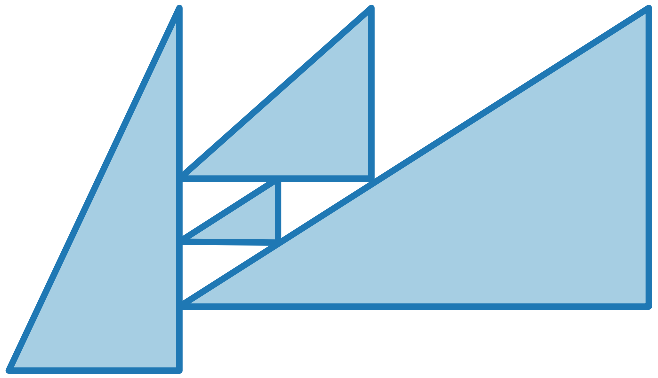
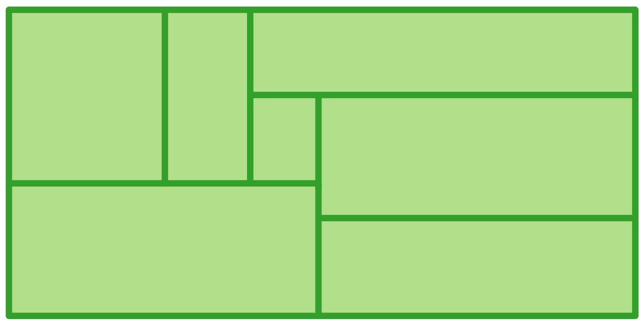
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Visualization of Graphs

Lecture 7:
Contact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals

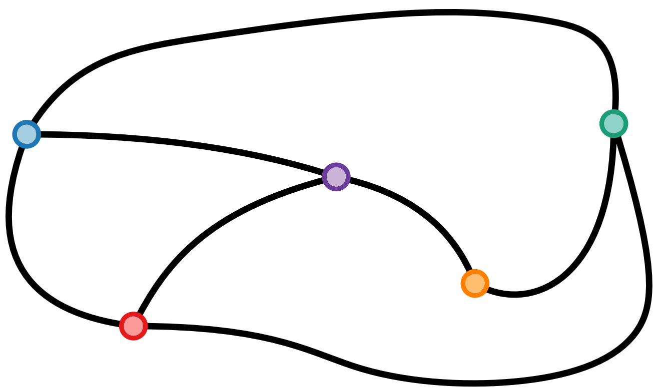


Johannes Zink



Intersection Representation of Graphs

In an **intersection representation** of a graph,
– each vertex is represented by a set



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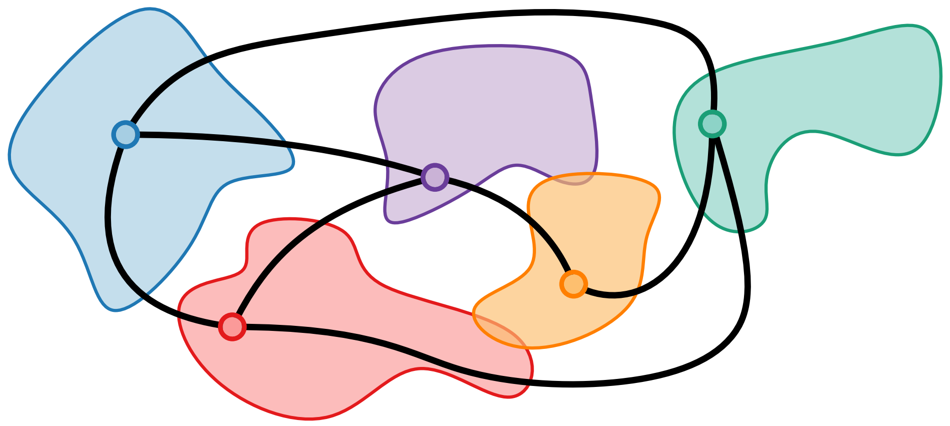

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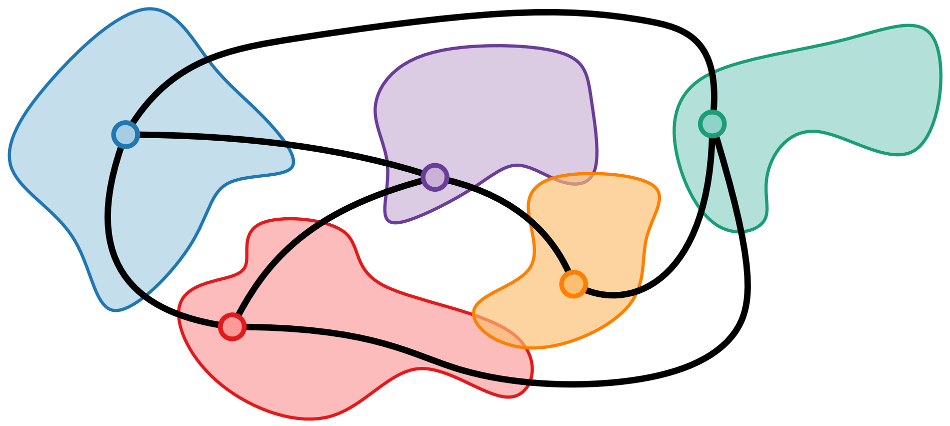

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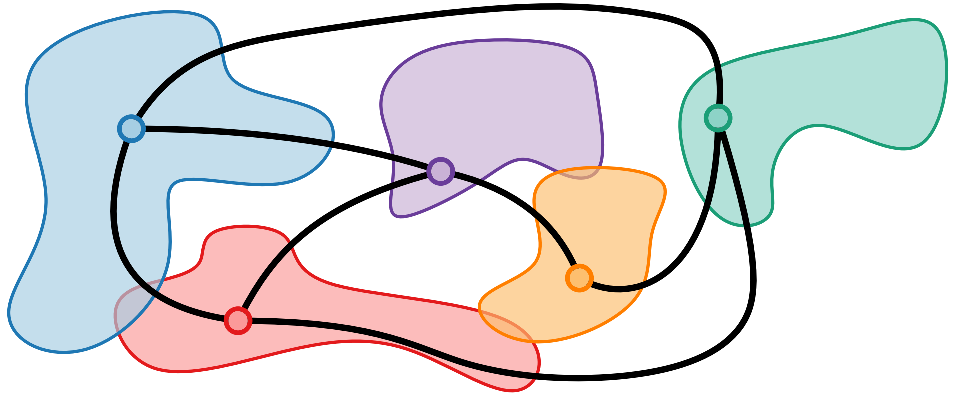

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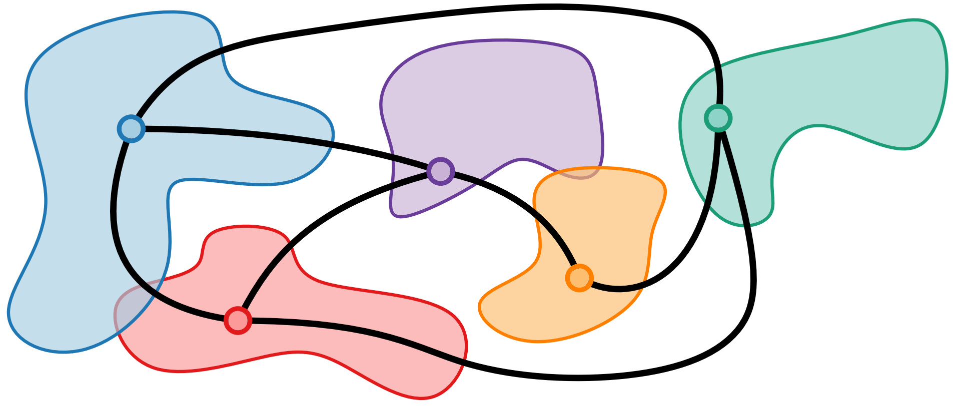

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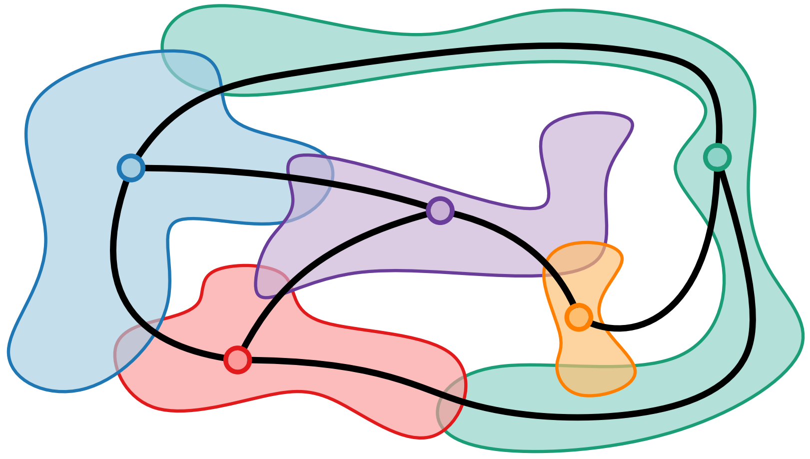

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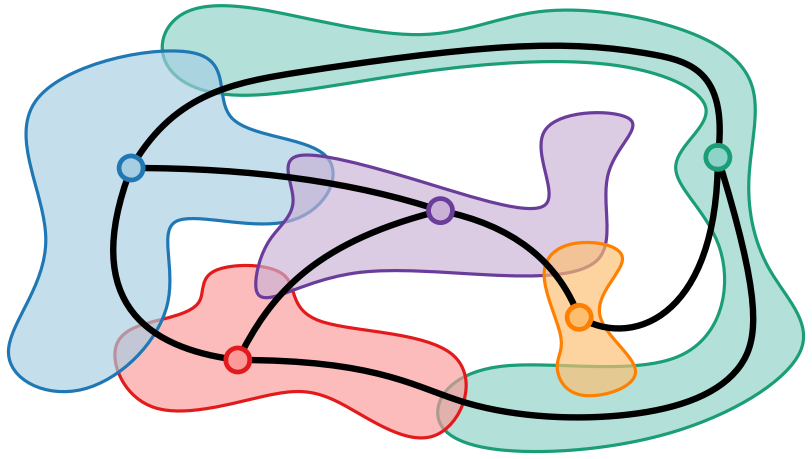
For a collection $\mathcal{S}$ of sets, the **intersection graph** $G(\mathcal{S})$ of $\mathcal{S}$ has vertex set $\mathcal{S}$ and edge set $\{\{S, S'\} \colon S, S' \in \mathcal{S}, S \neq S', \text{ and } S \cap S' \neq \emptyset\}$.



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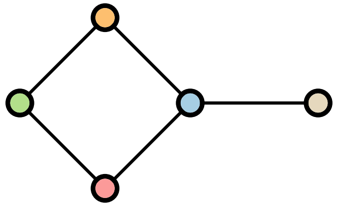

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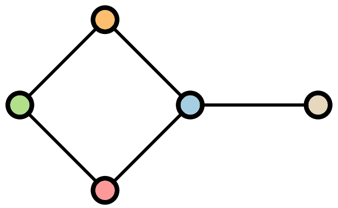

Represent each vertex v by a geometric object $S(v)$

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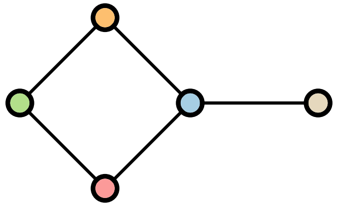

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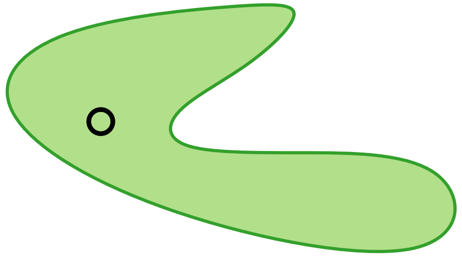

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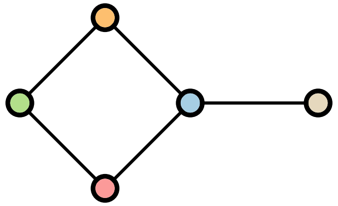

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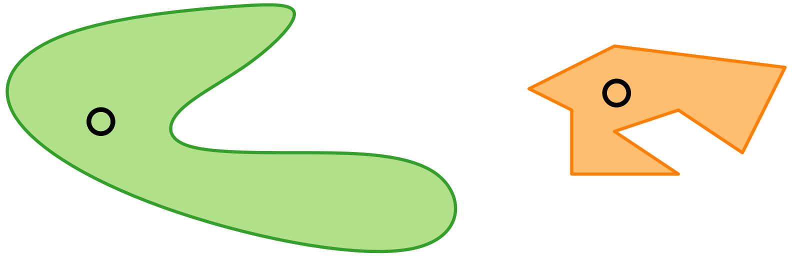

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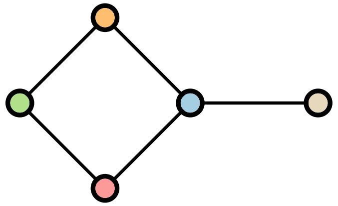

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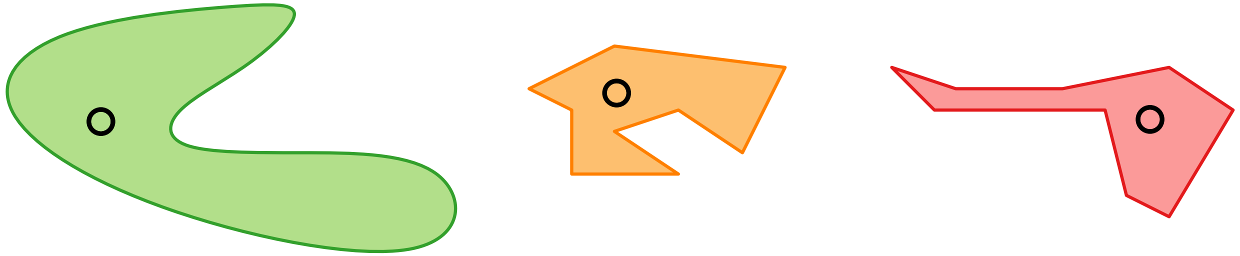

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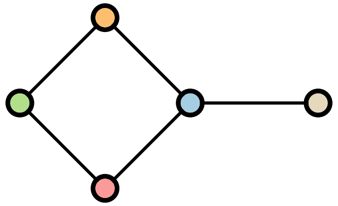

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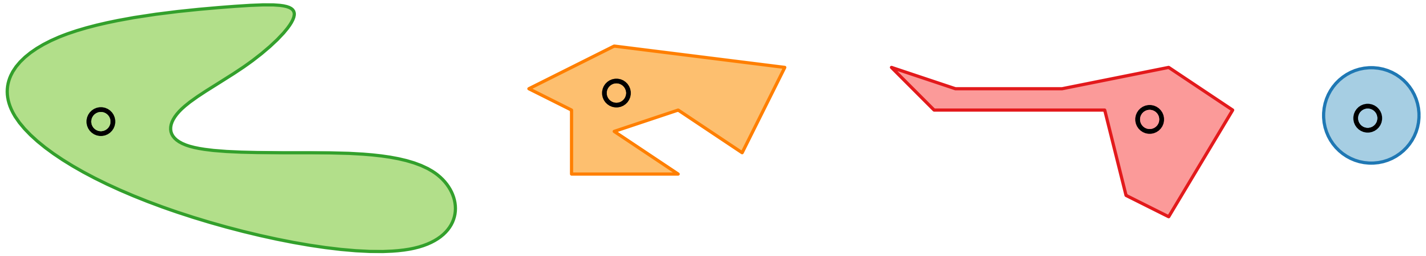

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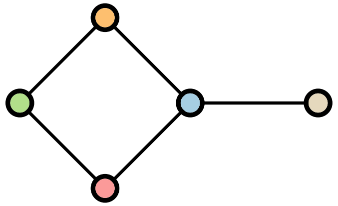

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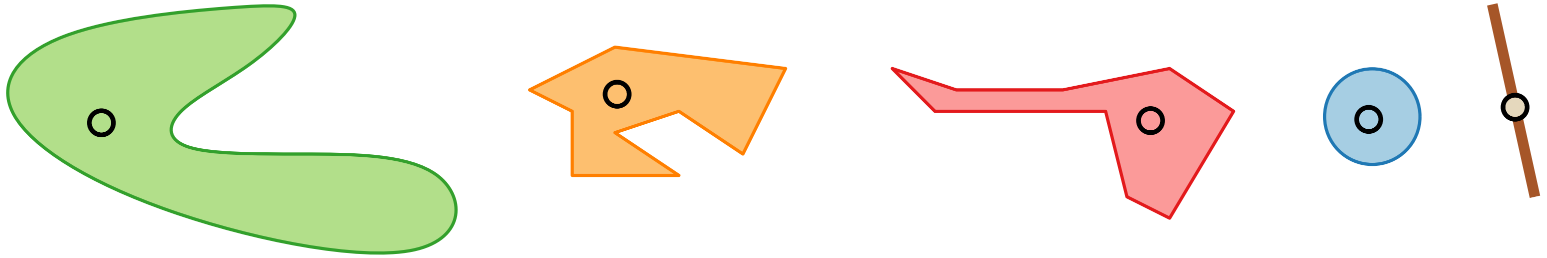

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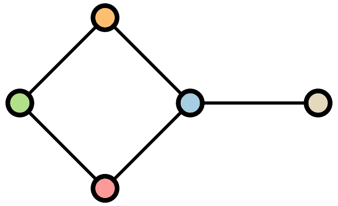

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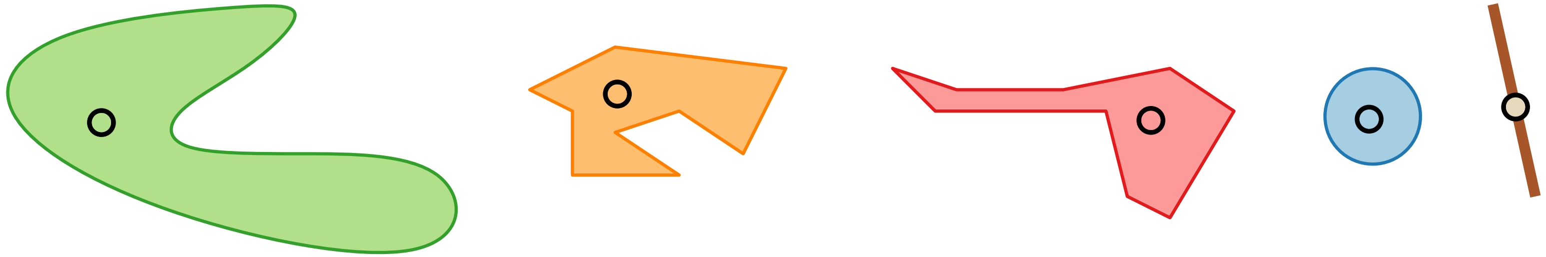

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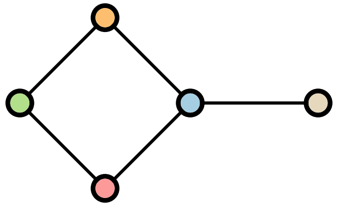

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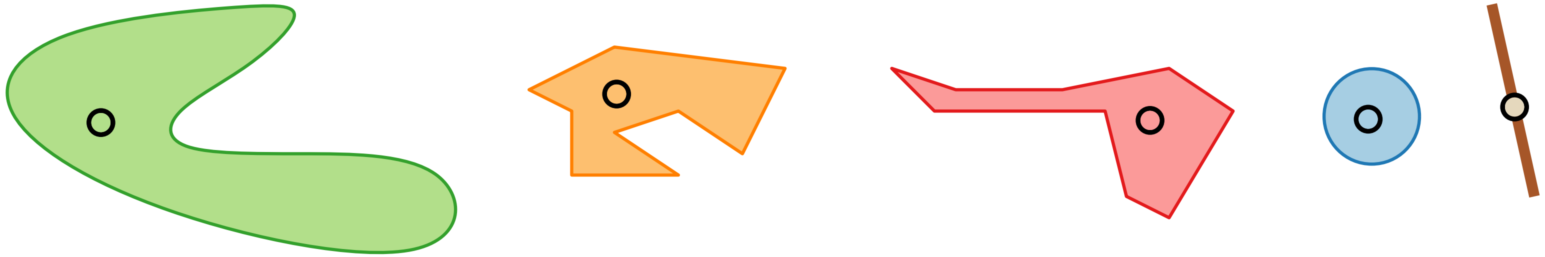

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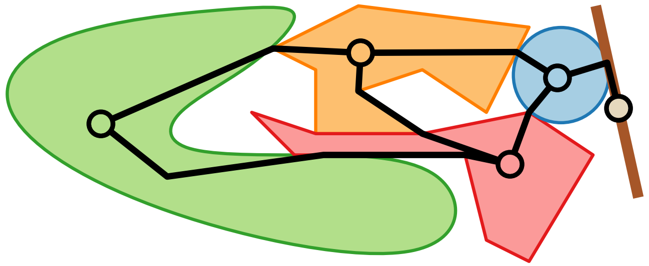

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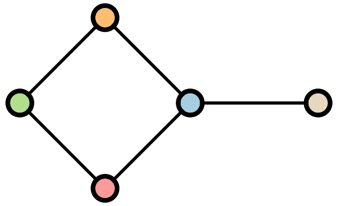

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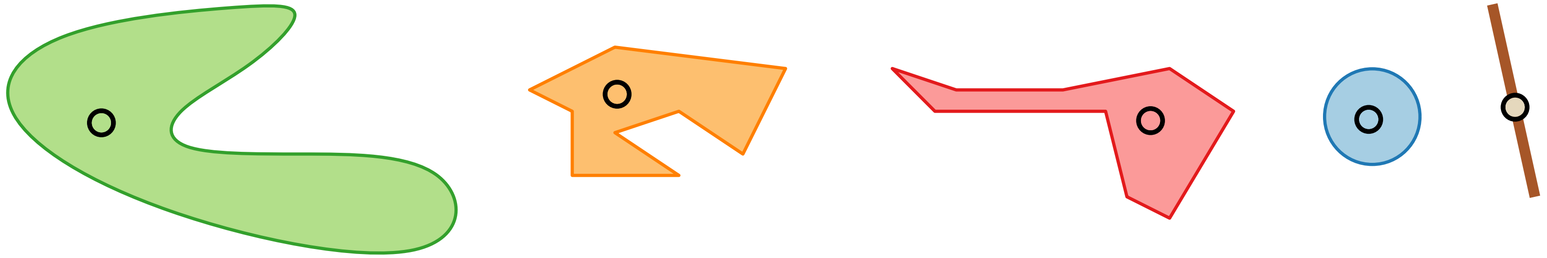

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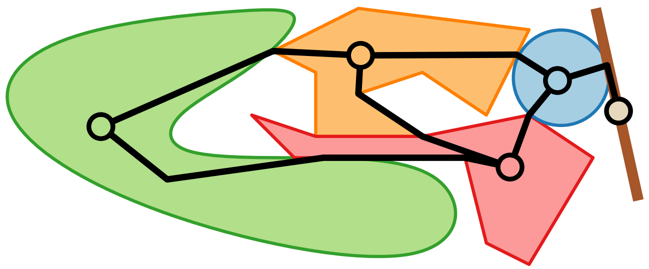

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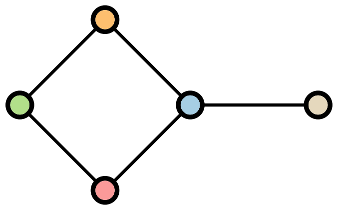

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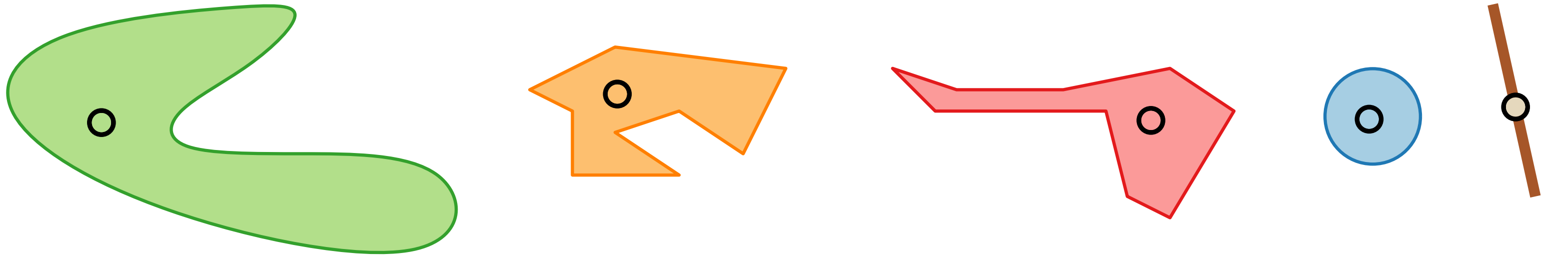

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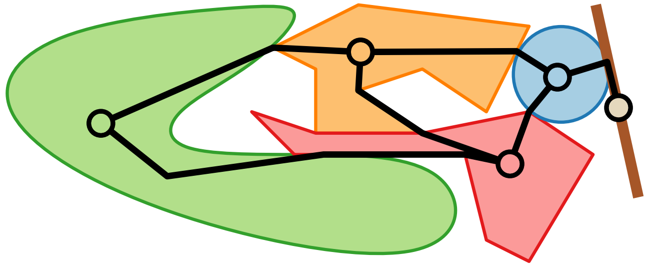

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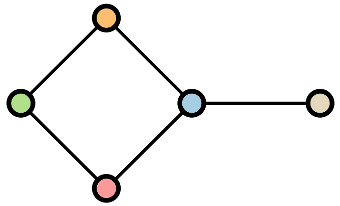

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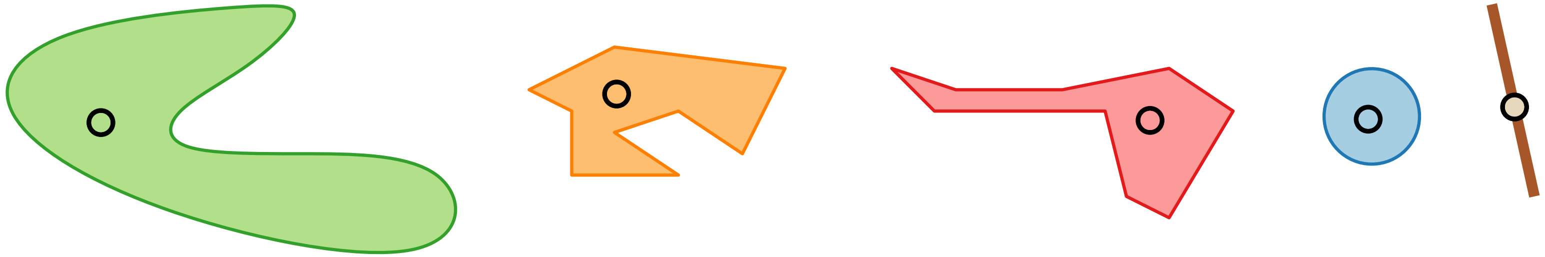

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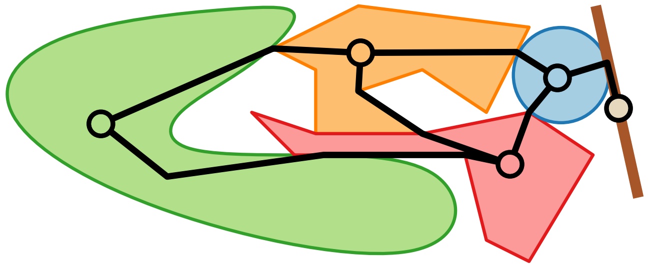

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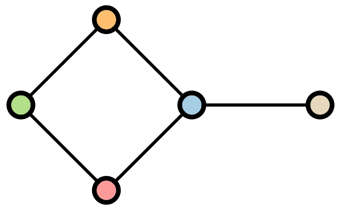

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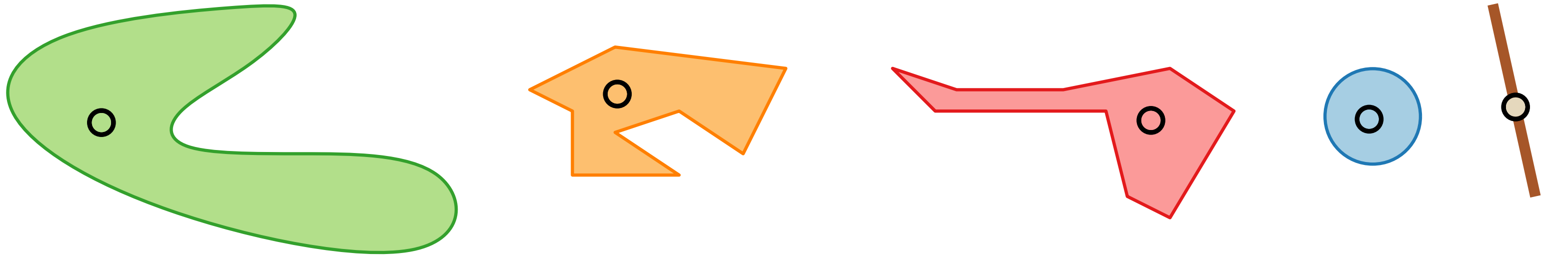

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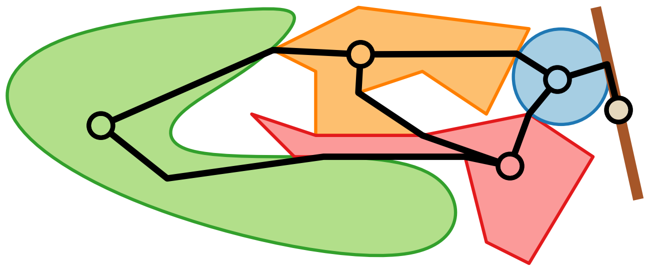

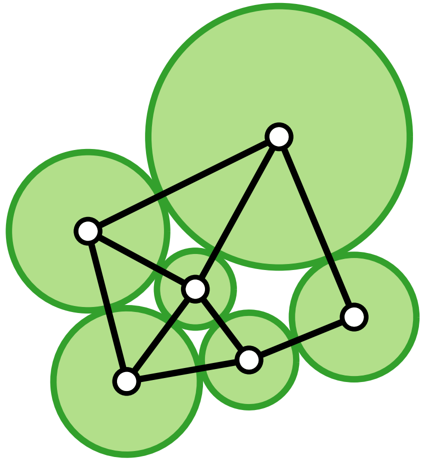

disks

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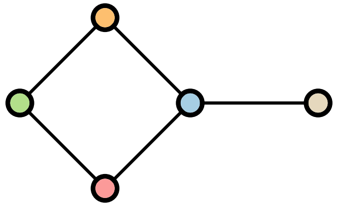

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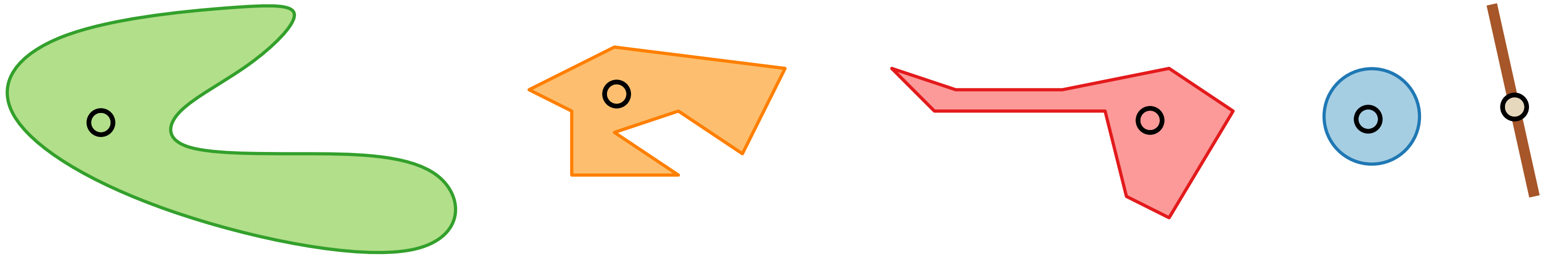

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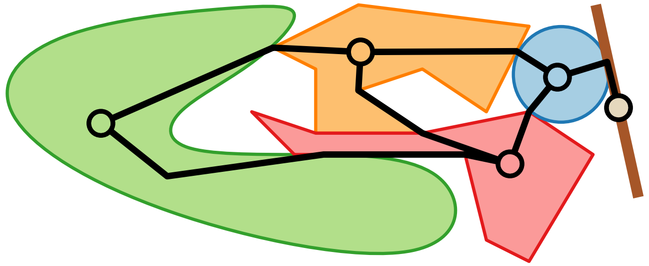

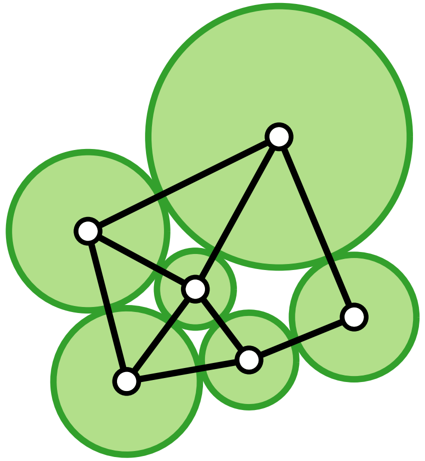

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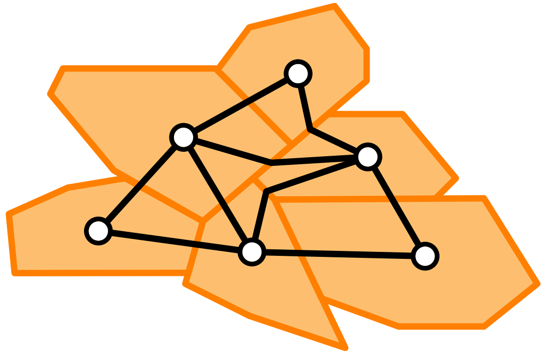

polygons

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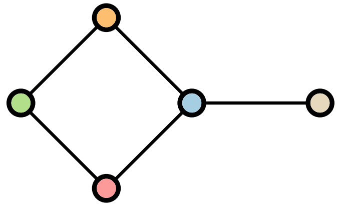
Let G be a graph.  Let $\mathcal{S}$ be a family of geometric objects (e.g., disks).

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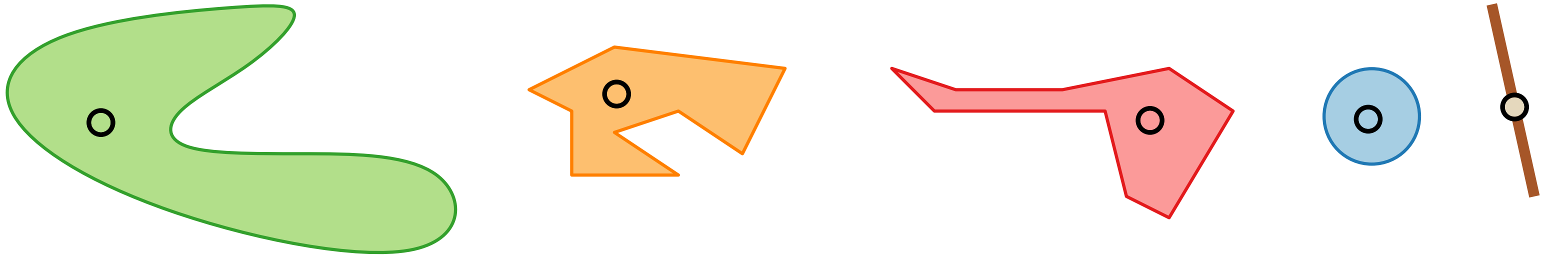
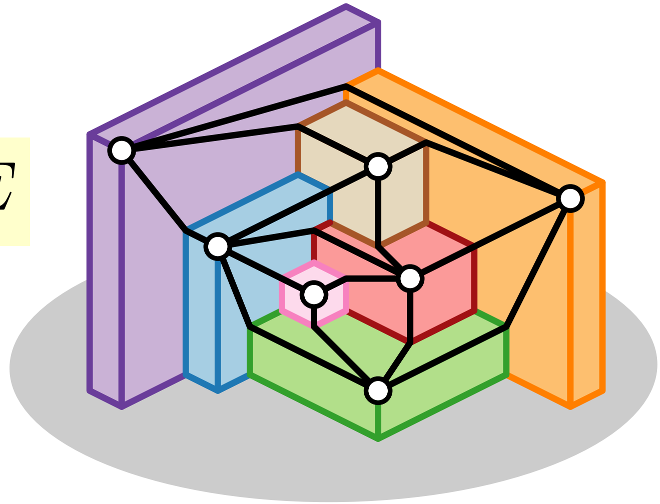


rectangular cuboids



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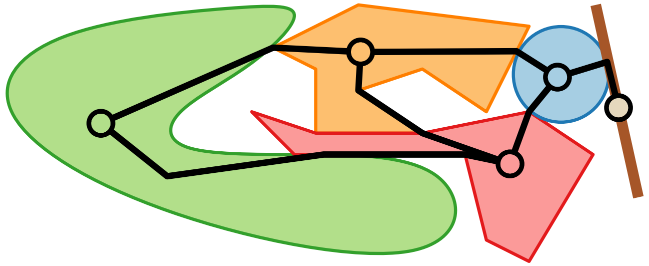

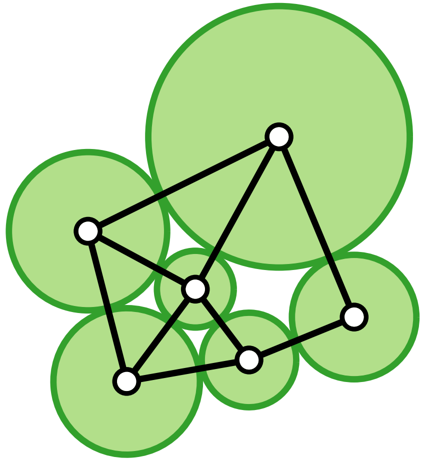

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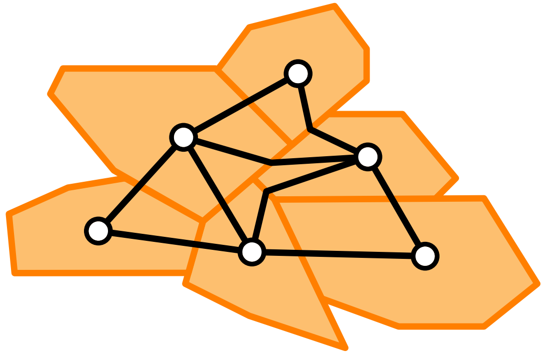

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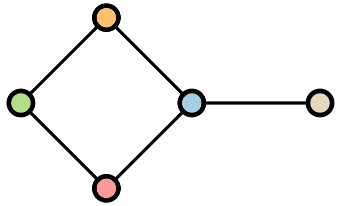

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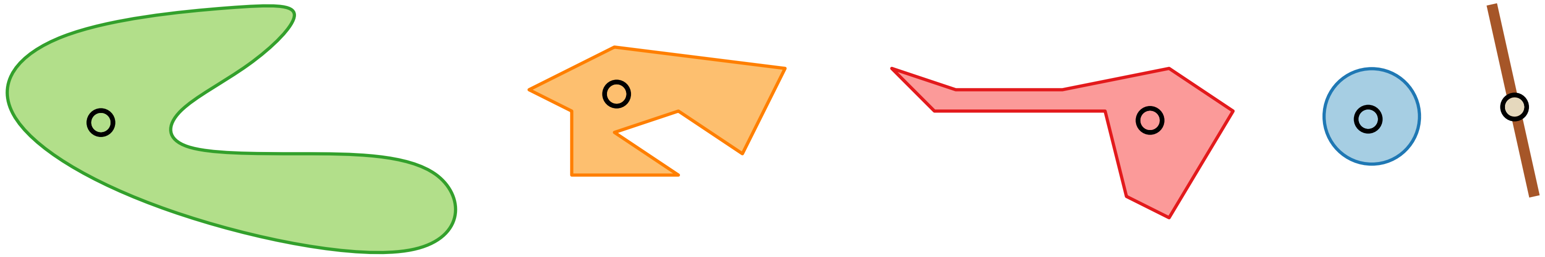

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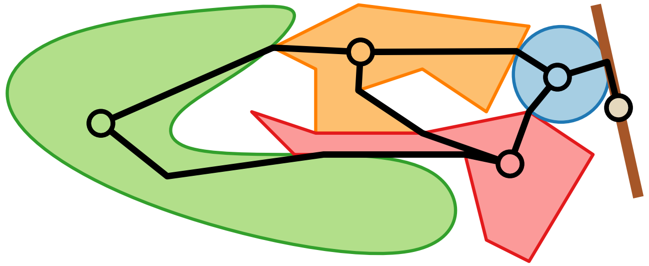
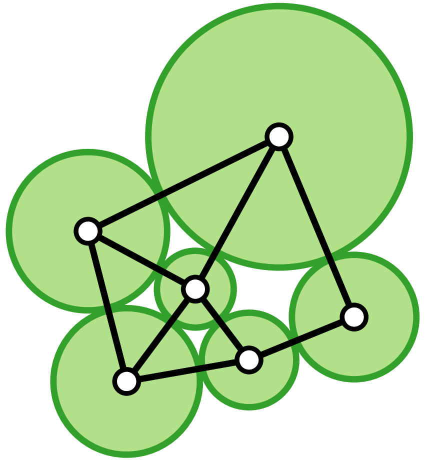


G is planar



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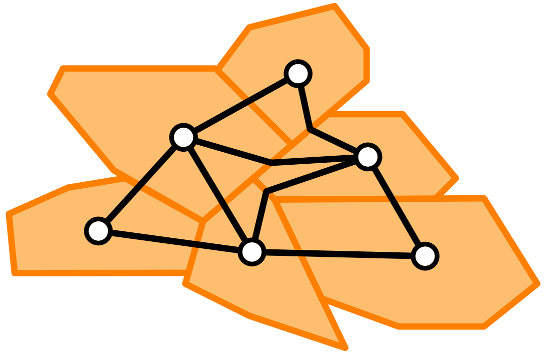
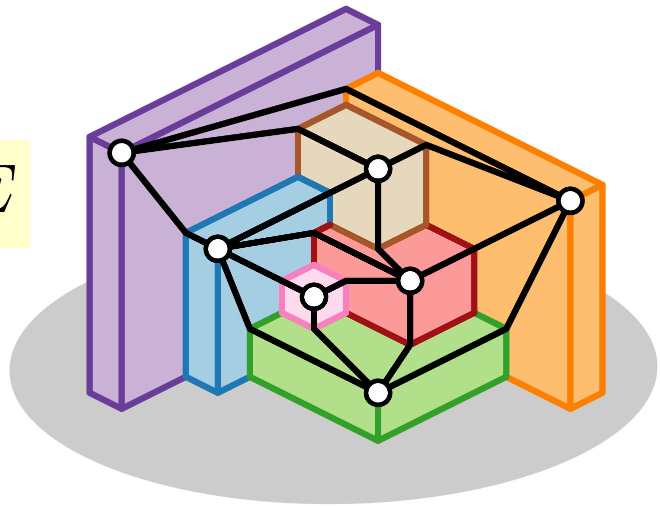


polygons

rectangular cuboids



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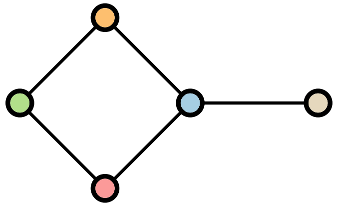

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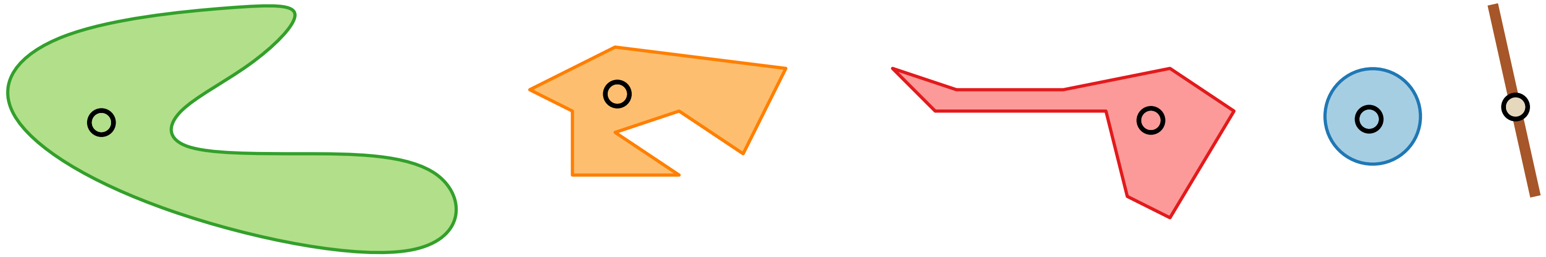

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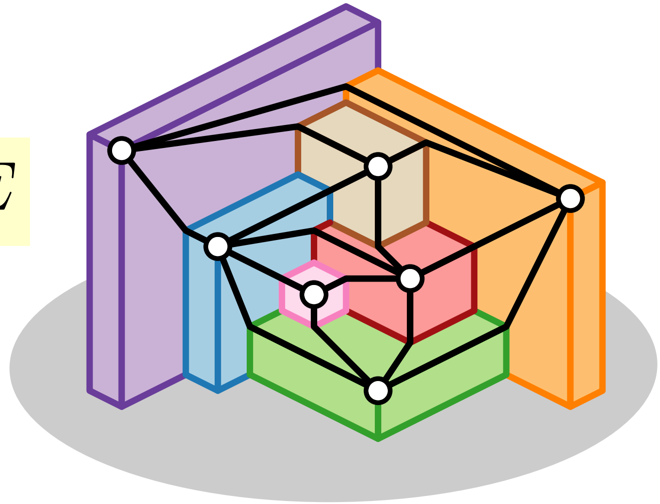

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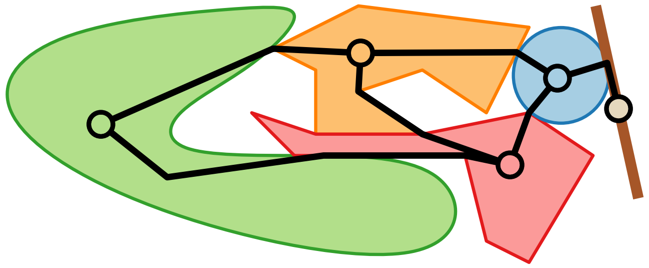

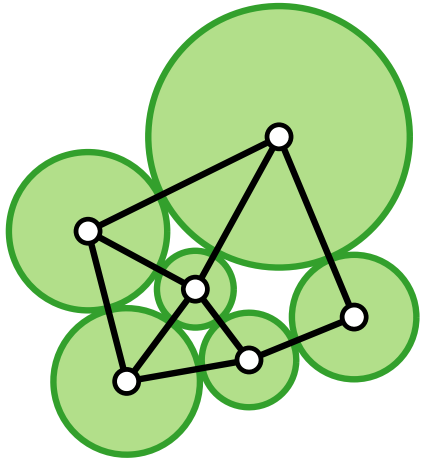

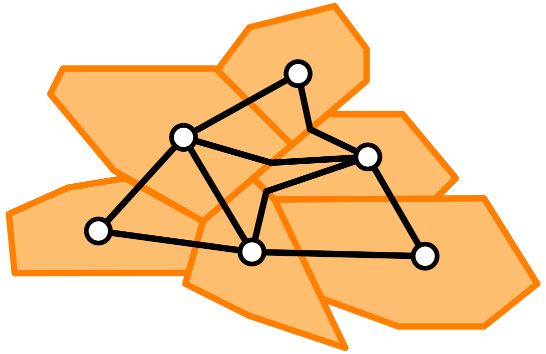

G is planar → disks [Koebe 1936]

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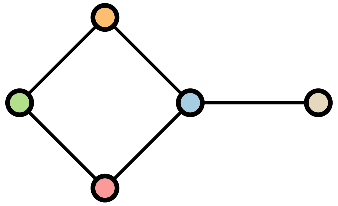

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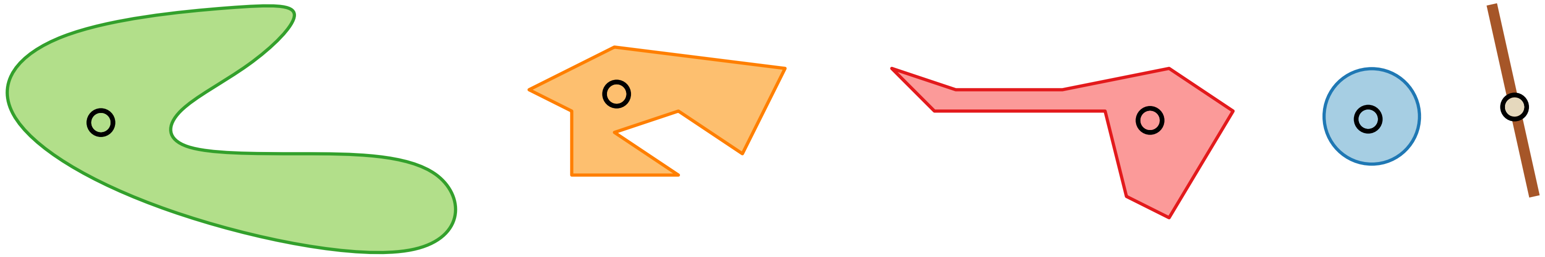
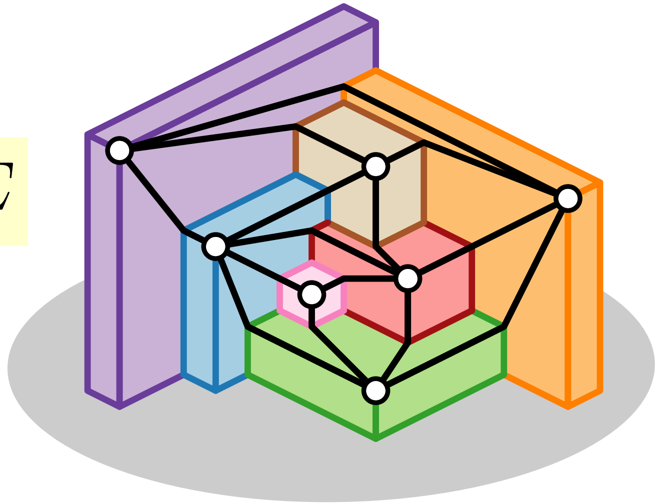
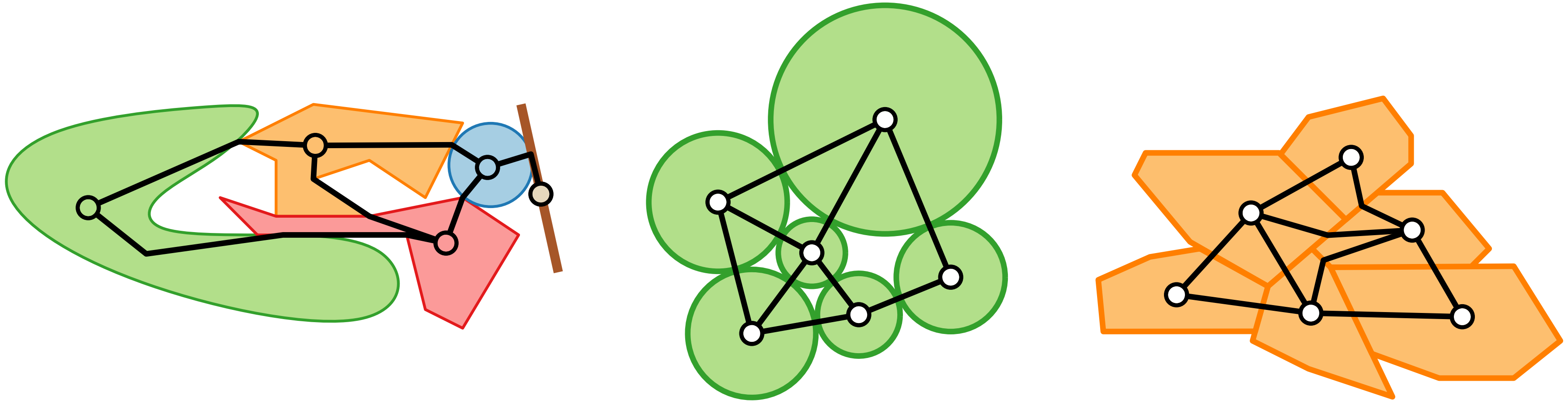


rectangular cuboids



In an **$\mathcal{S}$-contact representation** of G, $S(u)$ and $S(v)$ *touch* iff $uv \in E$

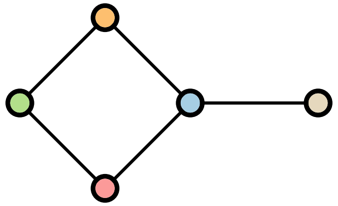


G is planar → disks → polygons

[Koebe 1936]

Contact Representation of Graphs

Let G be a graph.



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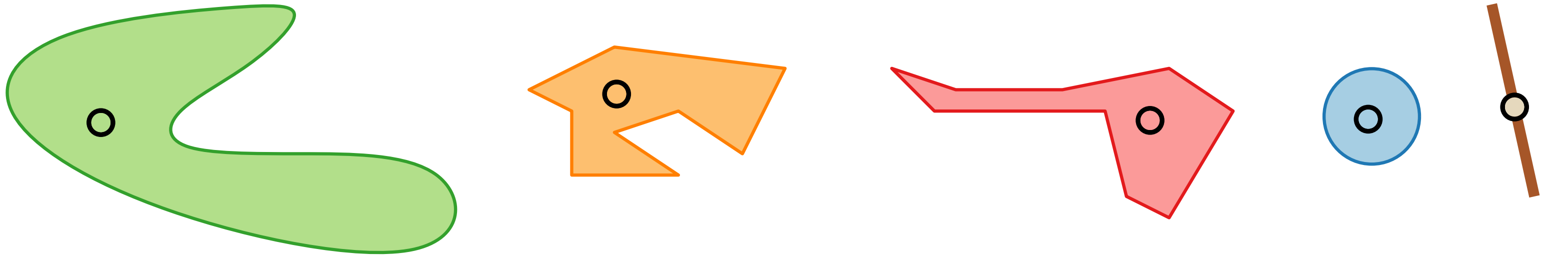

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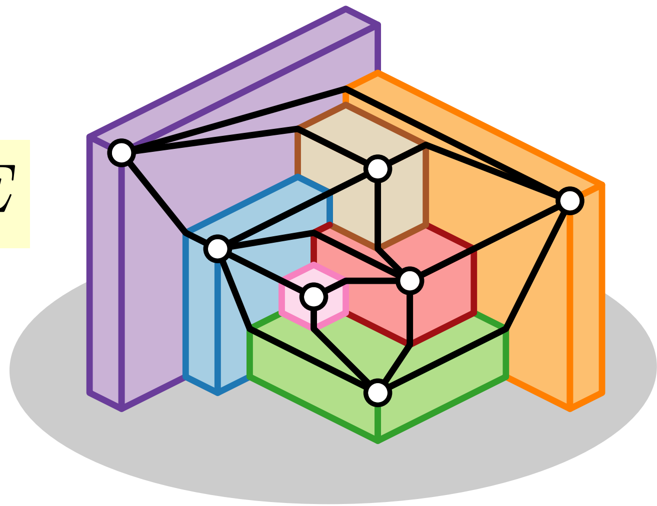

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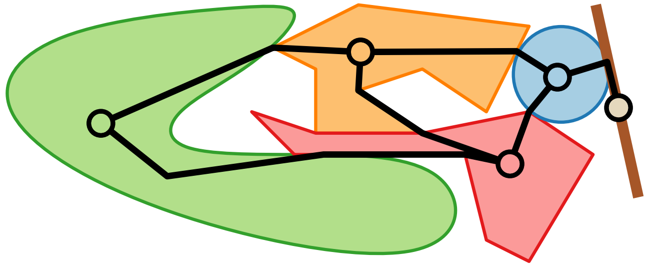

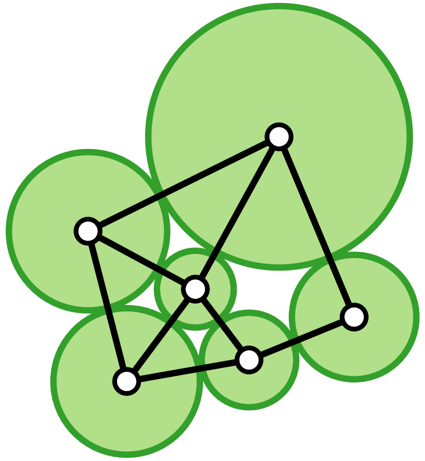

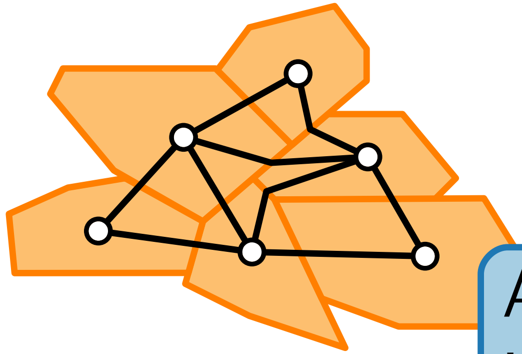

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A contact representation is an intersection representation with interior-disjoint sets.

Contact Representation of Planar Graphs

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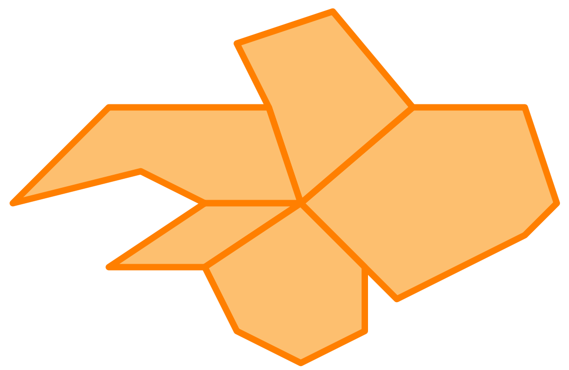

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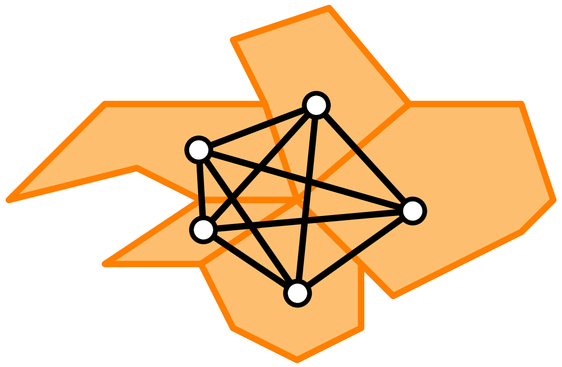

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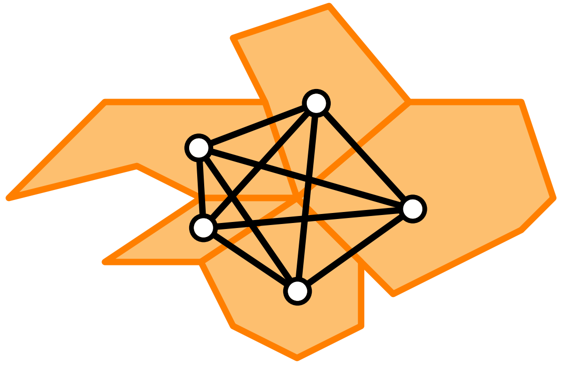

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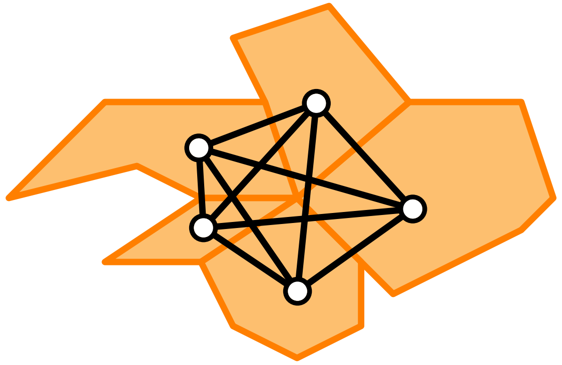

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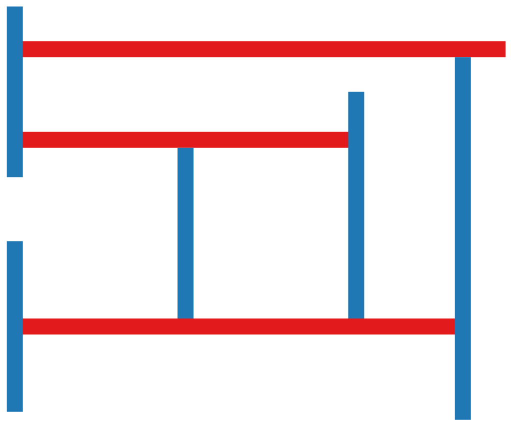

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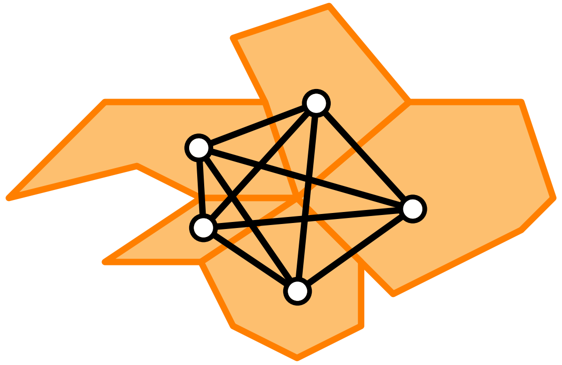

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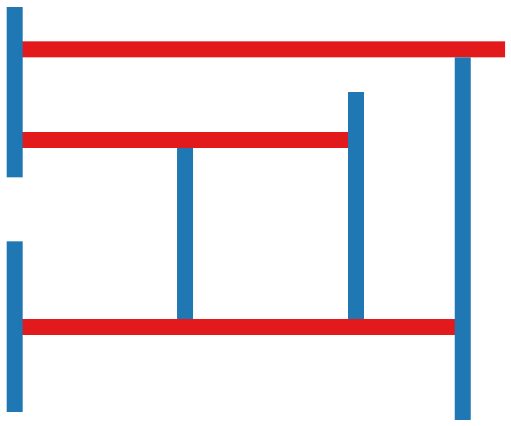

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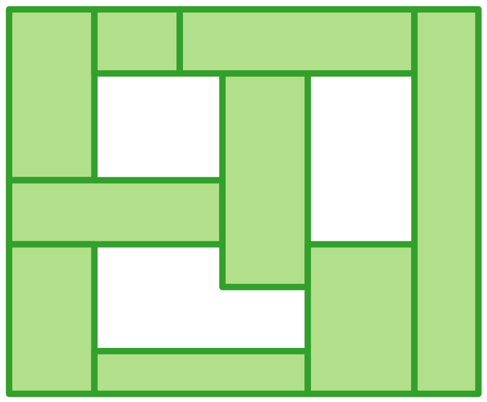

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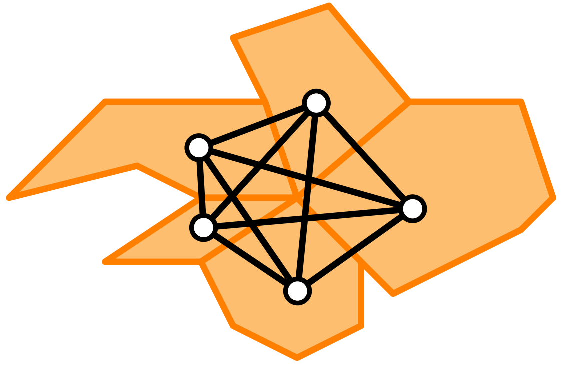

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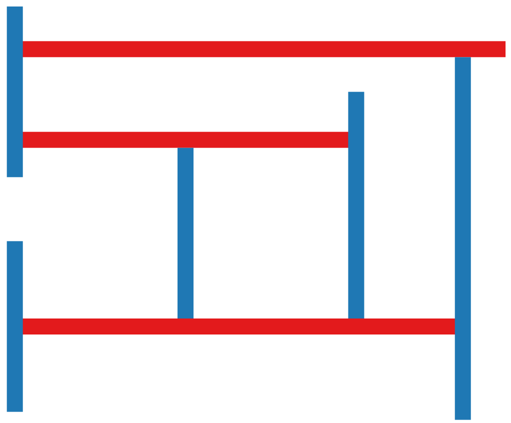

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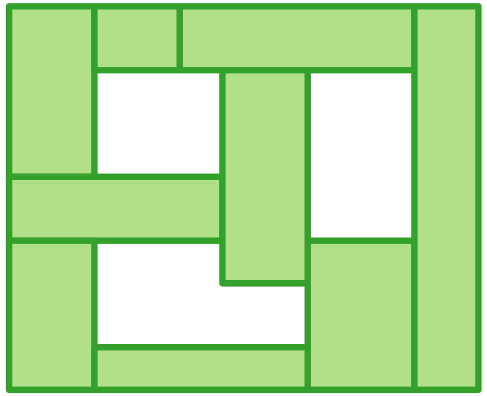

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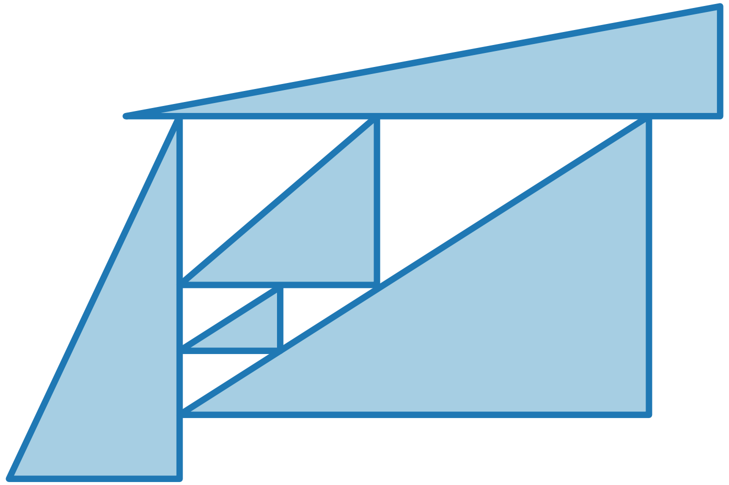

planar triangulations

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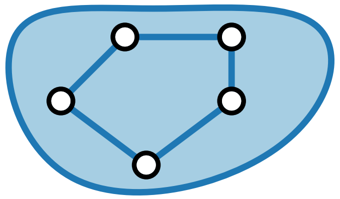

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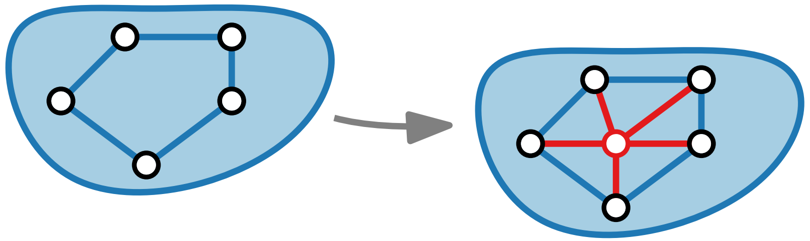

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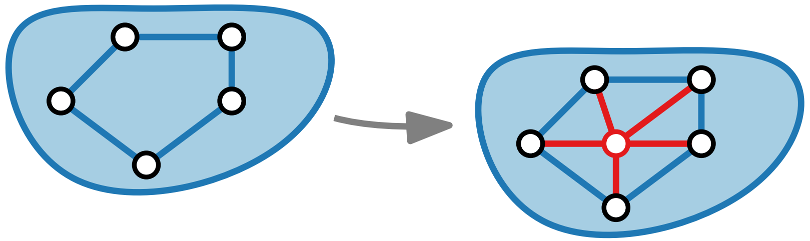

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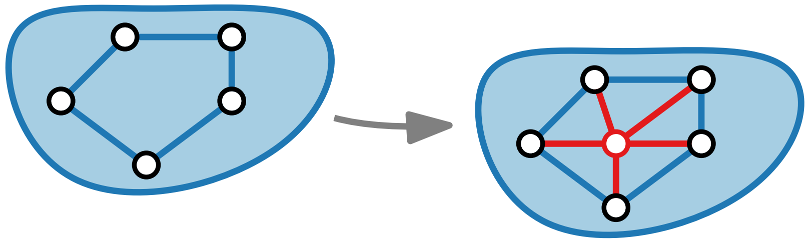

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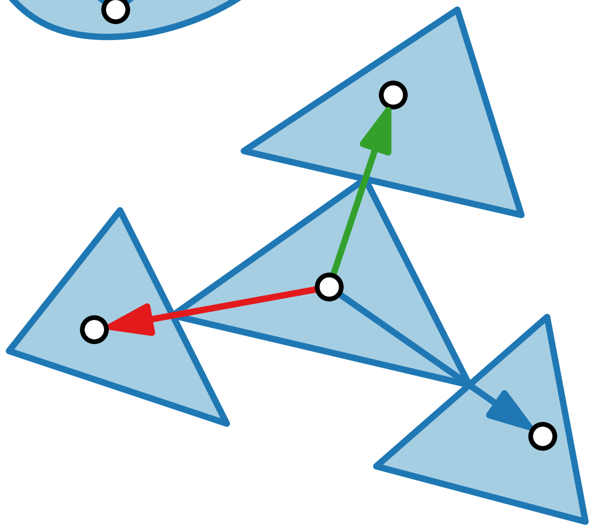

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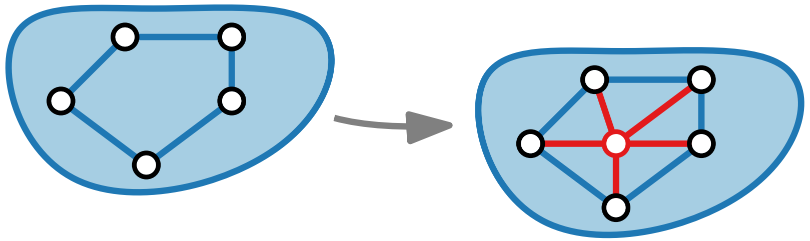

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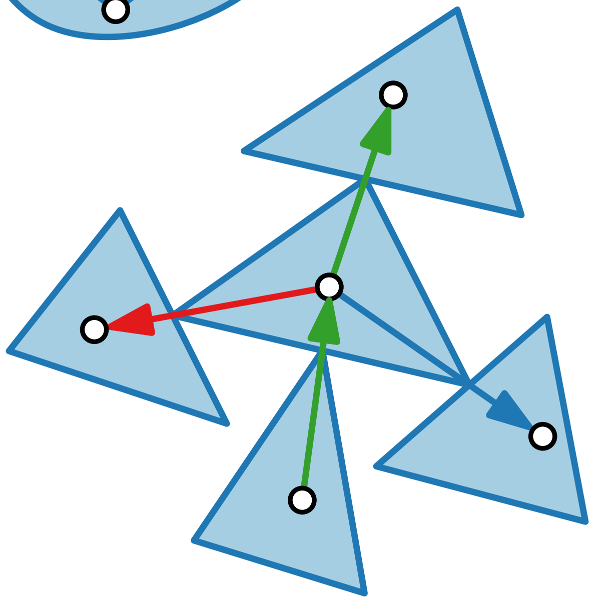

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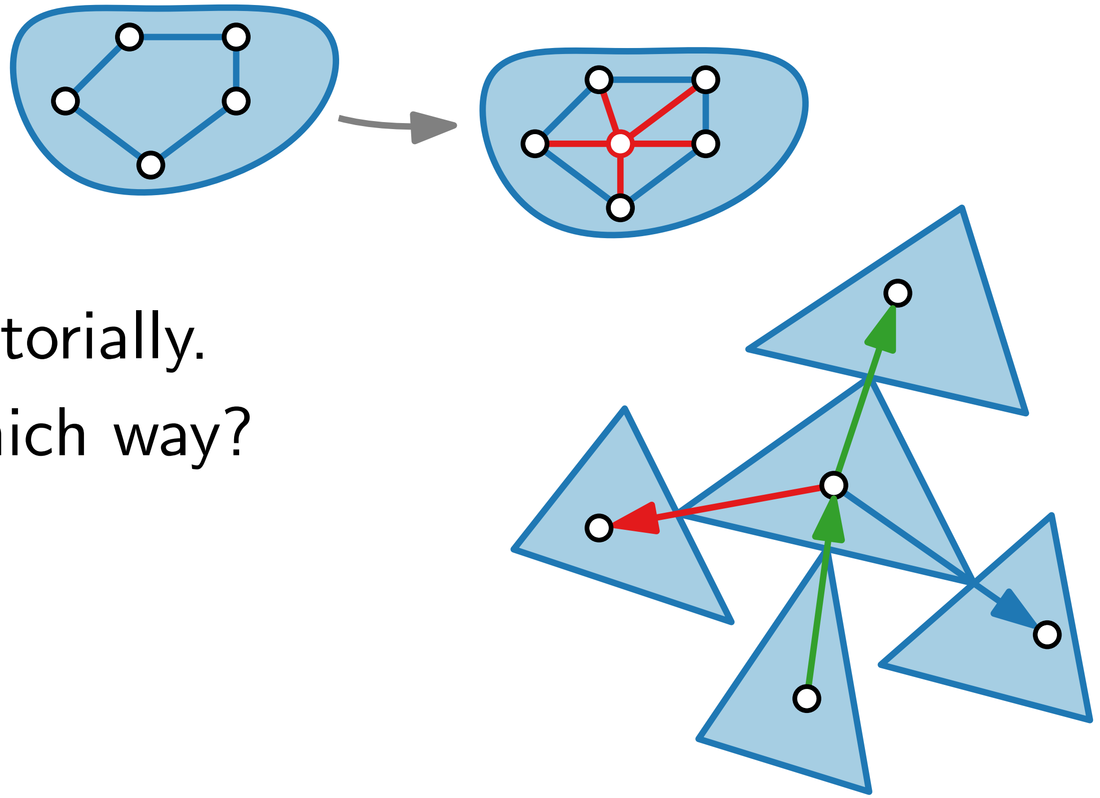

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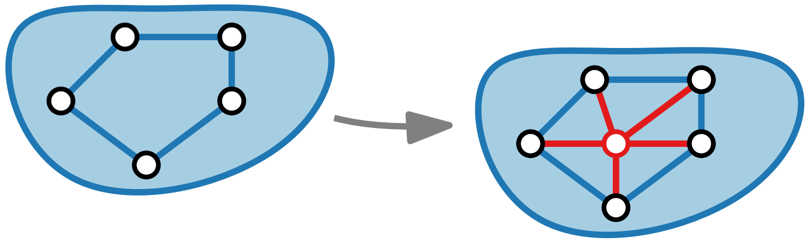
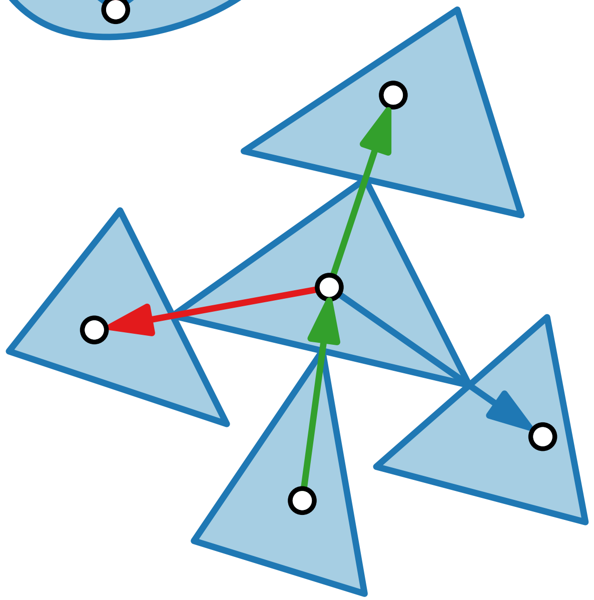
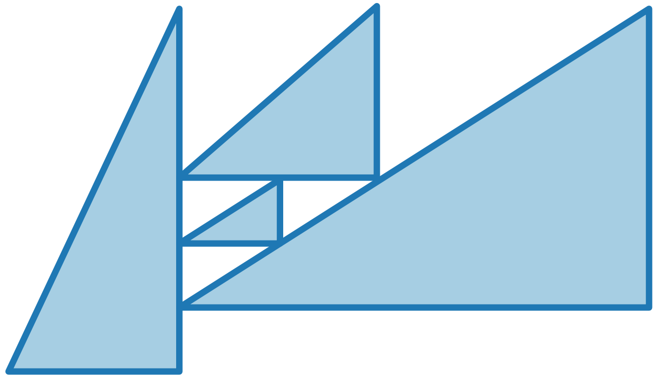


- Describe contact representation combinatorially.
  - Which objects touch each other in which way?
- Compute combinatorial description.
- Show that combinatorial description can be used to construct drawing.



This Lecture

Representation with right-triangles and corner contact:



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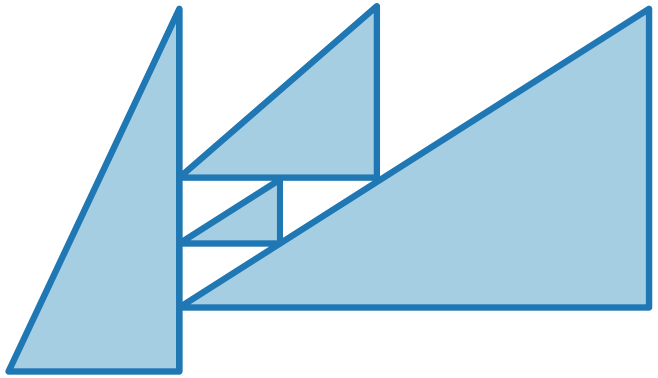

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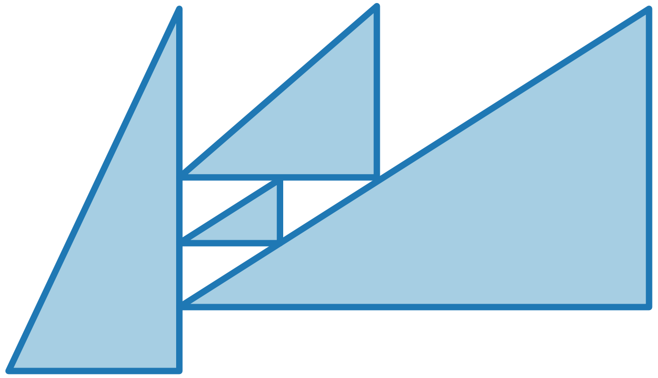

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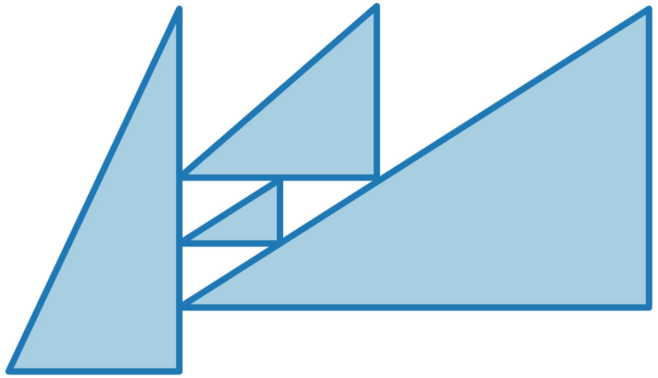

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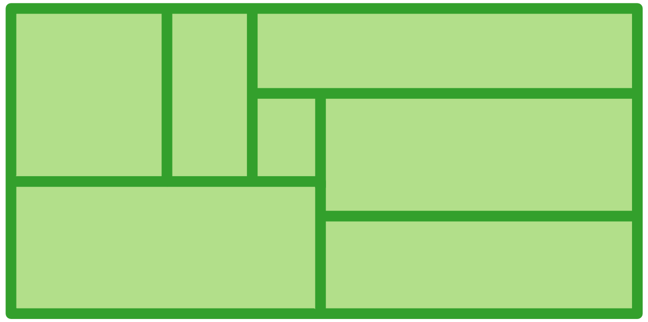

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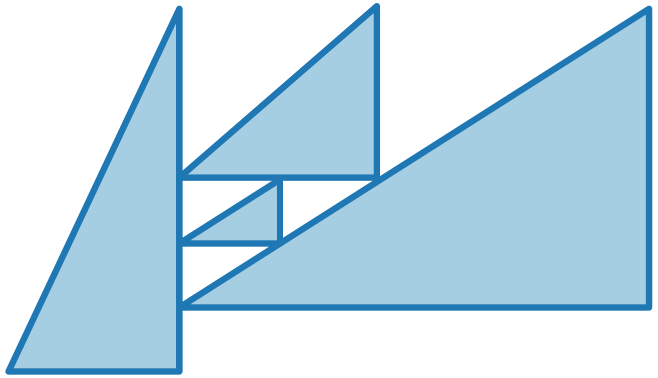

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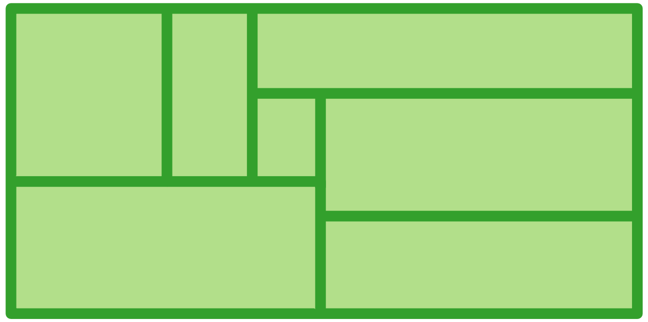

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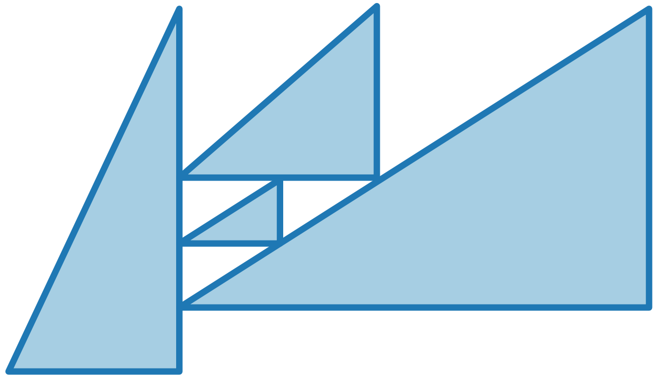

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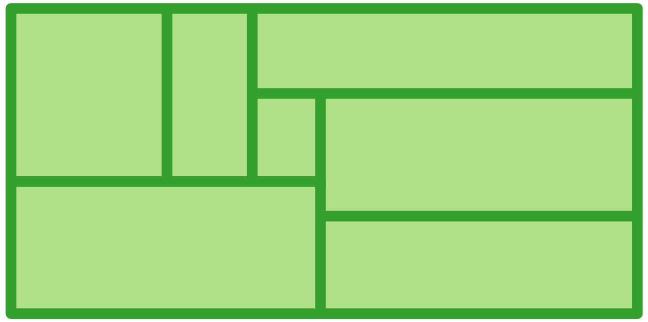

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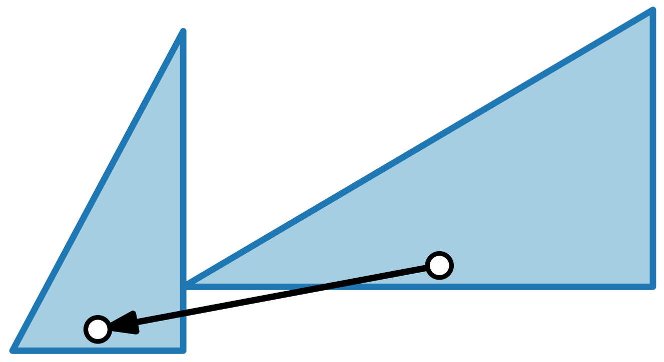

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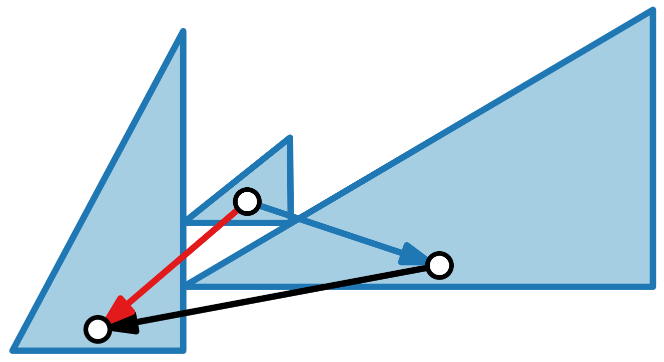

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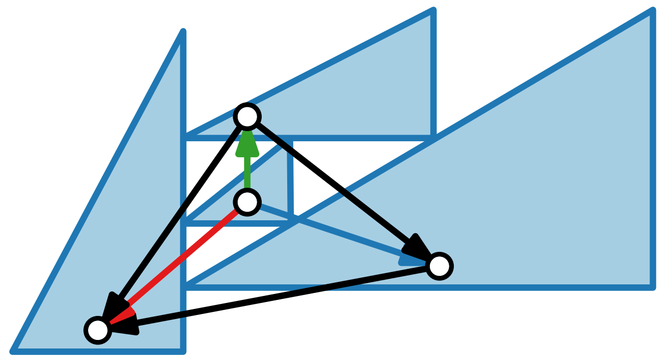

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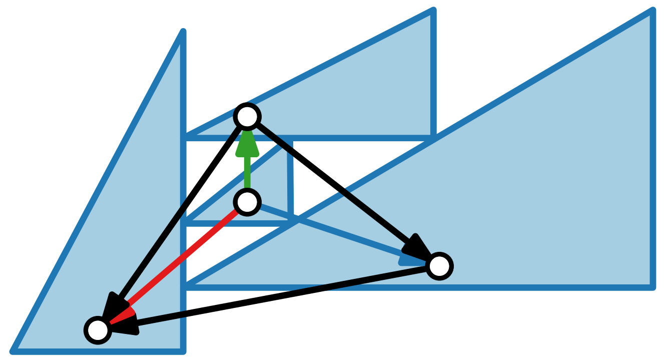

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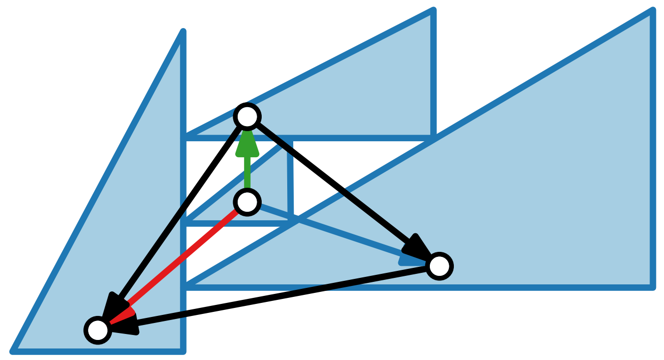

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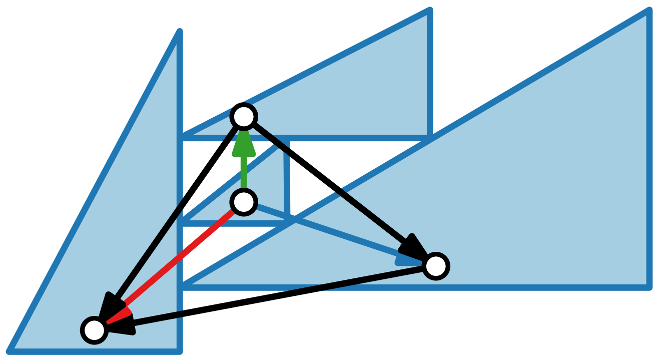

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- Outgoing edges in Schnyder forest indicate corner contacts.

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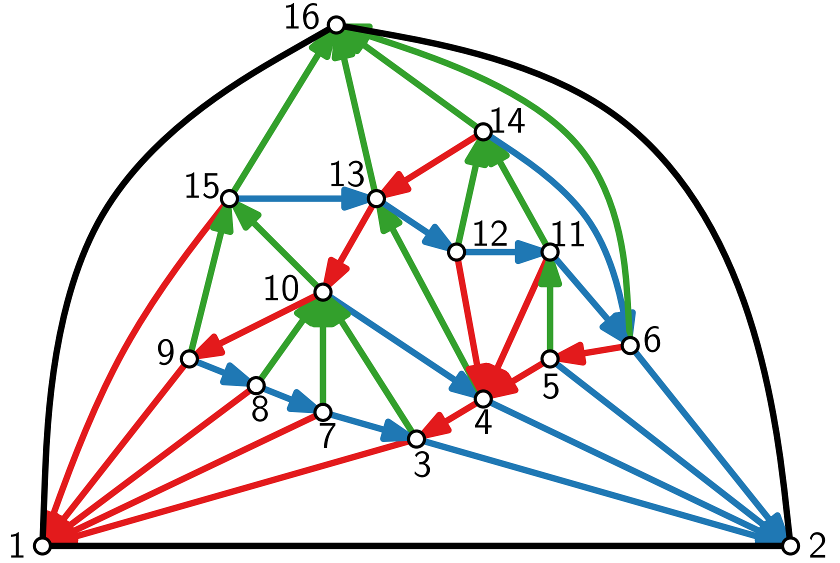

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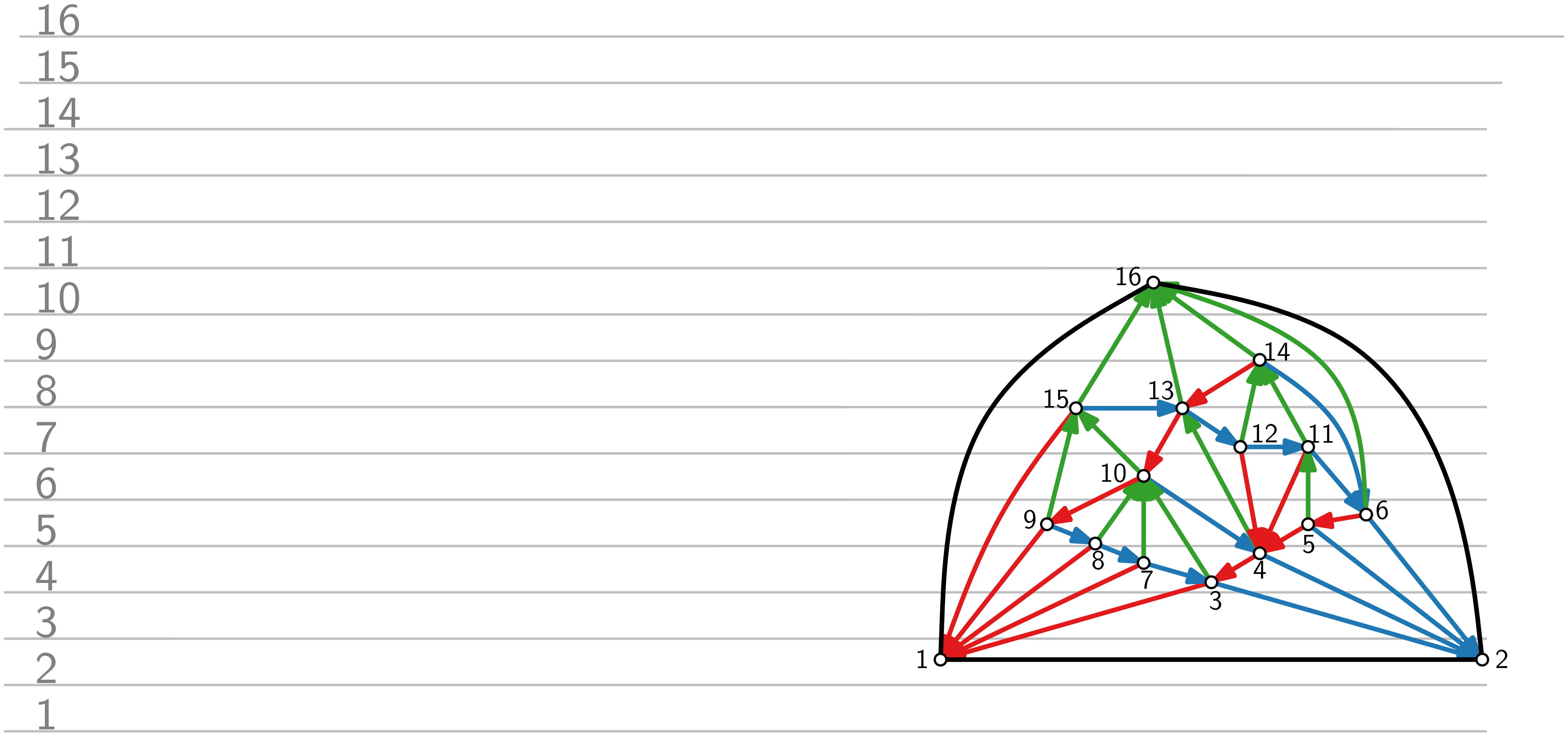

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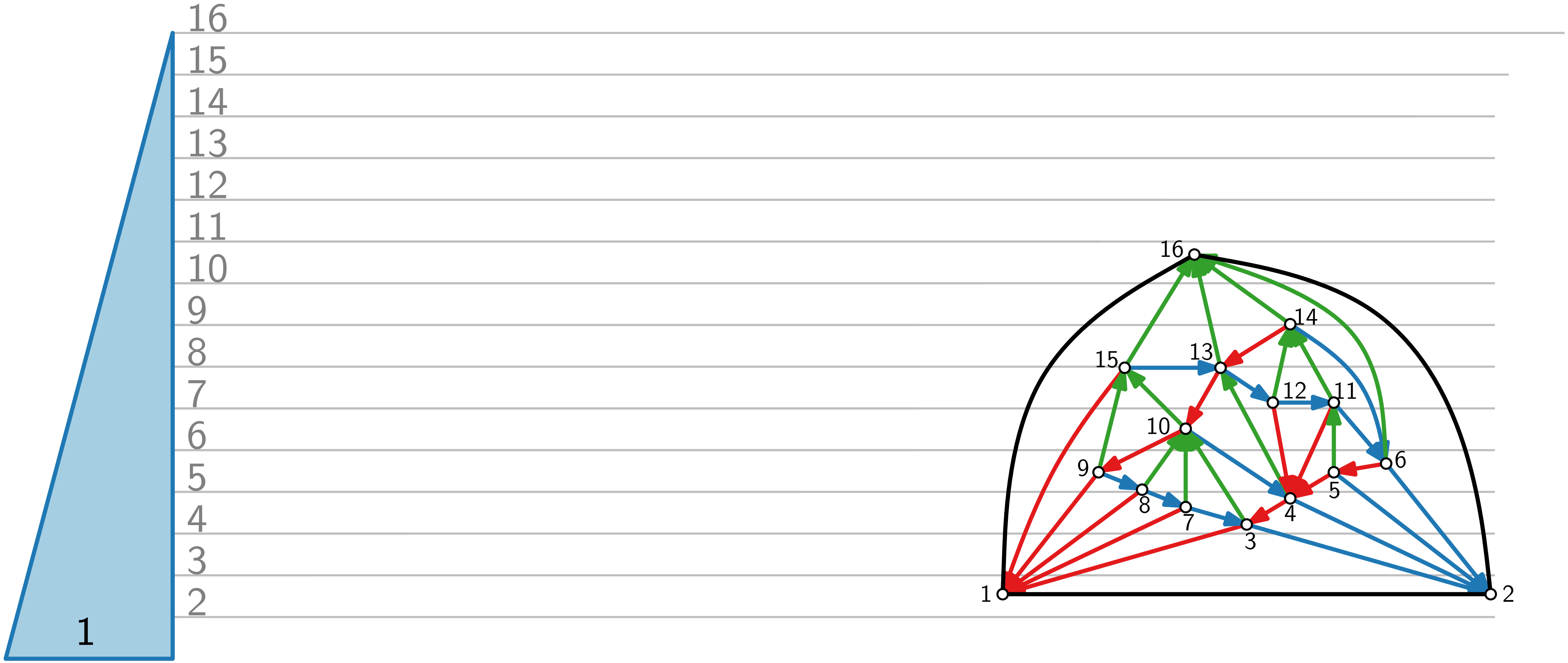

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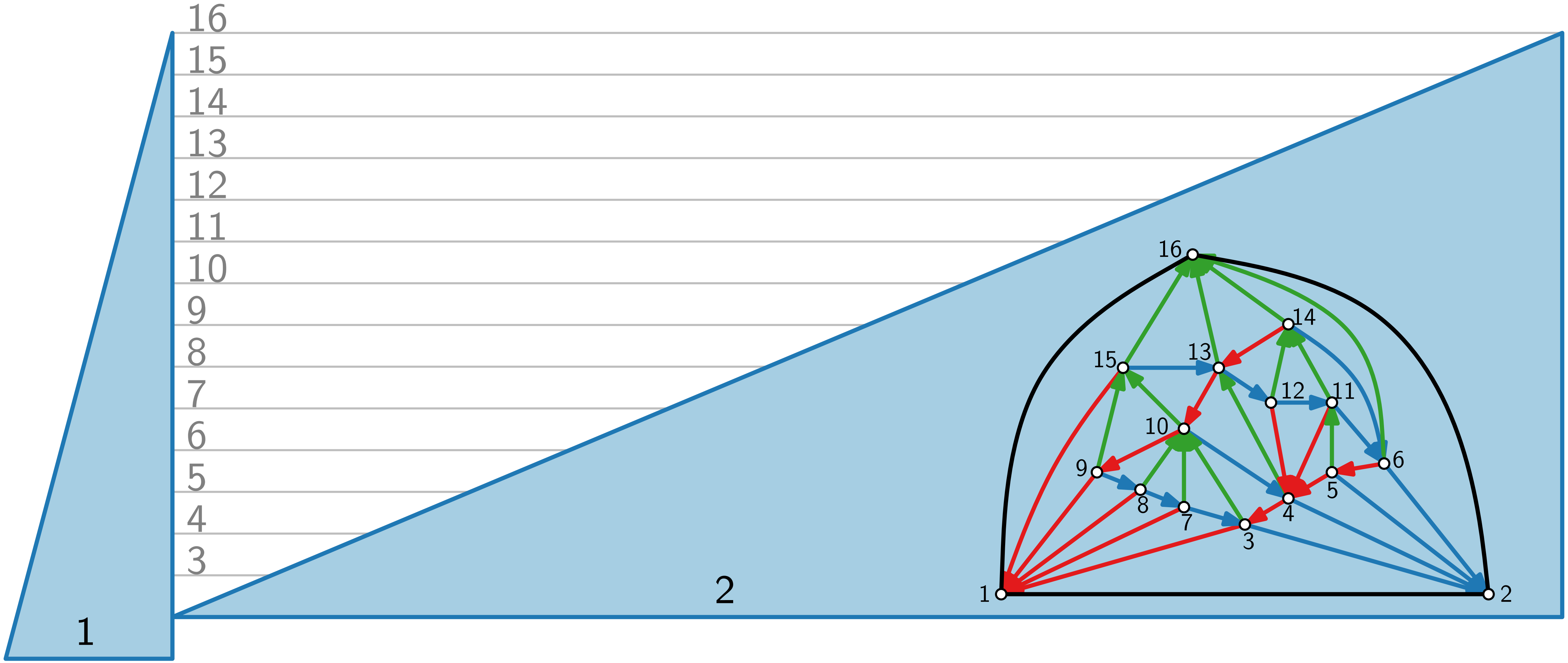

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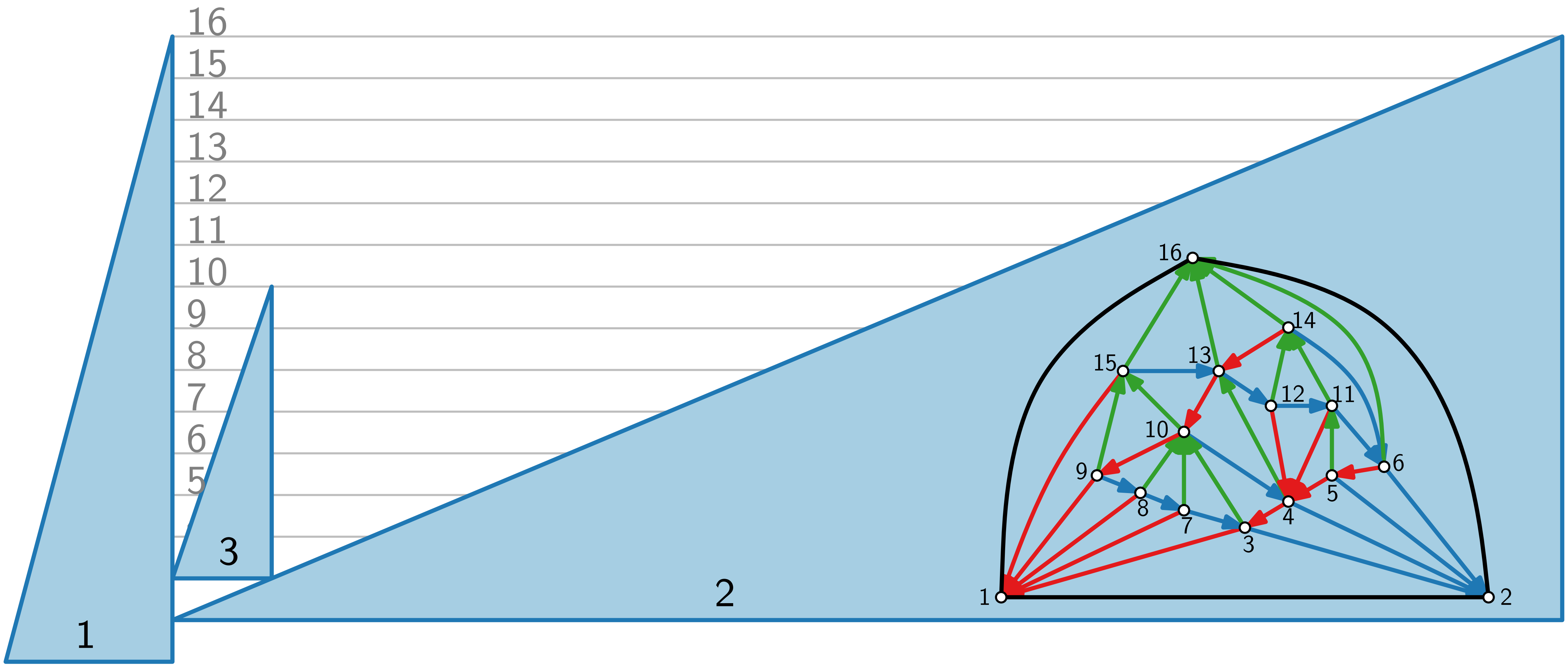

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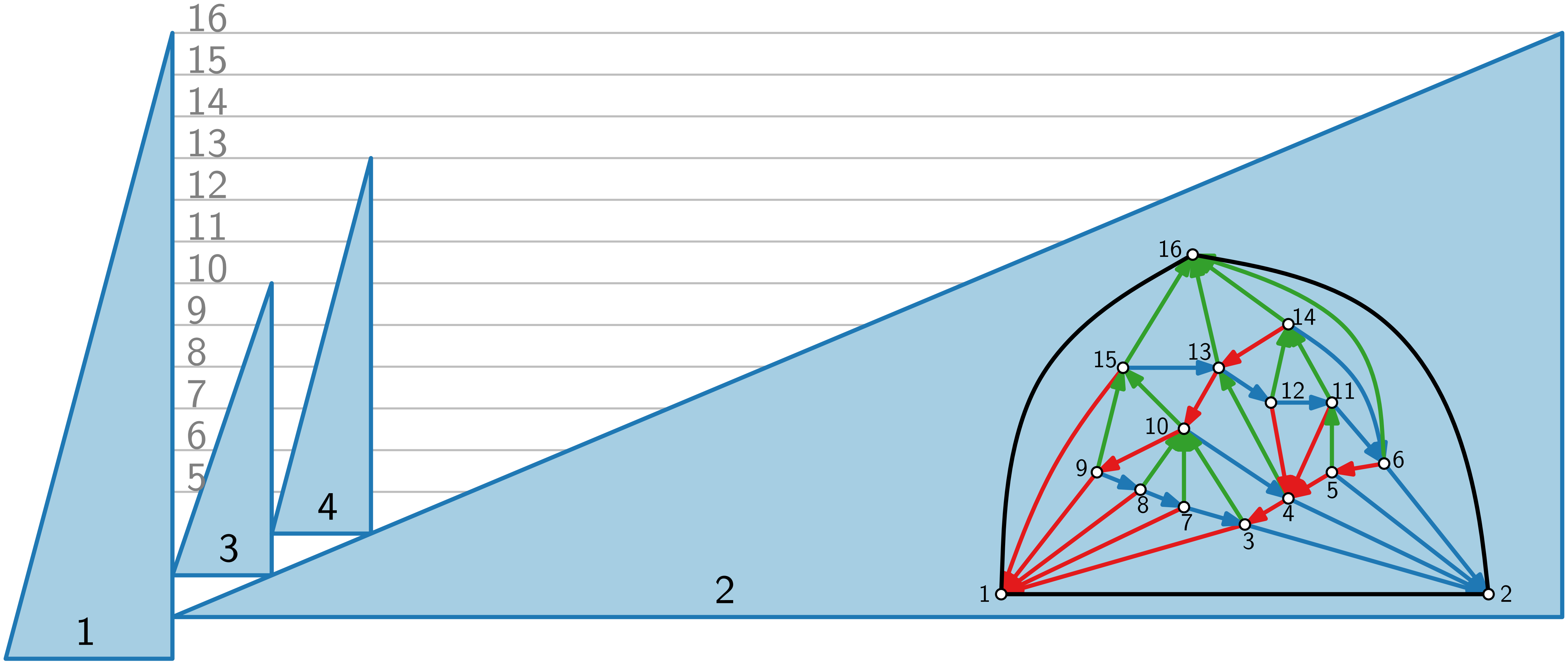

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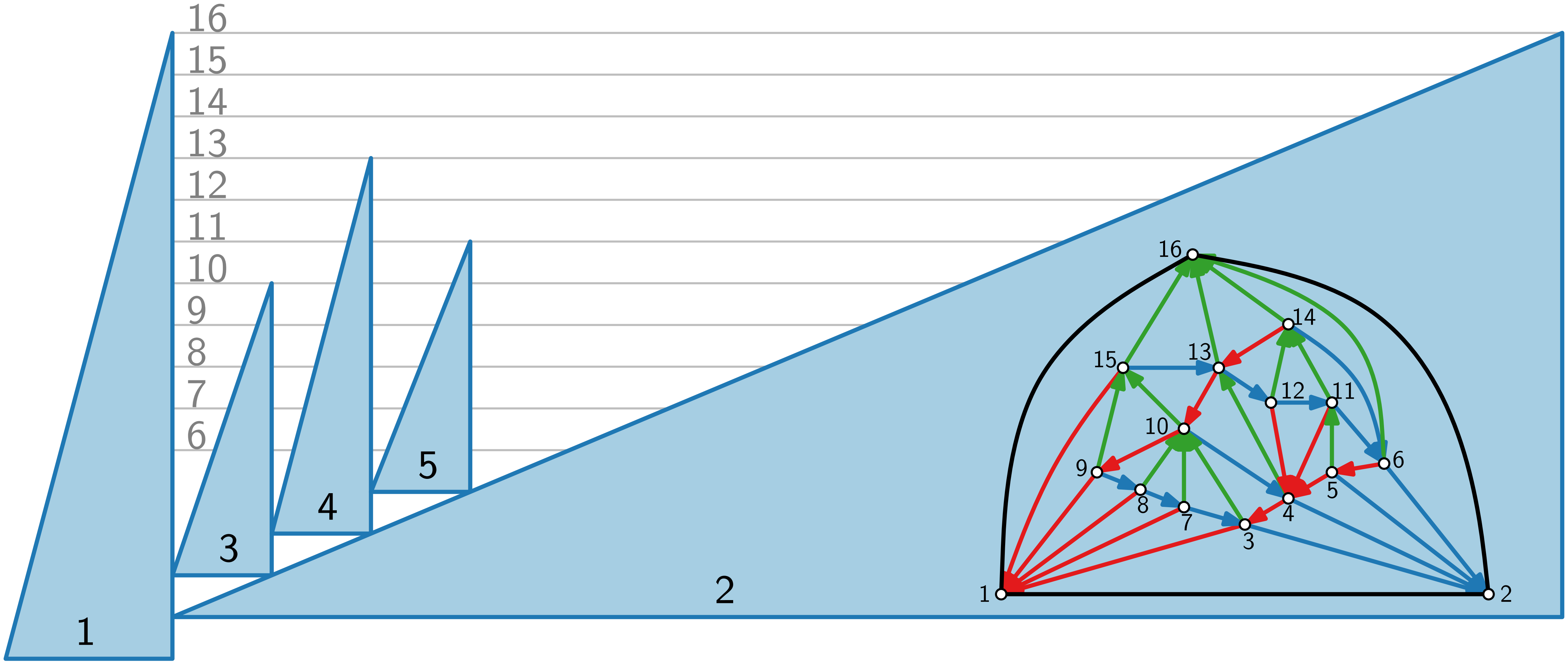

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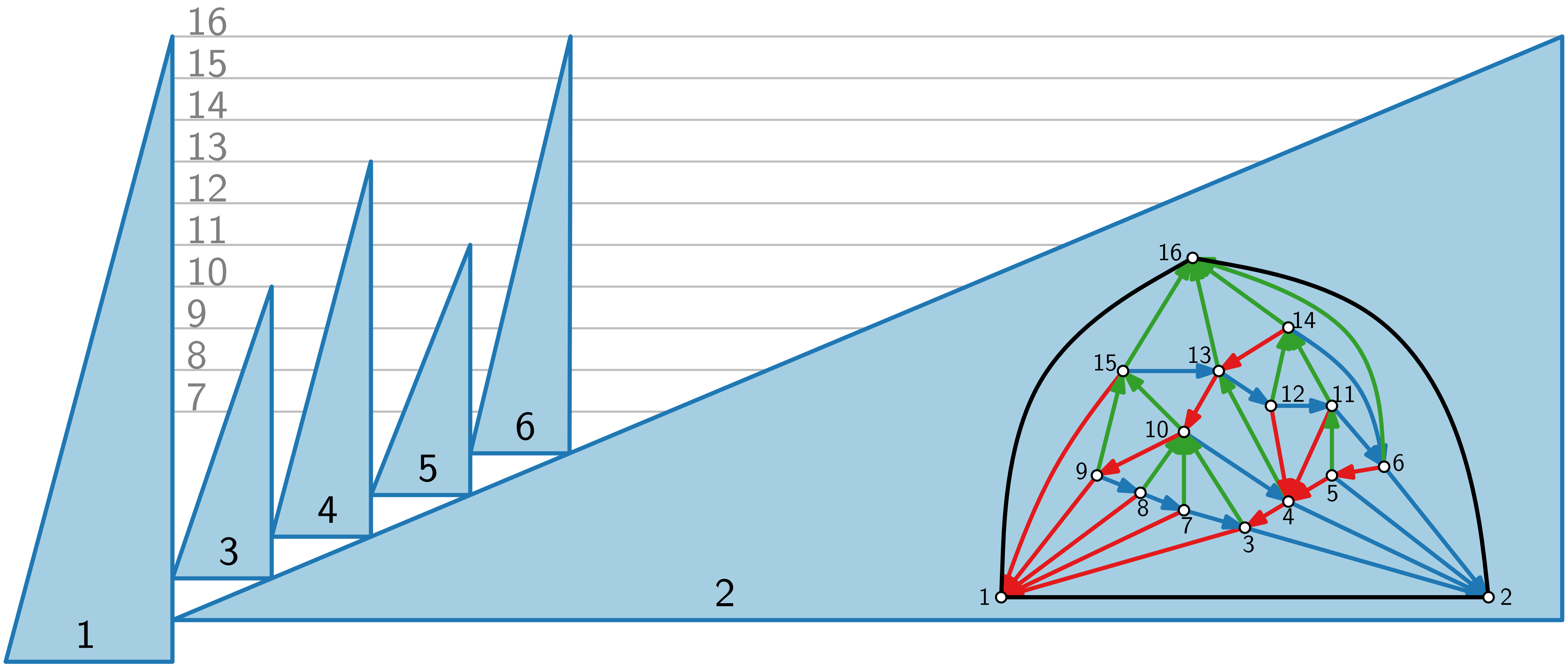

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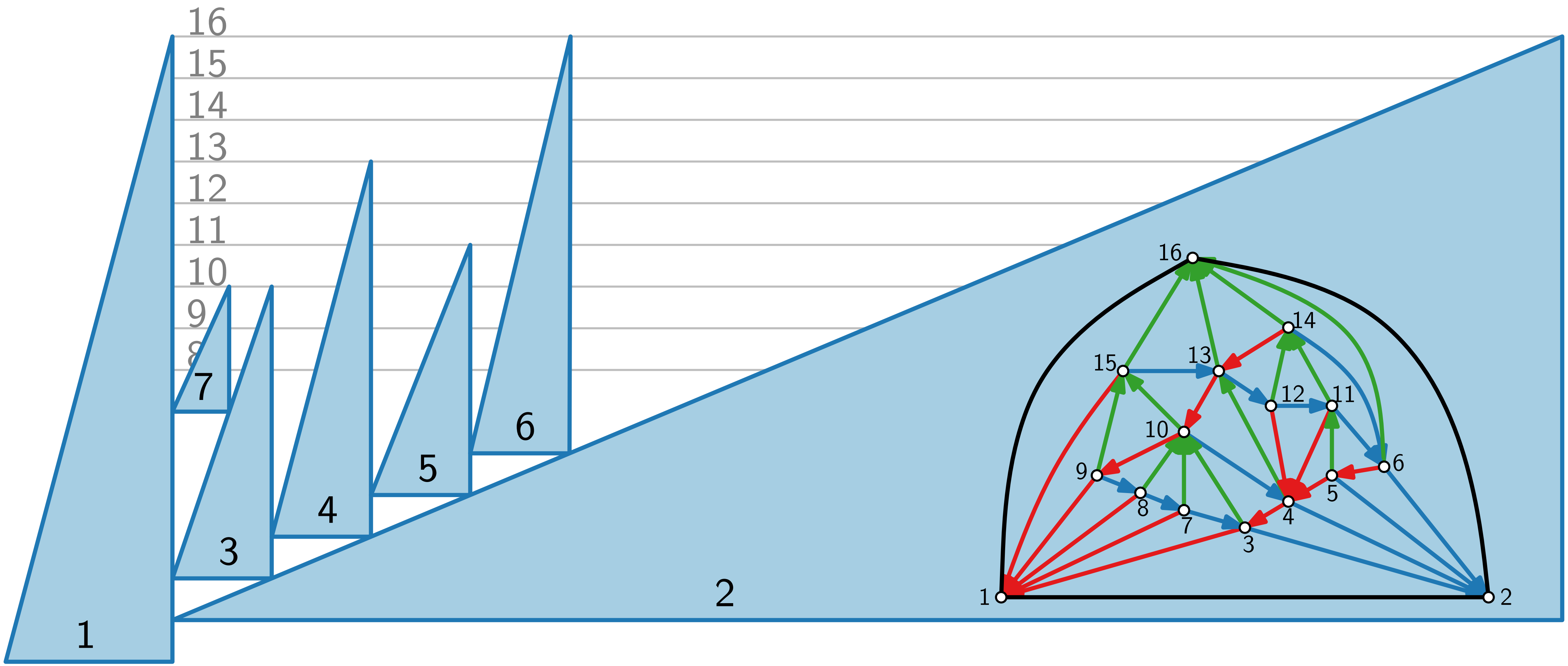

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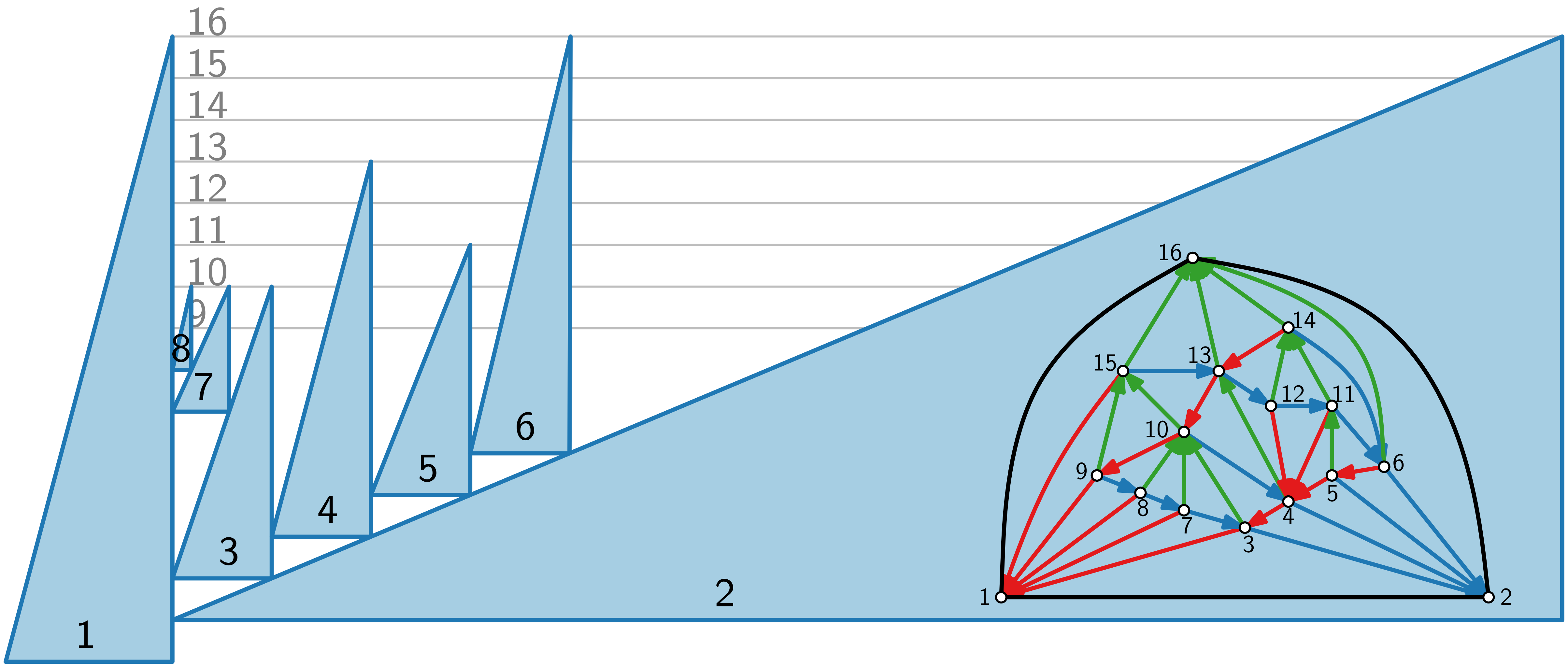

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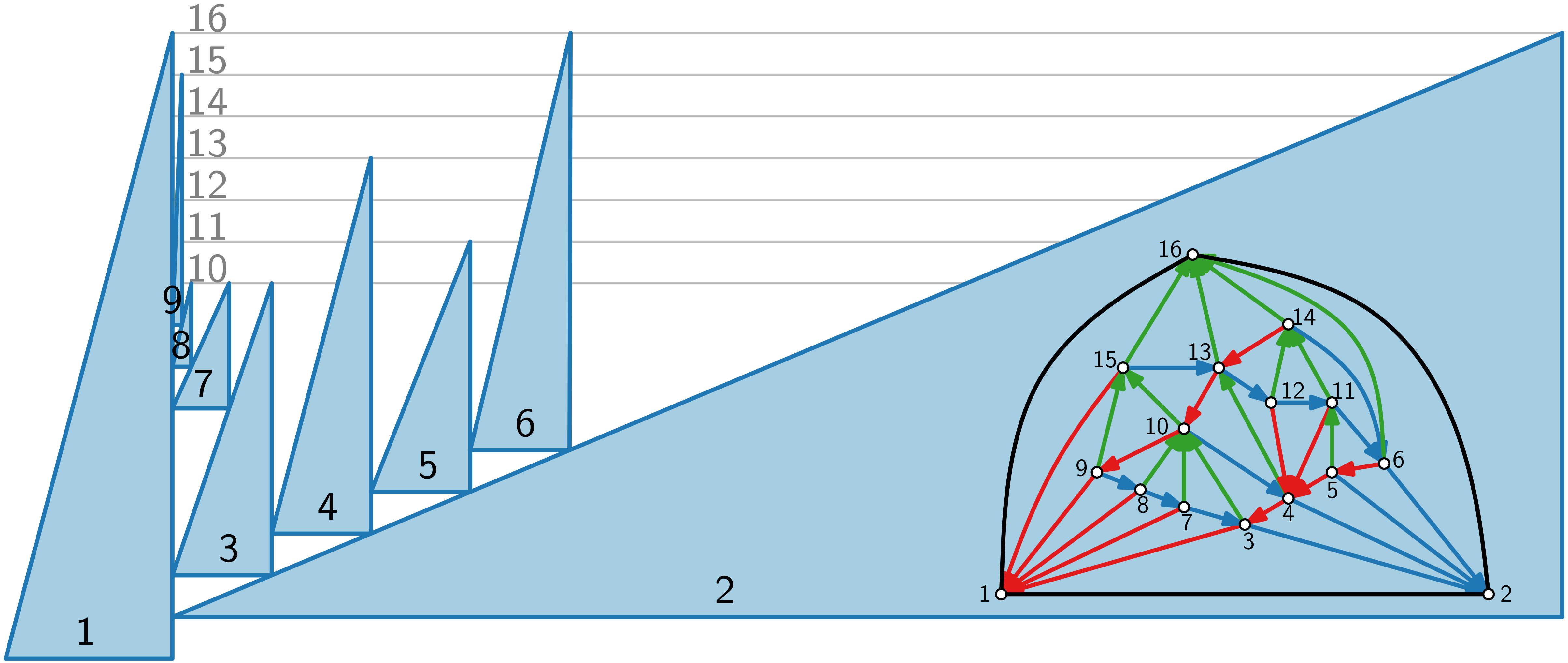

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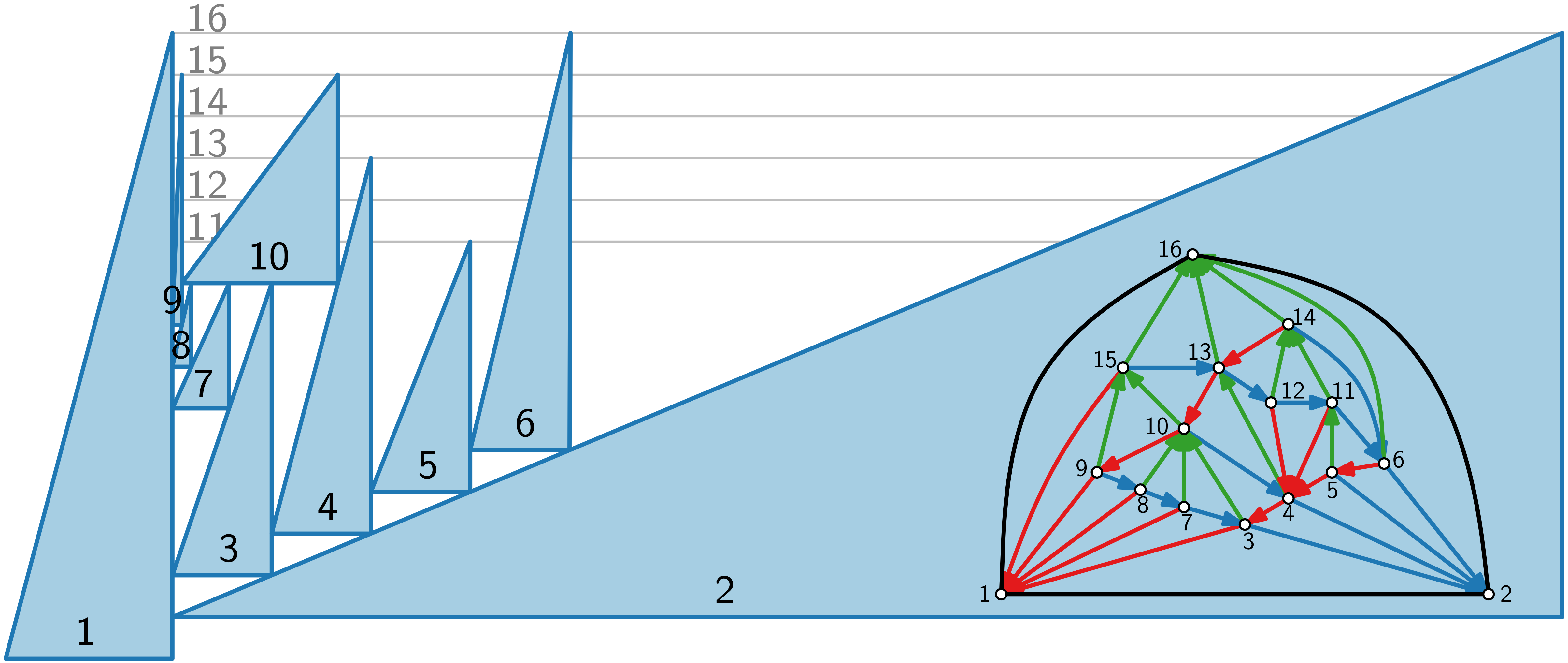

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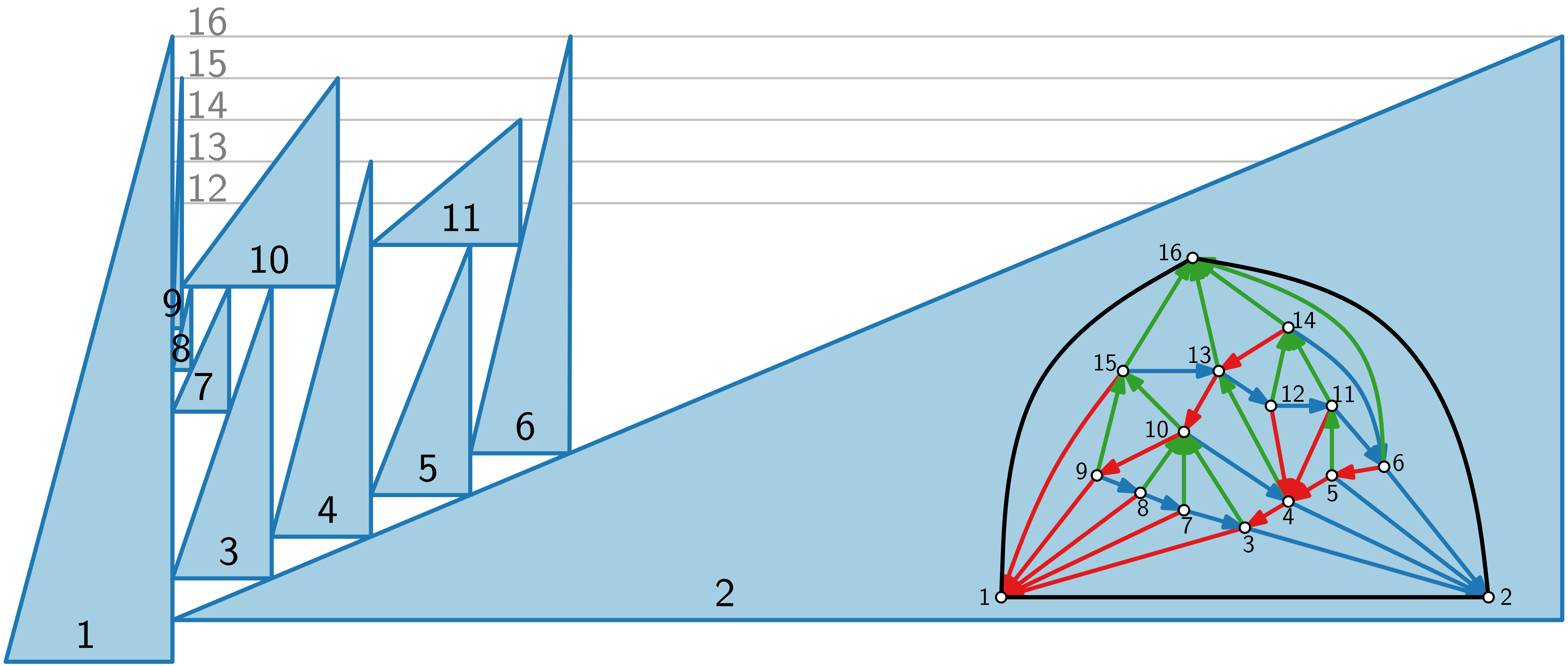
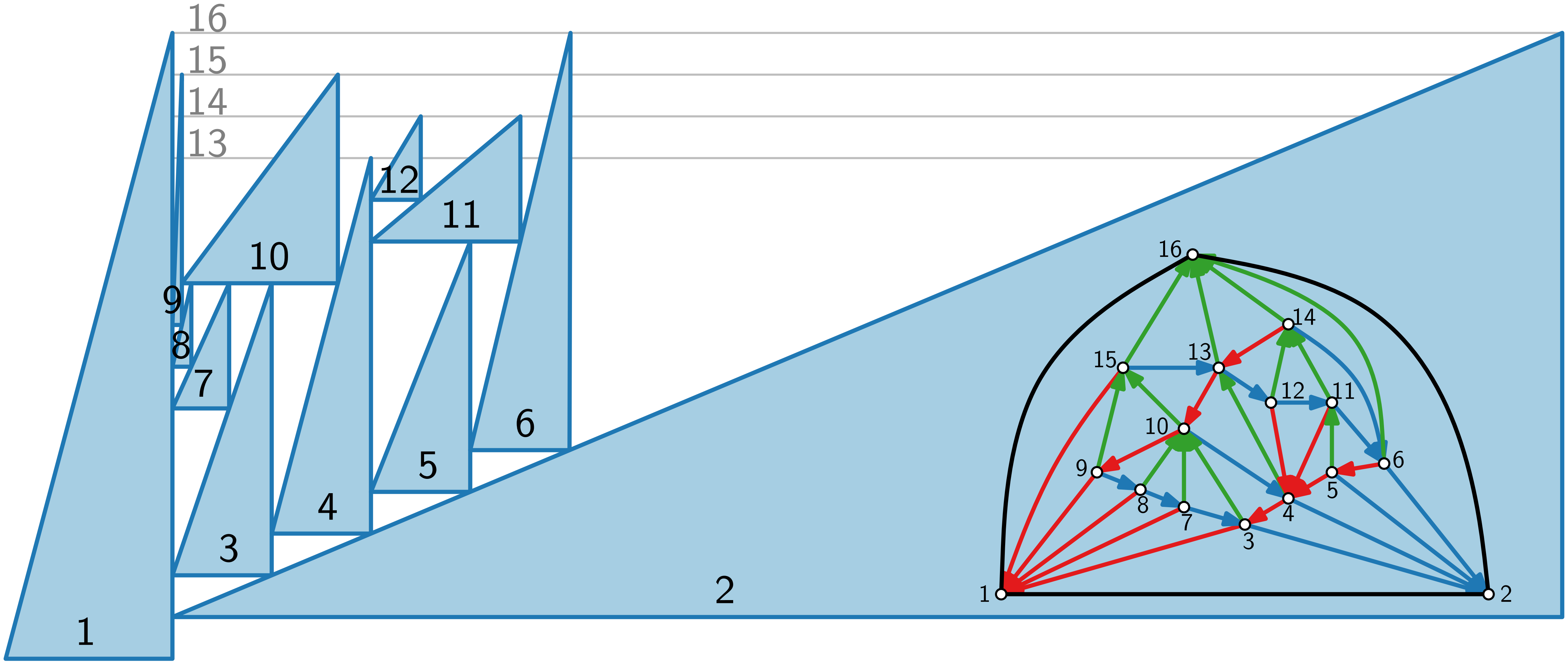
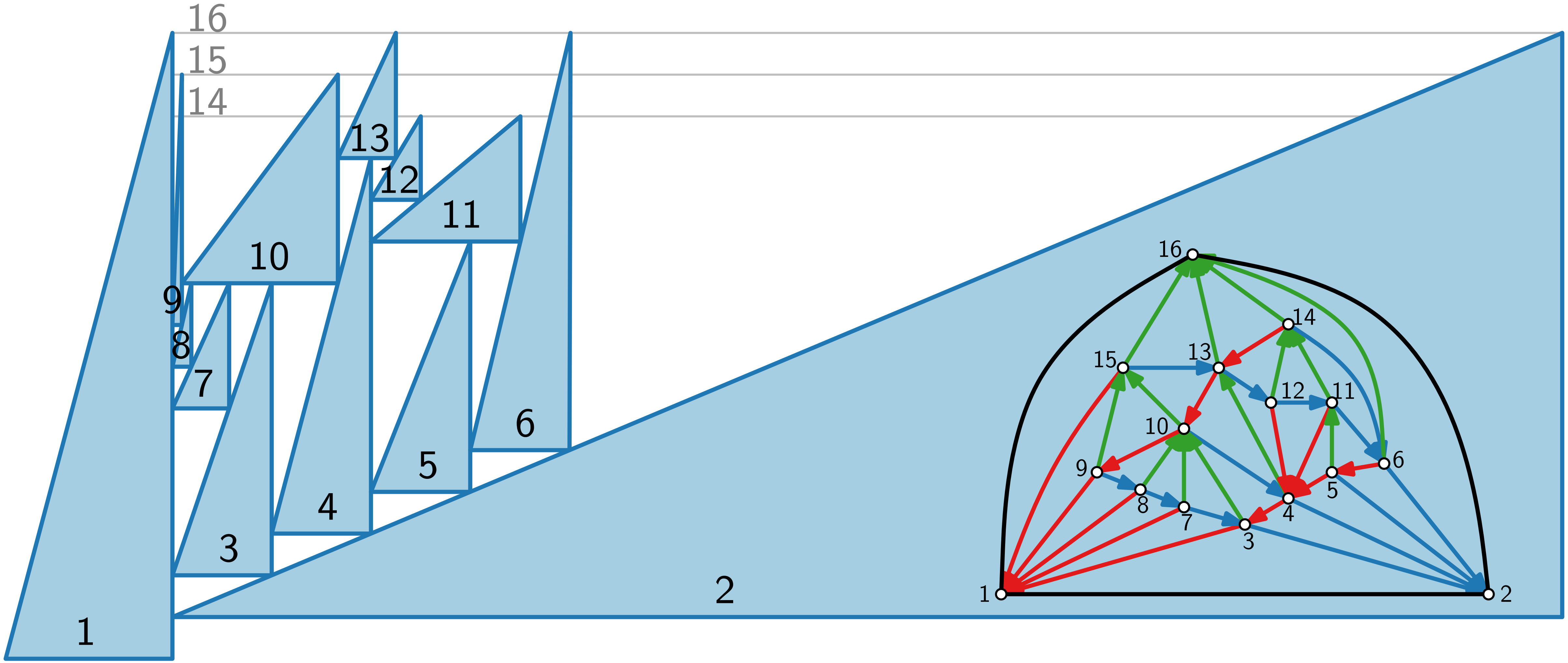


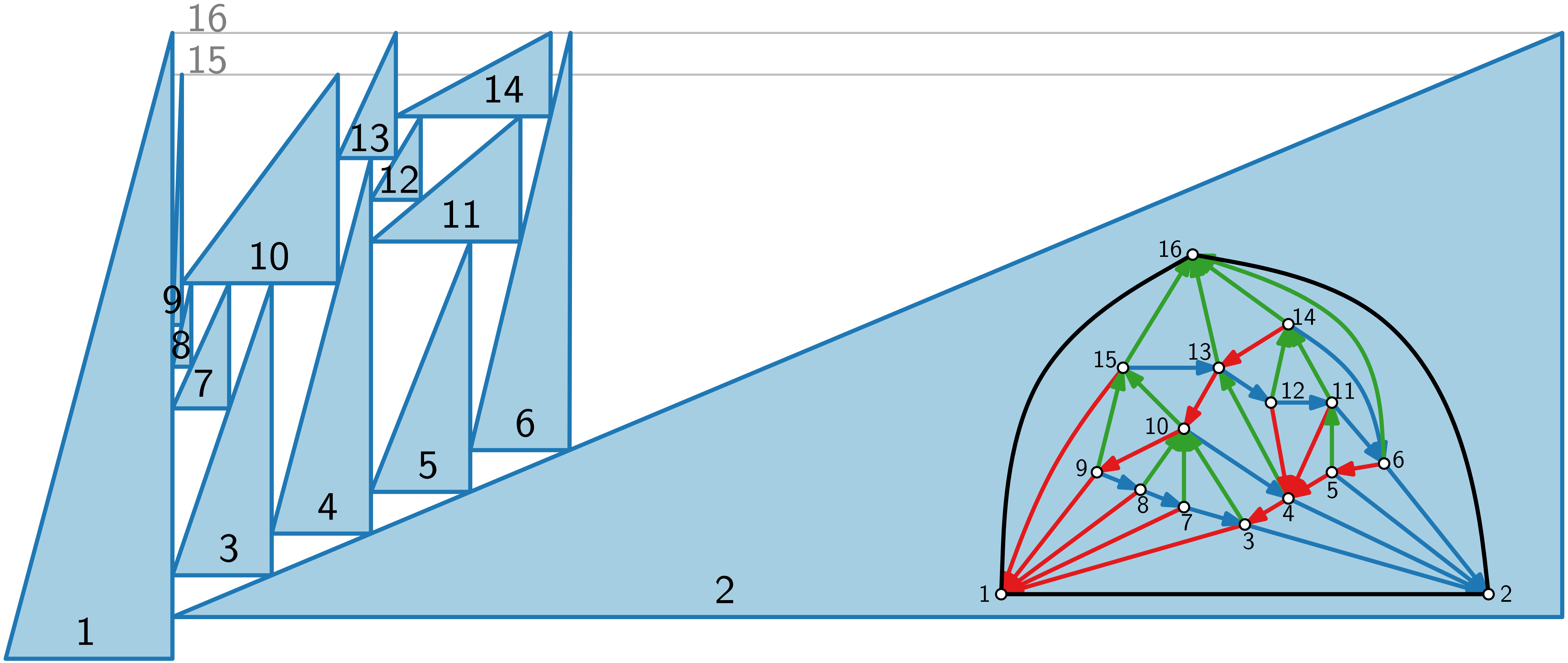
Triangle Contact Representation Example



Triangle Contact Representation Example



Triangle Contact Representation Example



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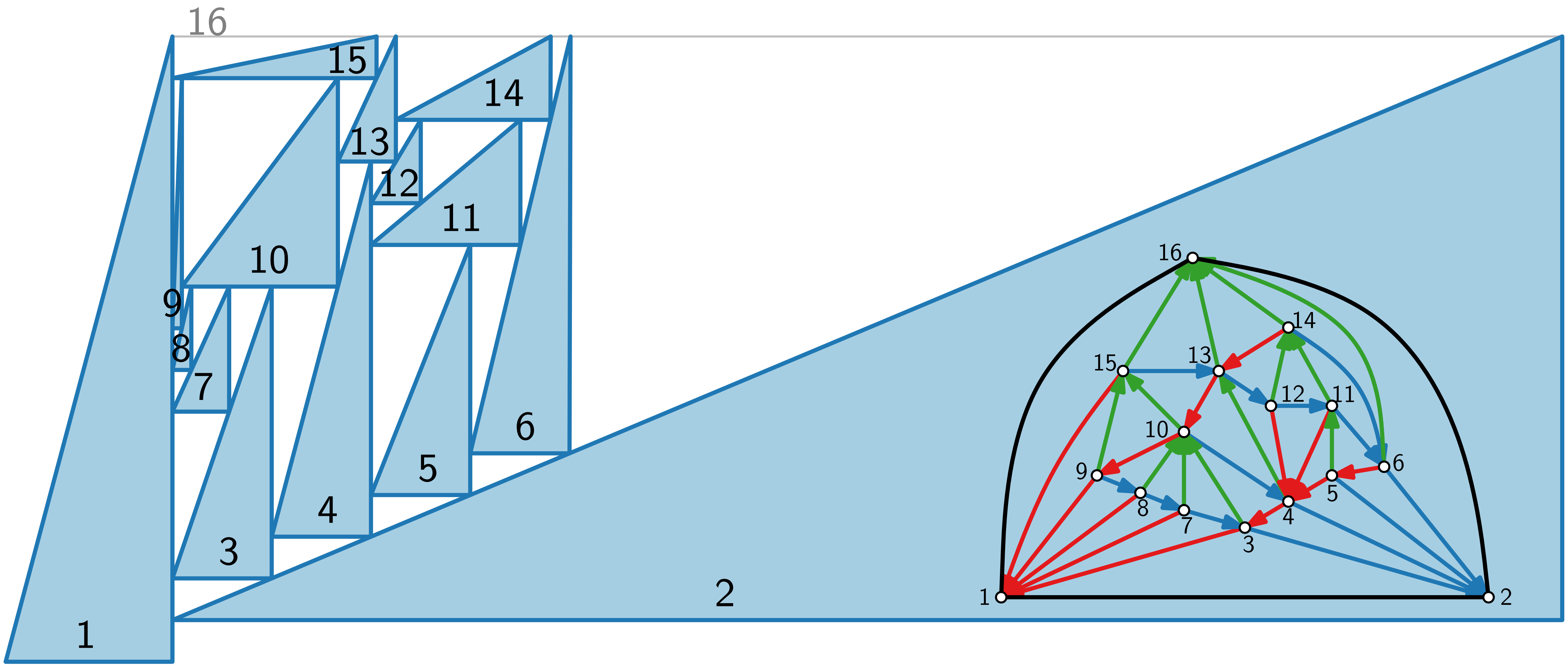
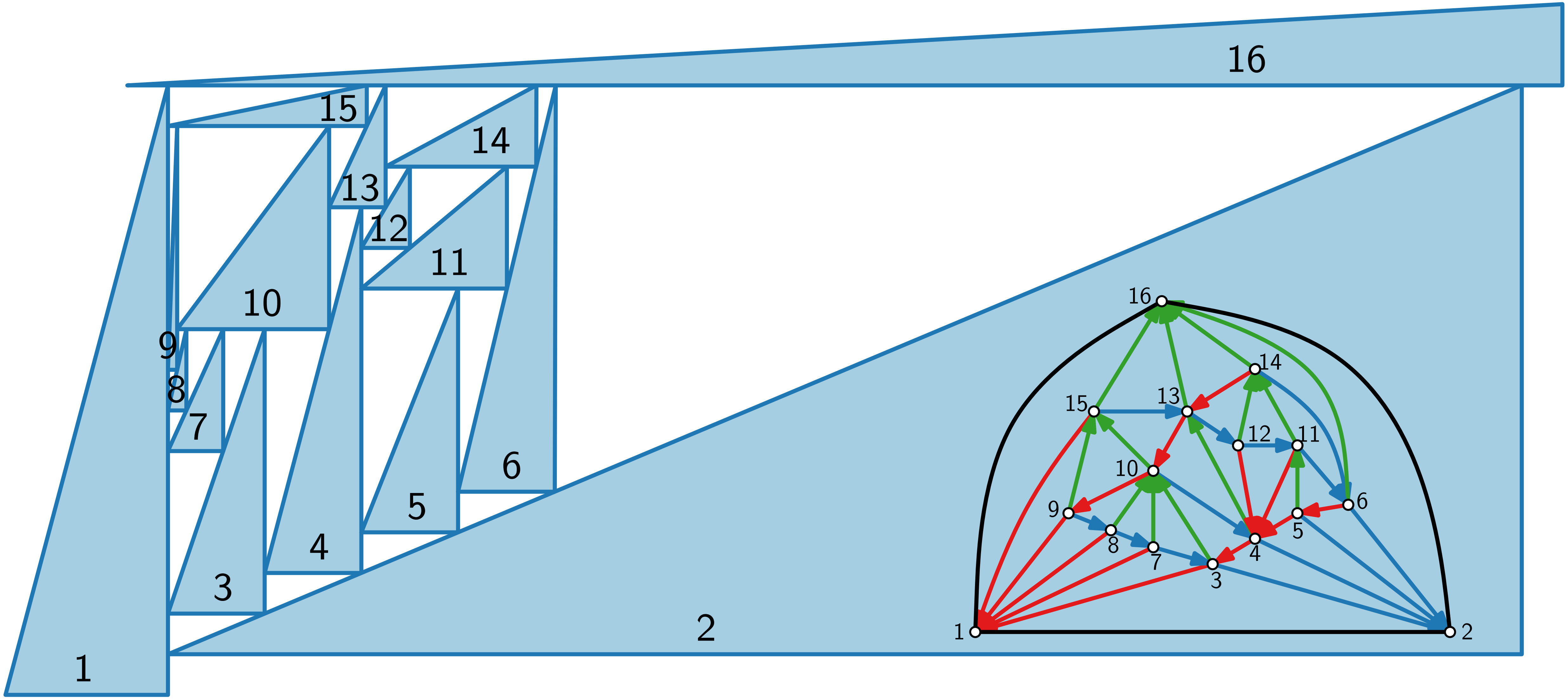
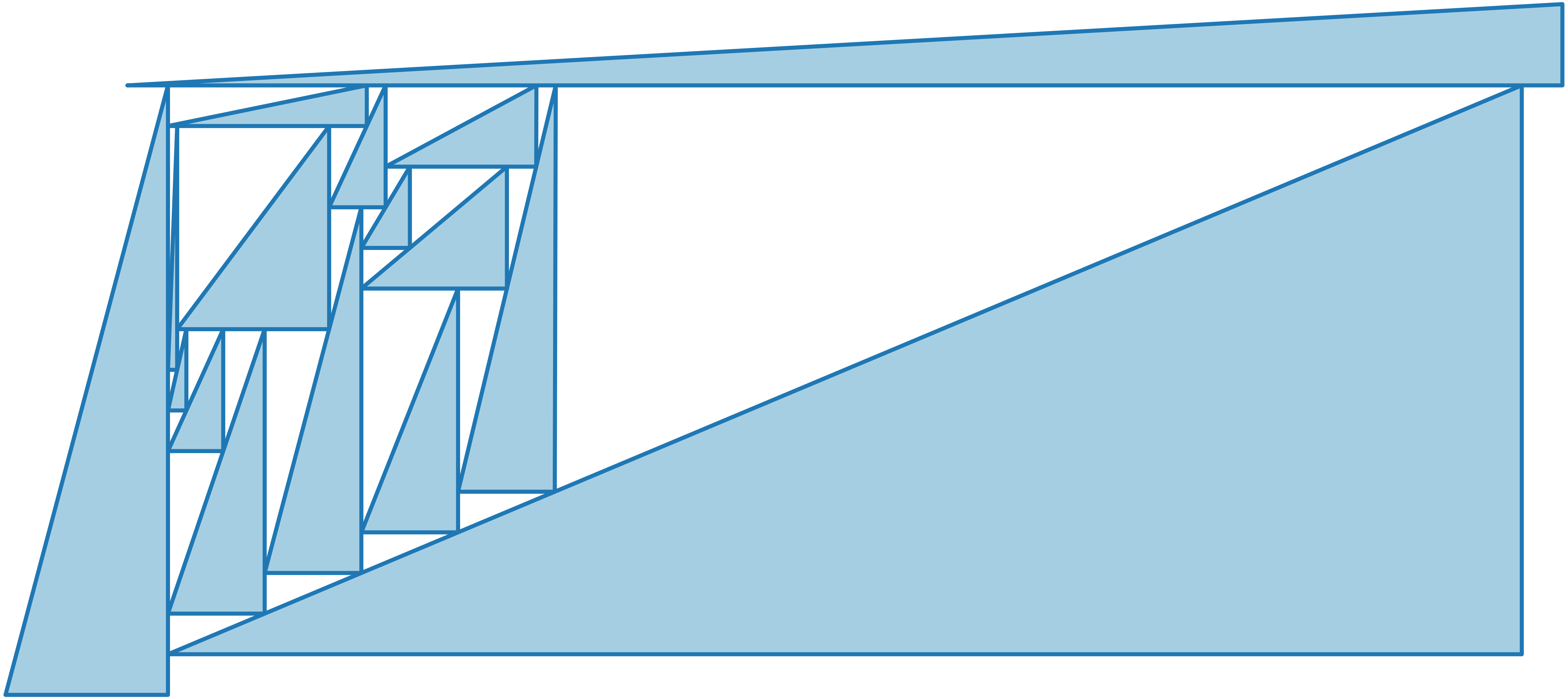


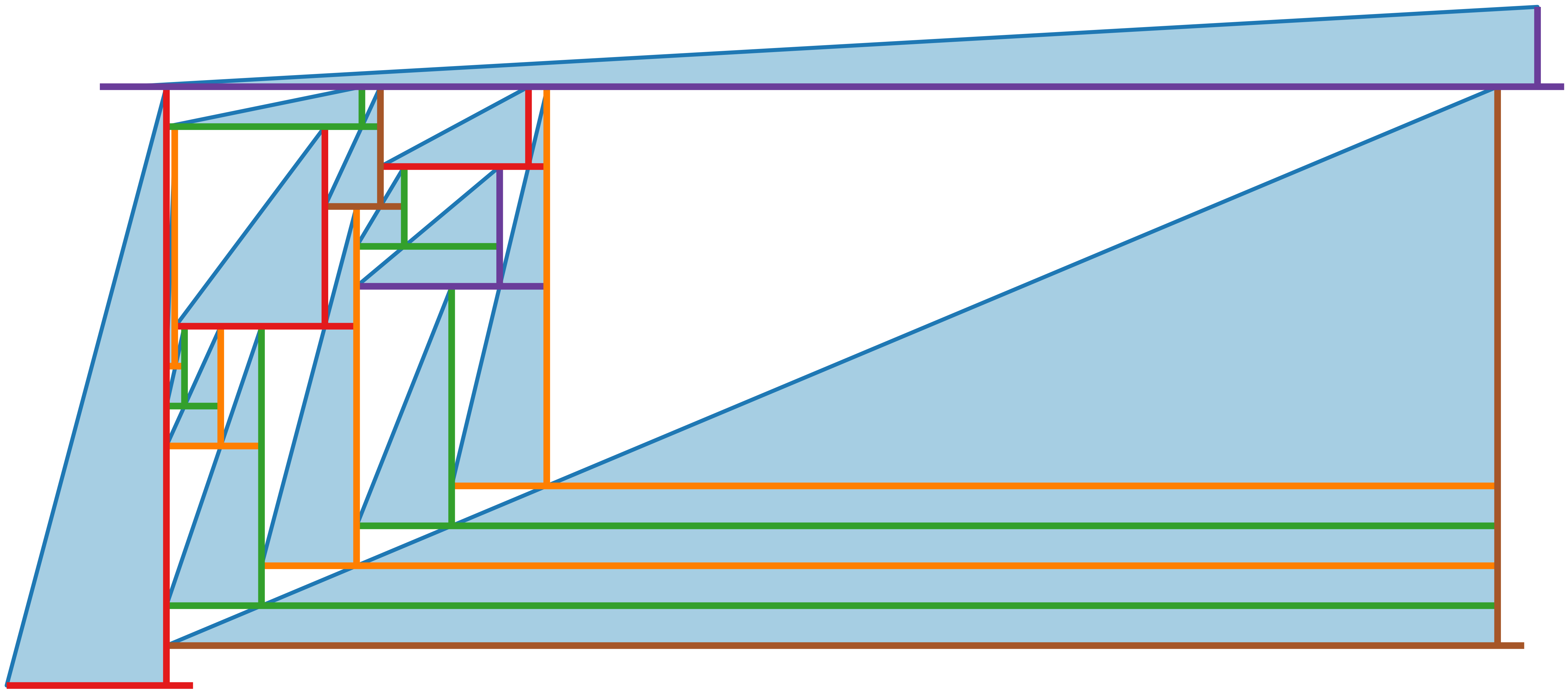
Triangle Contact Representation Example



T-shape Contact Representation



T-shape Contact Representation



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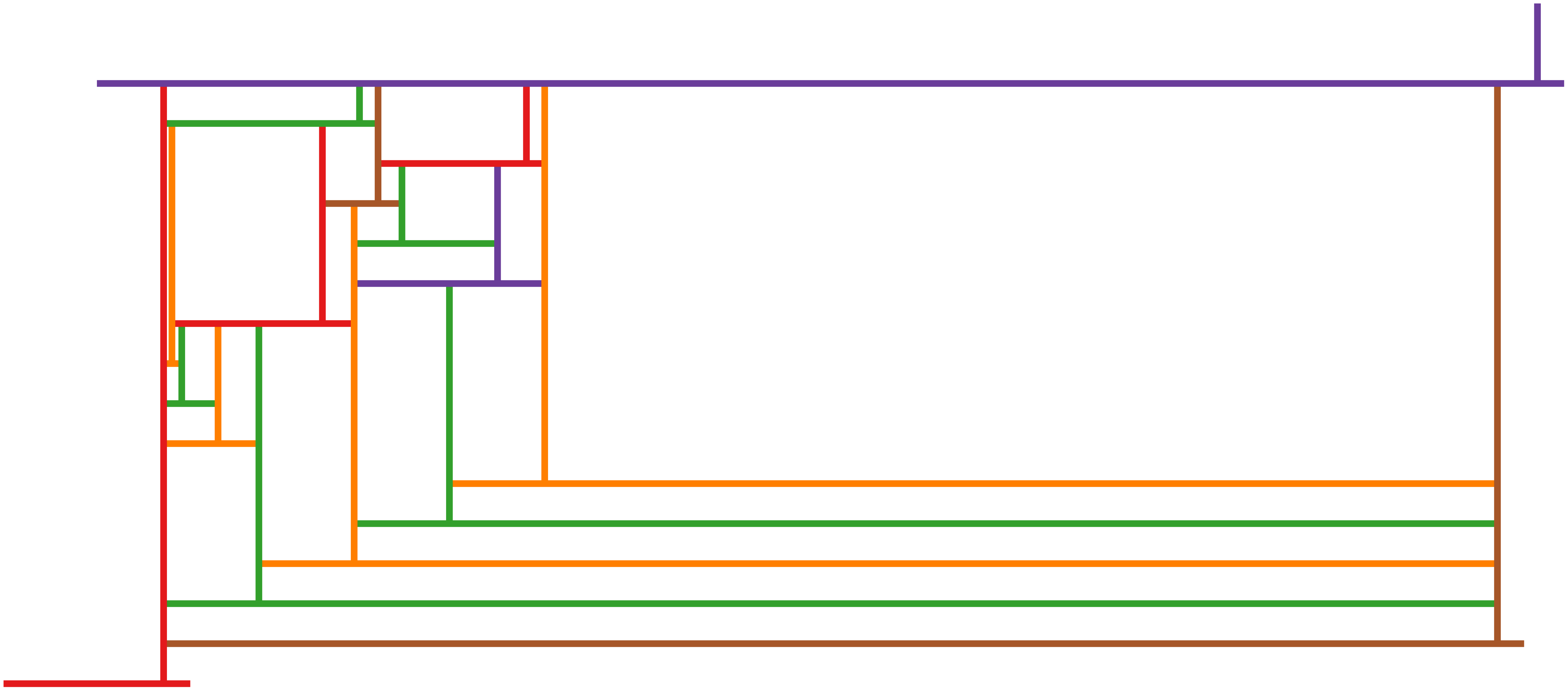

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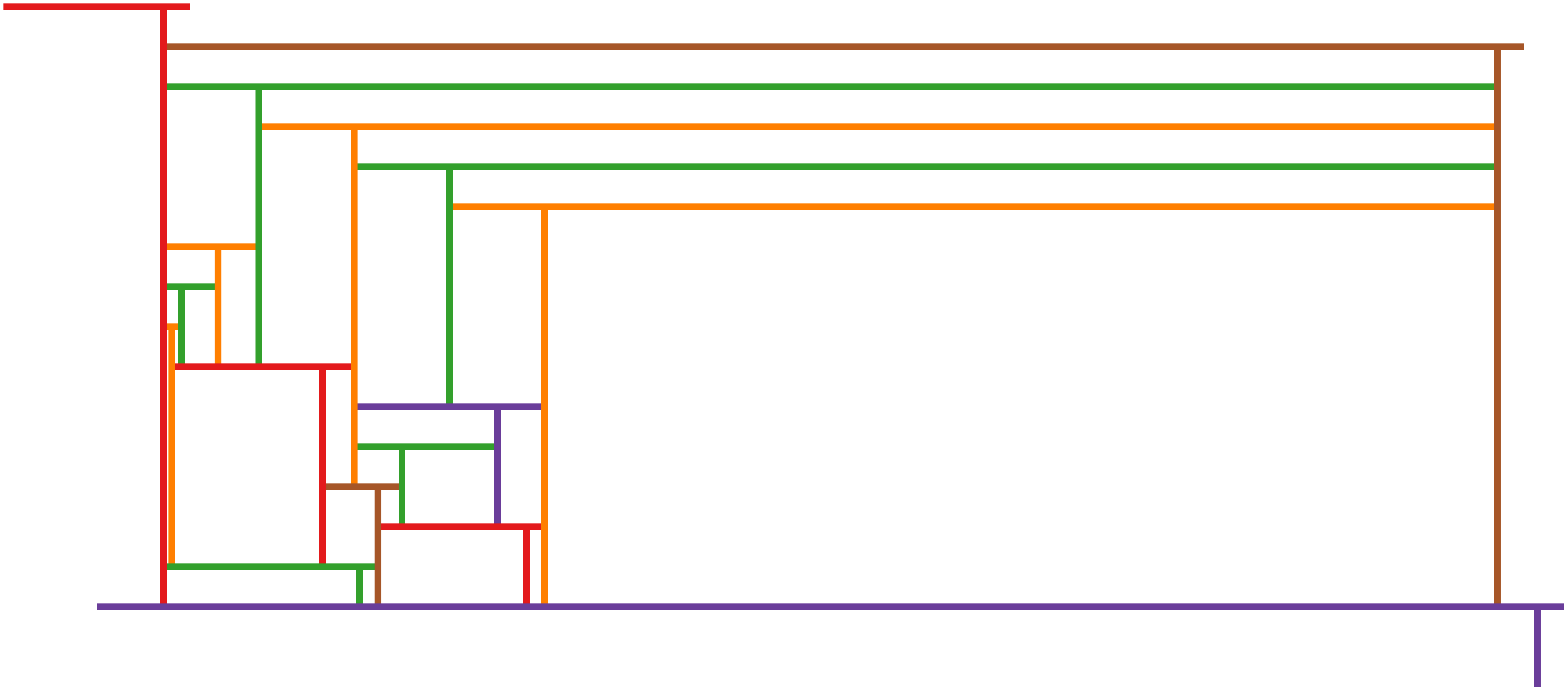
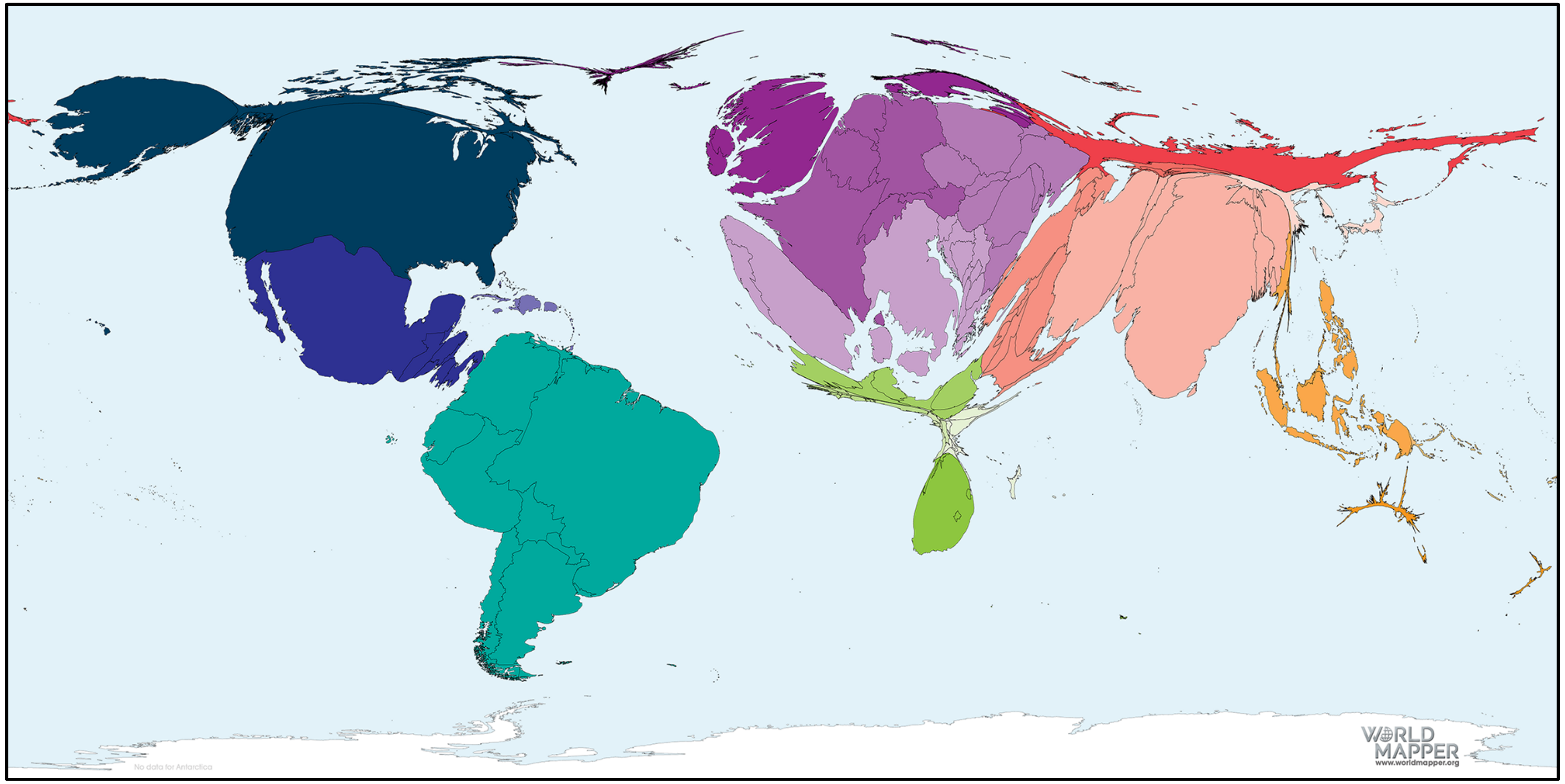


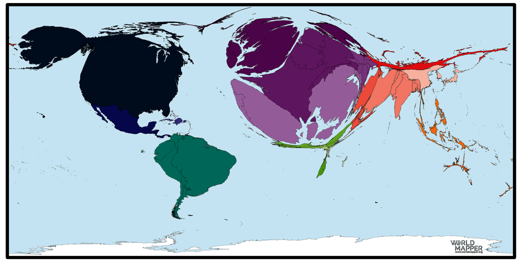
Cartograms

Cartograms



COVID19 reported deaths (January 1, 2021)

Cartograms



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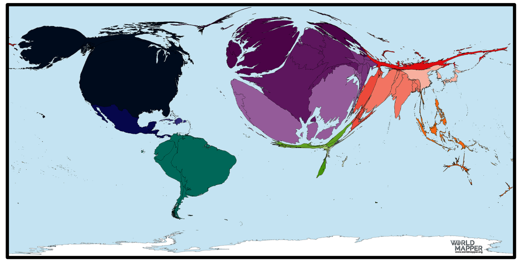
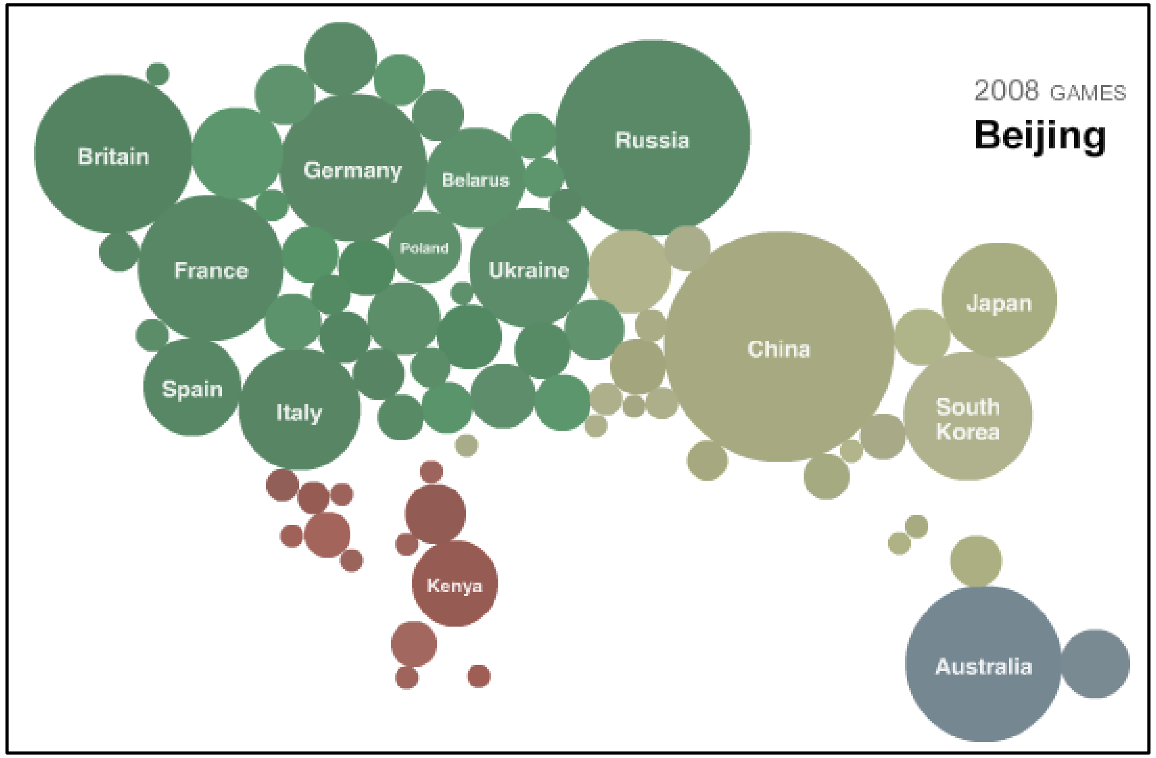
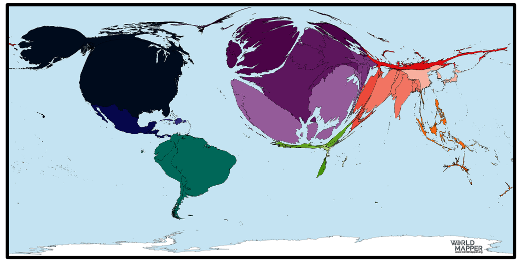




2008 GAMES
Beijing
Britain
Germany
Belarus
Russia
France
Poland
Ukraine
China
Japan
Spain
Italy
South Korea
Kenya
Australia

Cartograms



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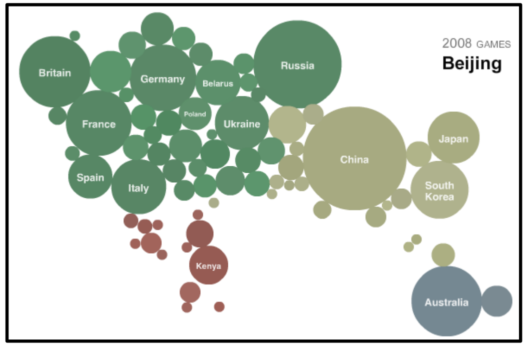

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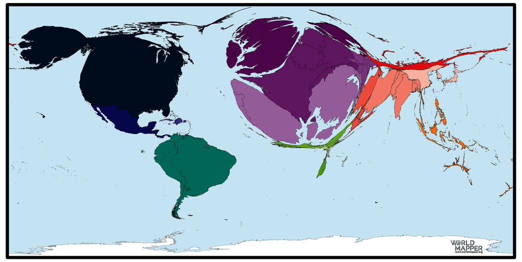

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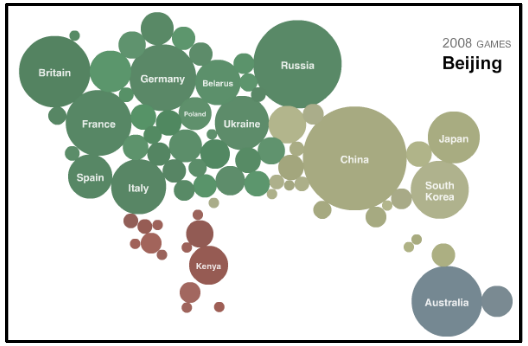

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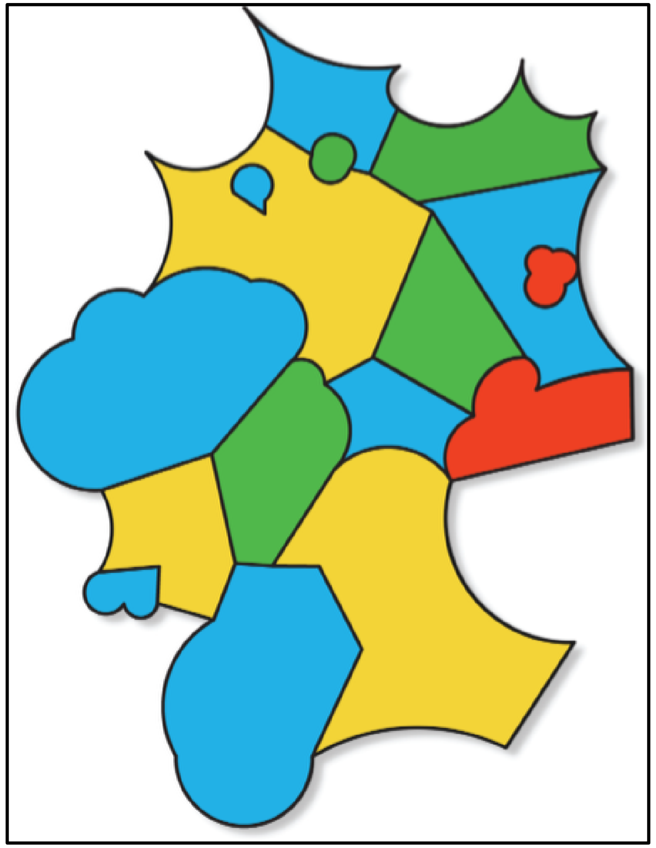

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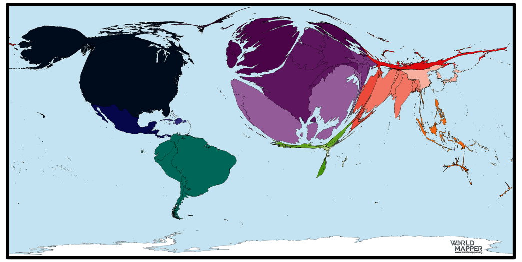

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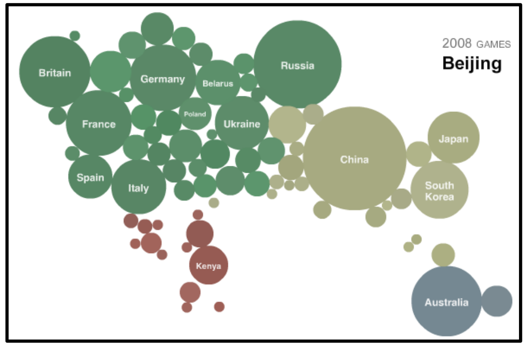

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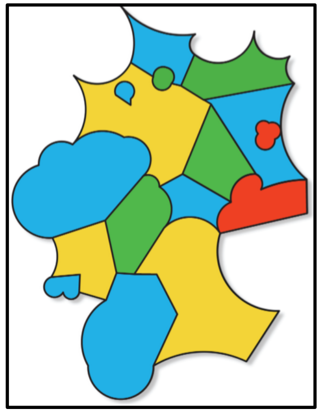

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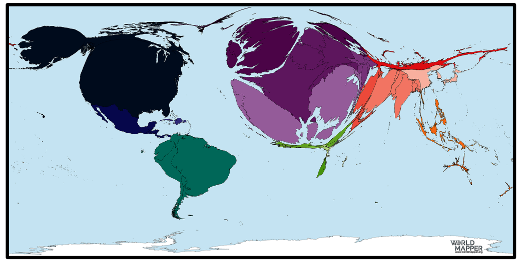

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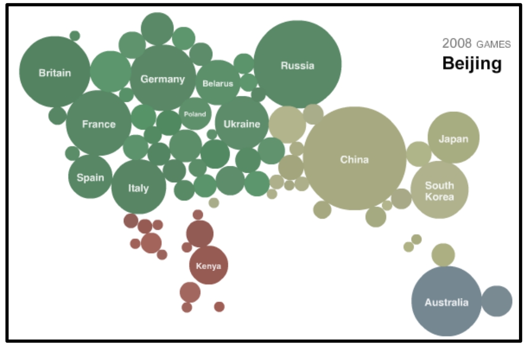

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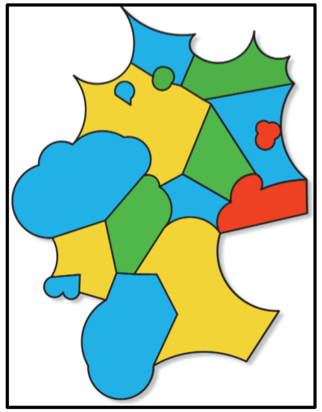

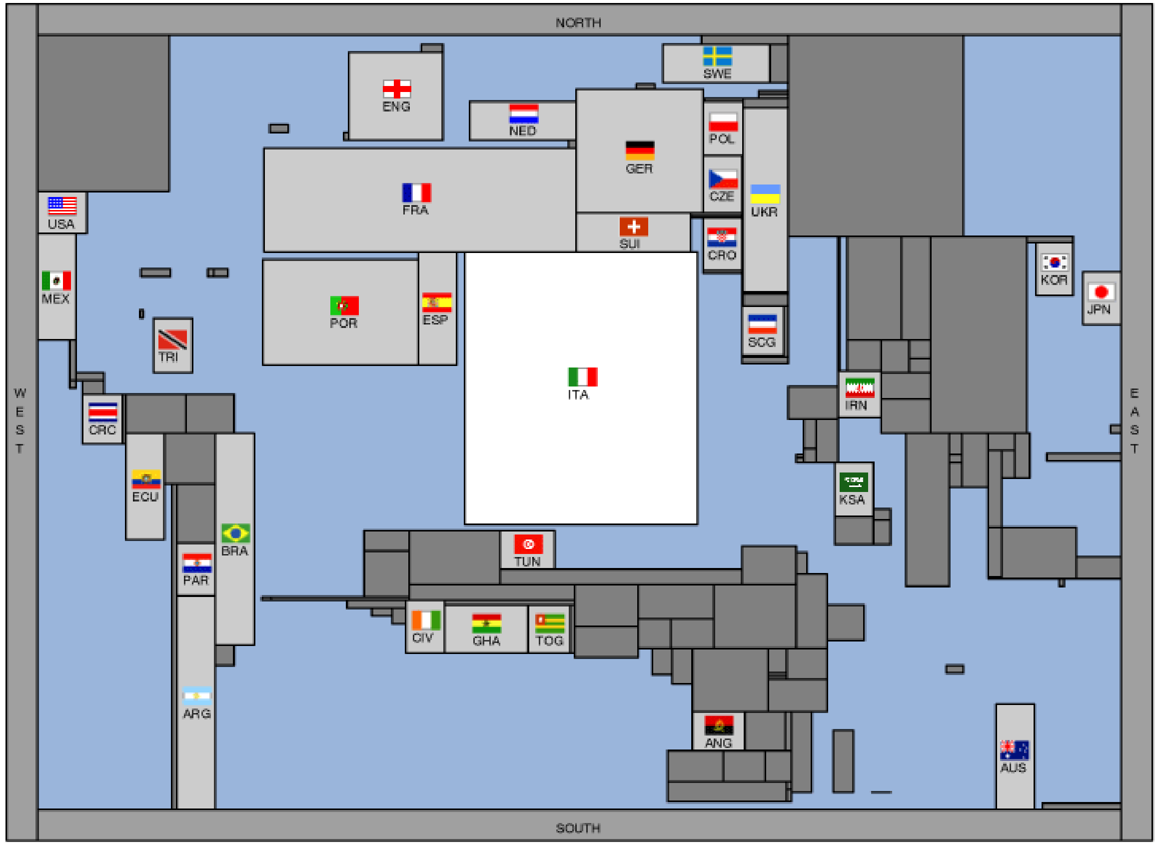

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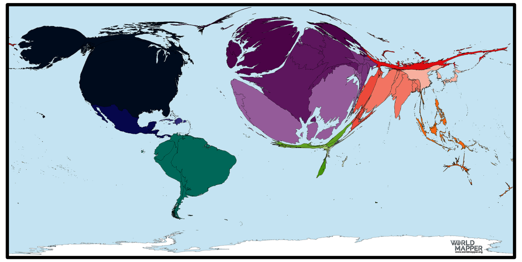

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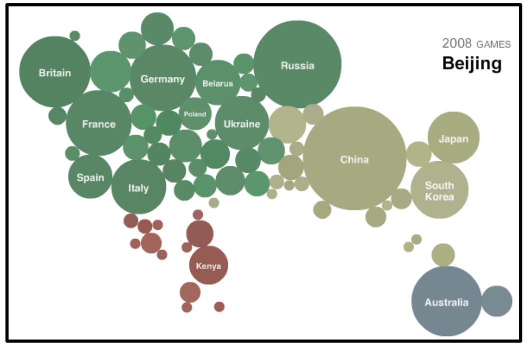

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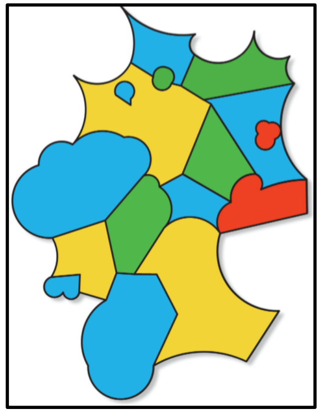

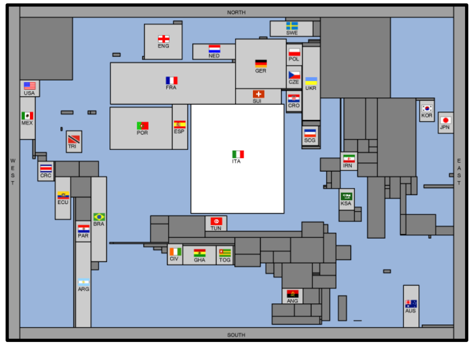

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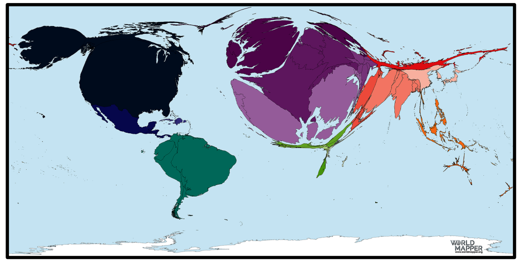

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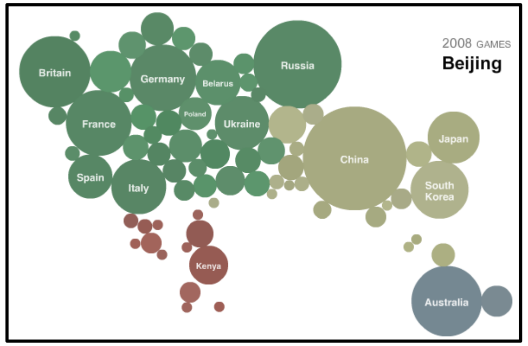

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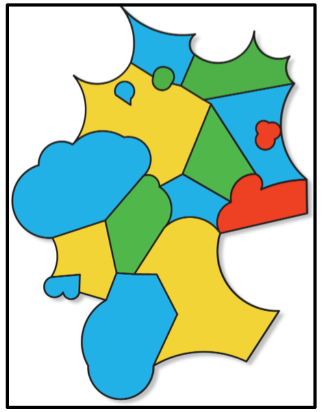

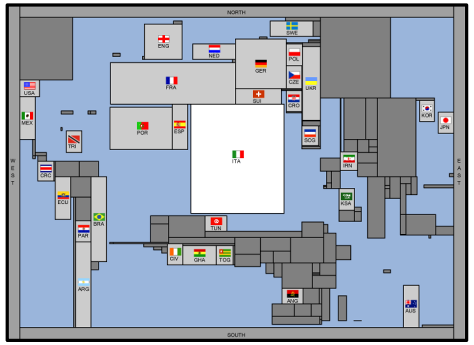

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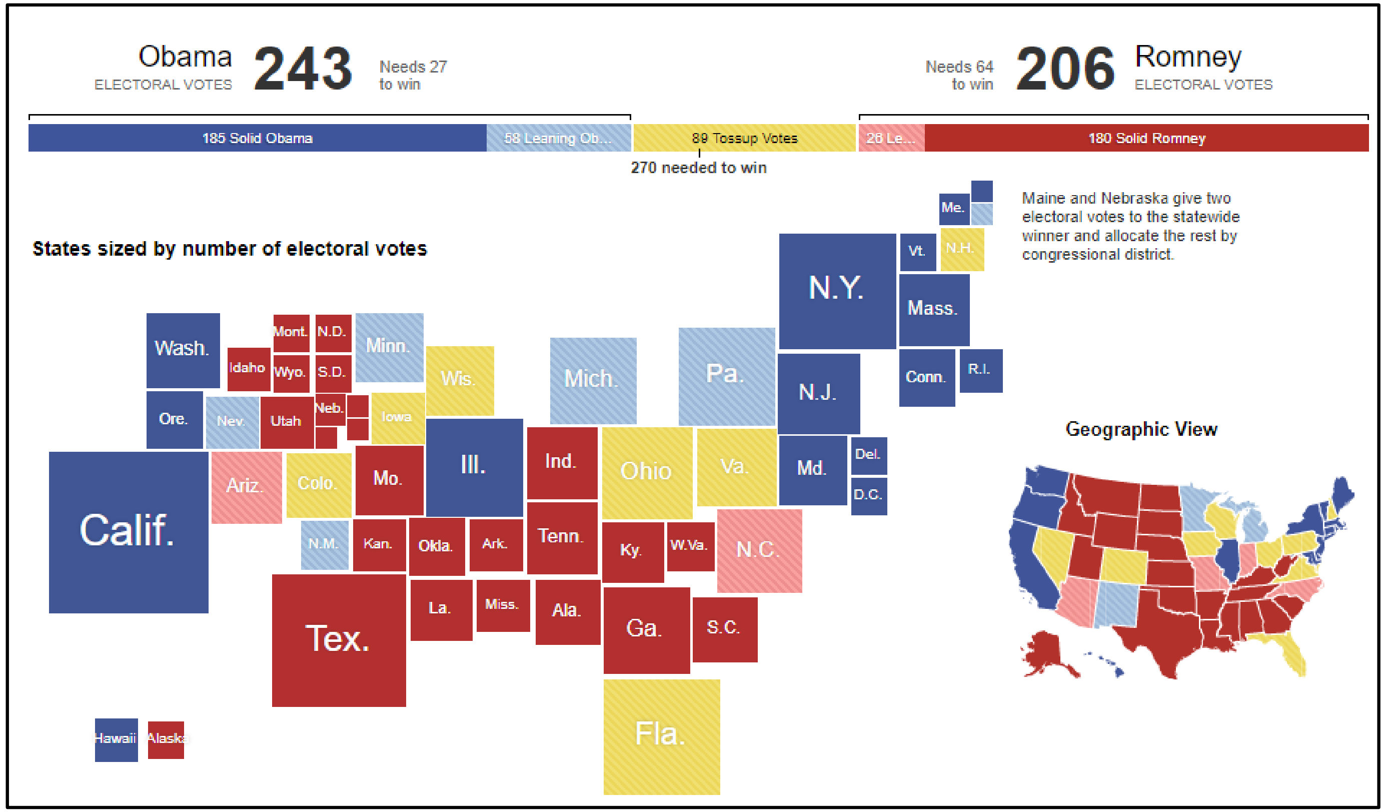

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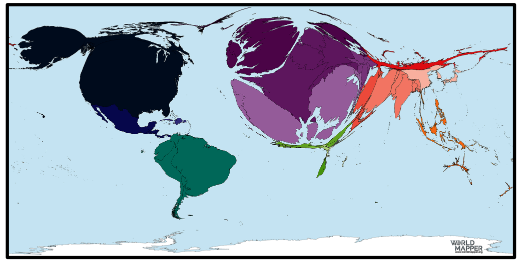

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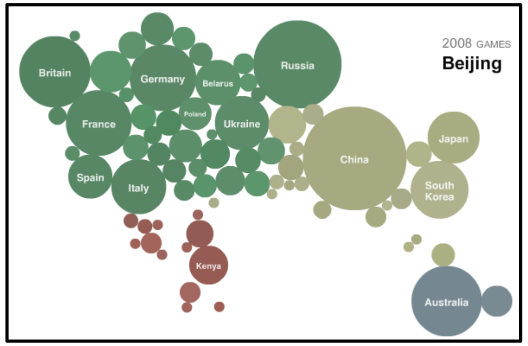

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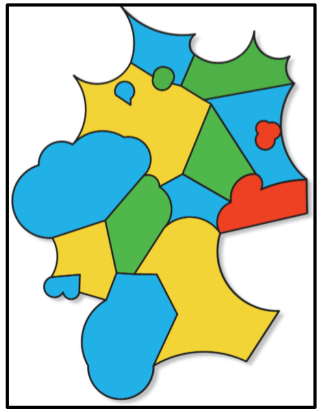

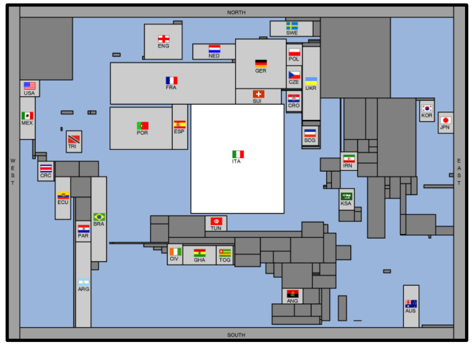

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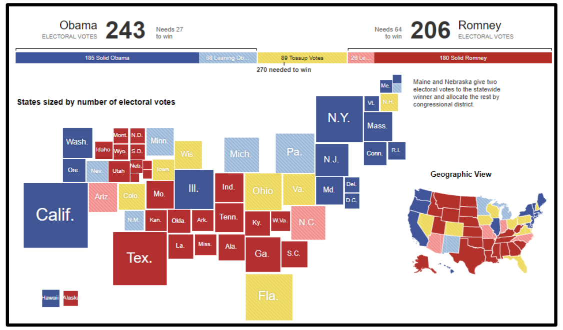

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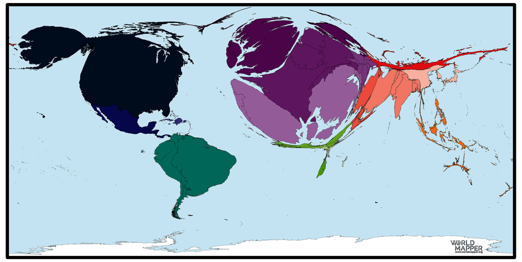

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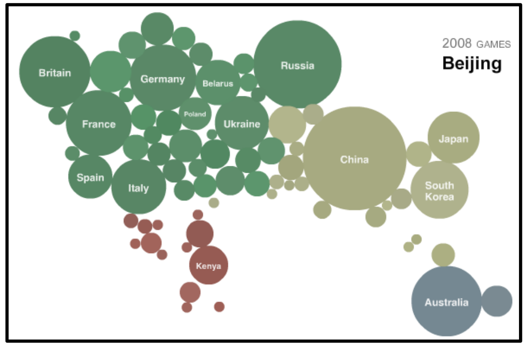

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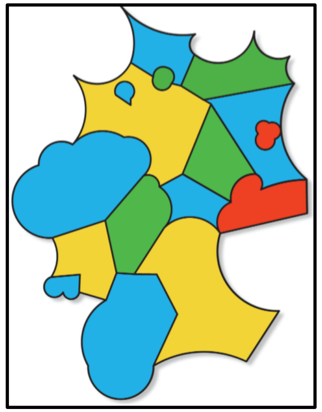

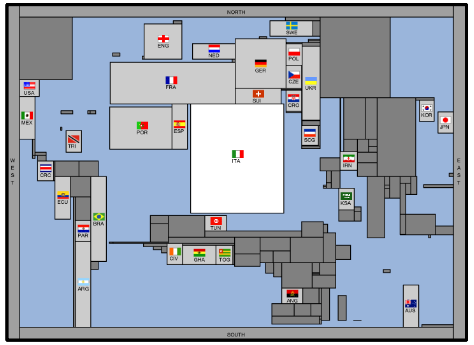

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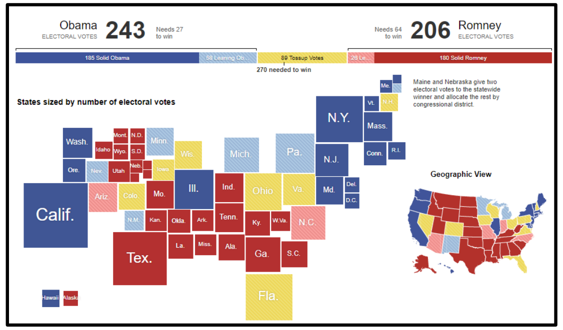

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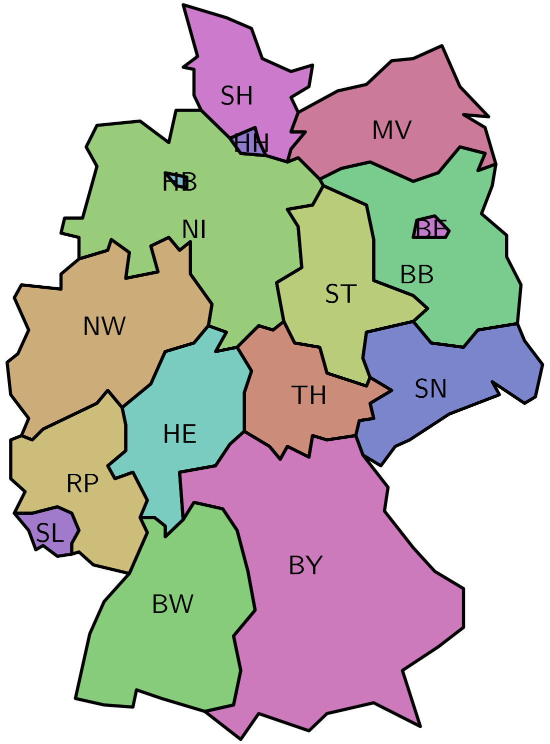

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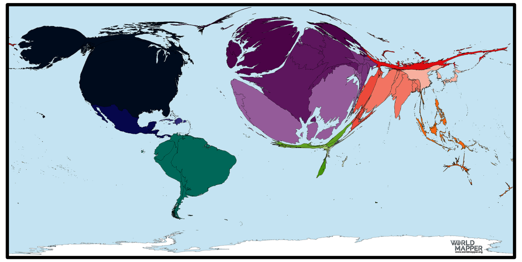

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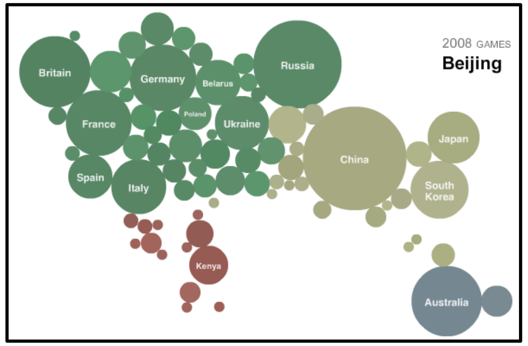

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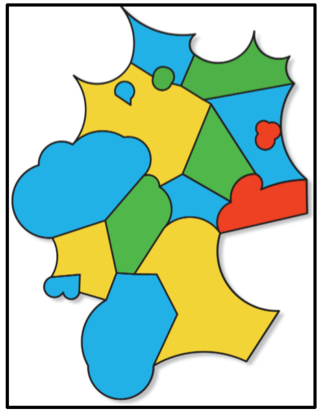

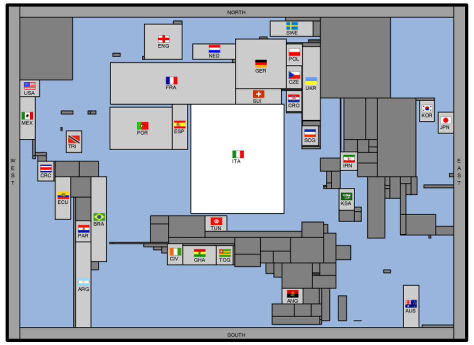

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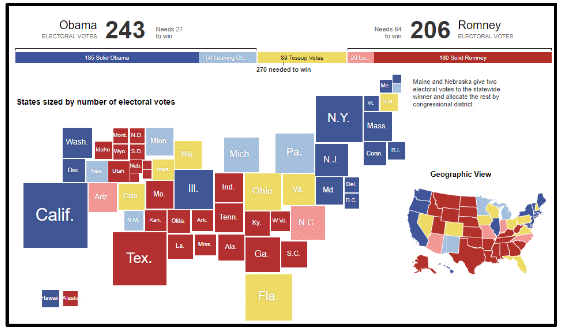

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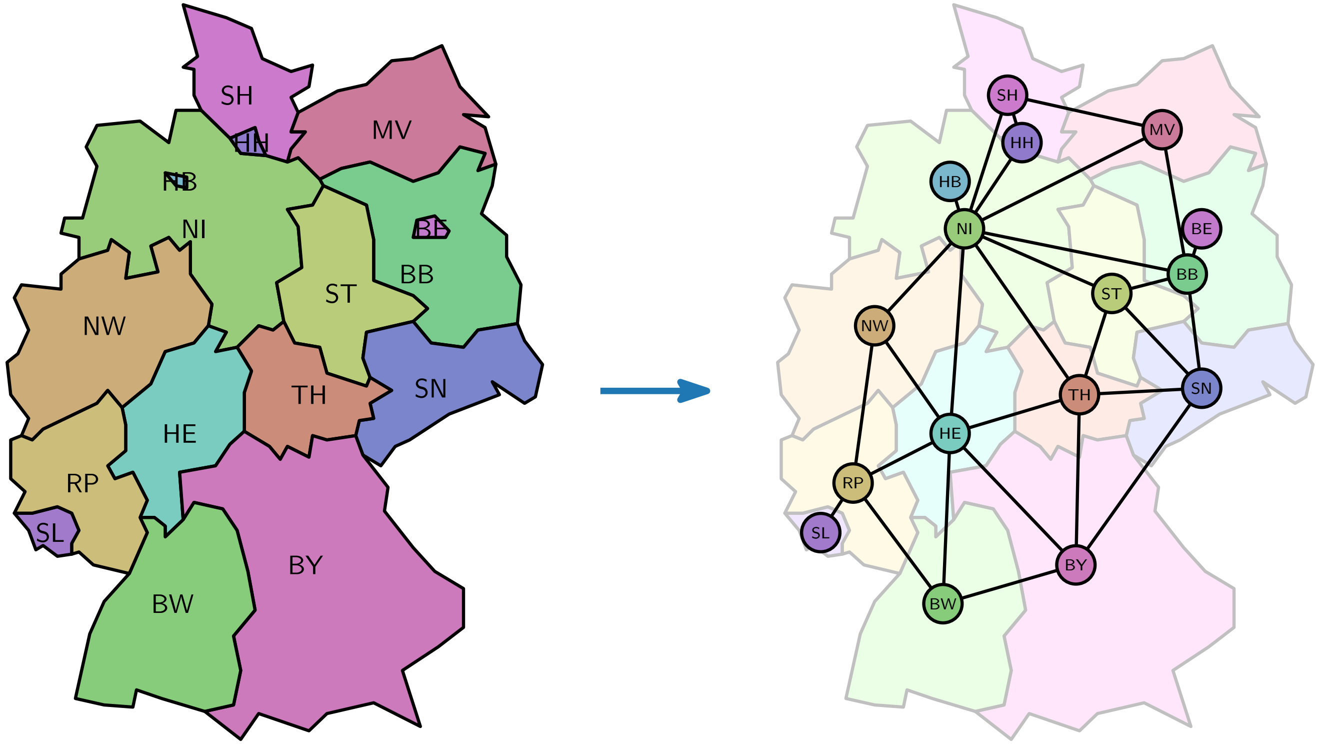

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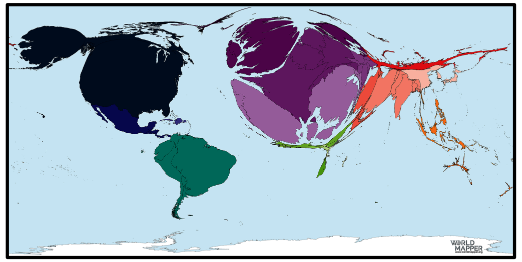

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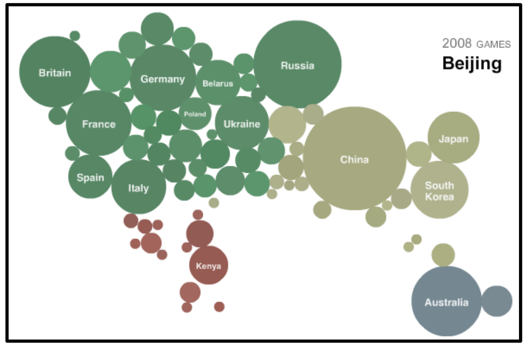

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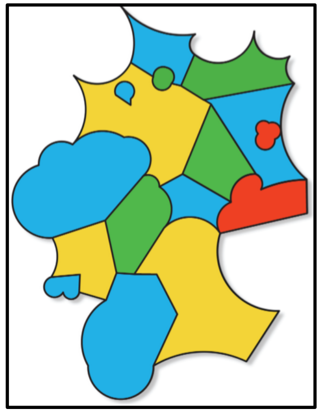

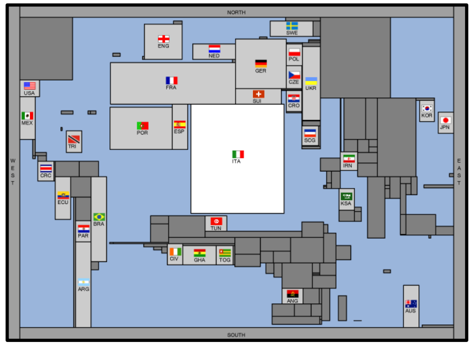

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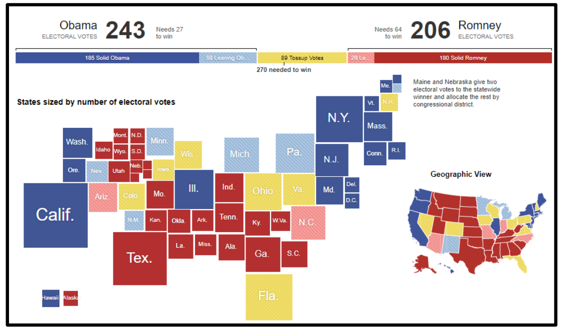

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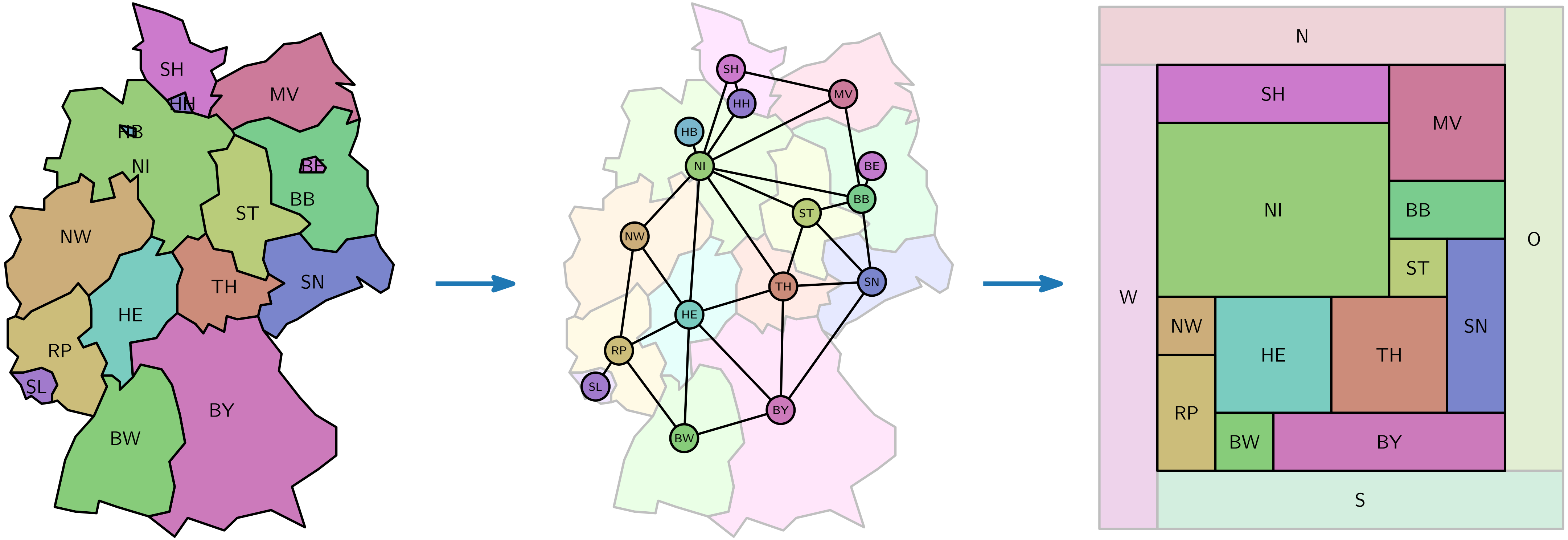
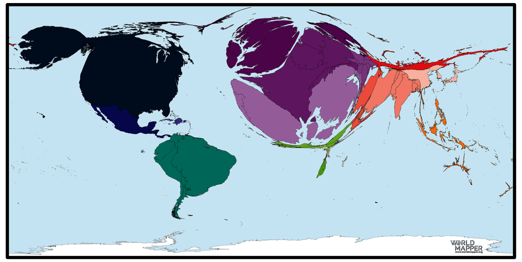


Cartograms



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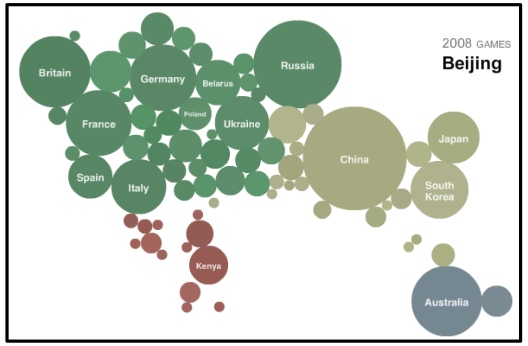

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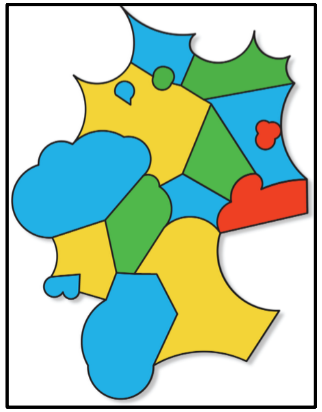

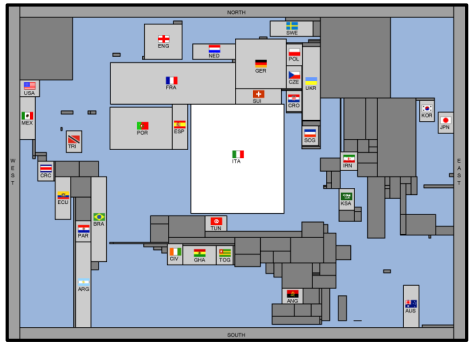

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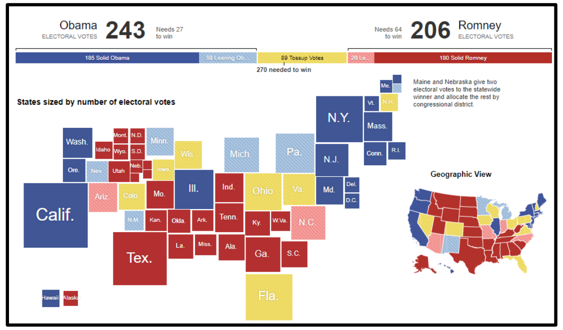

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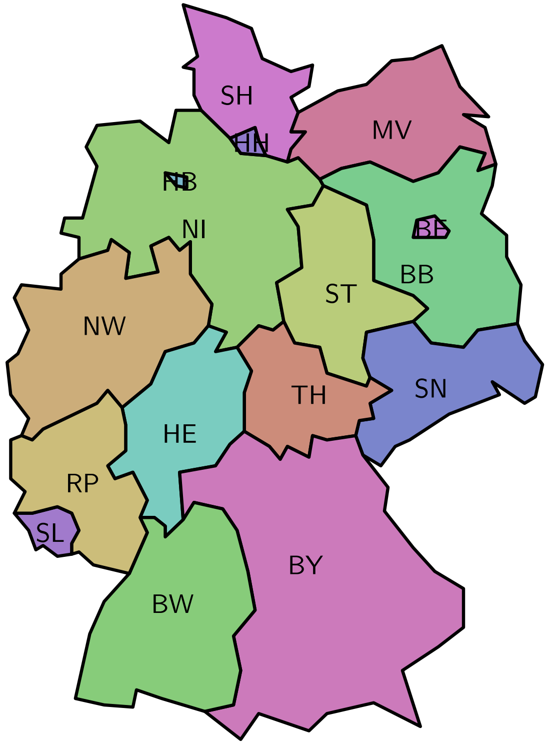
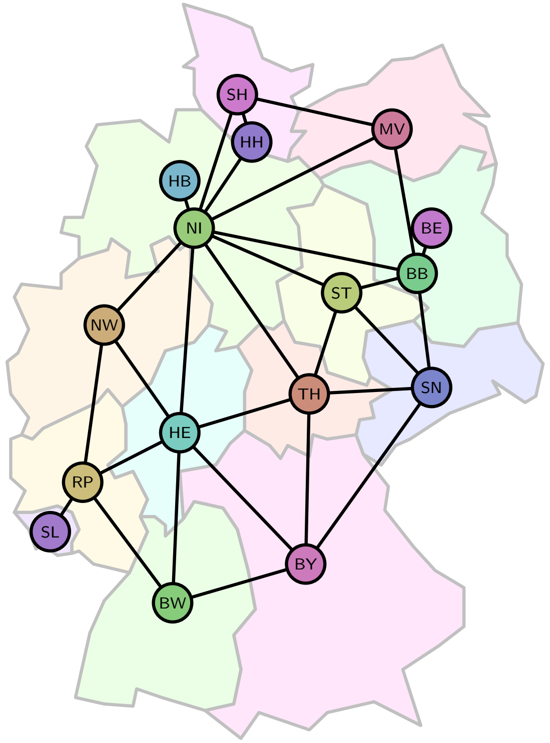
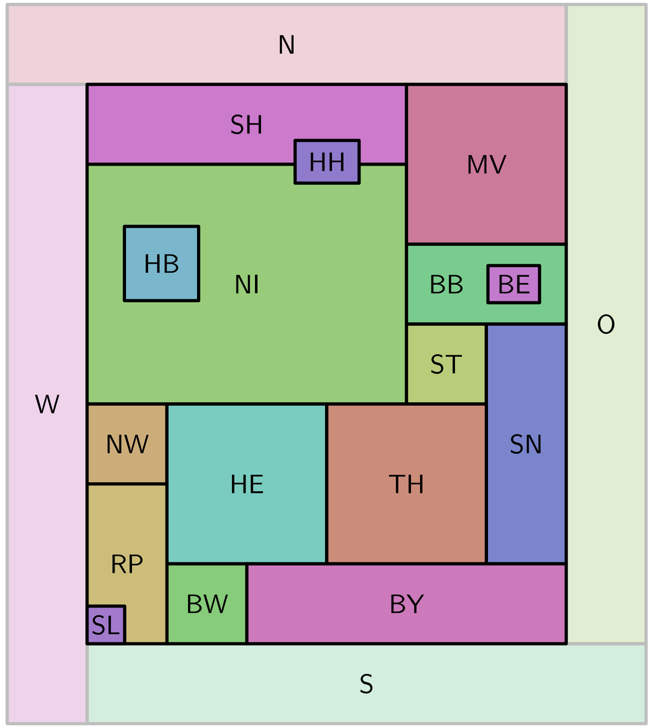
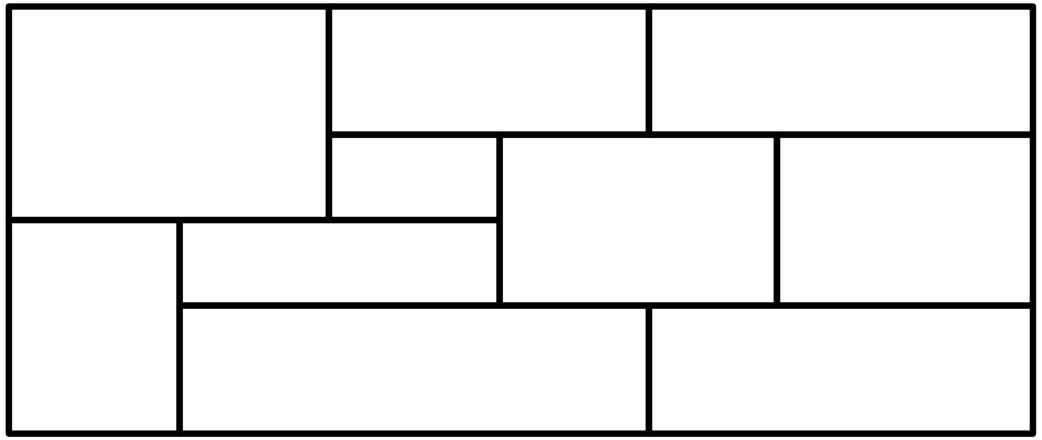




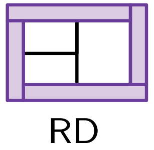


Rectangular Dual

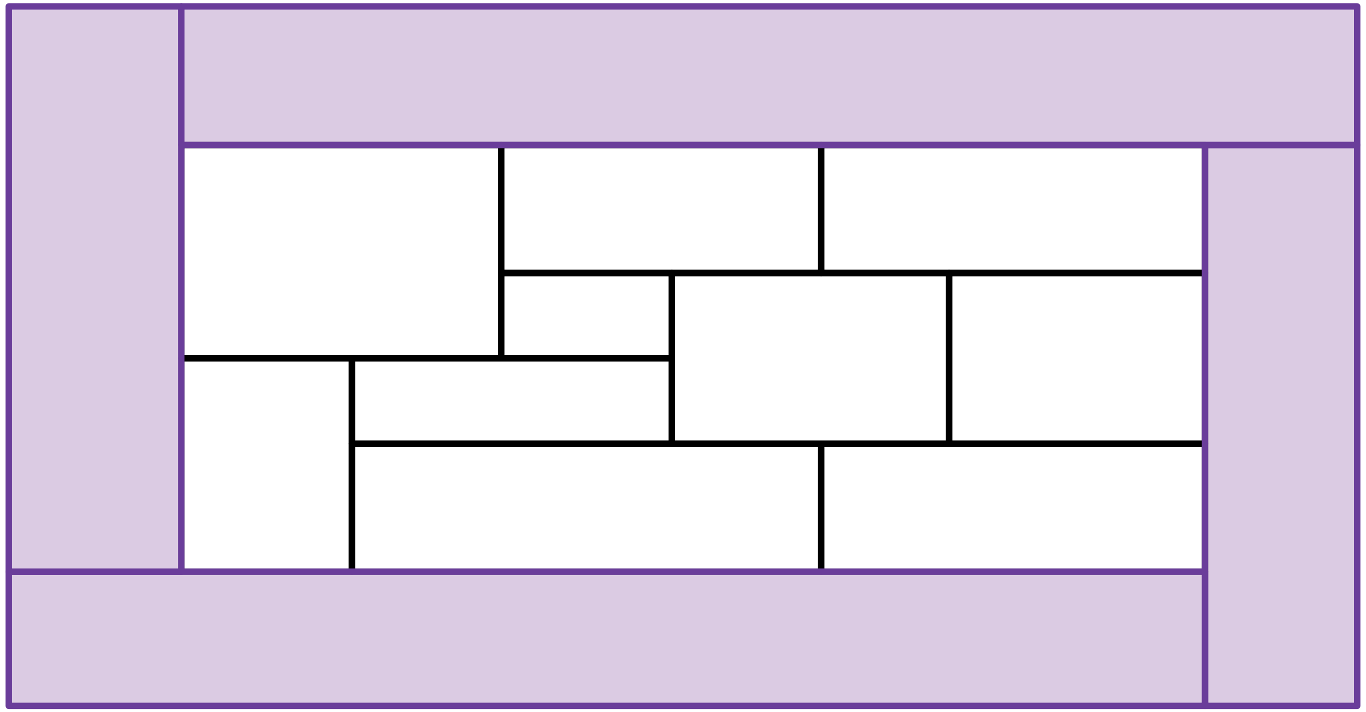
Rectangular Dual



Rectangular Dual



Rectangular Dual $\mathcal{R}$



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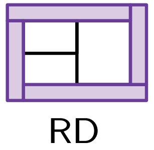
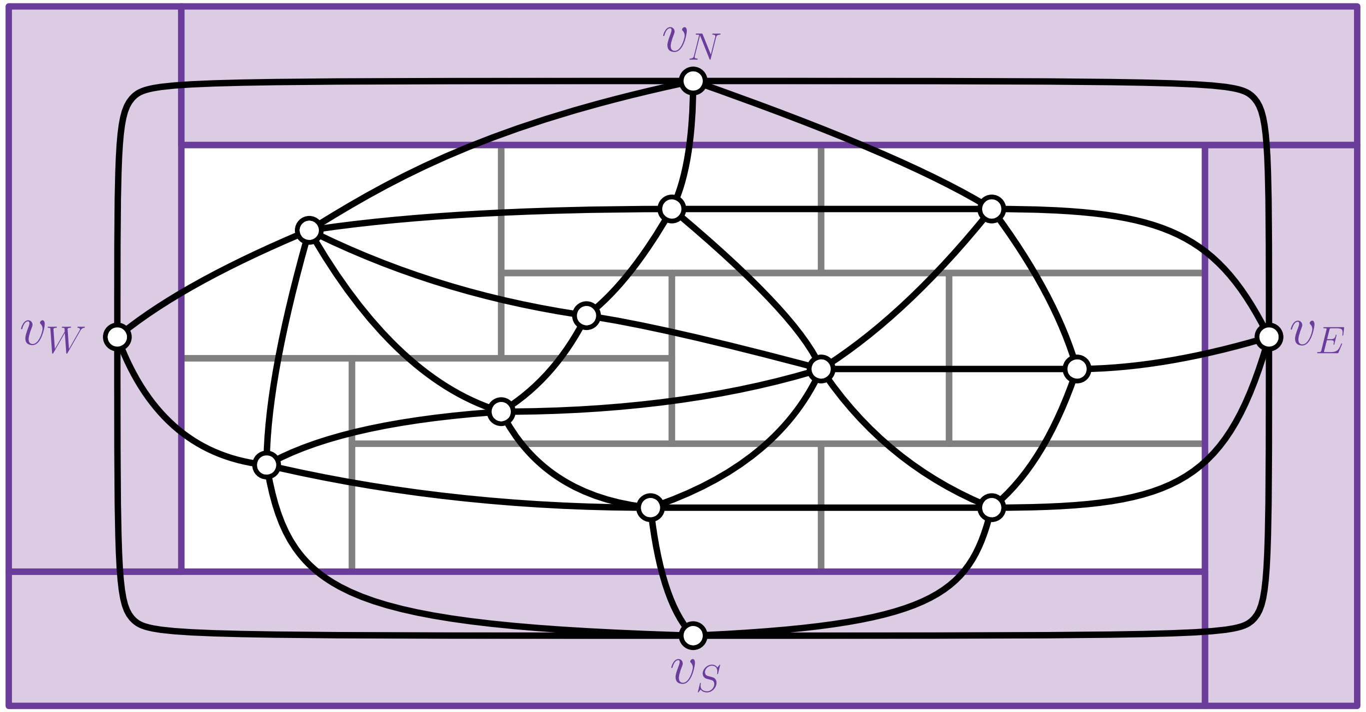


Rectangular Dual $\mathcal{R}$



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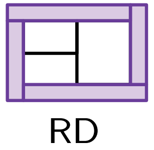
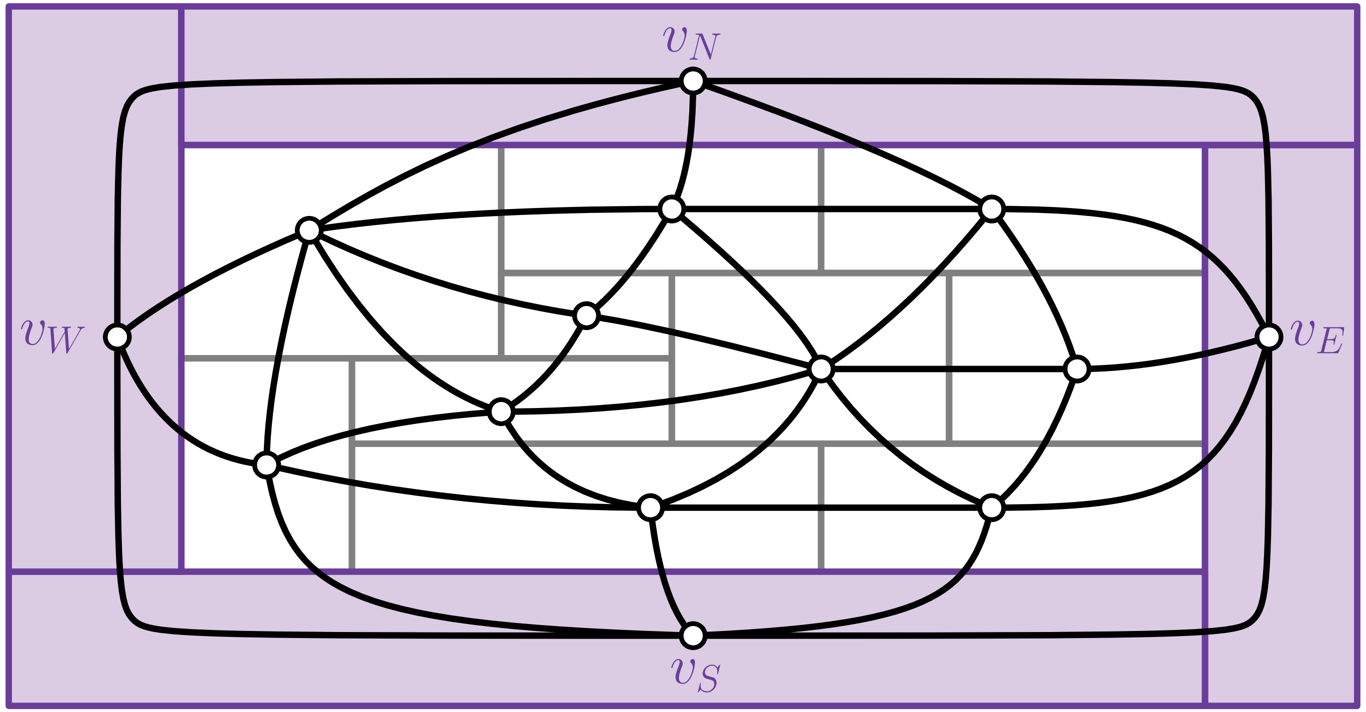
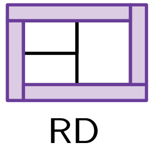


Rectangular Dual $\mathcal{R}$

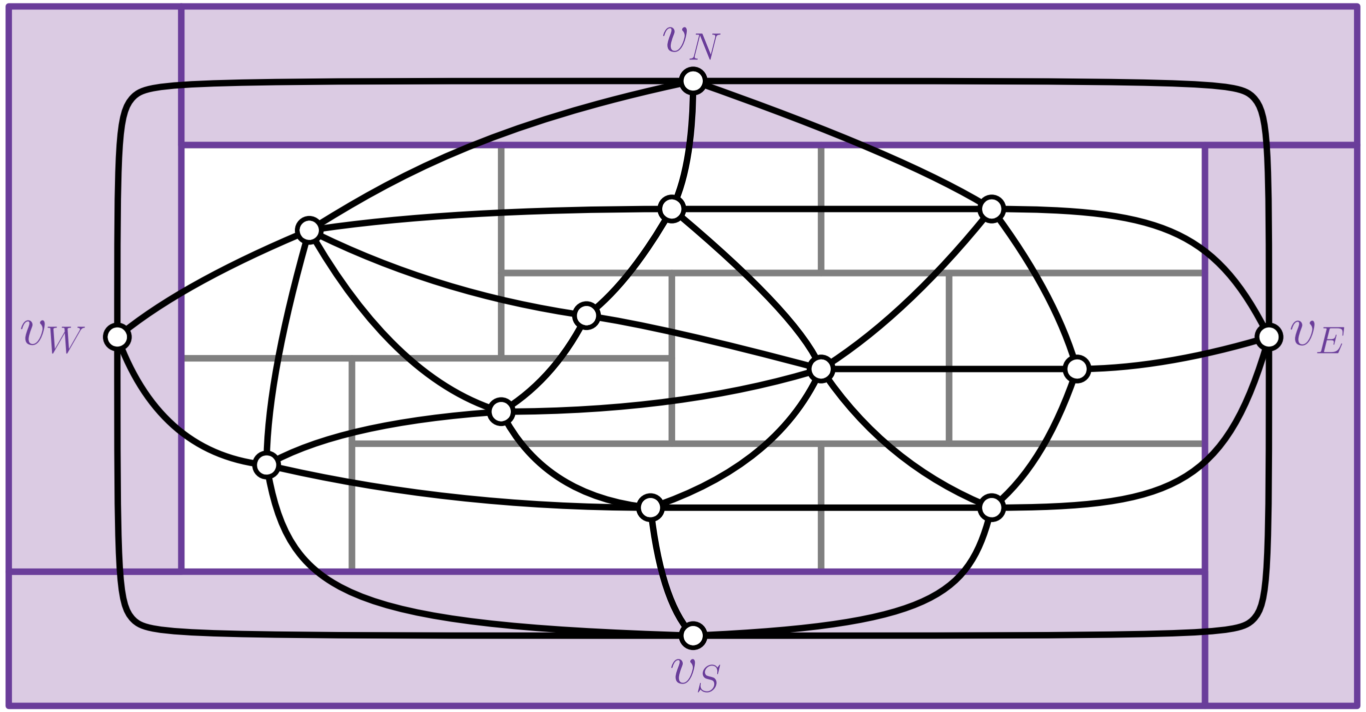


A **rectangular dual** of a graph G is a contact representation with axis-aligned rectangles s.t.

Rectangular Dual



Rectangular Dual $\mathcal{R}$



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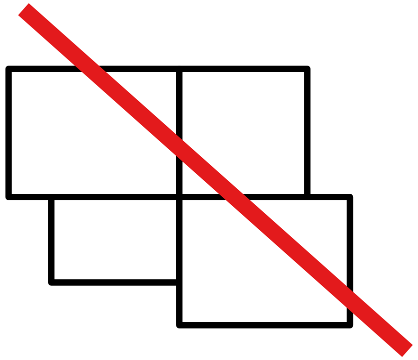

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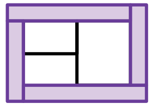

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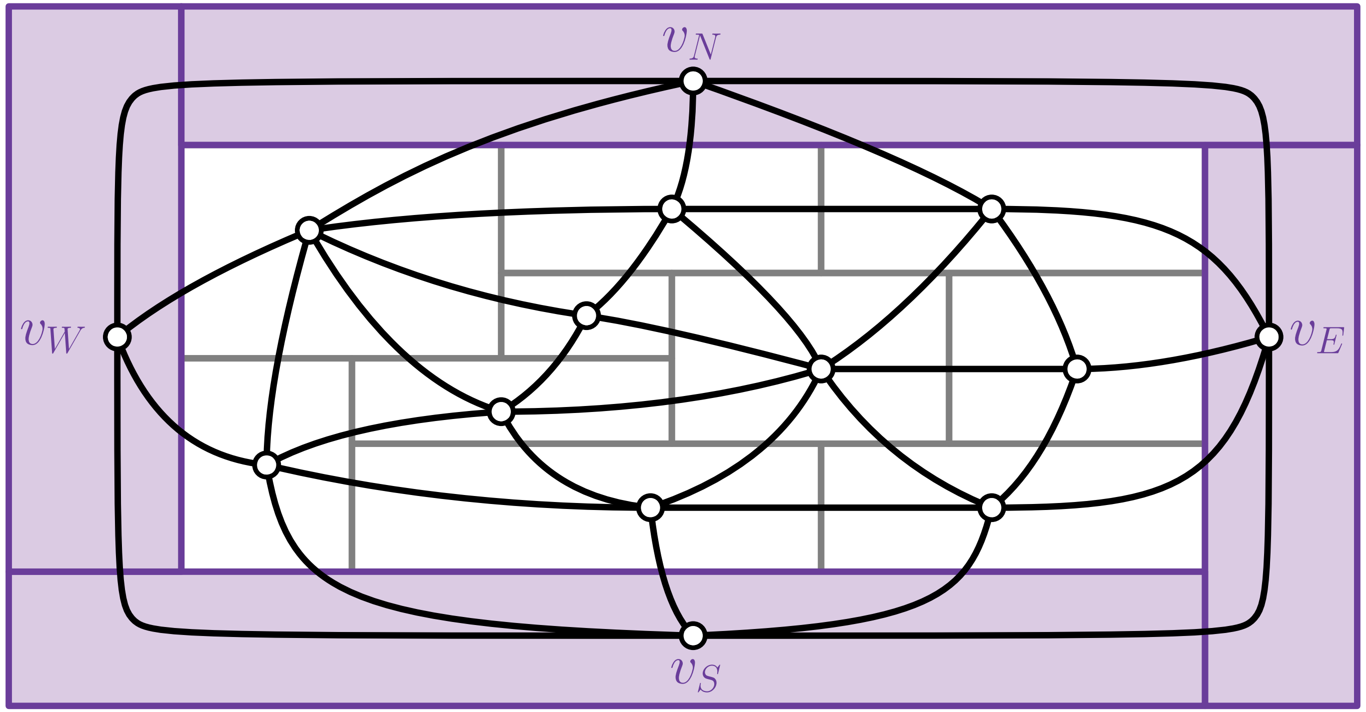

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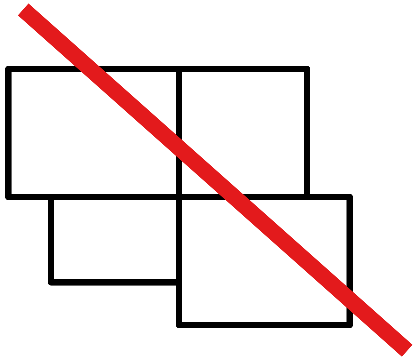

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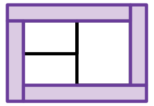

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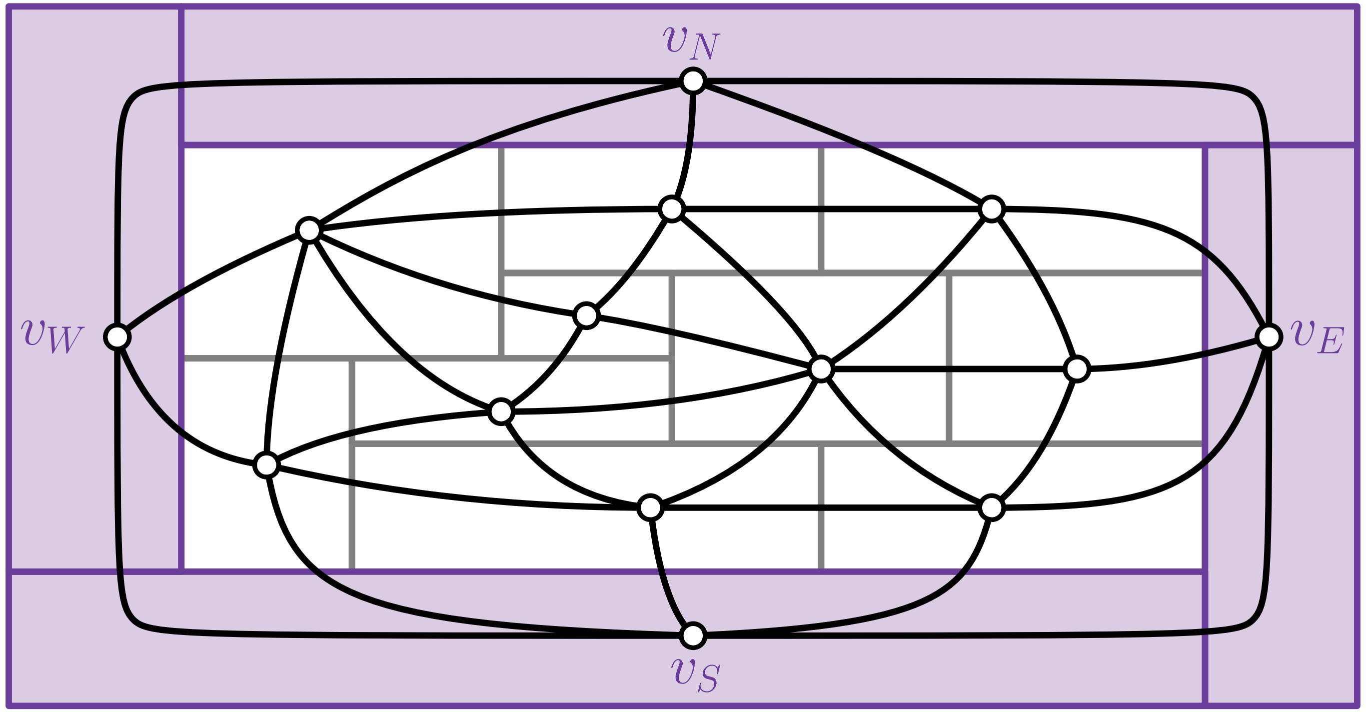

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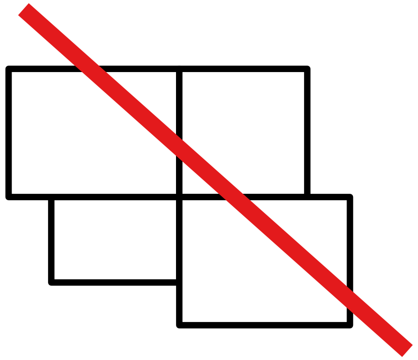

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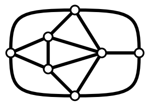
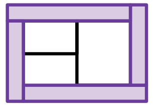


Properly Triangulated Planar Graph G



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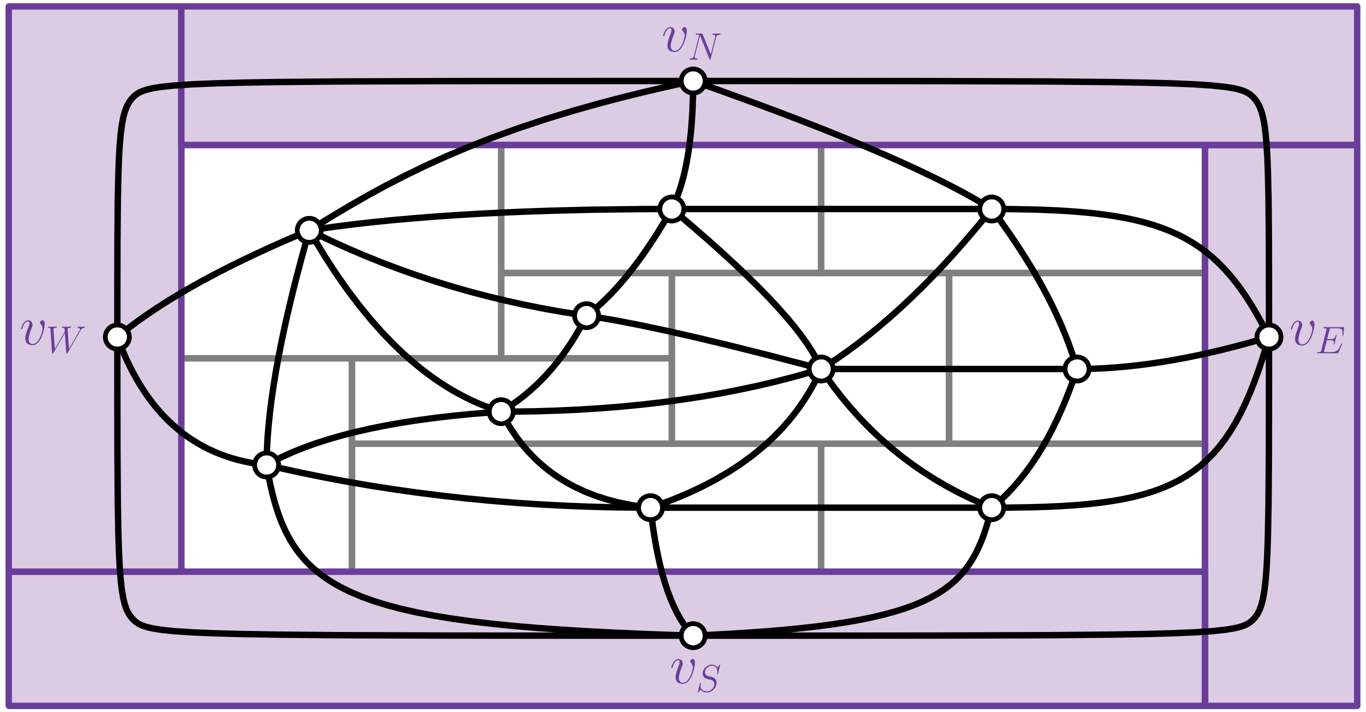

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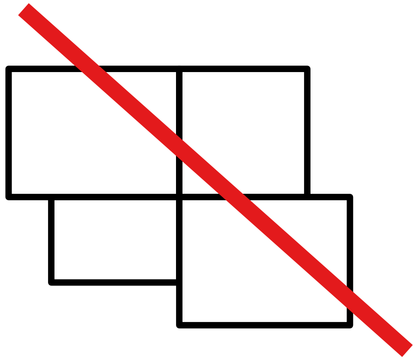

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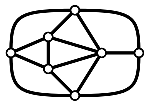

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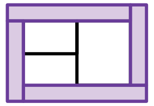

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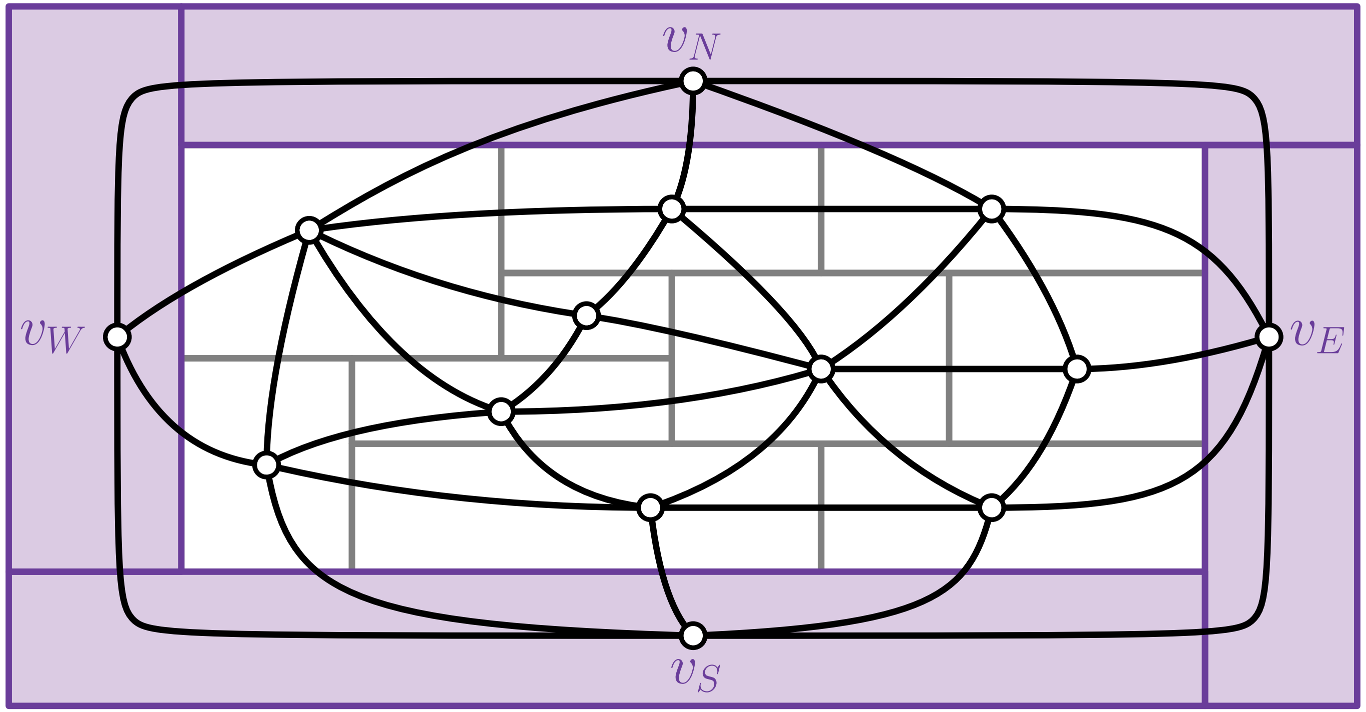

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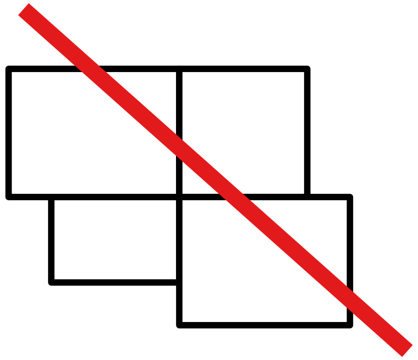

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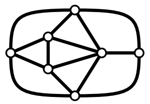

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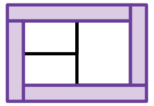

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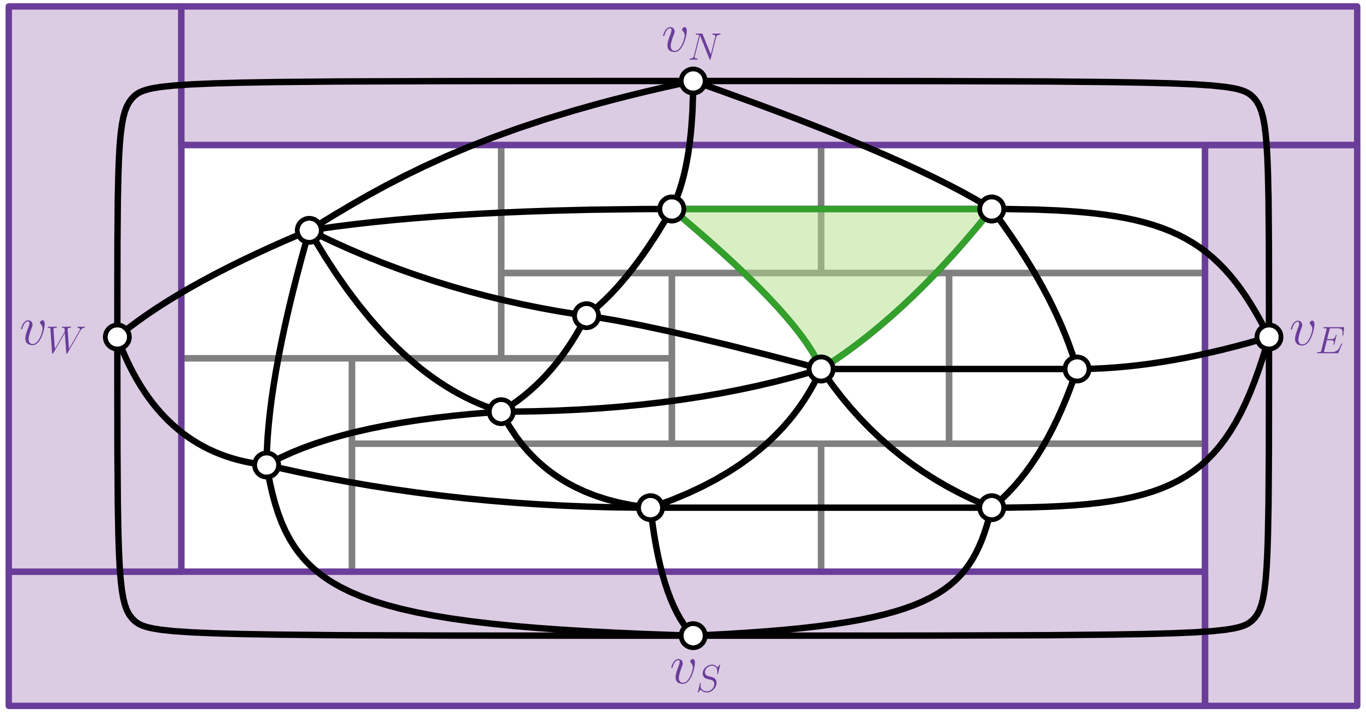

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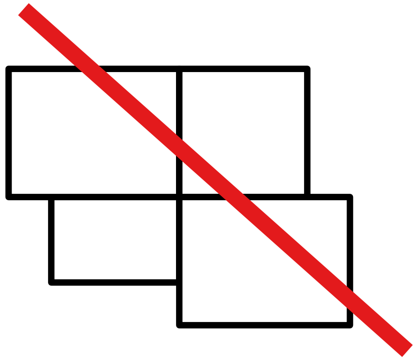

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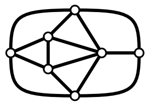

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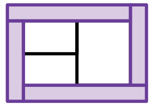

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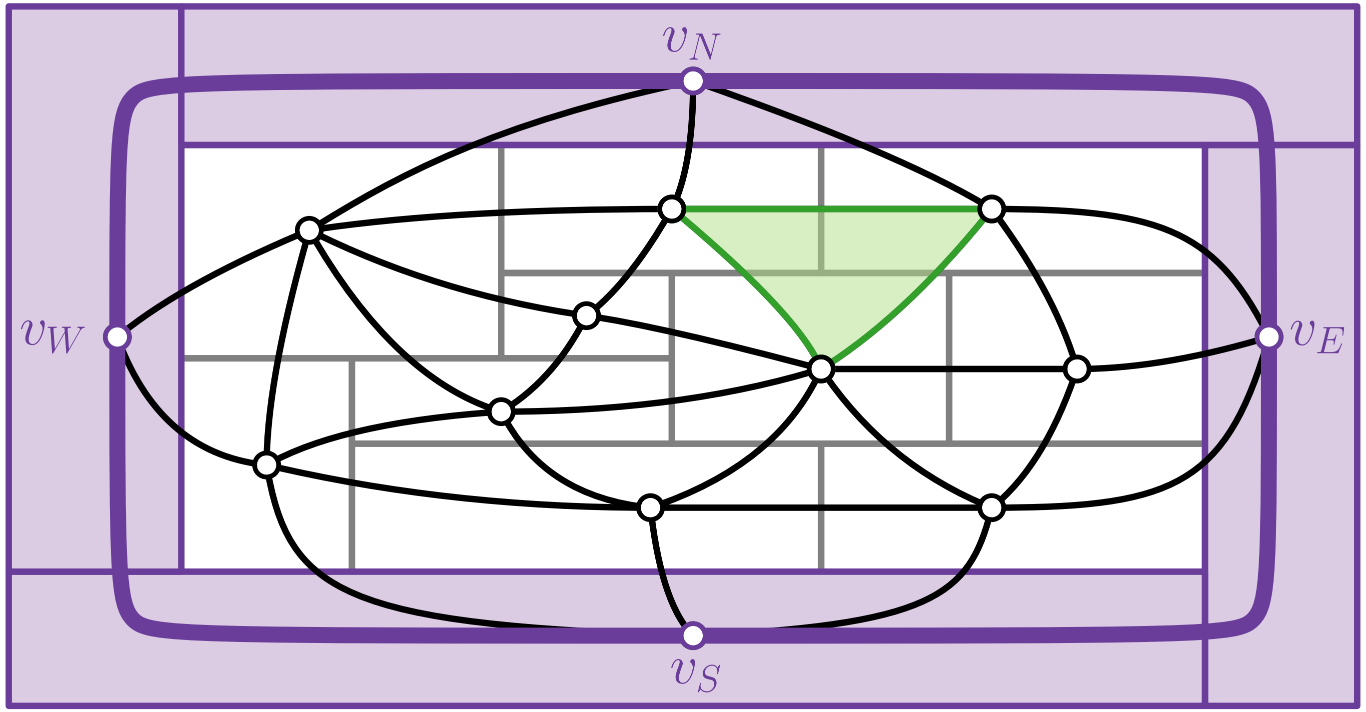

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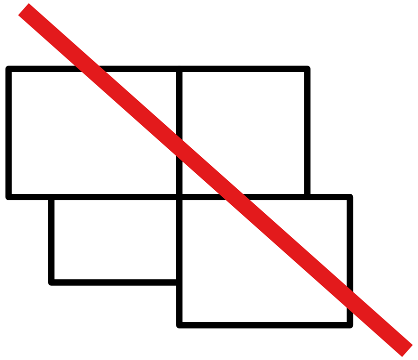

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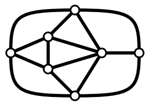

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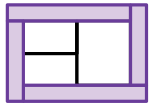
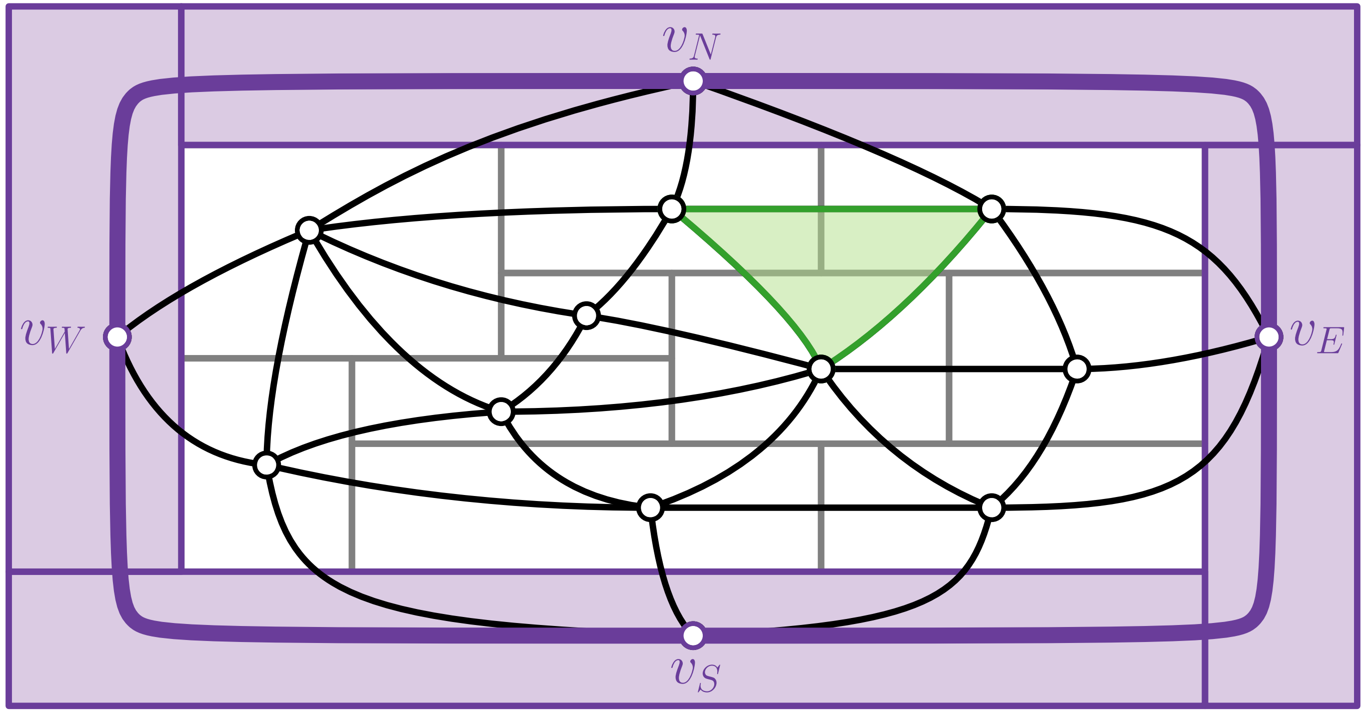


Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face



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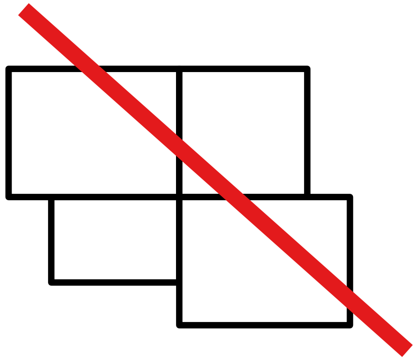

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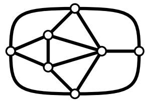
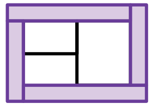


Properly Triangulated Planar Graph G



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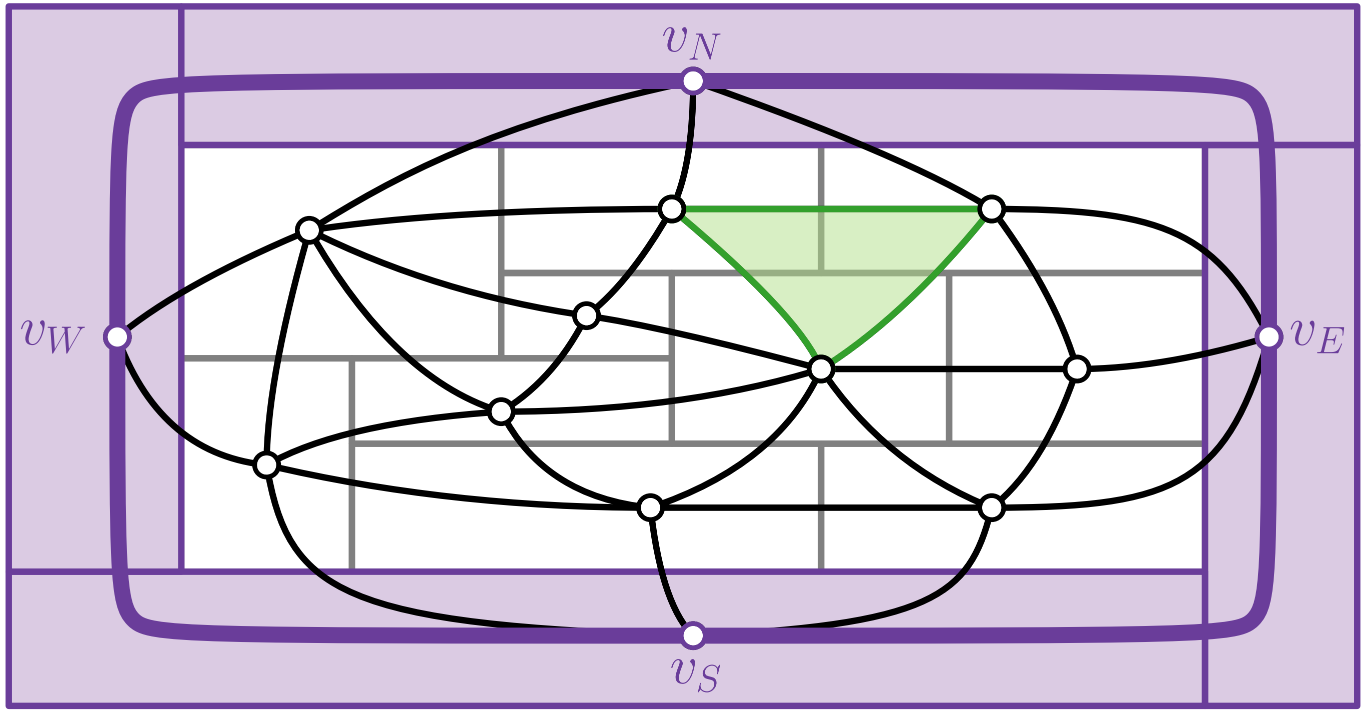

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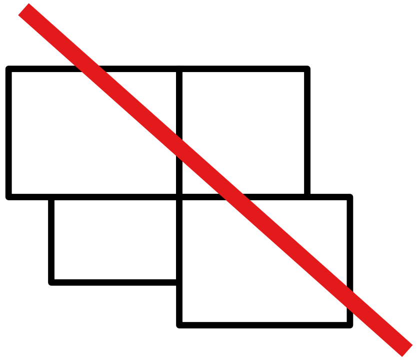

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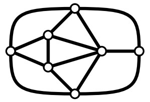

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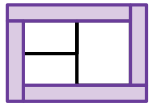

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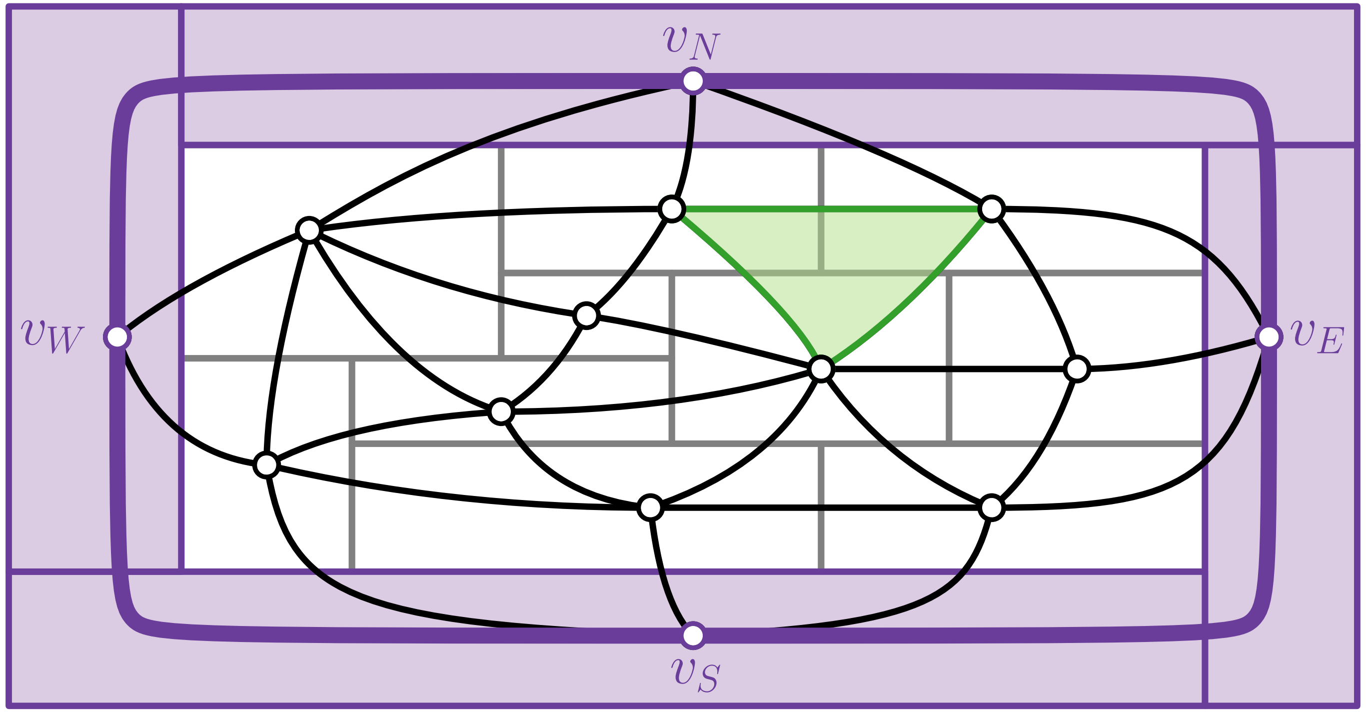

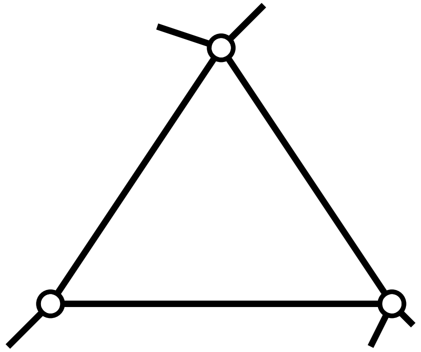

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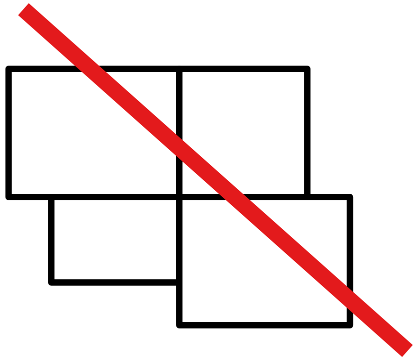

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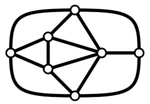
PTP

Properly Triangulated Planar Graph G

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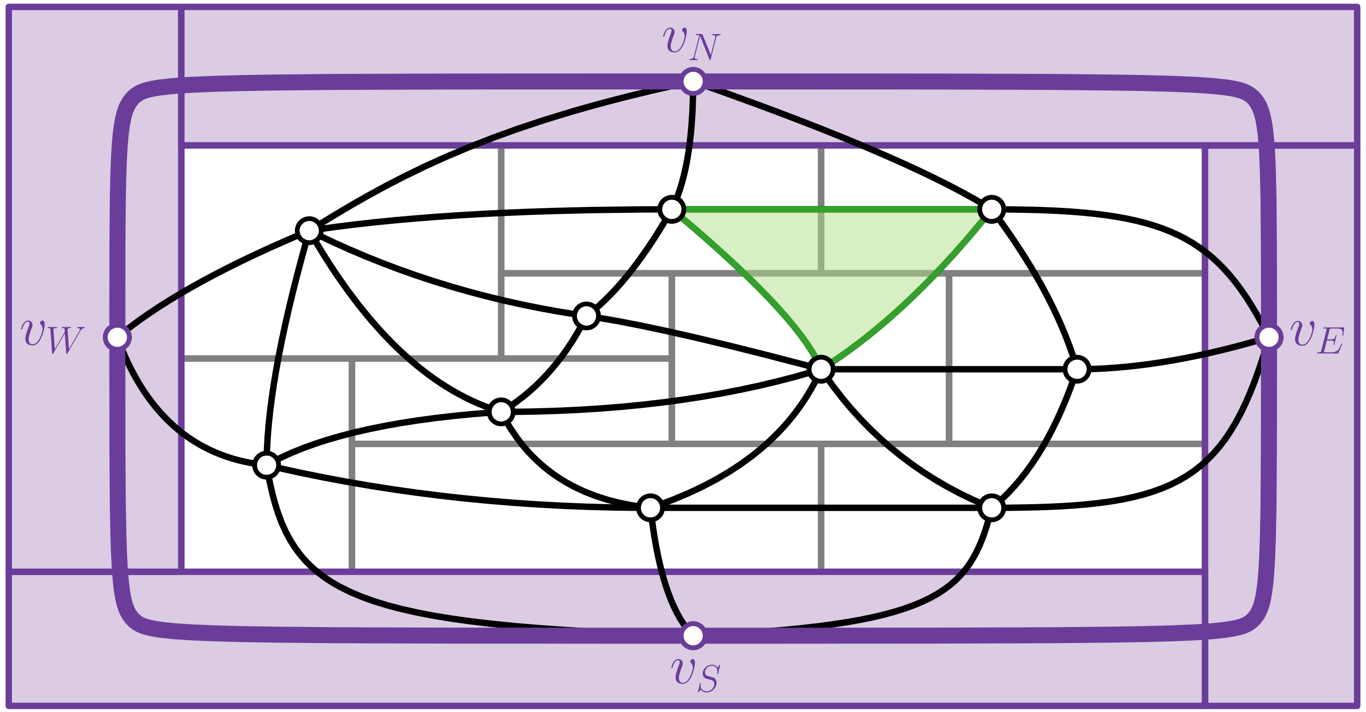

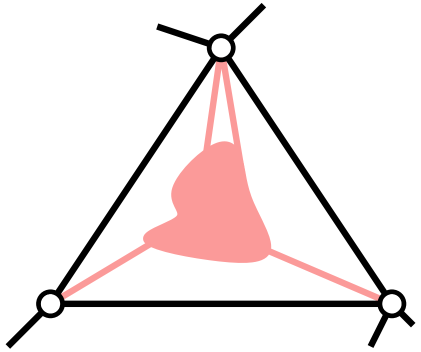

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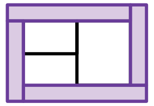
RD

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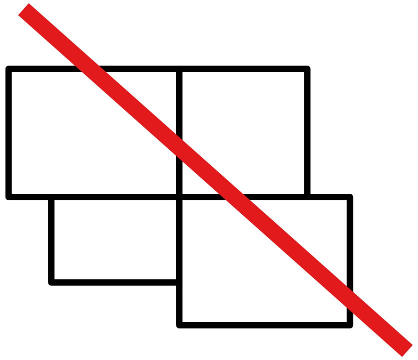

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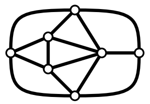
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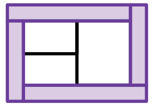
Properly Triangulated Planar Graph G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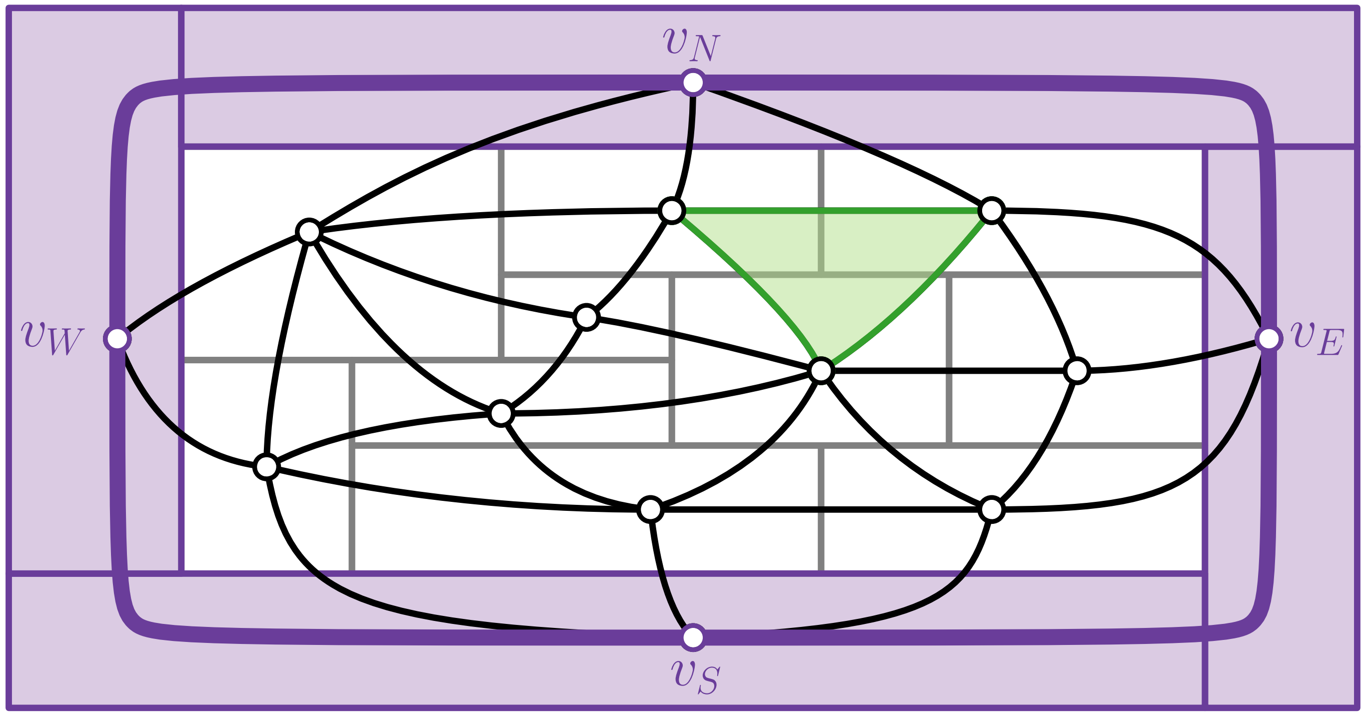

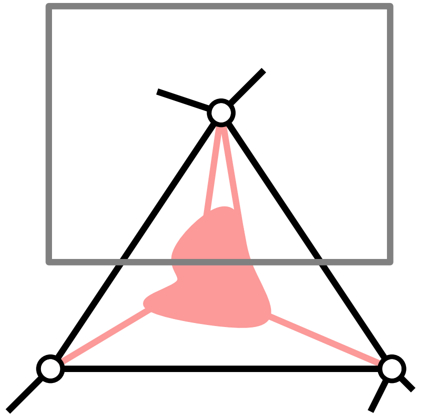

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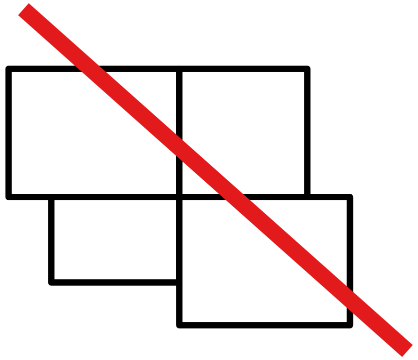

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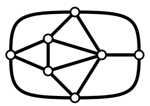
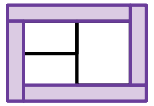


Properly Triangulated Planar Graph G



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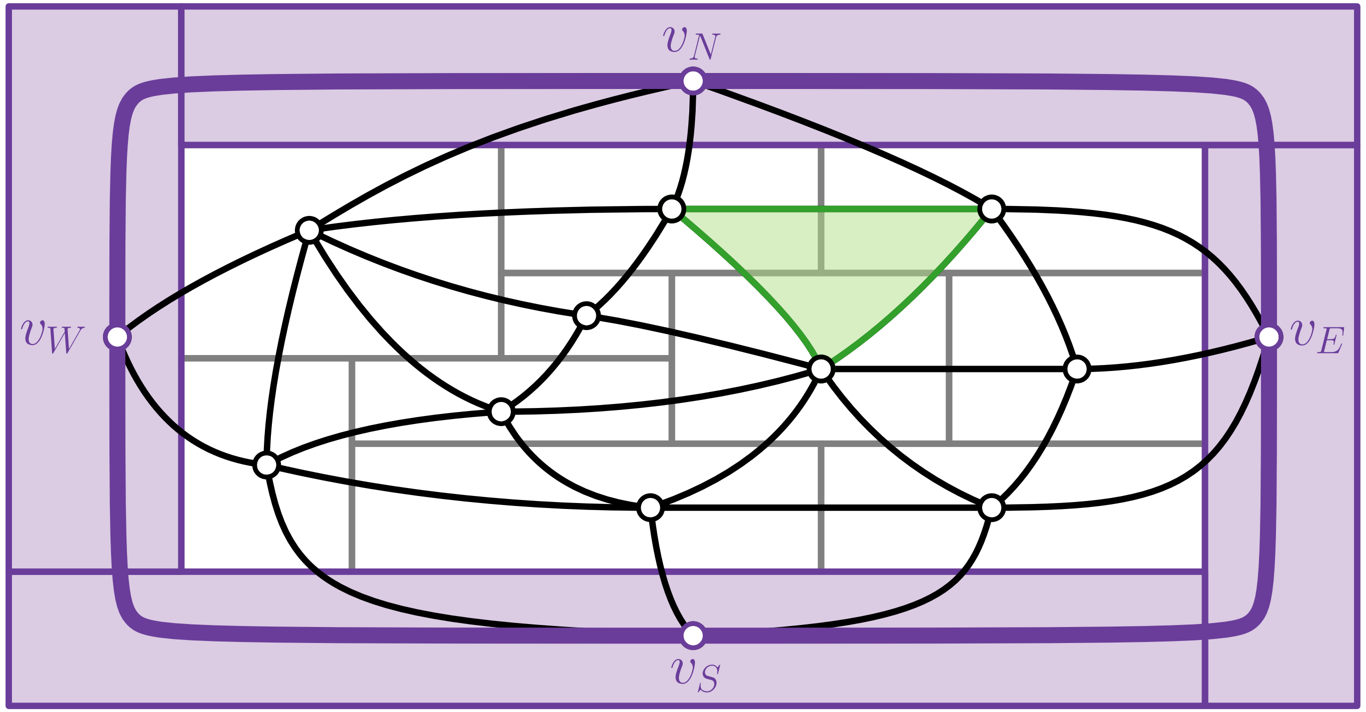

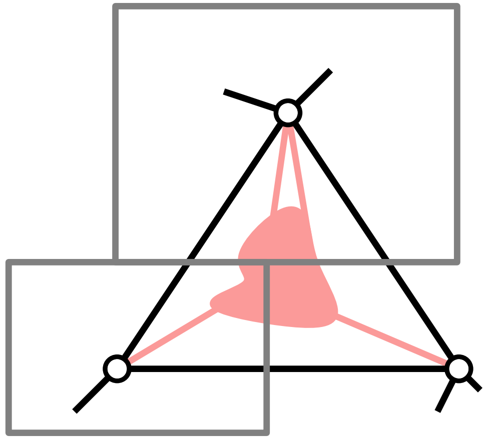

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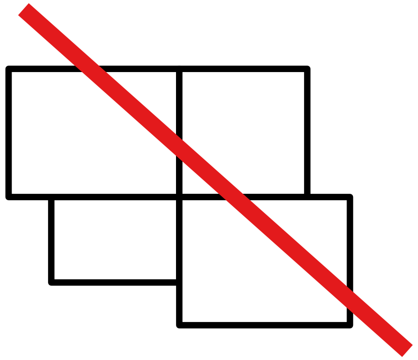

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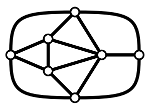
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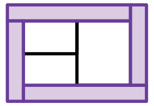
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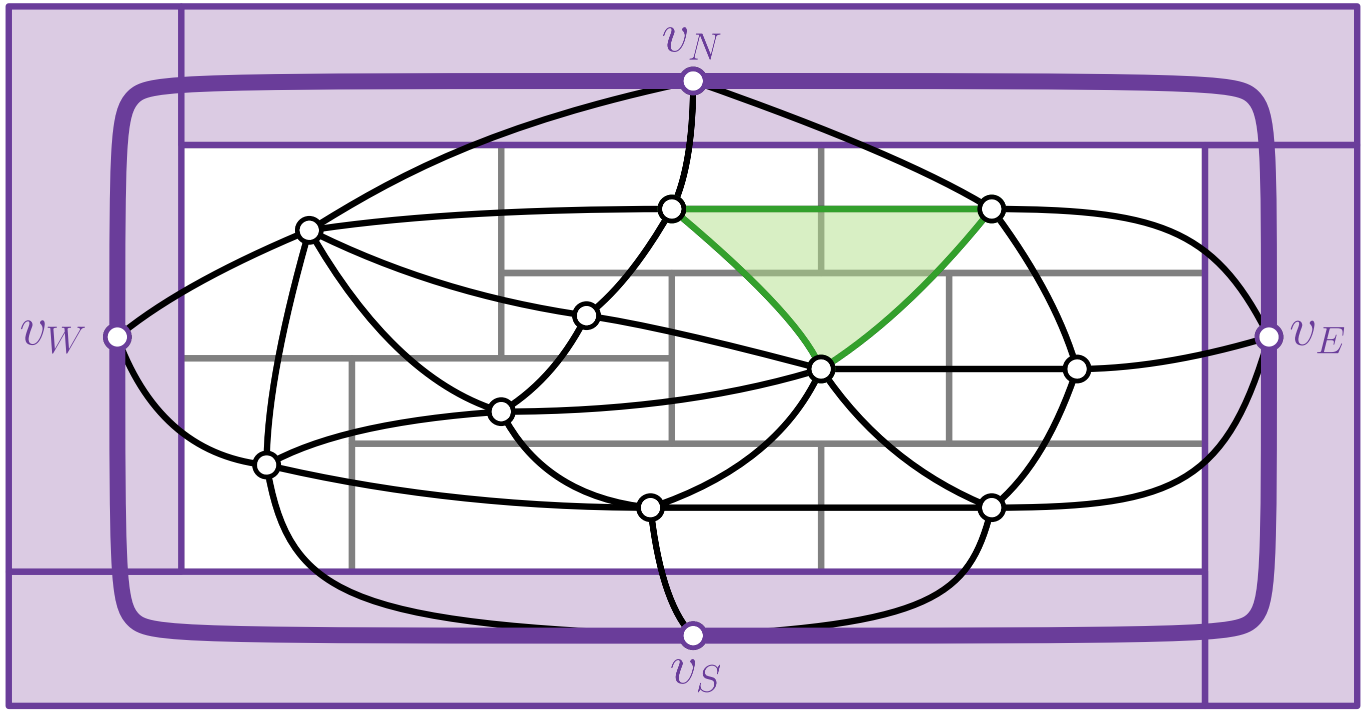

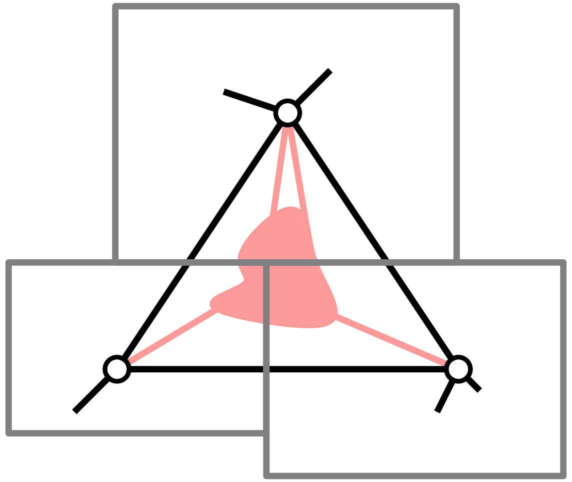
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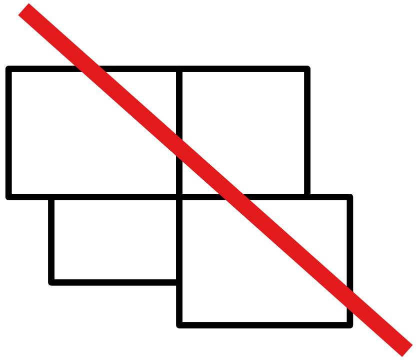

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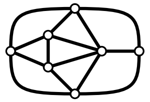
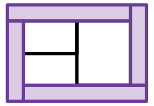


Properly Triangulated Planar Graph G



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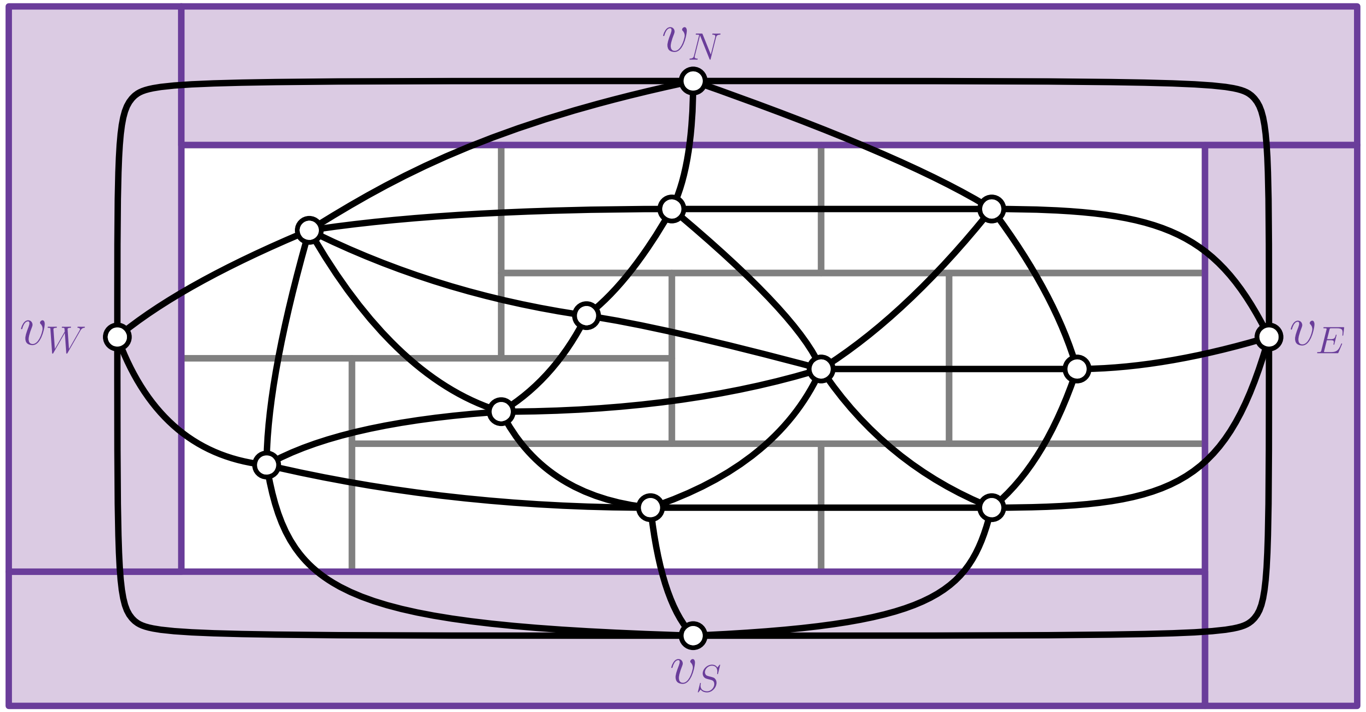

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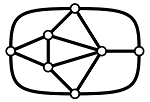
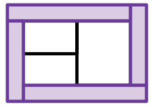
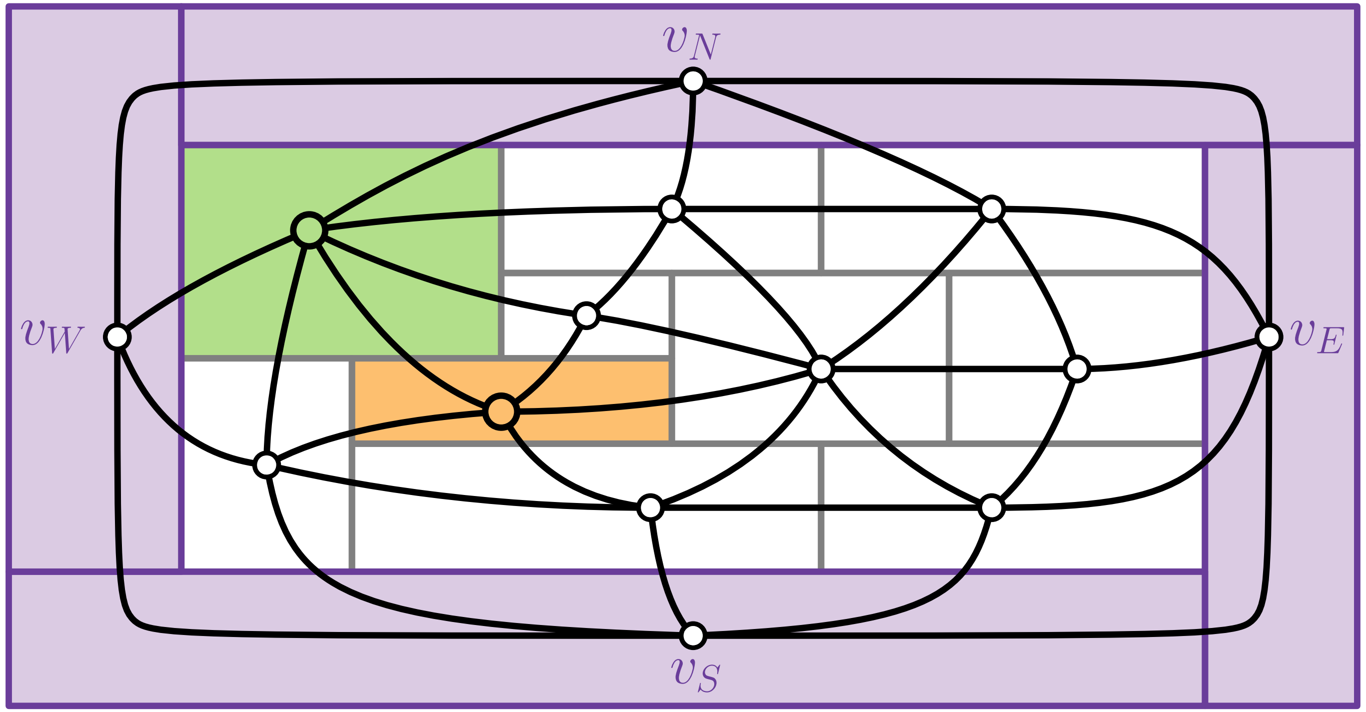


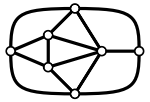
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$



Regular Edge Labeling



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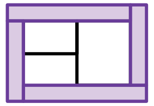

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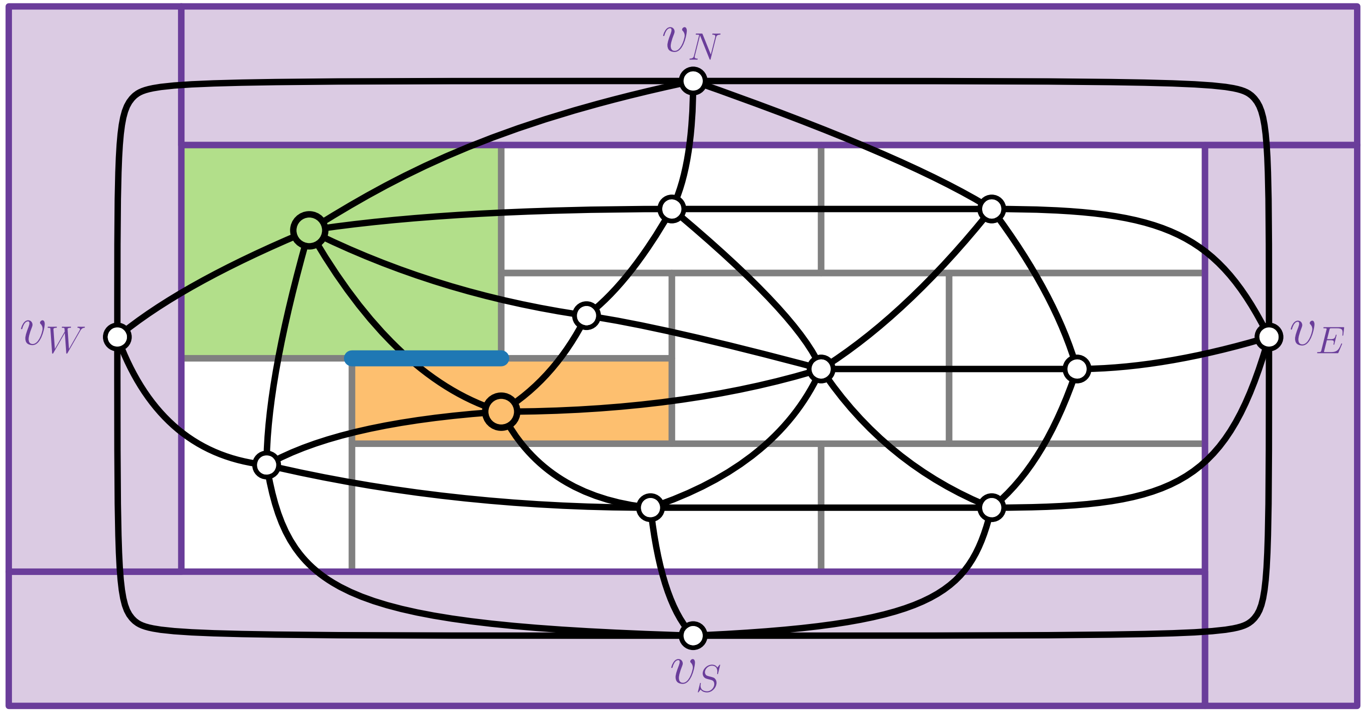

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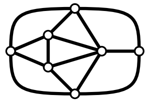

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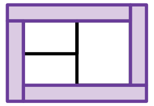
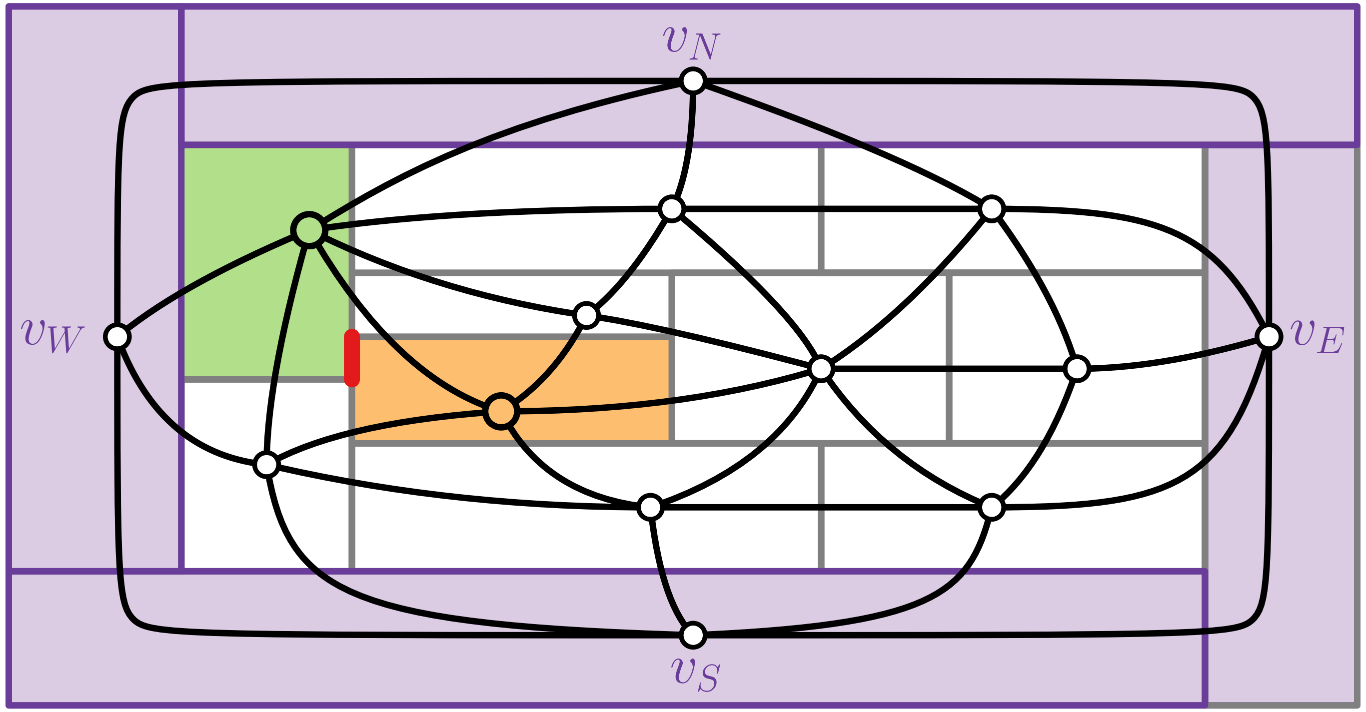


Rectangular Dual $\mathcal{R}$



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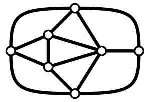

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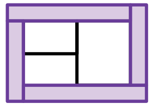
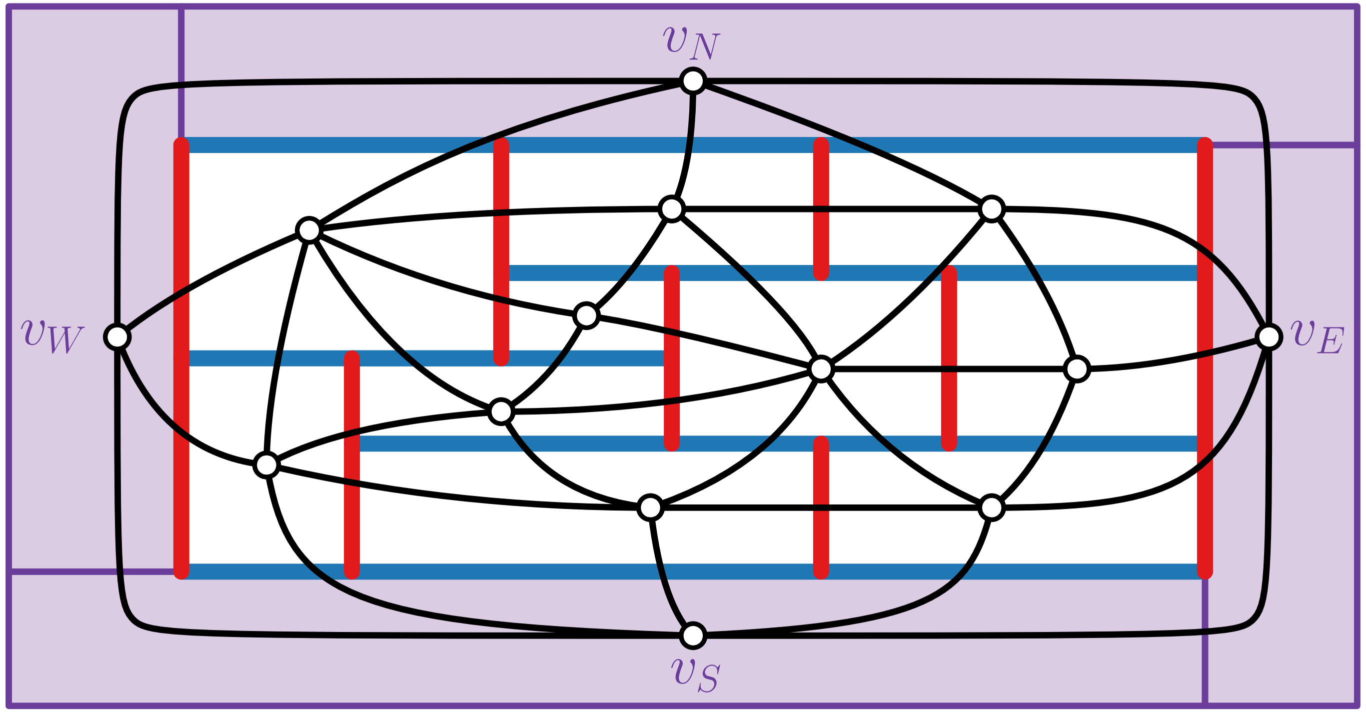


Rectangular Dual $\mathcal{R}$



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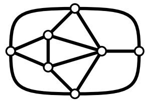

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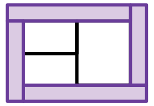
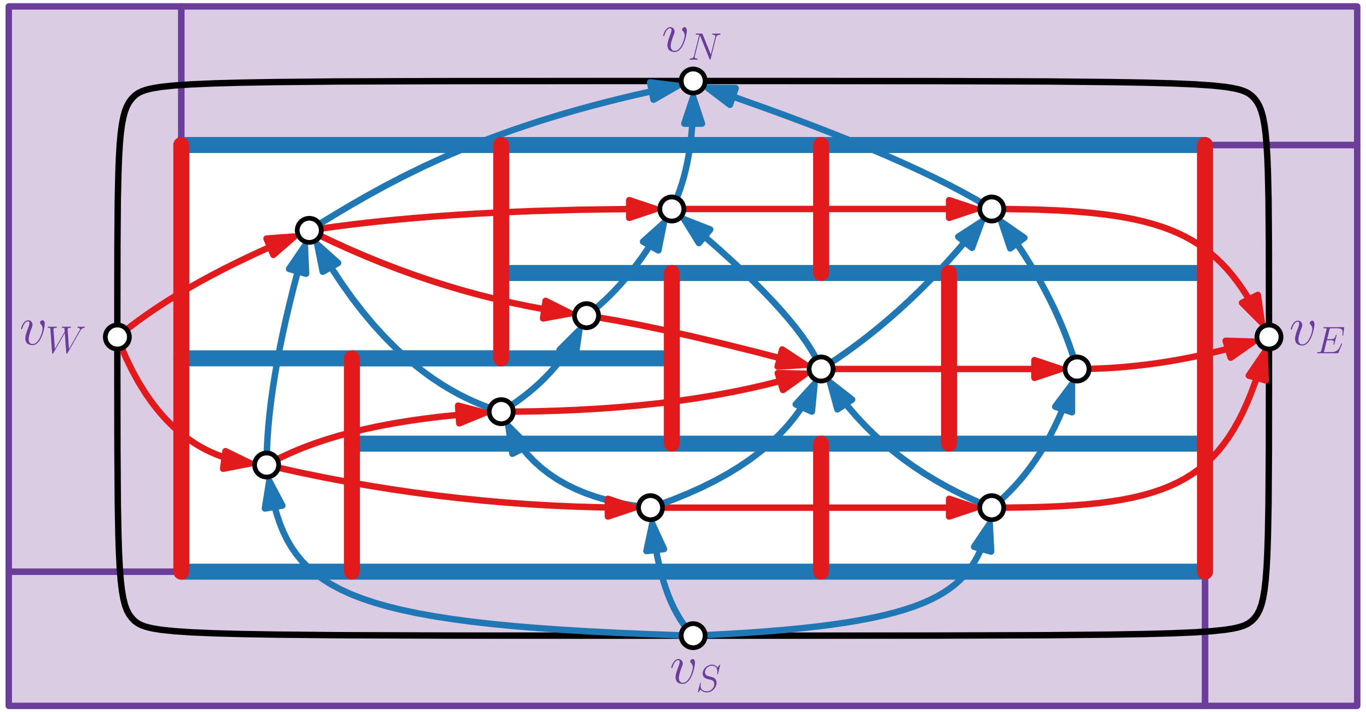
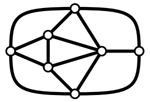


Rectangular Dual $\mathcal{R}$

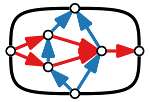


Regular Edge Labeling



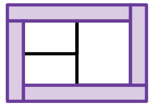
PTP

Properly Triangulated Planar Graph G



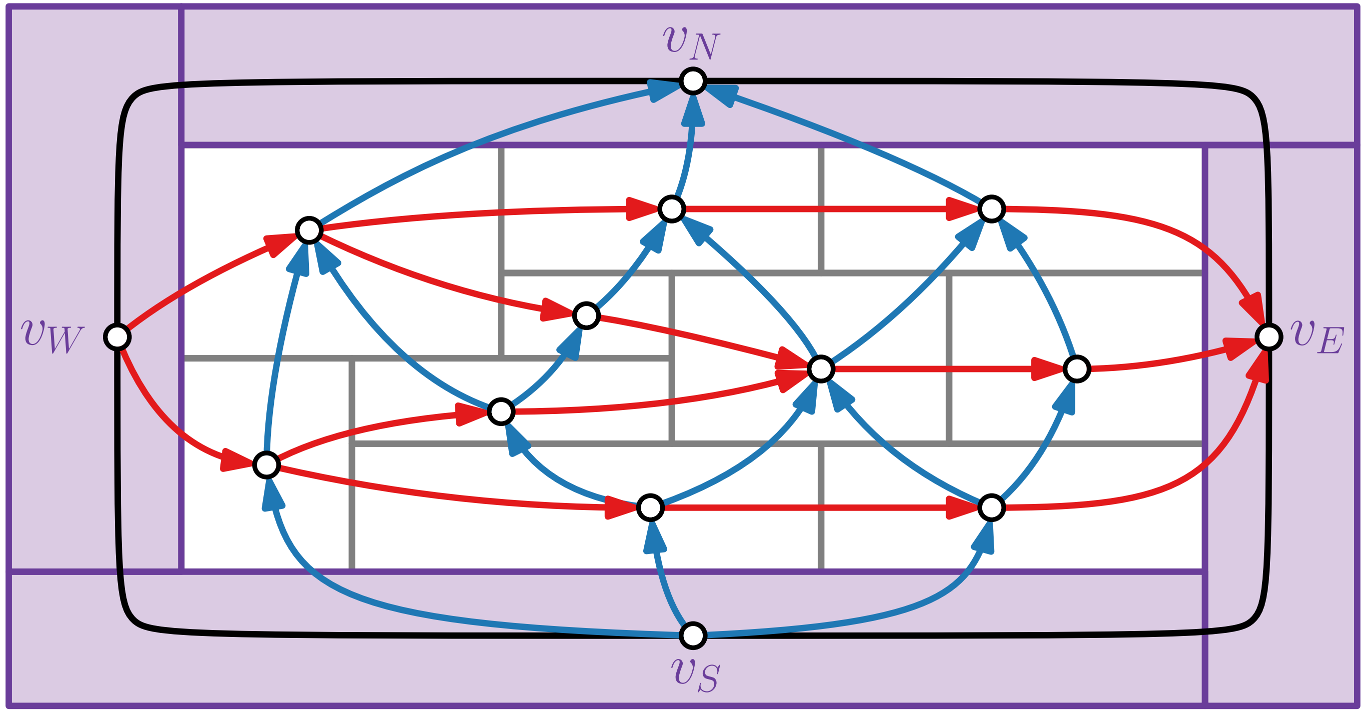
REL

Regular Edge Labeling

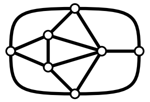


RD

Rectangular Dual $\mathcal{R}$



Regular Edge Labeling



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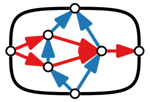

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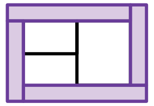
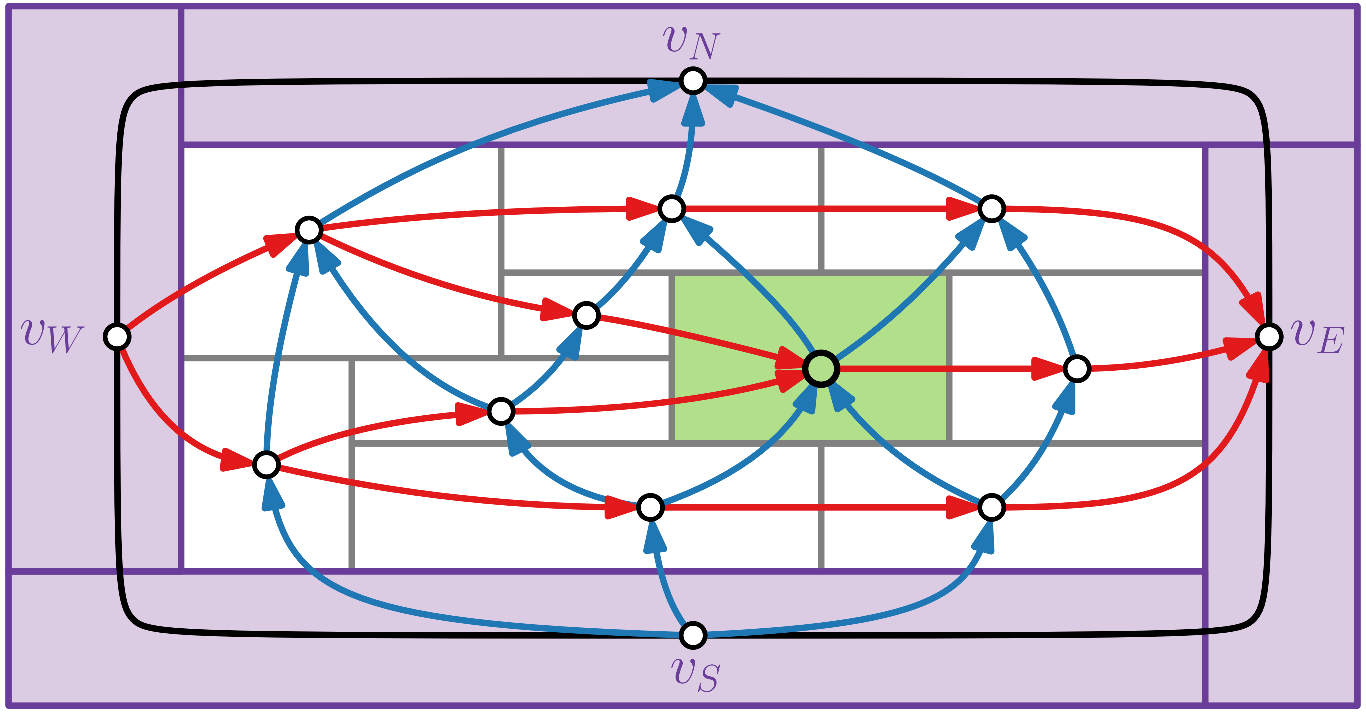
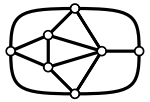


Rectangular Dual $\mathcal{R}$



Regular Edge Labeling


PTP

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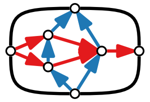
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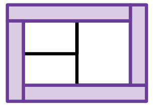
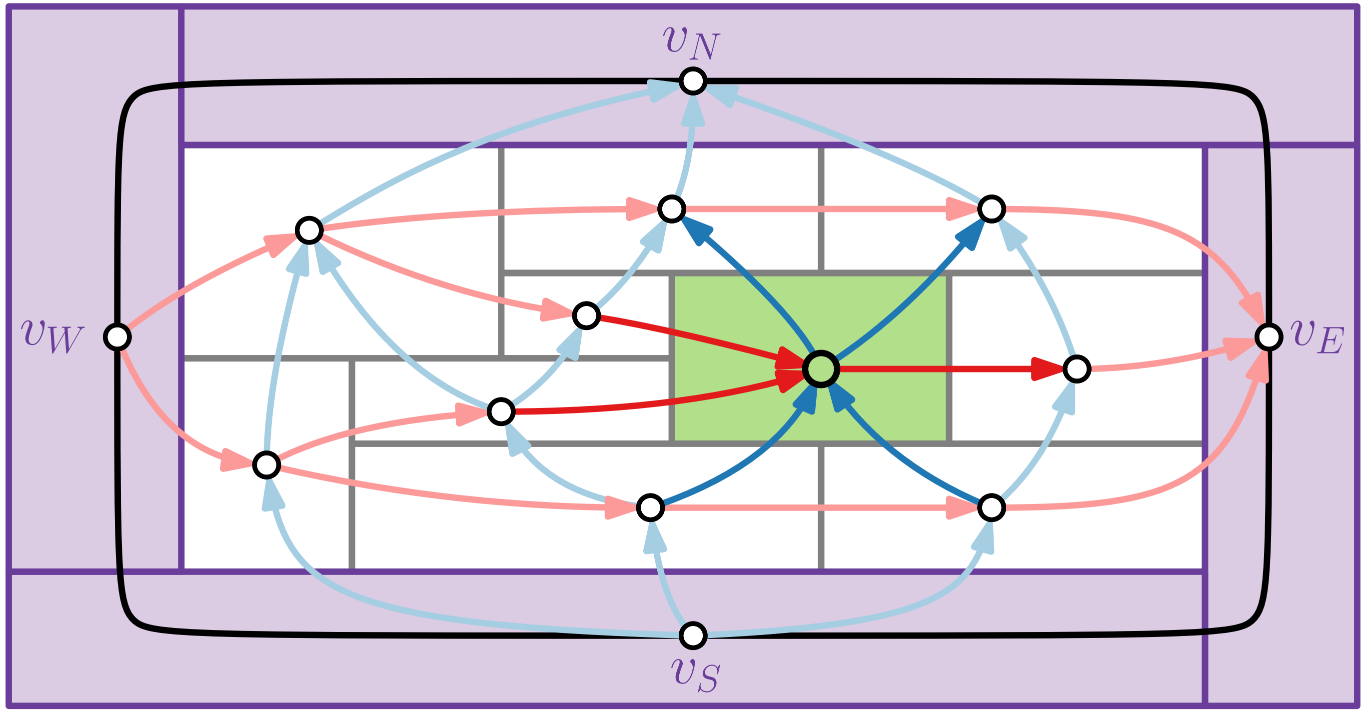

RD

Rectangular Dual $\mathcal{R}$



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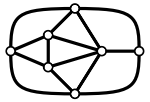
PTP

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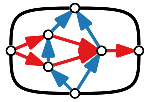
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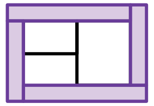
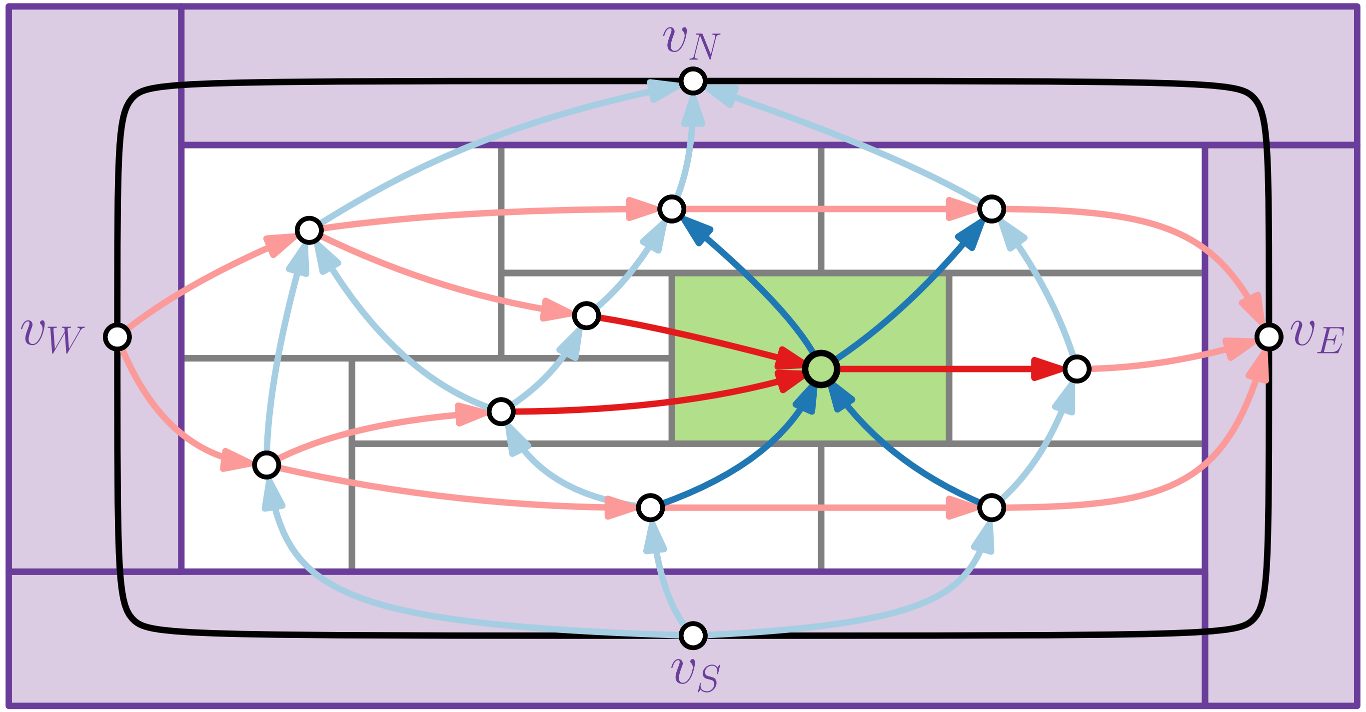
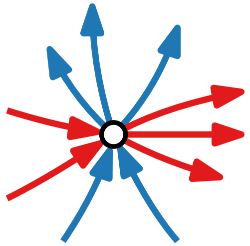

RD

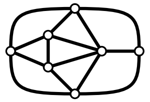
Rectangular Dual $\mathcal{R}$



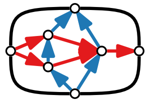


for every inner vertex

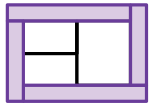
Regular Edge Labeling



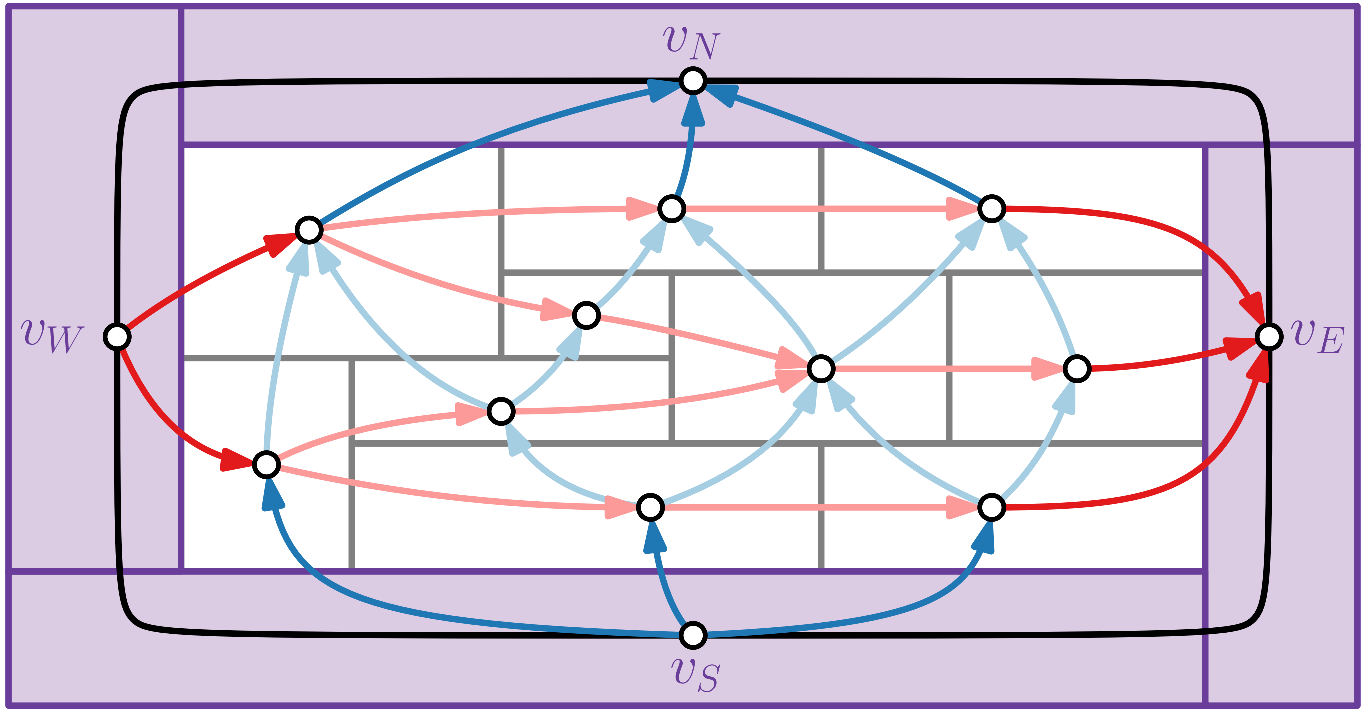
Properly Triangulated Planar Graph G

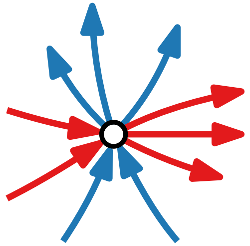


Regular Edge Labeling



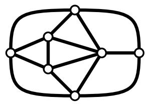
Rectangular Dual $\mathcal{R}$



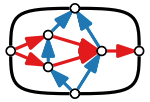


for every inner vertex

Regular Edge Labeling



Properly Triangulated Planar Graph G



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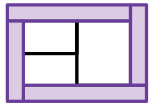
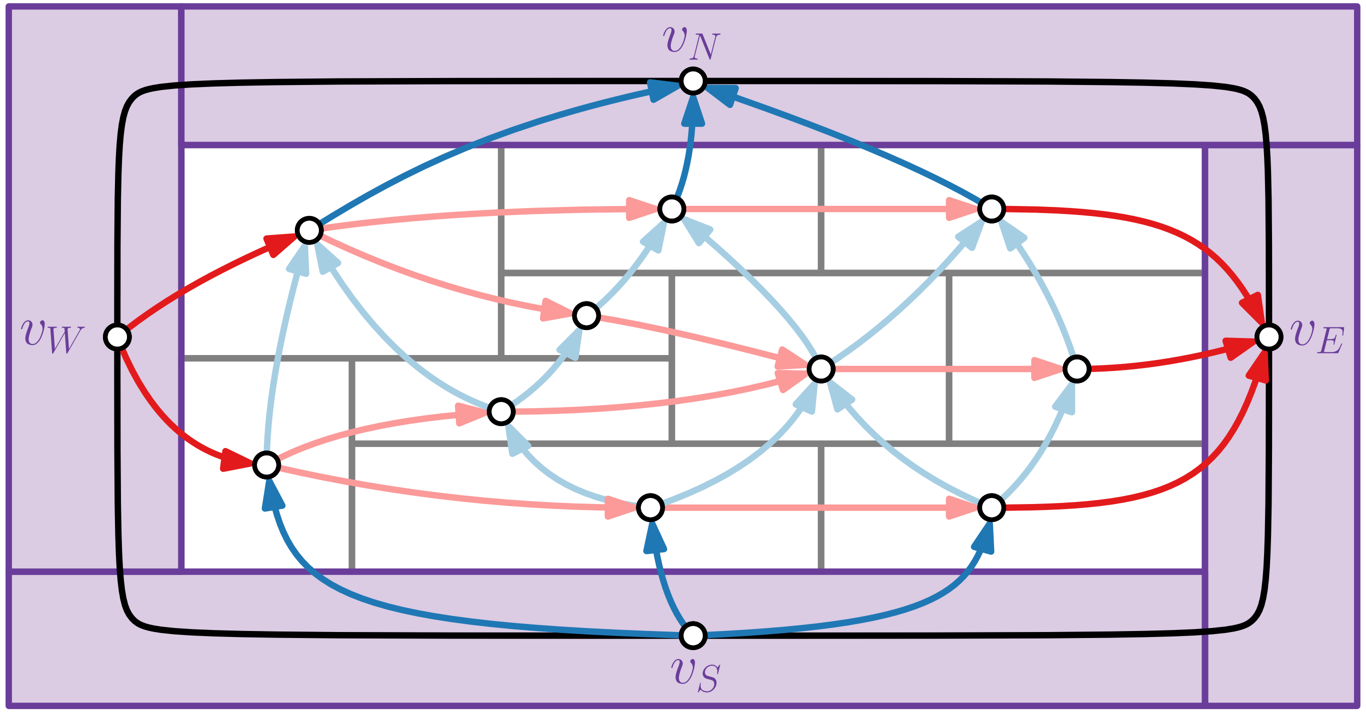
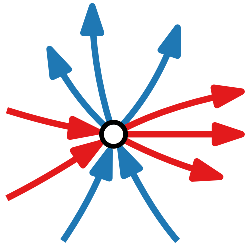


Rectangular Dual $\mathcal{R}$





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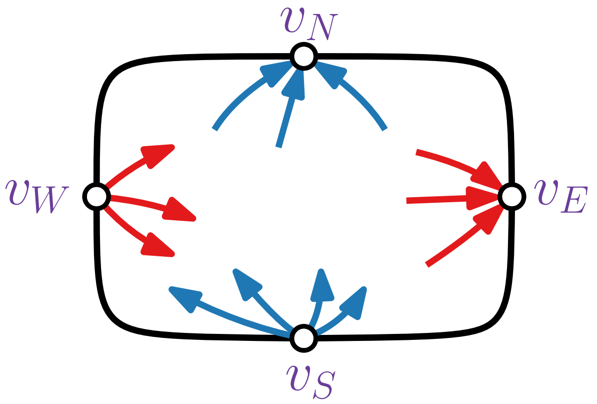
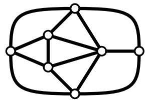
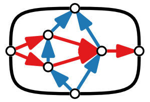


for four outer vertices

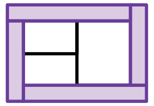
Regular Edge Labeling



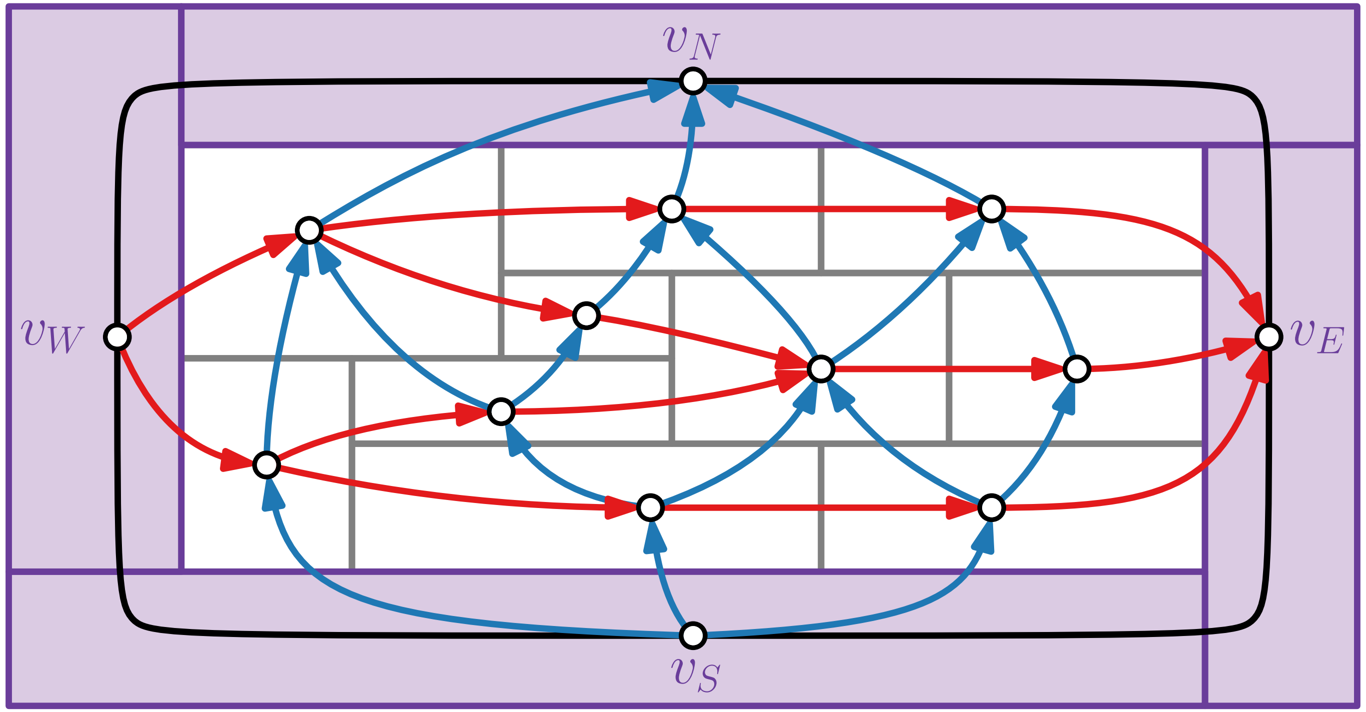
Properly Triangulated Planar Graph G

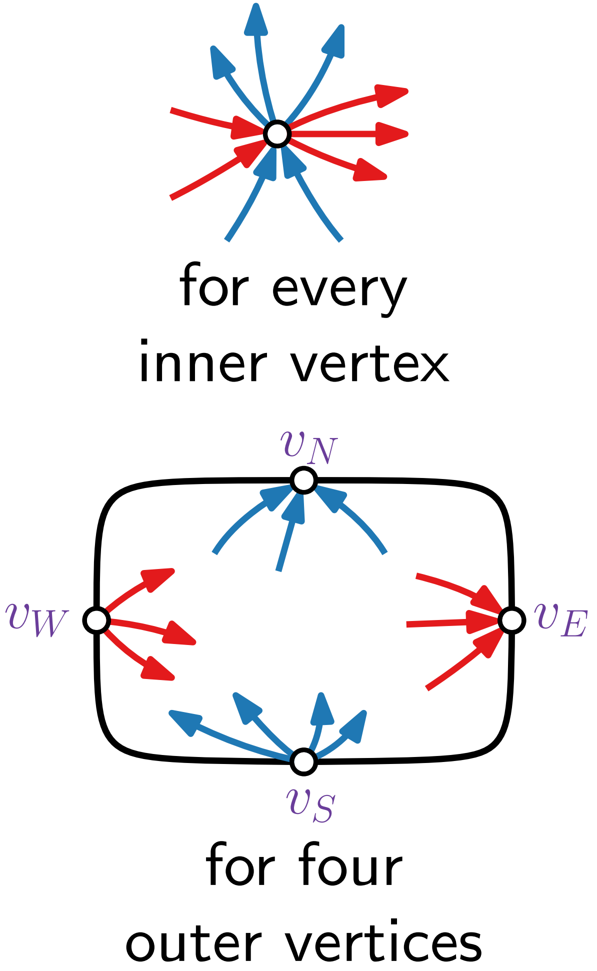


Regular Edge Labeling

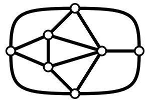


Rectangular Dual $\mathcal{R}$





[Kant, He '94]: In linear time



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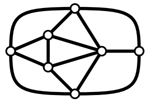
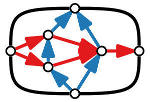
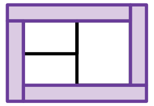
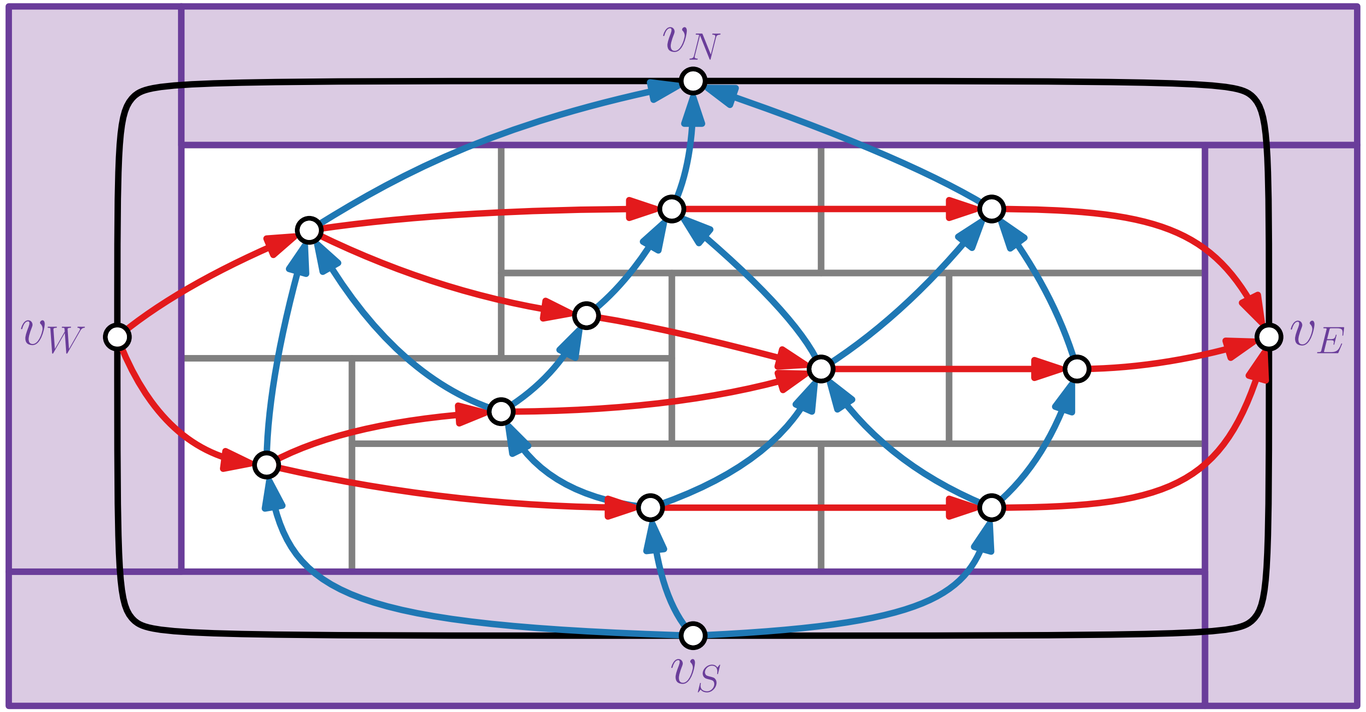
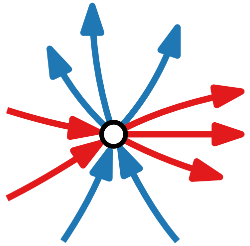

PTP
Properly Triangulated Planar Graph G

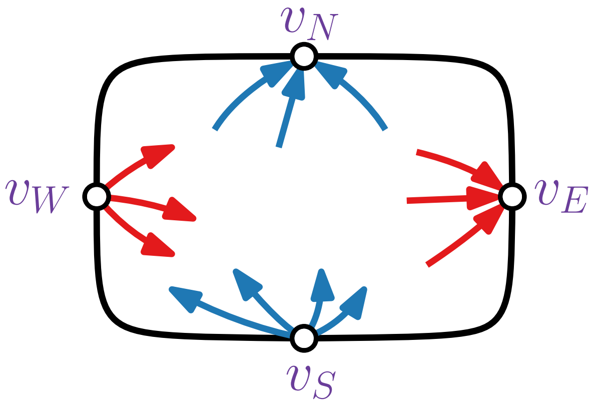

REL
Regular Edge Labeling


RD
Rectangular Dual $\mathcal{R}$





for every inner vertex



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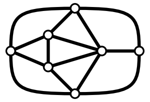
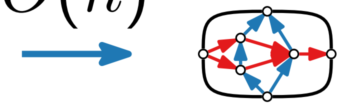
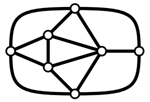

PTP

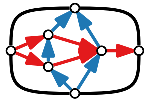
$O(n)$


REL

Regular Edge Labeling



Properly Triangulated Planar Graph G



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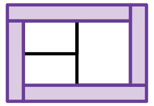
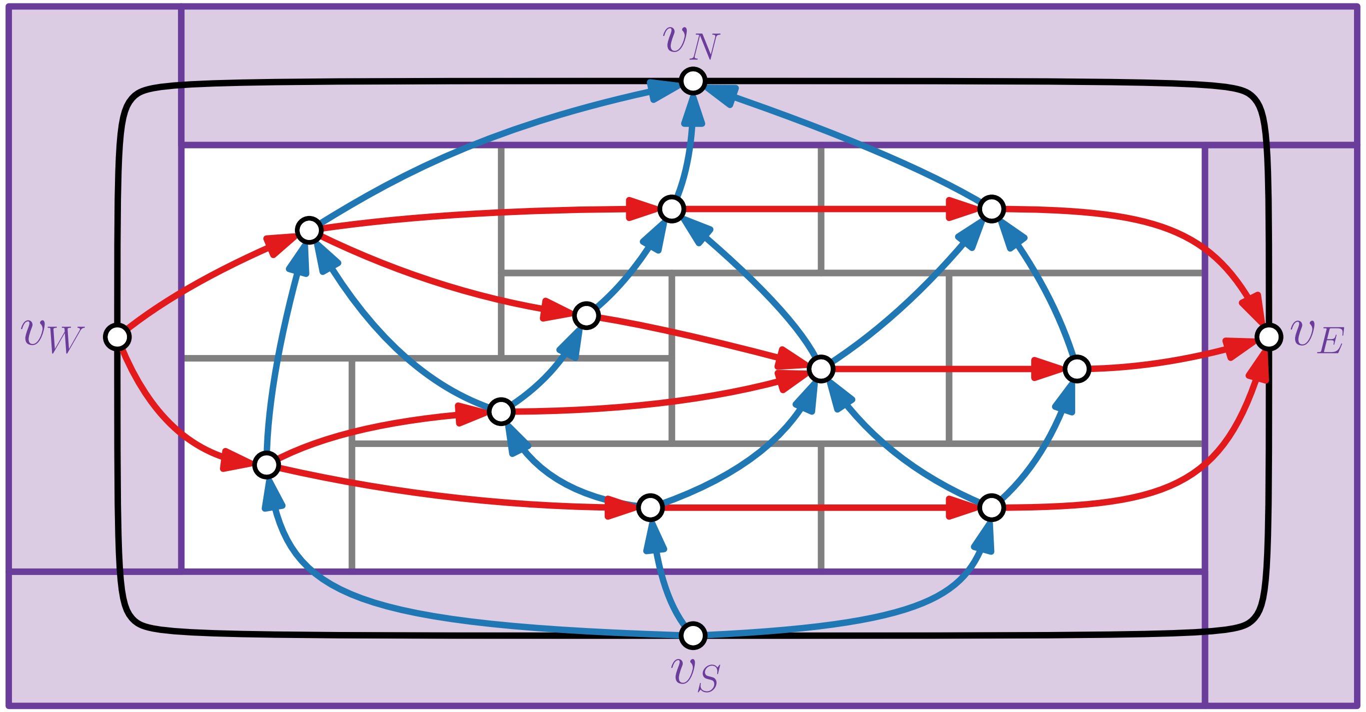
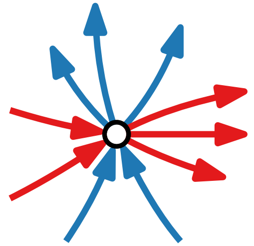


Rectangular Dual $\mathcal{R}$





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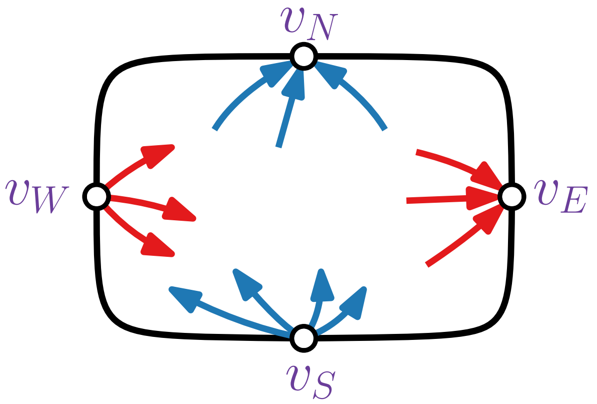

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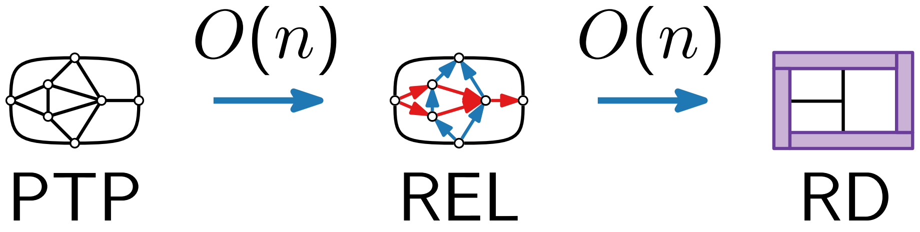

Refined Canonical Order

Theorem.
Let G be a PTP graph.

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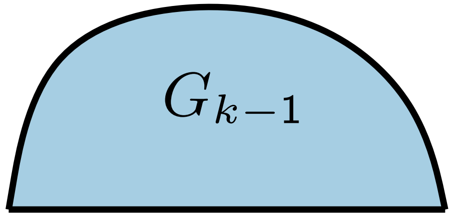

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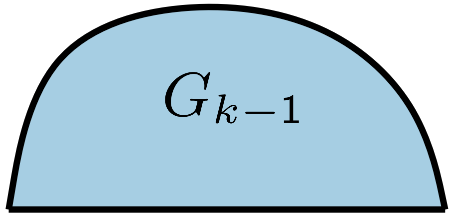

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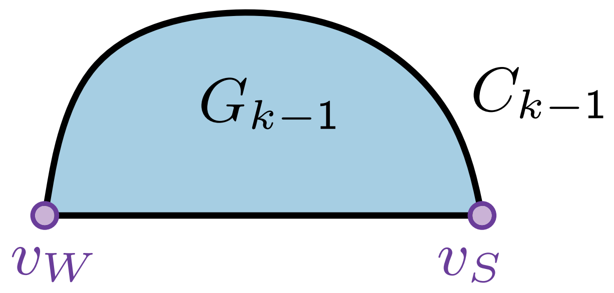

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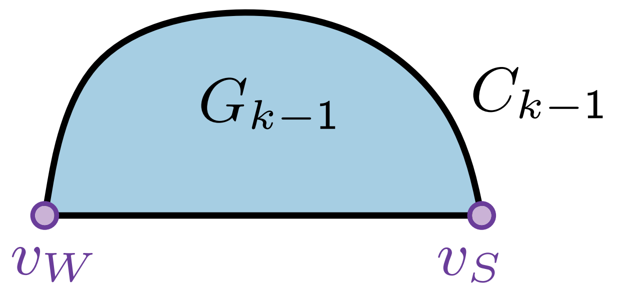

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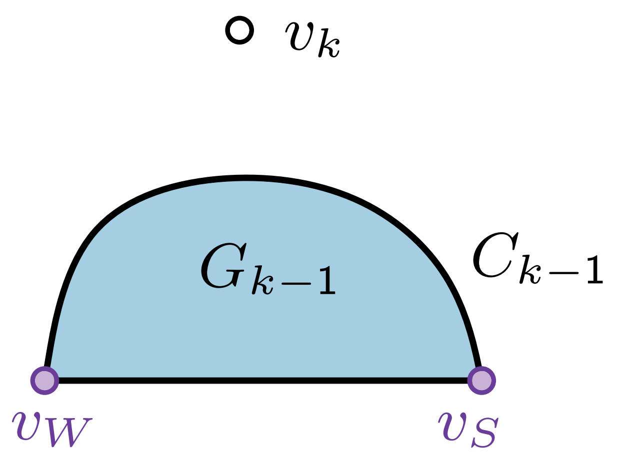

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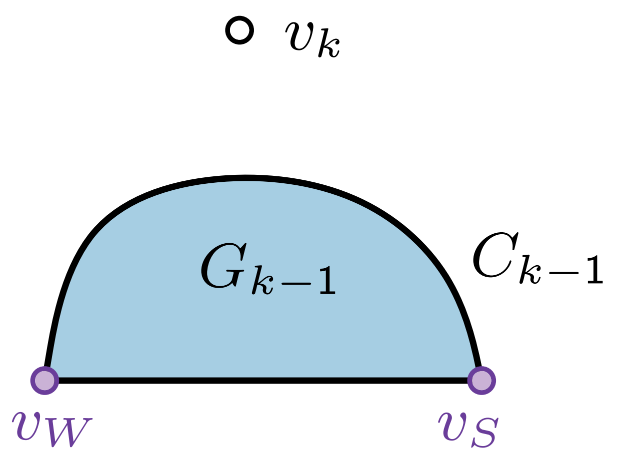

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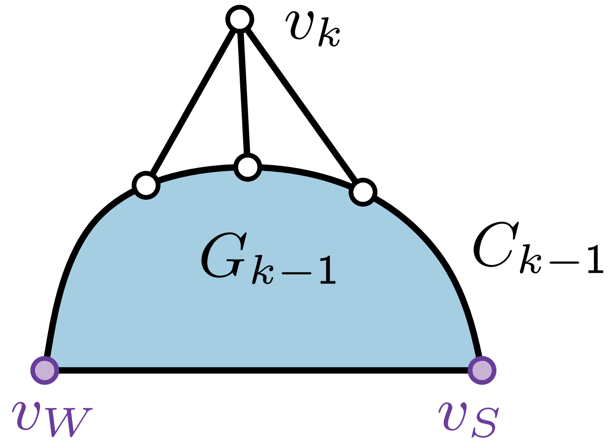

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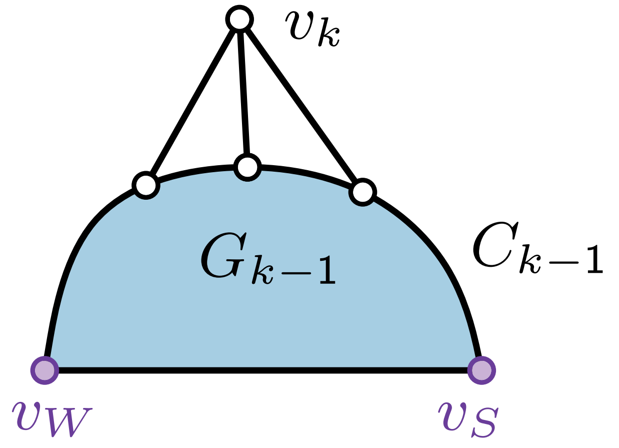

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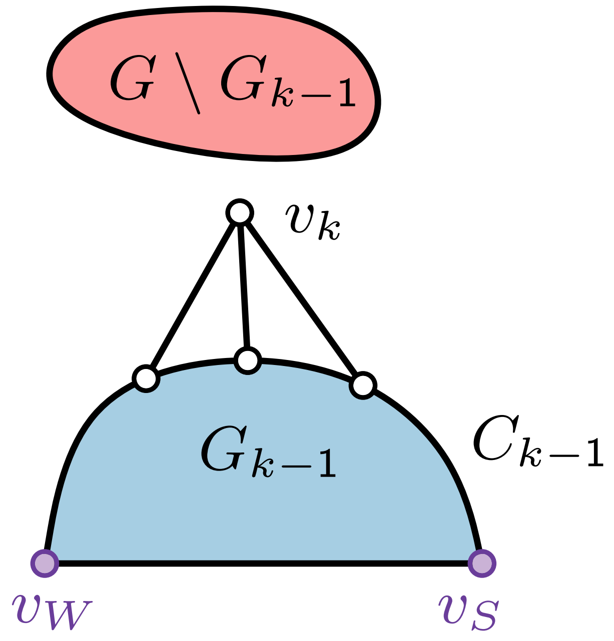

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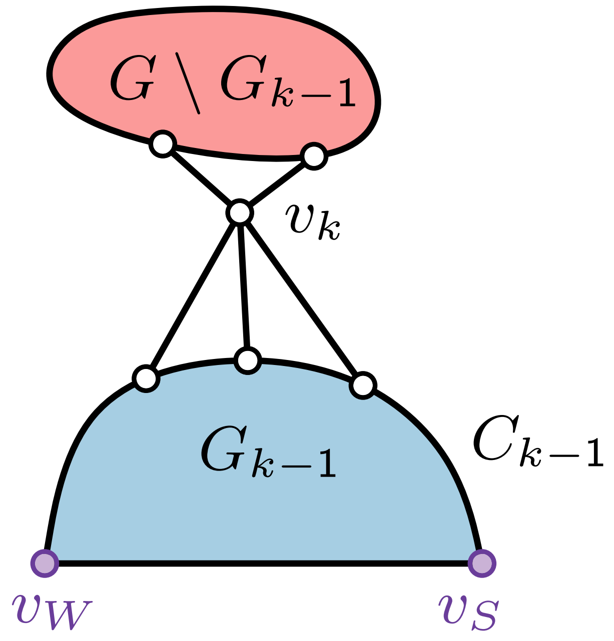

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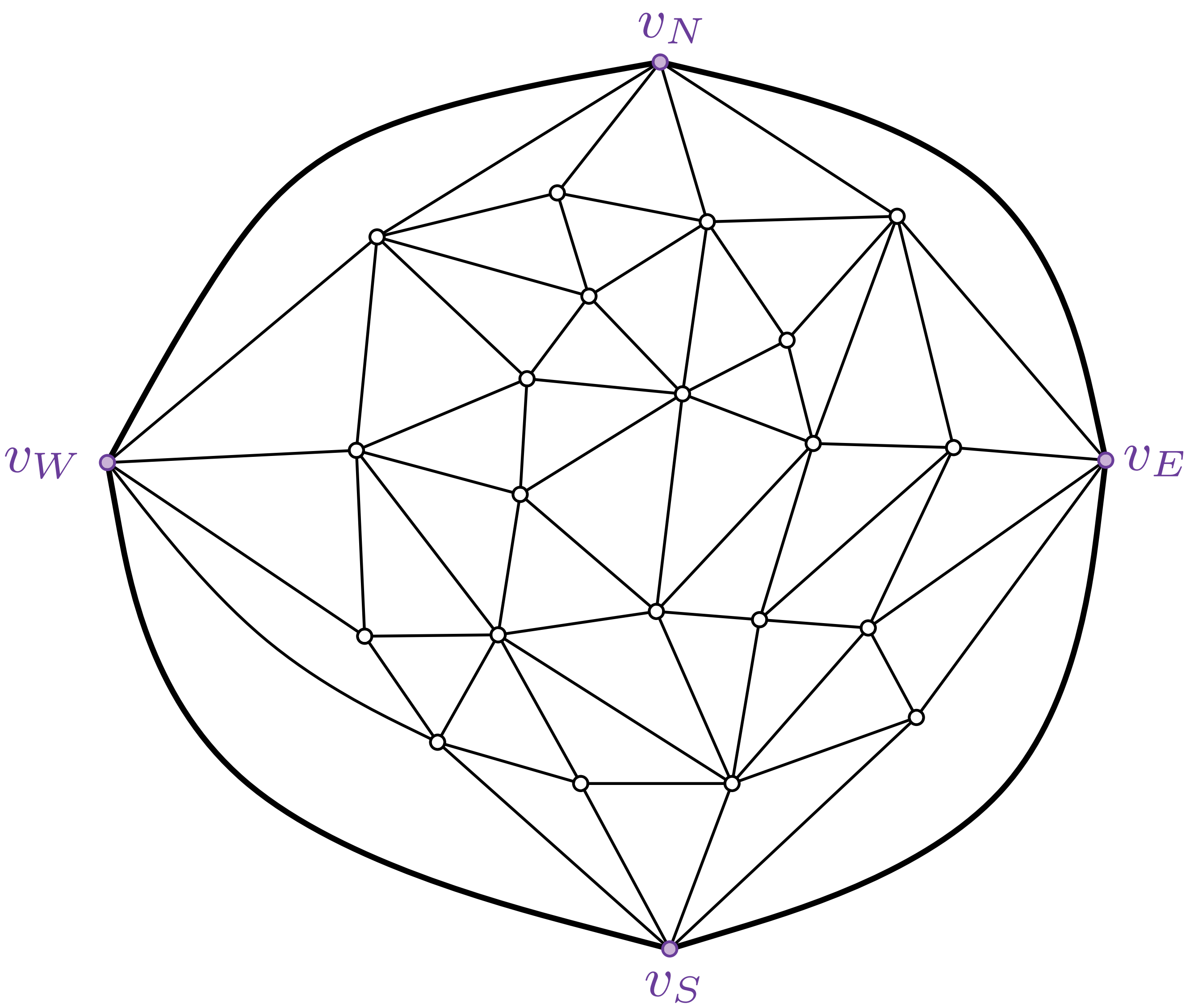

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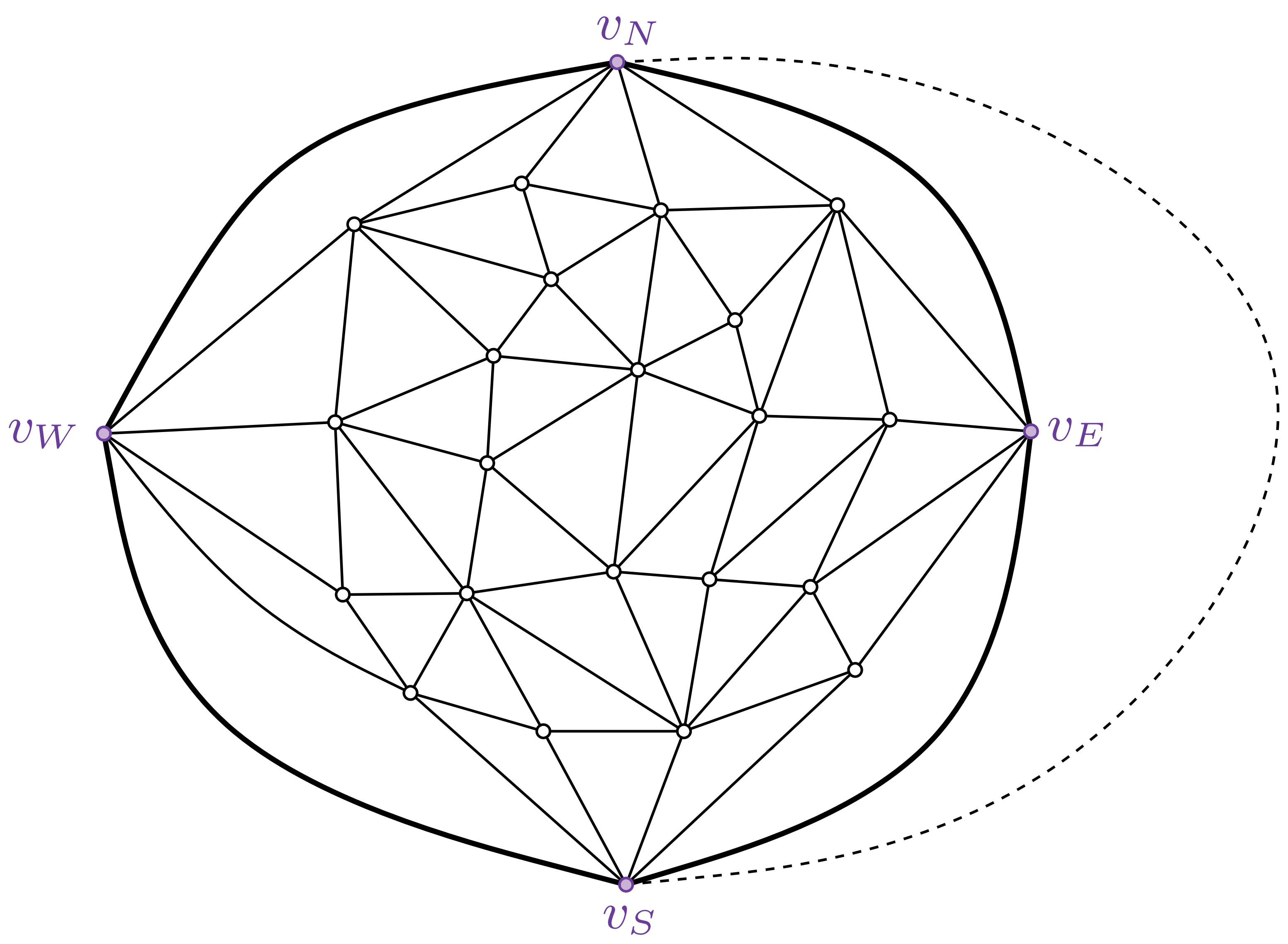

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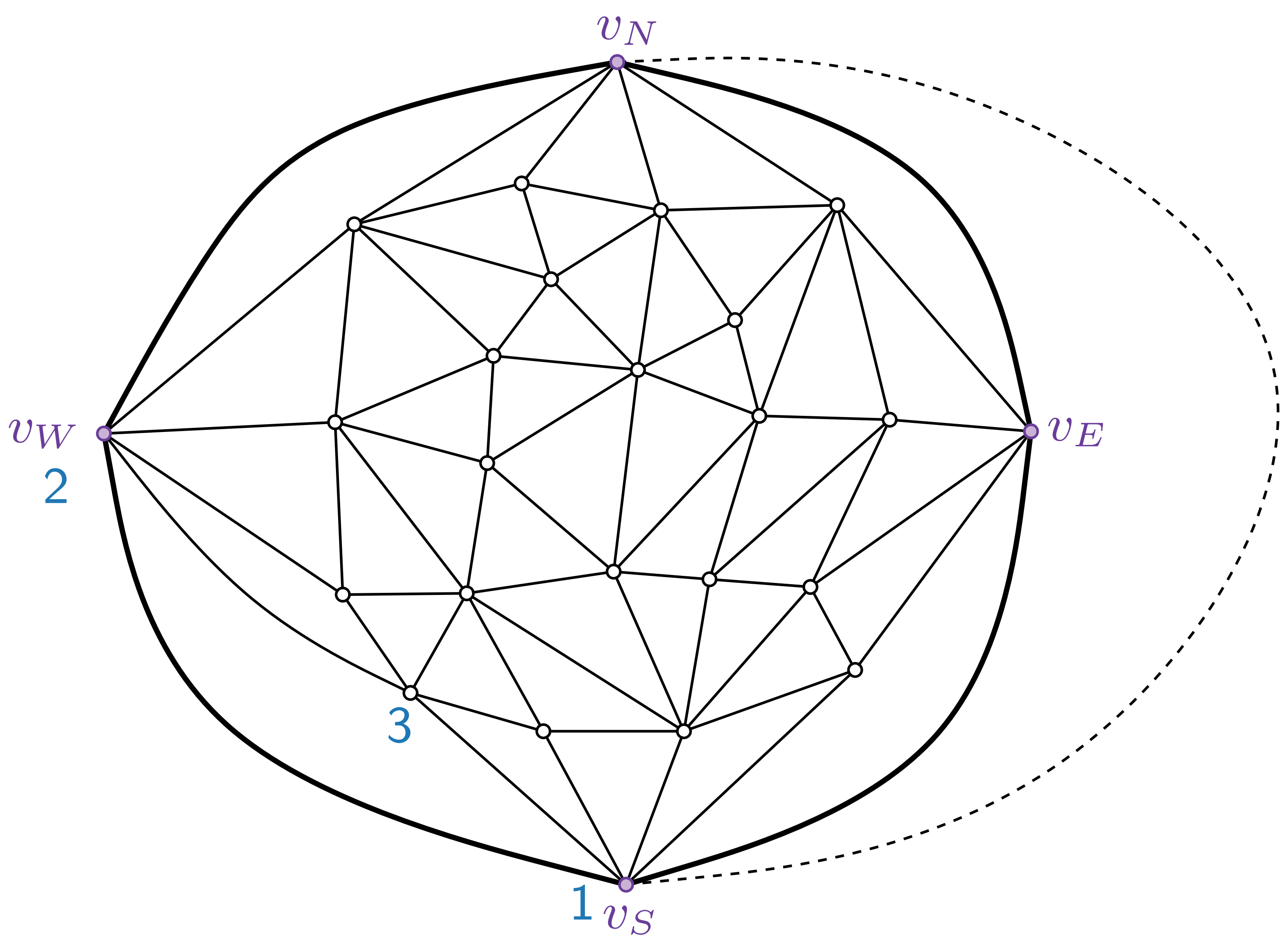

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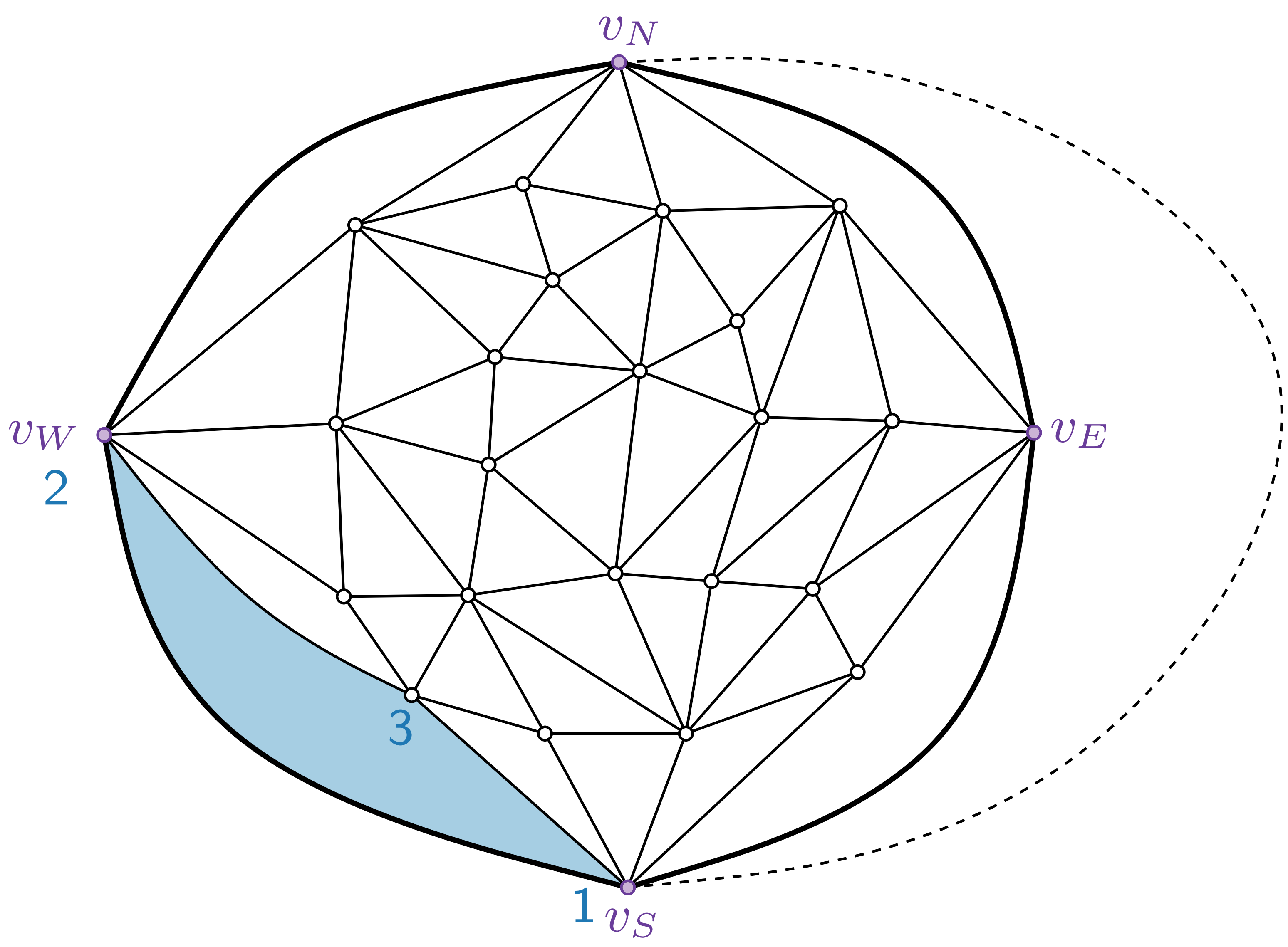

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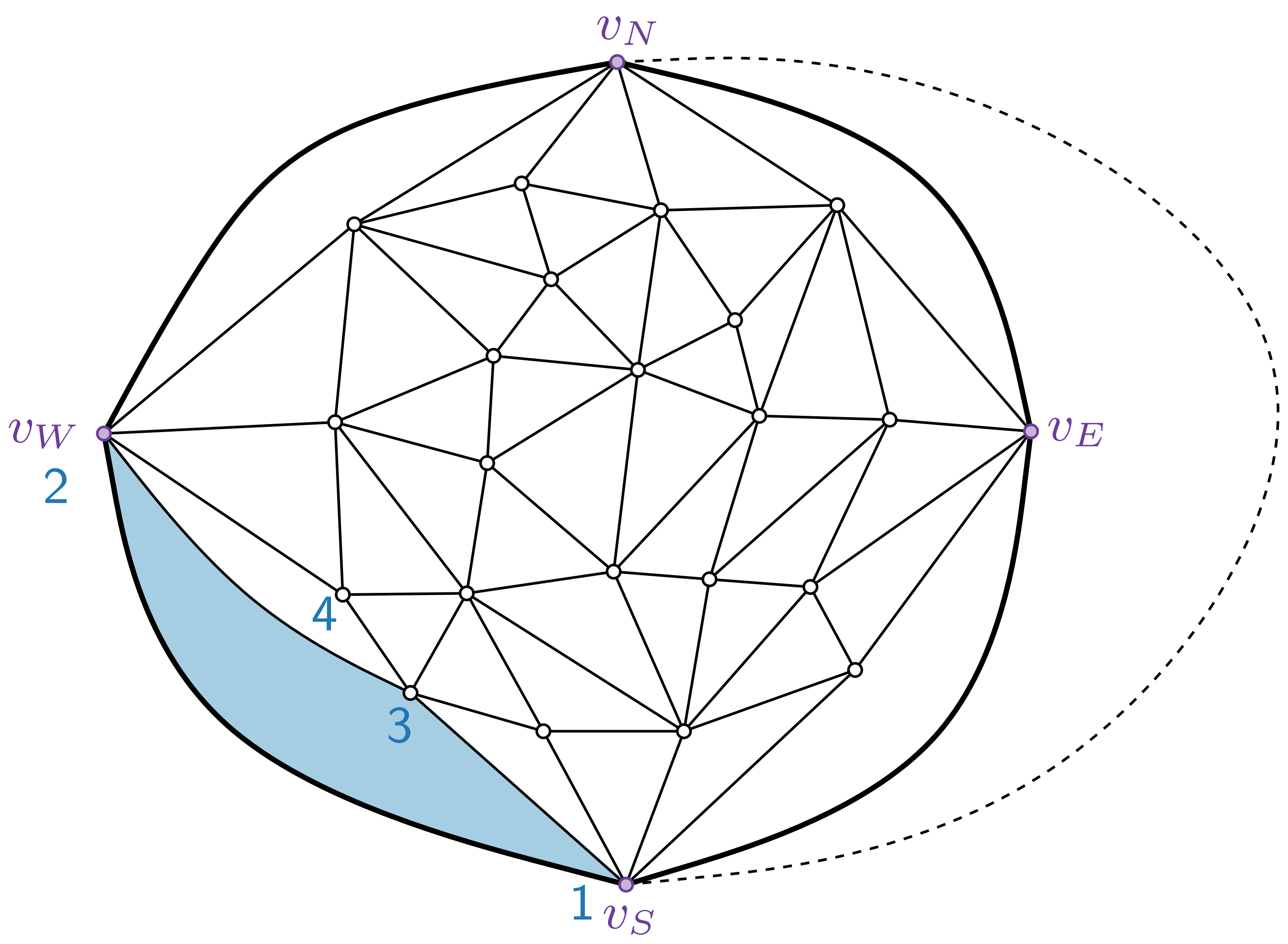

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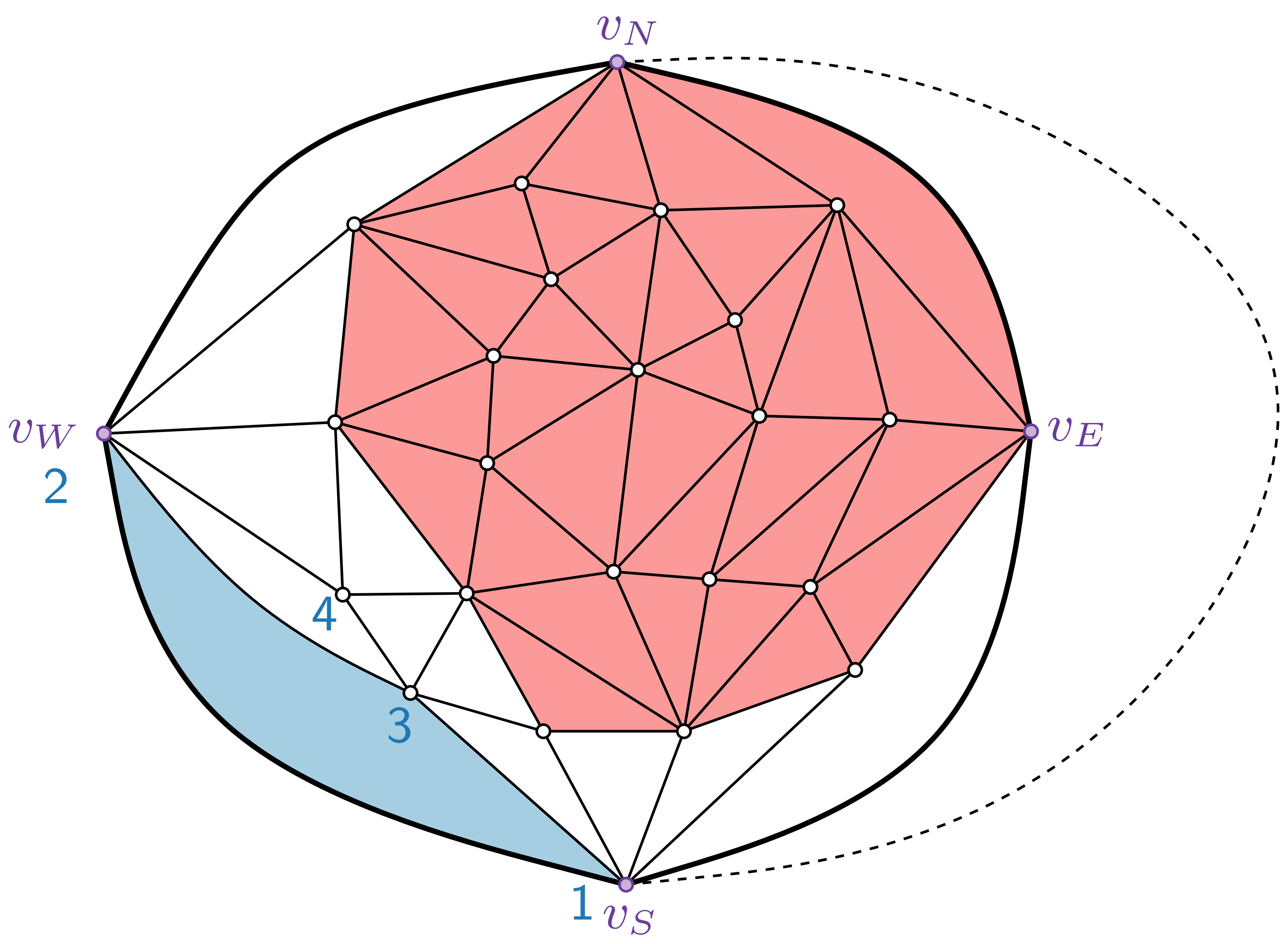

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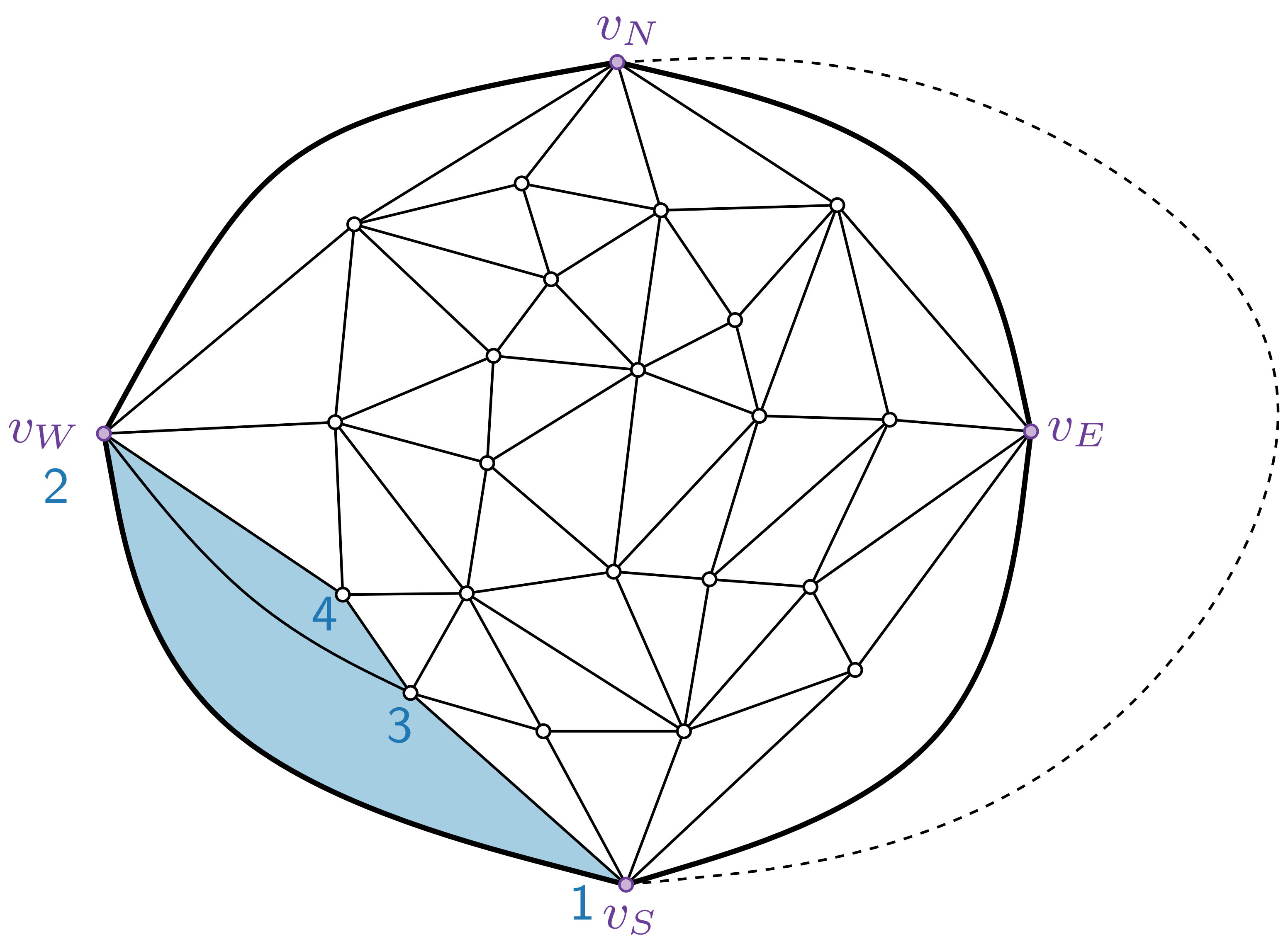

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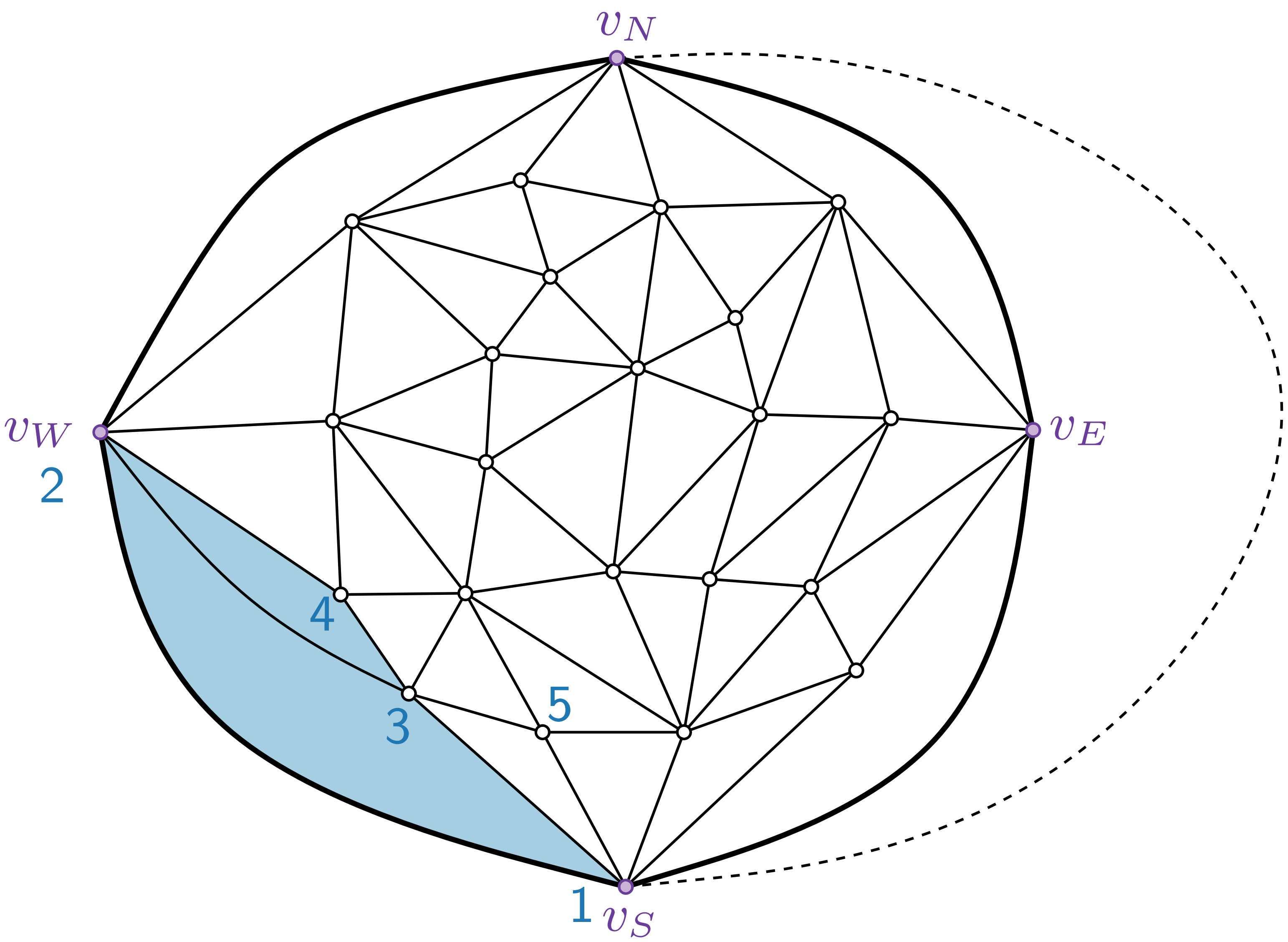

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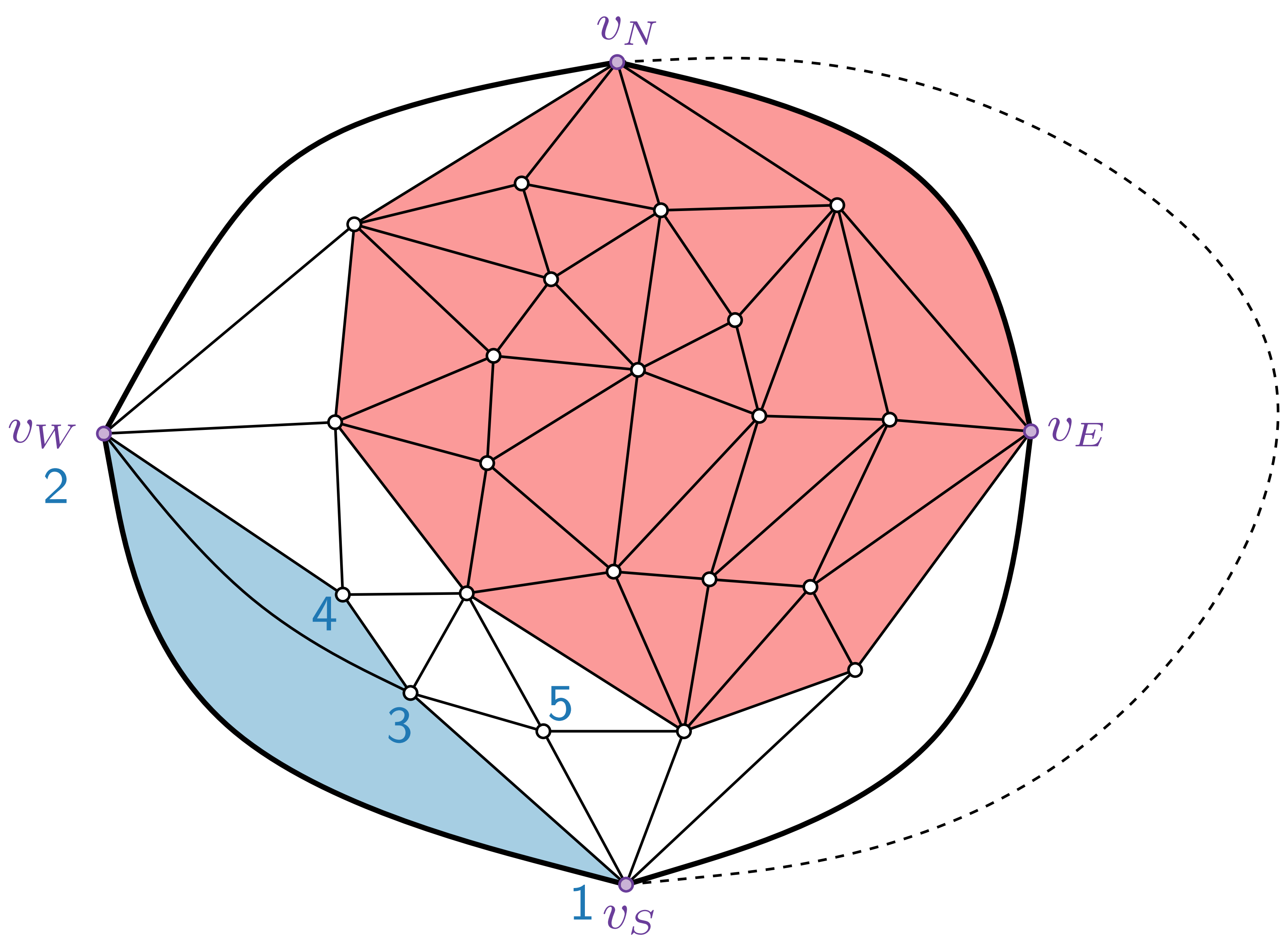

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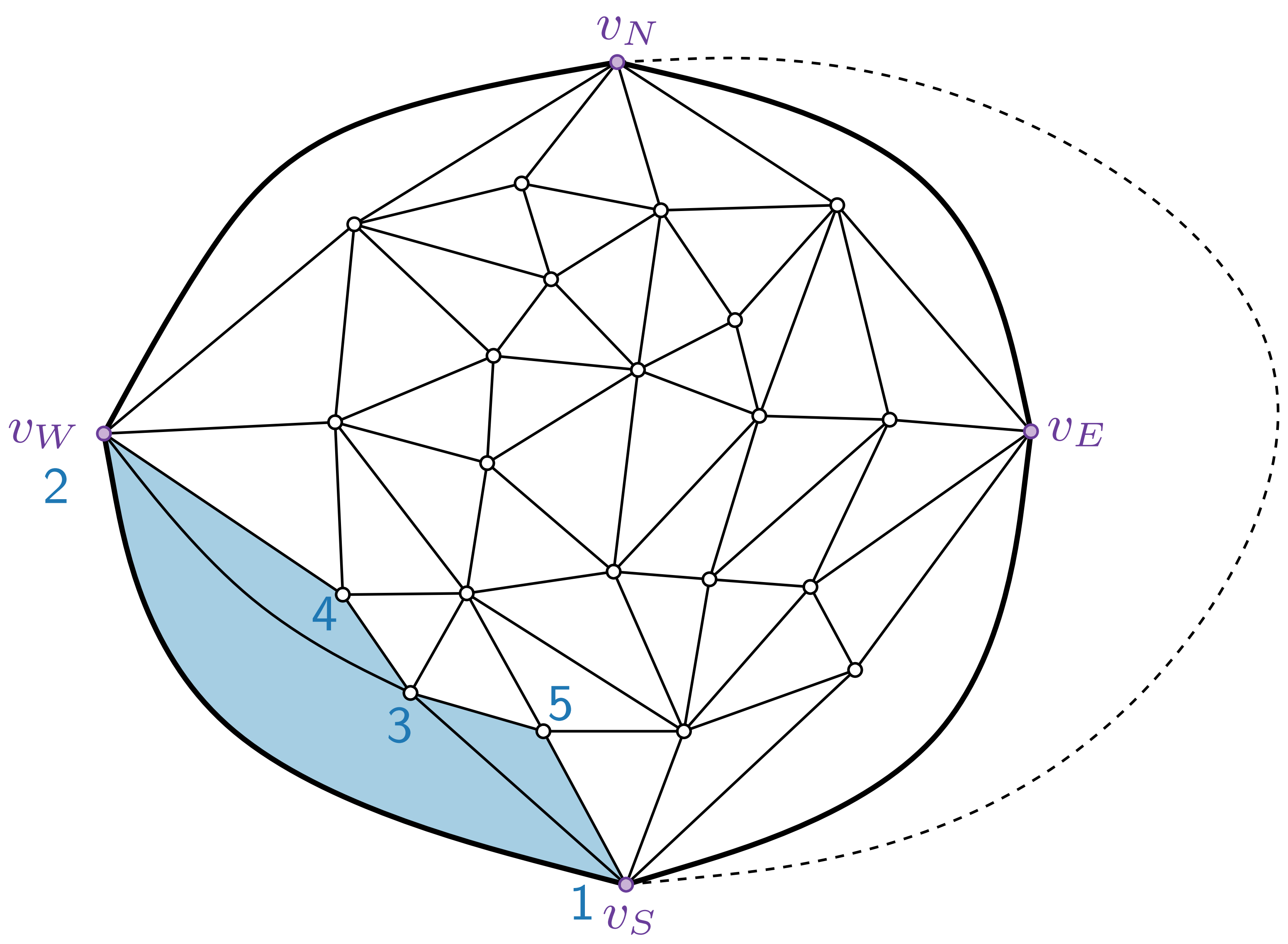

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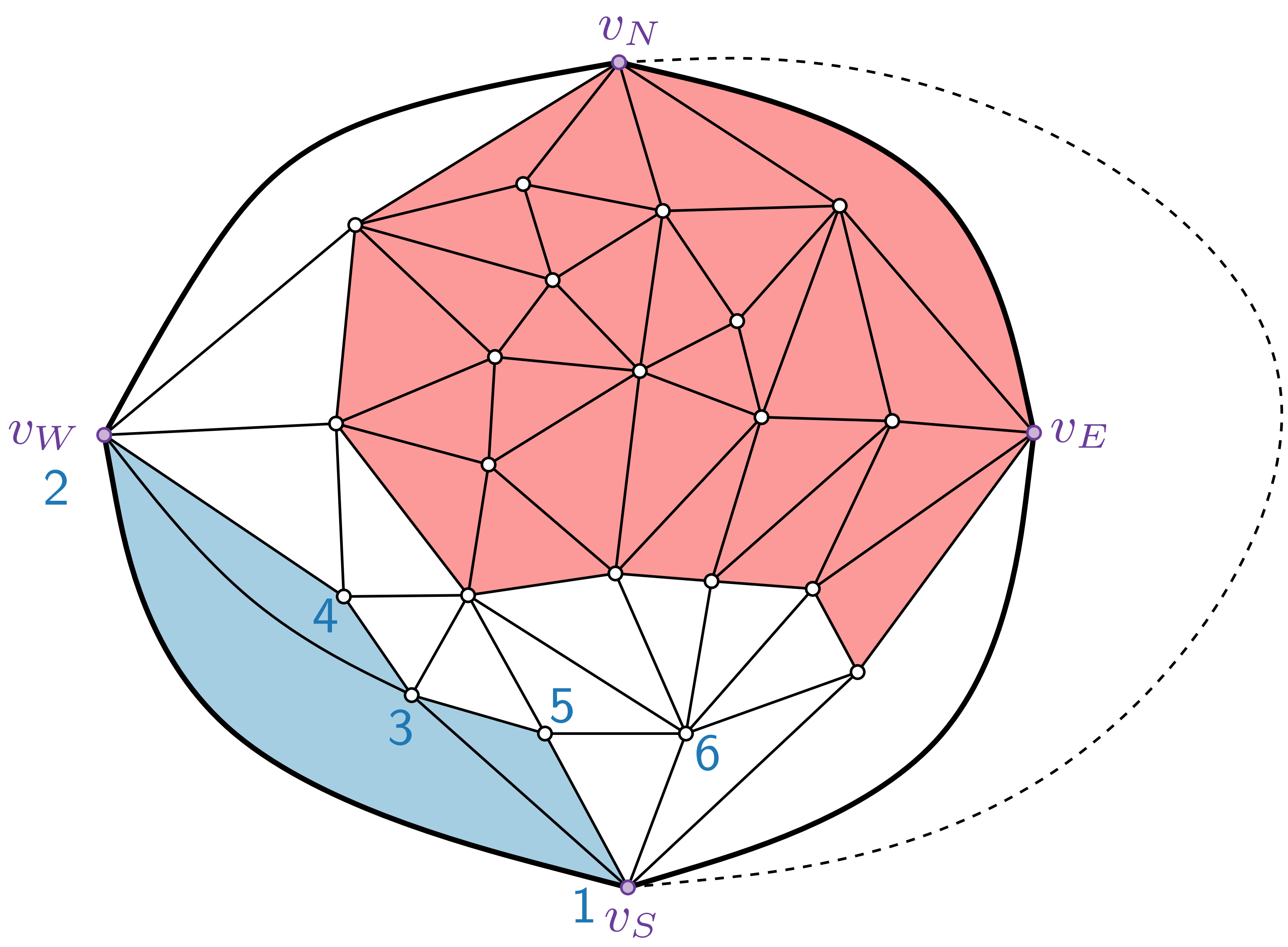

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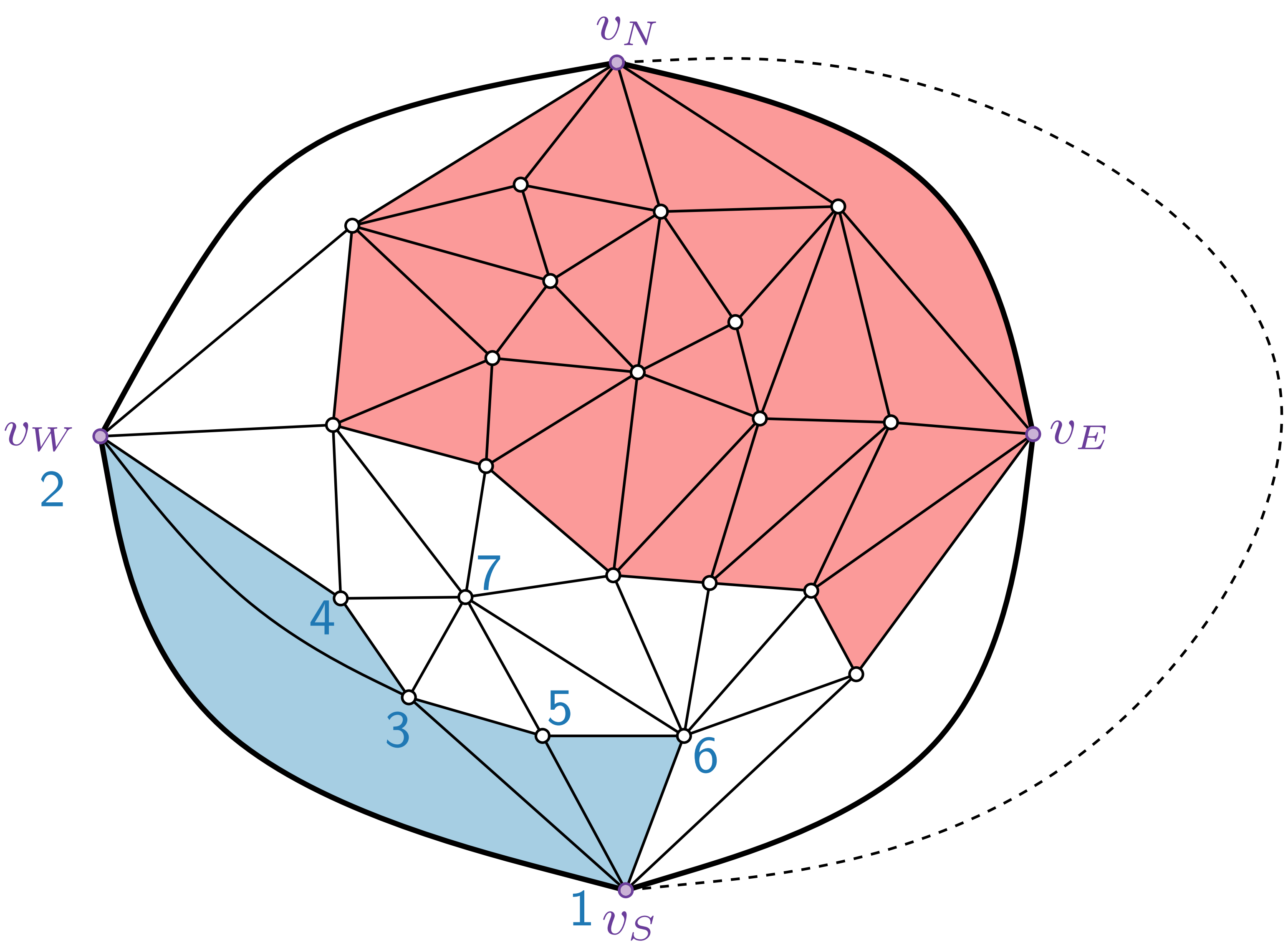

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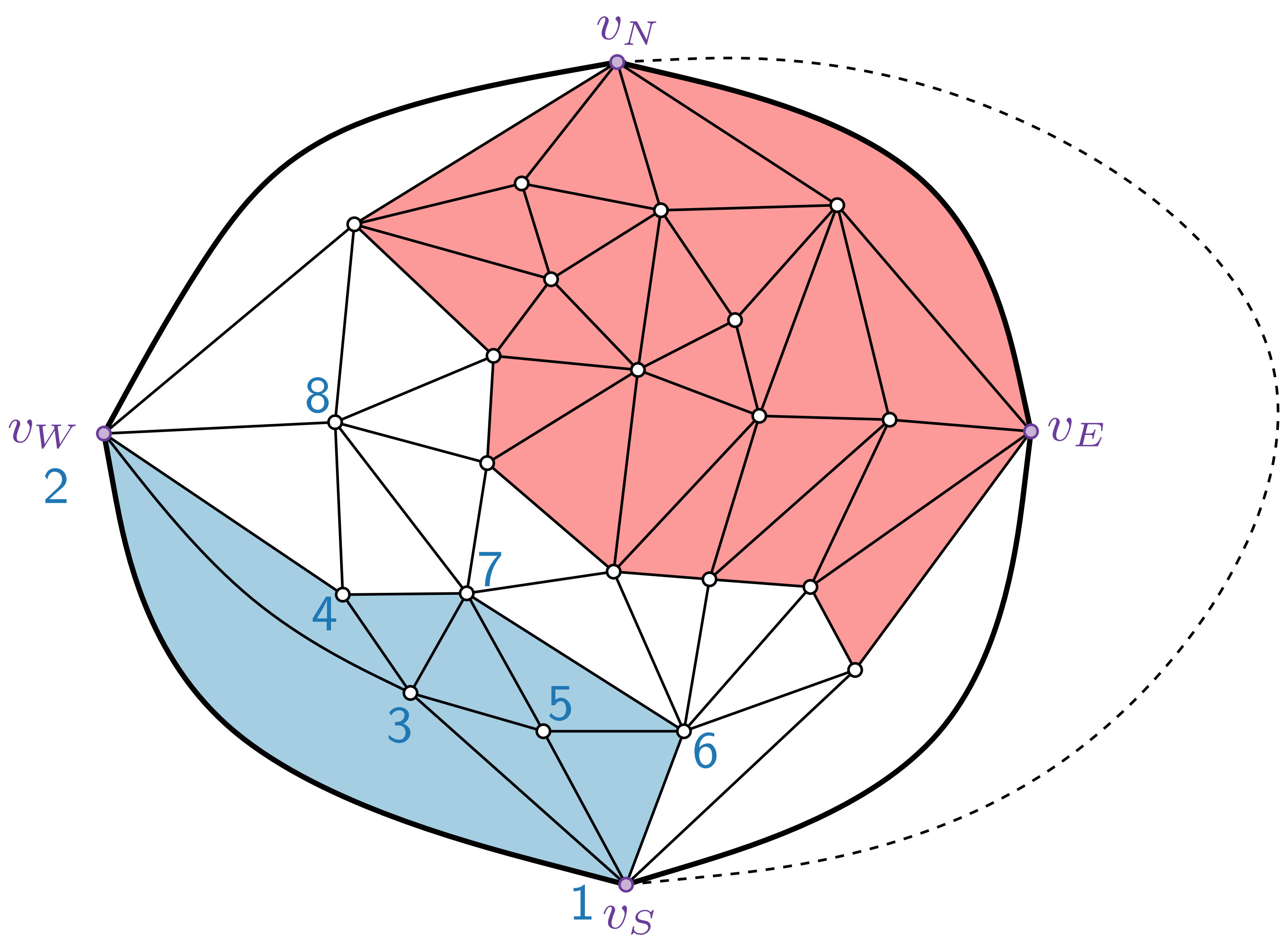

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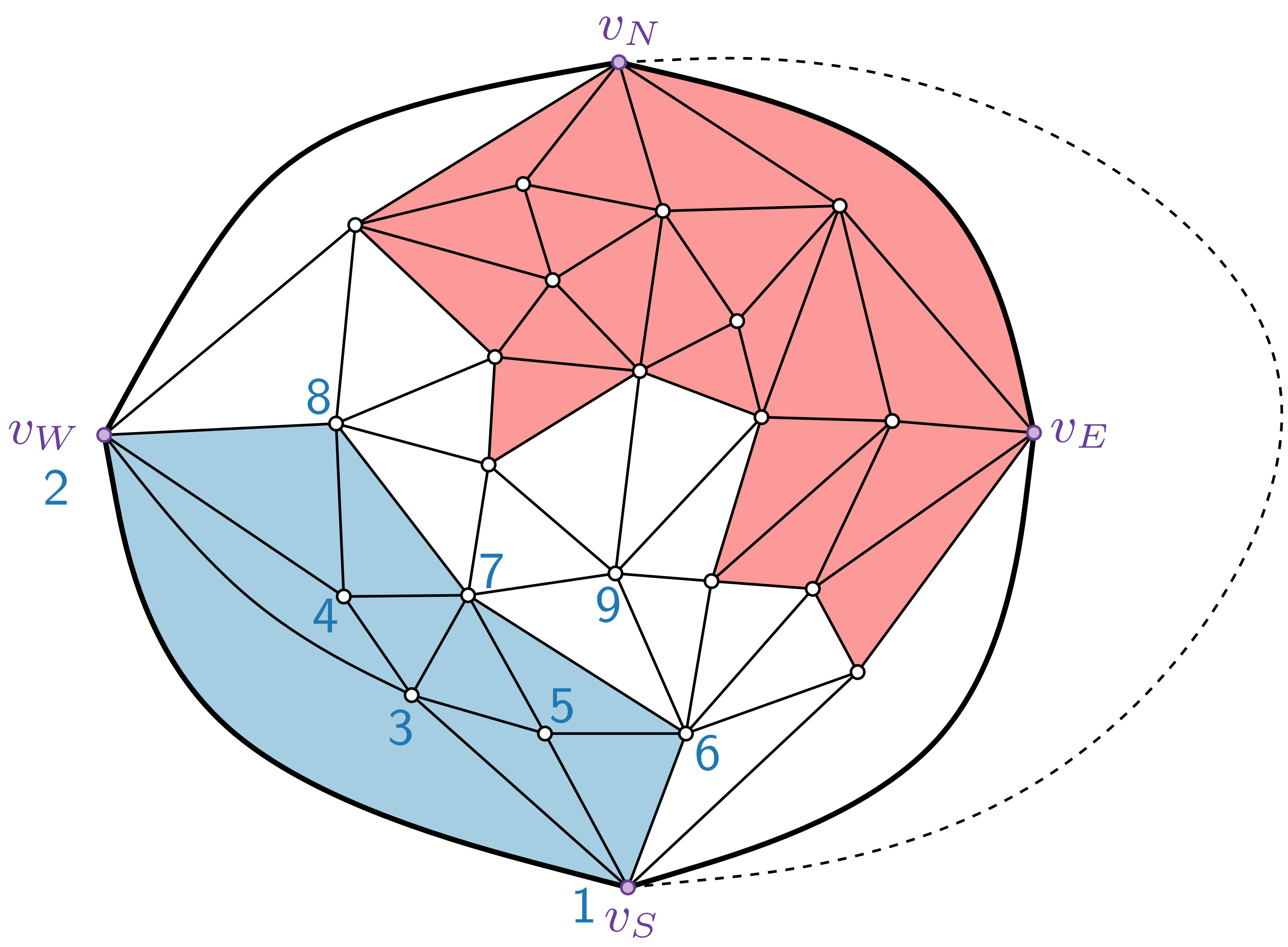

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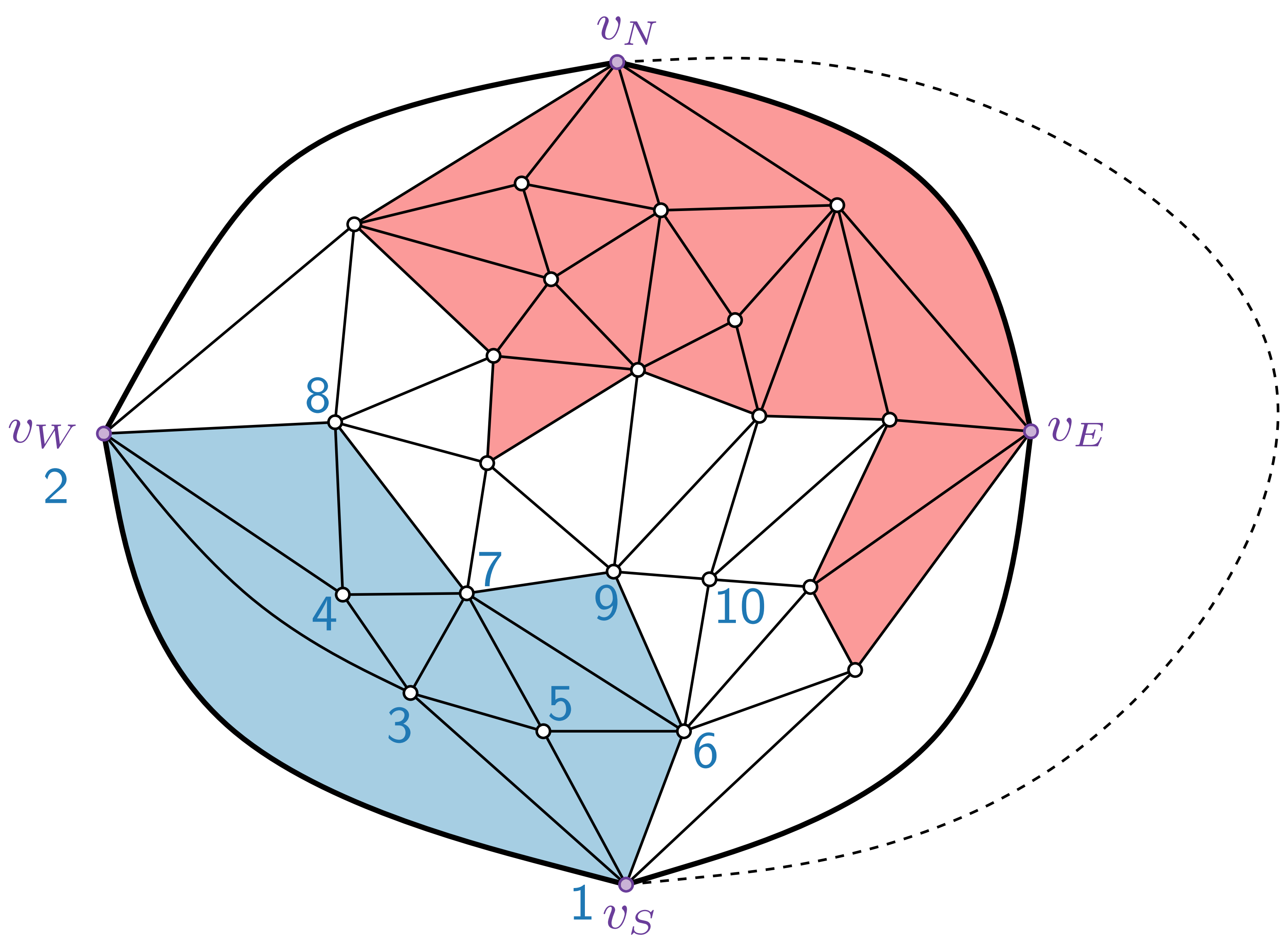

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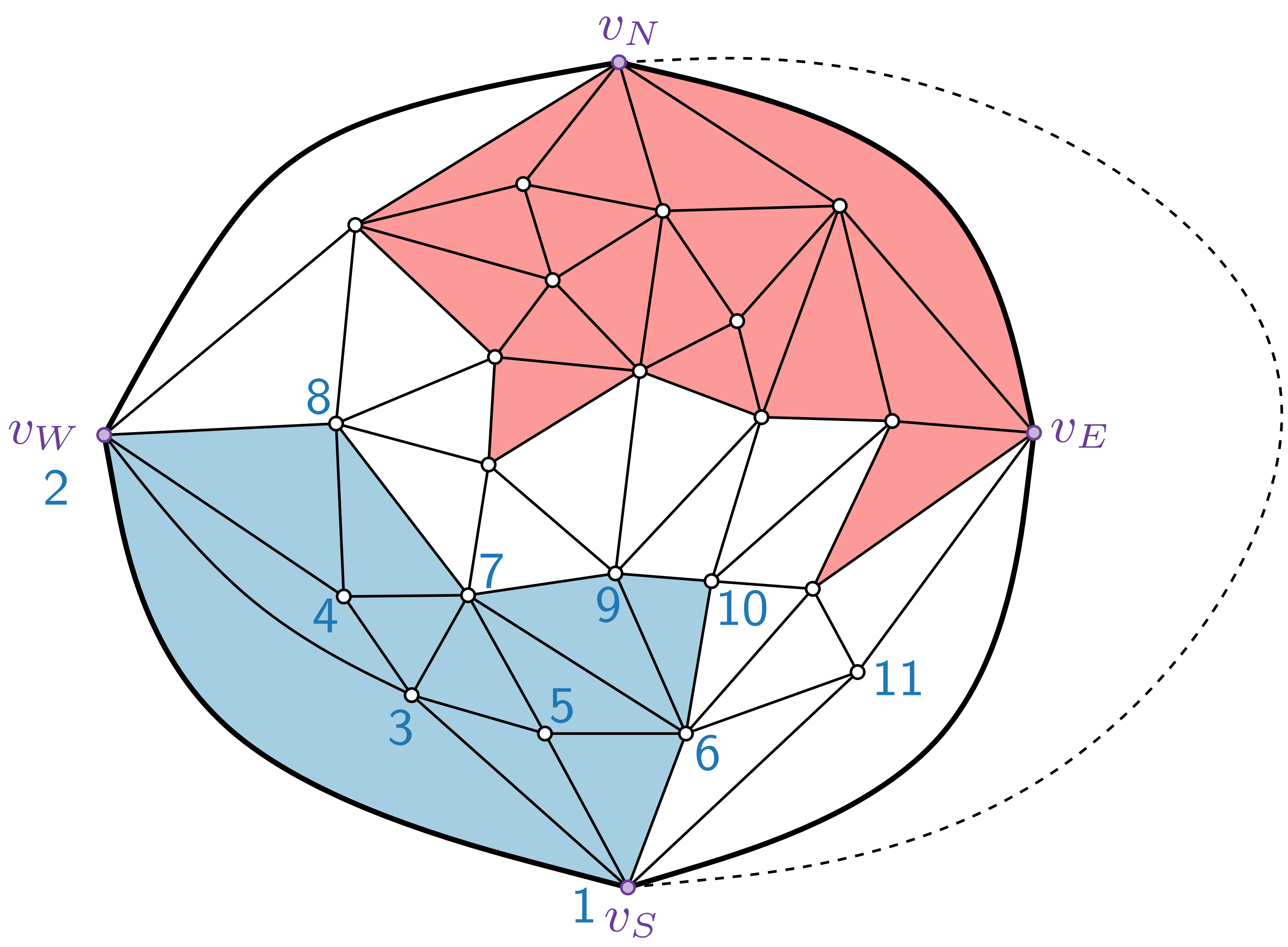

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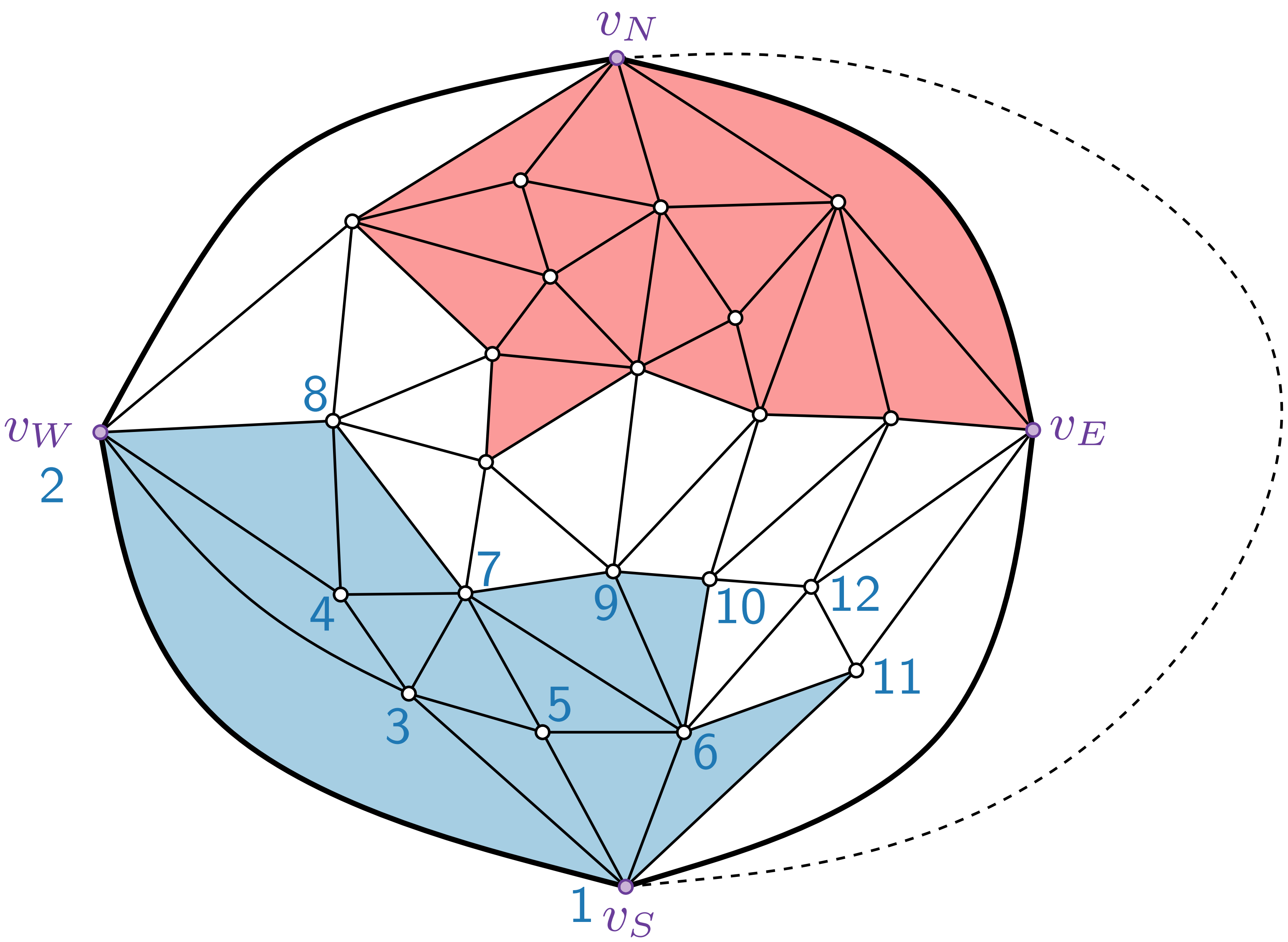

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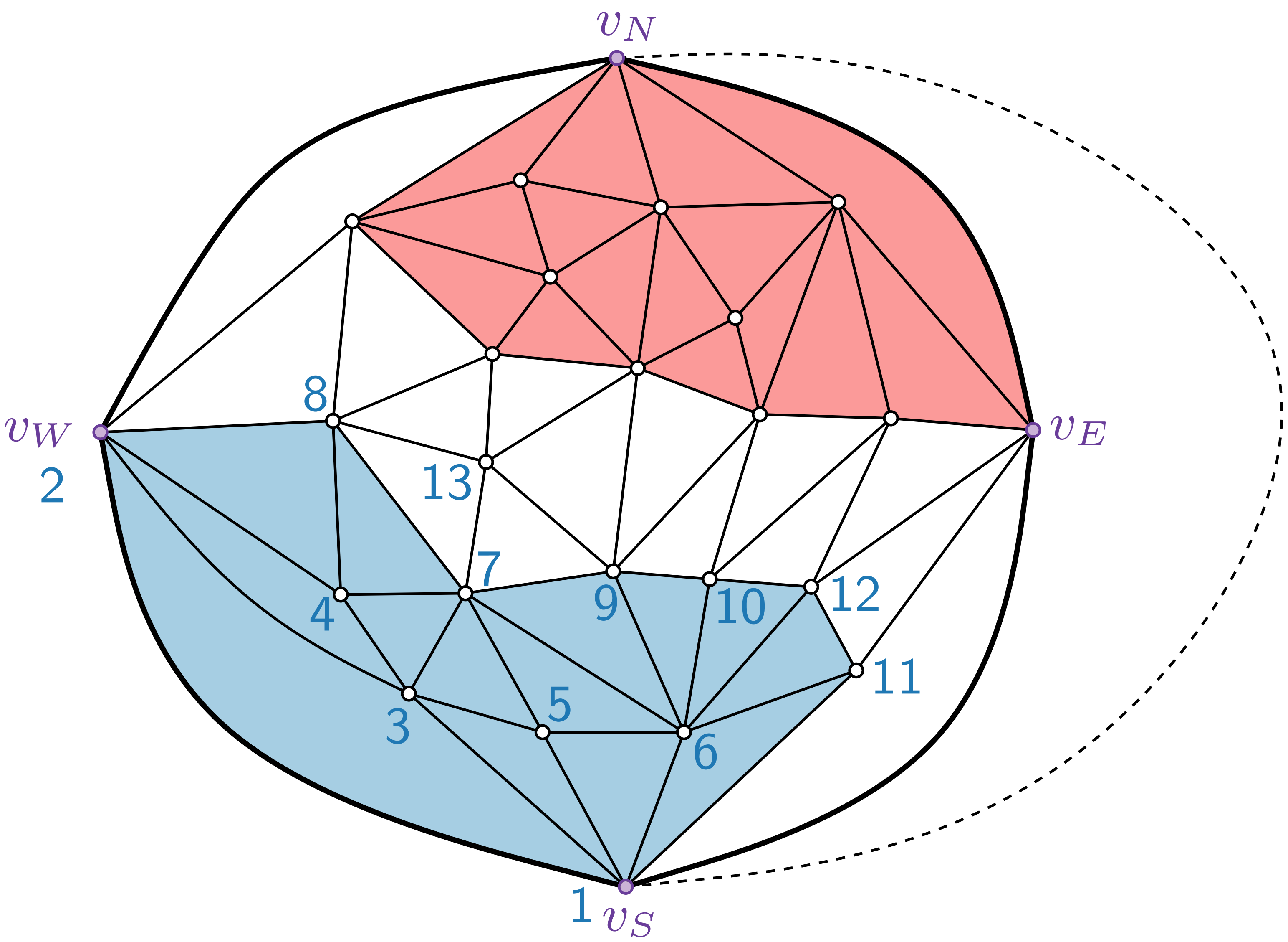

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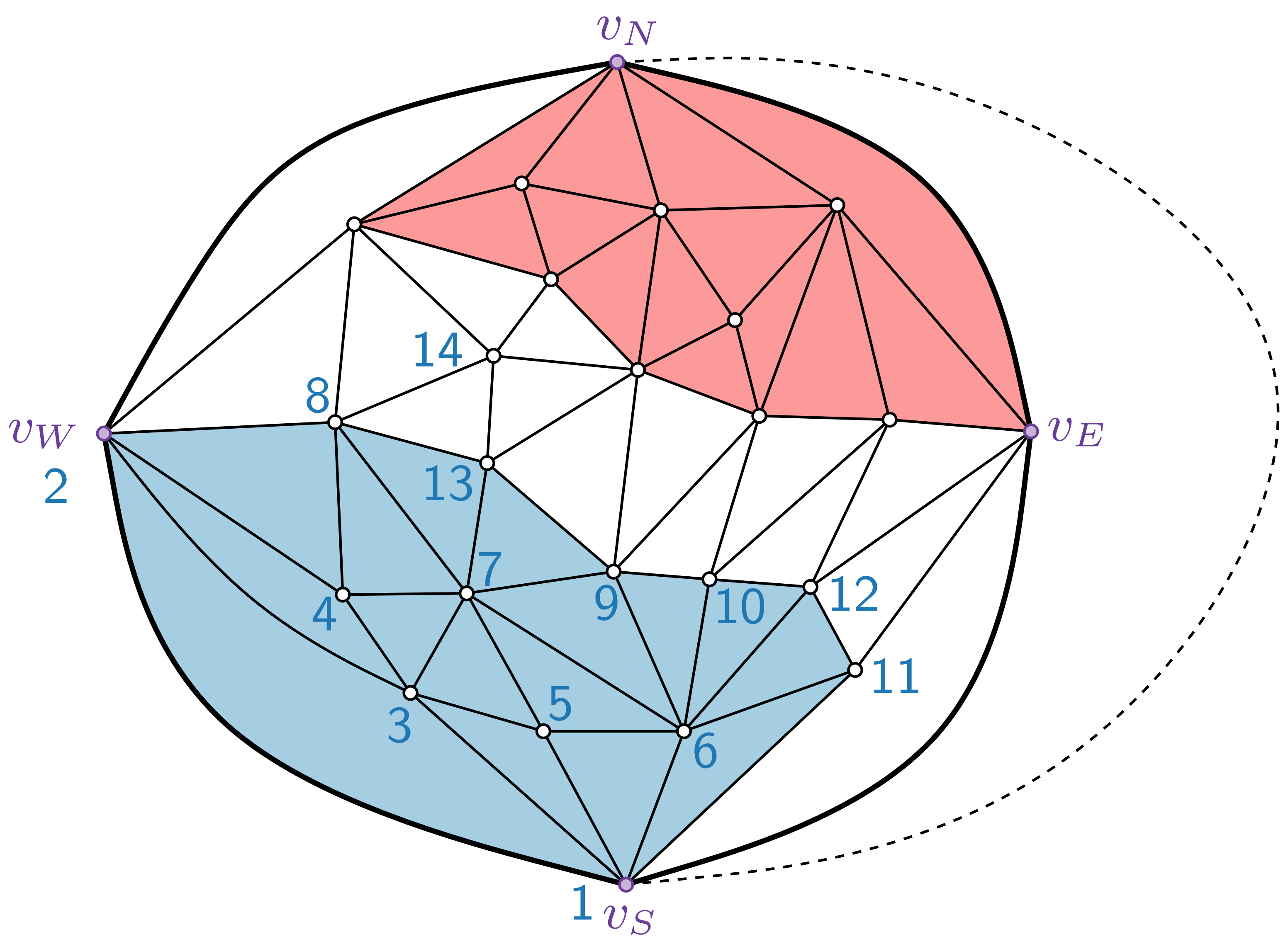

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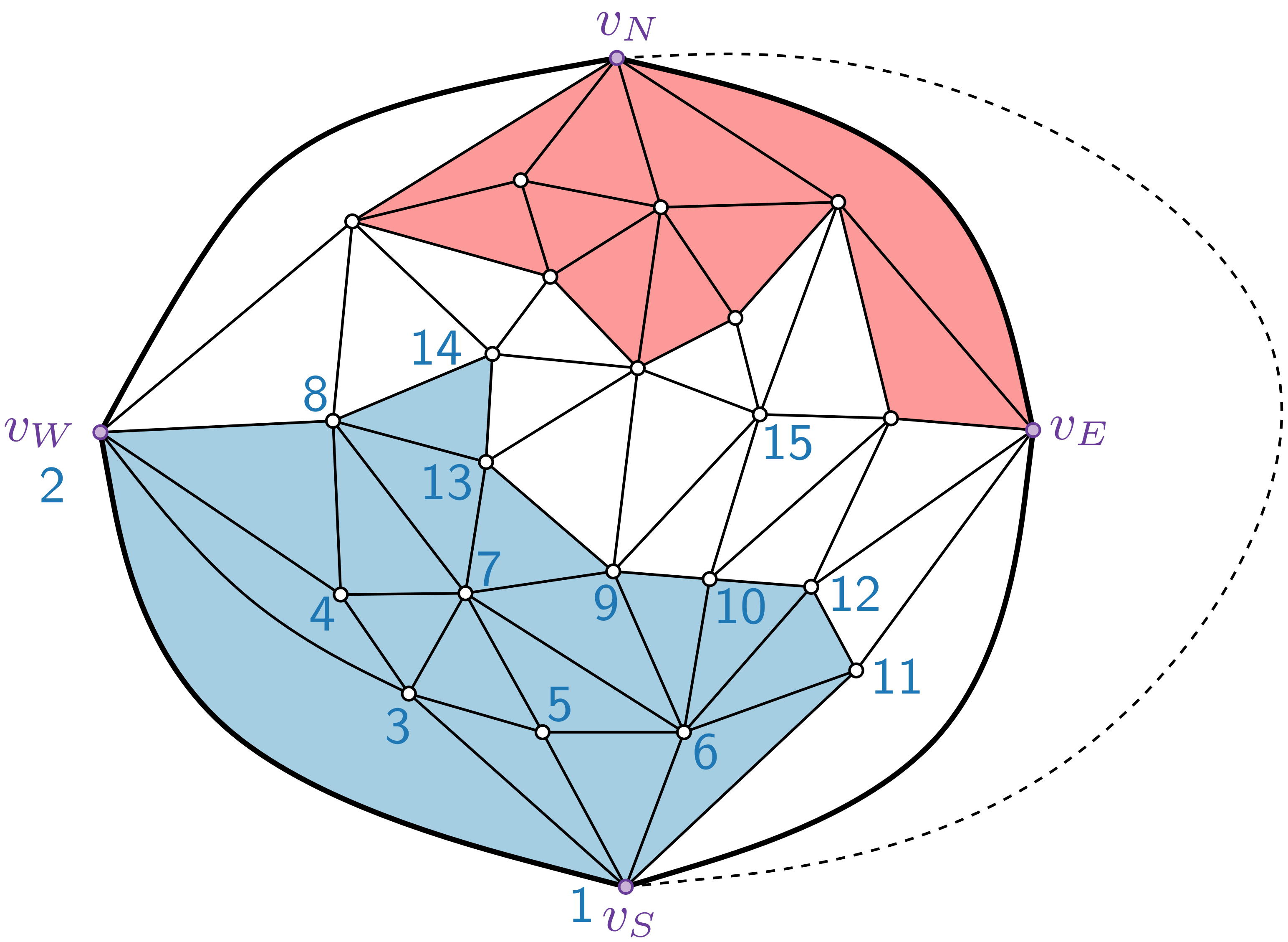

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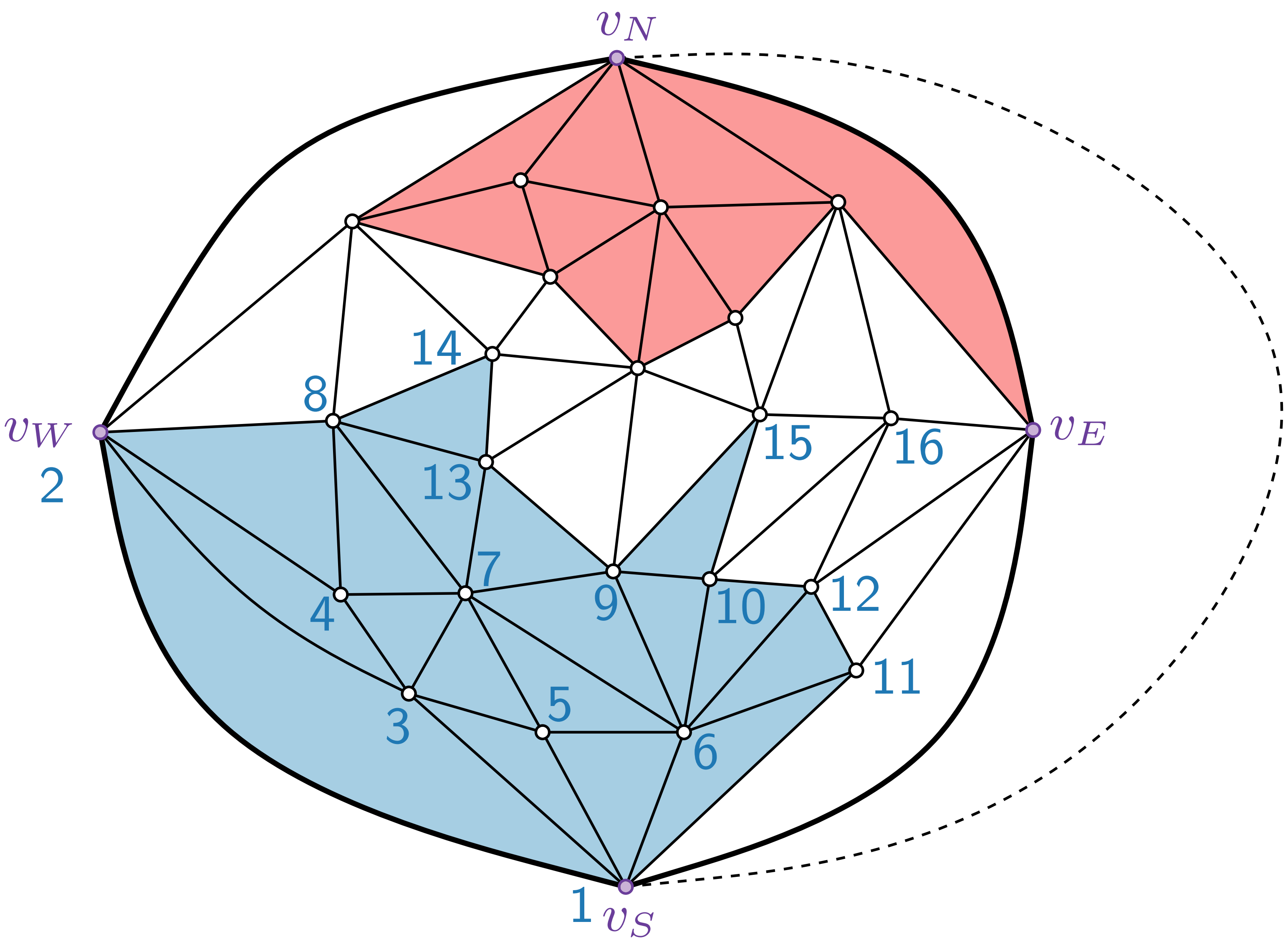

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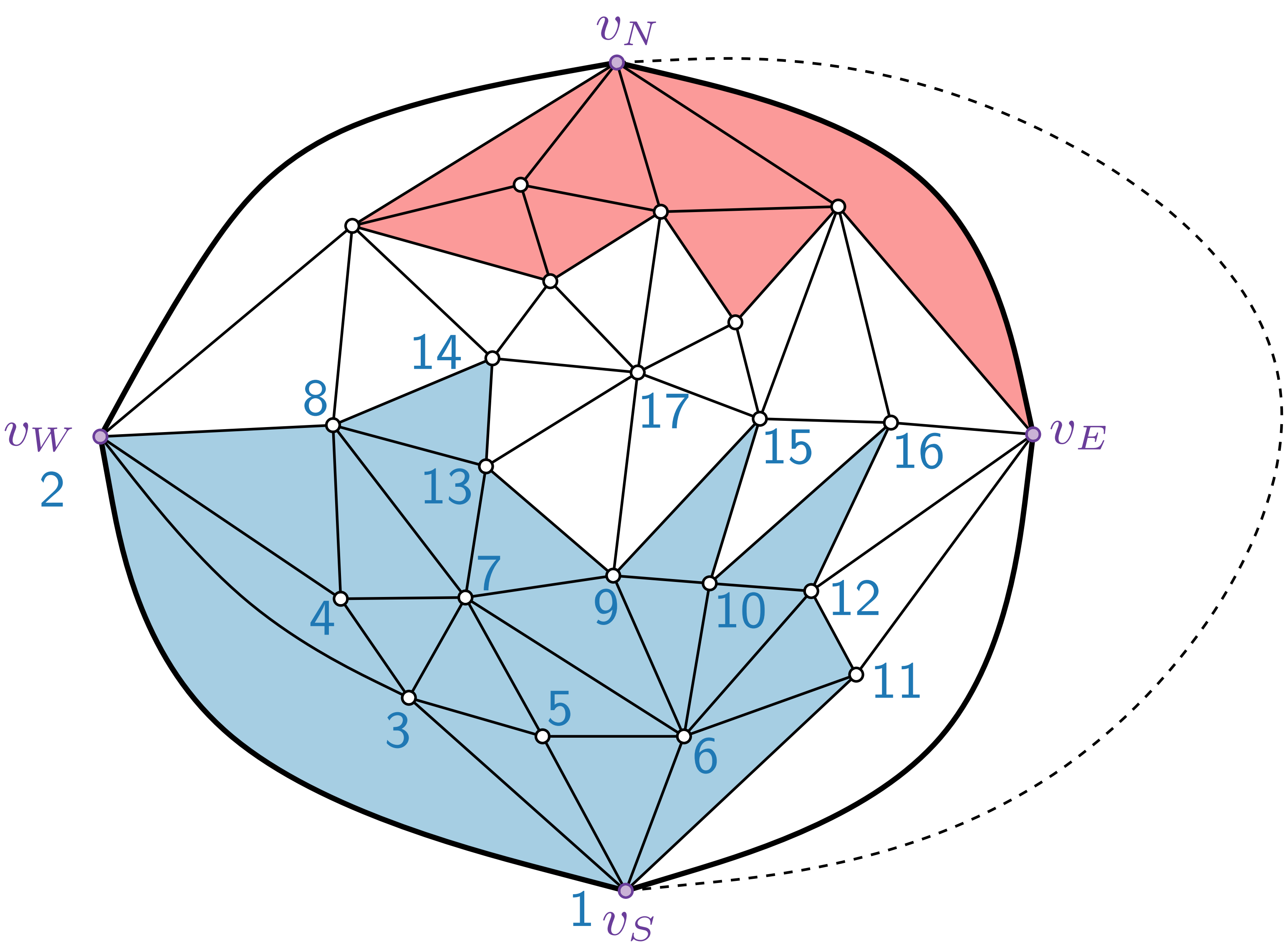

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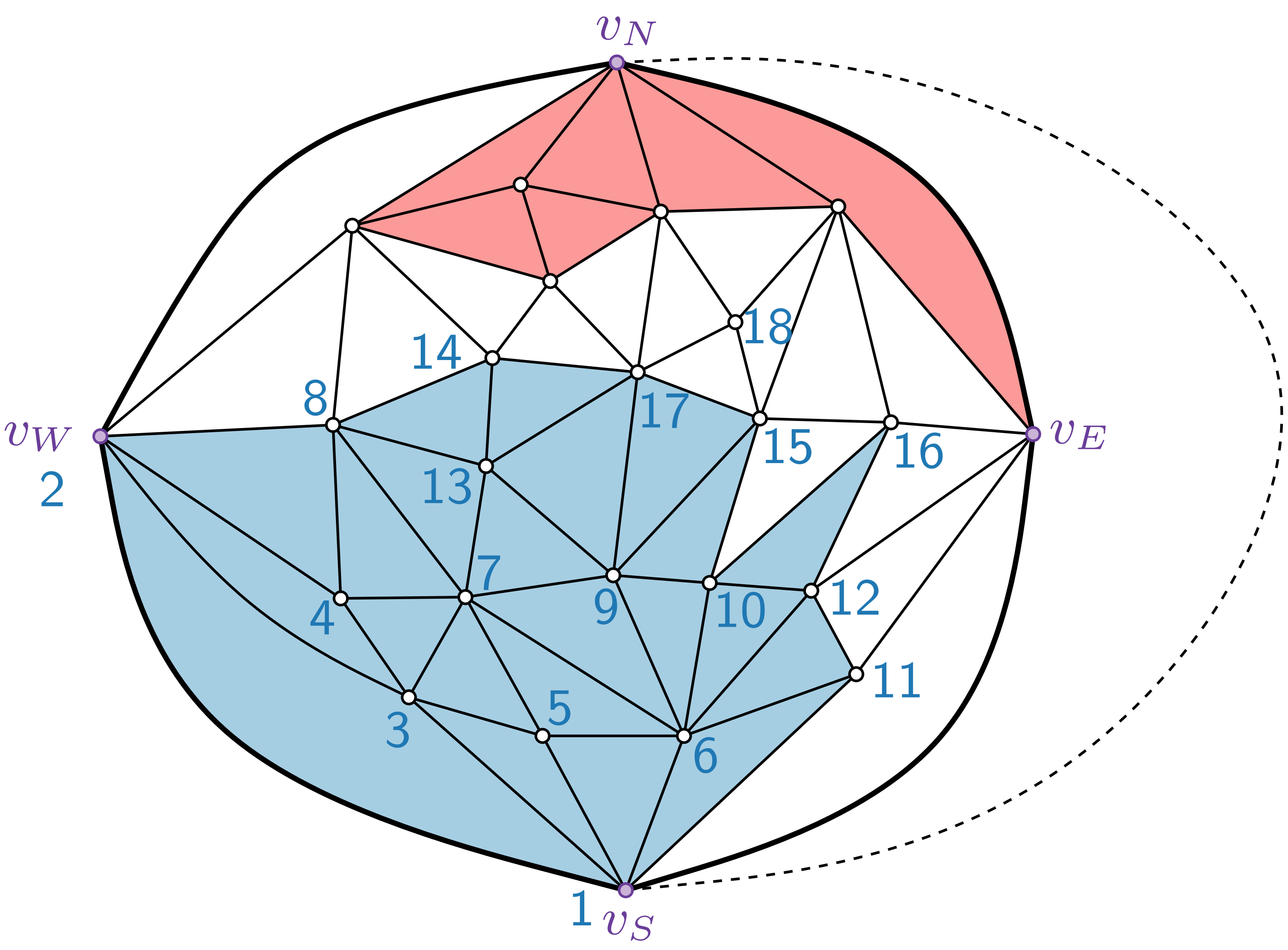

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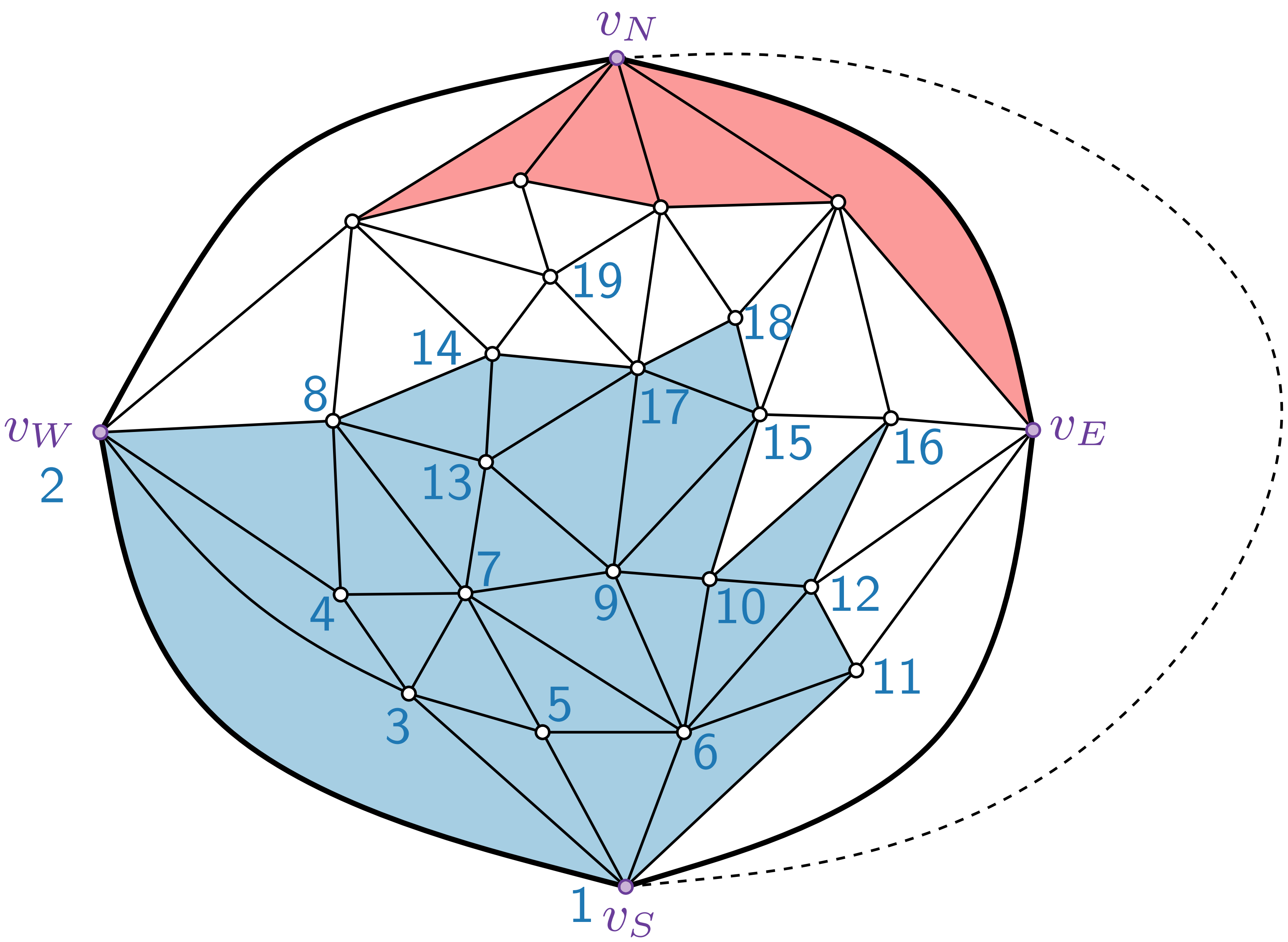

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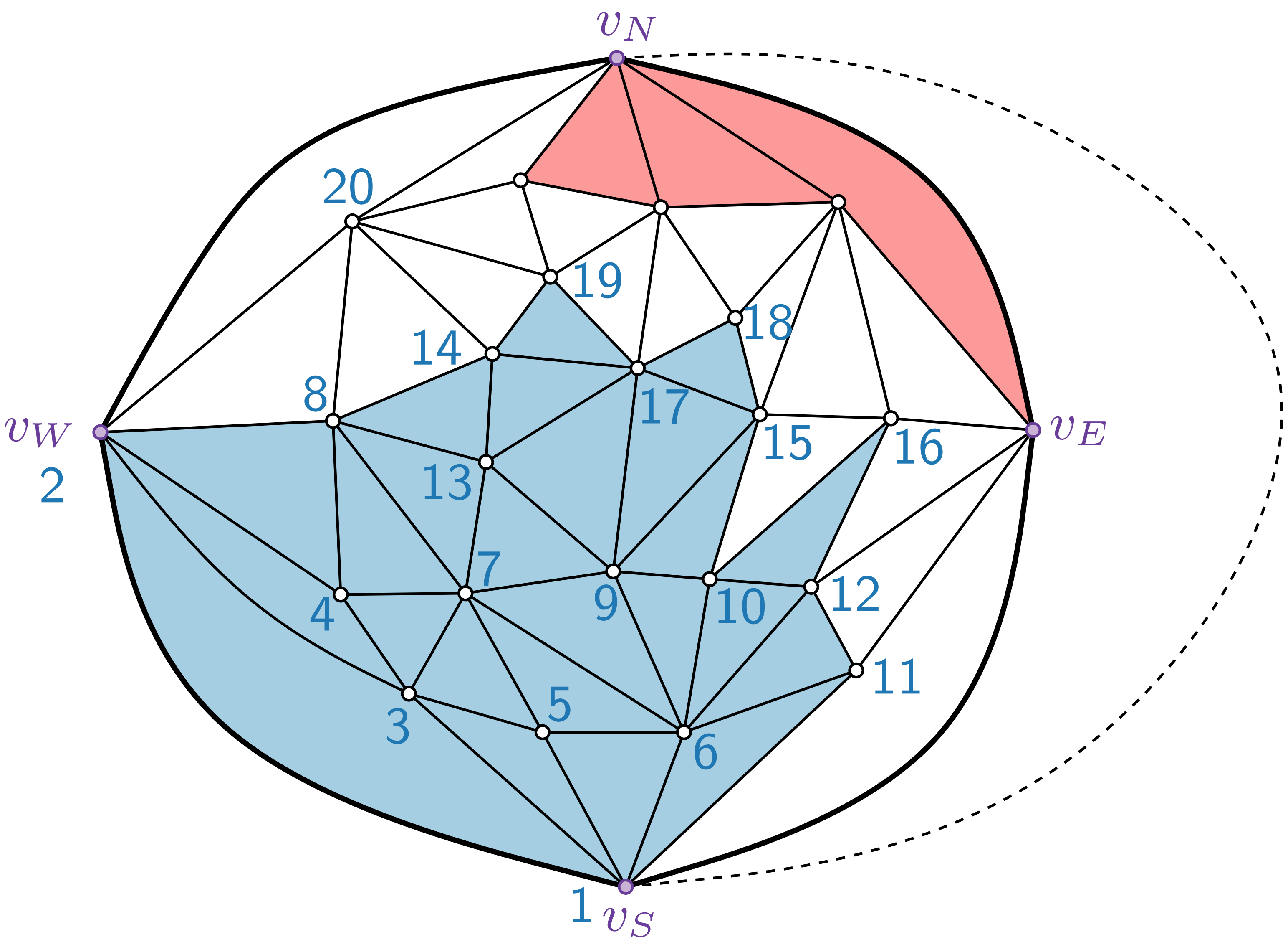

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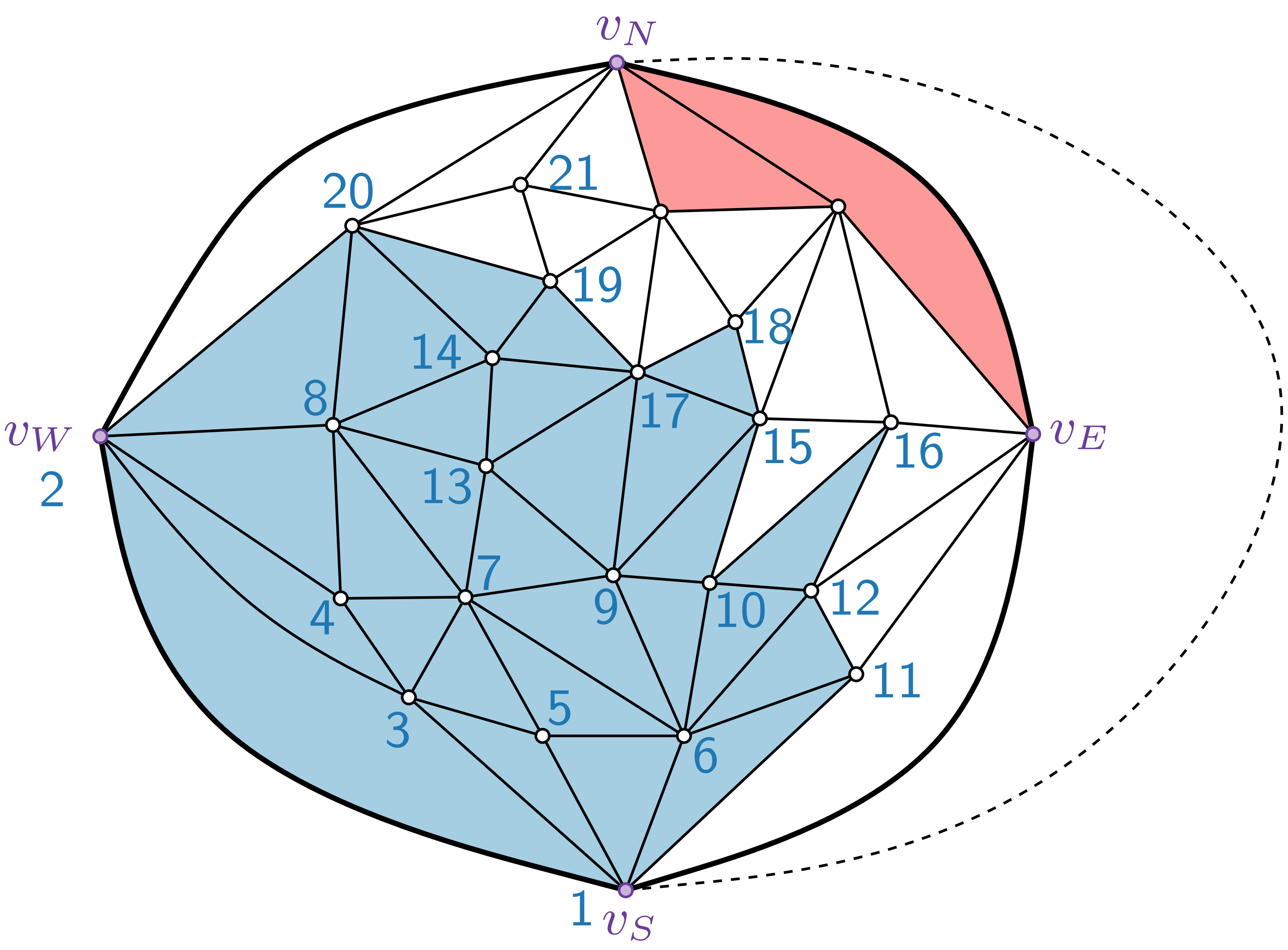

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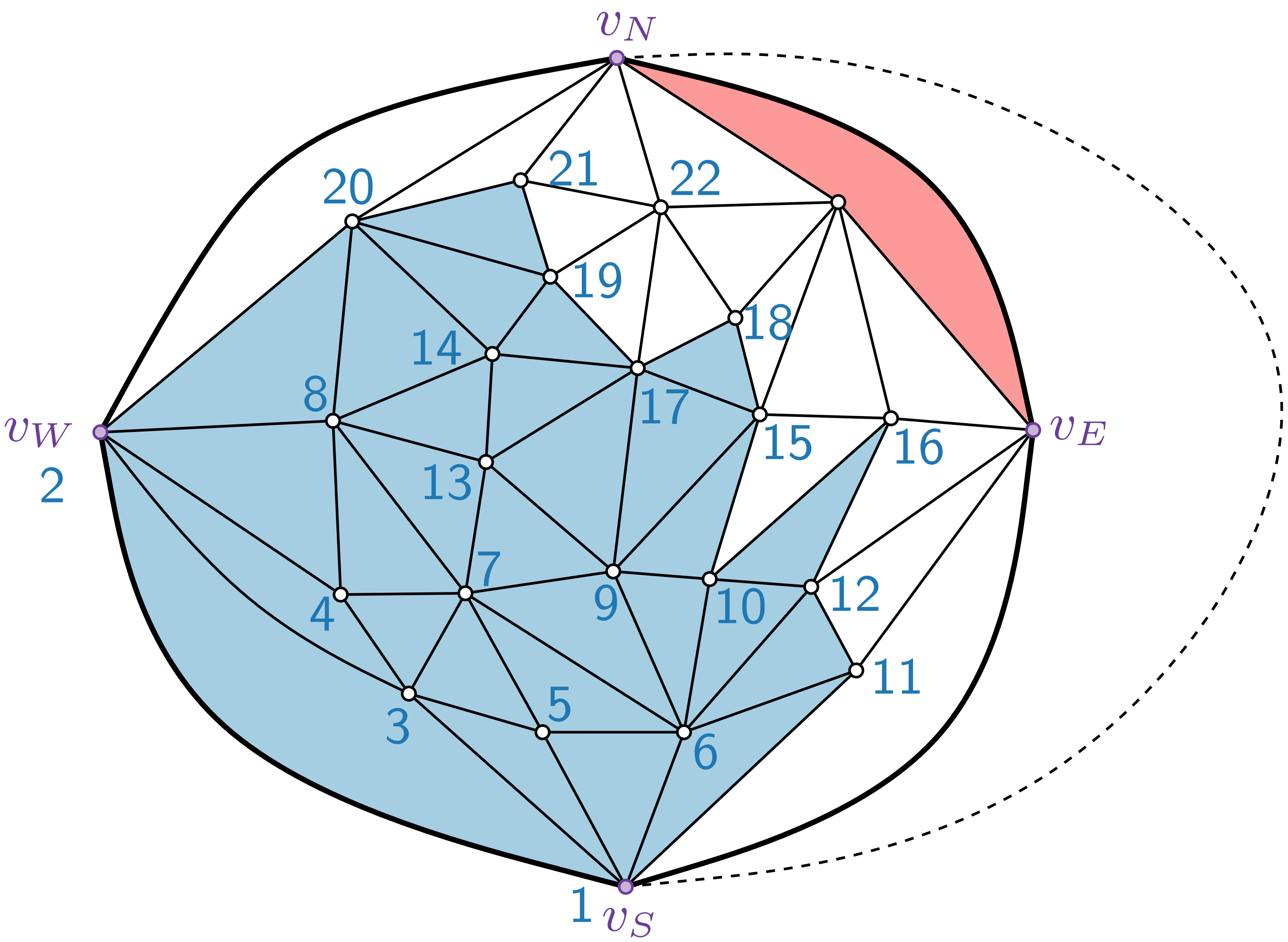

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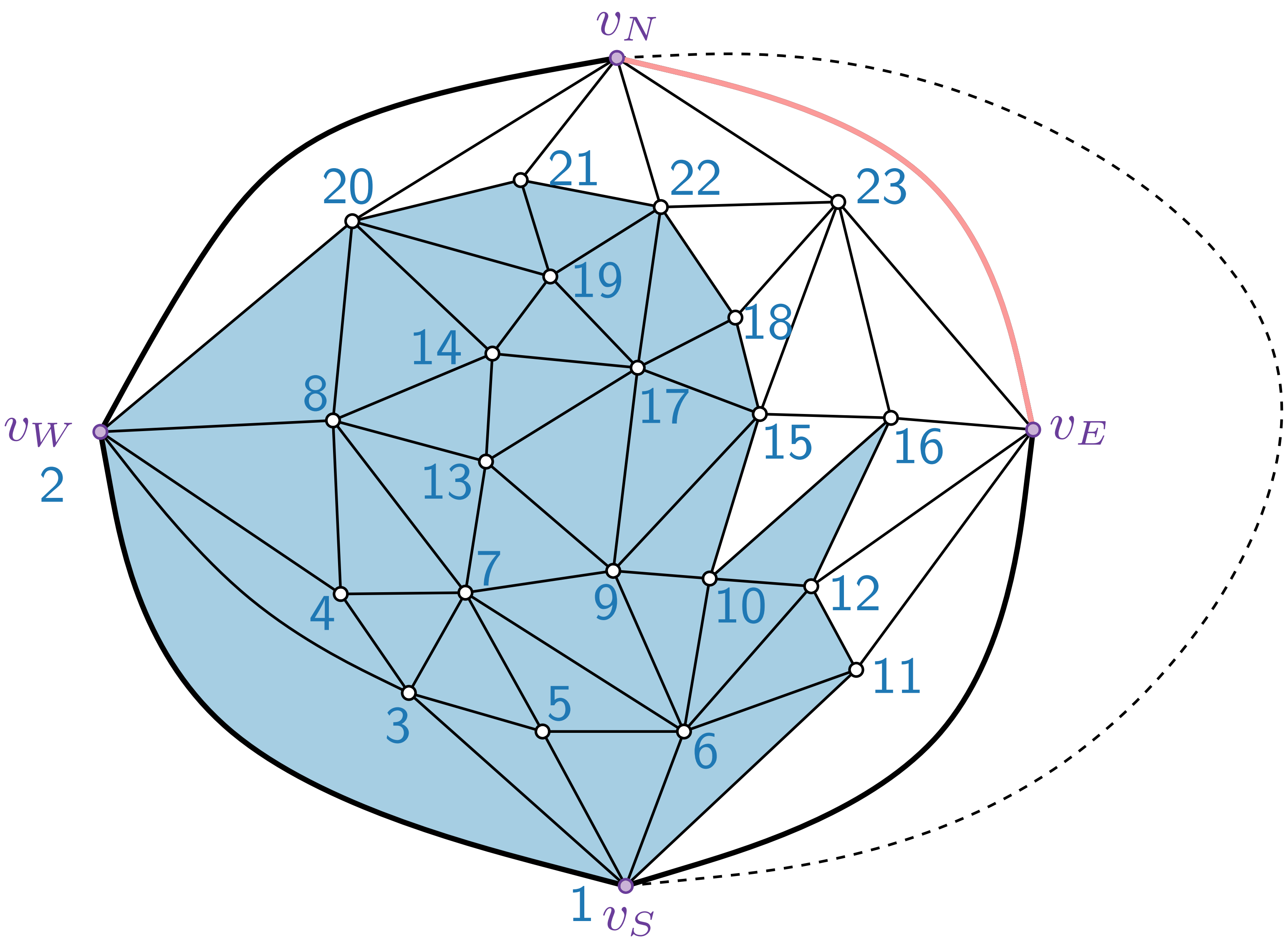

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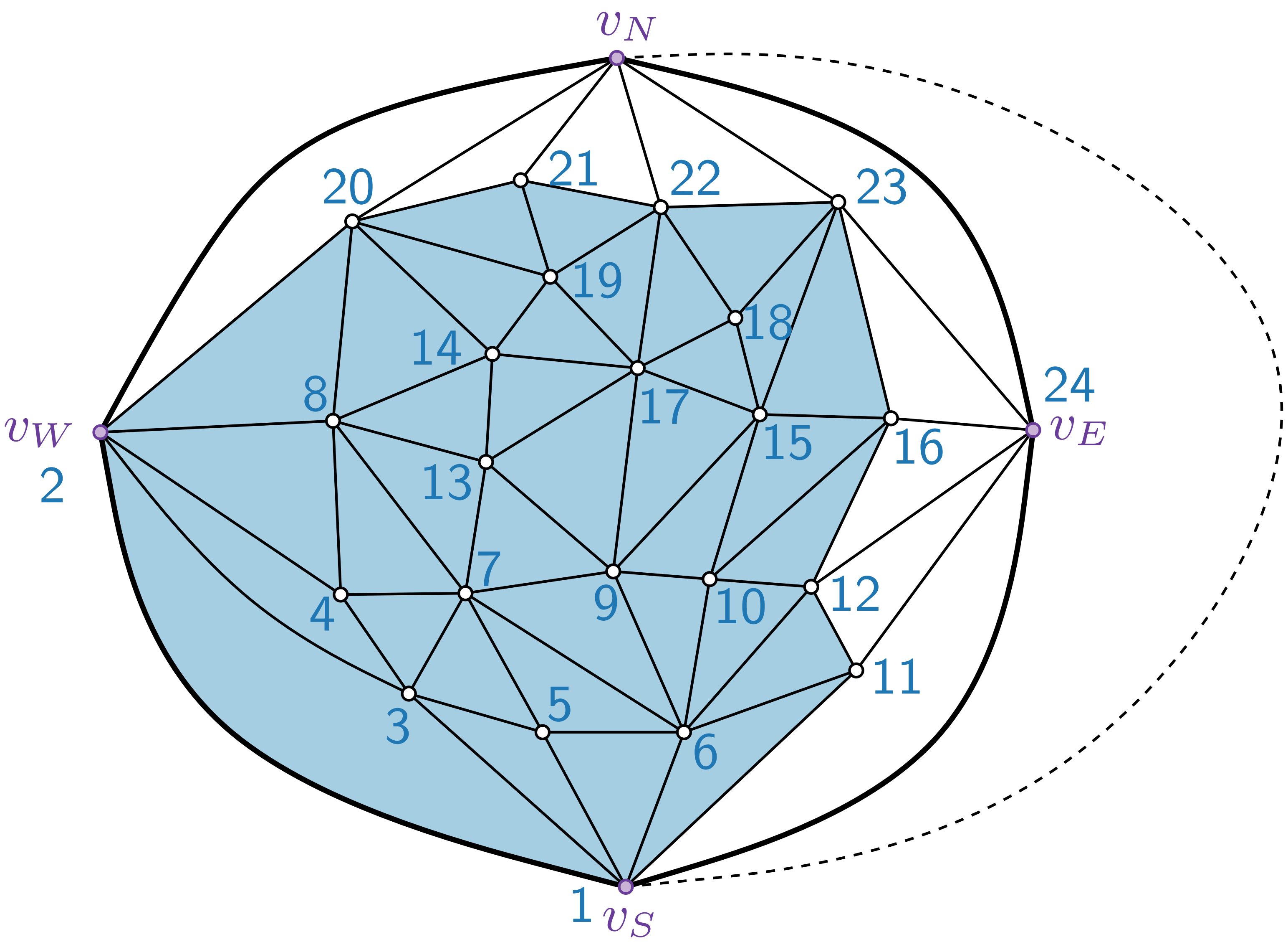

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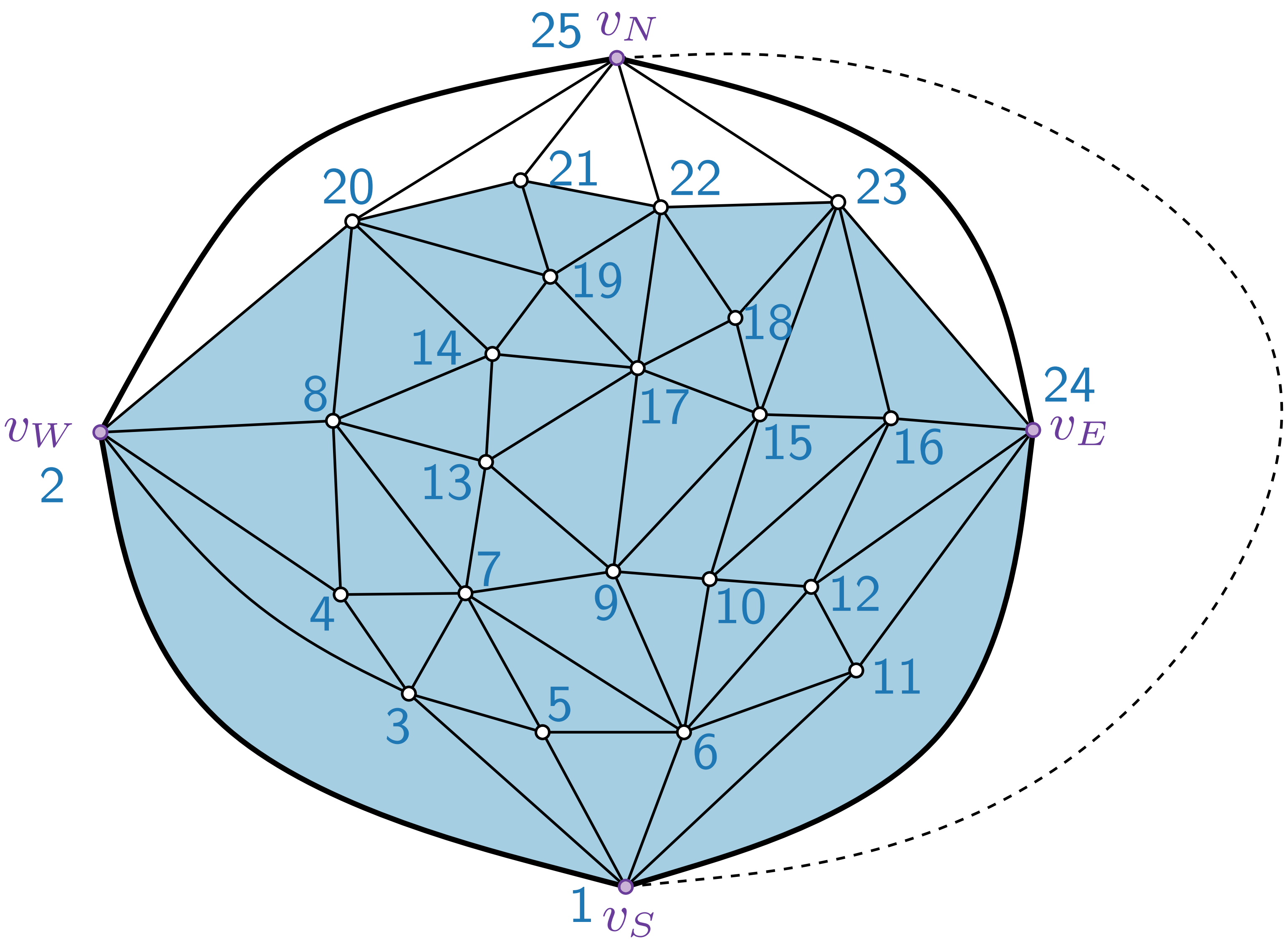

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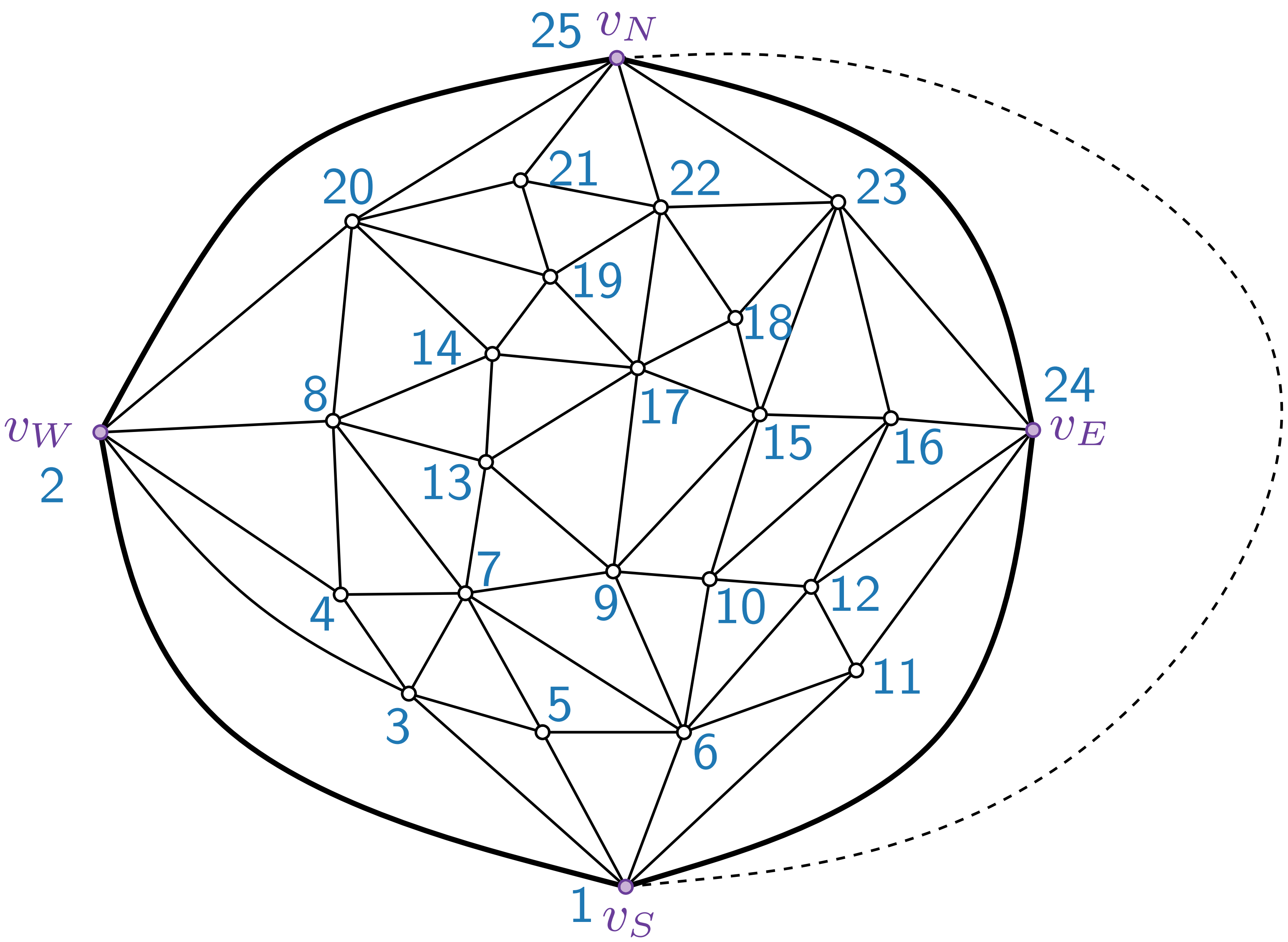

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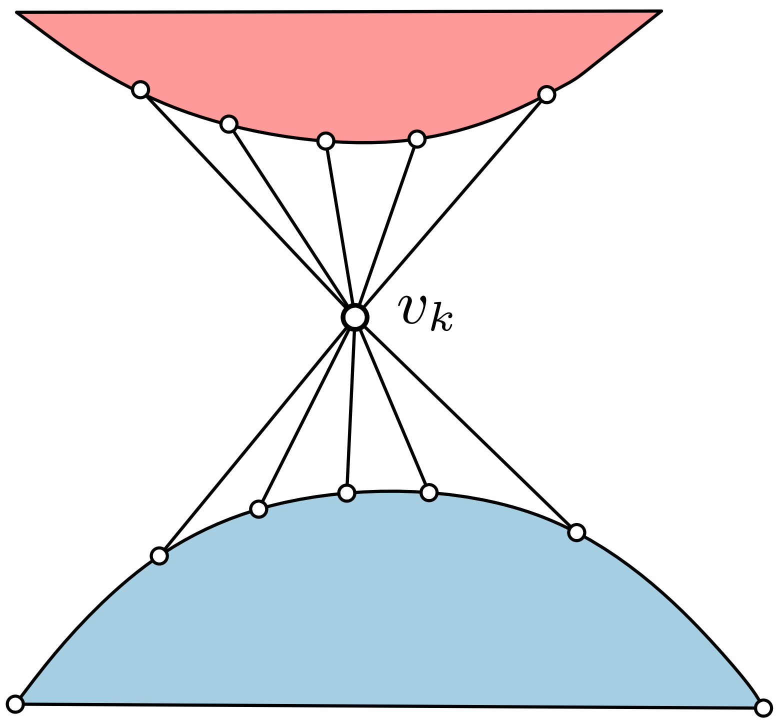

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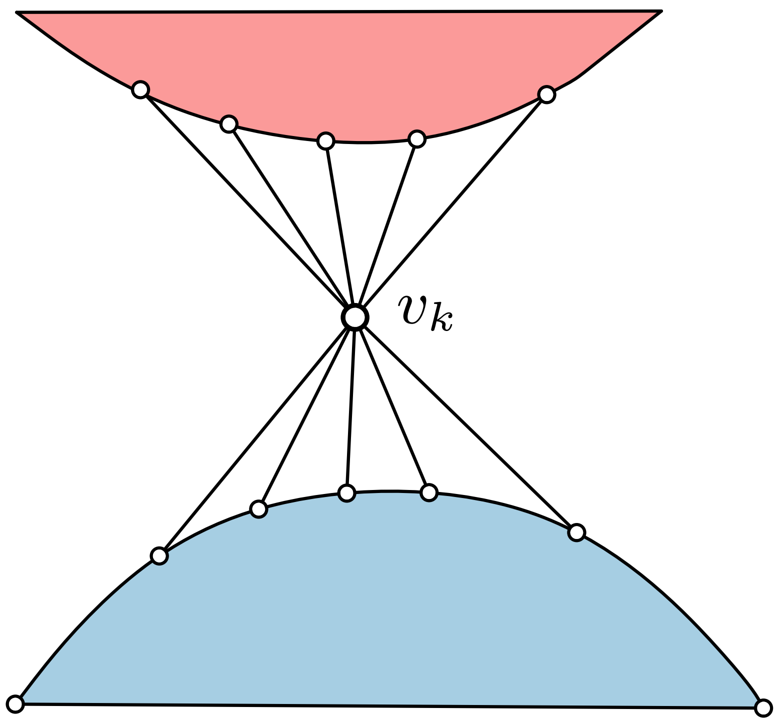

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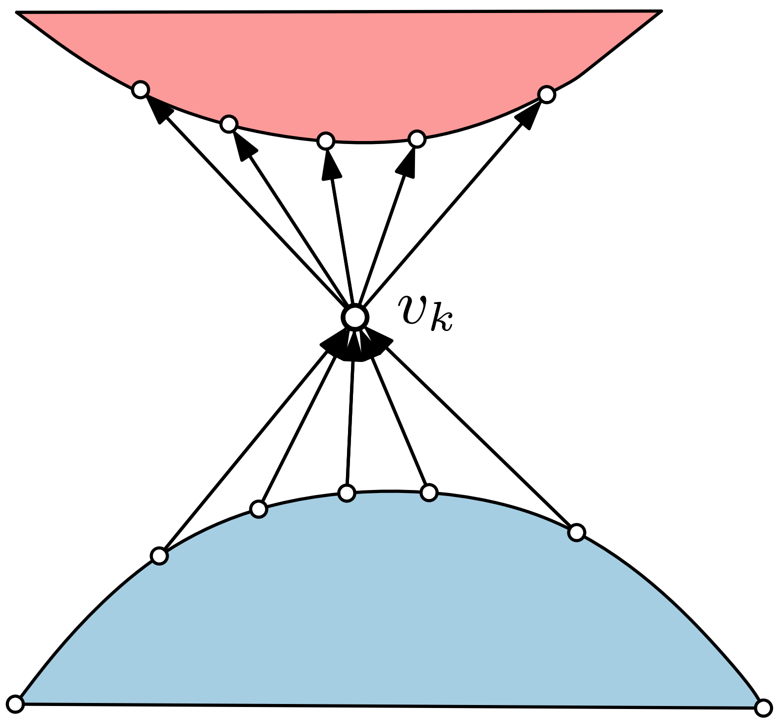

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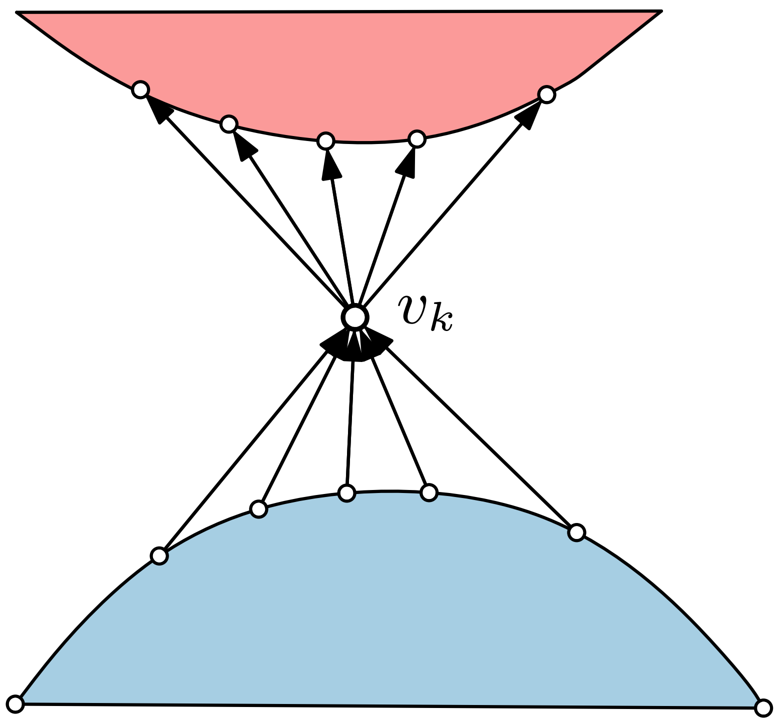

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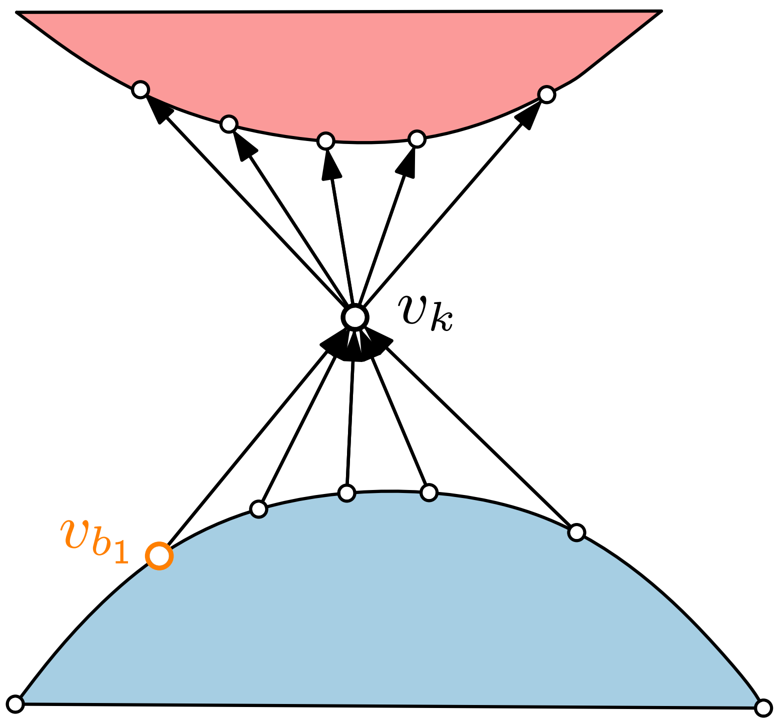

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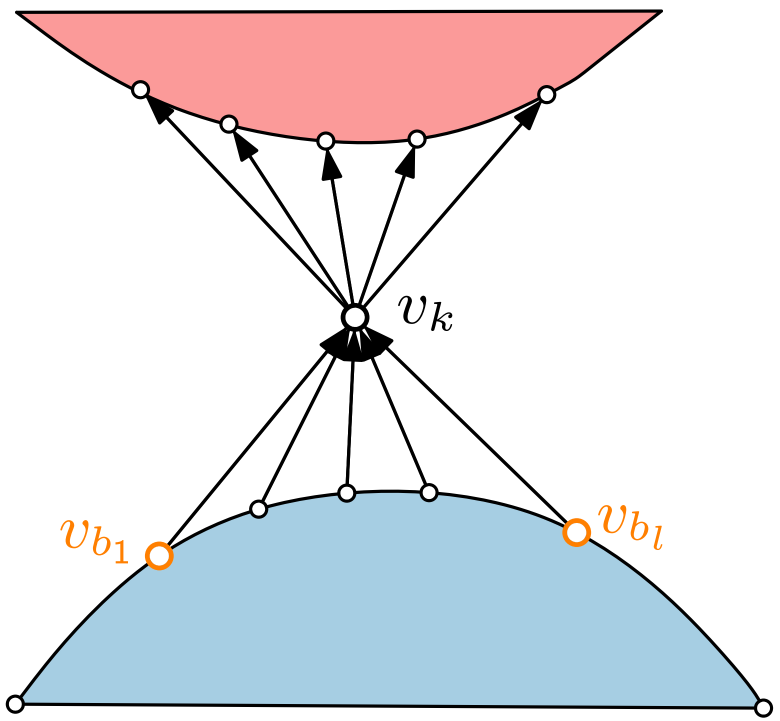

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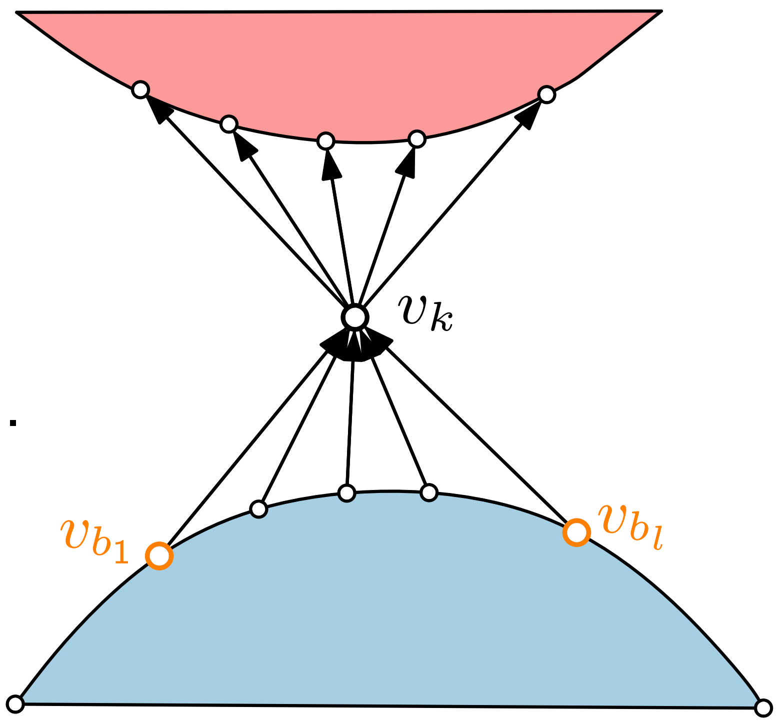

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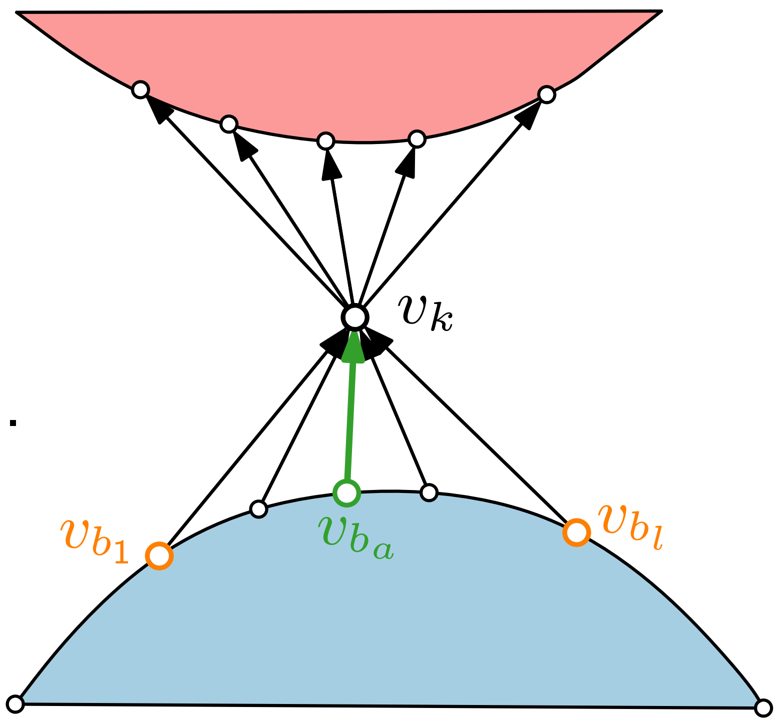

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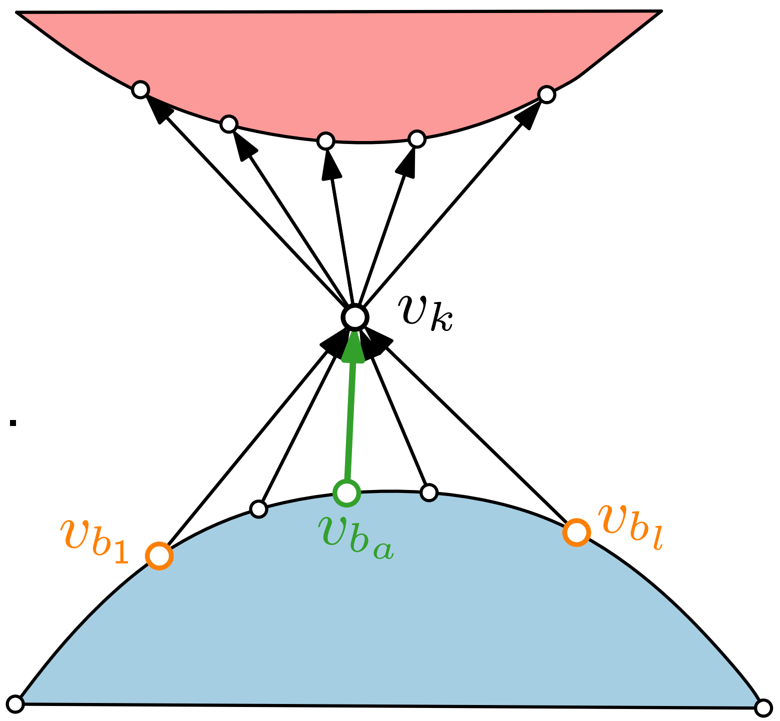

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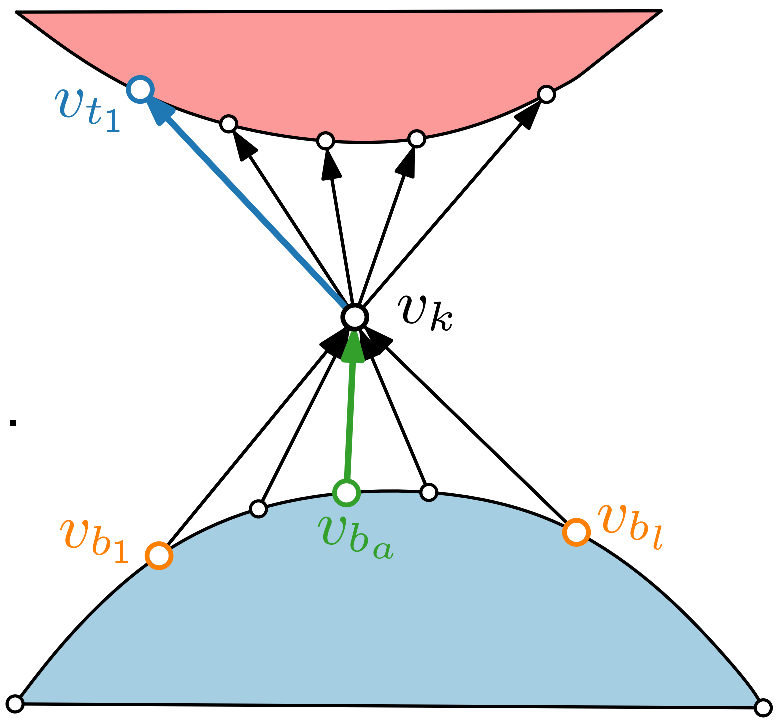

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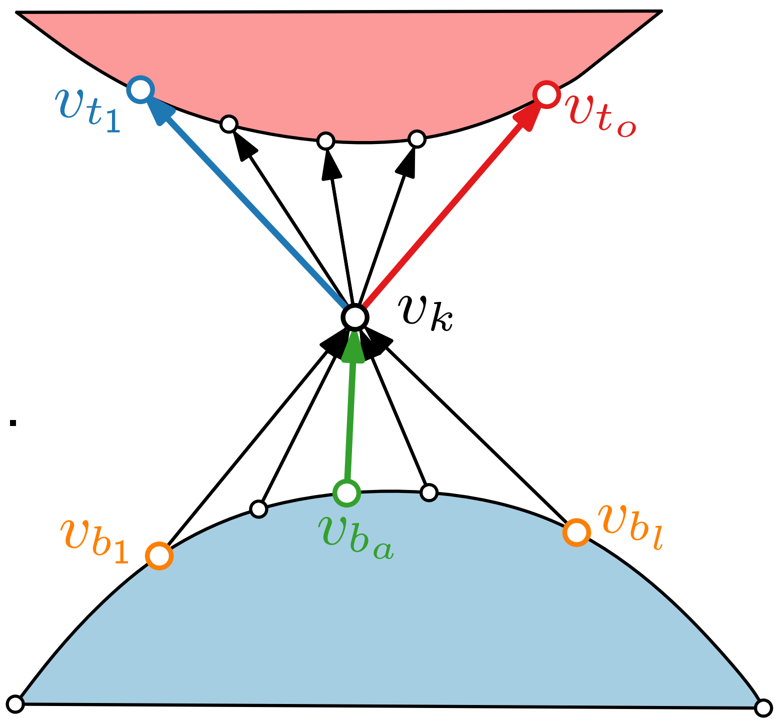

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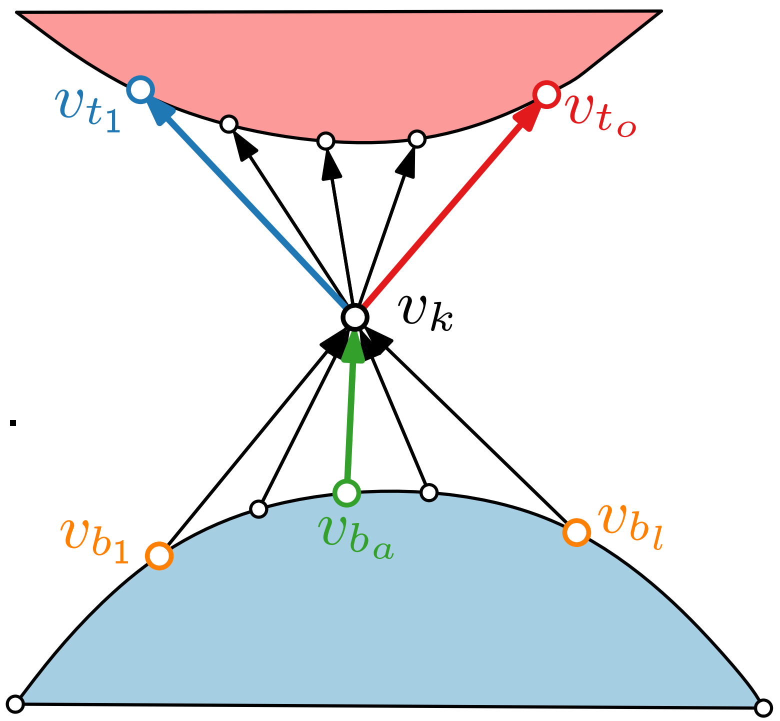

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Refined Canonical Order $\to$ REL

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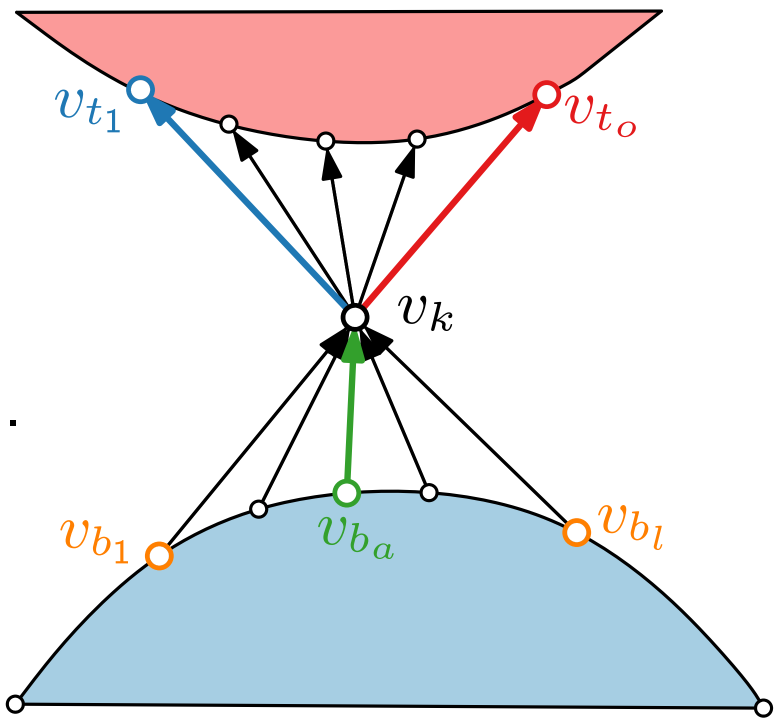

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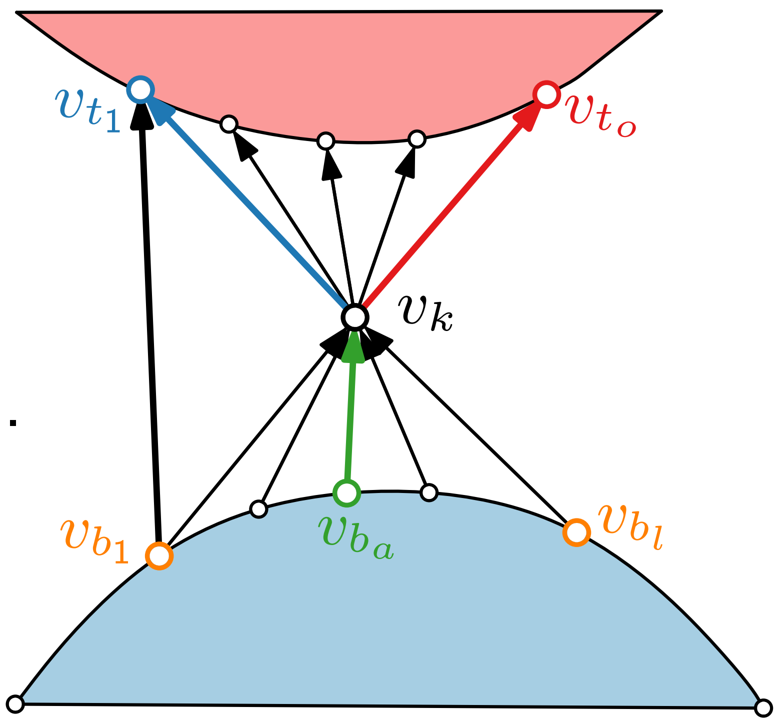

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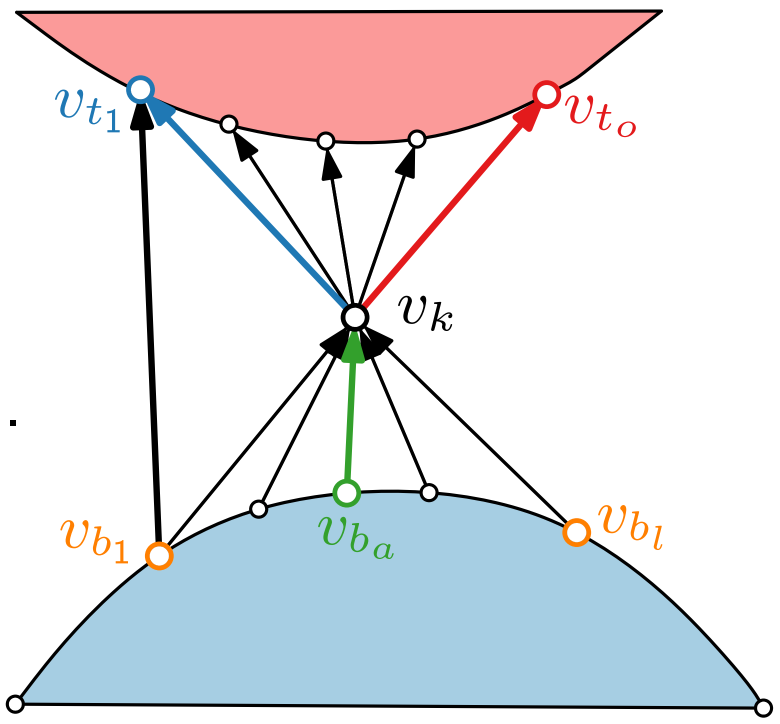

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Refined Canonical Order → REL

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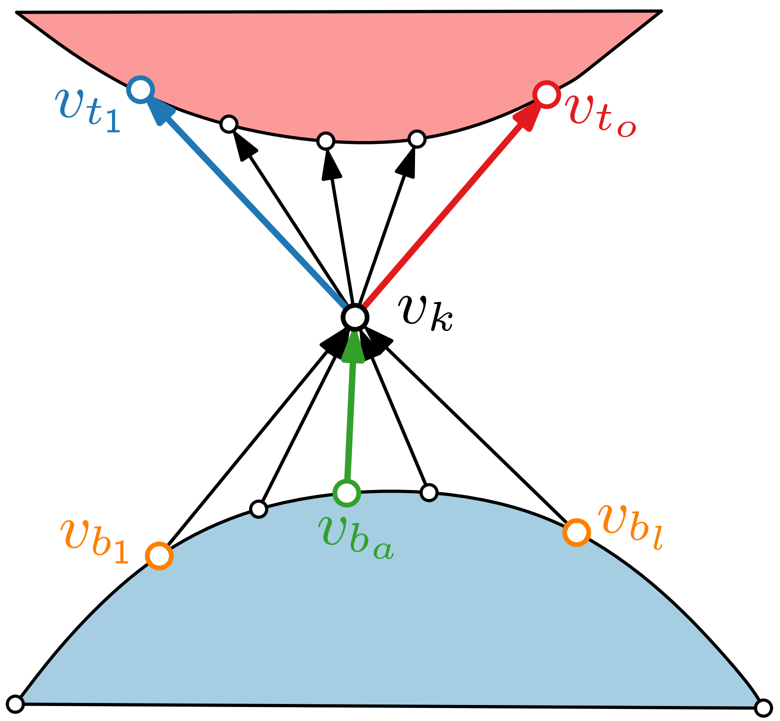

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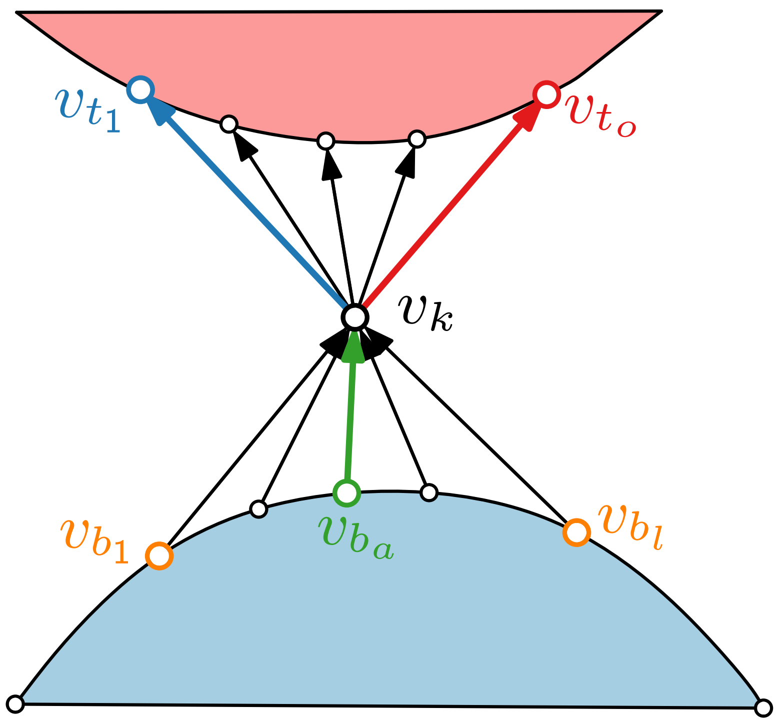

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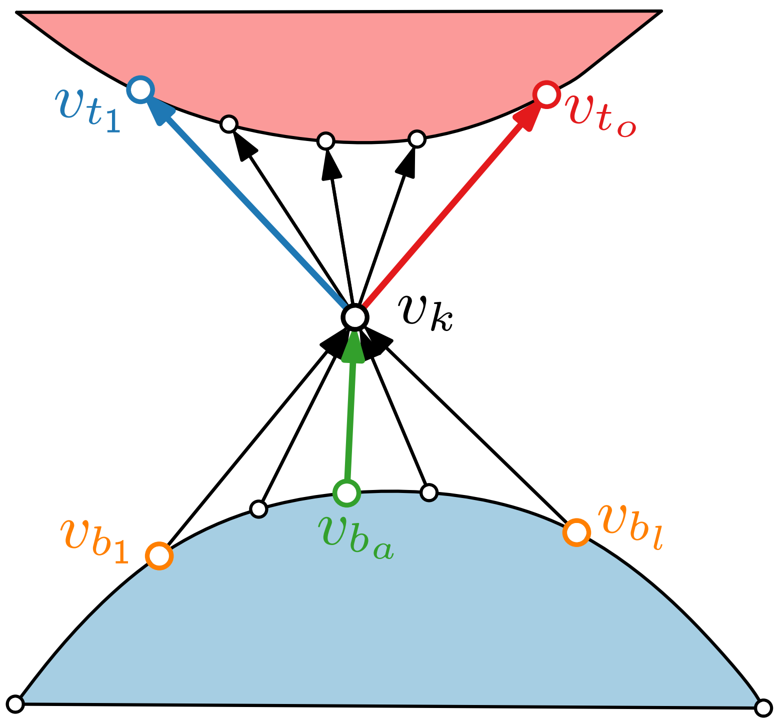

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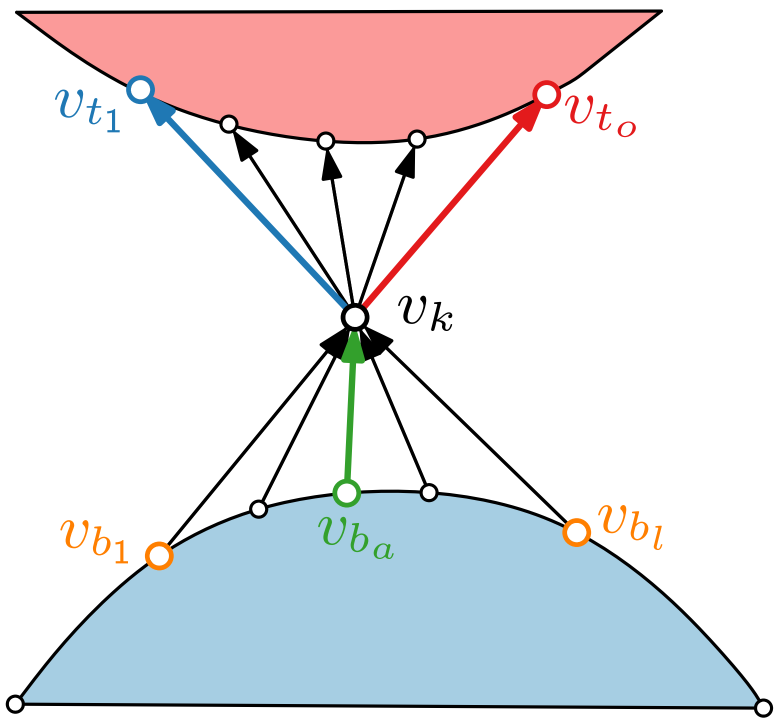

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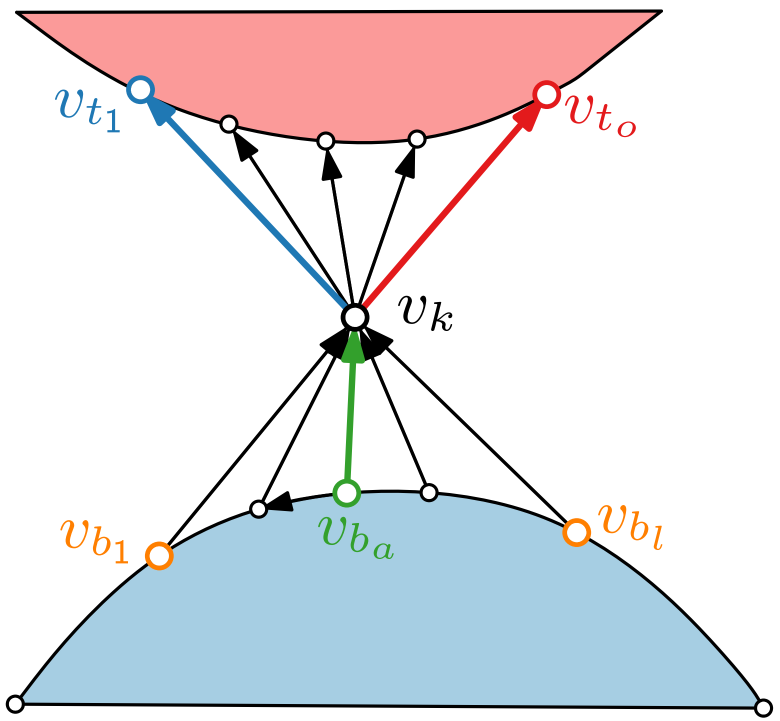

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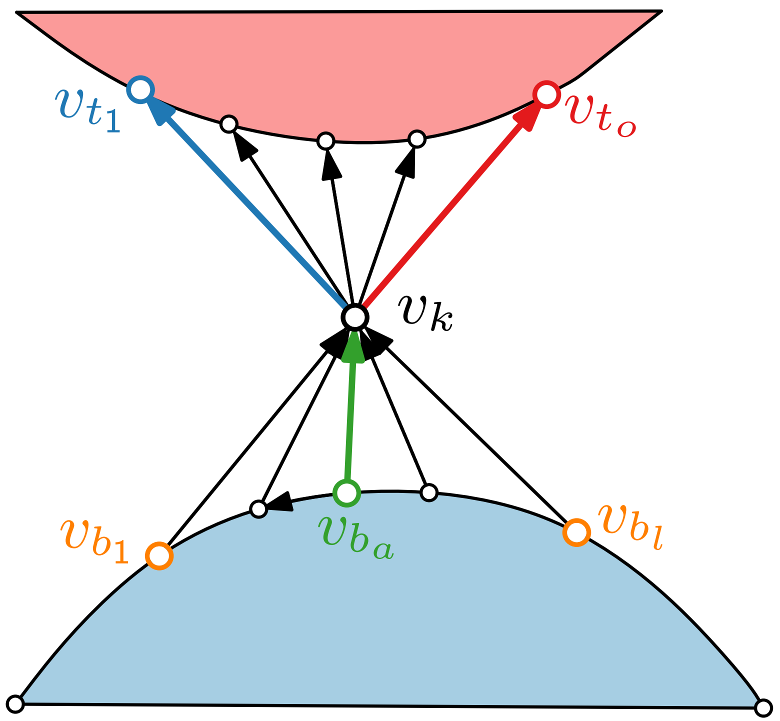

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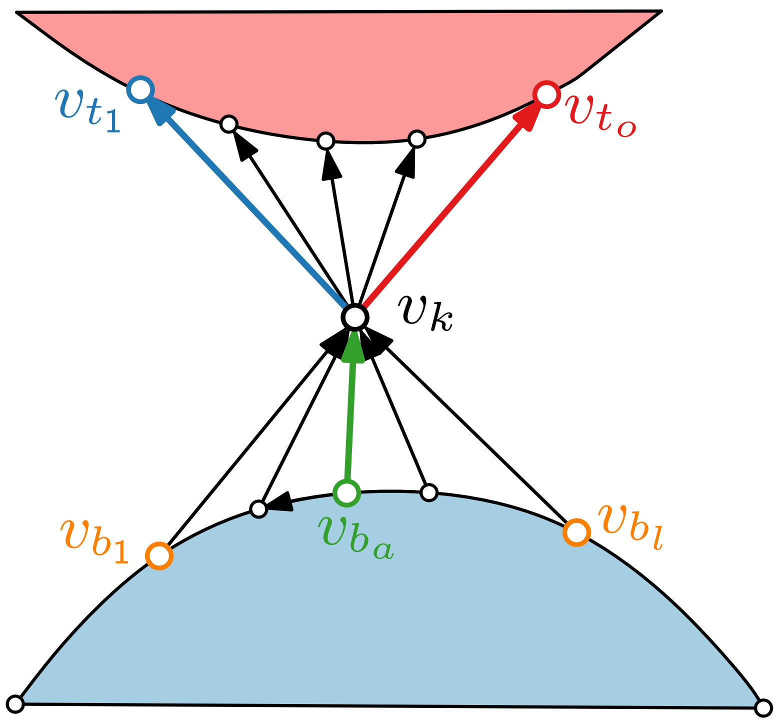

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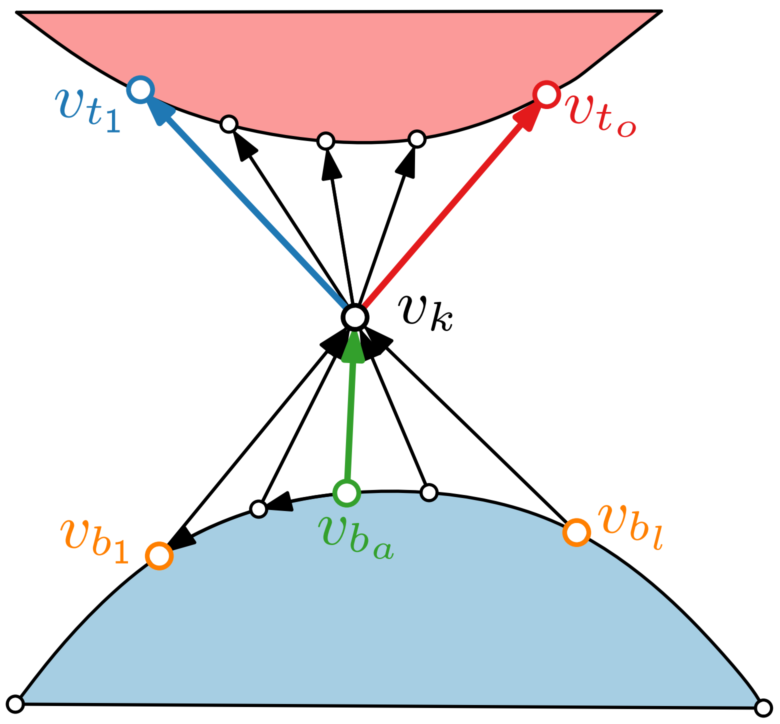

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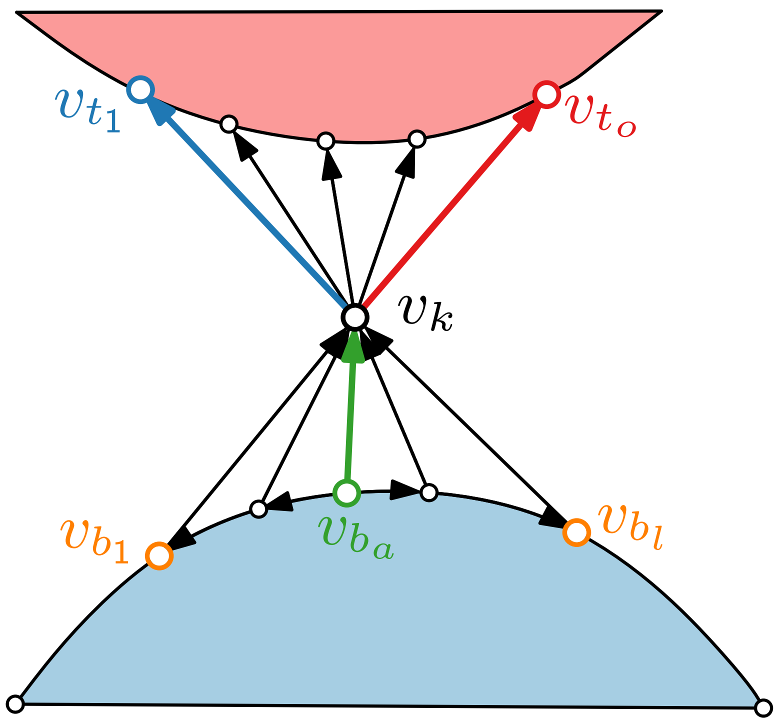

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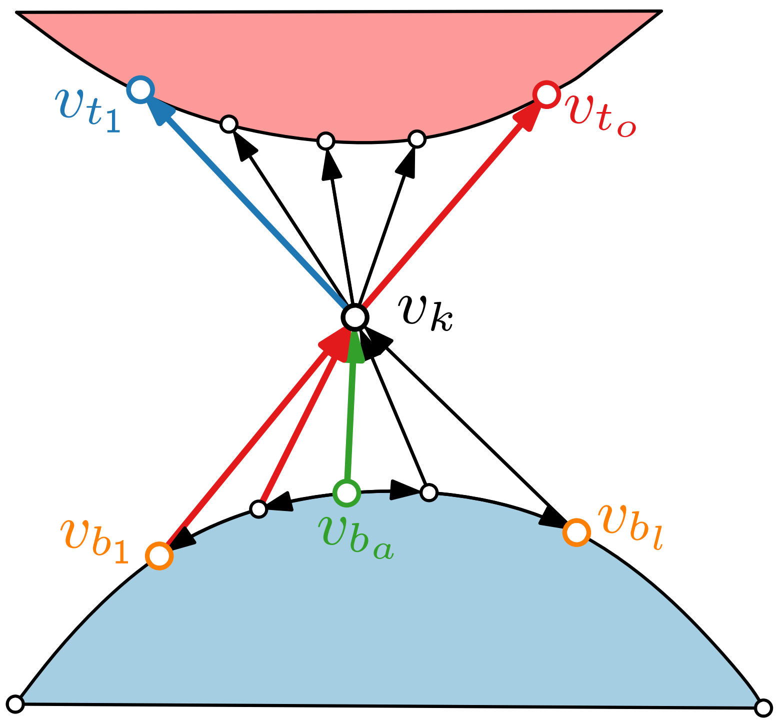

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- Edges (v_{b_i}, v_k), $1 \leq i < a-1$, are right edges.

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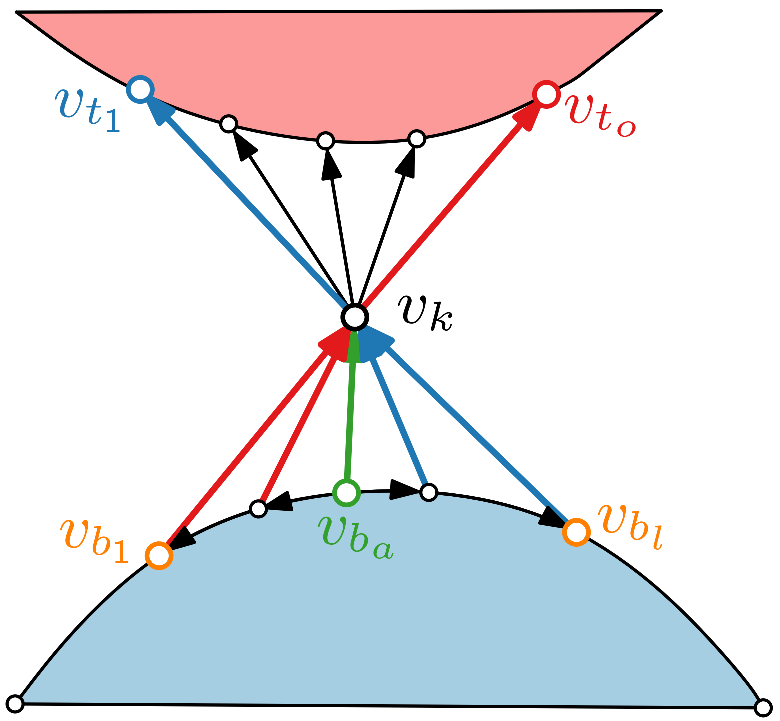
- Similarly, (v_{b_i}, v_k), for $a+1 \leq i \leq l$, are left edges.



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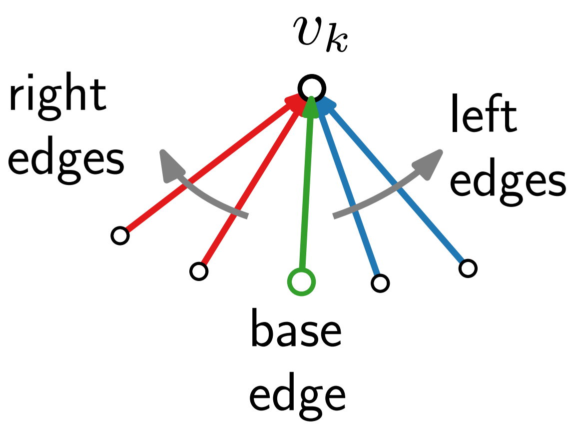

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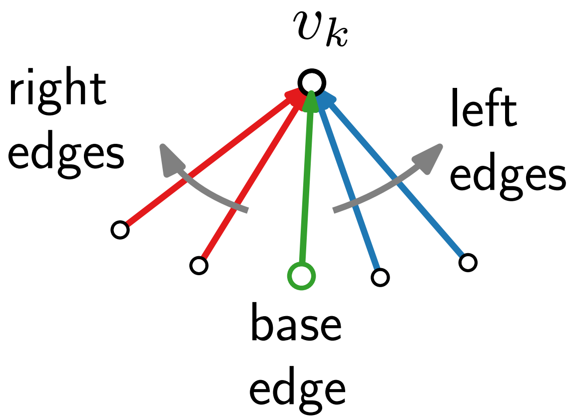

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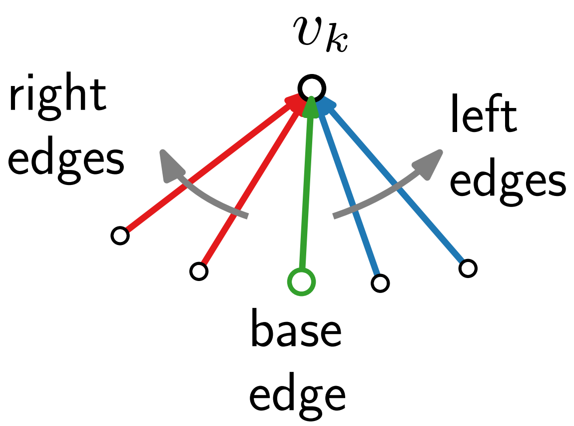

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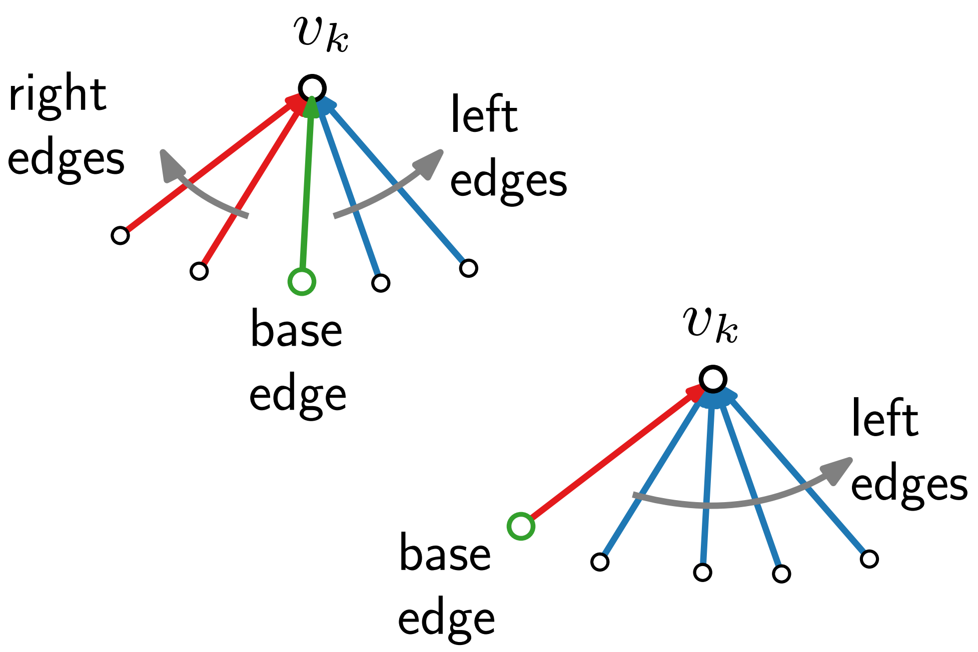

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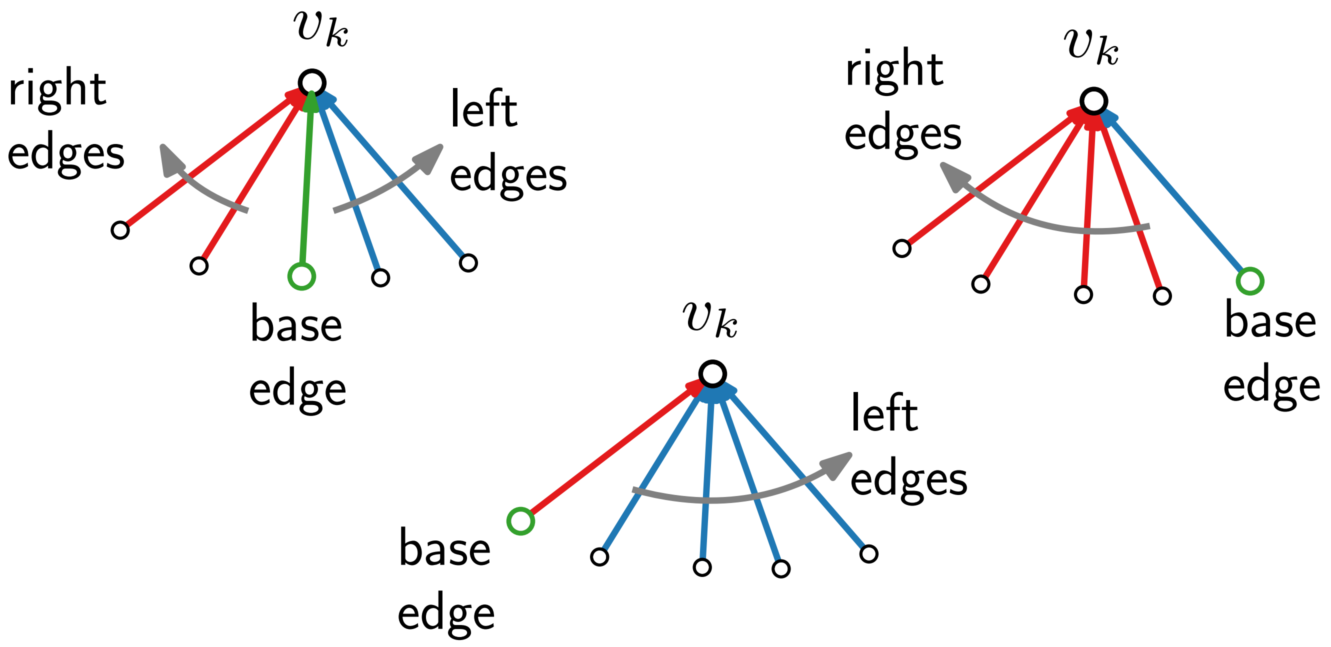

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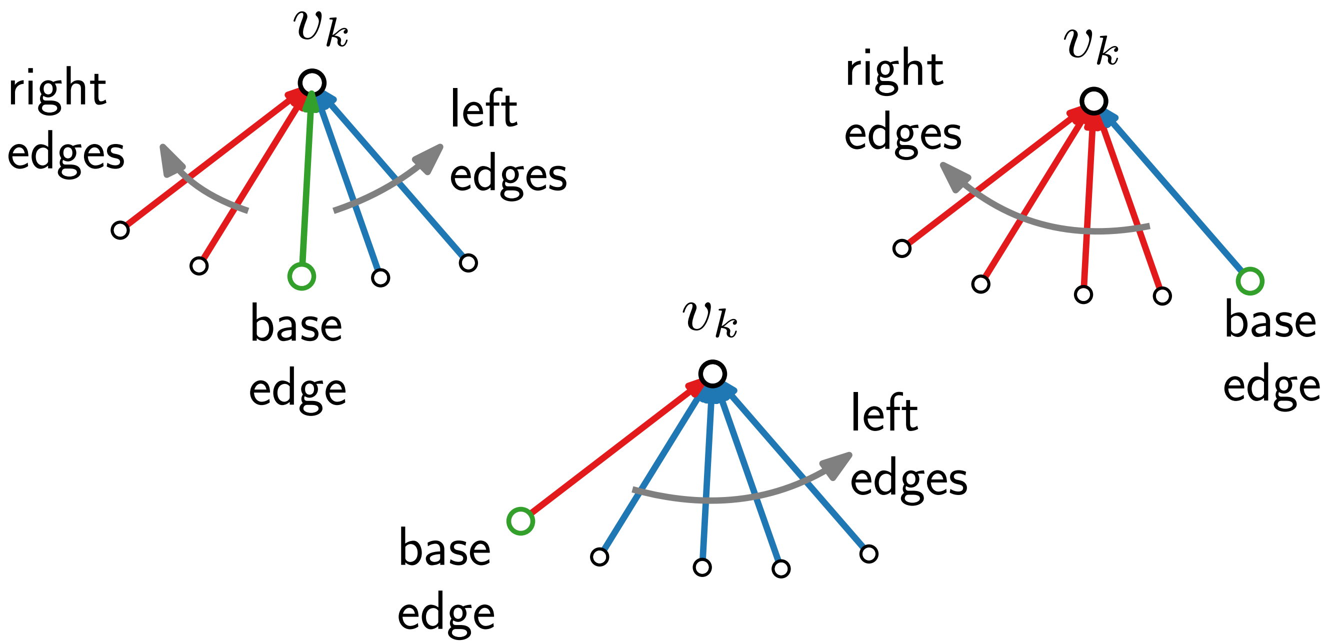

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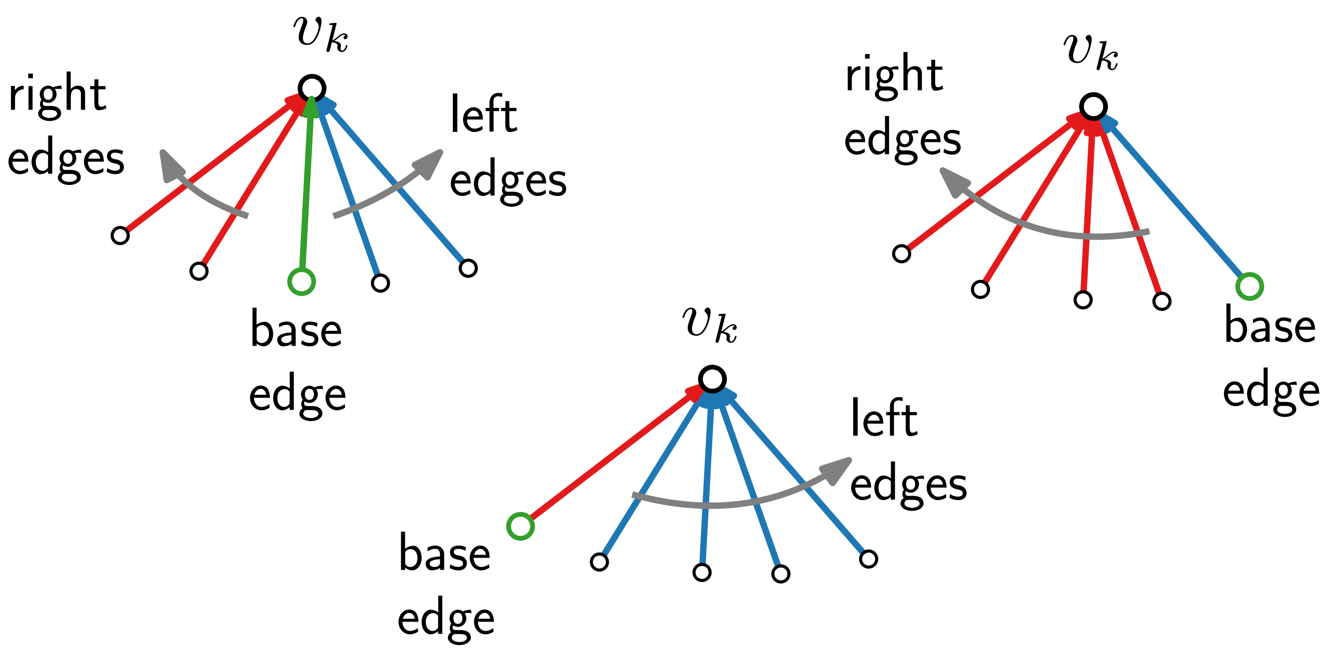

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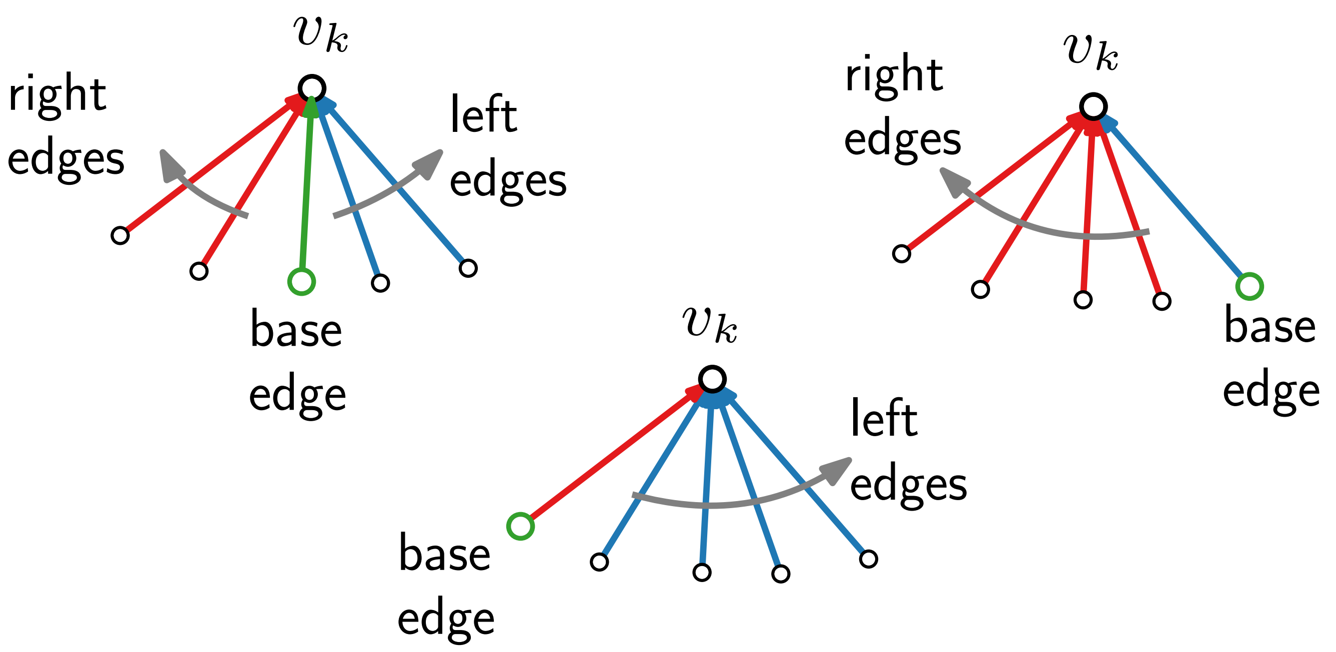
Refined Canonical Order → REL



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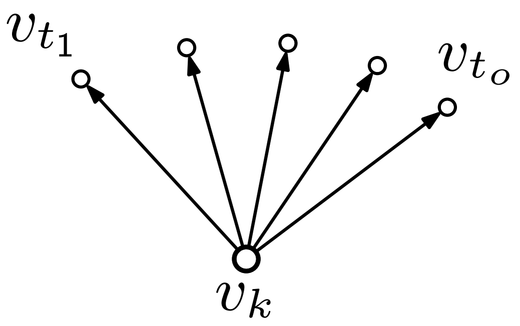

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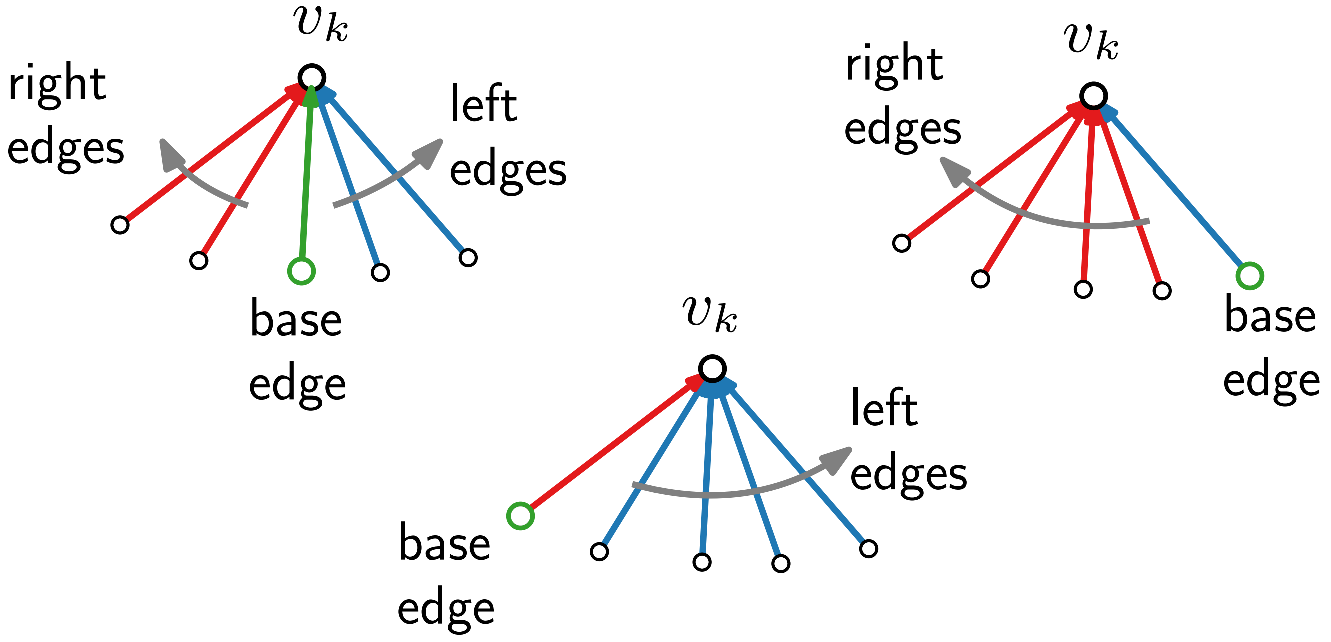

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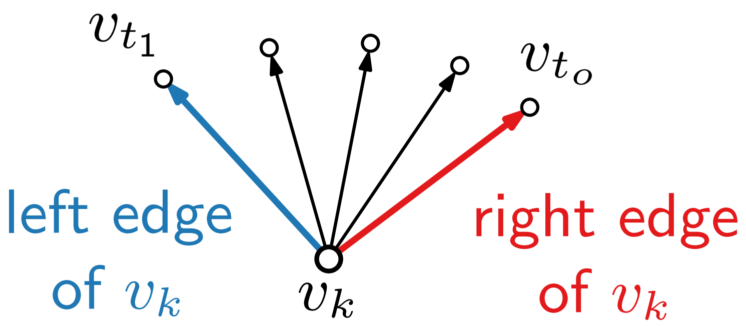

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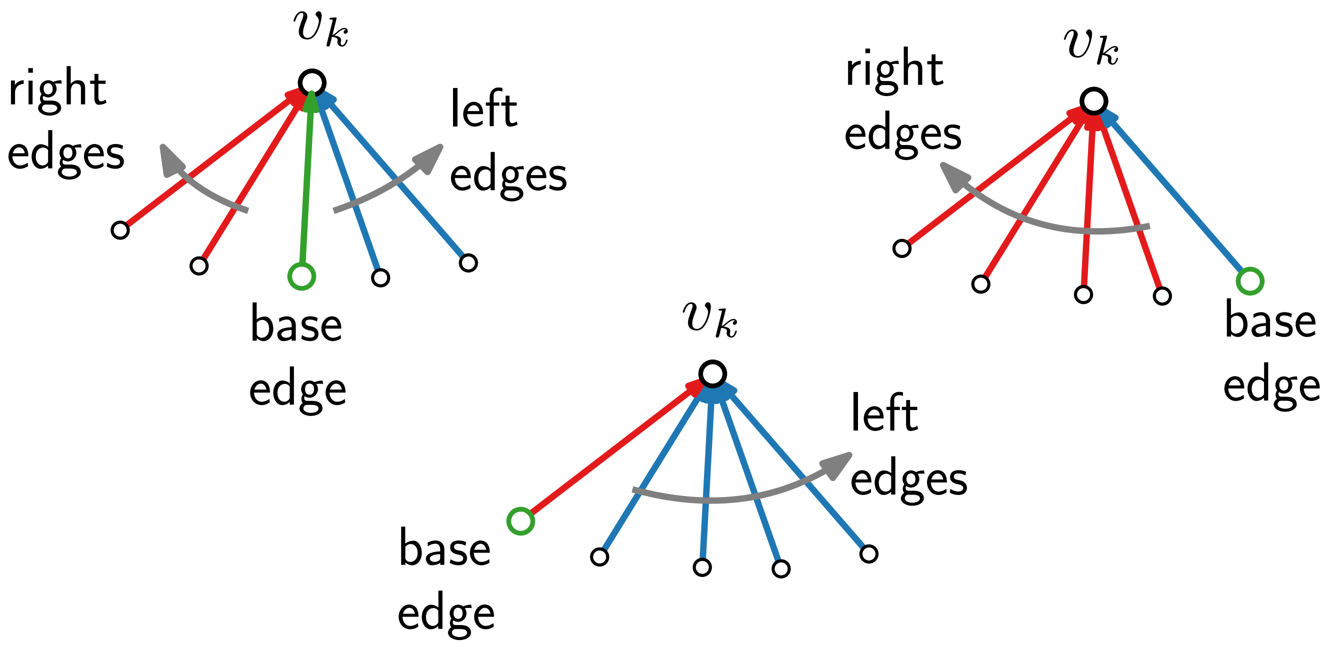

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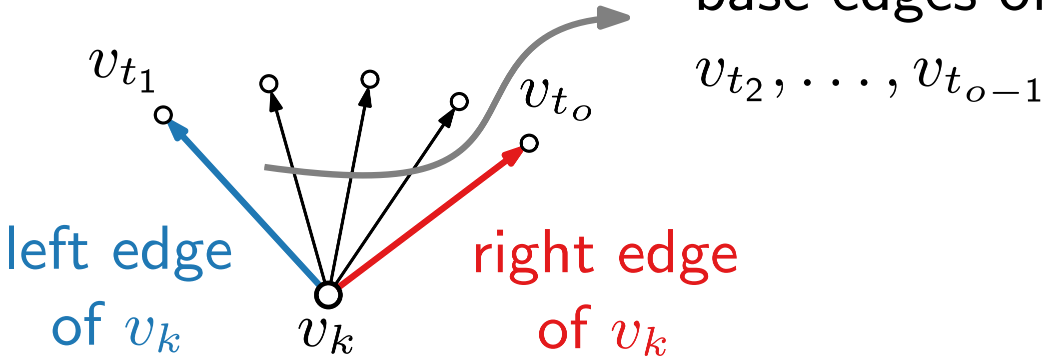

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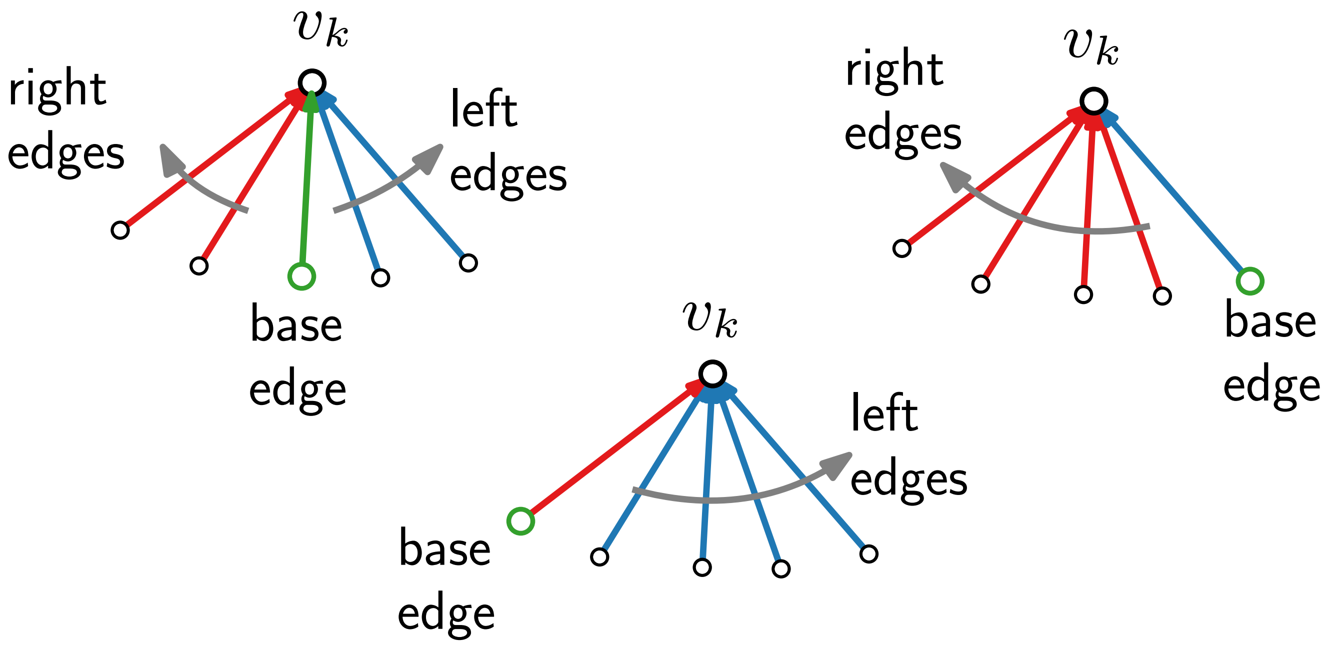

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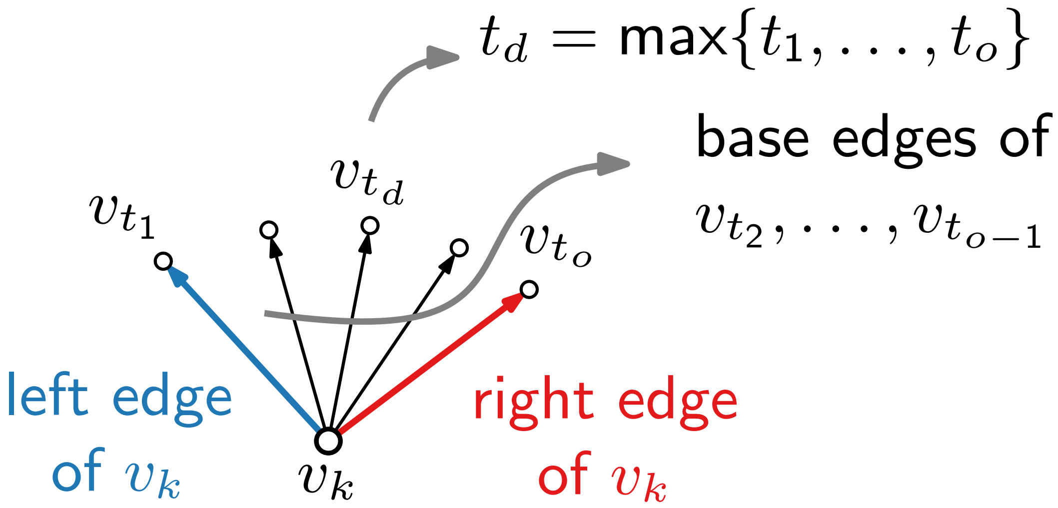

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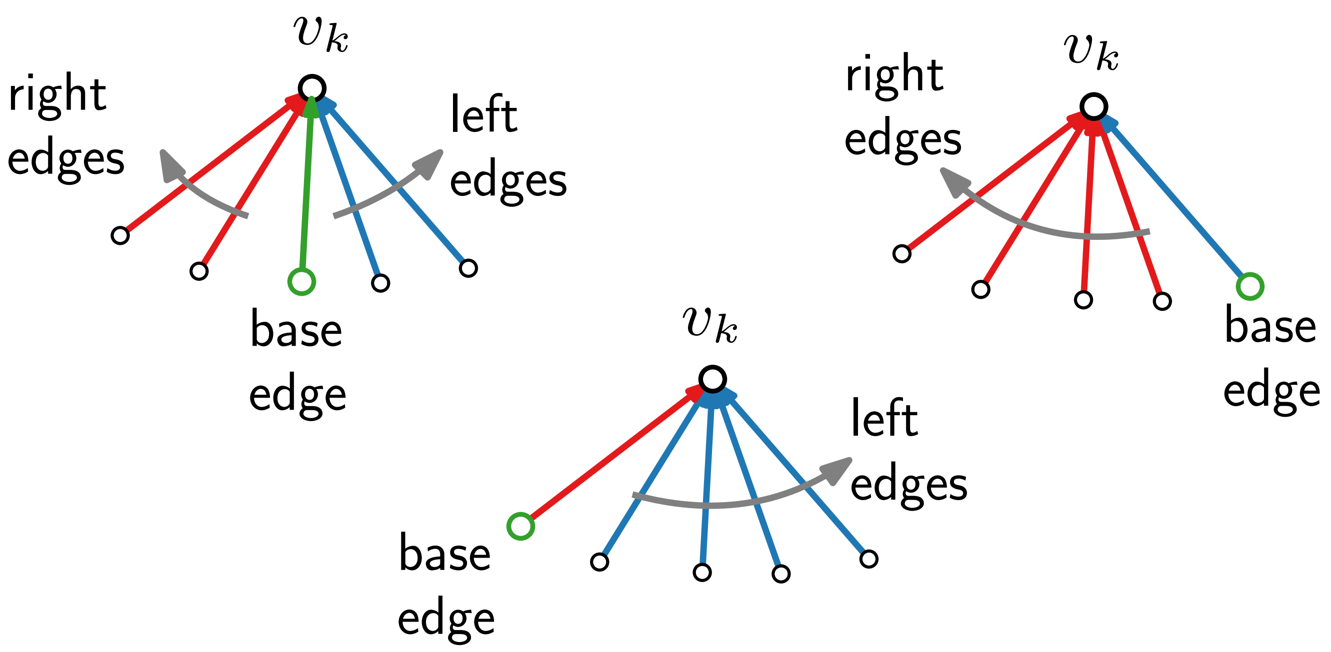

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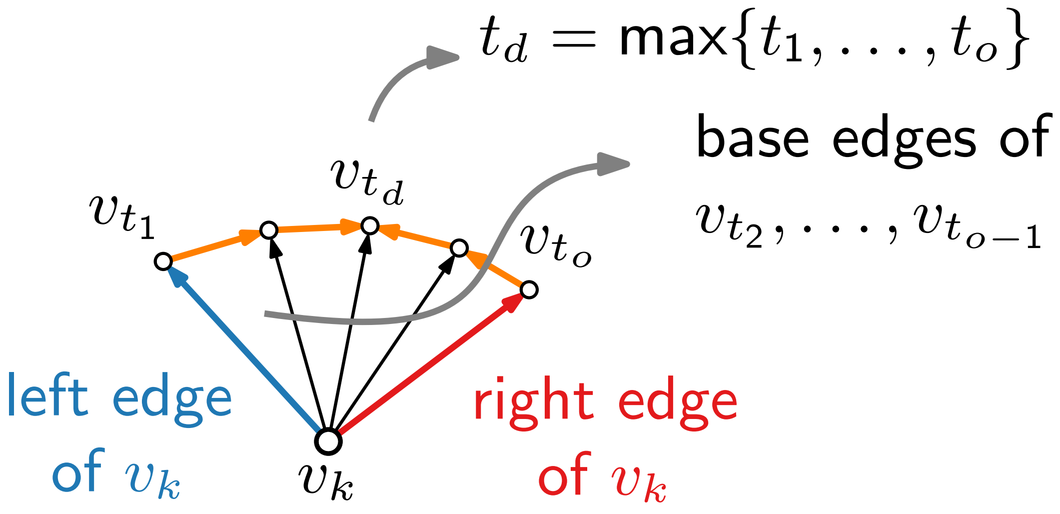

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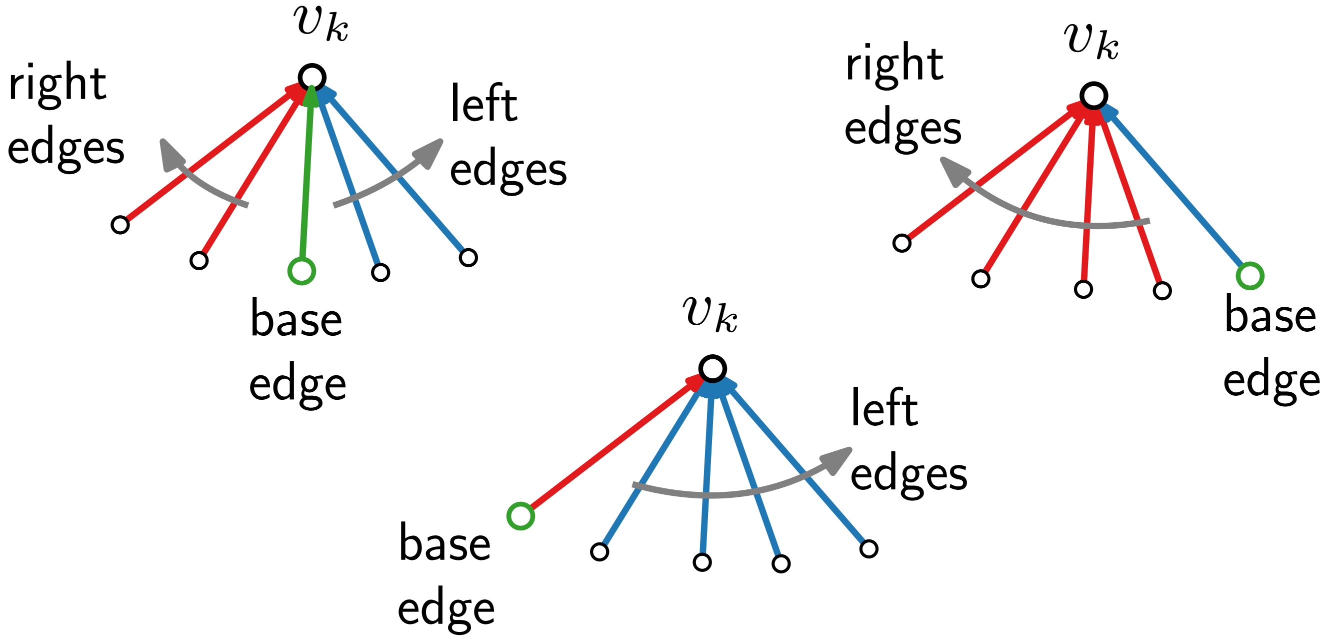
Refined Canonical Order → REL



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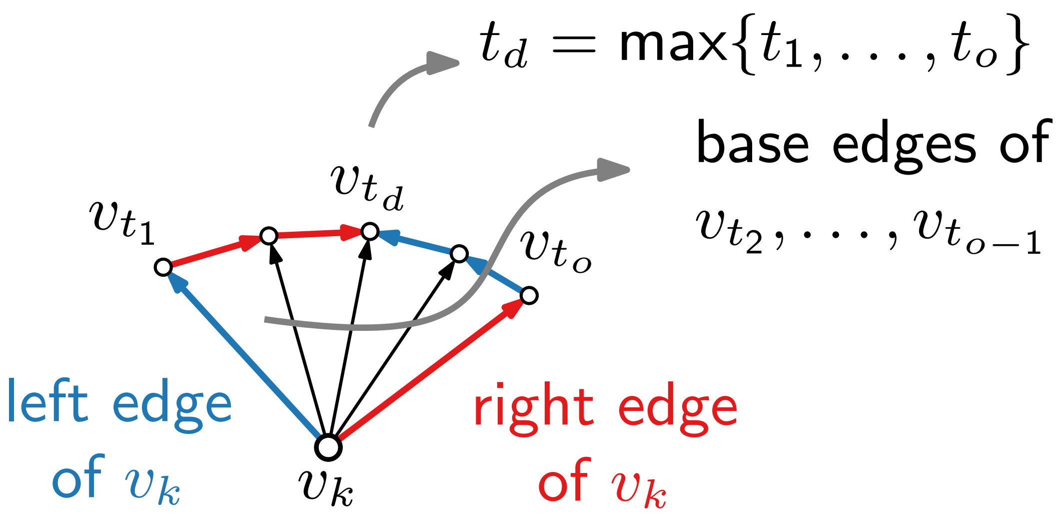

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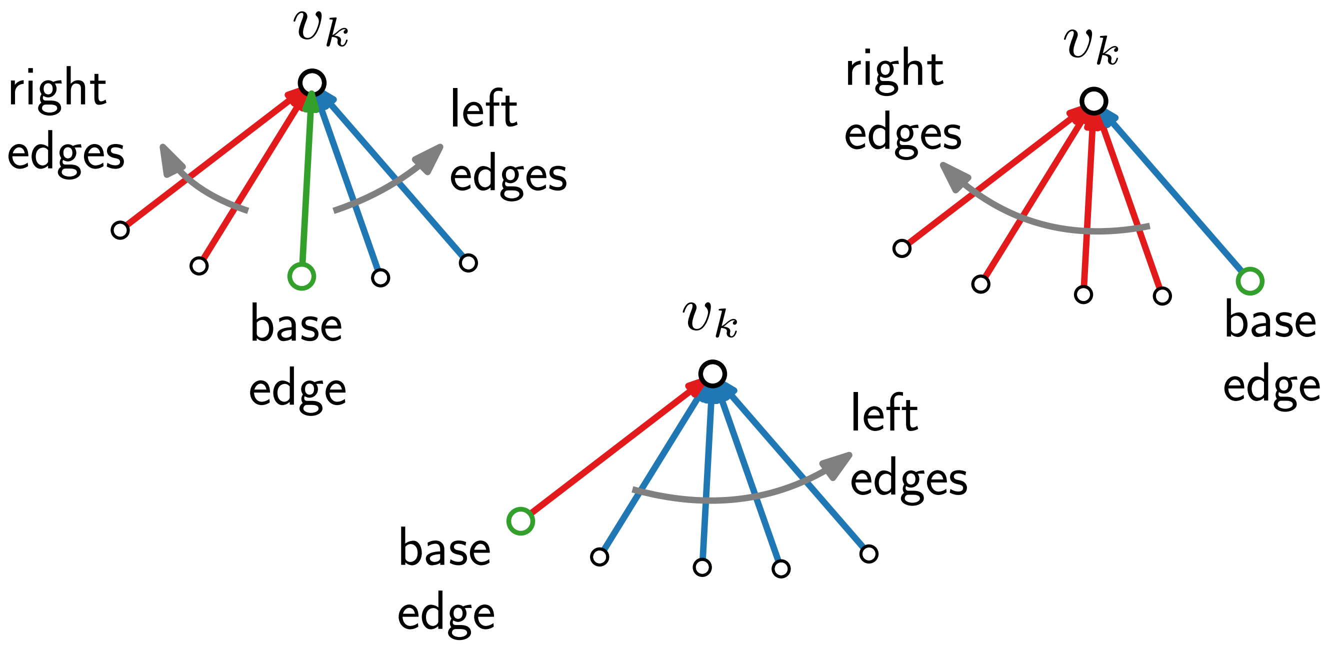

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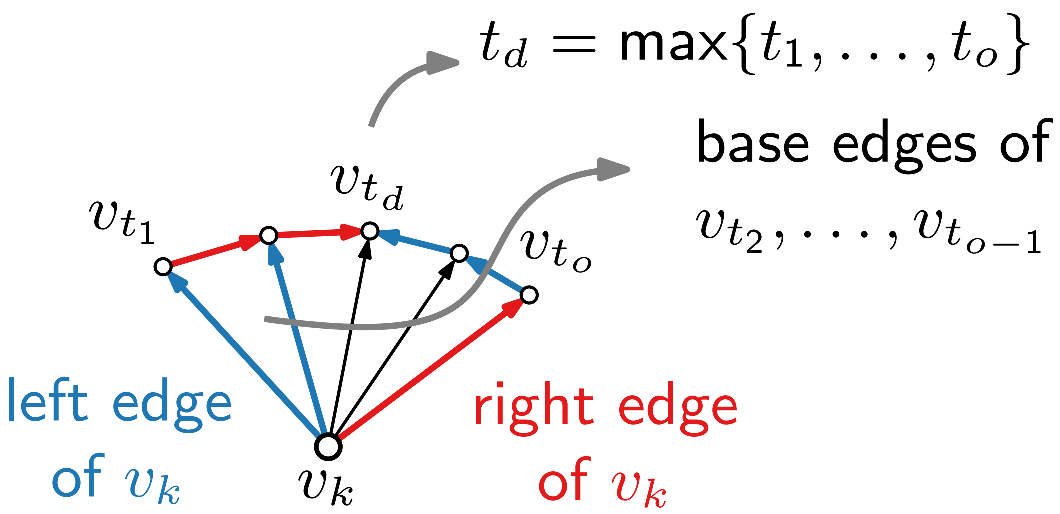

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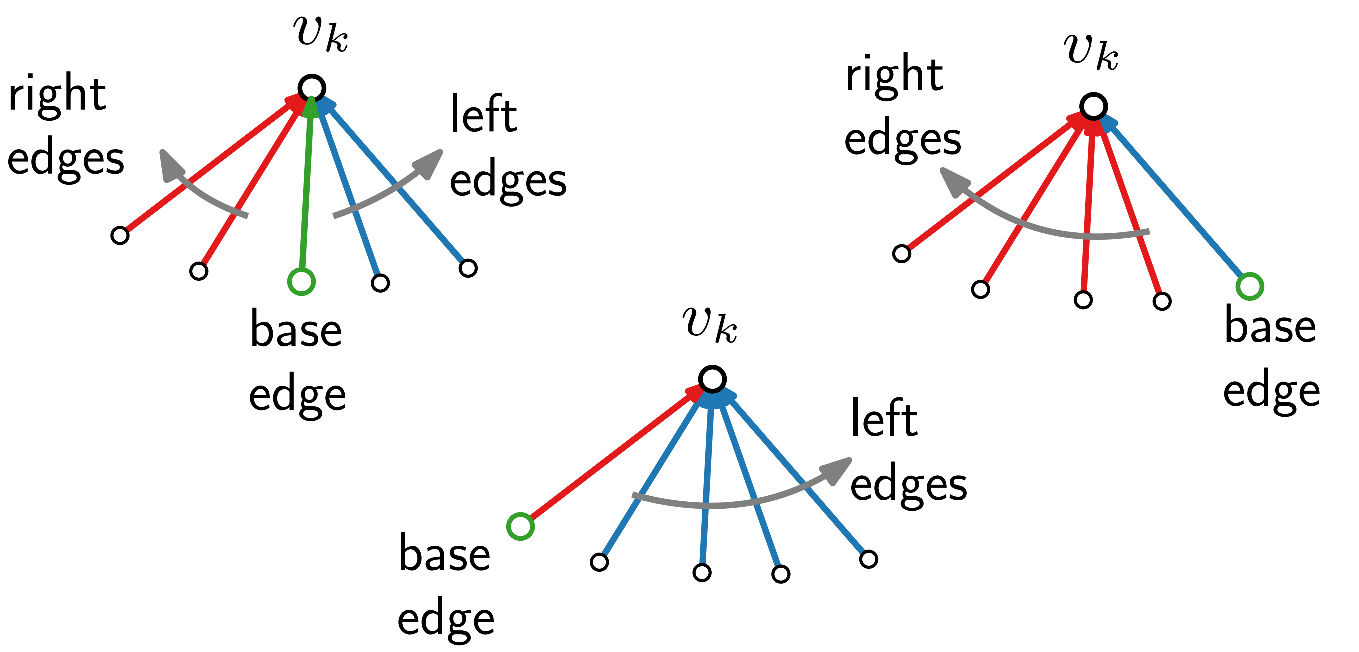
Refined Canonical Order → REL



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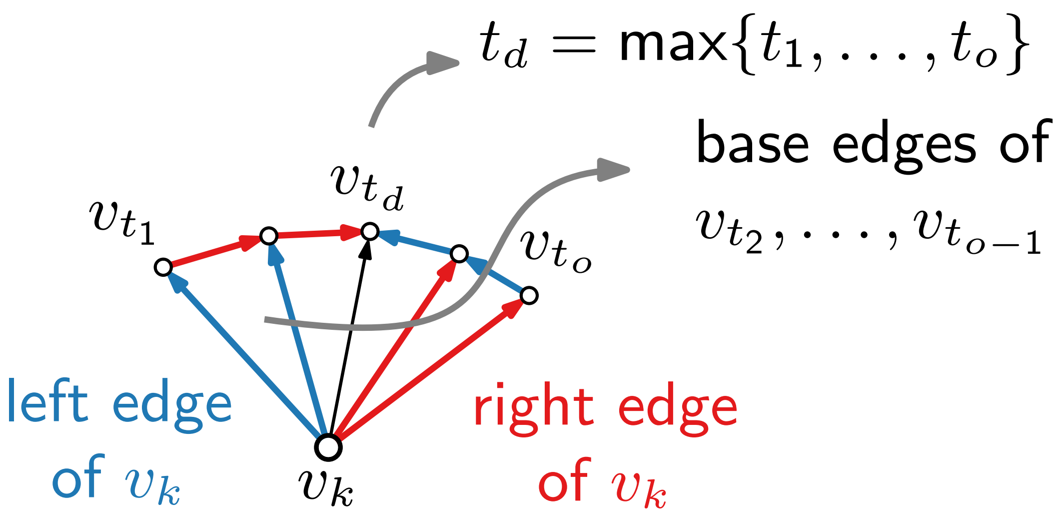

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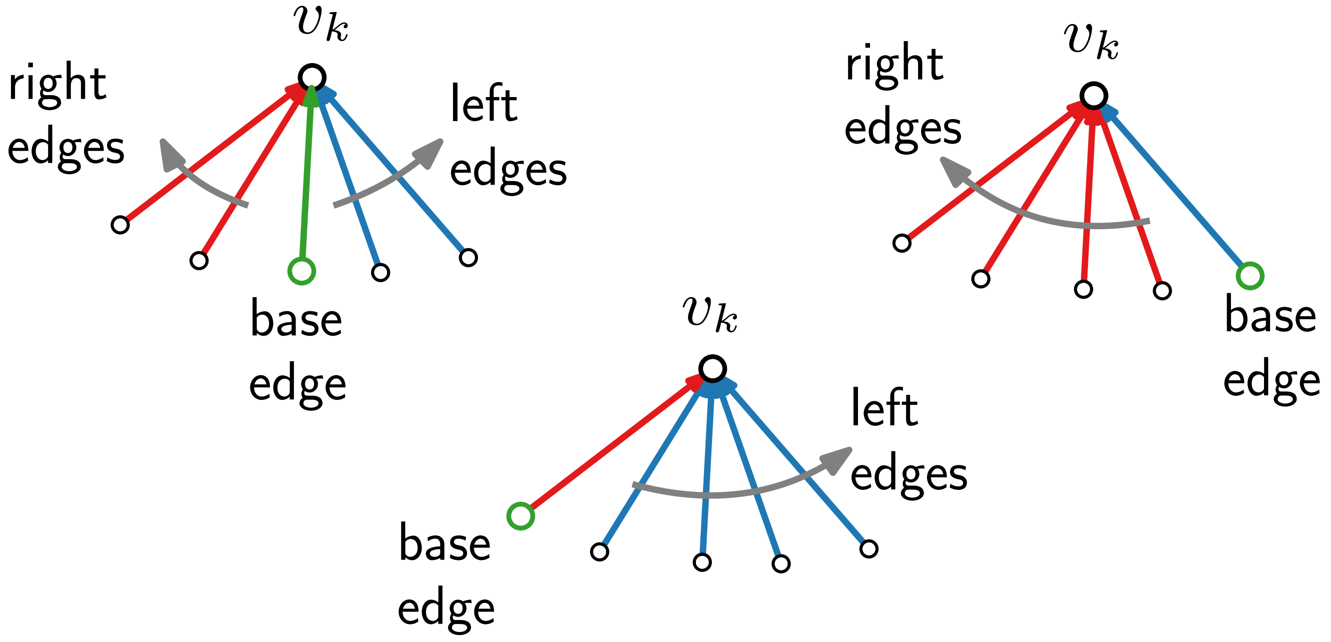
Refined Canonical Order → REL



Coloring.

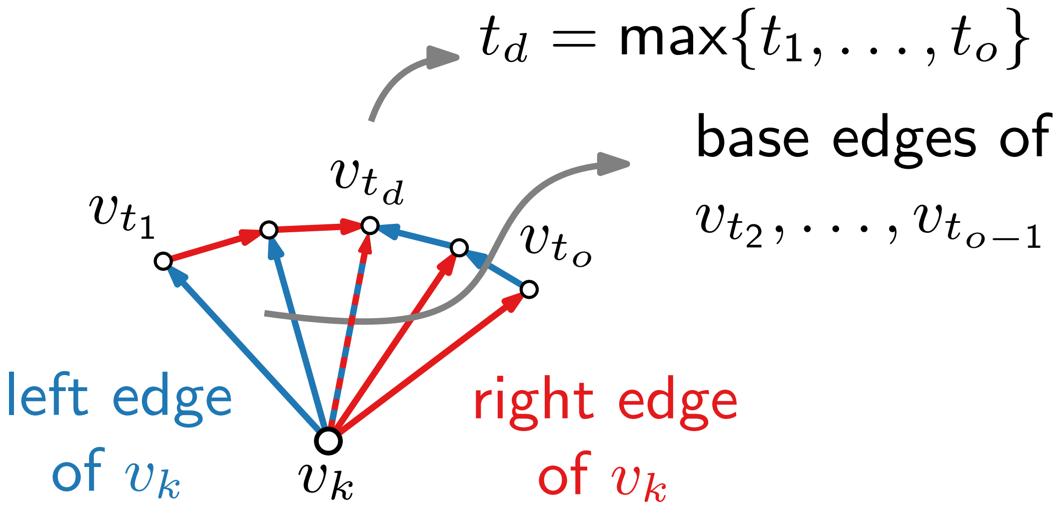
- Color right (left) edges in red (blue).
- Color a base edge (v_{b_i}, v_k) red if $i = 1$ and blue if $i = l$ and otherwise arbitrarily.

Let T_r be the red edges and T_b the blue edges.

> **Lemma 3.**
> $\{T_r, T_b\}$ is a regular edge labeling.

Proof.

$t_o \geq 2$



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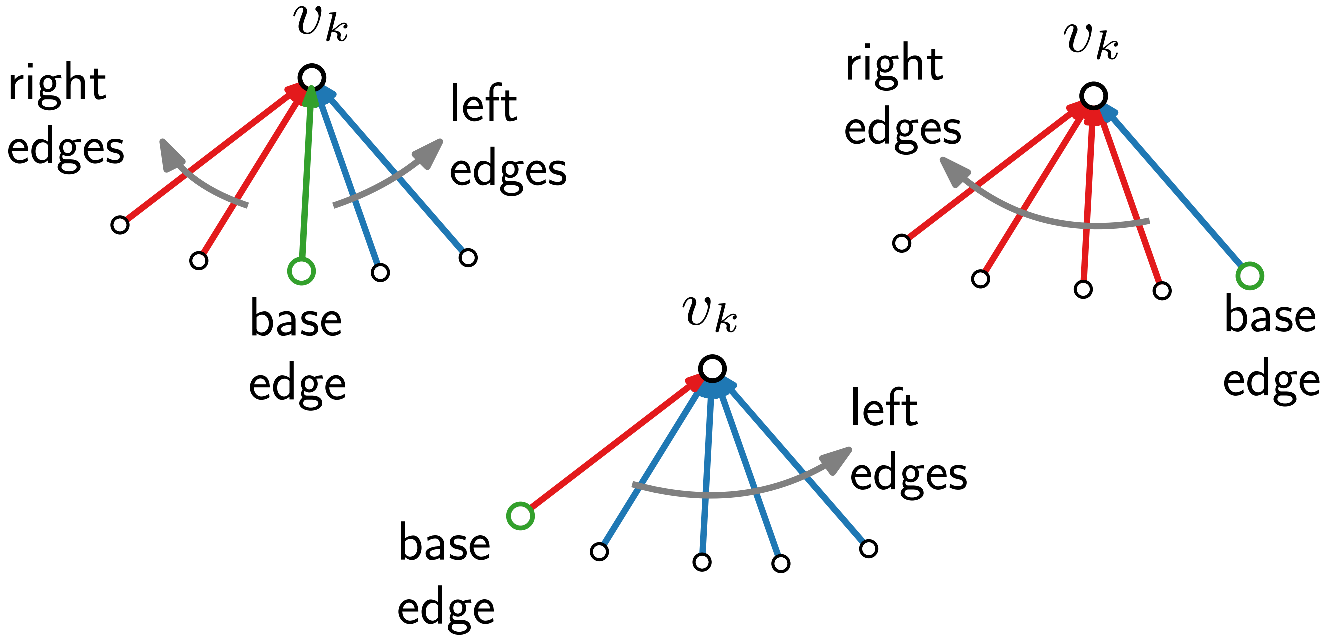

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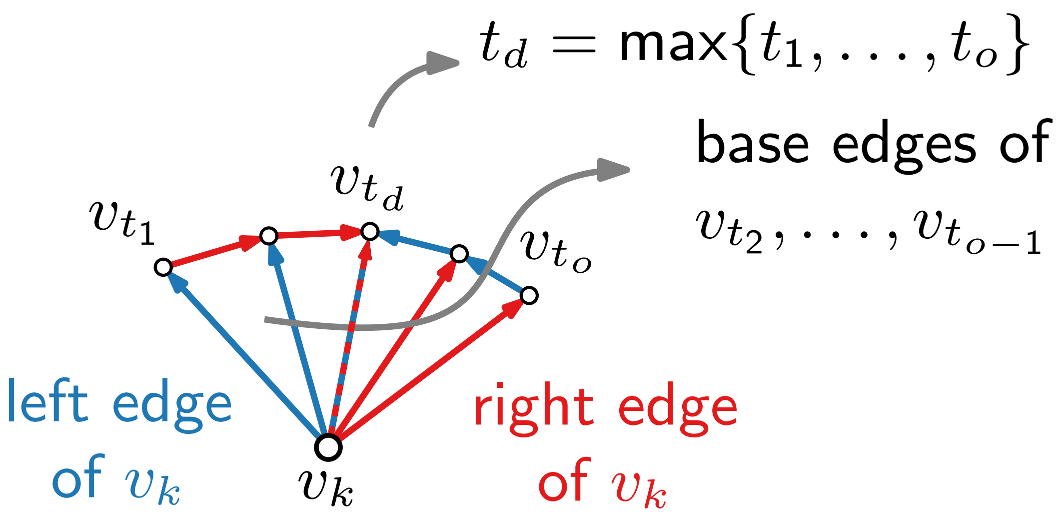

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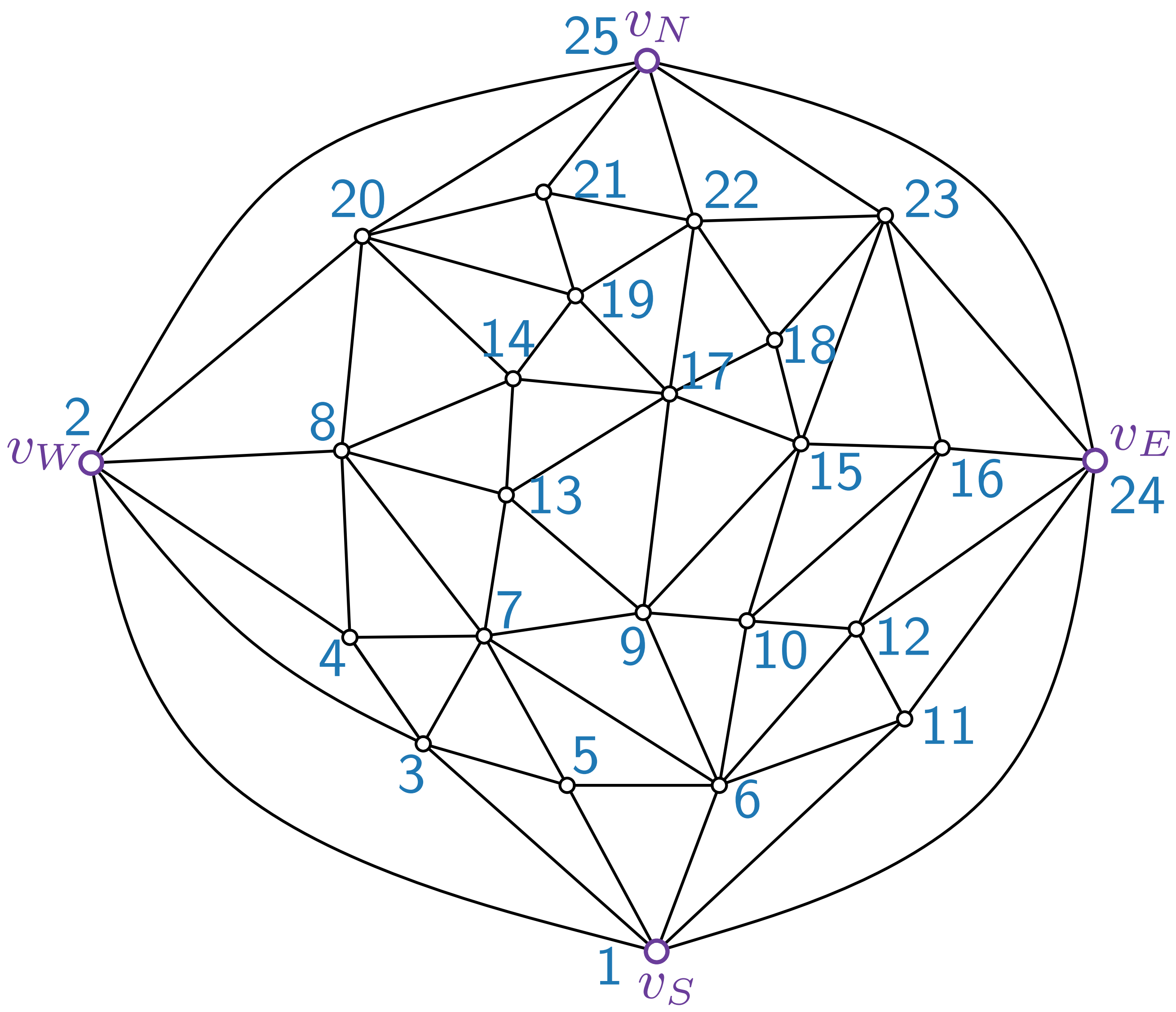
⇒ Circular order of outgoing edges at v_k correct.

From REL to st-Digraphs to Coordinates



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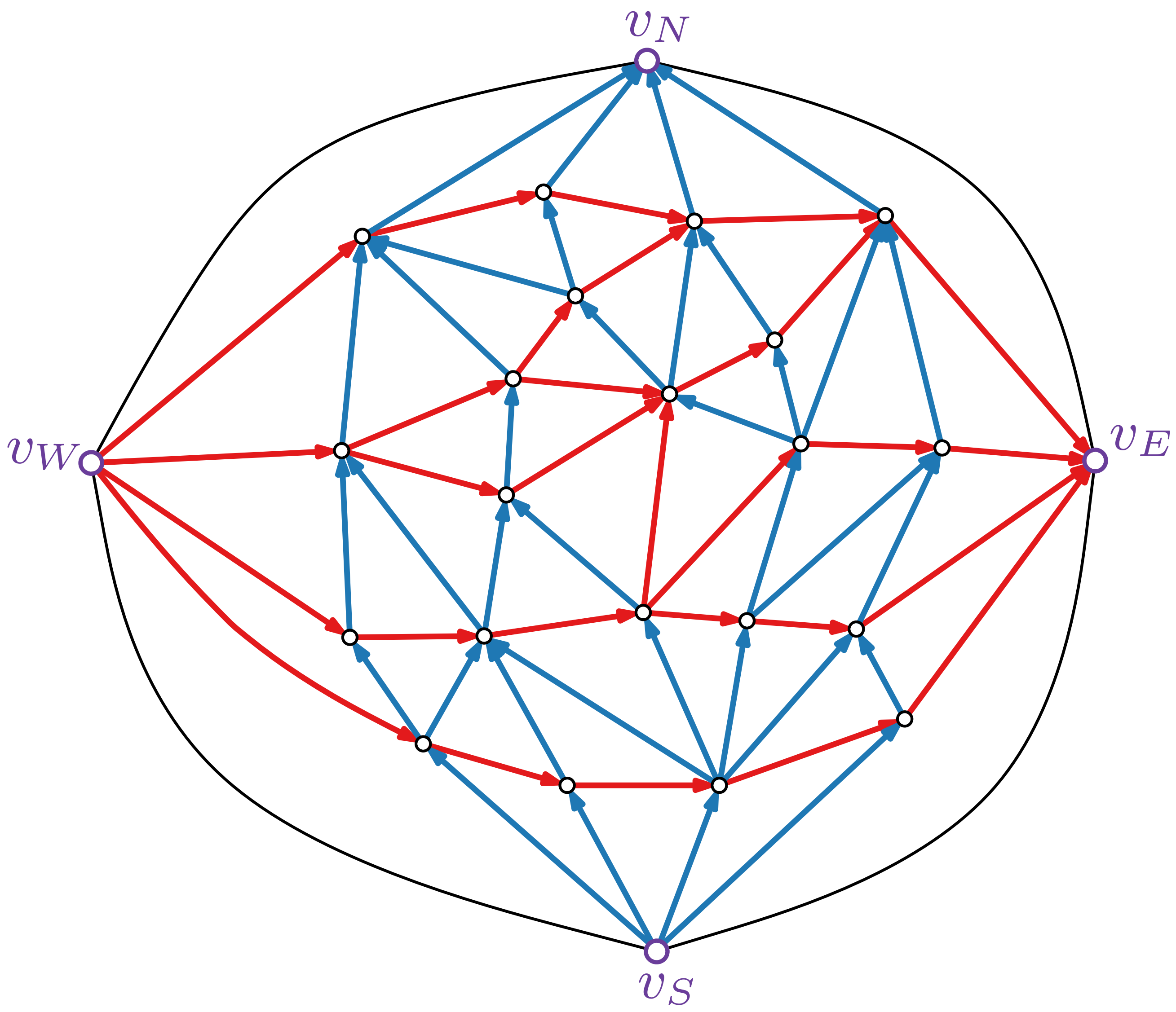

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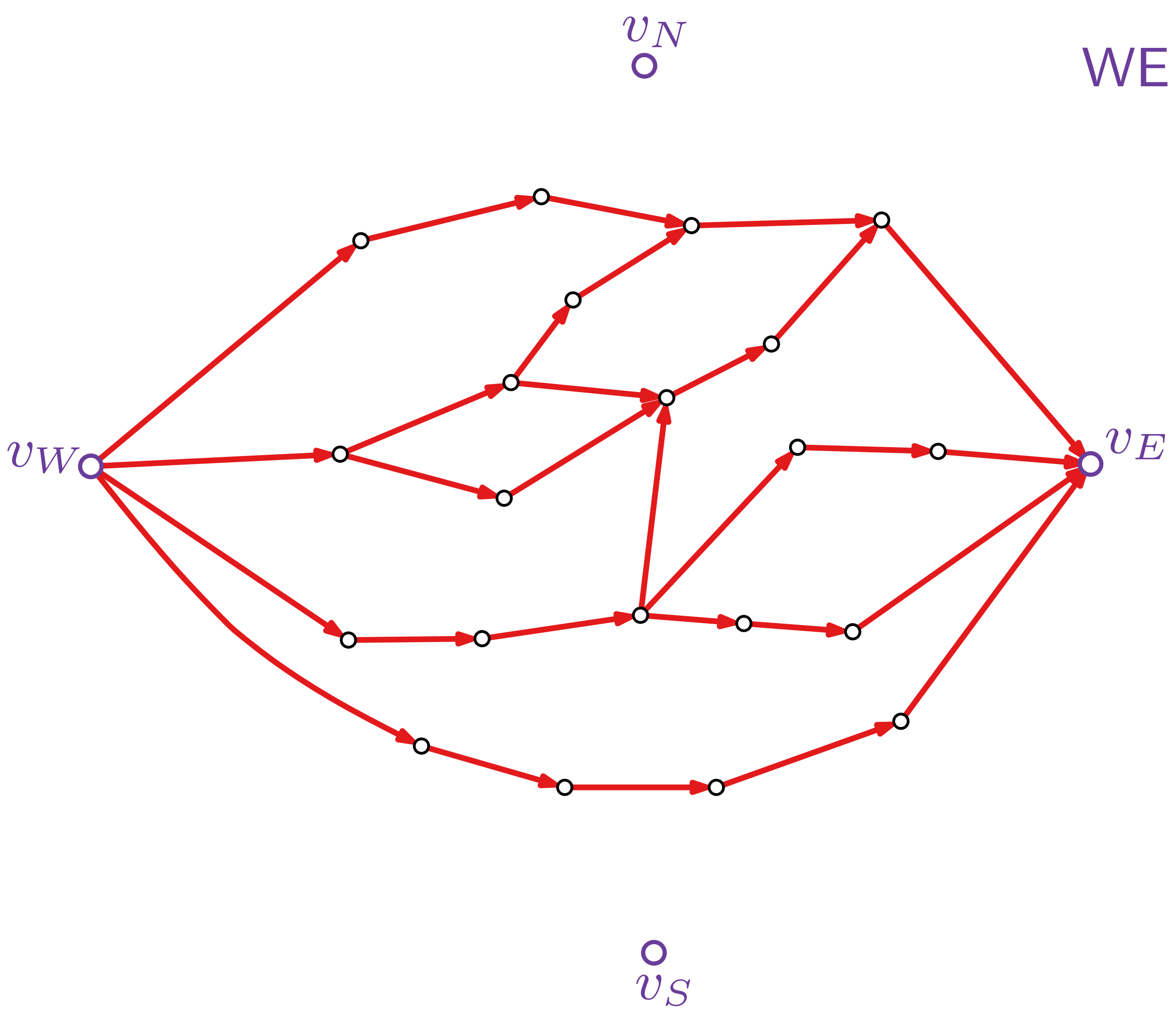
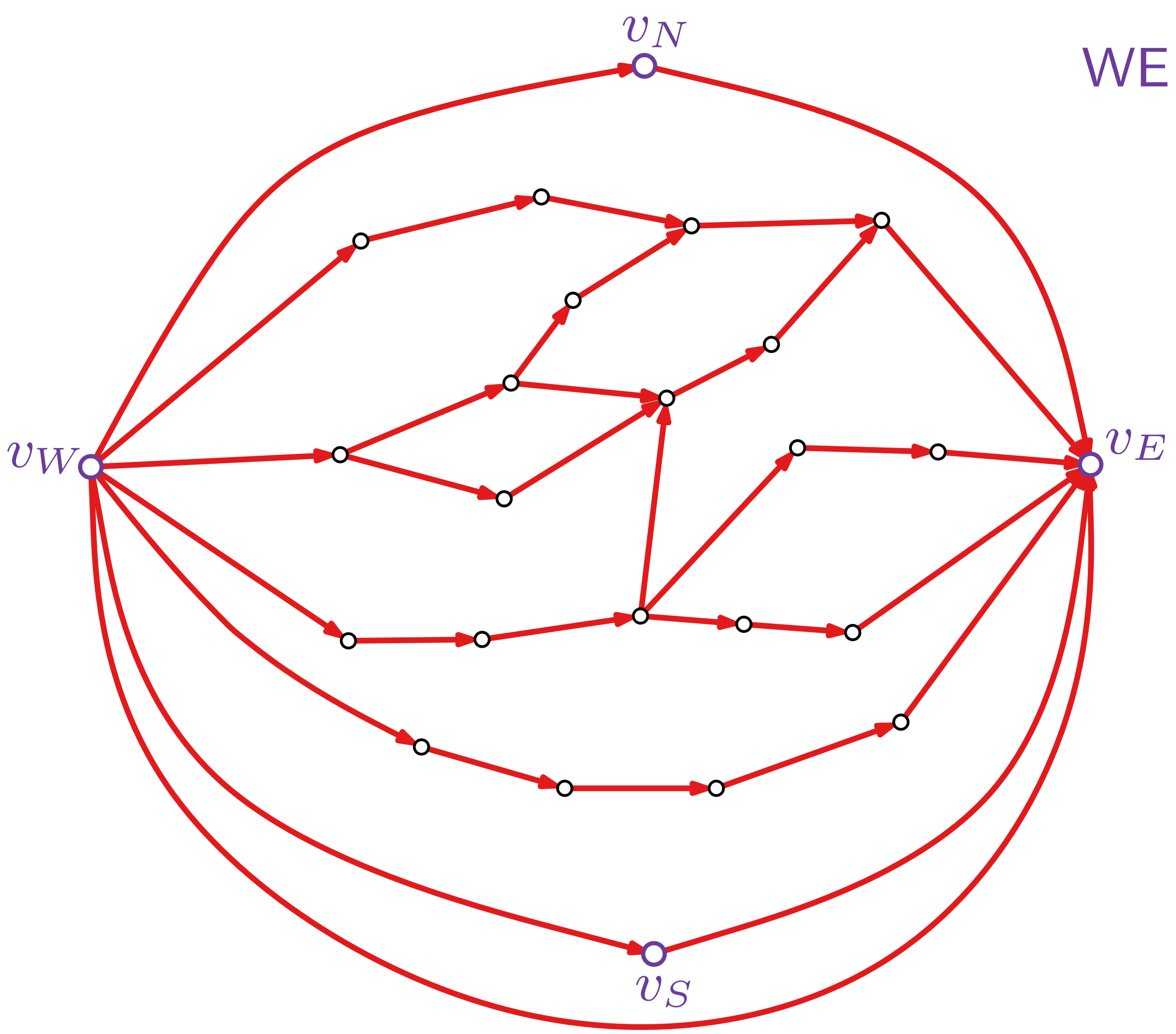


WE network G_{hor}

From REL to st-Digraphs to Coordinates



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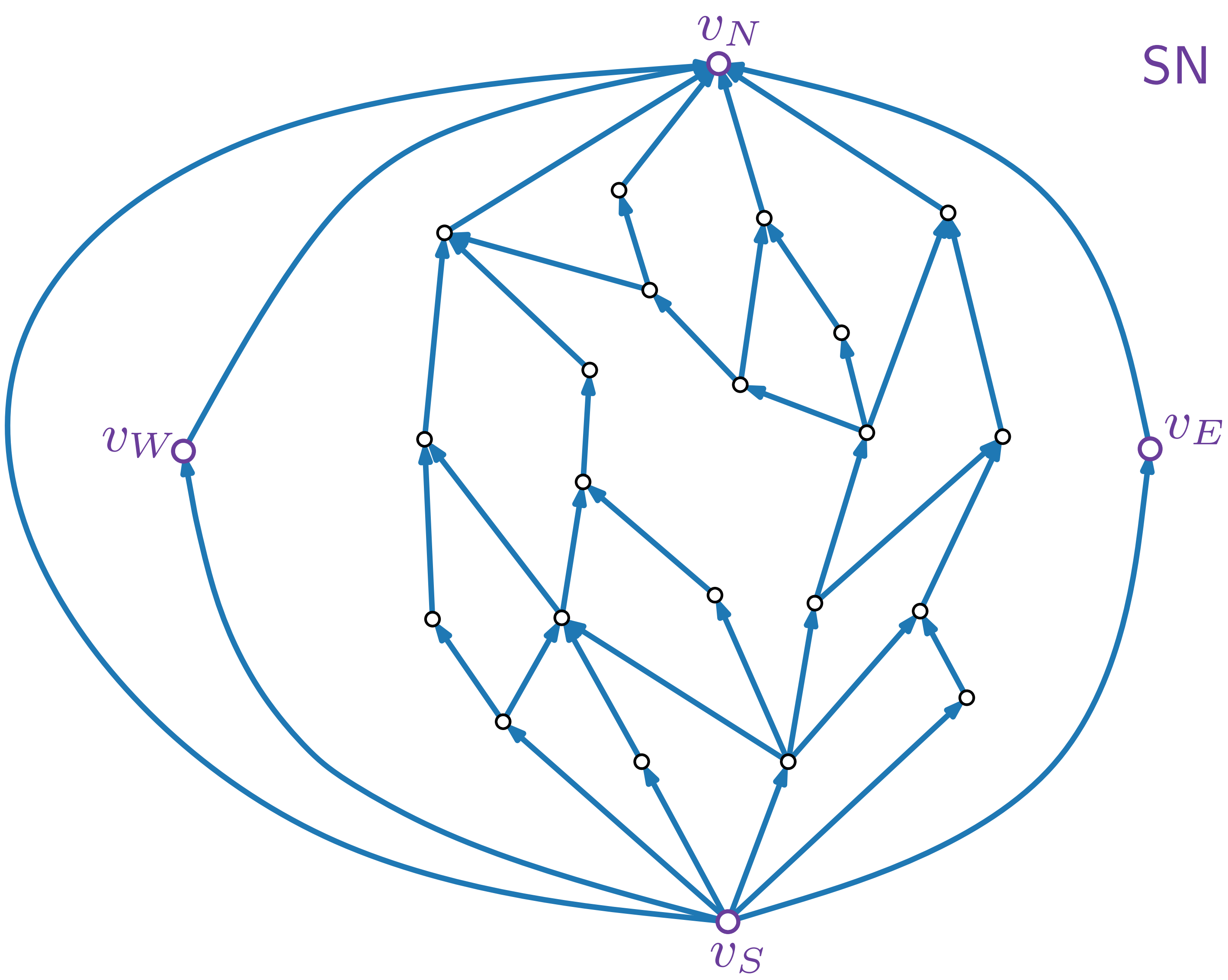

SN network G_{ver}

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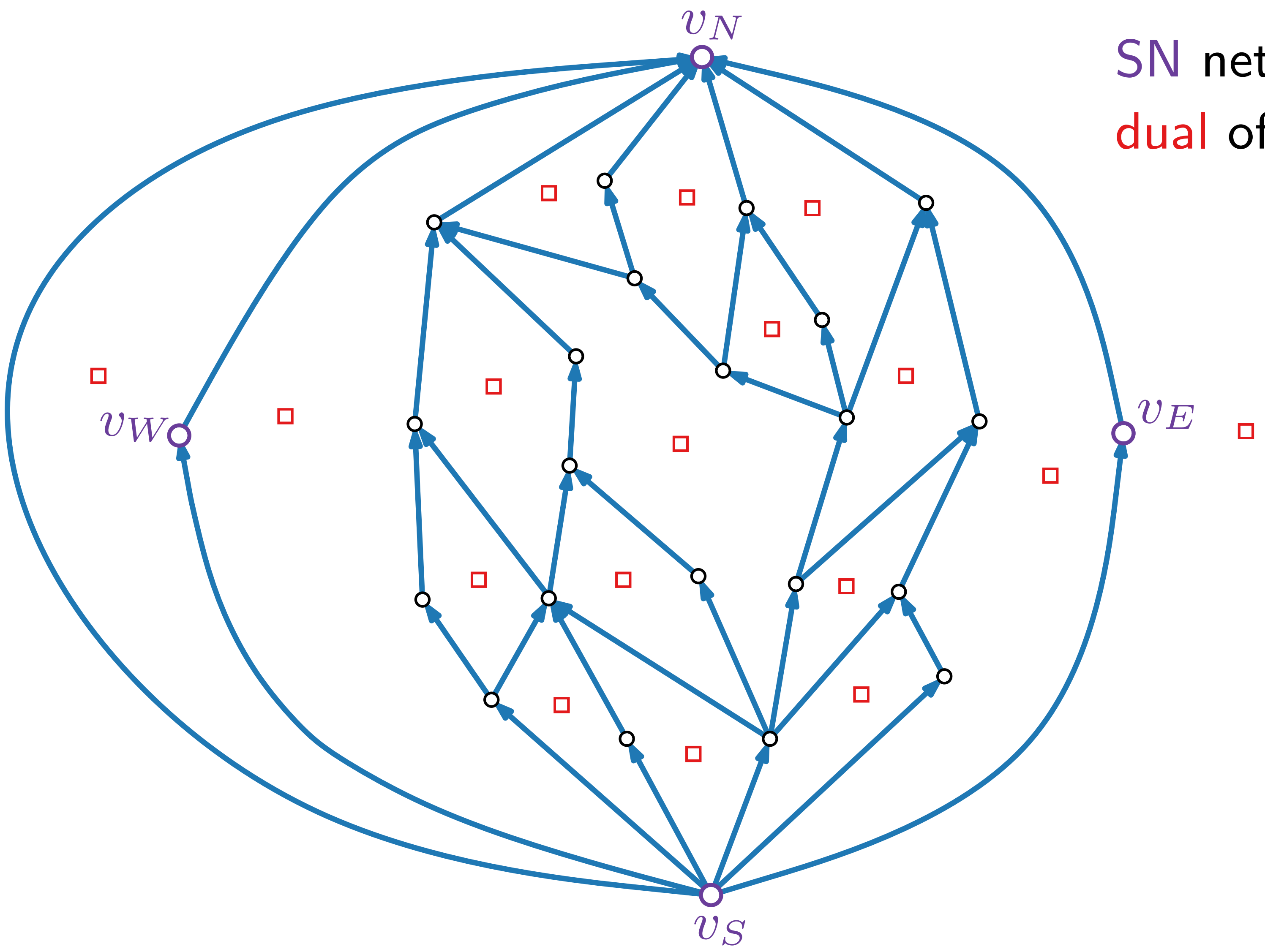

SN network G_{ver}

dual of G_{ver}

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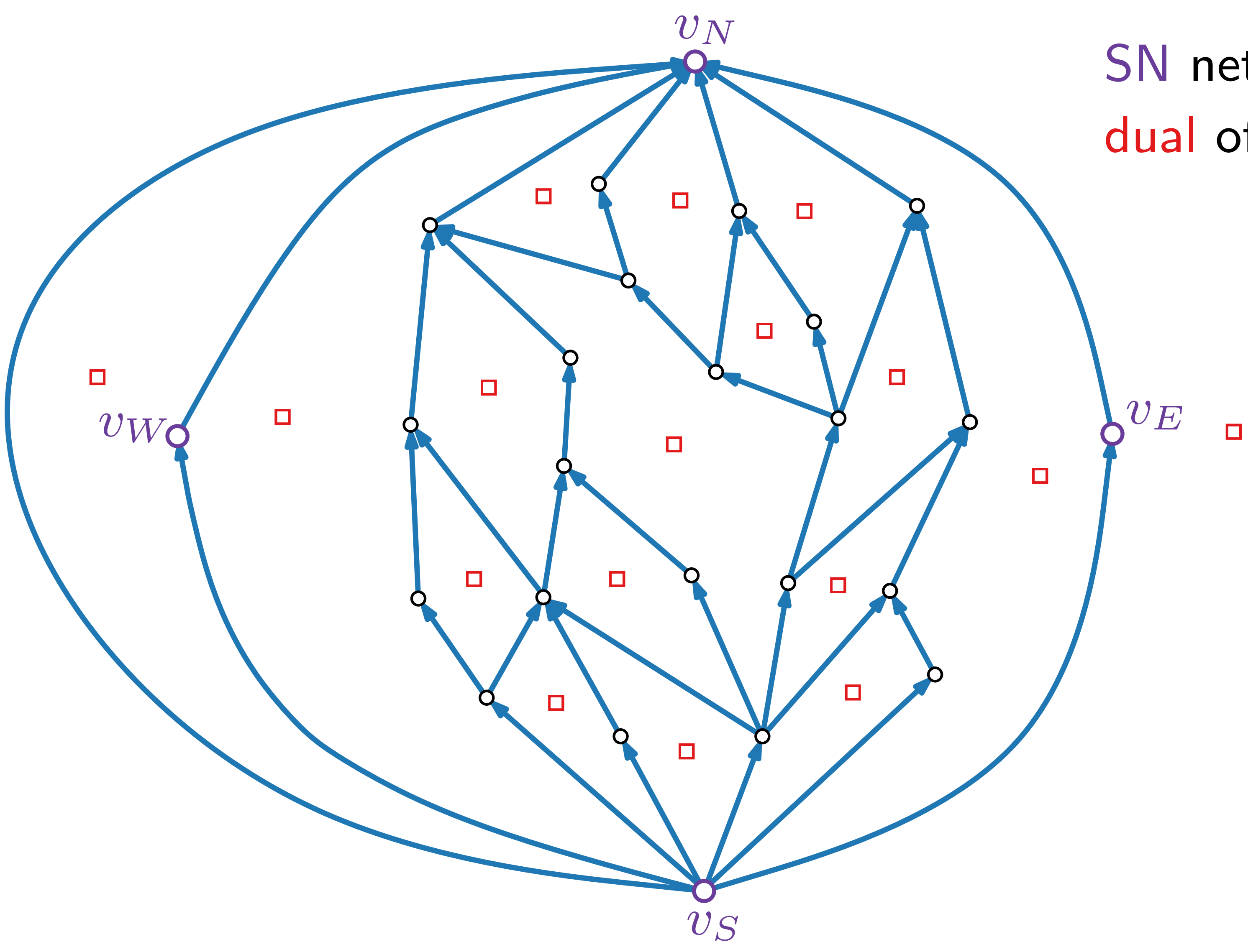

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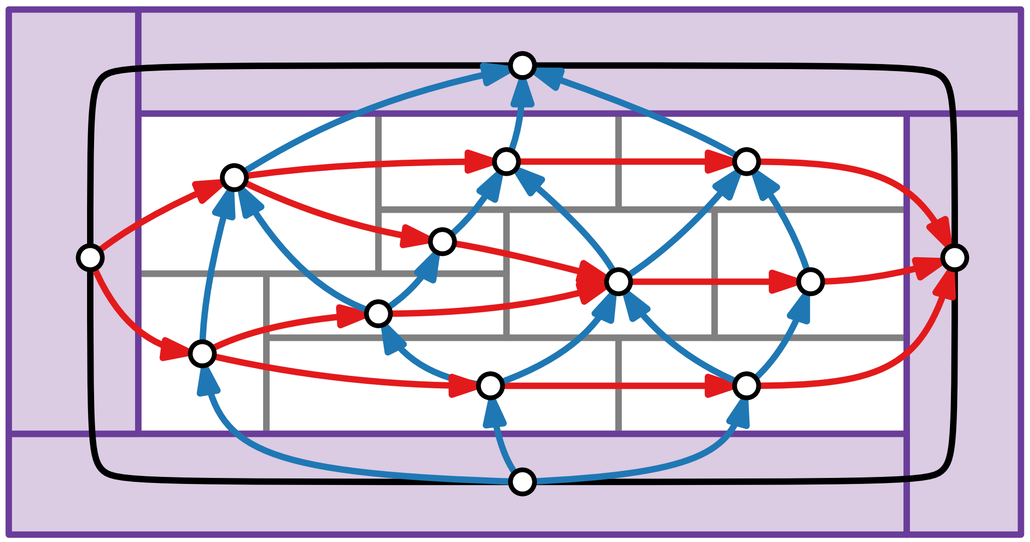

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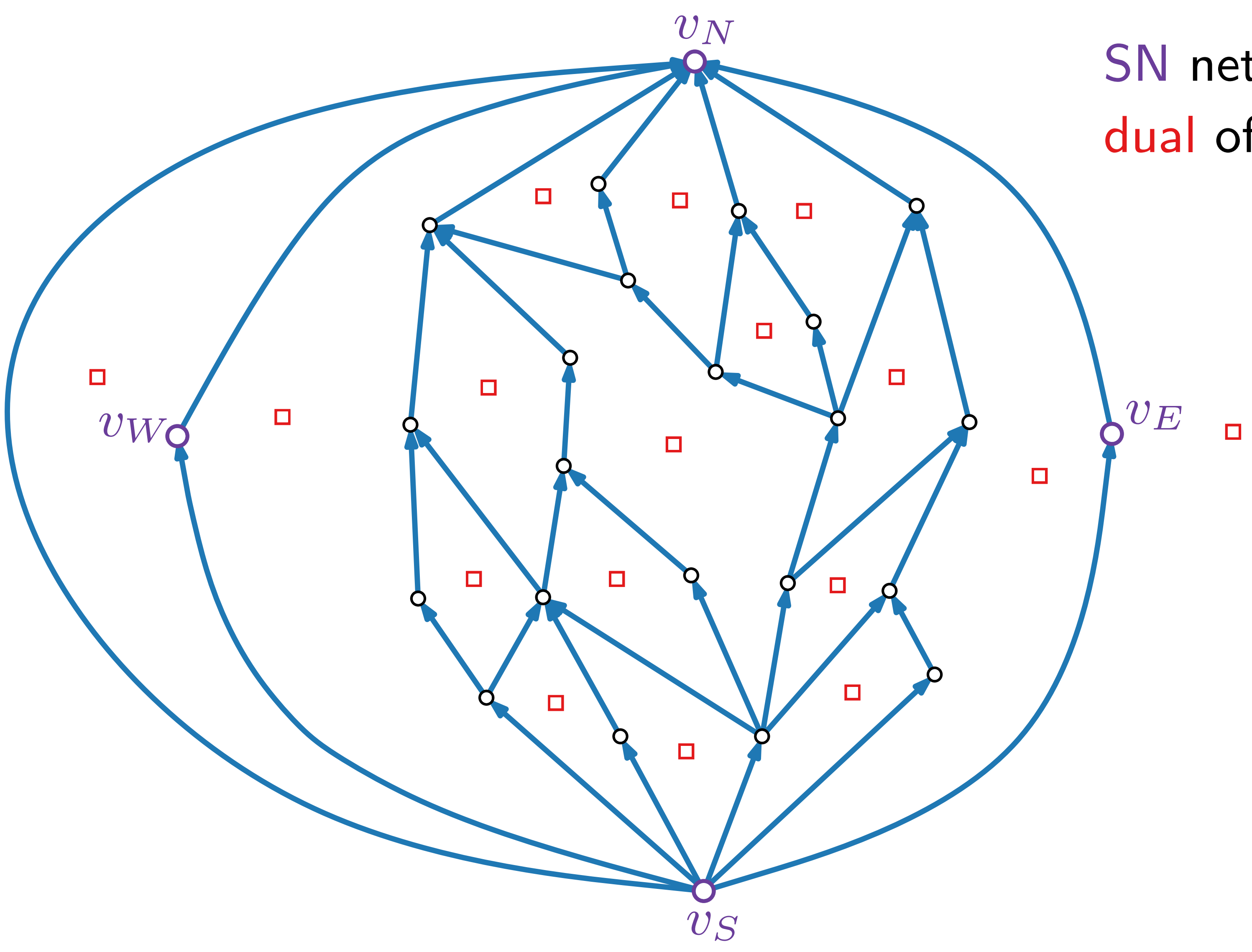

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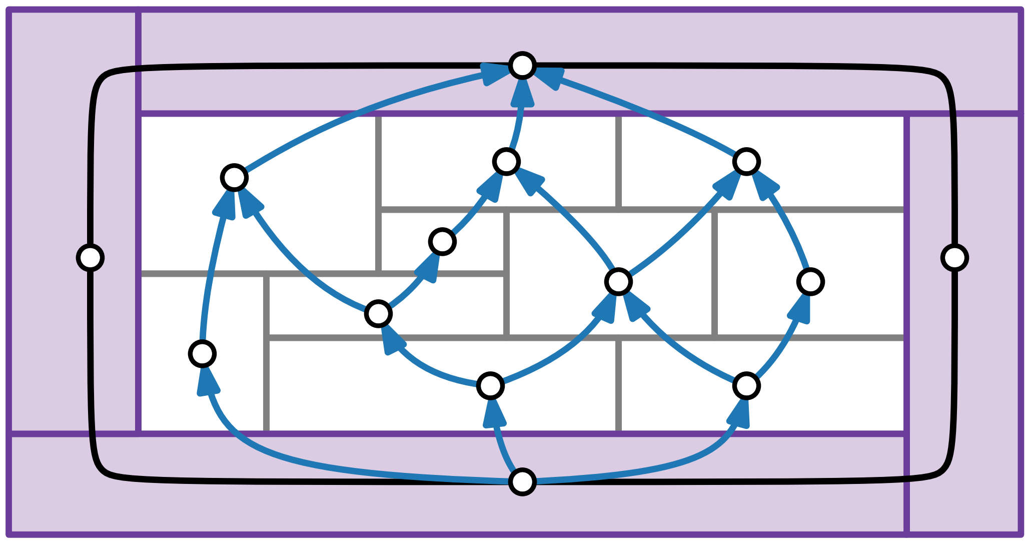

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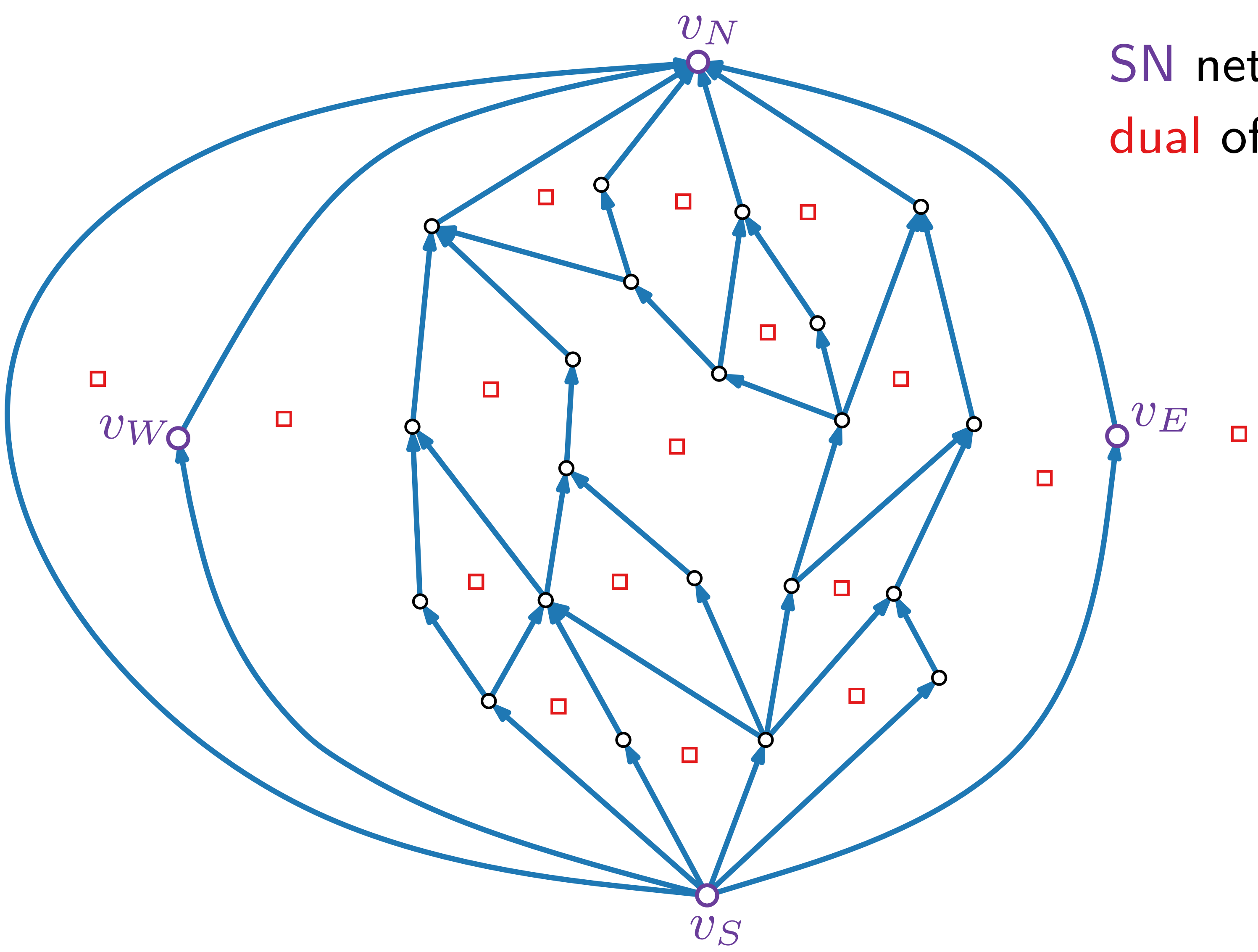

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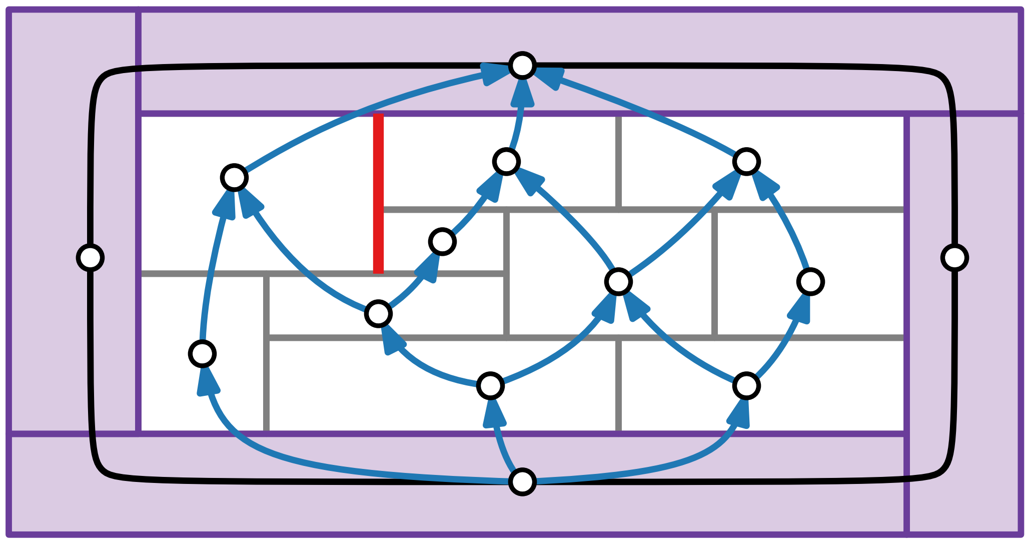

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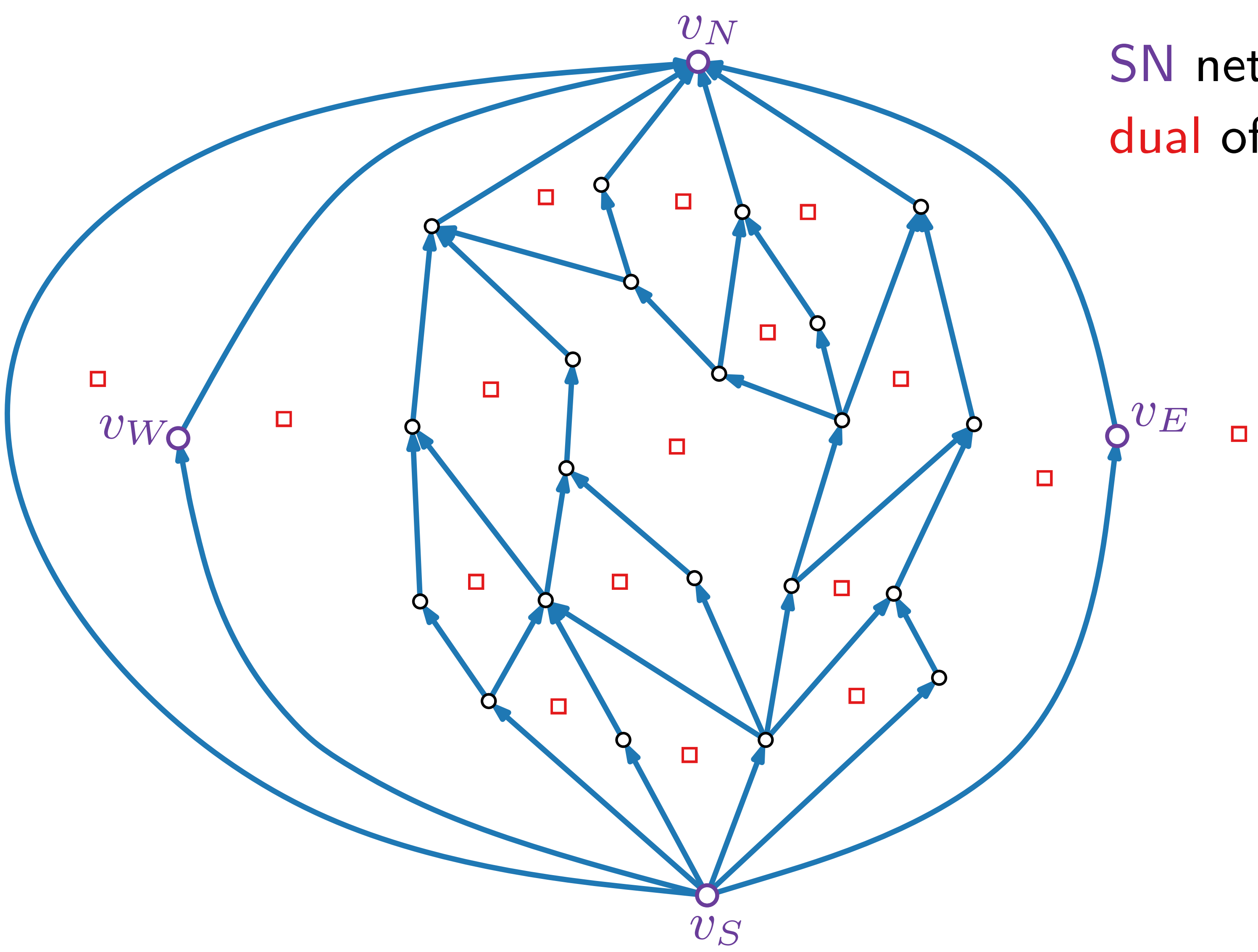

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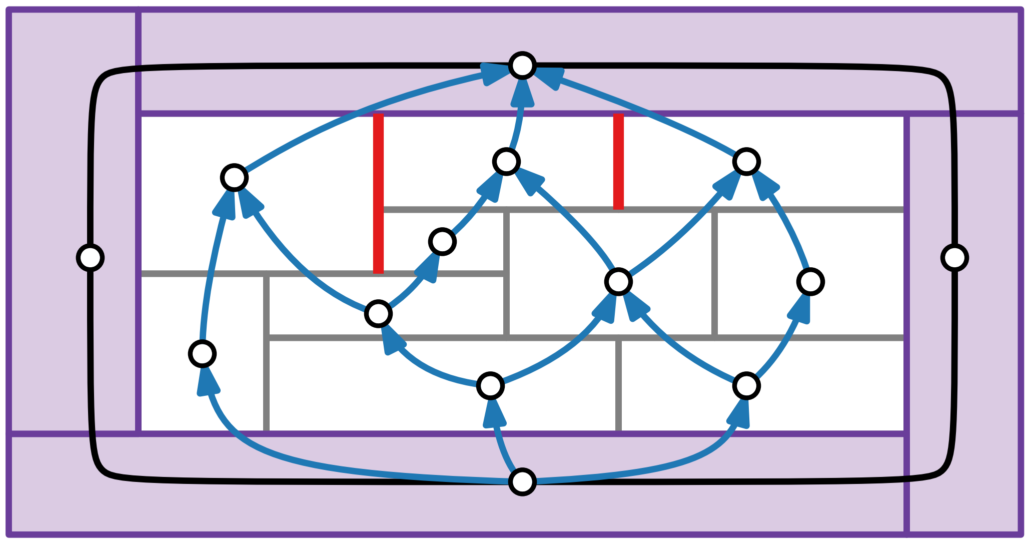

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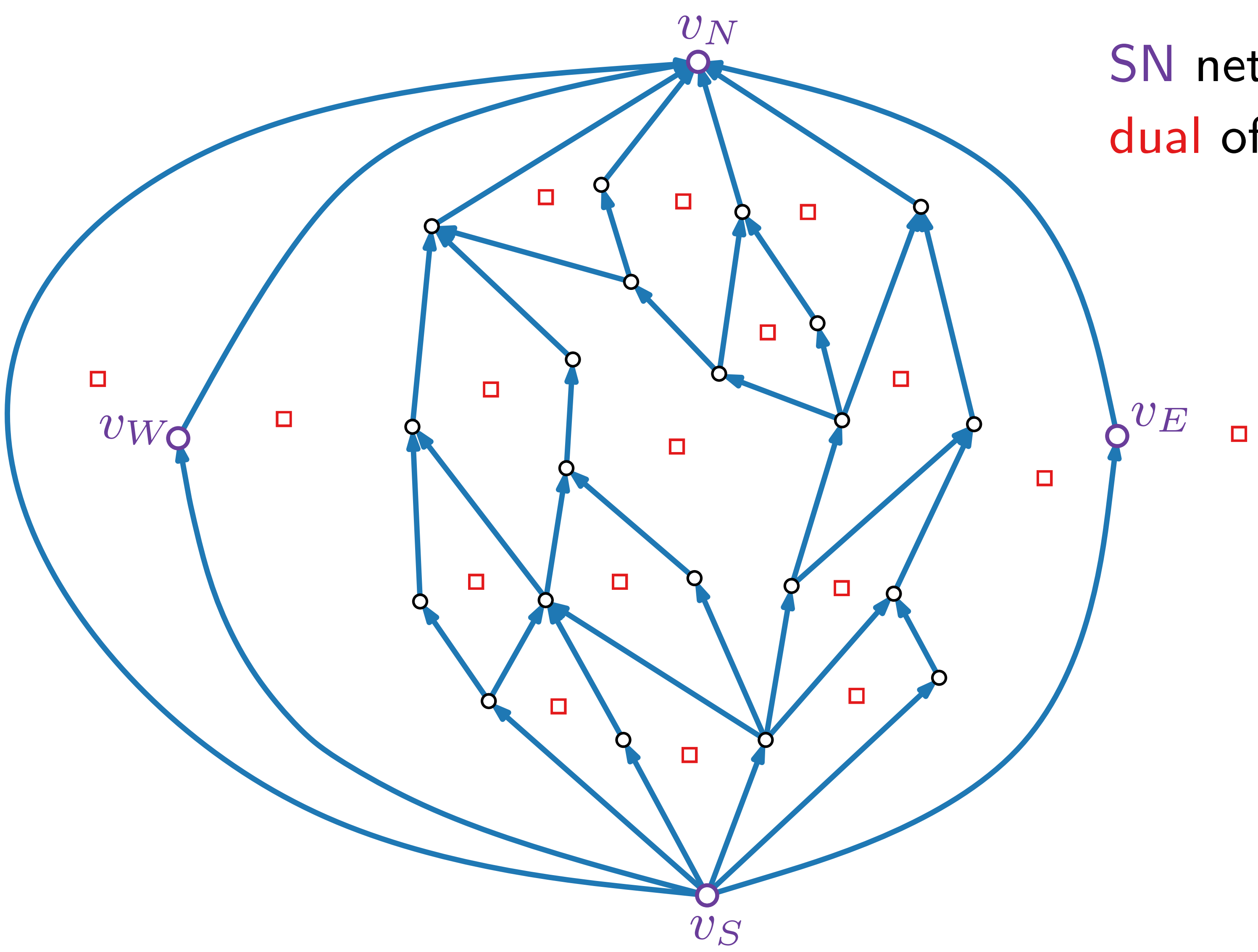

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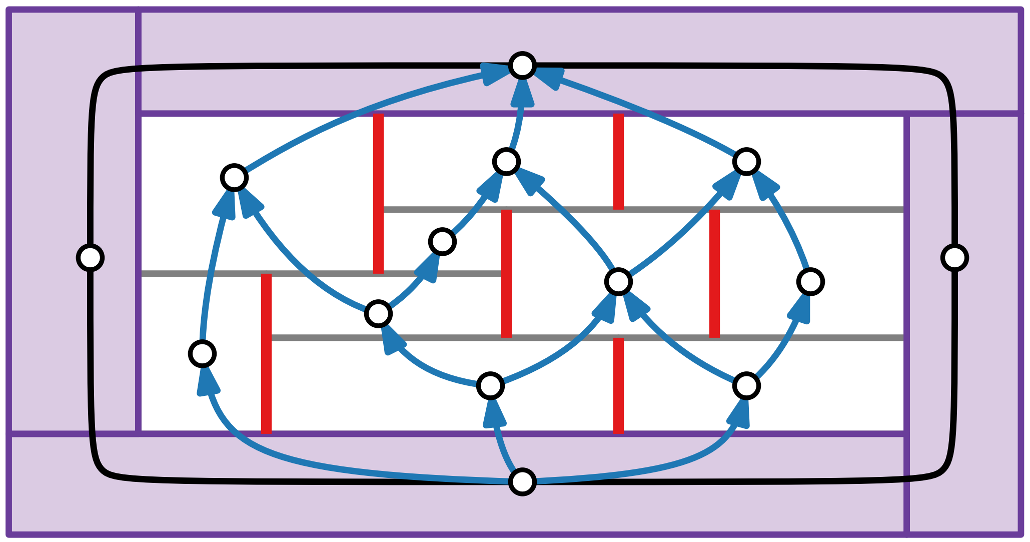

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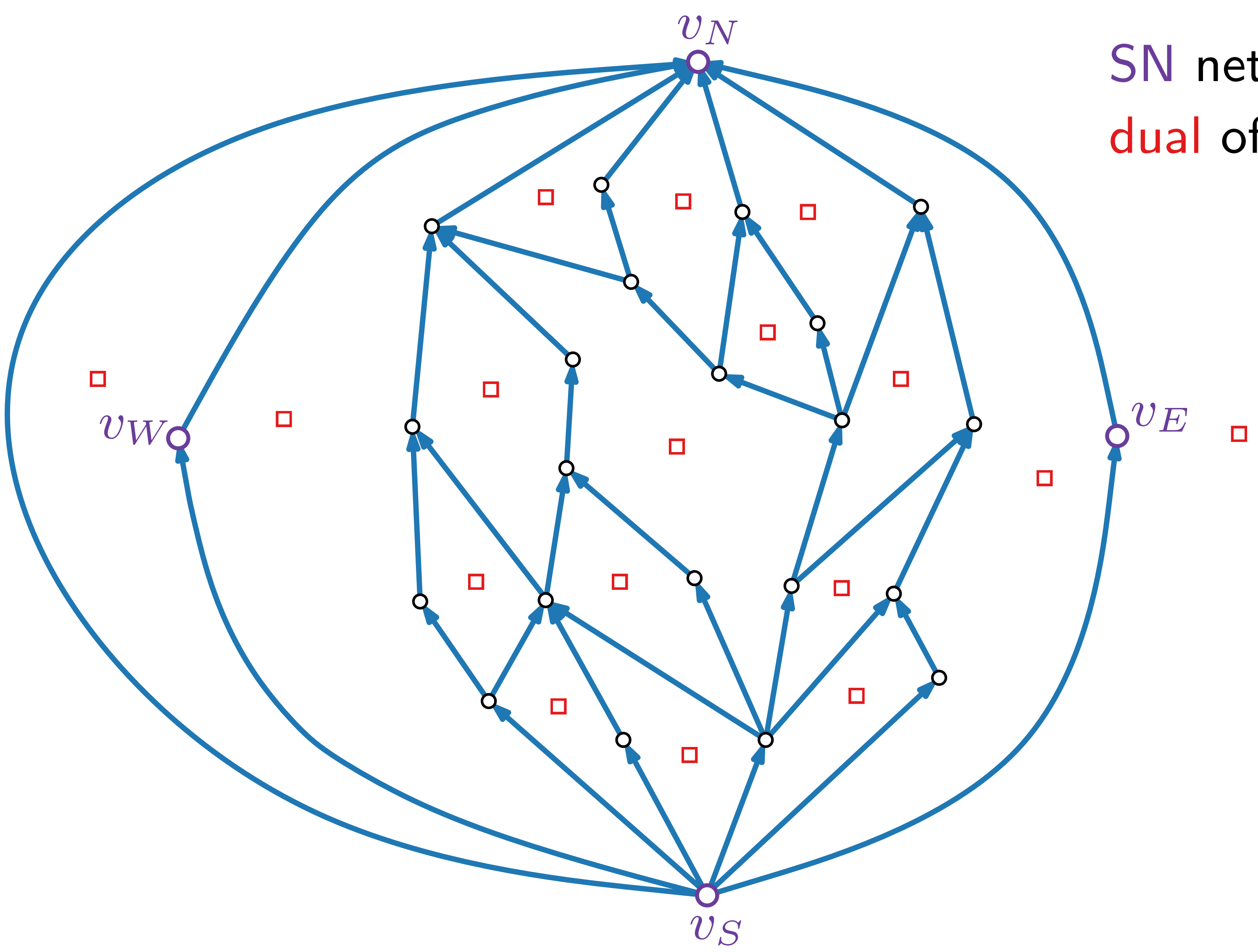

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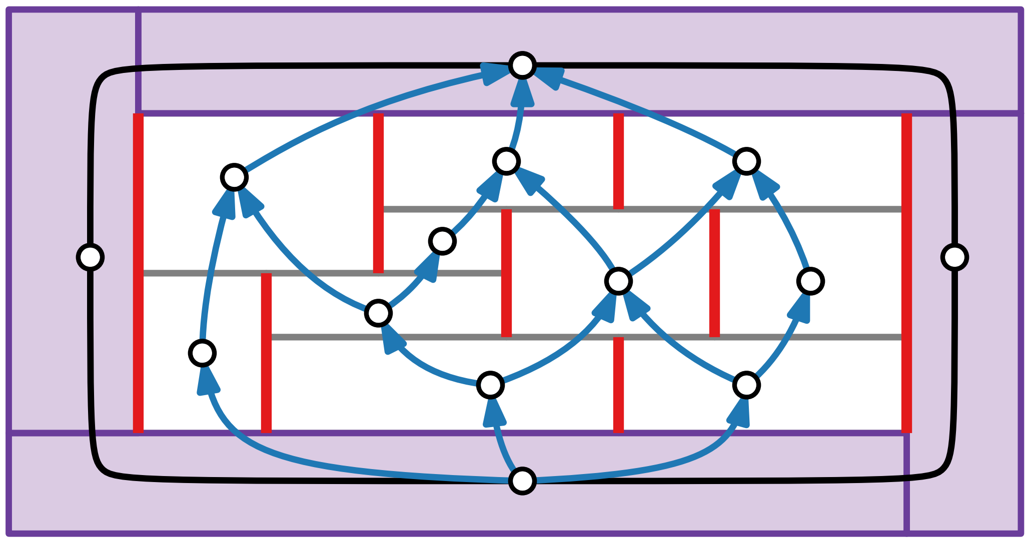

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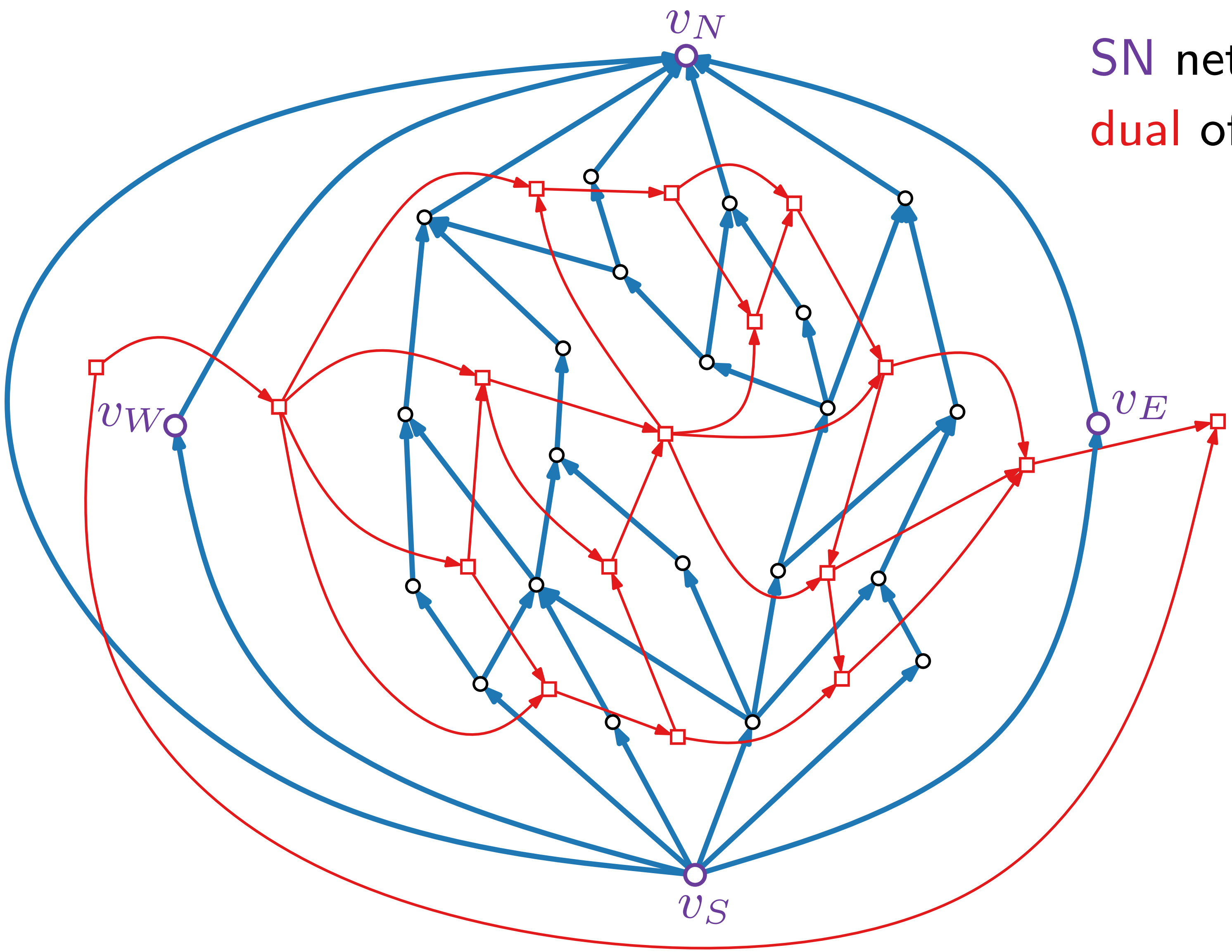

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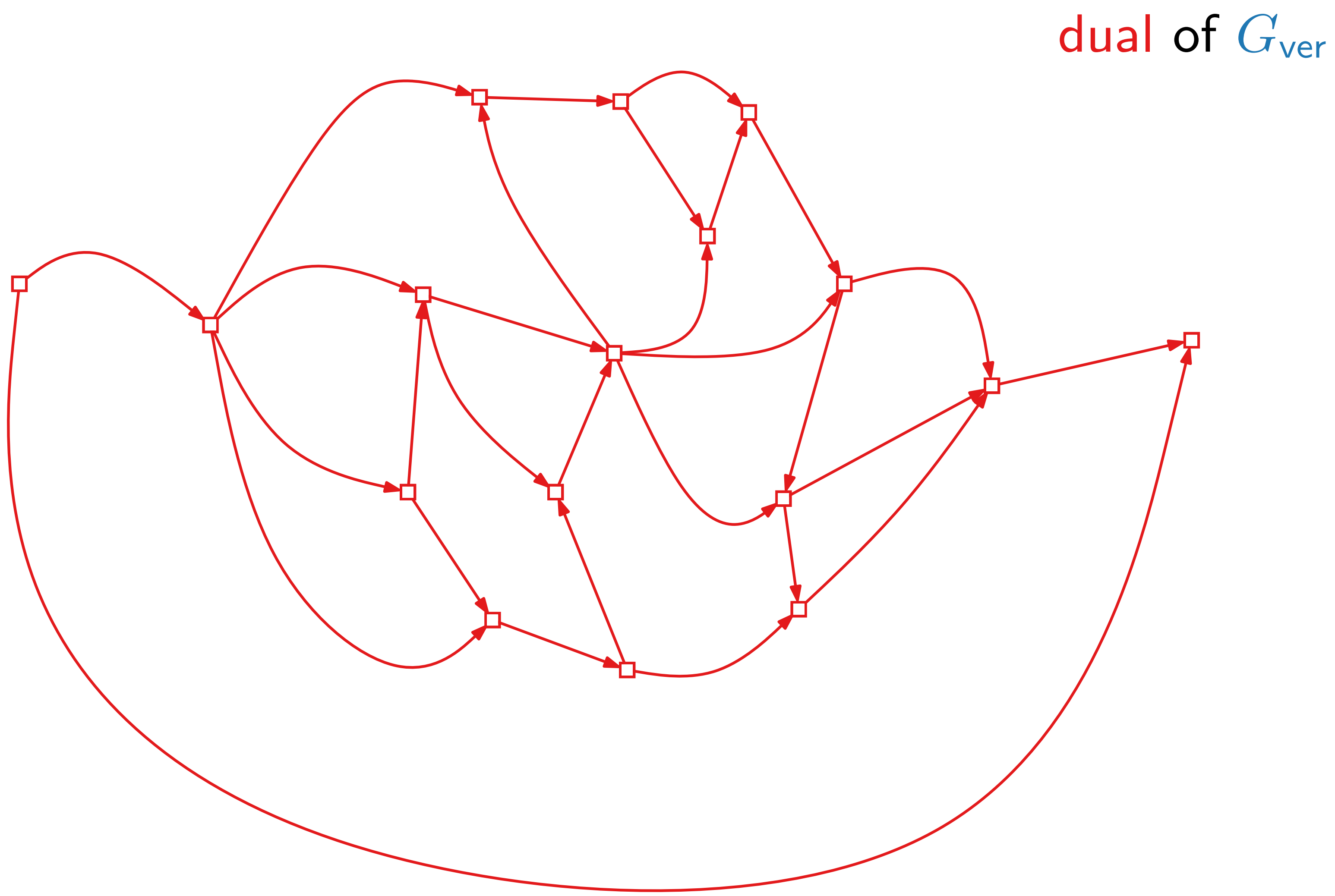

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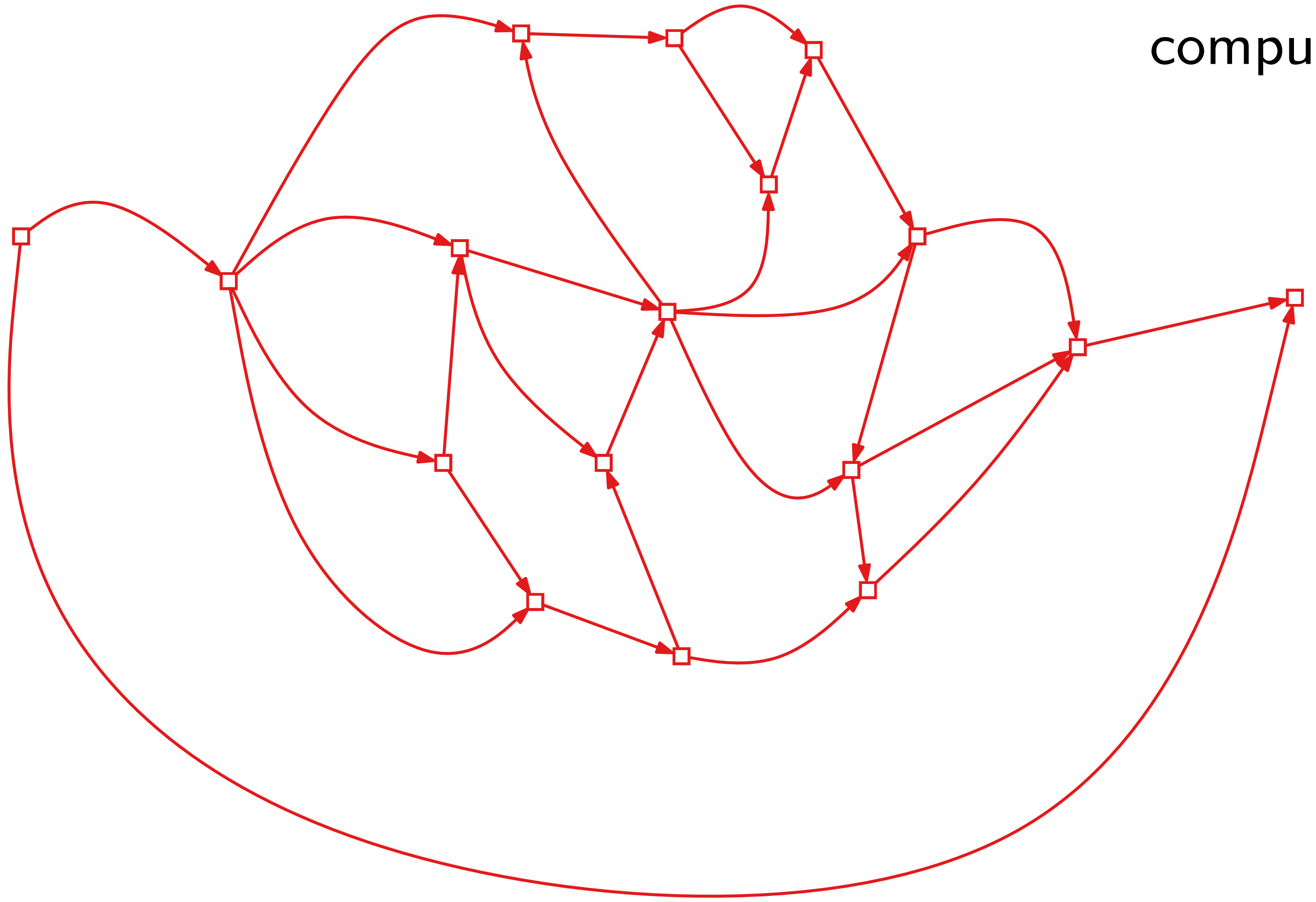

dual of G_{ver}

compute topological order

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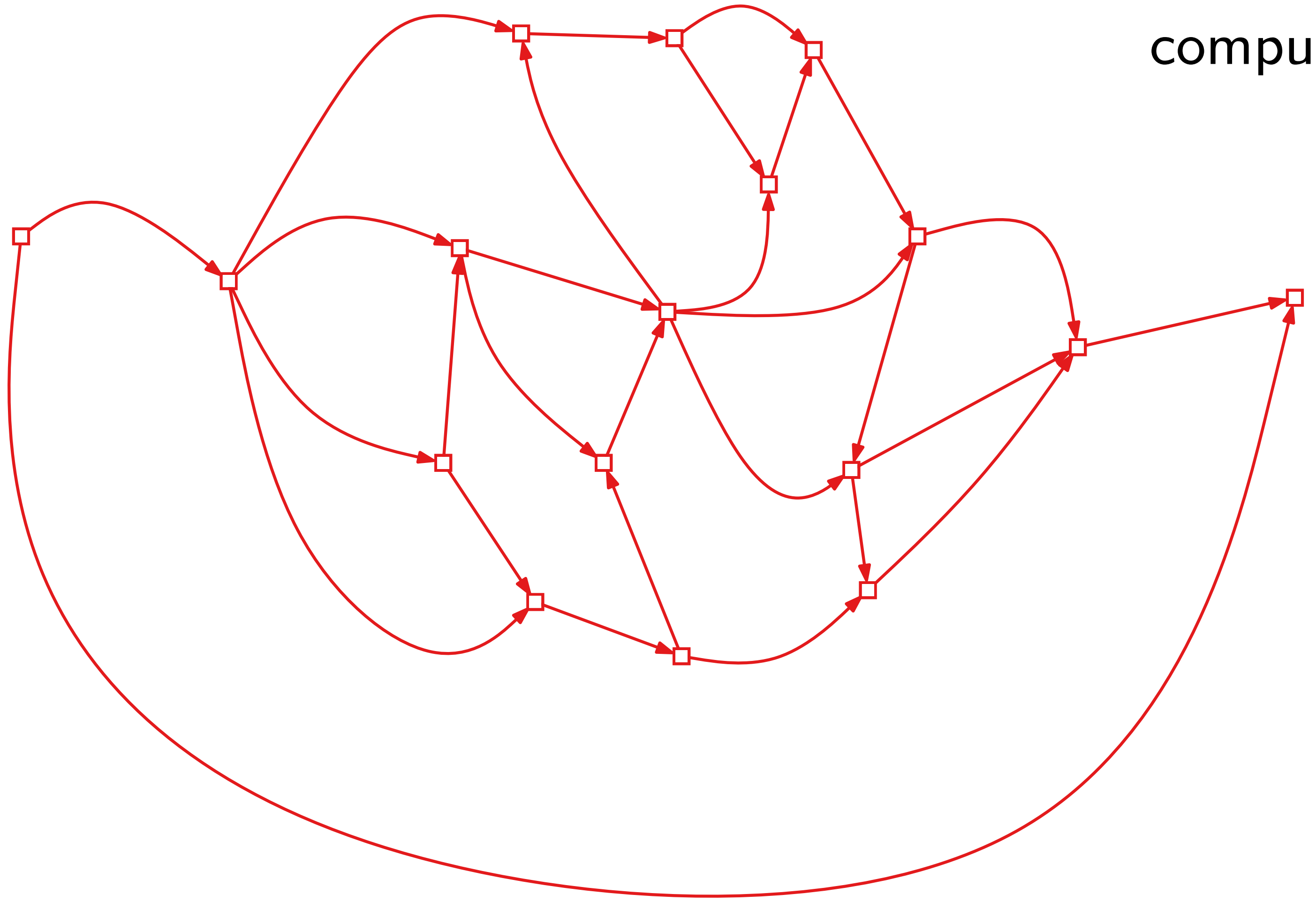

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i.e., linear order of all vertices such that all directed edges point in the same direction

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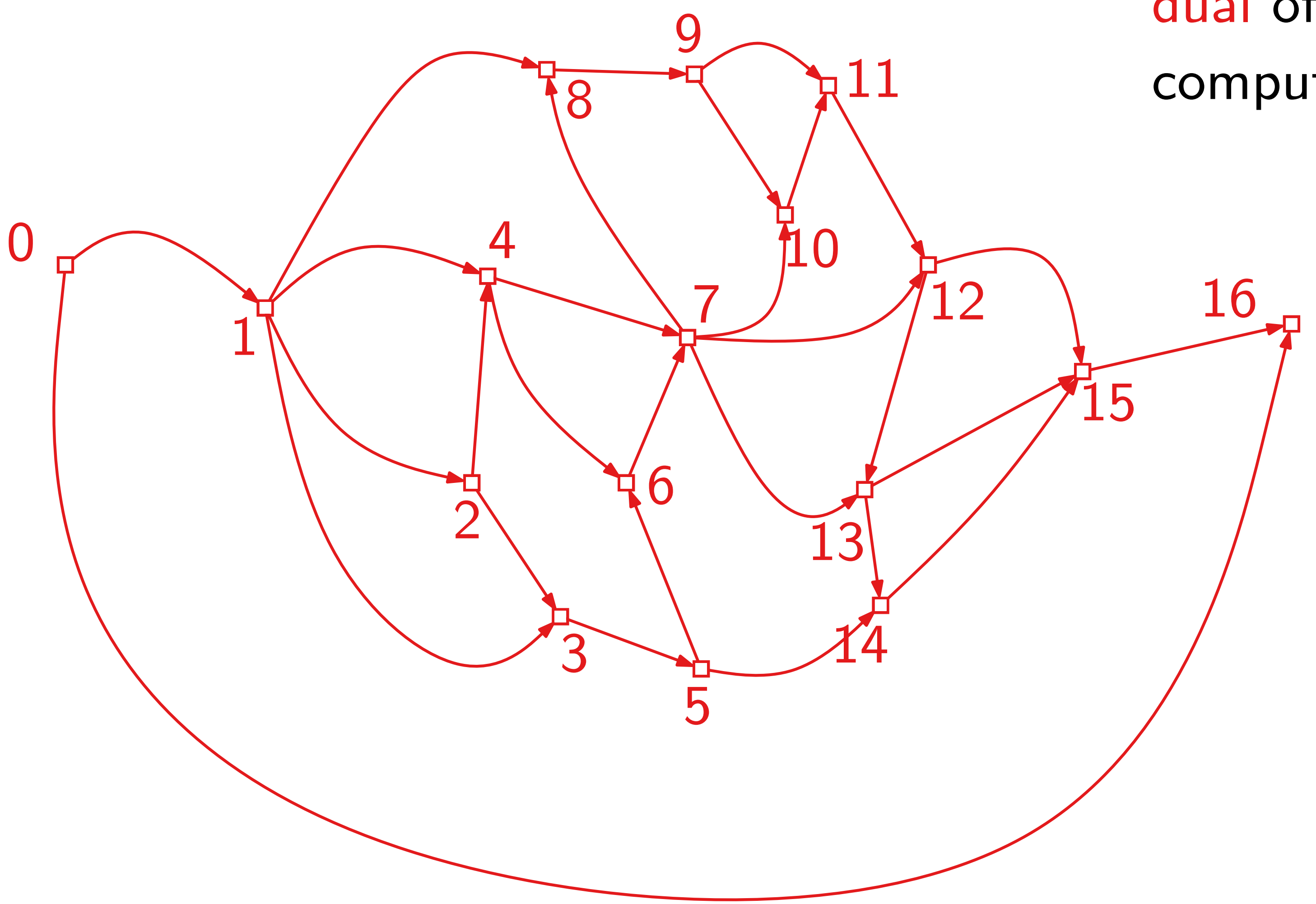

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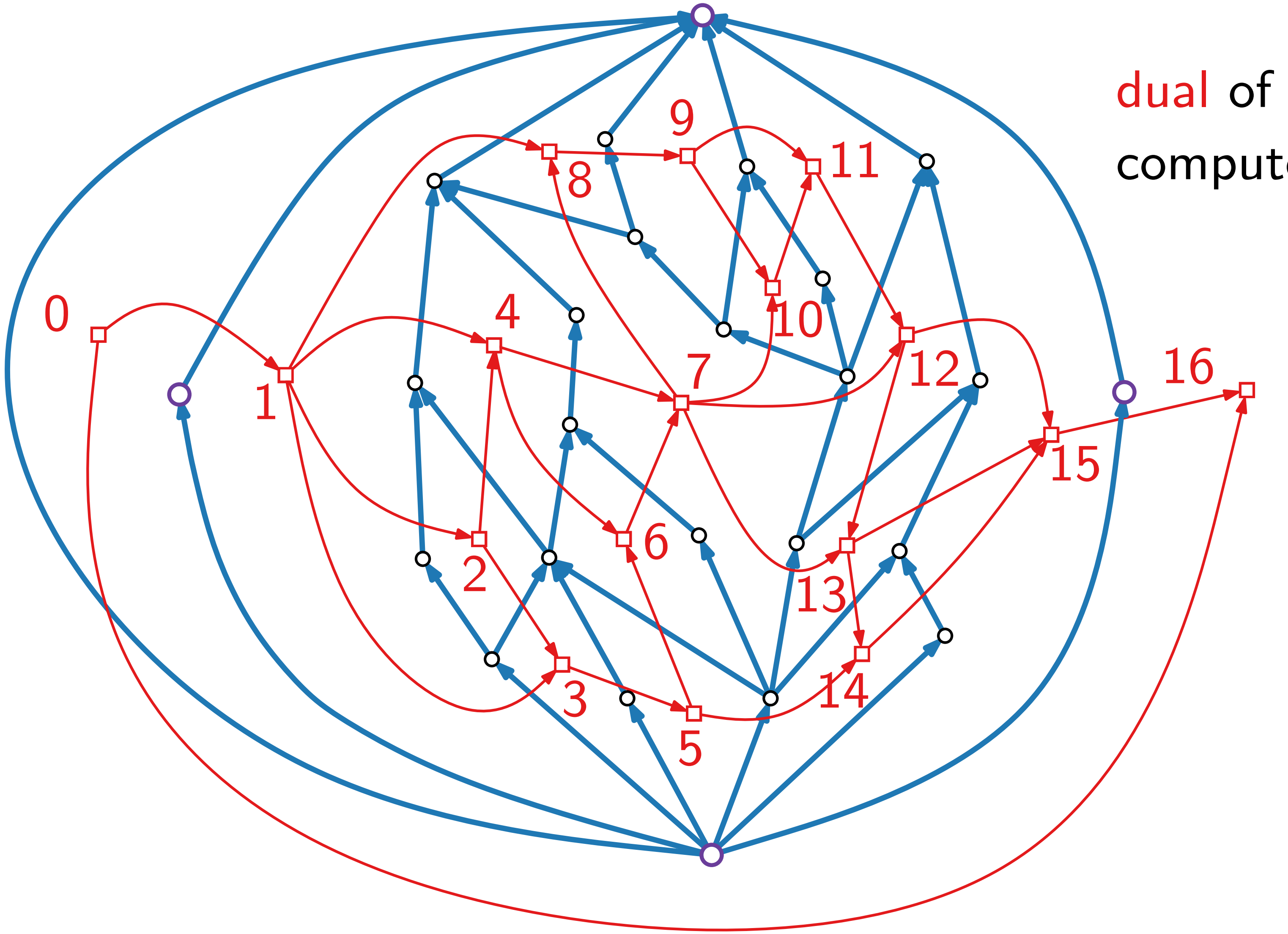

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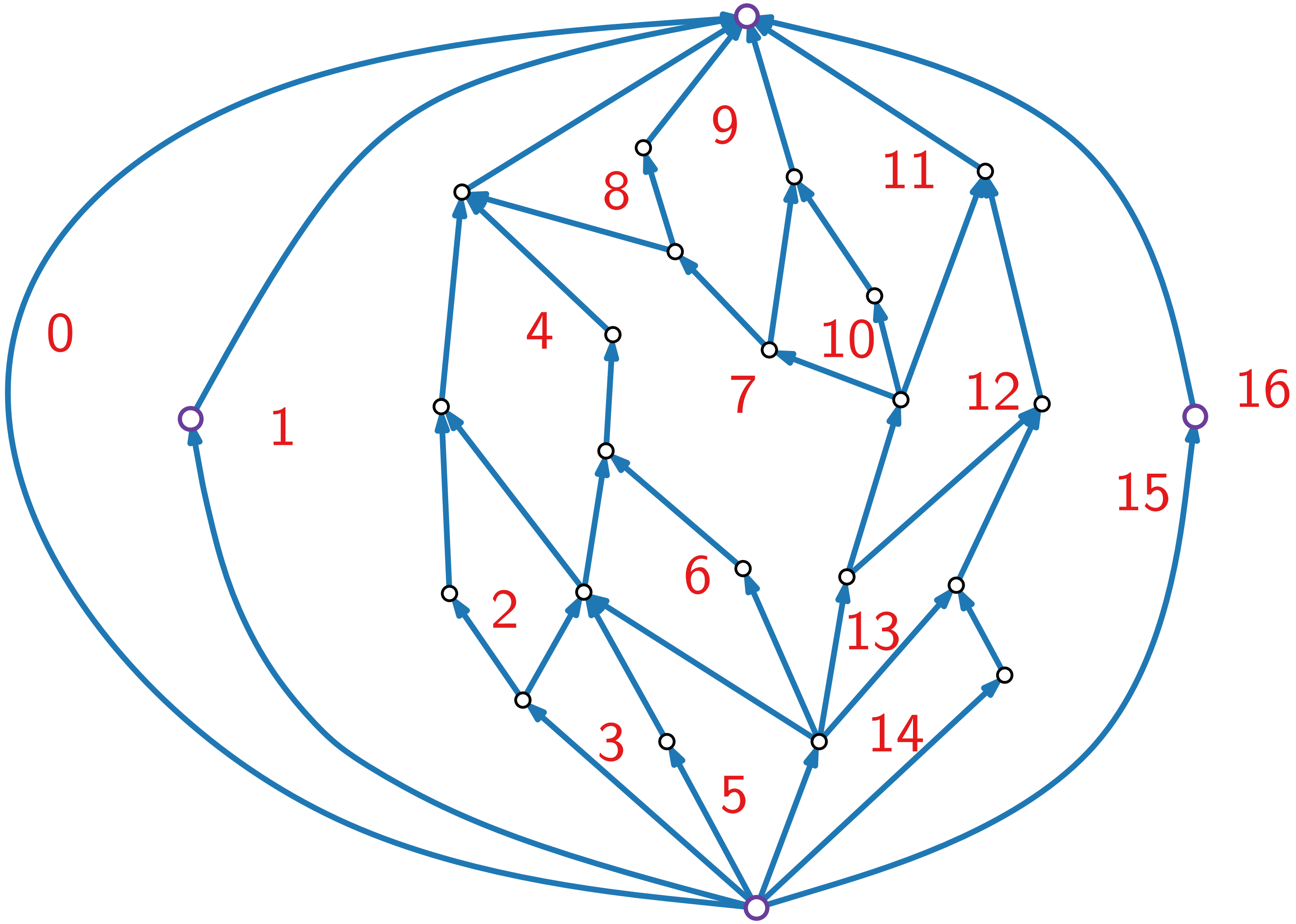

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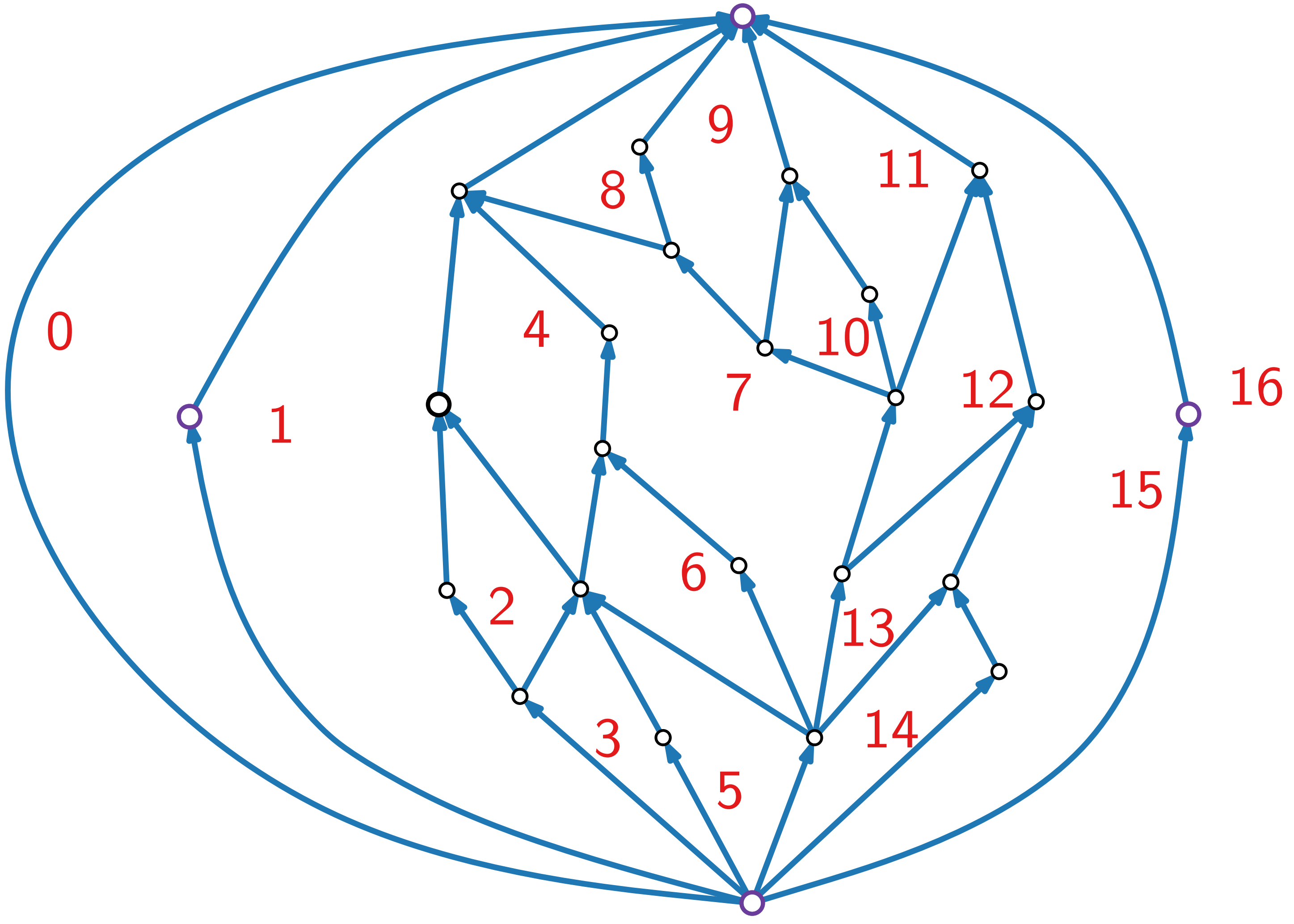

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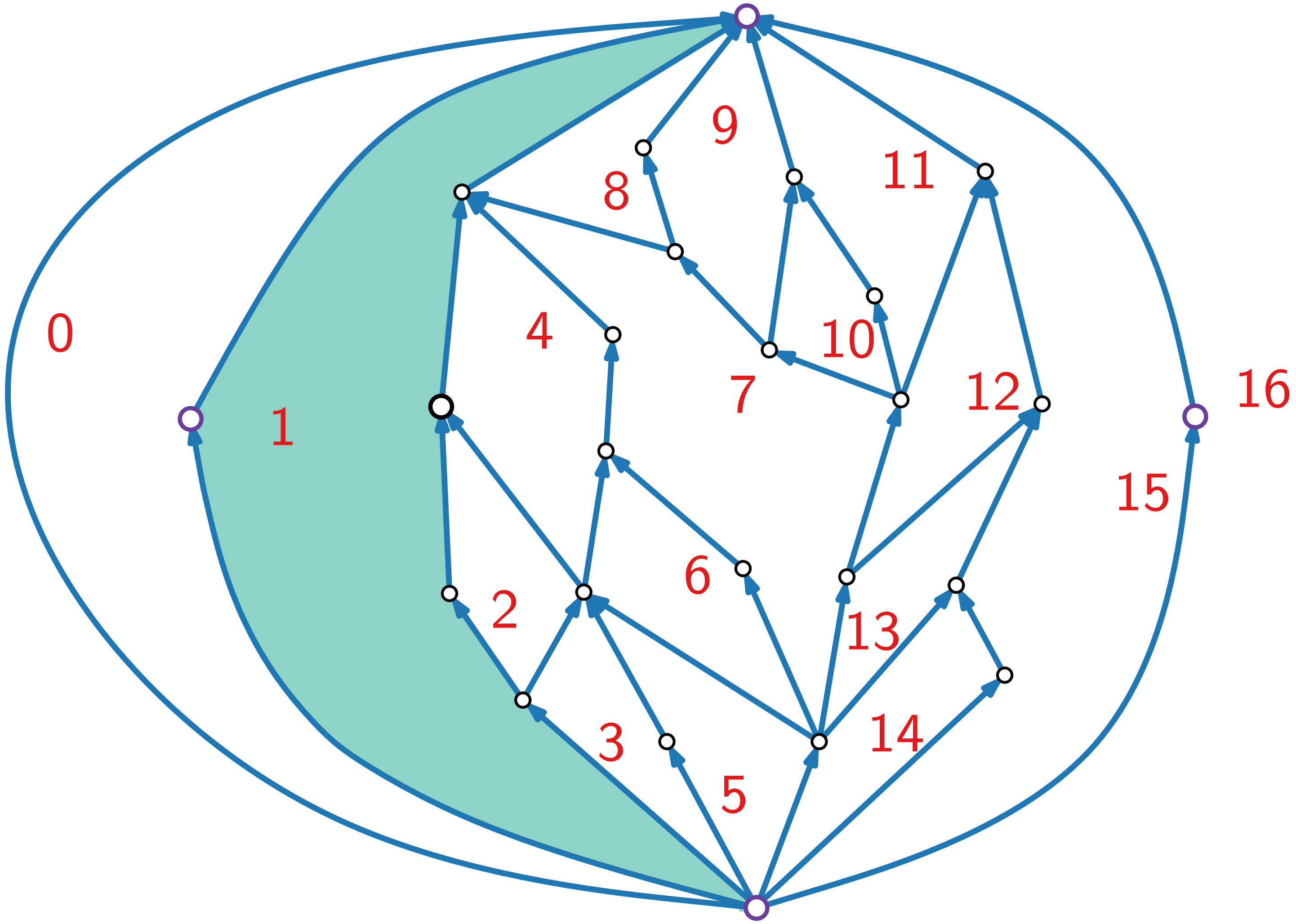

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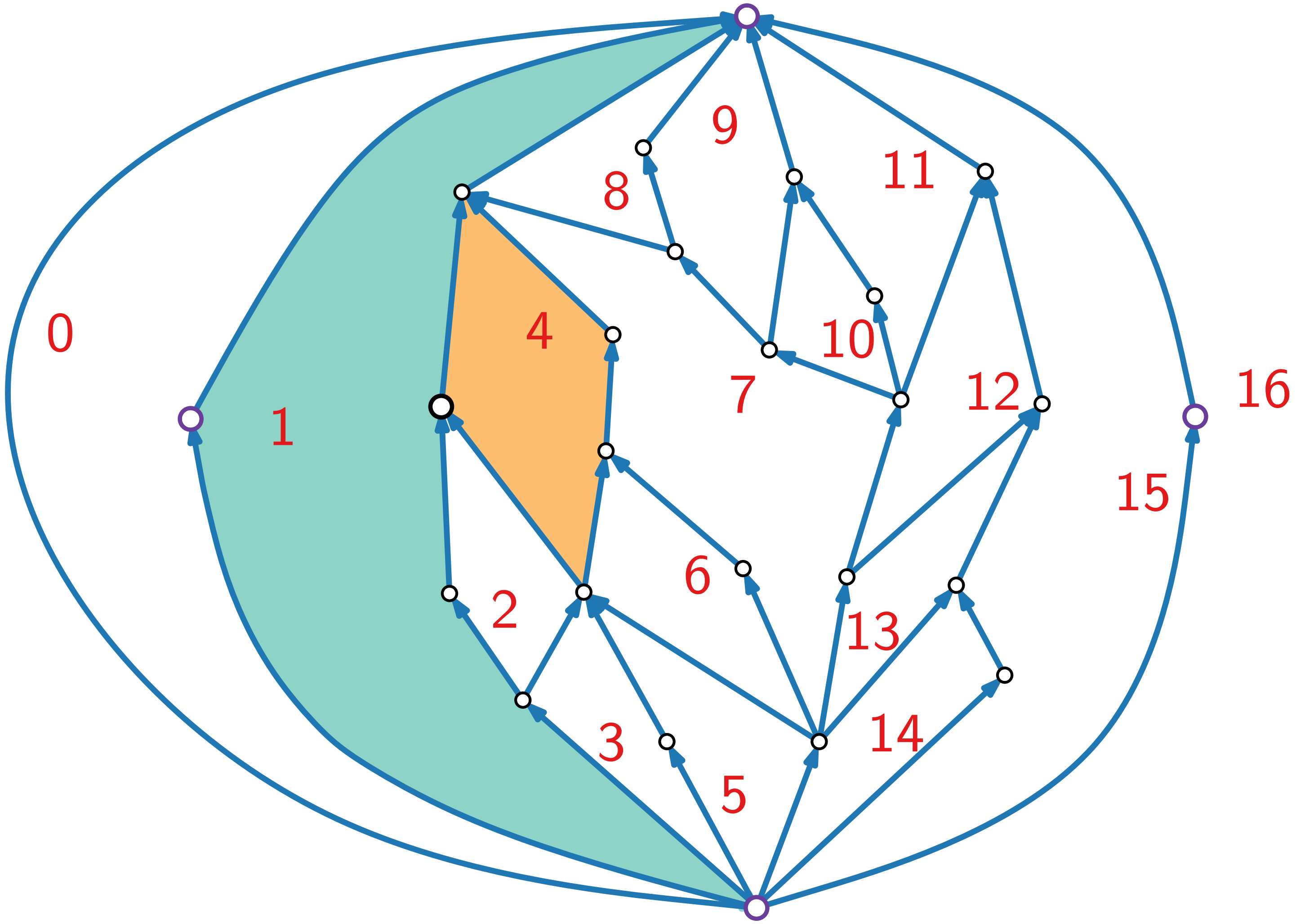

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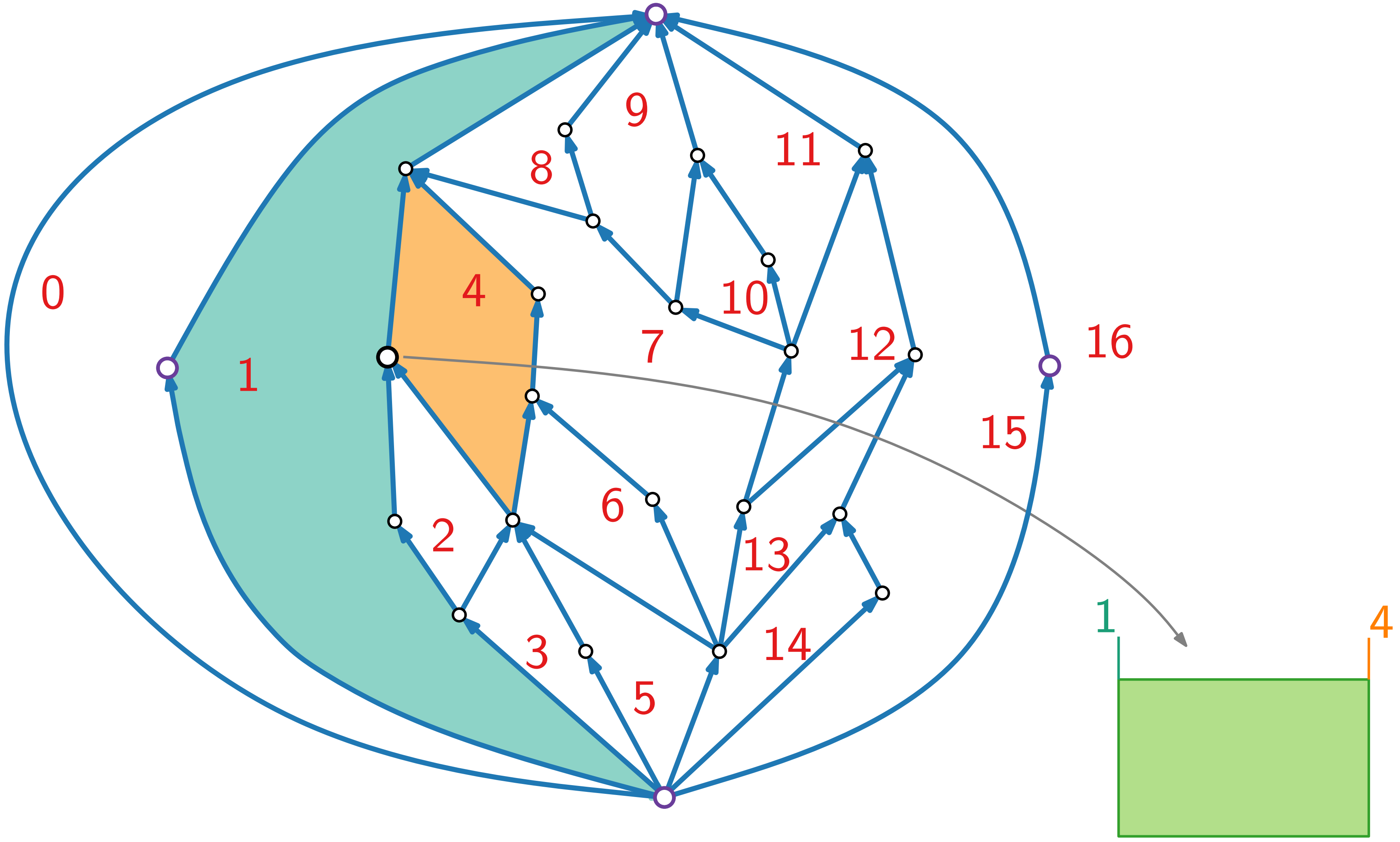

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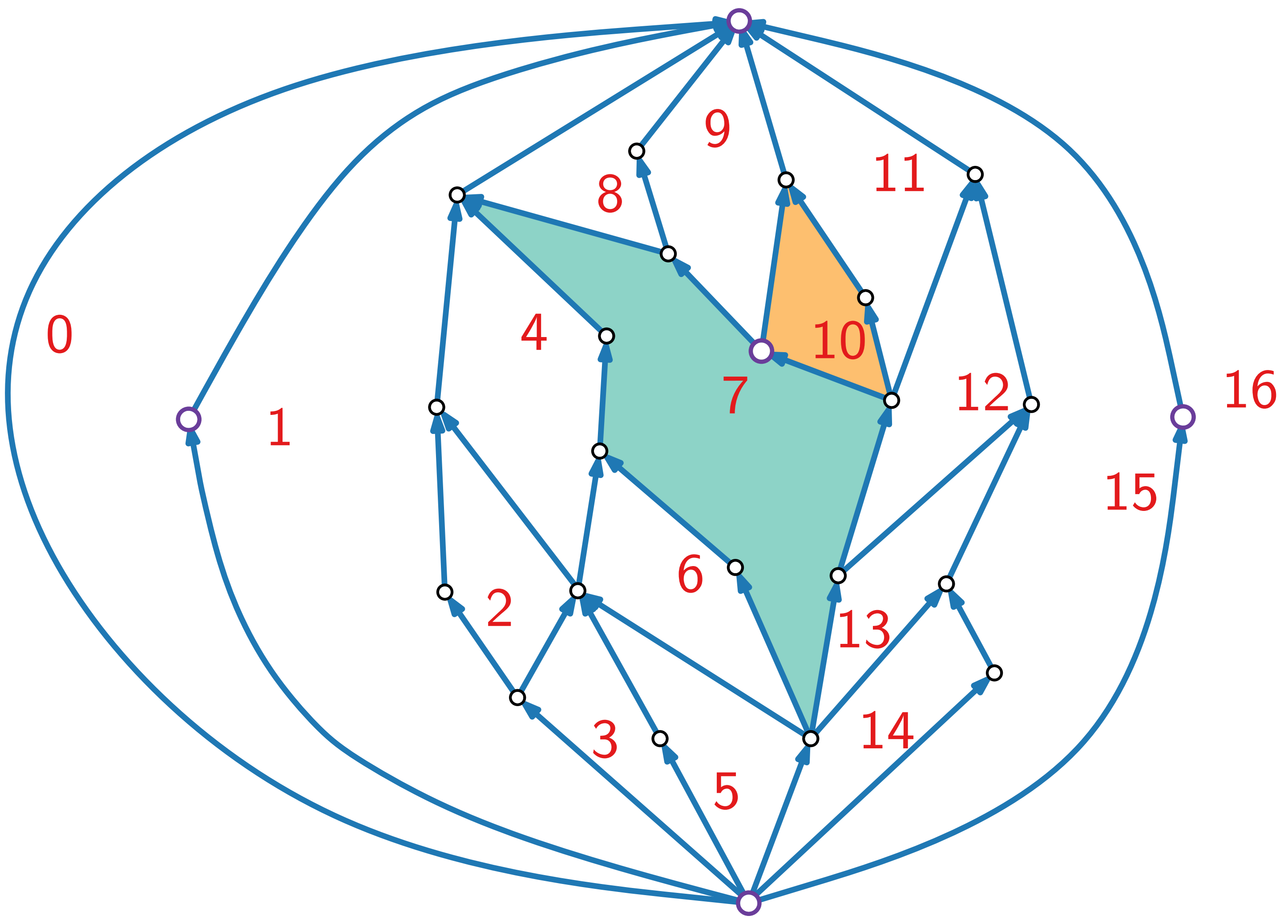

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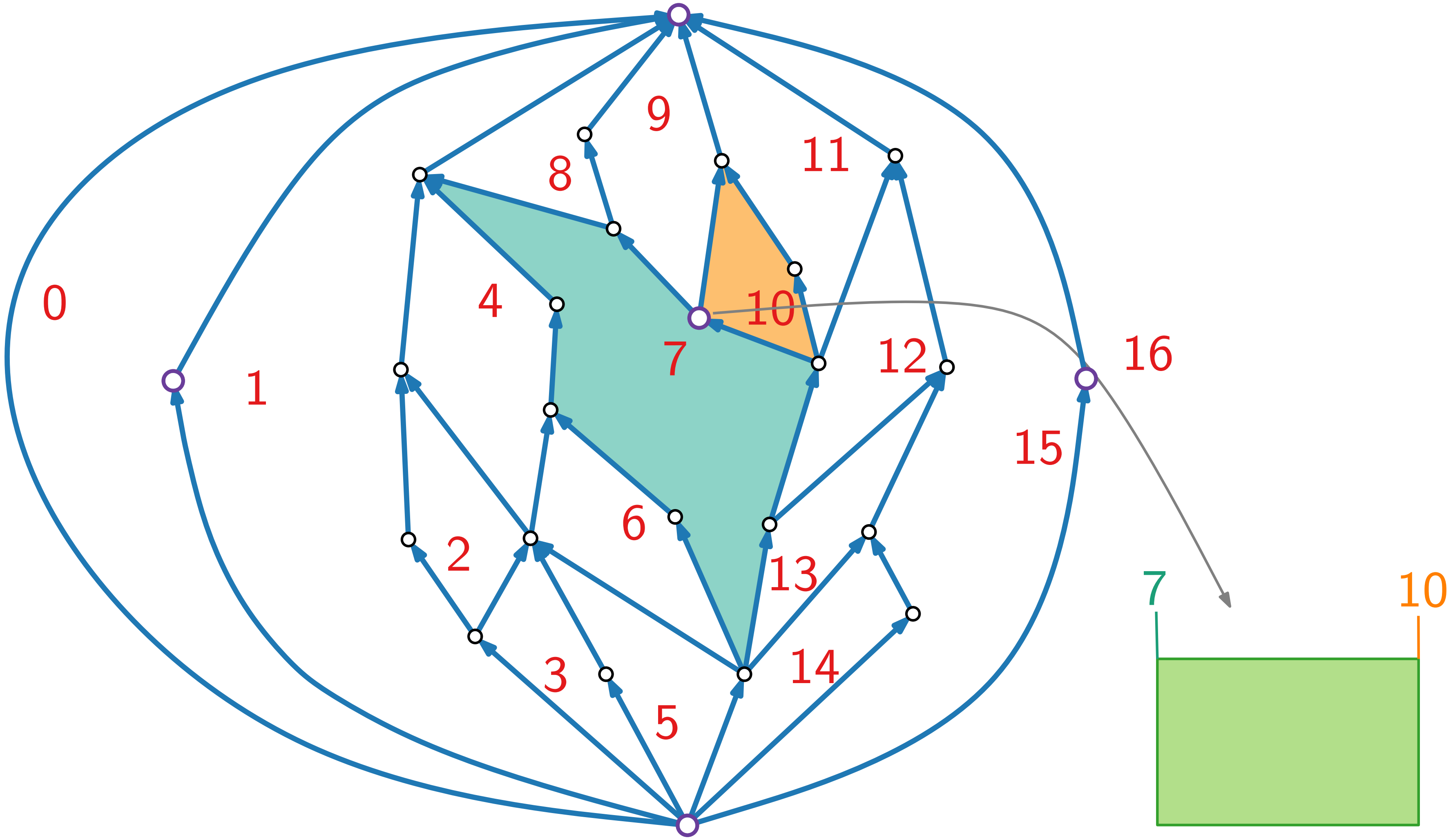

Rectangular Dual Algorithm

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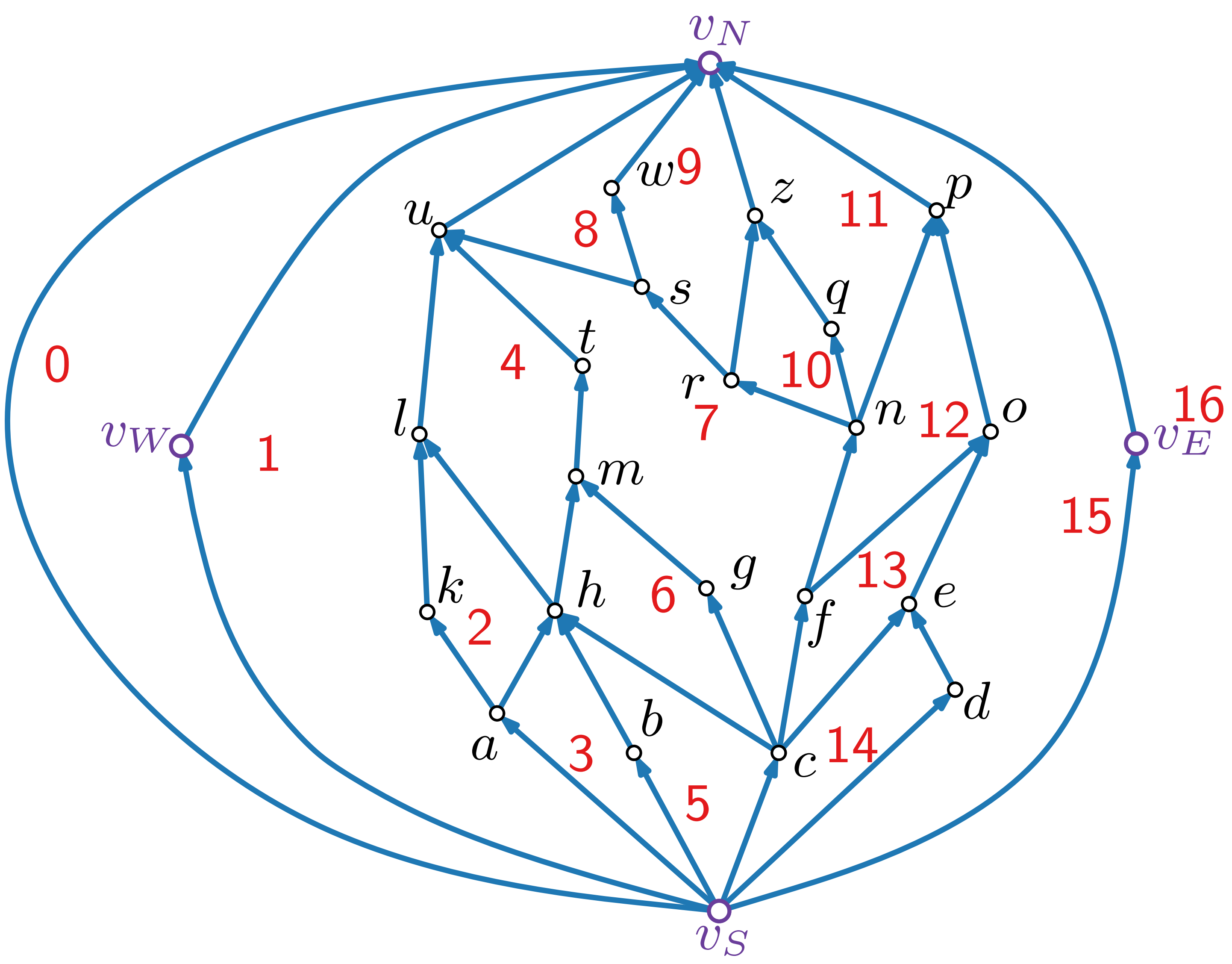
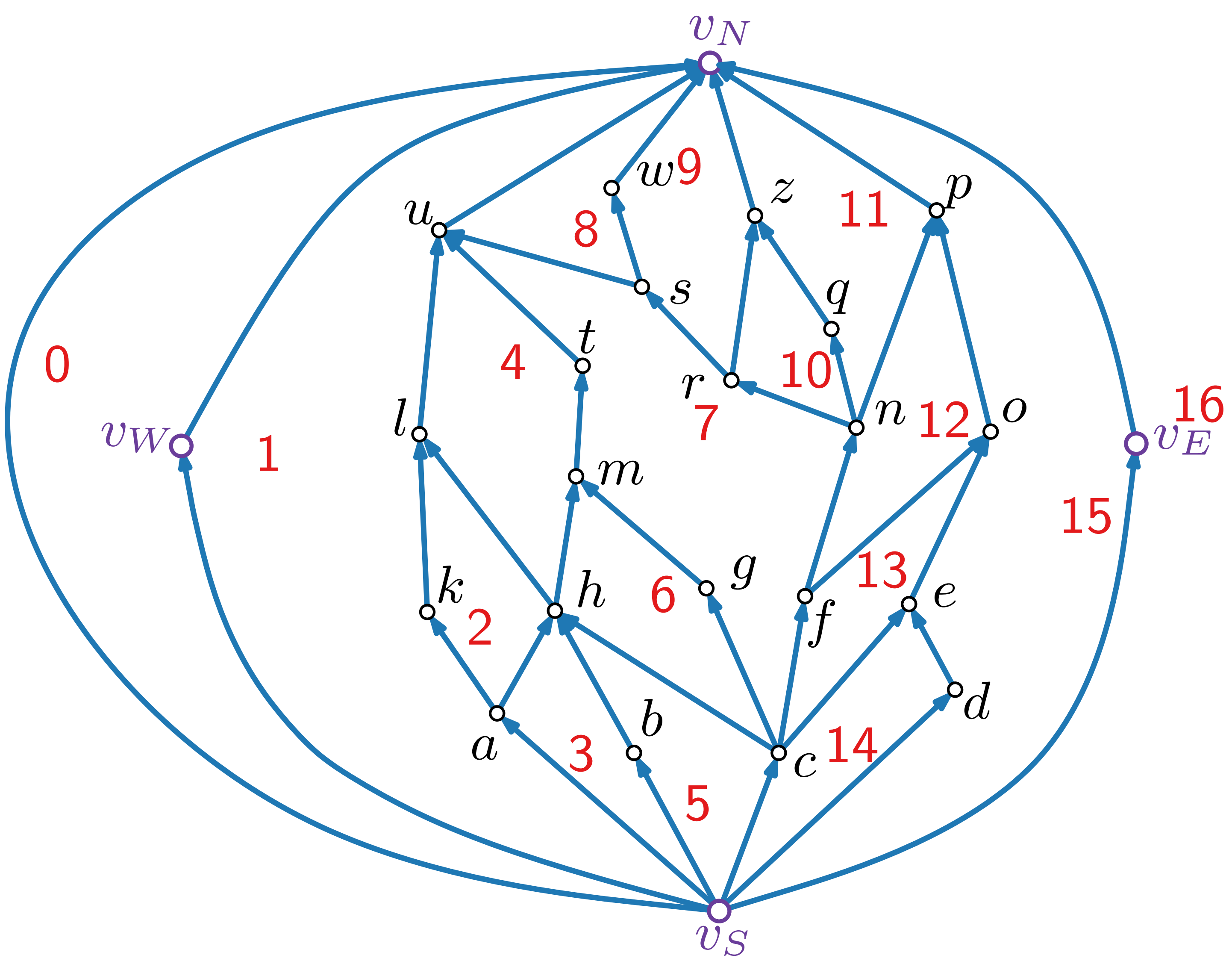
- For each $v \in V$, let $R(v) = [x_1(v), x_2(v)] \times [y_1(v), y_2(v)]$.

Reading off Coordinates to Get Rectangular Dual



Reading off Coordinates to Get Rectangular Dual

$x_1(v_N) = 0,\ x_2(v_N) = 15$



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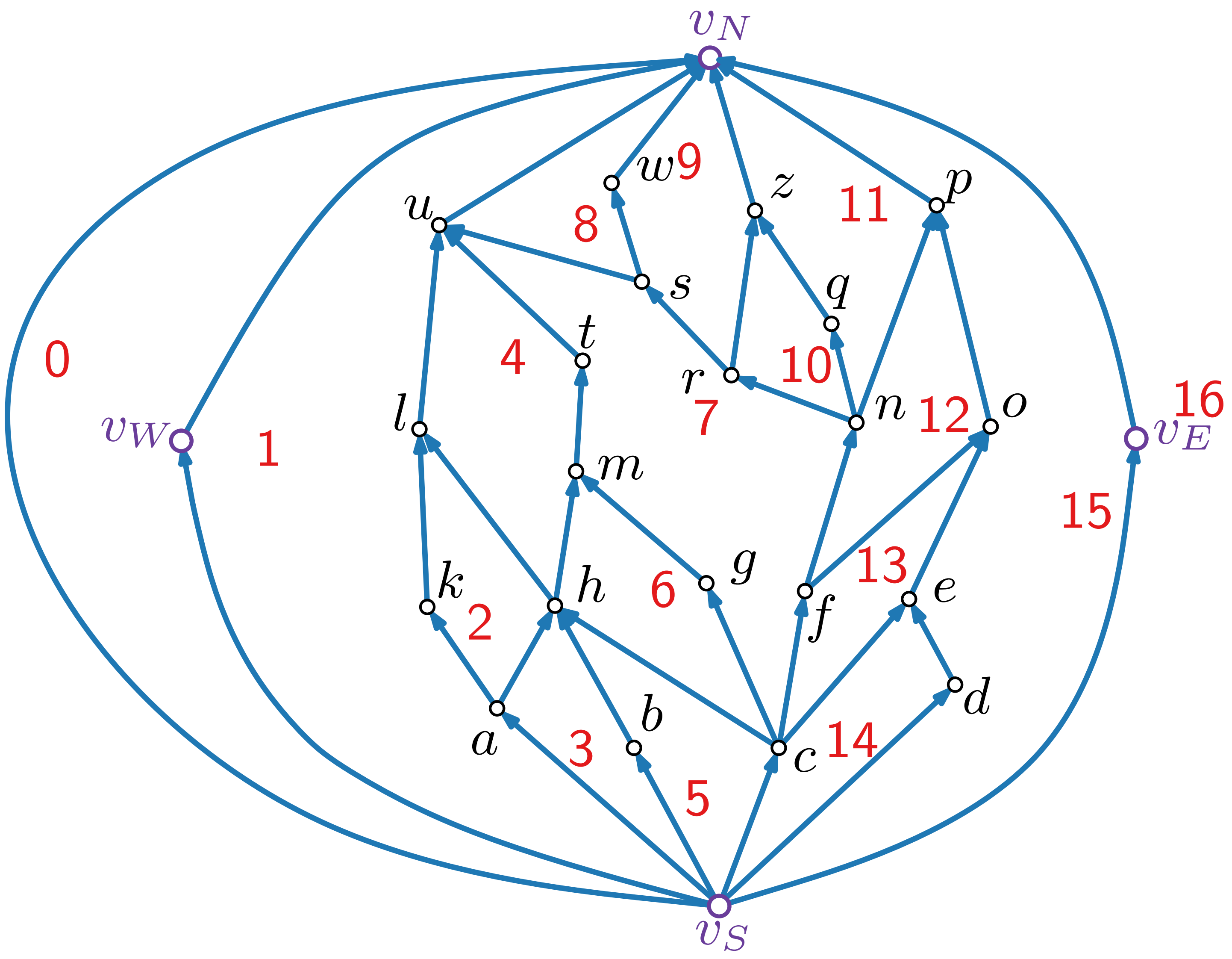
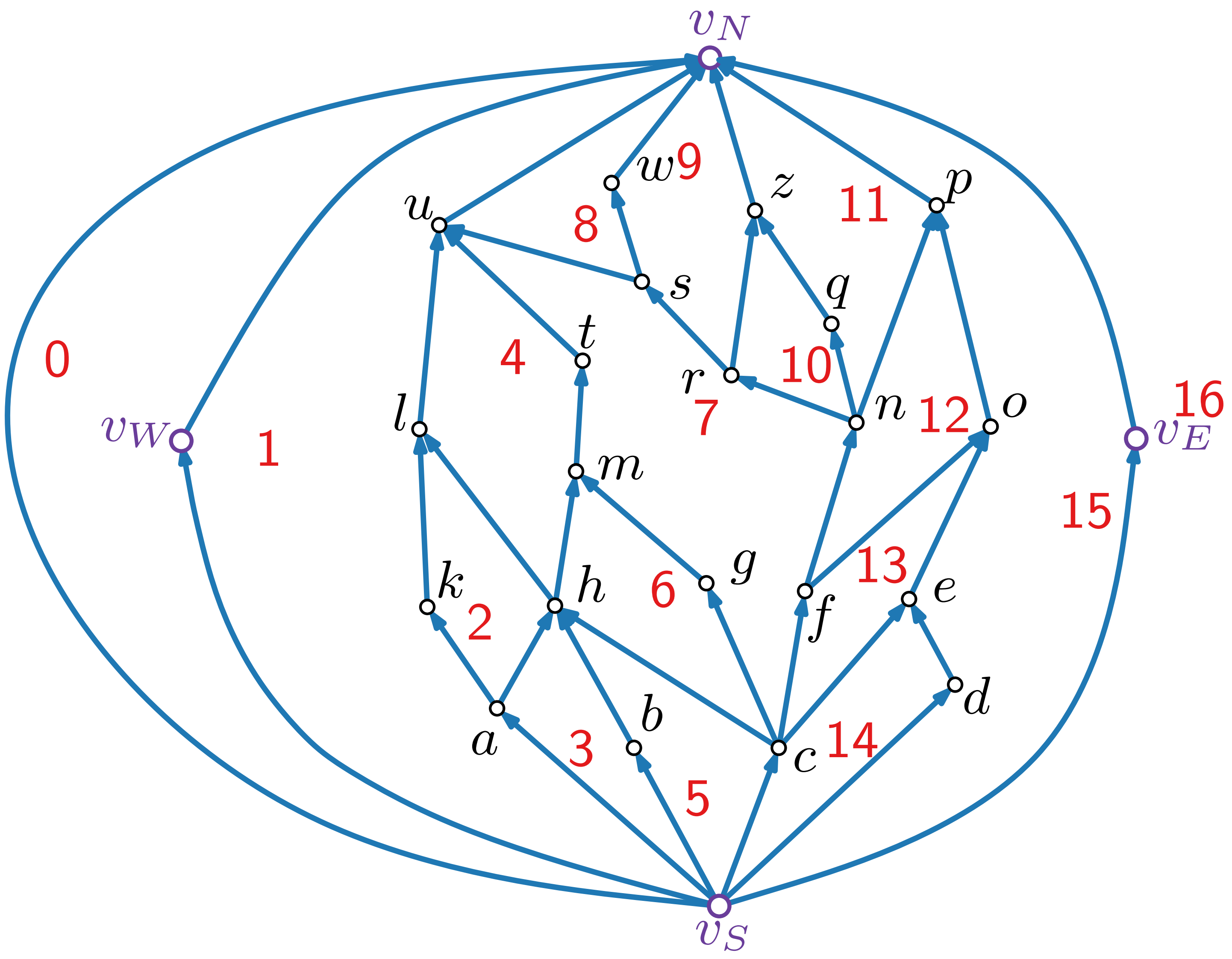


$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$

Reading off Coordinates to Get Rectangular Dual



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$x_1(v_W) = 0, x_2(v_W) = 1$

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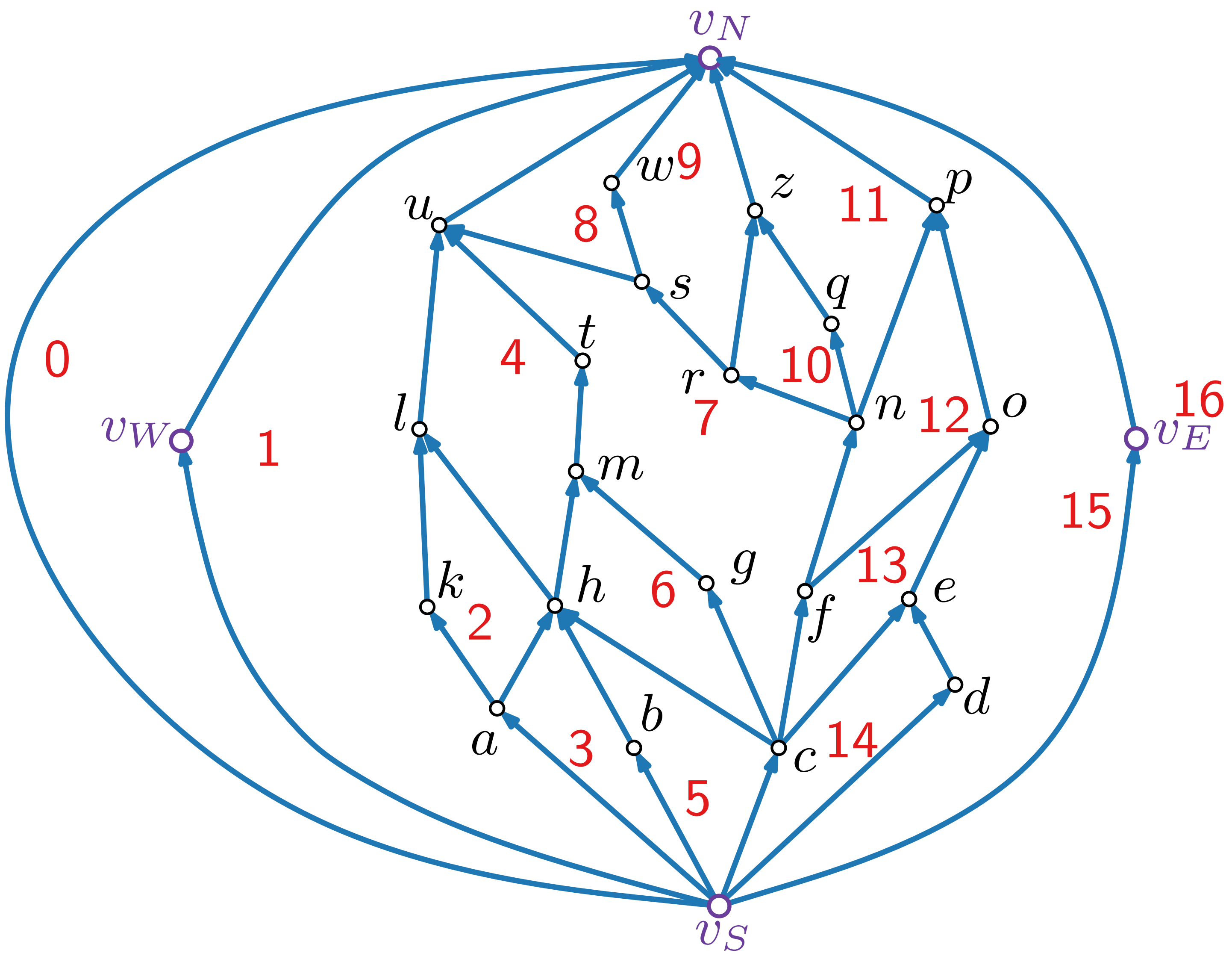

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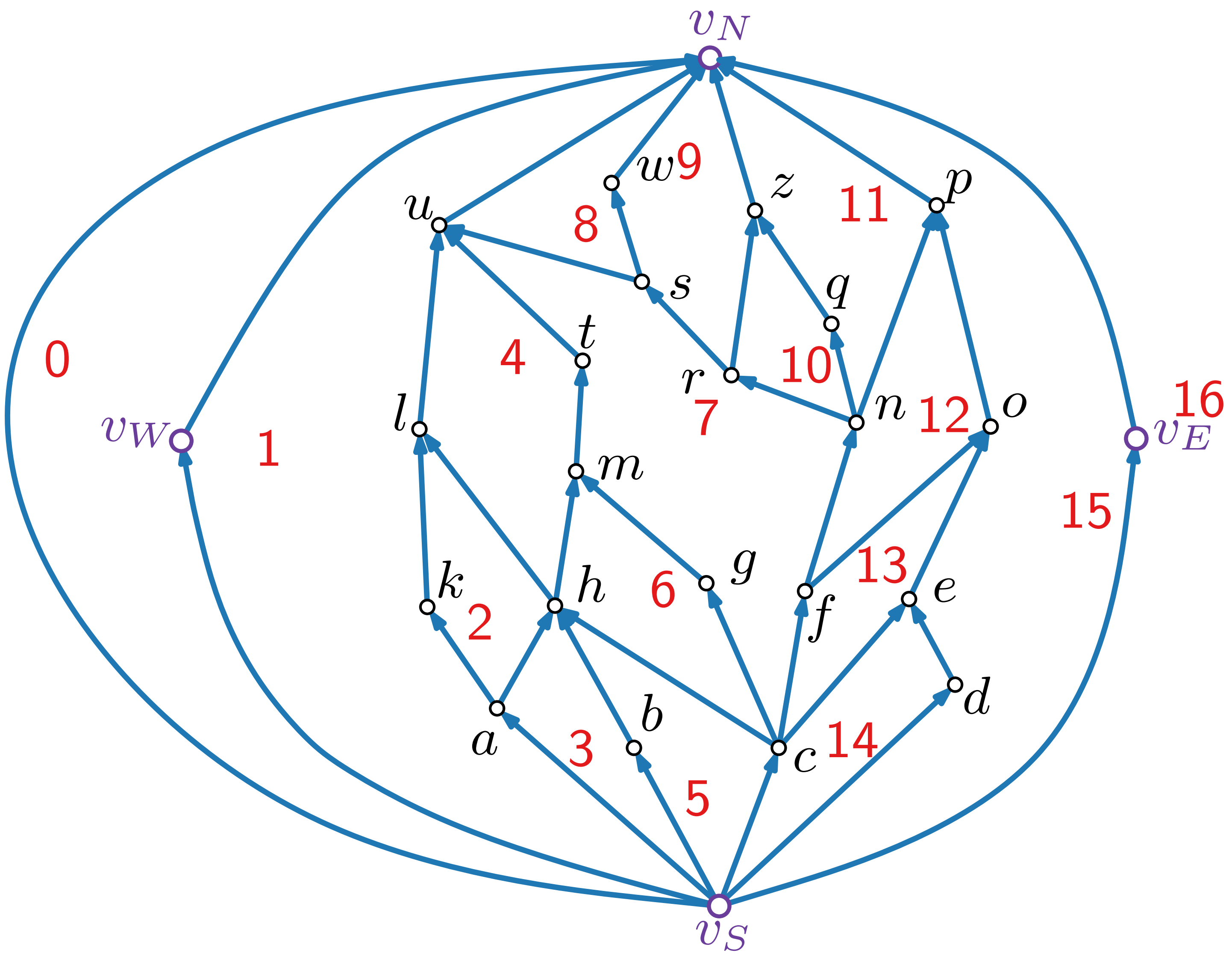
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$

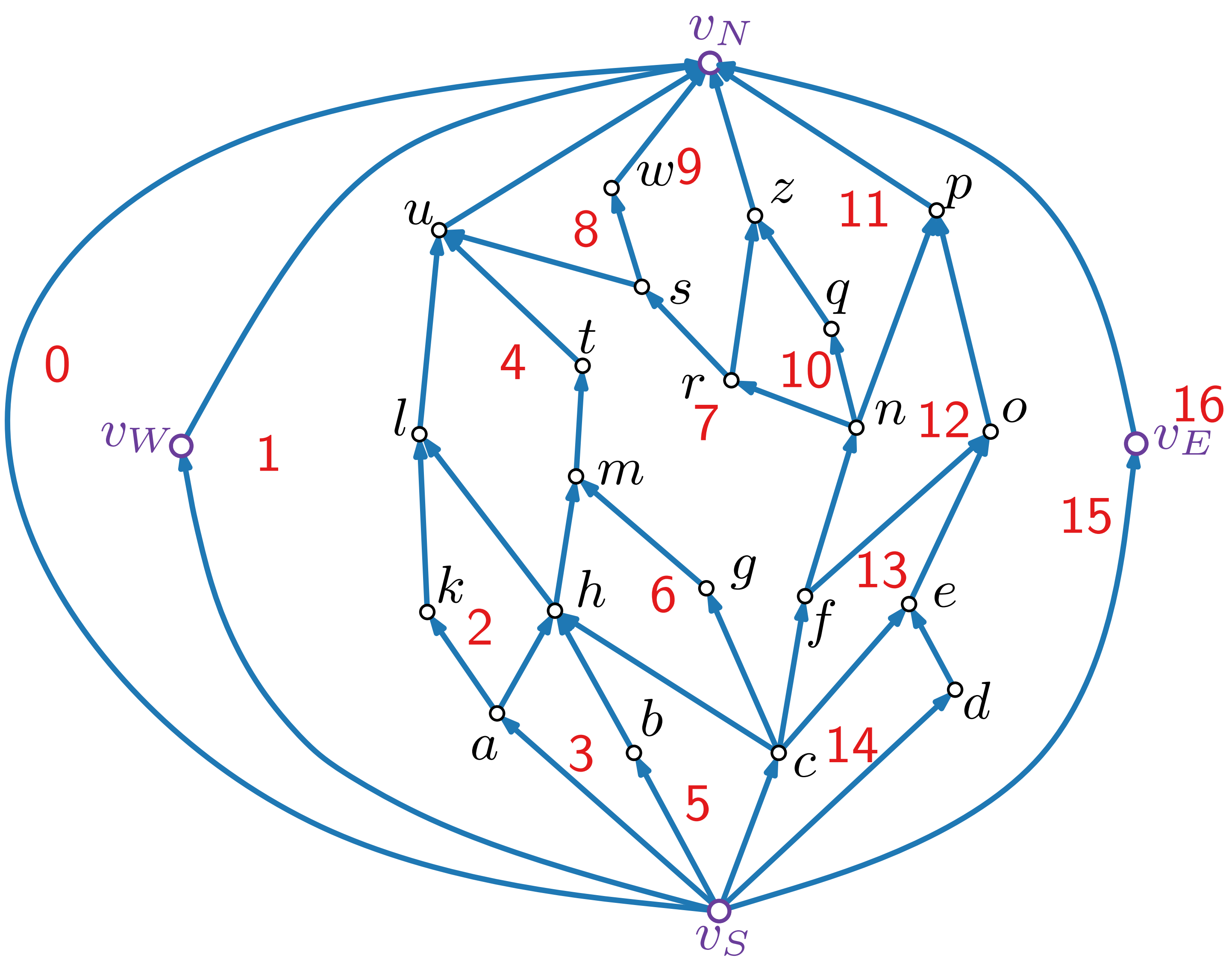
$x_1(v_S) = 1,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

Reading off Coordinates to Get Rectangular Dual



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$x_1(v_S) = 1,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

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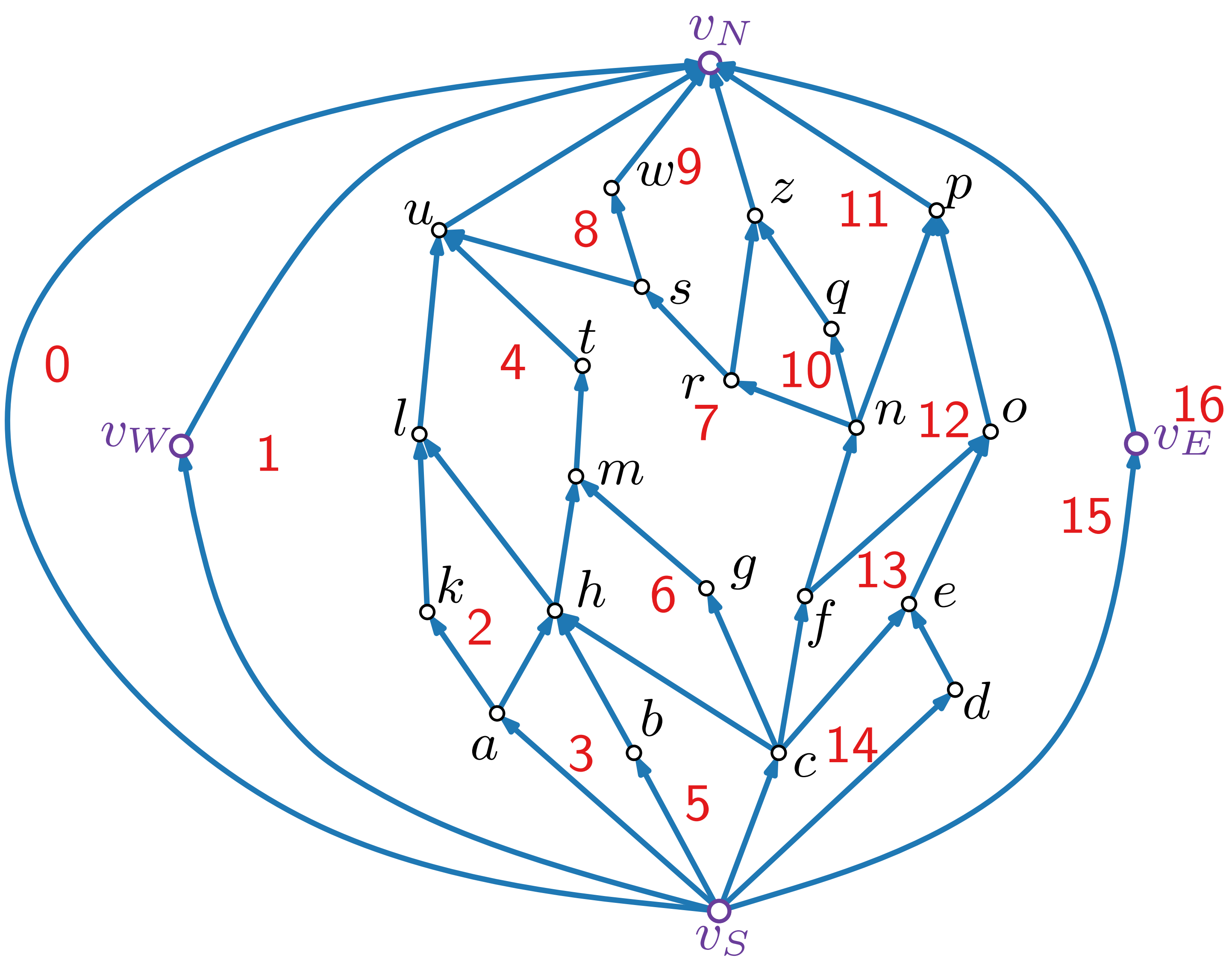
$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

Reading off Coordinates to Get Rectangular Dual



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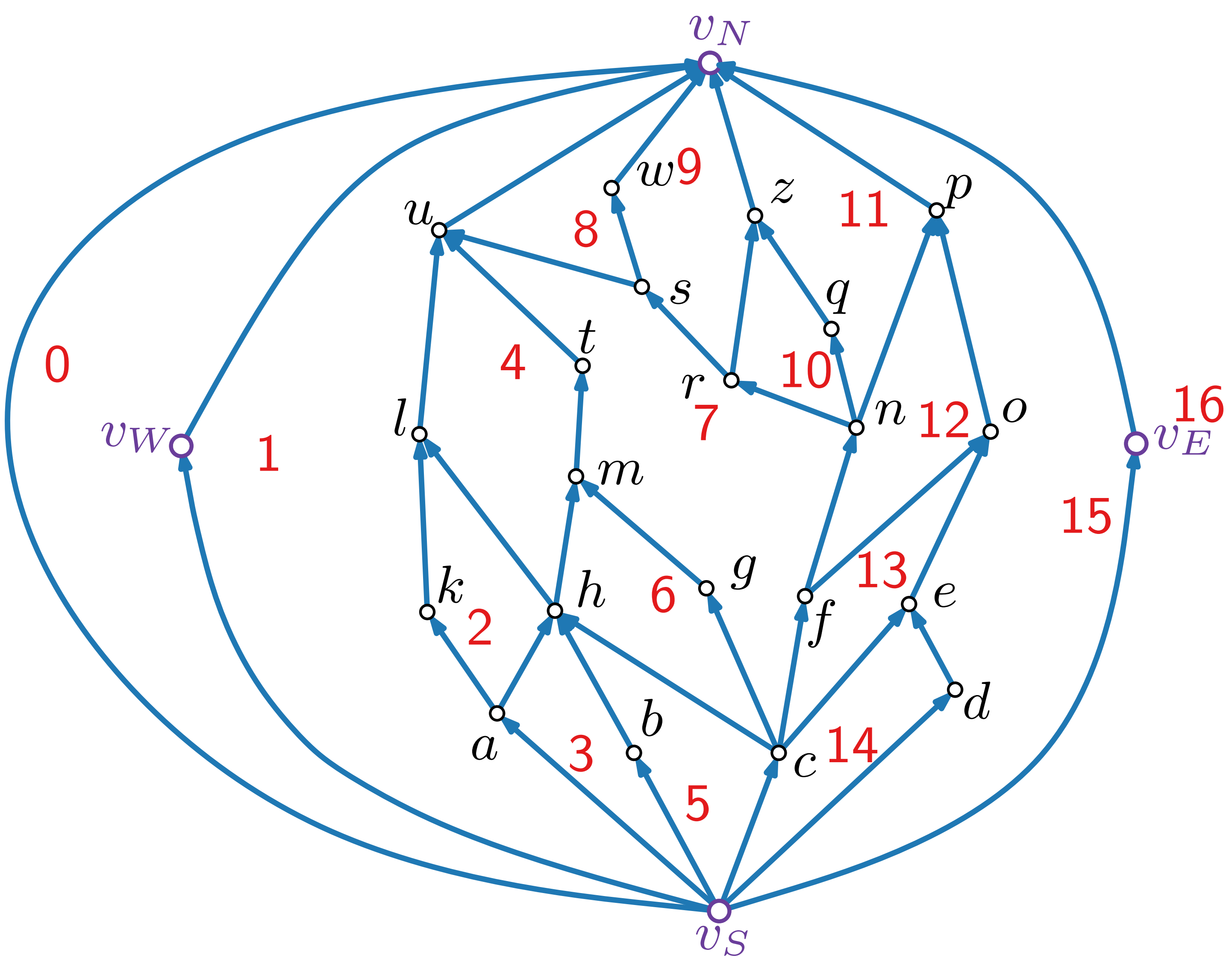
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$

Reading off Coordinates to Get Rectangular Dual



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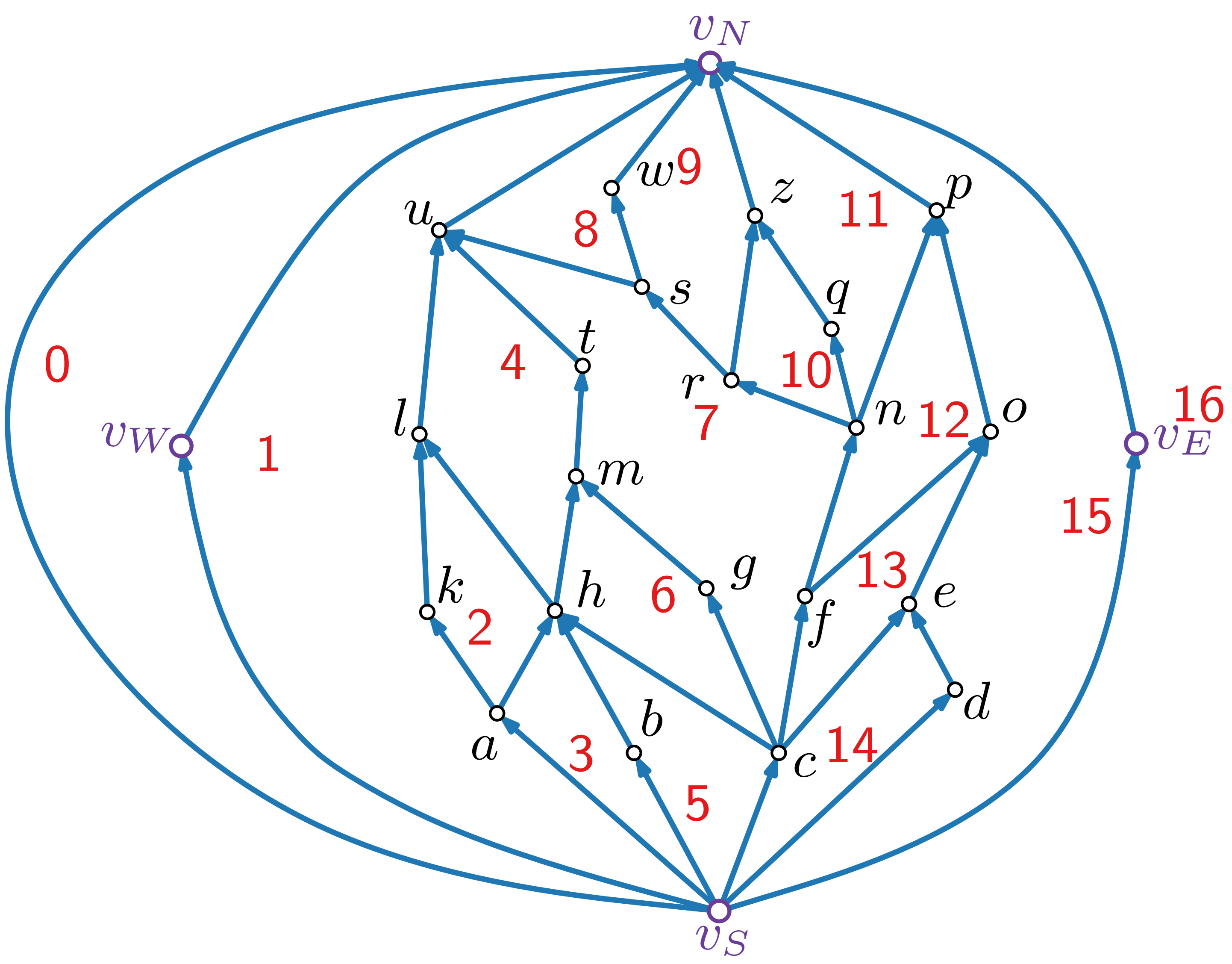
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$

Reading off Coordinates to Get Rectangular Dual



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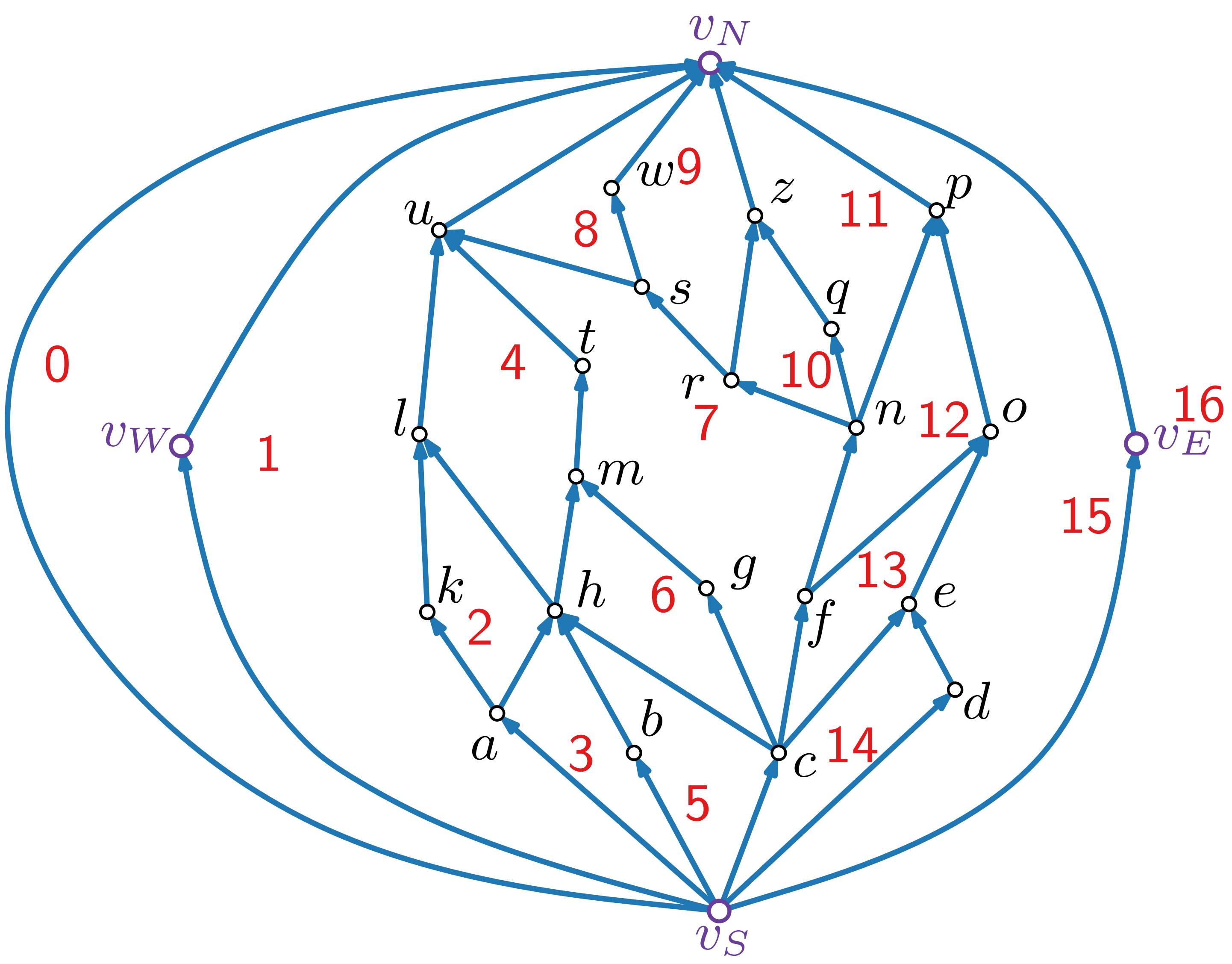
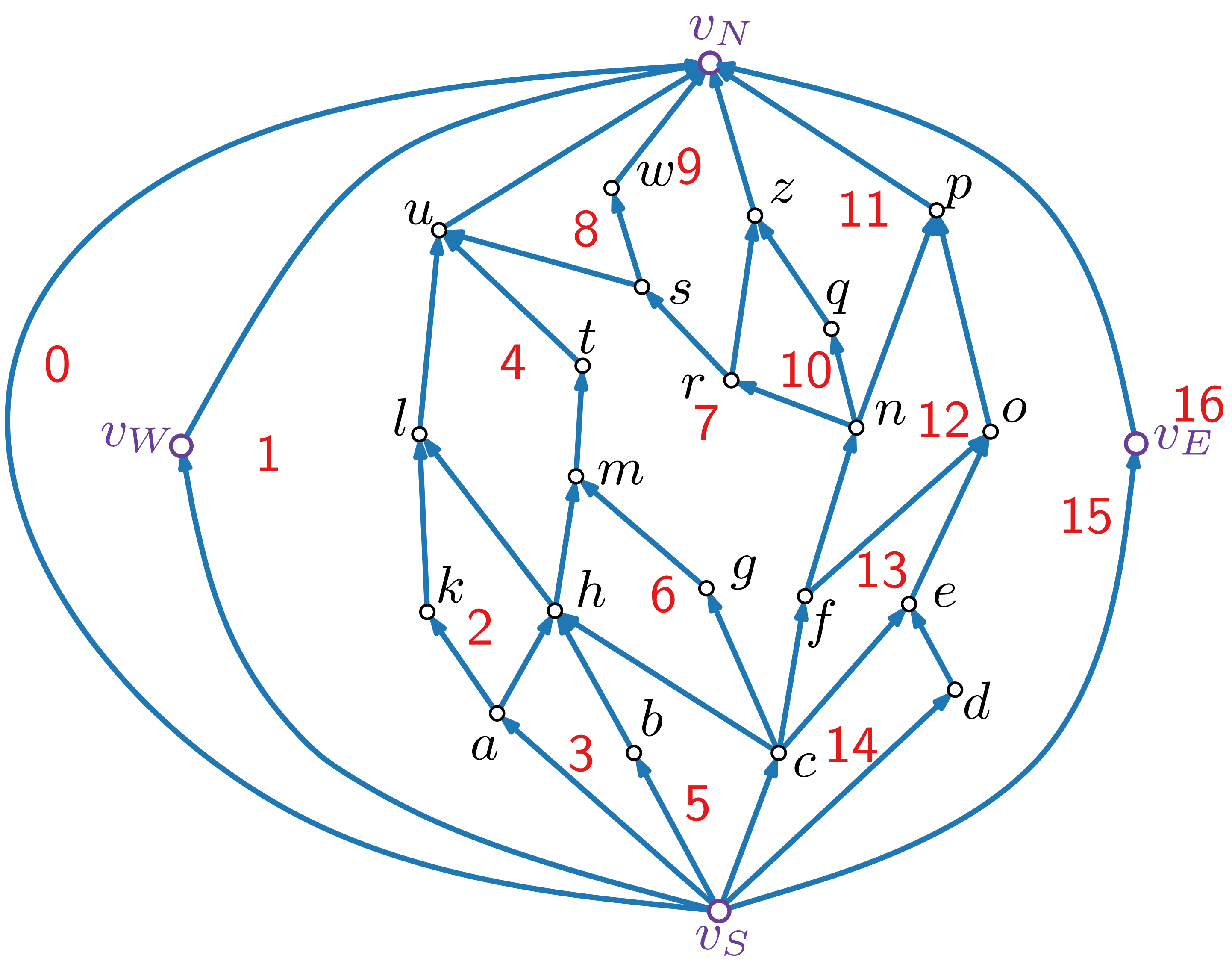
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

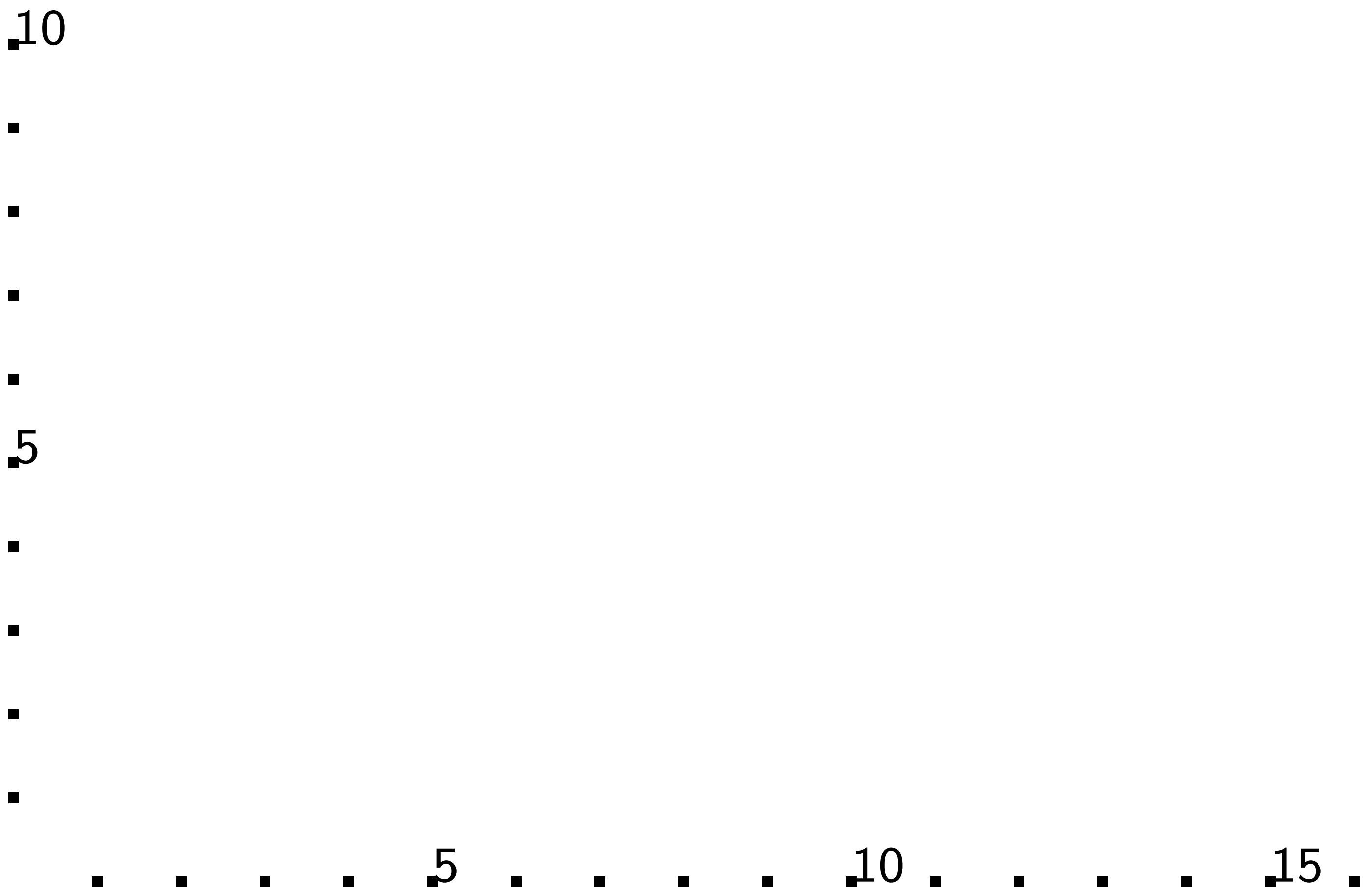
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

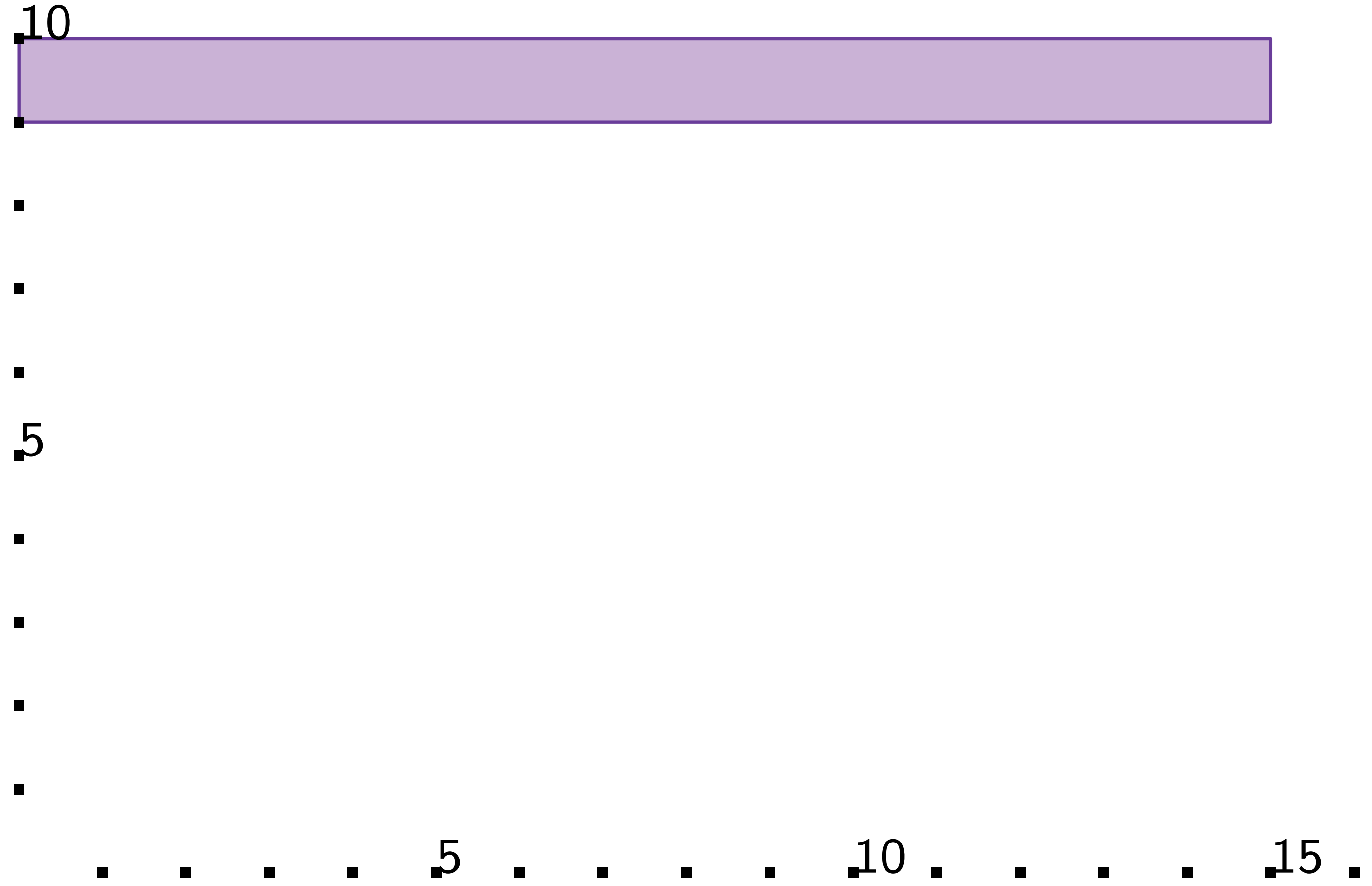
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

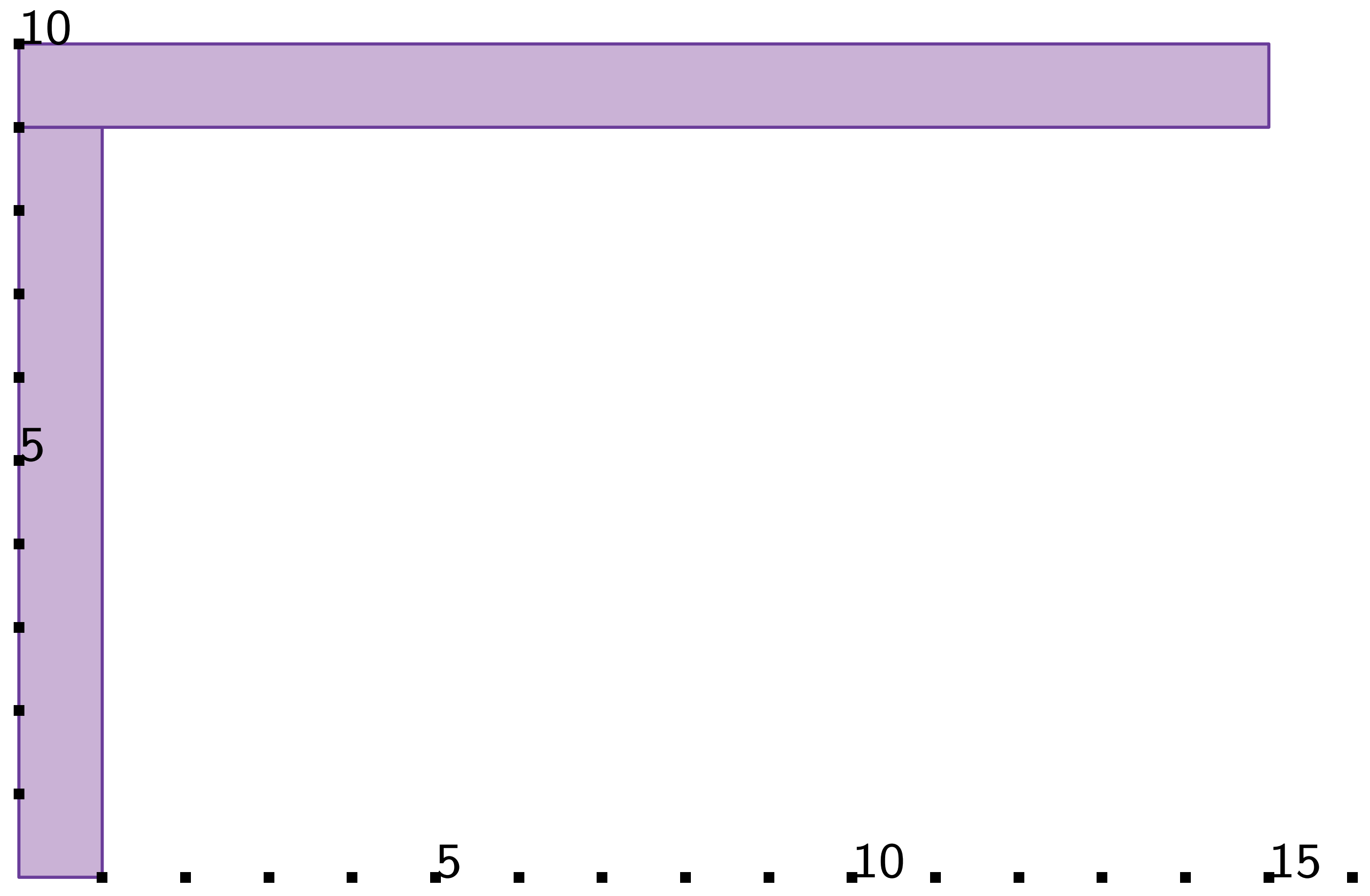
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

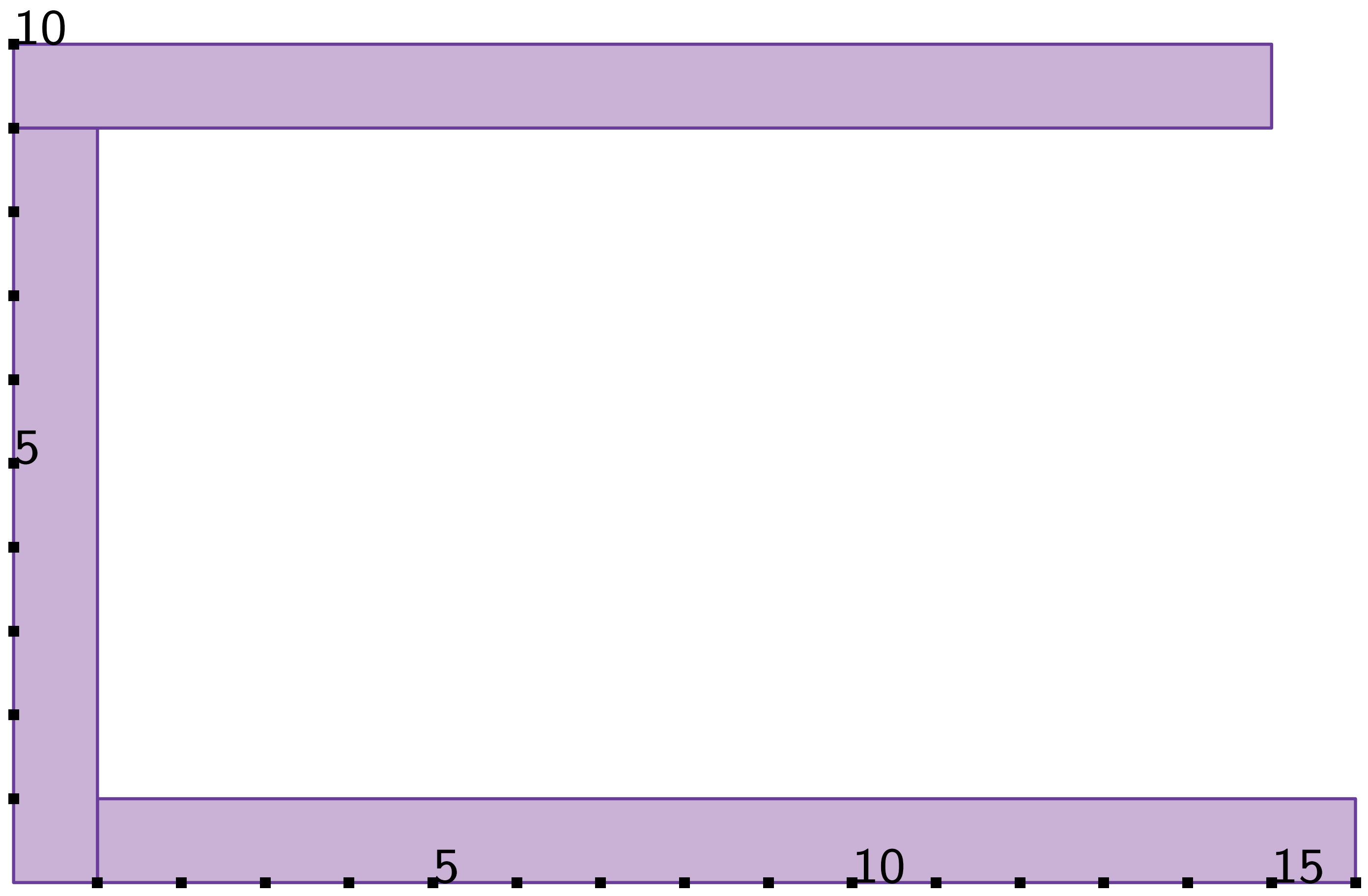
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

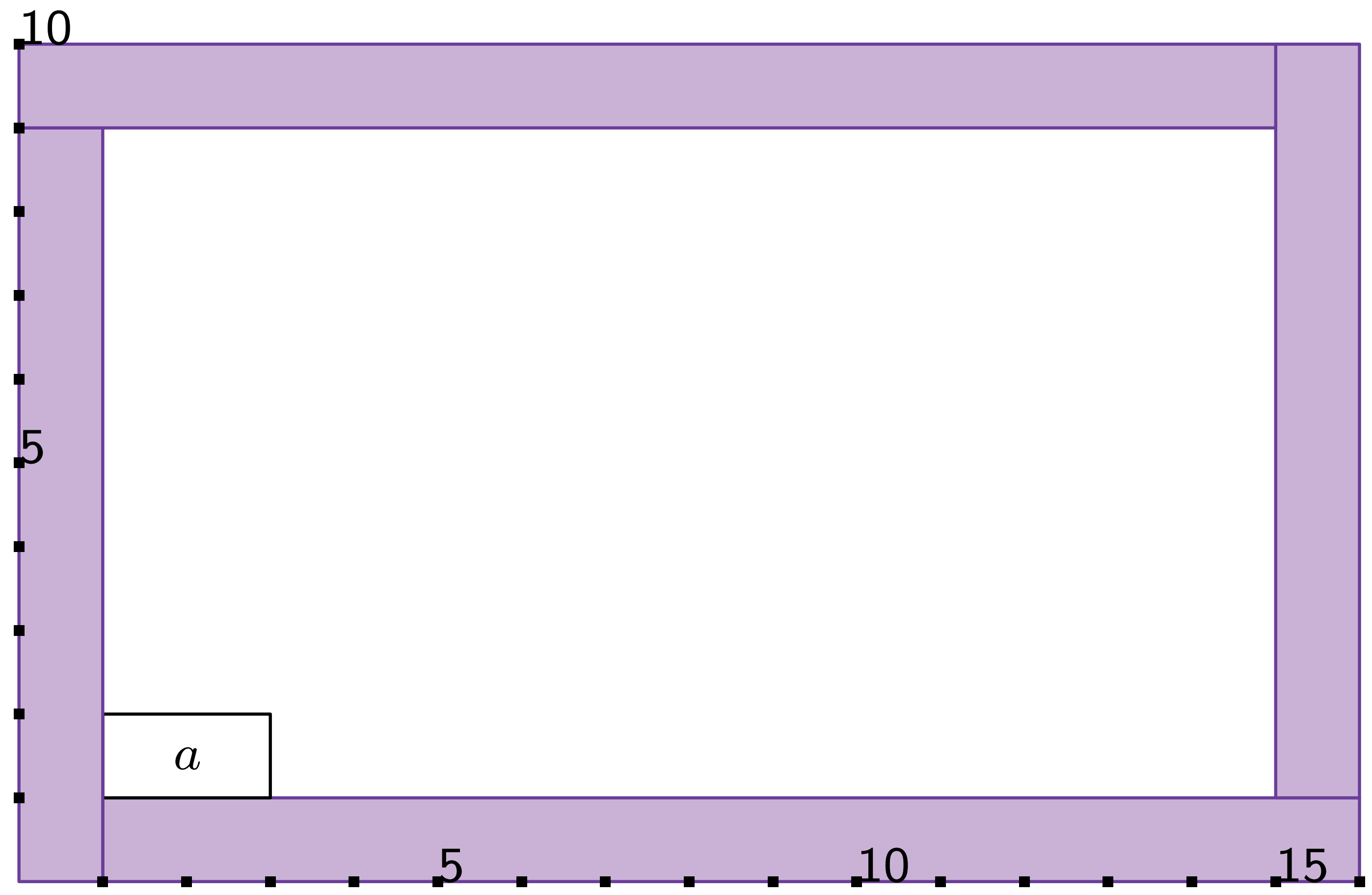
$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

Reading off Coordinates to Get Rectangular Dual

$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

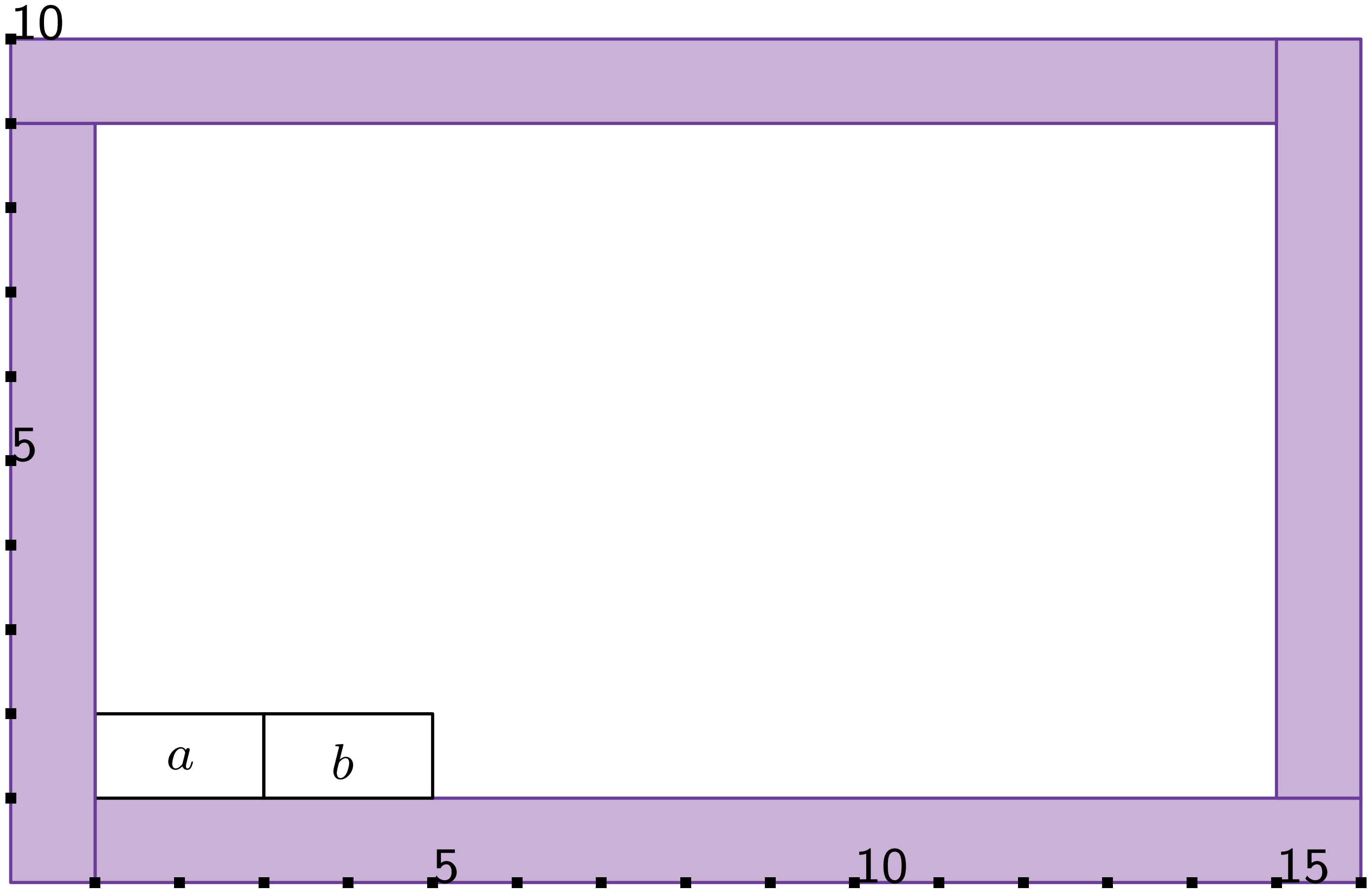
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

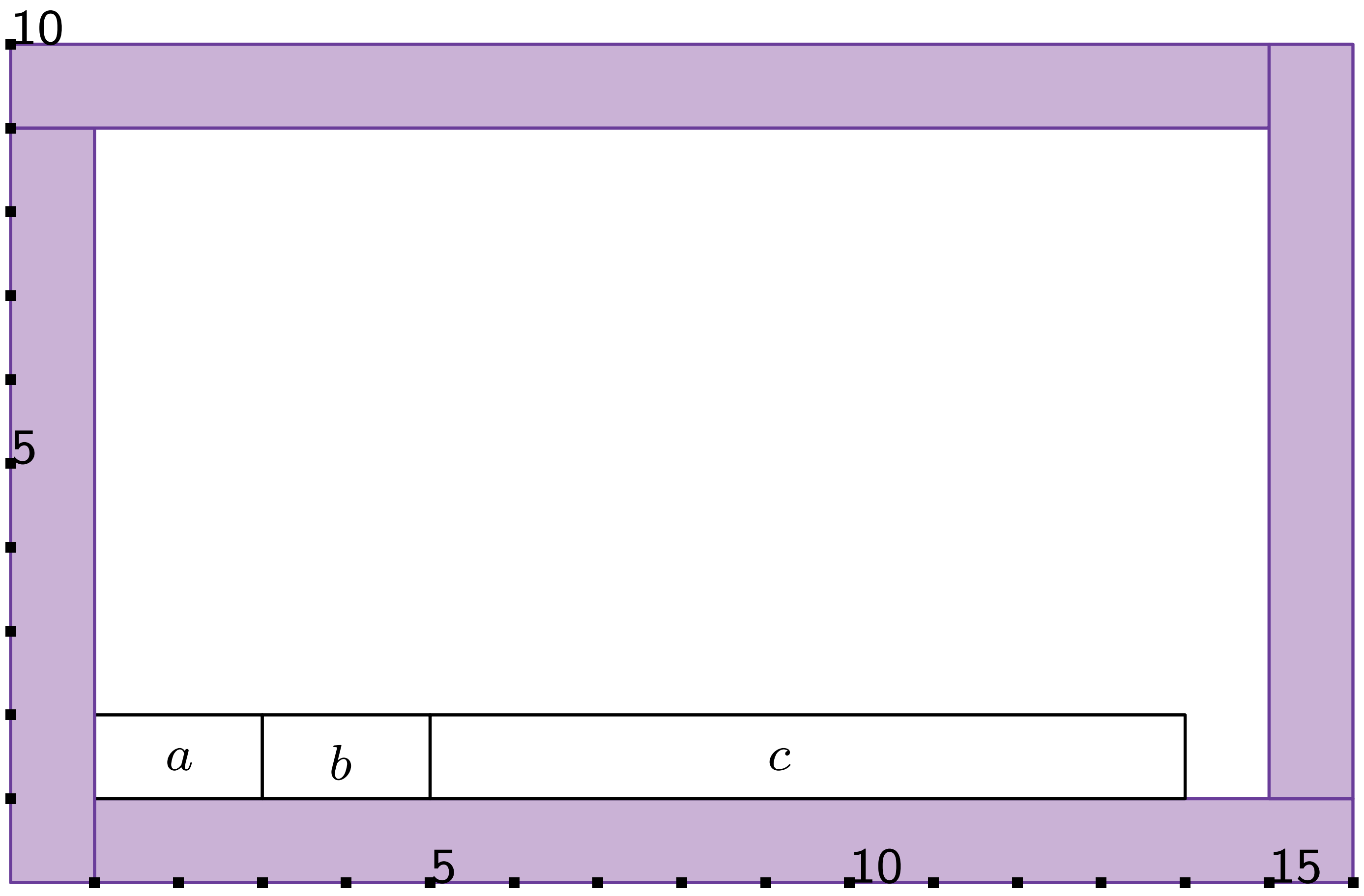
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

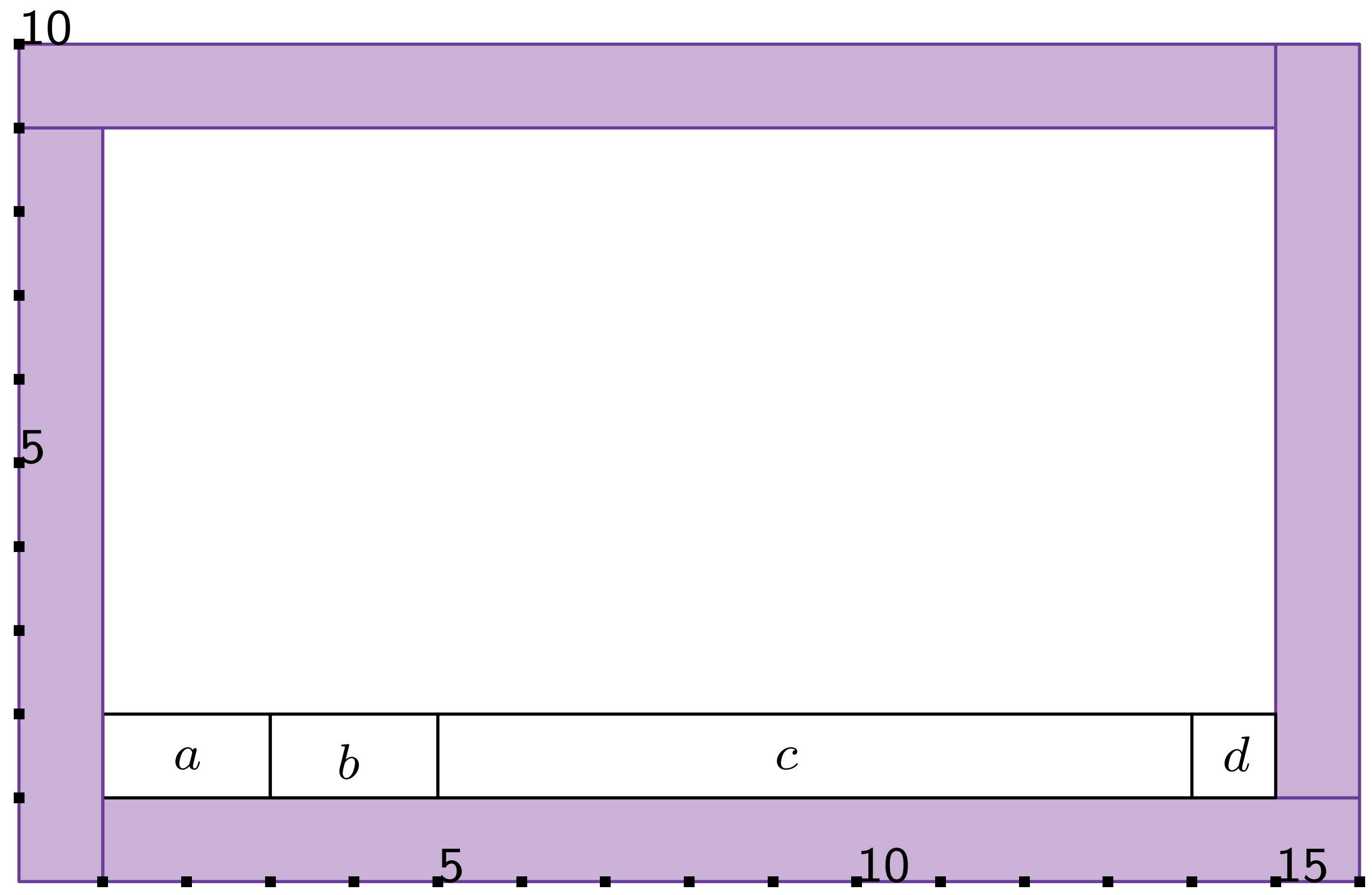
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

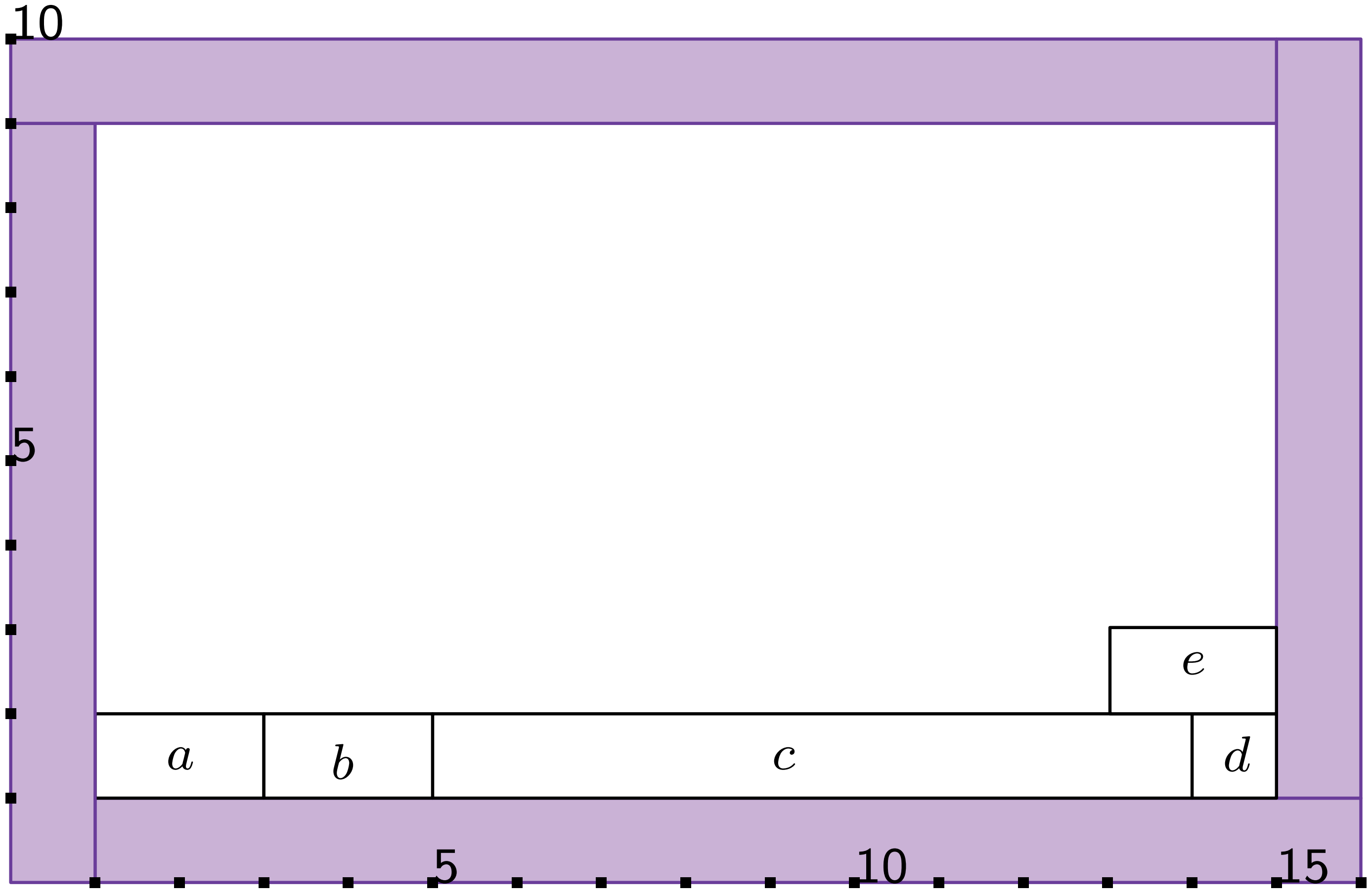
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

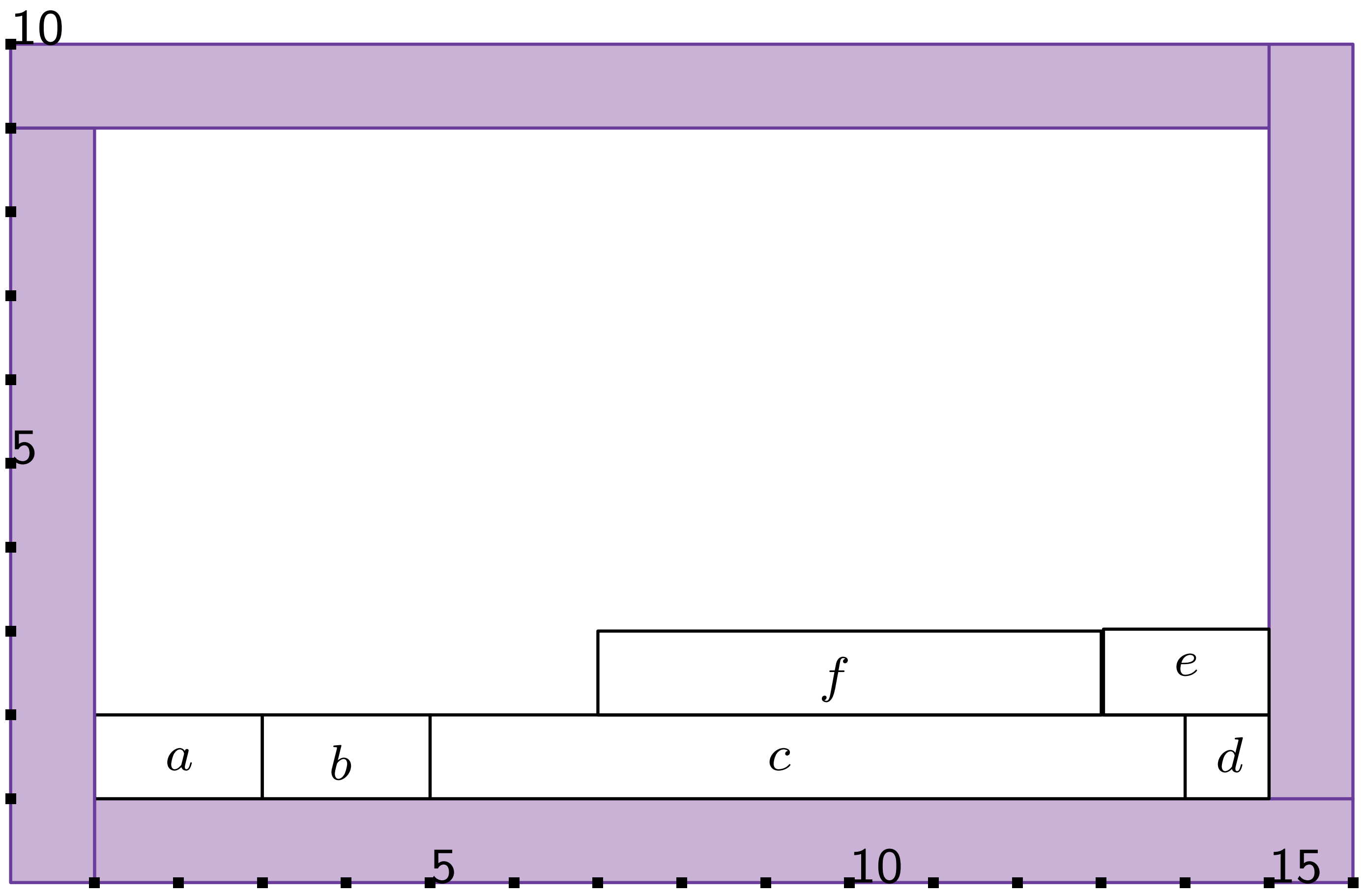
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

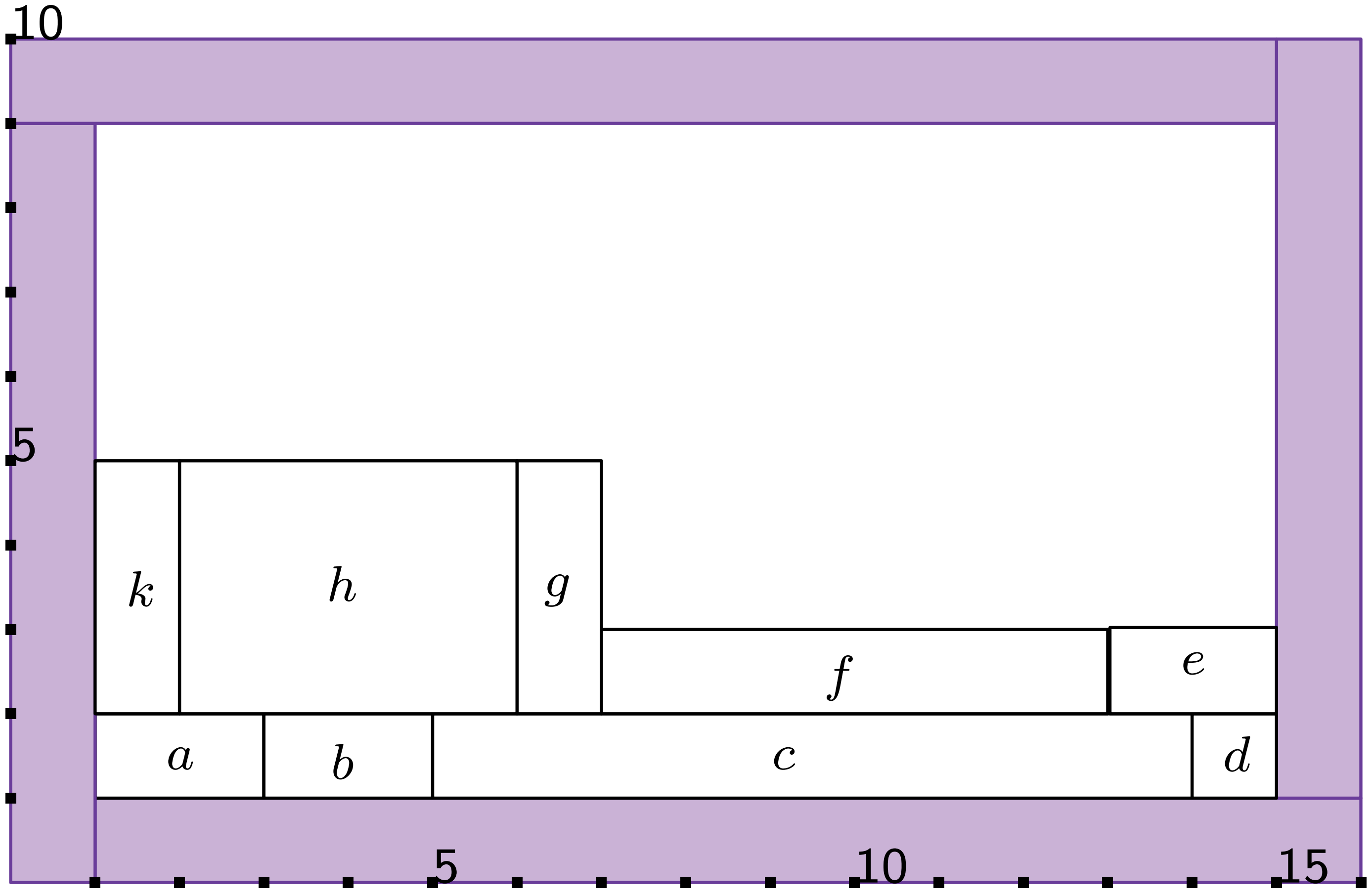
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

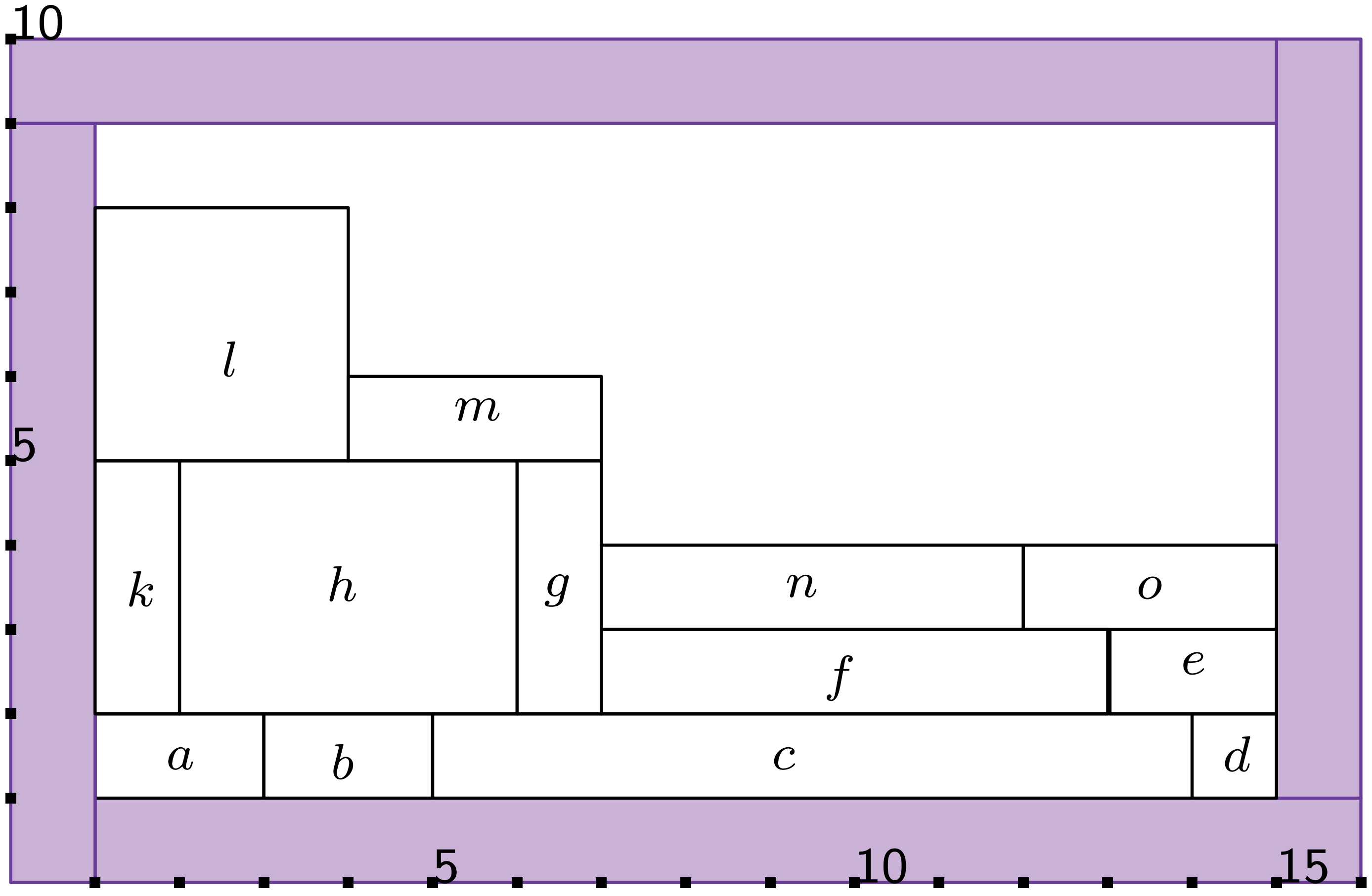
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

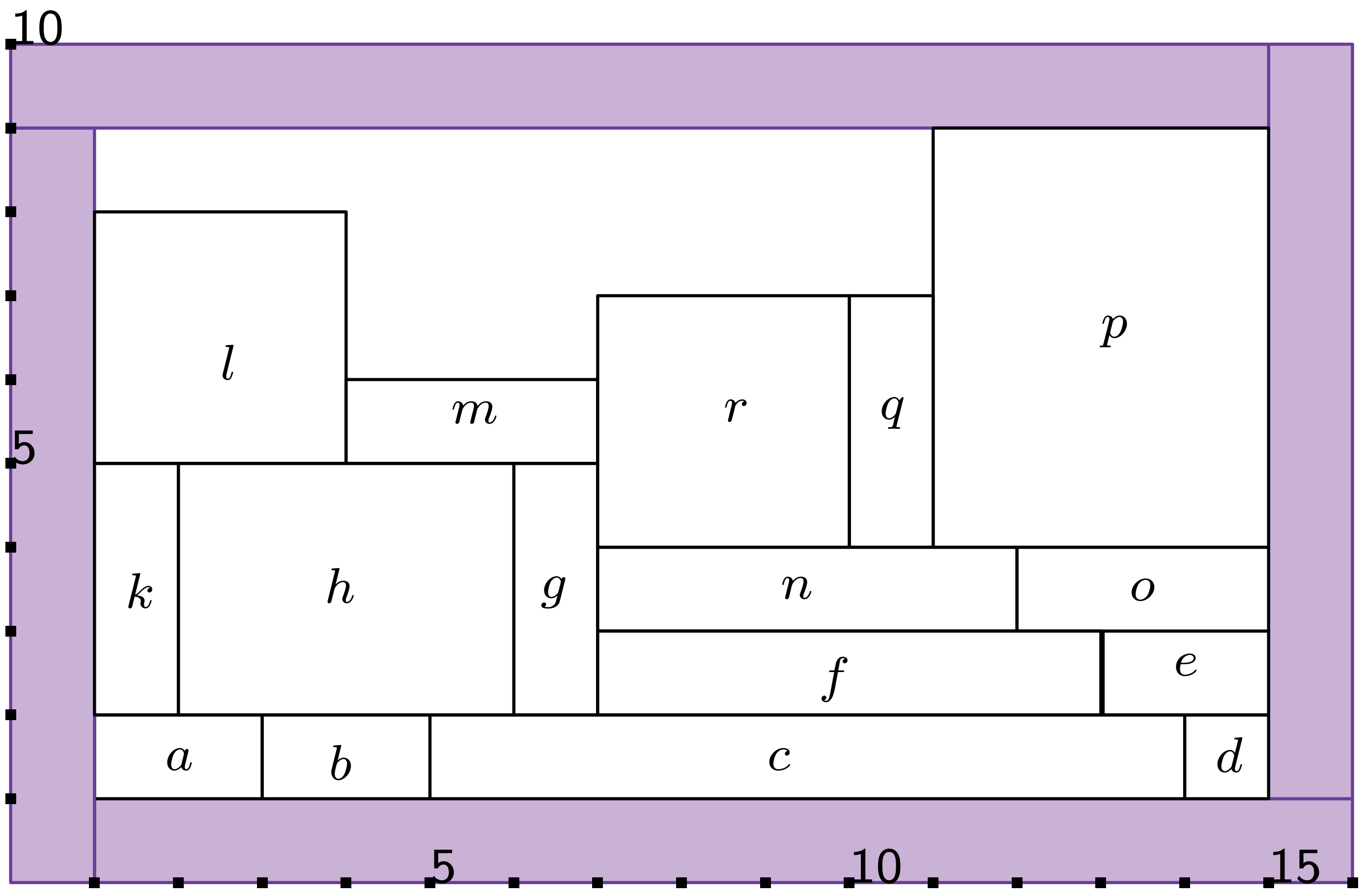
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

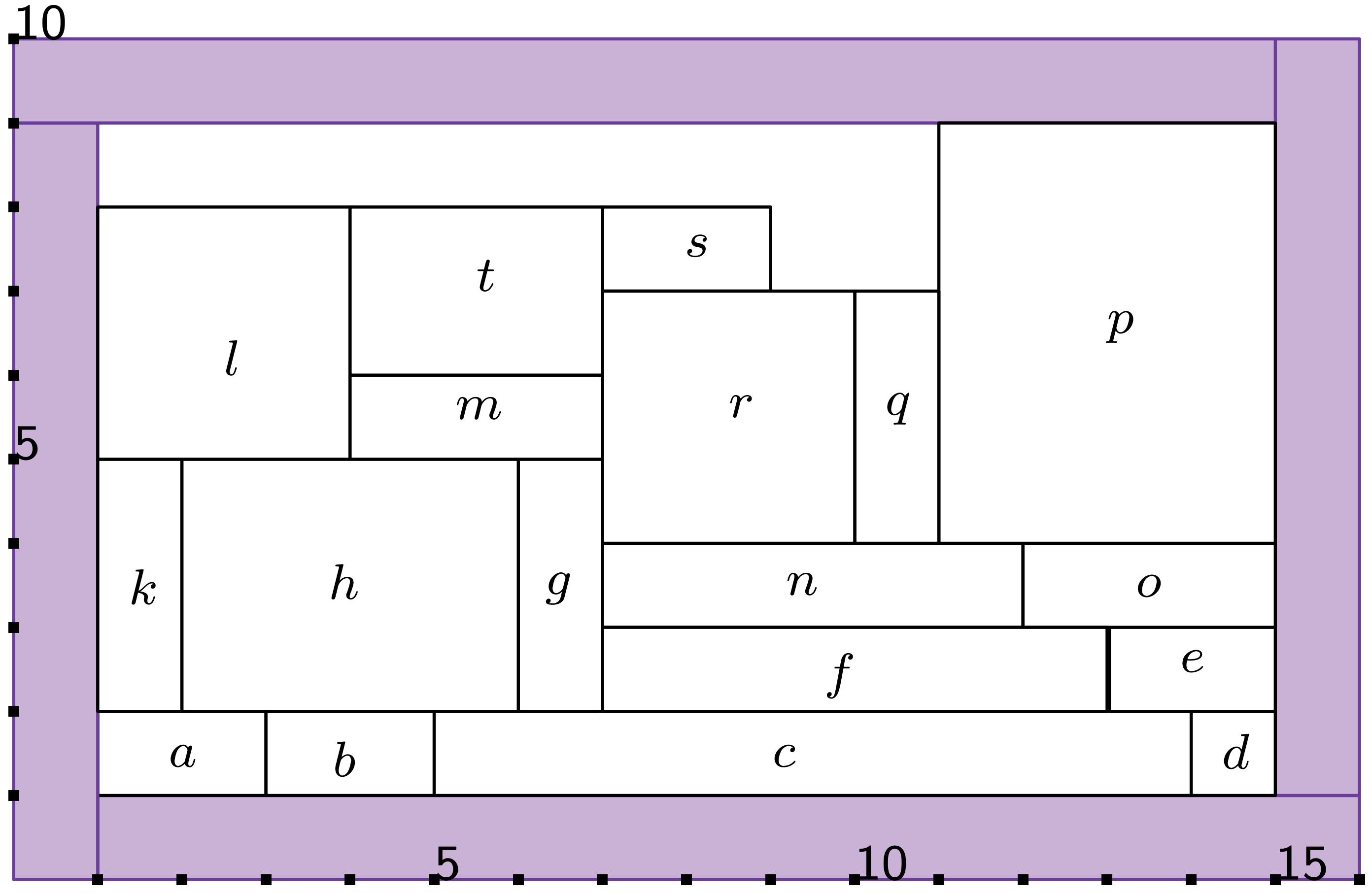
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

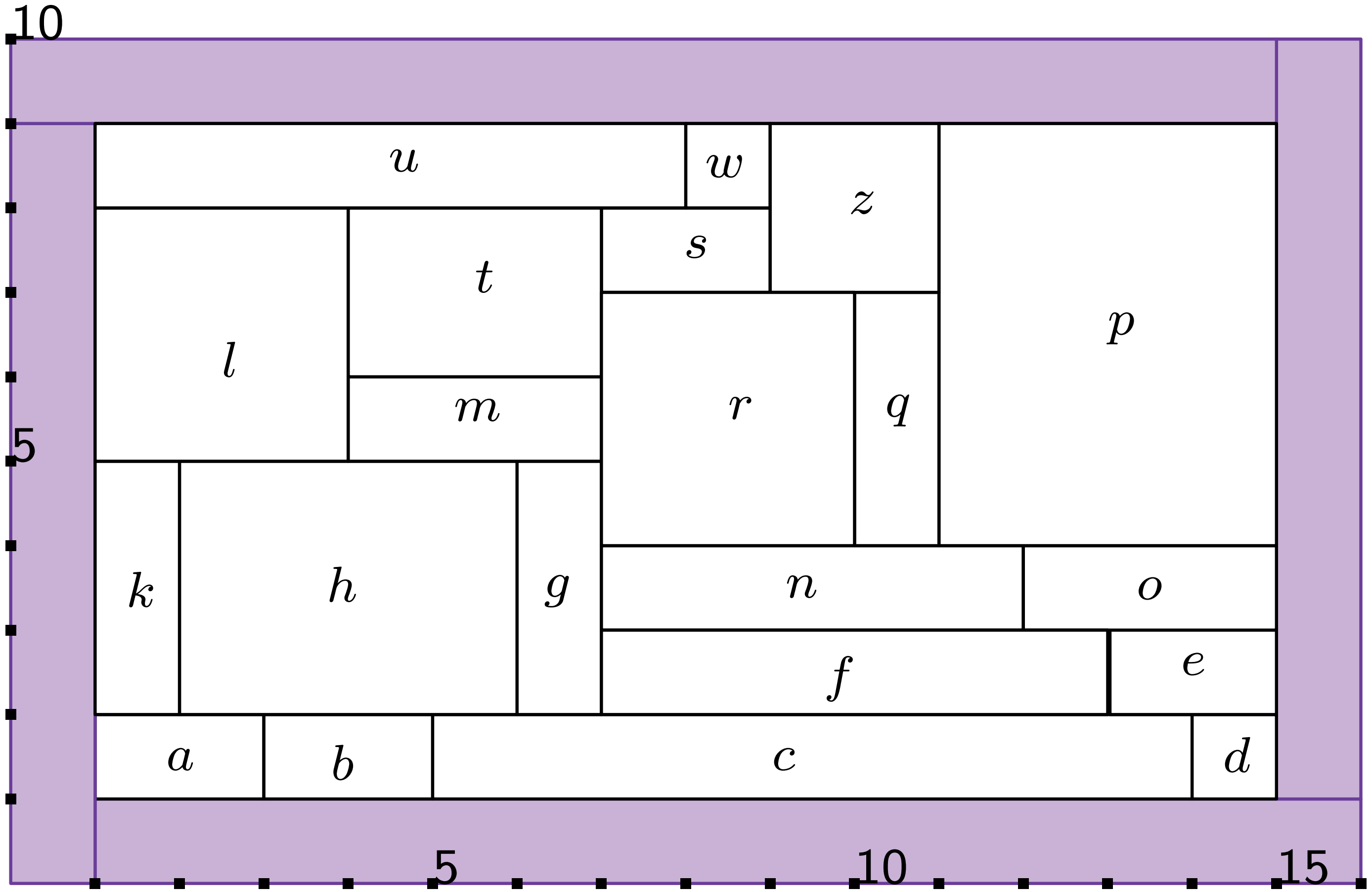
Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

Reading off Coordinates to Get Rectangular Dual



$x_1(v_N) = 0,\ x_2(v_N) = 15$
$x_1(v_S) = 1,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
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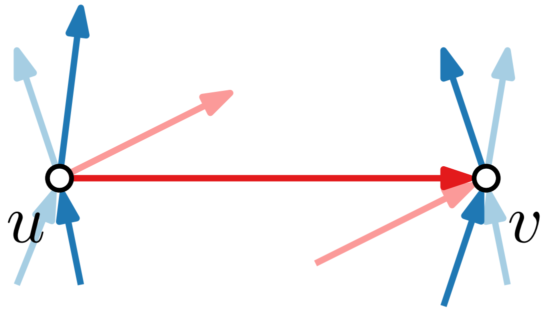

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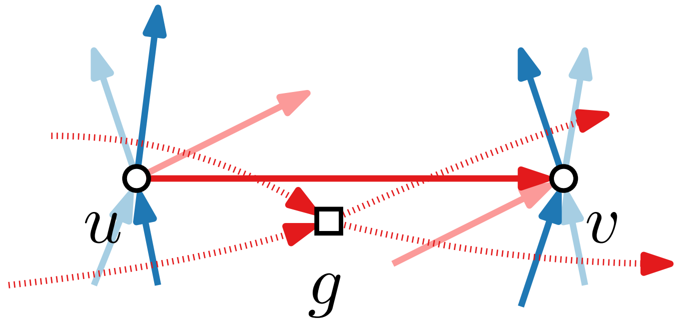

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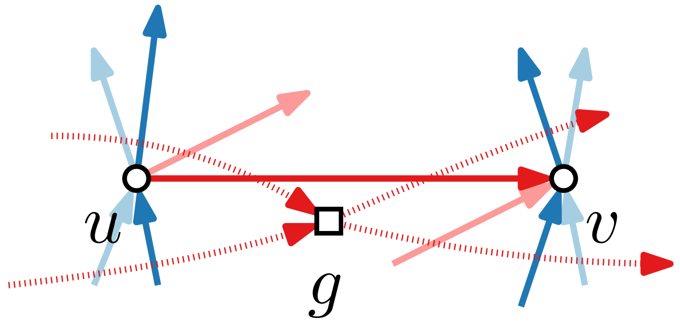

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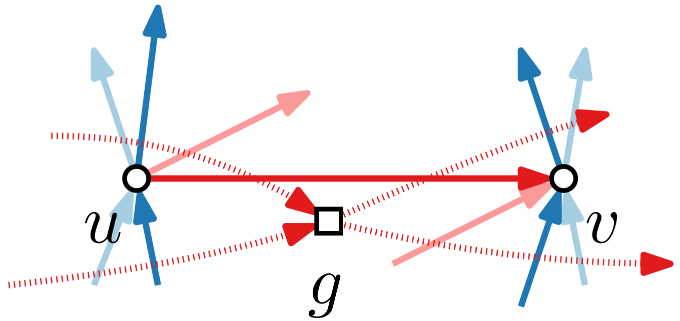

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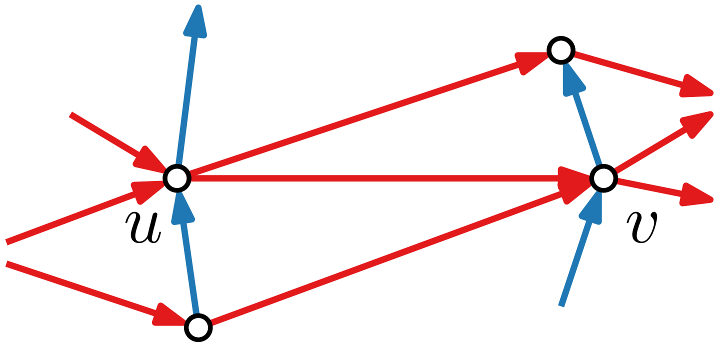
- and the vertical segments of their rectangles overlap



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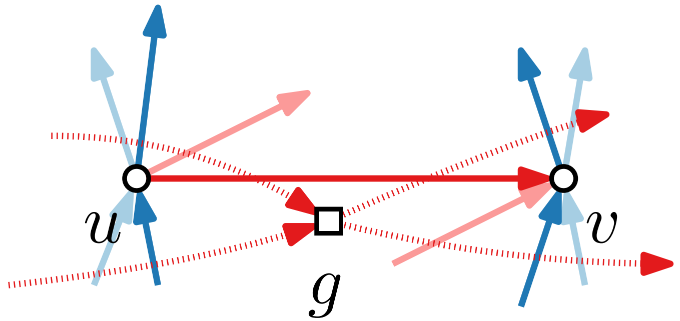

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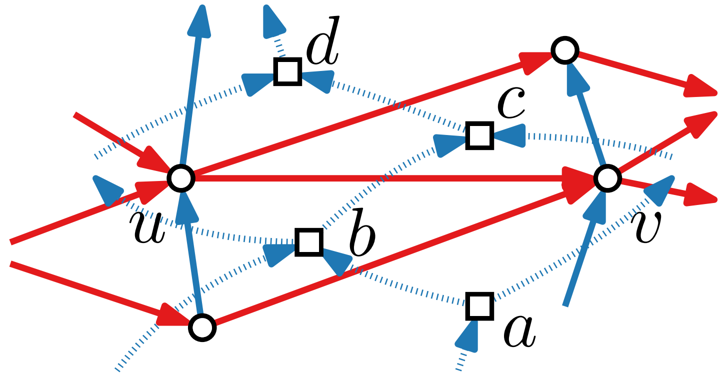

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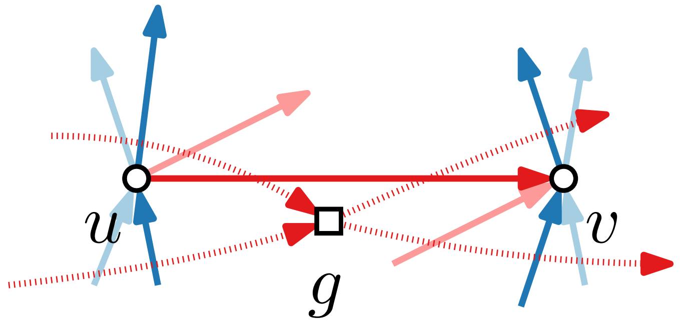

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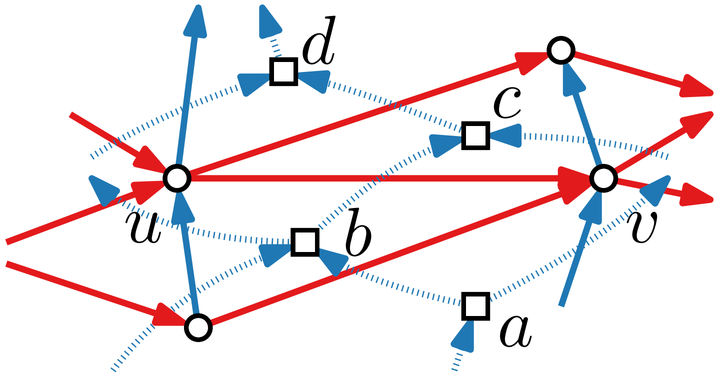

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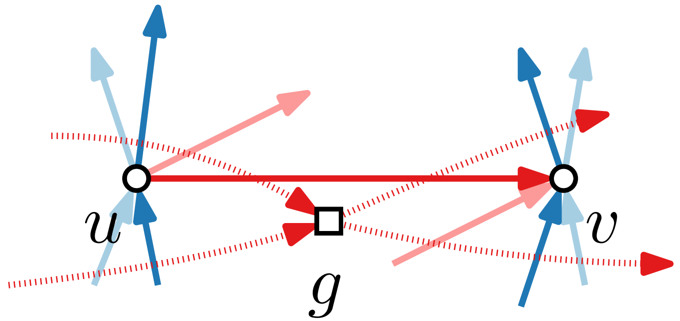

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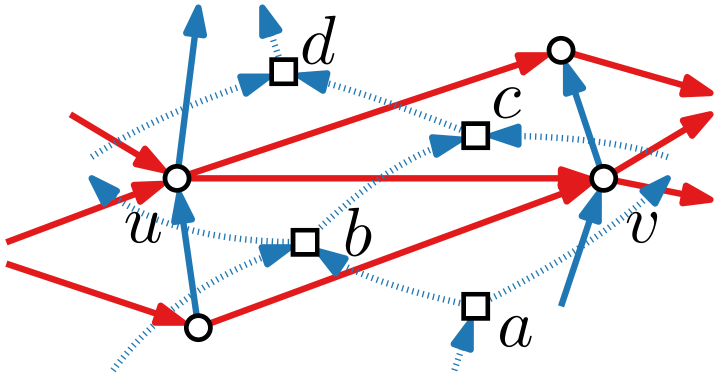

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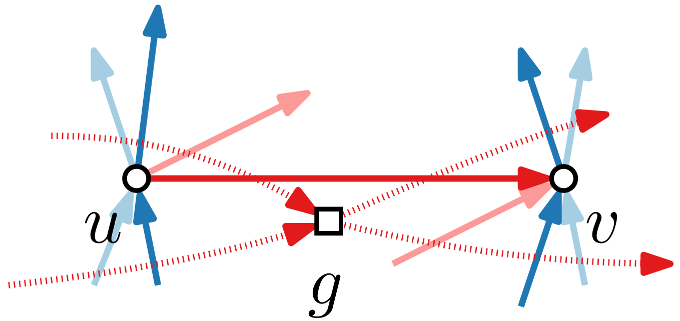

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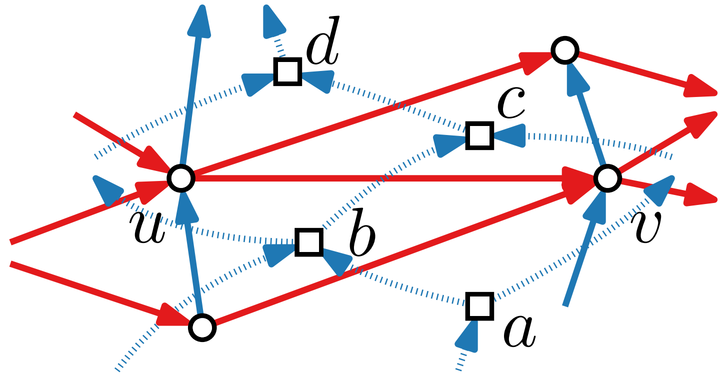

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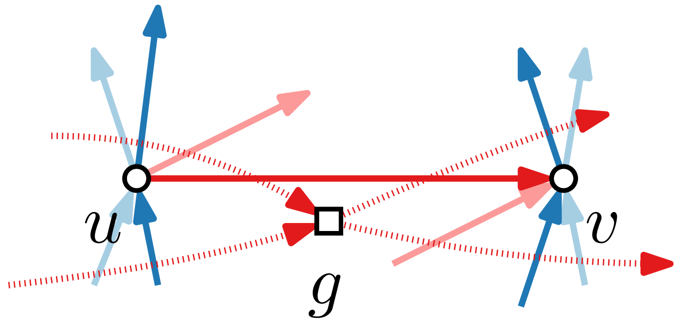

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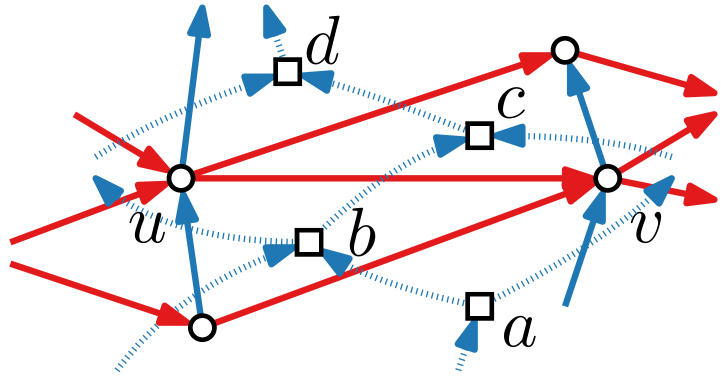

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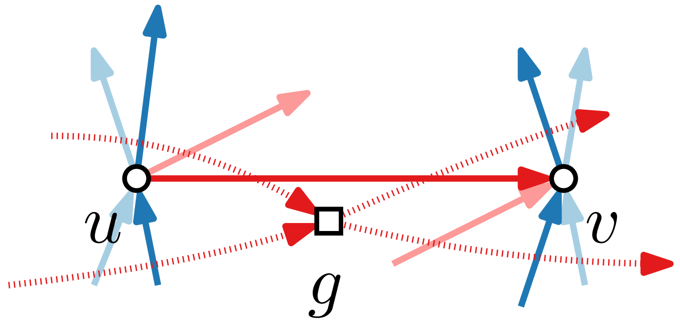

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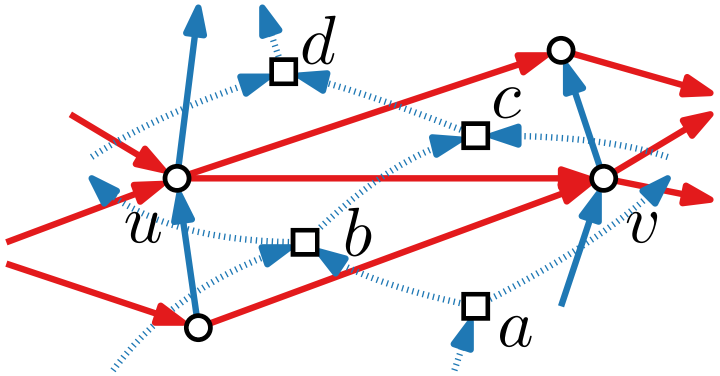

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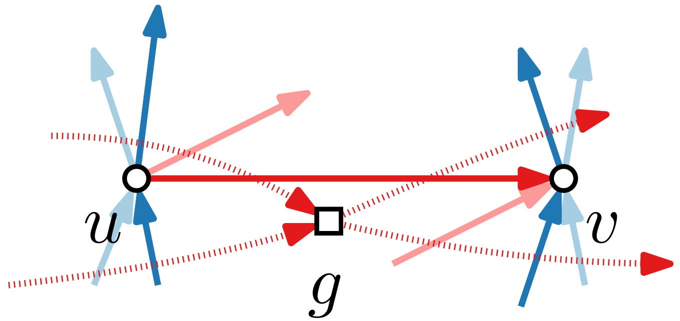

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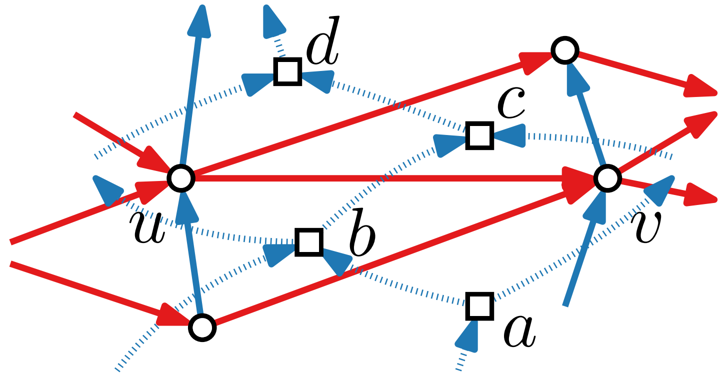

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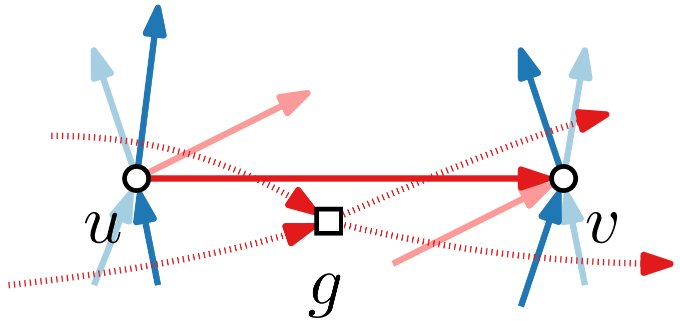

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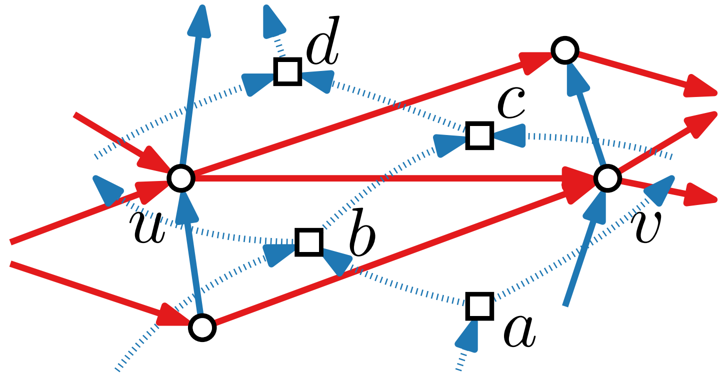

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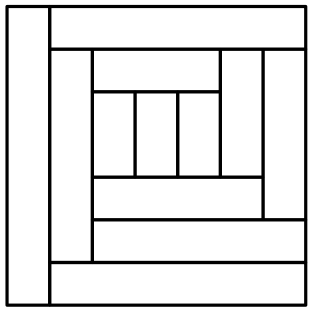

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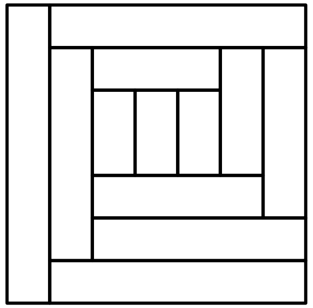

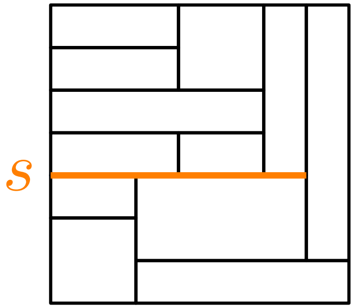

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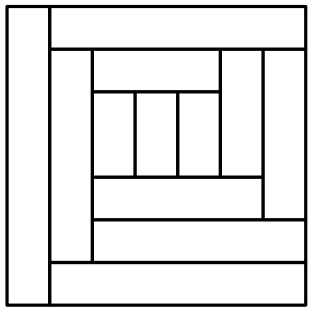

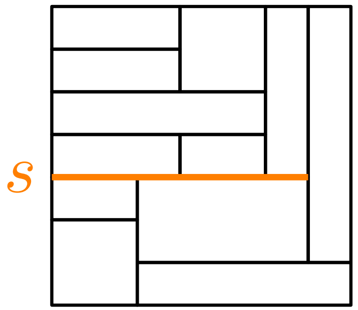

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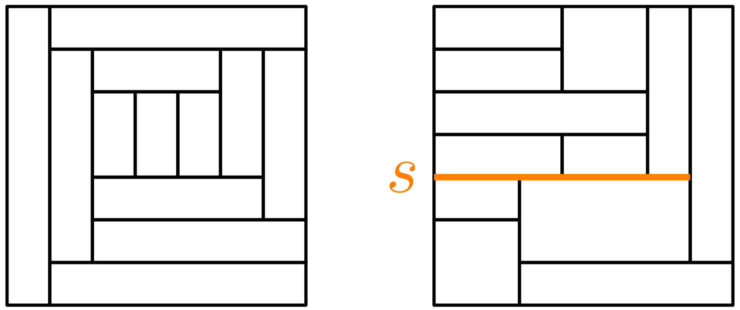

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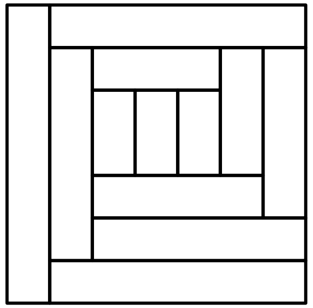

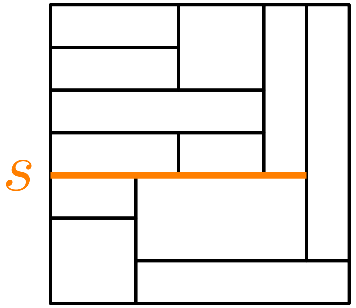

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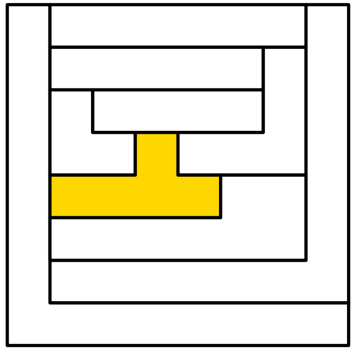

Literature

Construction of triangle contact representations based on

- [de Fraysseix, Ossona de Mendez, Rosenstiehl ’94] On Triangle Contact Graphs

Construction of rectangular dual based on

- [He ’93] On Finding the Rectangular Duals of Planar Triangulated Graphs
- [Kant, He ’94] Two algorithms for finding rectangular duals of planar graphs

and originally from

- [Koźmiński, Kinnen ’85] Rectangular Duals of Planar Graphs