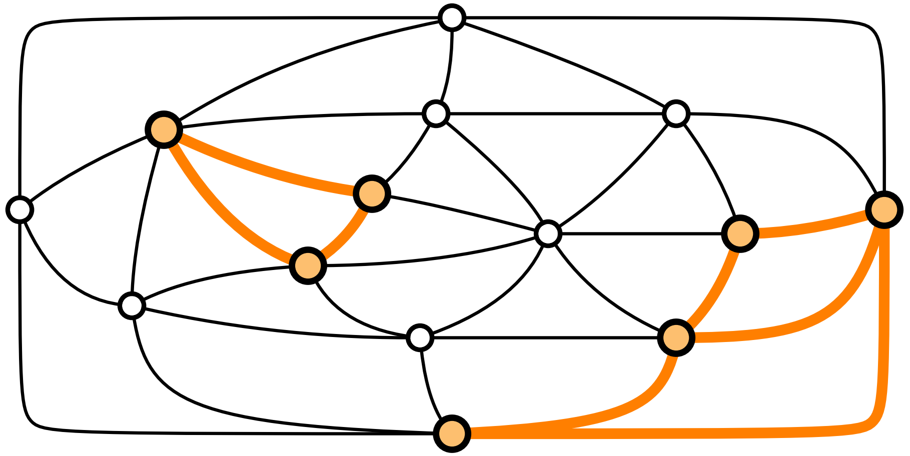

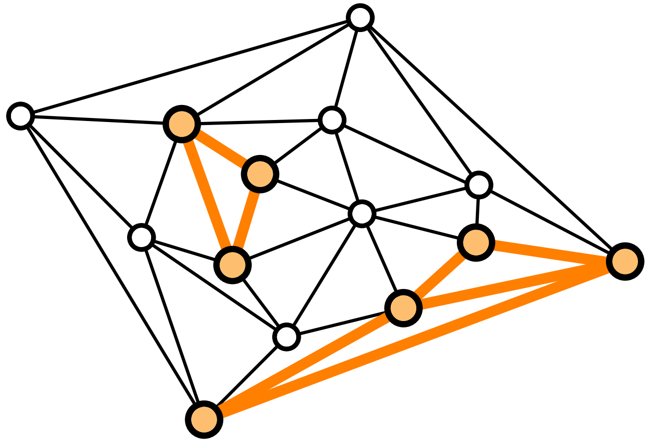

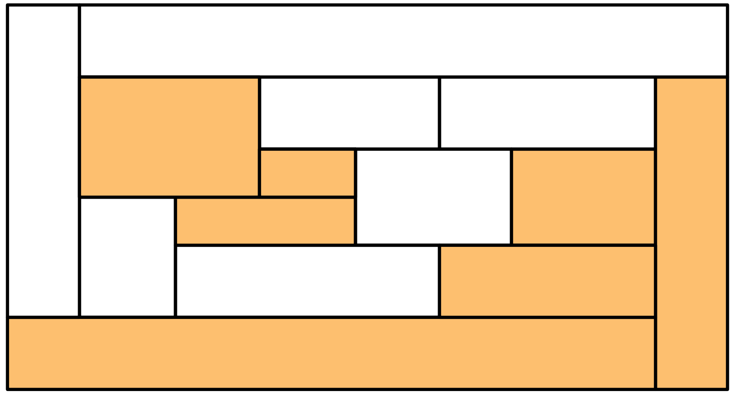

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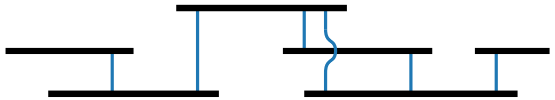

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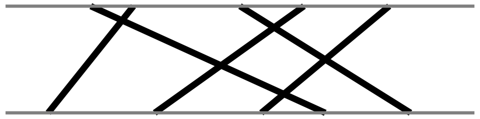

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NP-hard for:

- planar straight-line drawings

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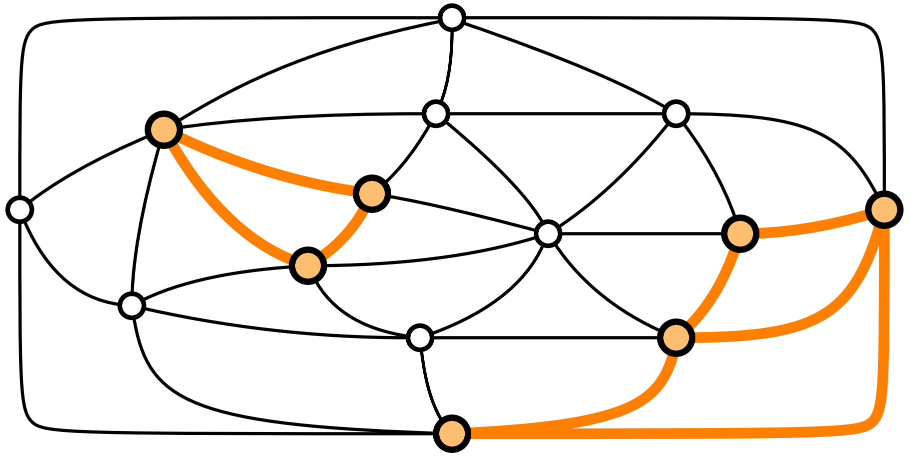

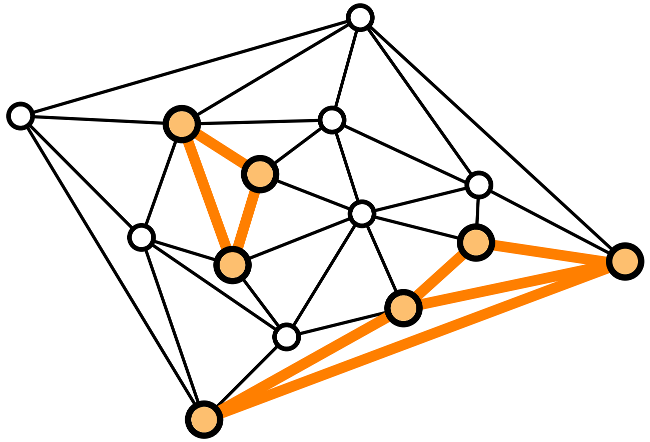

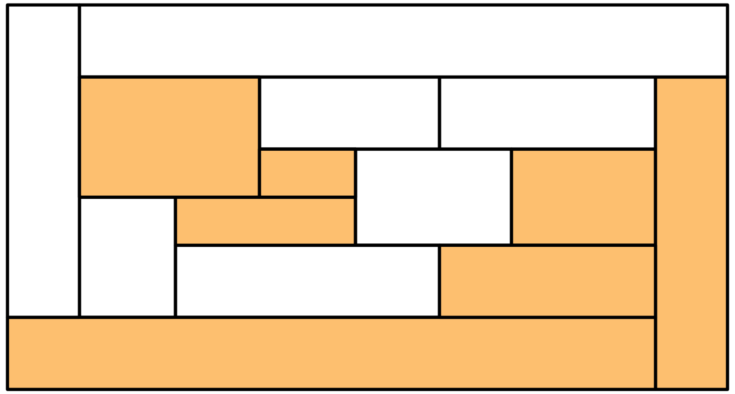

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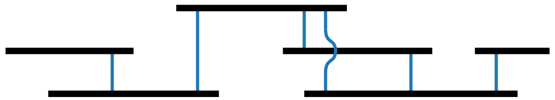

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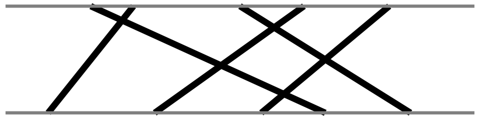

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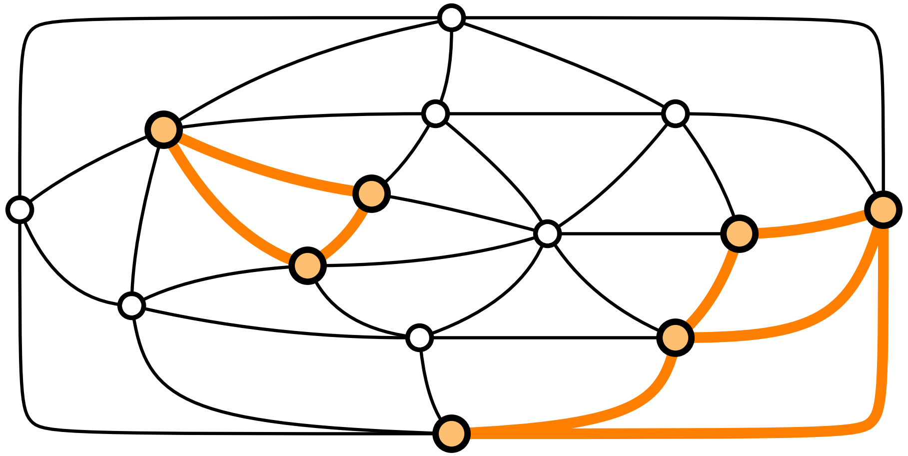

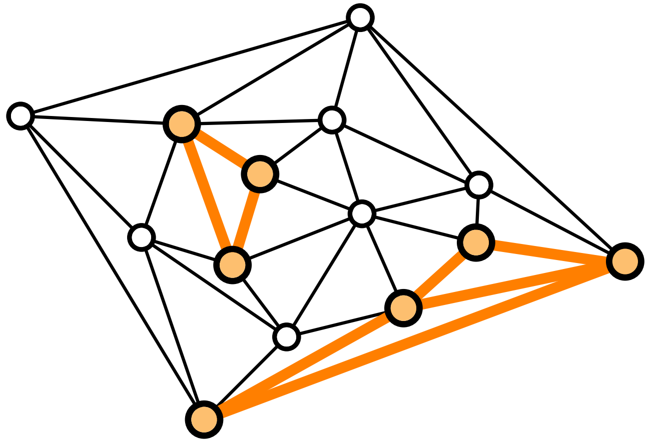

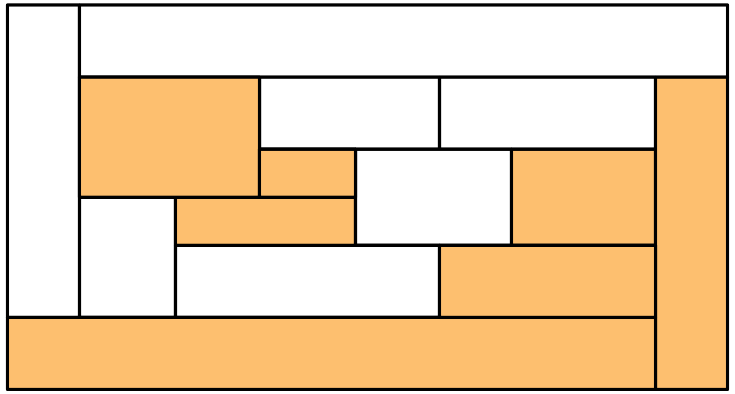

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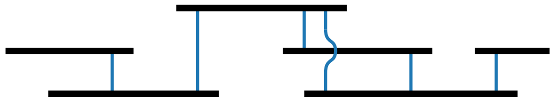

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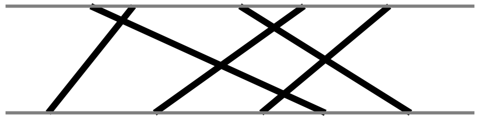

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NP-hard for:

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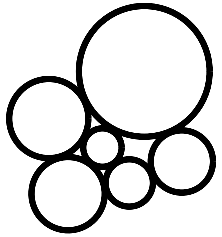

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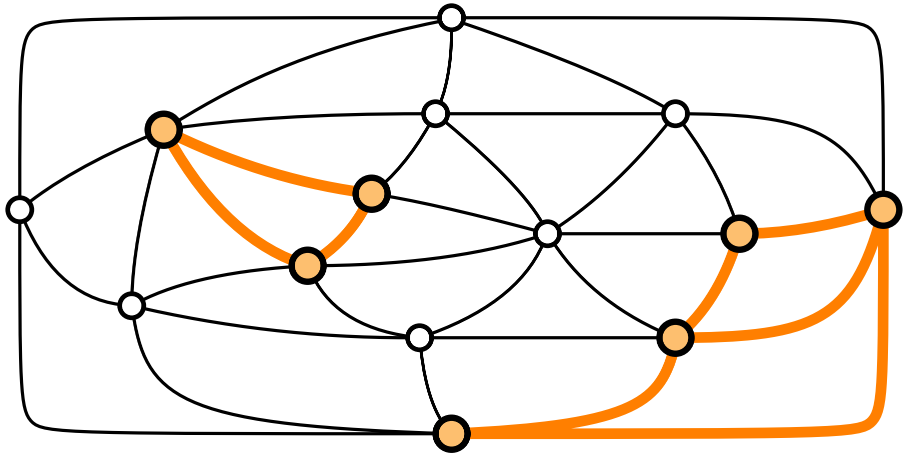

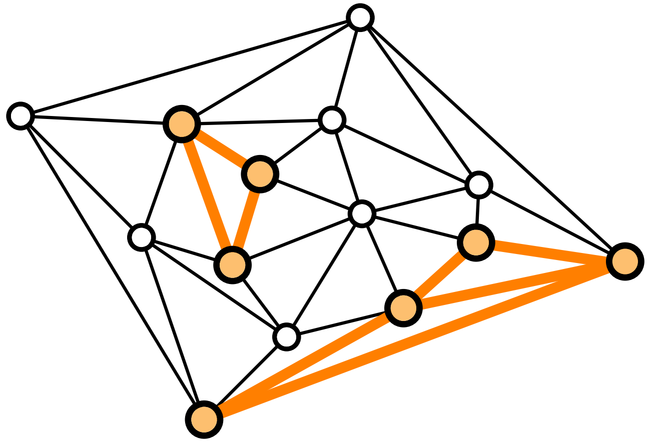

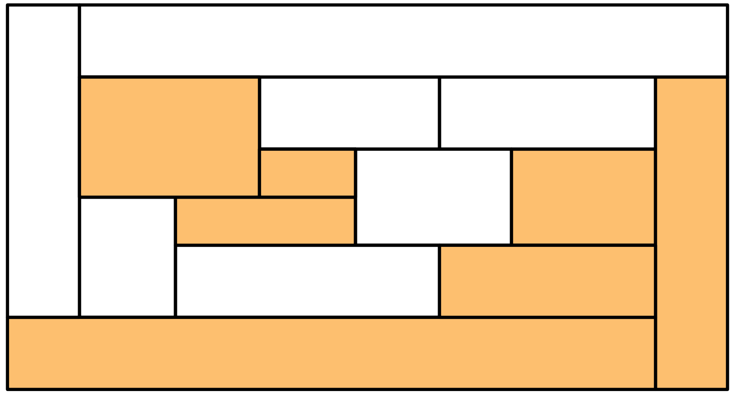

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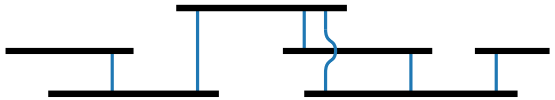

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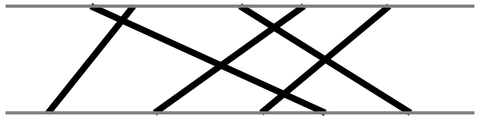

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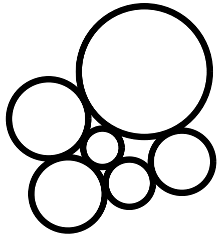

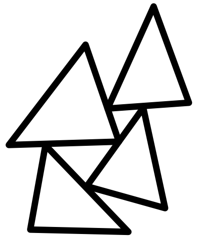

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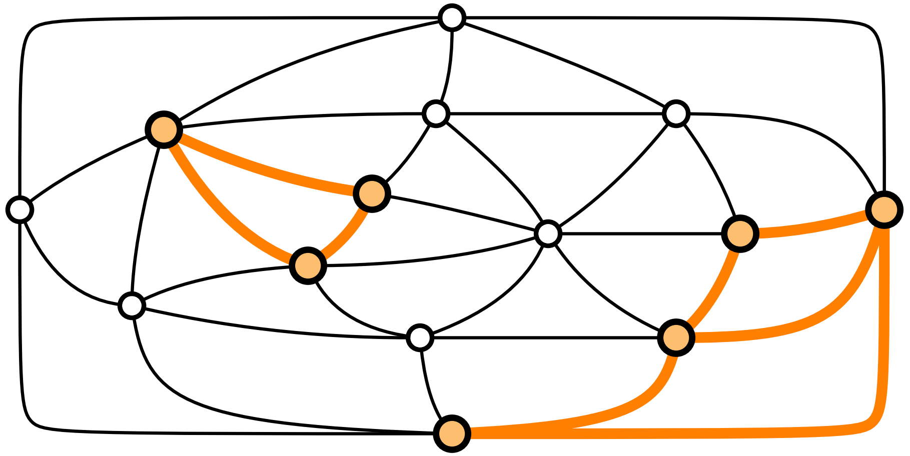
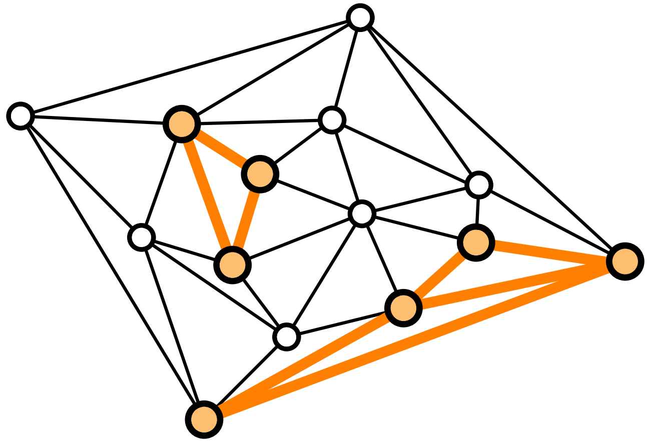
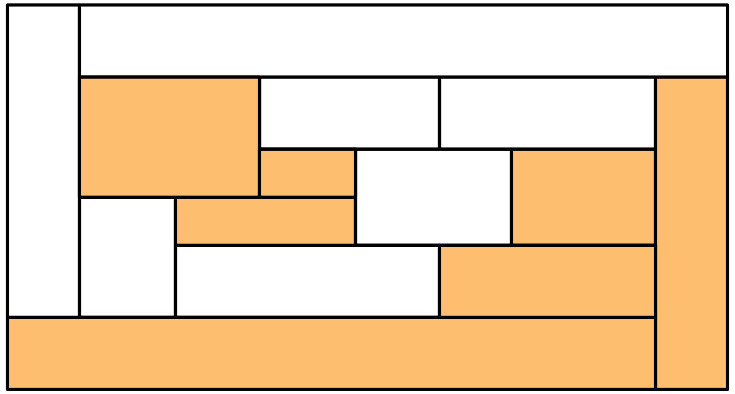
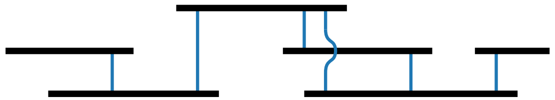




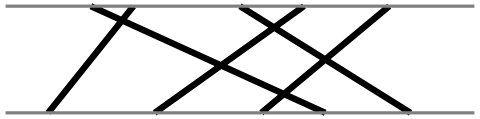


Polytime for:

- (unit) interval graphs



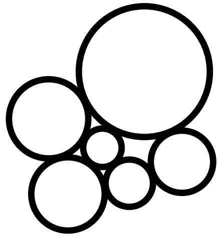
- permutation graphs

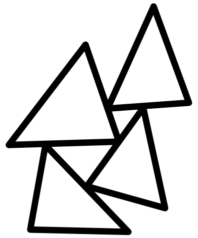


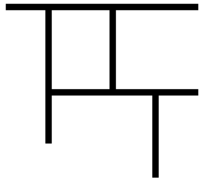
- circle graphs

NP-hard for:

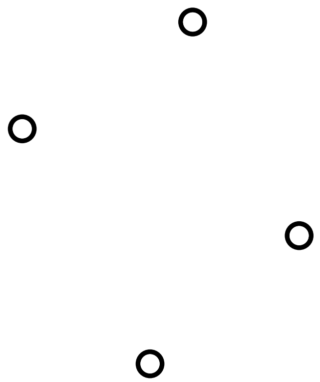
- planar straight-line drawings
- contacts of
  - disks
  - triangles
  - orthogonal segments







Bar Visibility Representation



Bar Visibility Representation

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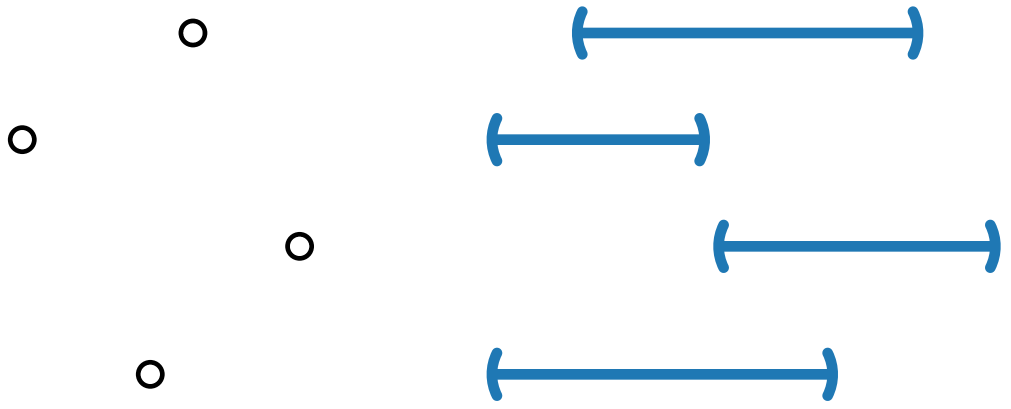

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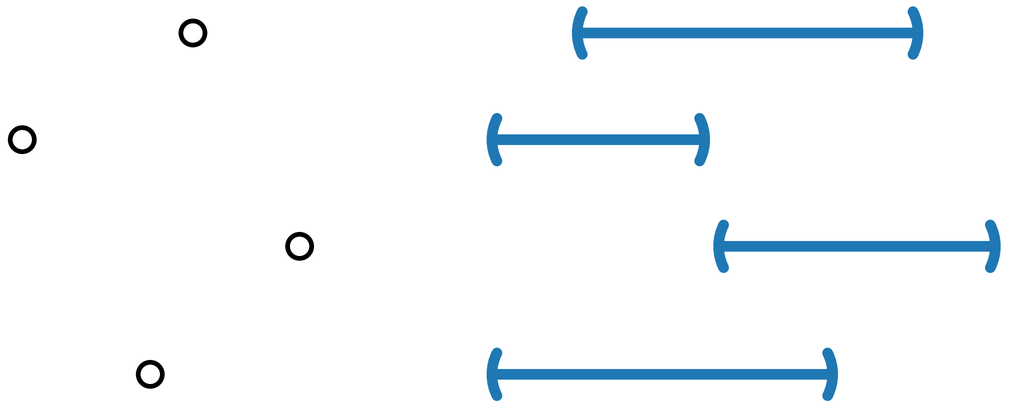

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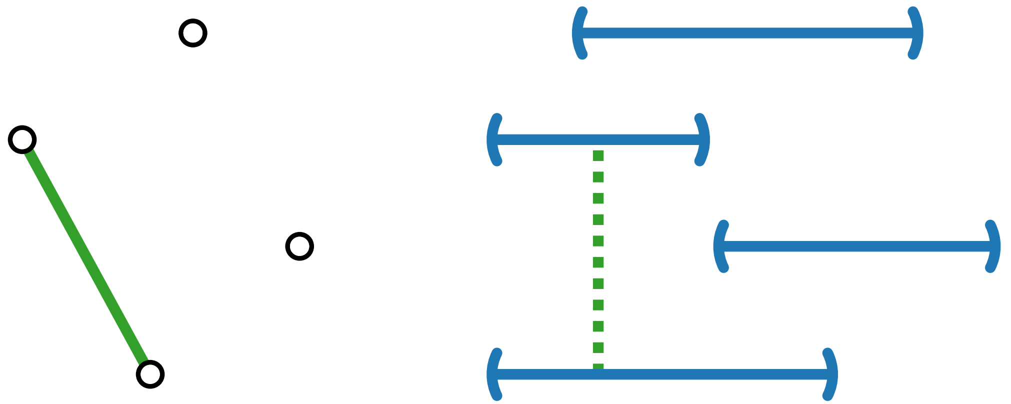

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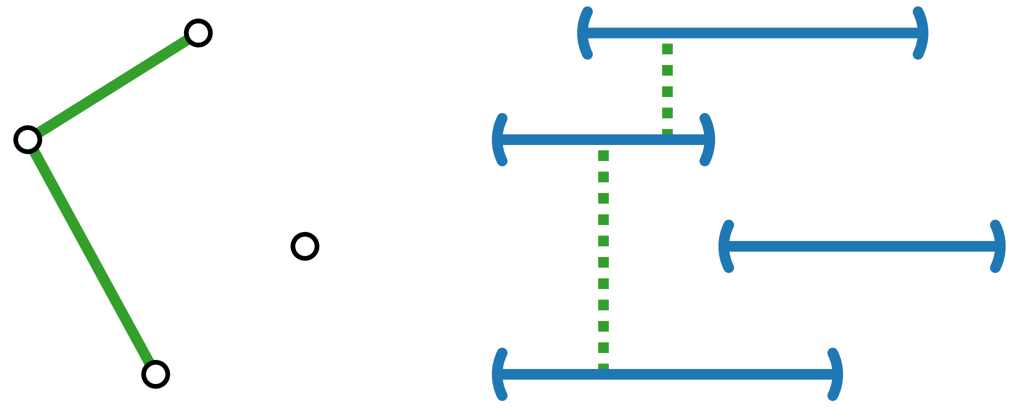

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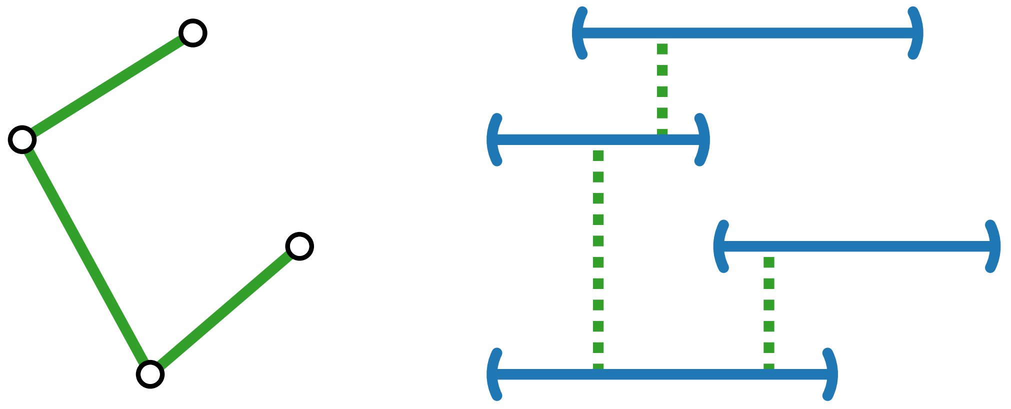

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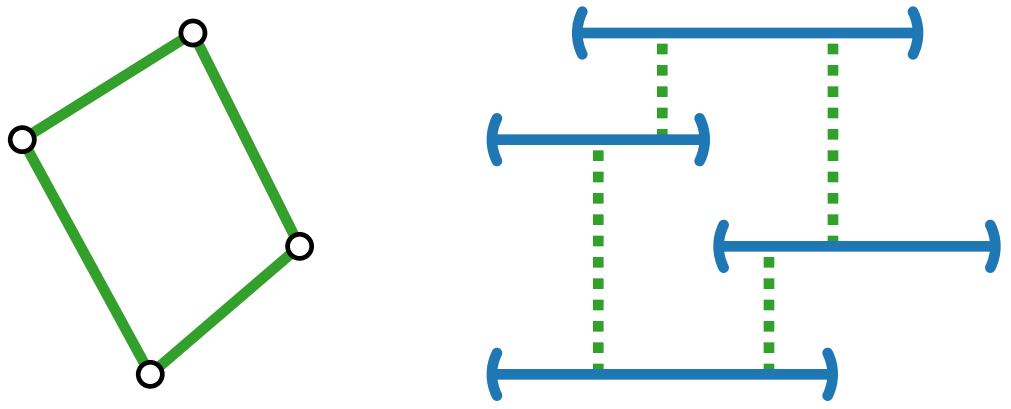

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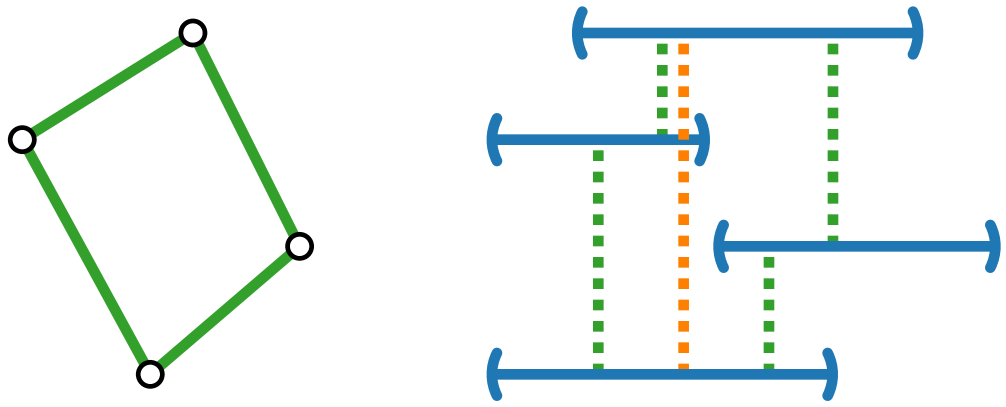

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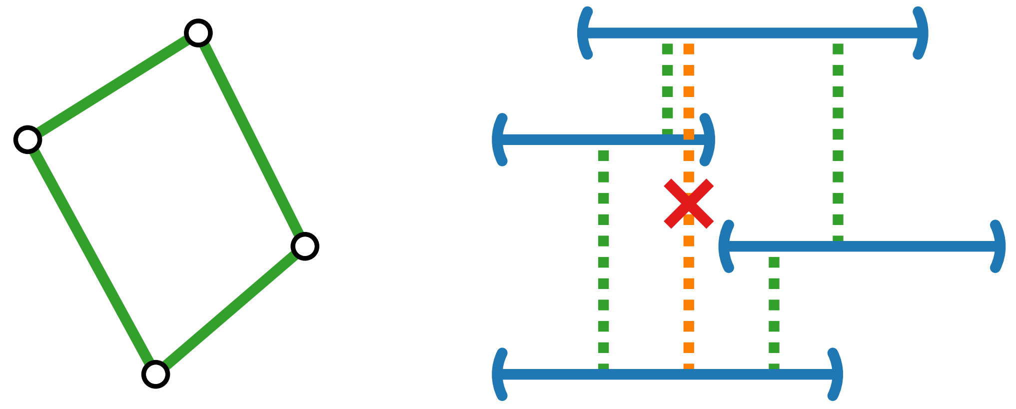

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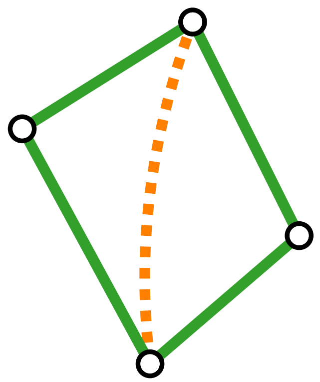

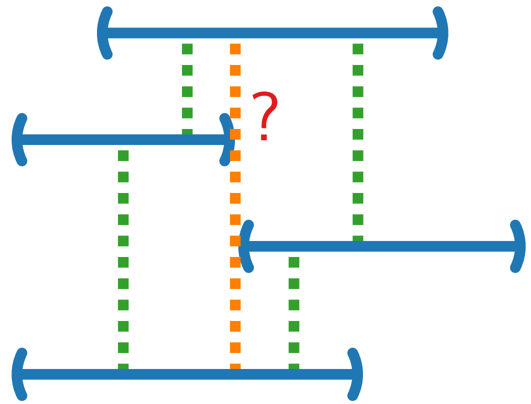

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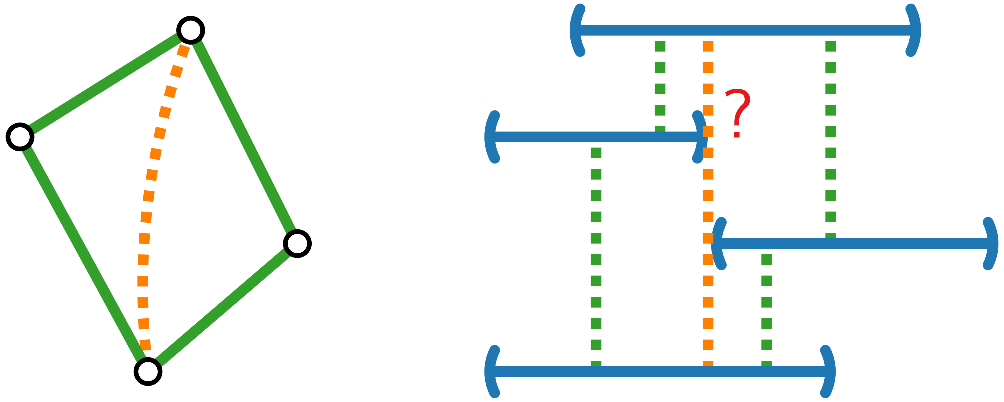

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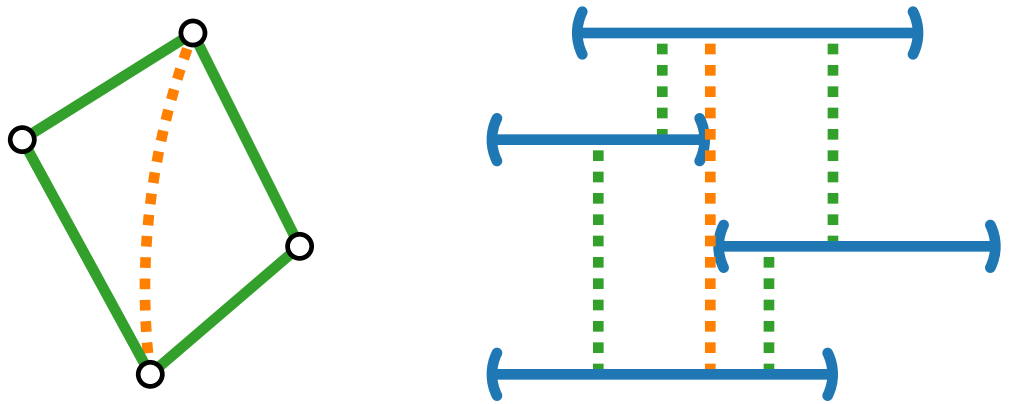

Models.

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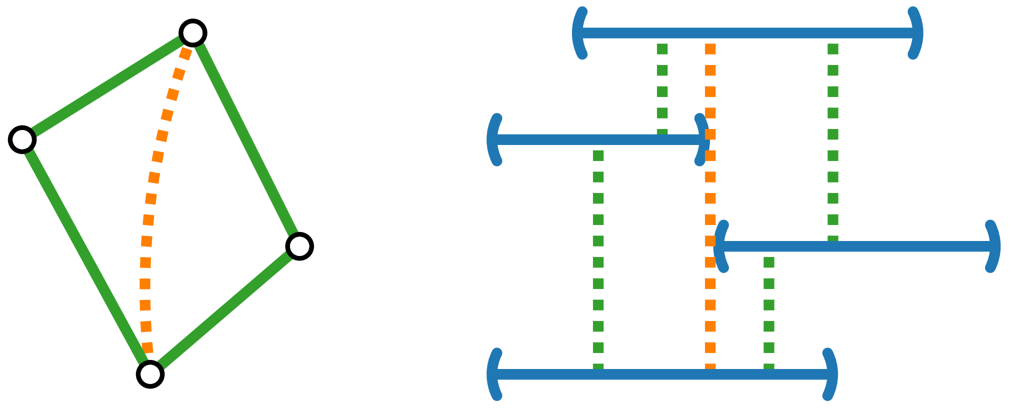

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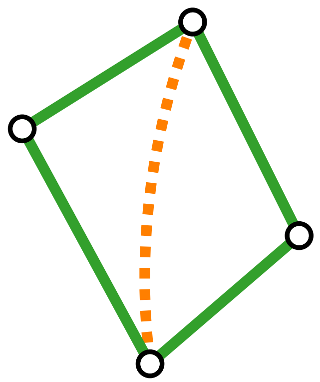

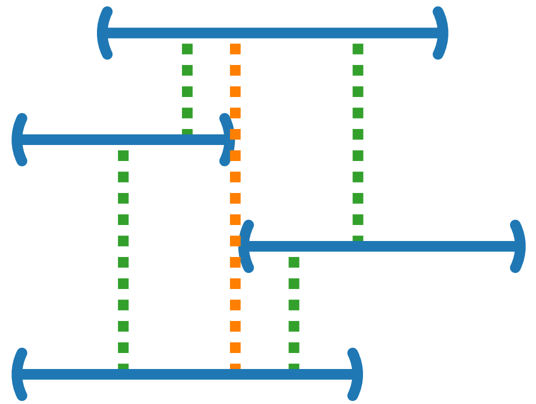

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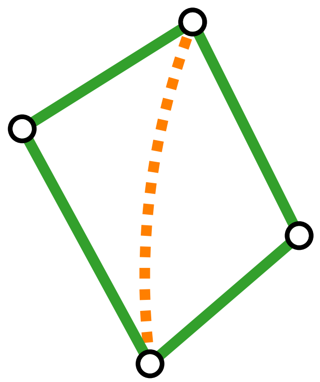

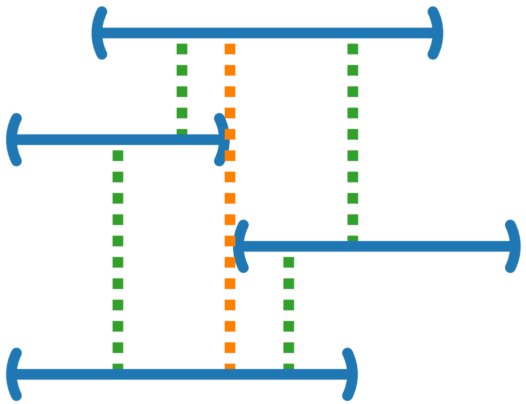

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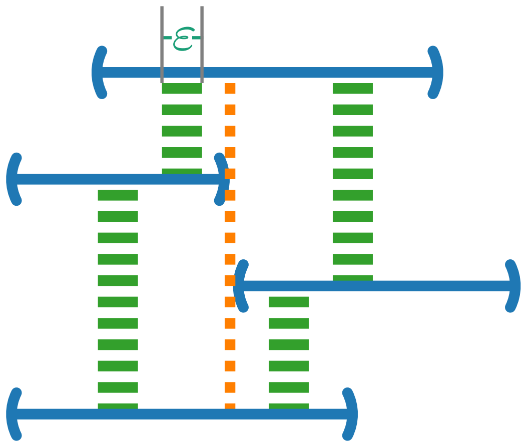

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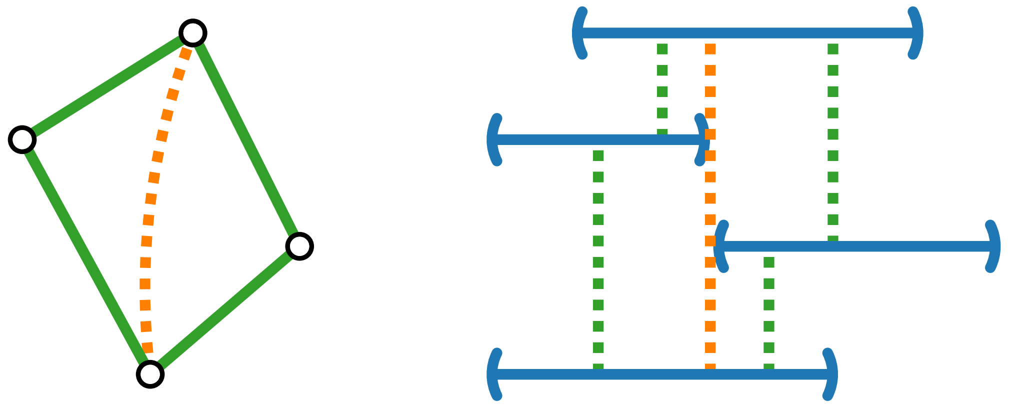

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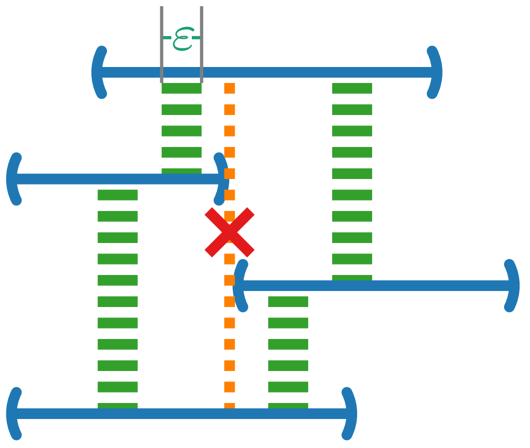

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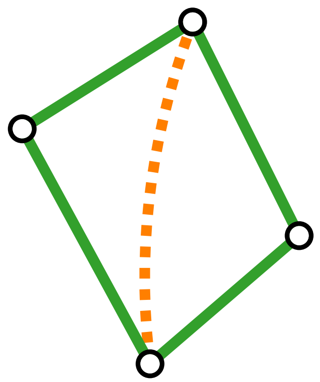

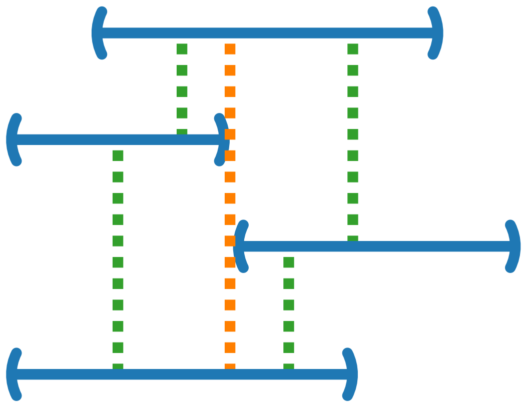

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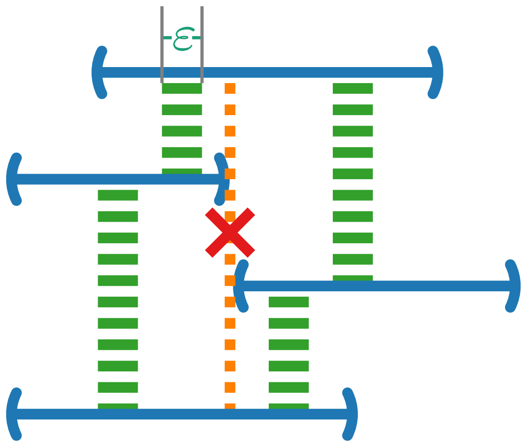

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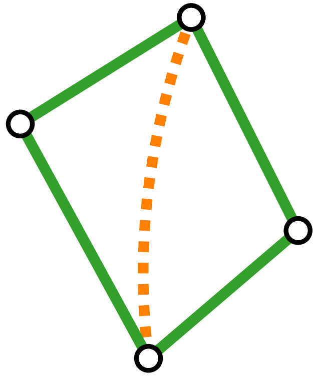

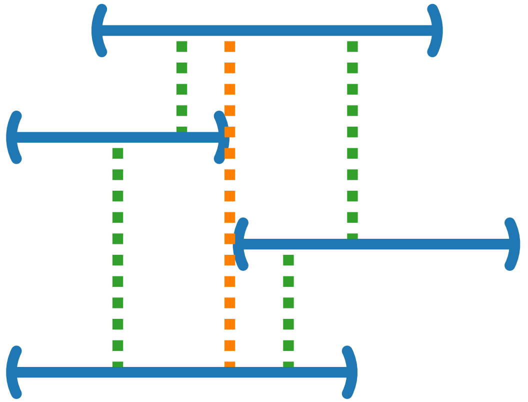

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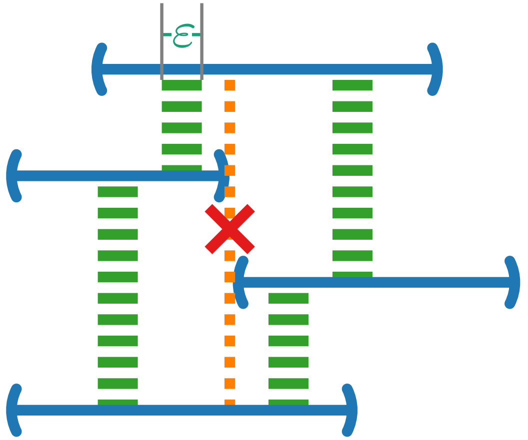
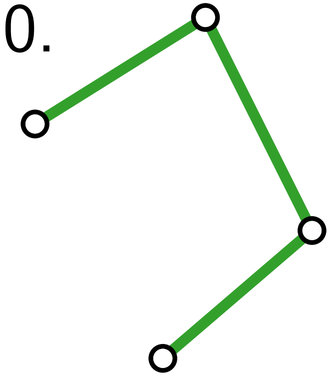
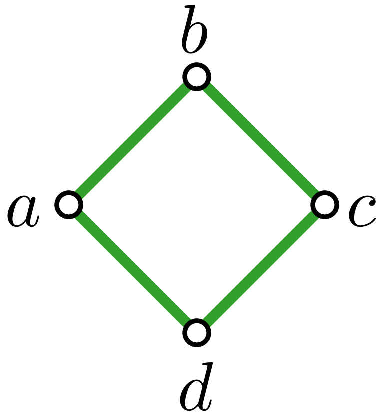
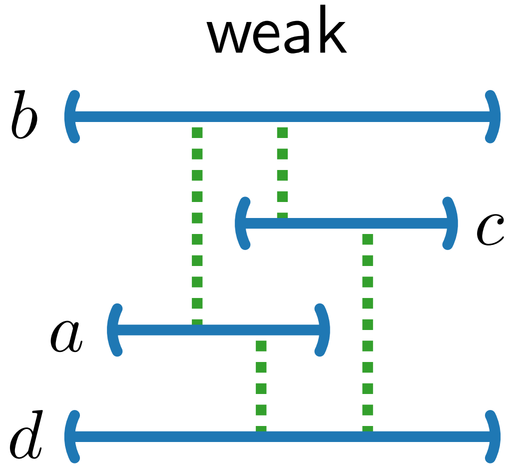




Problems





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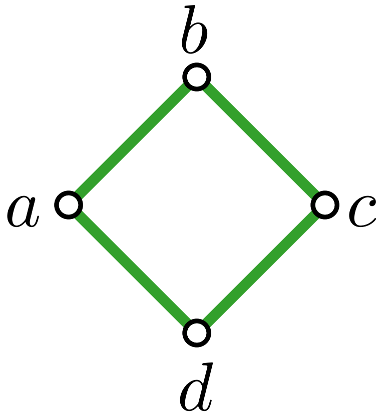

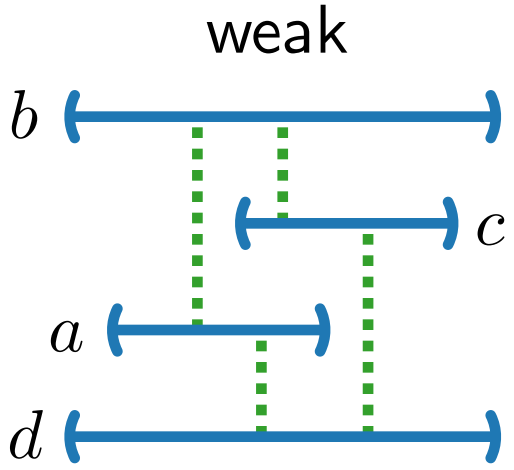

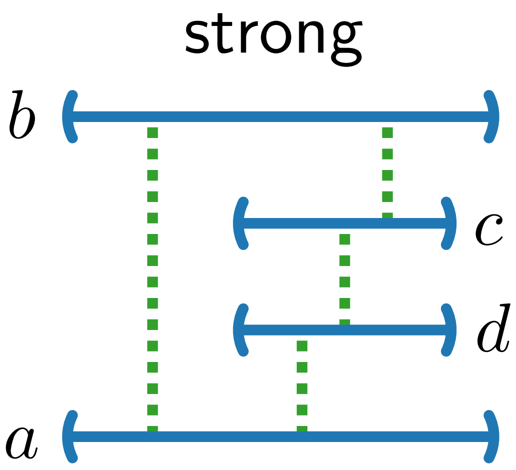

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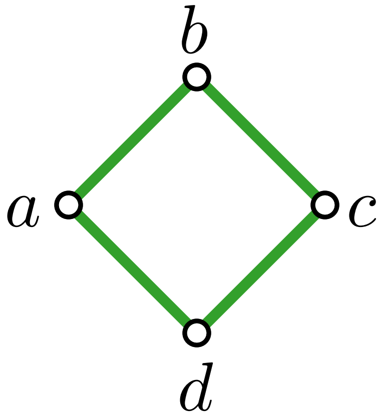

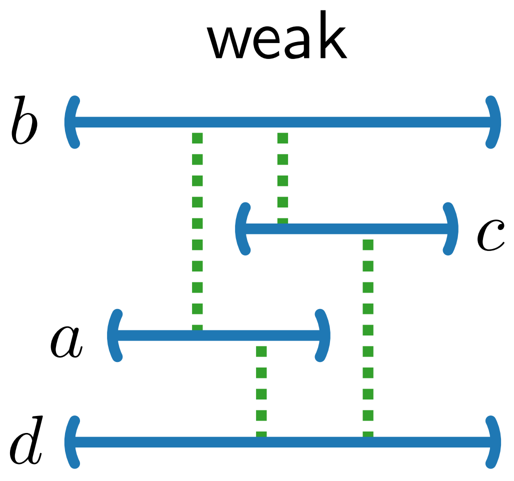

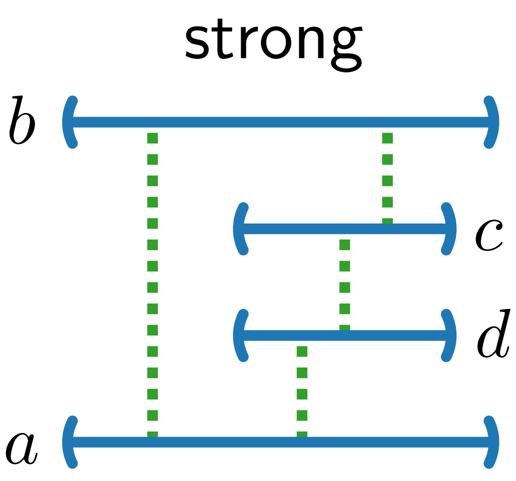

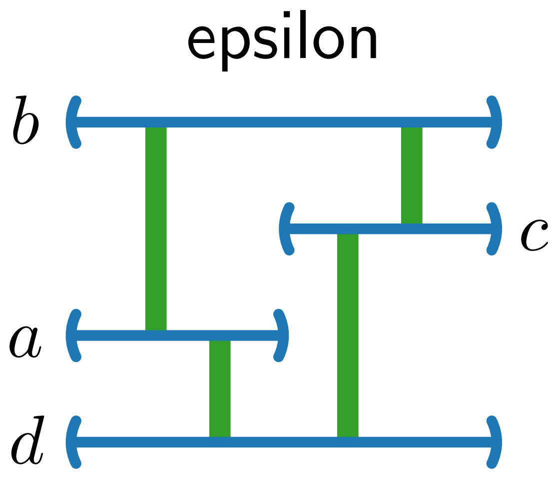

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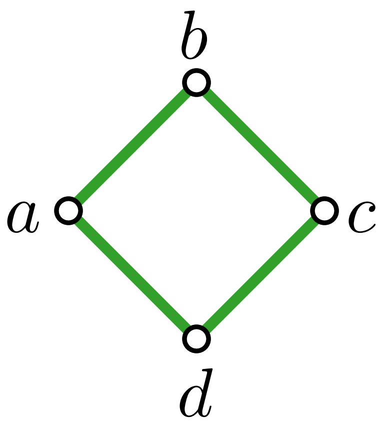

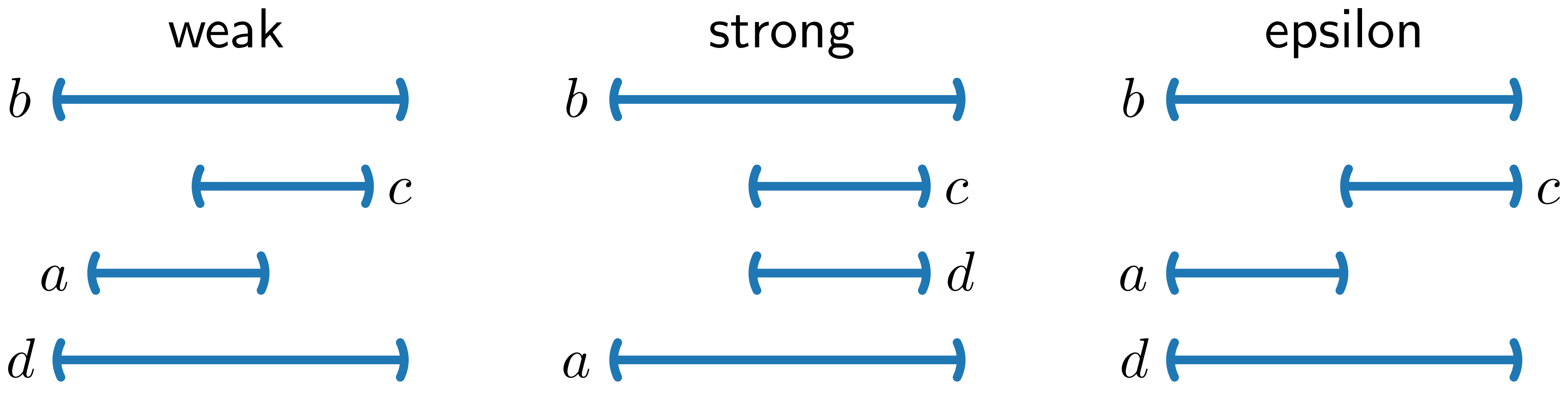

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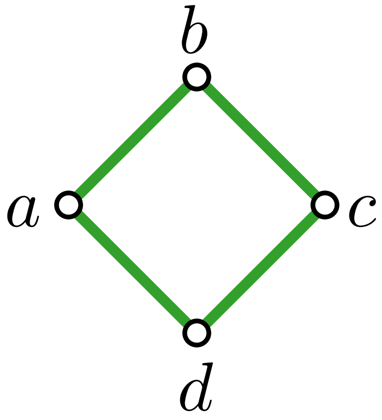

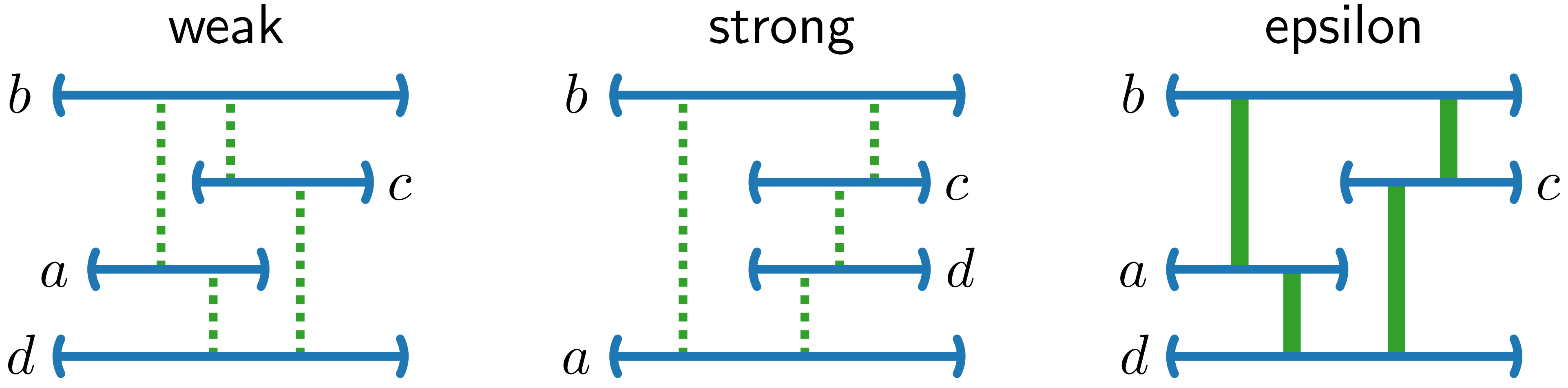

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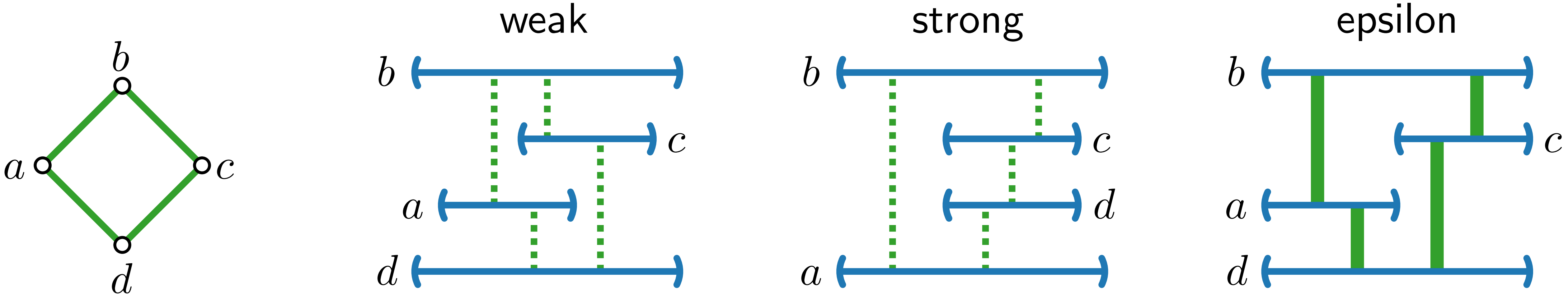
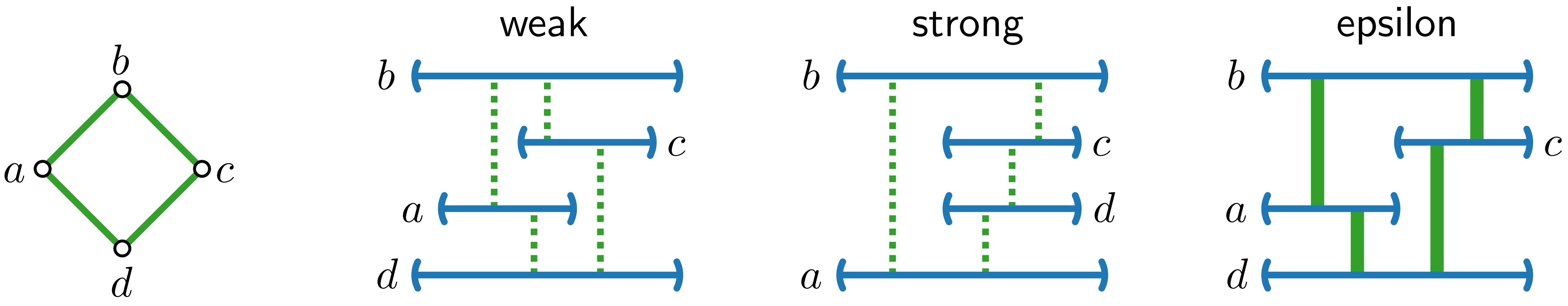


Recognition Problem.
Given a graph G, **decide** whether there exists a weak/strong/ε bar visibility representation ψ of G.

Problems



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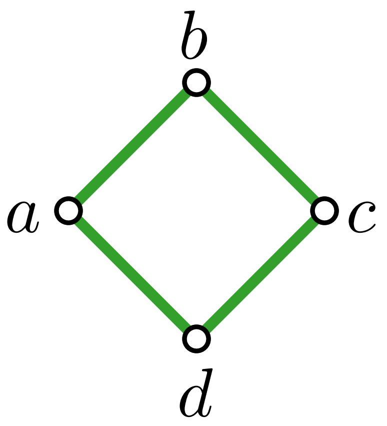

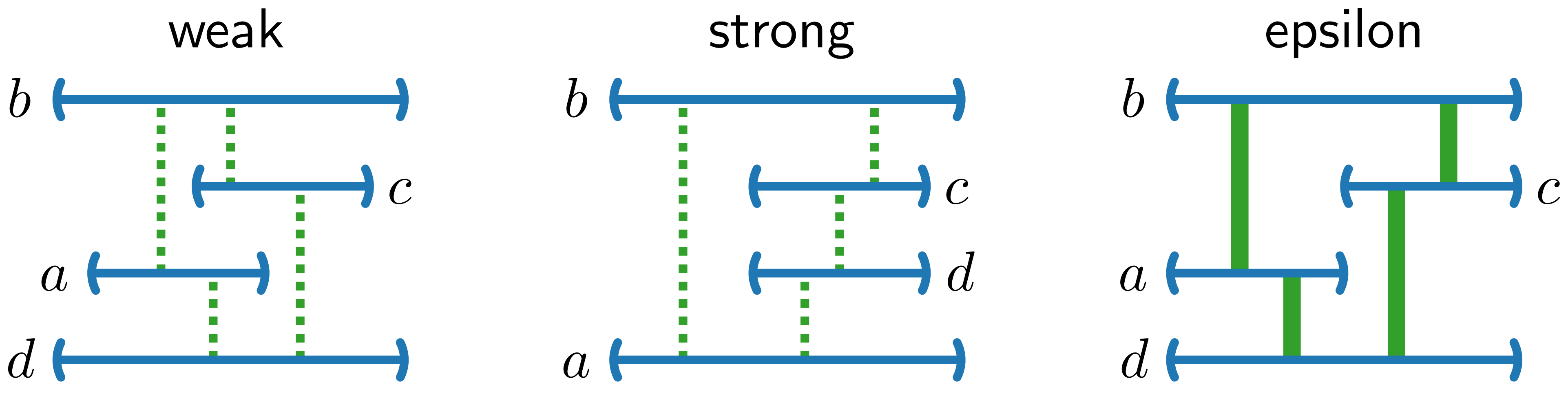

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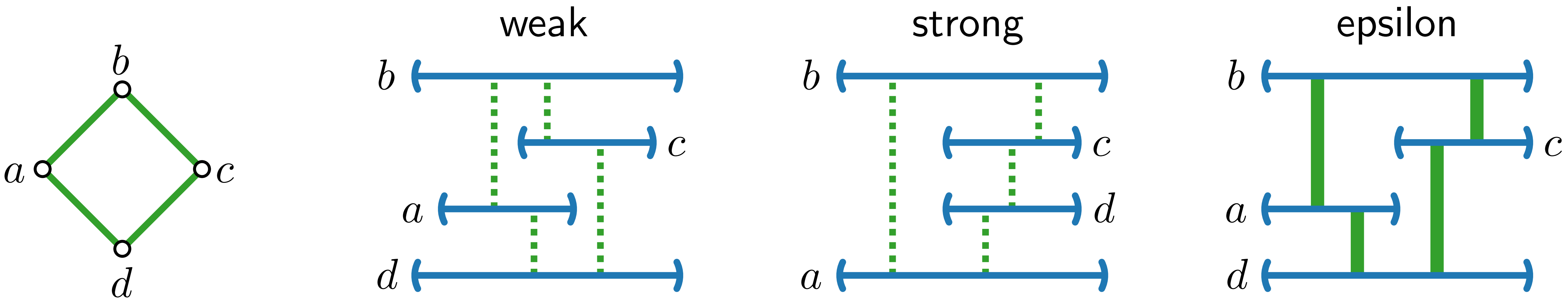
Partial Representation Extension Problem.
Given a graph G and a **set of bars ψ' of $V' \subseteq V(G)$**, **decide** whether there exists a weak/strong/ε bar visibility representation ψ of G **where $\psi|_{V'} = \psi'$** (and **construct** ψ if a representation exists).

Background



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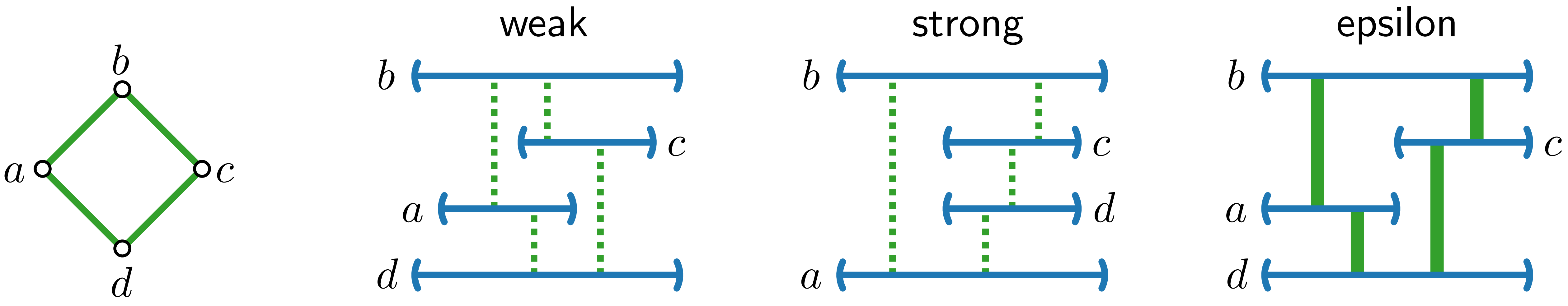

Weak Bar Visibility.

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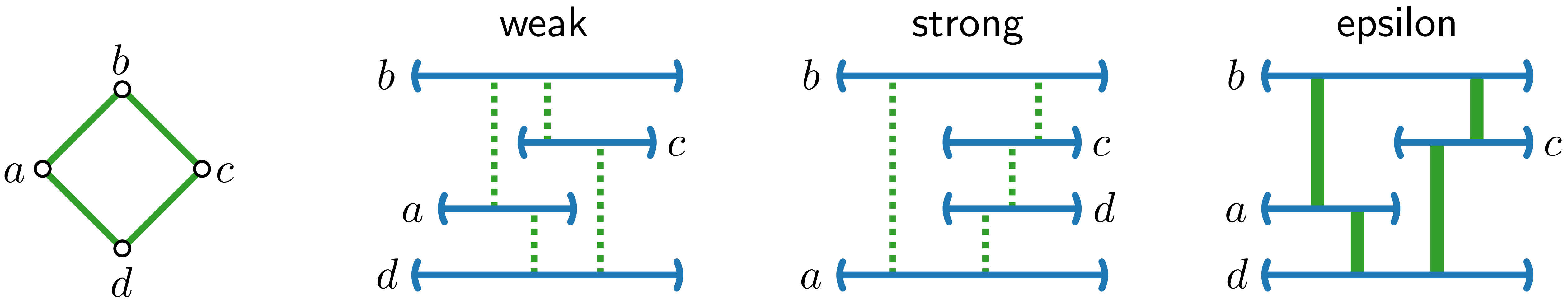

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- Exactly all planar graphs [Tamassia & Tollis ’86; Wismath ’85]

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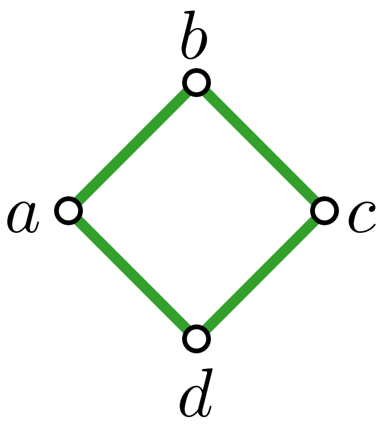

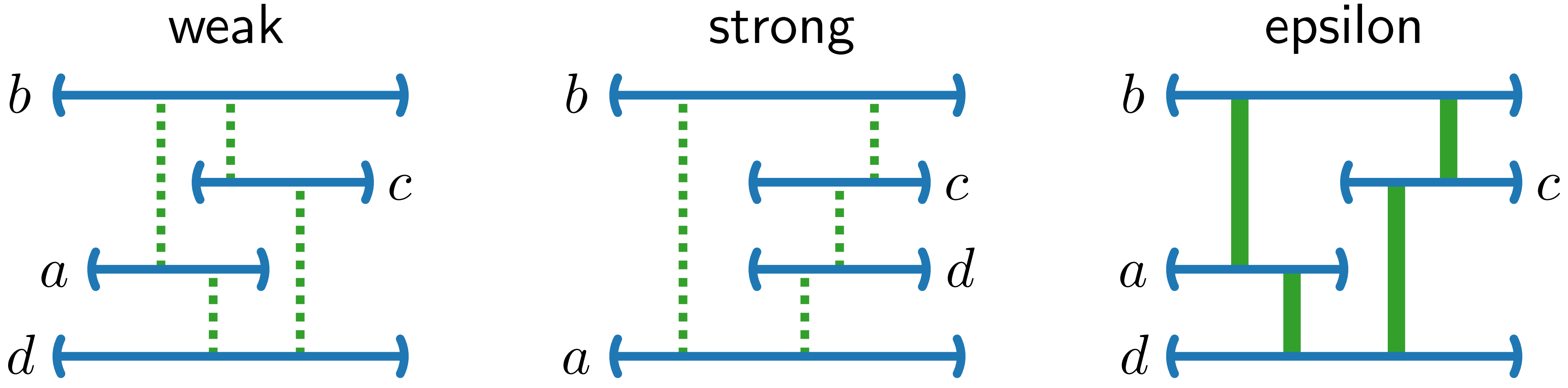

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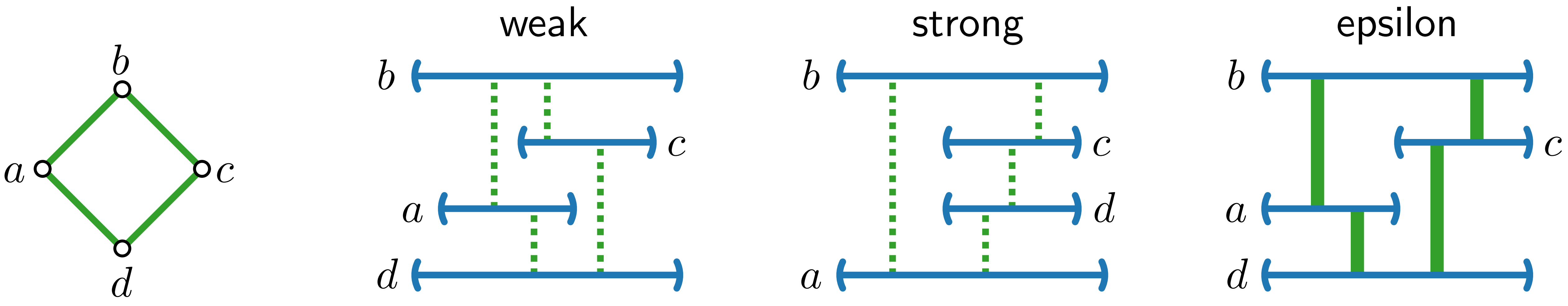

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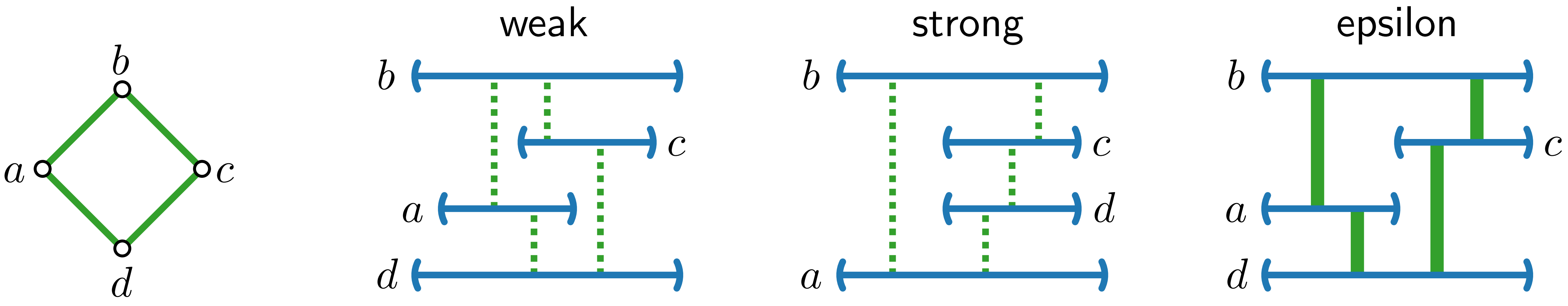

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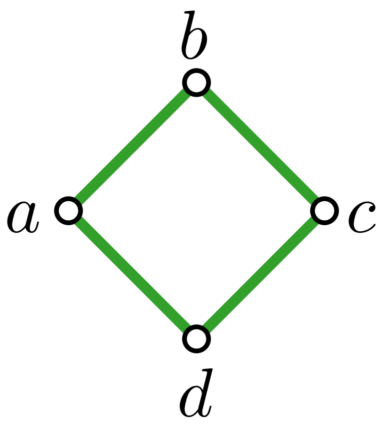

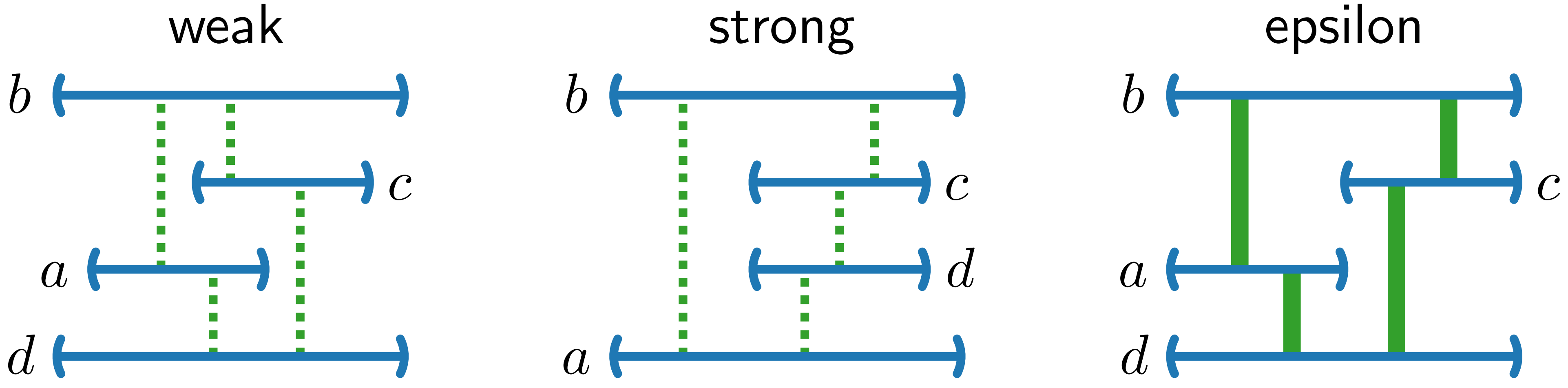

Weak Bar Visibility.

- Exactly all planar graphs [Tamassia & Tollis '86; Wismath '85]
- Linear time recognition and construction [T&T '86]
- Representation extension is NP-complete [Chaplick et al. '14]

Strong Bar Visibility.

- NP-complete to recognize [Andreae '92]

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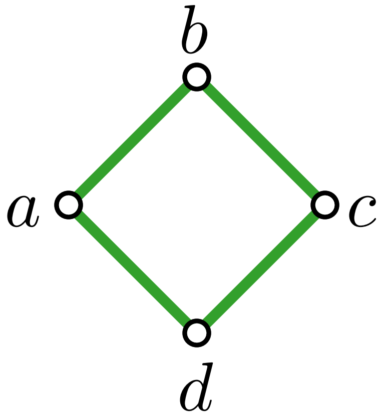

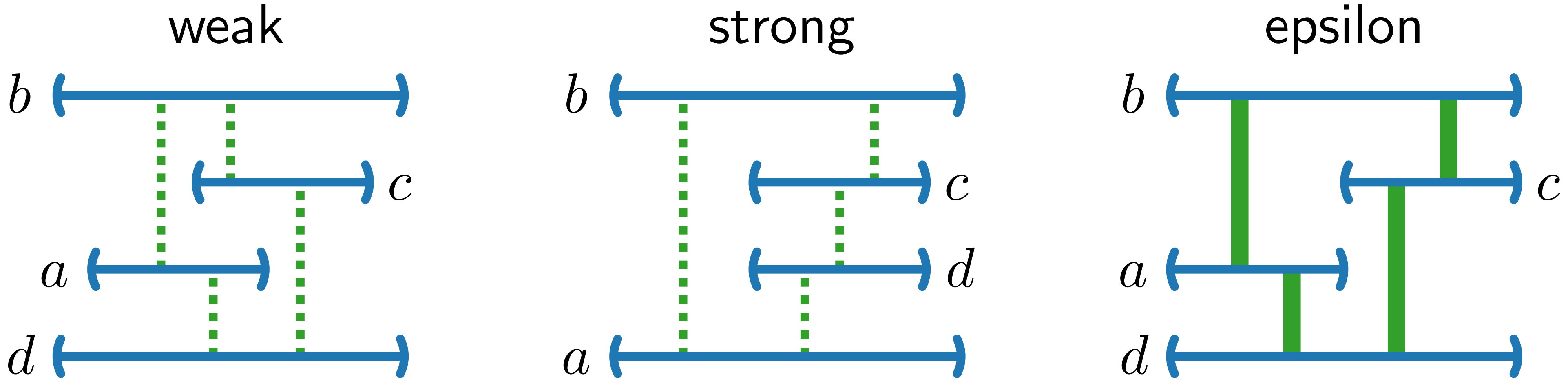

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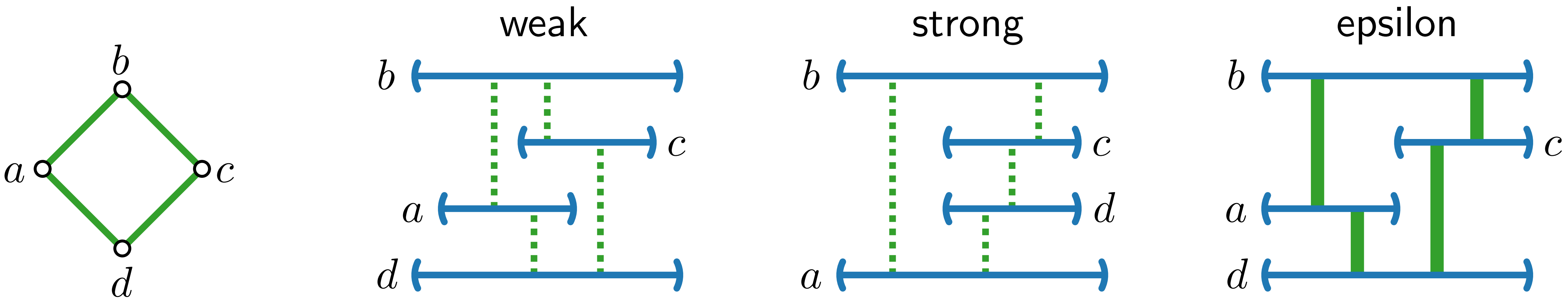

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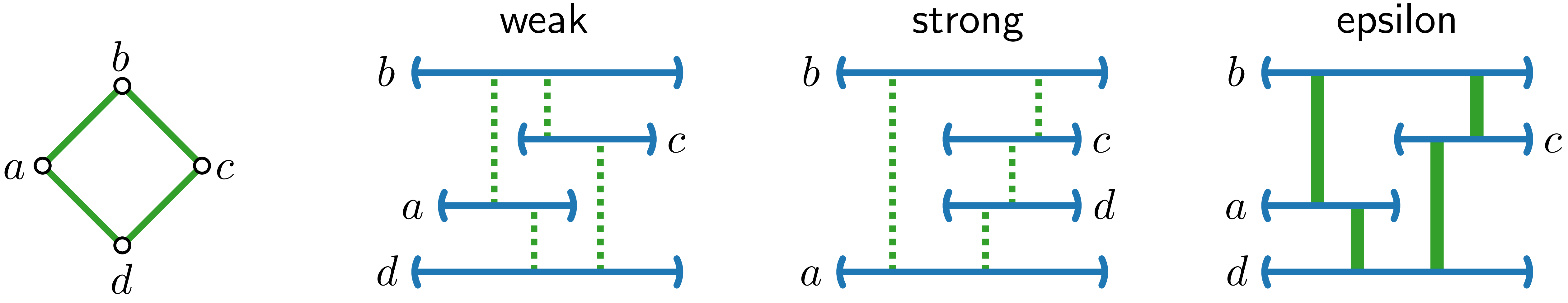

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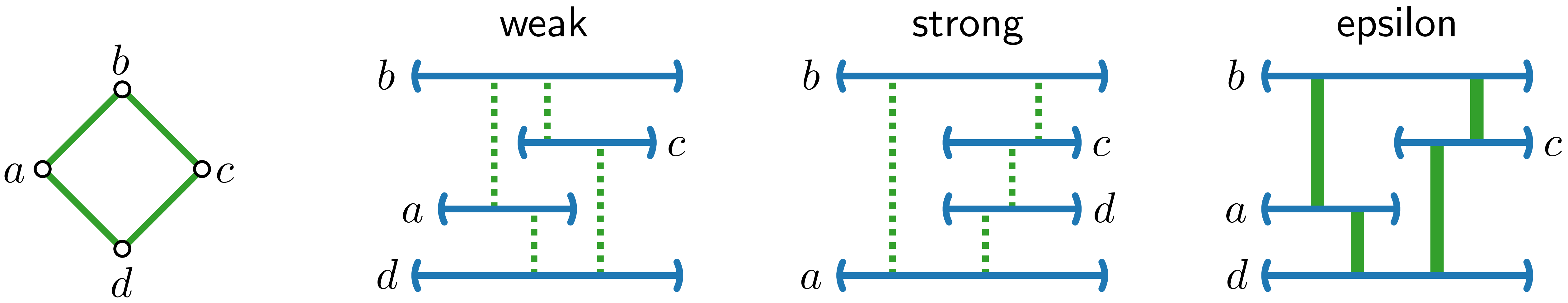

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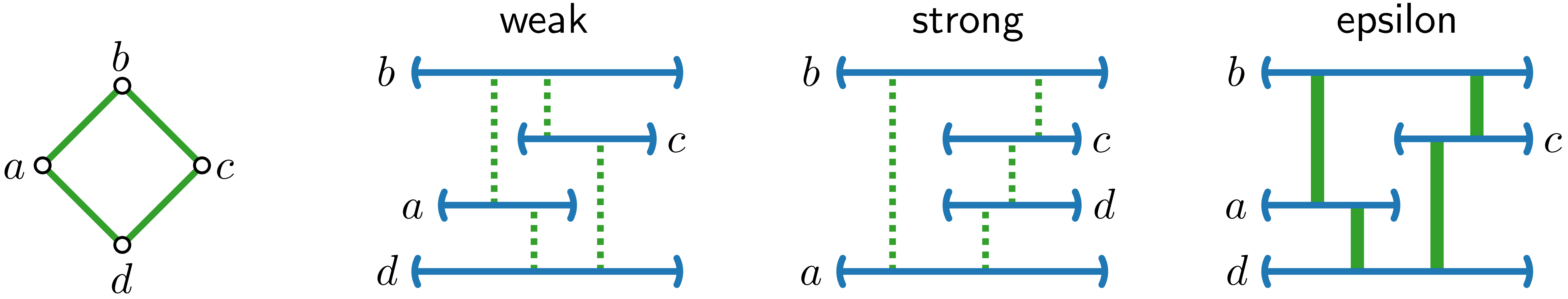

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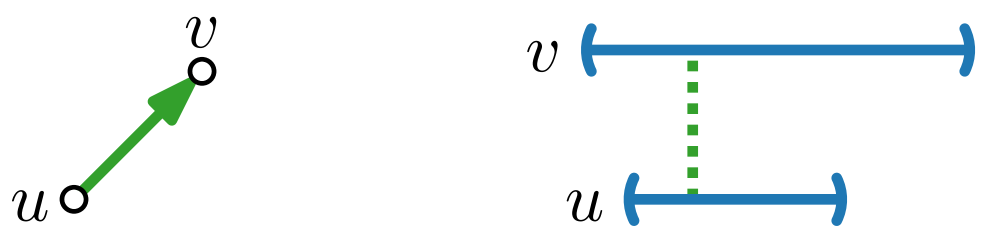

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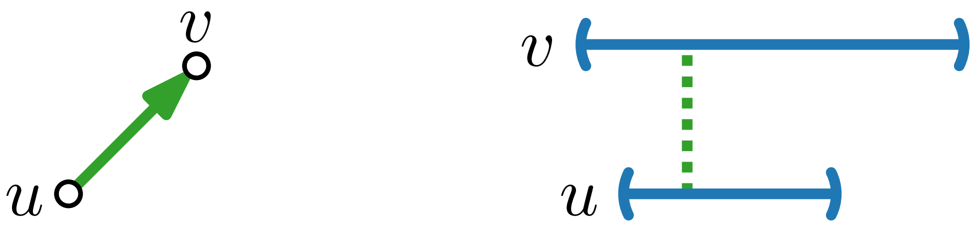

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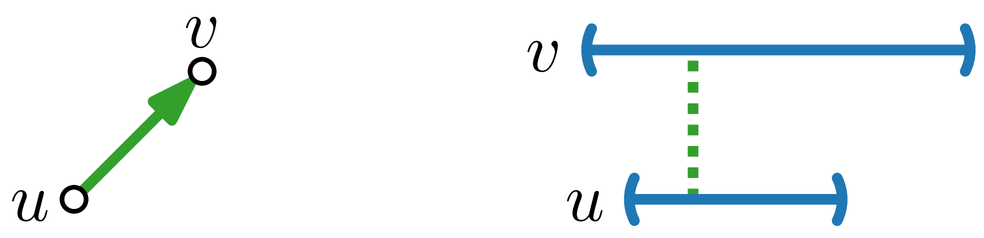

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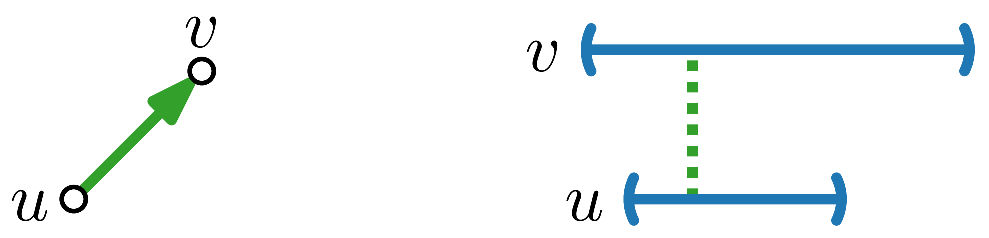

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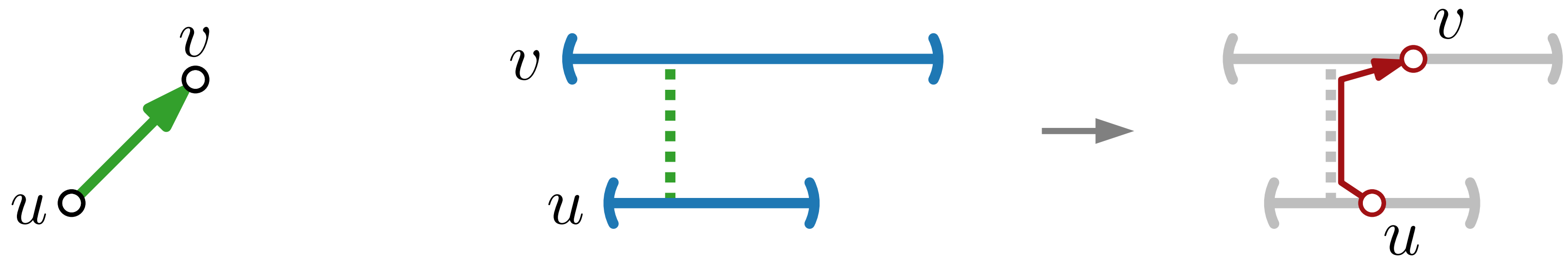

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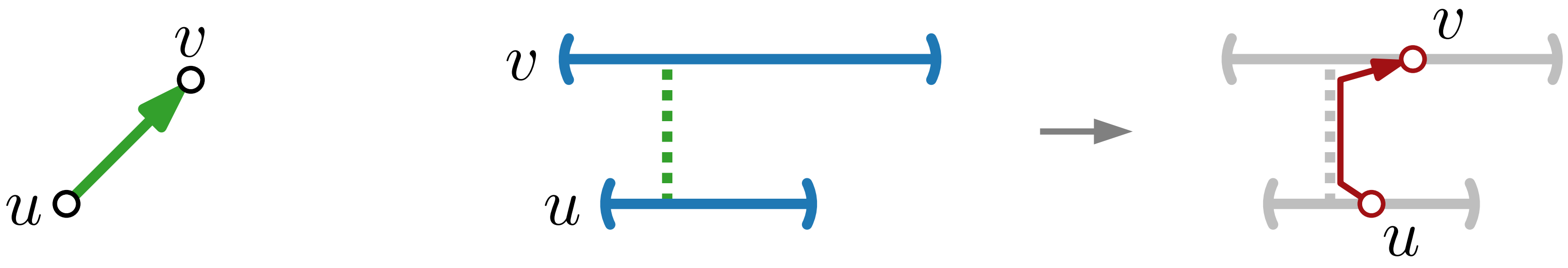

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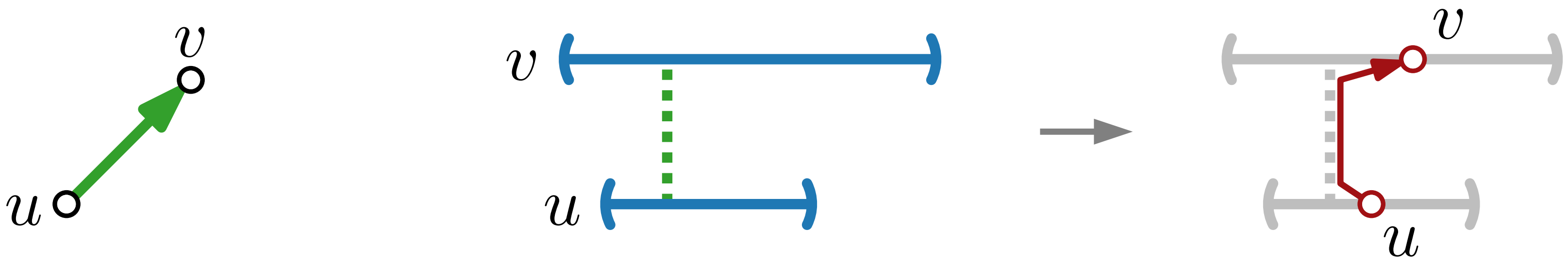

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Next, we consider ε-bar visibility representations of specific directed graphs ($\rightarrow$ st-graphs)

ε-Bar Visibility and st-Graphs

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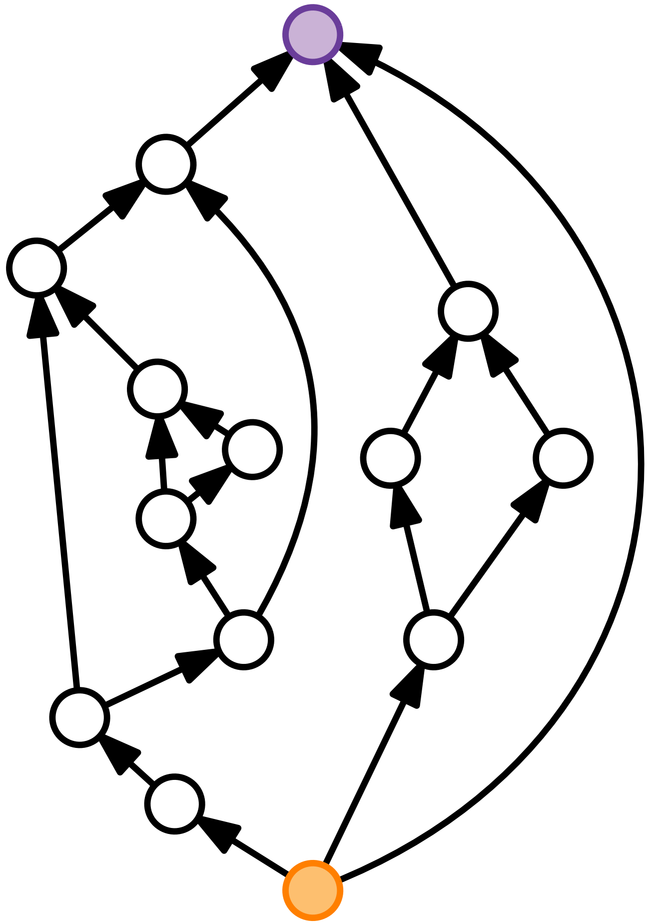

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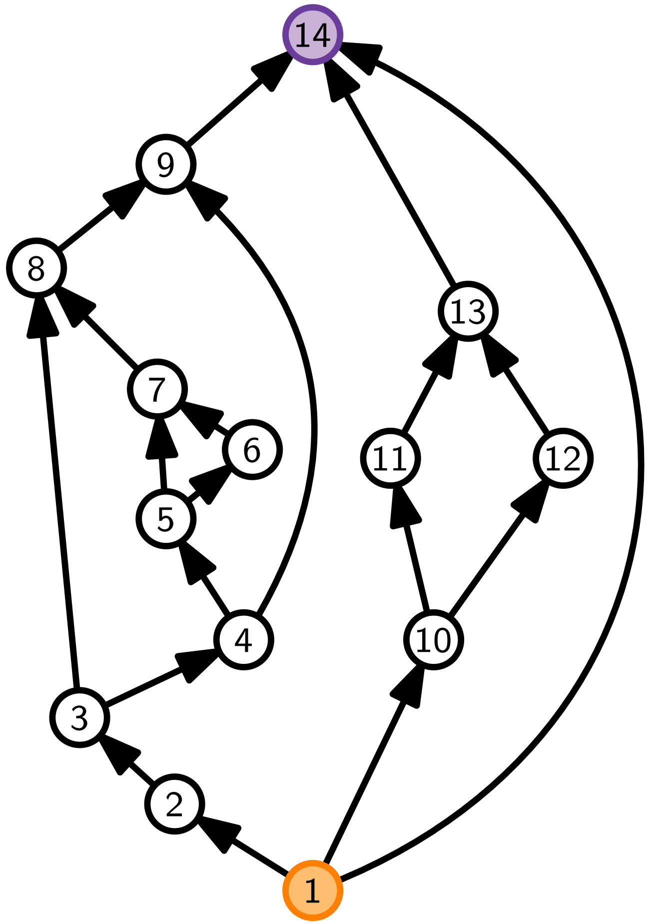

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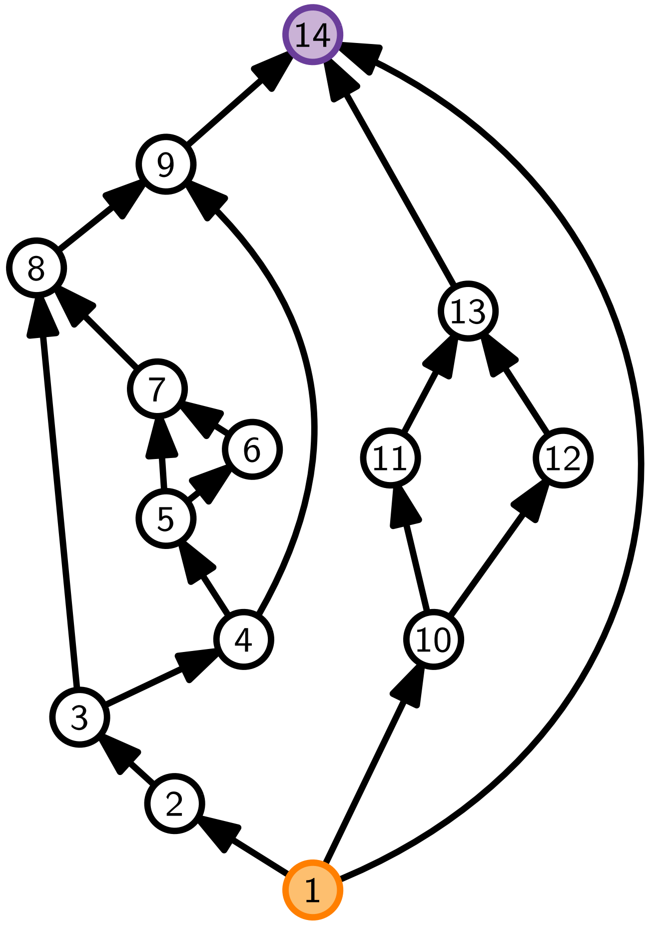

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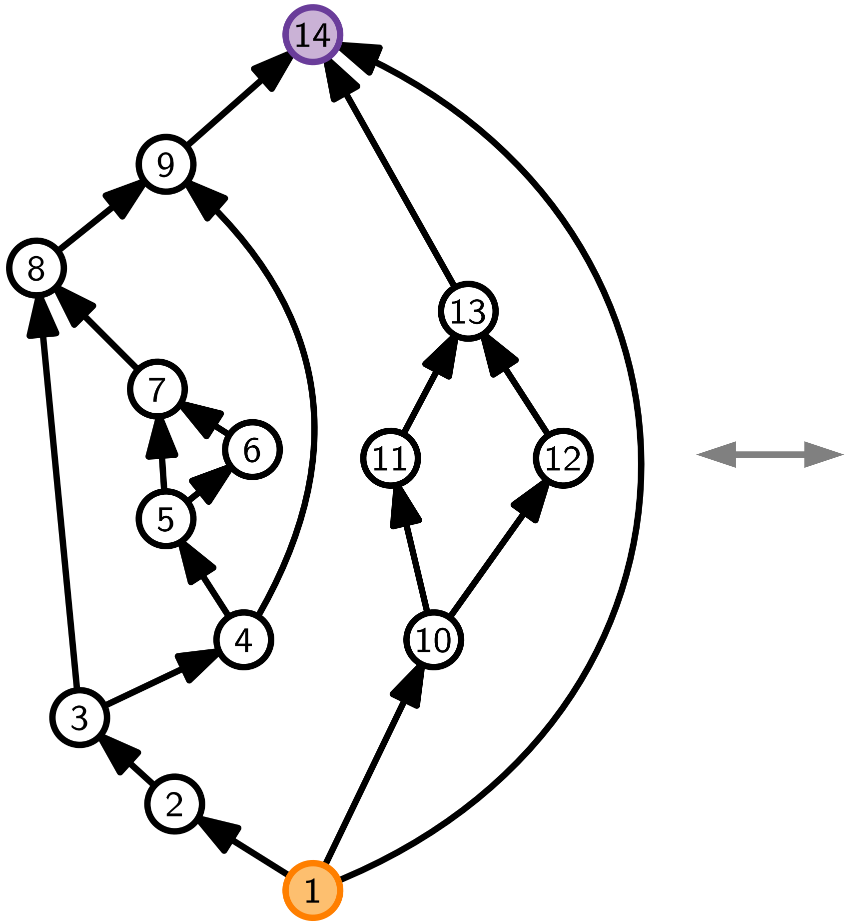

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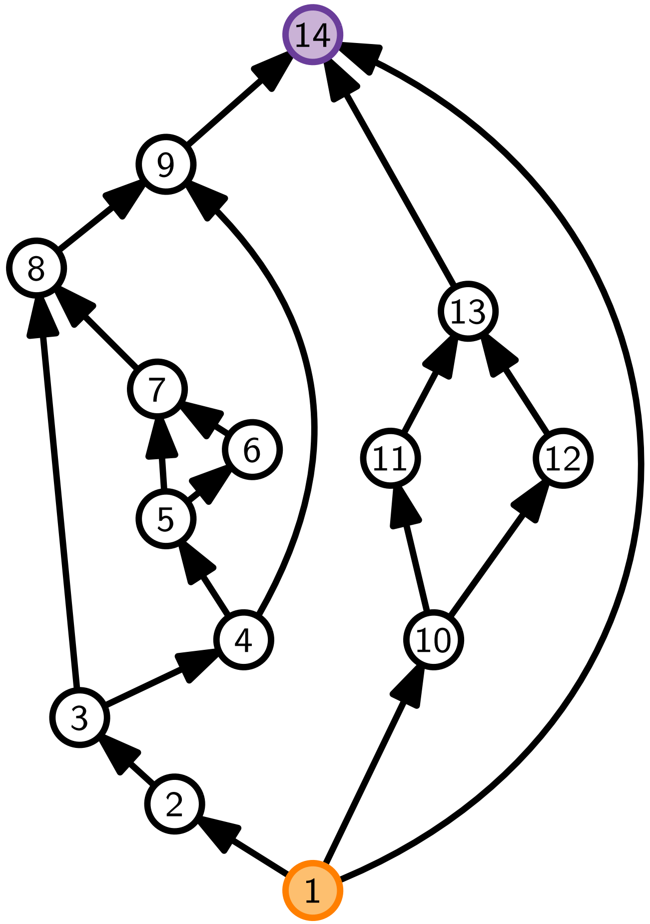

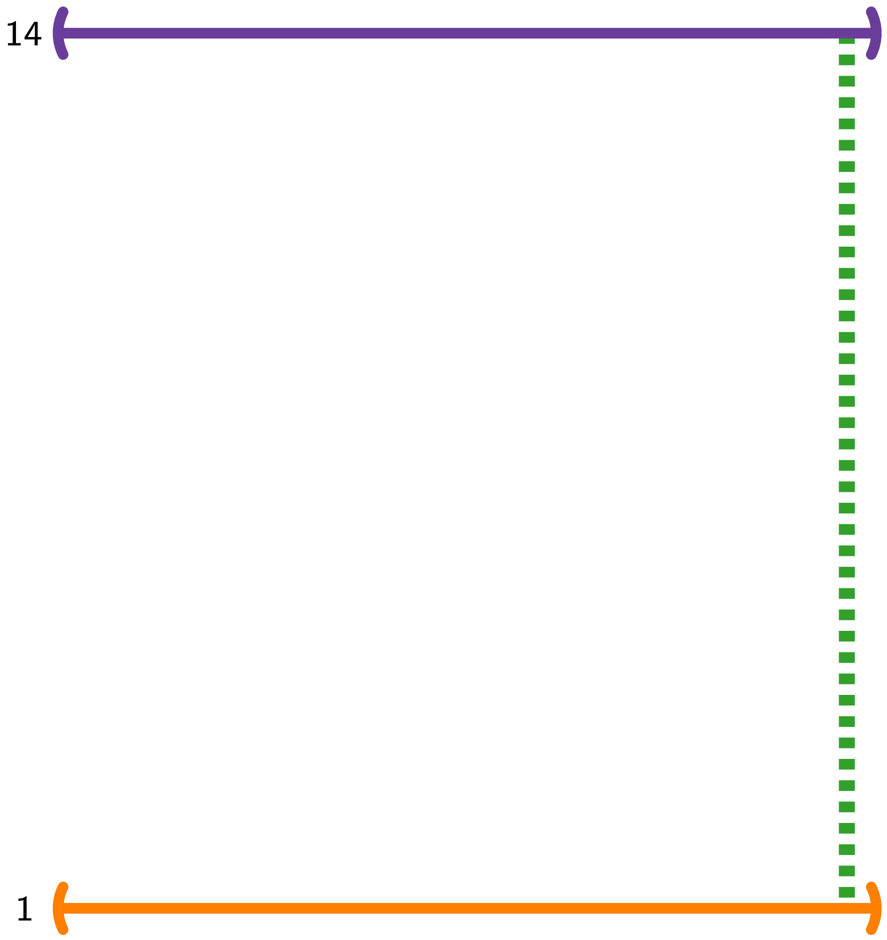

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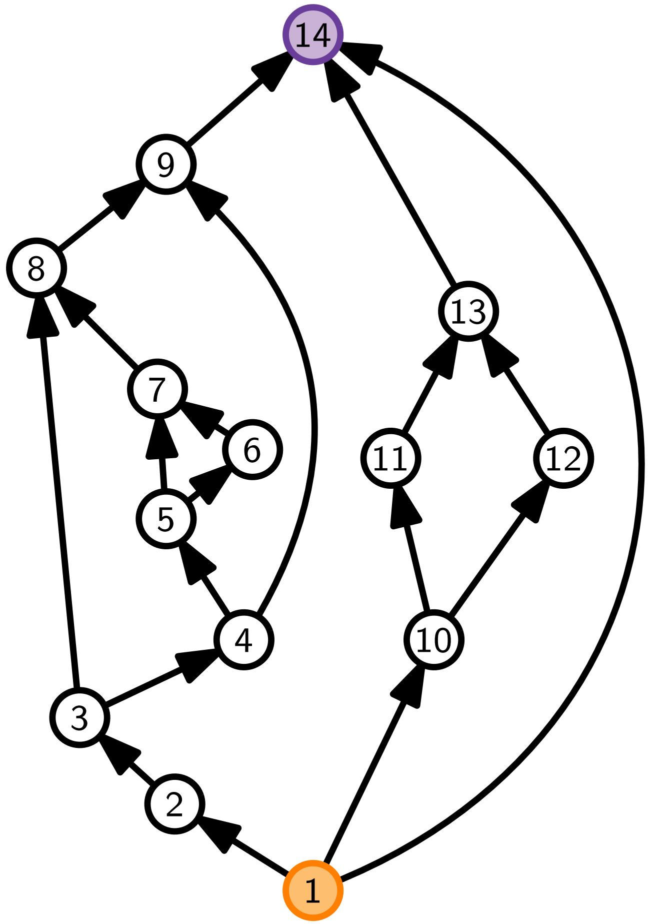

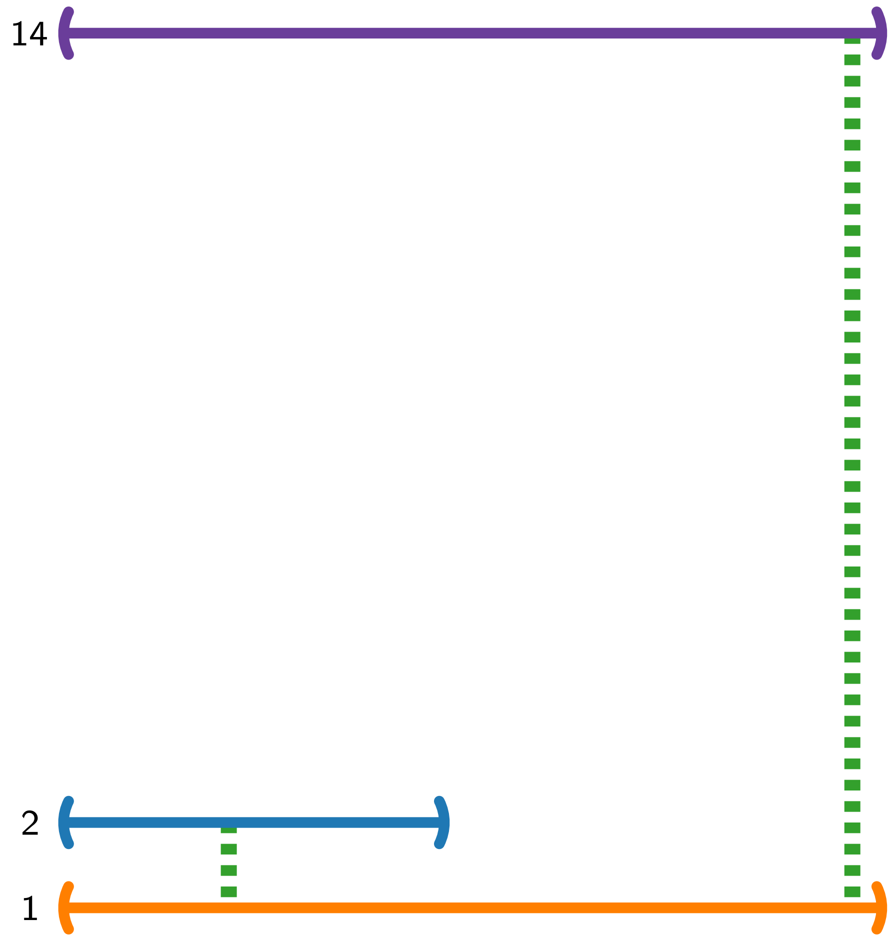

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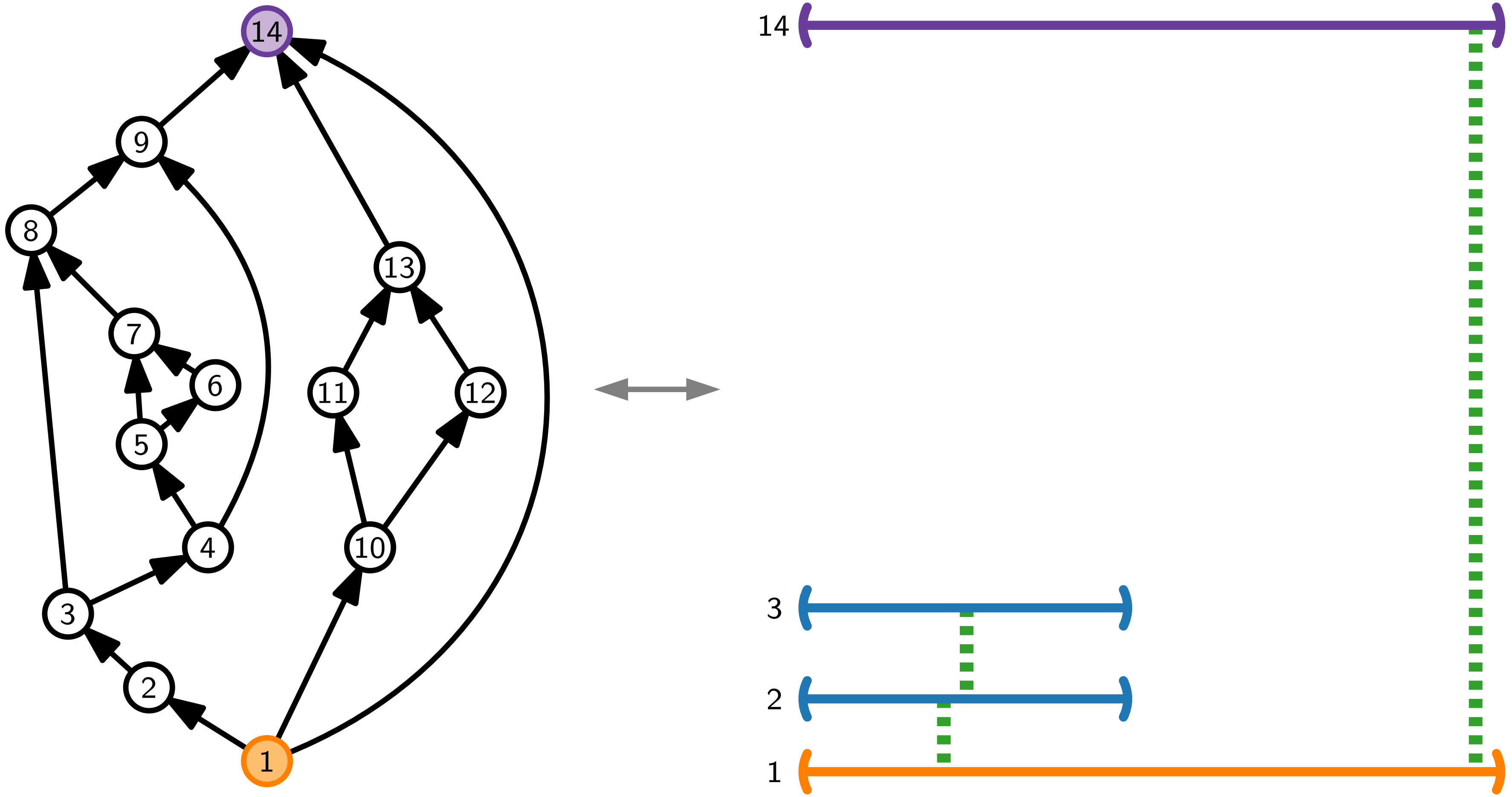

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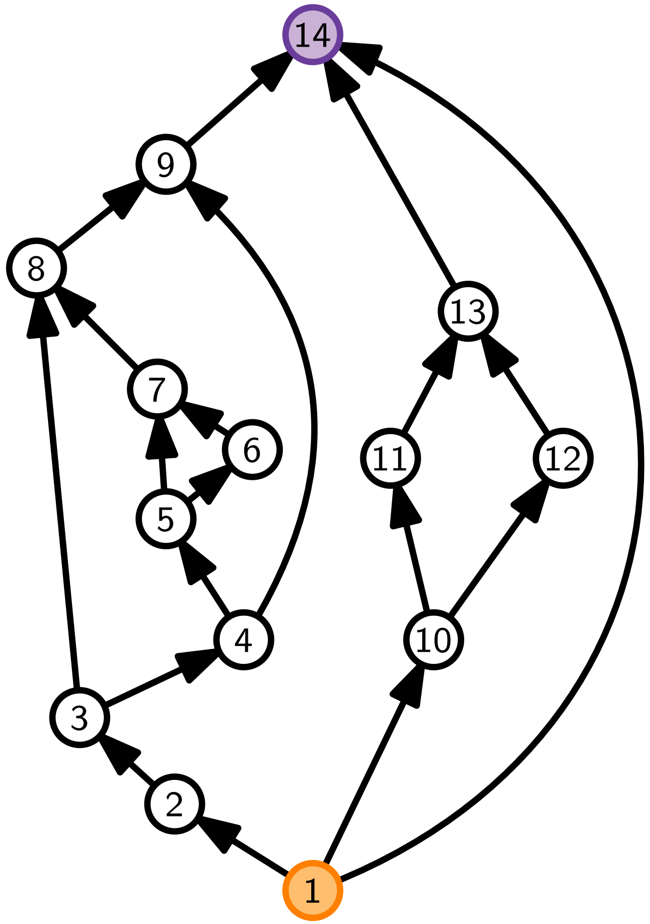

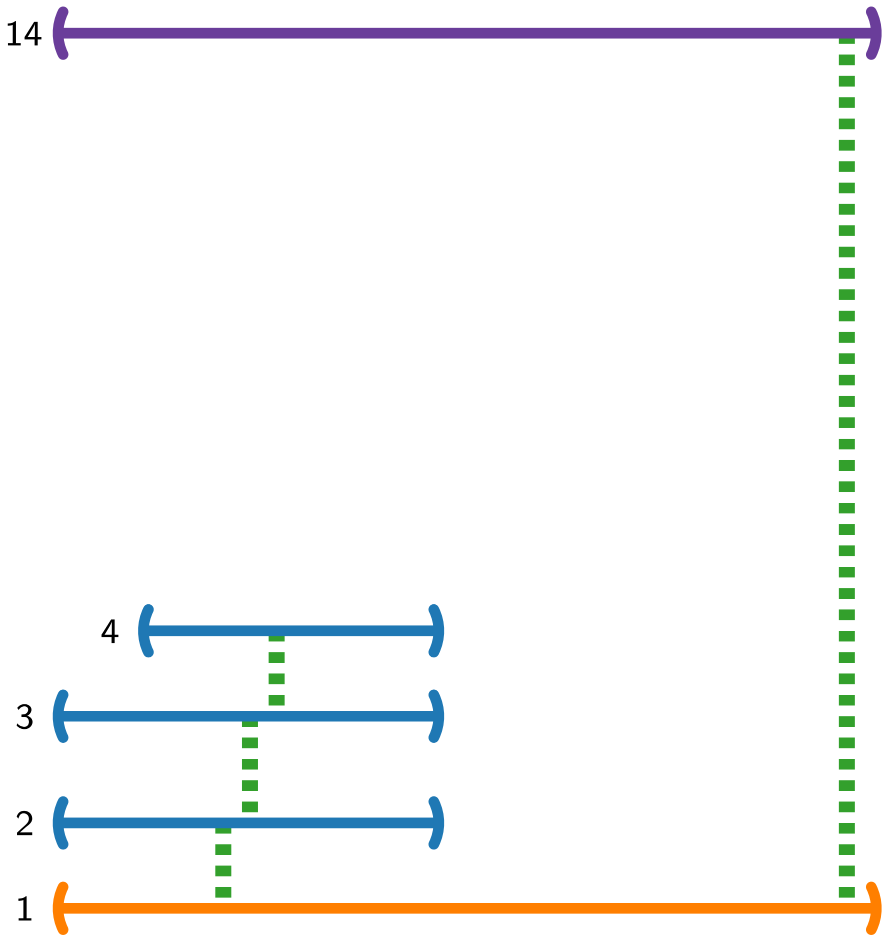

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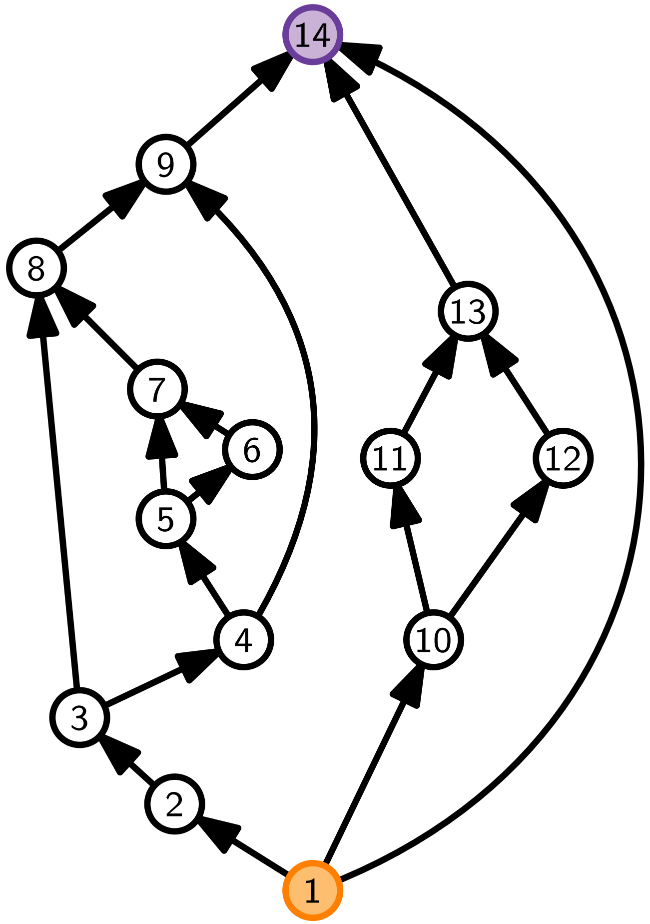

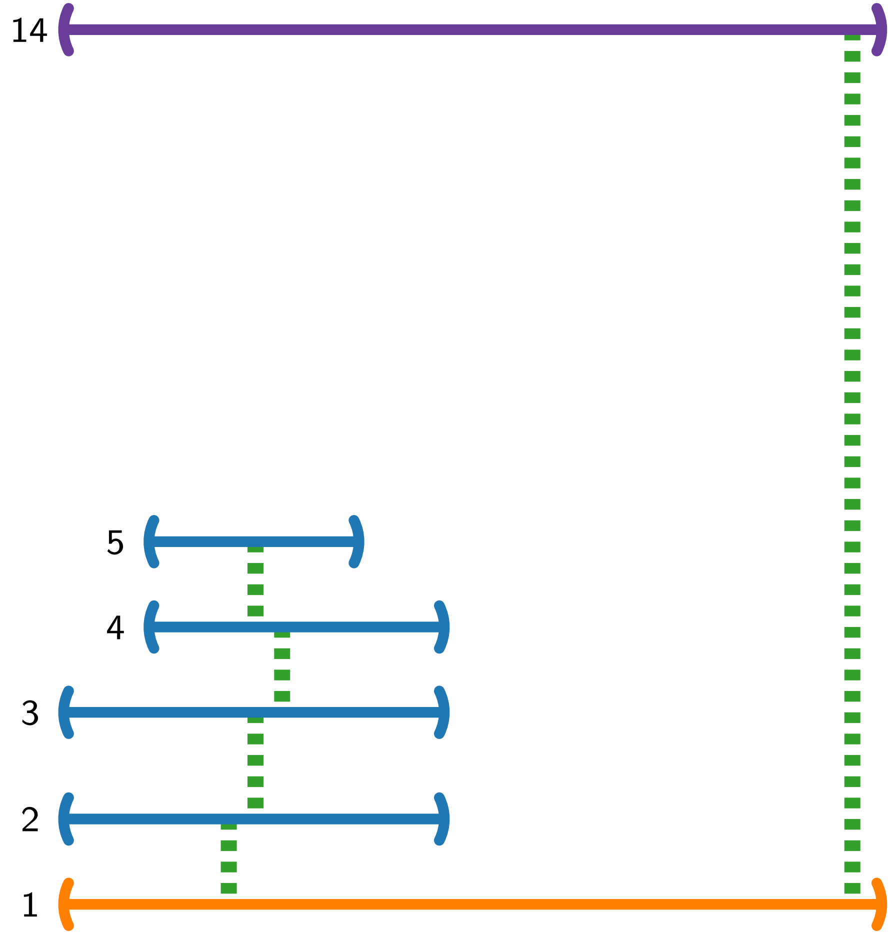

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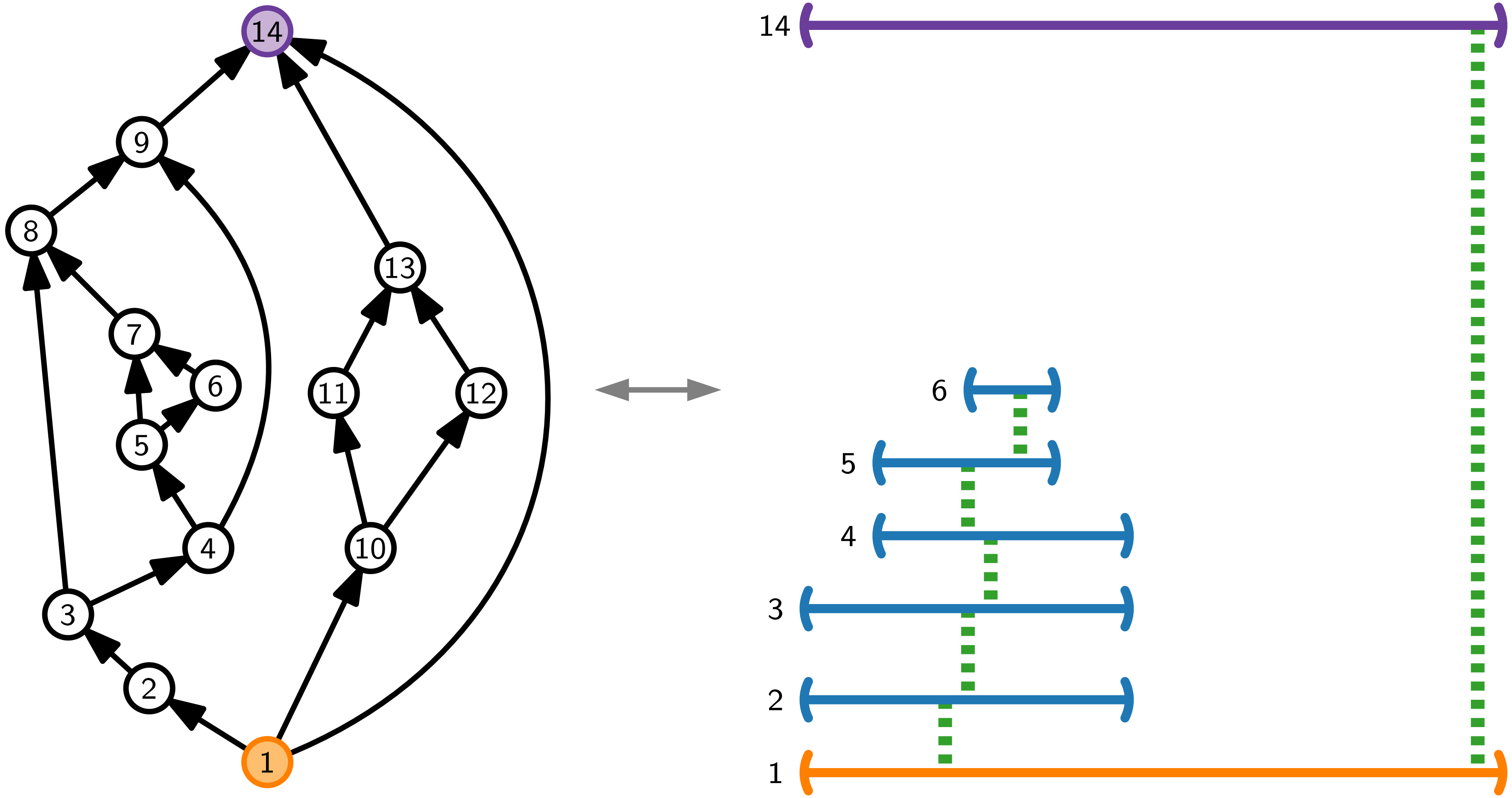

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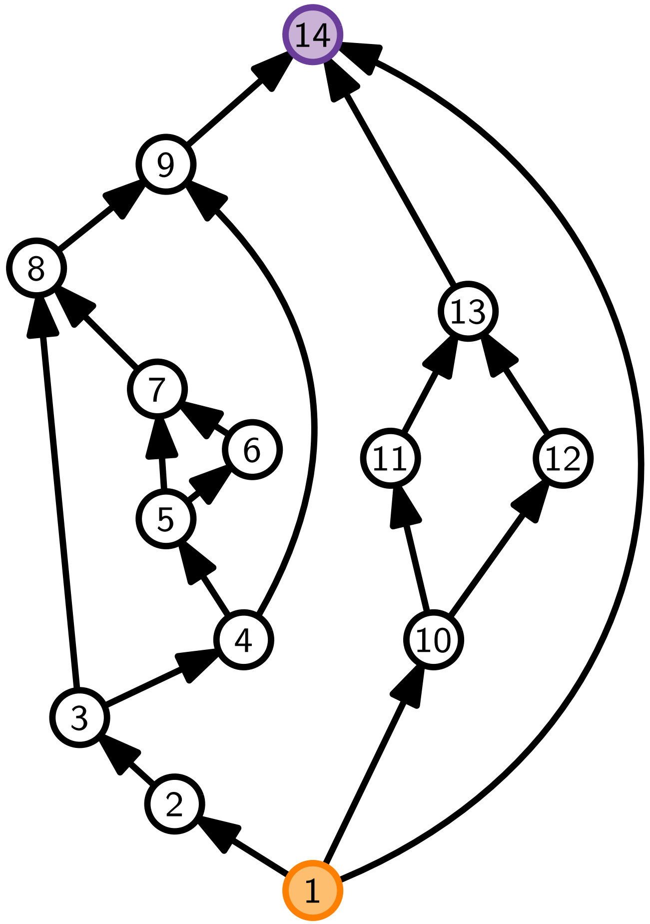

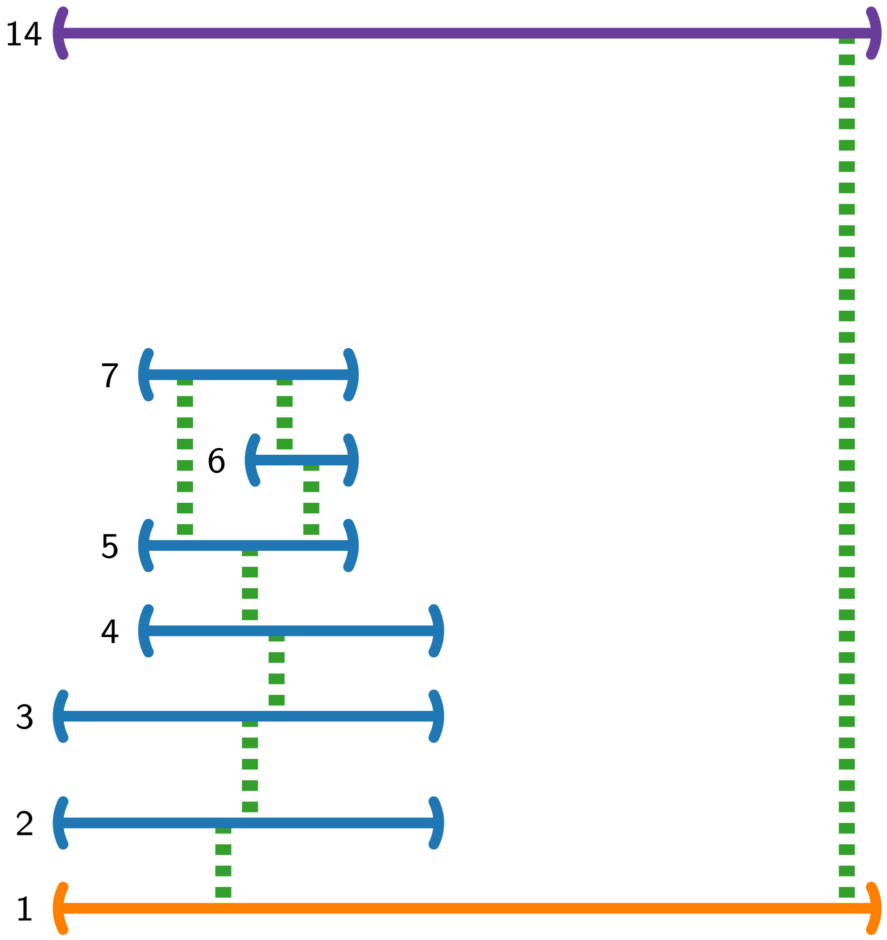

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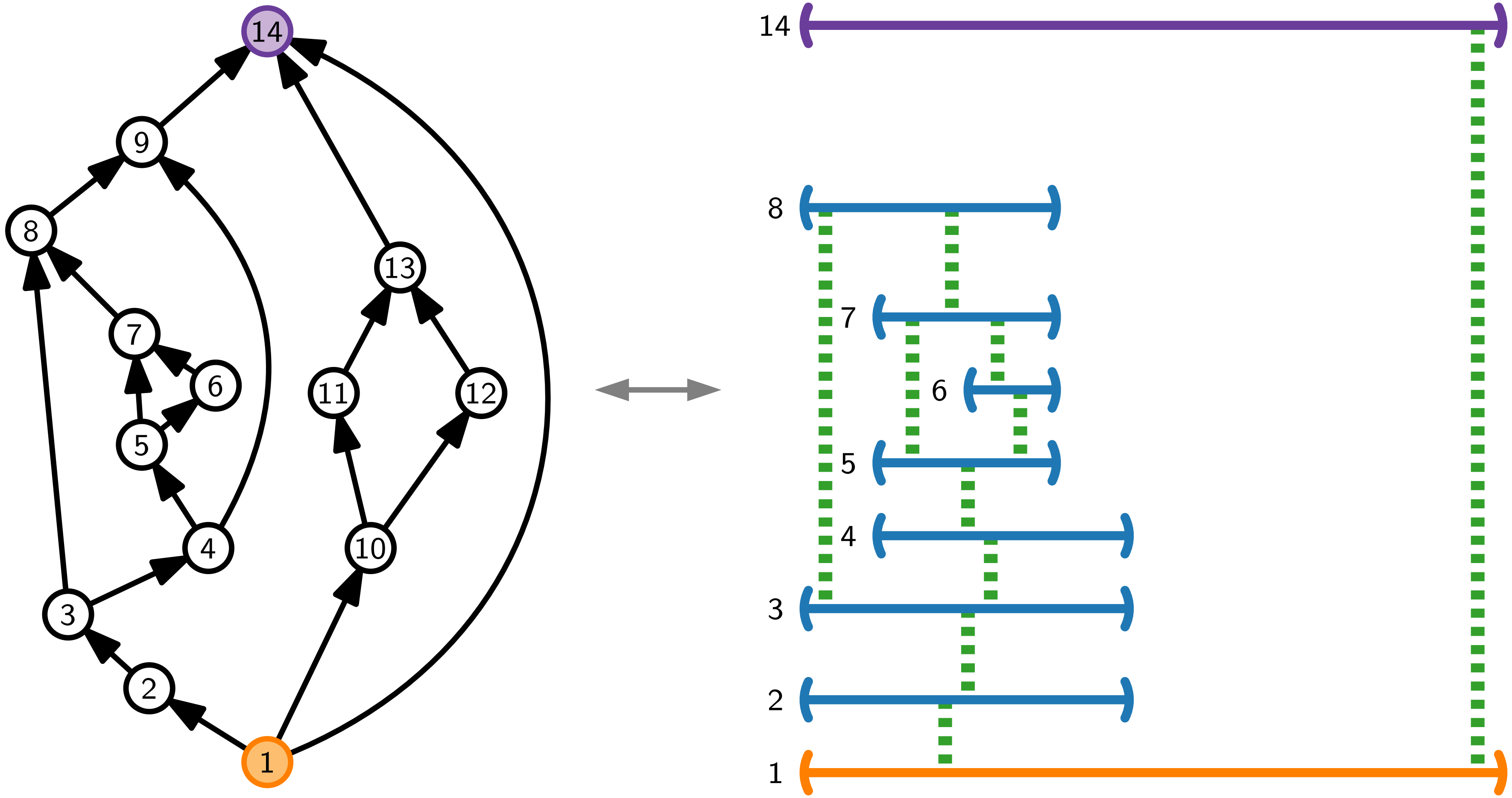

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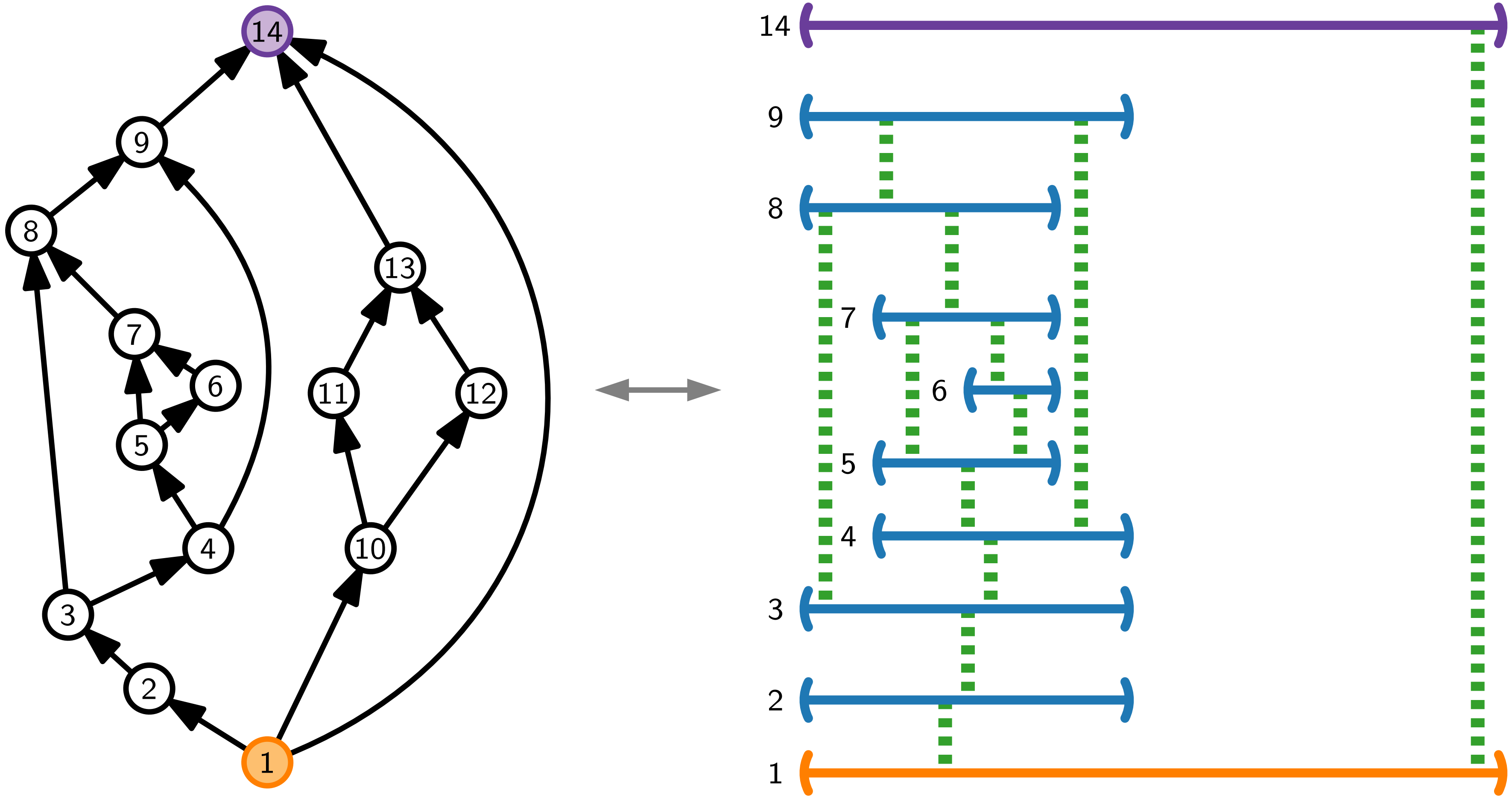

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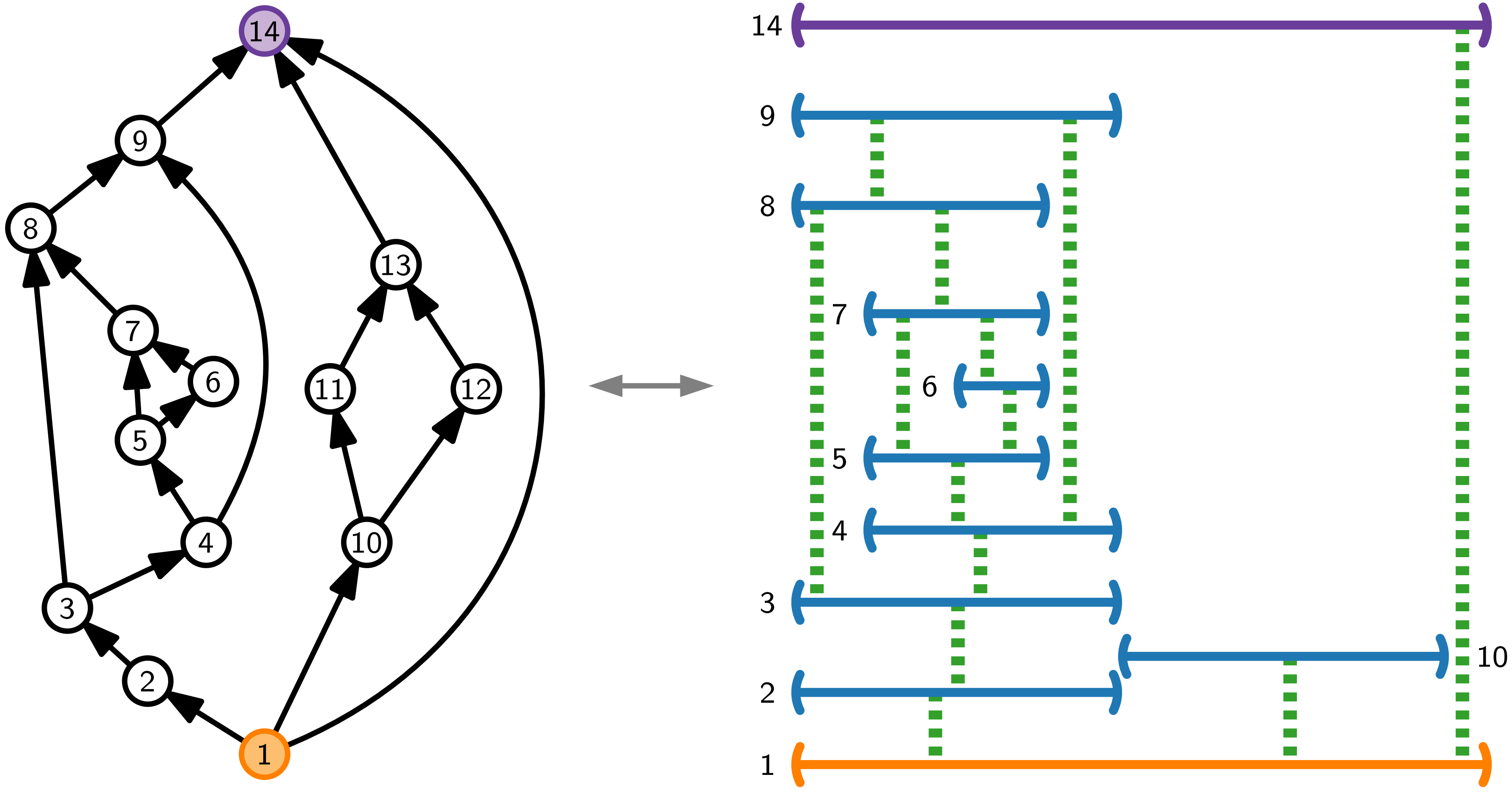

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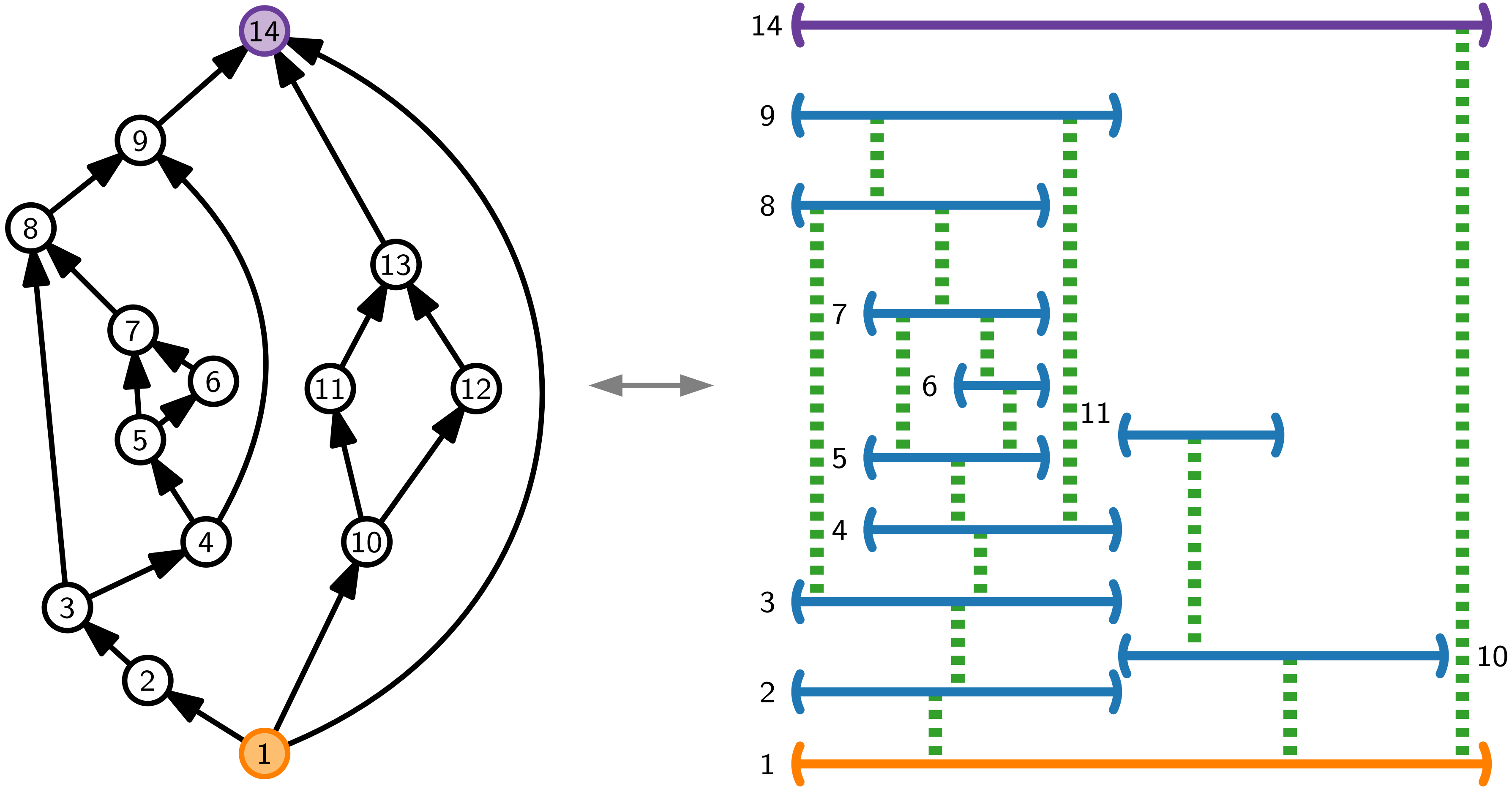

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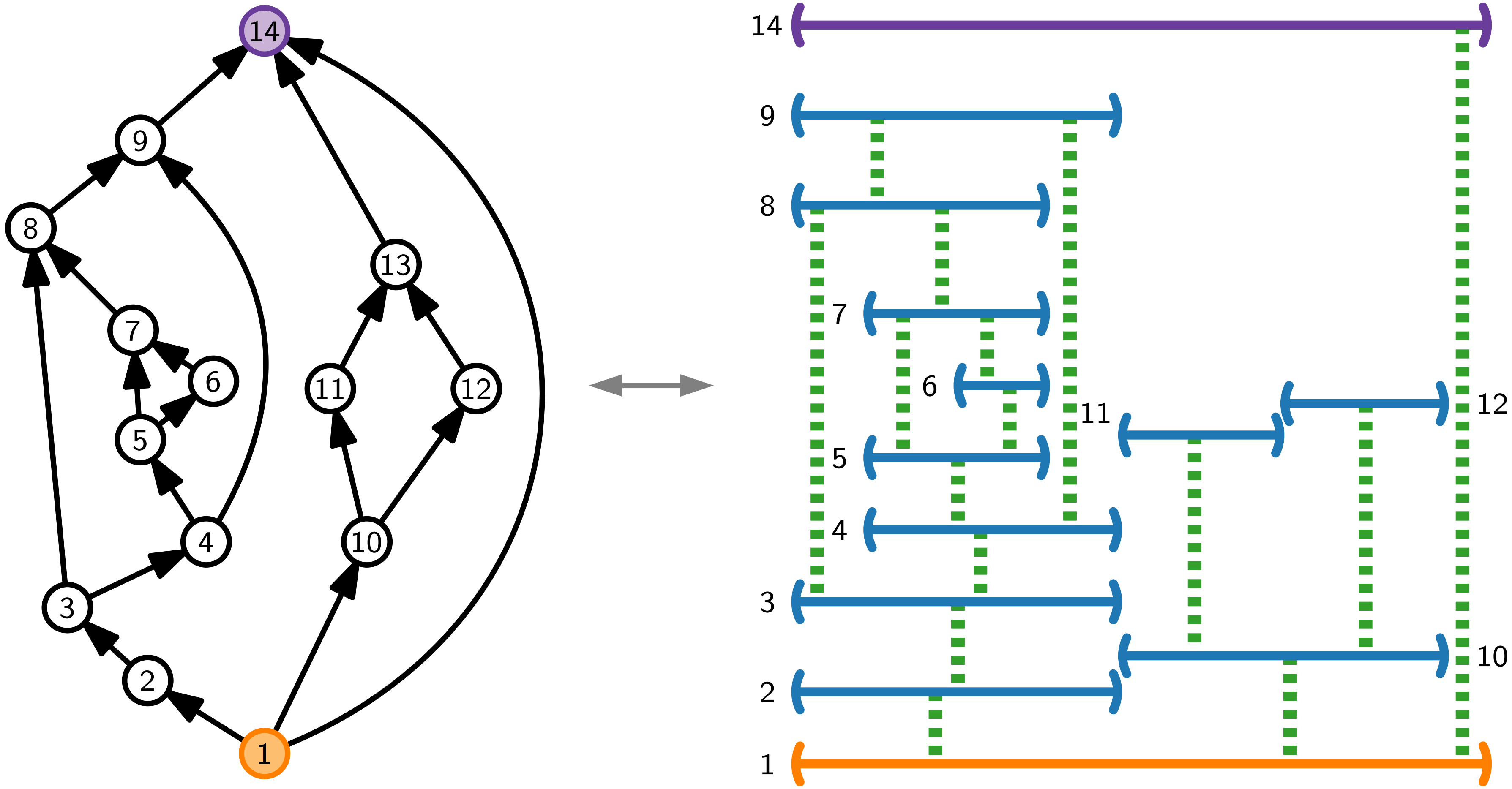

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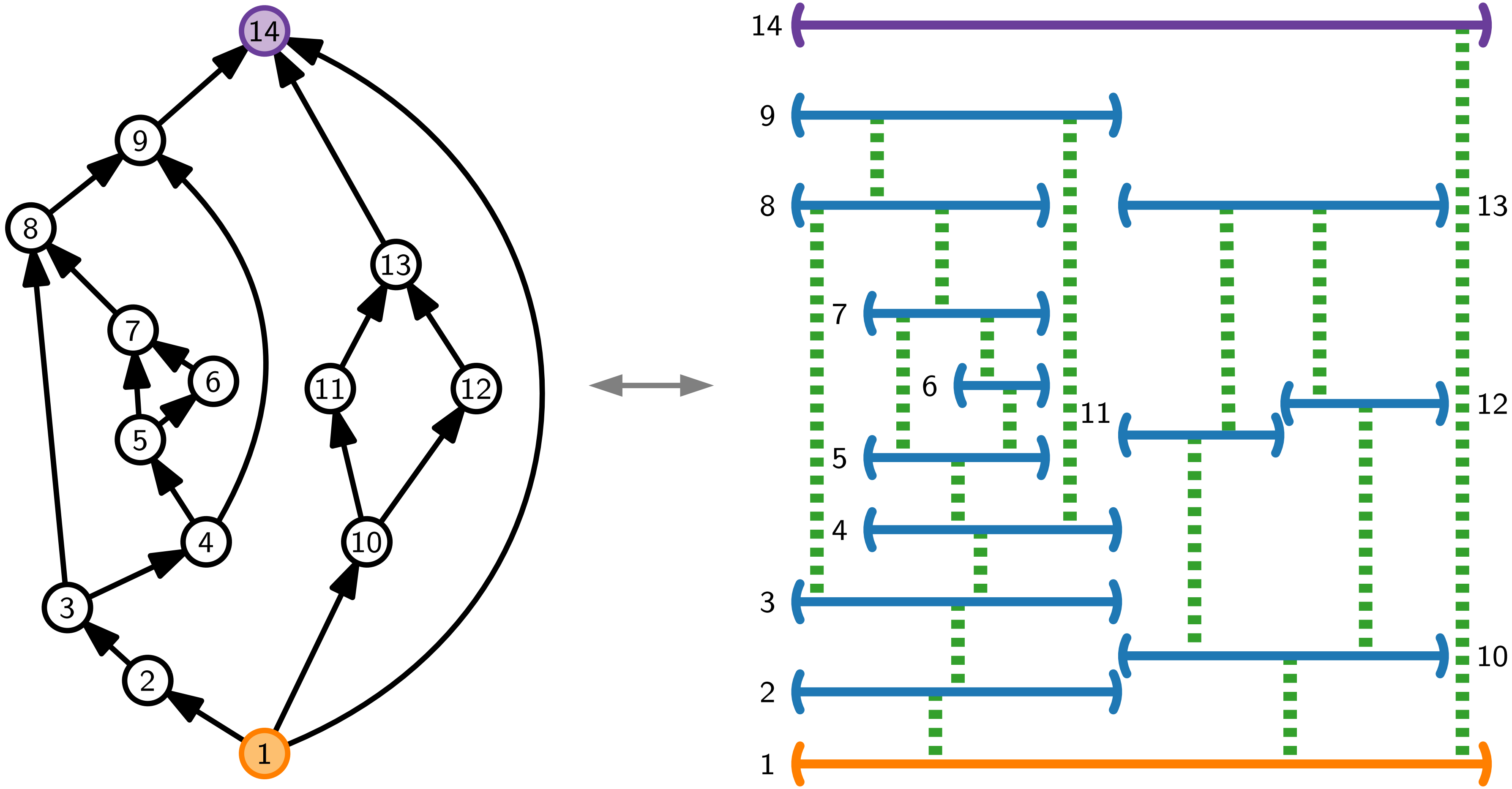

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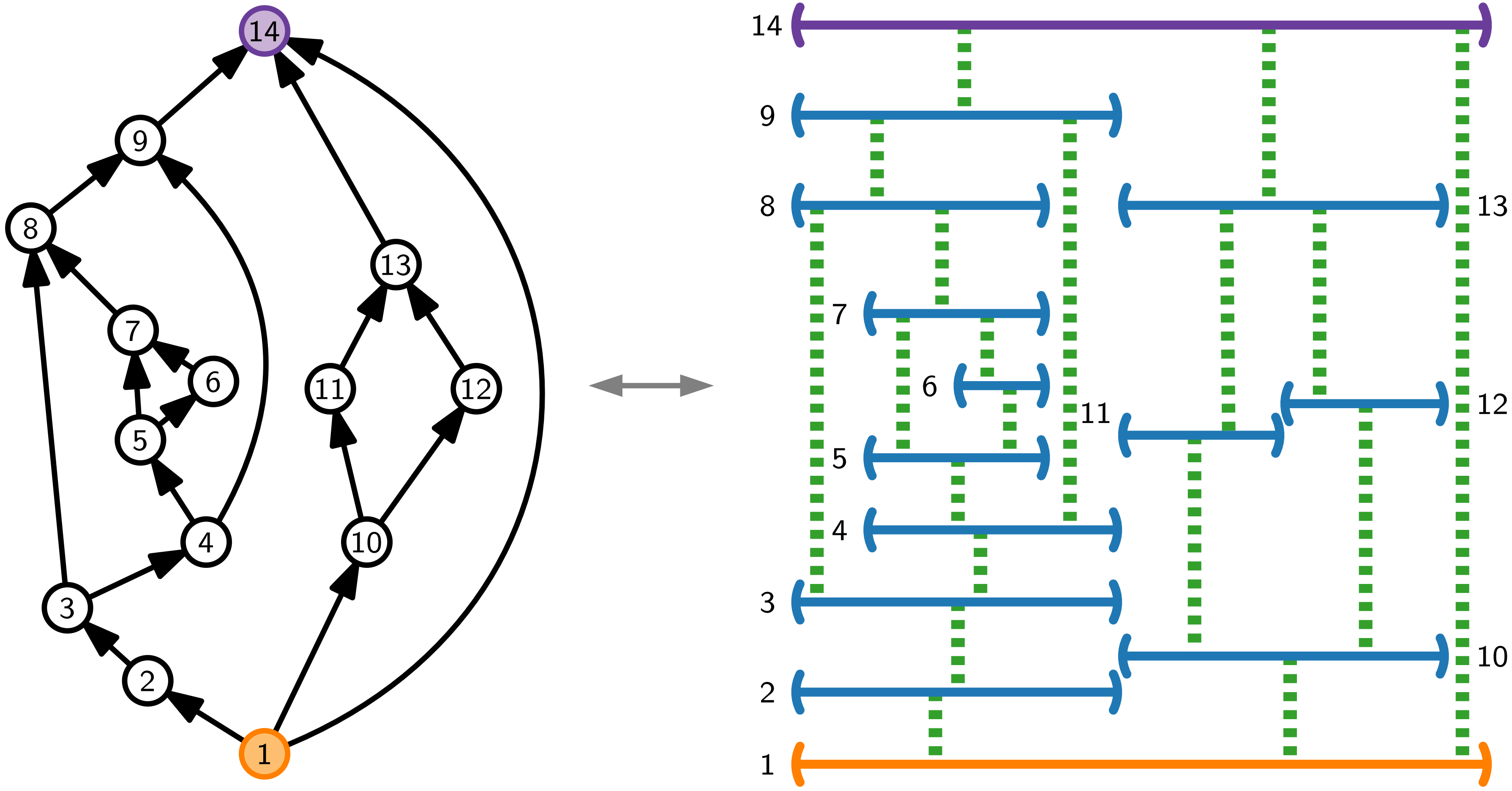

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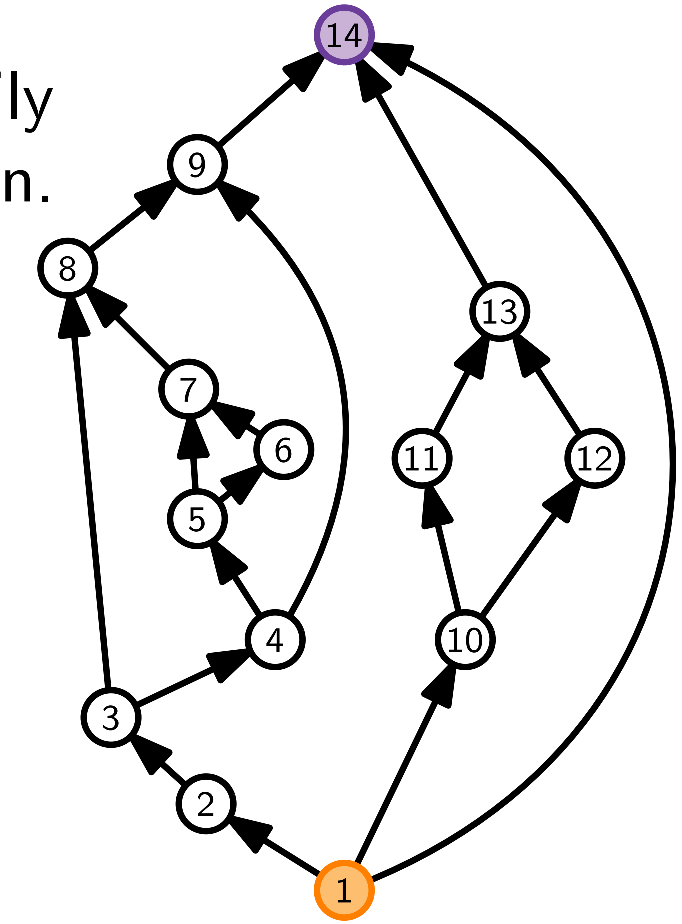

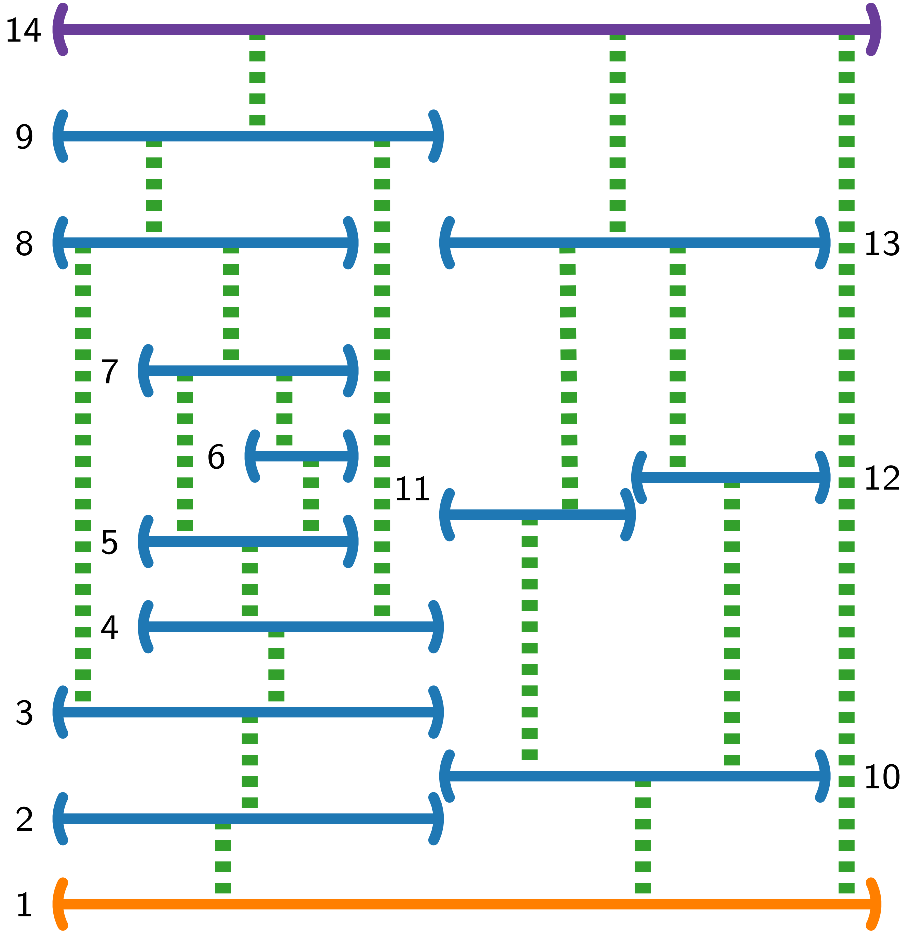

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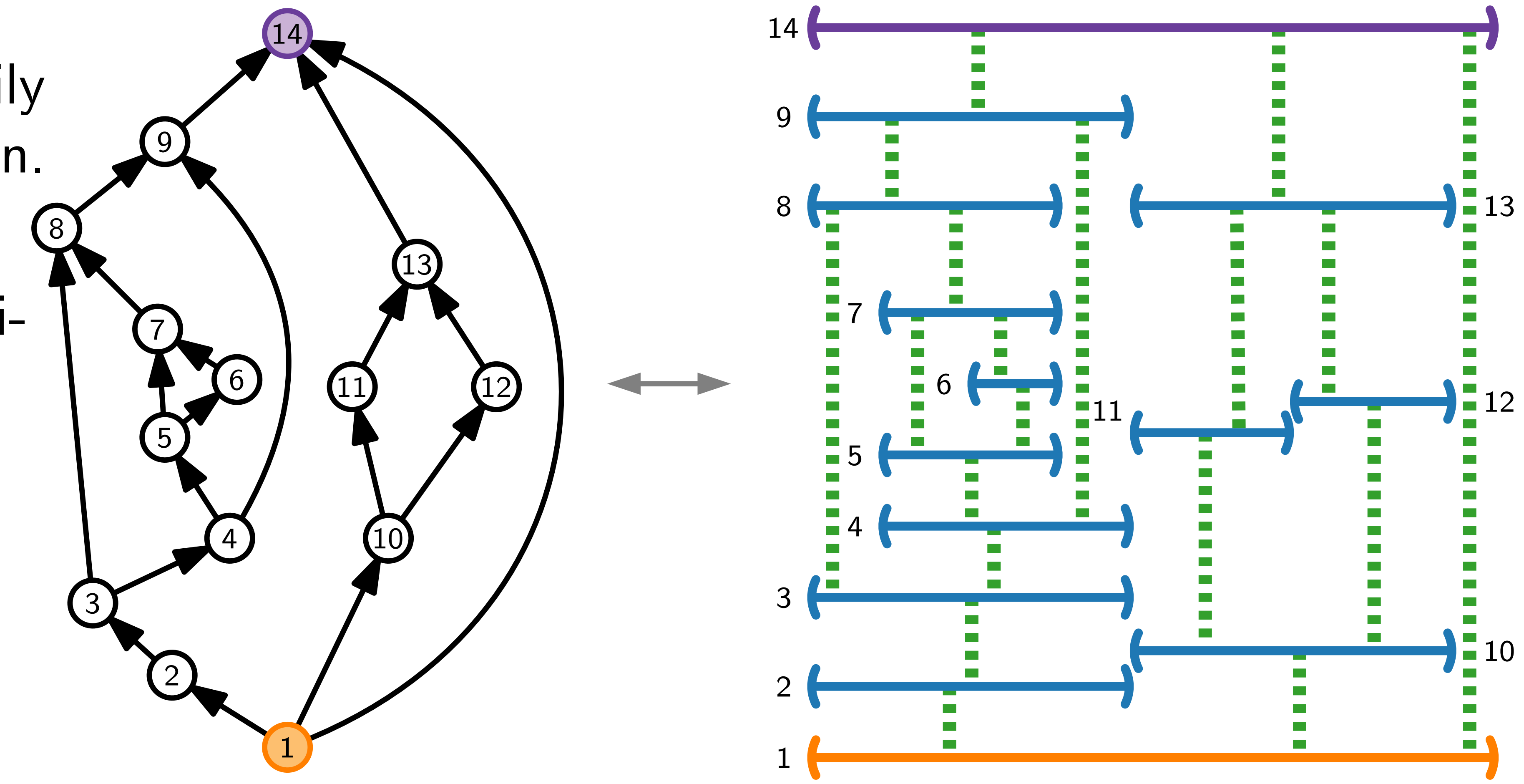

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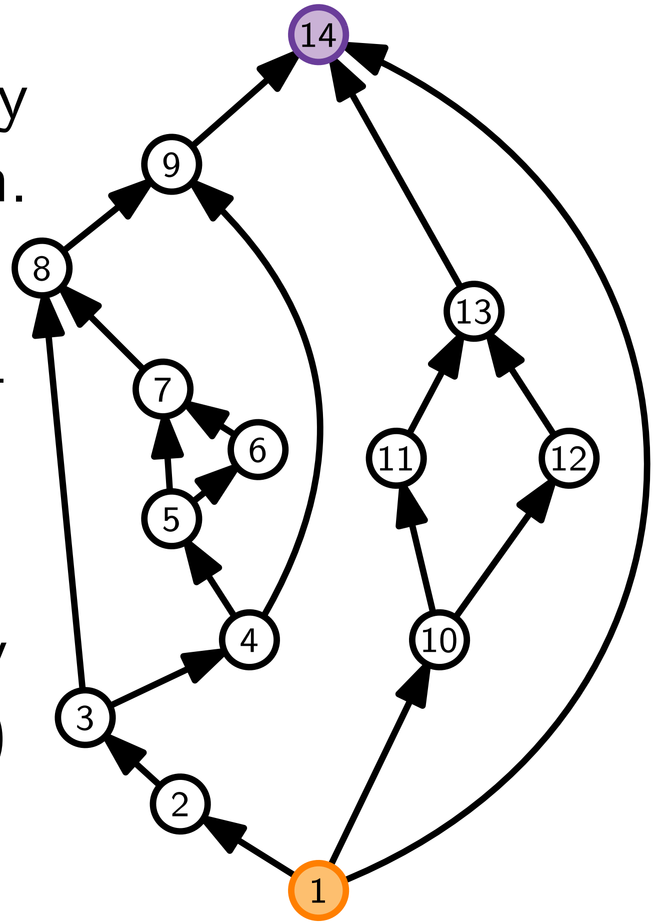

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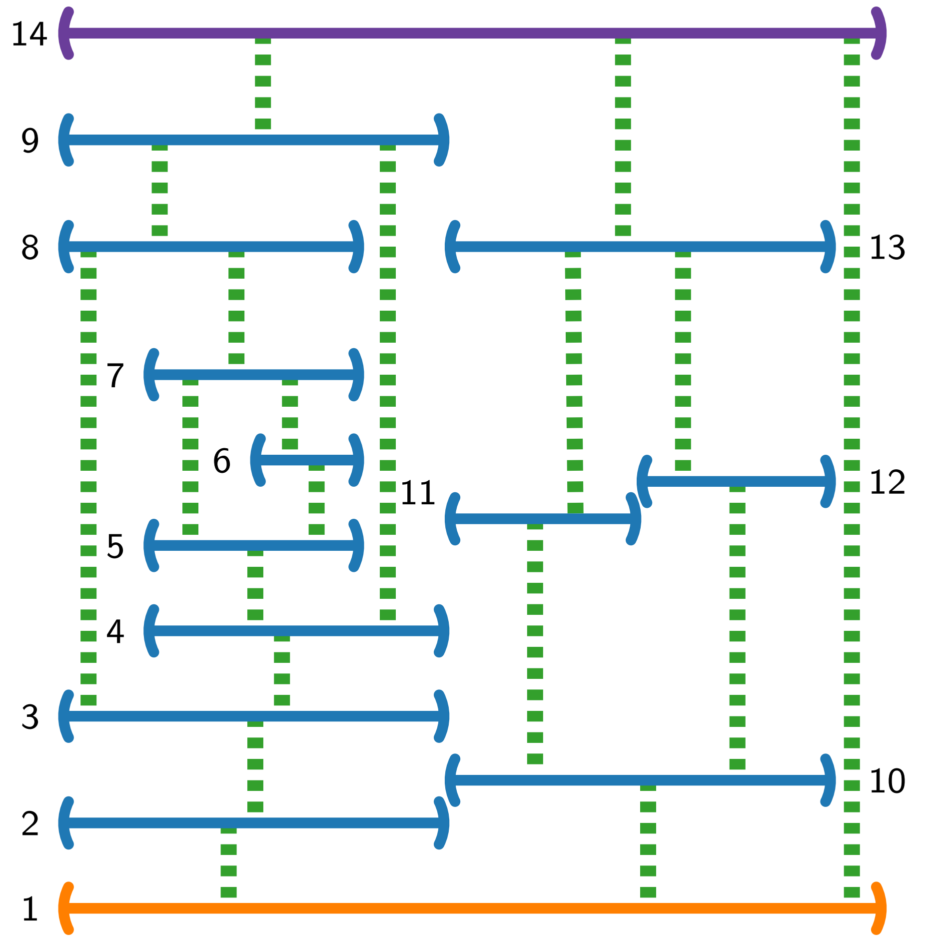

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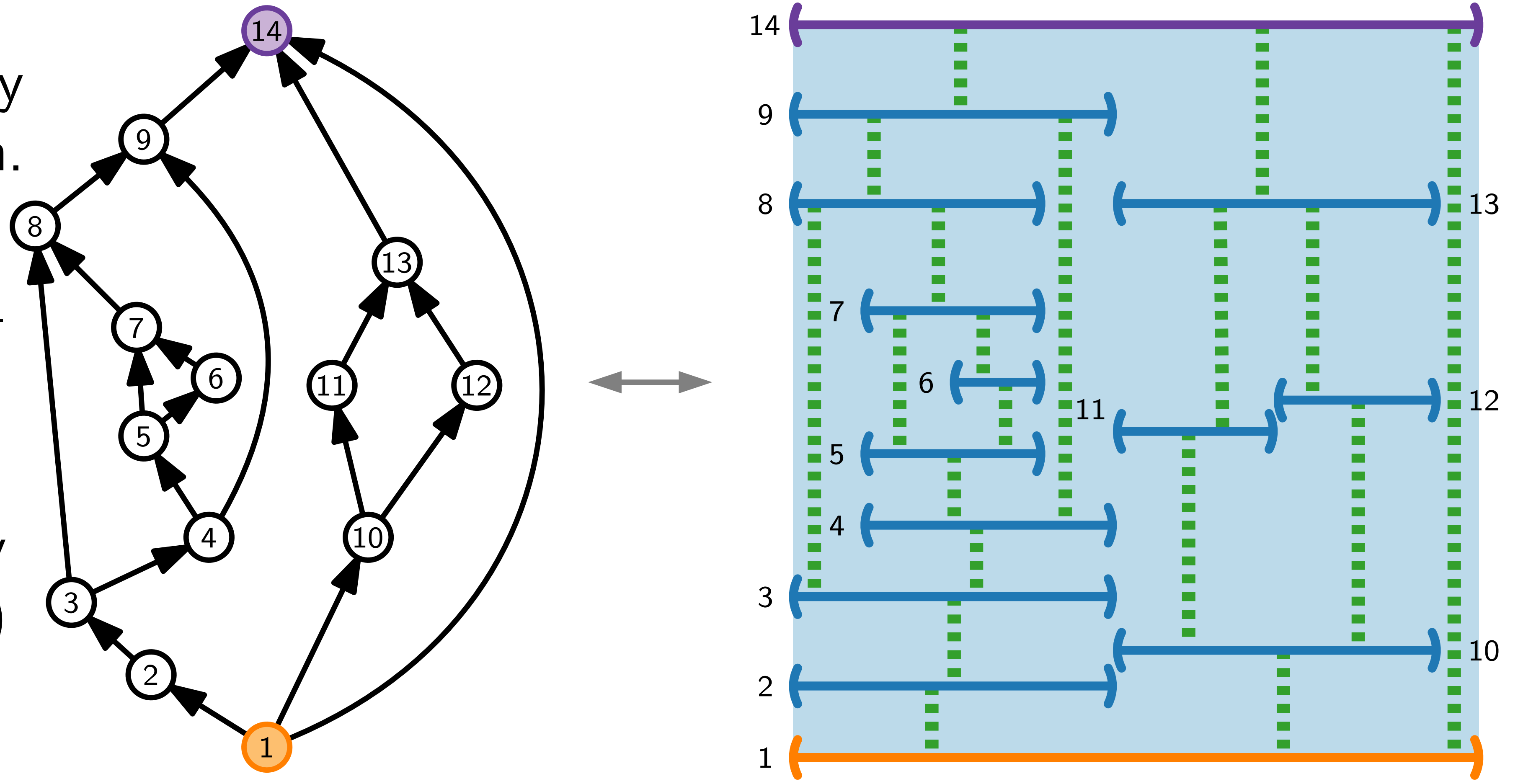

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Rectangular ε-bar visibility representation extension can be solved in $\mathcal{O}(n \log^2 n)$ time for st-graphs.

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Theorem 3. [Chaplick et al. '18]
ε-bar visibility representation extension is NP-complete for (series-parallel) st-graphs when restricted to the **integer grid** (or if any fixed $\varepsilon > 0$ is specified).

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- Dynamic program via SPQR-trees
- Easier version: $\mathcal{O}(n^2)$

Theorem 2. [Chaplick et al. '18]
ε-bar visibility representation extension is NP-complete.

- Reduction from Planar Monotone 3-SAT

Theorem 3. [Chaplick et al. '18]
ε-bar visibility representation extension is NP-complete for (series-parallel) st-graphs when restricted to the **integer grid** (or if any fixed $\varepsilon > 0$ is specified).

- Reduction from 3-Partition

SPQR-Tree

- An **SPQR-tree** T is a decomposition of a planar graph G by **separation pairs**.

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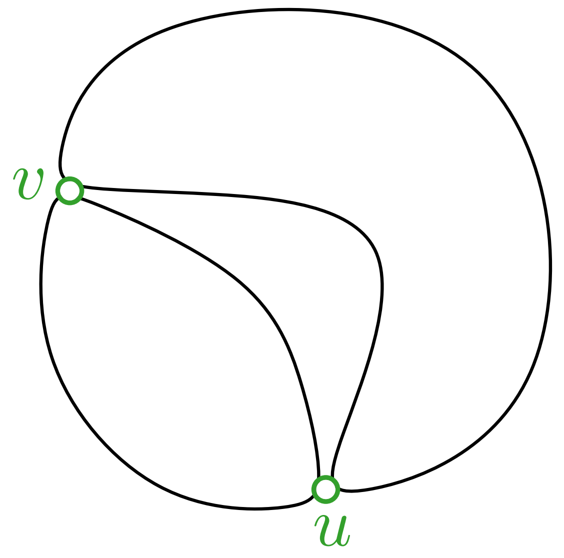

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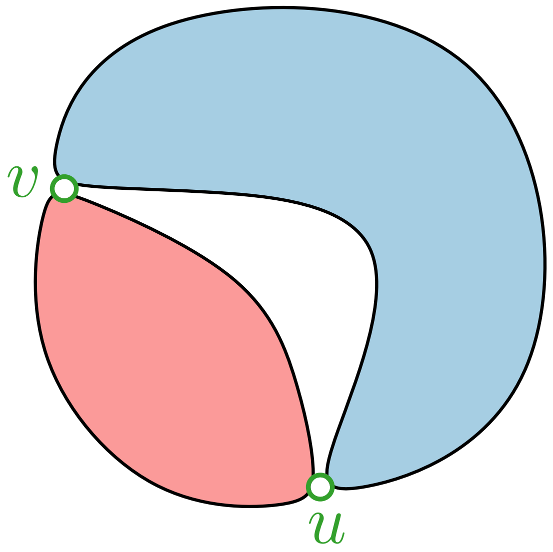

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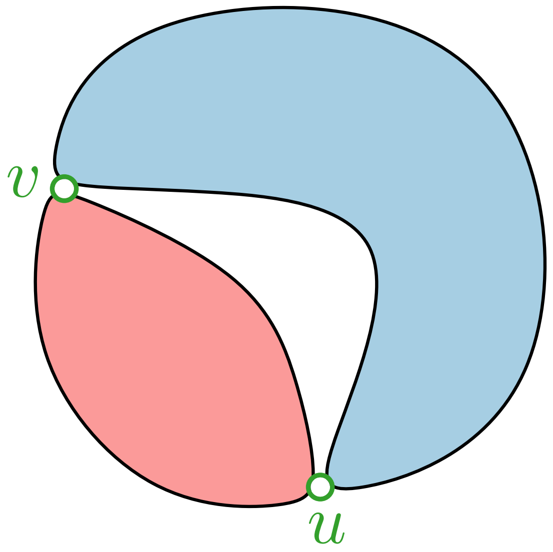

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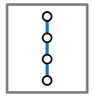

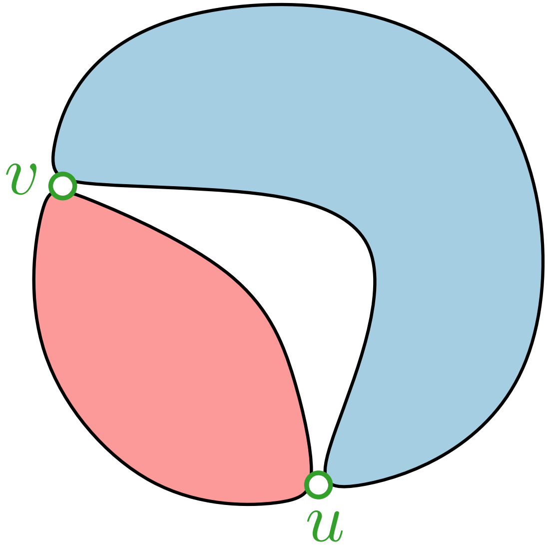

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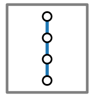
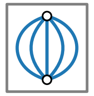
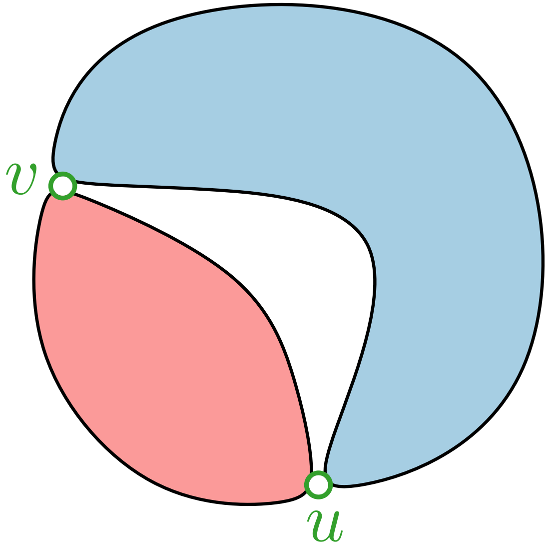
  - **S**-nodes represent a series composition 
  - **P**-nodes represent a parallel composition 



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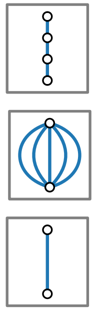

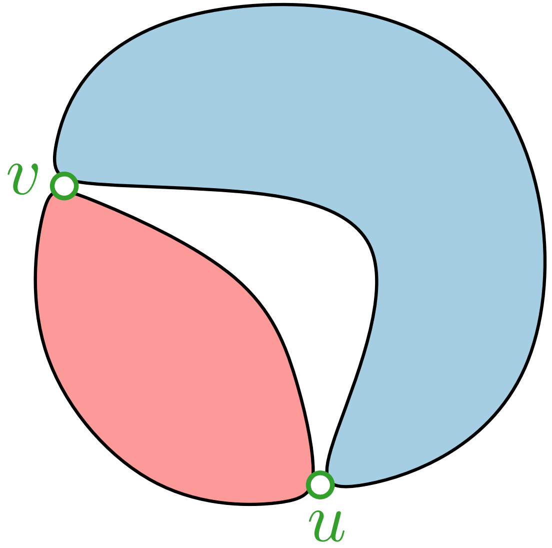

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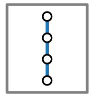
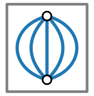
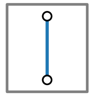
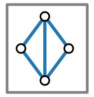
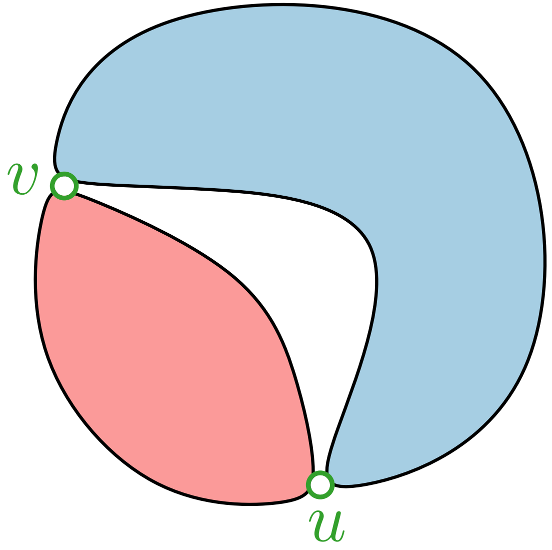
  - **S**-nodes represent a series composition 
  - **P**-nodes represent a parallel composition 
  - **Q**-nodes represent a single edge 
  - **R**-nodes represent 3-connected (*rigid*) subgraphs 



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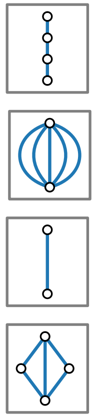

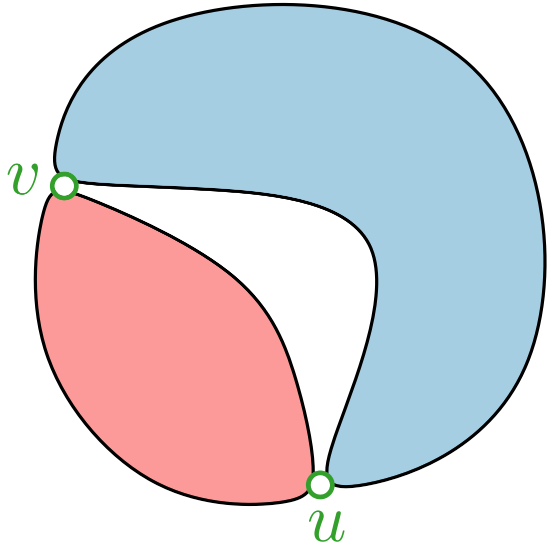

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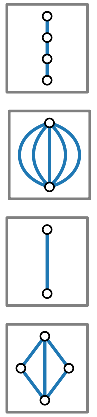

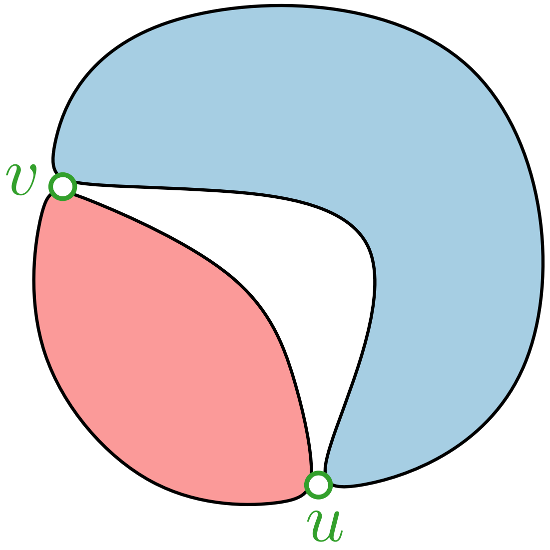

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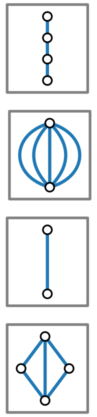

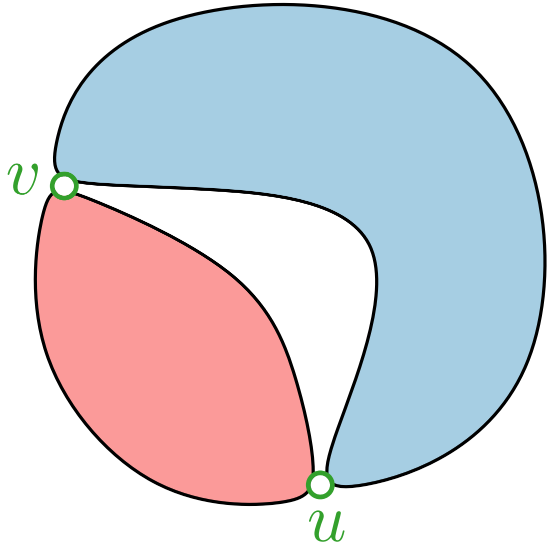

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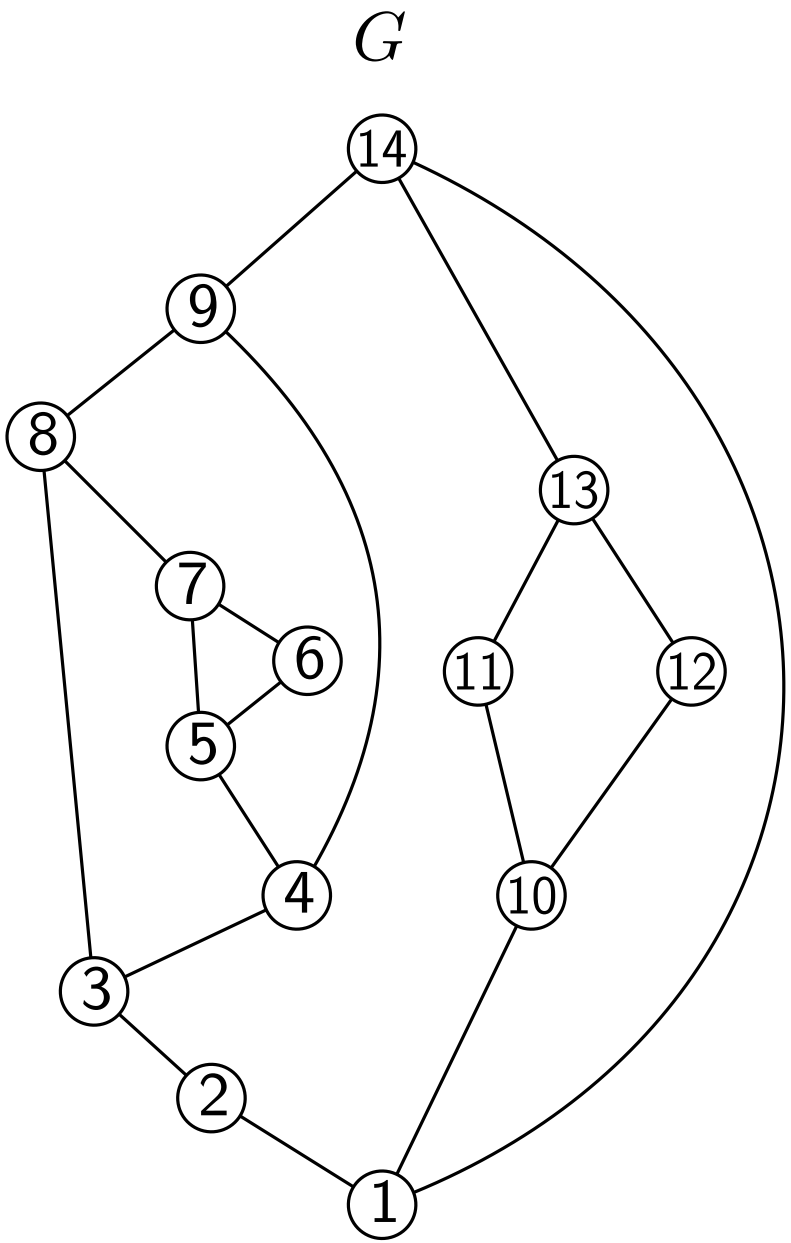
- T can be computed in $\mathcal{O}(n)$ time. [Gutwenger, Mutzel '01]

SPQR-Tree Example



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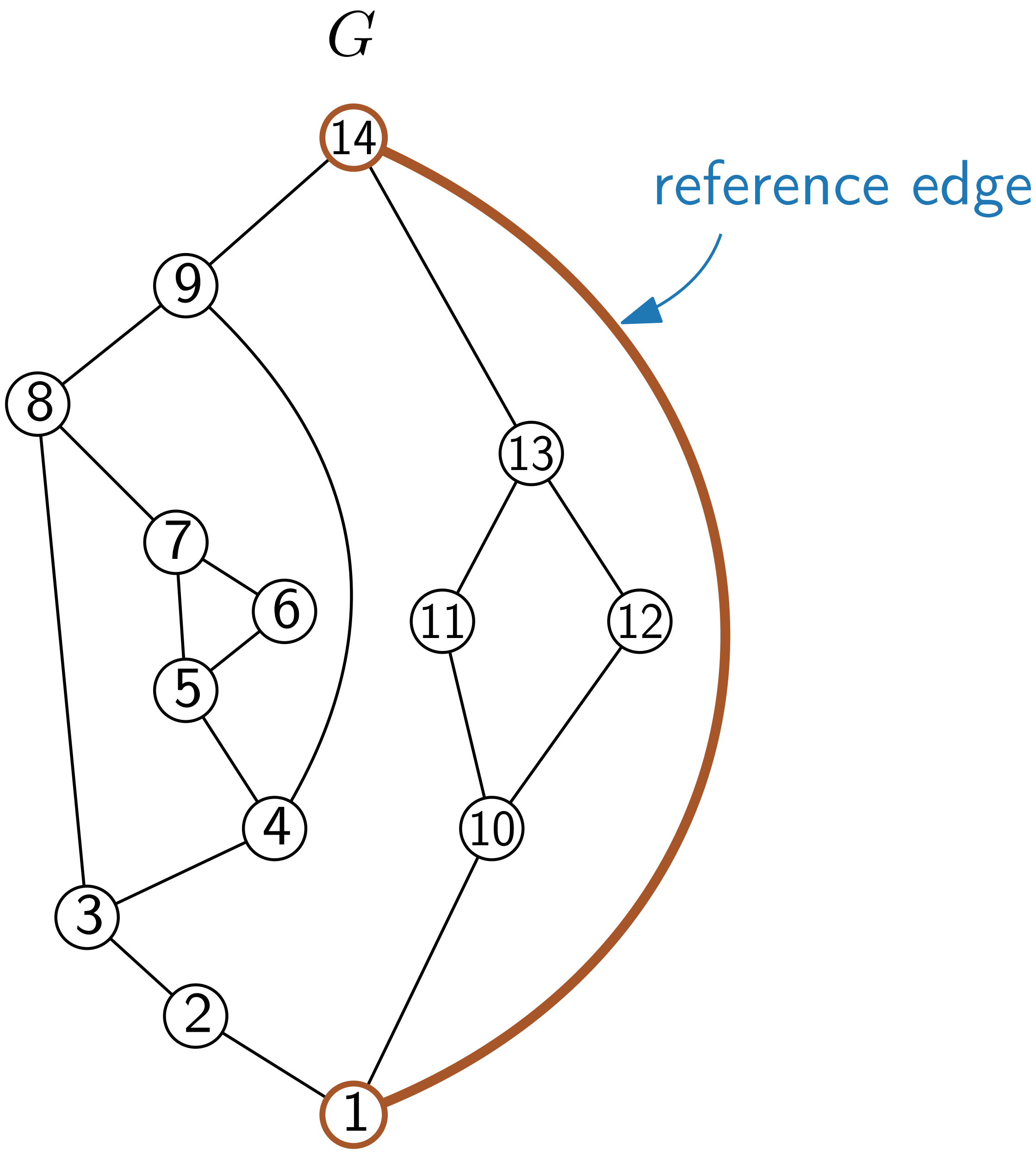

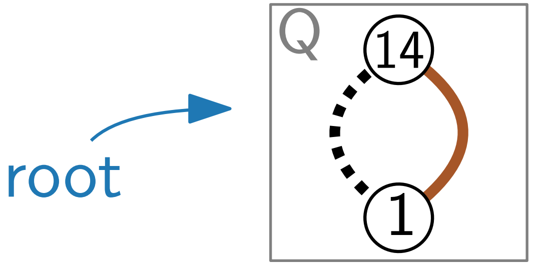

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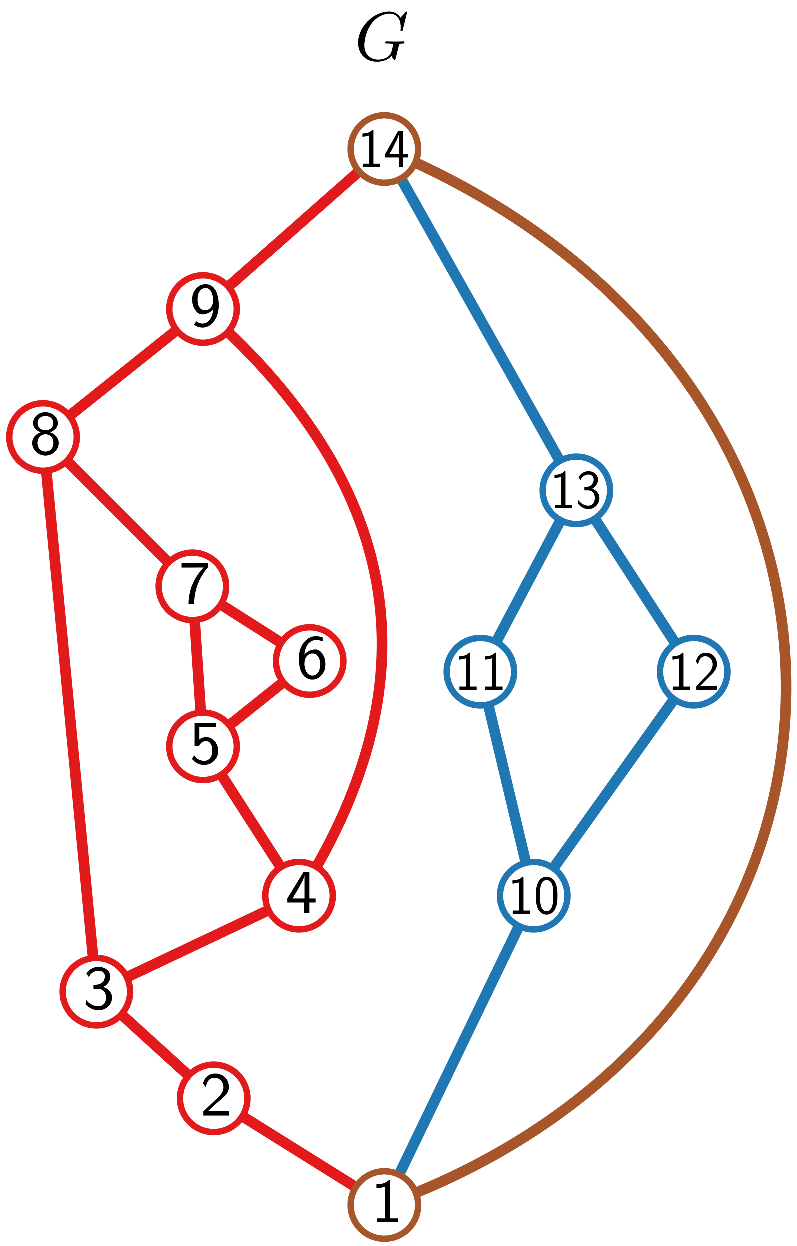

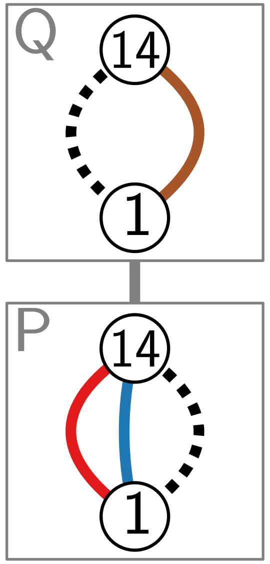

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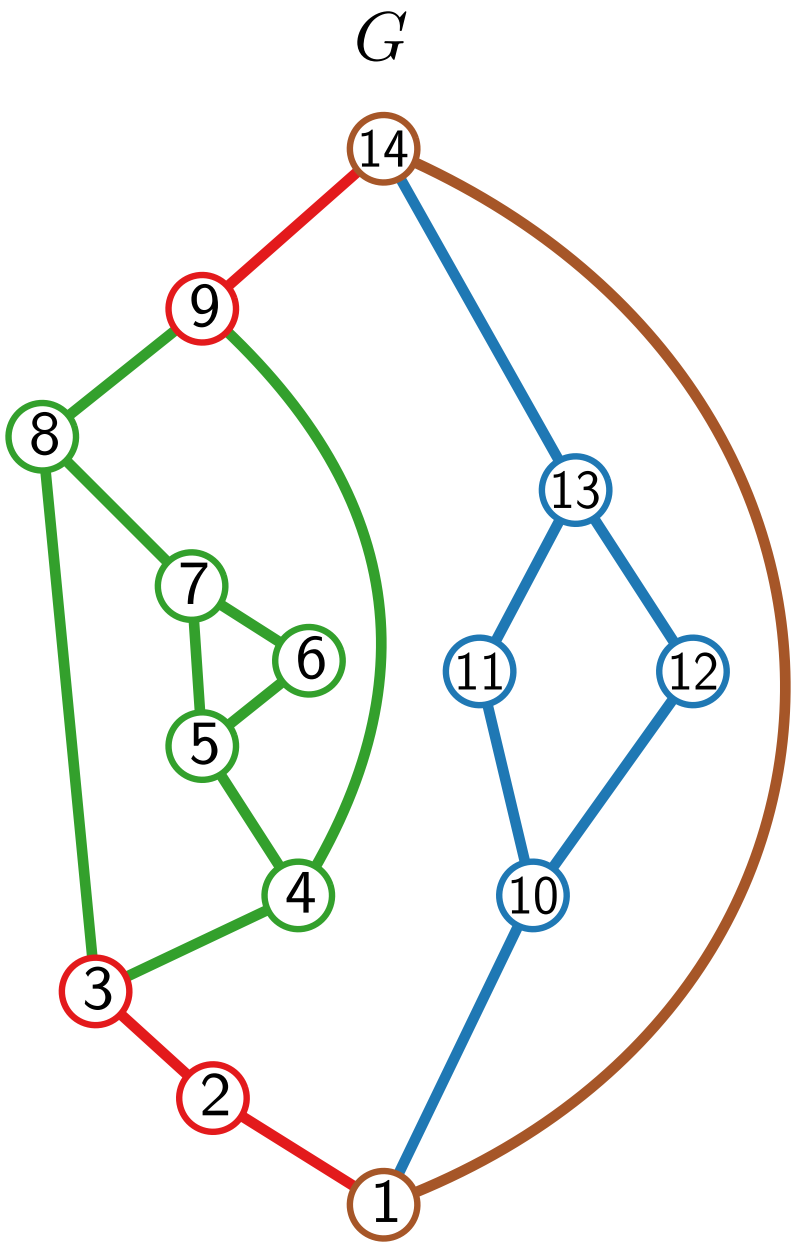

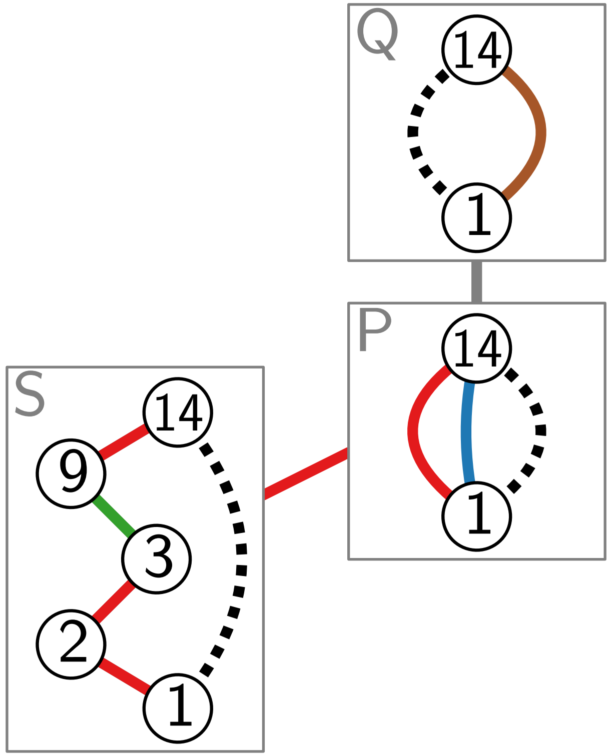

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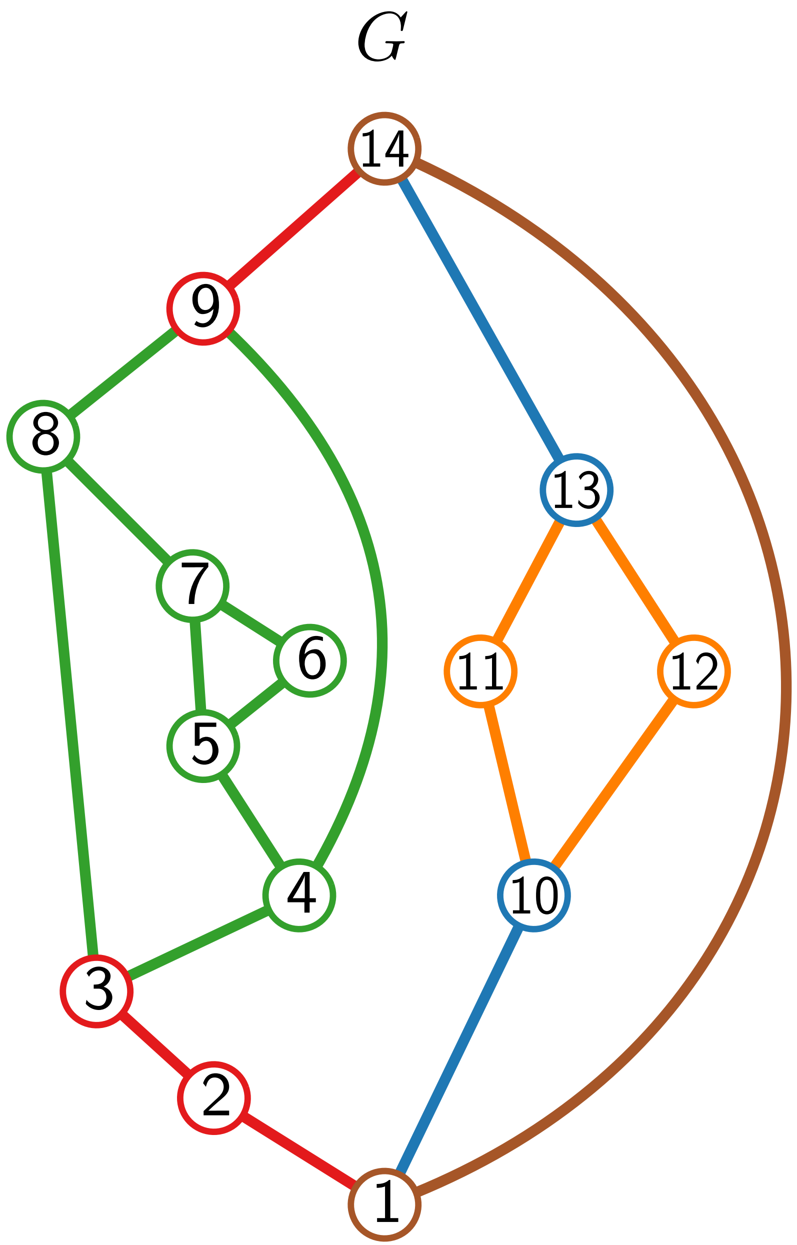

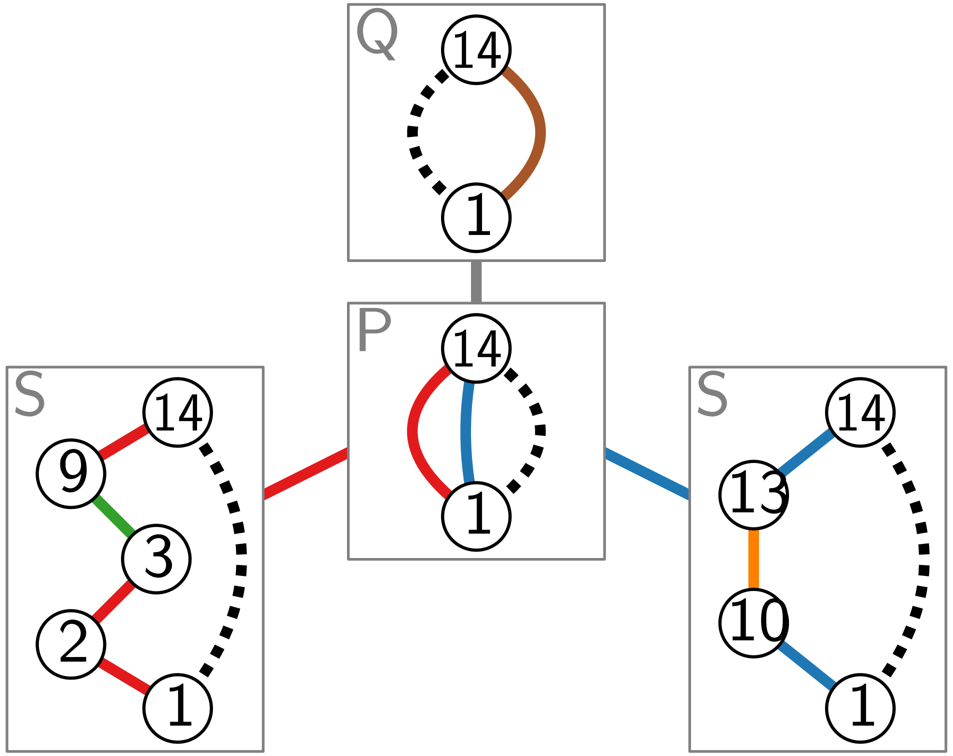

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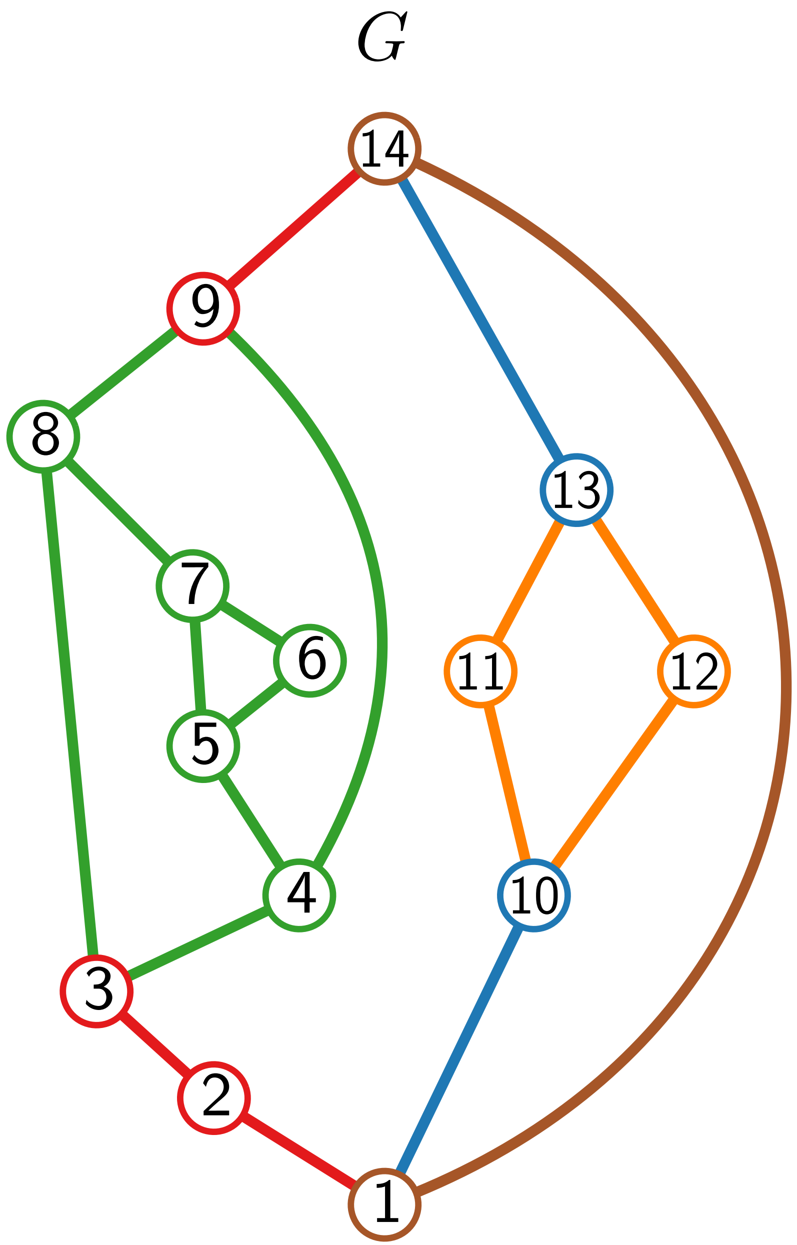

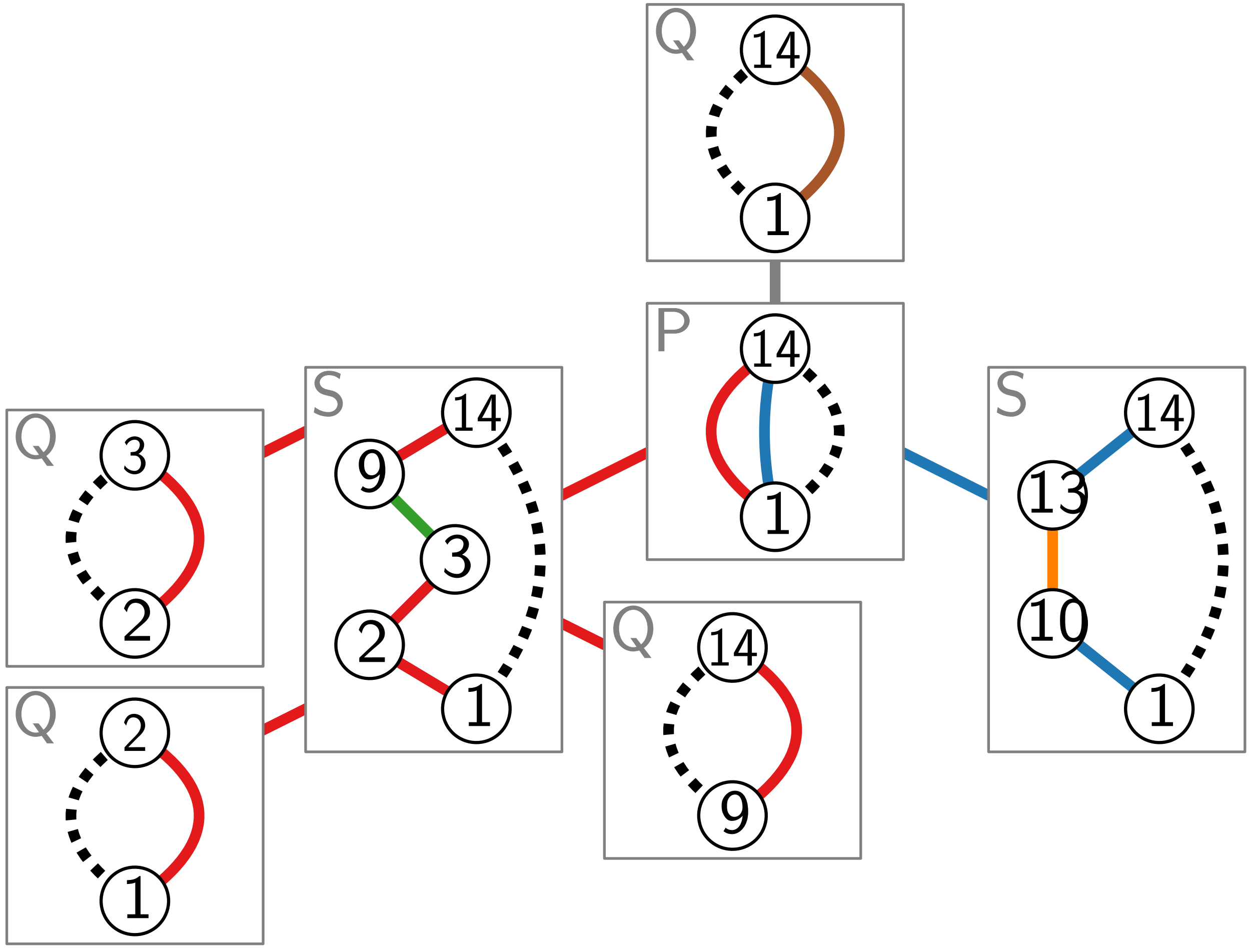

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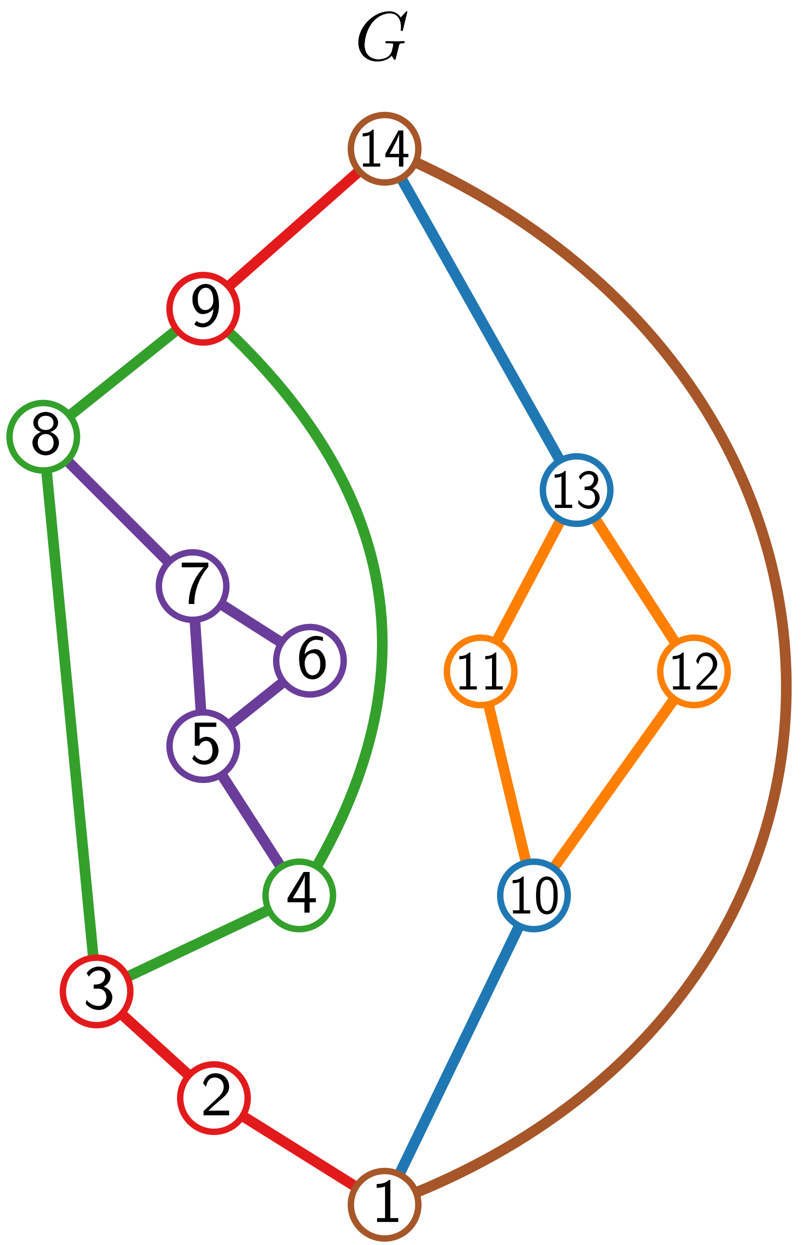

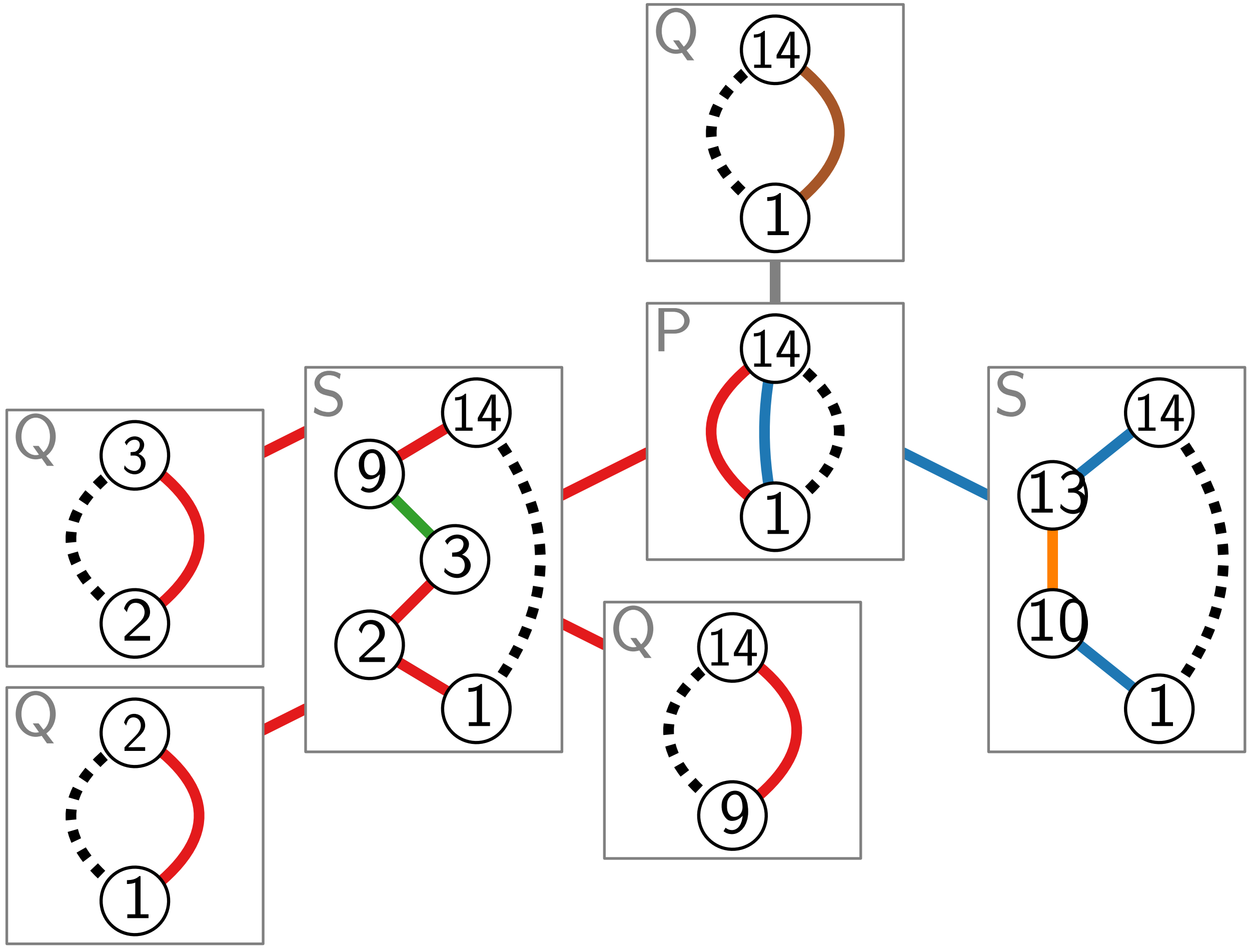

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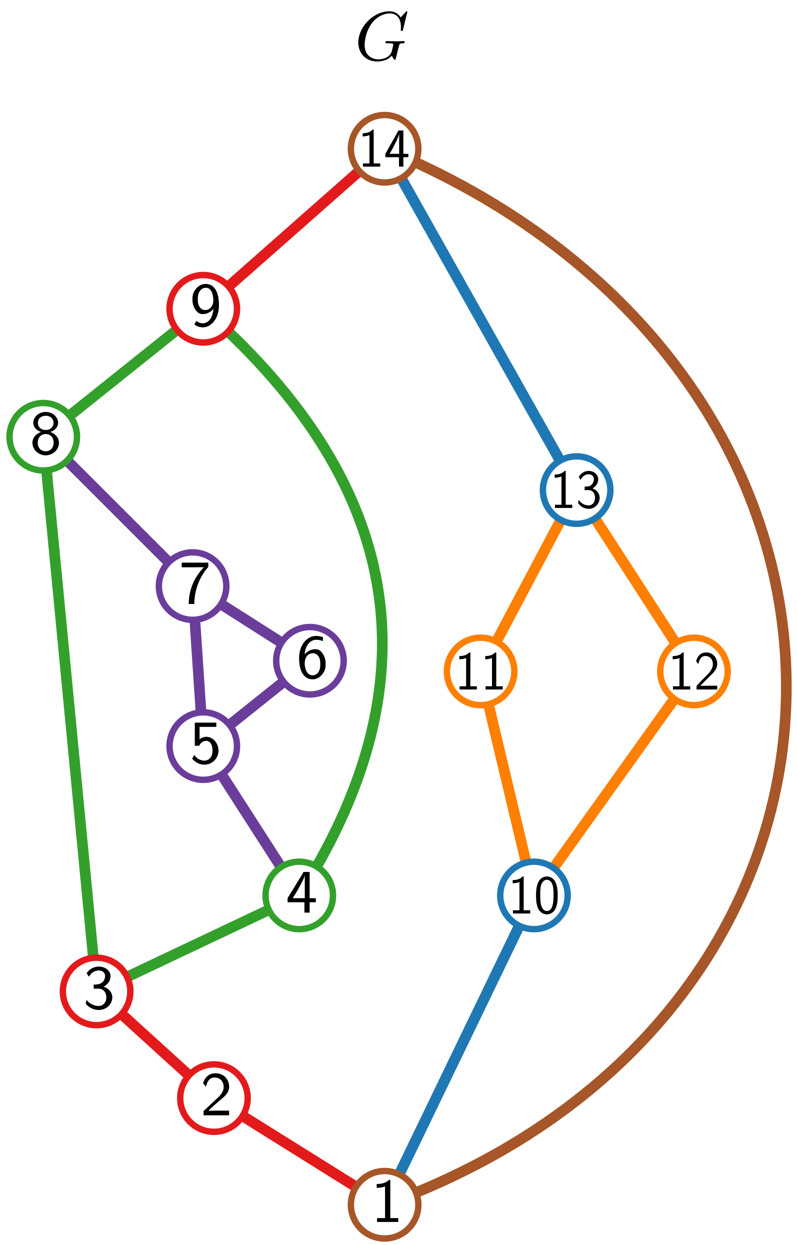

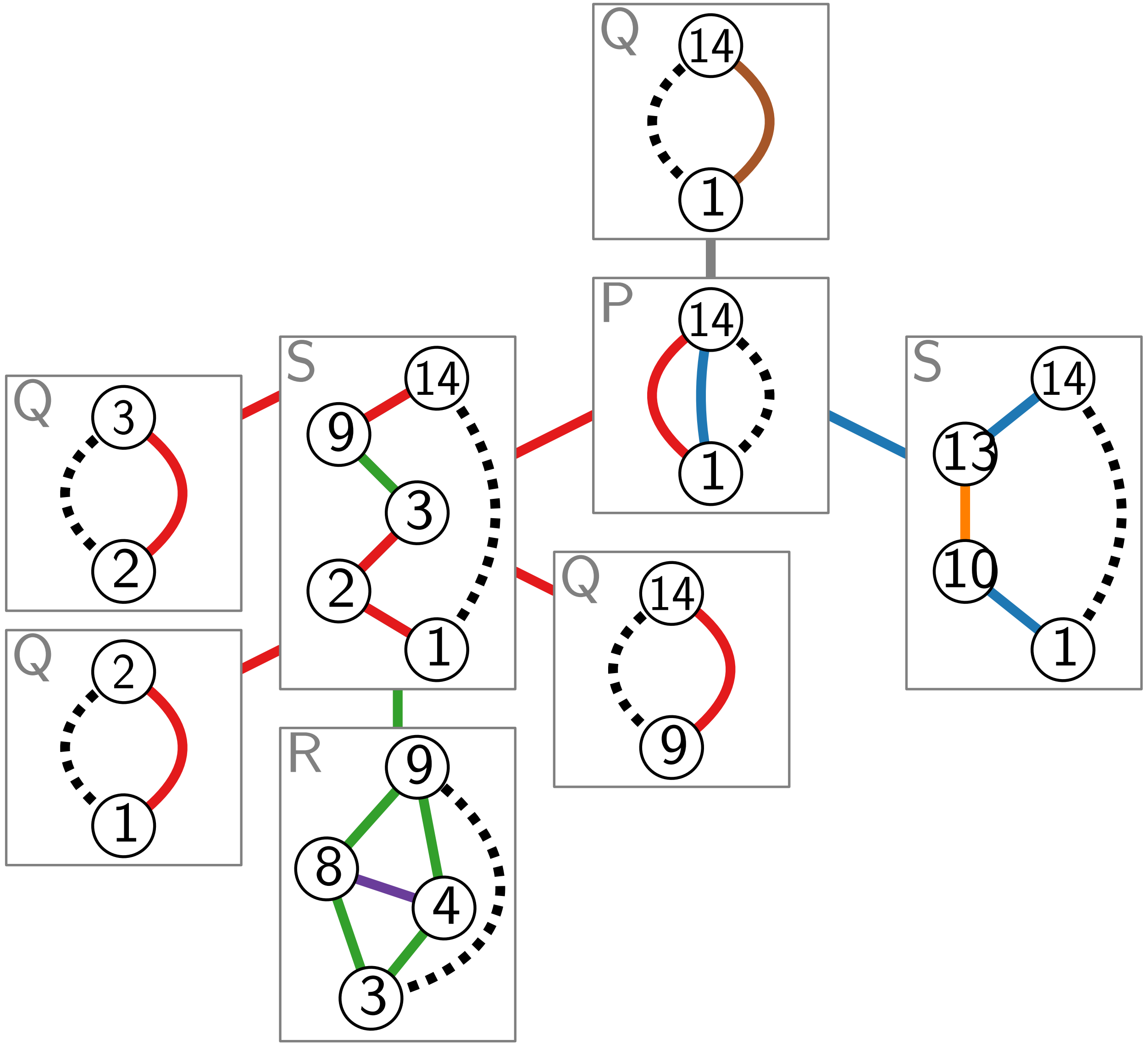

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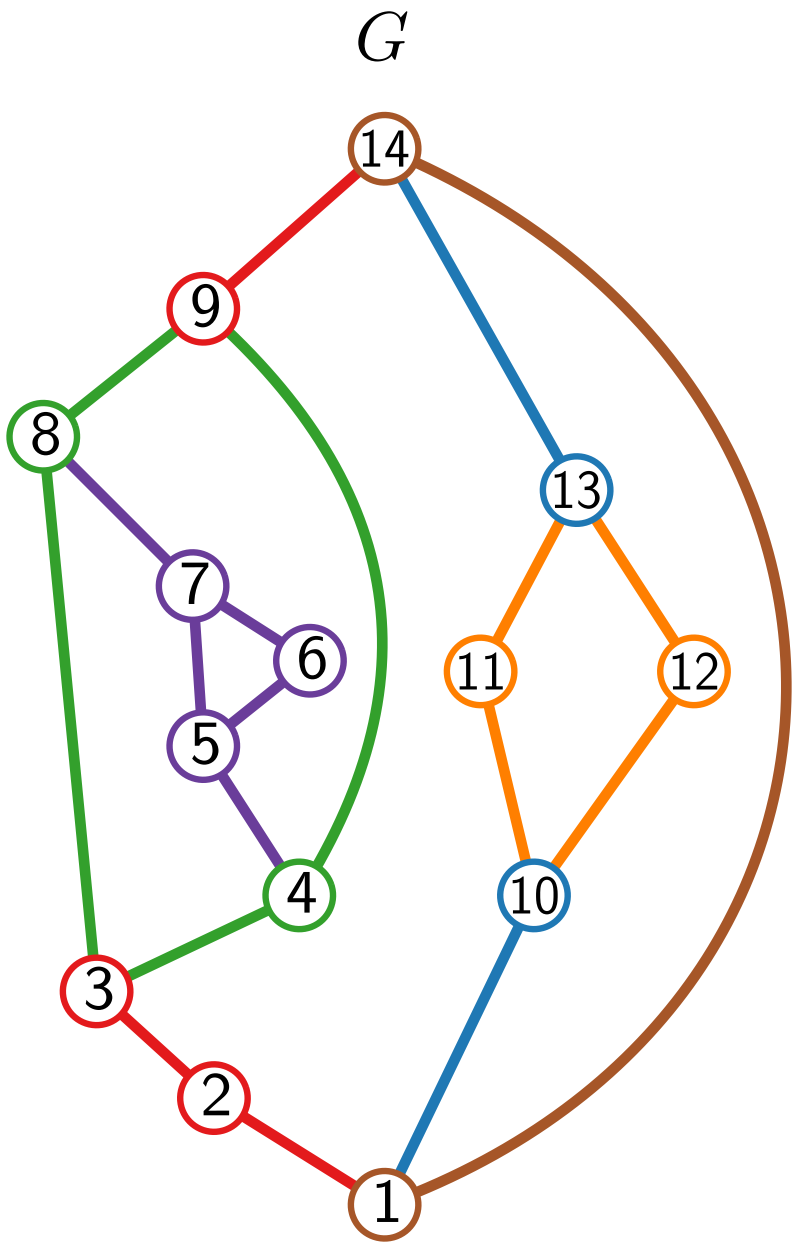

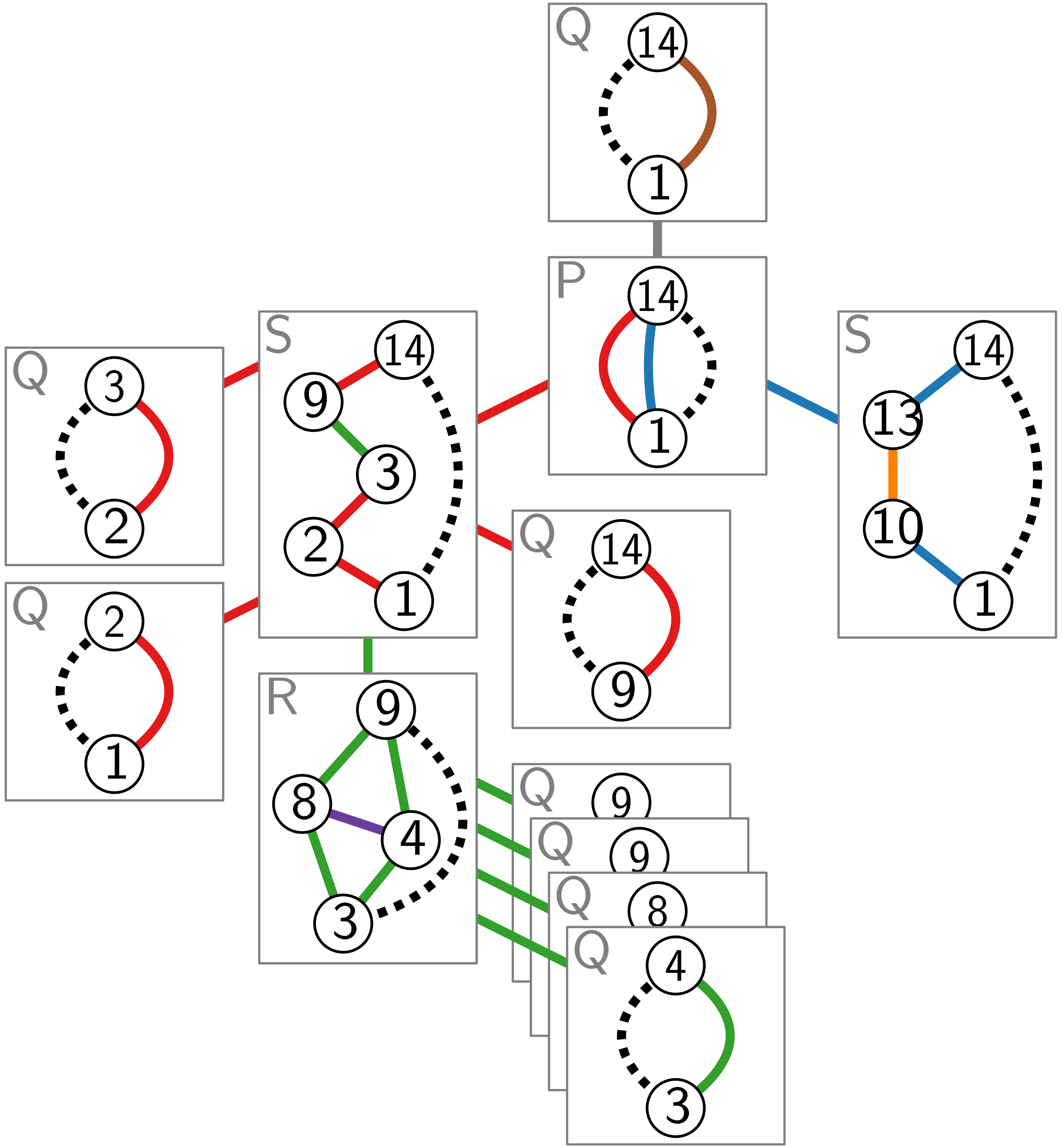

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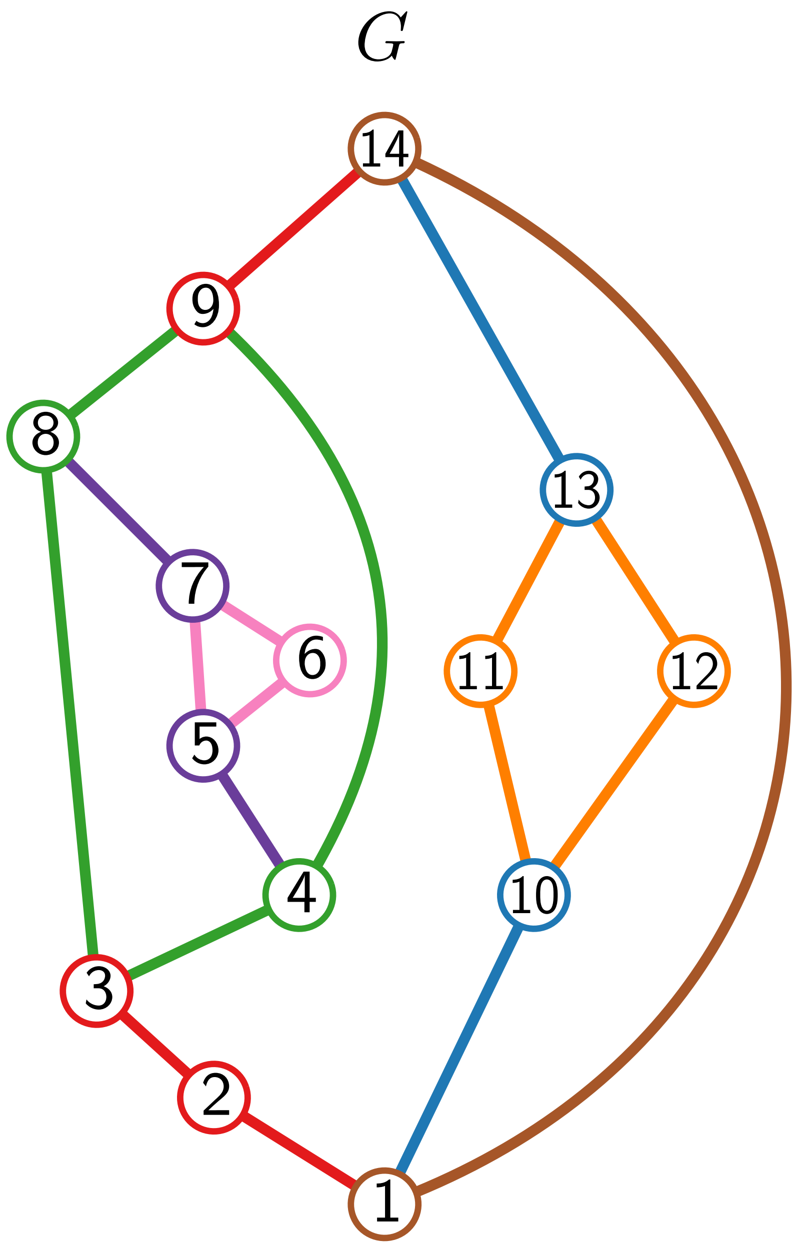

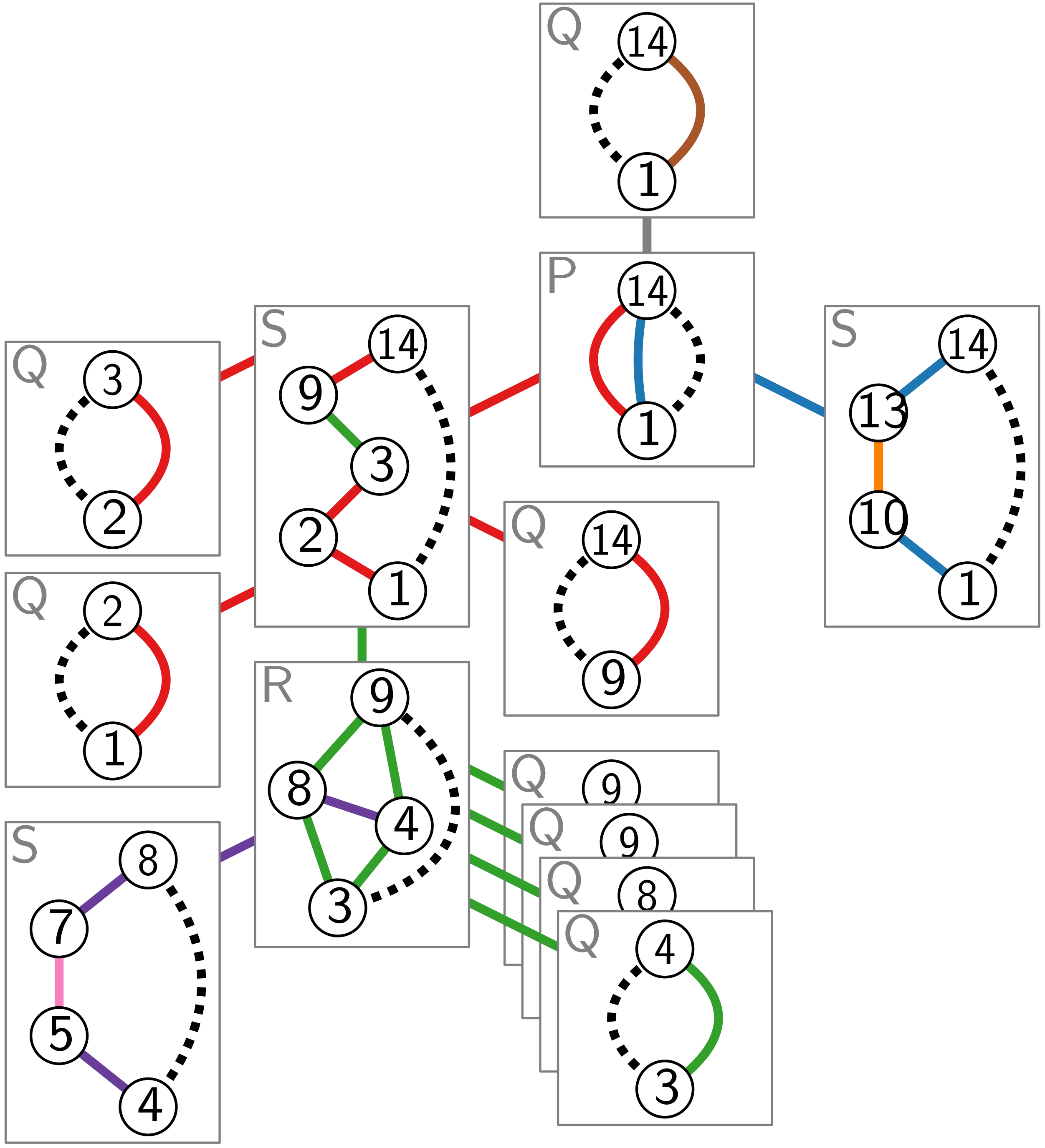

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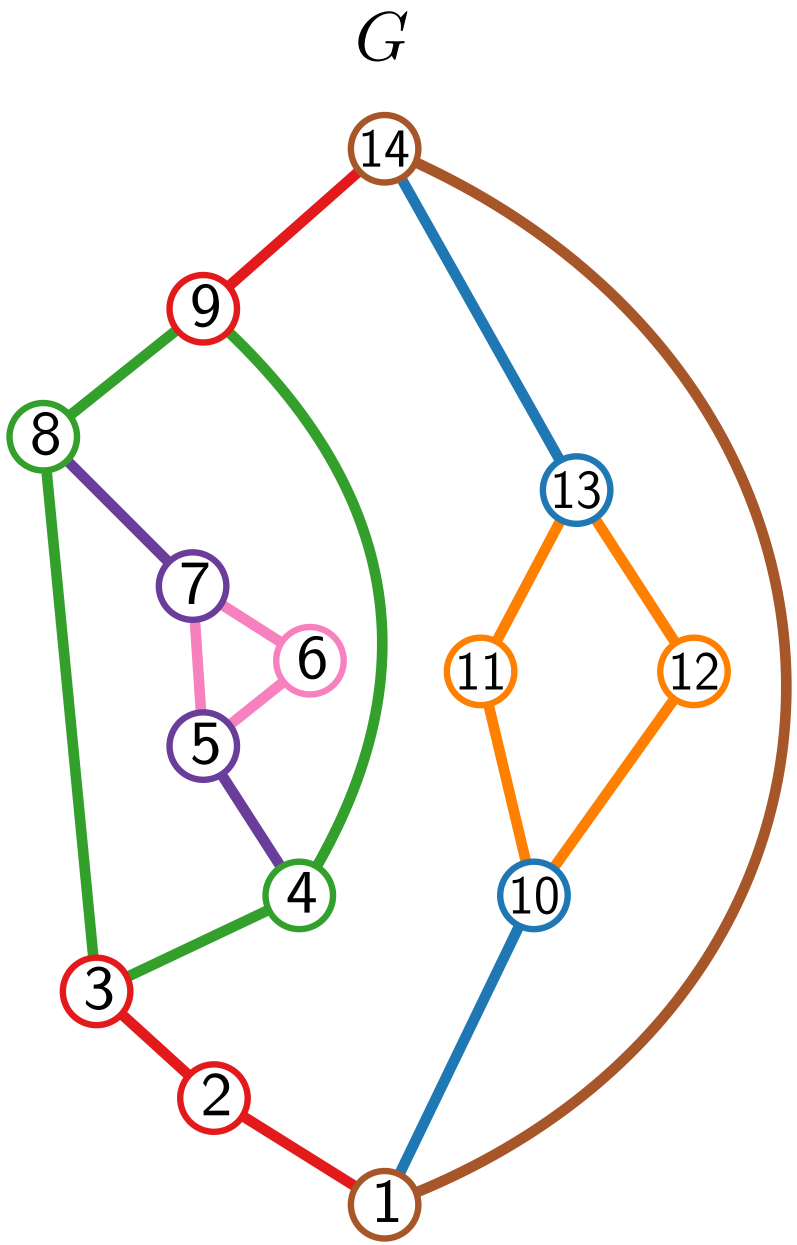

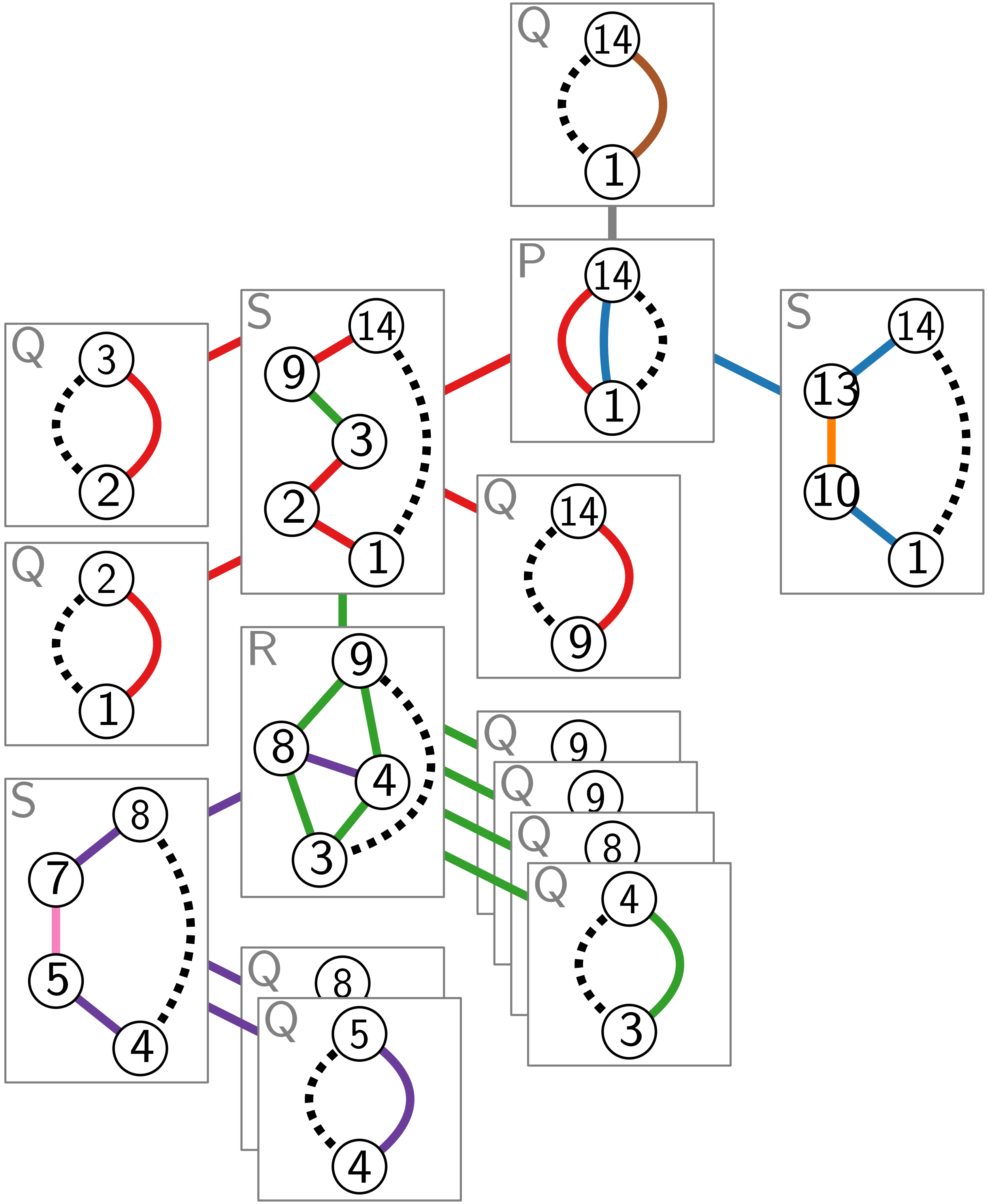

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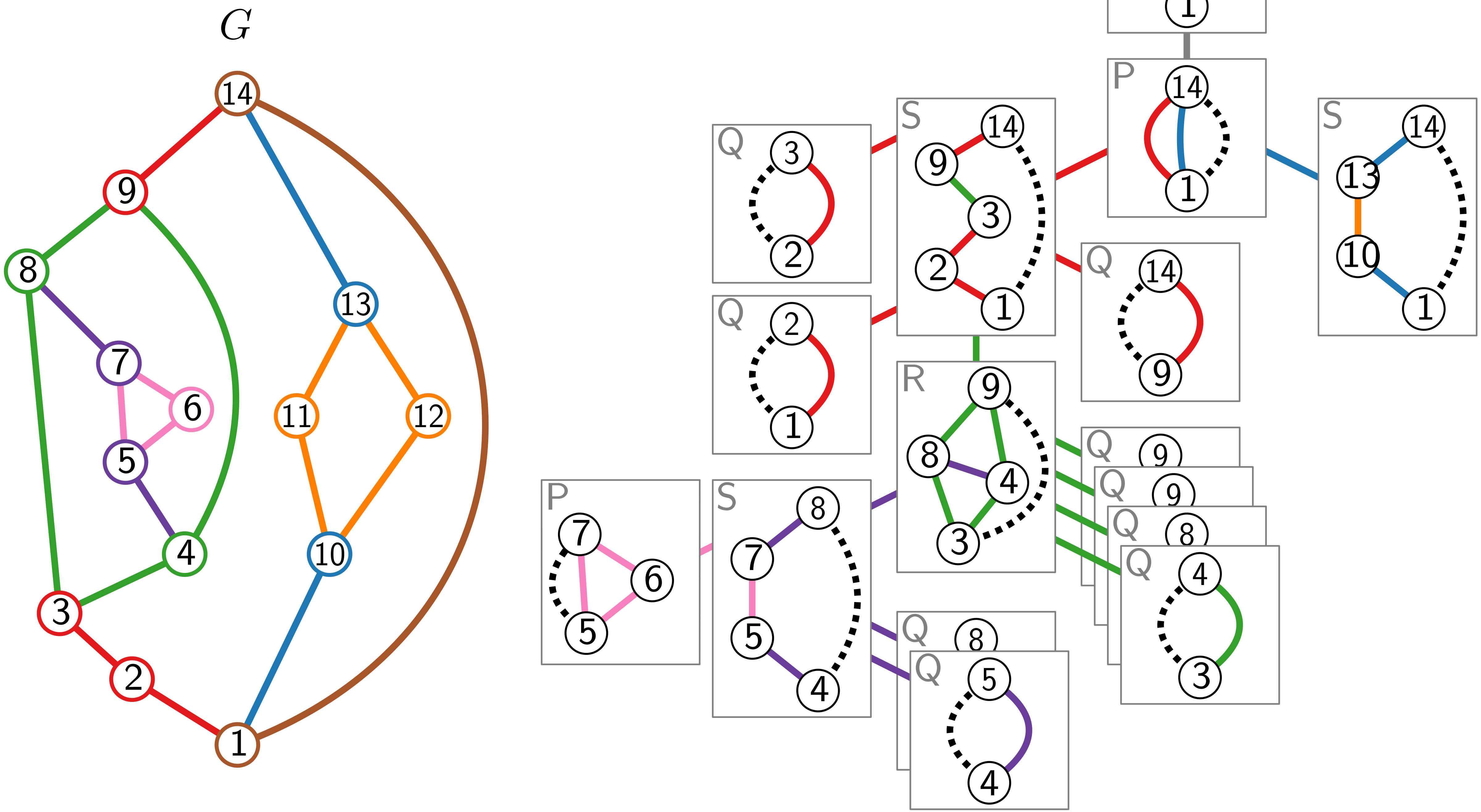

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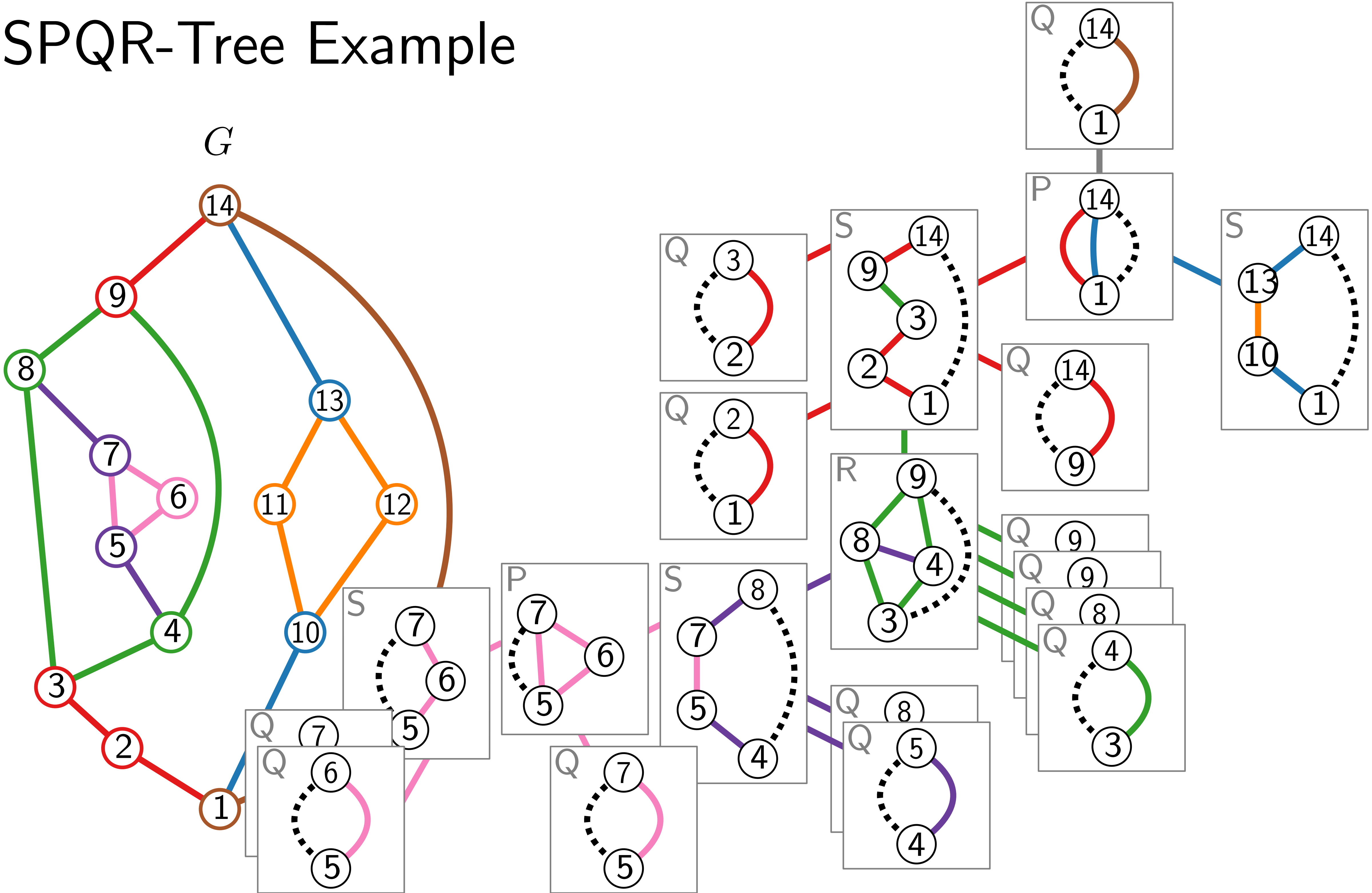

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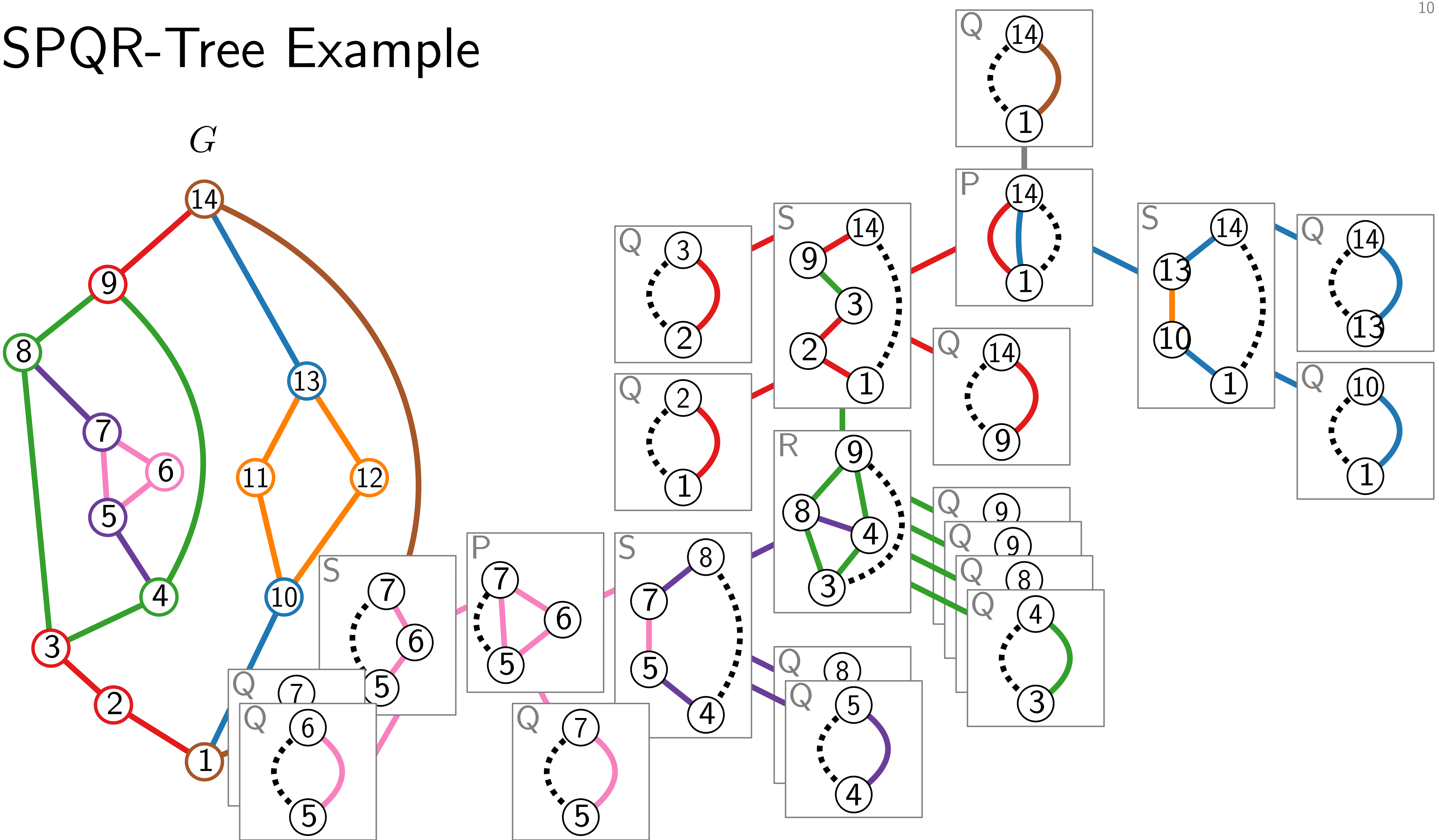

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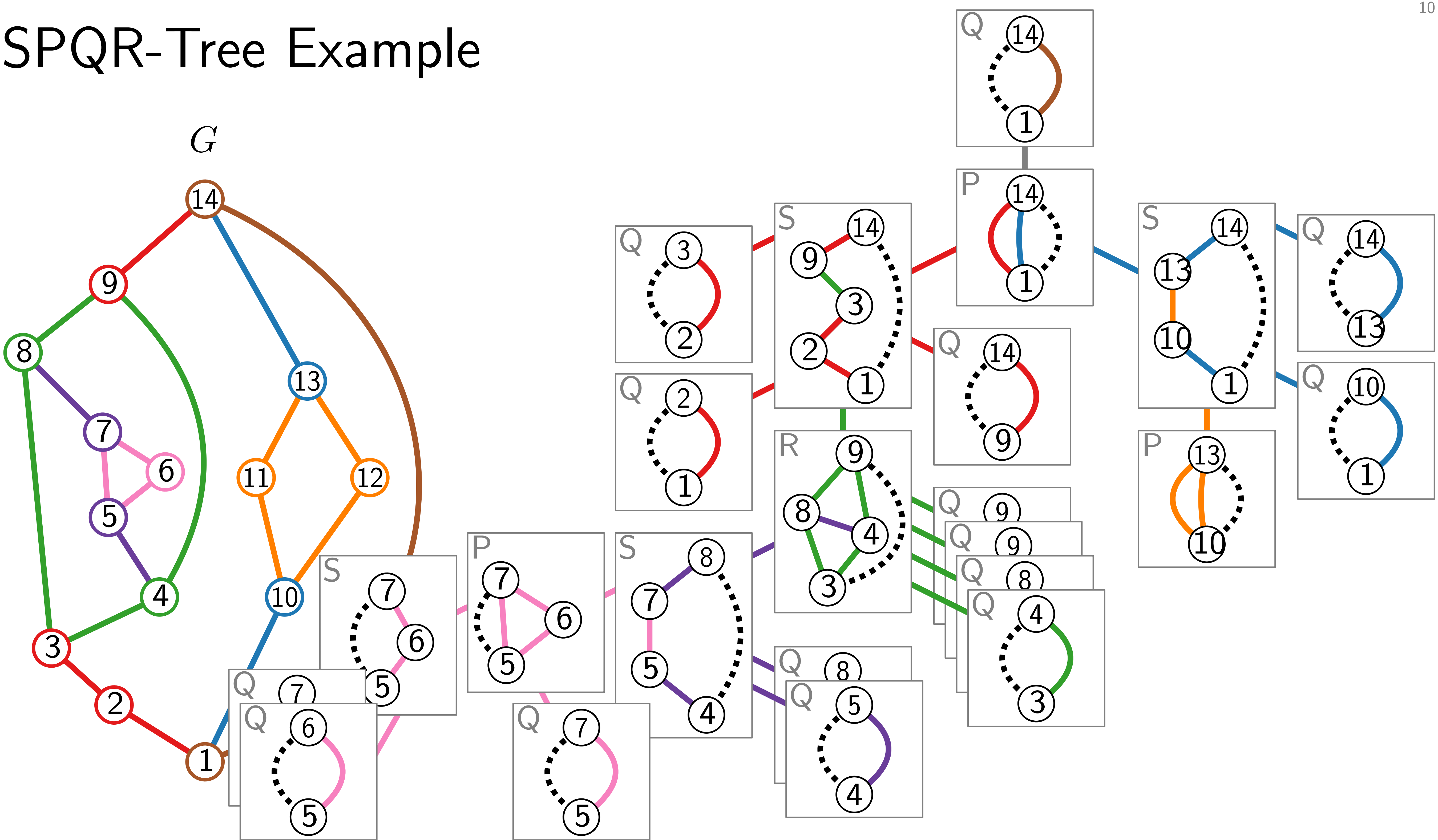

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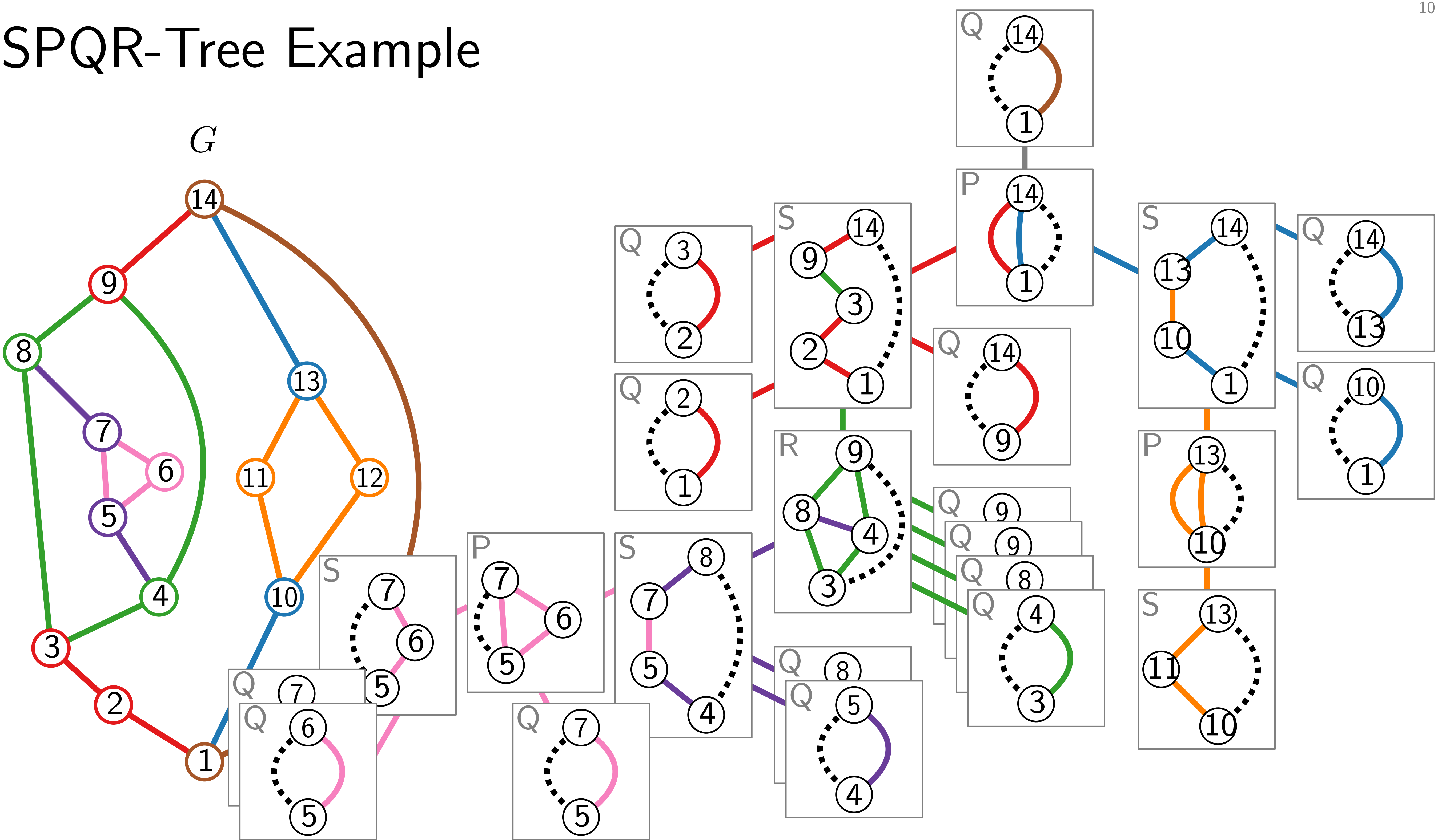

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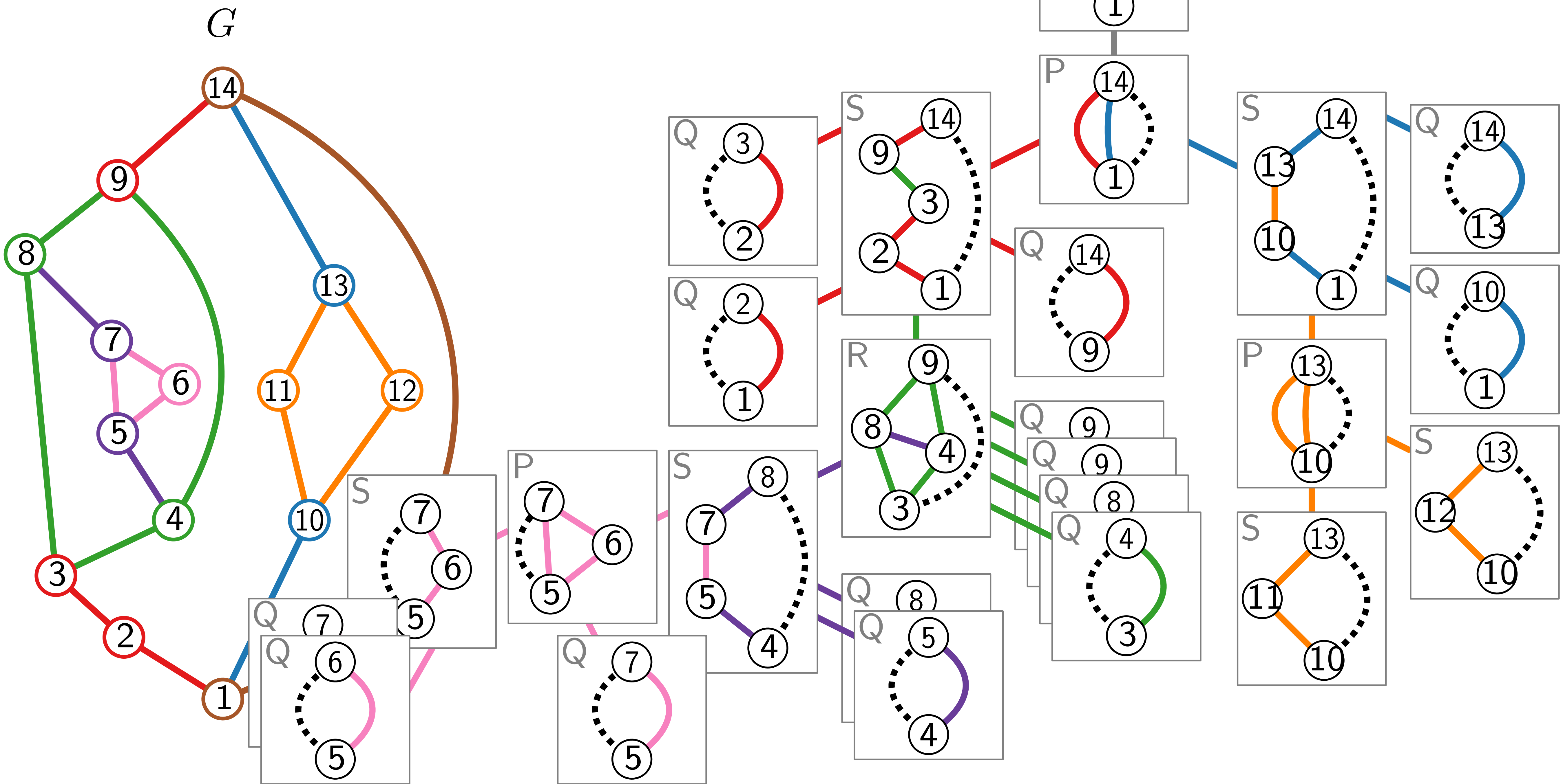

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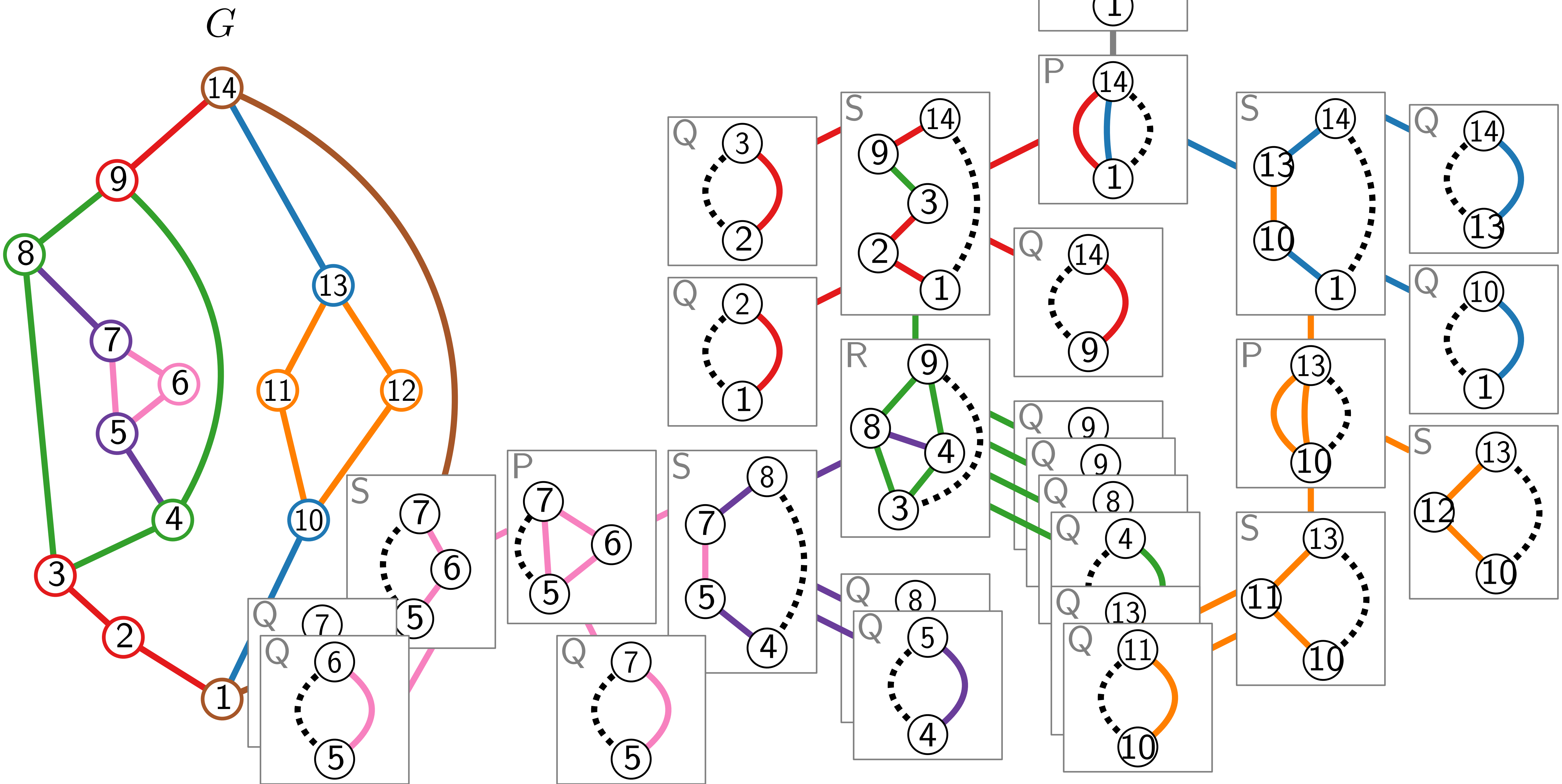

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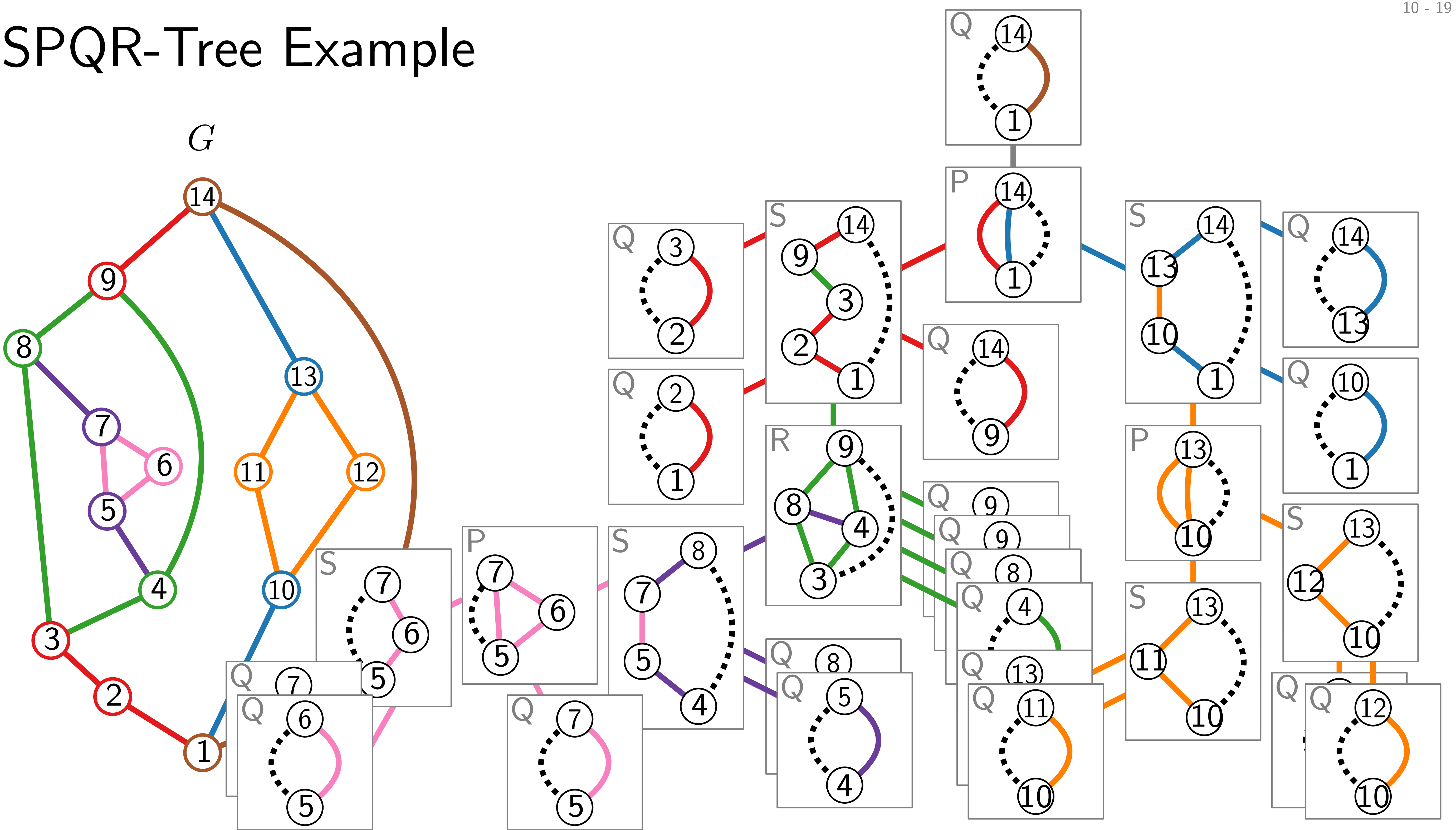
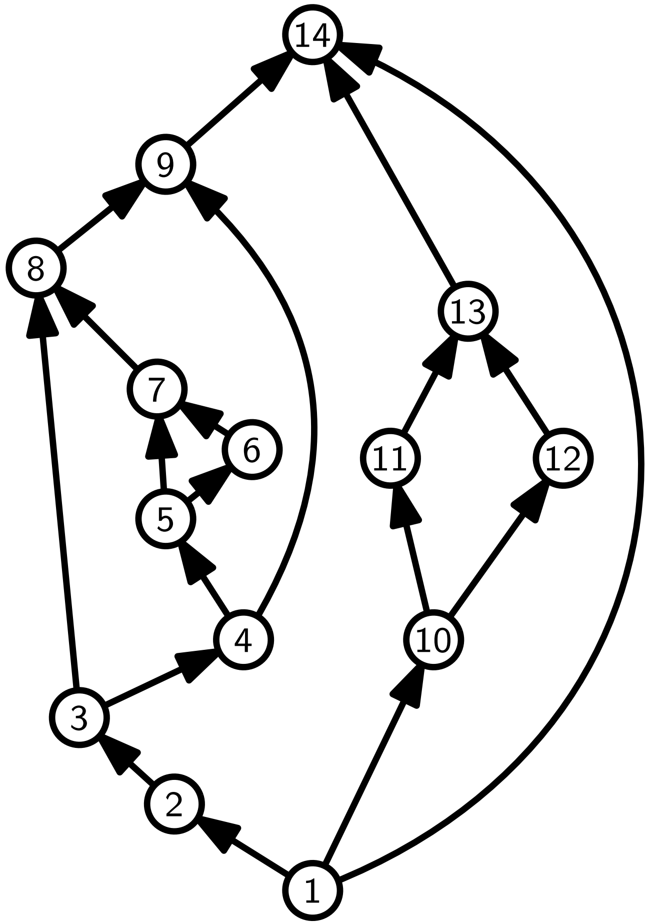


Representation Extension for st-Graphs

Theorem 1'.
Rectangular ε-bar visibility representation extension can be solved in $\mathcal{O}(n^2)$ time for st-graphs.



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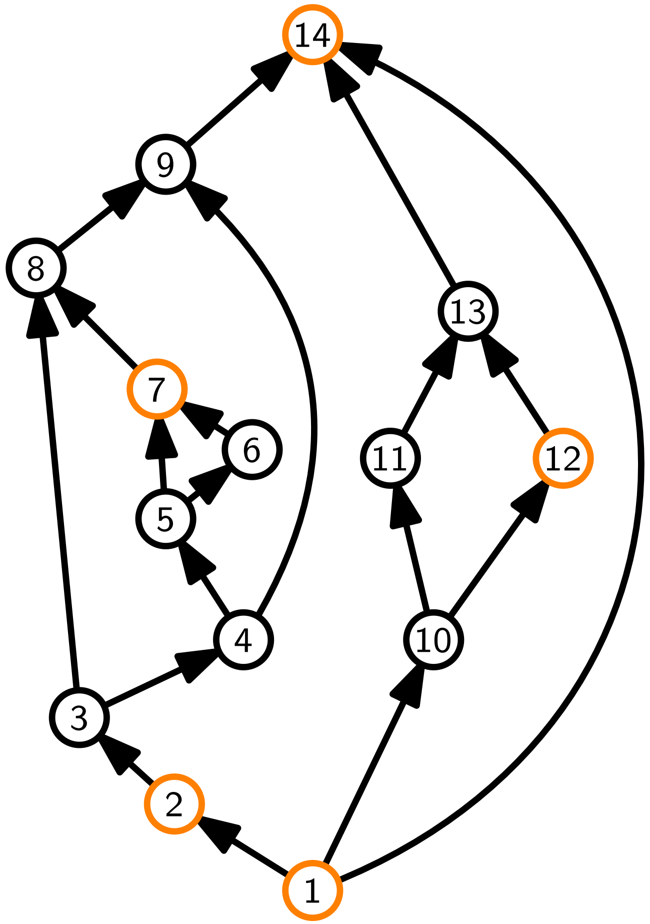

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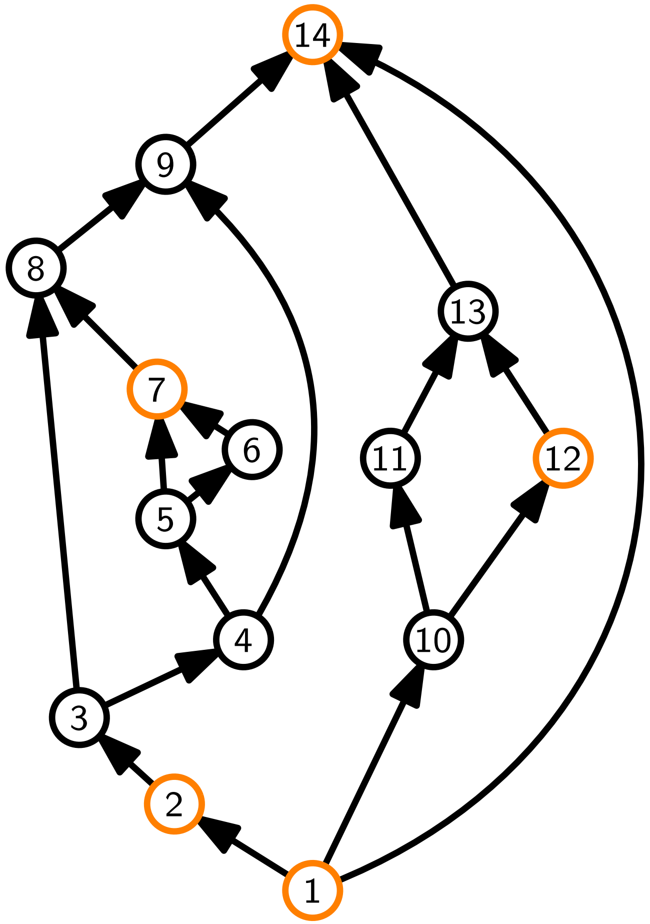

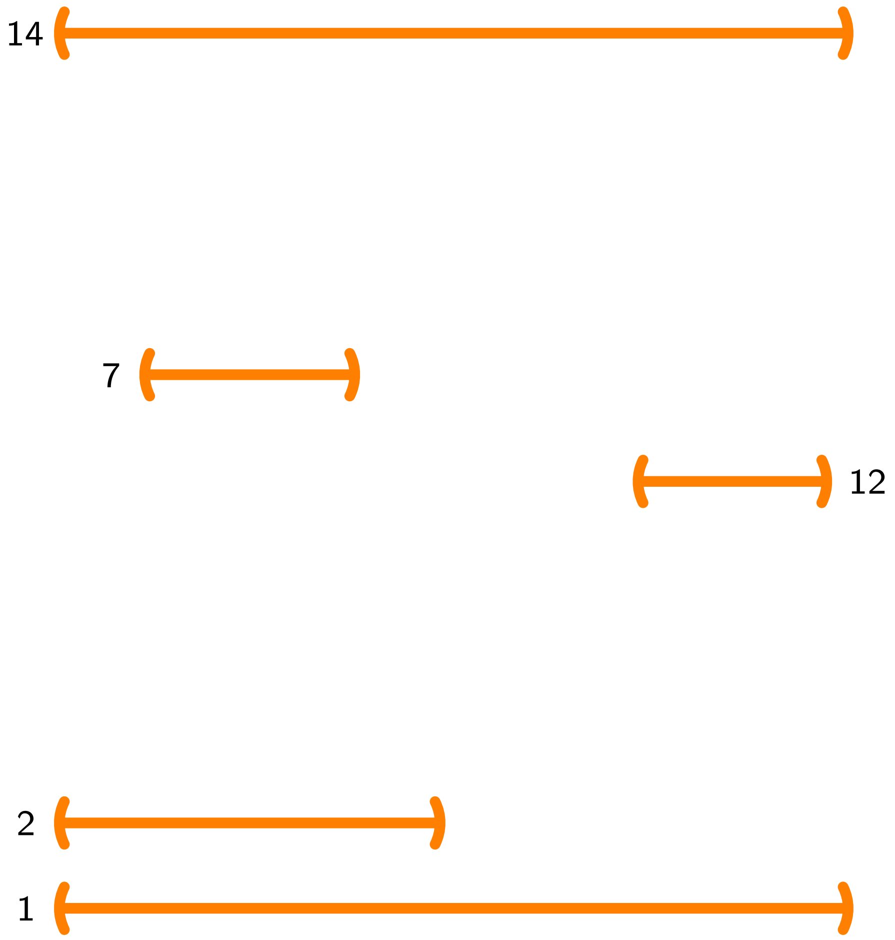

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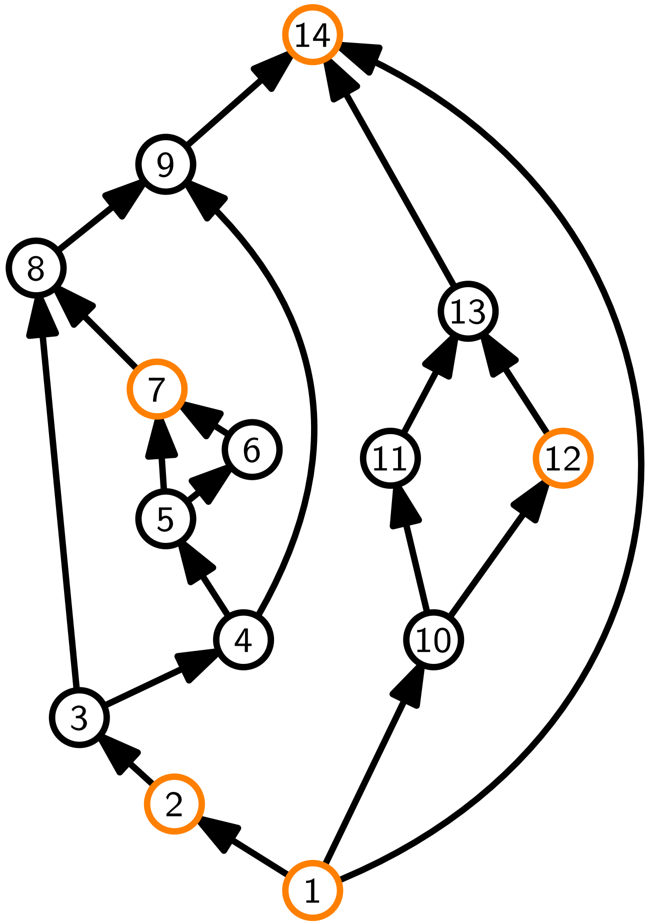

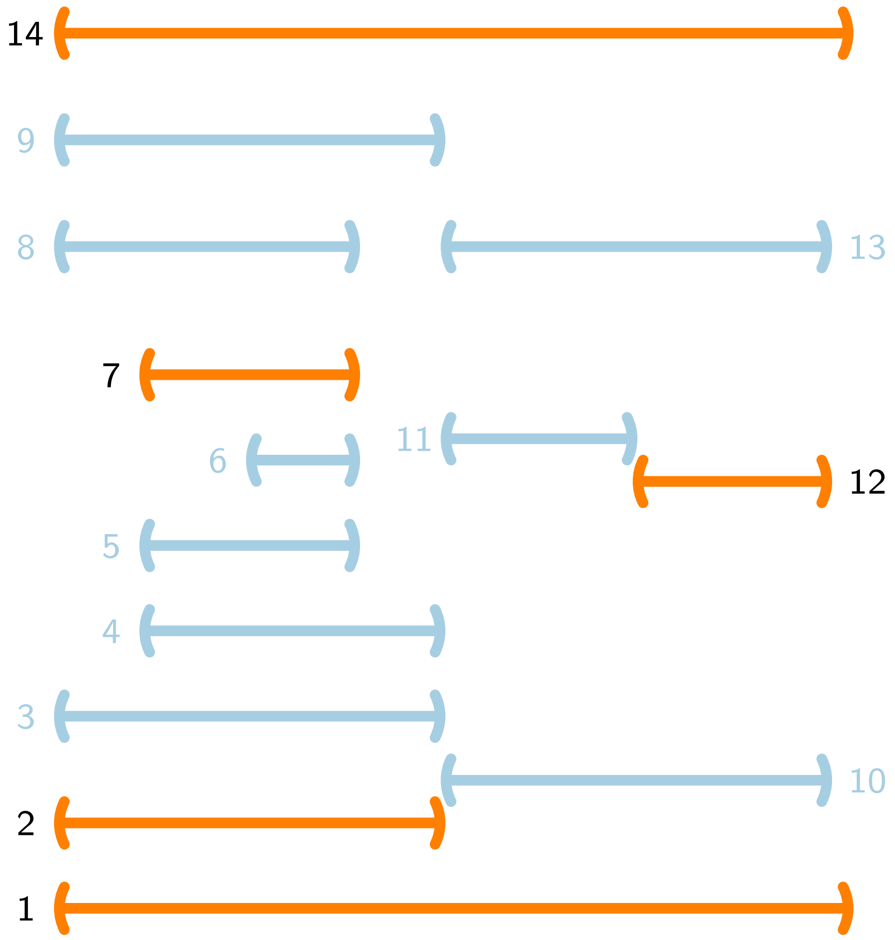

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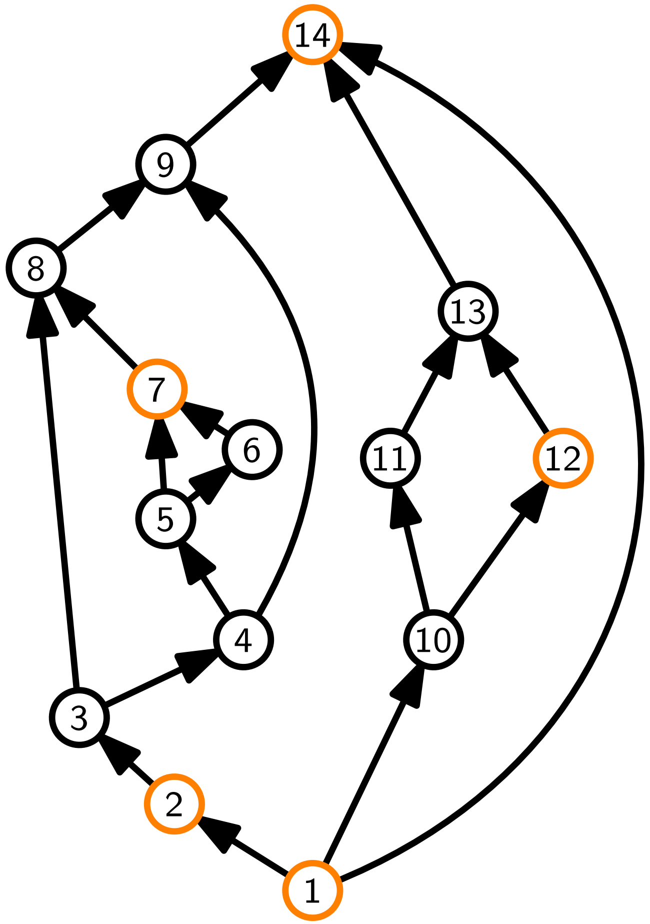

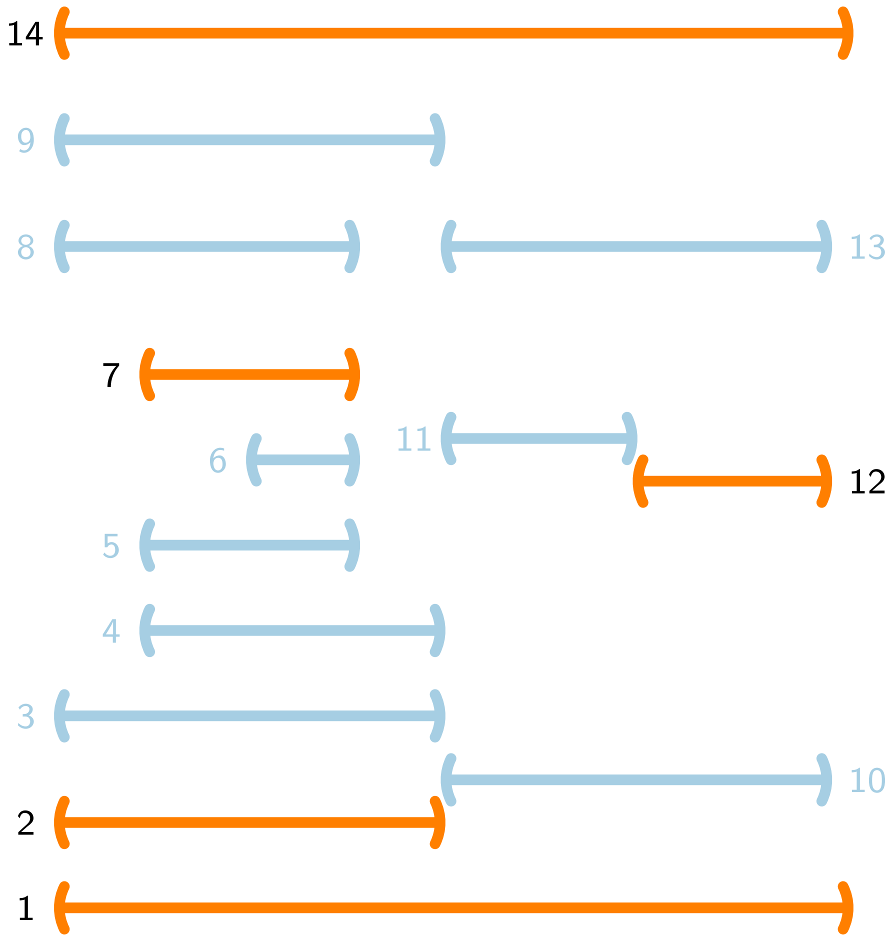

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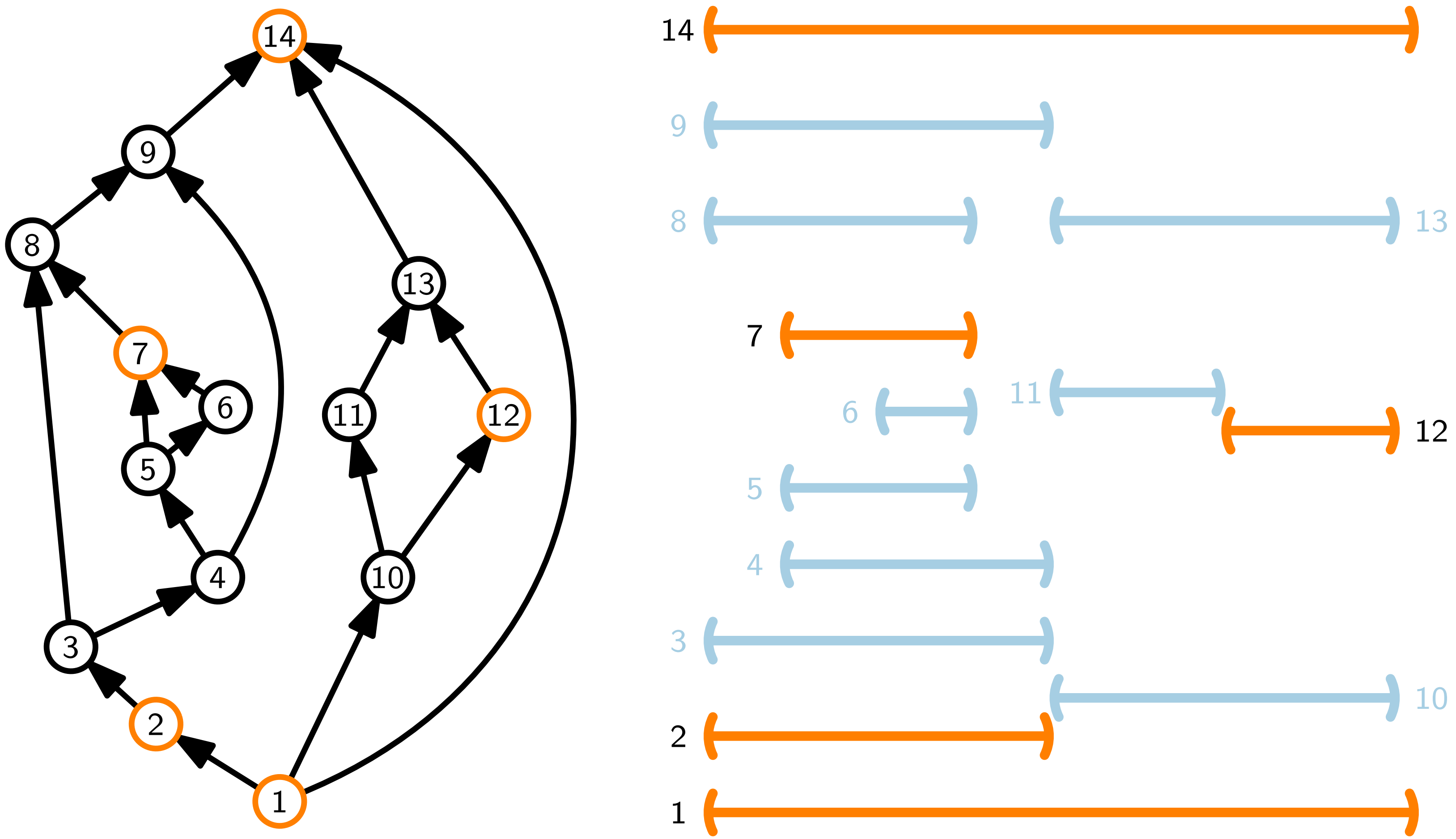

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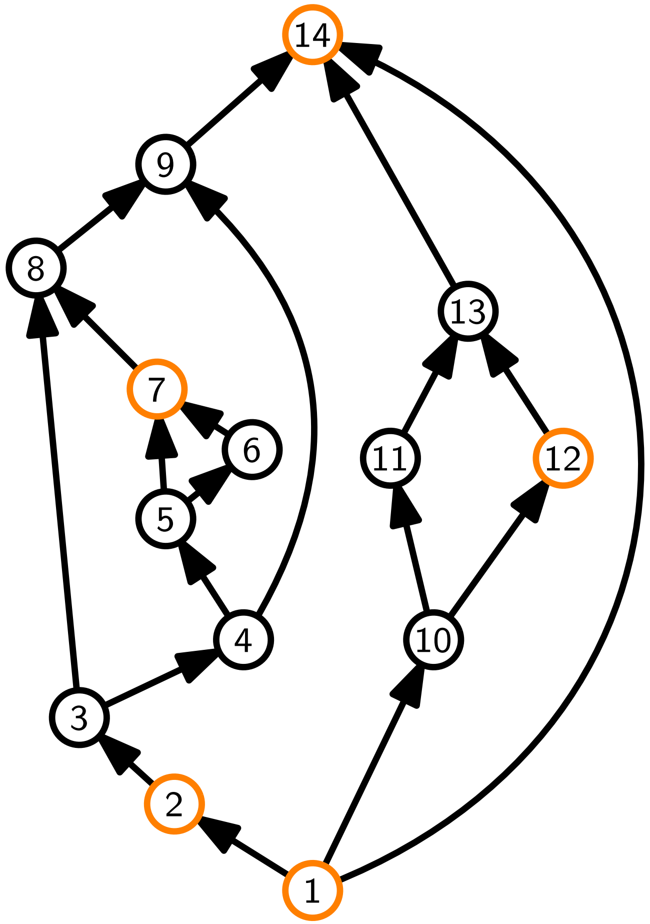

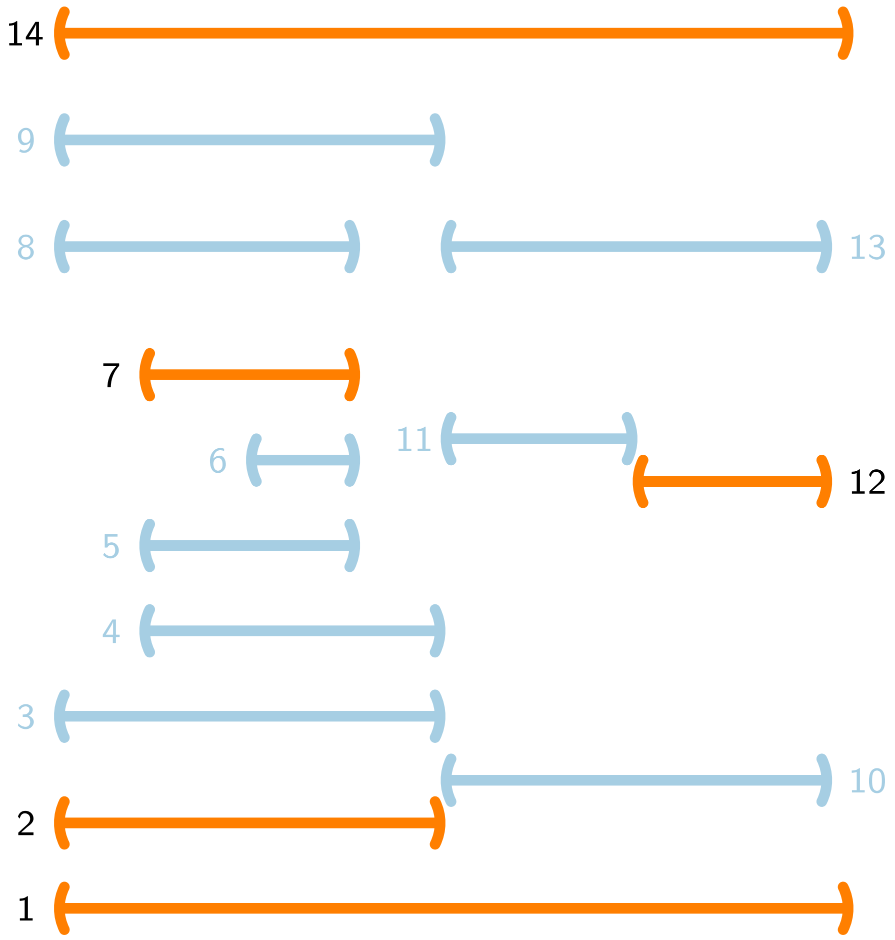

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- Solve problems for **S**-, **P**-, and **R**-nodes

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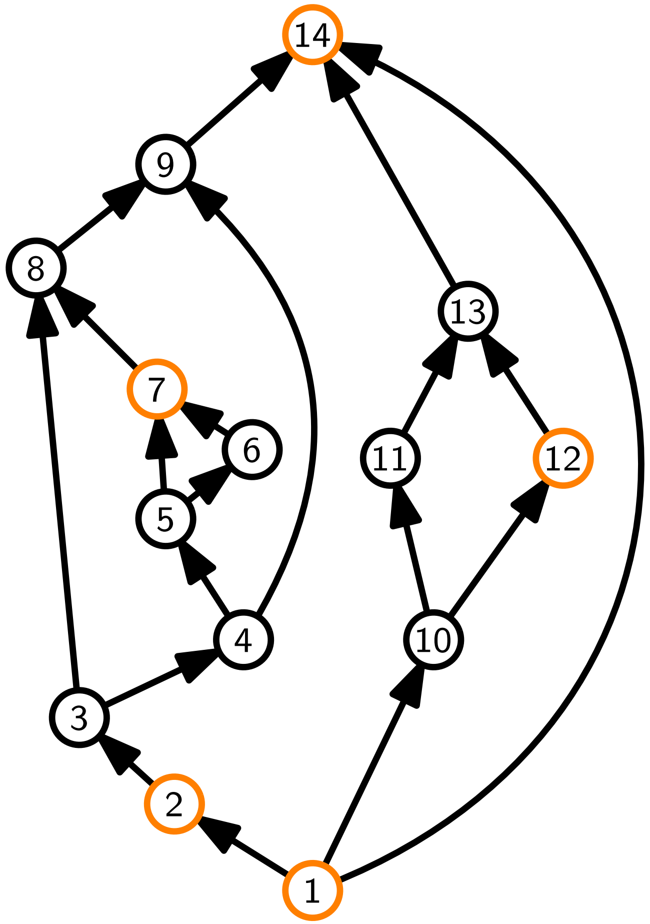

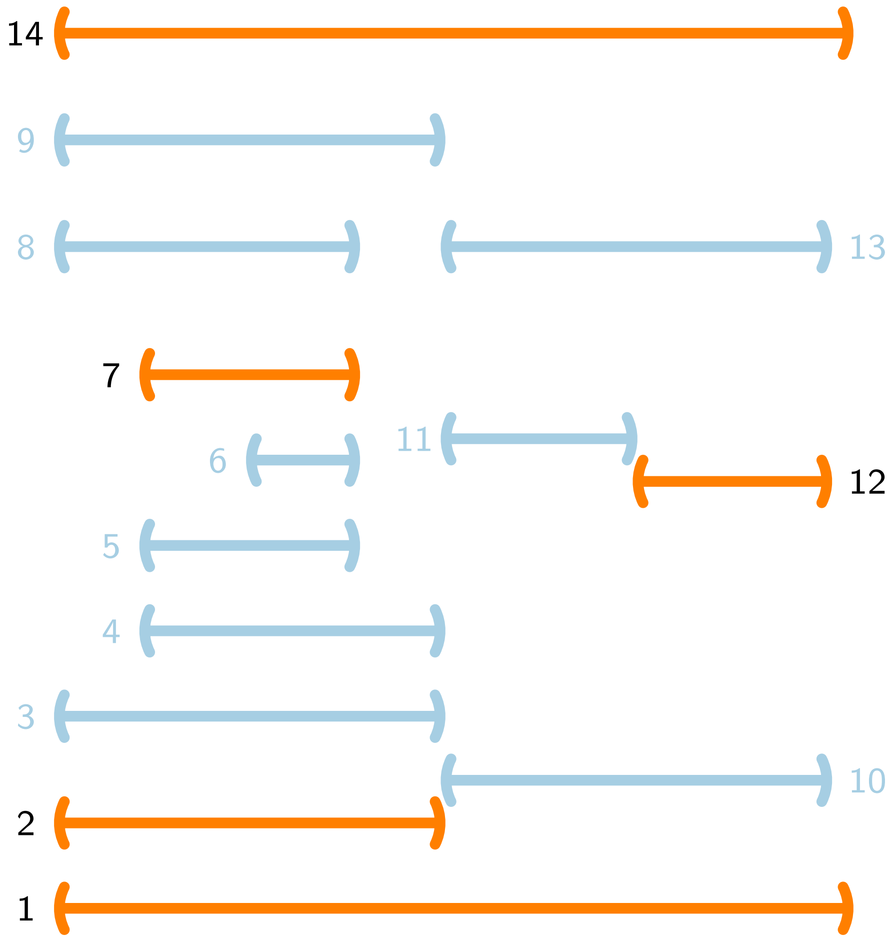

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- Look at connection to SPQR-trees – tiling
- Solve problems for **S**-, **P**-, and **R**-nodes
- Dynamic program via SPQR-tree

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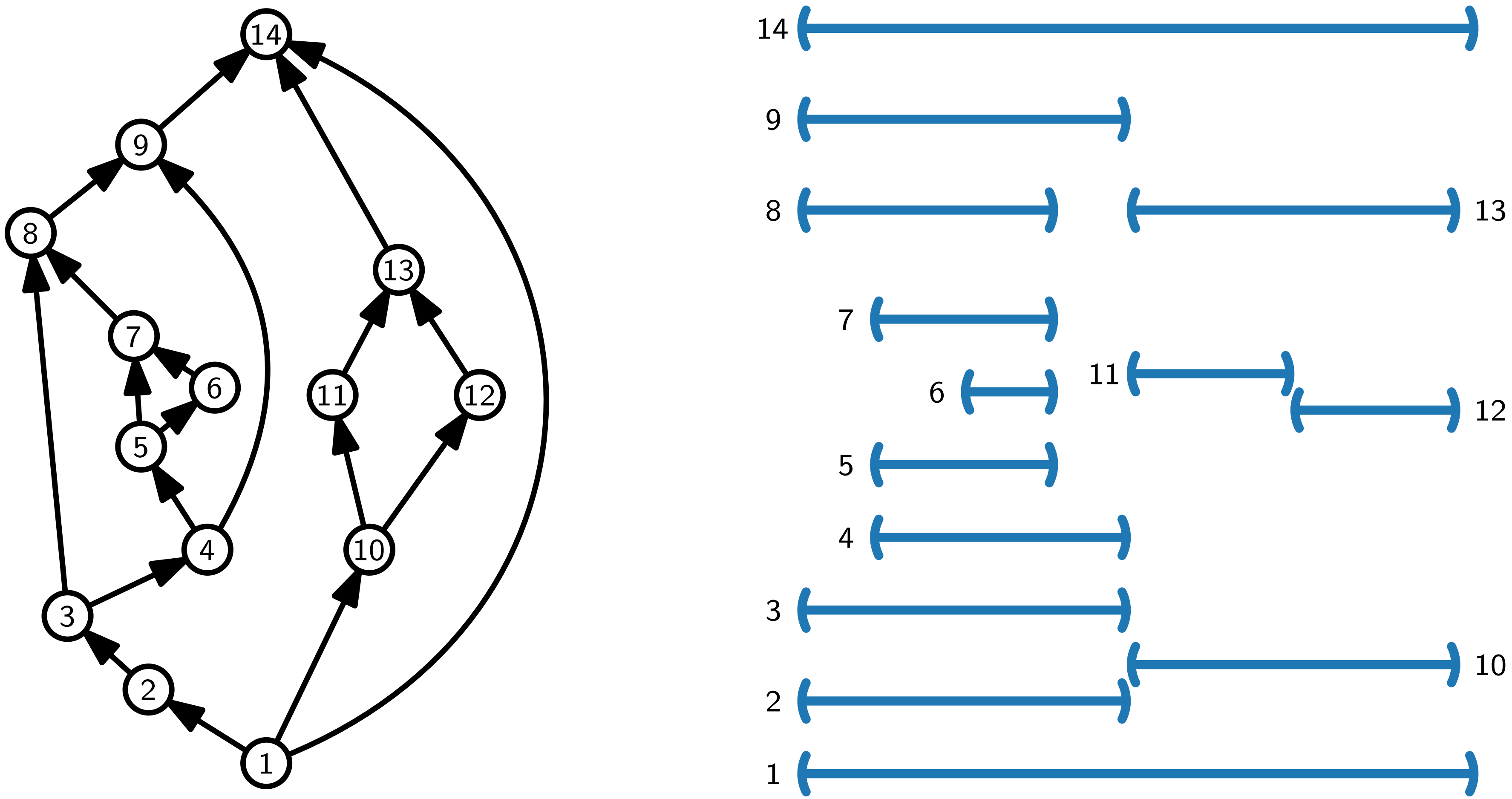
But Why Do SPQR-Trees Help?



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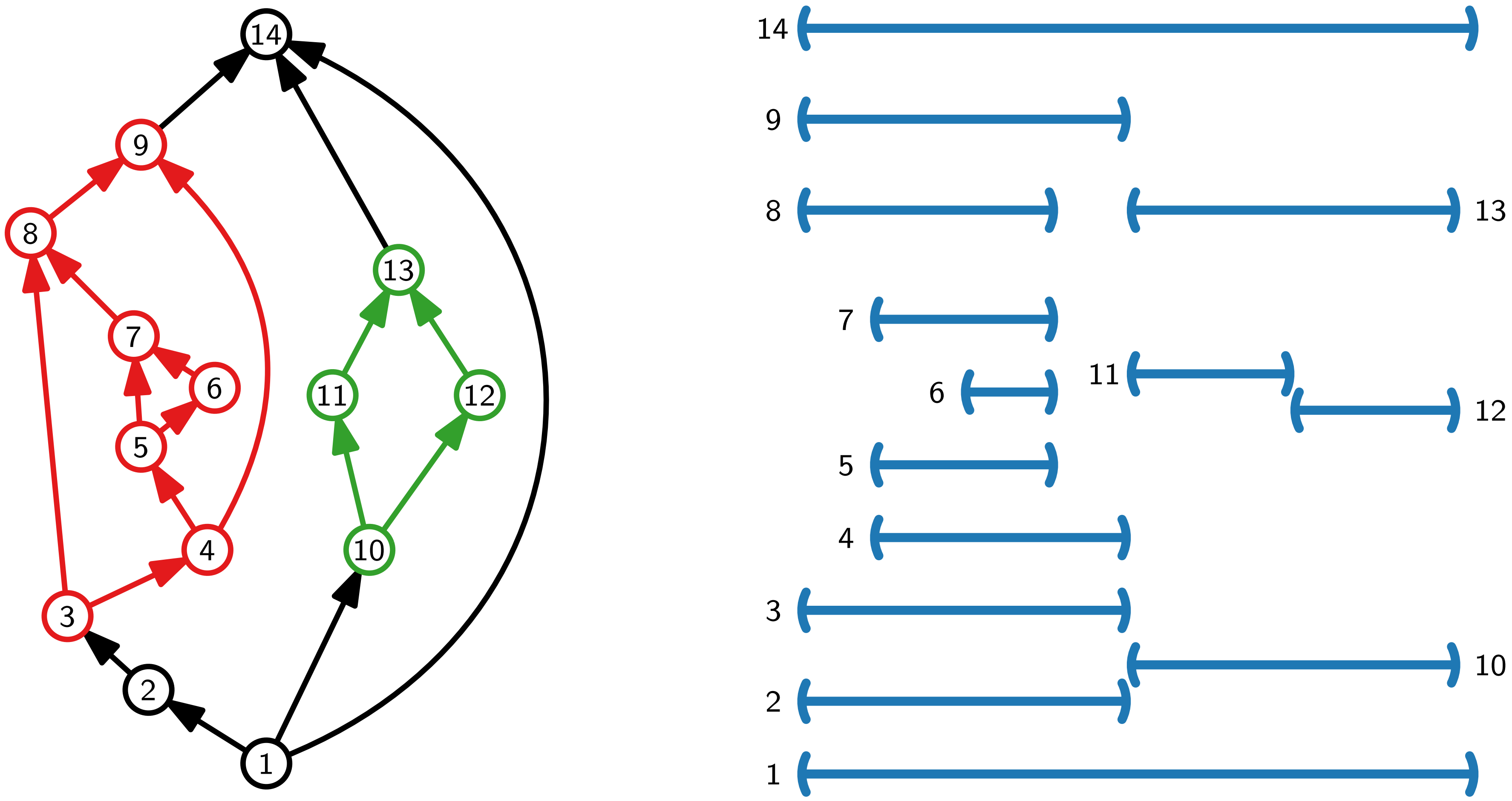

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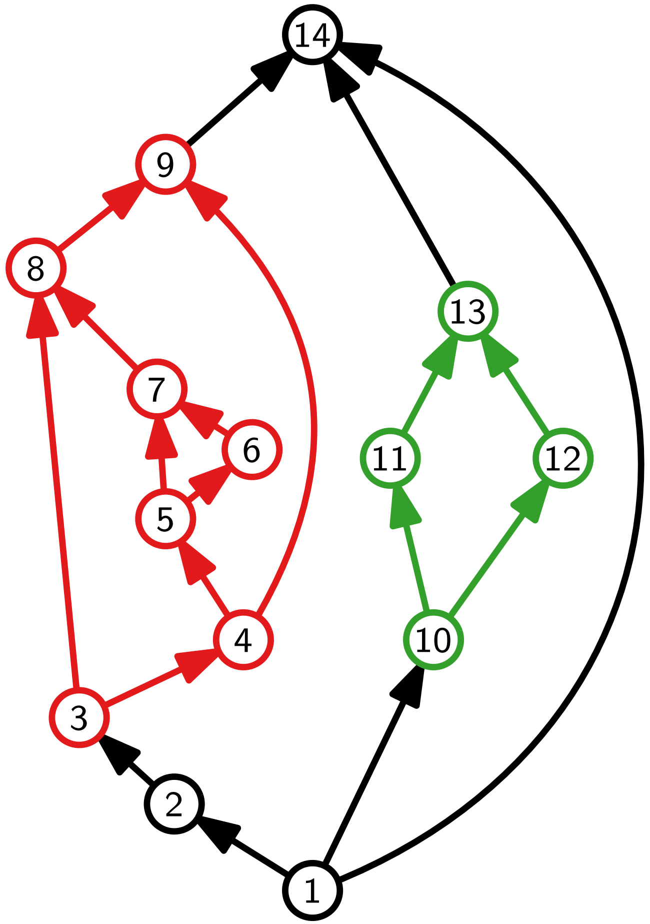

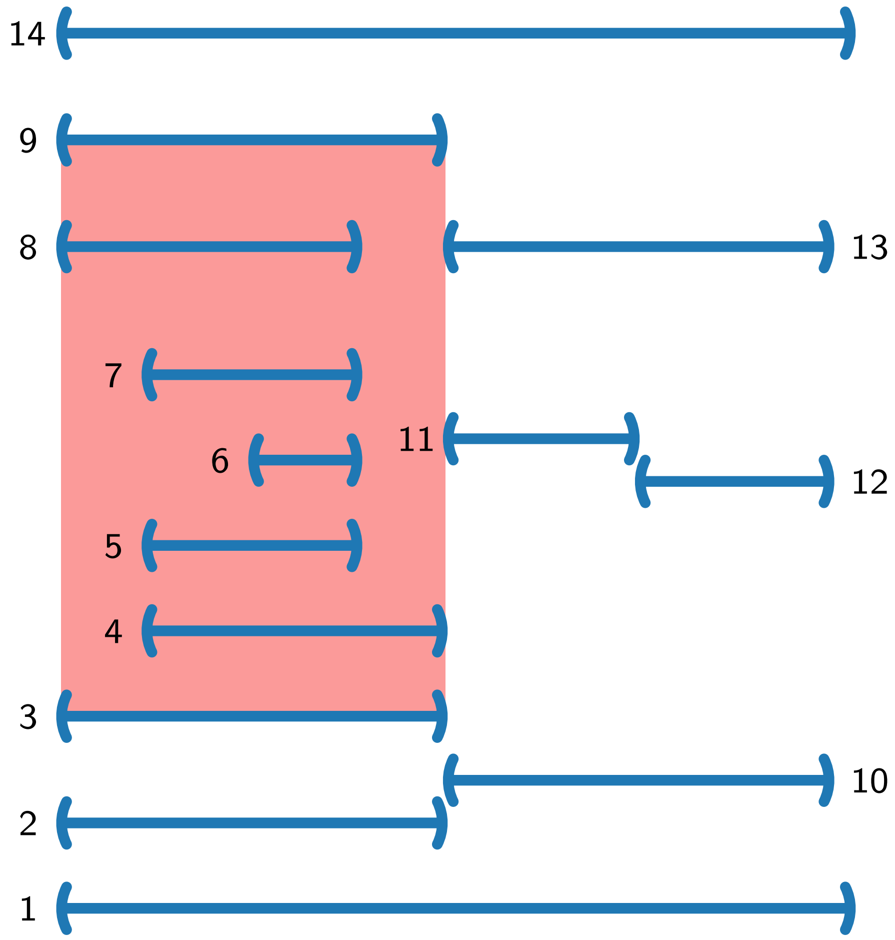

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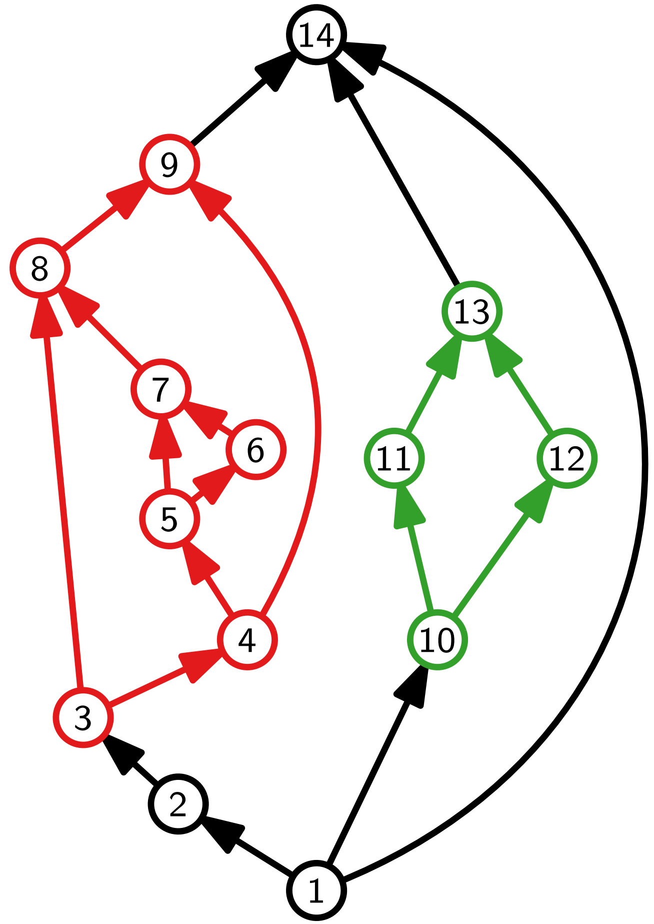

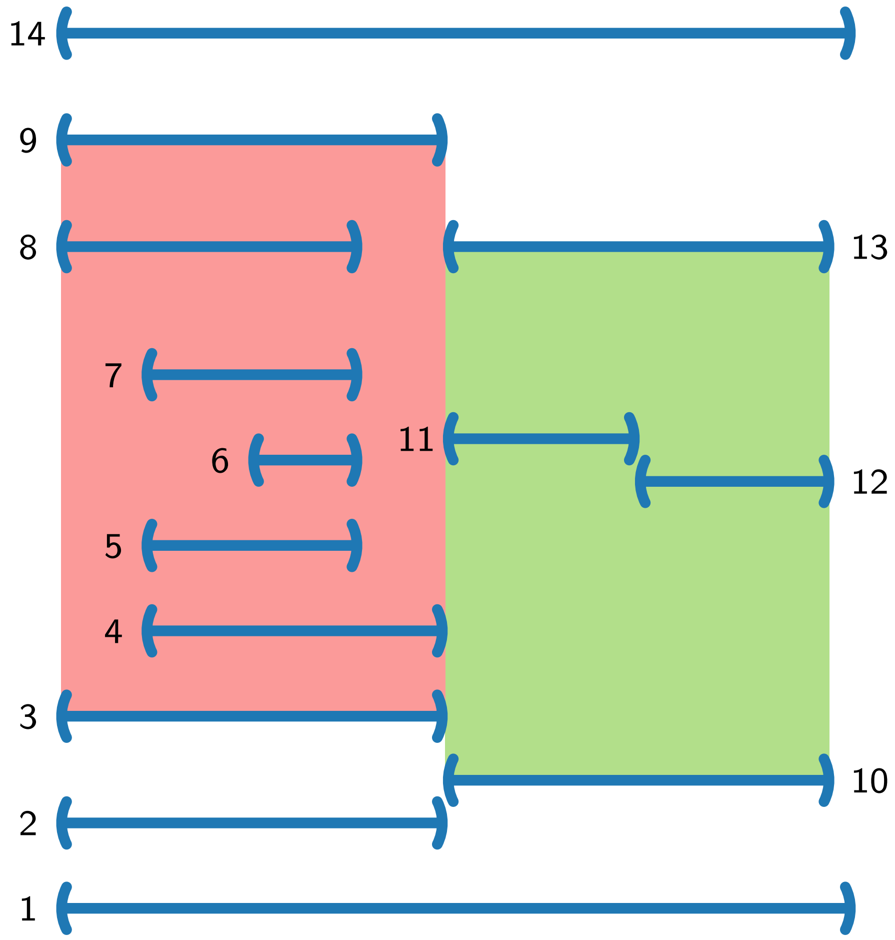

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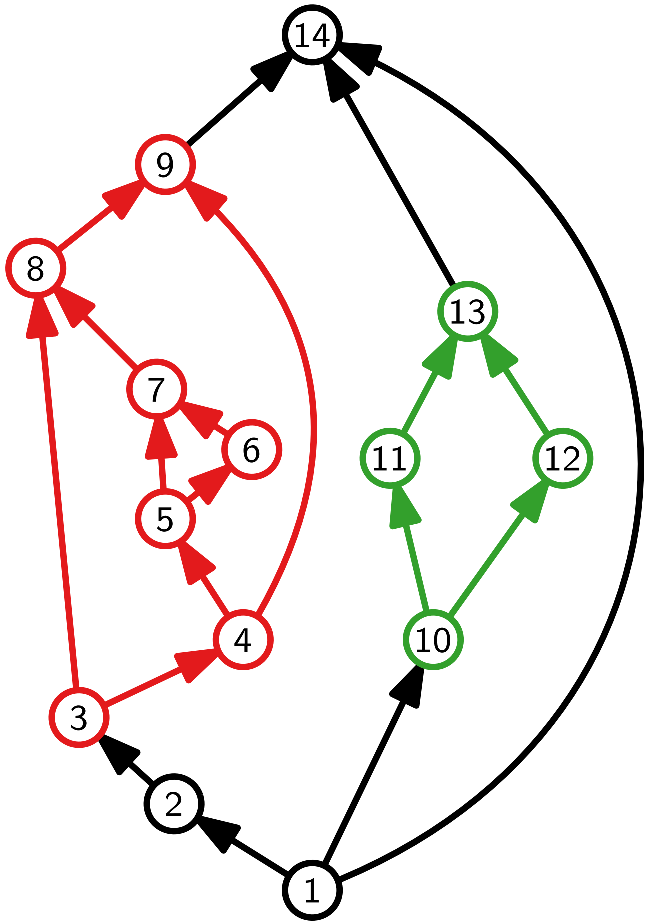

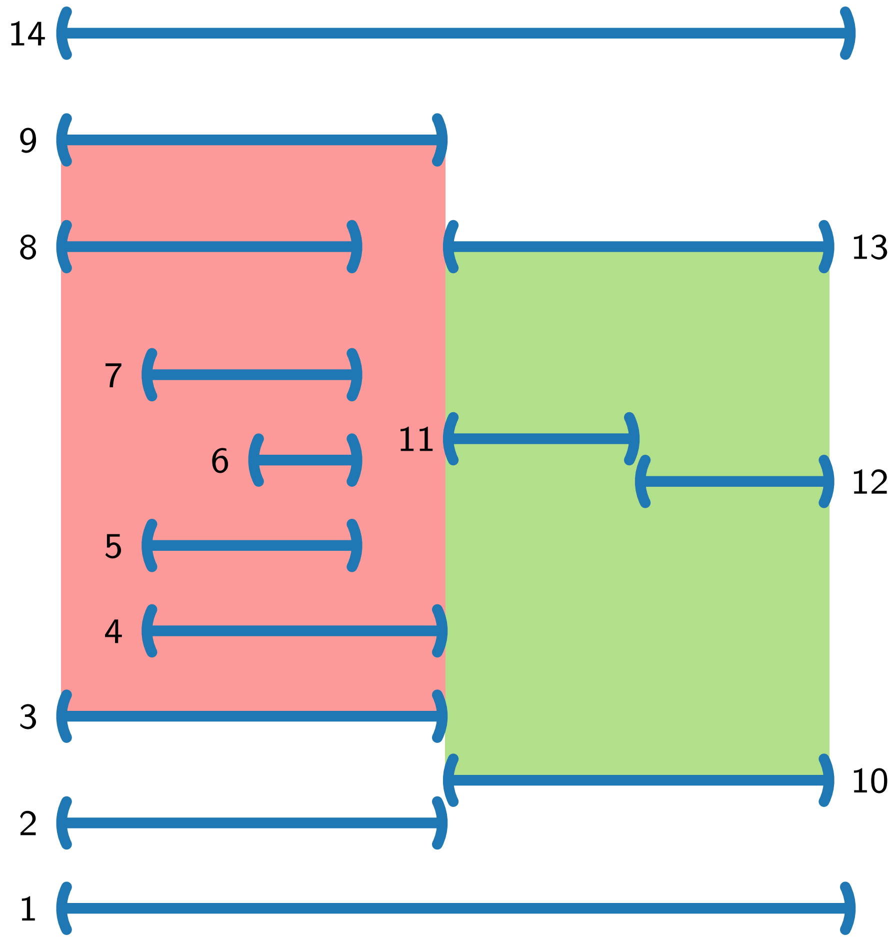

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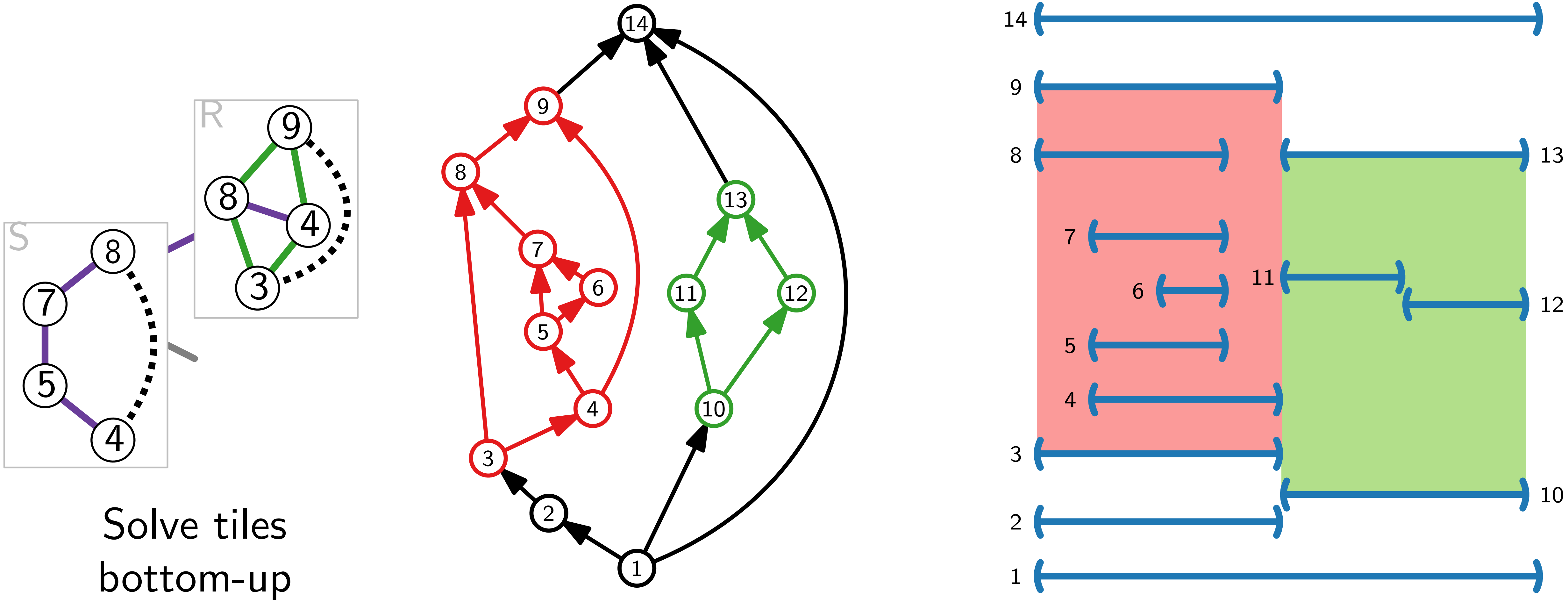
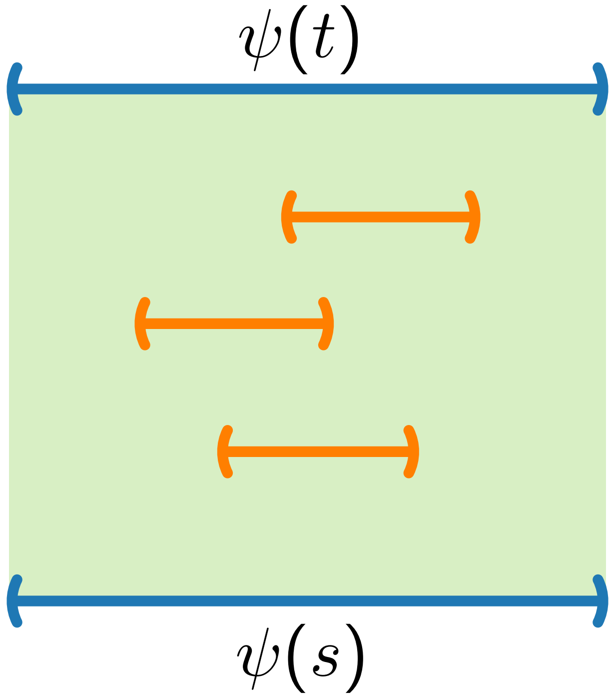


Solve tiles bottom-up

Tiles

Convention. **Orange** bars are from the partial representation



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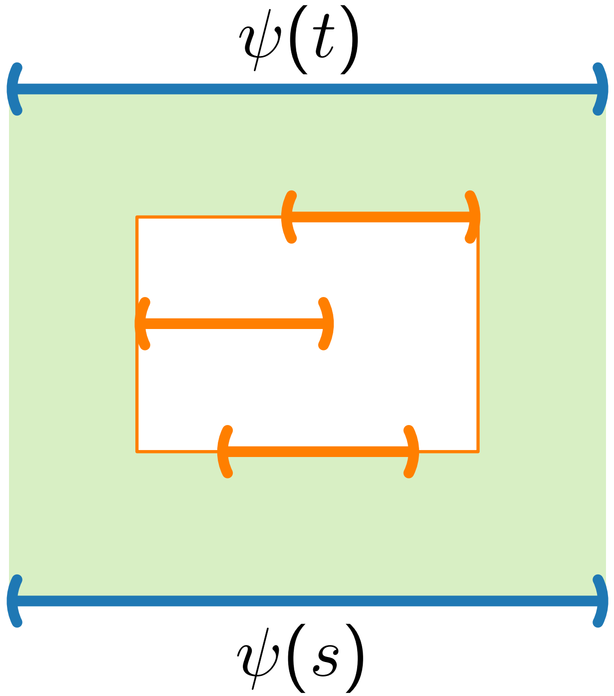

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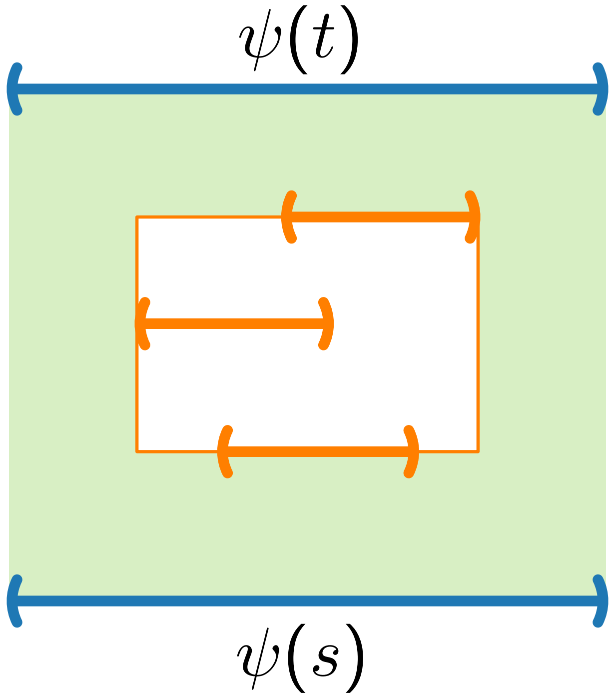

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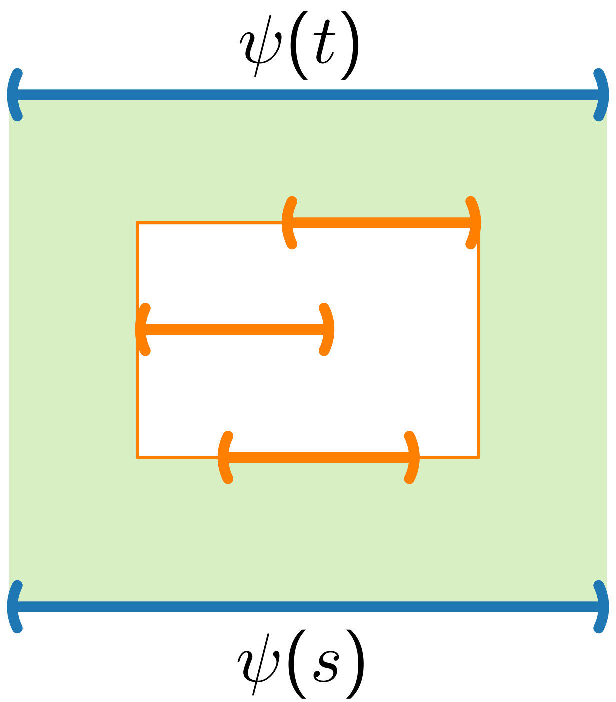

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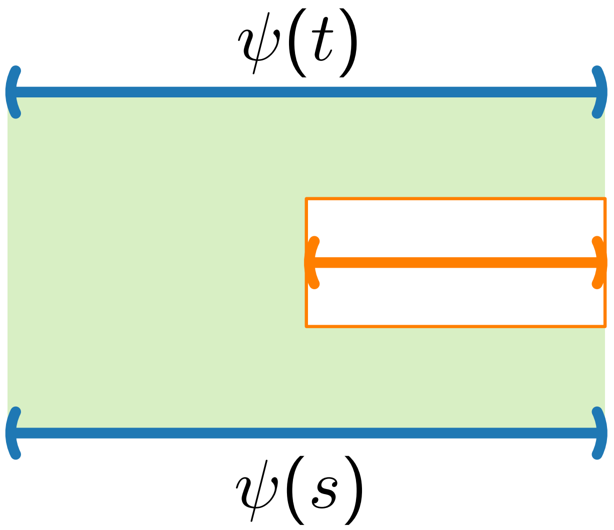
How many different **types** of tiles are there?

Types of Tiles



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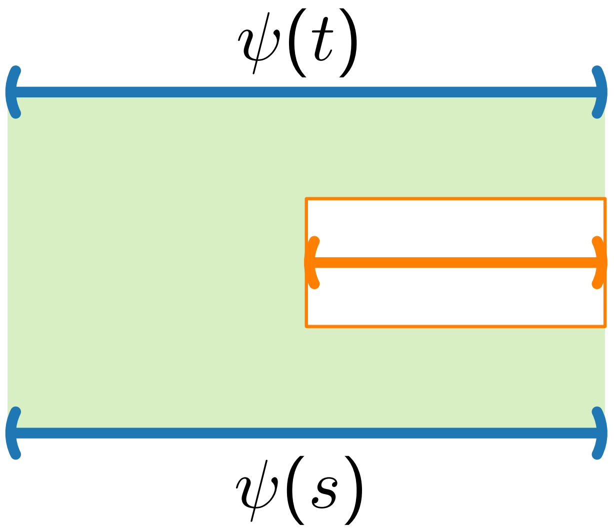

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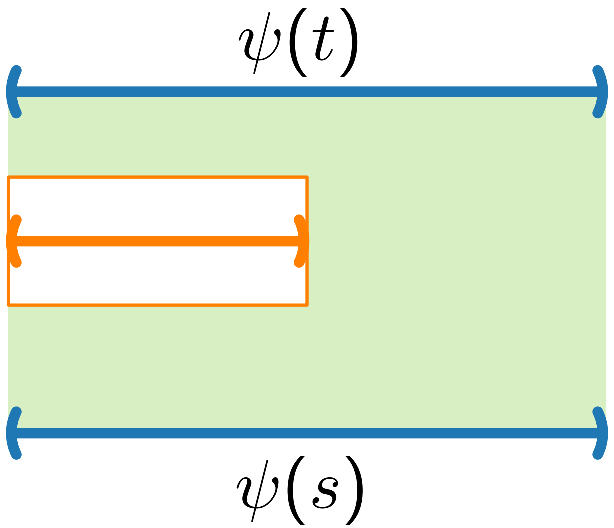

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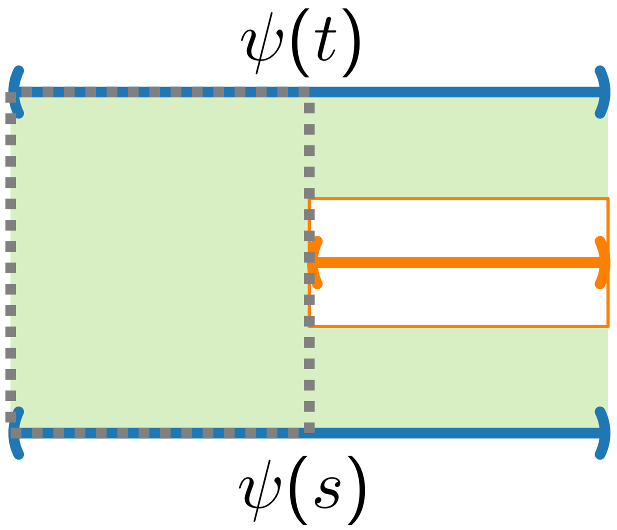

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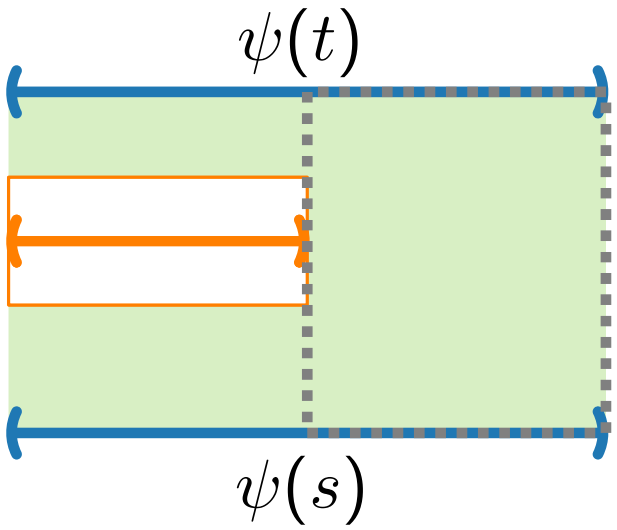

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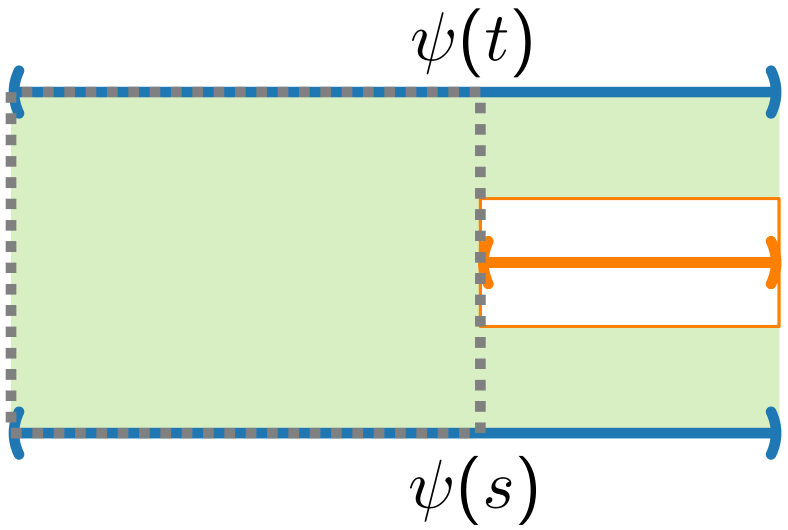

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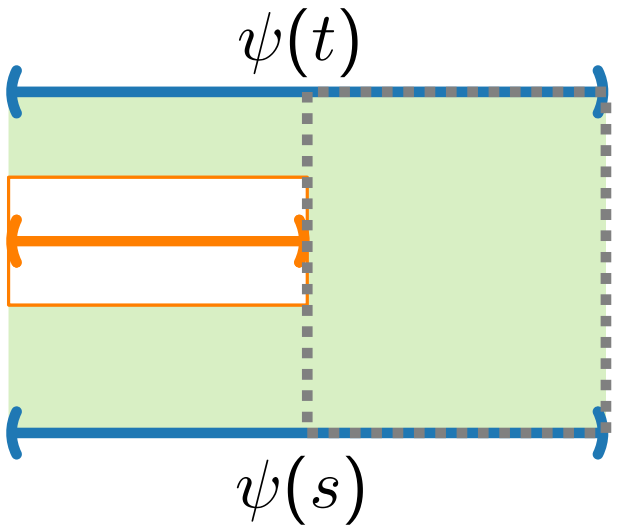

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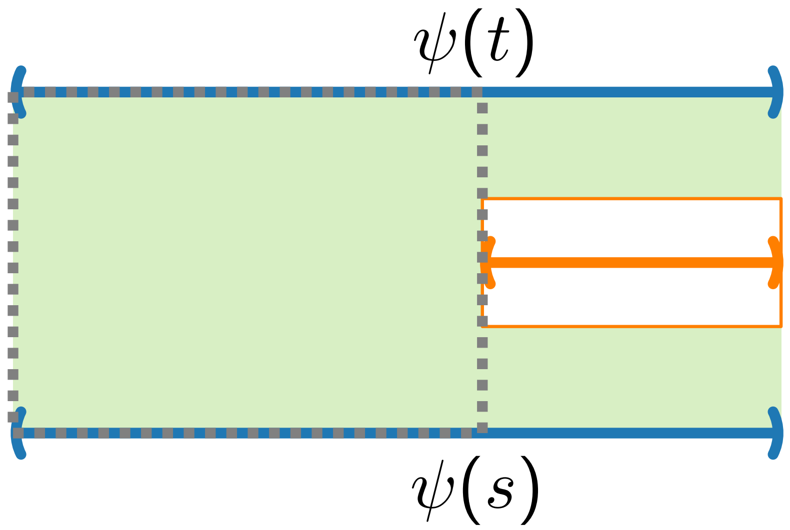

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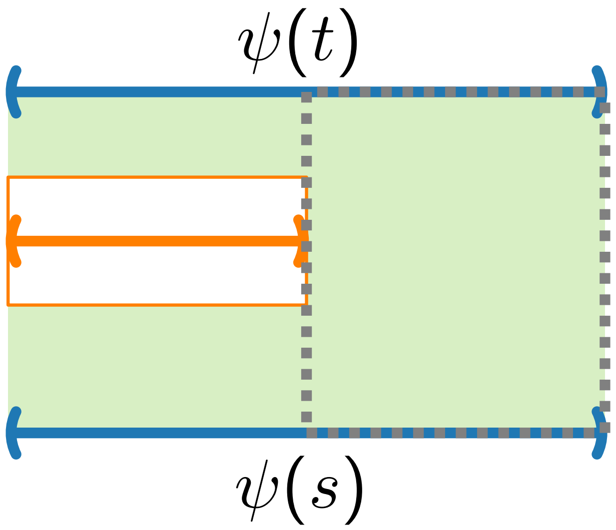

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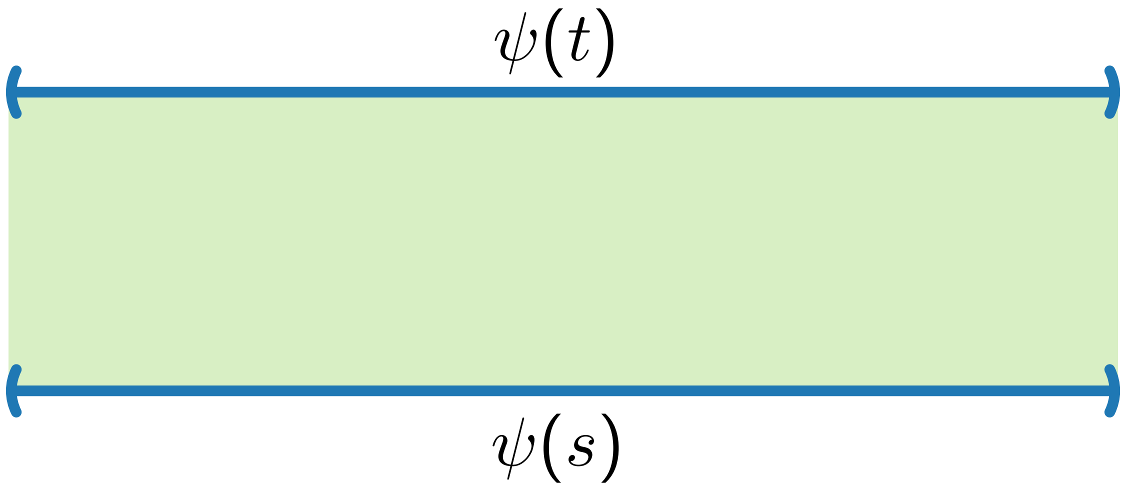
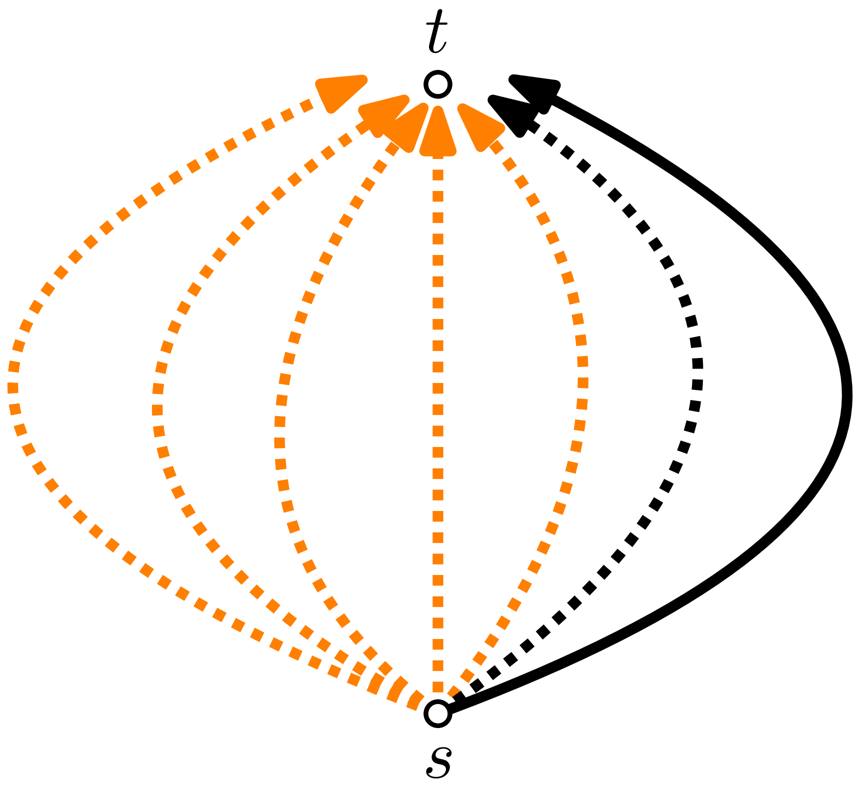
Four different types: **FF, FL, LF, LL**

P-Nodes





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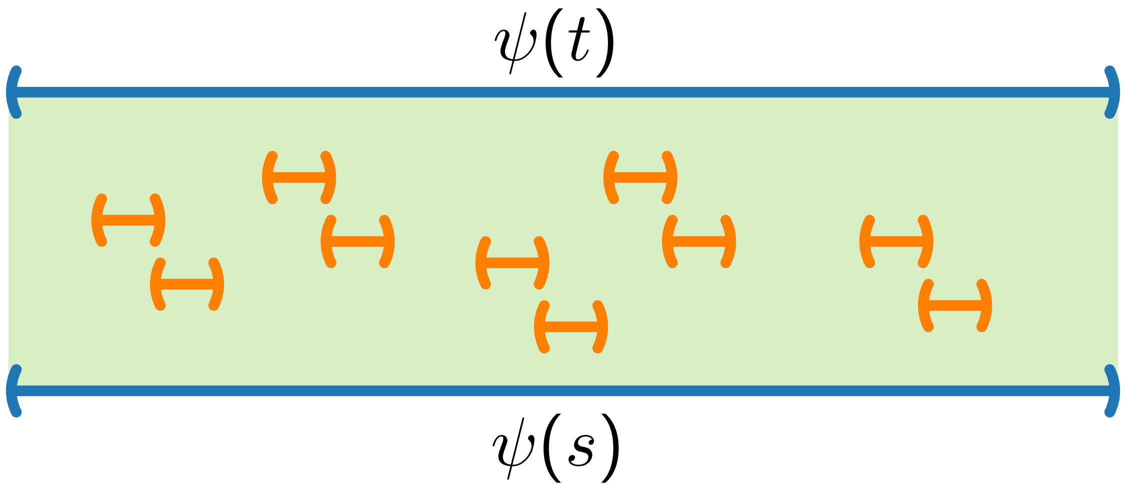

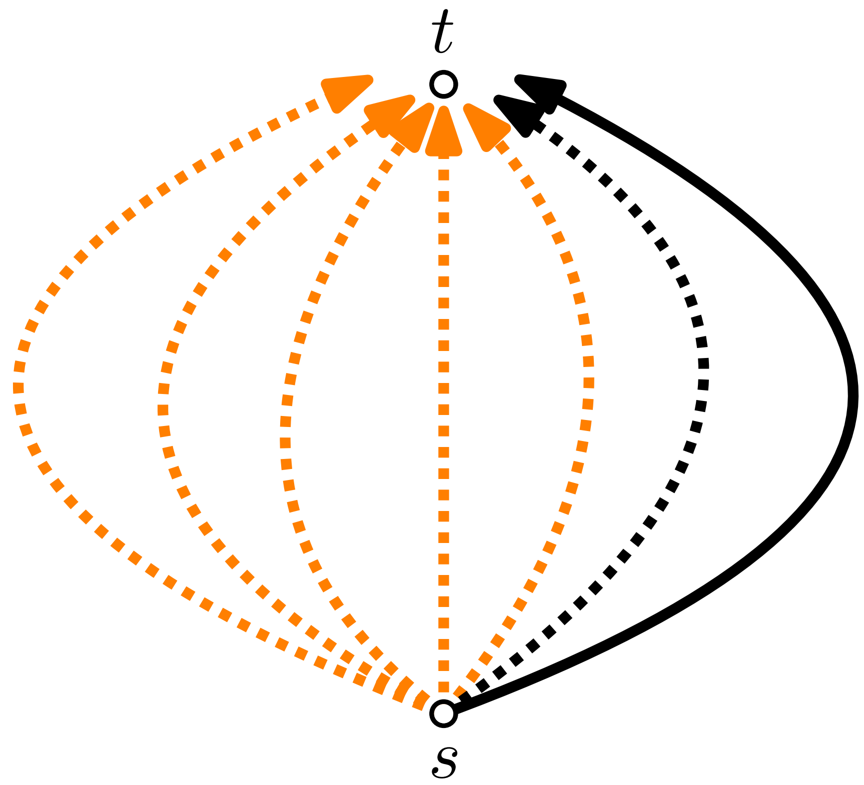

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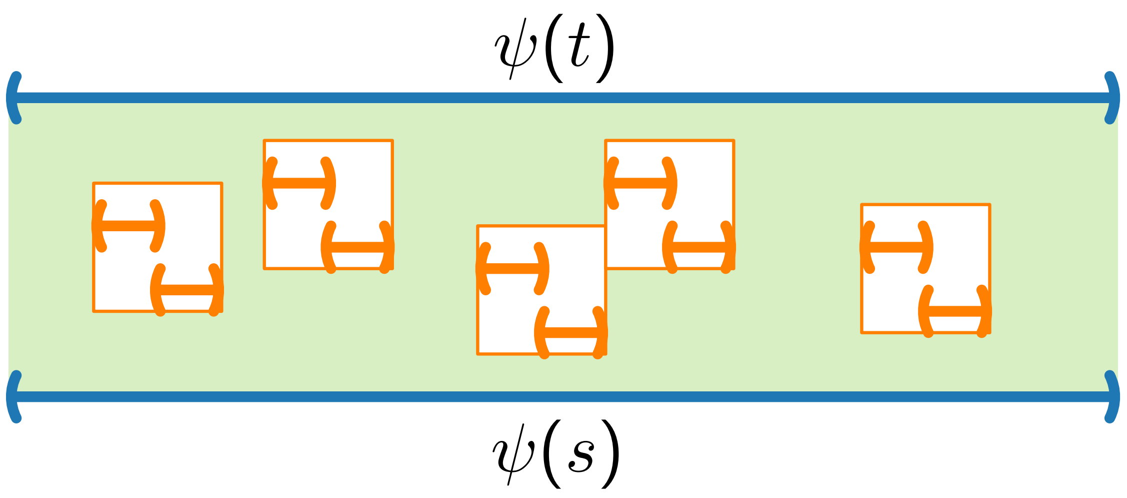

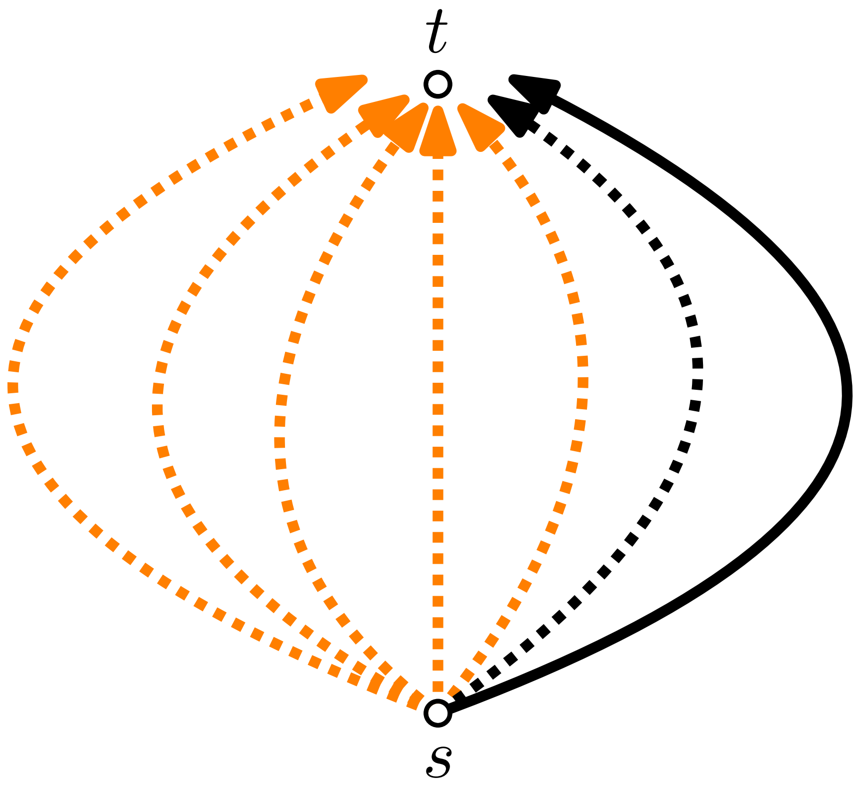

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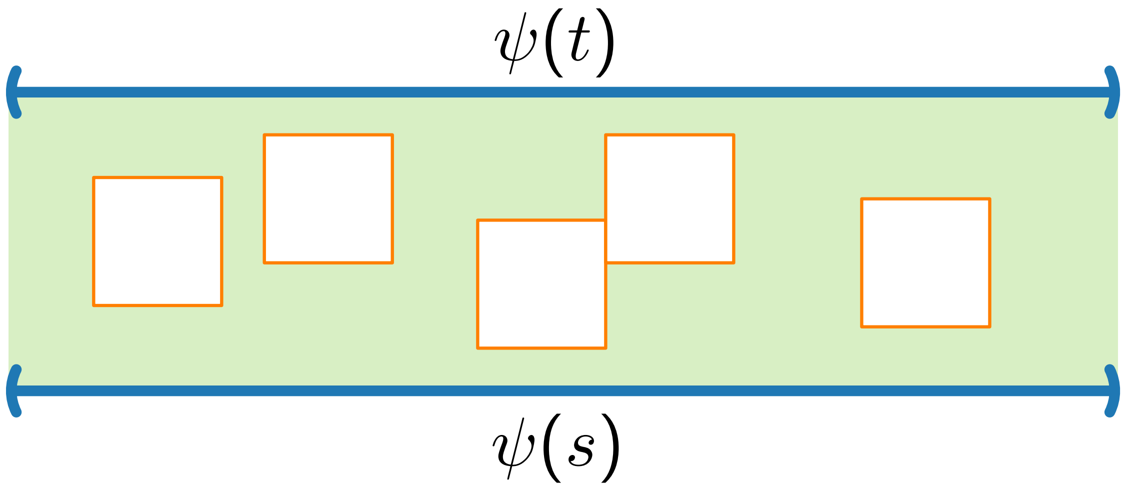

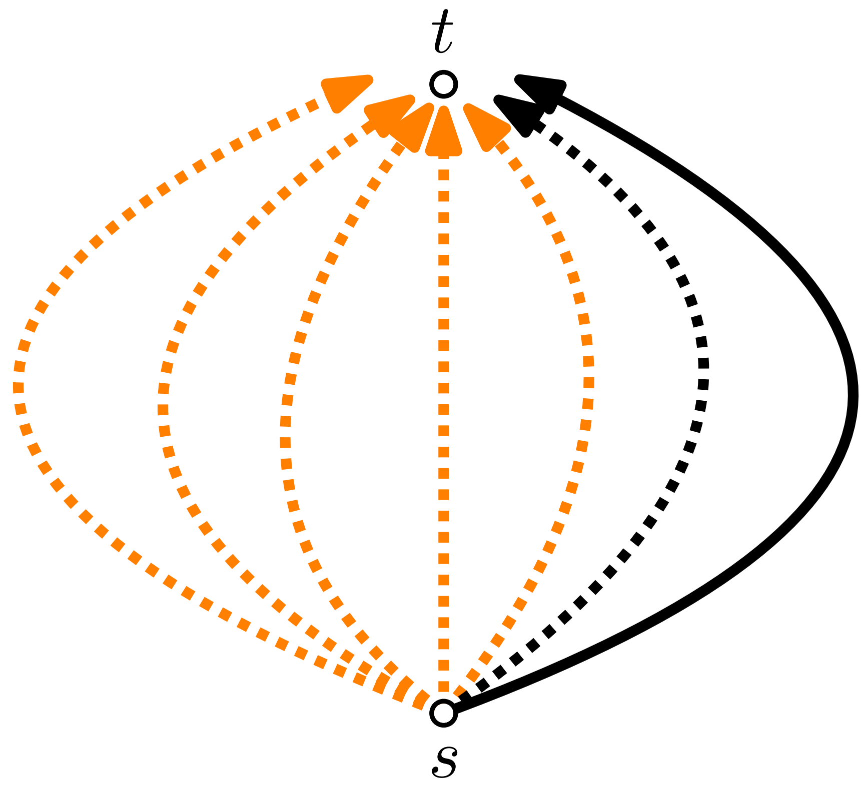

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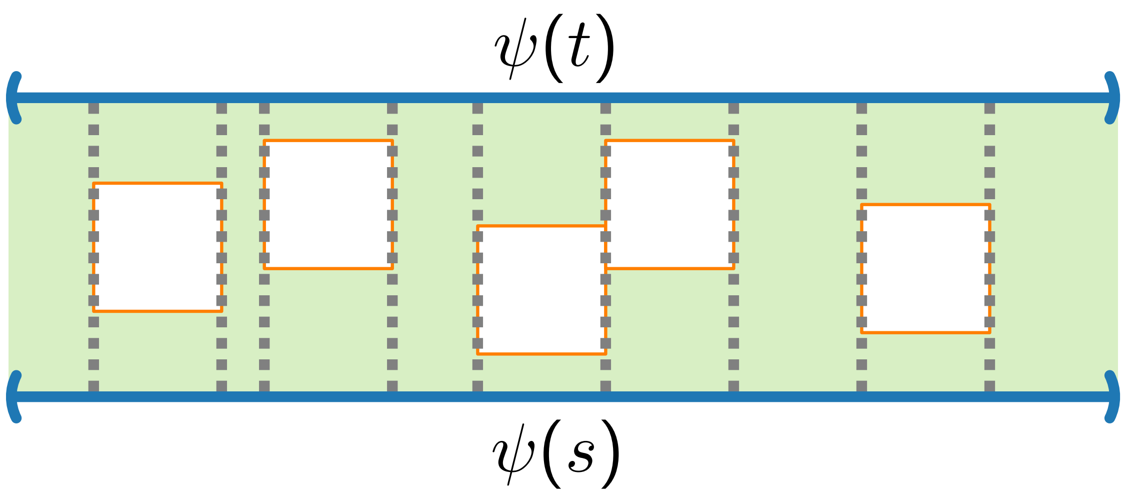

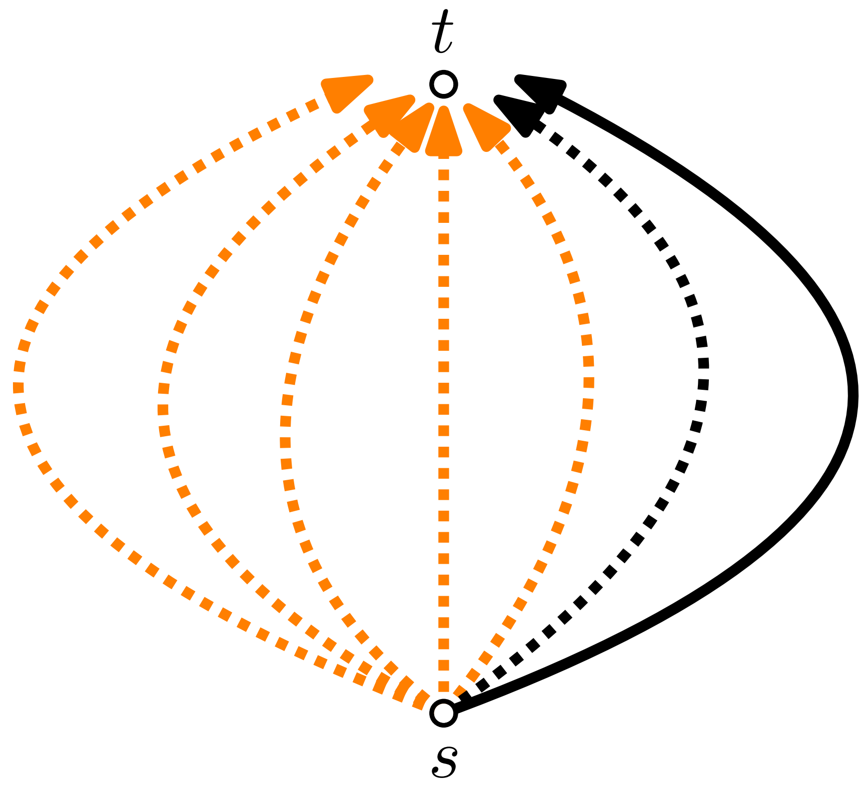

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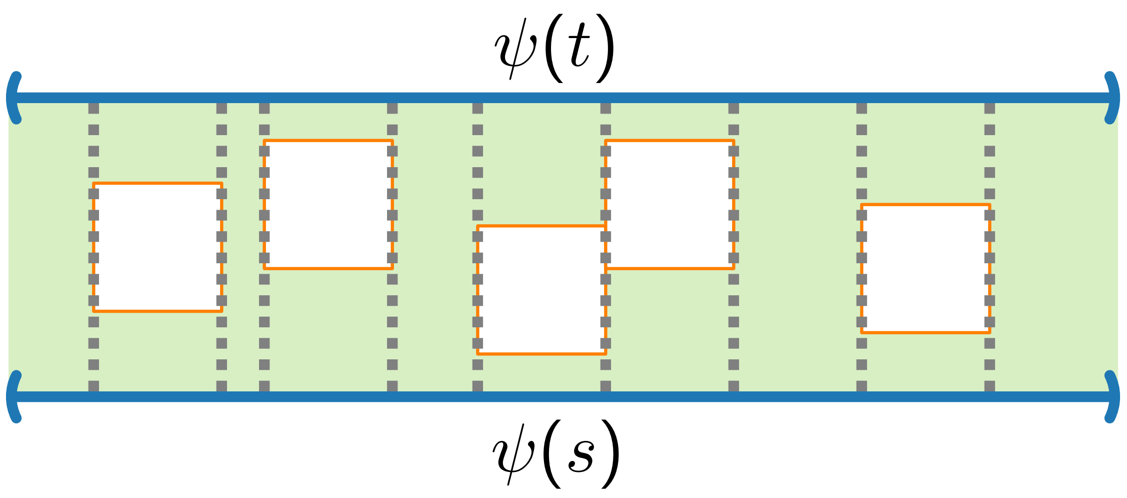

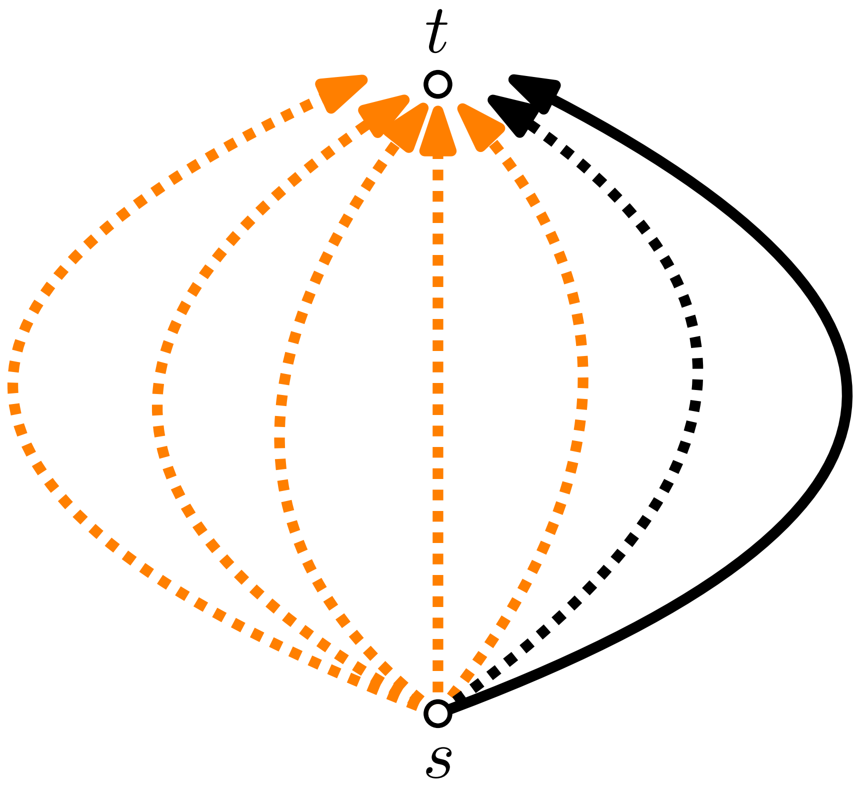

- Children of **P**-node with prescribed bars occur in given left-to-right order

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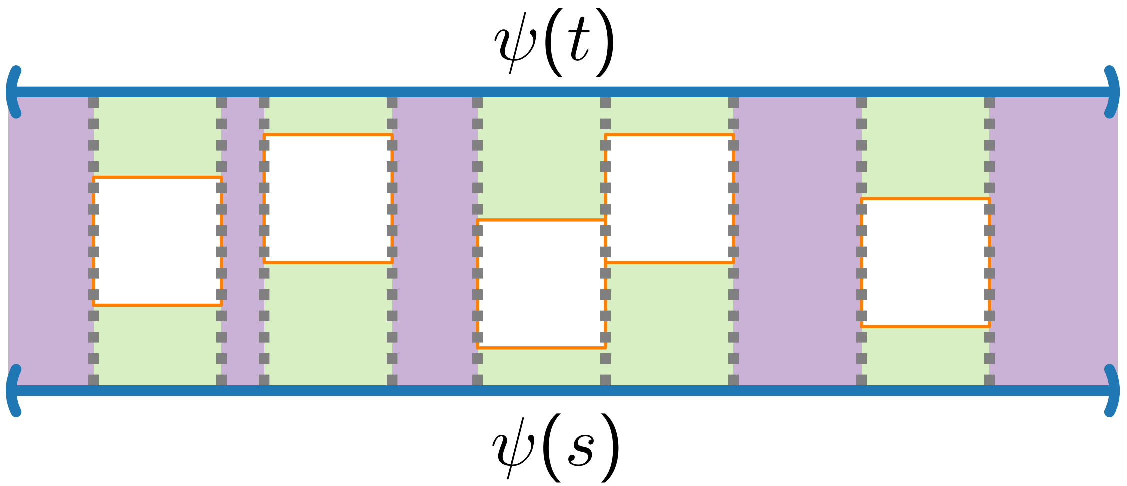

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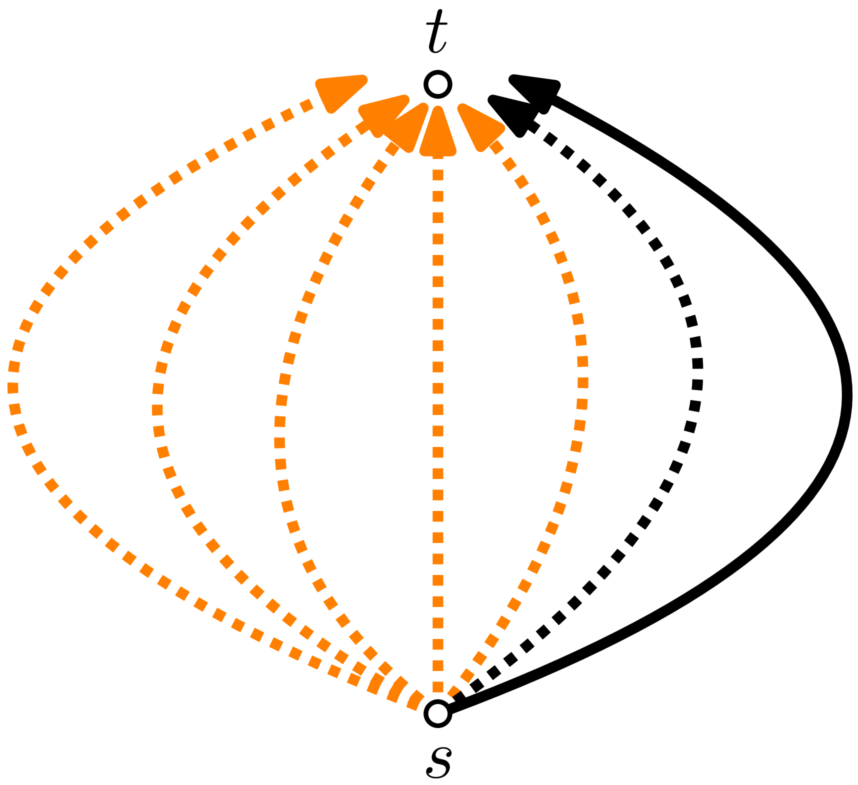

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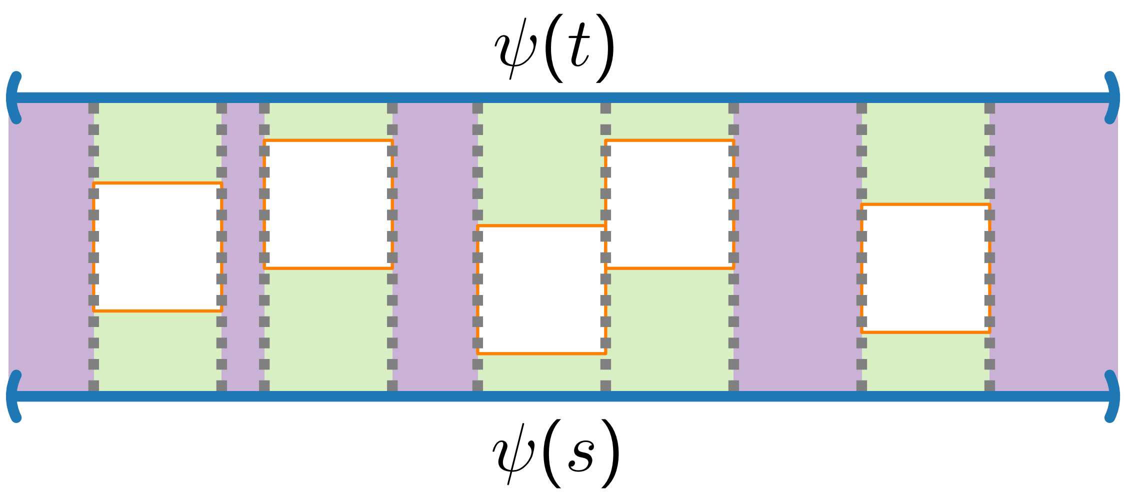

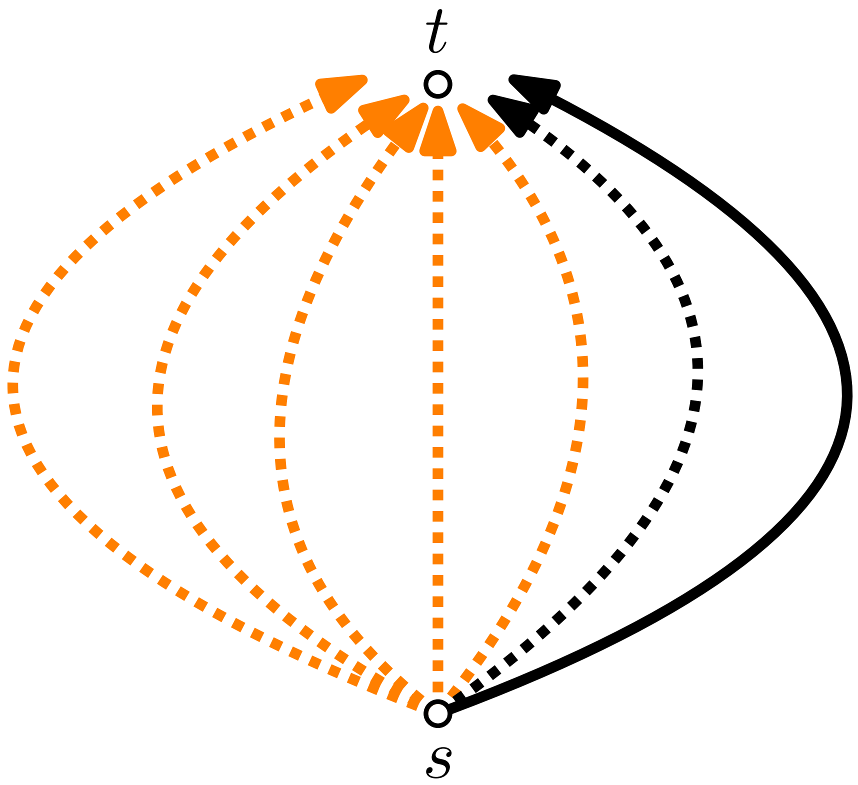

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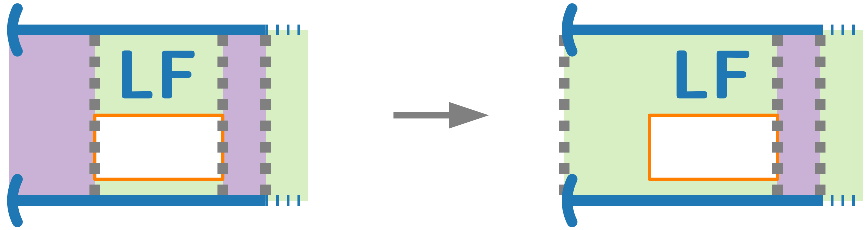
Greedily *fill* the gaps by preferring to “stretch” the children with prescribed bars.



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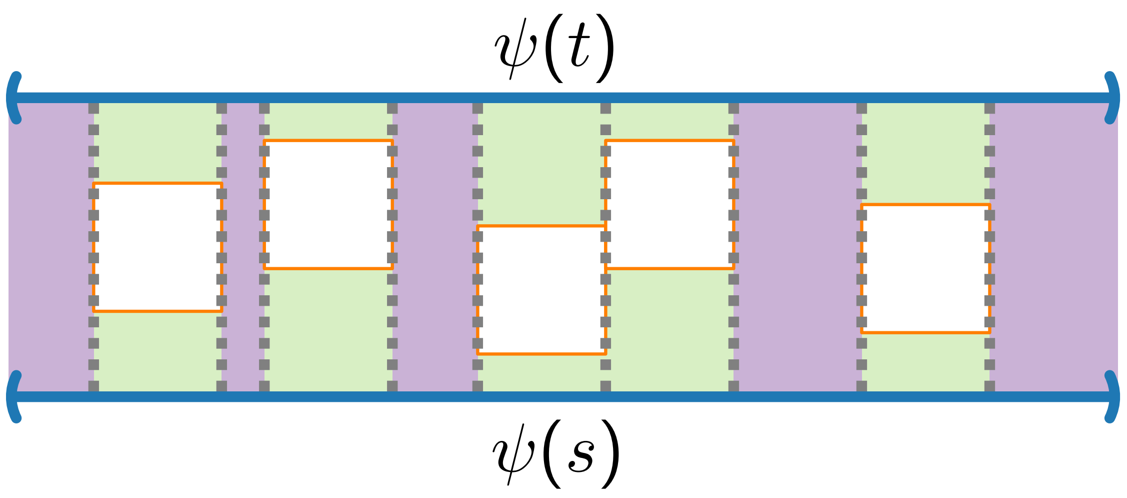

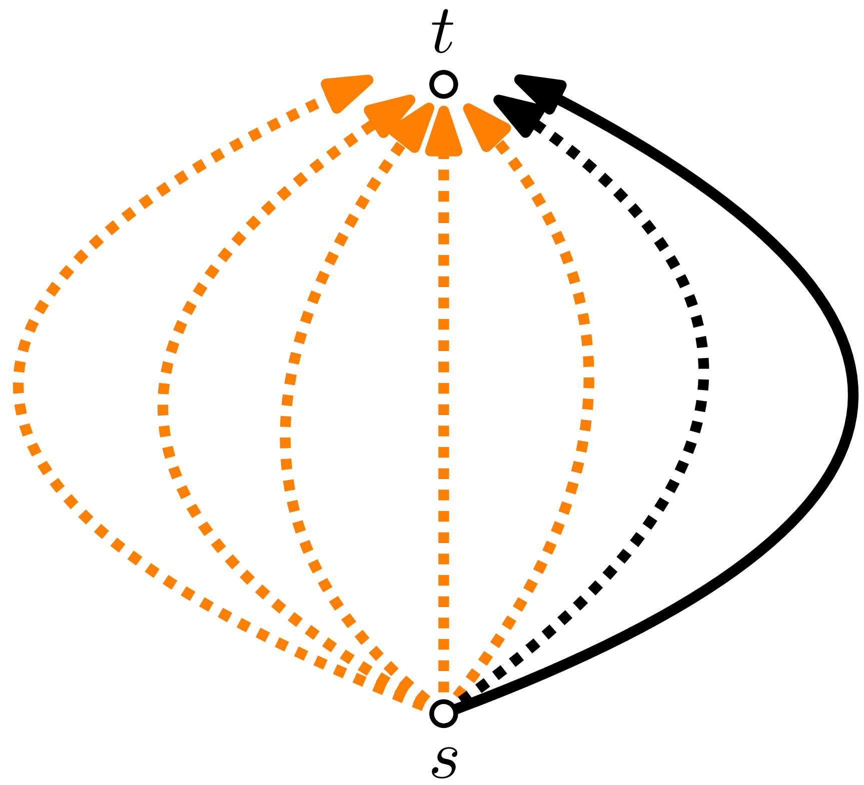

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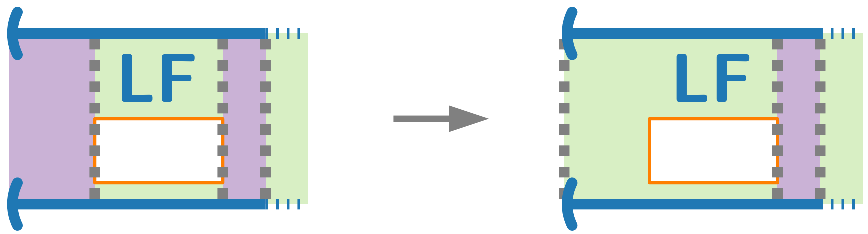
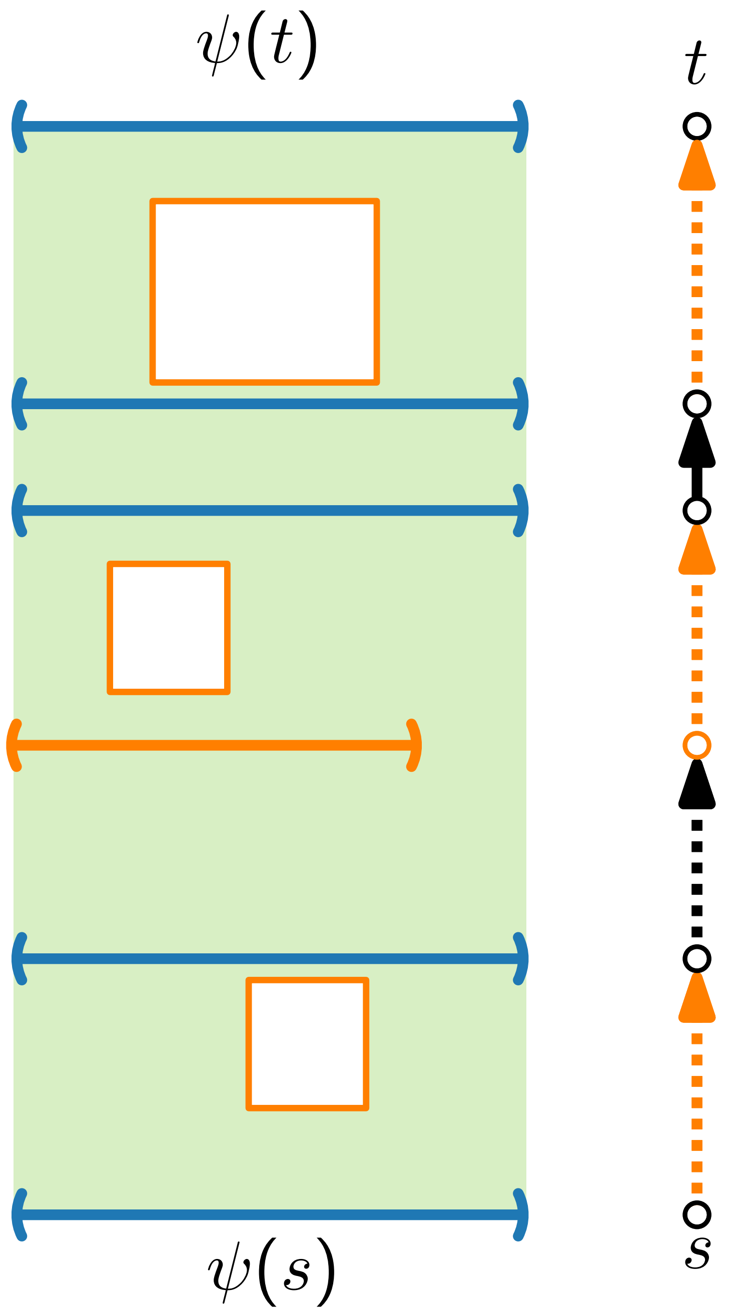


Outcome.
After processing, we must know the valid types for the corresponding subgraphs.

S-Nodes



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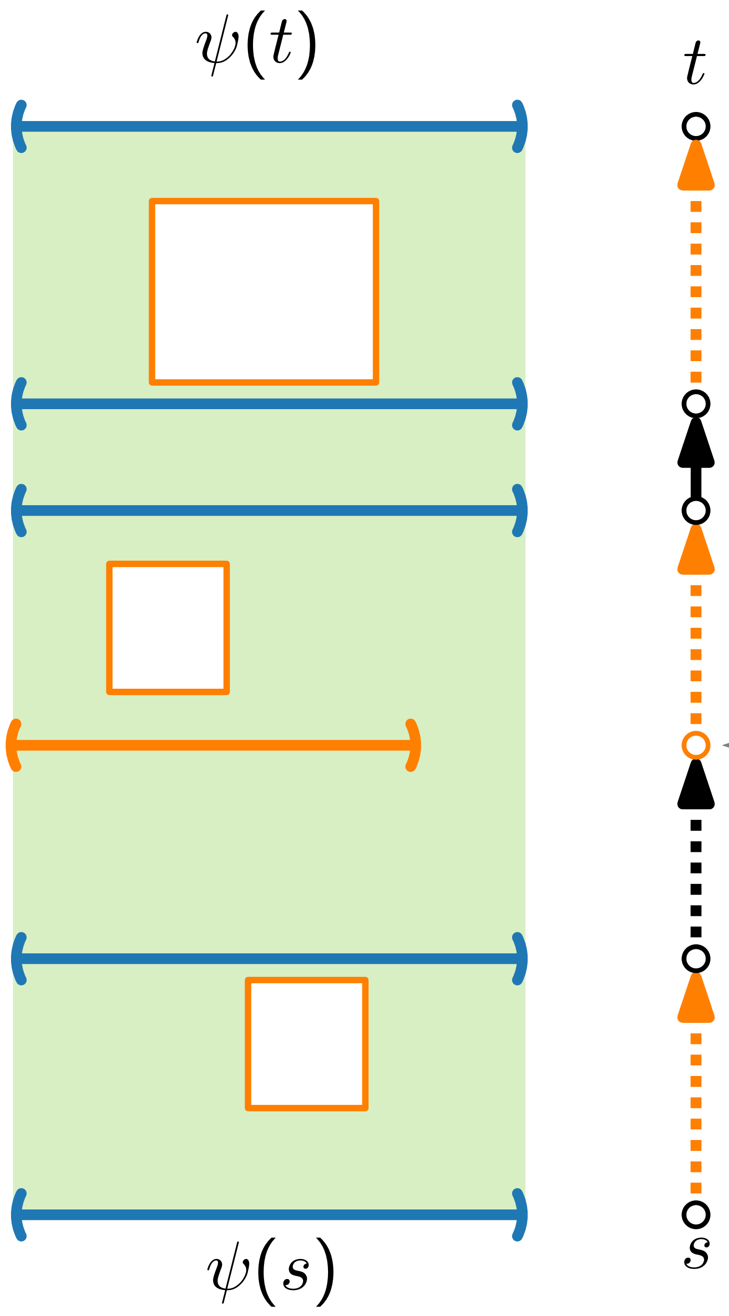

This fixed vertex means we can only make a **F**ixed-**F**ixed representation!

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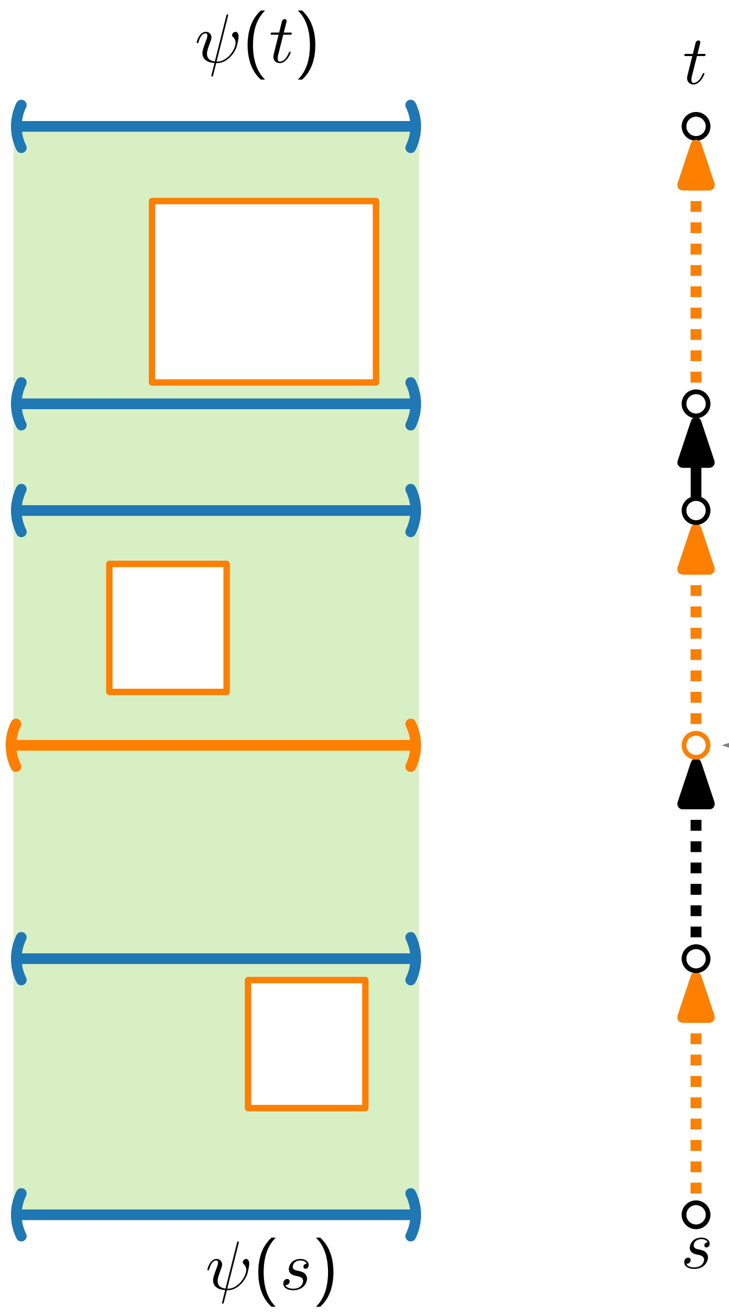

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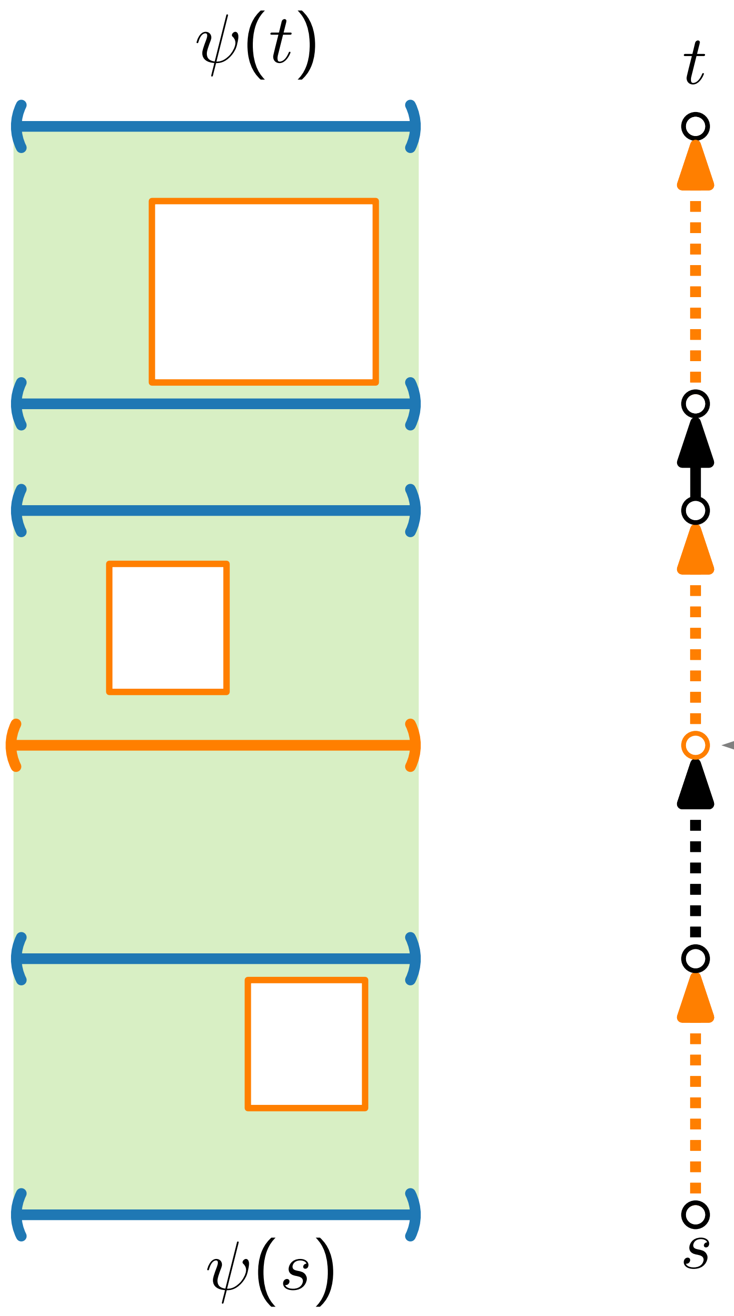

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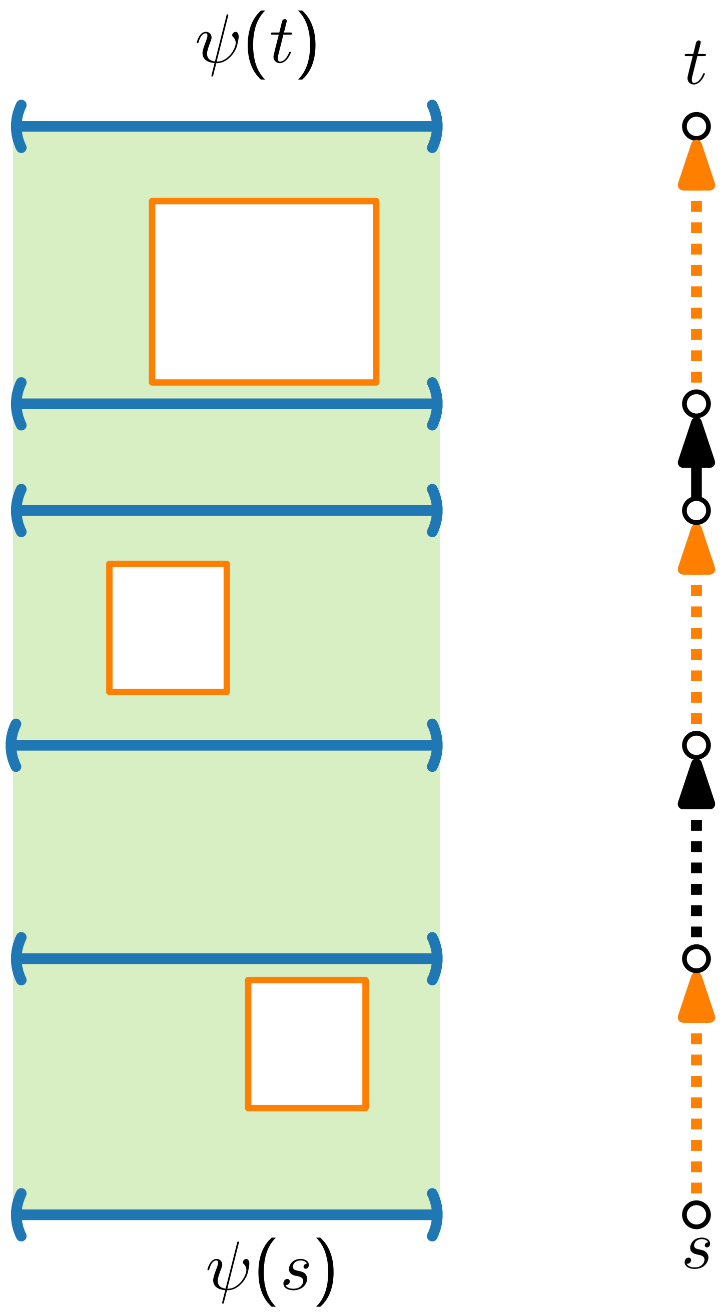

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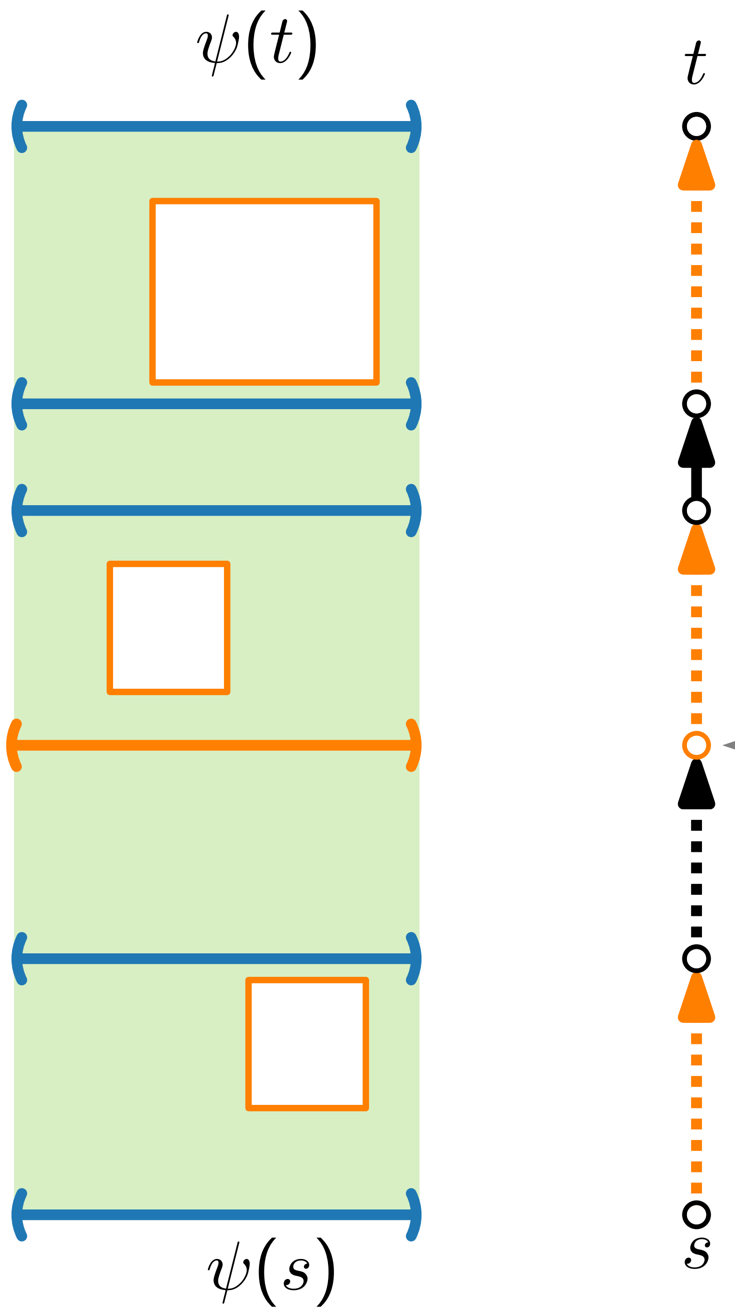
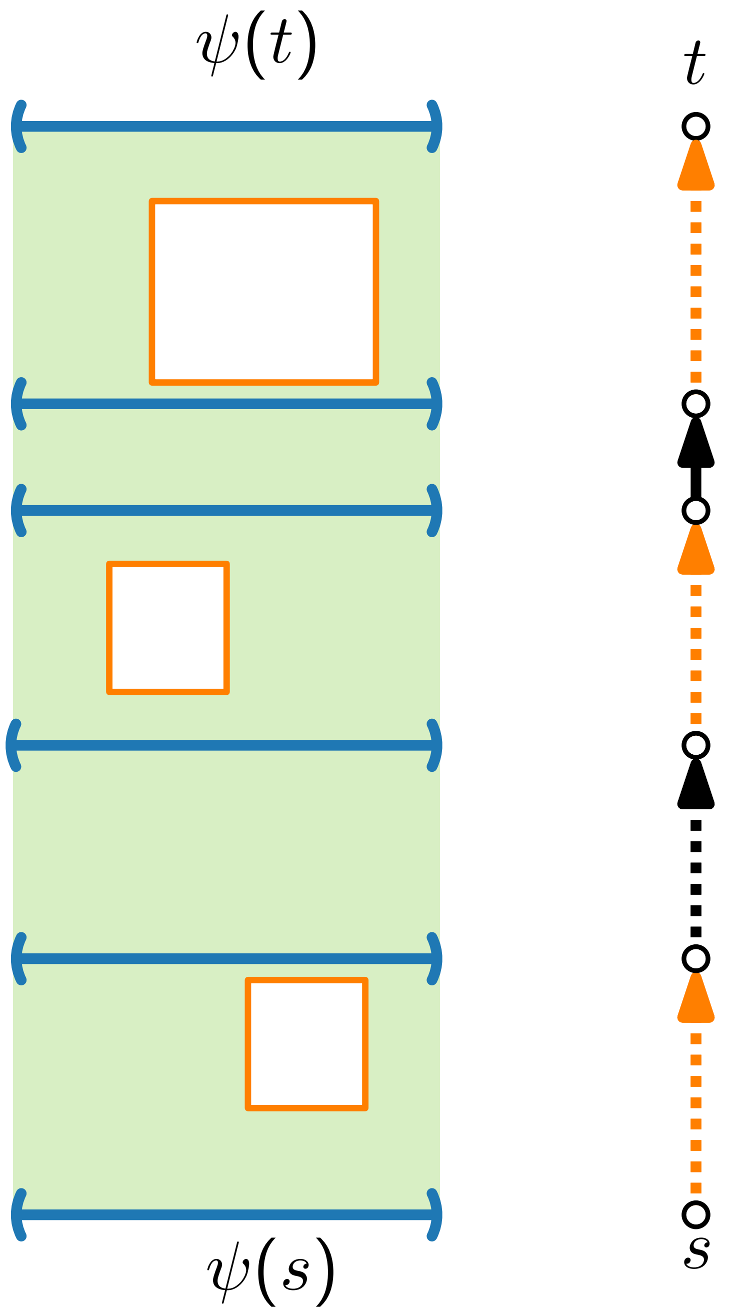
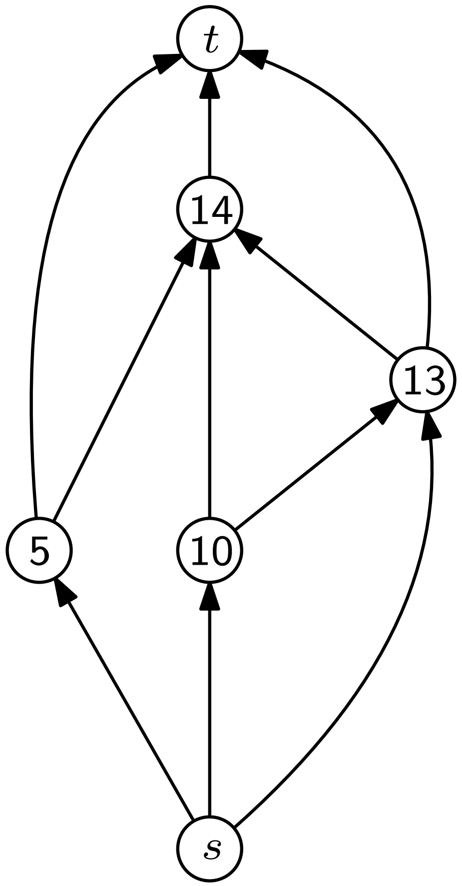


This fixed vertex means we can only make a **F**ixed-**F**ixed representation!

Here we have a chance to make all (**LL, FL, LF, FF**) types.



R-Nodes



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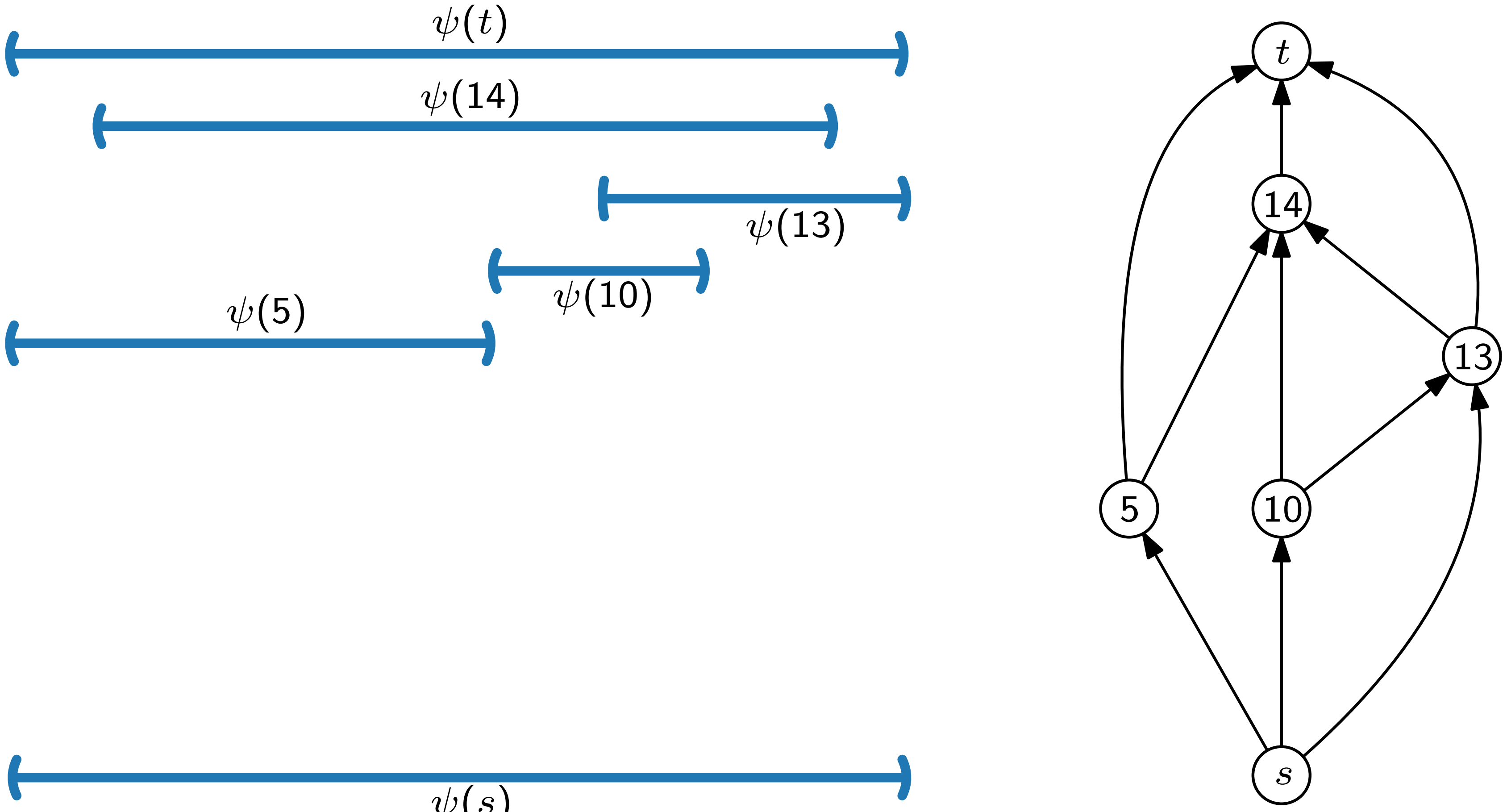

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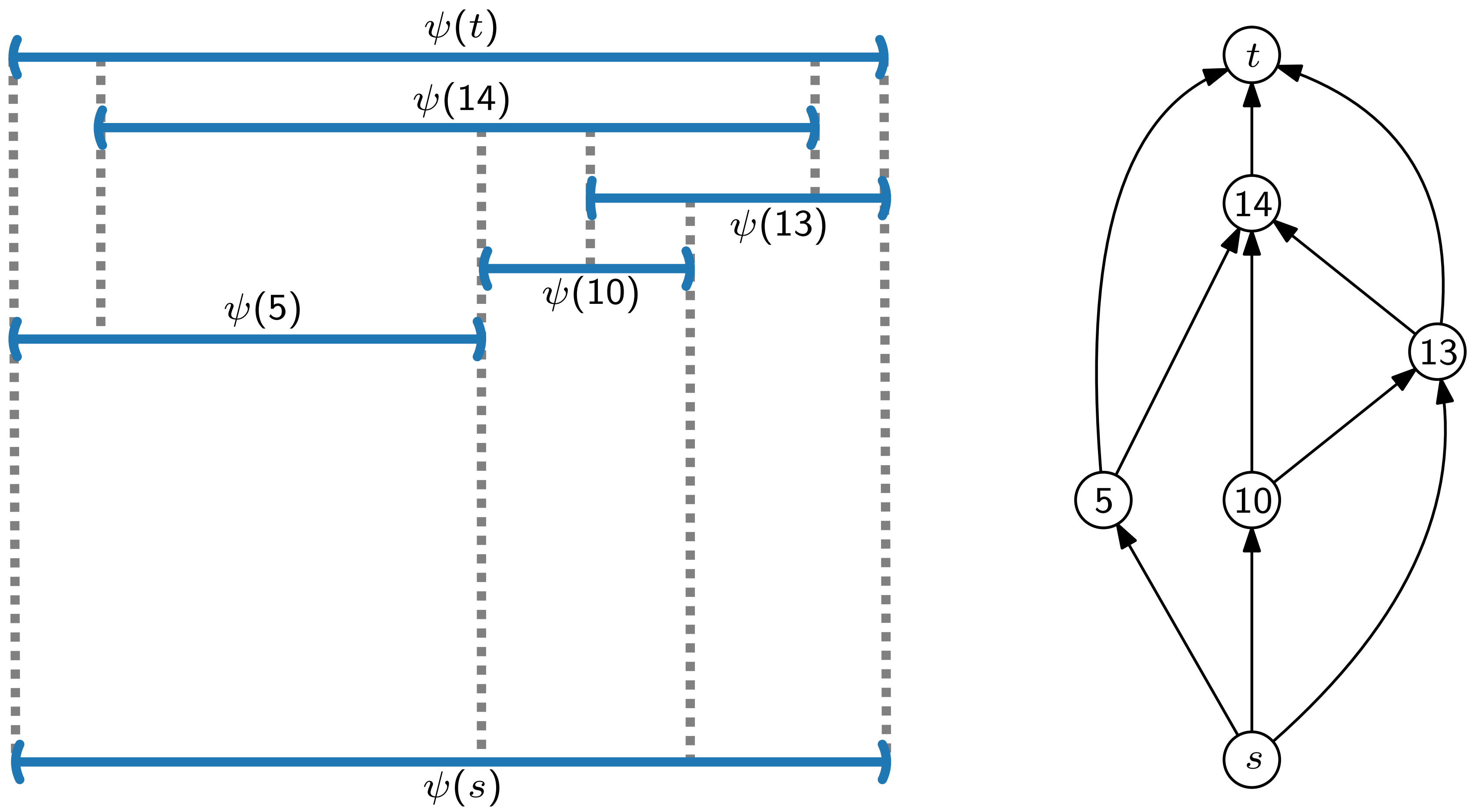

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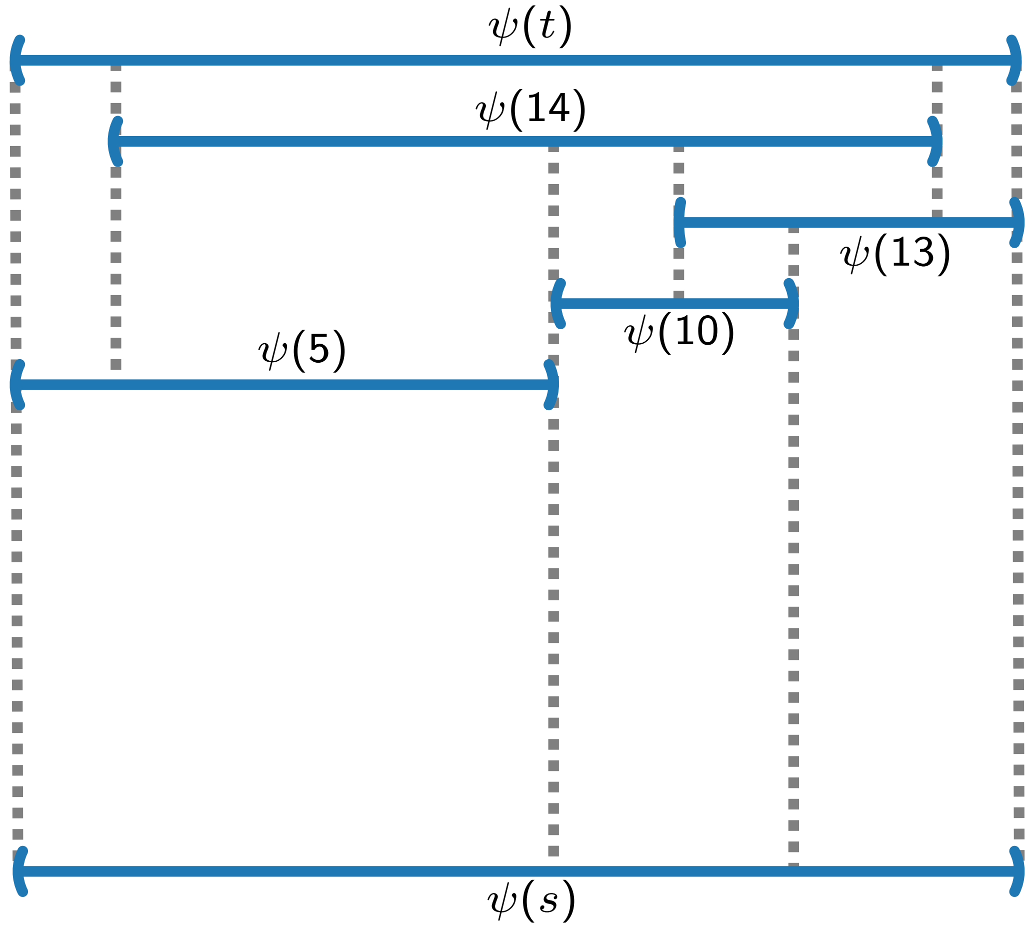

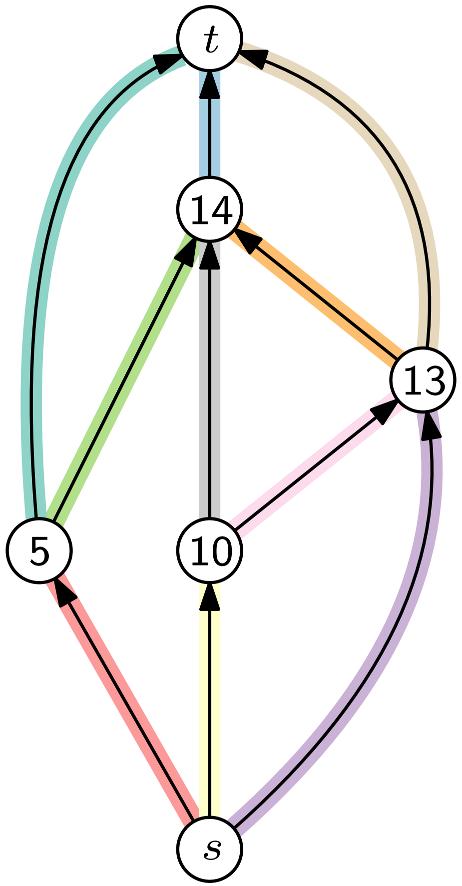

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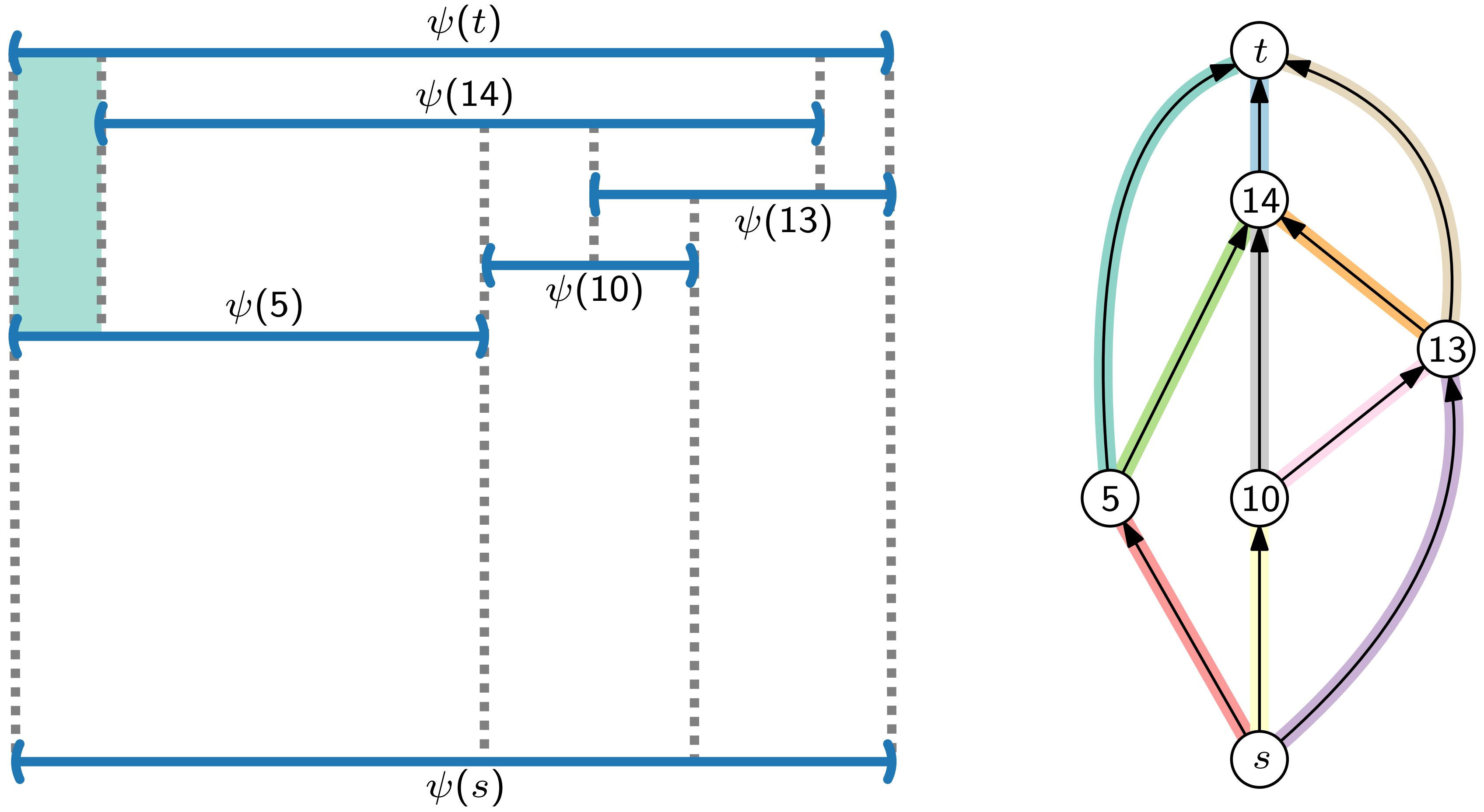

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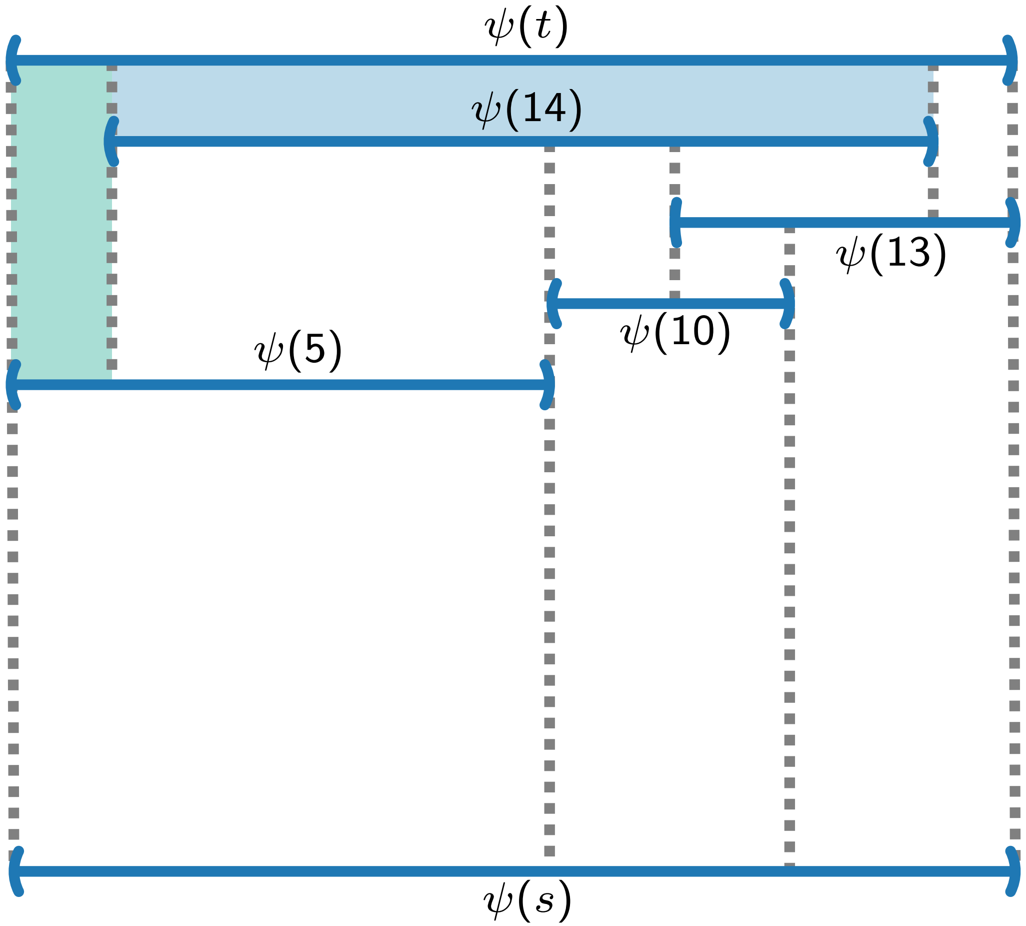

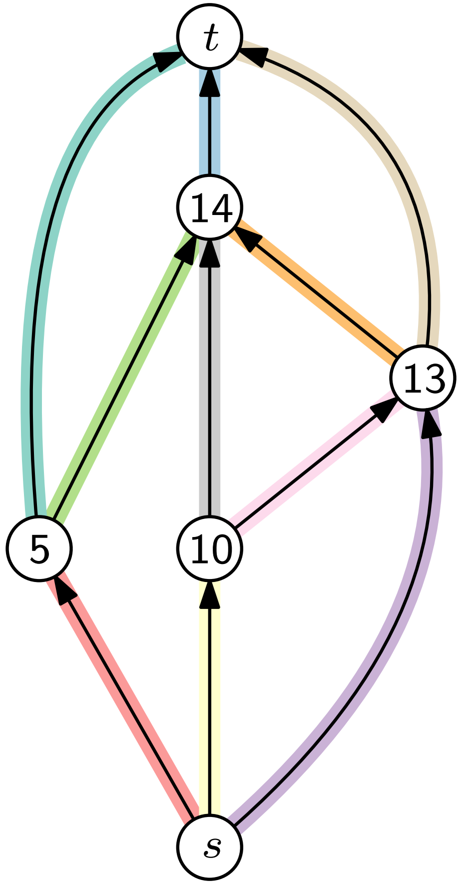

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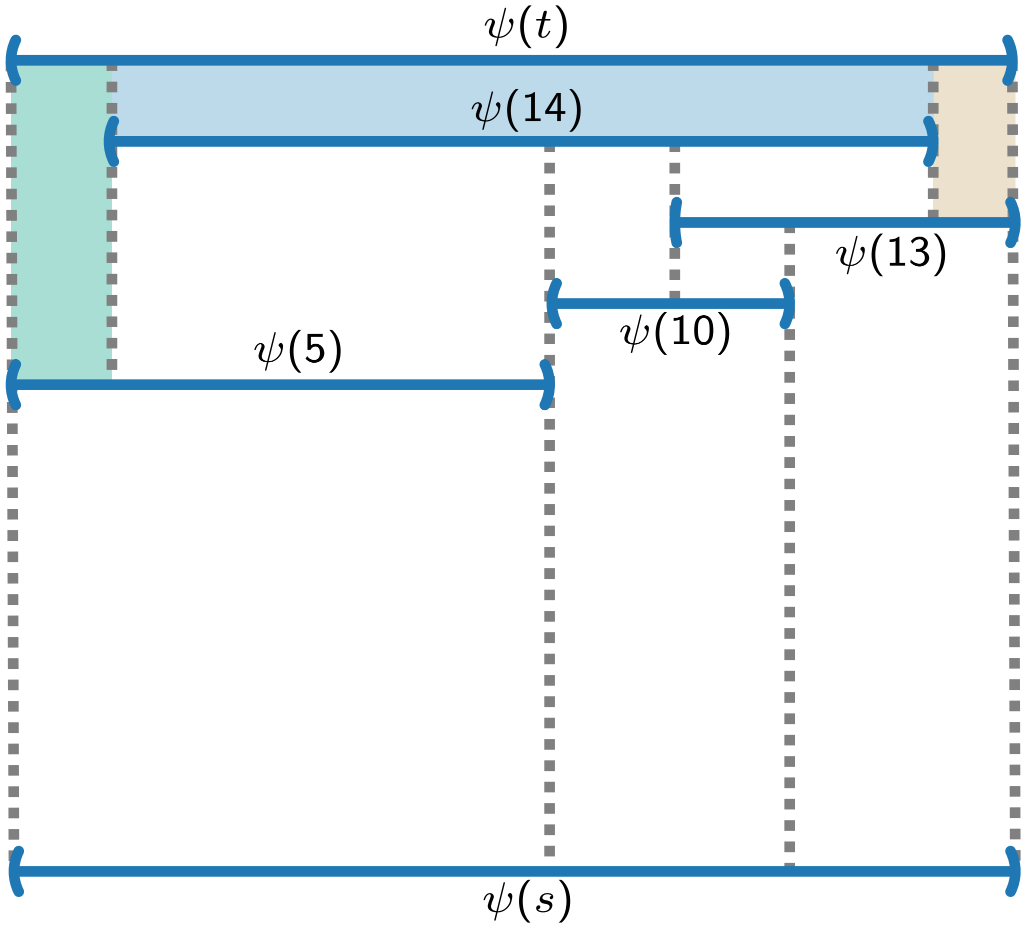
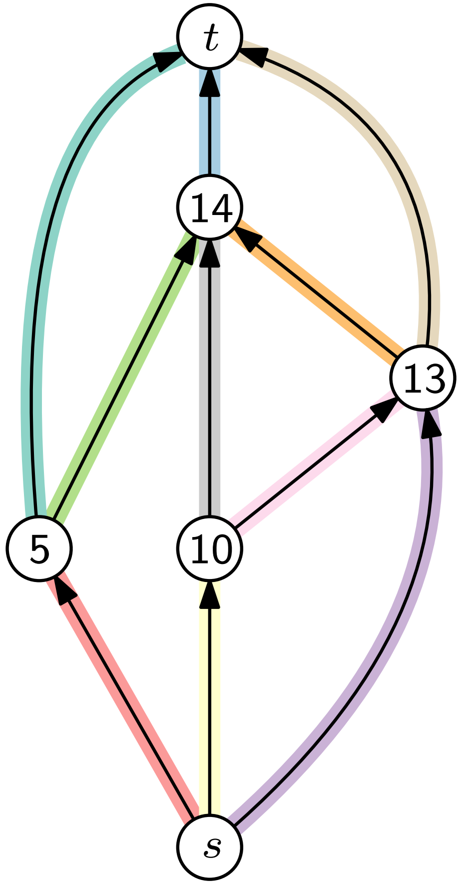
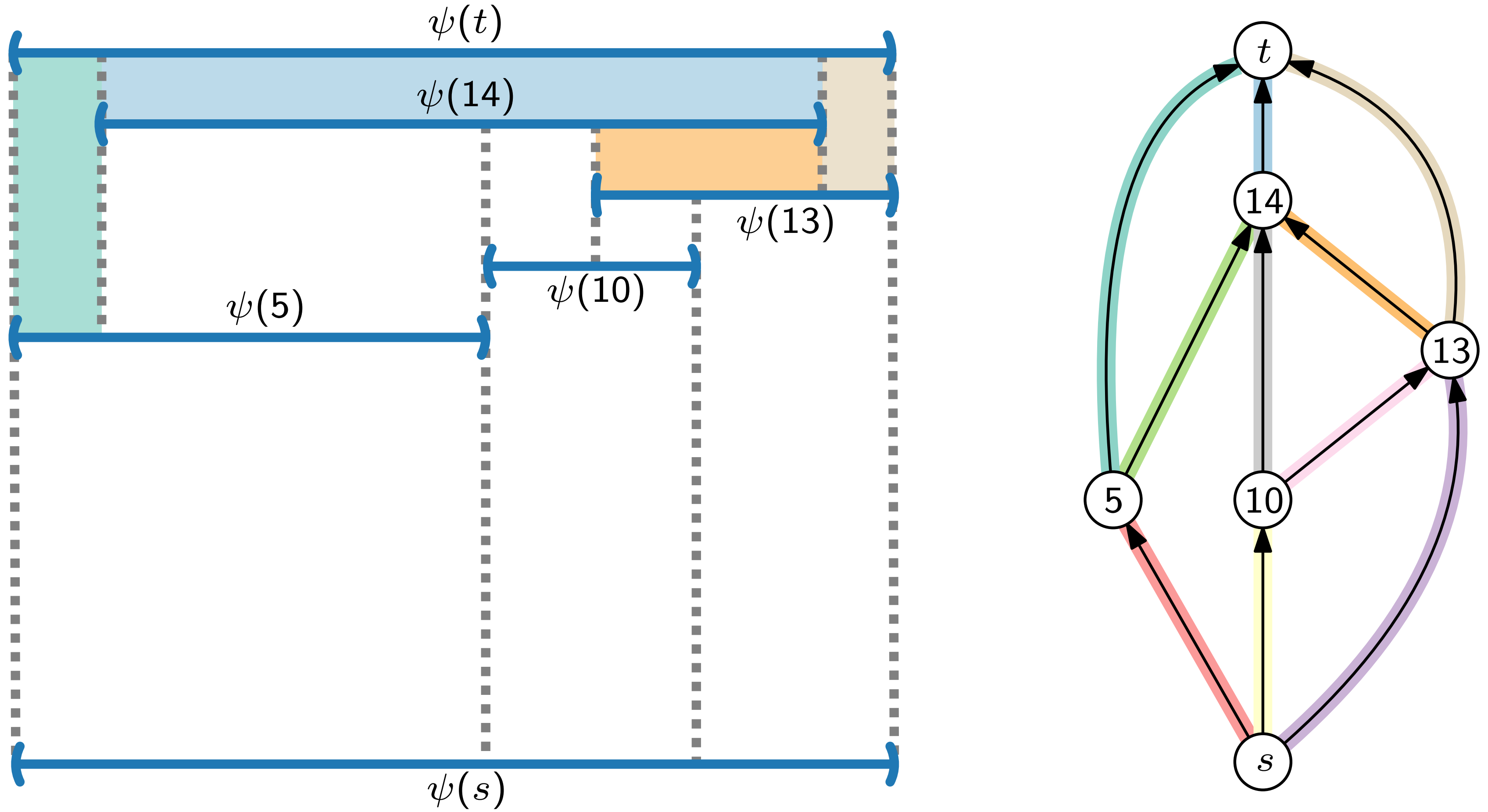




R-Nodes



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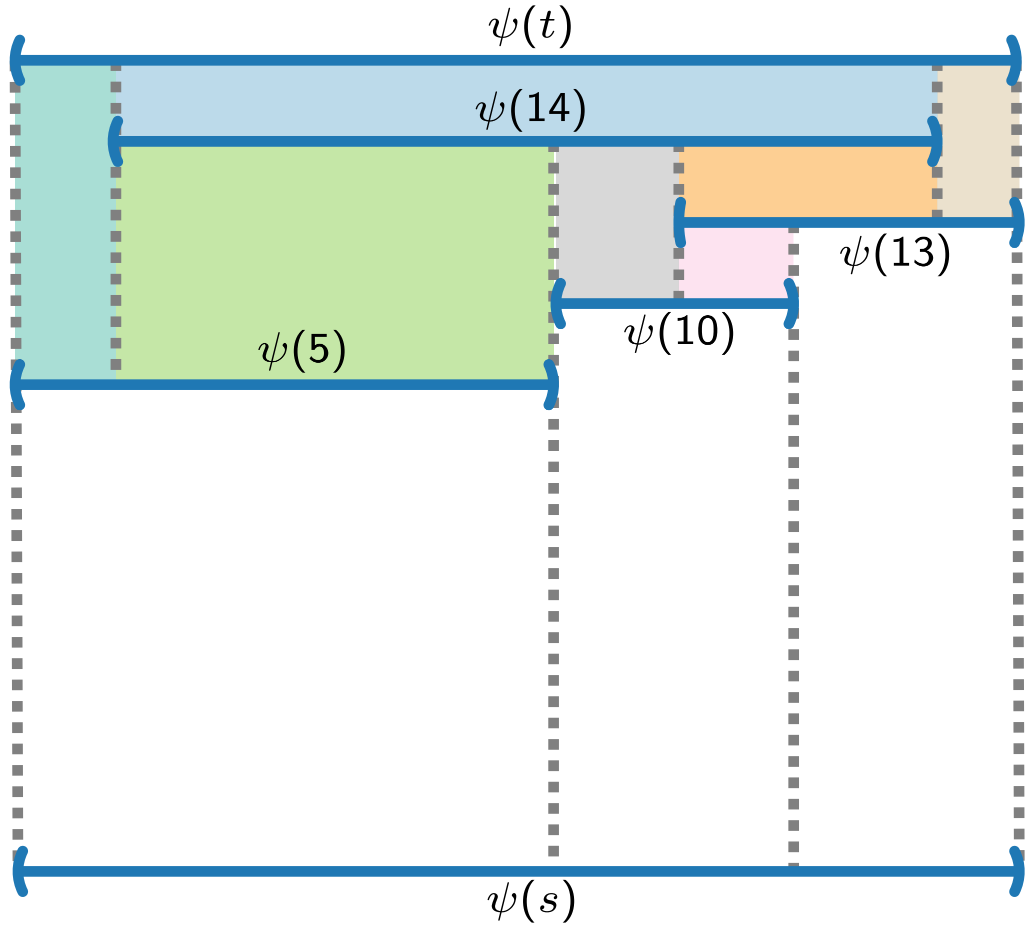

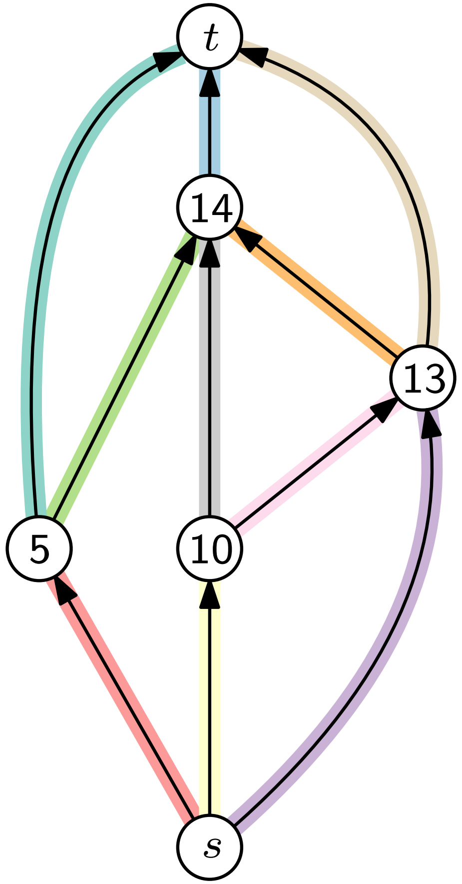

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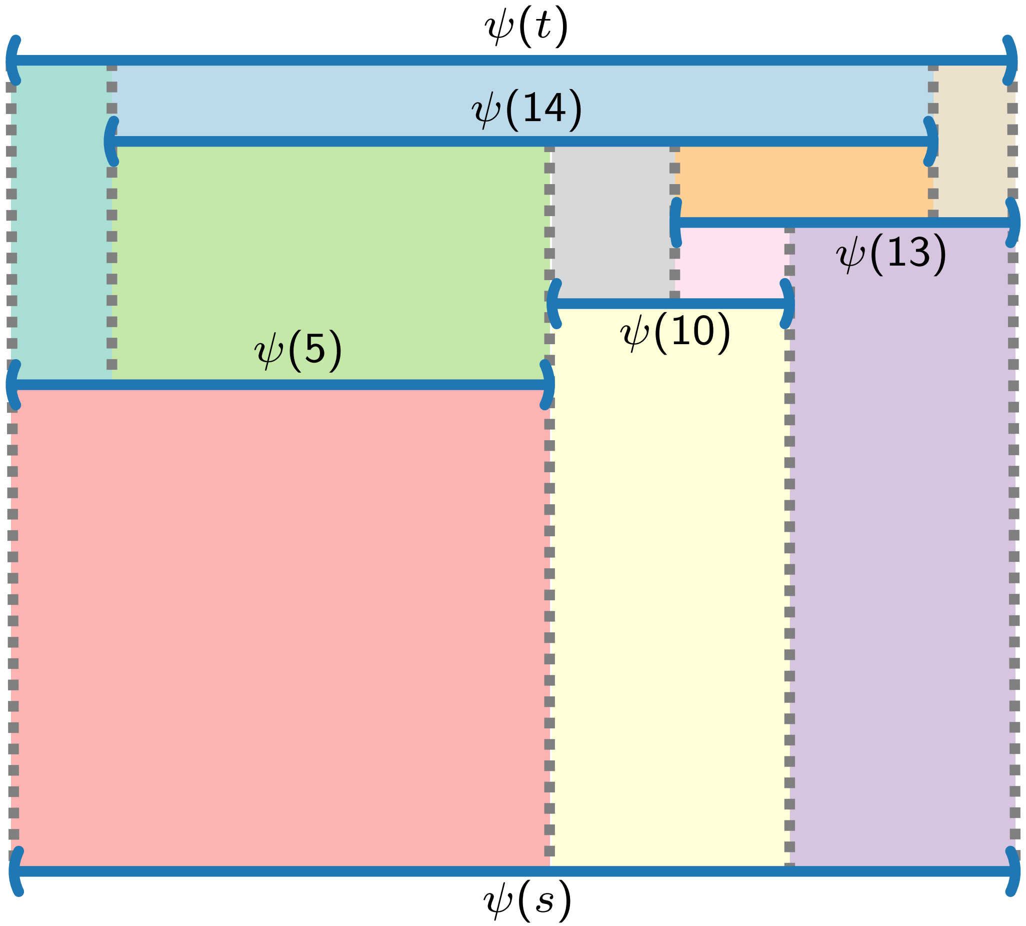

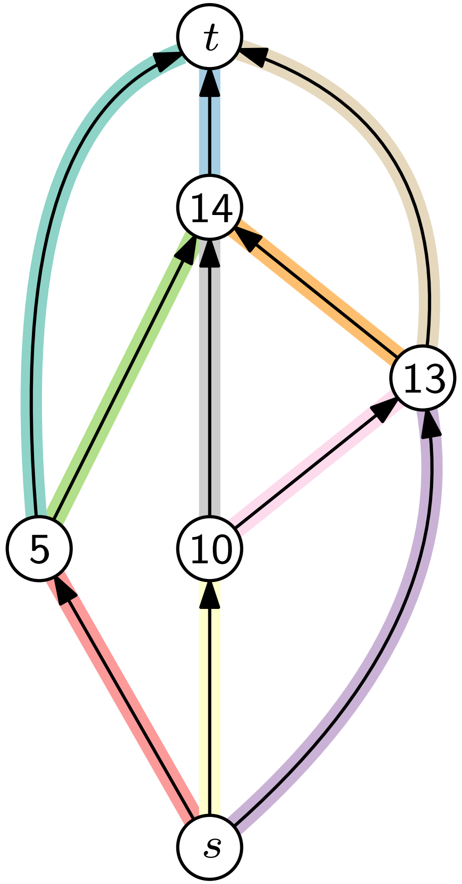

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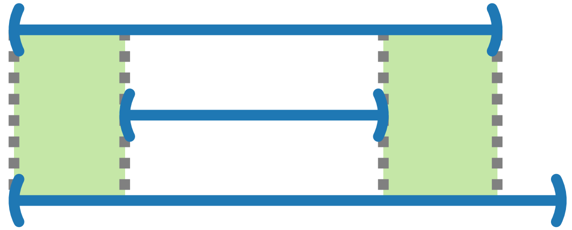

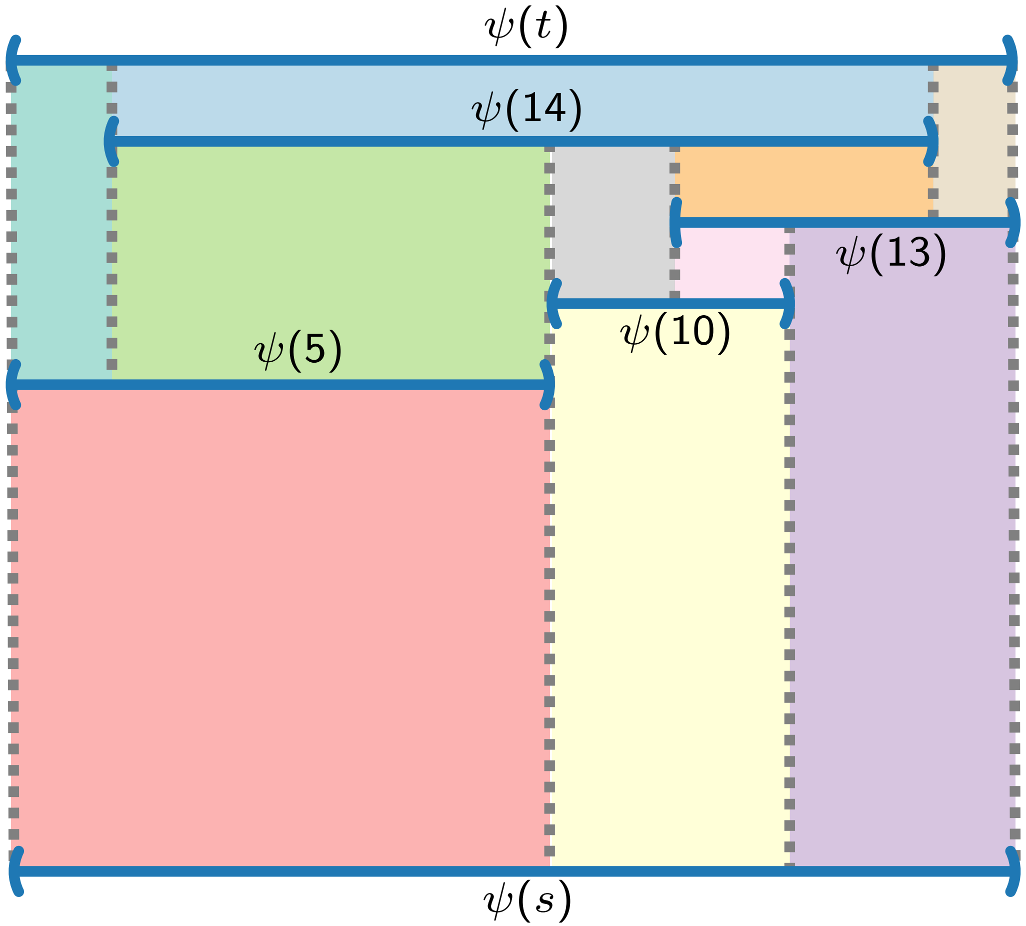

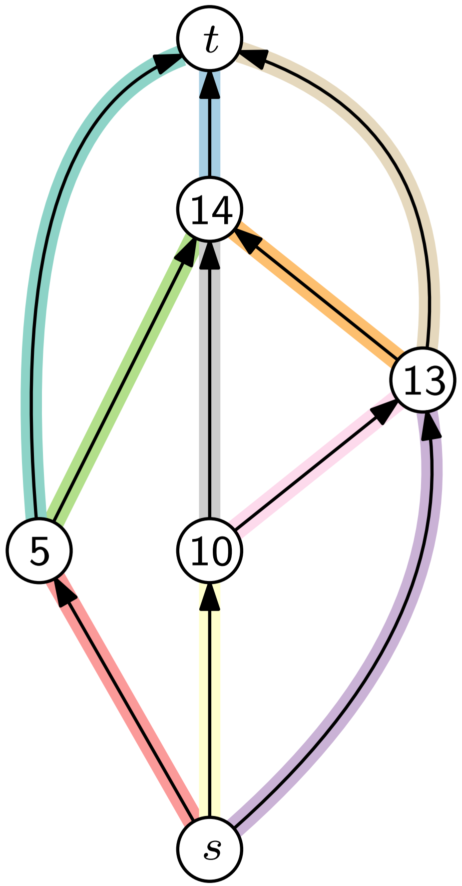

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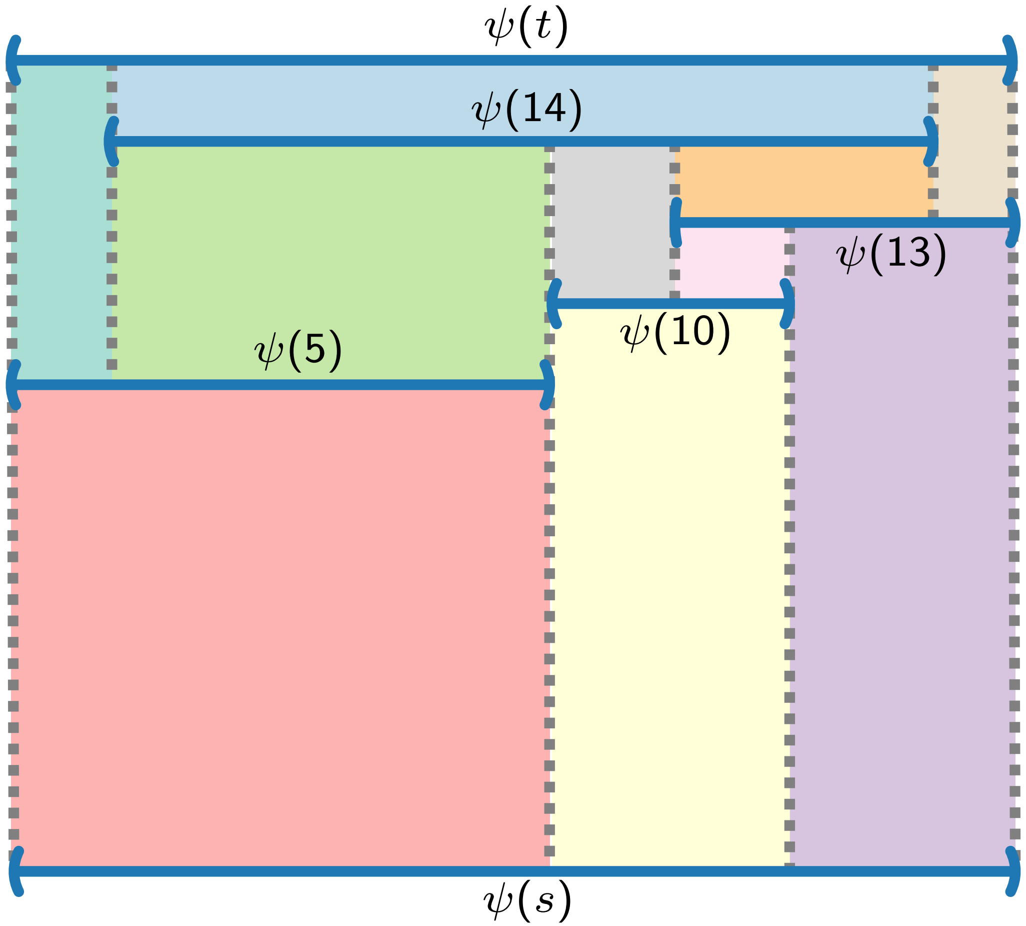

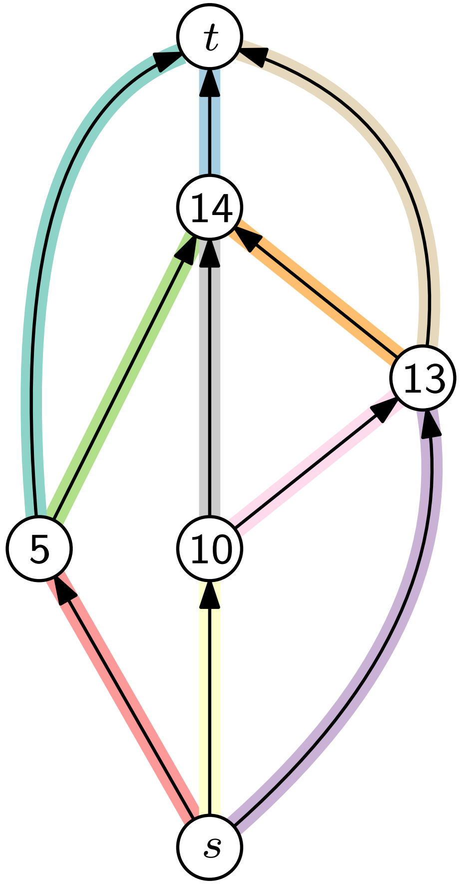

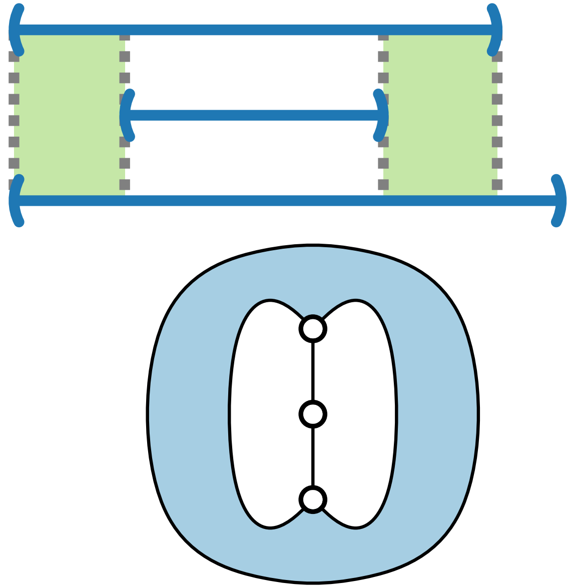

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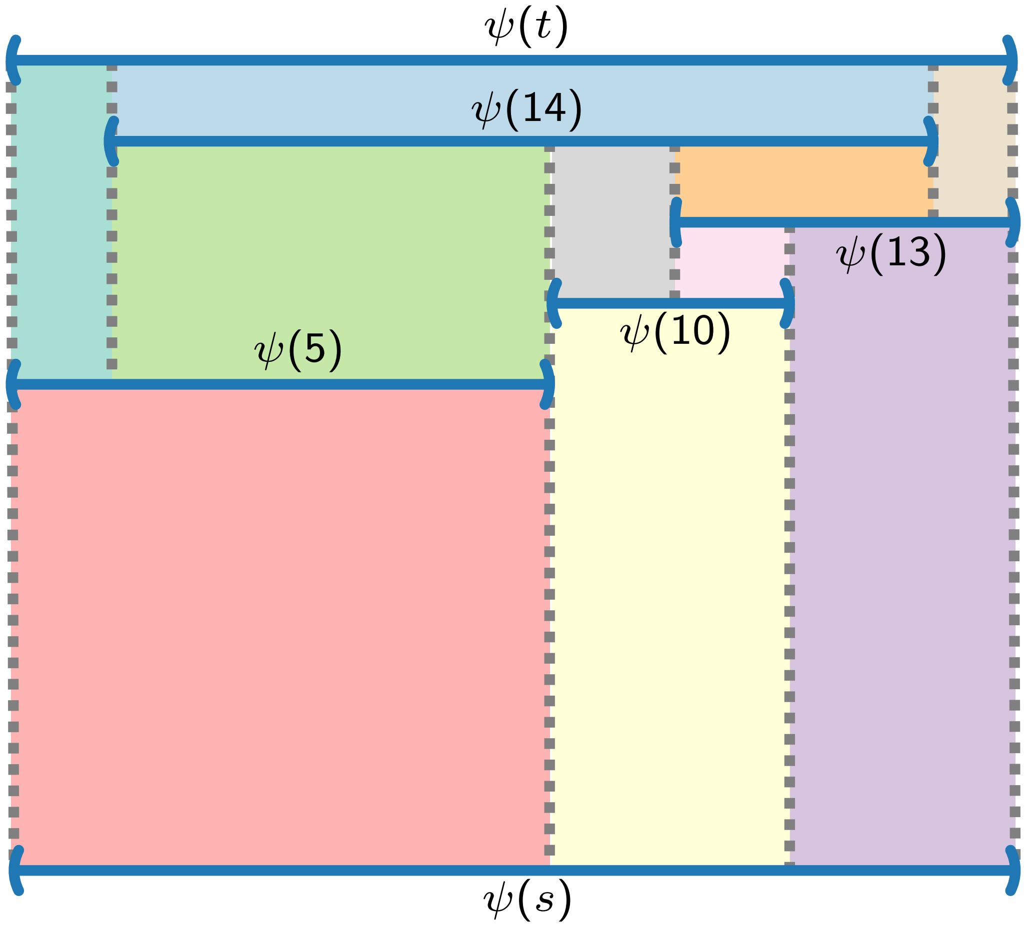

$\psi(t)$
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$\psi(13)$
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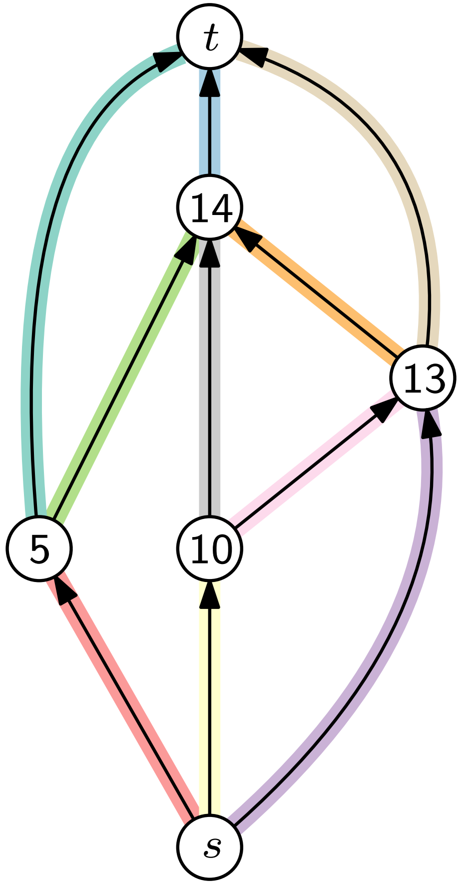
$\psi(5)$
$\psi(s)$



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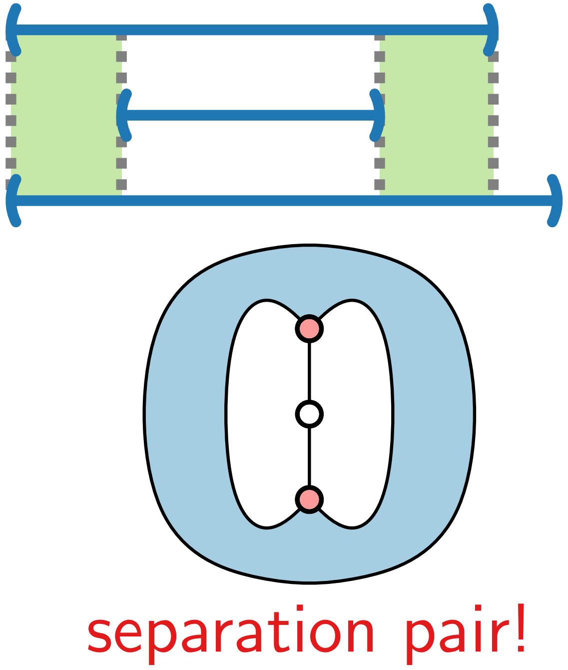
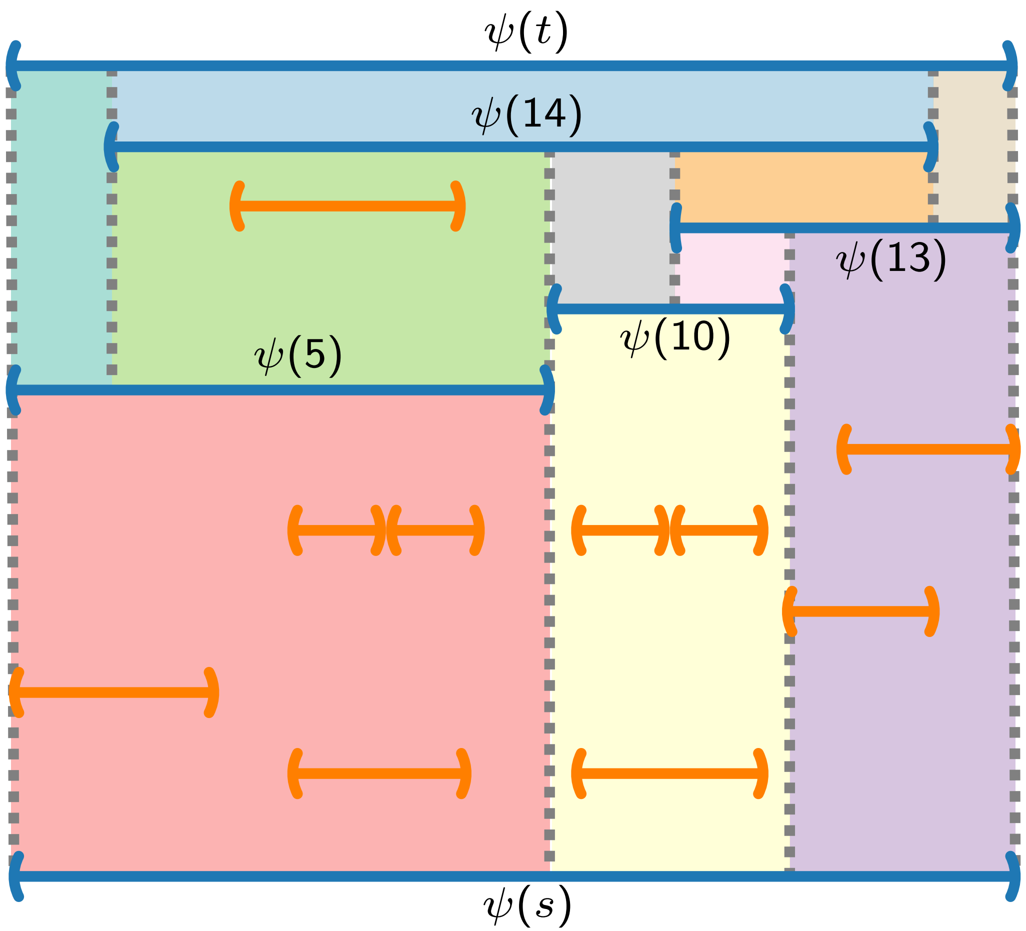
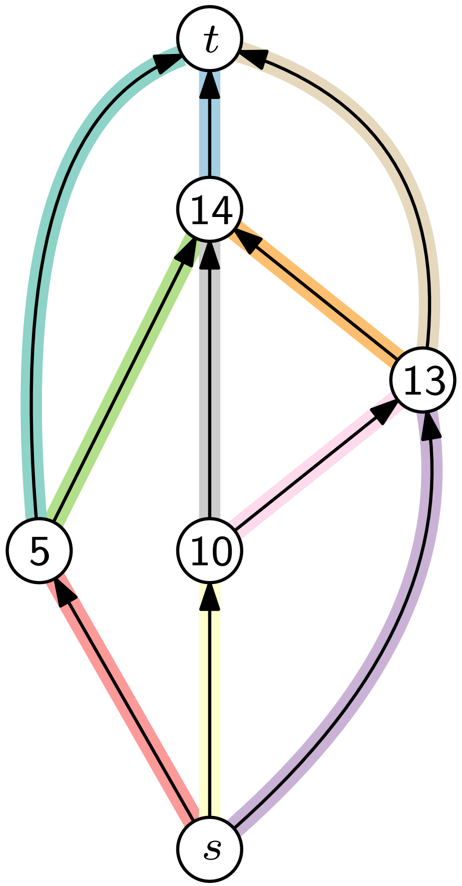
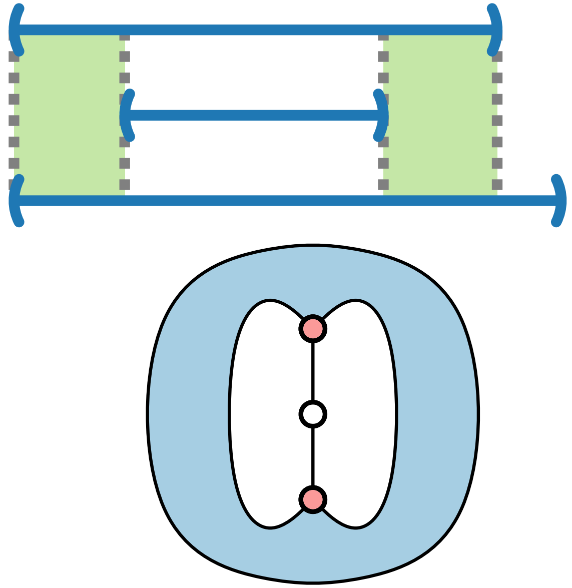


separation pair!

R-Nodes







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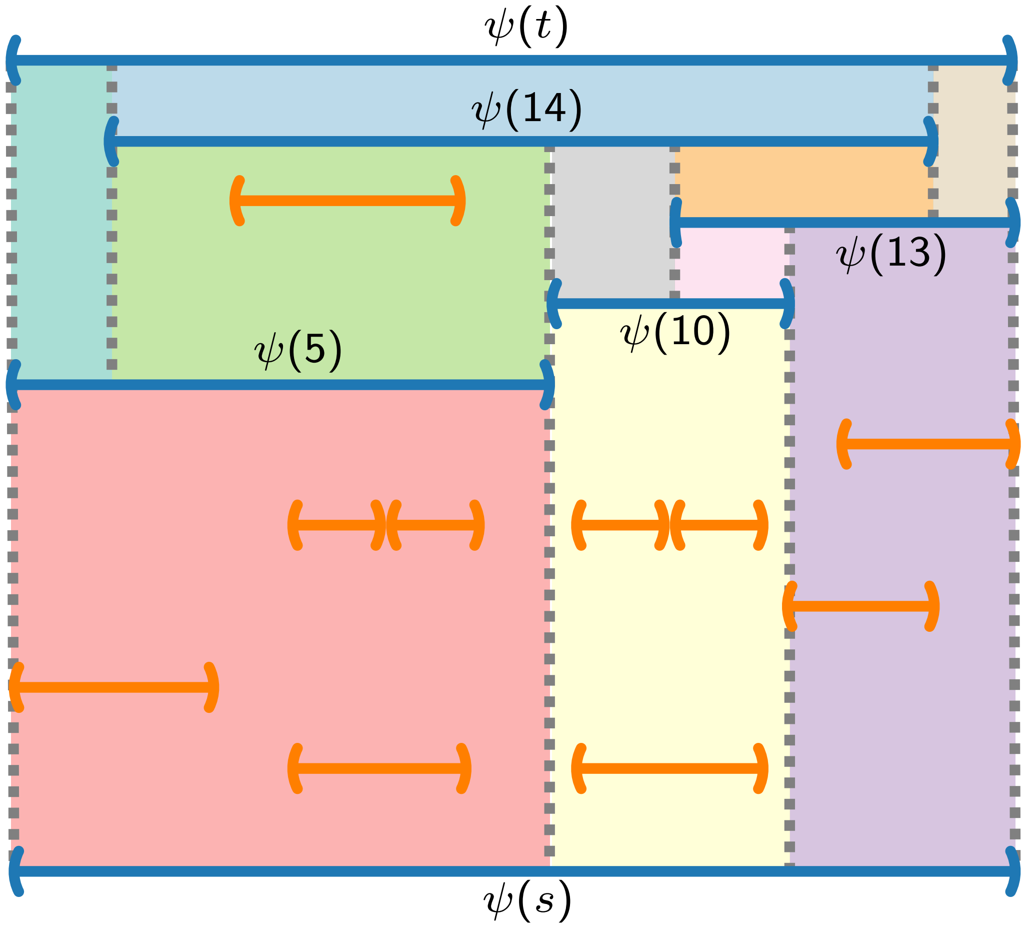

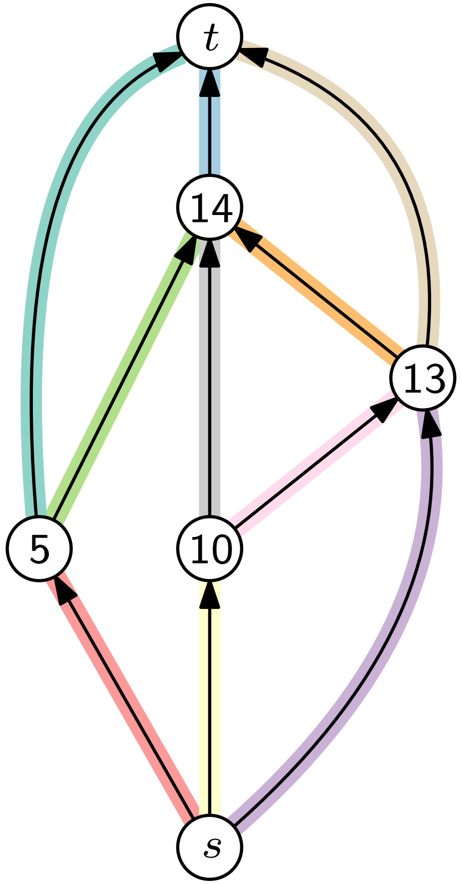

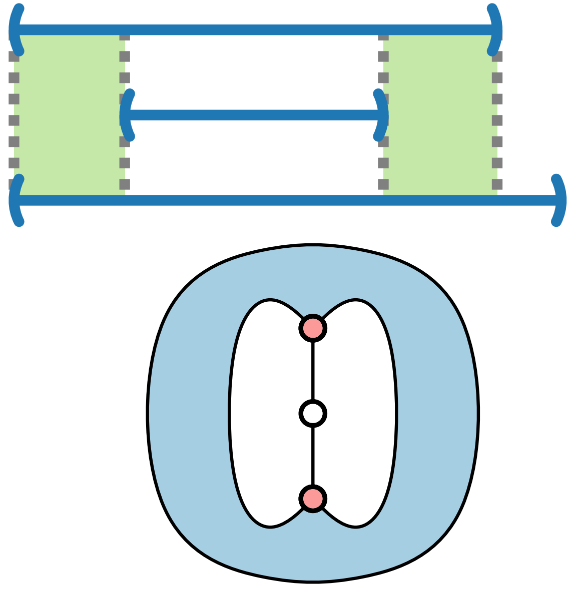

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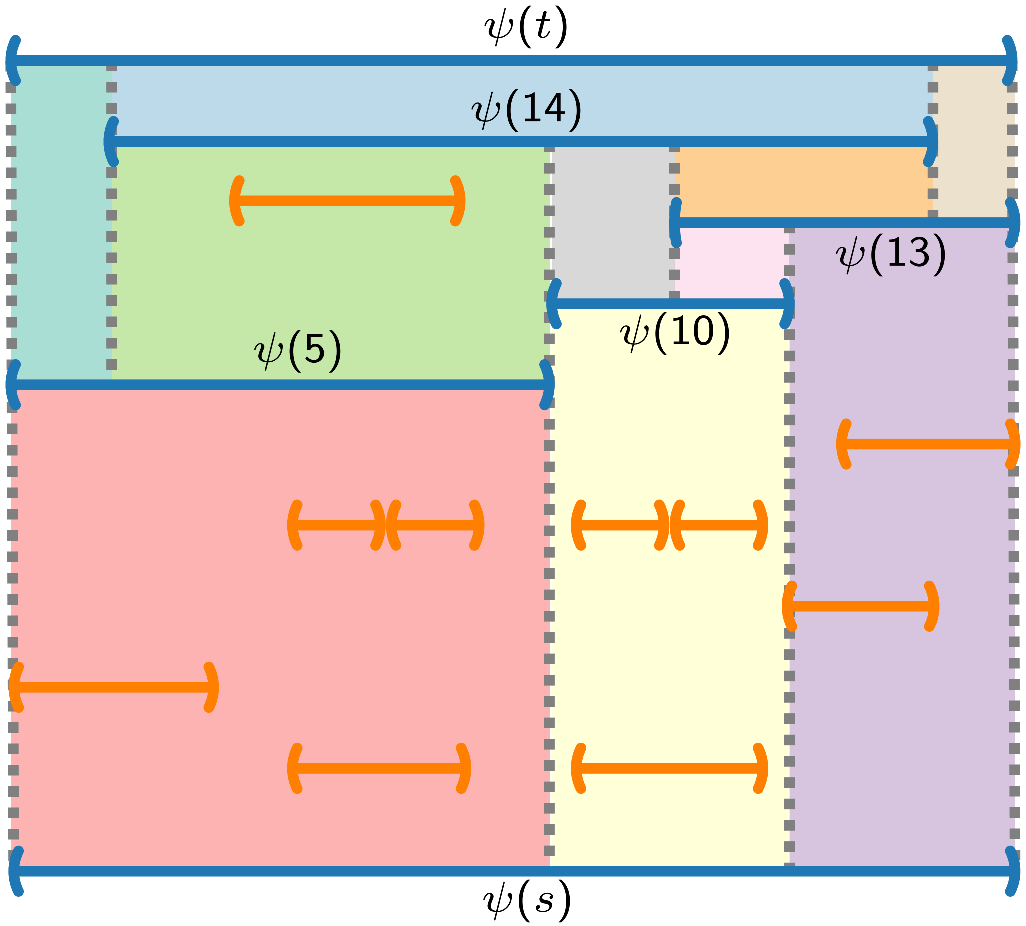

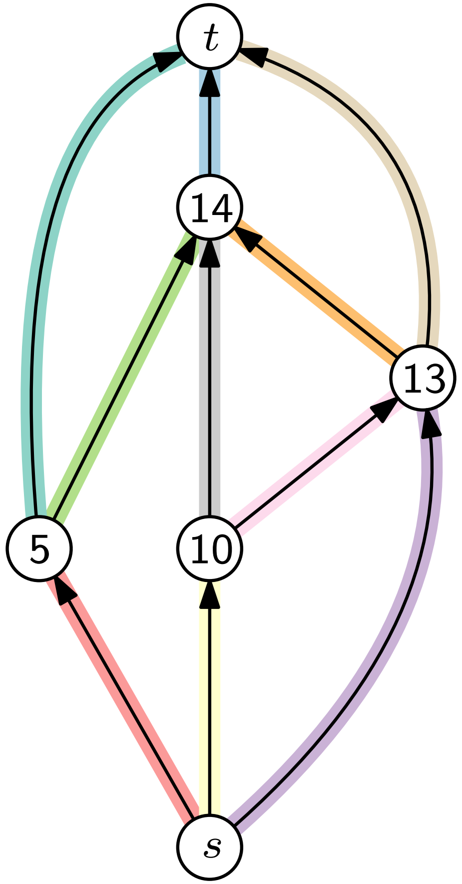

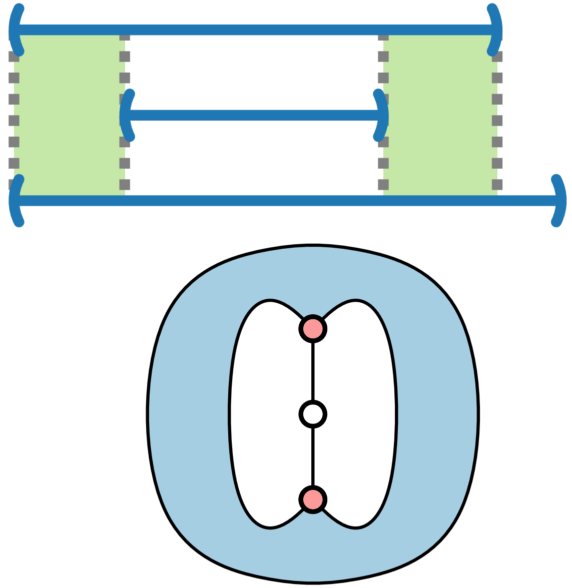

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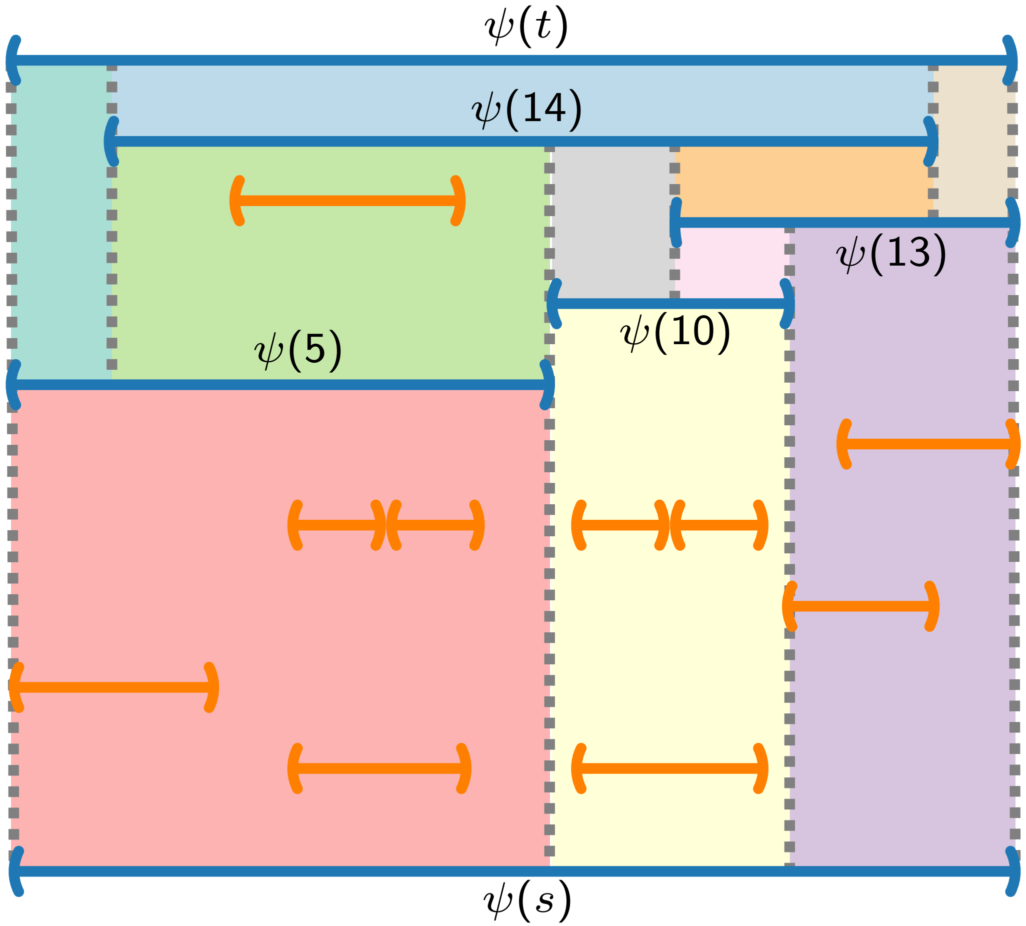

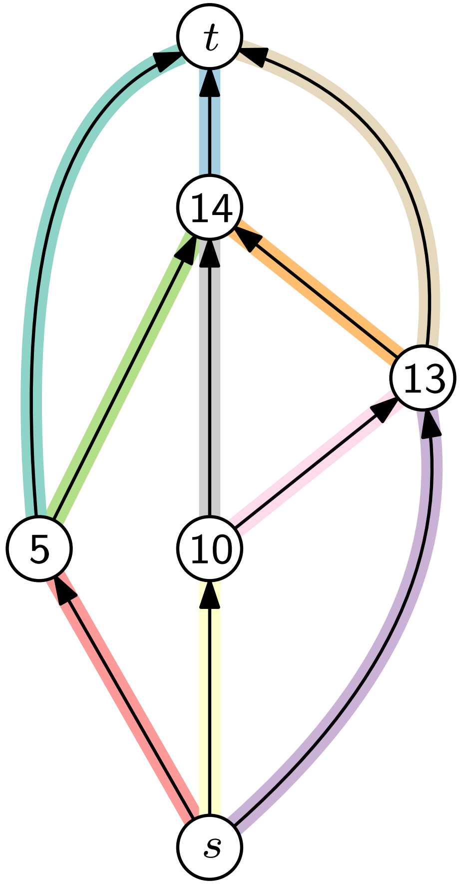

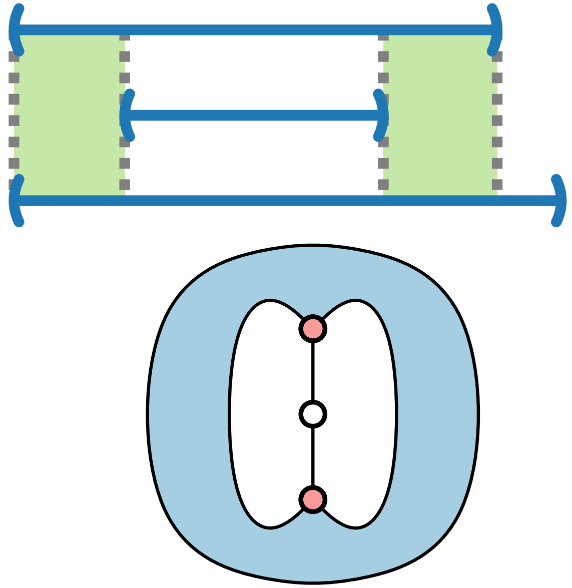

separation pair!

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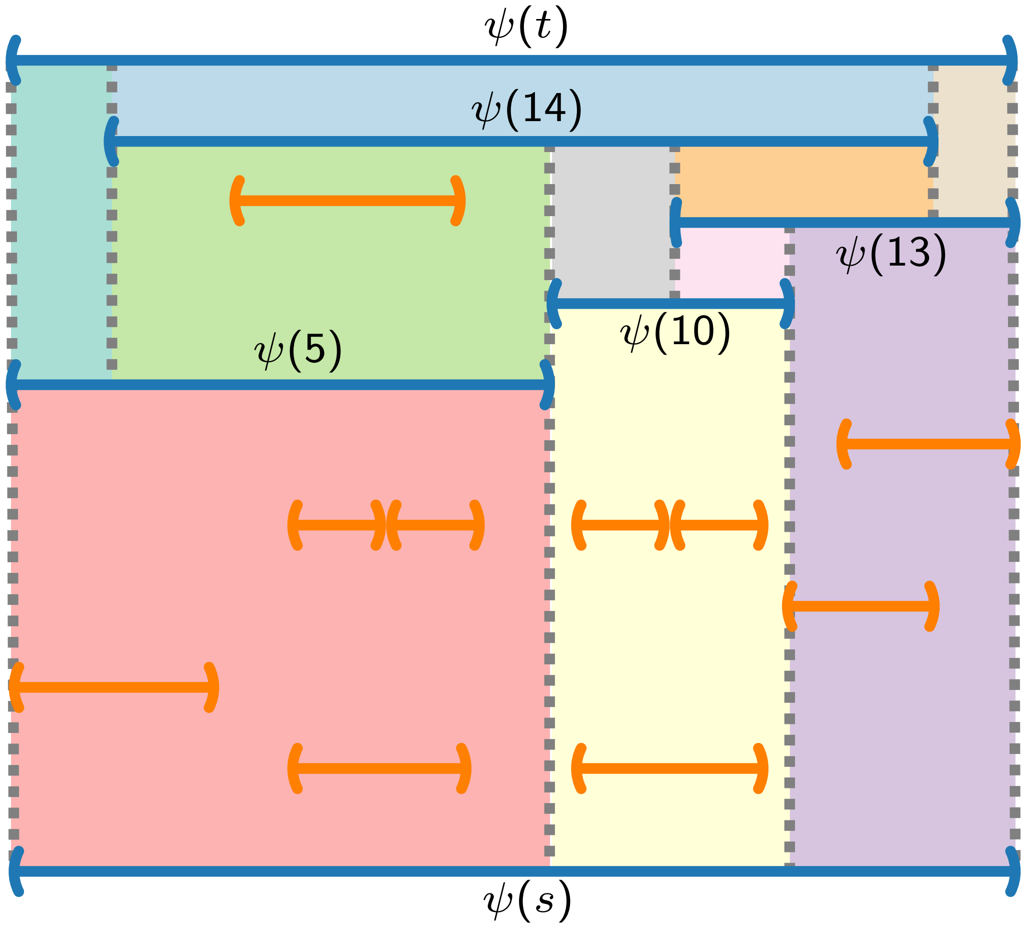

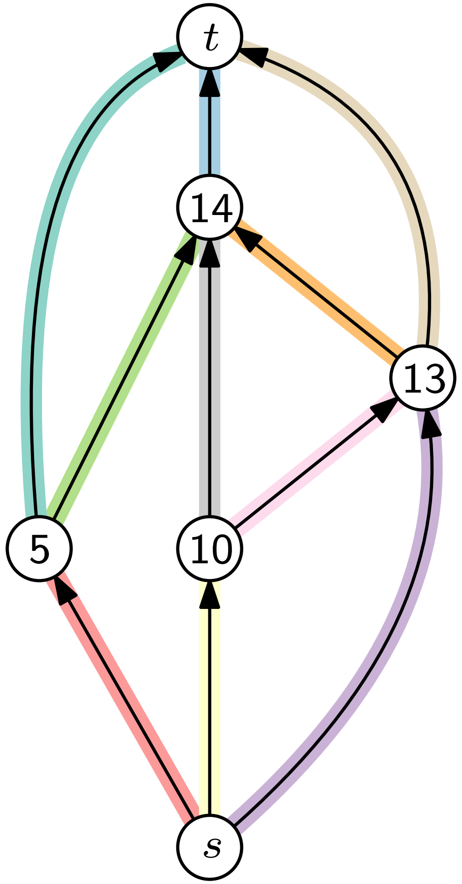

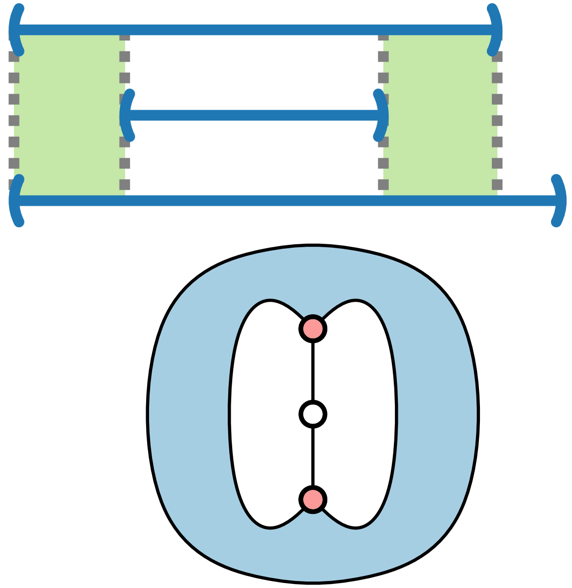

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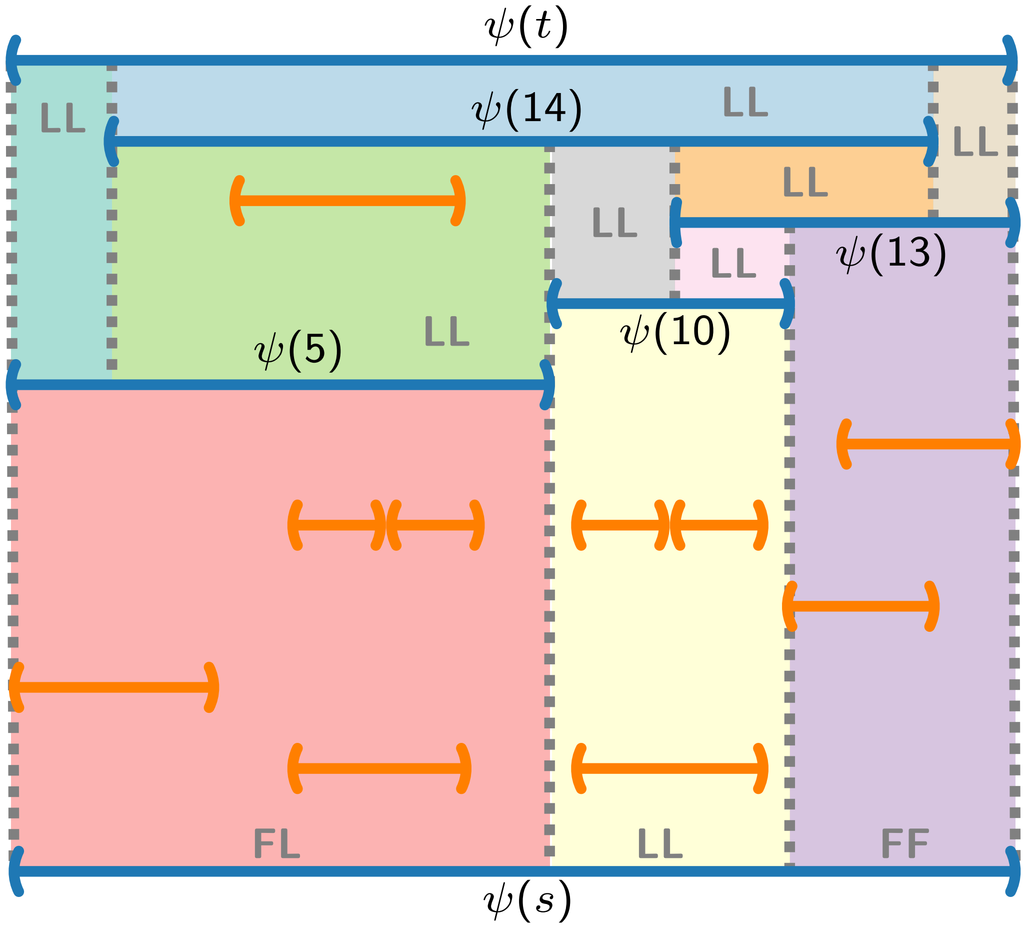

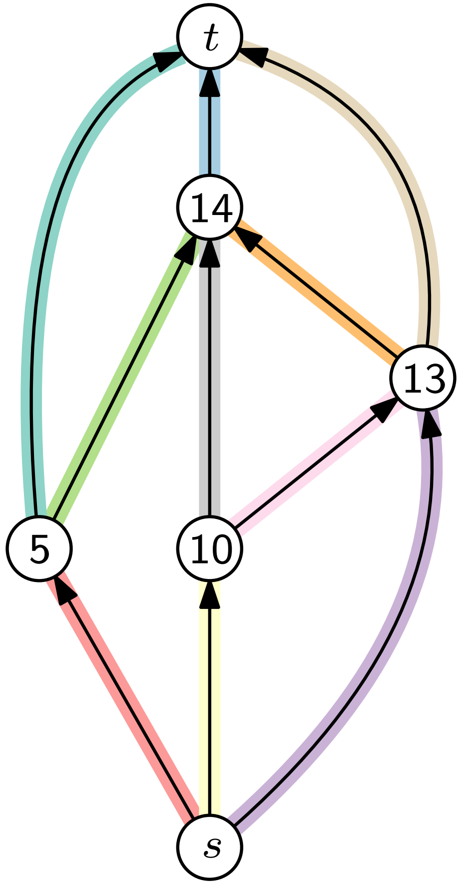

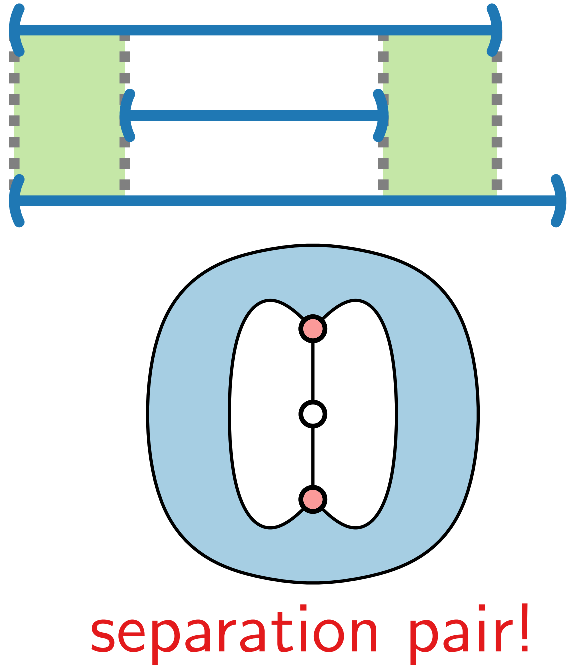

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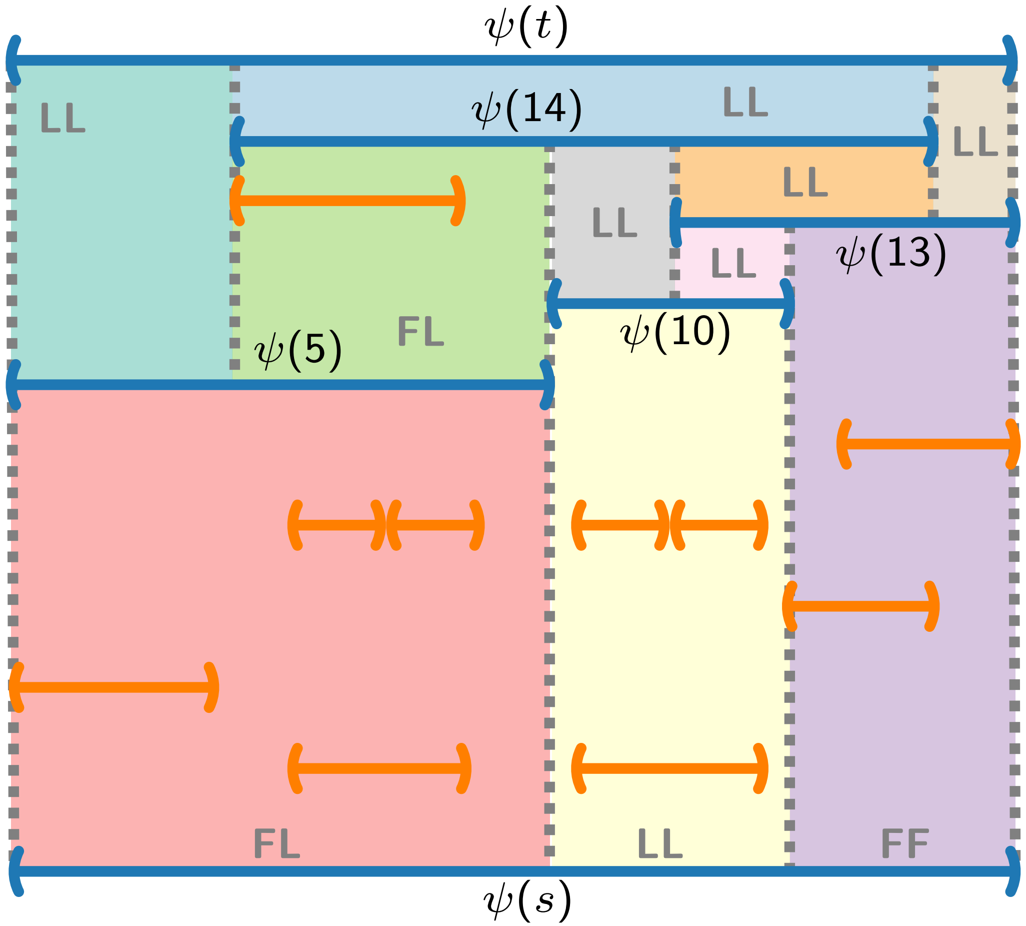

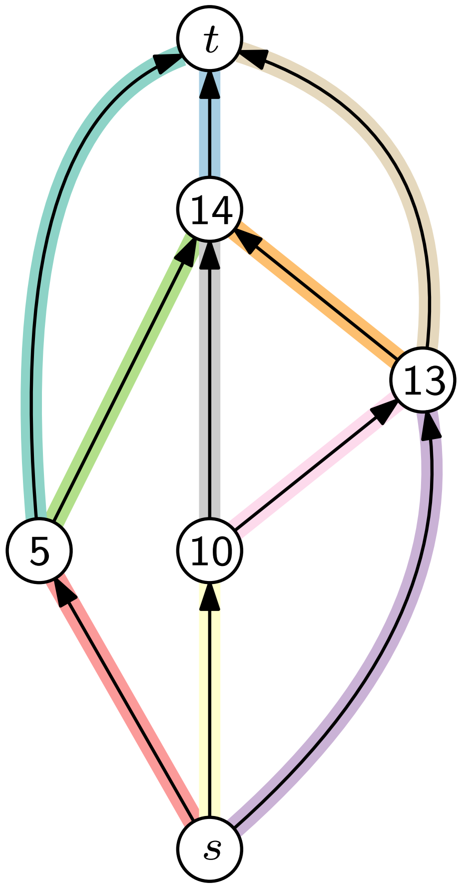

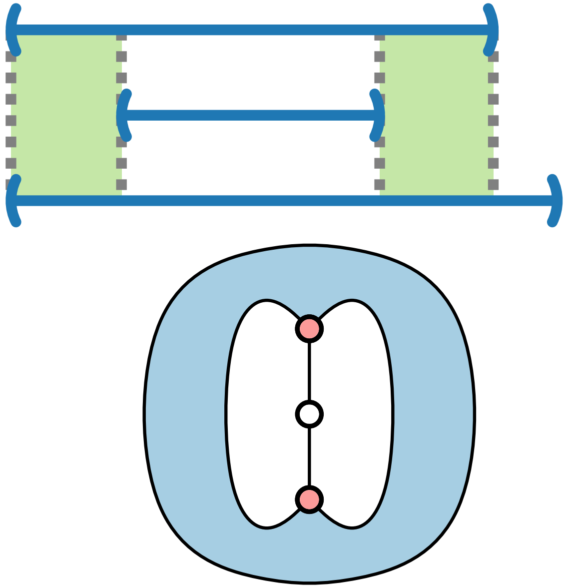

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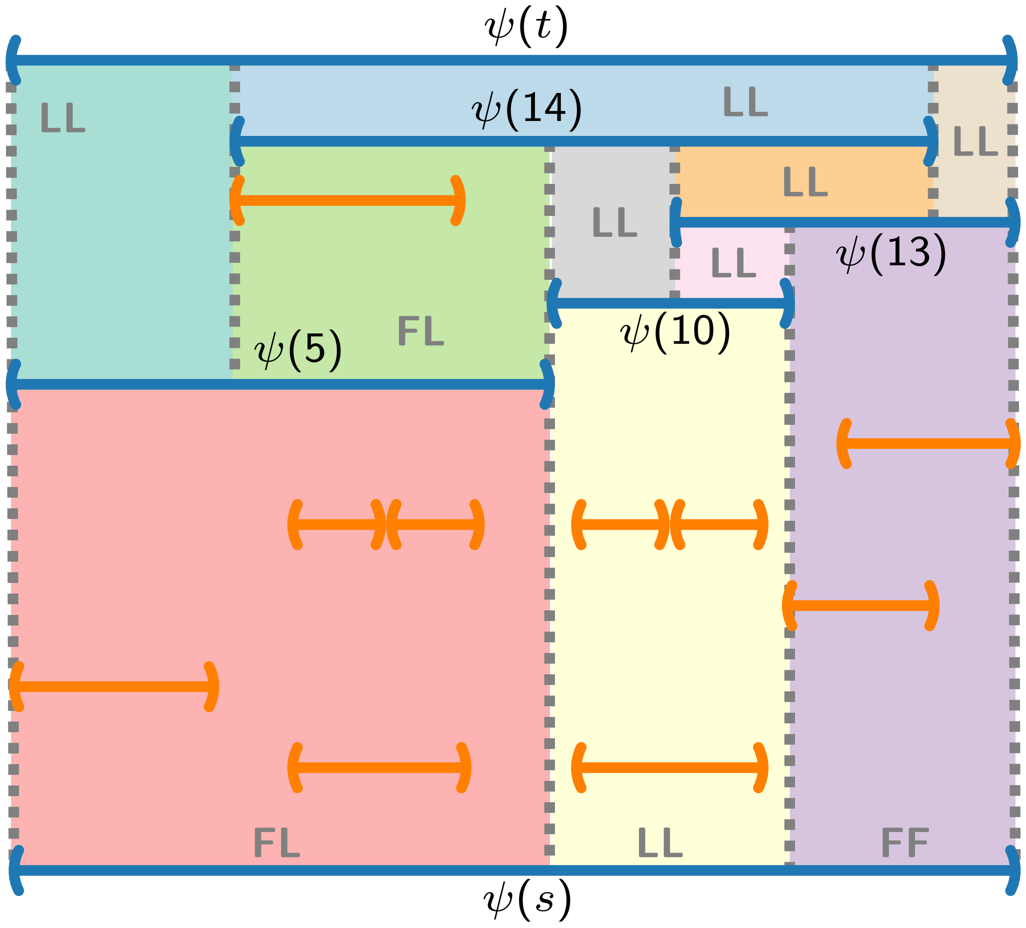

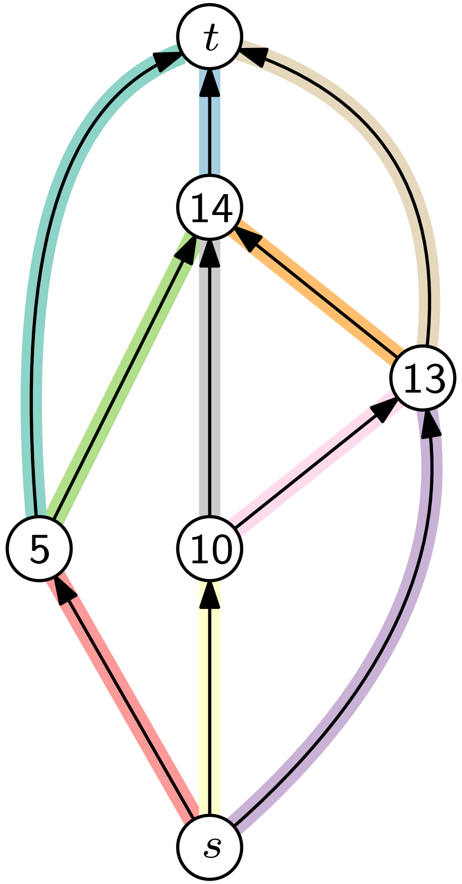

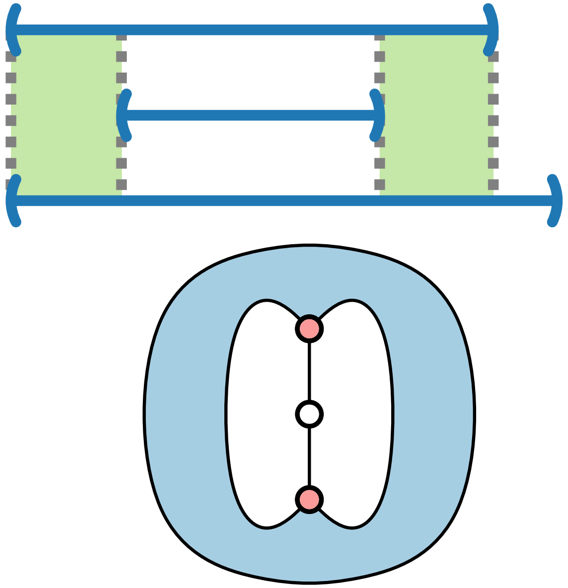

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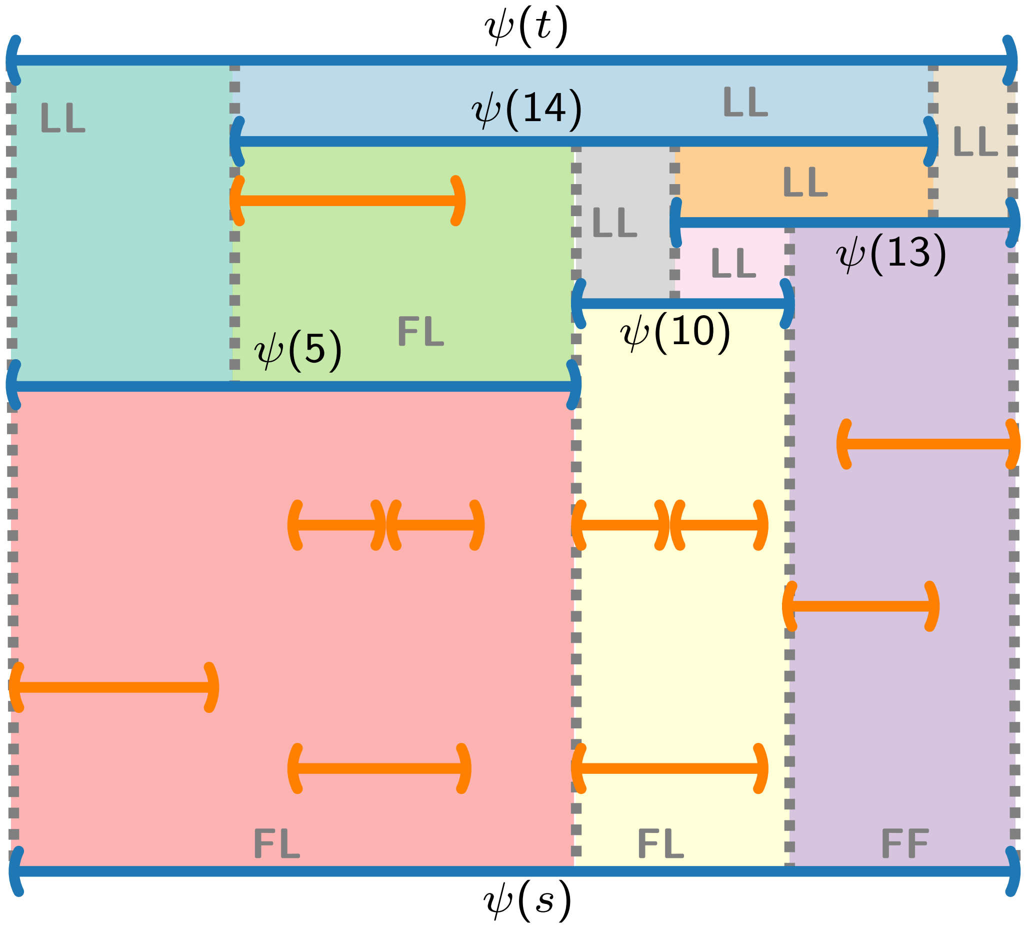

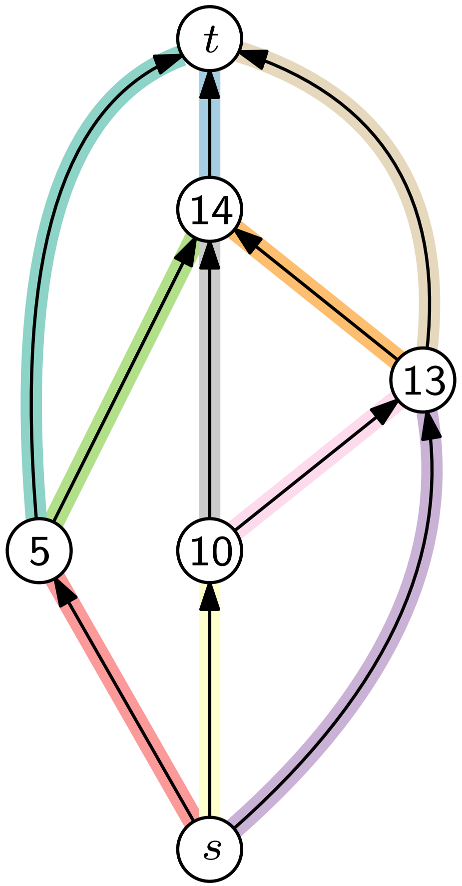

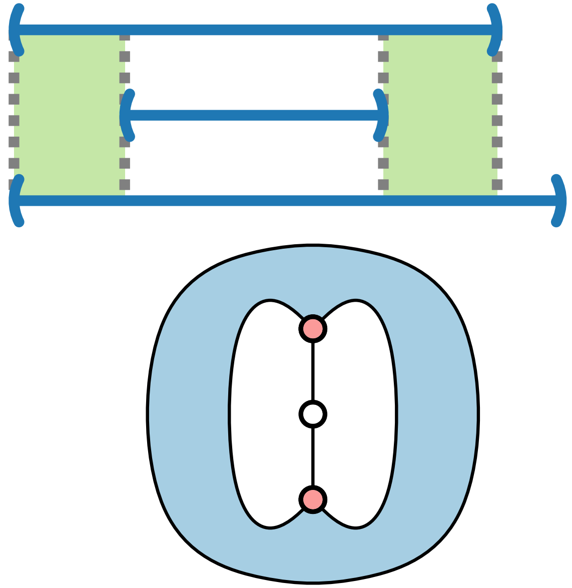

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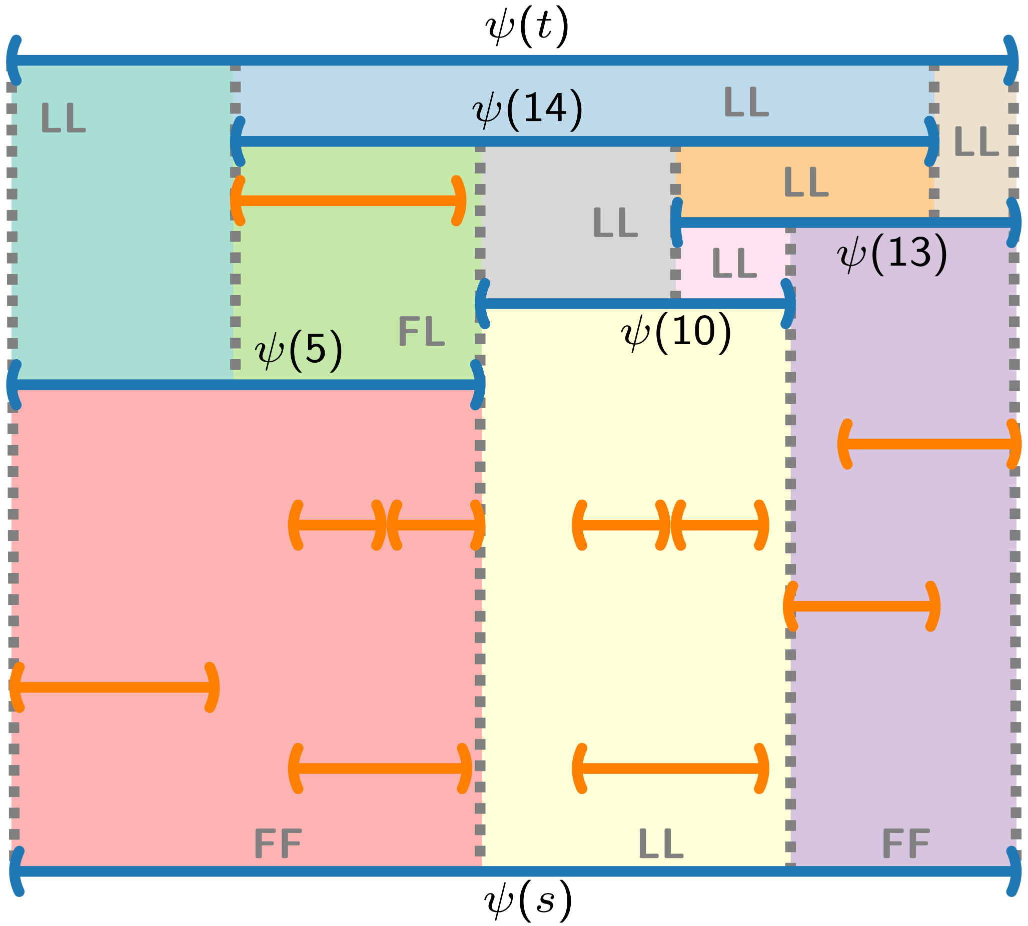

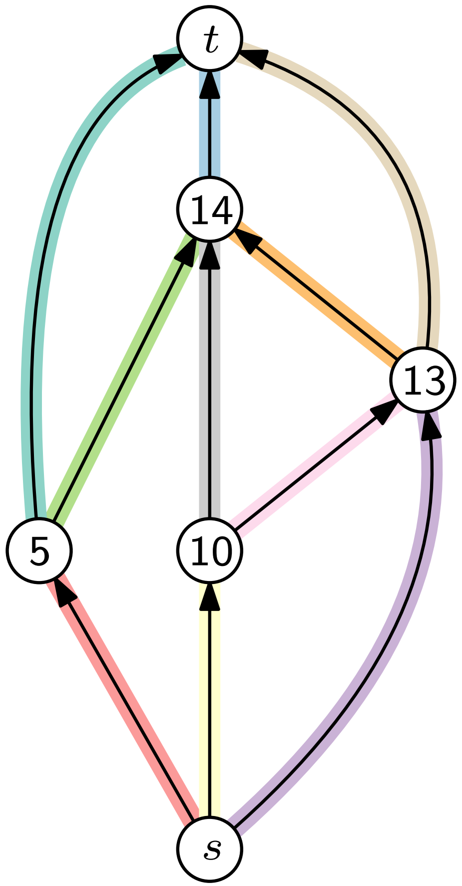

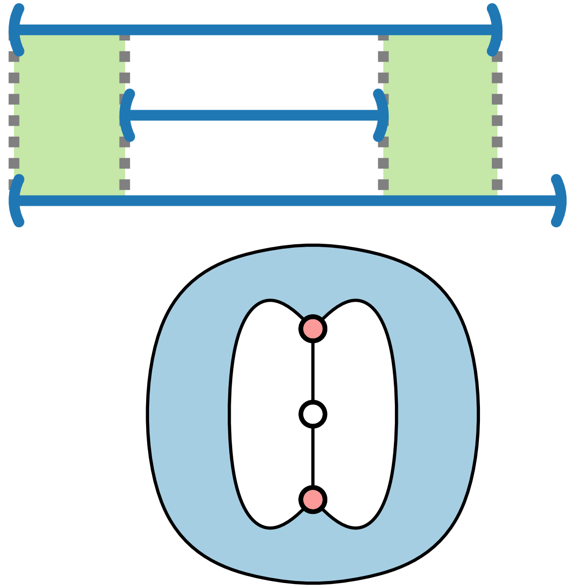

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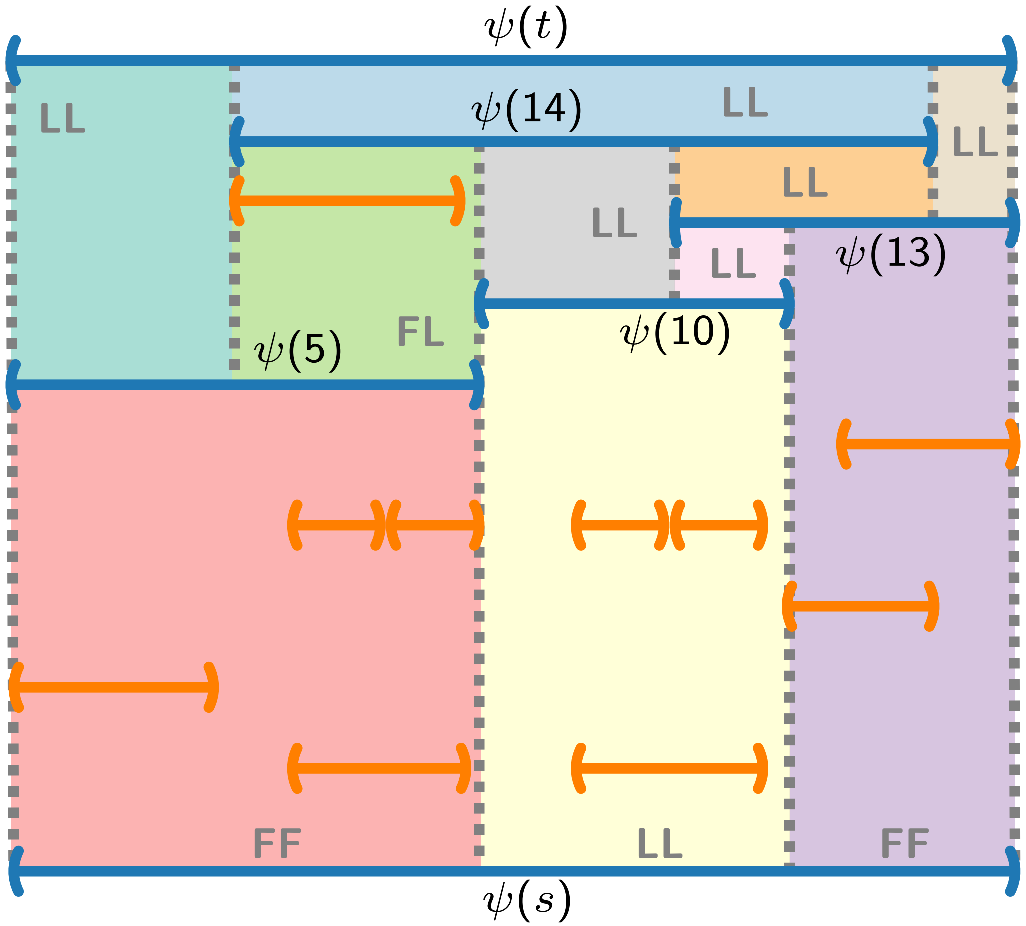

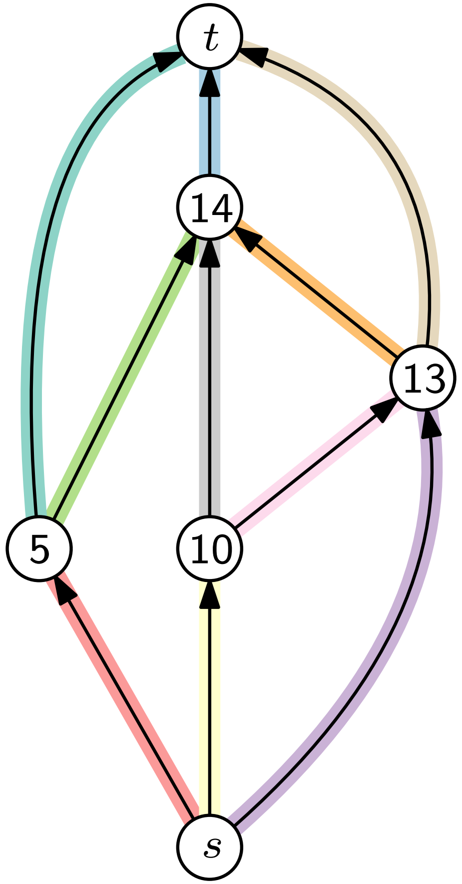

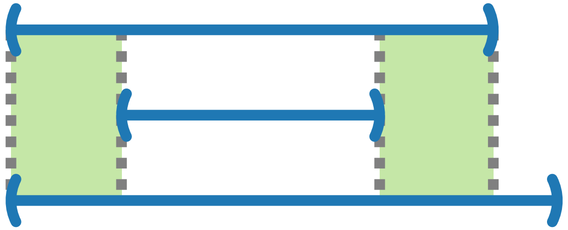

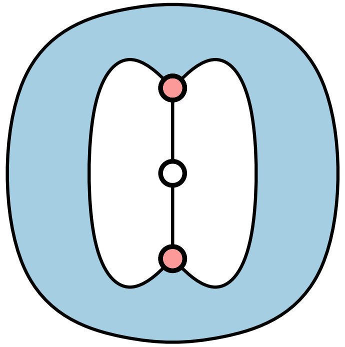

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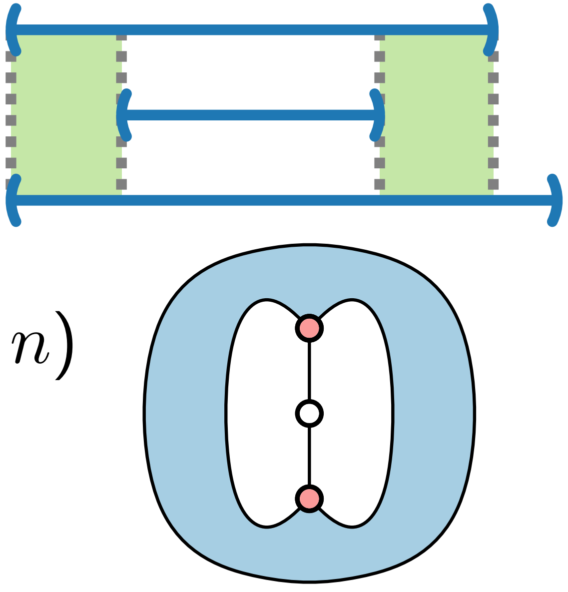

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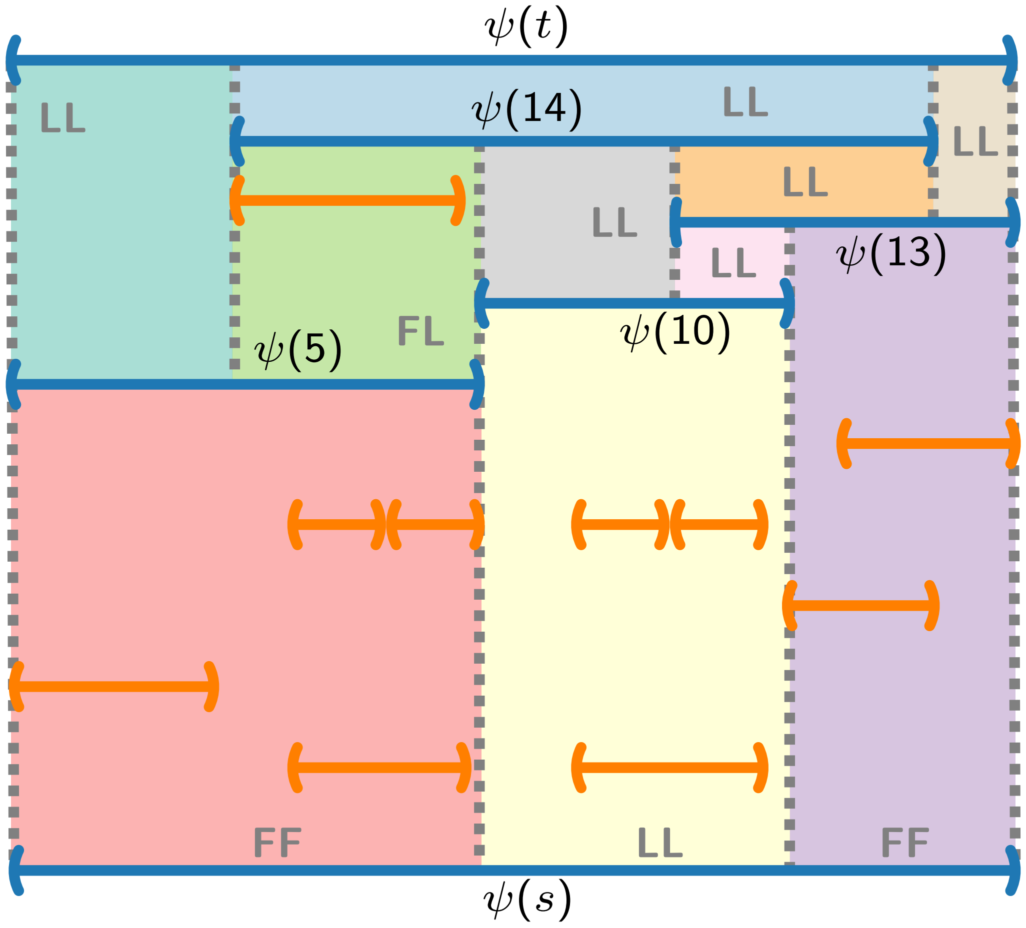

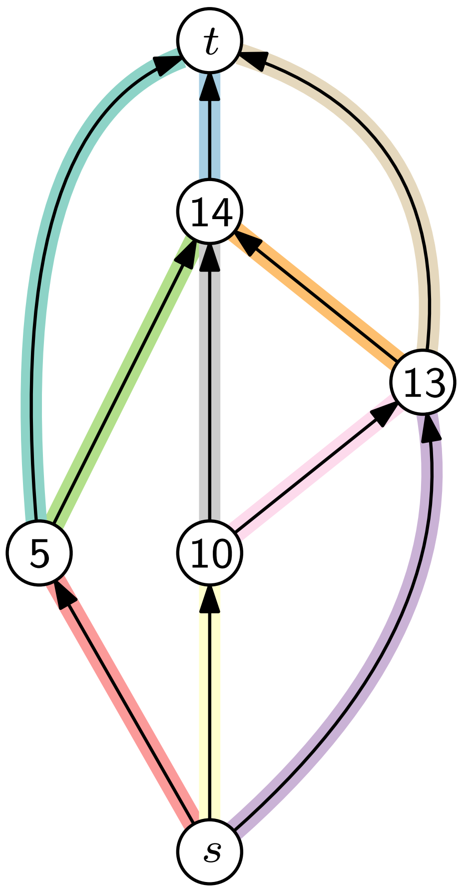

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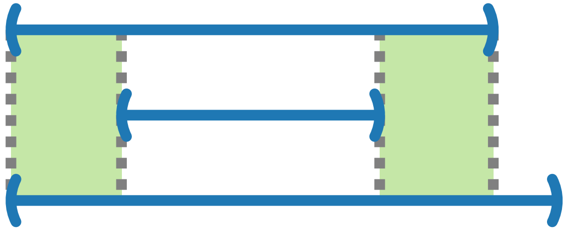

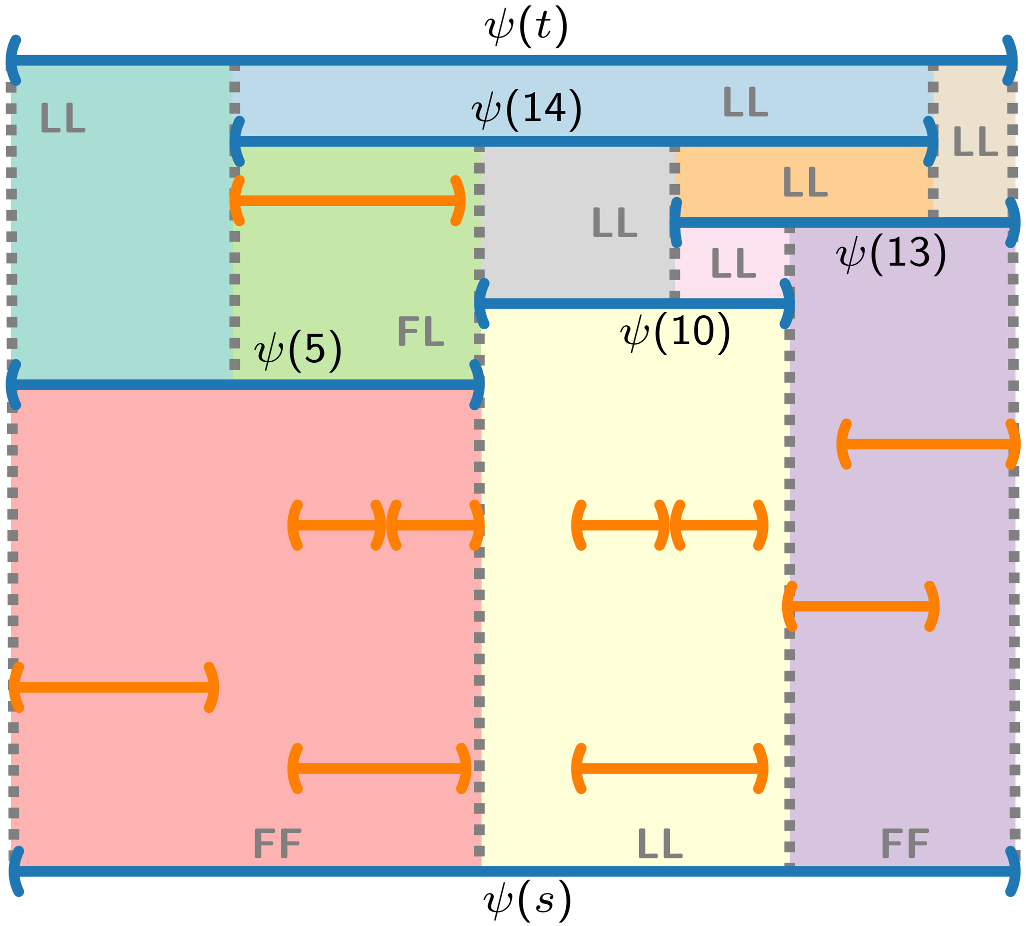

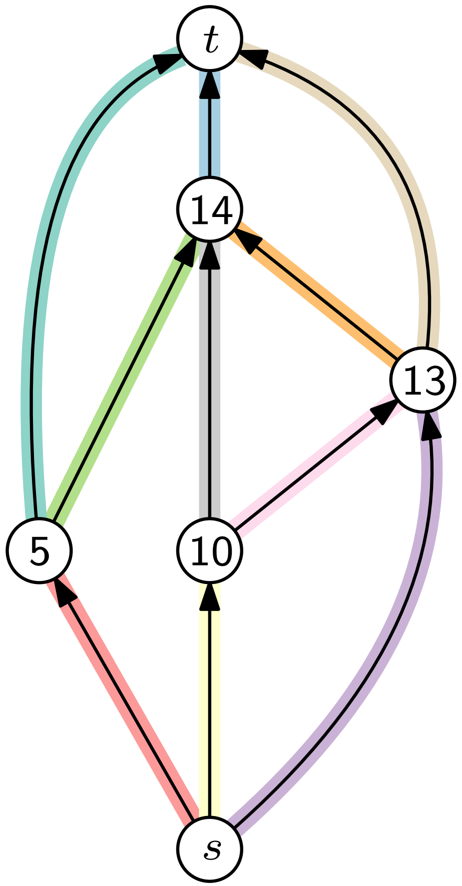

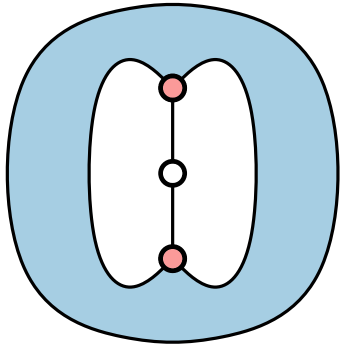

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- Finding a satisfying assignment of a 2-SAT formula can be done in linear time!

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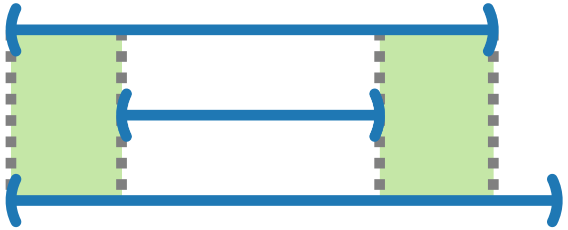

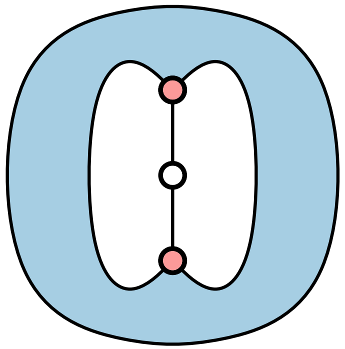

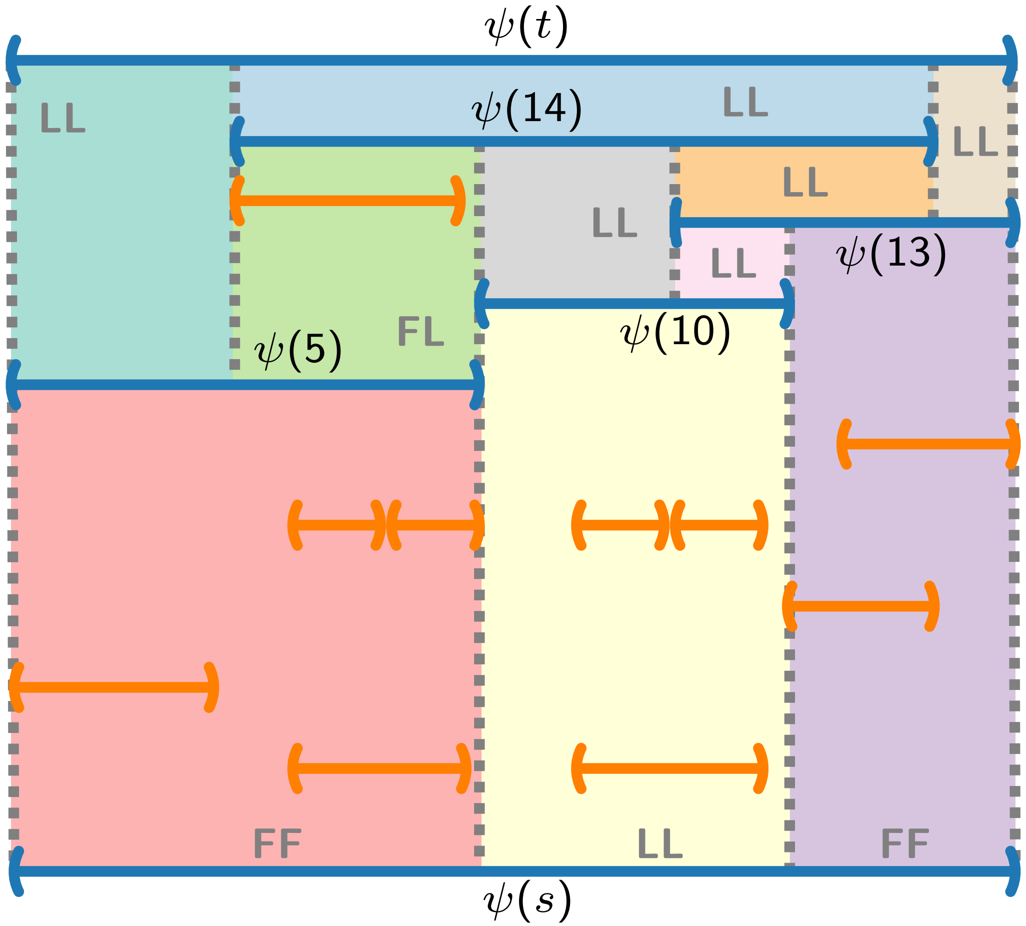

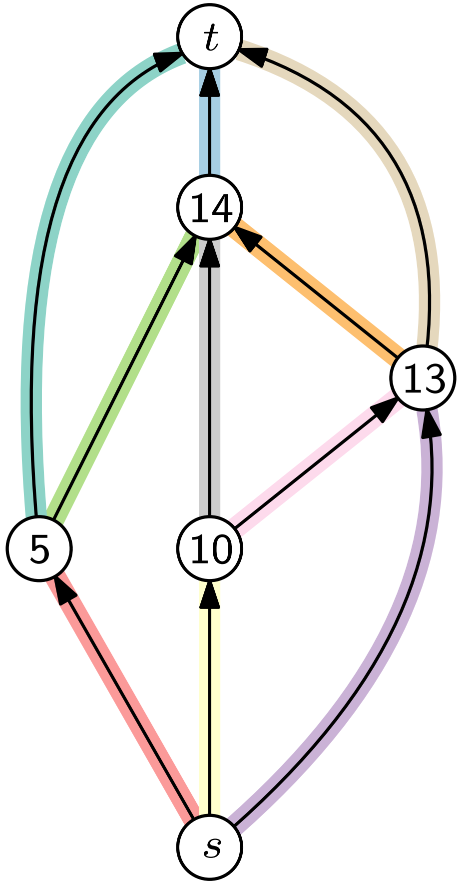

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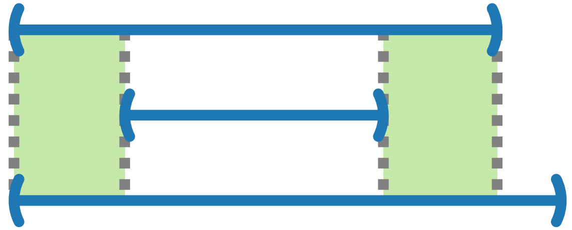

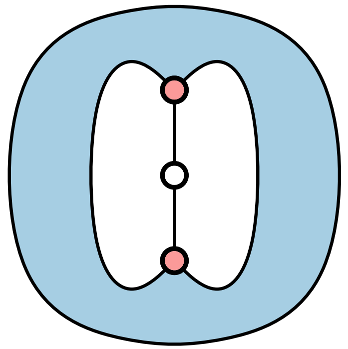

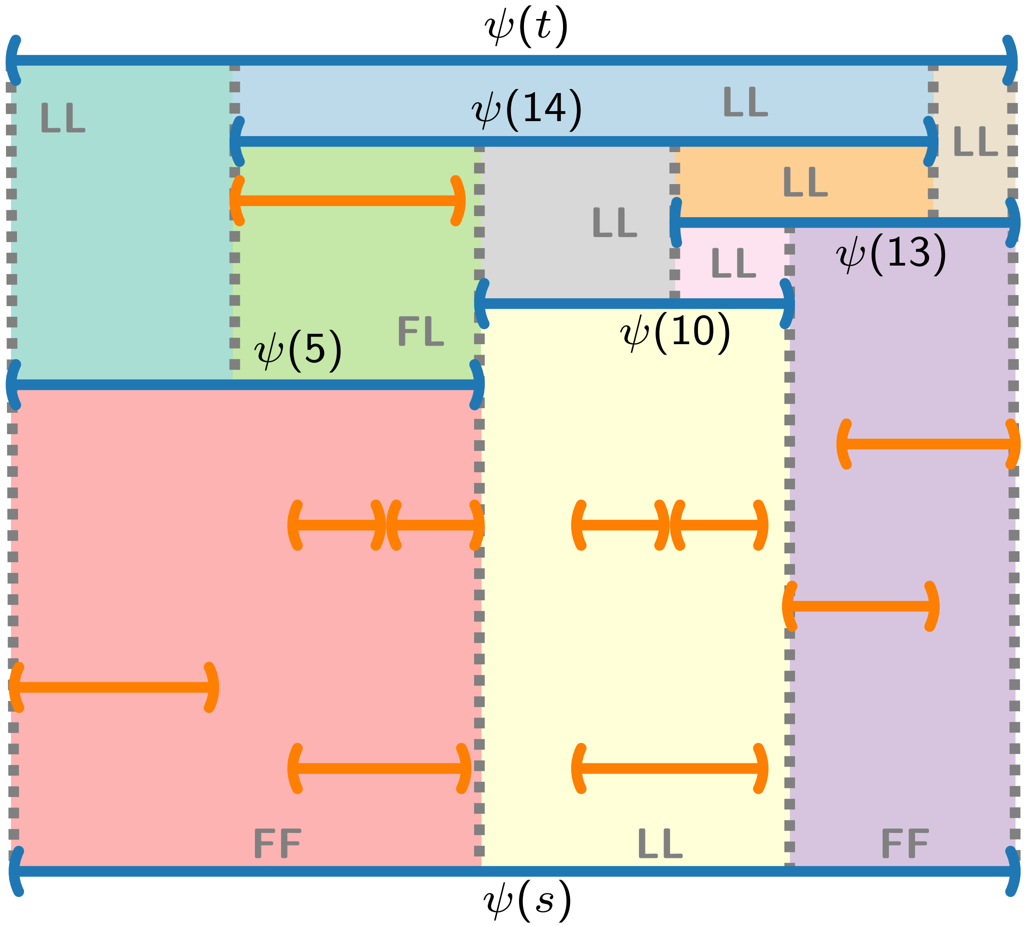

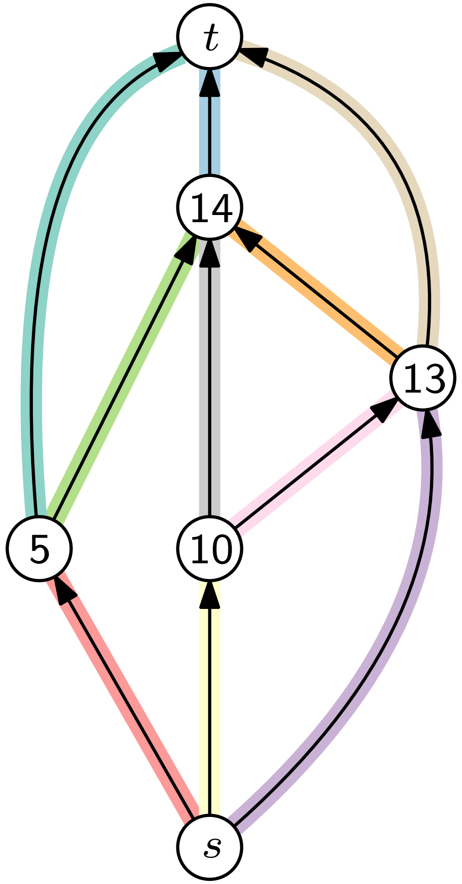

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NP-Hardness of RepExt in the General Case

Theorem 2.
ε-Bar visibility representation extension is NP-complete.

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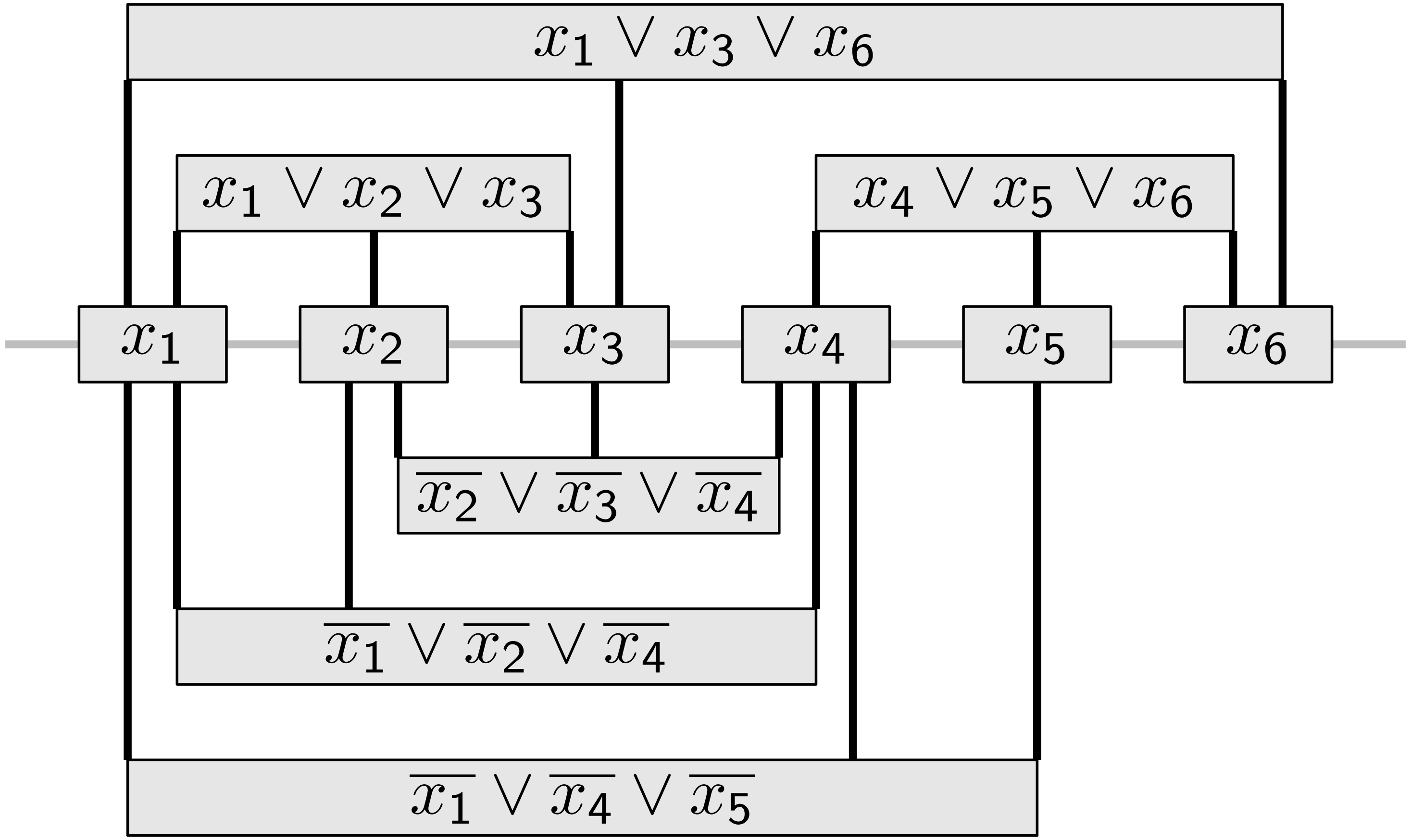

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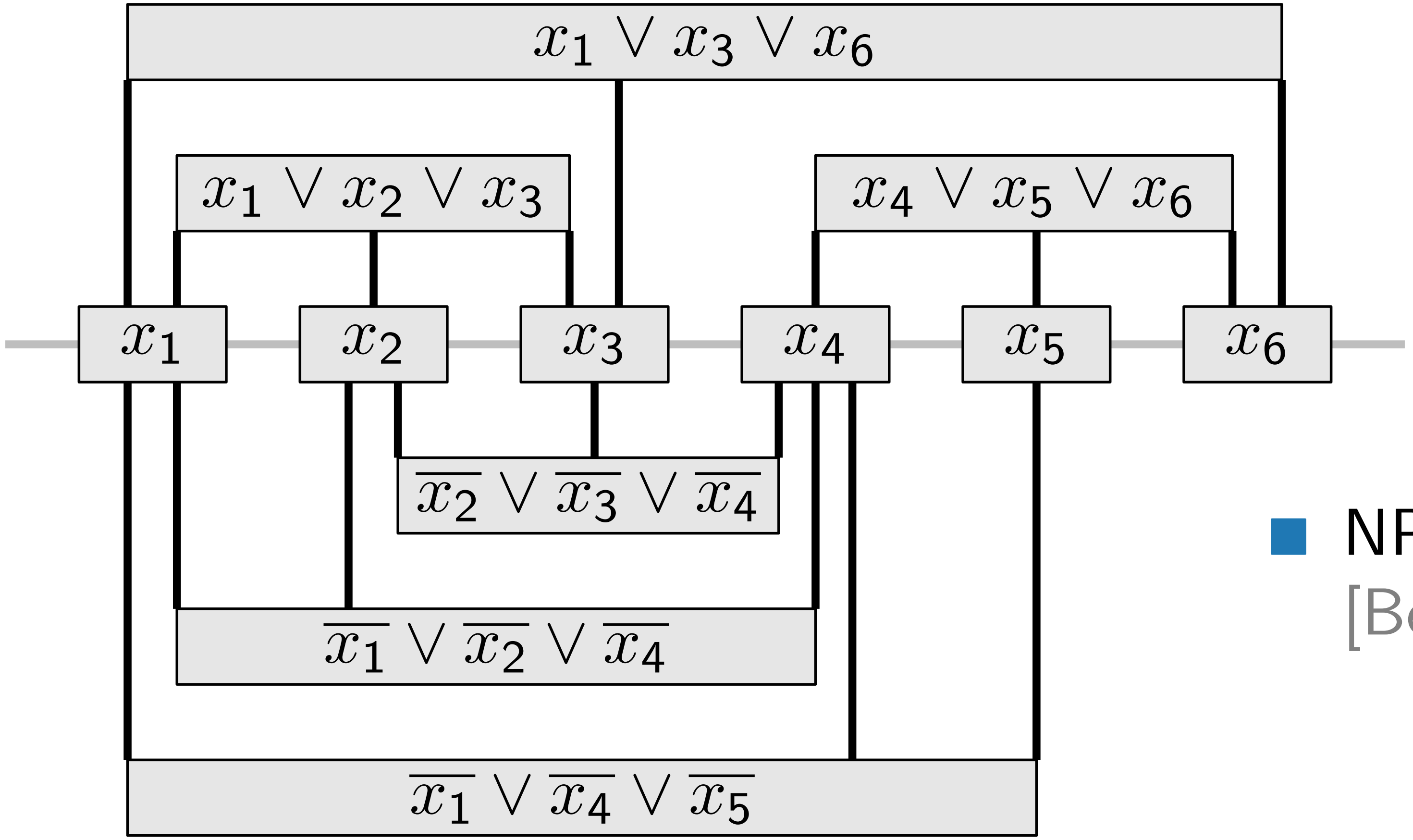
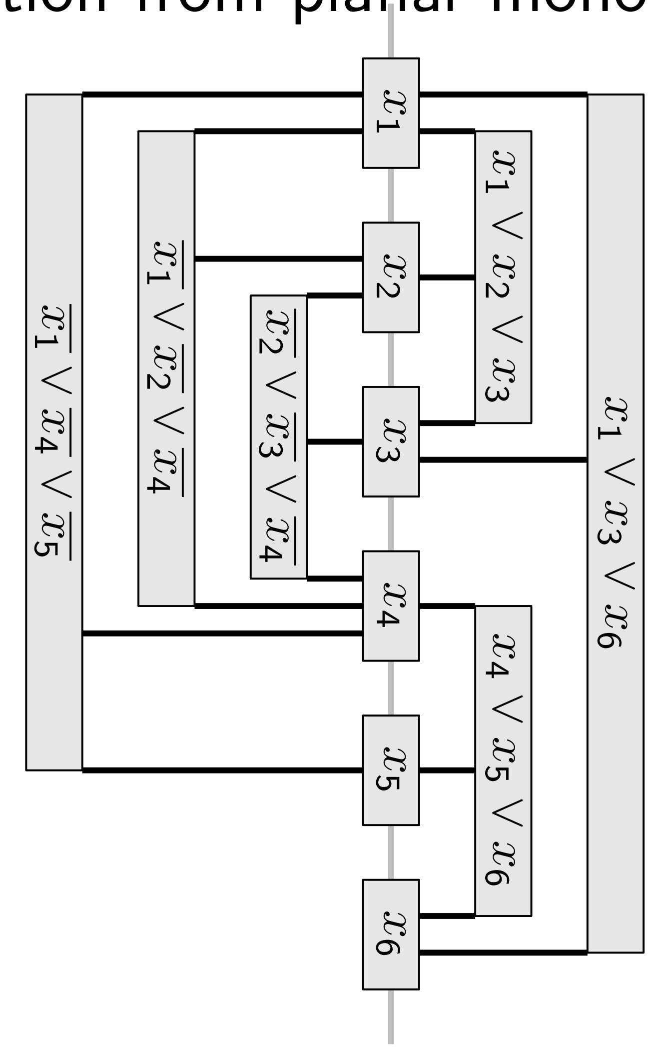


- NP-complete
  [Berg & Khosravi '10]

NP-Hardness of RepExt in the General Case

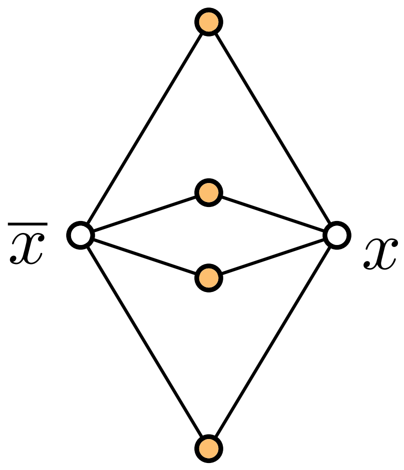
Theorem 2.
ε-Bar visibility representation extension is NP-complete.

- Reduction from planar monotone 3-SAT



- NP-complete
  [Berg & Khosravi '10]

Variable Gadget



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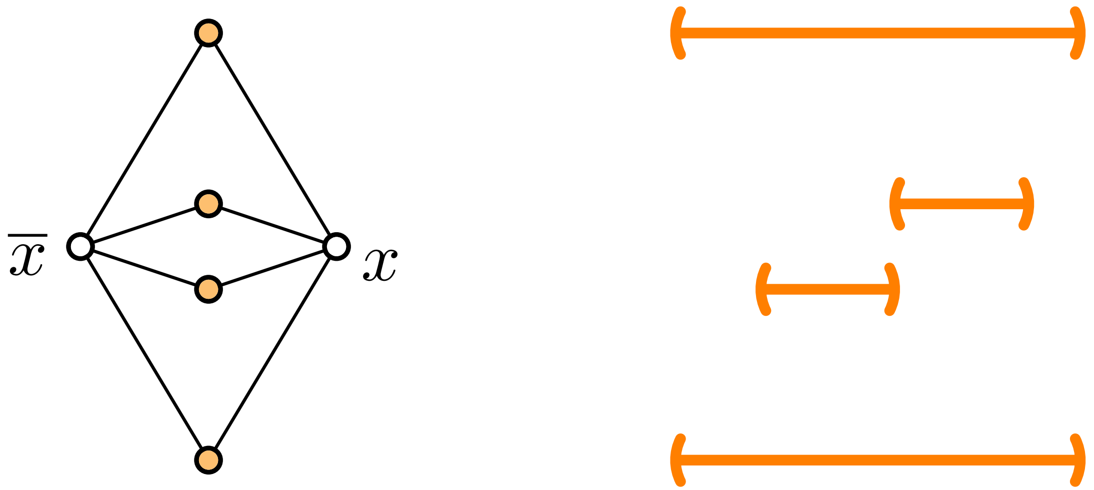

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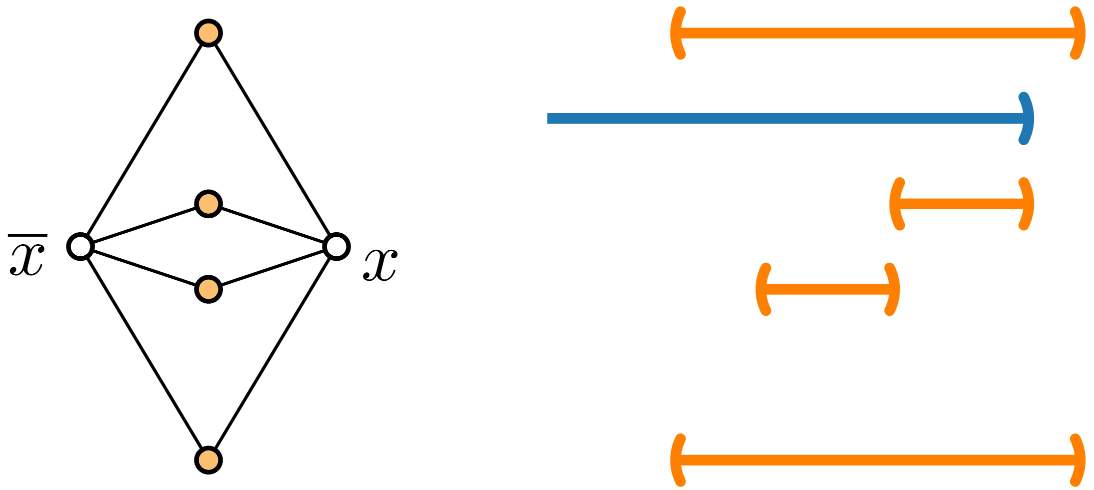

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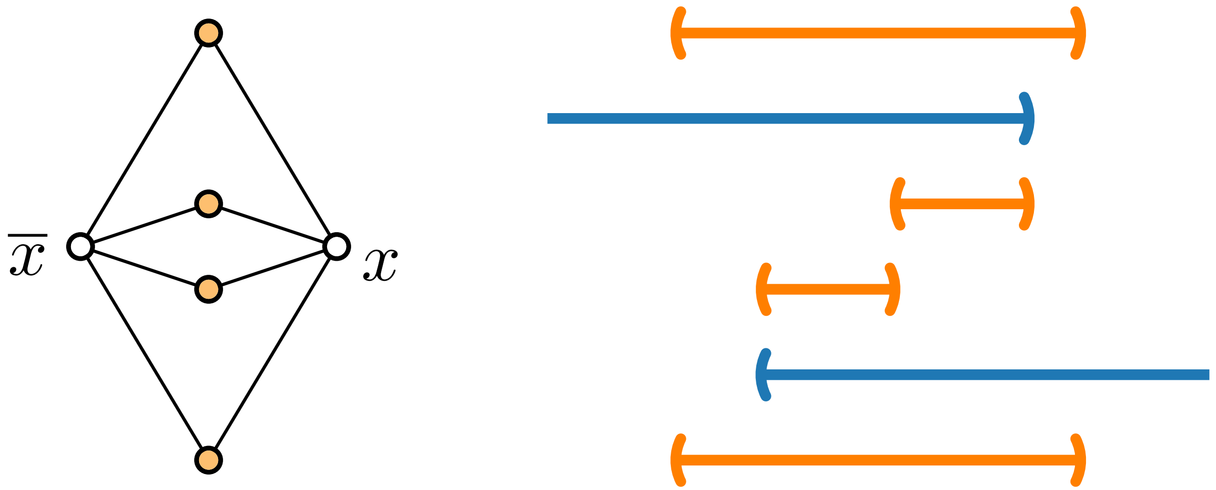

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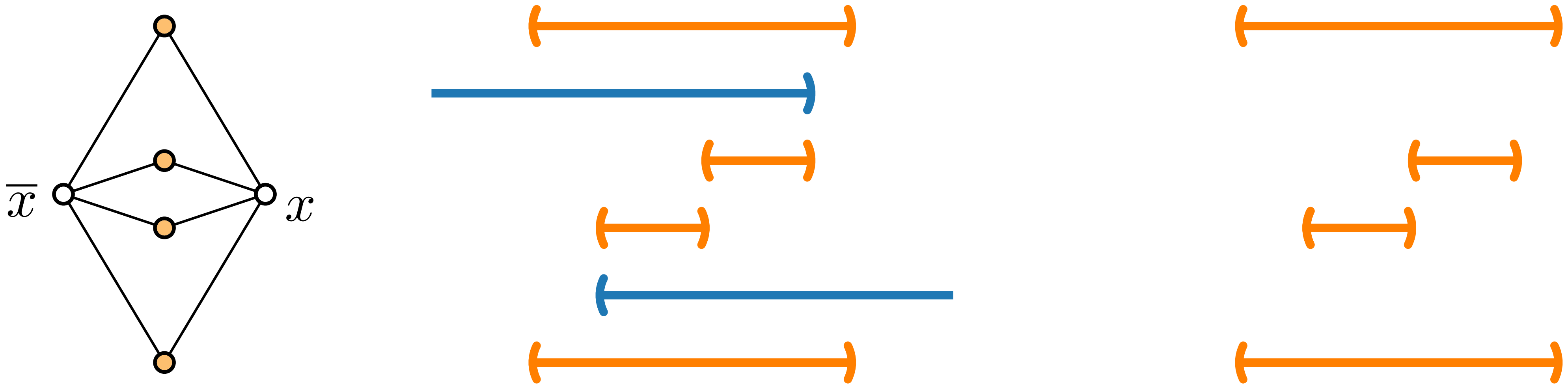

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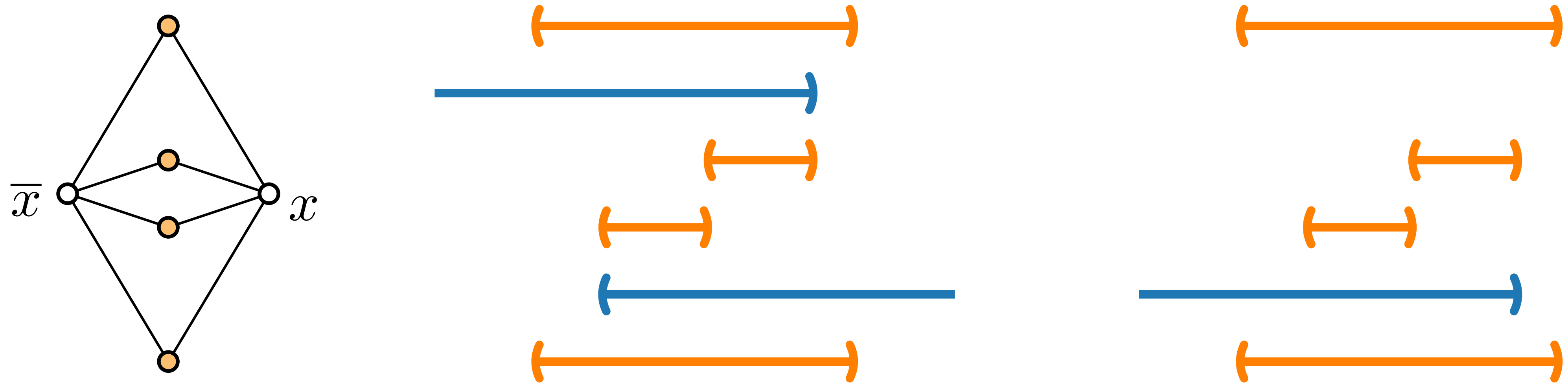

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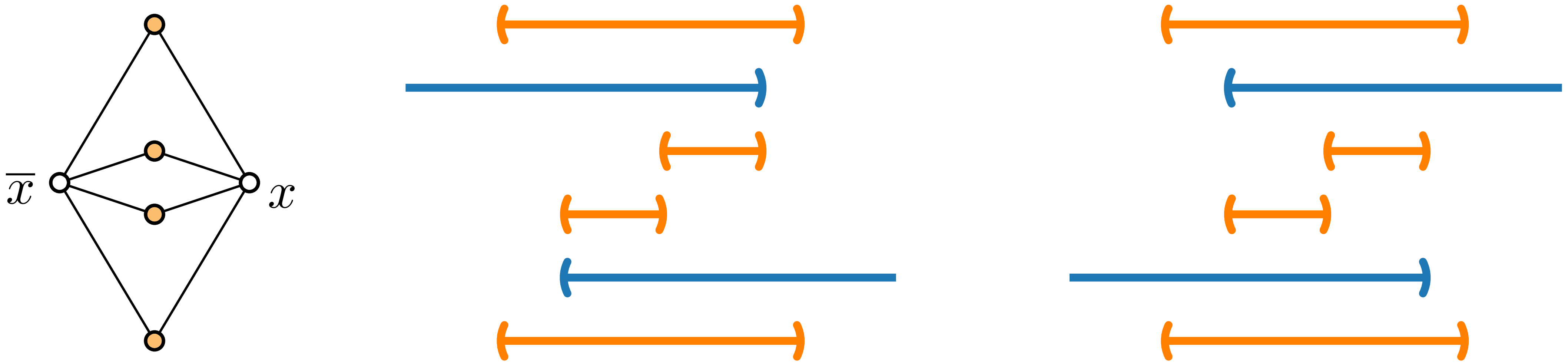

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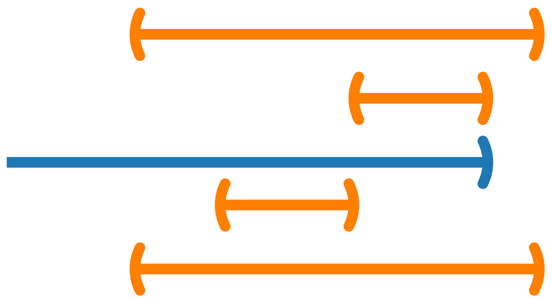

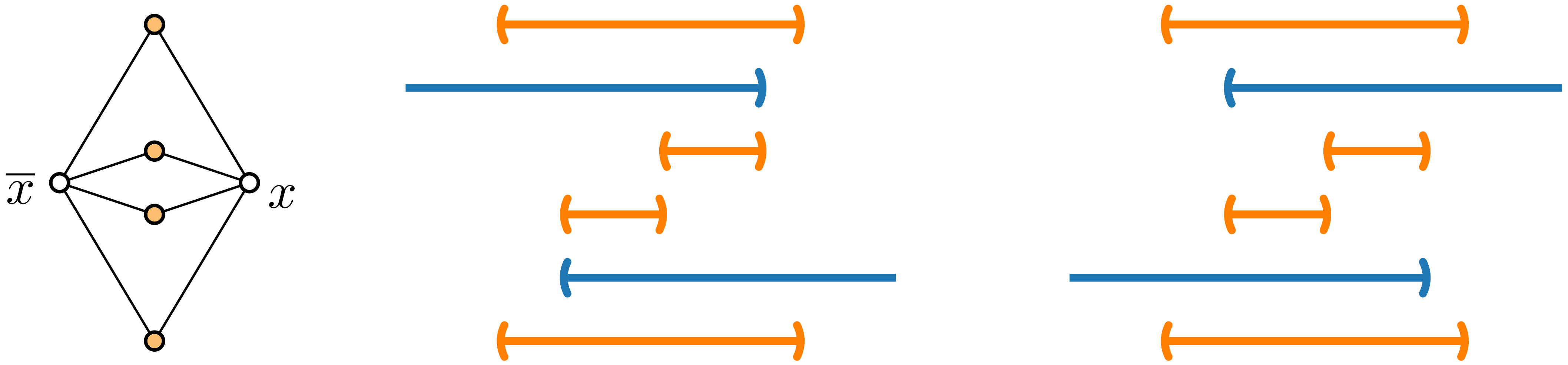

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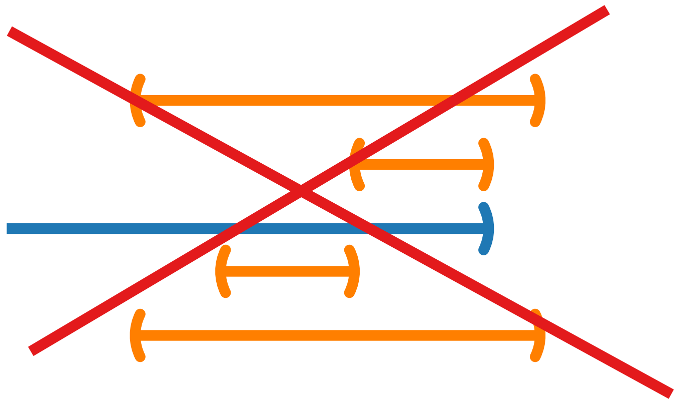

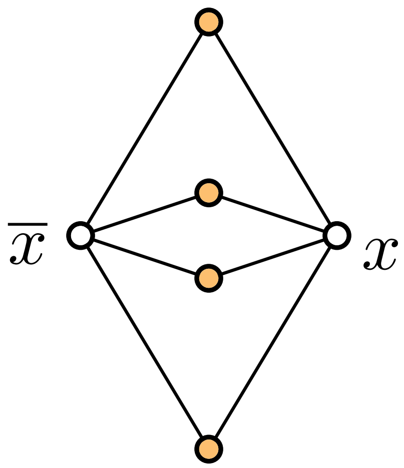

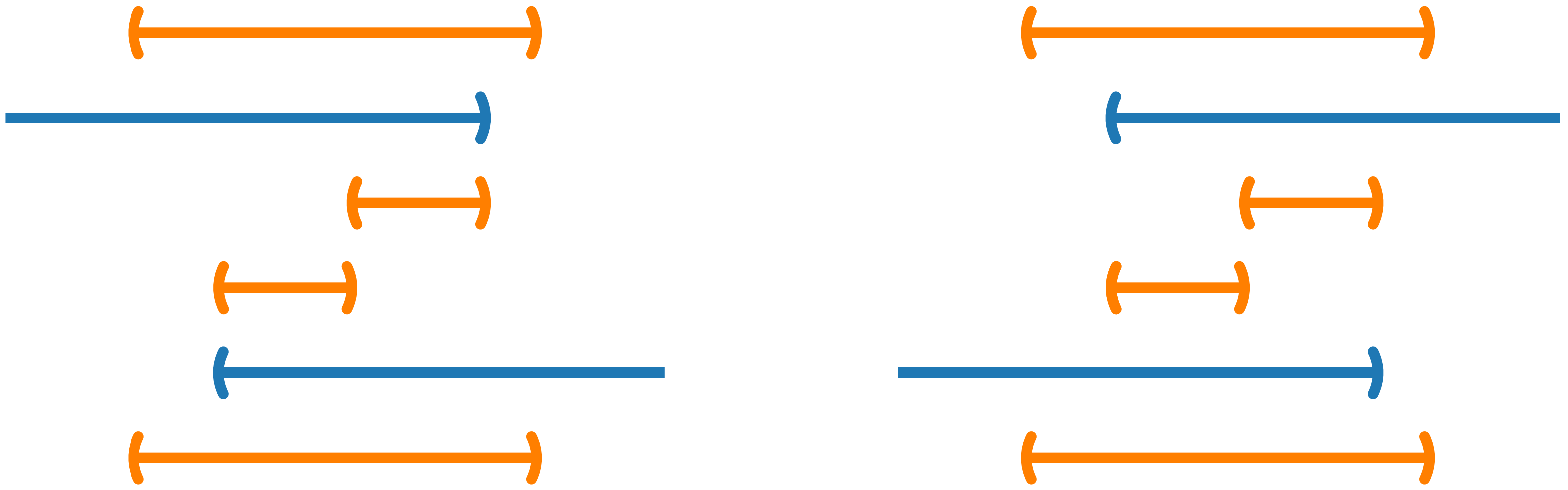

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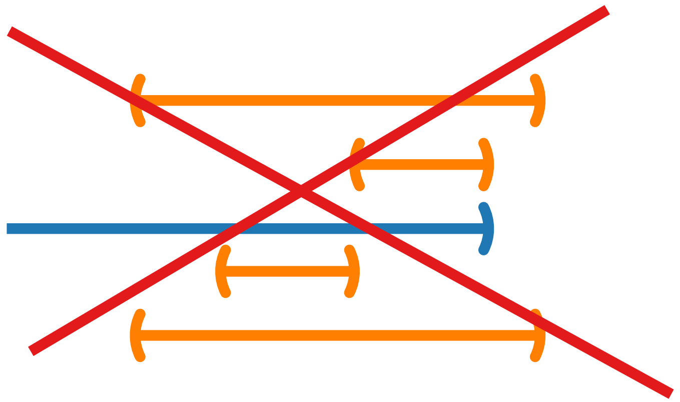

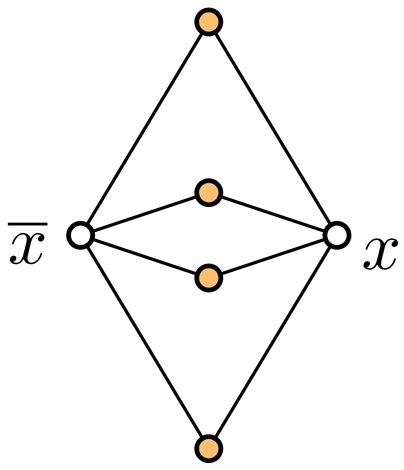

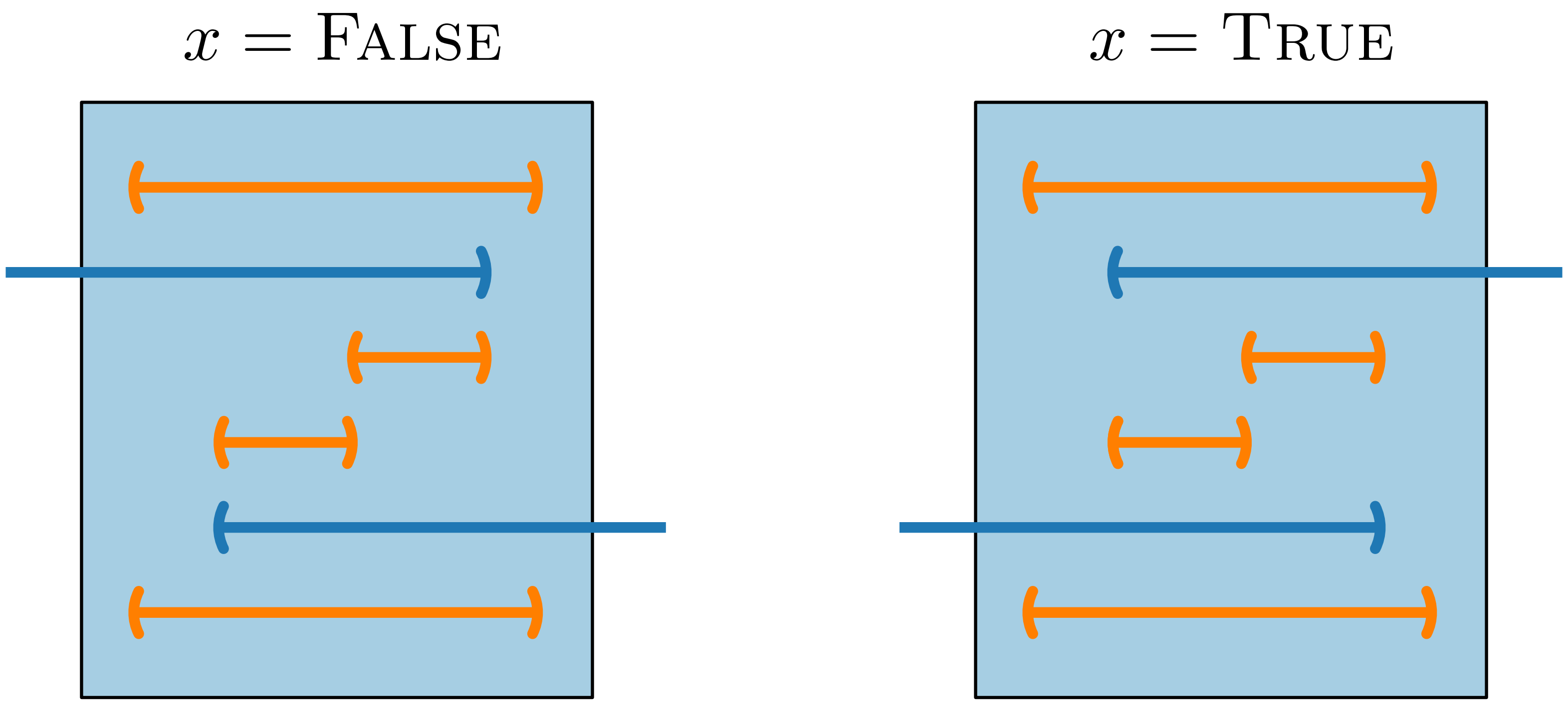

Clause Gadget

$$x \lor y \lor z$$

Clause Gadget

$x \vee y \vee z$

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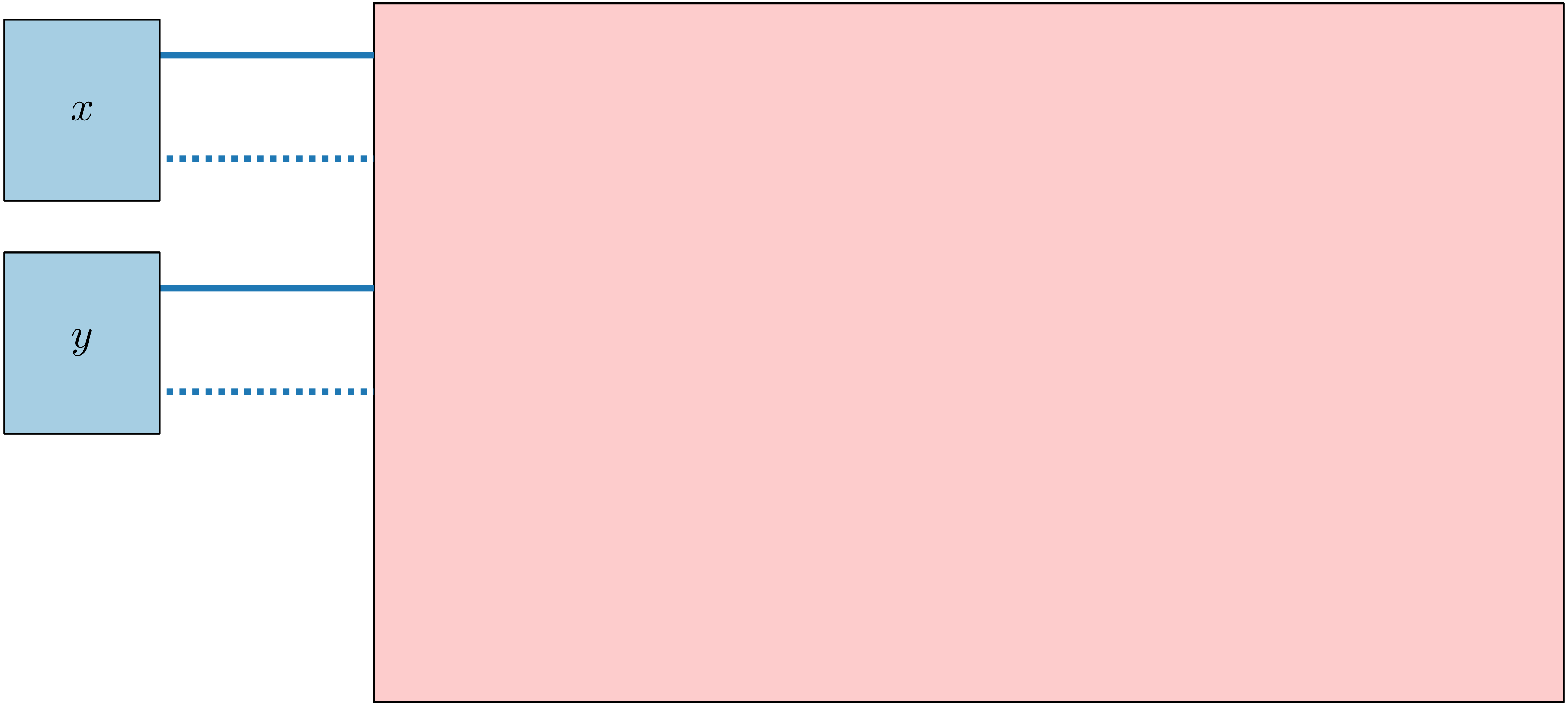

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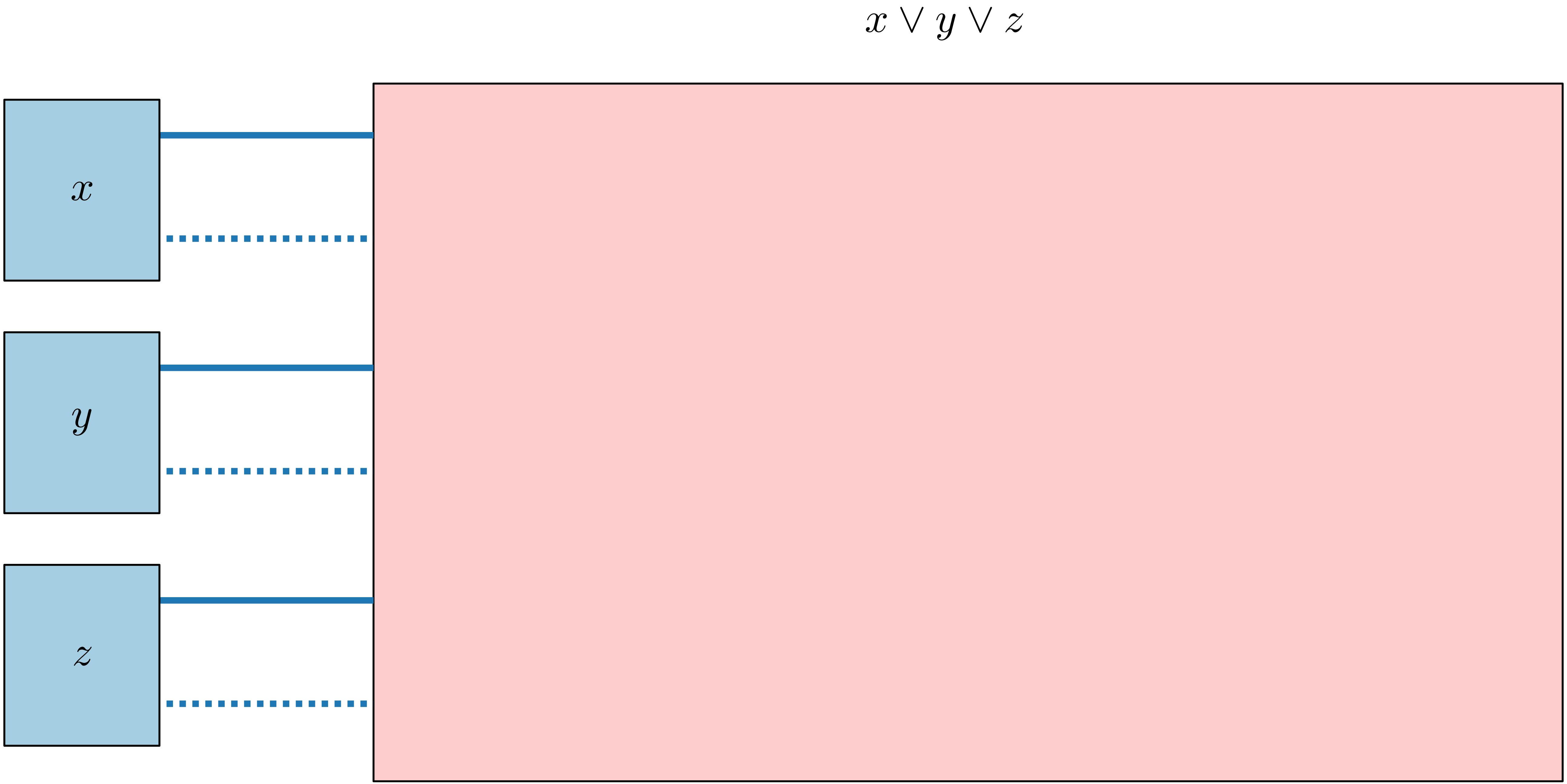

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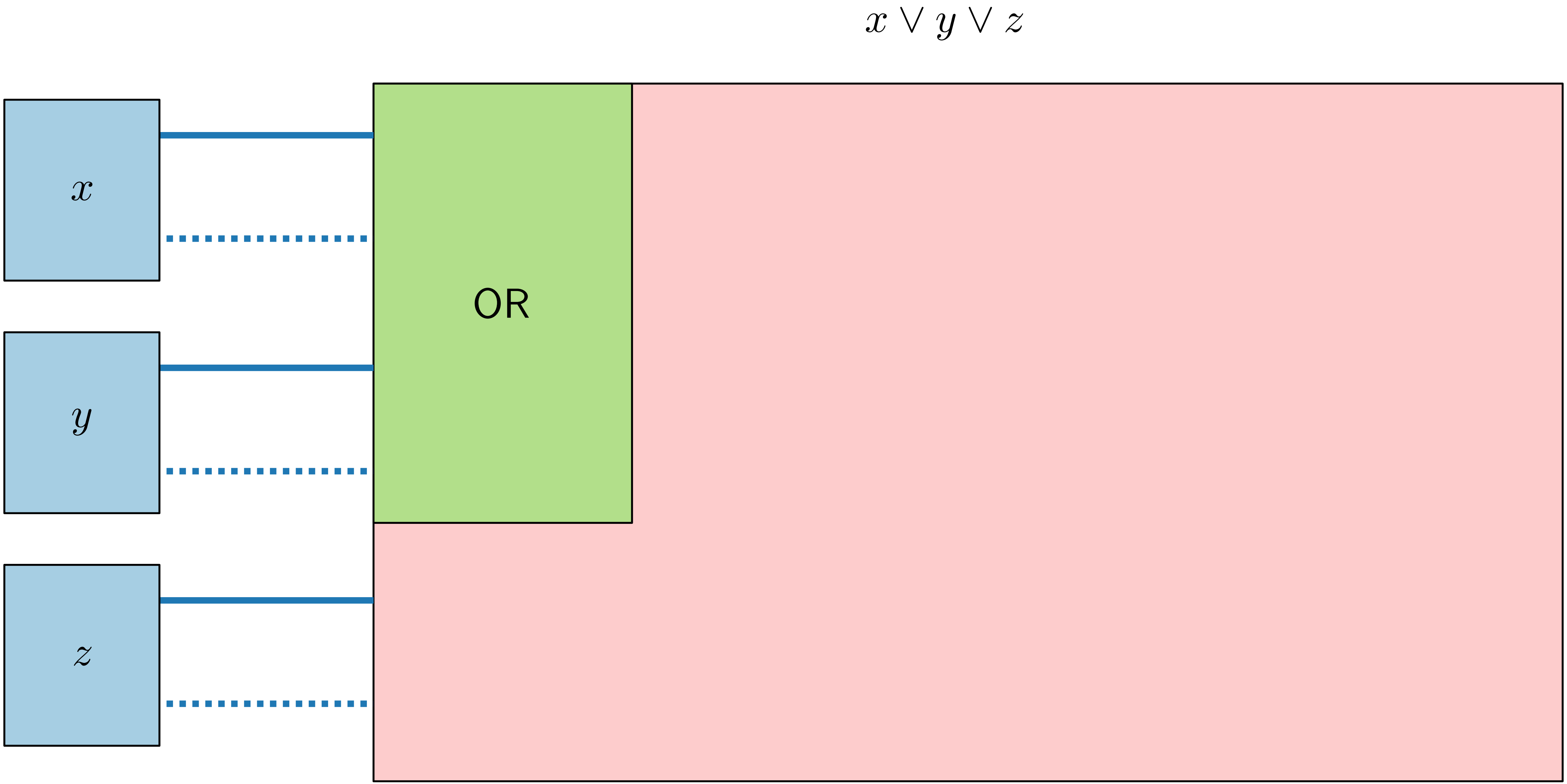

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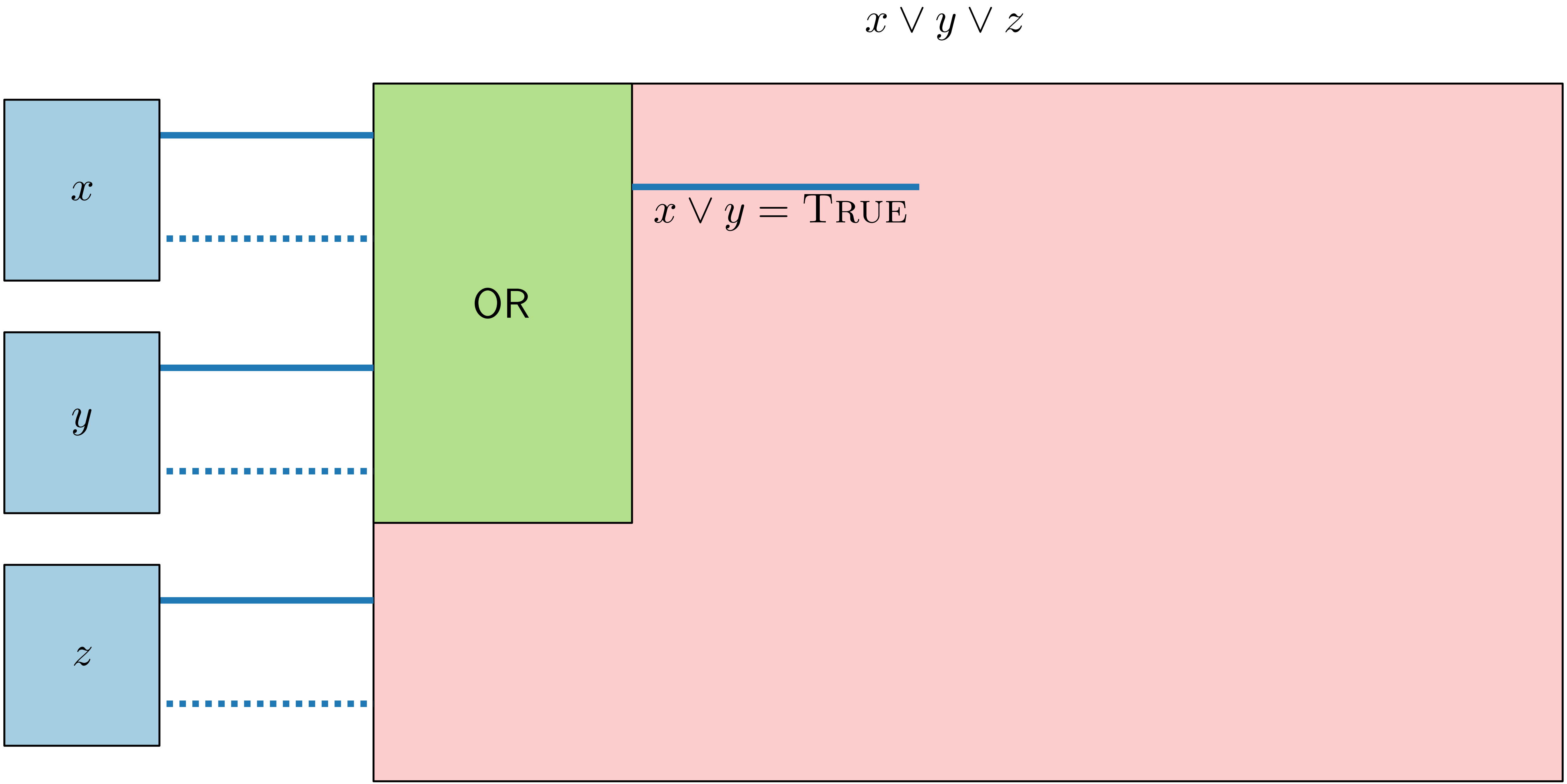

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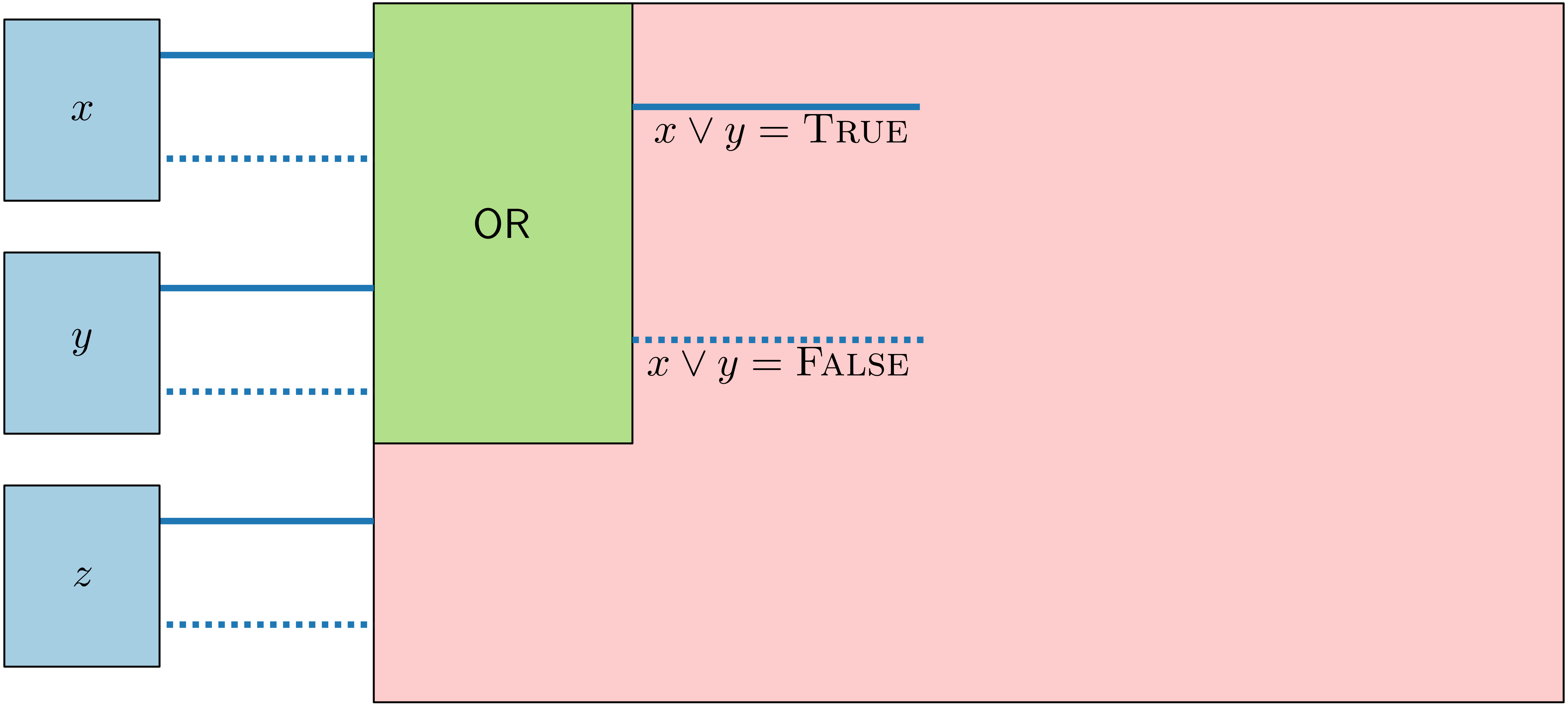

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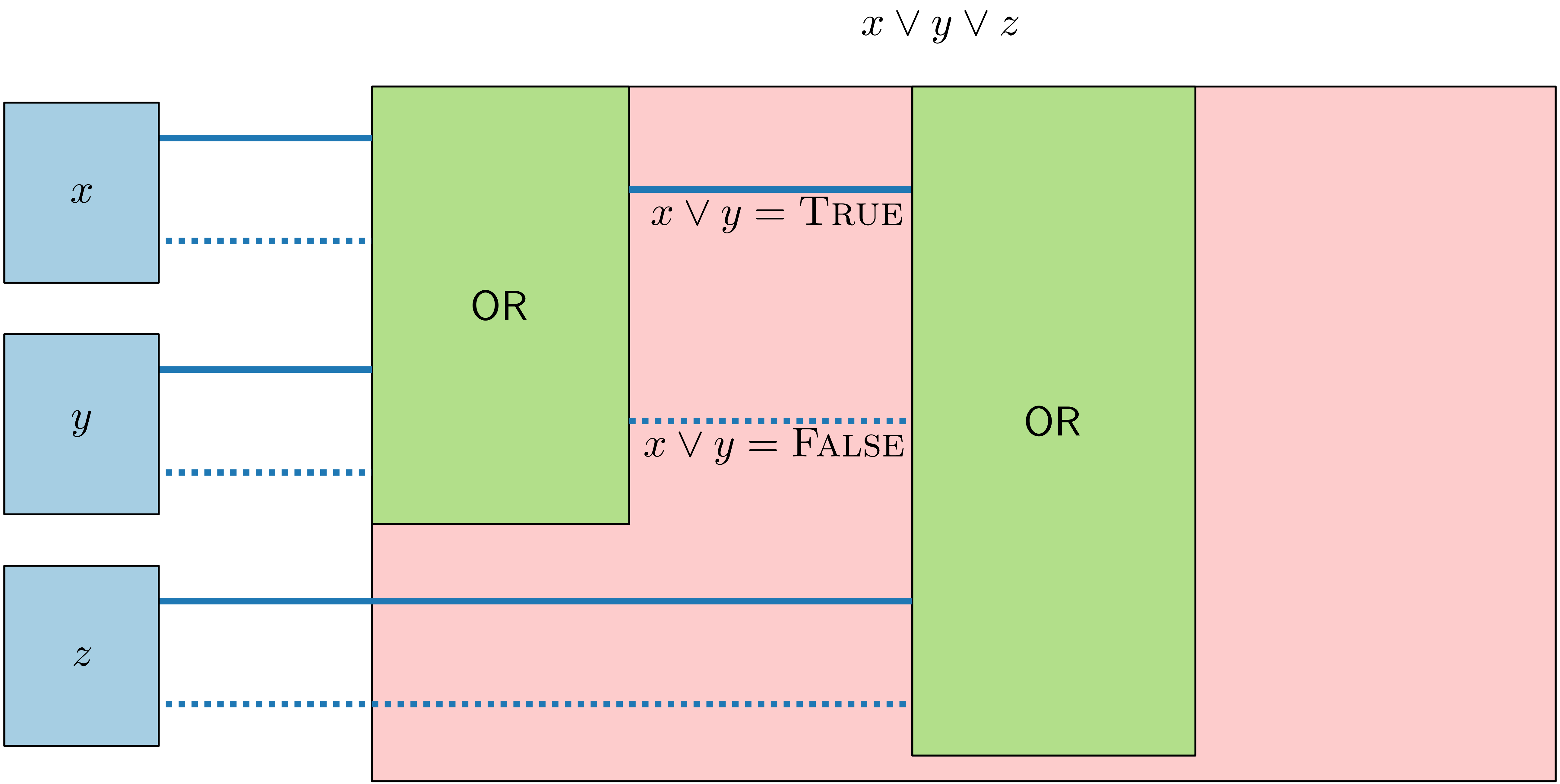

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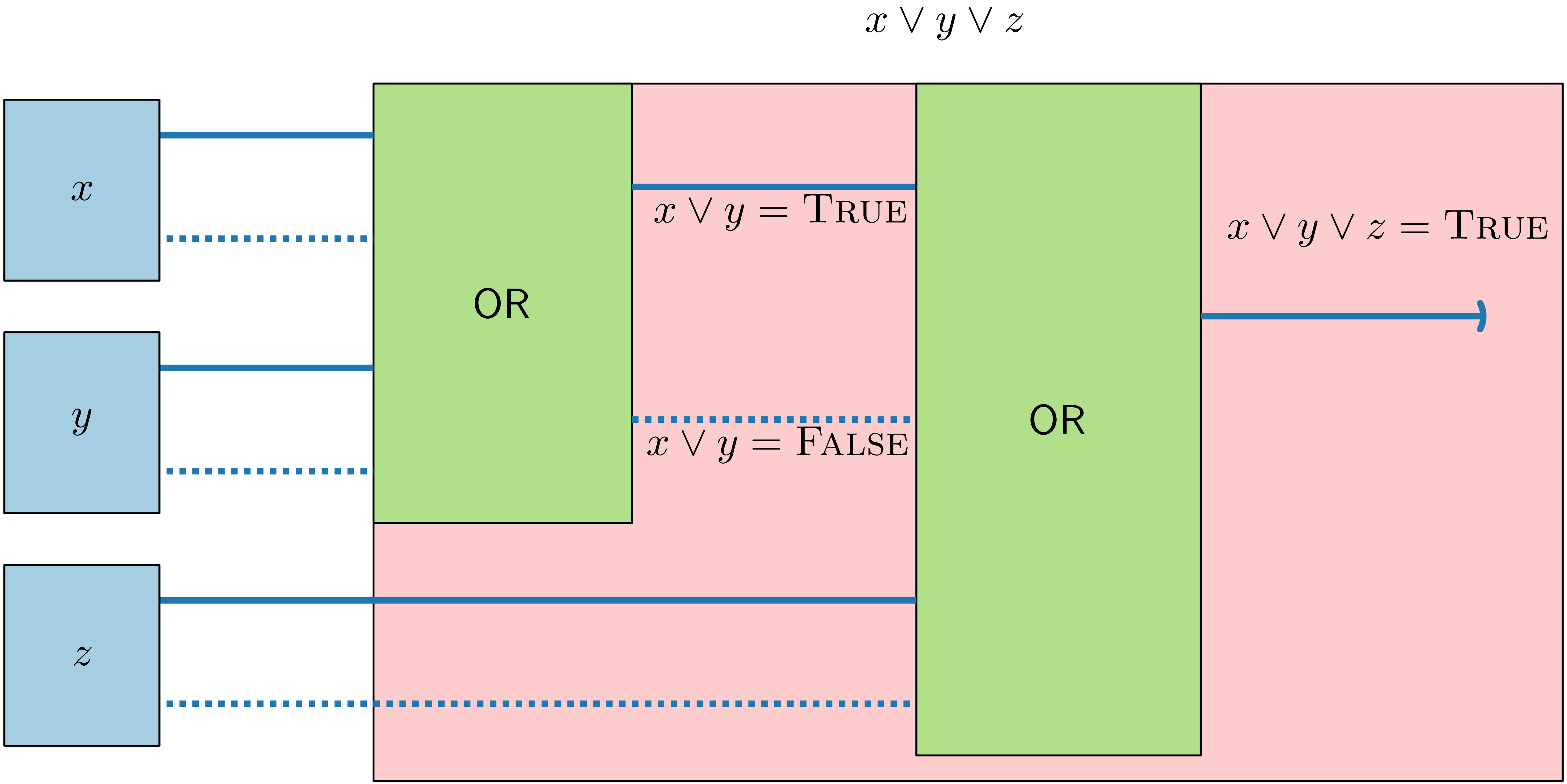

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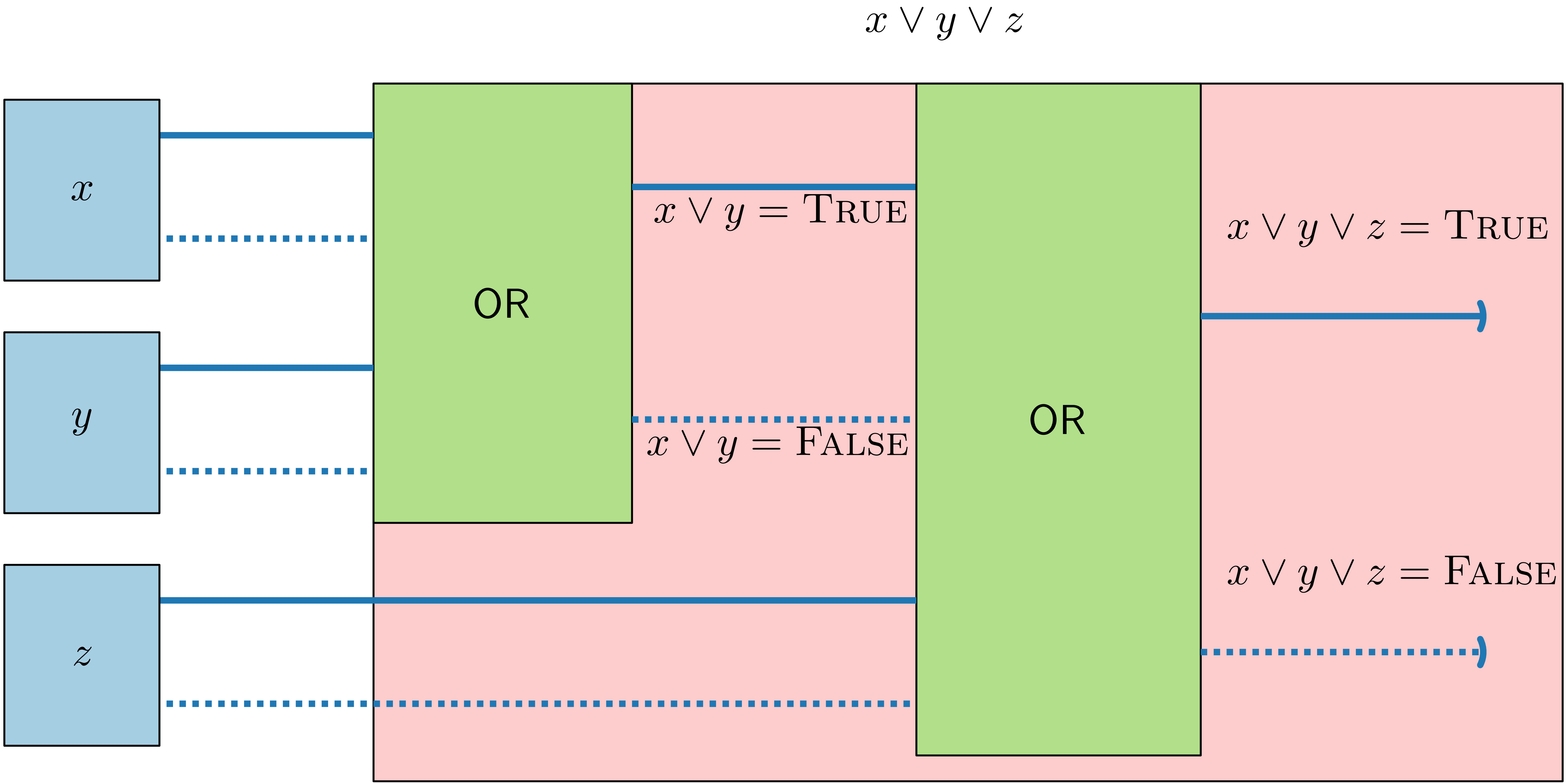
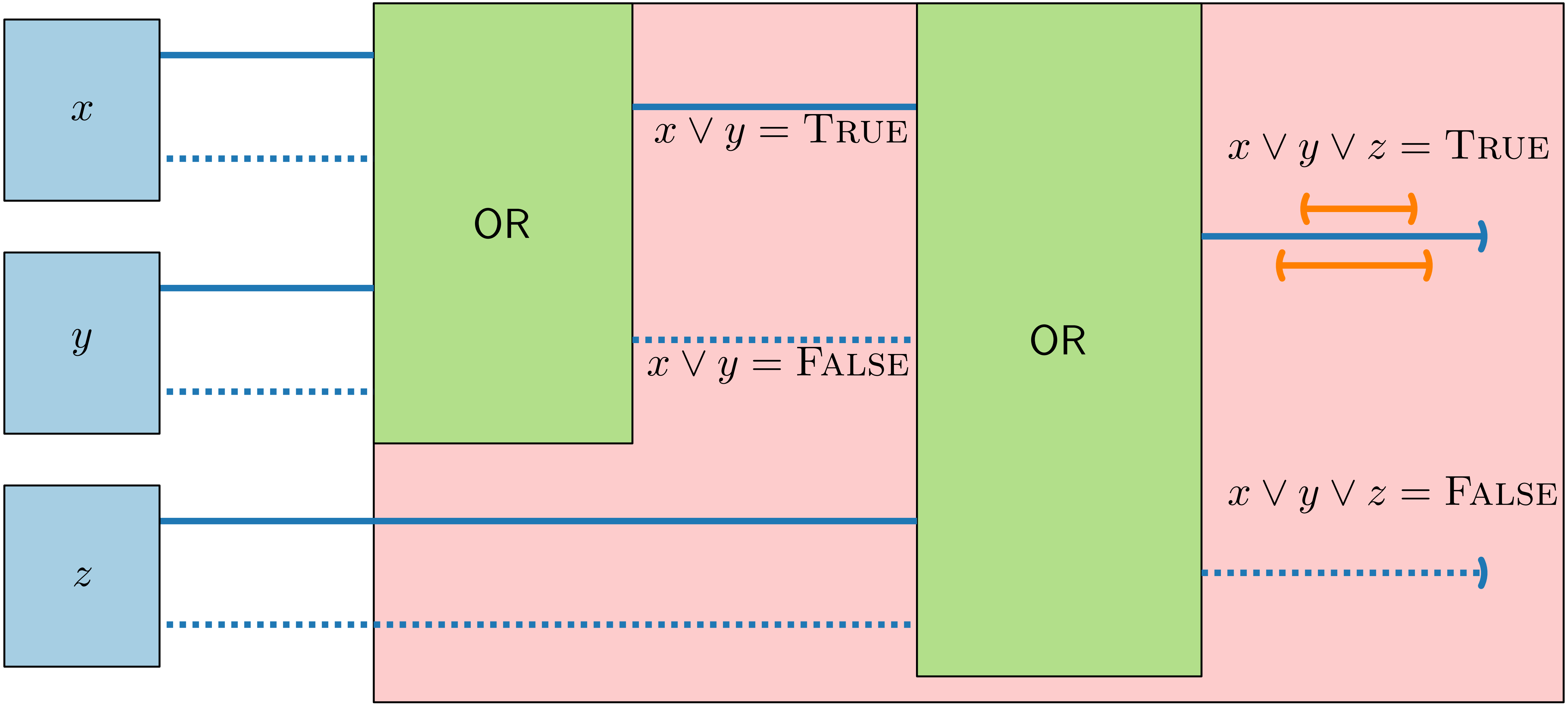


Clause Gadget

$x \lor y \lor z$



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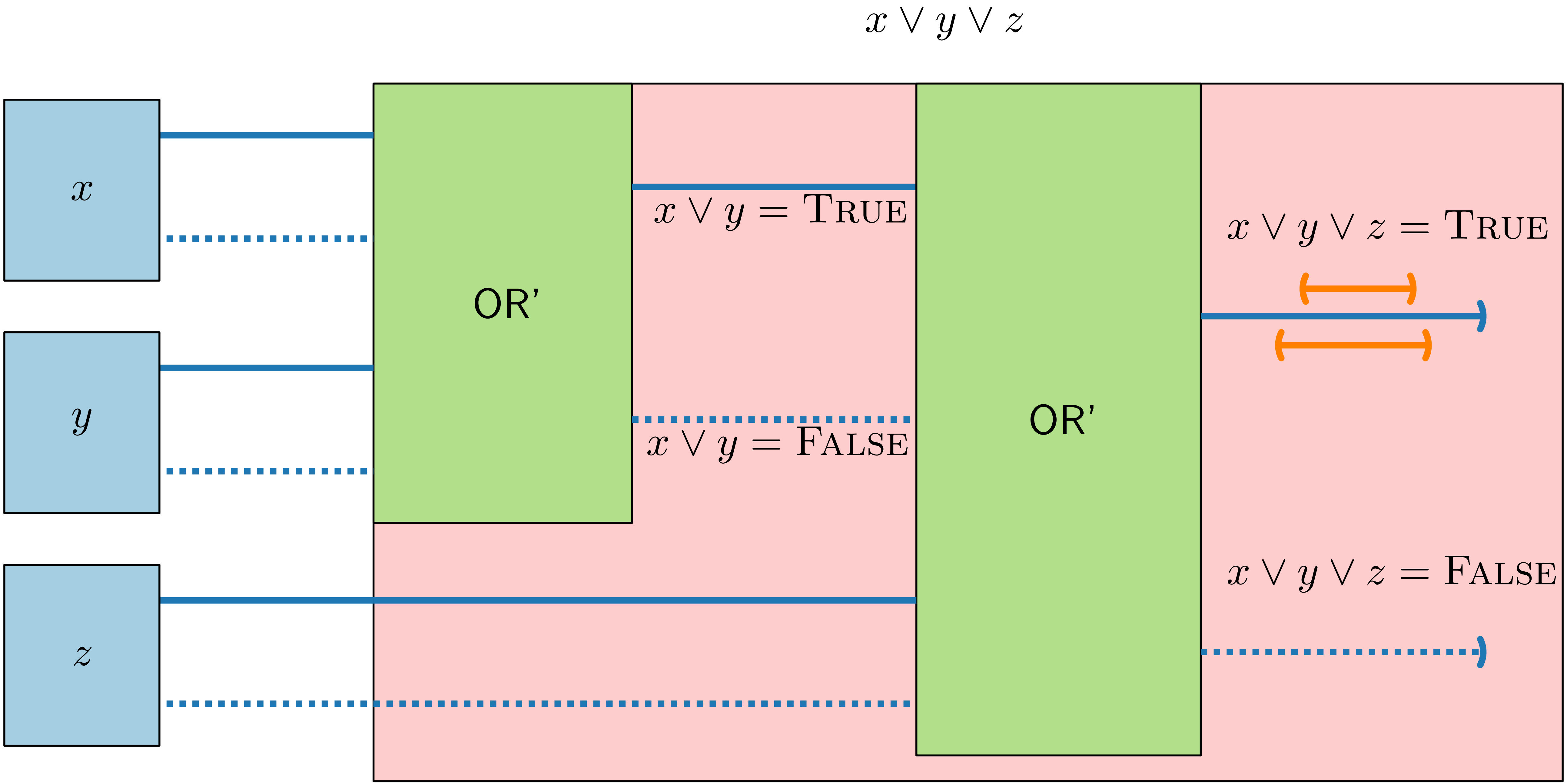

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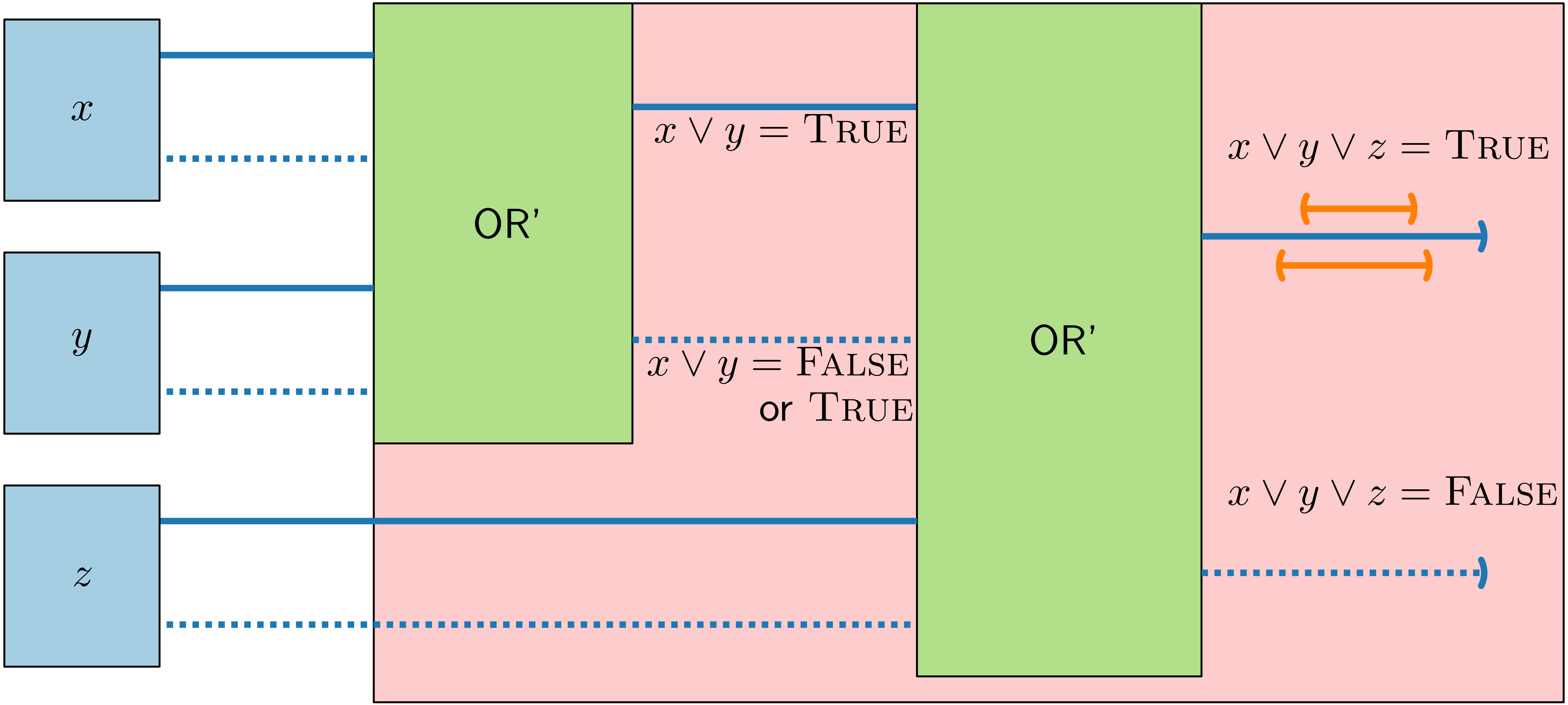

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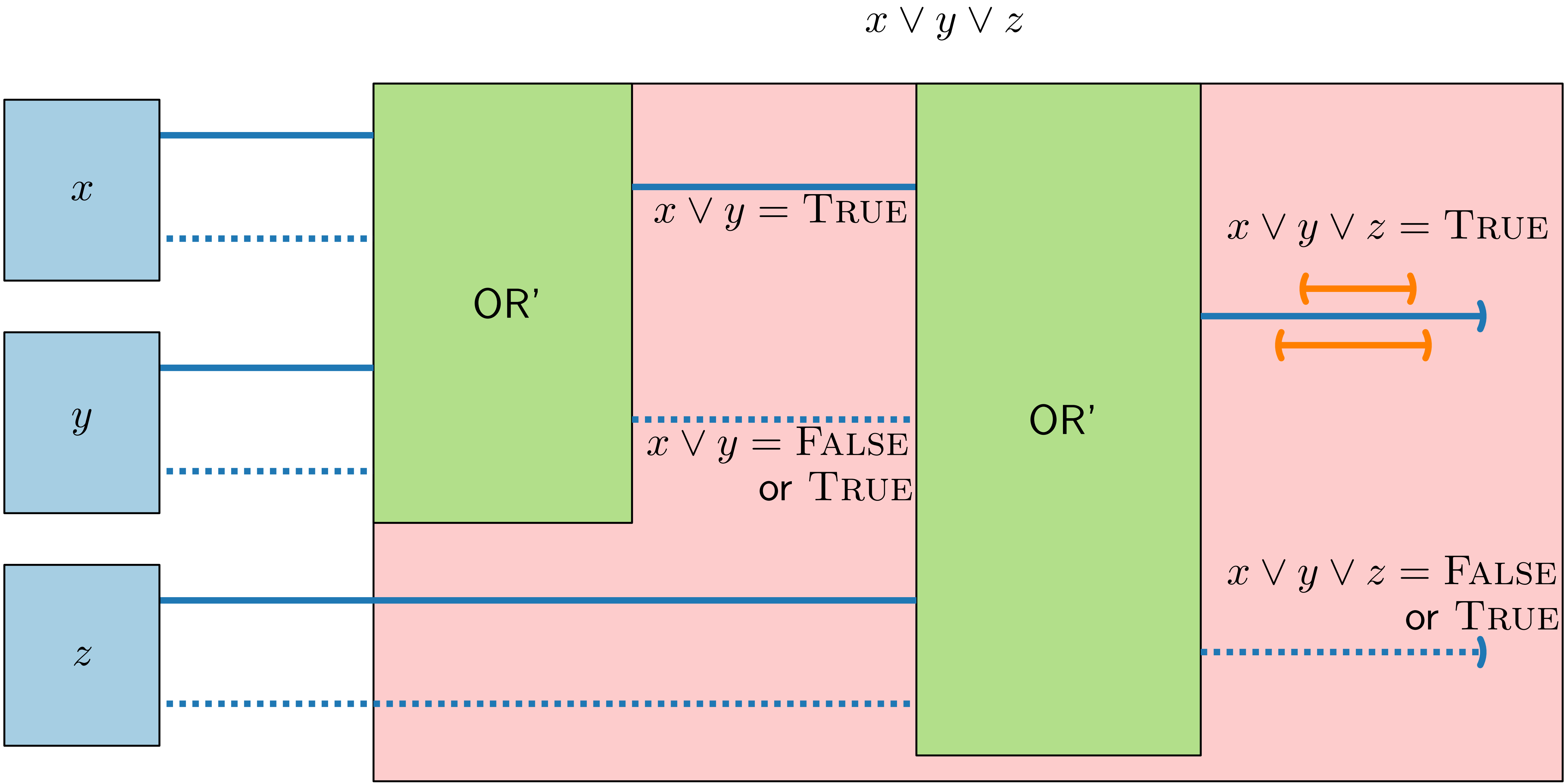
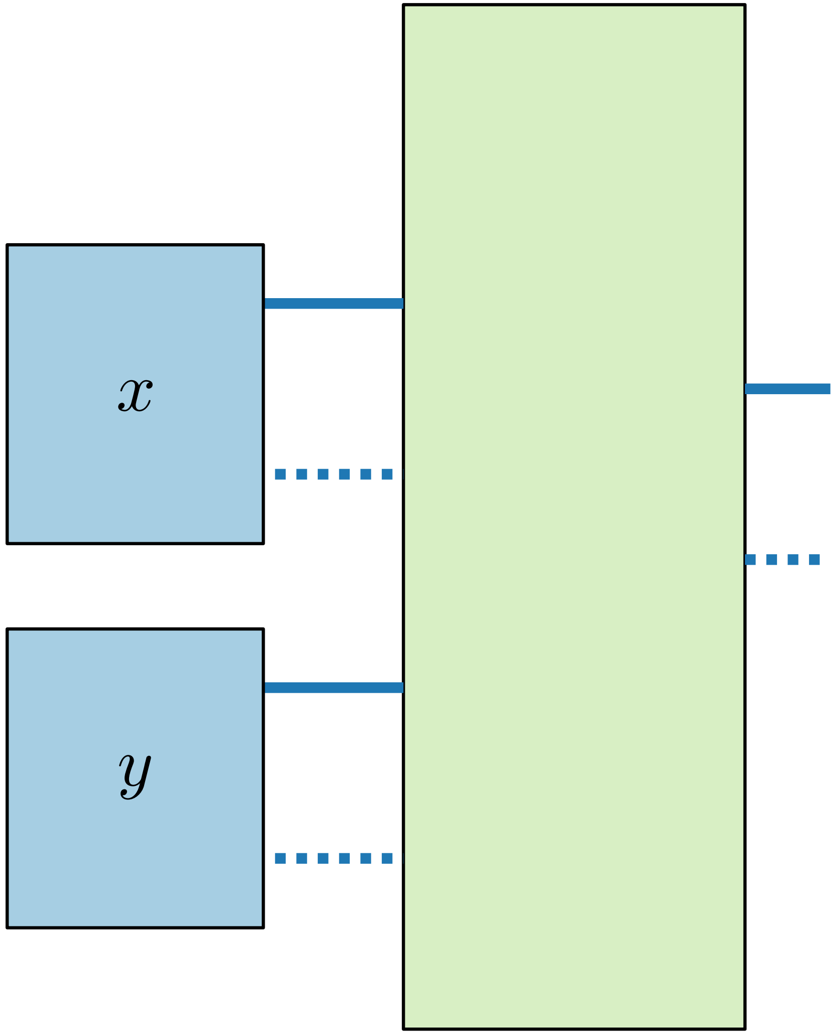


OR' Gadget



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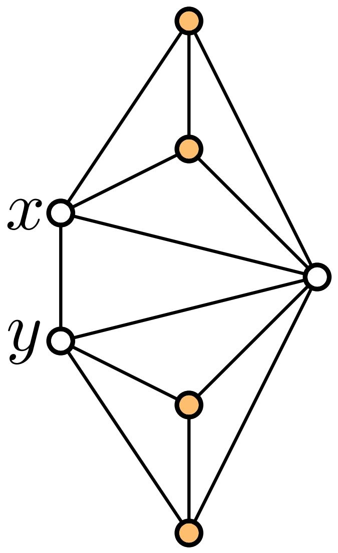

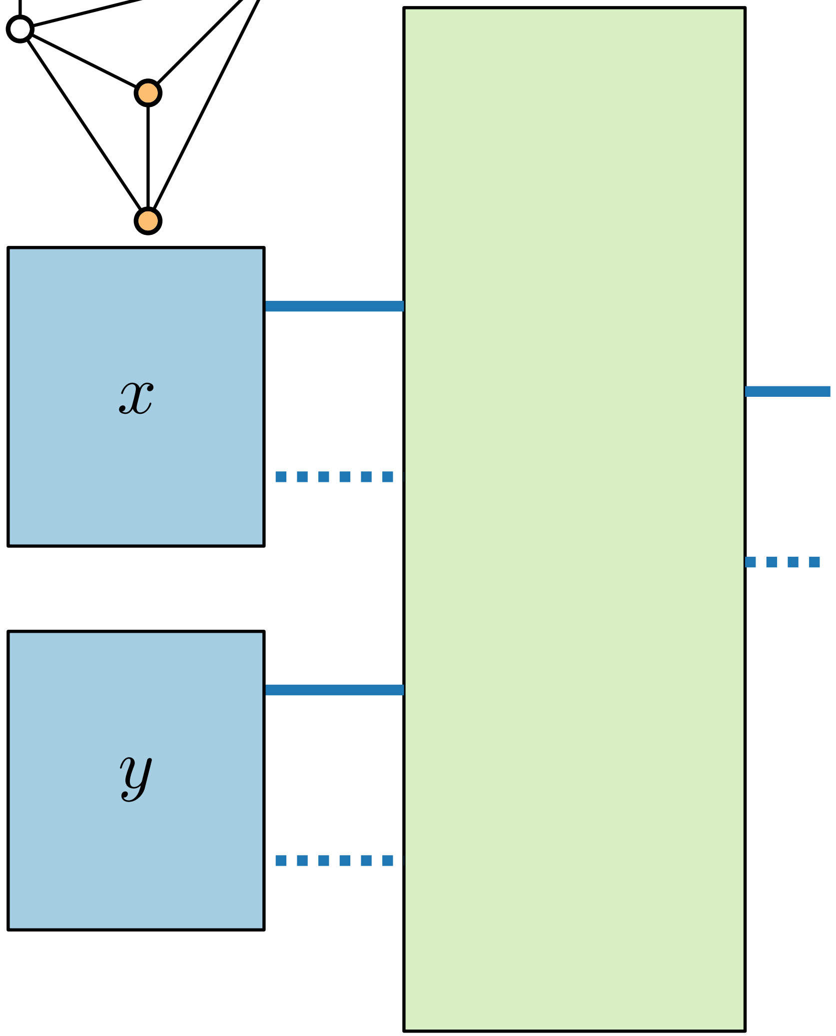

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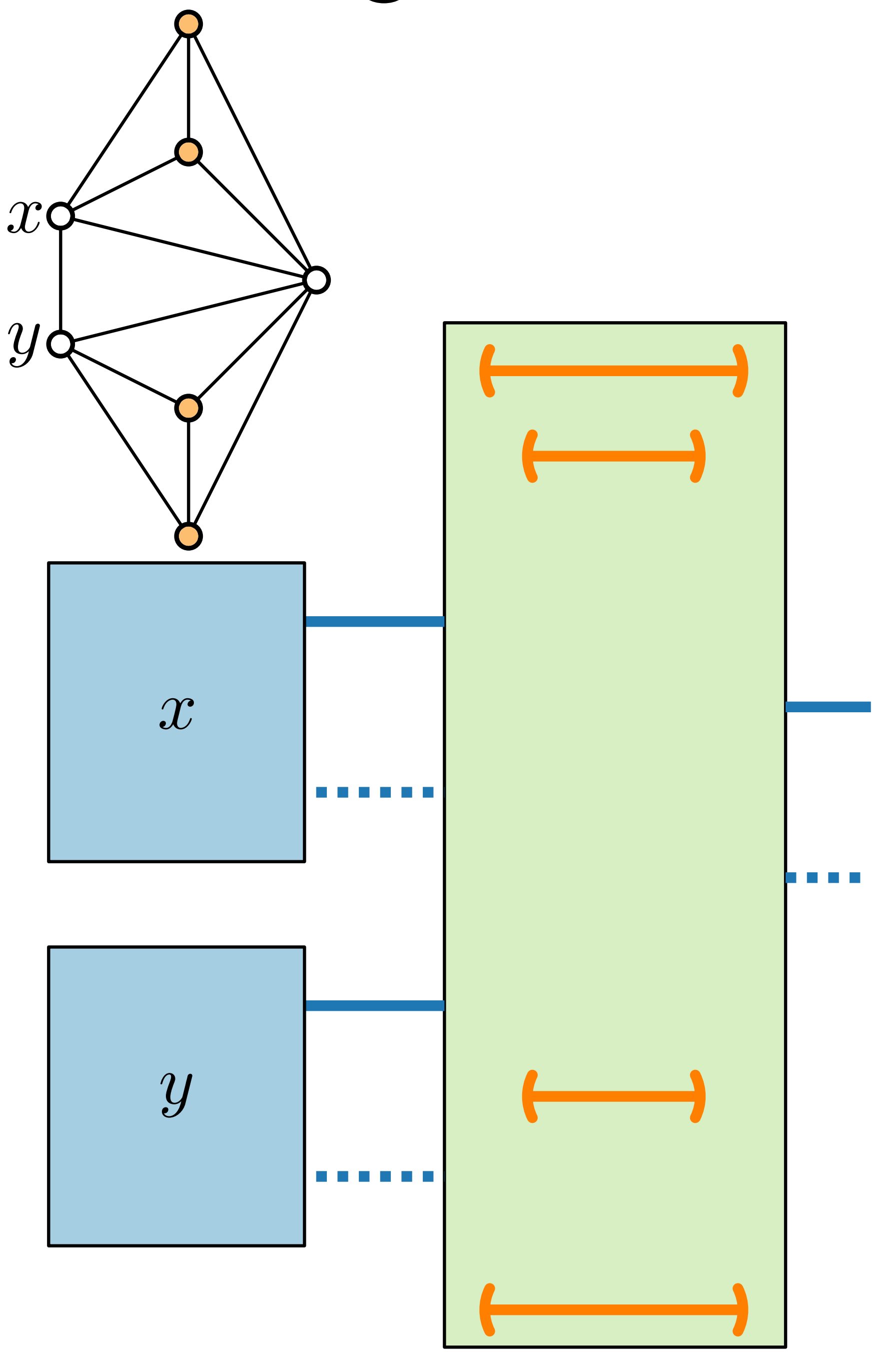

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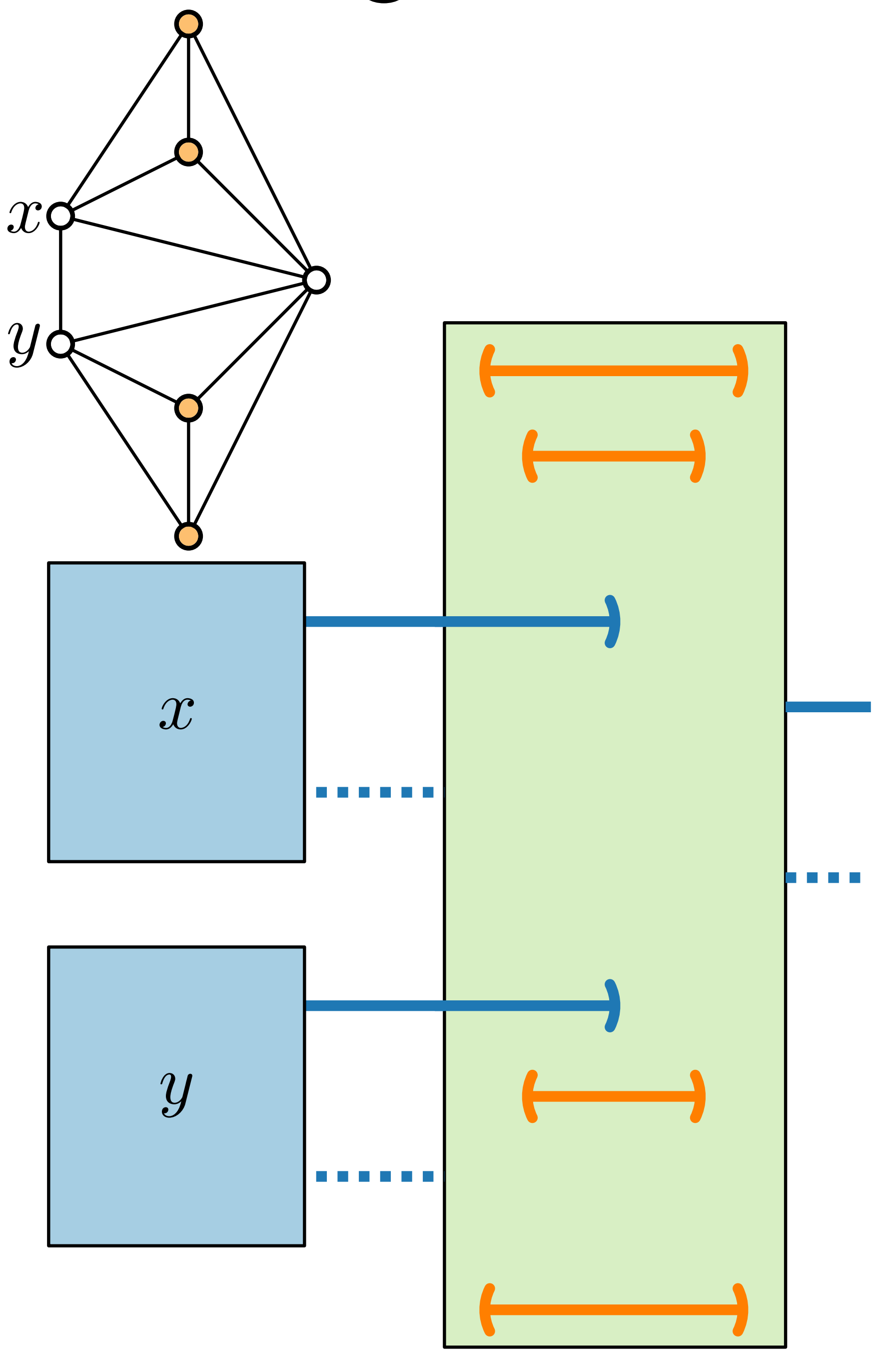

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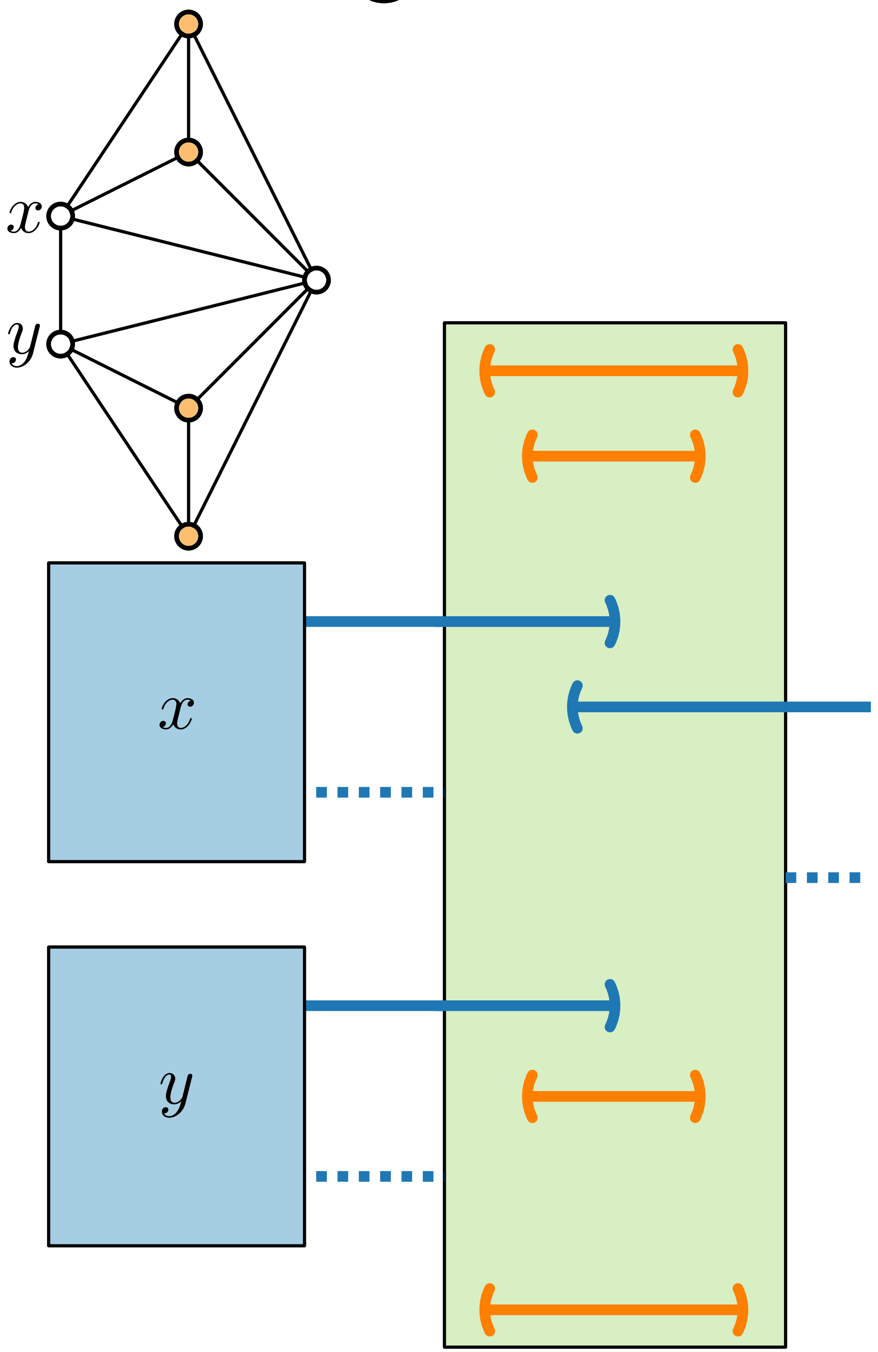

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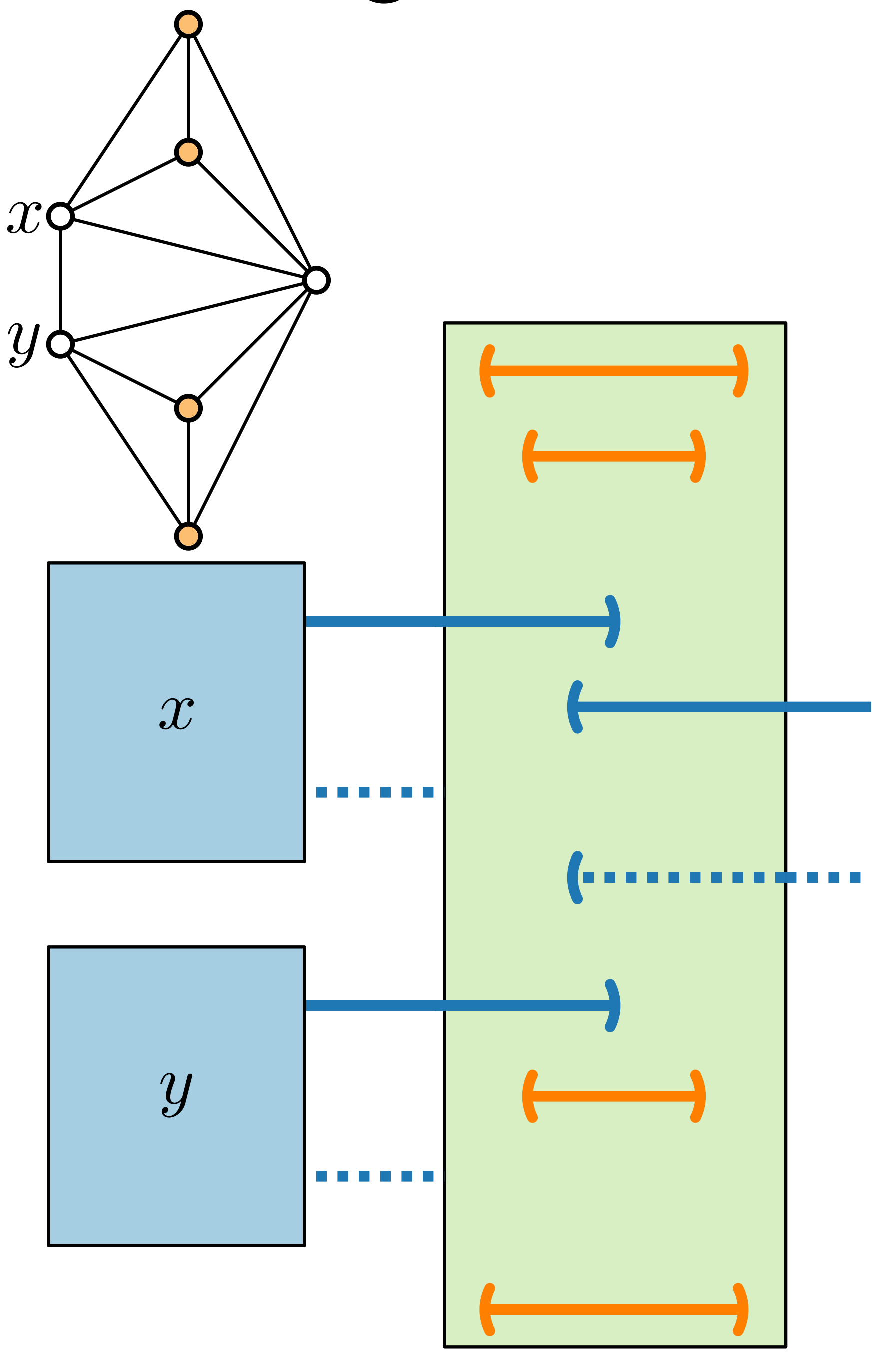

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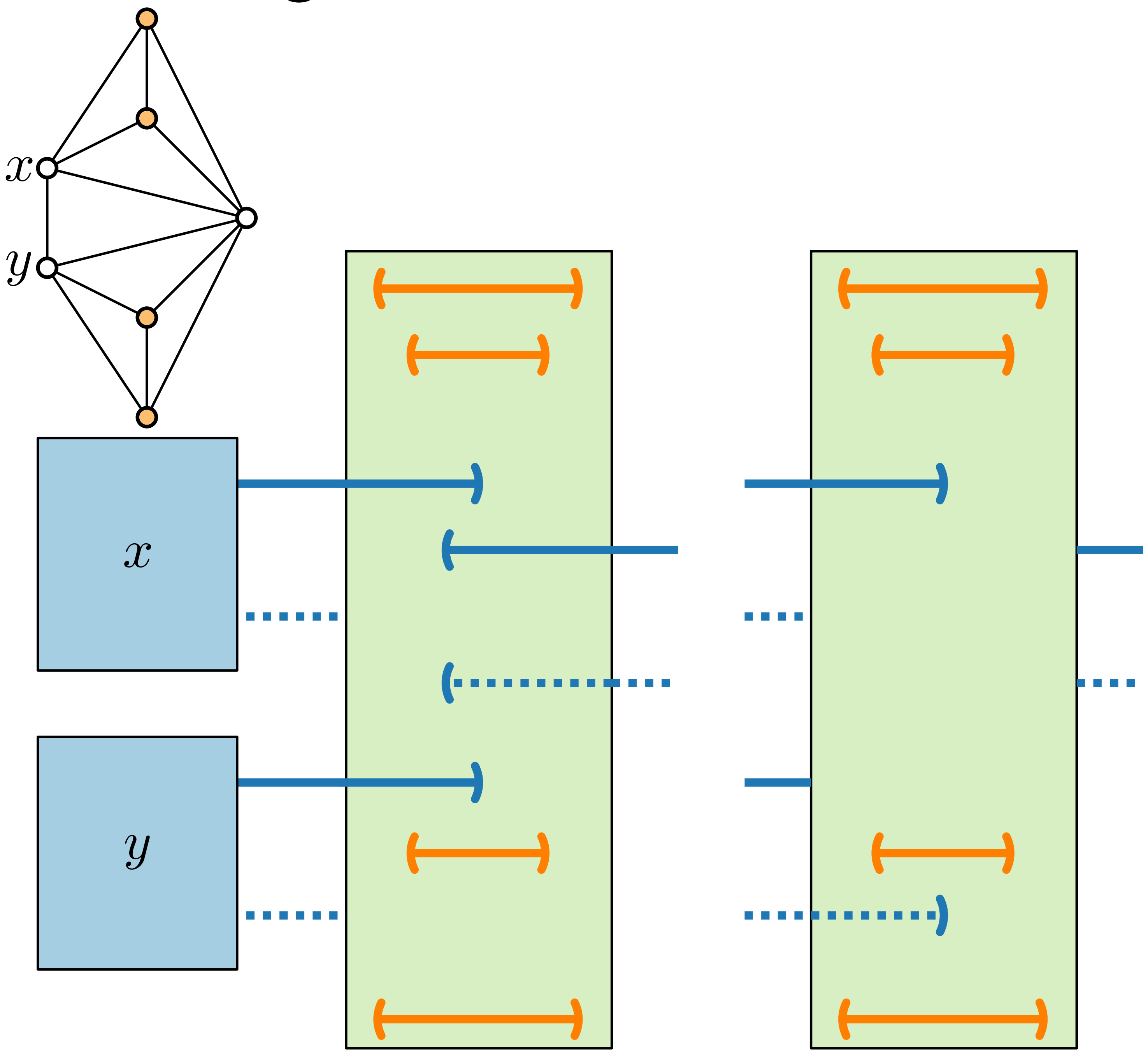

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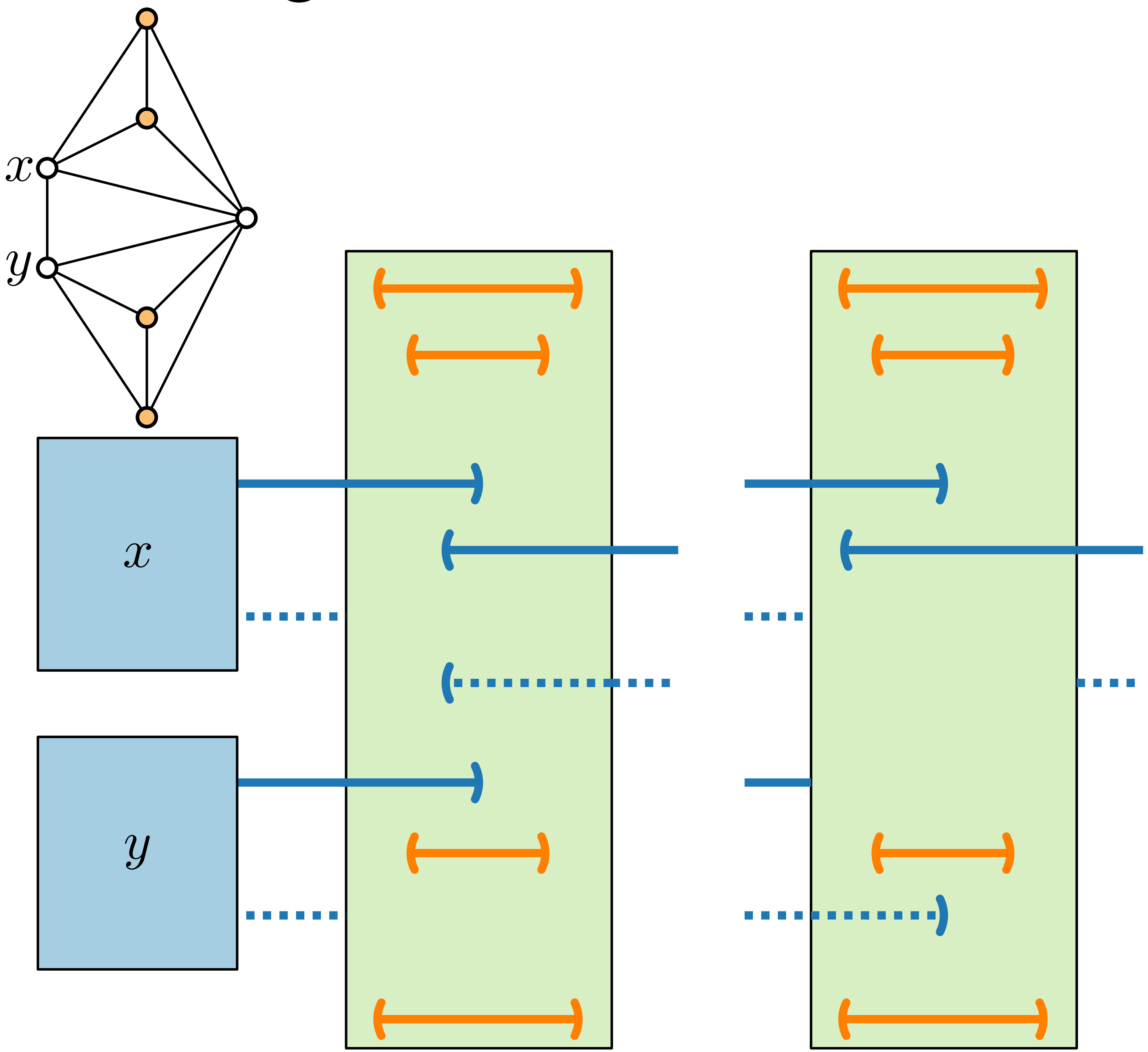

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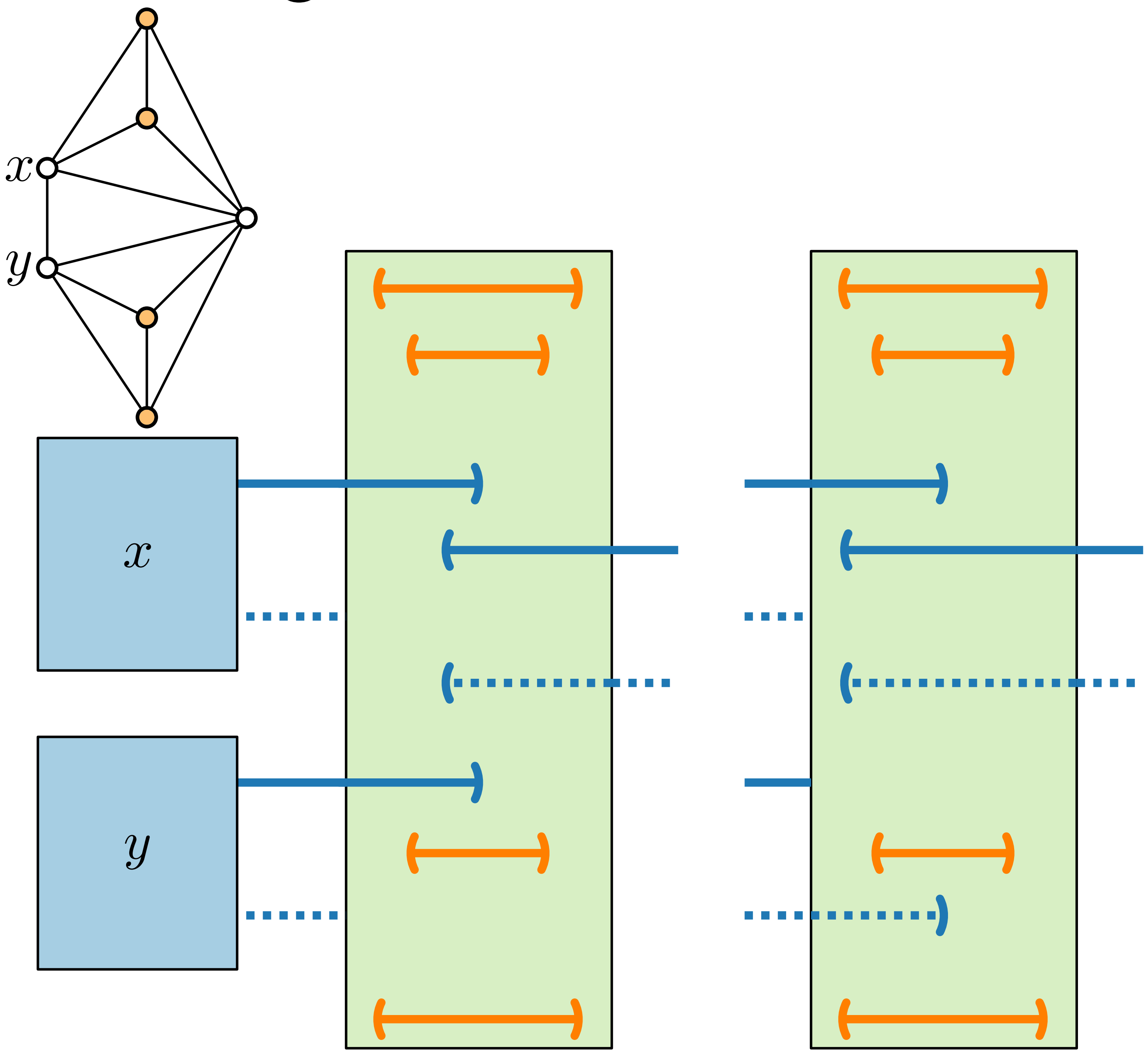

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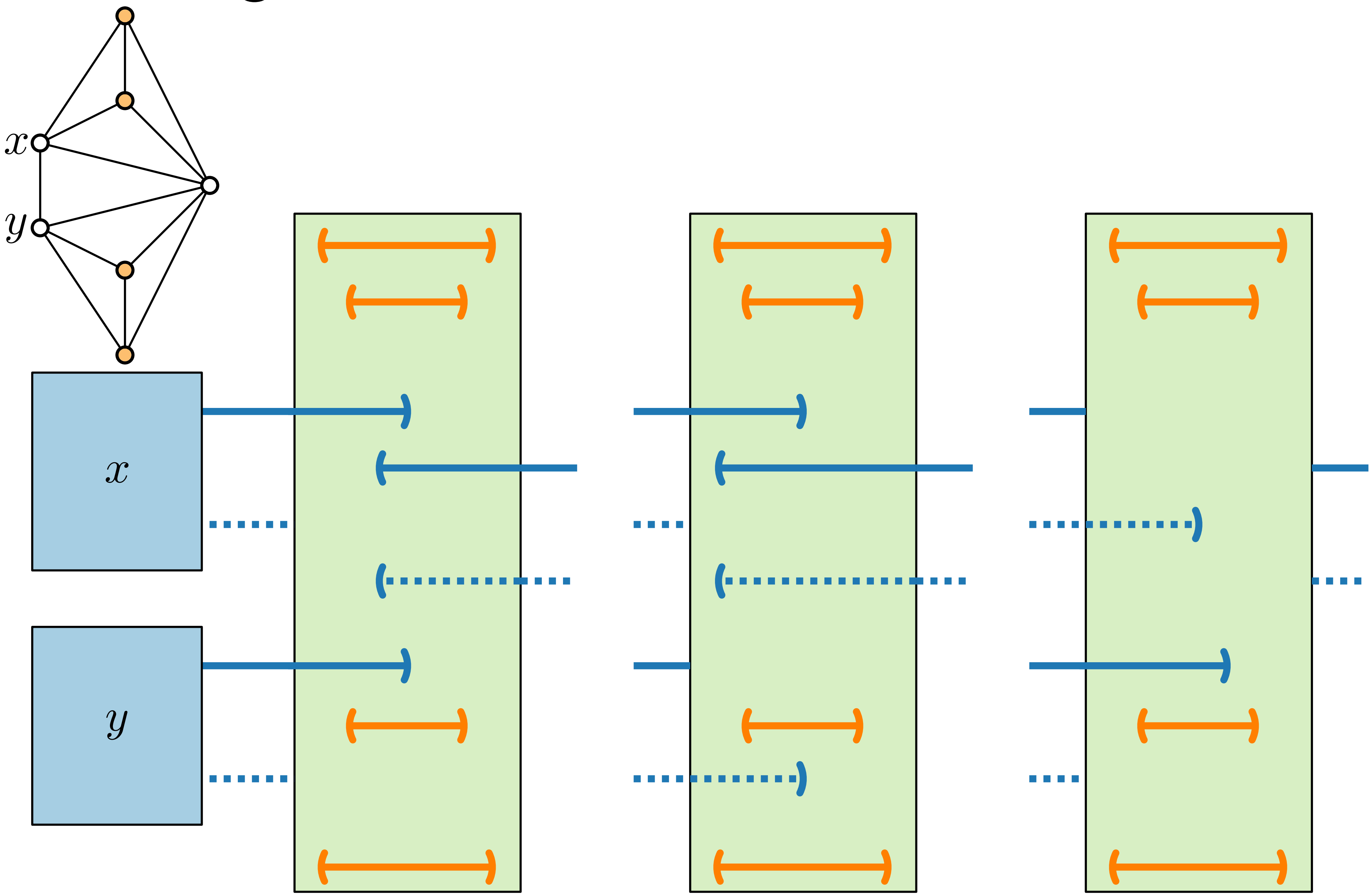

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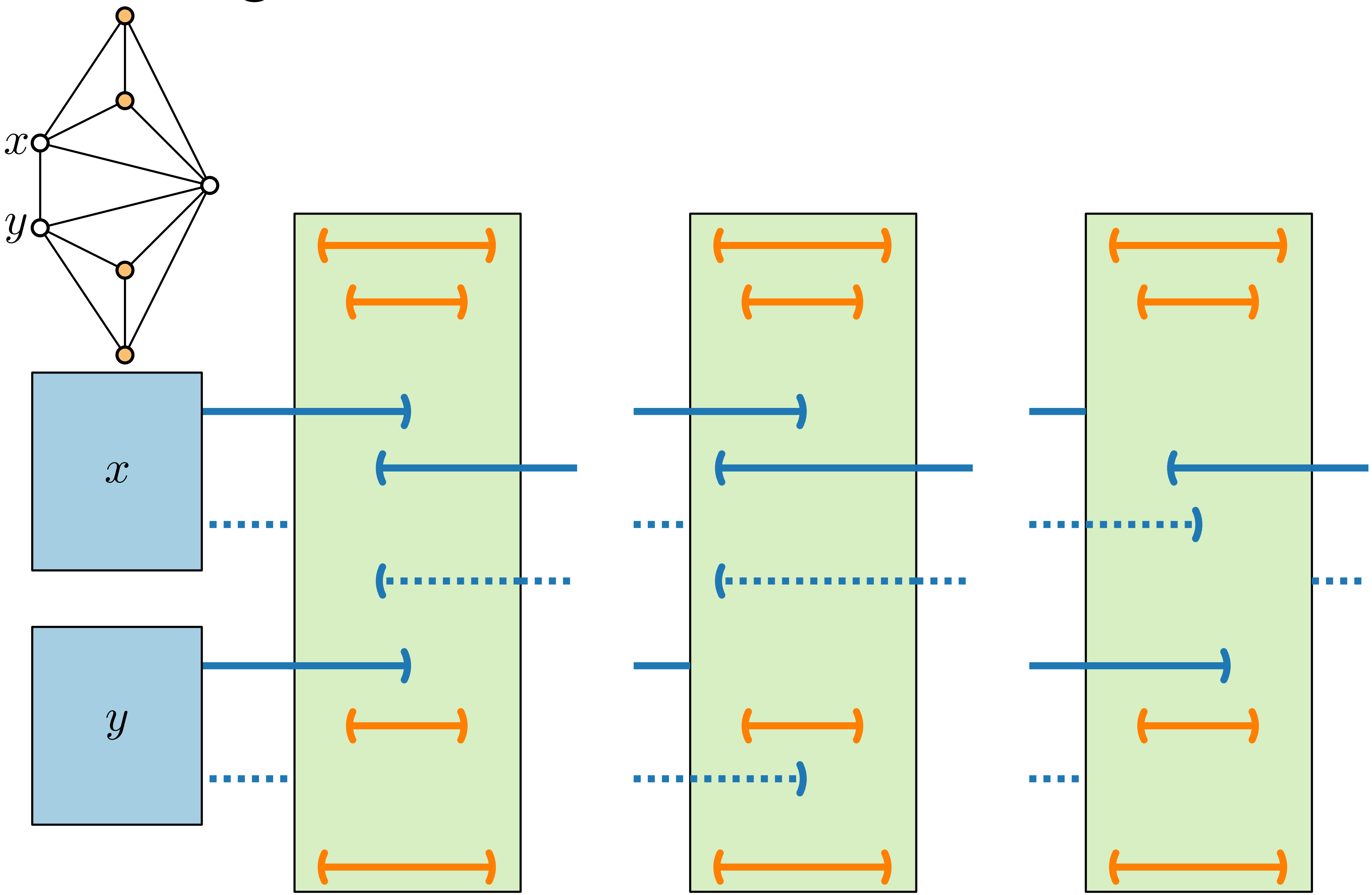

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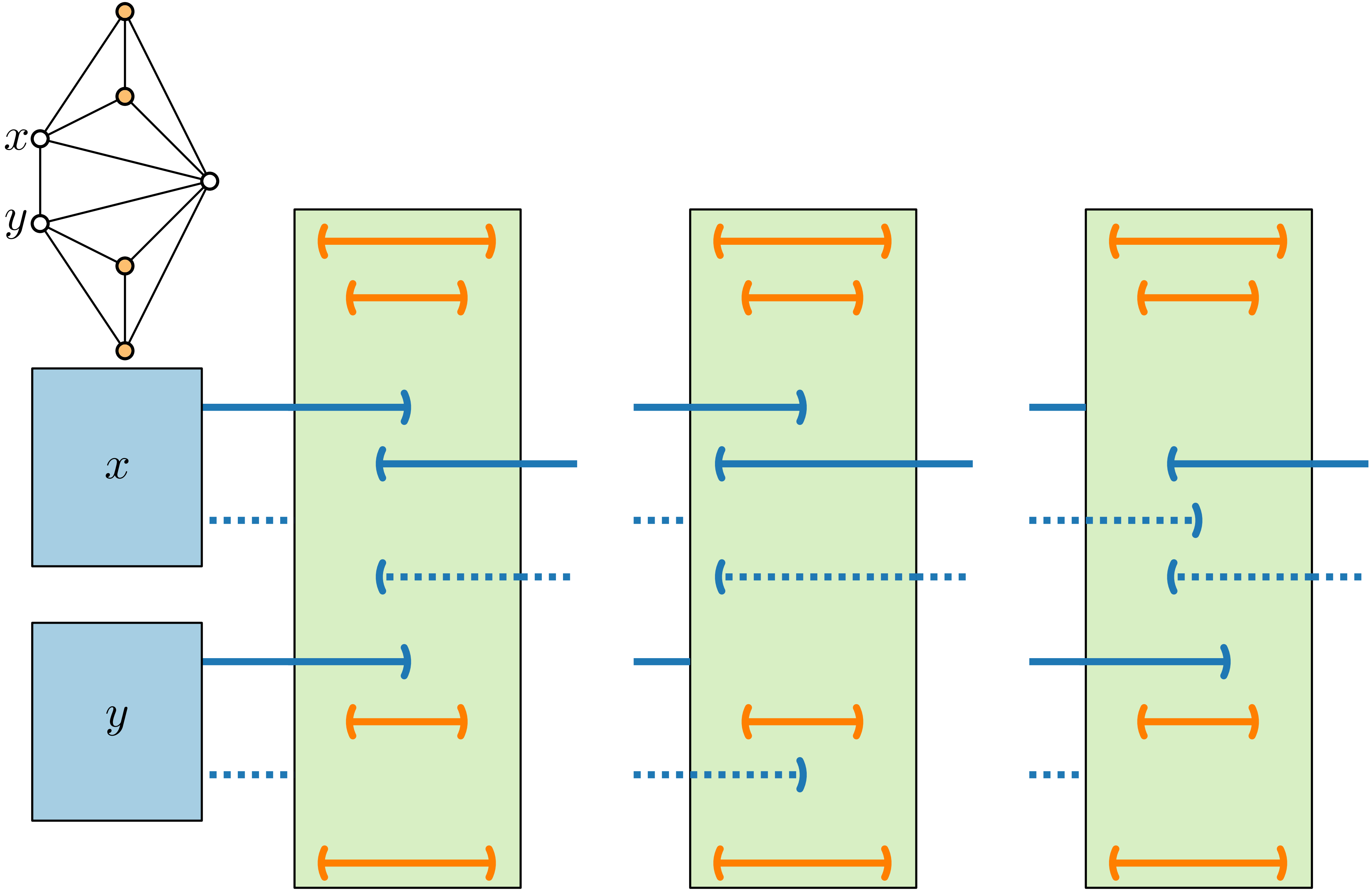

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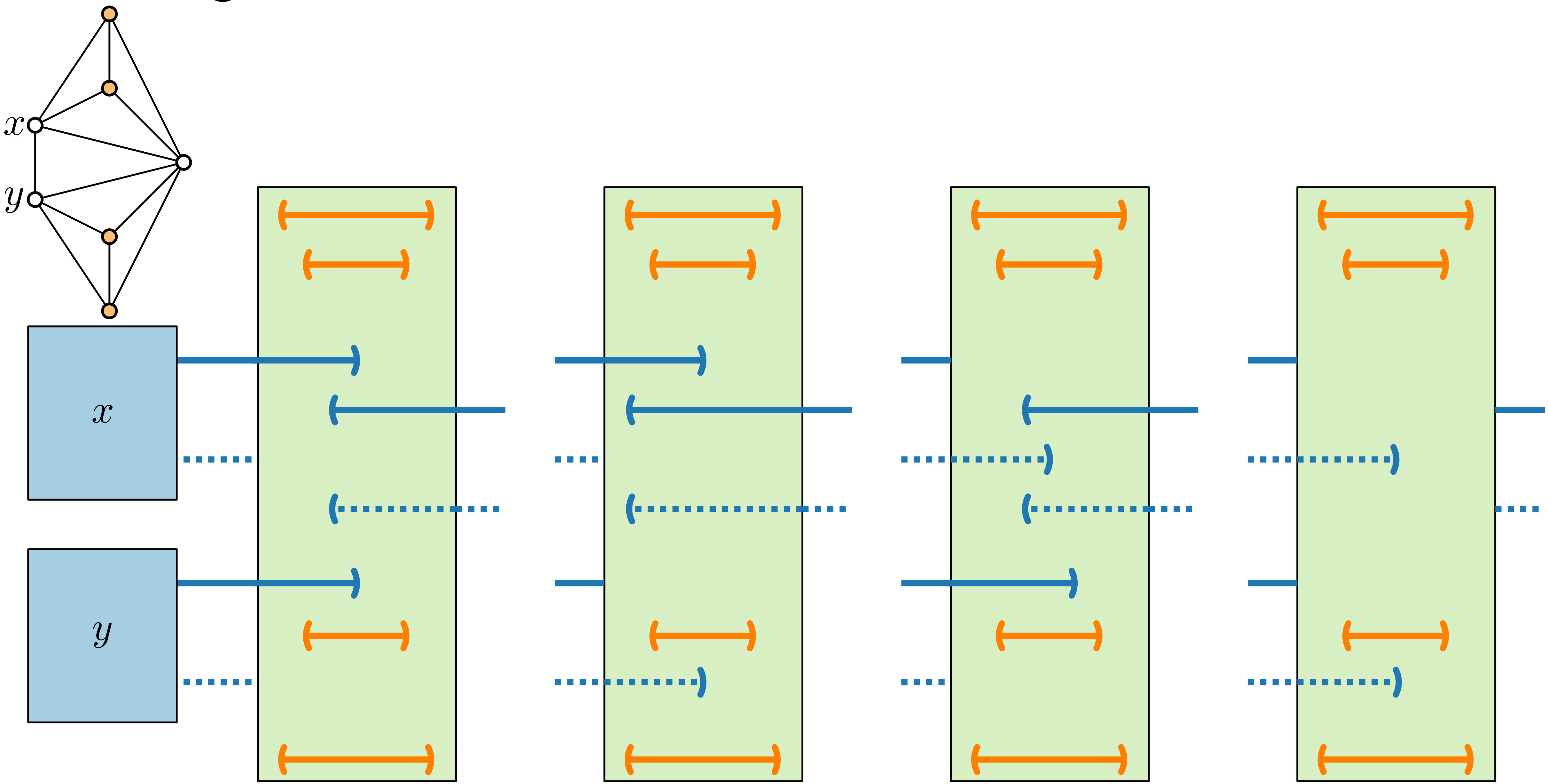

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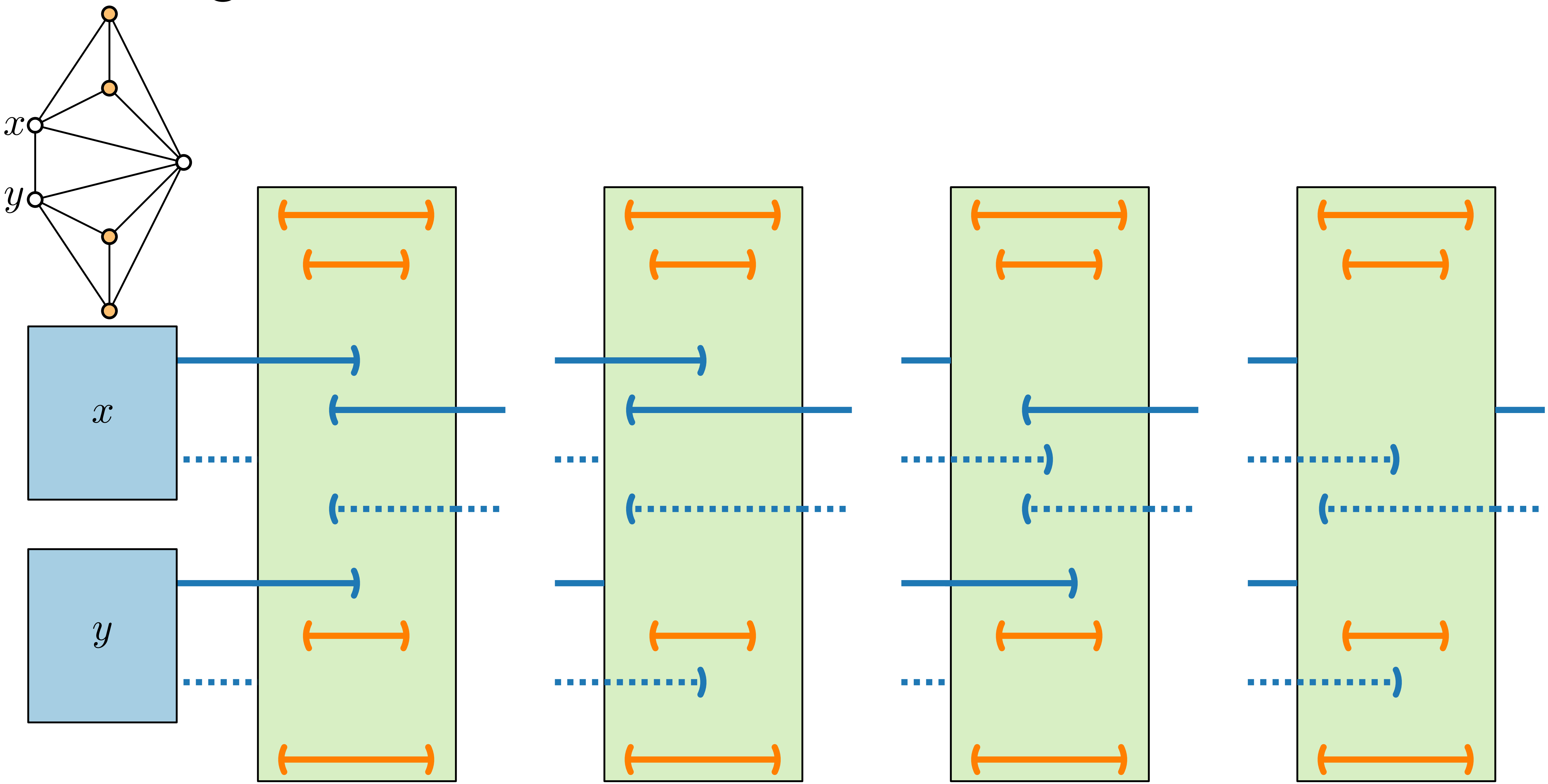

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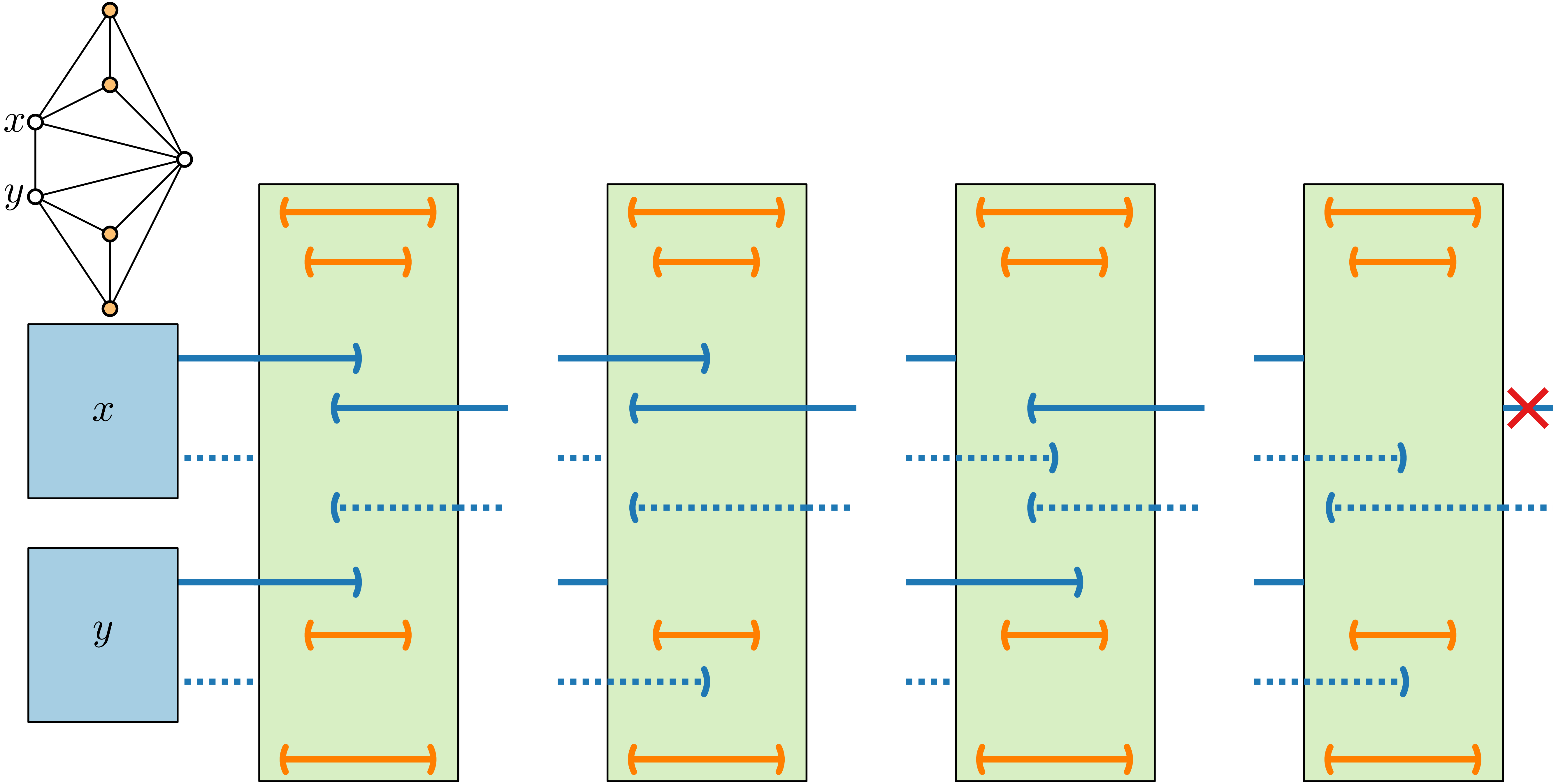

Discussion

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- Can **strong** bar visibility recognition / representation extension be solved in polynomial time for st-graphs?

Literature

Main source:

- [Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18]
  The Partial Visibility Representation Extension Problem

Referenced papers:

- [Gutwenger, Mutzel '01] A Linear Time Implementation of SPQR-Trees
- [Wismath '85] Characterizing bar line-of-sight graphs
- [Tamassia, Tollis '86] Algorithms for visibility representations of planar graphs
- [Andreae '92] Some results on visibility graphs
- [Chaplick, Dorbec, Kratchovíl, Montassier, Stacho '14]
  Contact representations of planar graphs: Extending a partial representation is hard