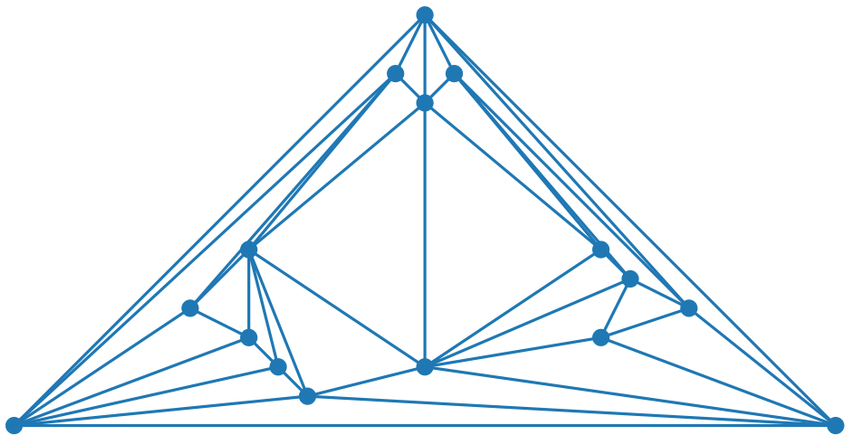
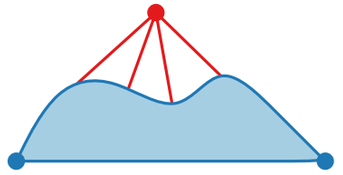
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

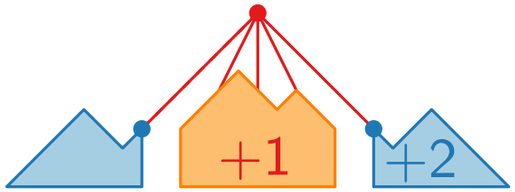
Visualization of Graphs

Lecture 3:
Straight-Line Drawings of Planar Graphs I:
Canonical Ordering and the Shift Method

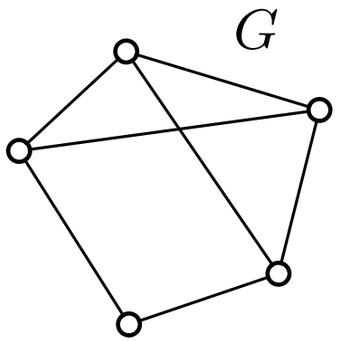


Johannes Zink





Planar Graphs



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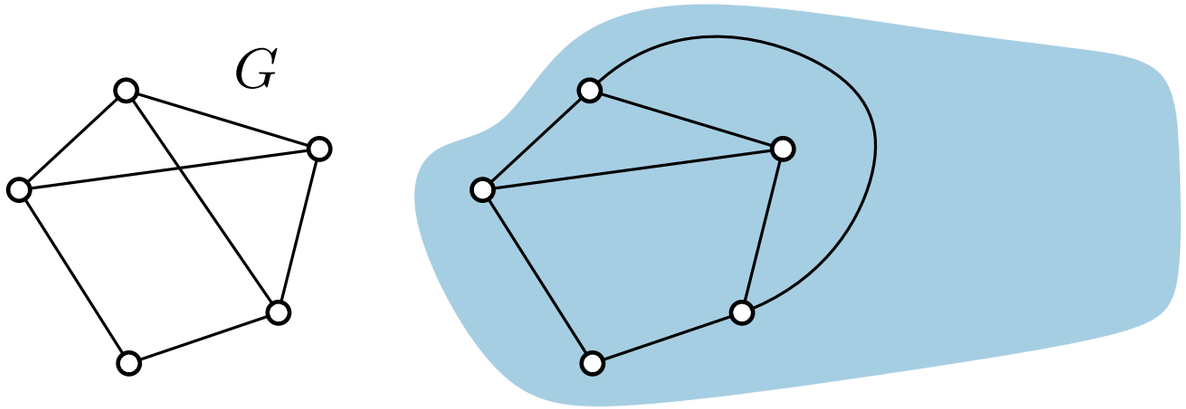

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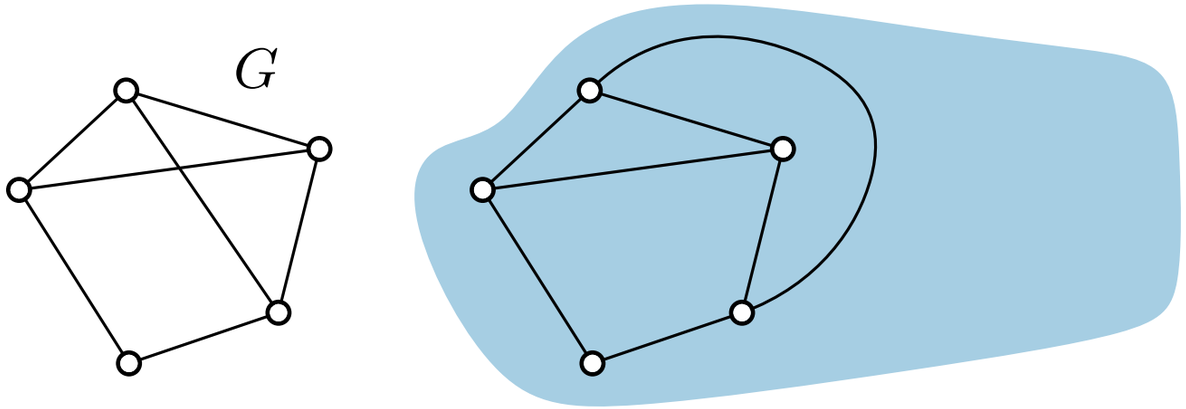

G is **planar**:
it can be drawn in such a way that no two edges intersect each other.

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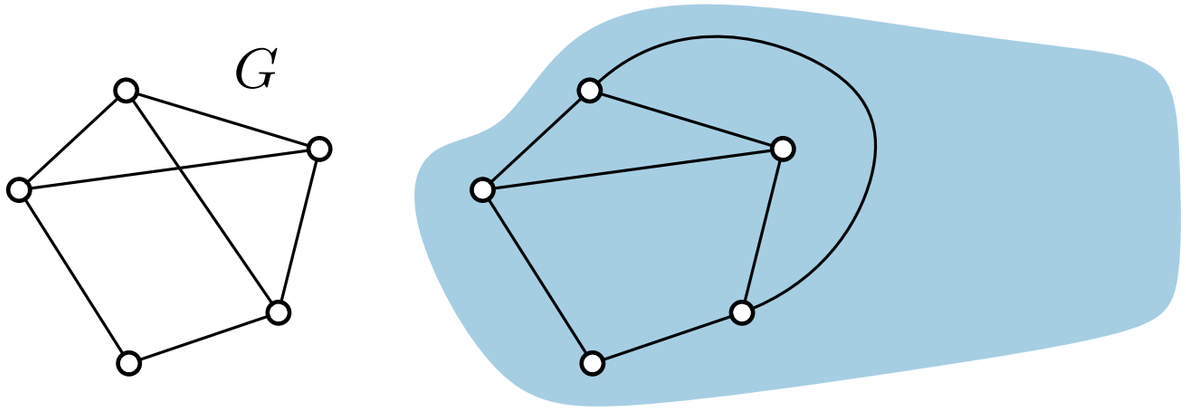

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planar embedding:
clockwise orientation of adjacent vertices around each vertex

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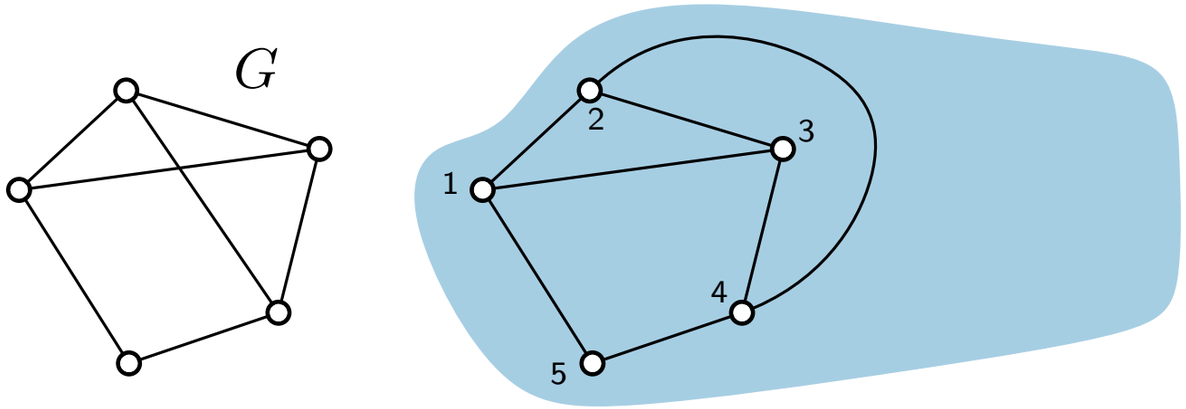

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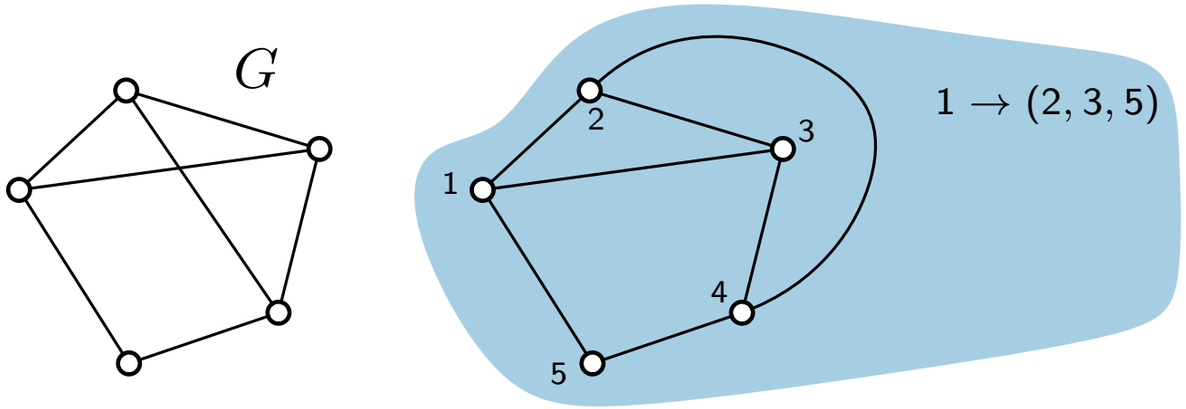

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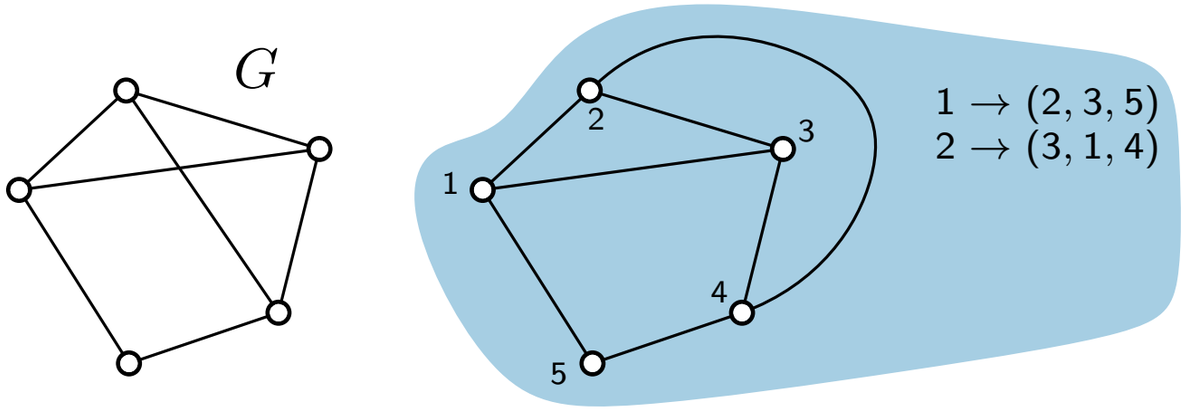

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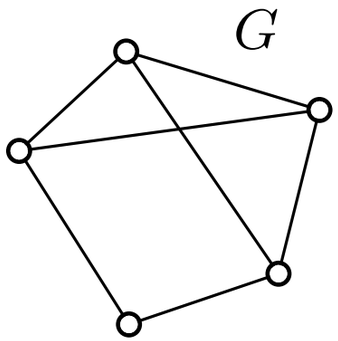

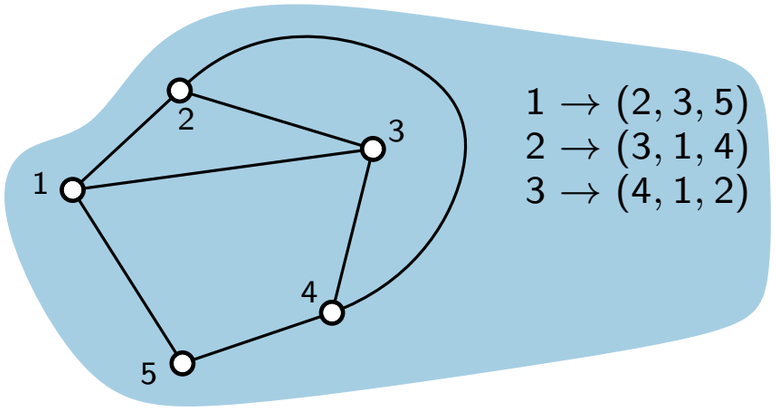

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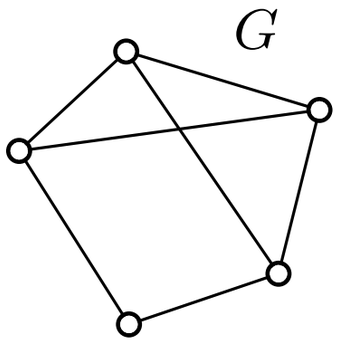

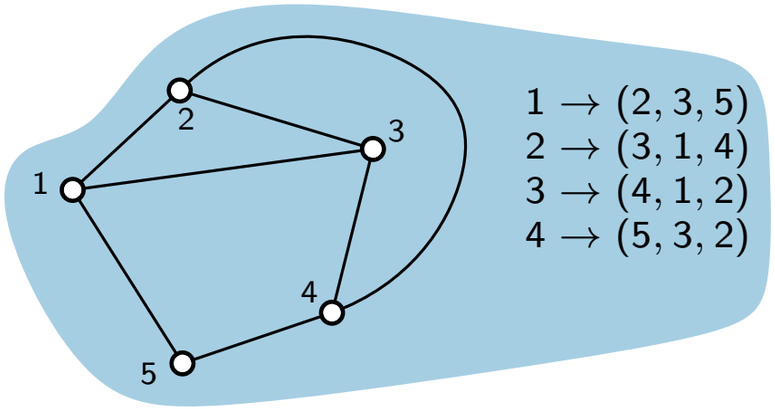

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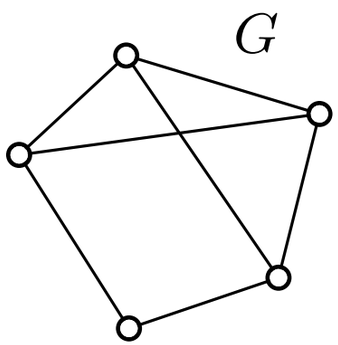

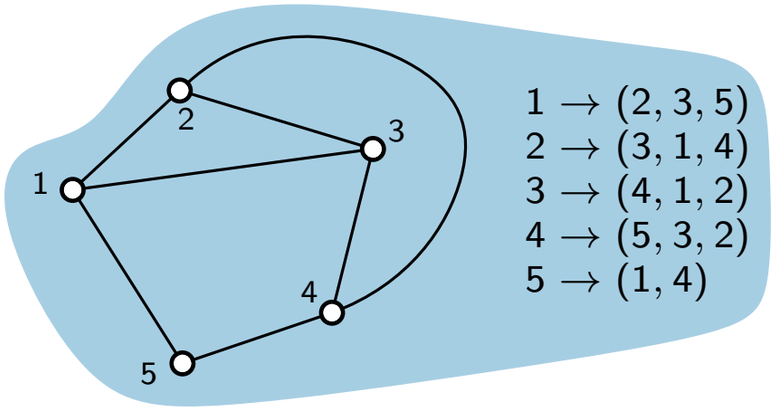

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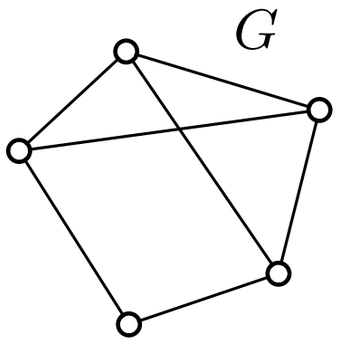

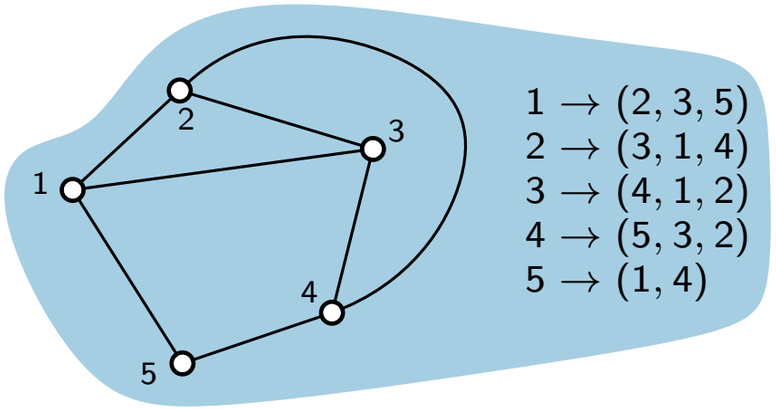

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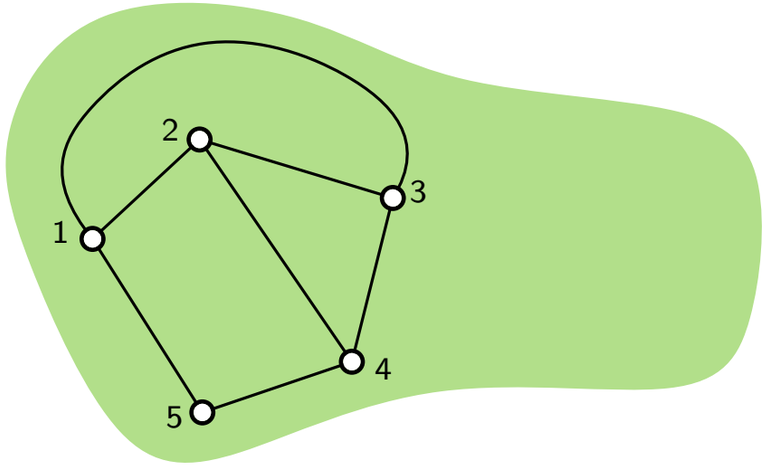

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A planar graph can have many planar embeddings.

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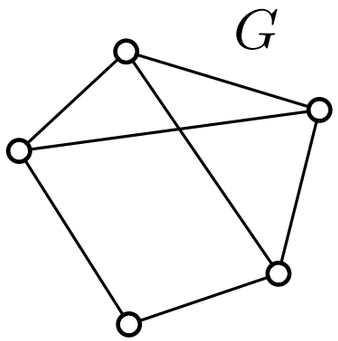

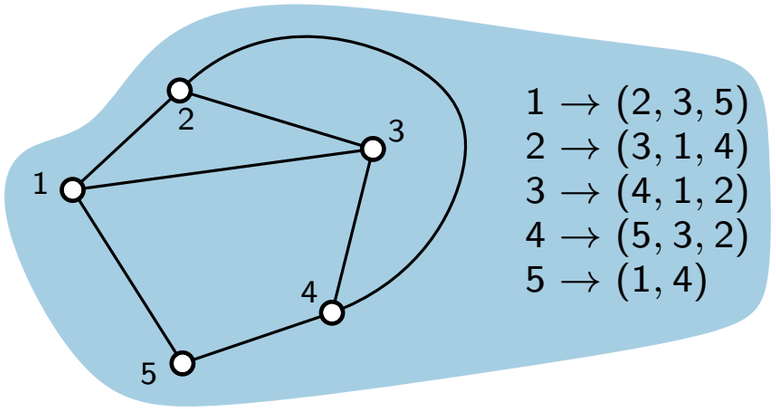

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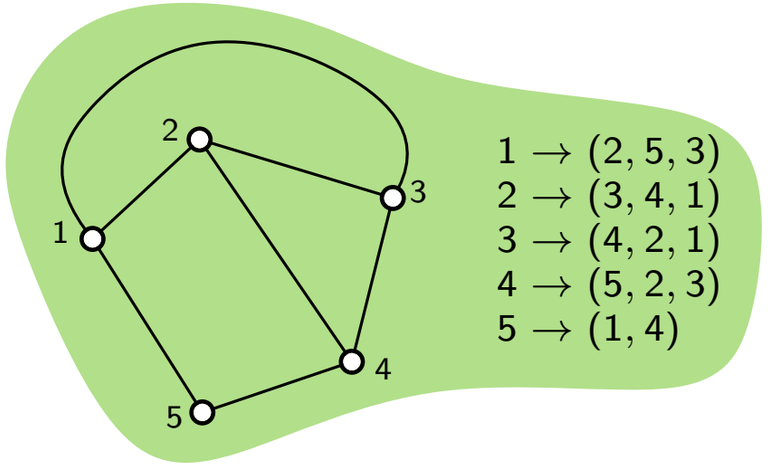

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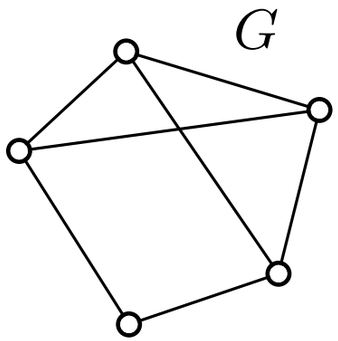

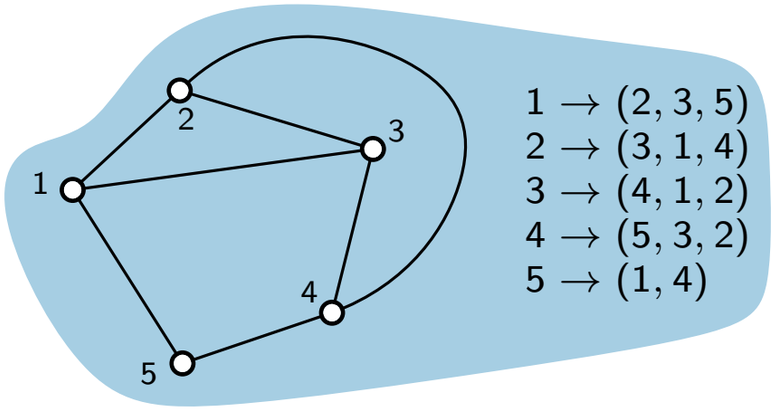

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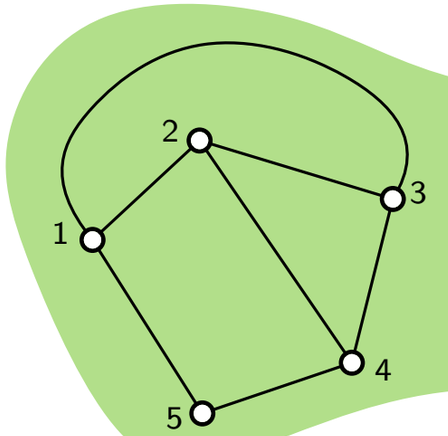

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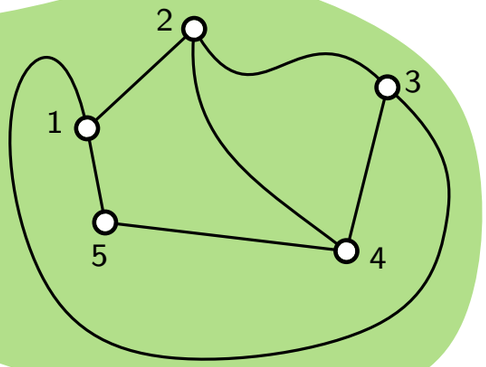

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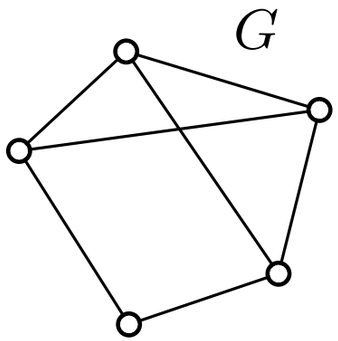

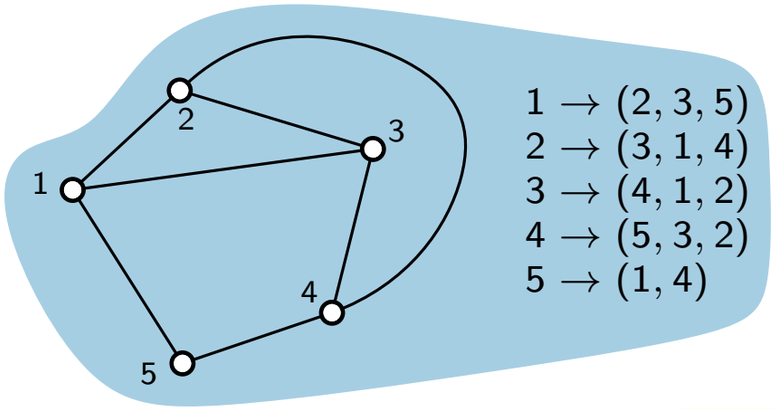

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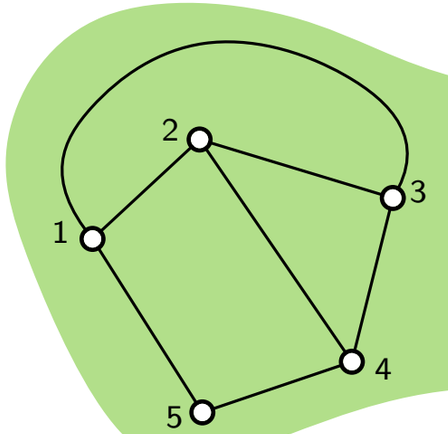

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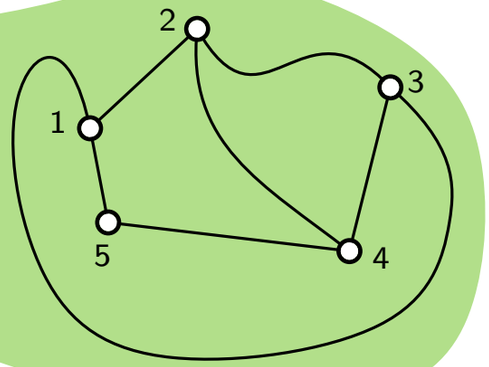

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planar embedding:
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faces: Connected region of the plane bounded by edges

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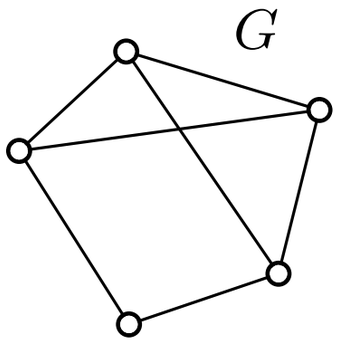

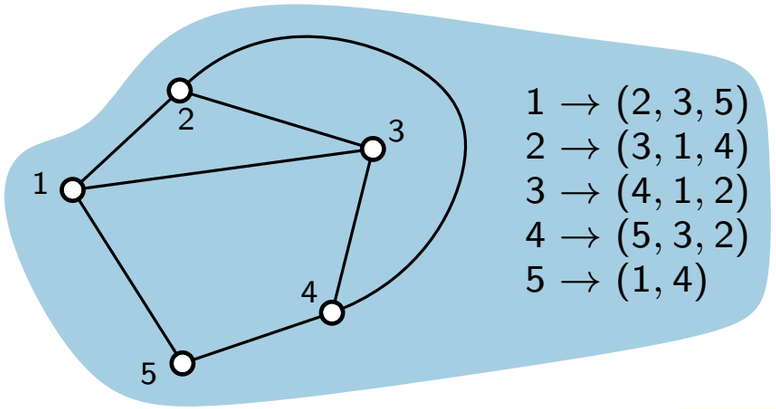

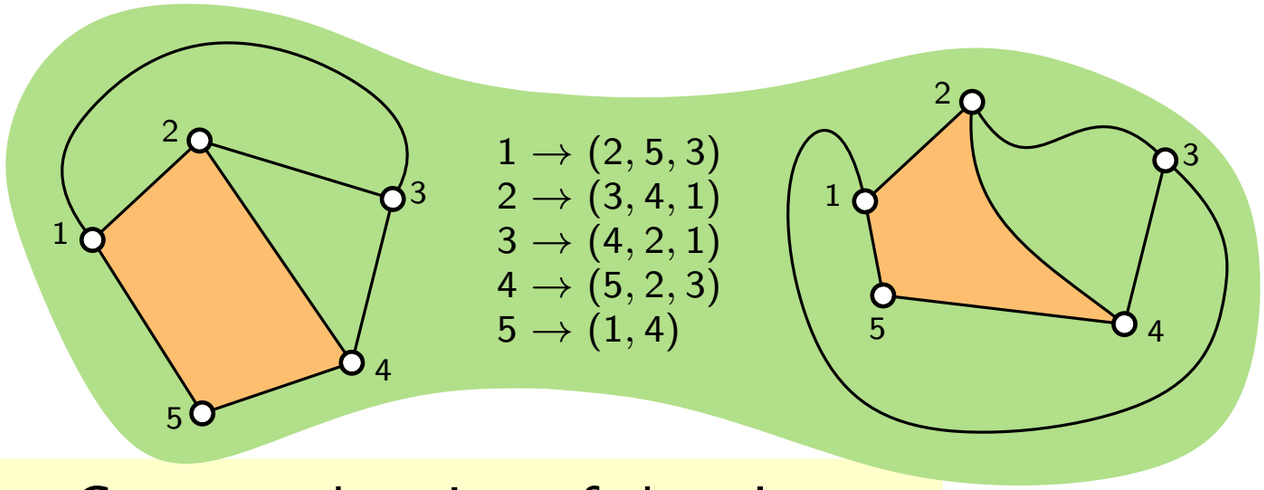

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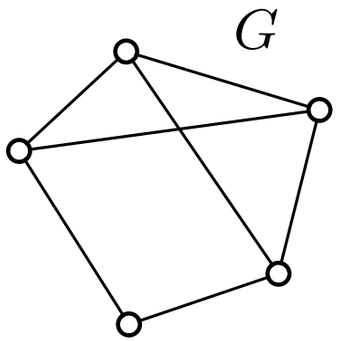

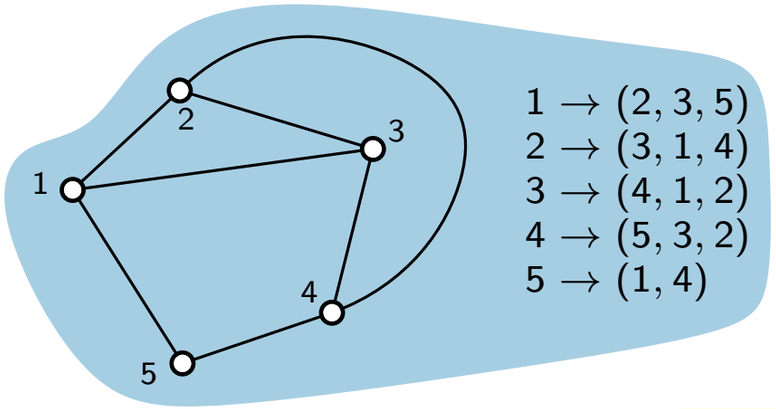

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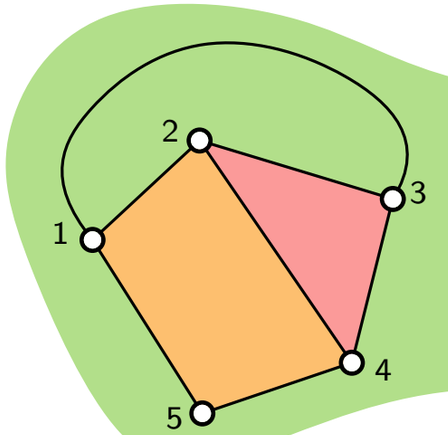

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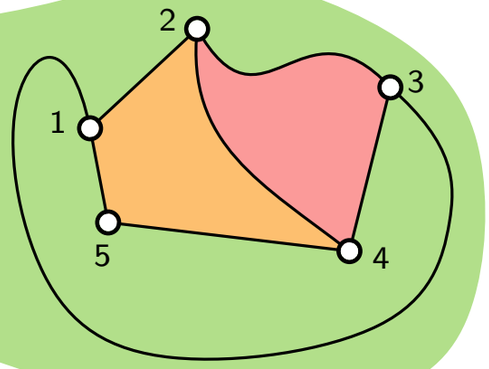

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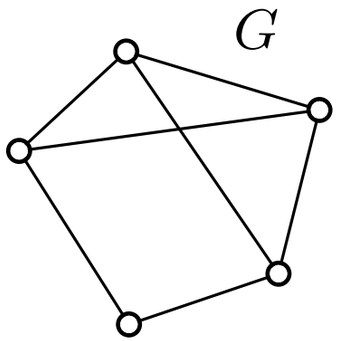

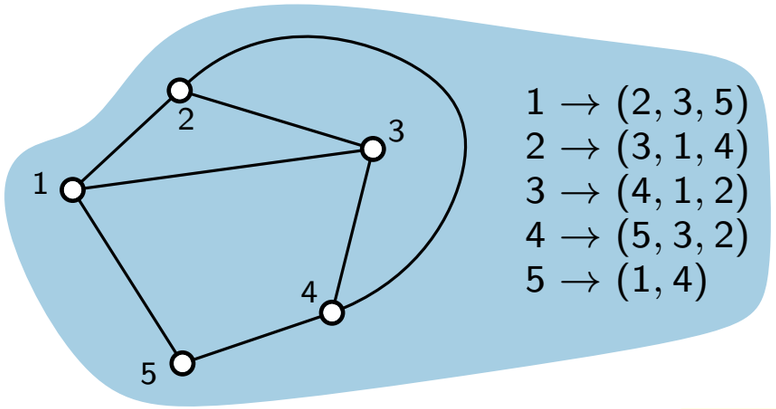

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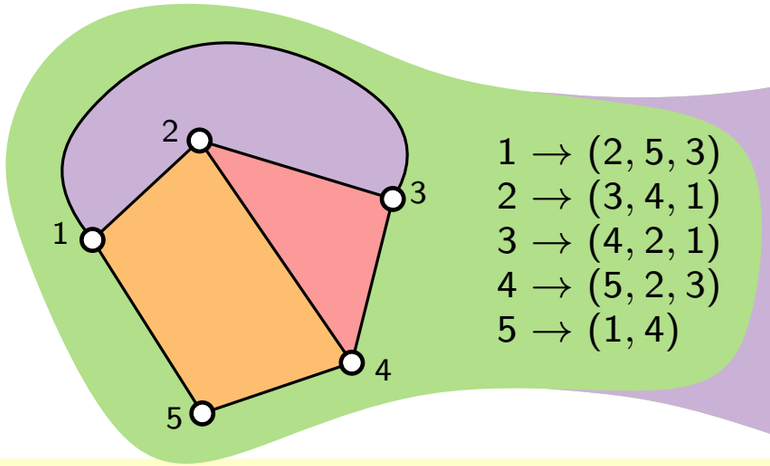

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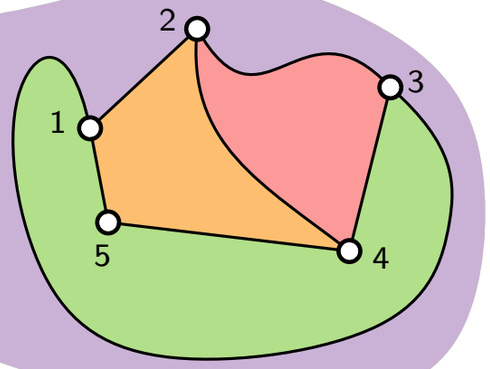

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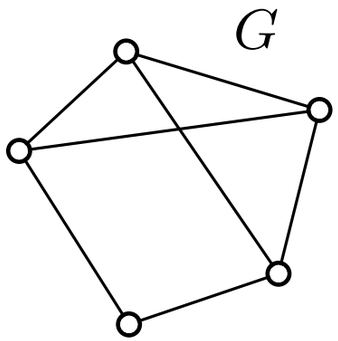
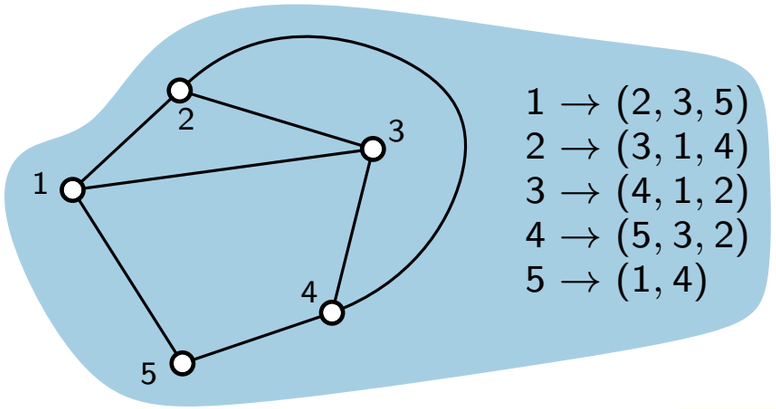
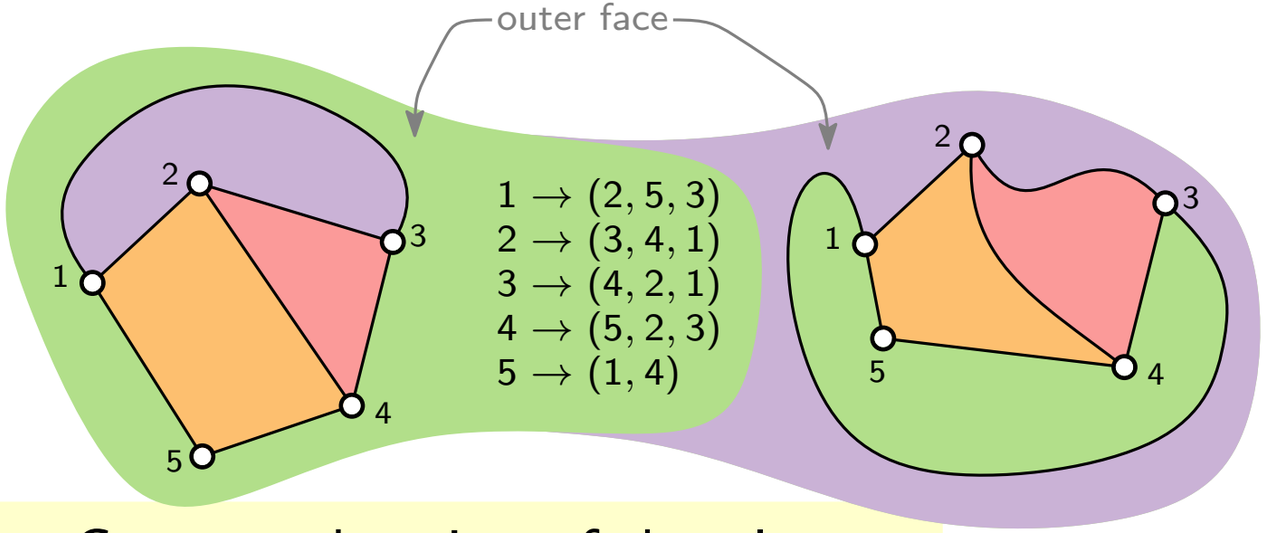




$1 \to (2, 3, 5)$
$2 \to (3, 1, 4)$
$3 \to (4, 1, 2)$
$4 \to (5, 3, 2)$
$5 \to (1, 4)$



$1 \to (2, 5, 3)$
$2 \to (3, 4, 1)$
$3 \to (4, 2, 1)$
$4 \to (5, 2, 3)$
$5 \to (1, 4)$

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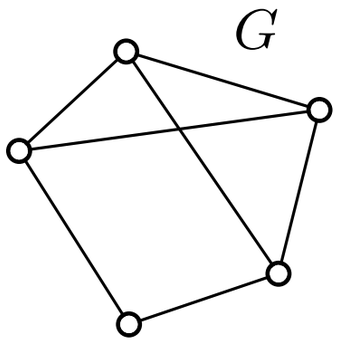

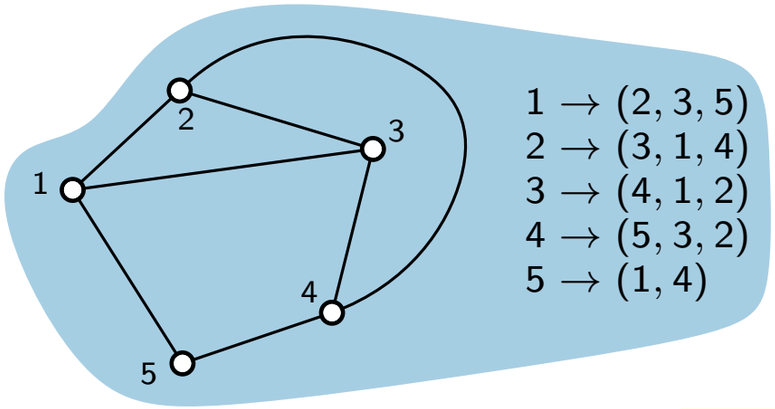

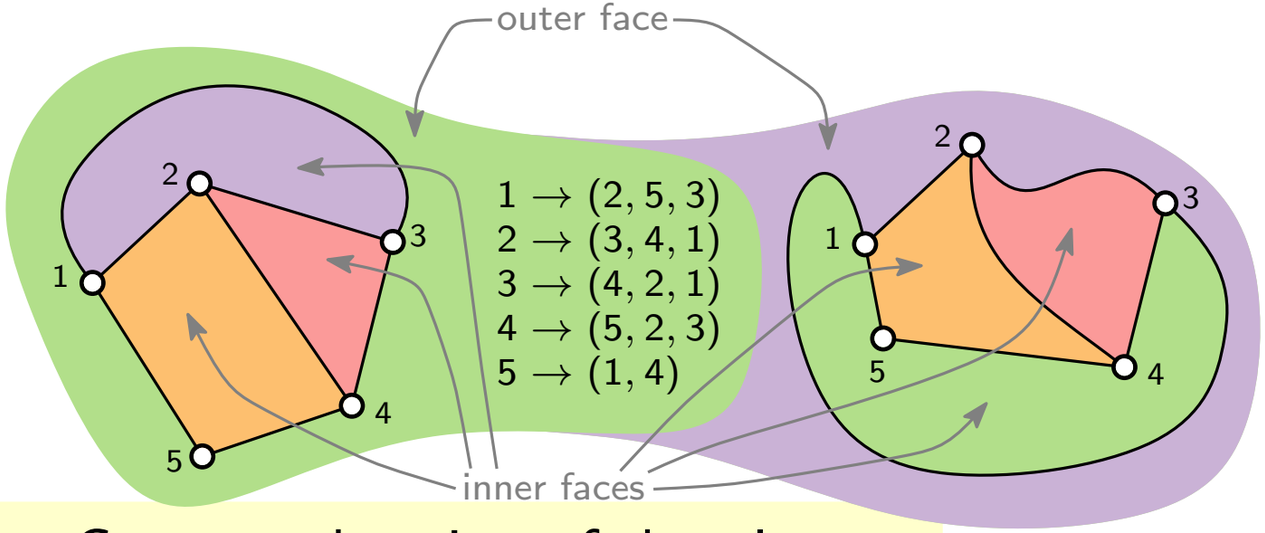

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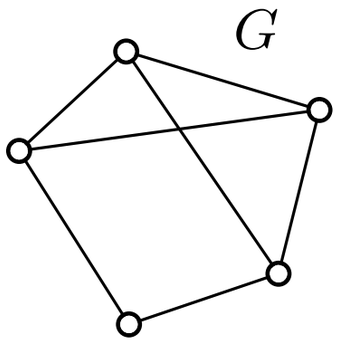

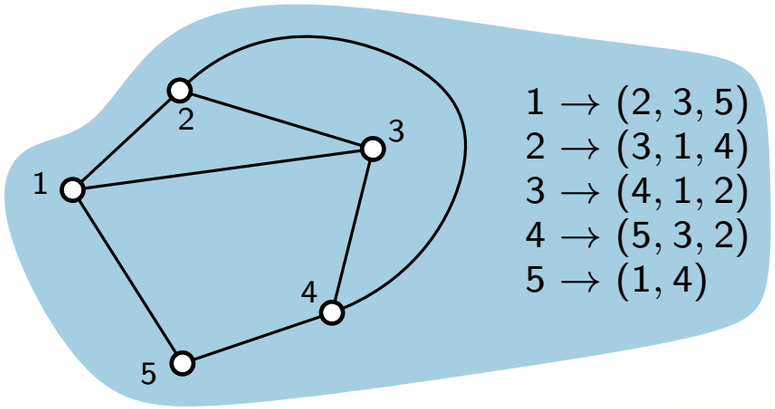

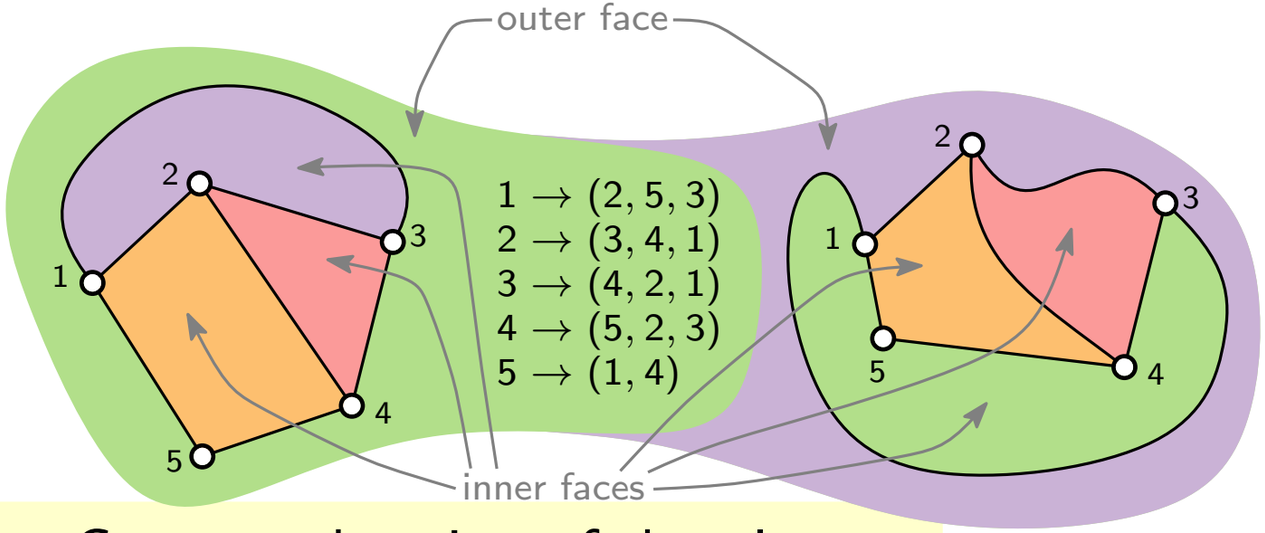

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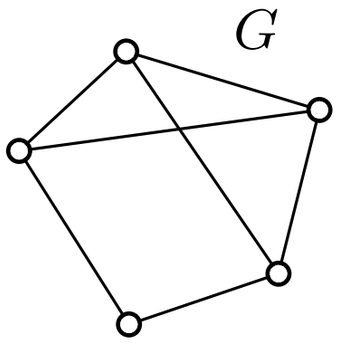

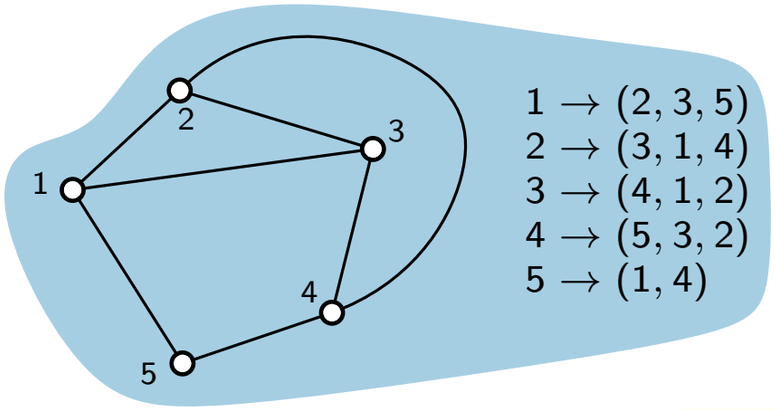

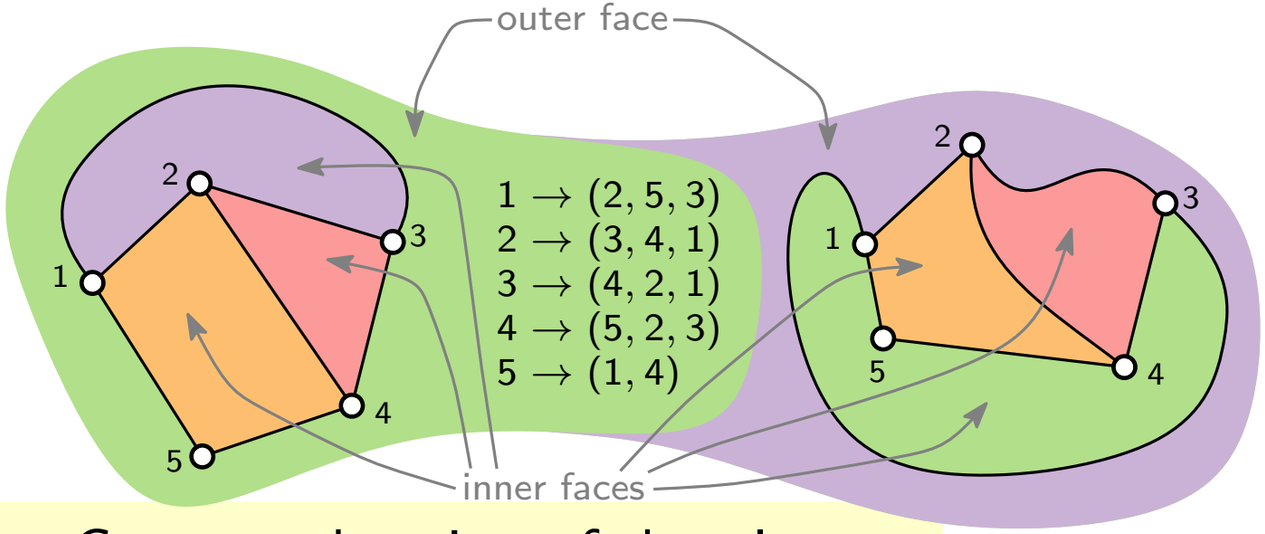

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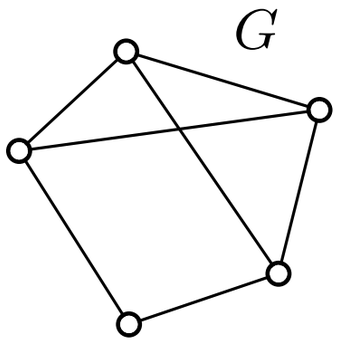

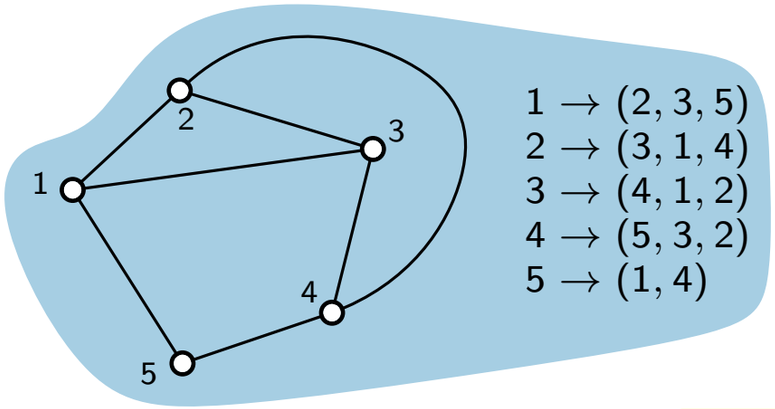

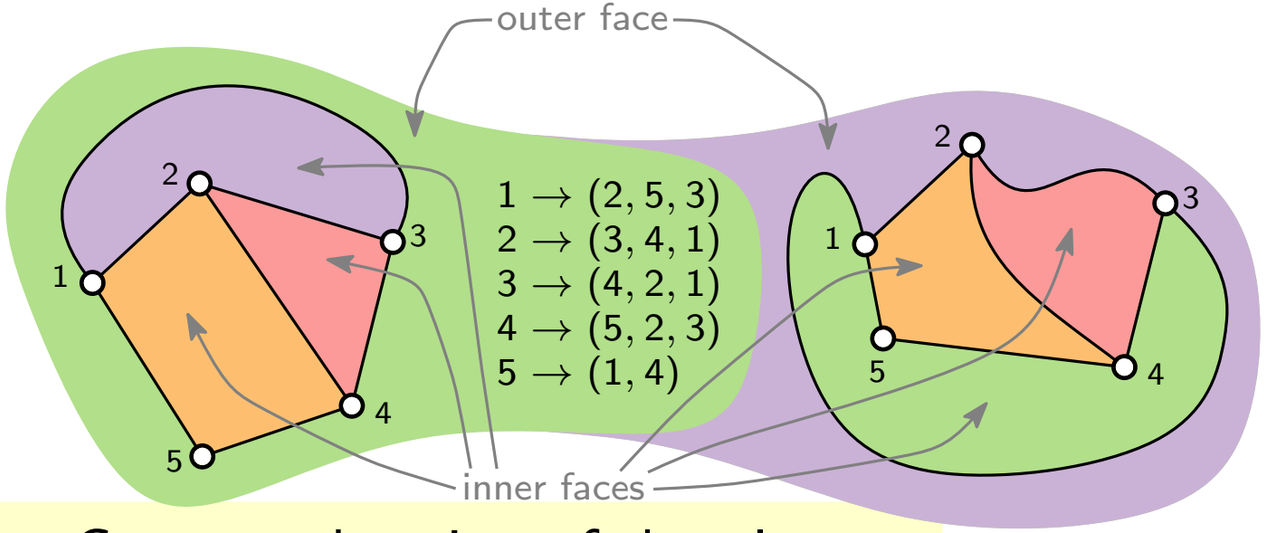

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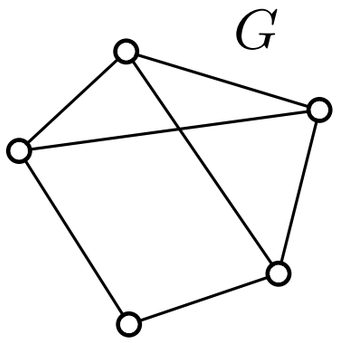

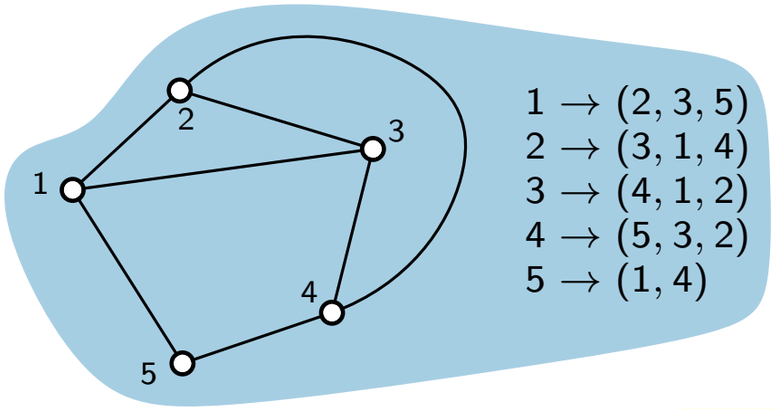

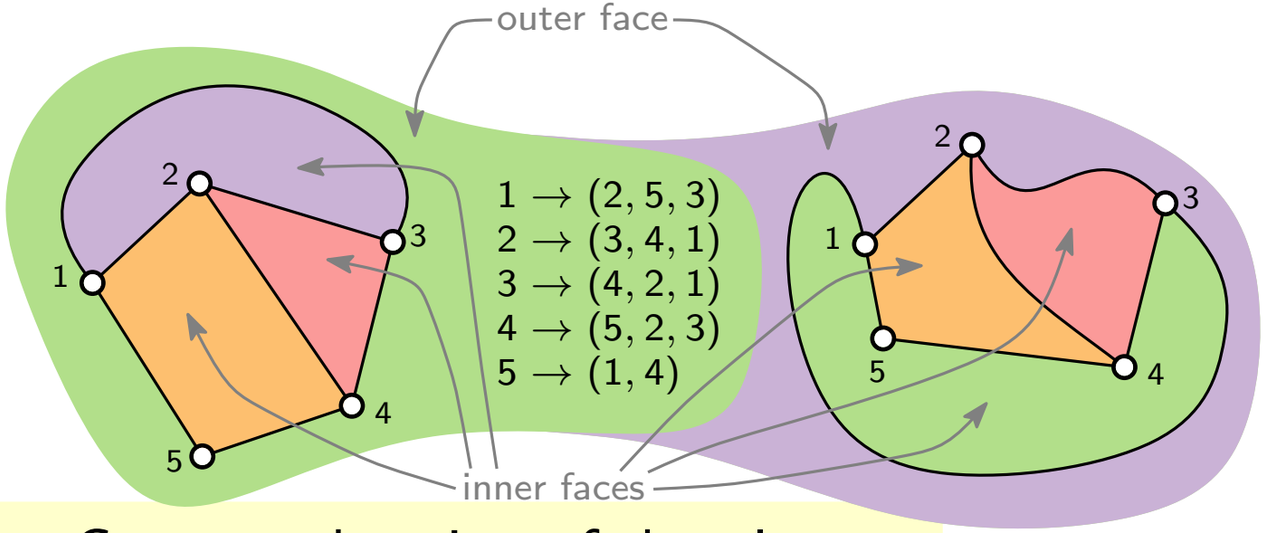

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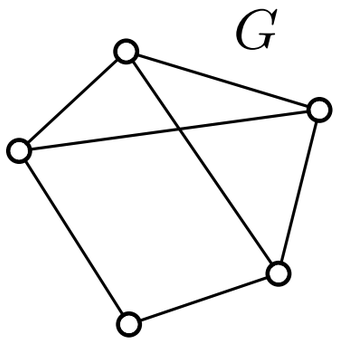

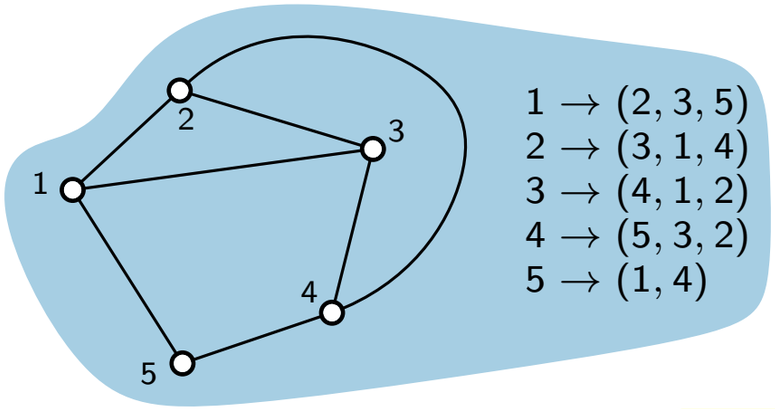

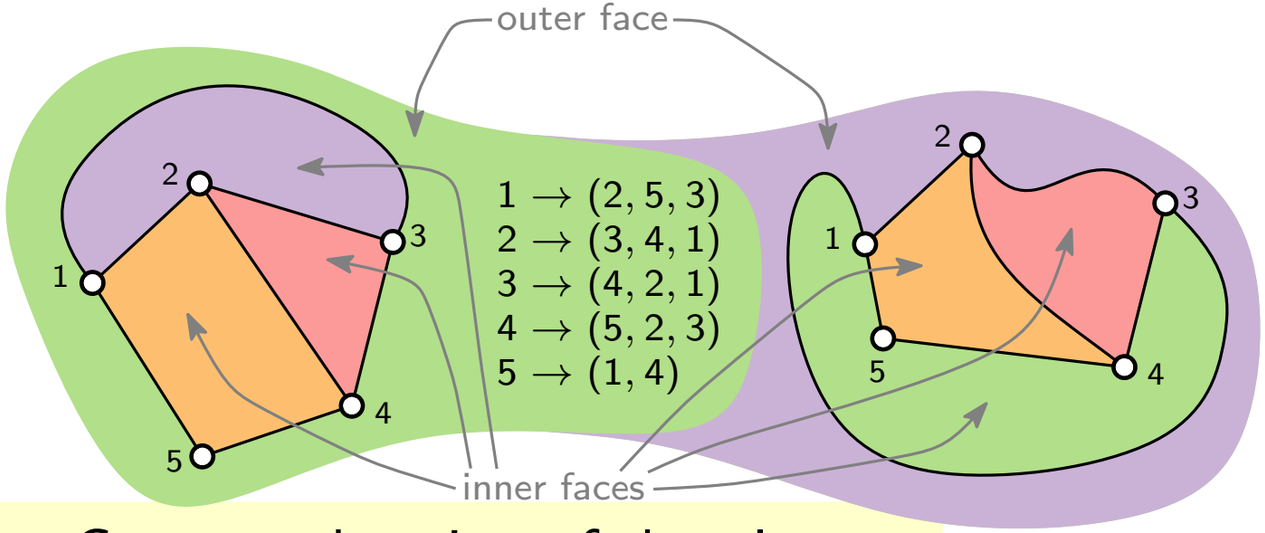

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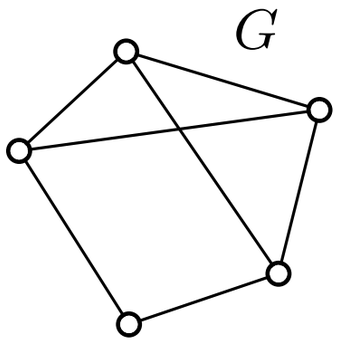

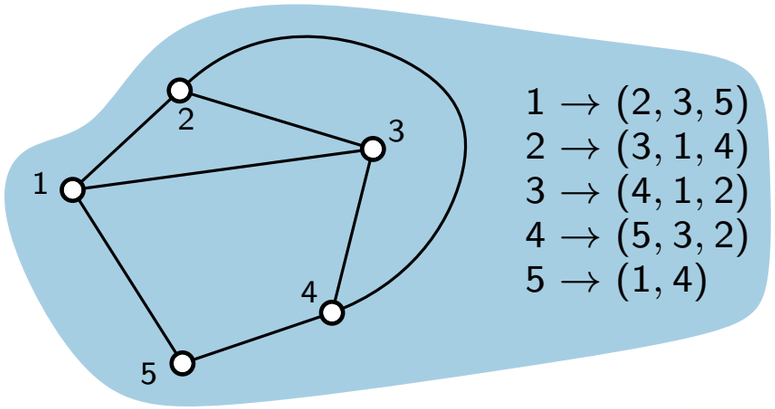

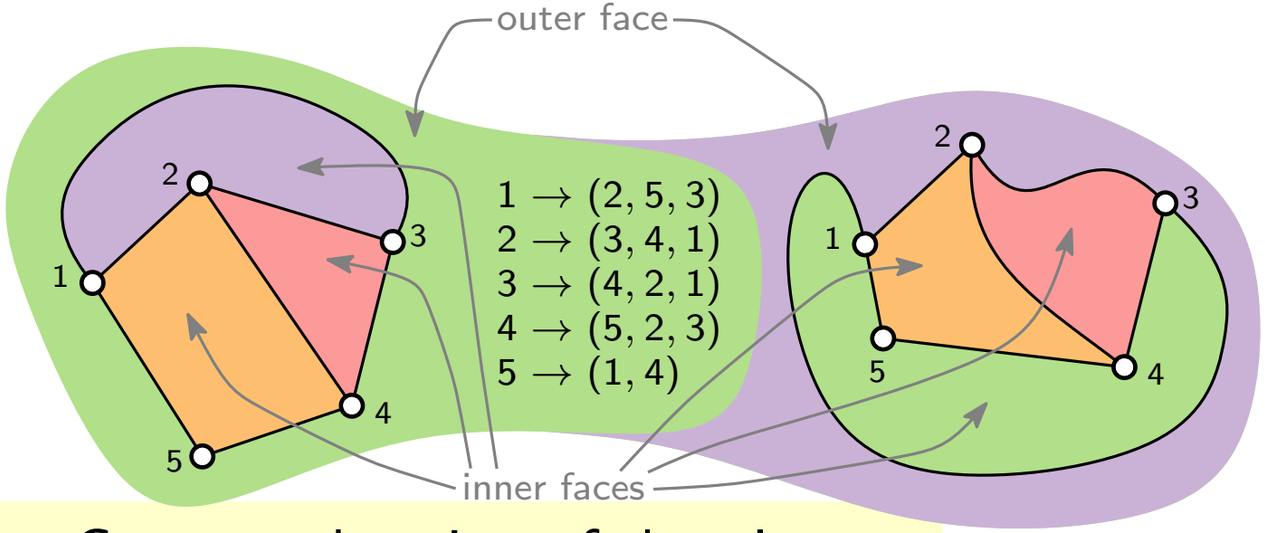

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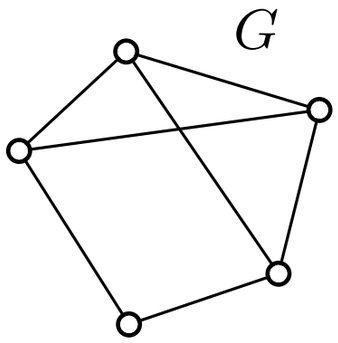

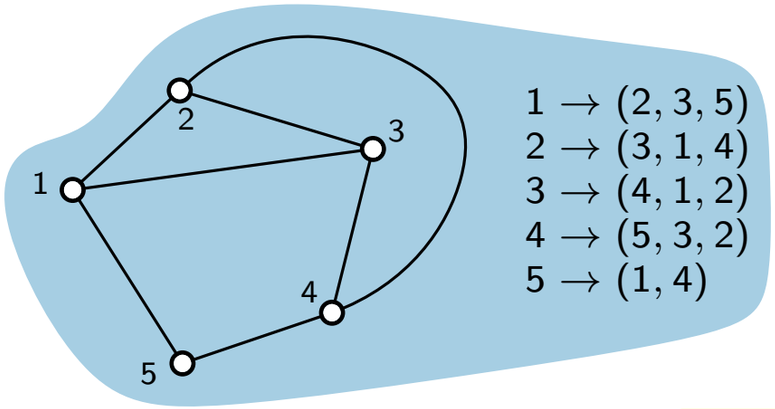

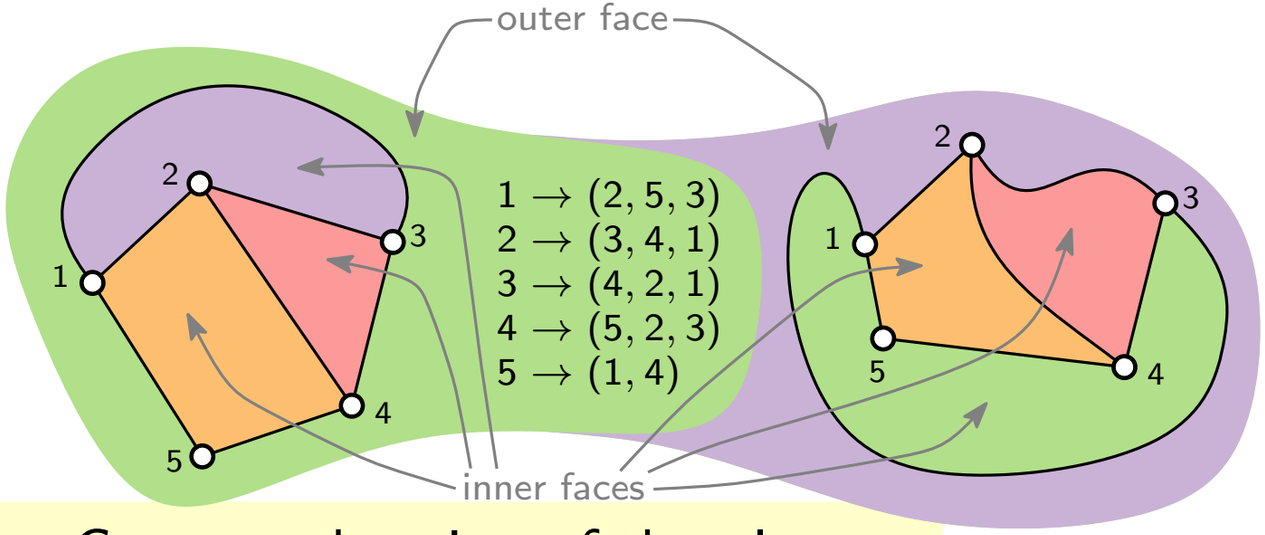

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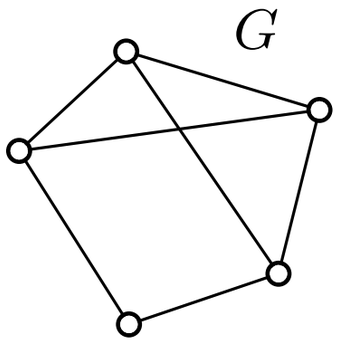

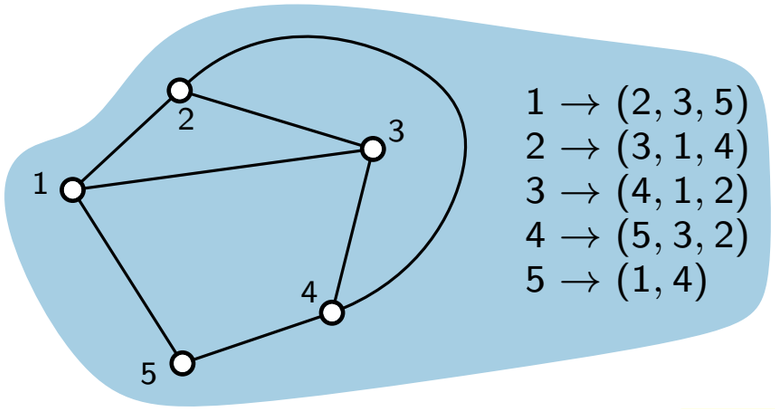

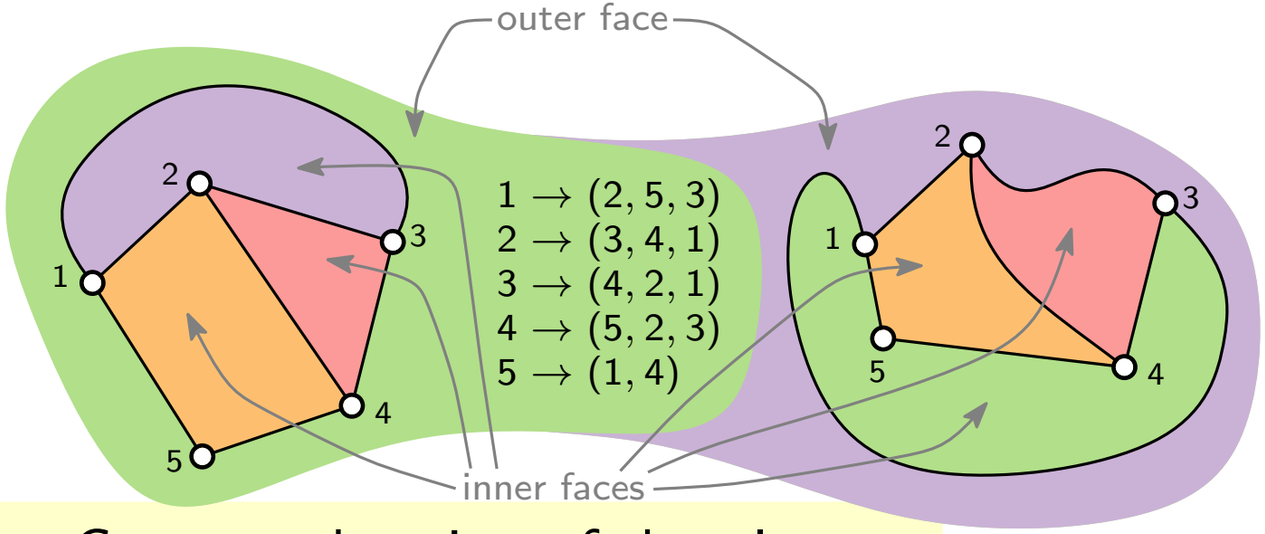

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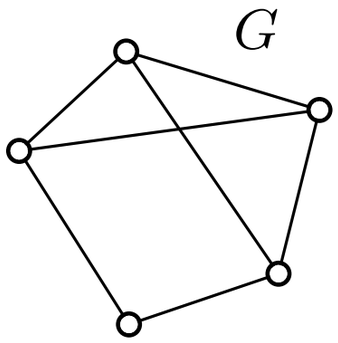

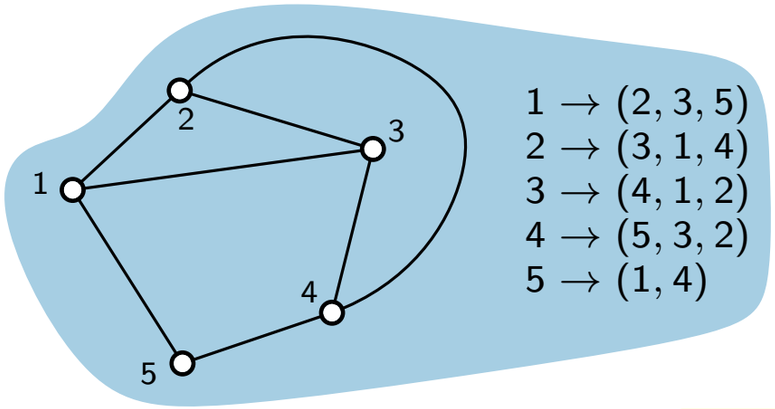

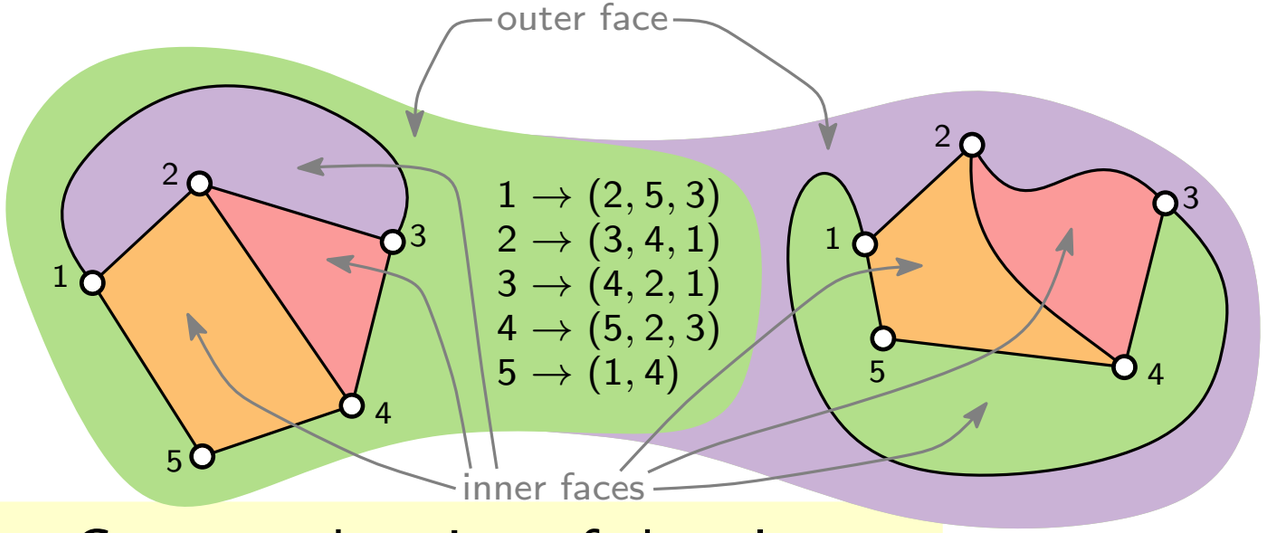

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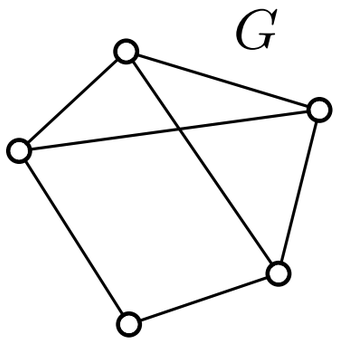

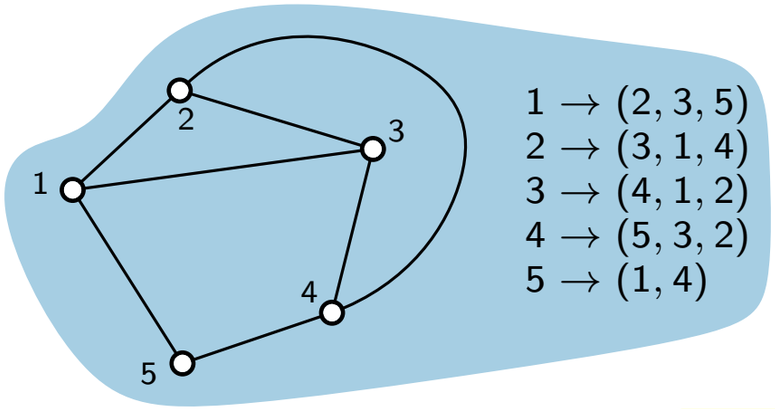

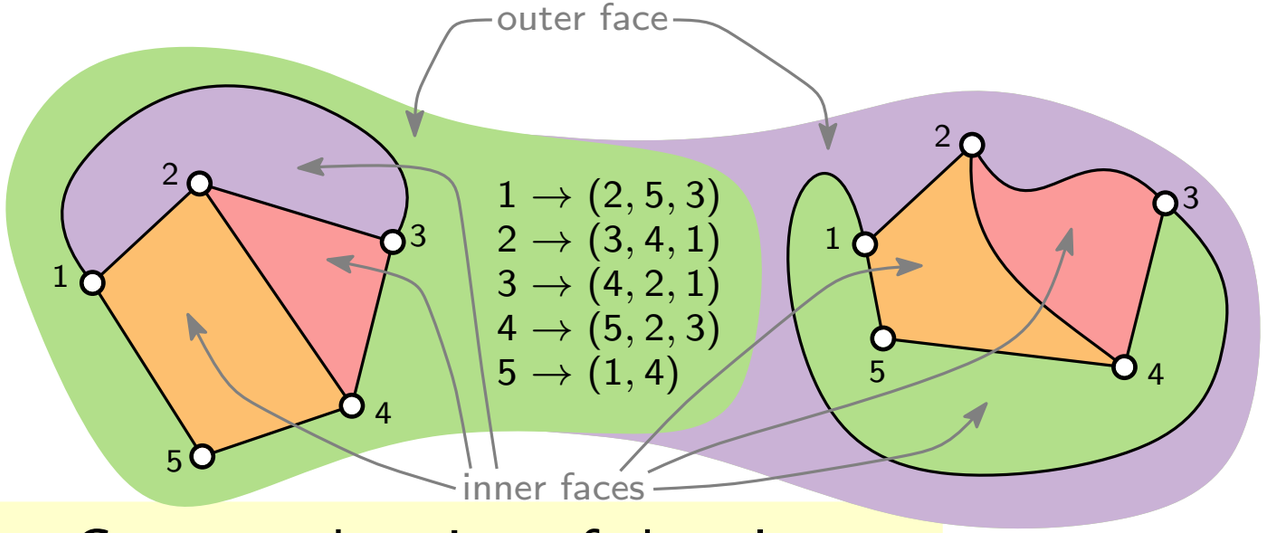

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$m \geq 1 \Rightarrow$ add some edge e

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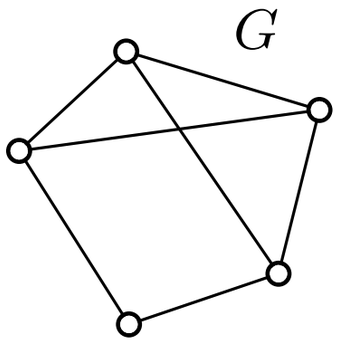

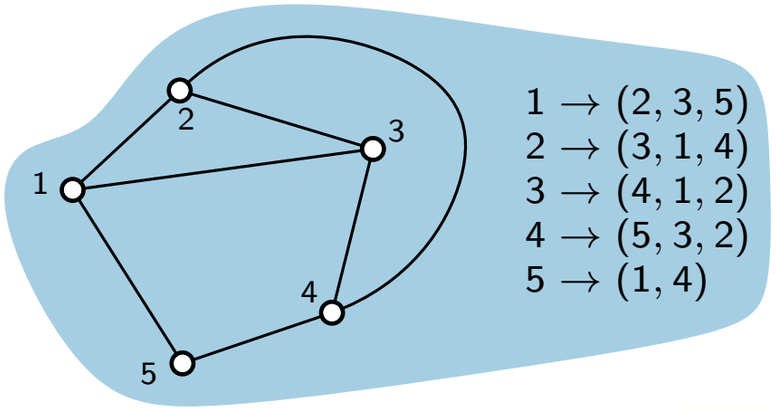

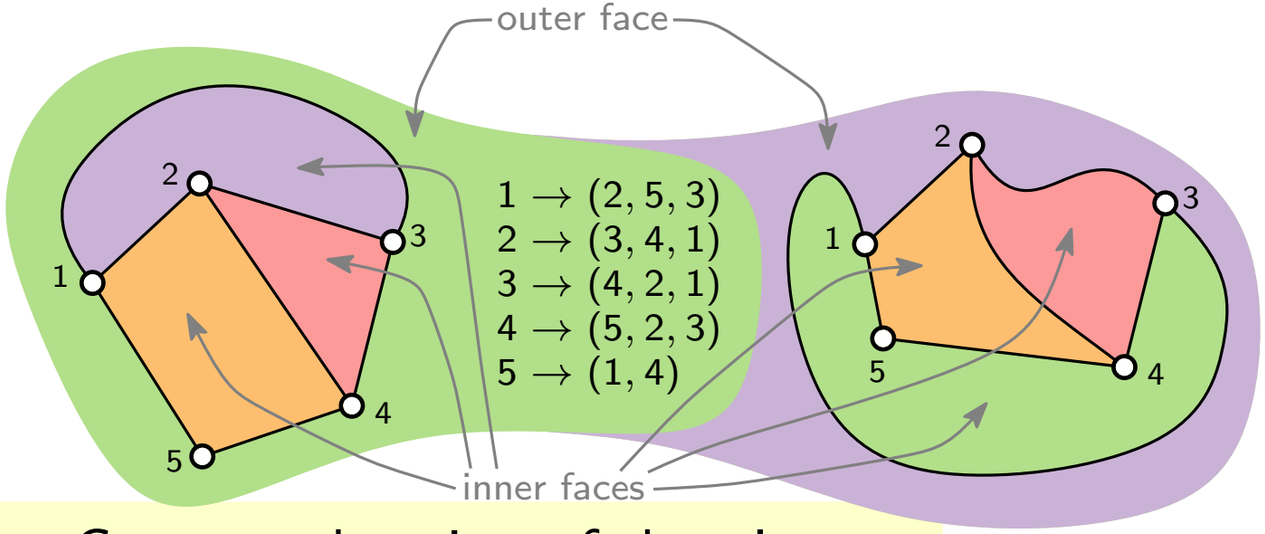

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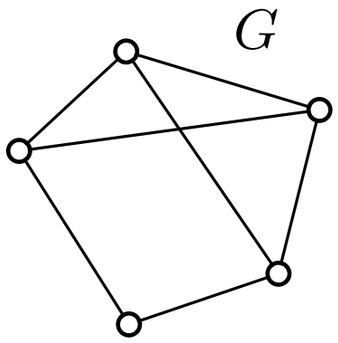

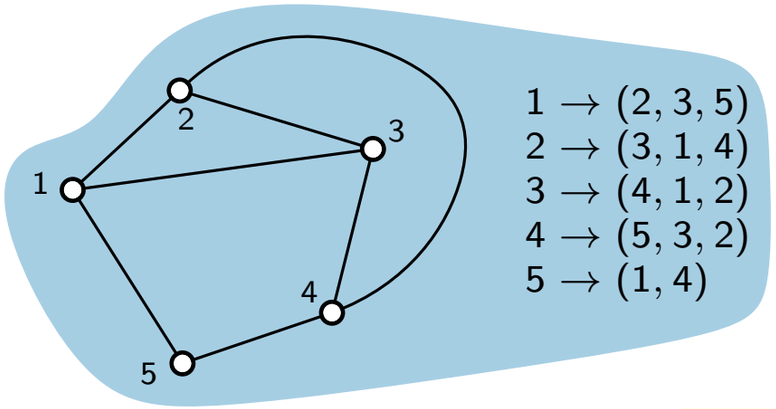

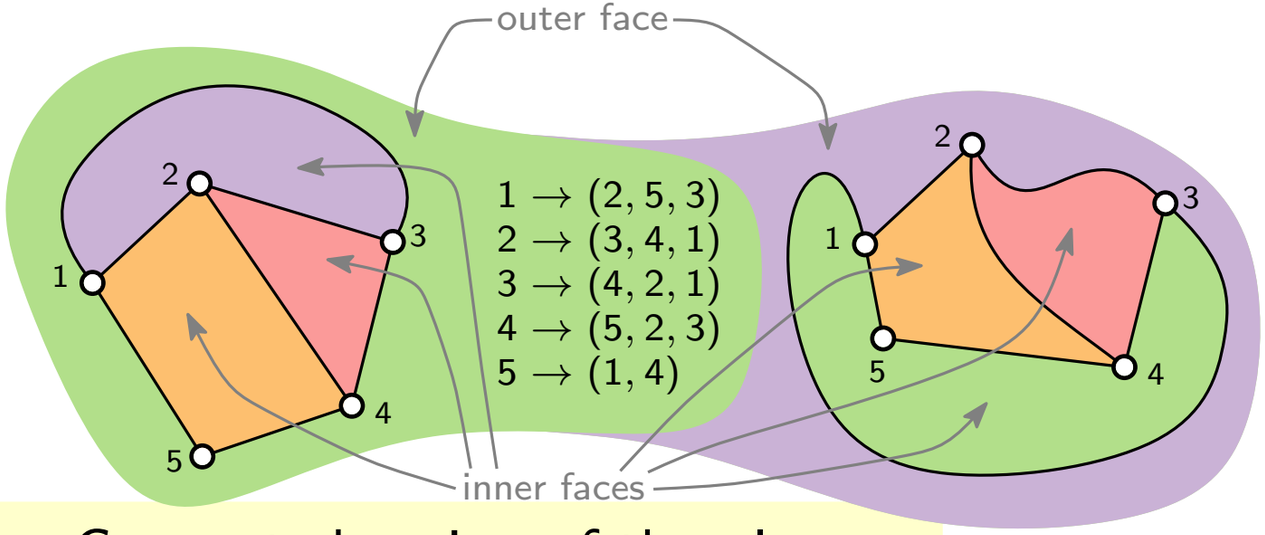

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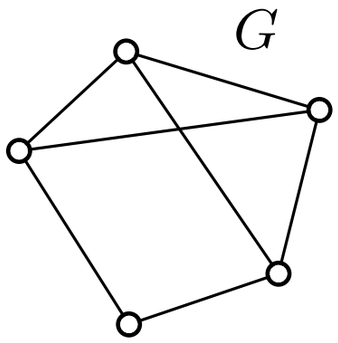

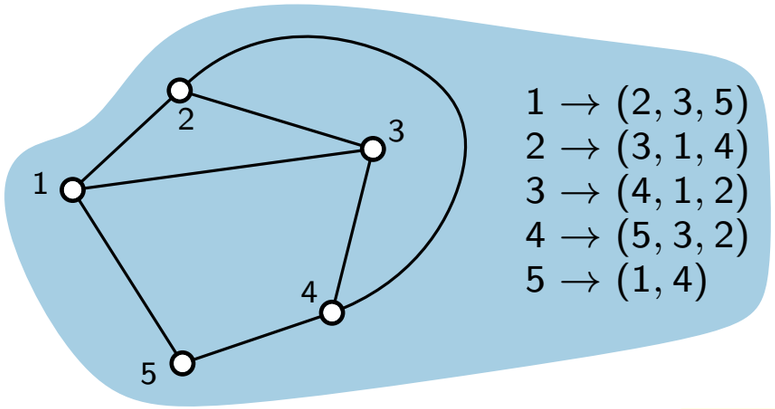

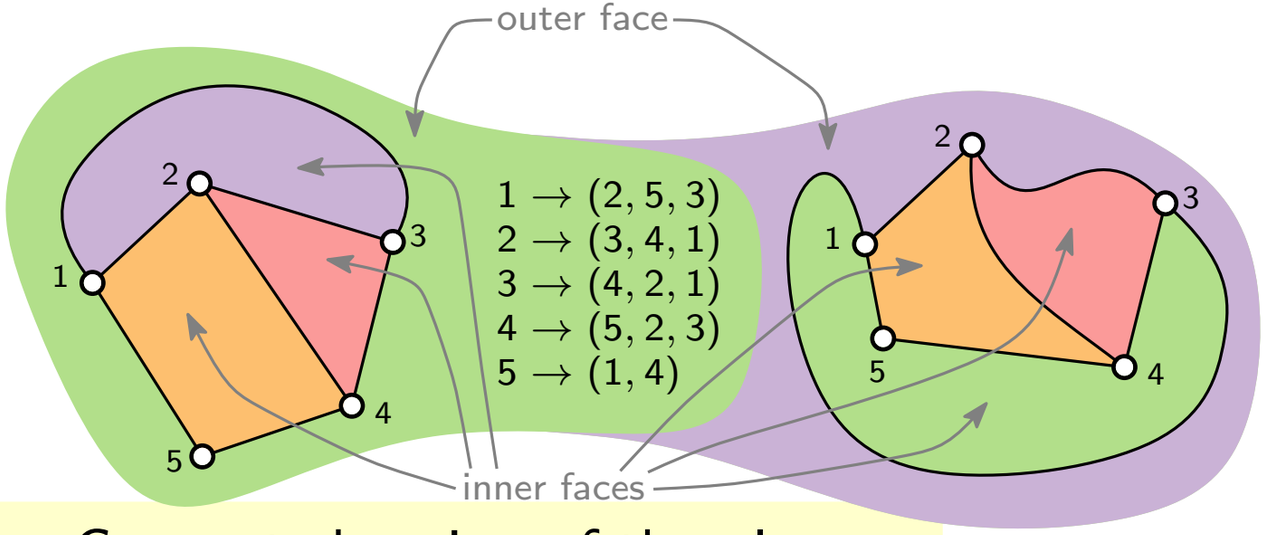

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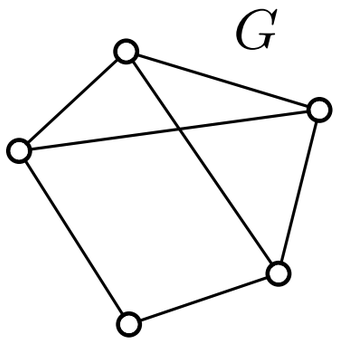

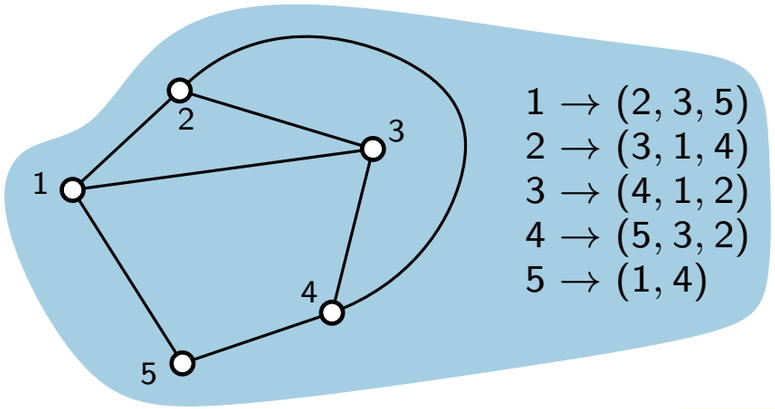

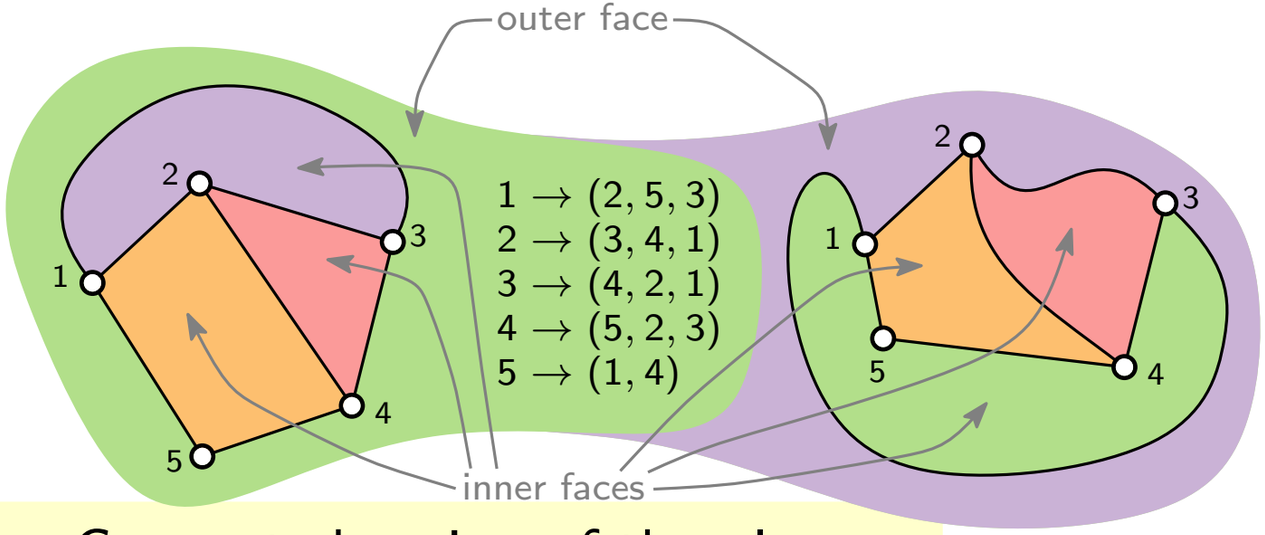

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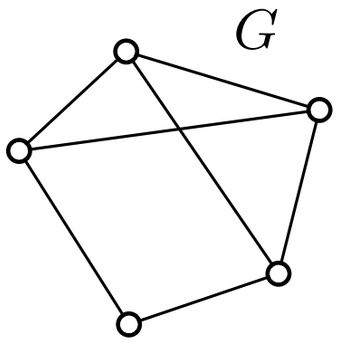

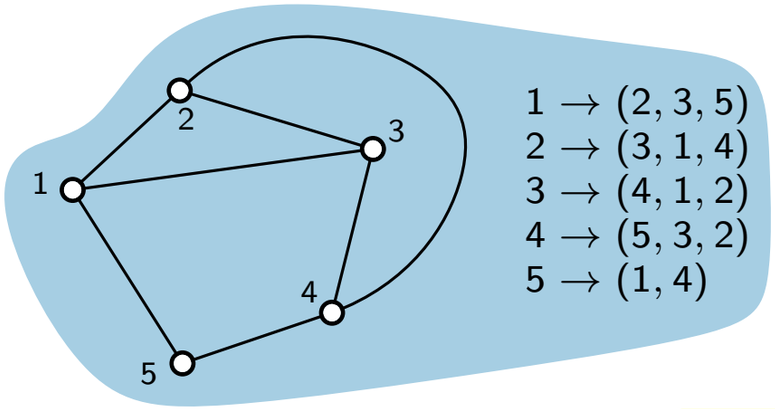

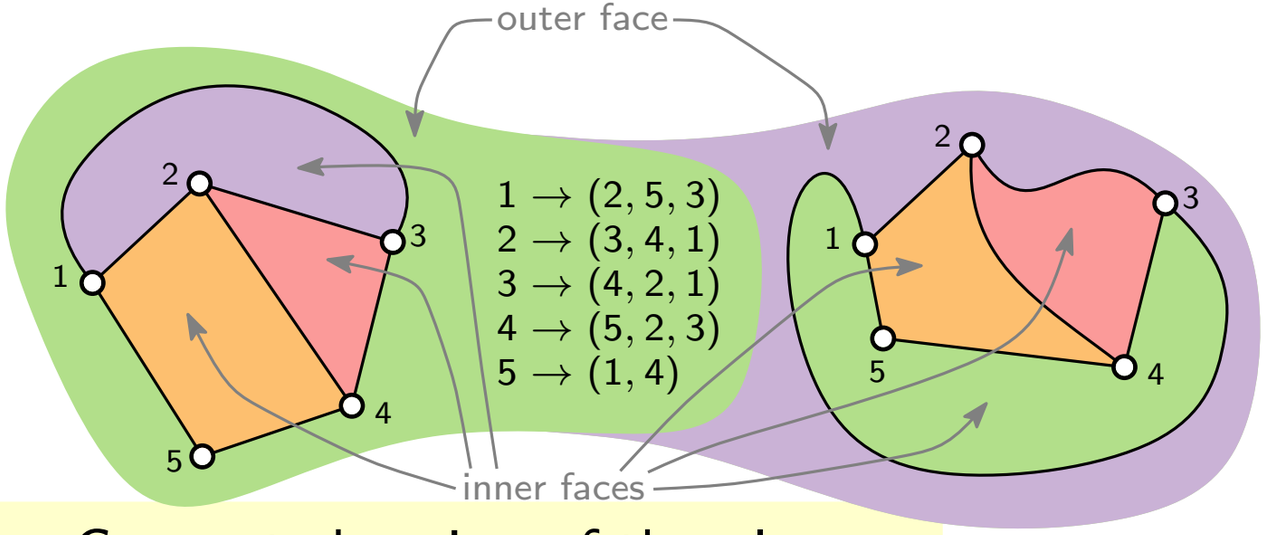

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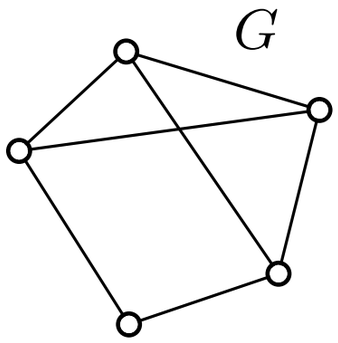

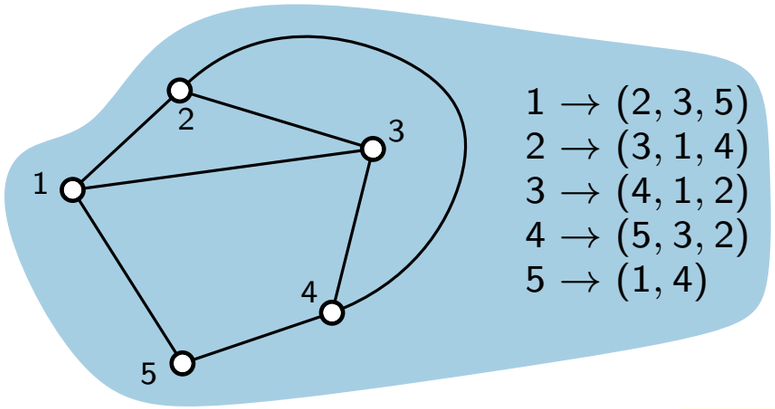

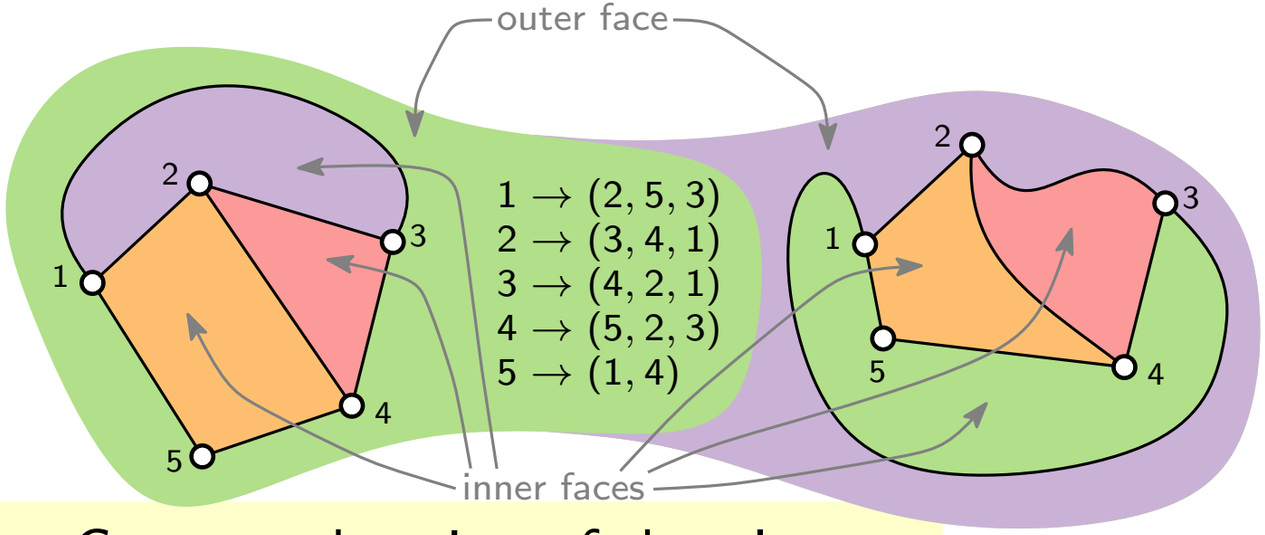

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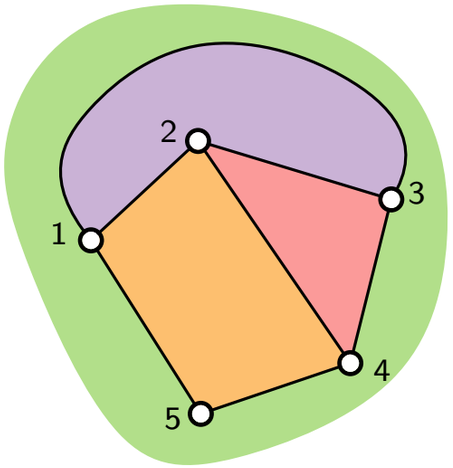

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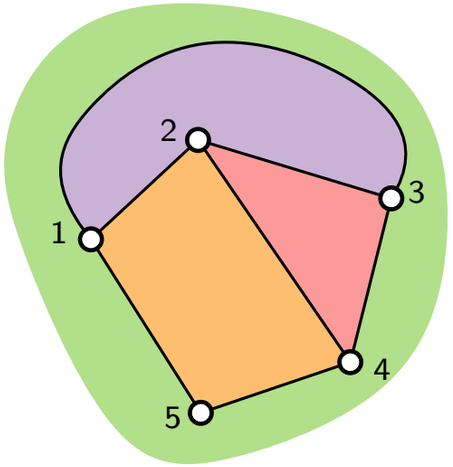

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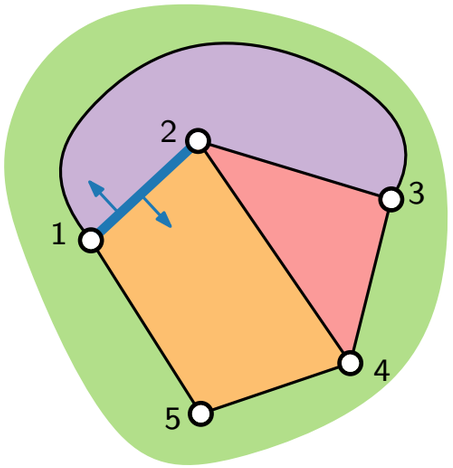

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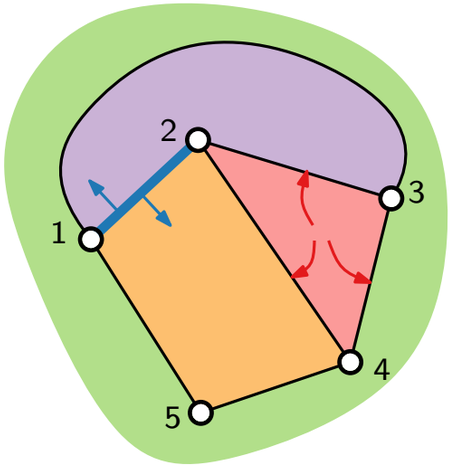

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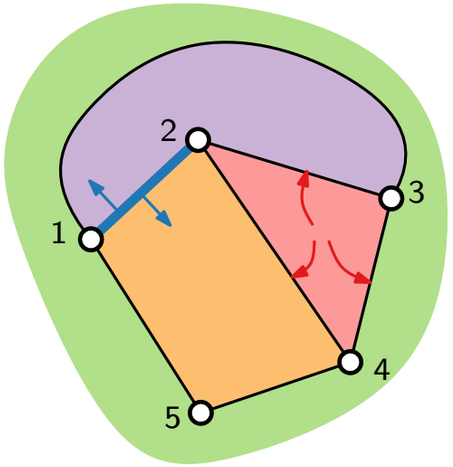

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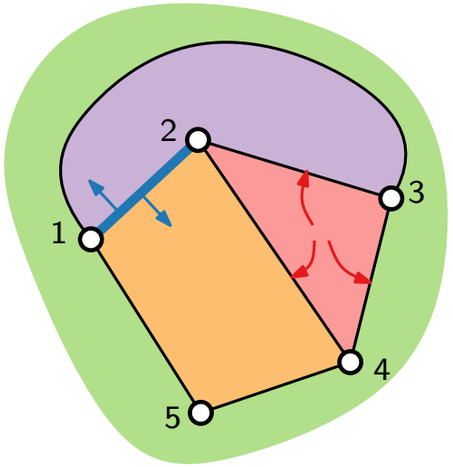

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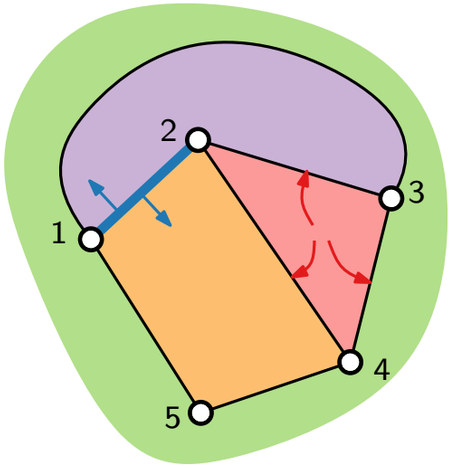

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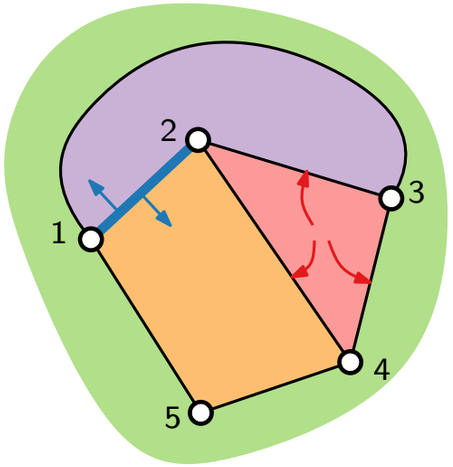

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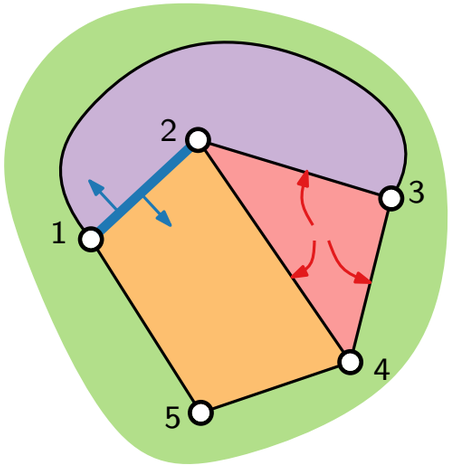

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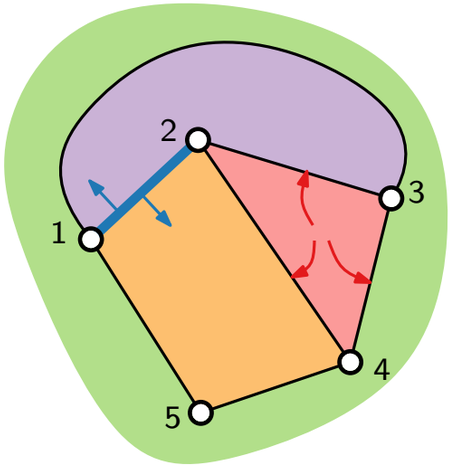

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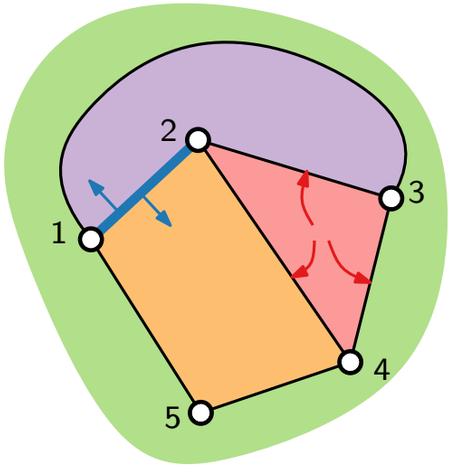

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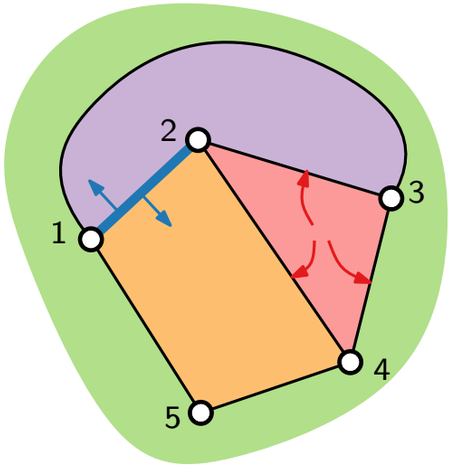

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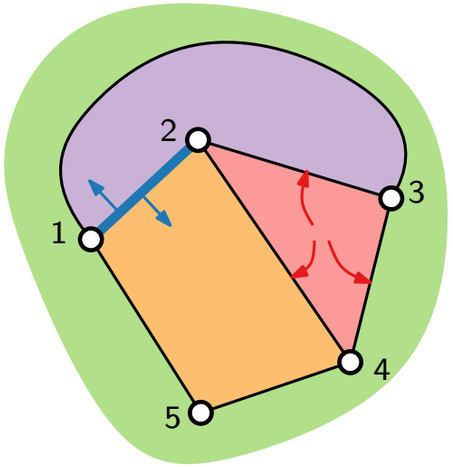

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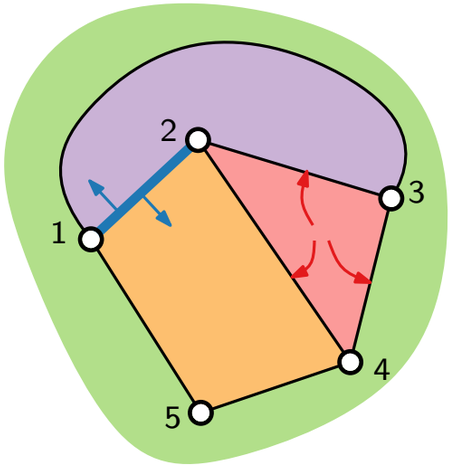

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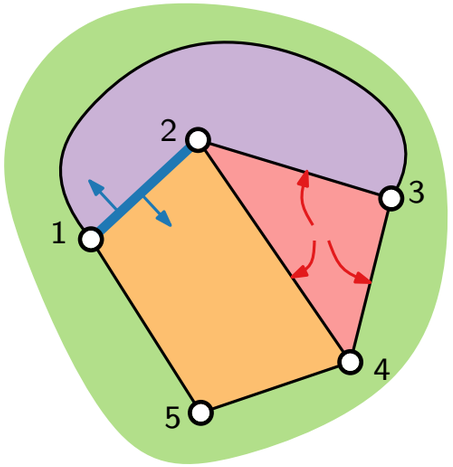

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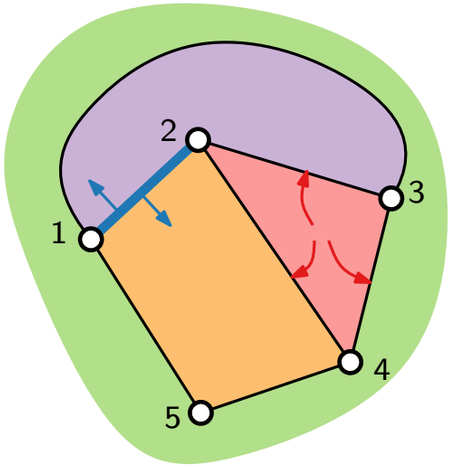

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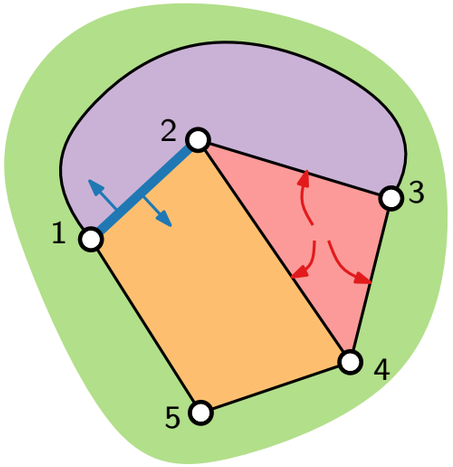

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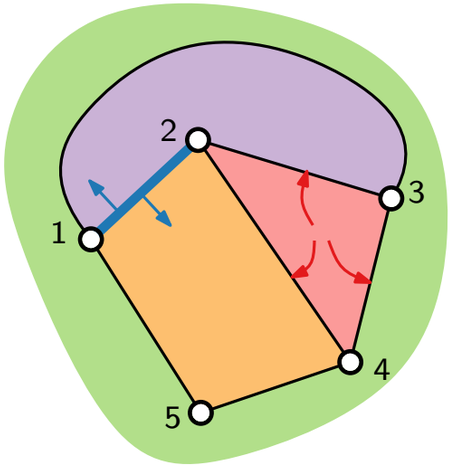

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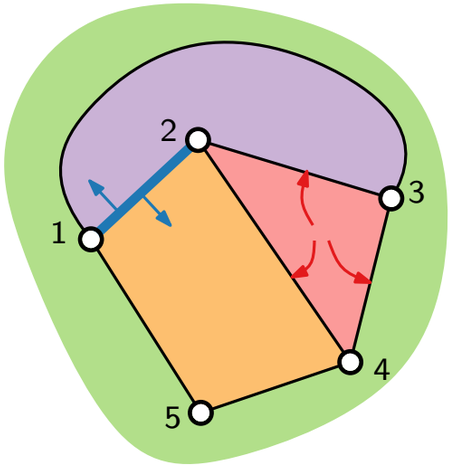

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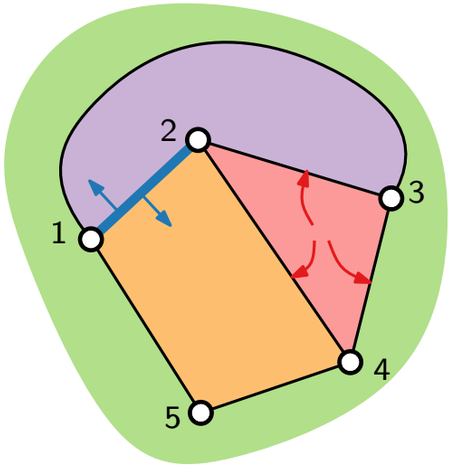

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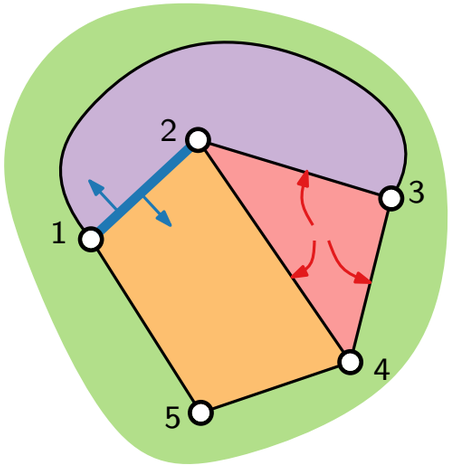

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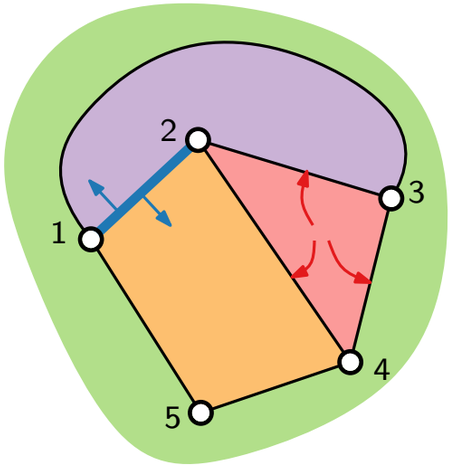

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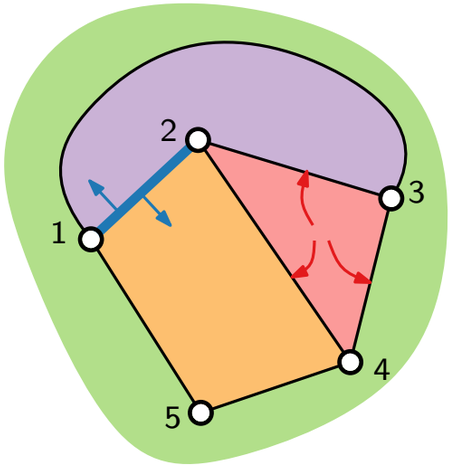

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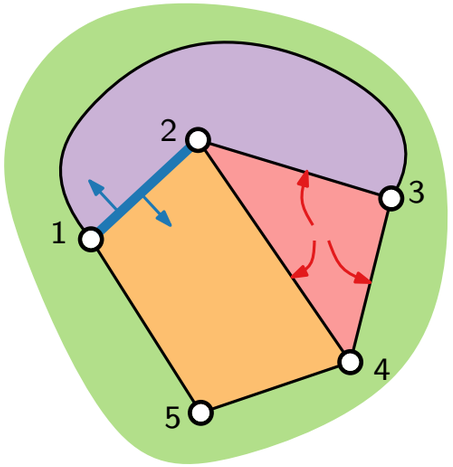

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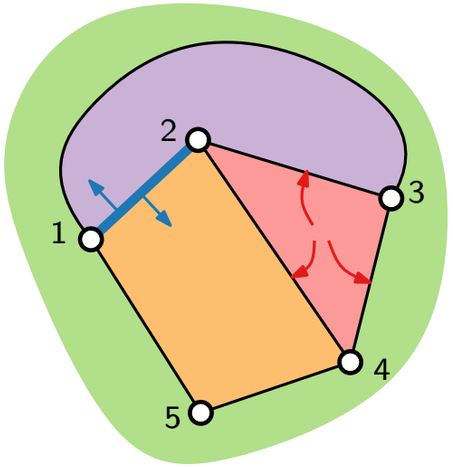

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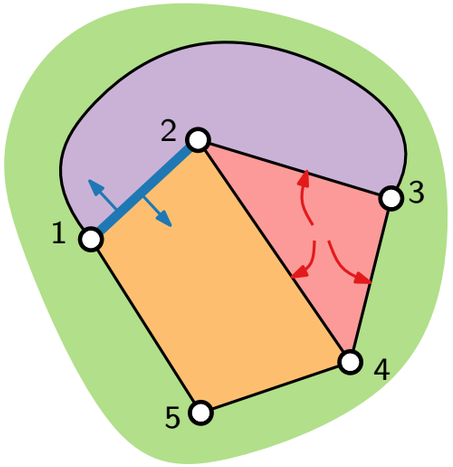

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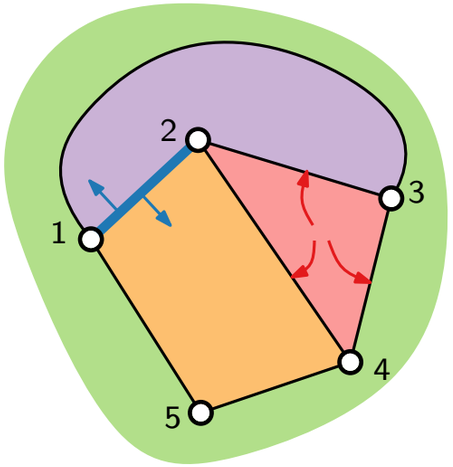

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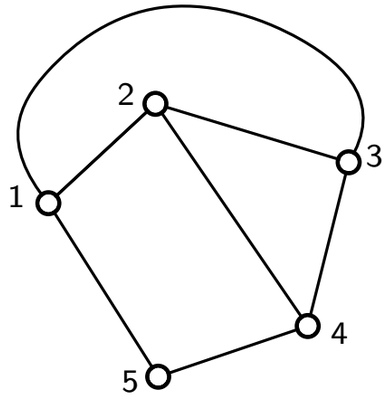

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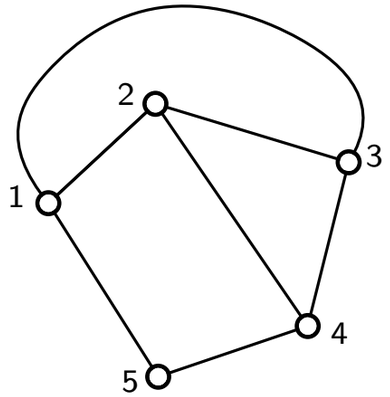

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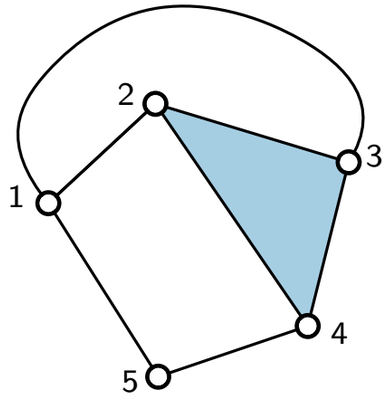

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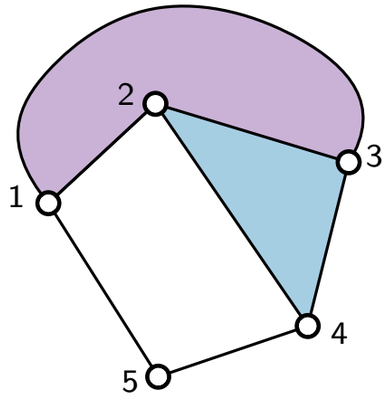

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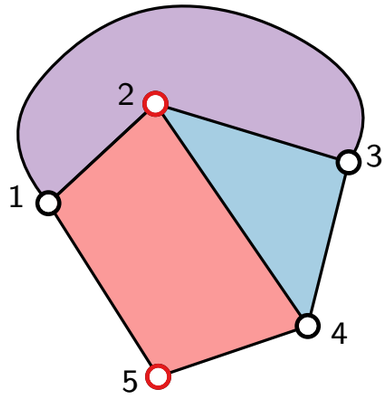

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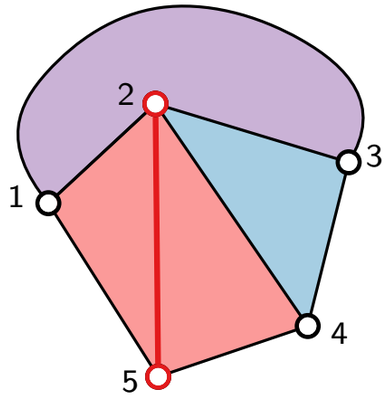

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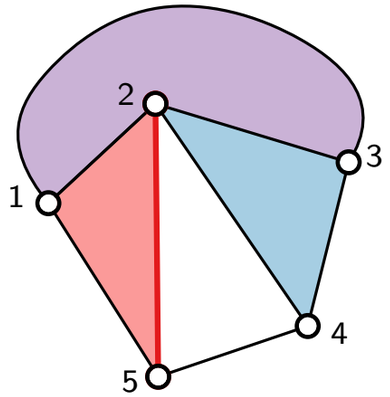

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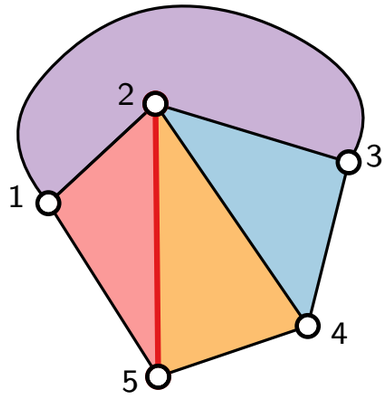

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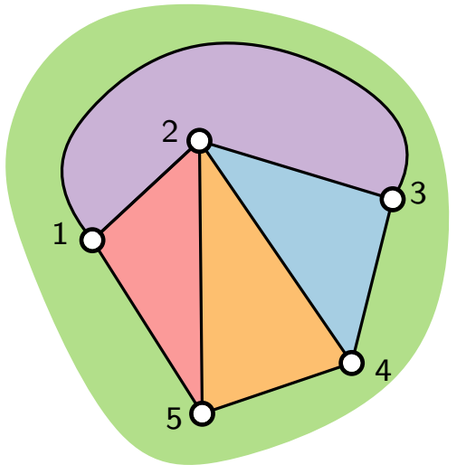

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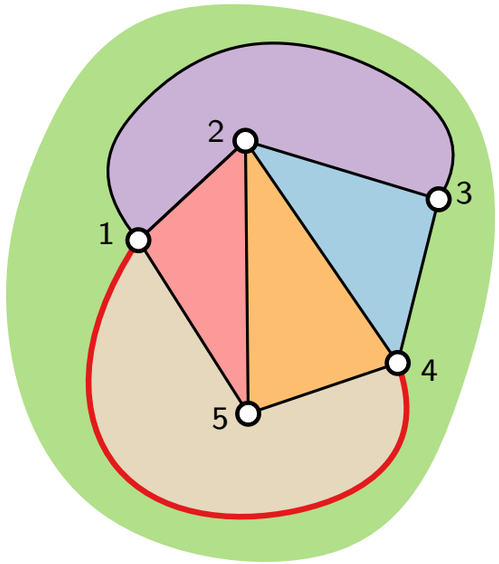

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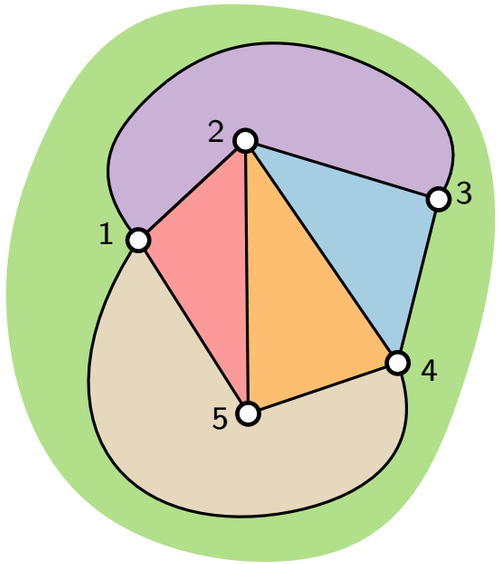

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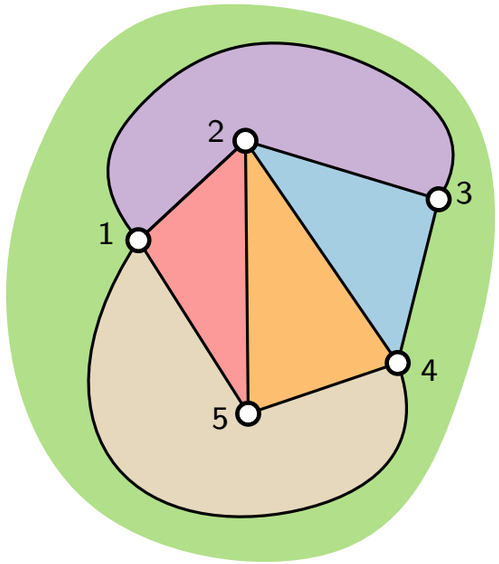

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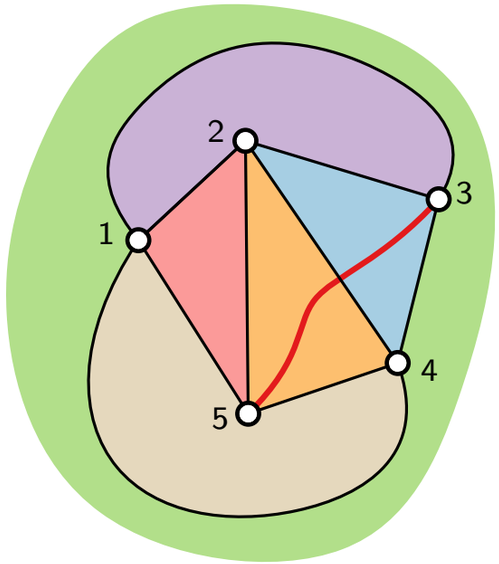

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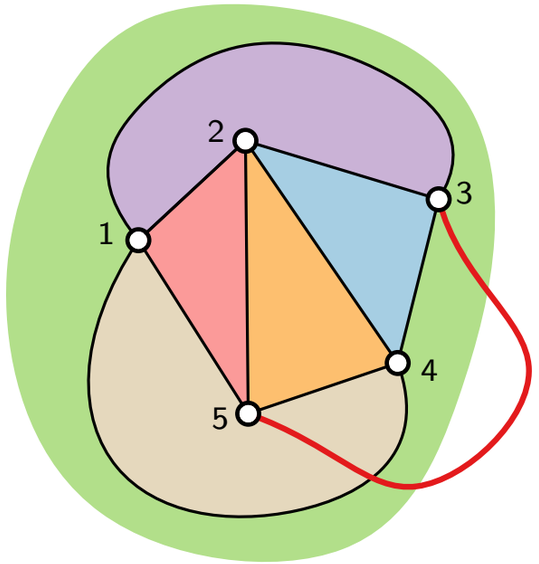

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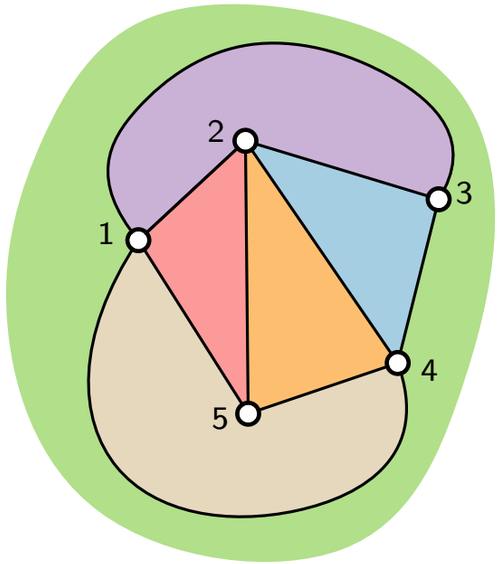

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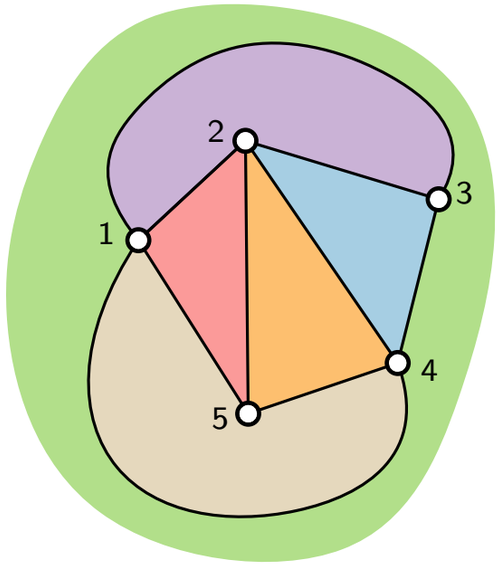

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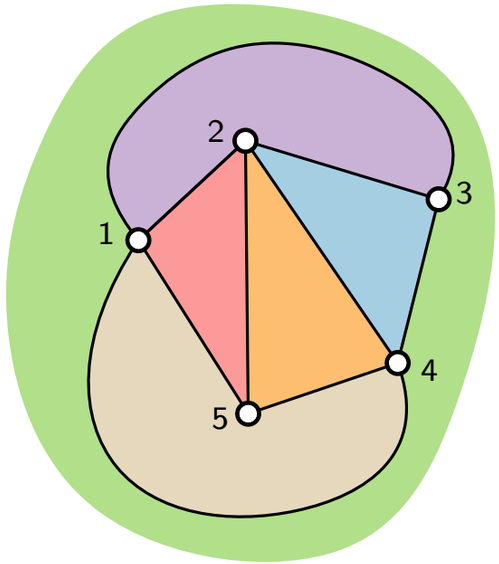

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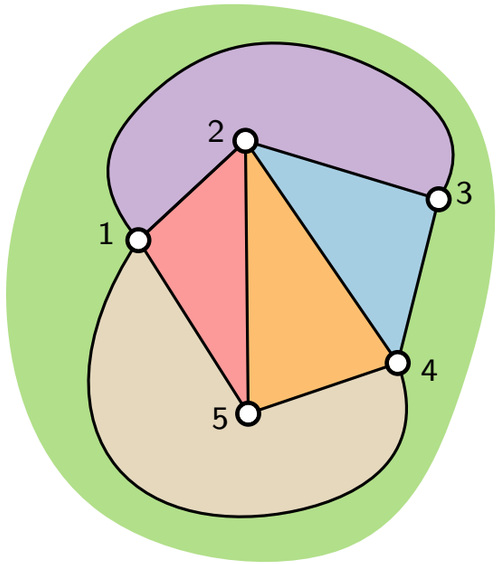

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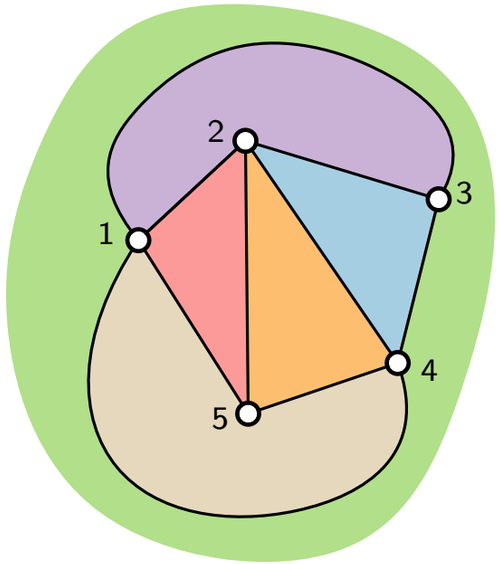

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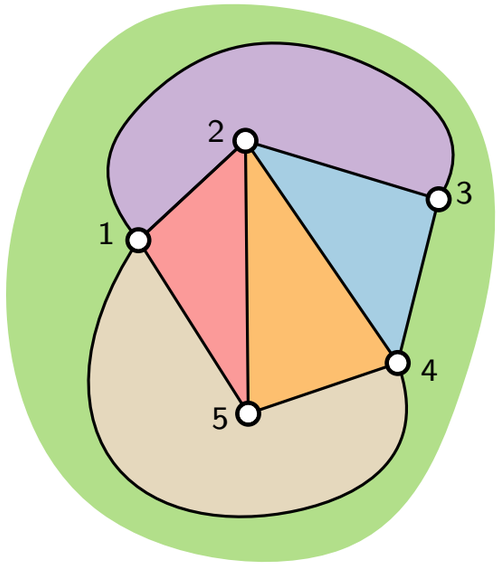

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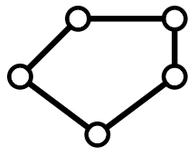

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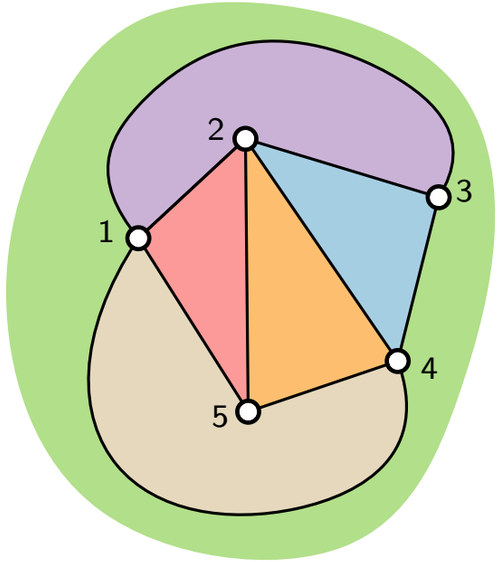

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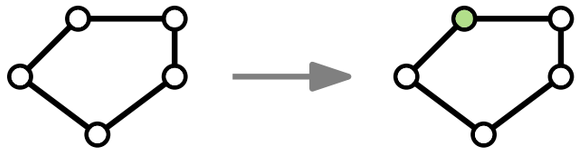

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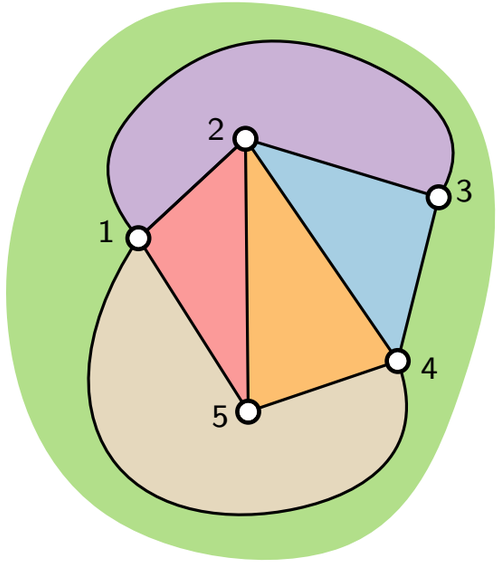

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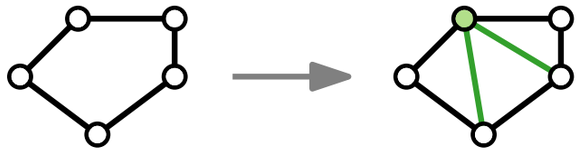

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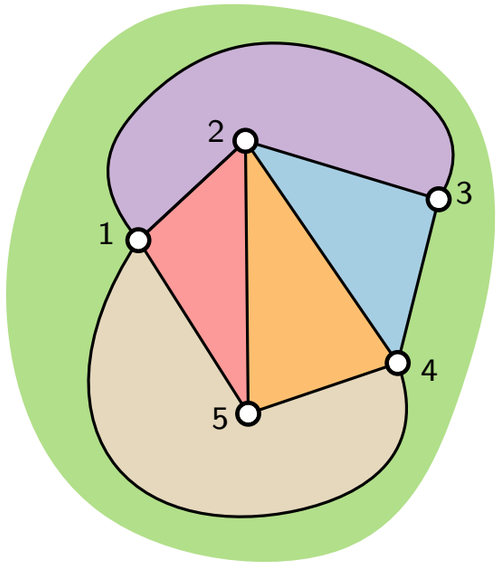

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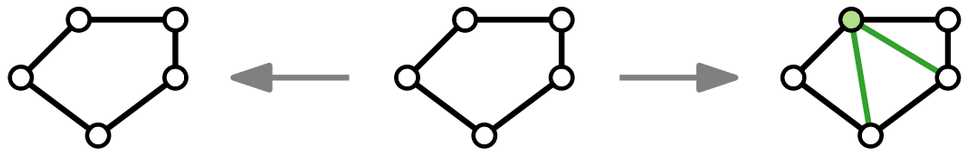

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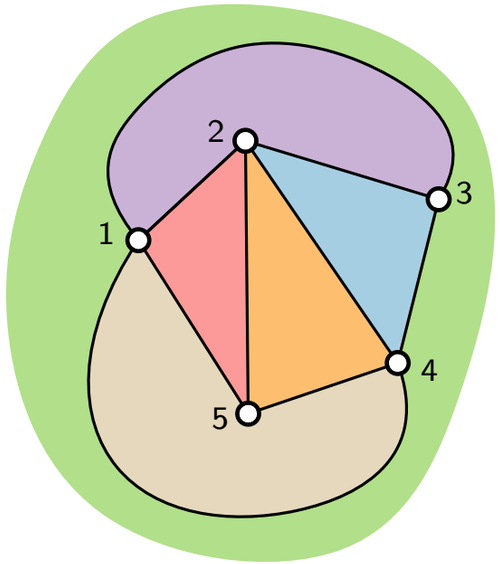

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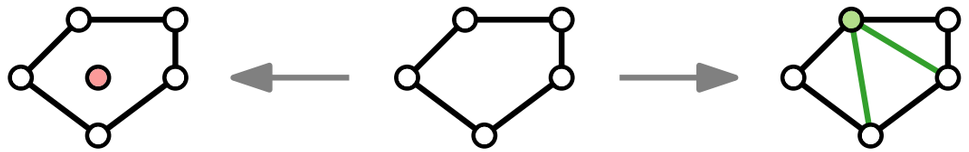

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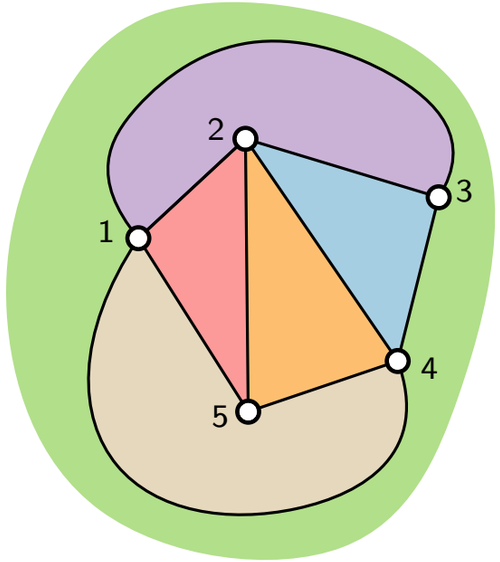

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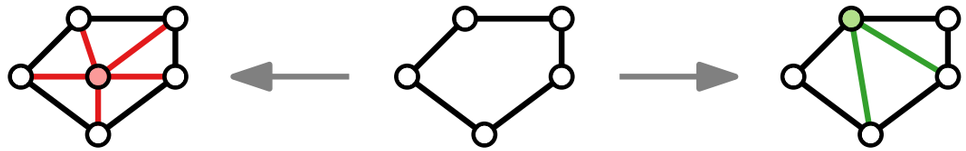

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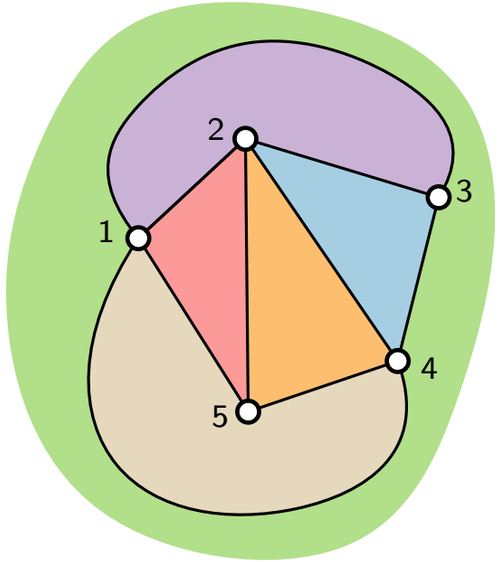

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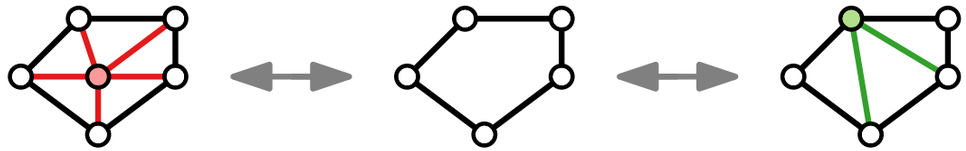

Motivation

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The Aesthetics of Graph Visualization

3.2. Edge Placement Heuristics

By far the most agreed-upon edge placement heuristic is to *minimize the number of edge crossings* in a graph [BMRW98, Har98, DH96, Pur02, TR05, TBB88]. The importance of avoiding edge crossings has also been extensively validated in terms of user preference and performance (see Section 4). Similarly, based on perceptual principles, it is beneficial to *minimize the number of edge bends* within a graph [Pur02, TR05, TBB88]. Edge bends make edges more difficult to follow because an edge with a sharp bend is more likely to be perceived as two separate objects. This leads to the heuristic of *keeping edge bends uniform* with respect to the bend's position on the edge and its angle [TR05]. If an edge must be bent to satisfy other aesthetic criteria, the angle of the bend should be as little as possible, and the bend placement should evenly divide the edge.

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- Area

Towards Straight-Line Drawings

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Characterization

Towards Straight-Line Drawings

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Towards Straight-Line Drawings

Theorem. [Kuratowski 1930]
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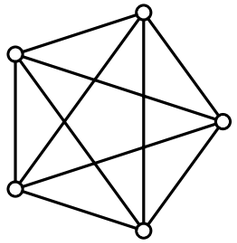

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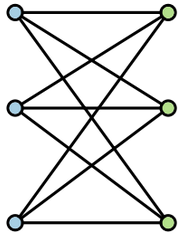

$K_{3,3}$

Characterization

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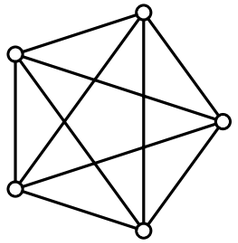

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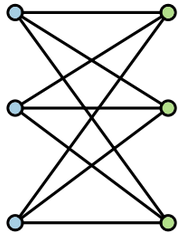

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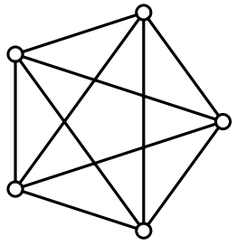

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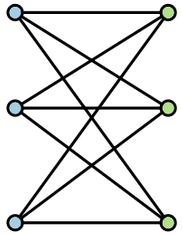

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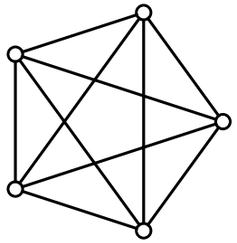

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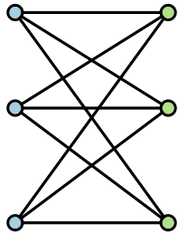

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Towards Straight-Line Drawings

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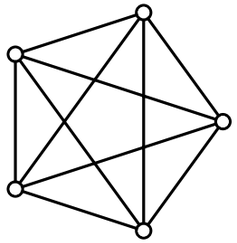

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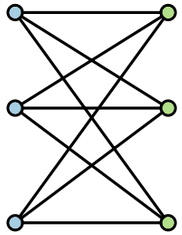

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The algorithms implied by these theorems produce drawings whose area is **not** bounded by any polynomial in n.

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Theorem. [De Fraysseix, Pach, Pollack '90]
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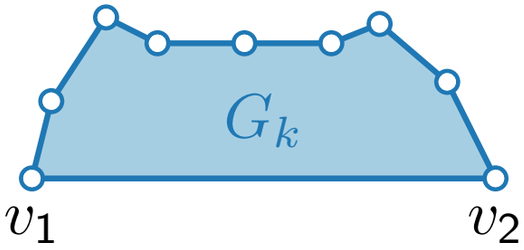

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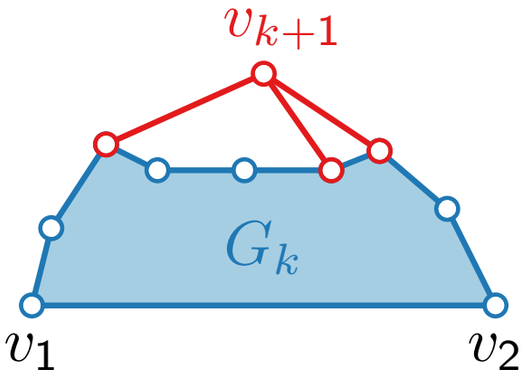

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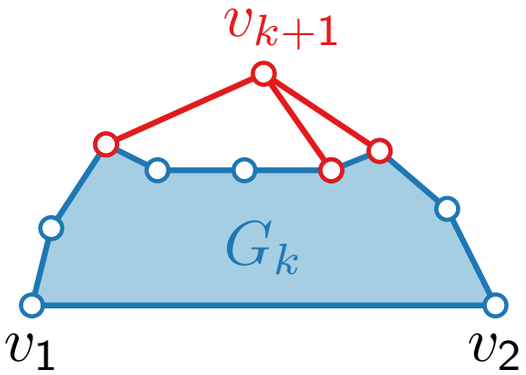

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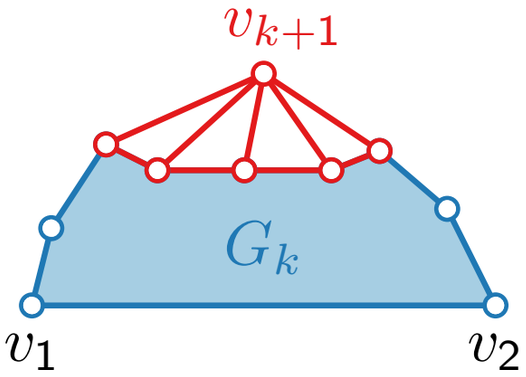

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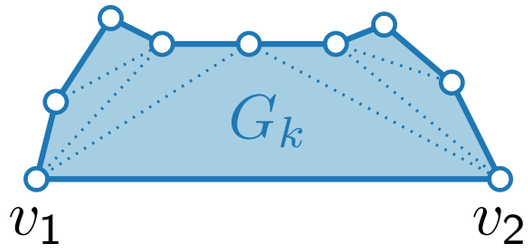

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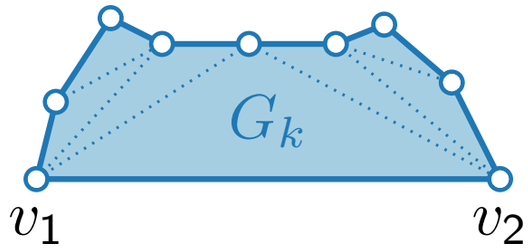

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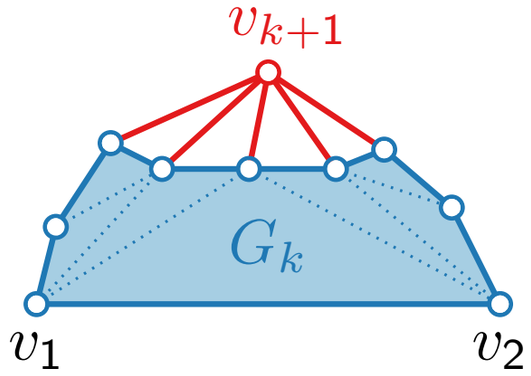

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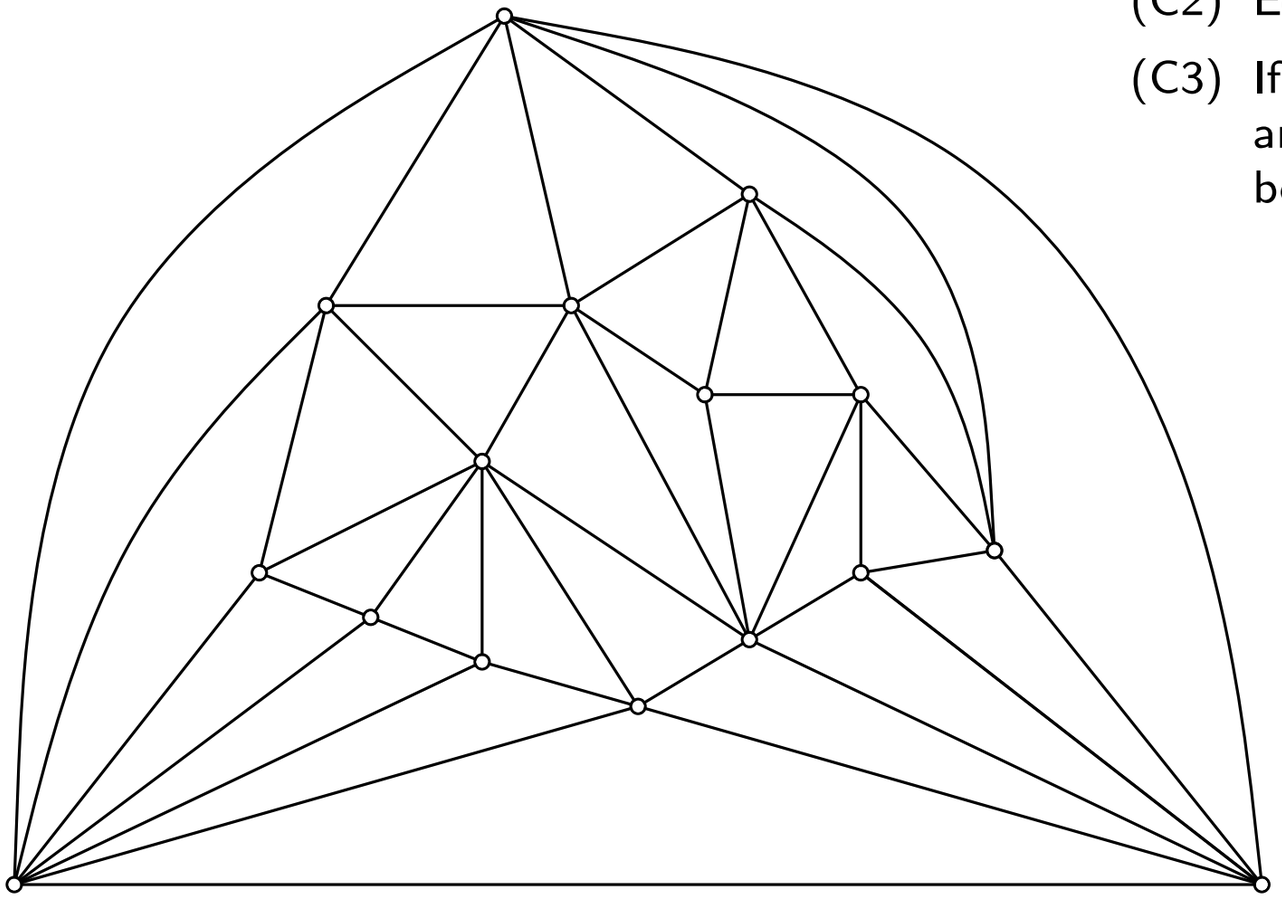

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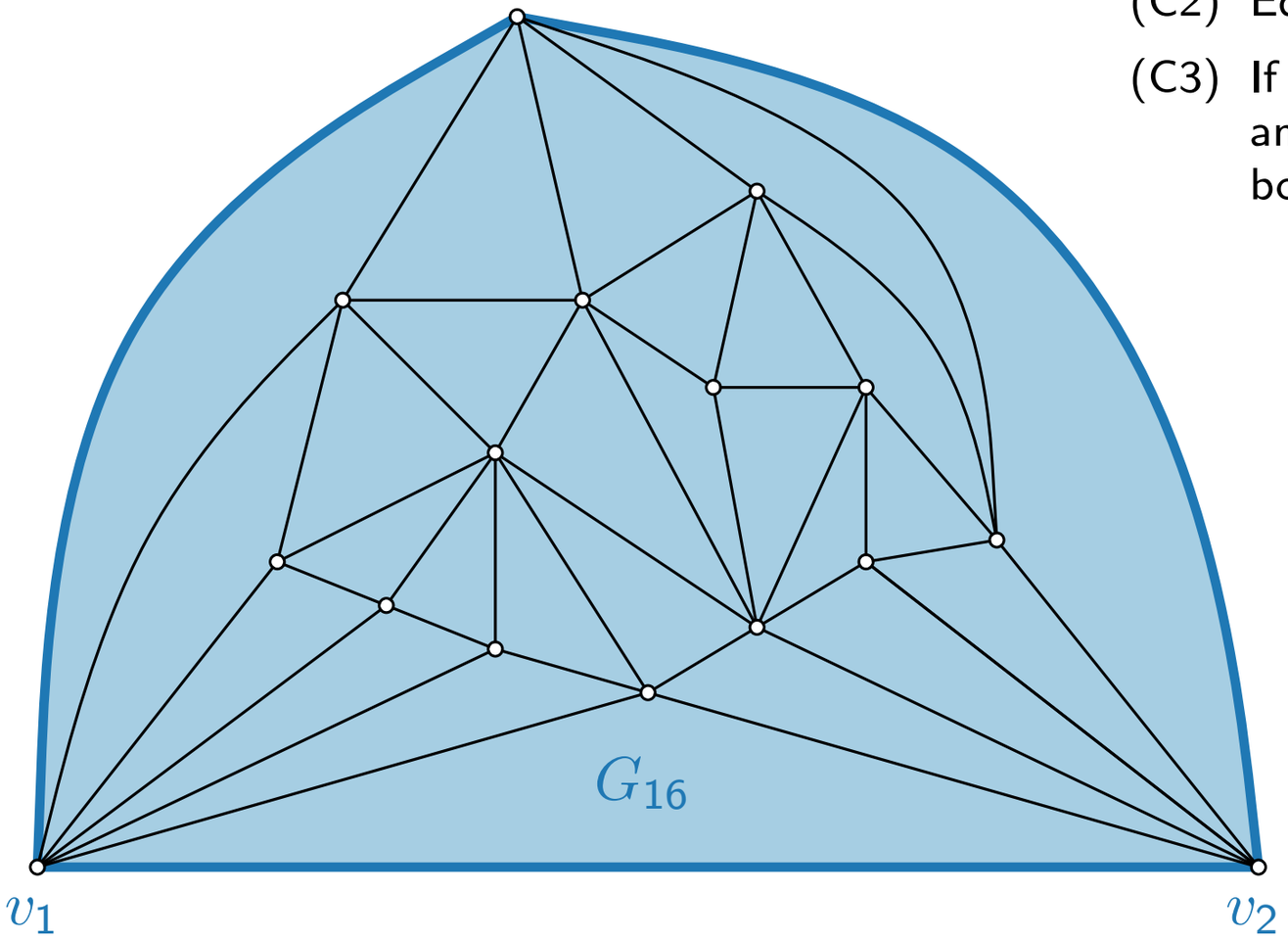

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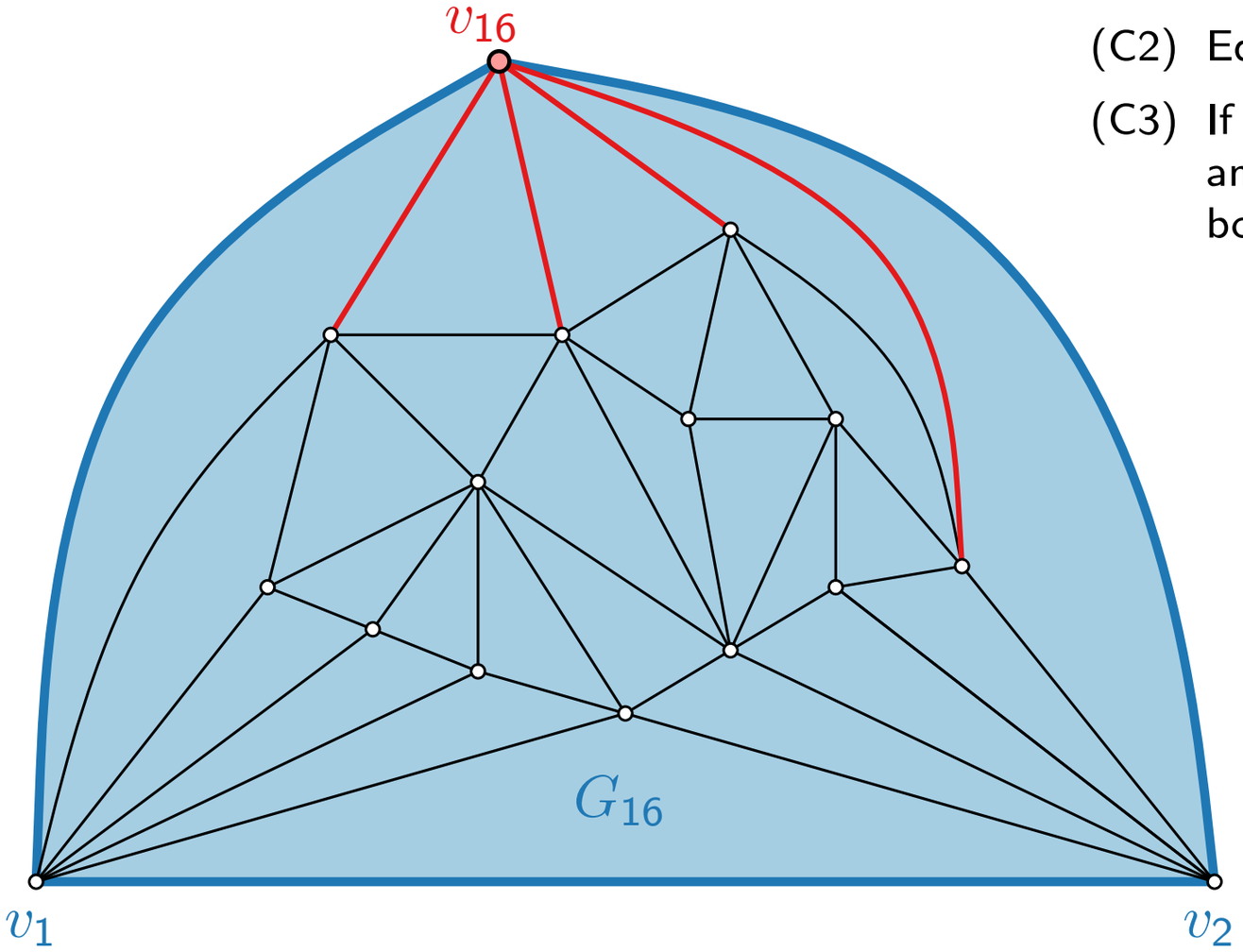

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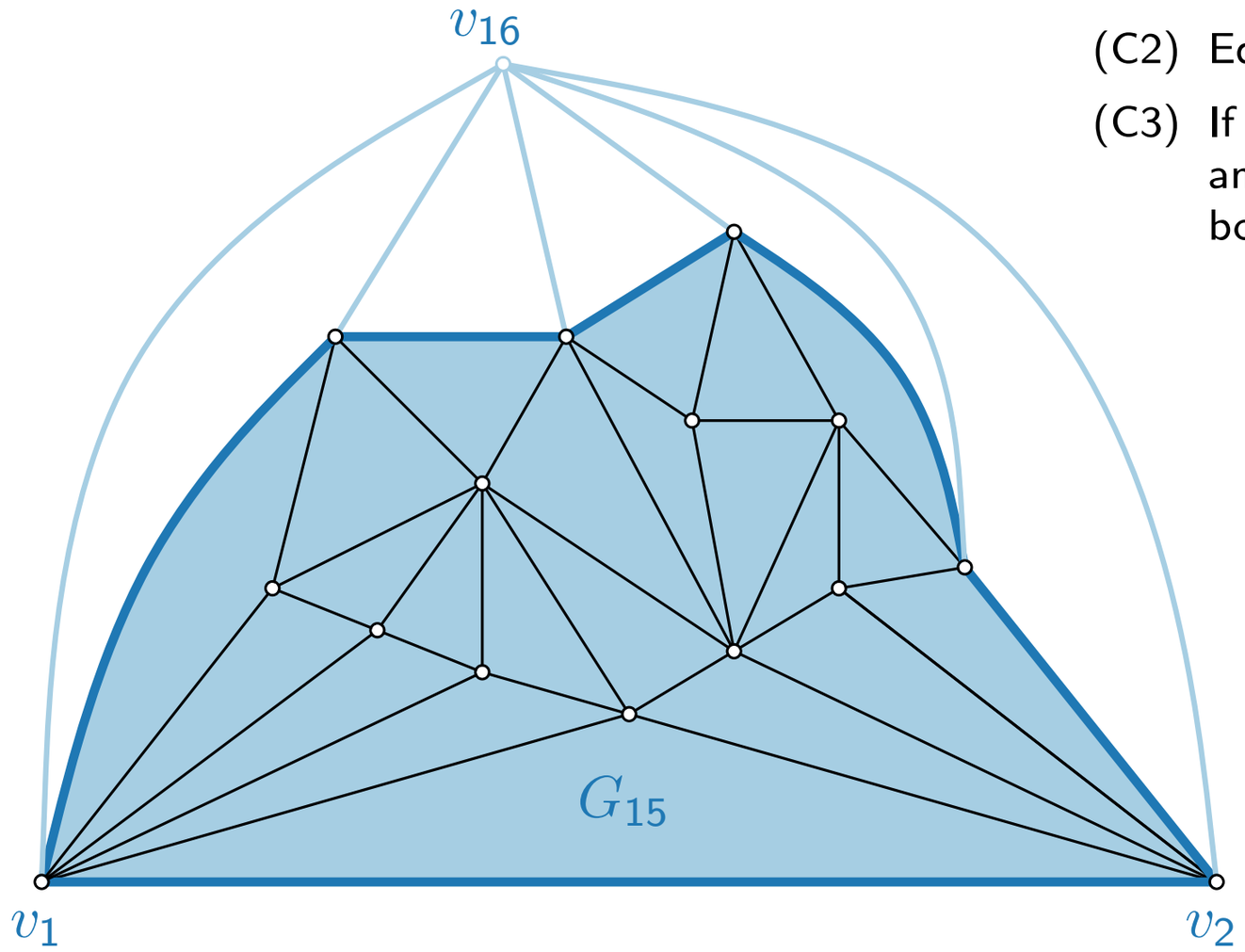

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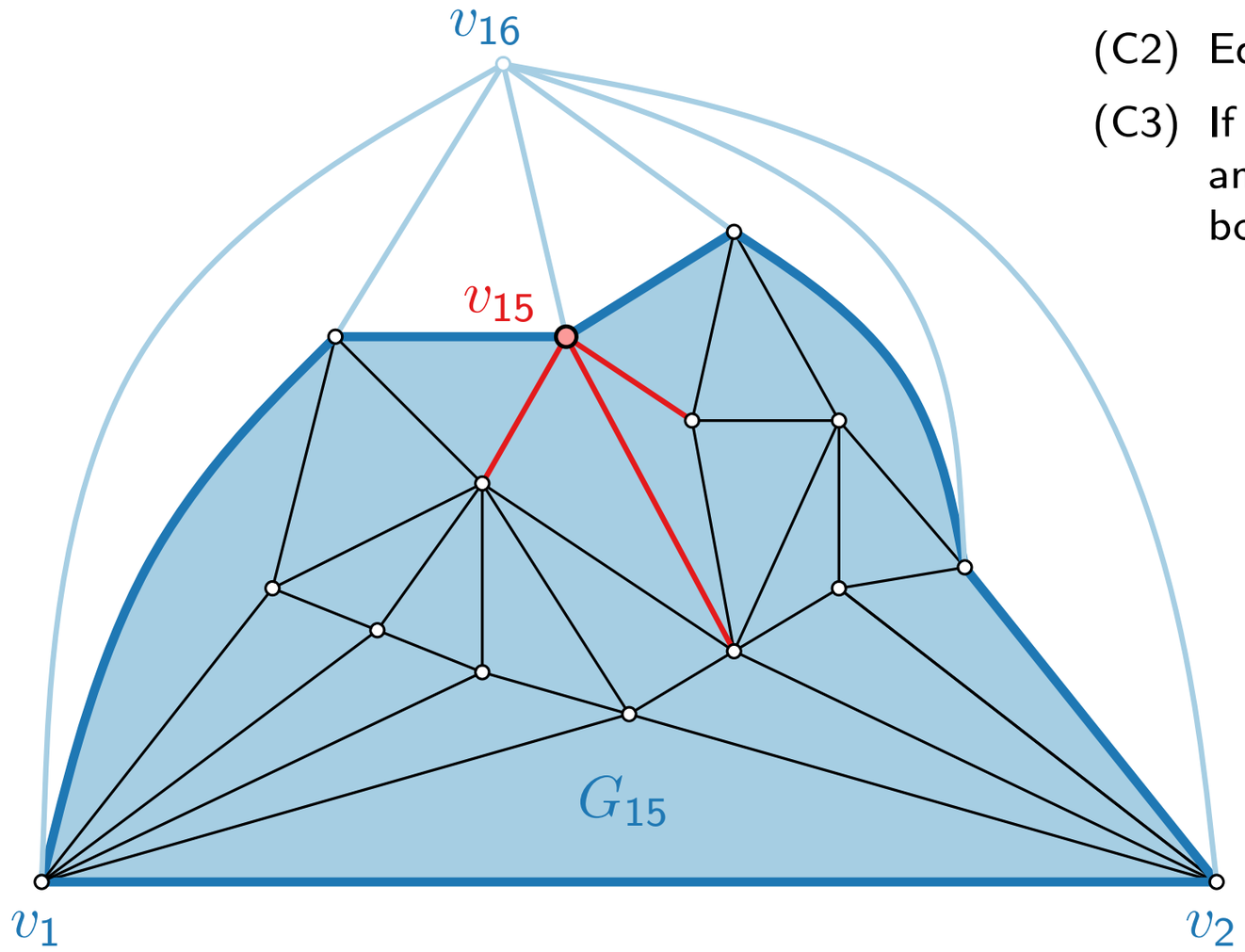

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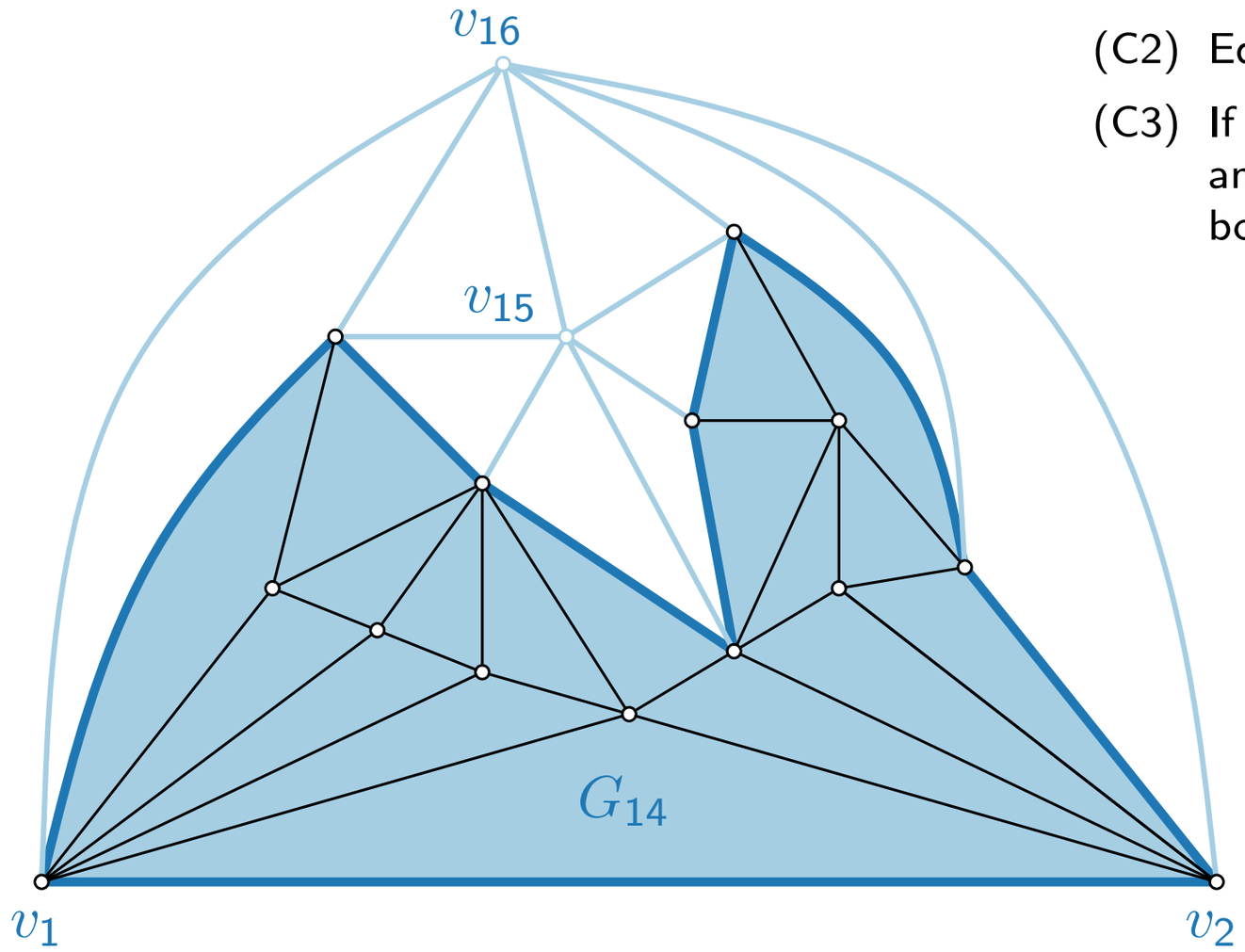

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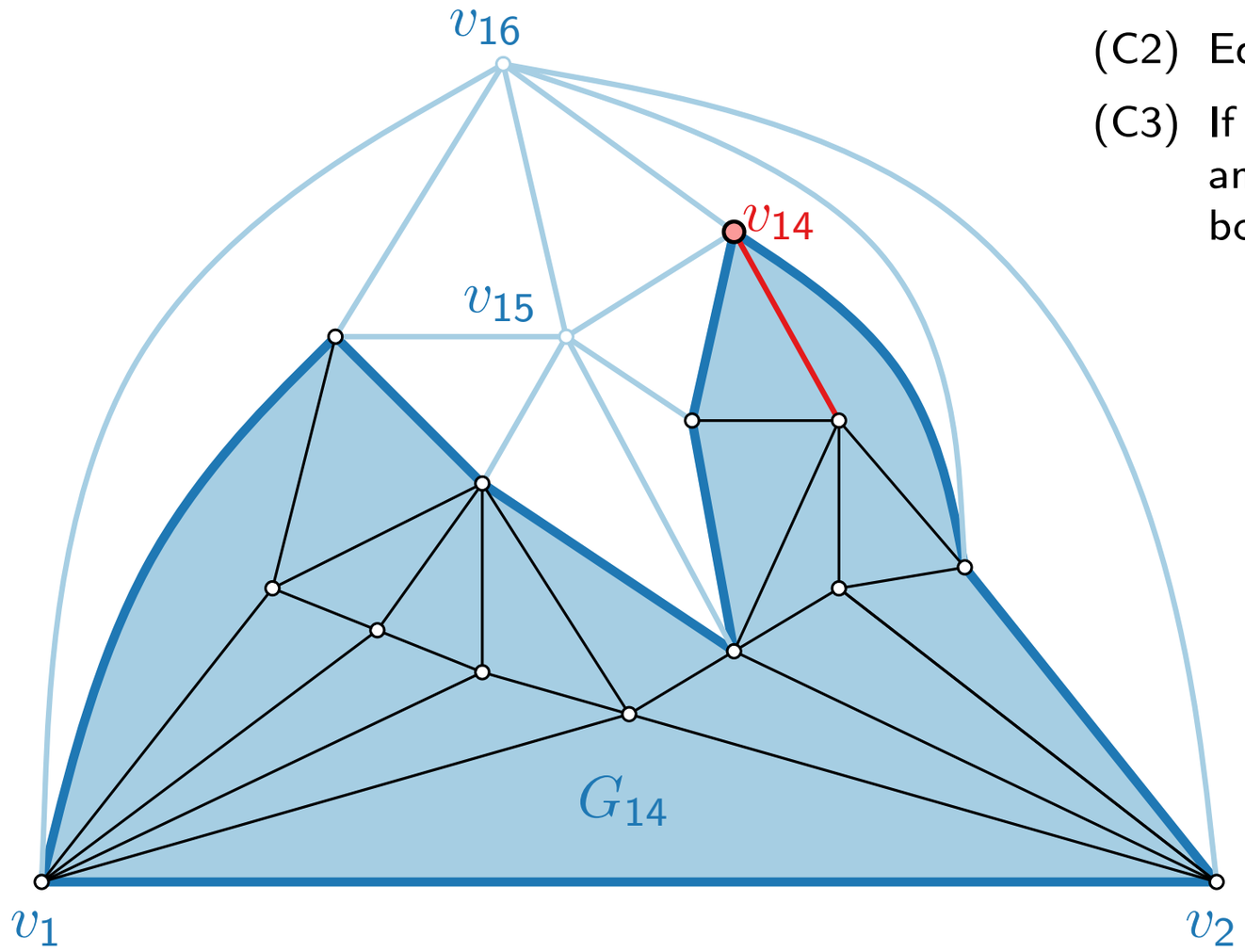

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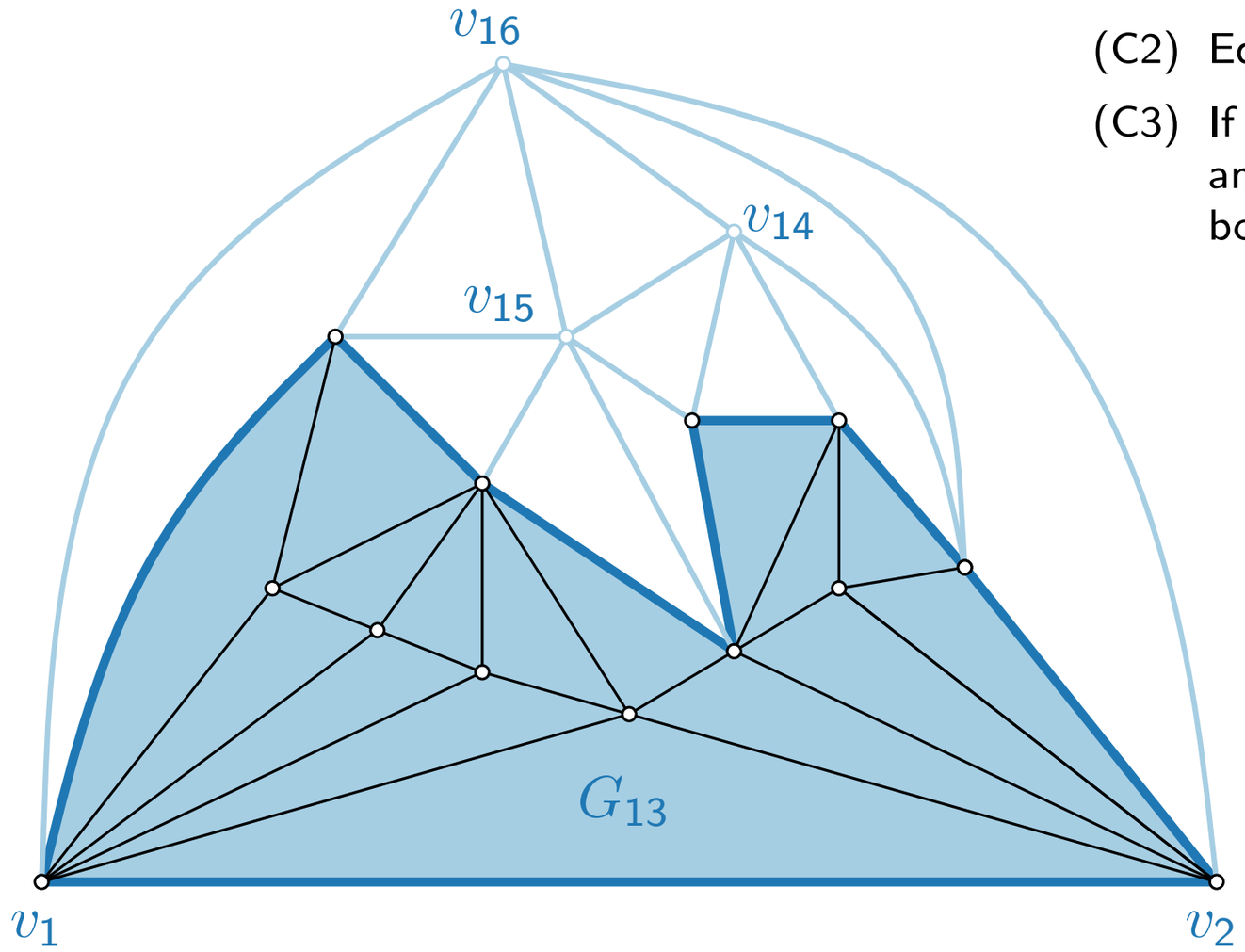

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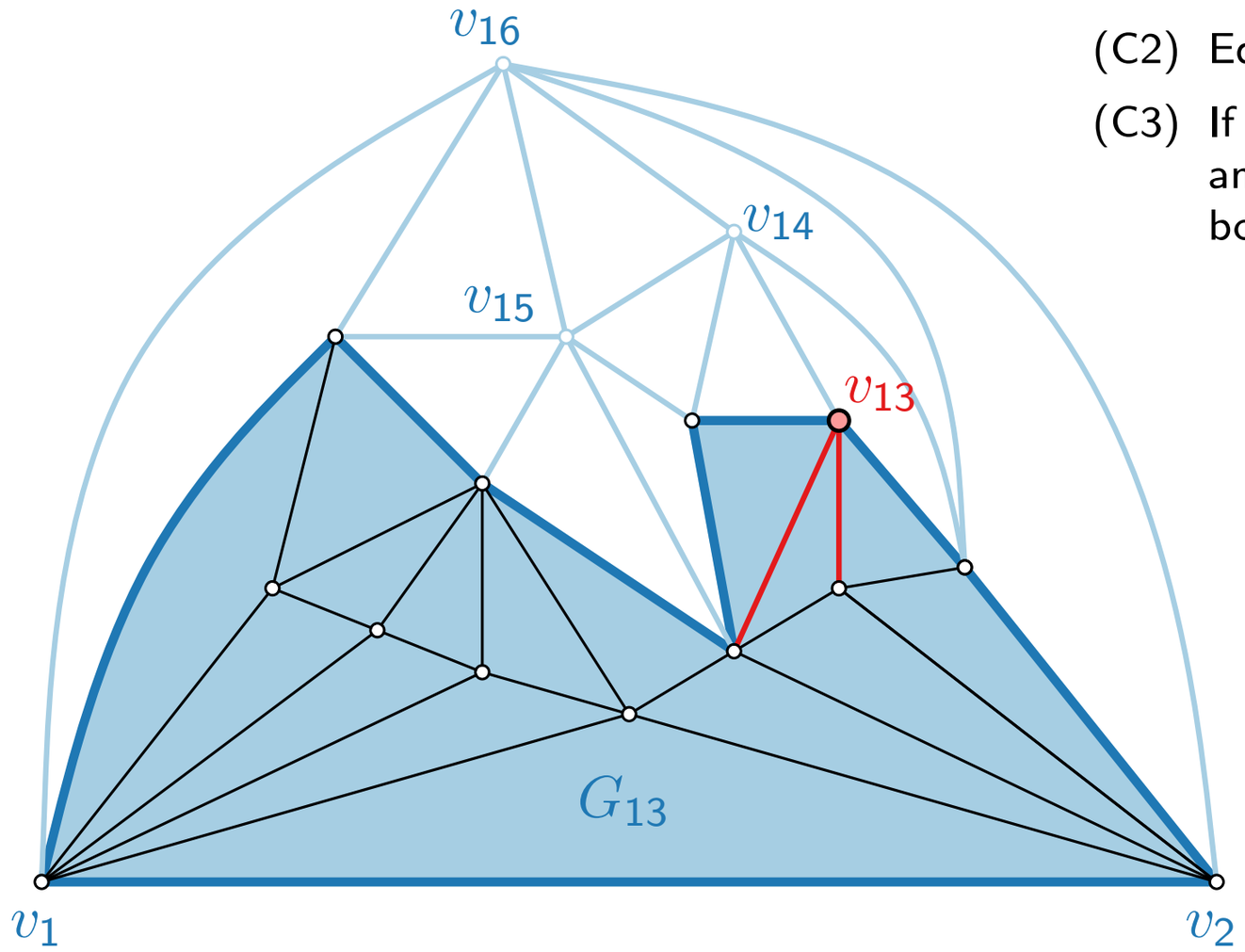

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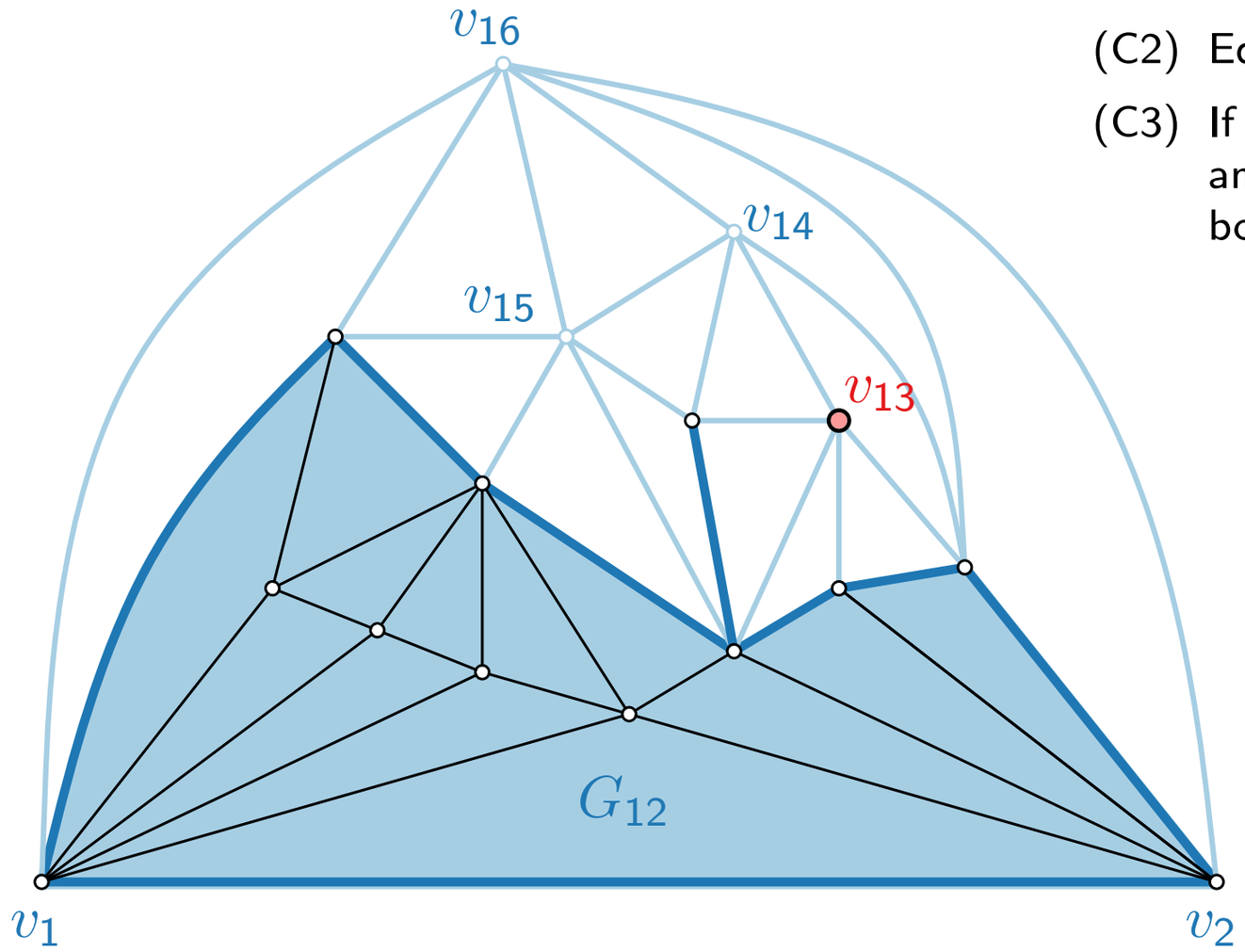

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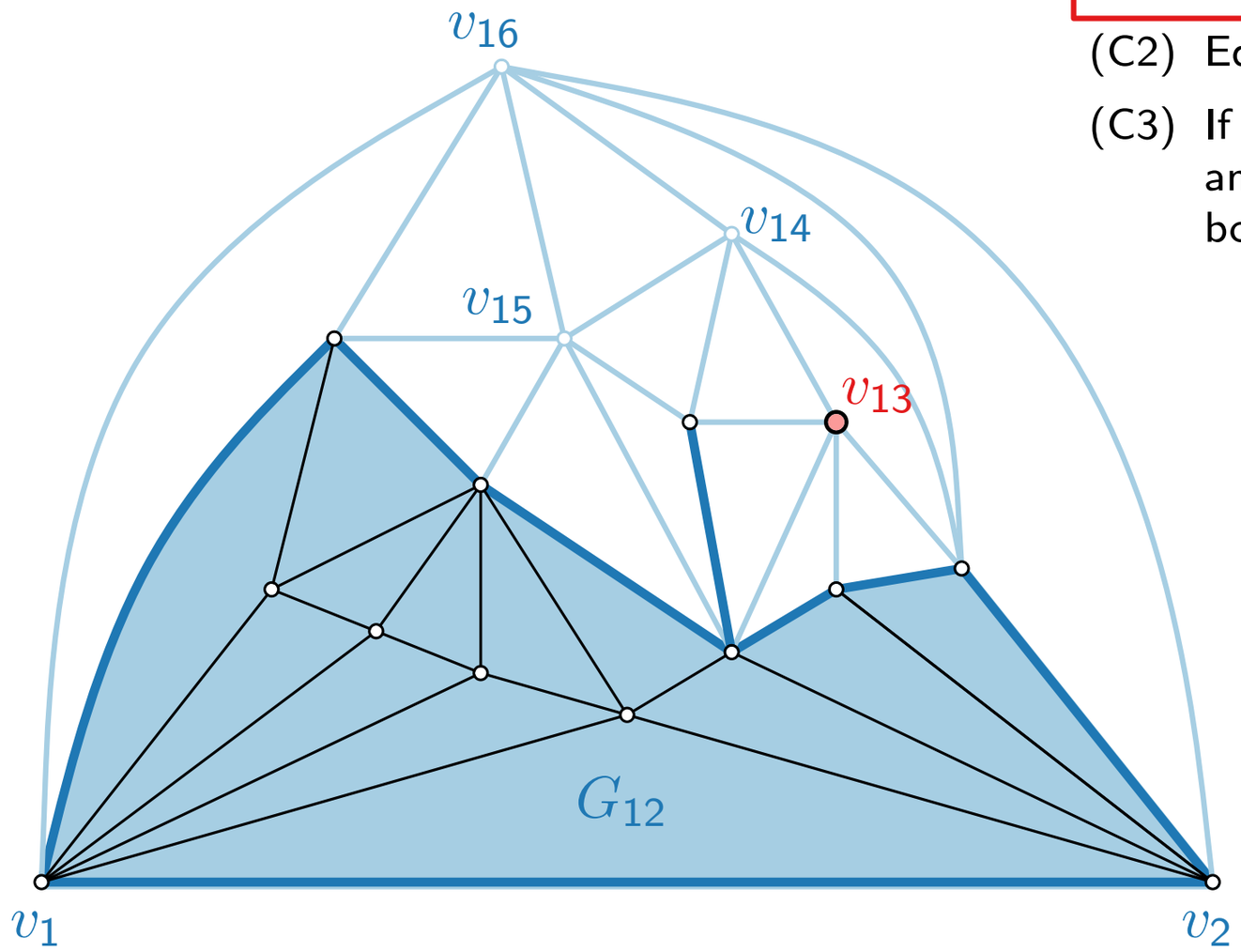

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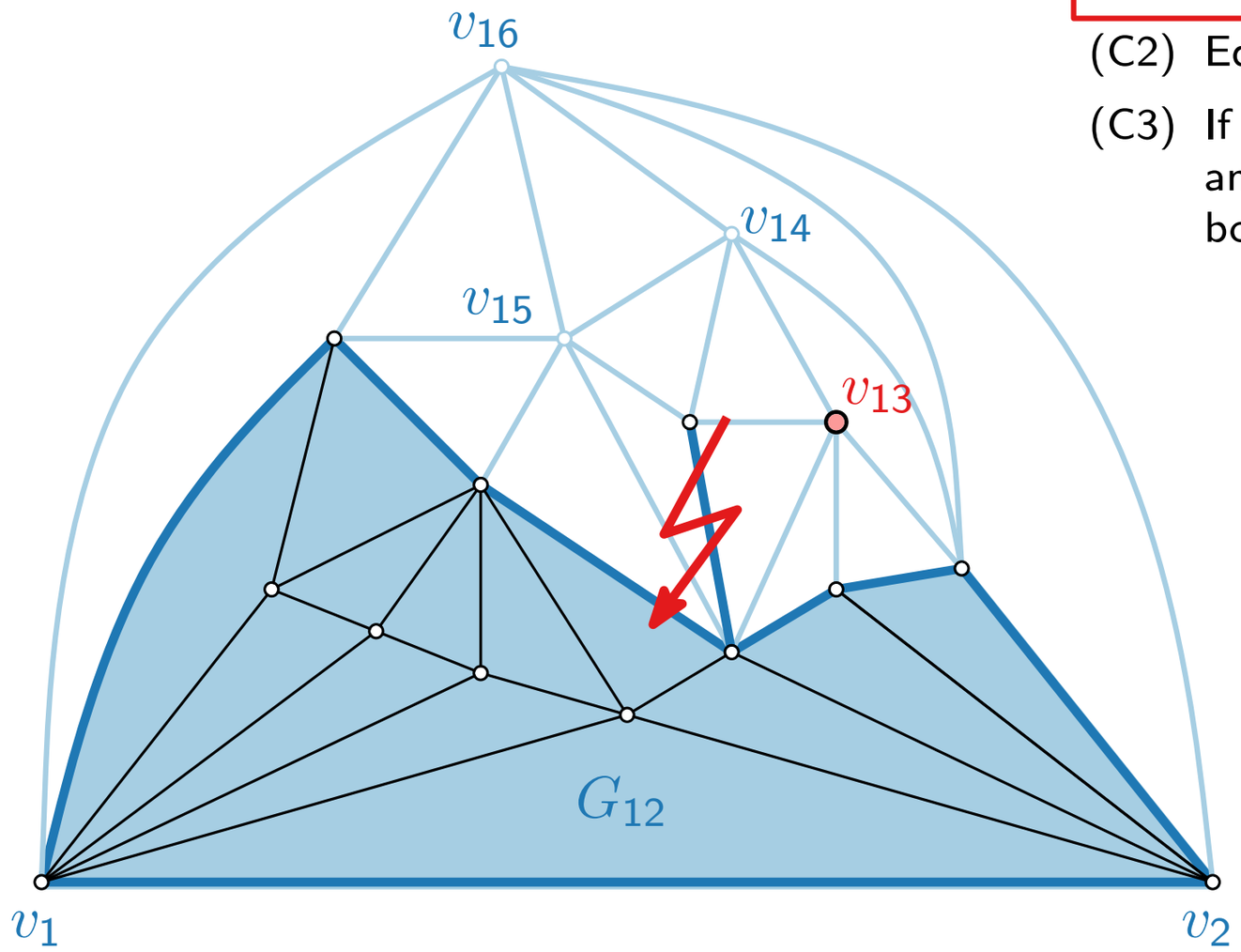

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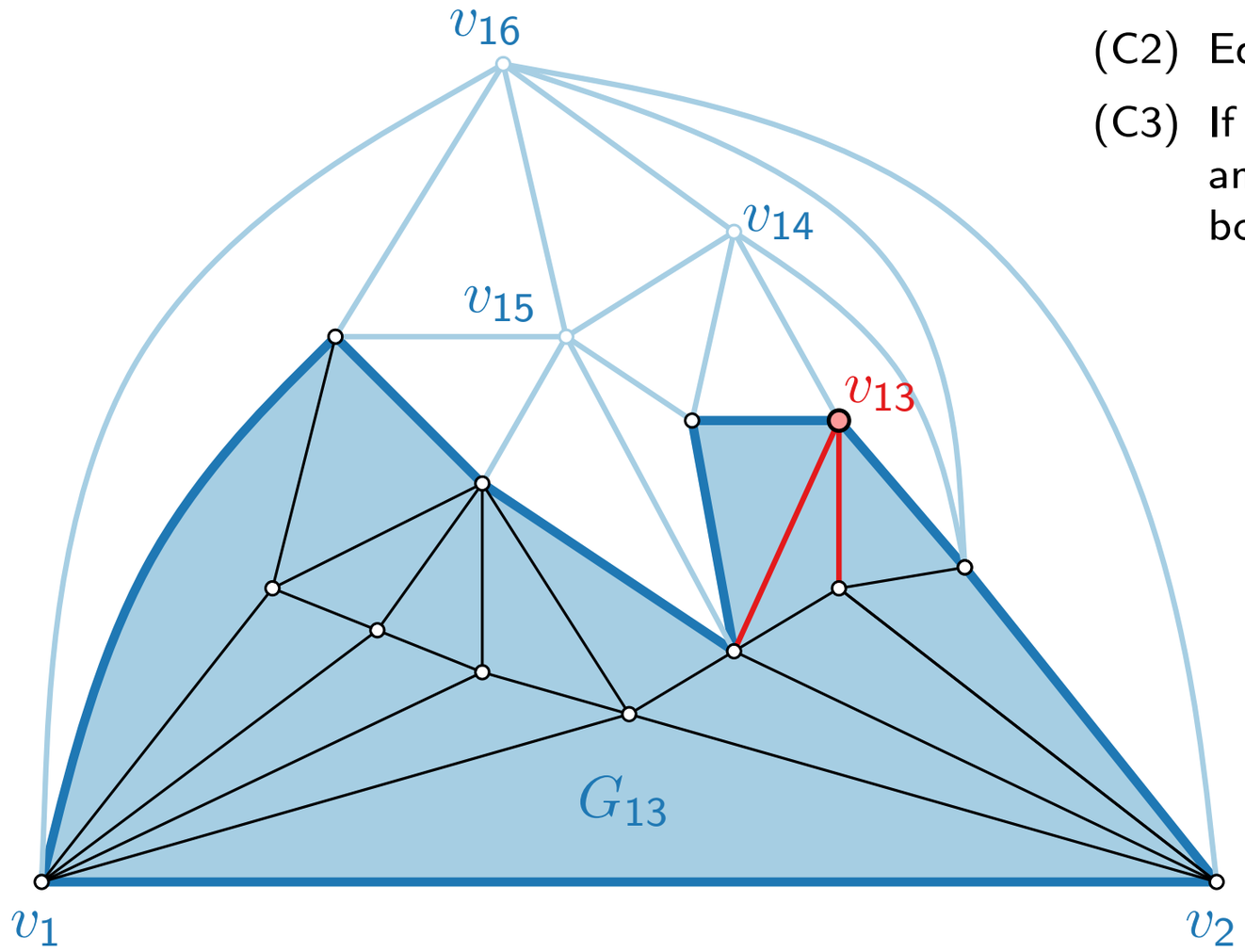

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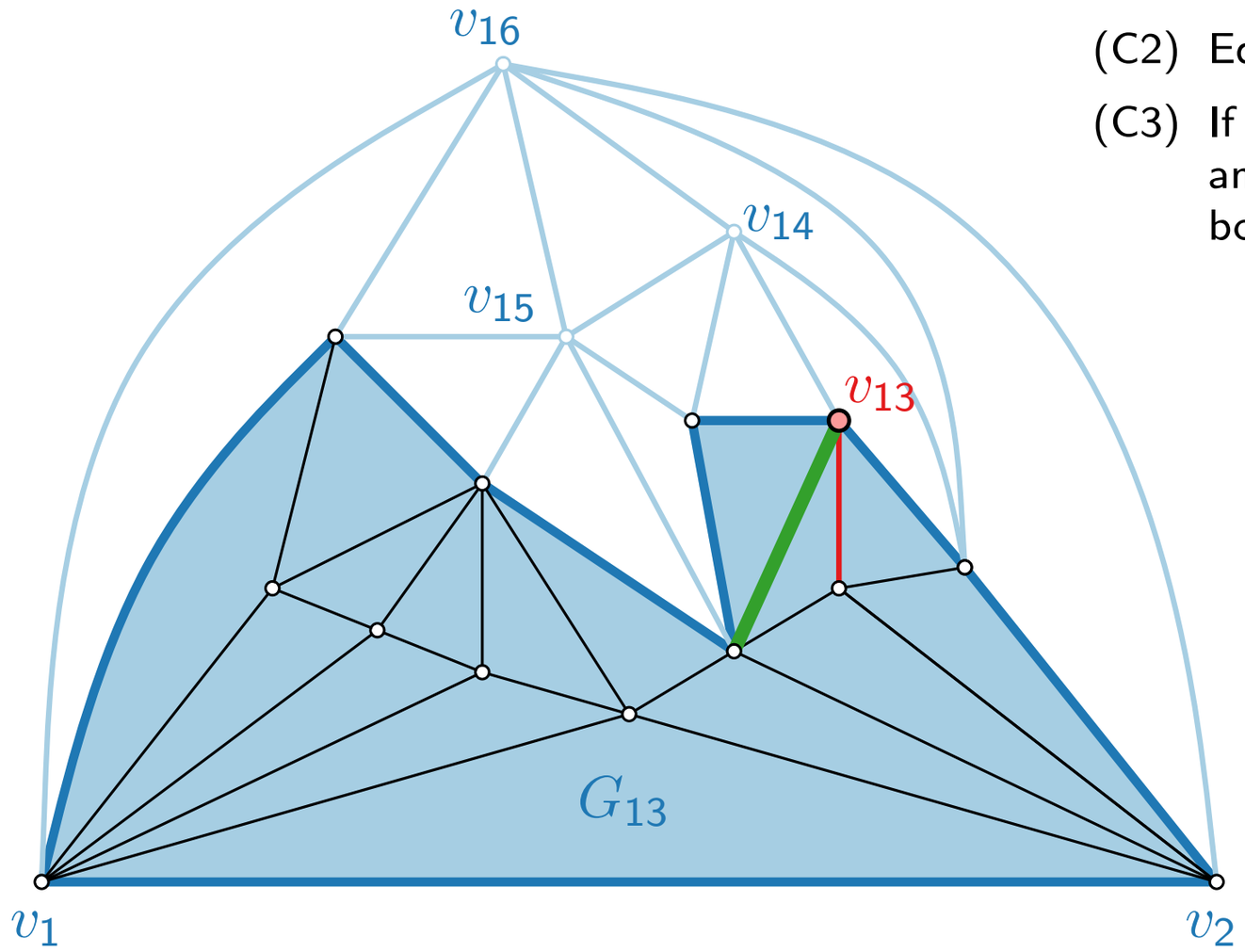

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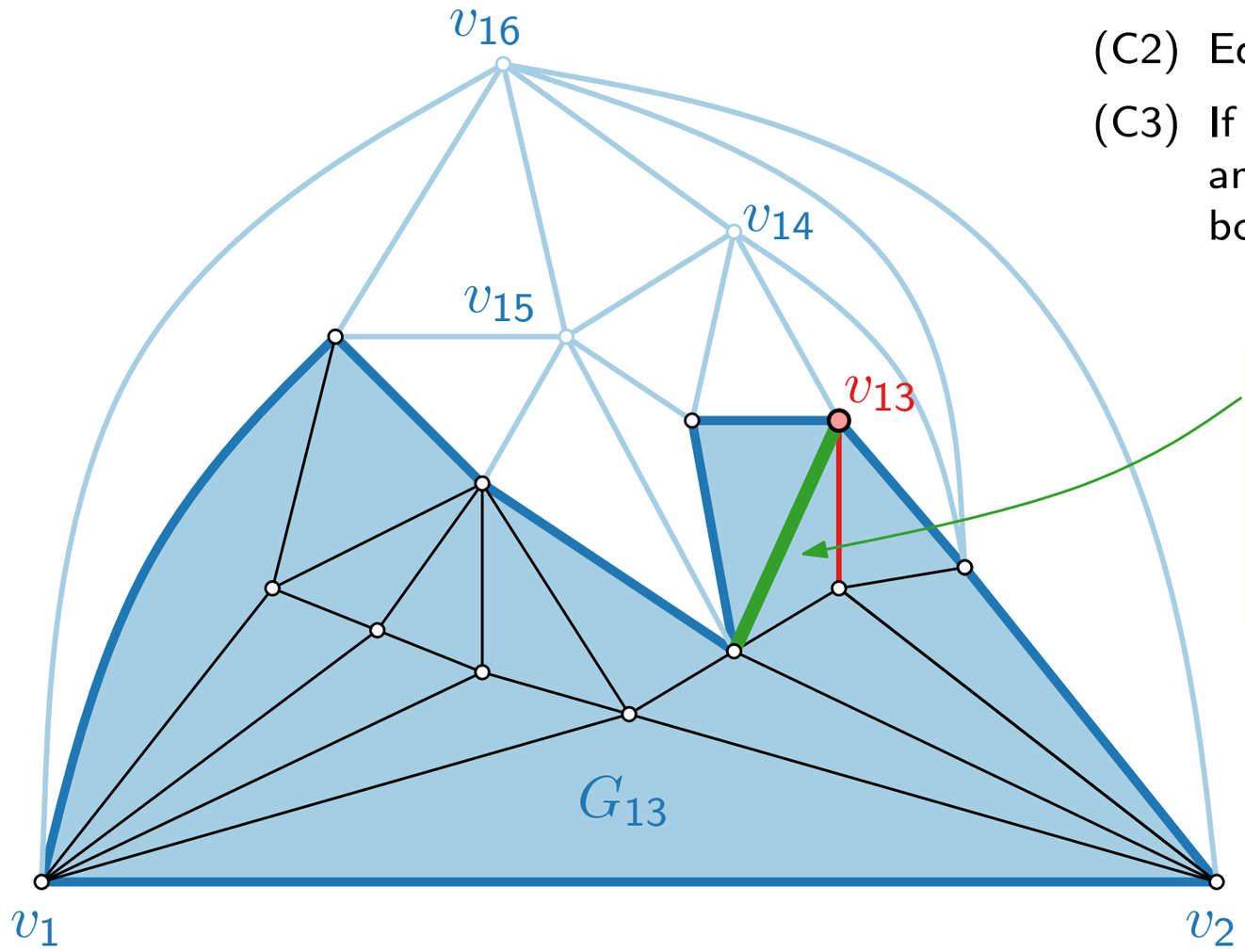

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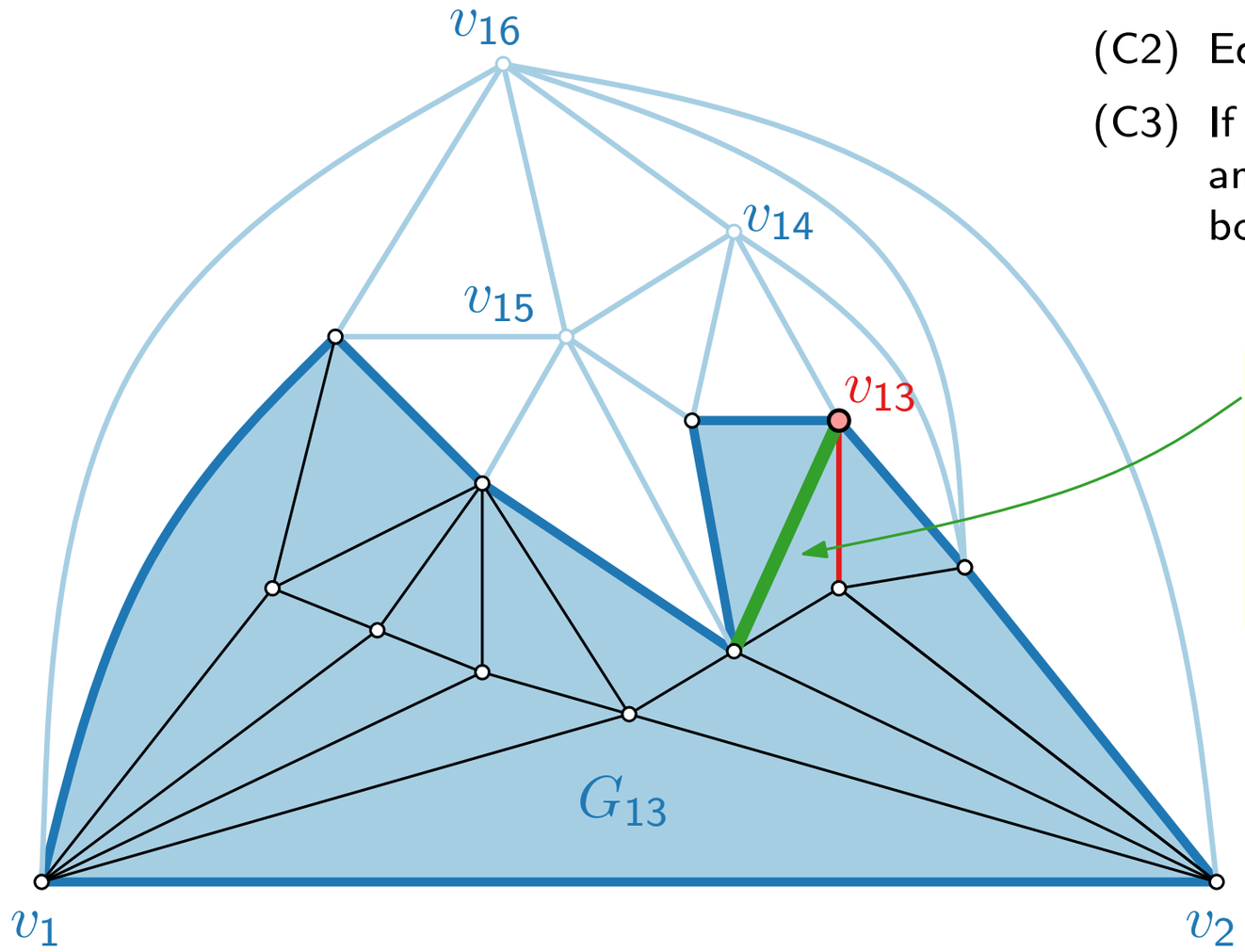

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chord:
edge joining two non-adjacent vertices in a cycle

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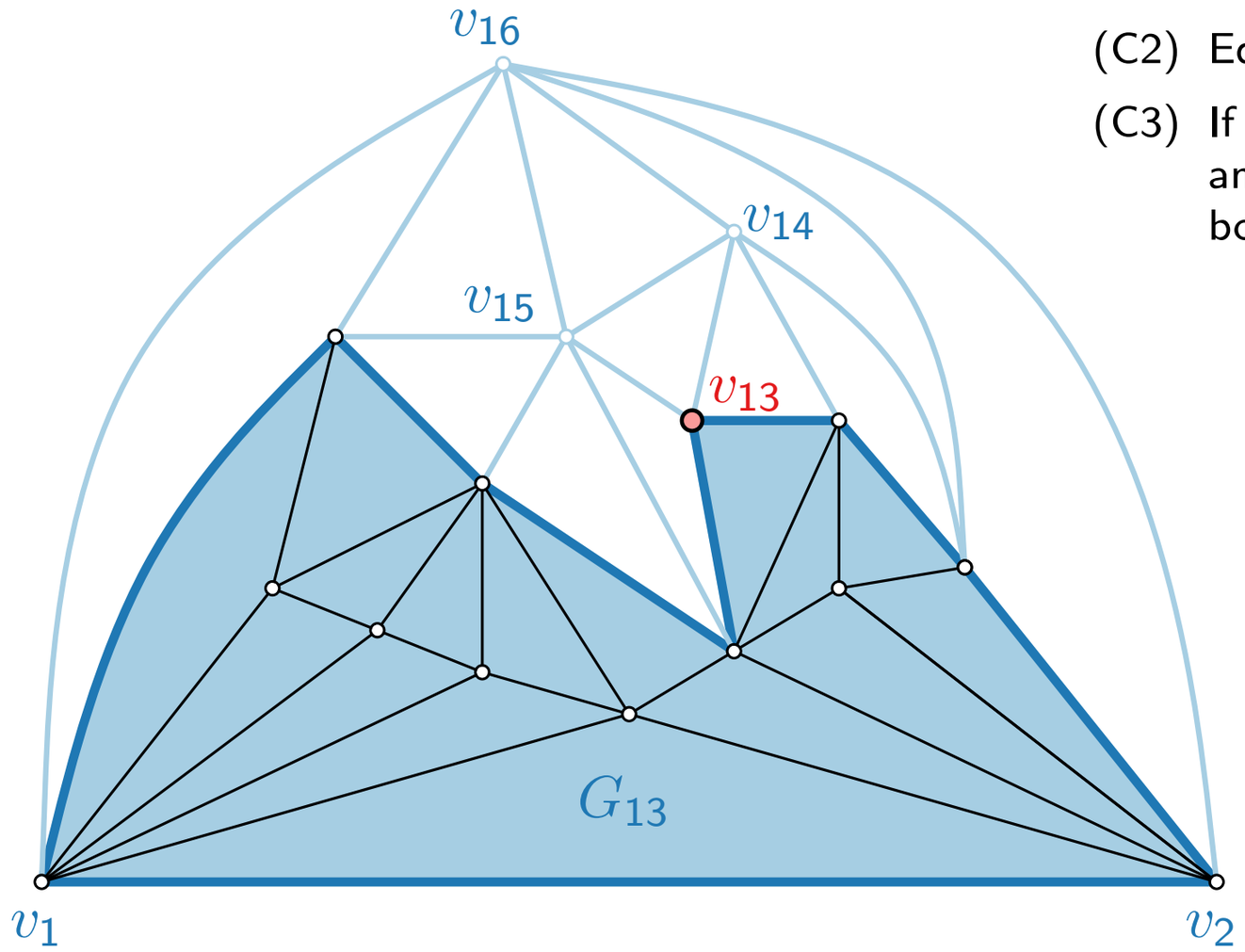

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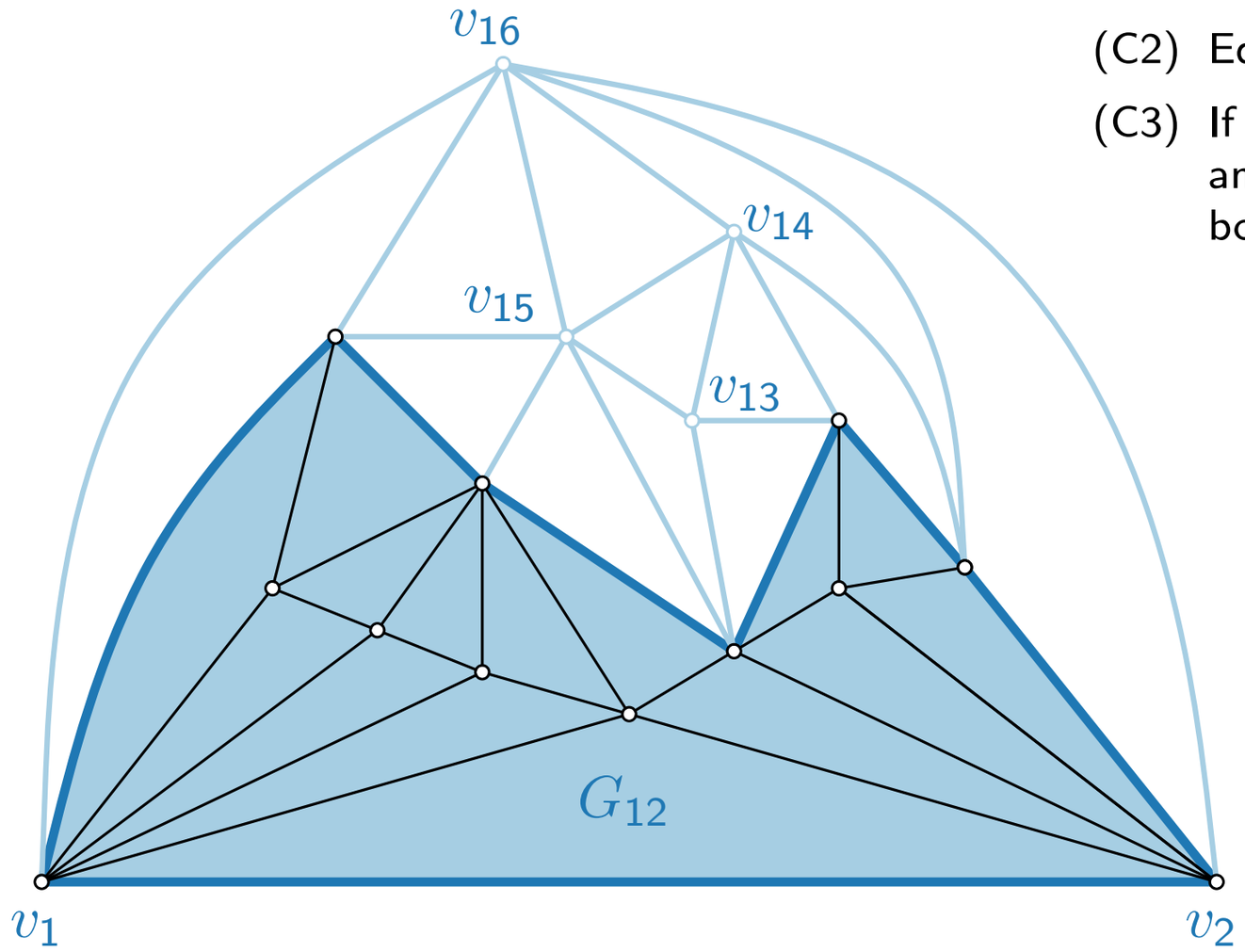

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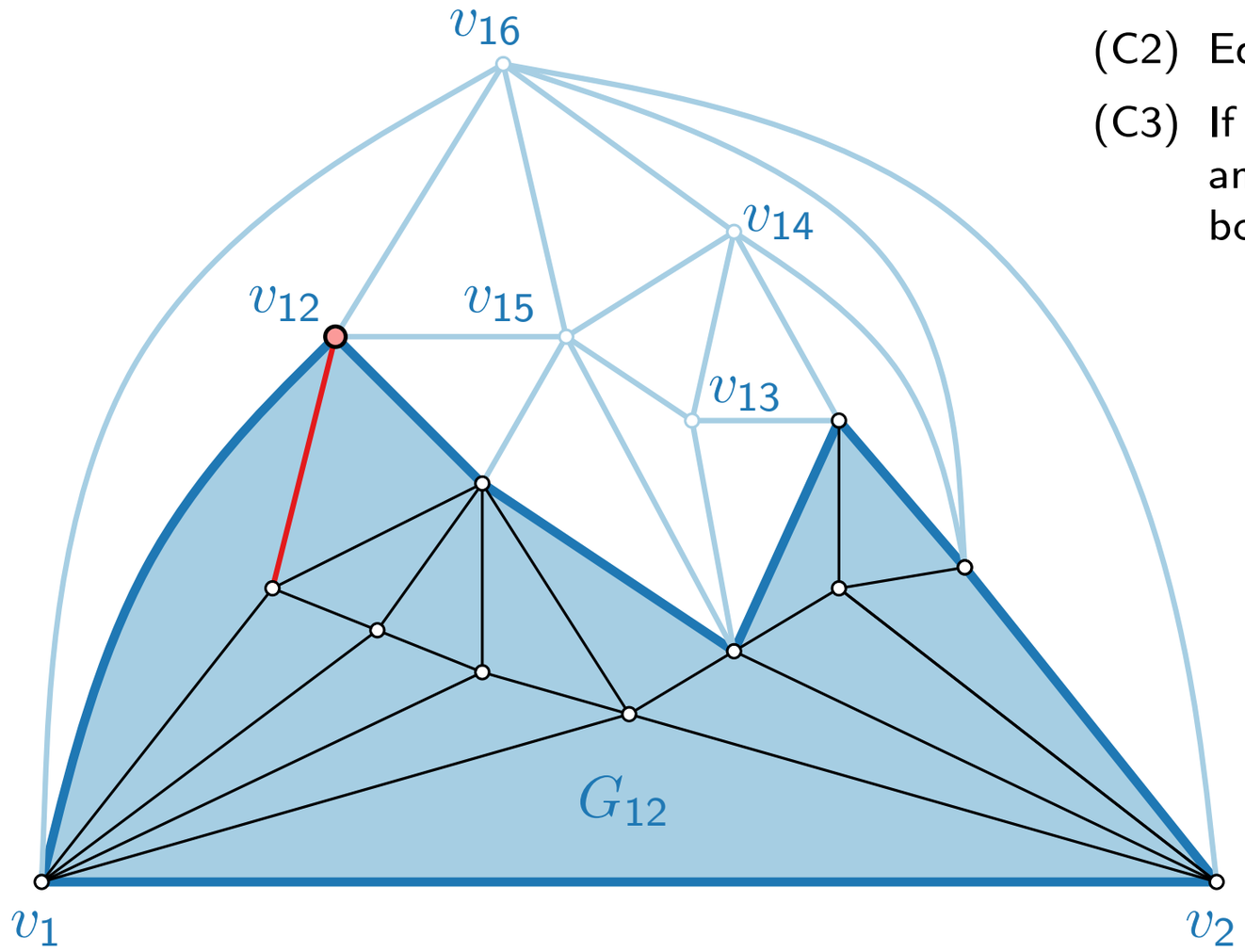

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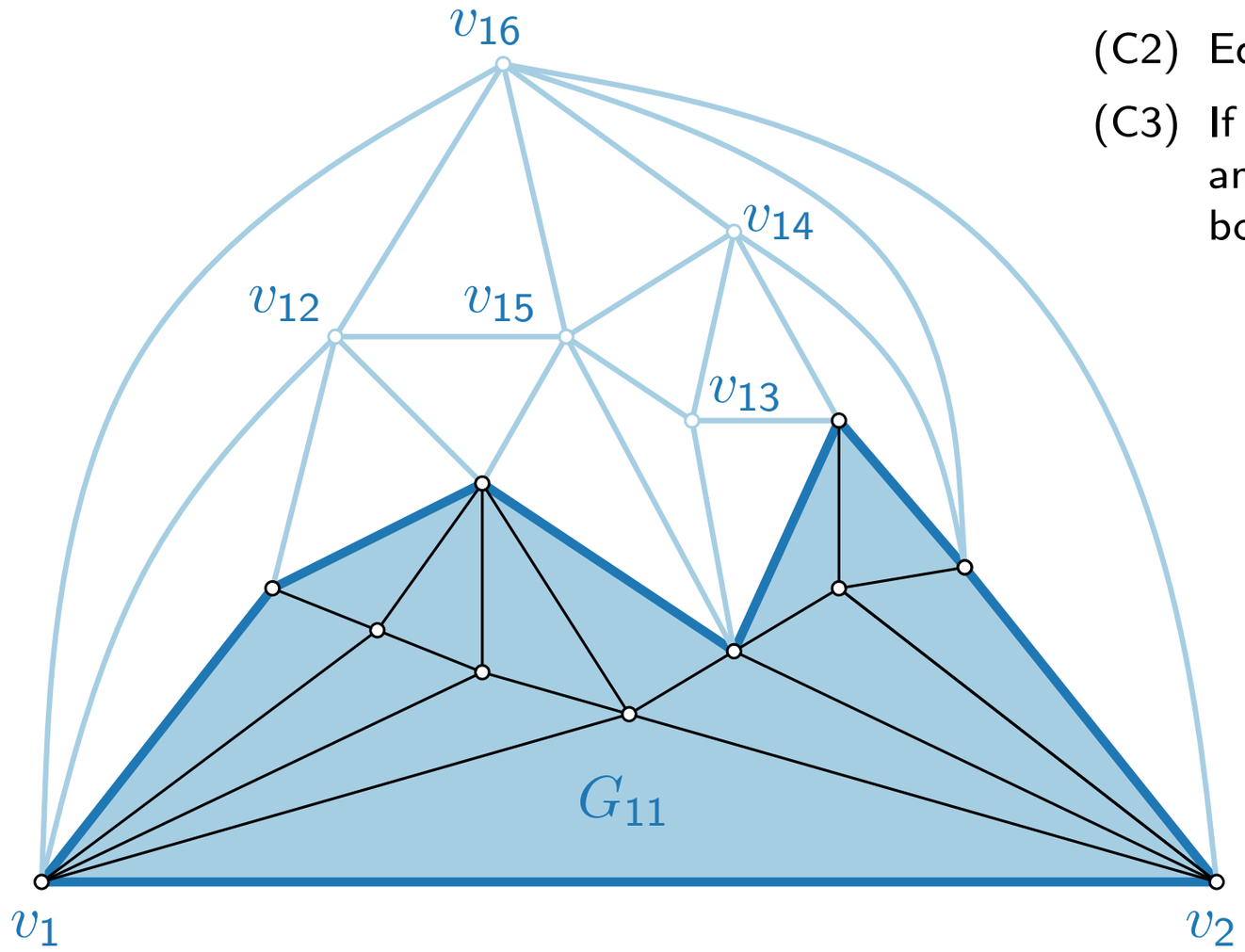

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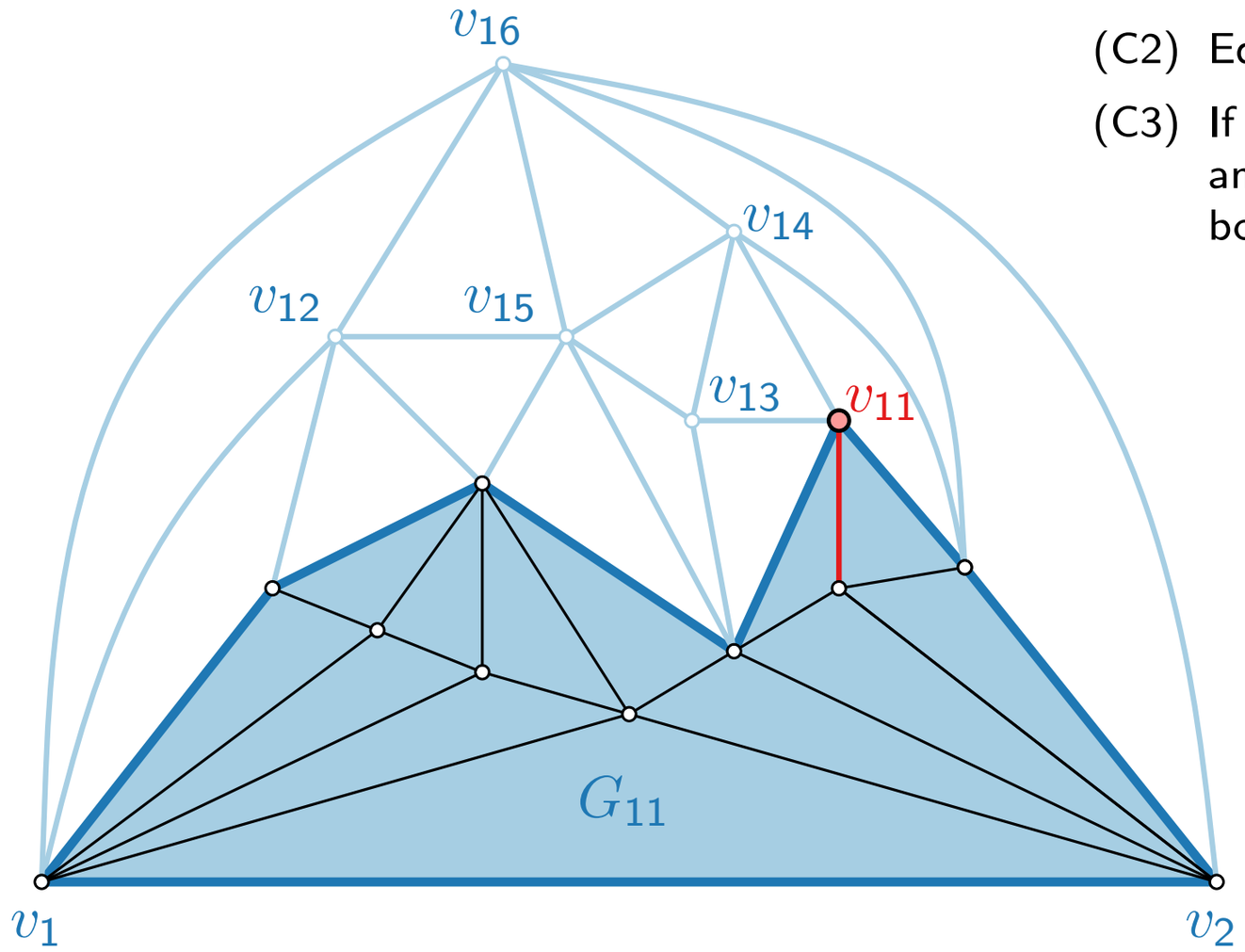

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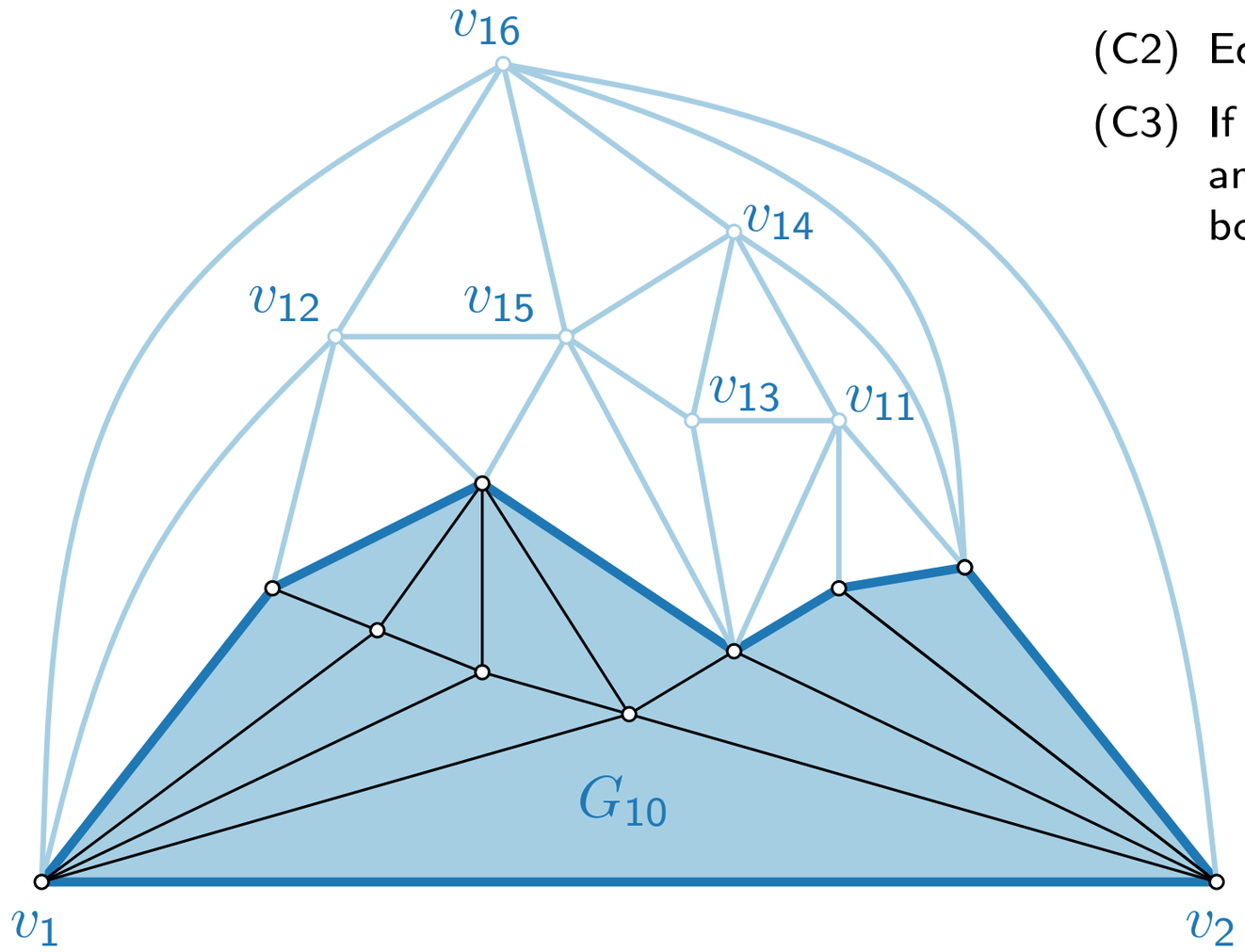

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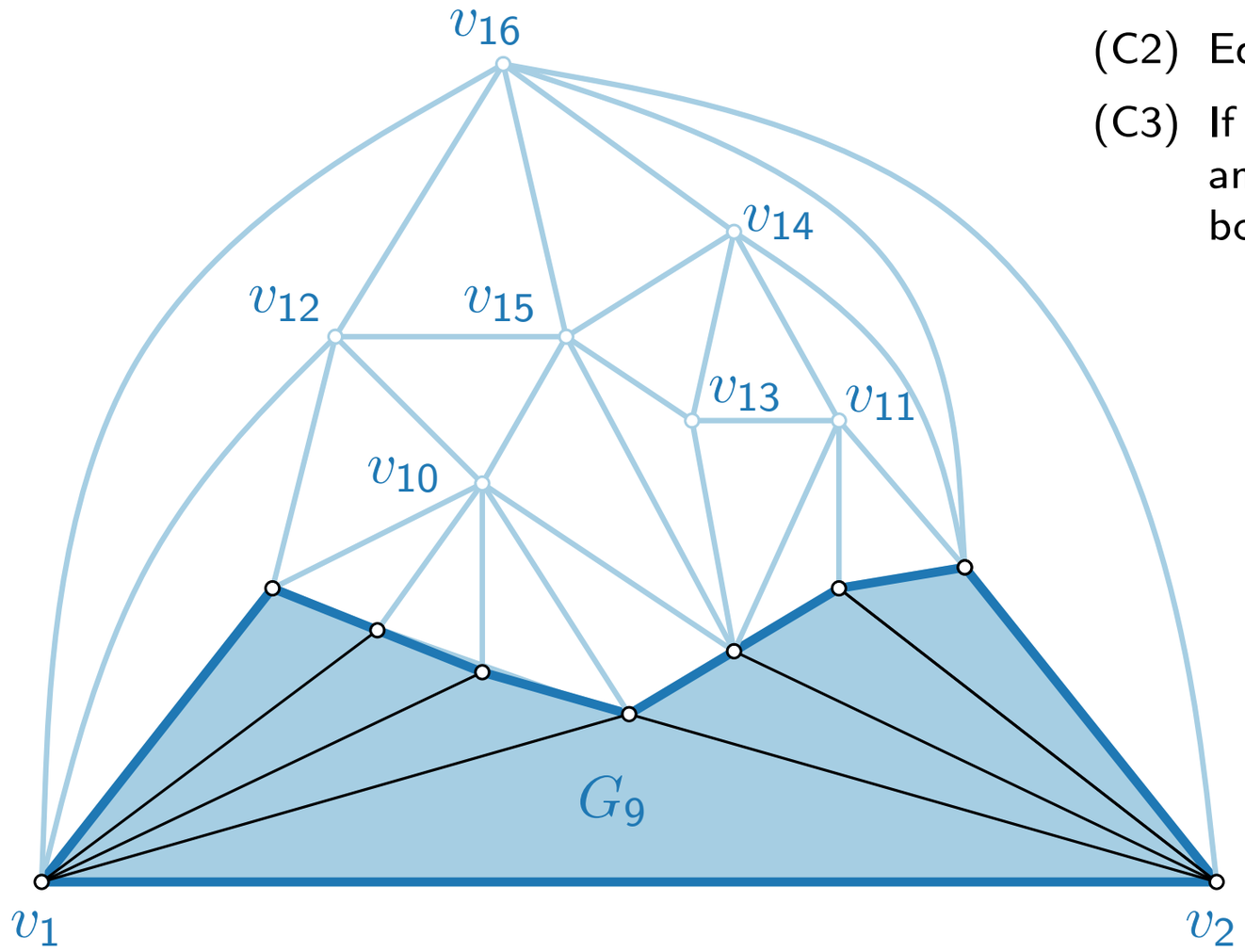

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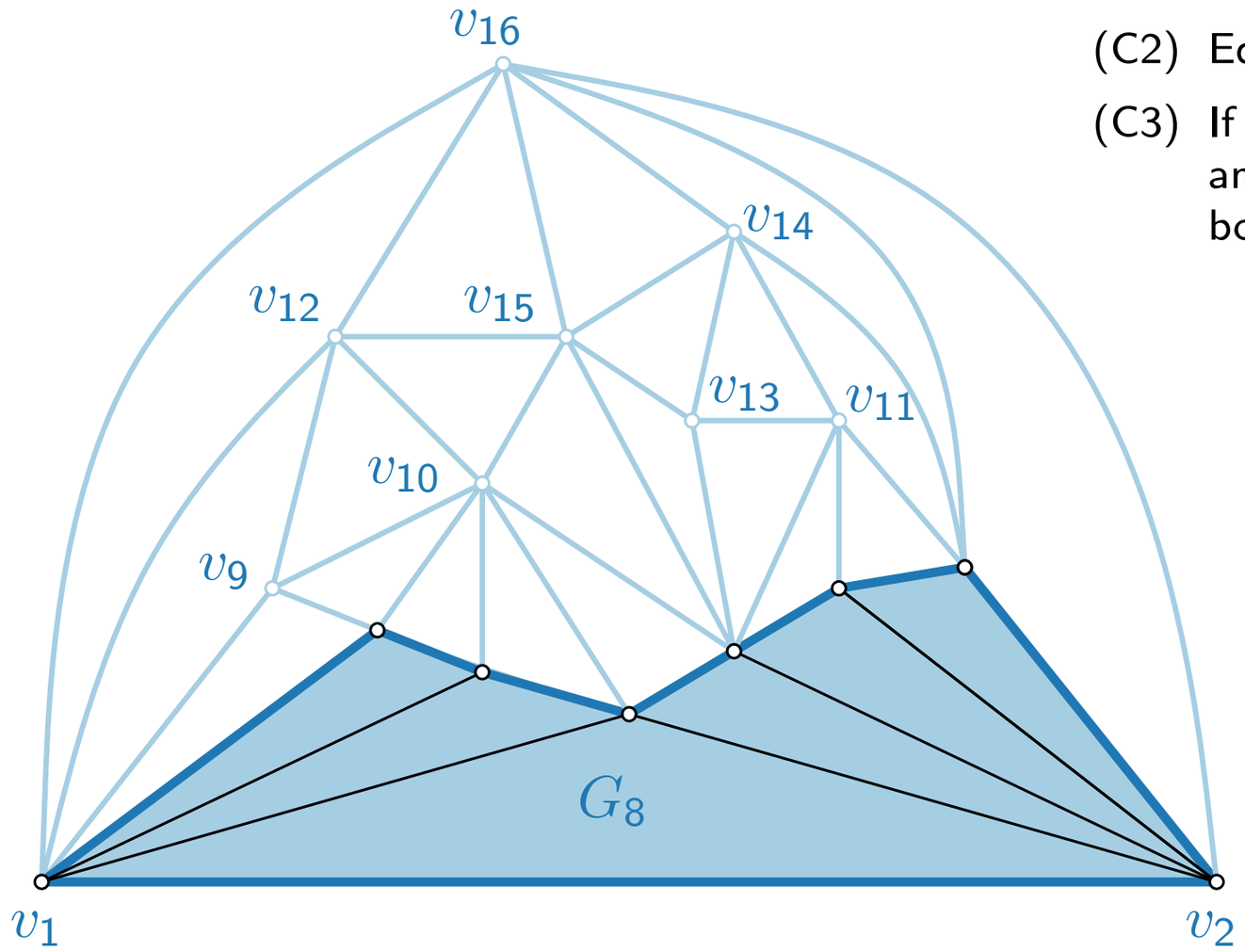

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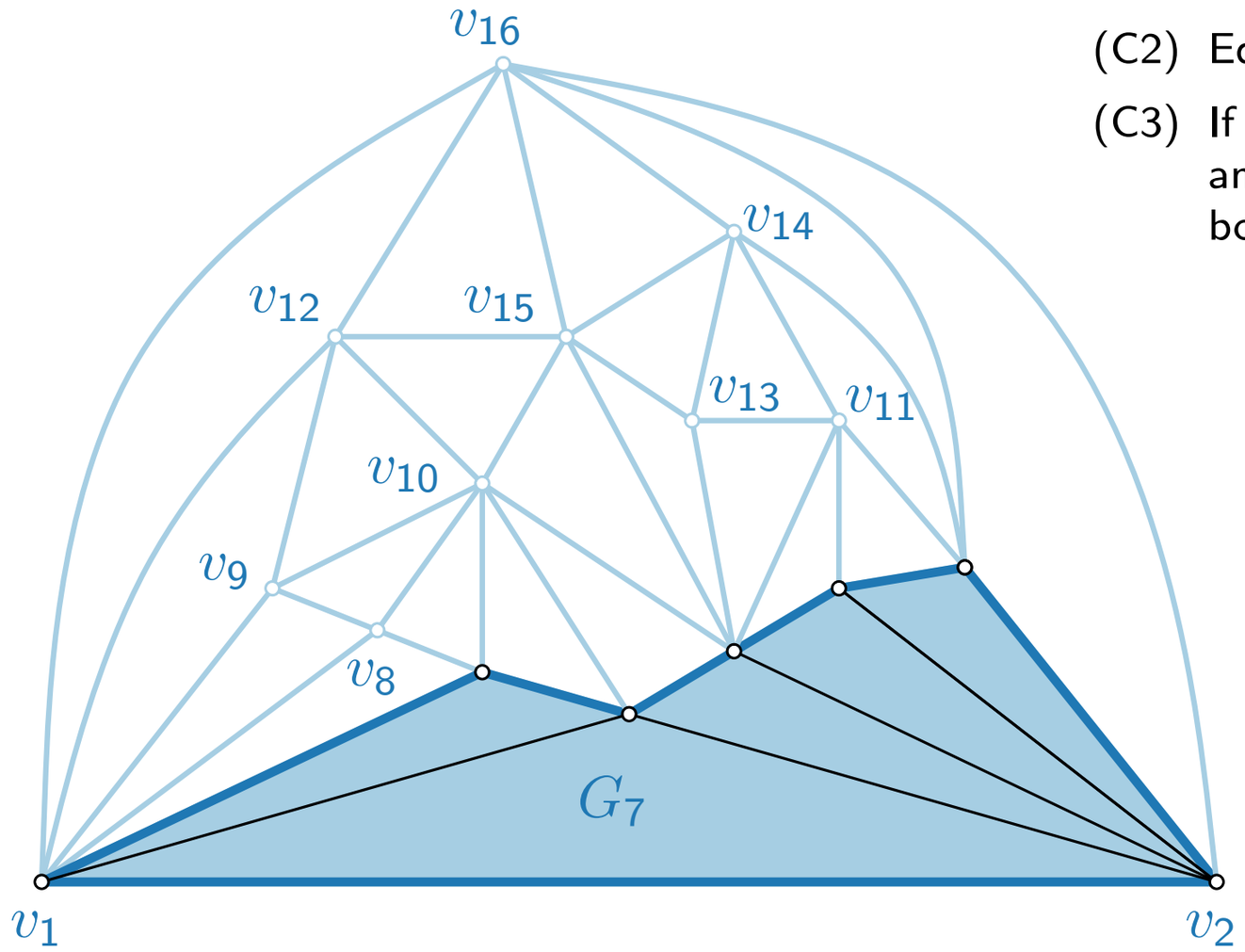

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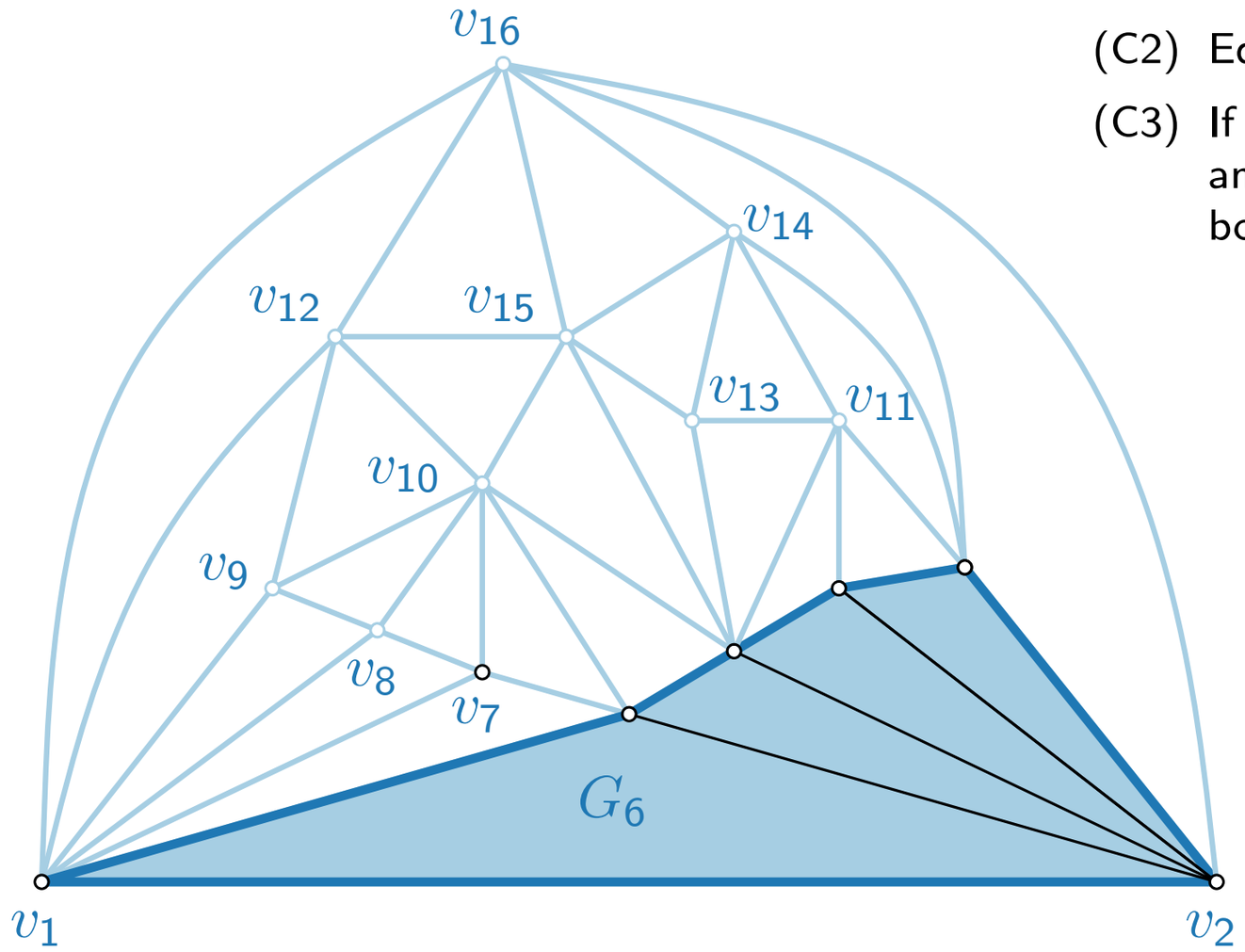

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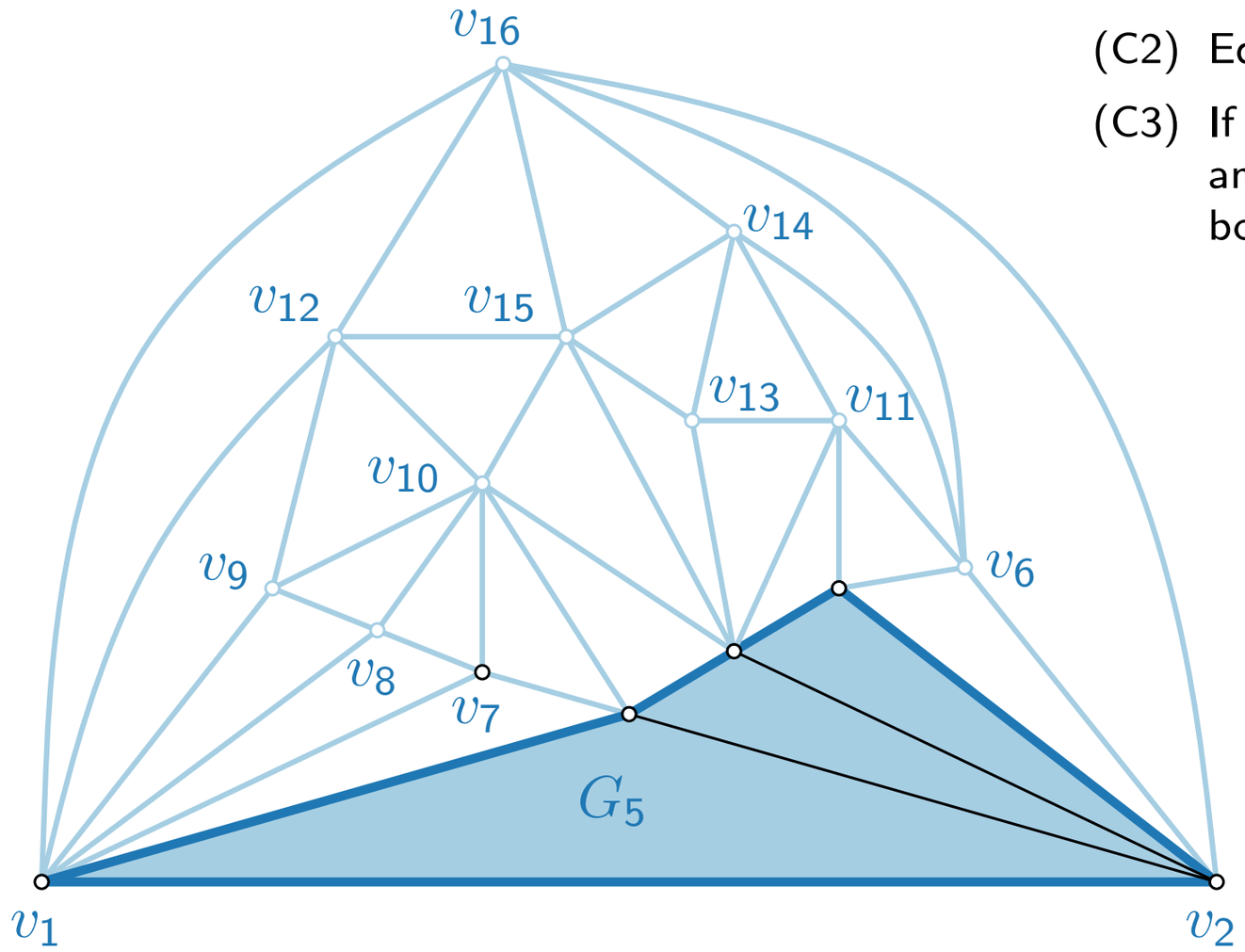

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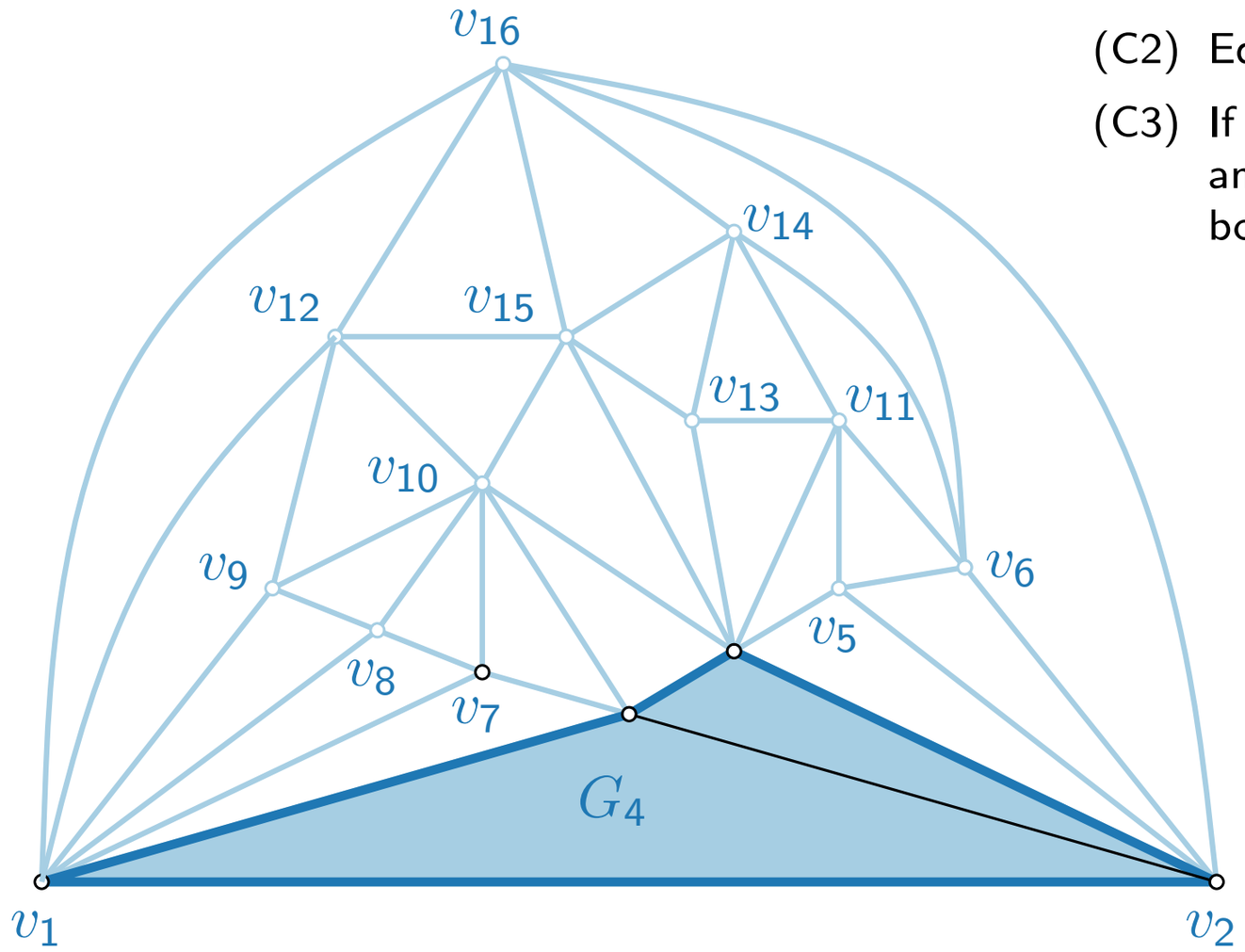

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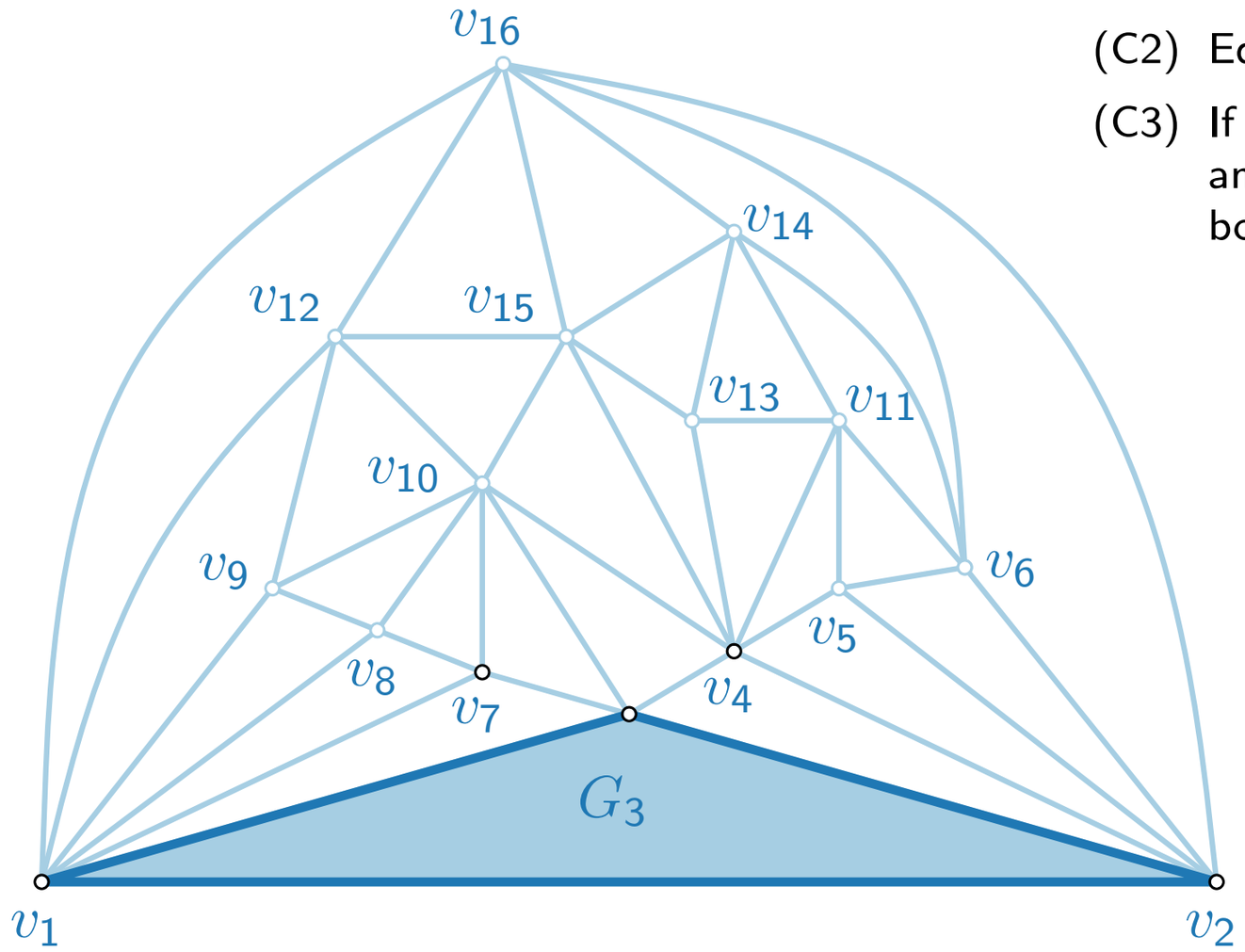

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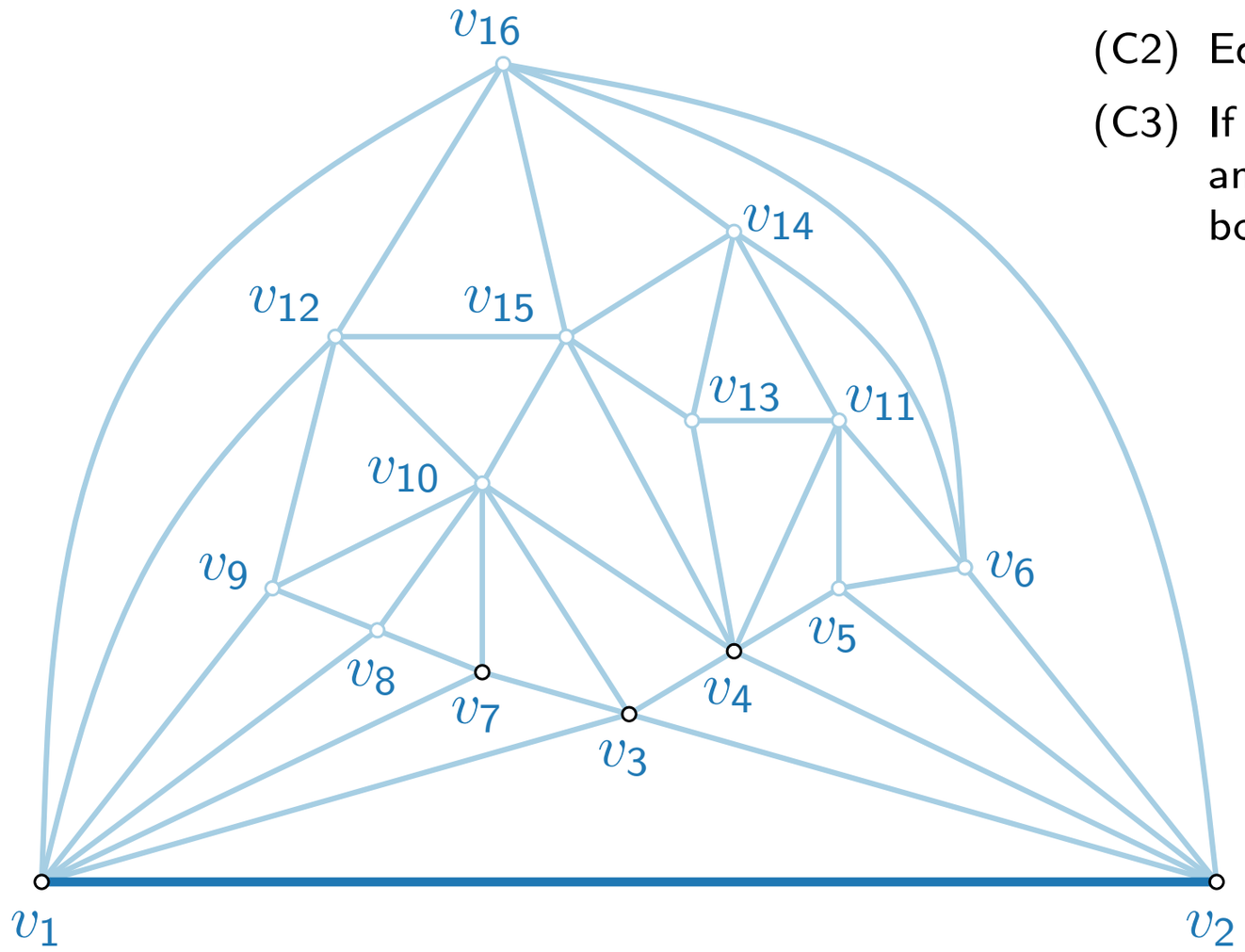

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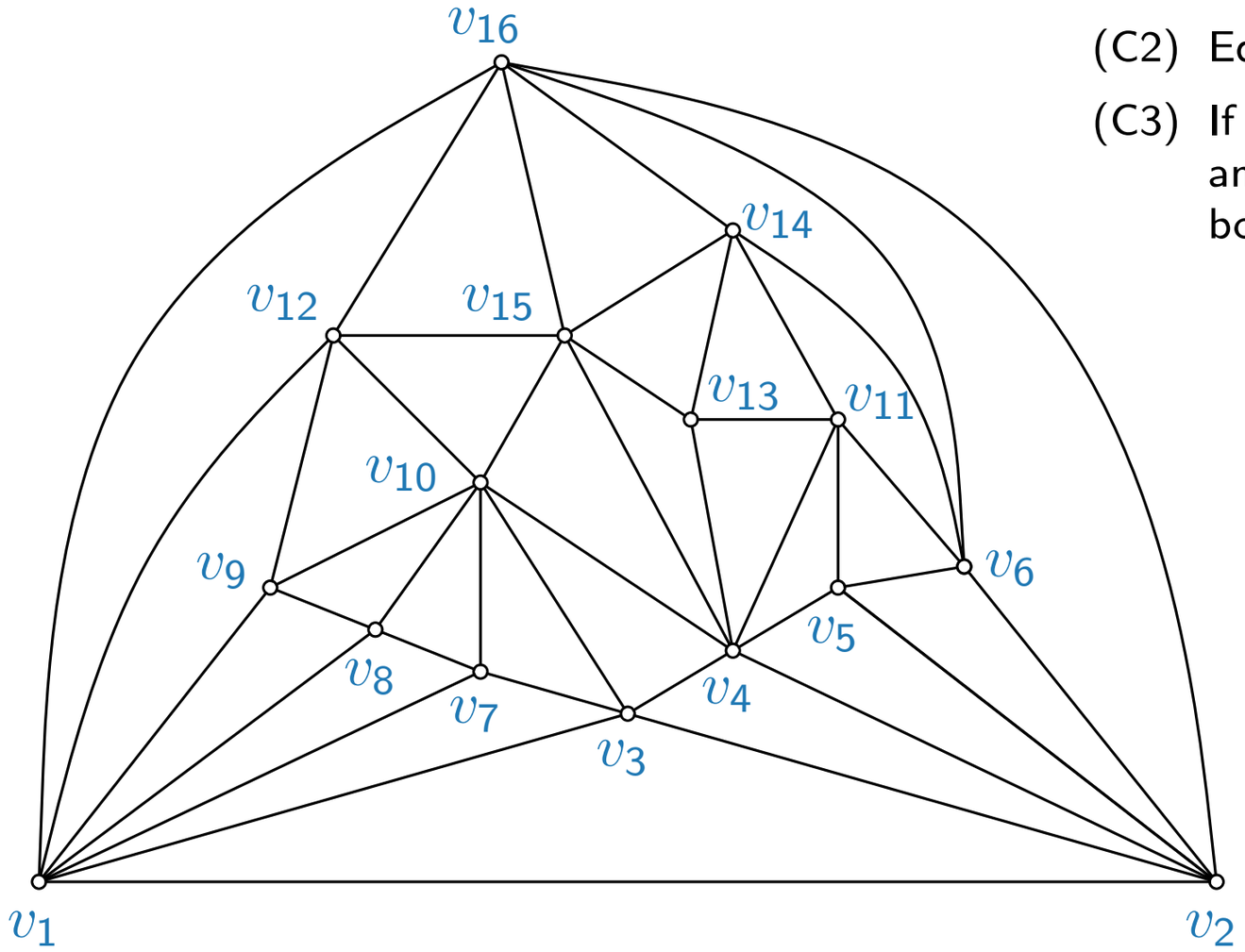

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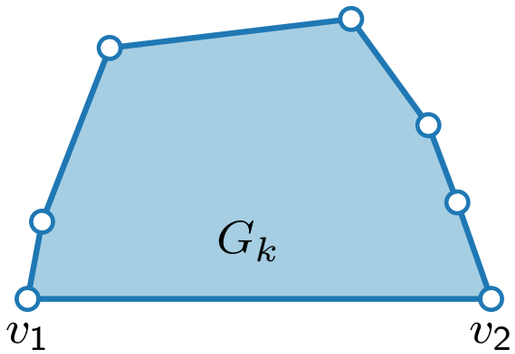

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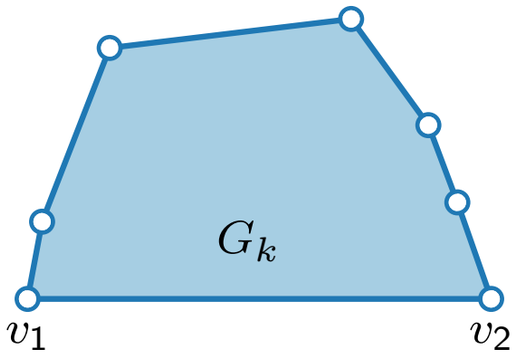

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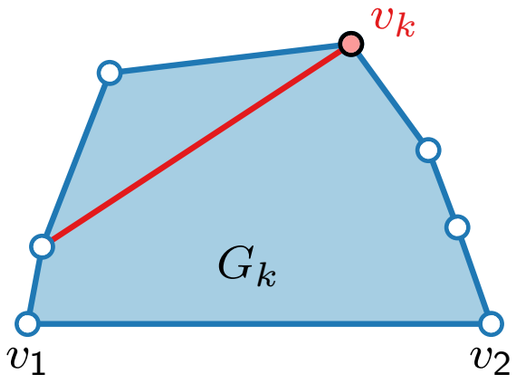

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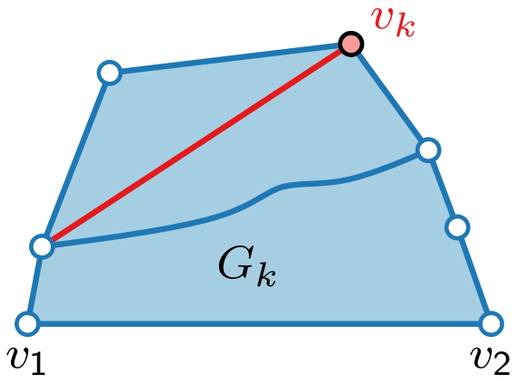

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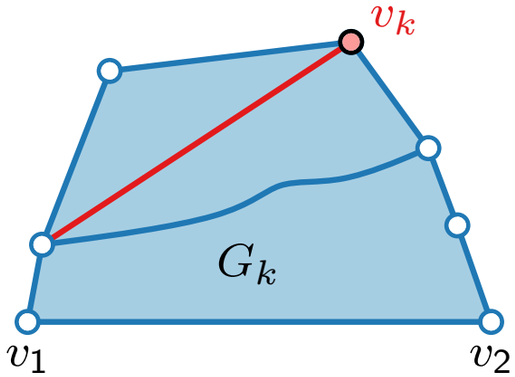

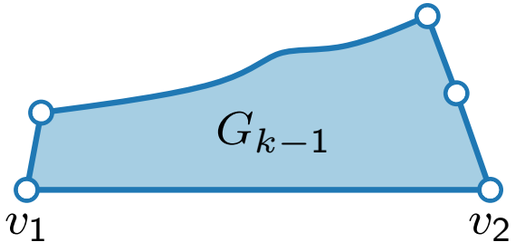

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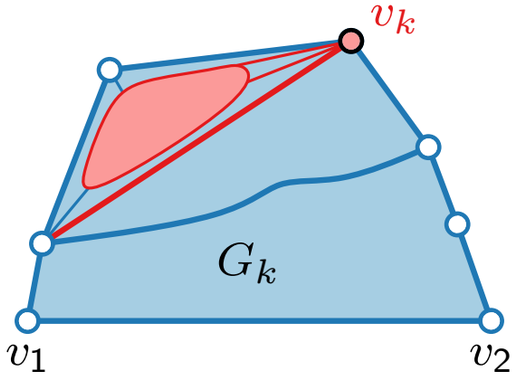

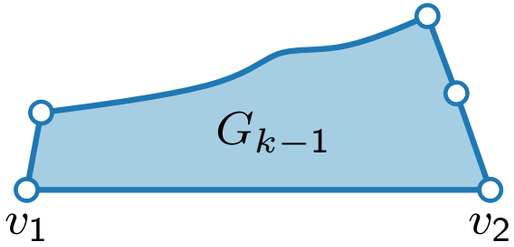

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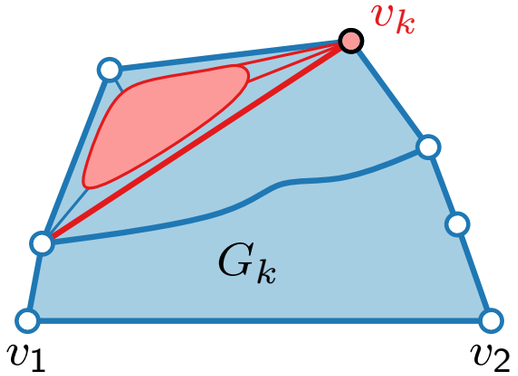

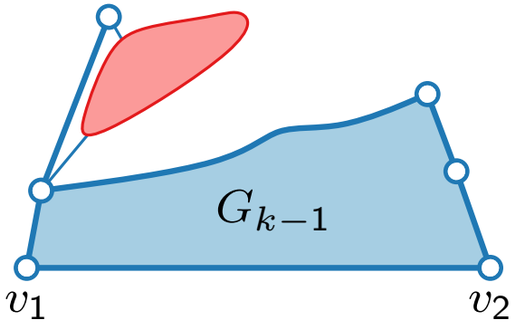

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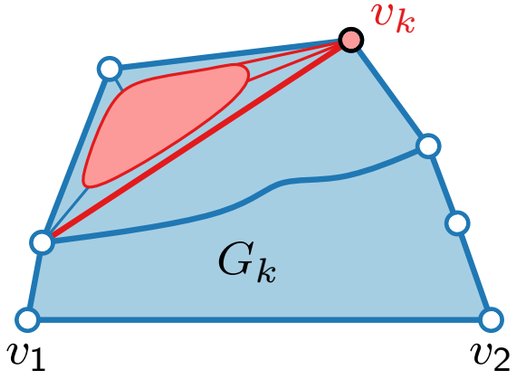

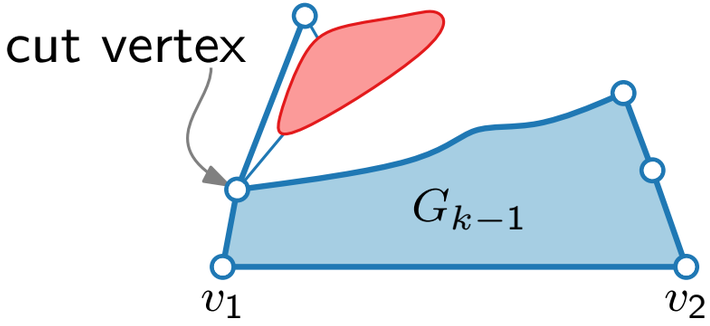

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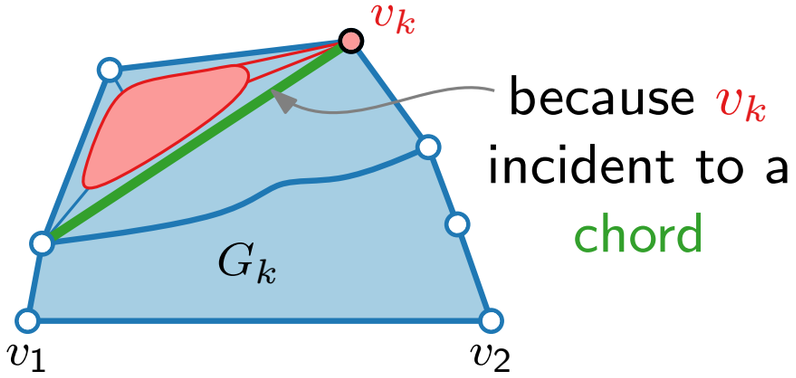

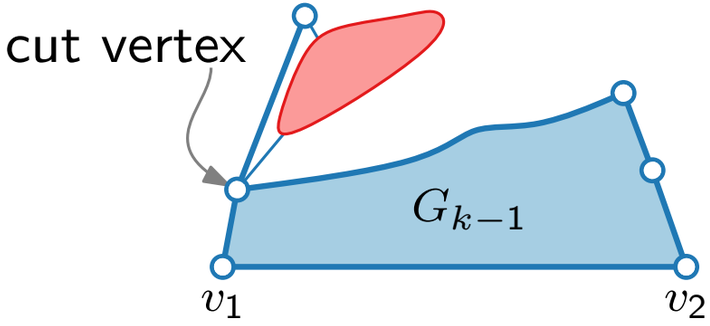

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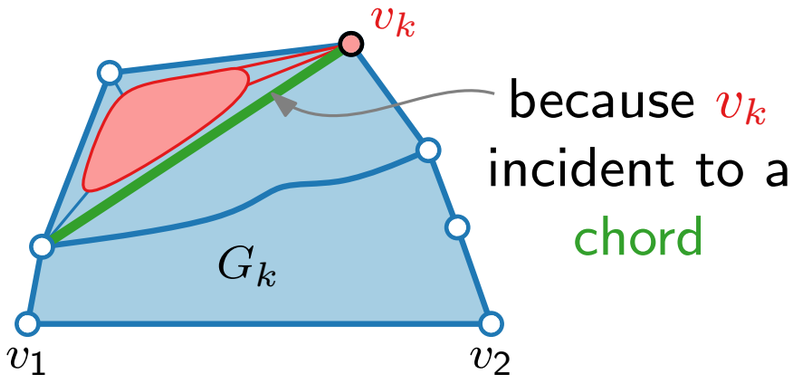

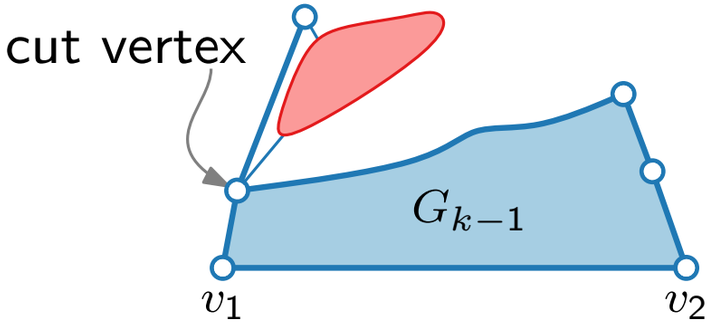

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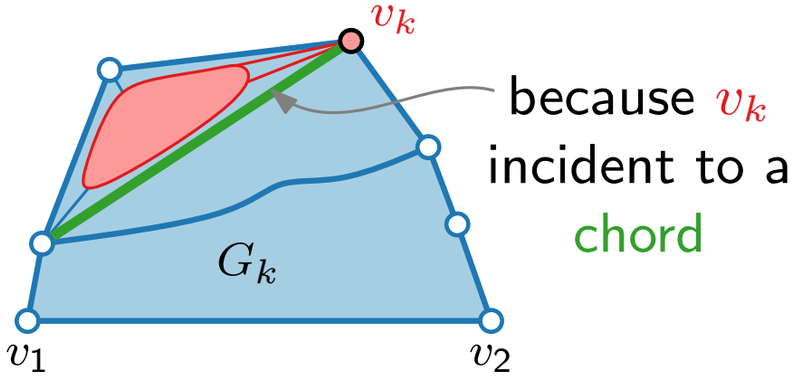

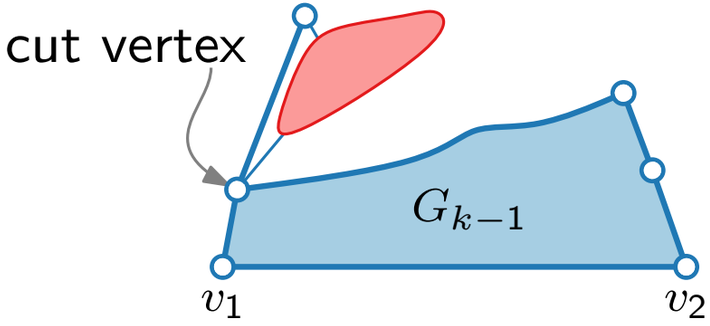

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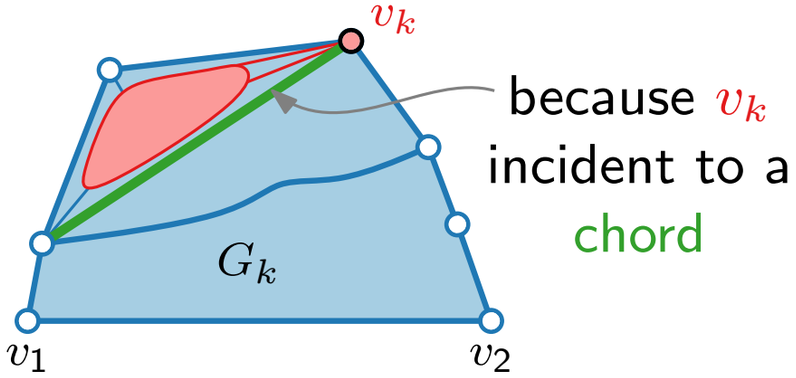

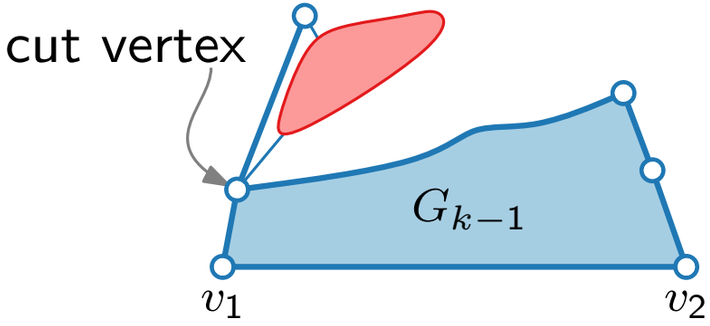

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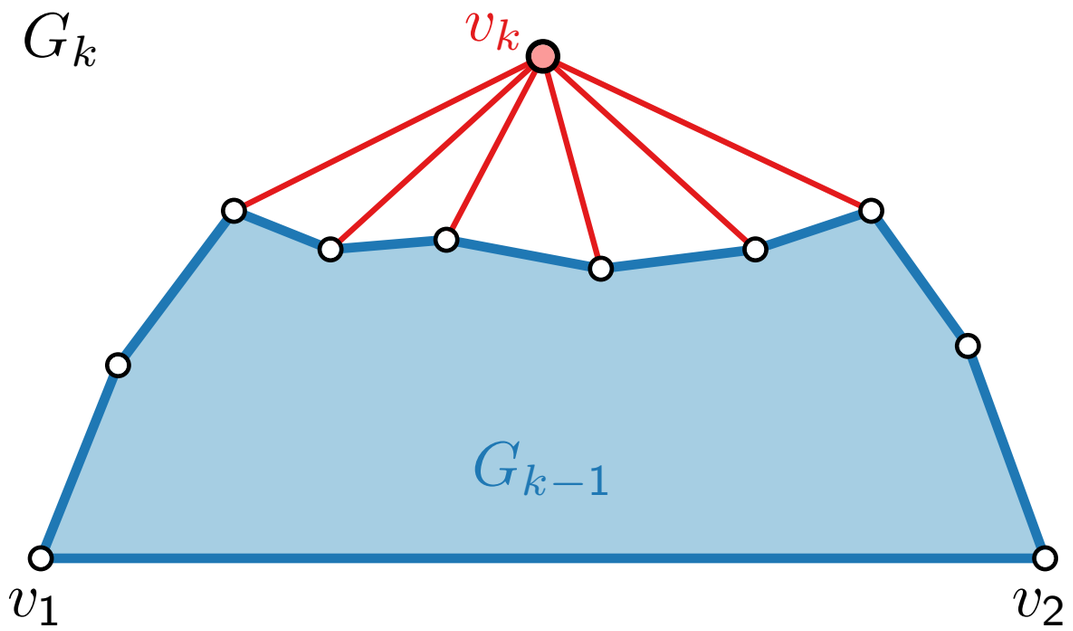

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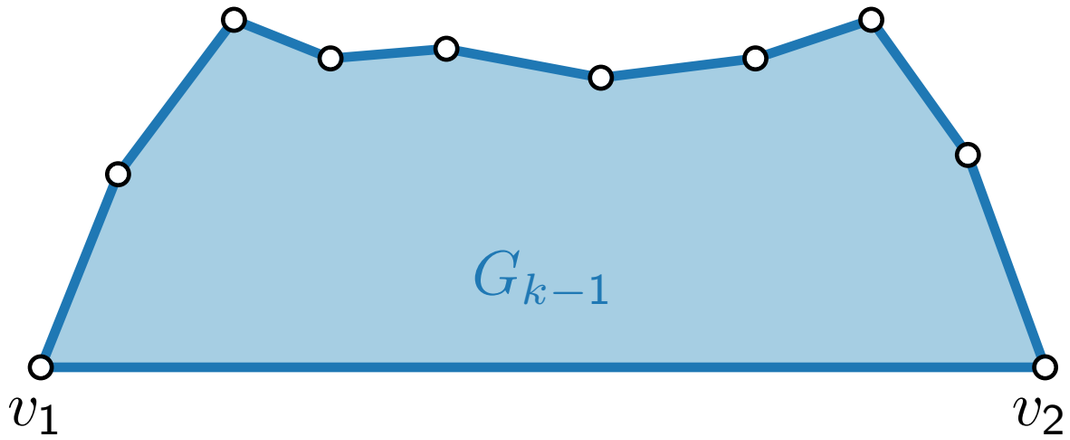

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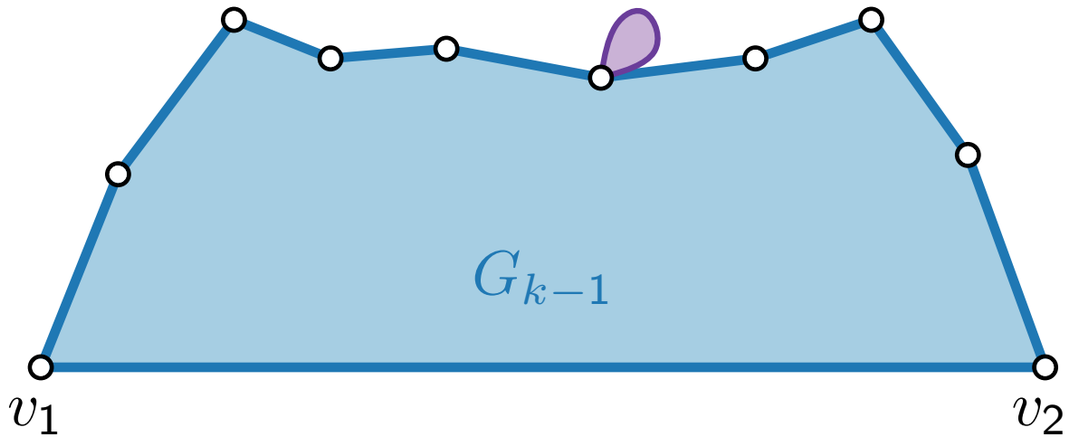

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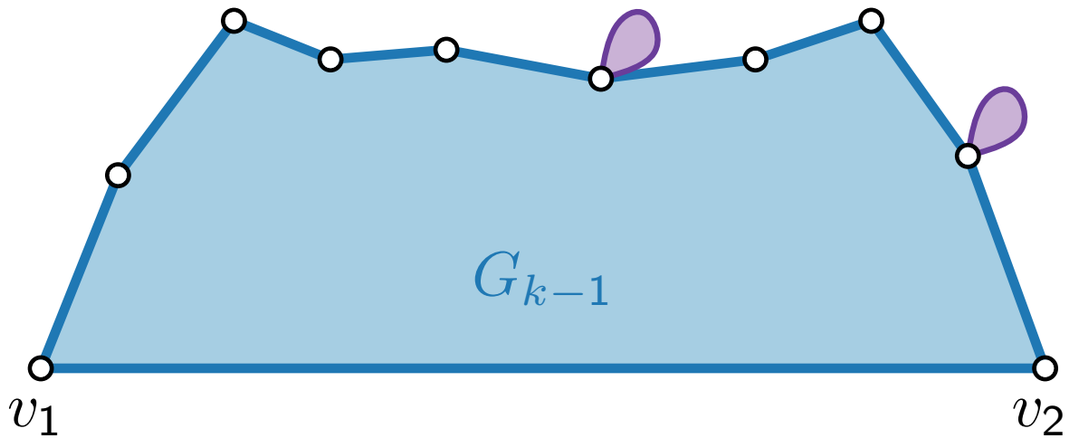

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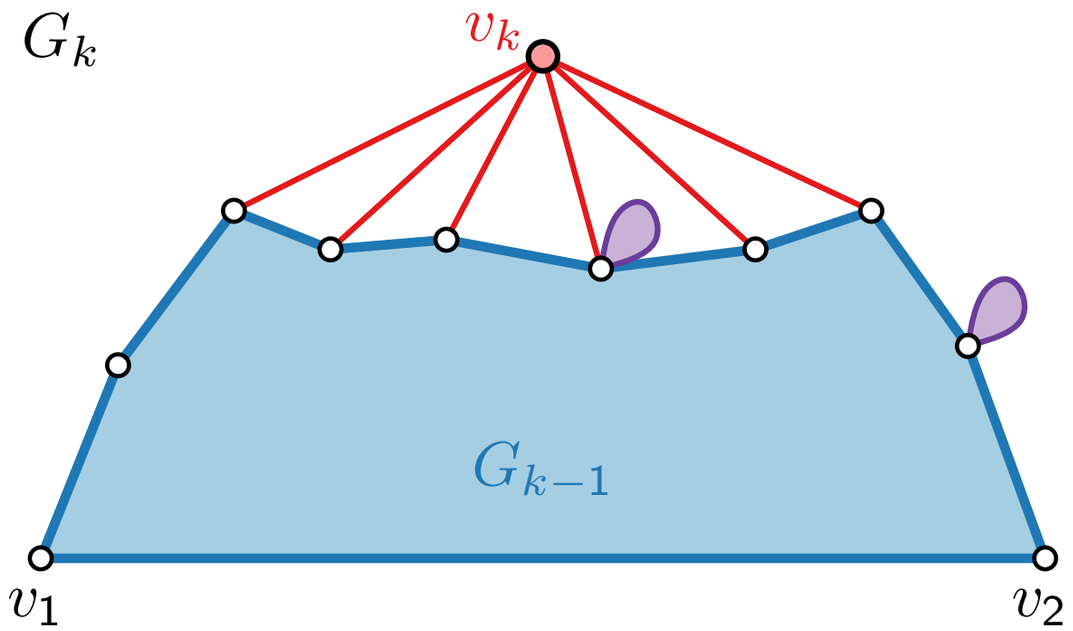

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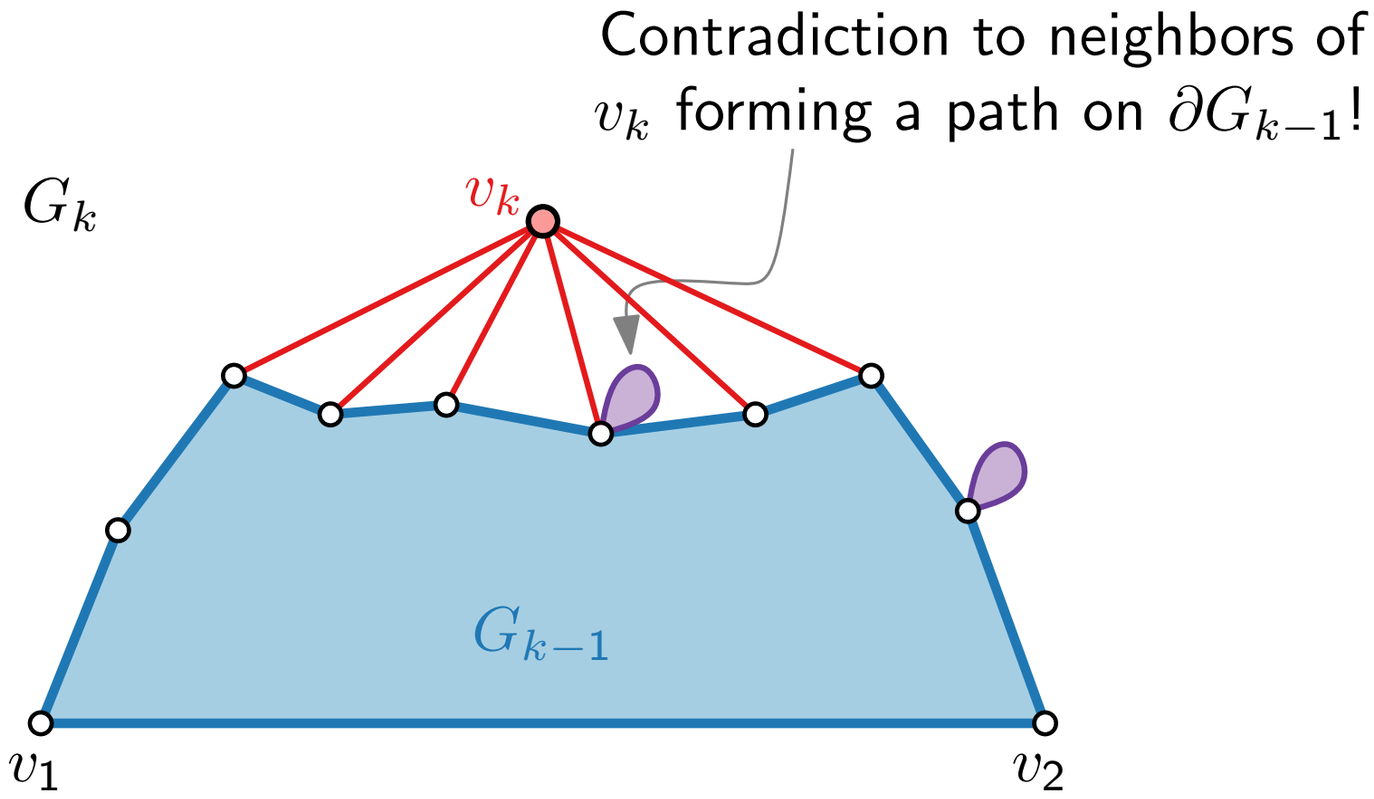

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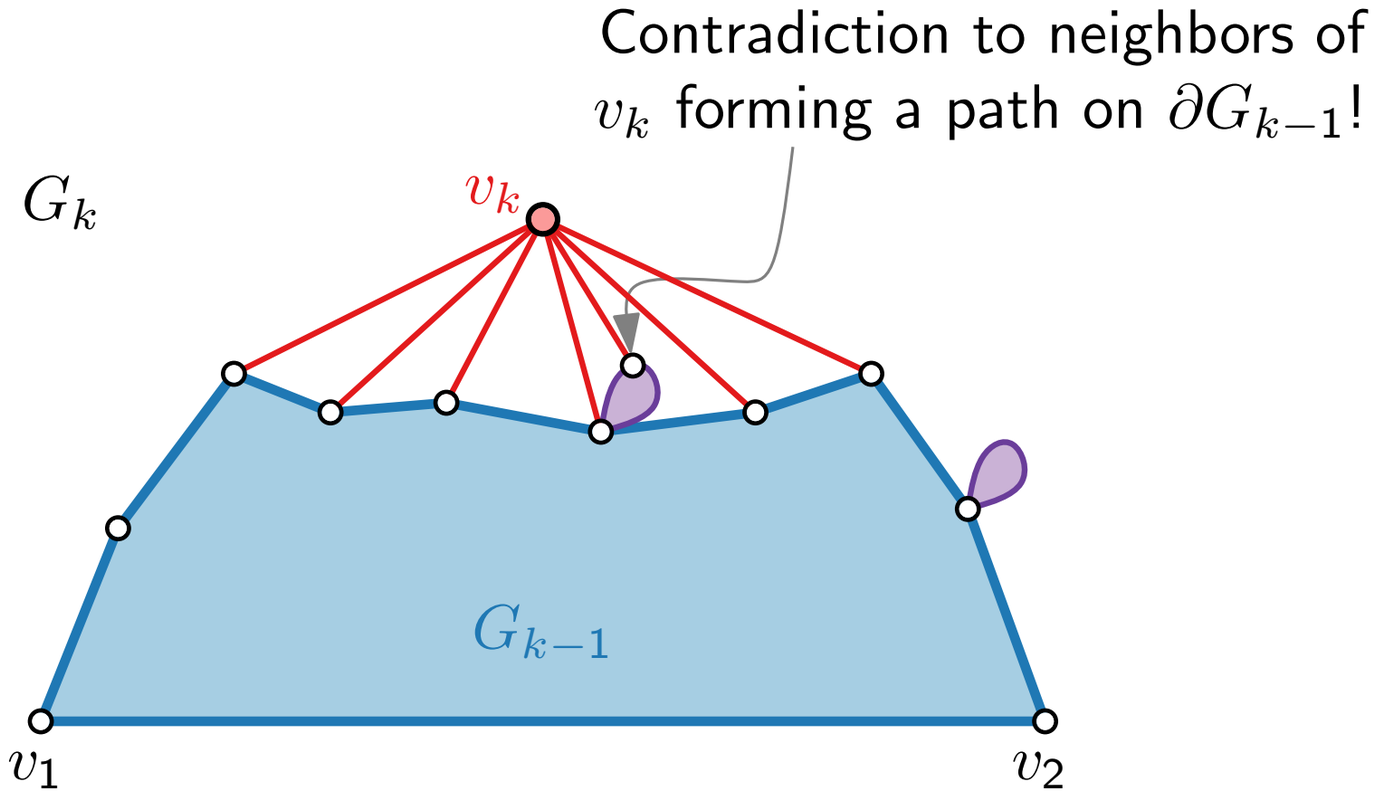

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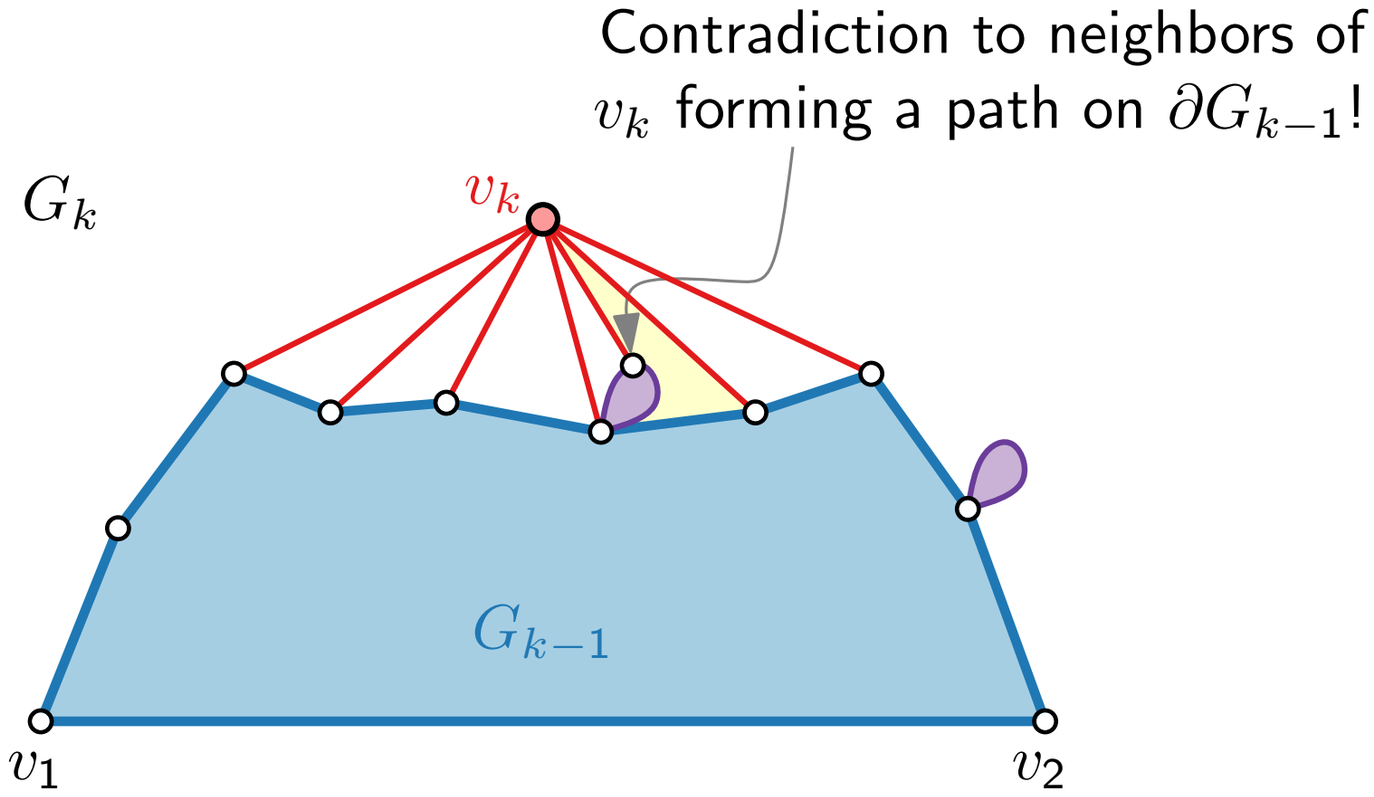

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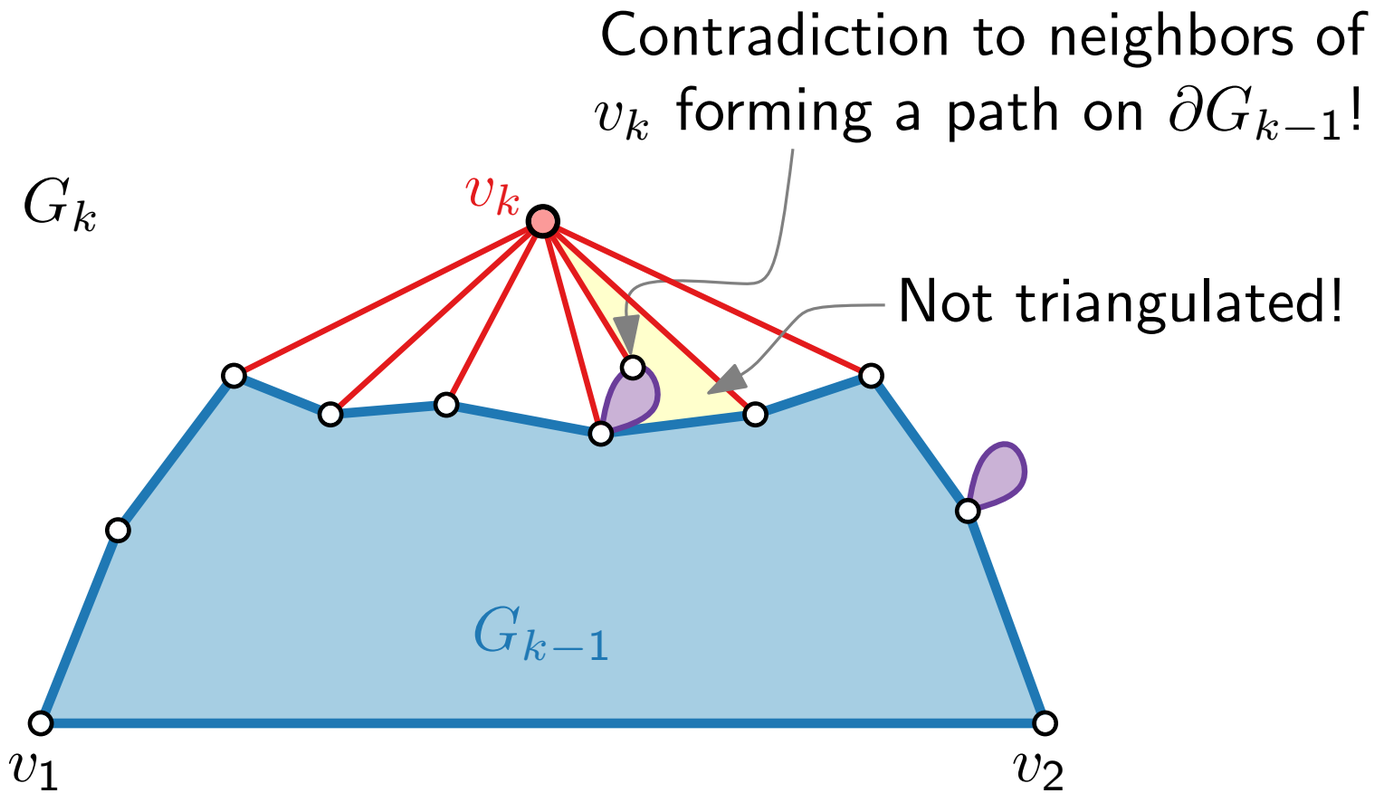

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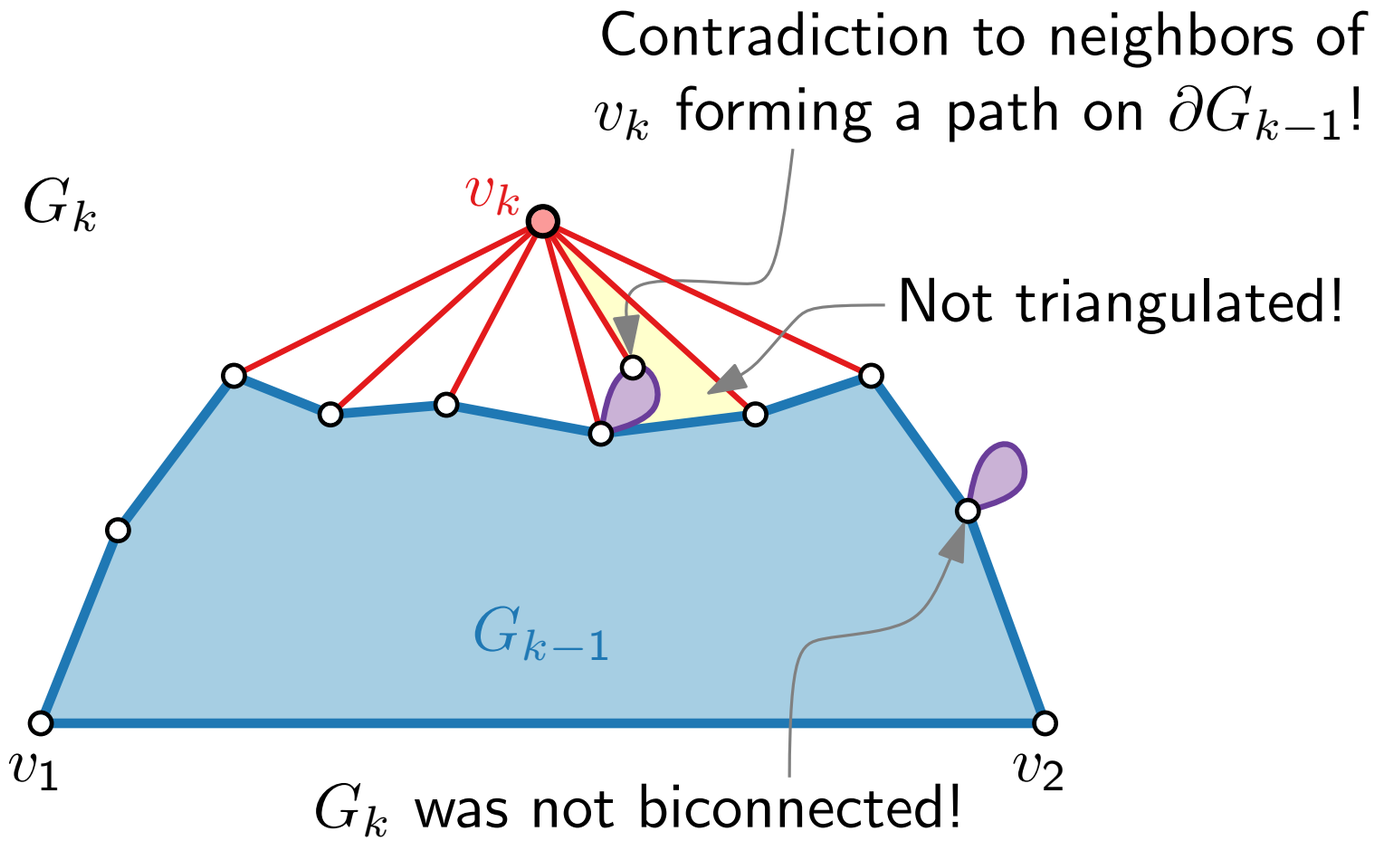

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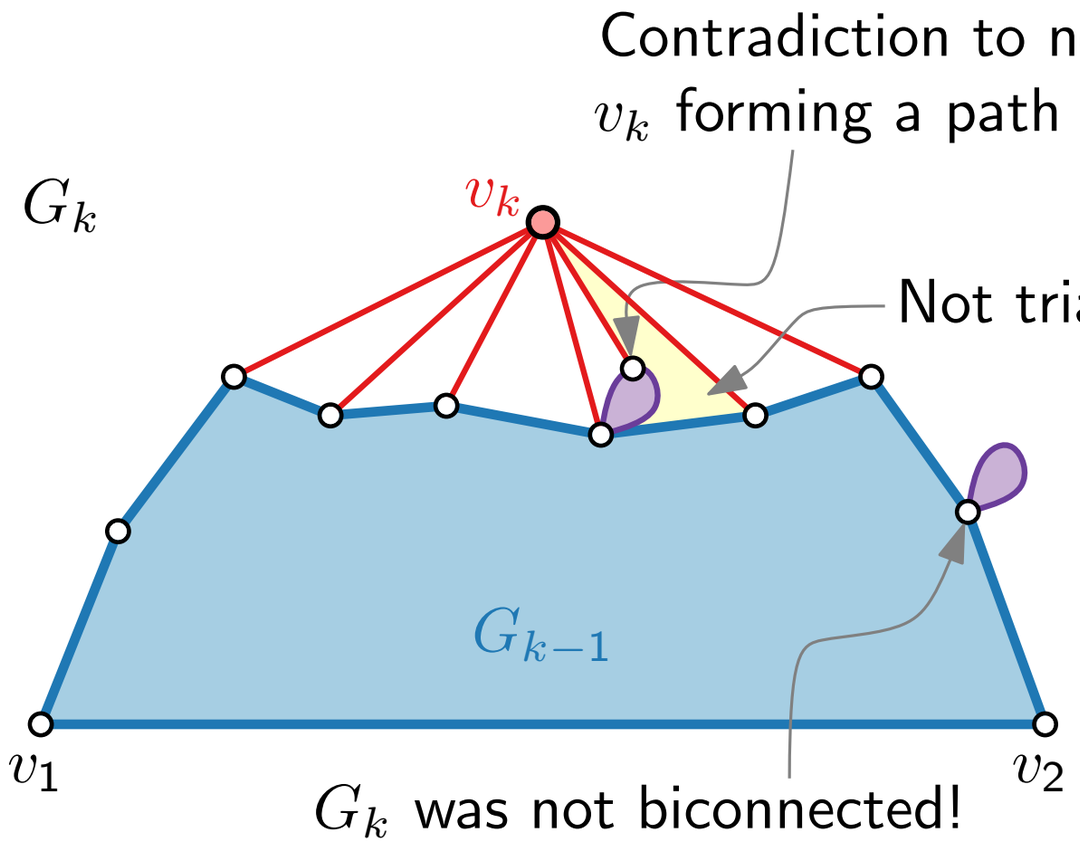

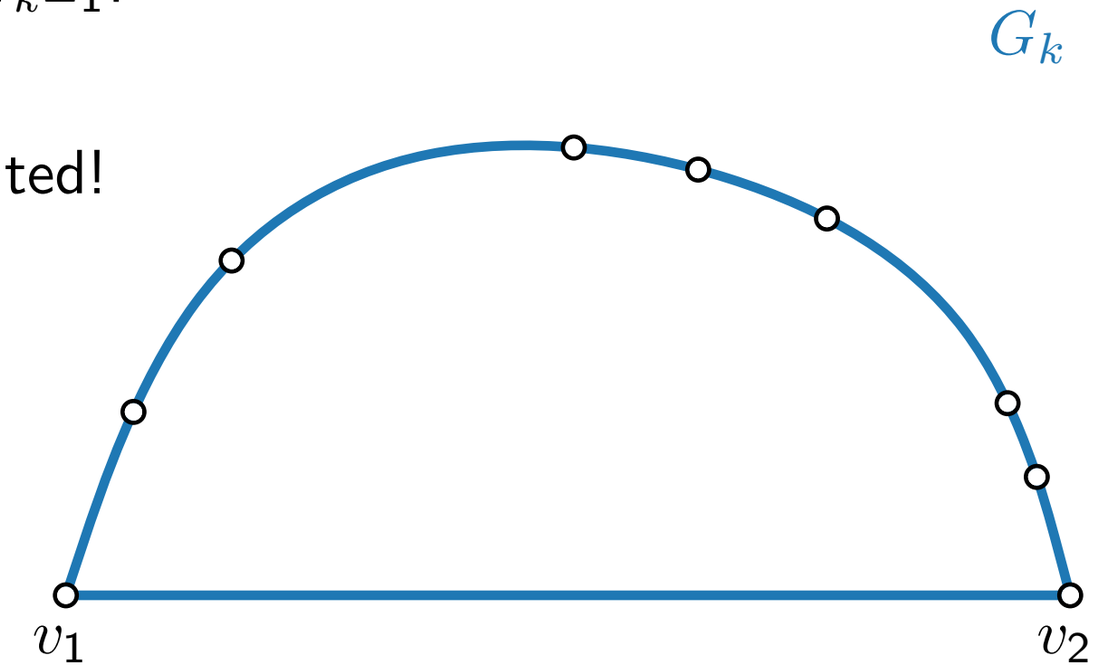

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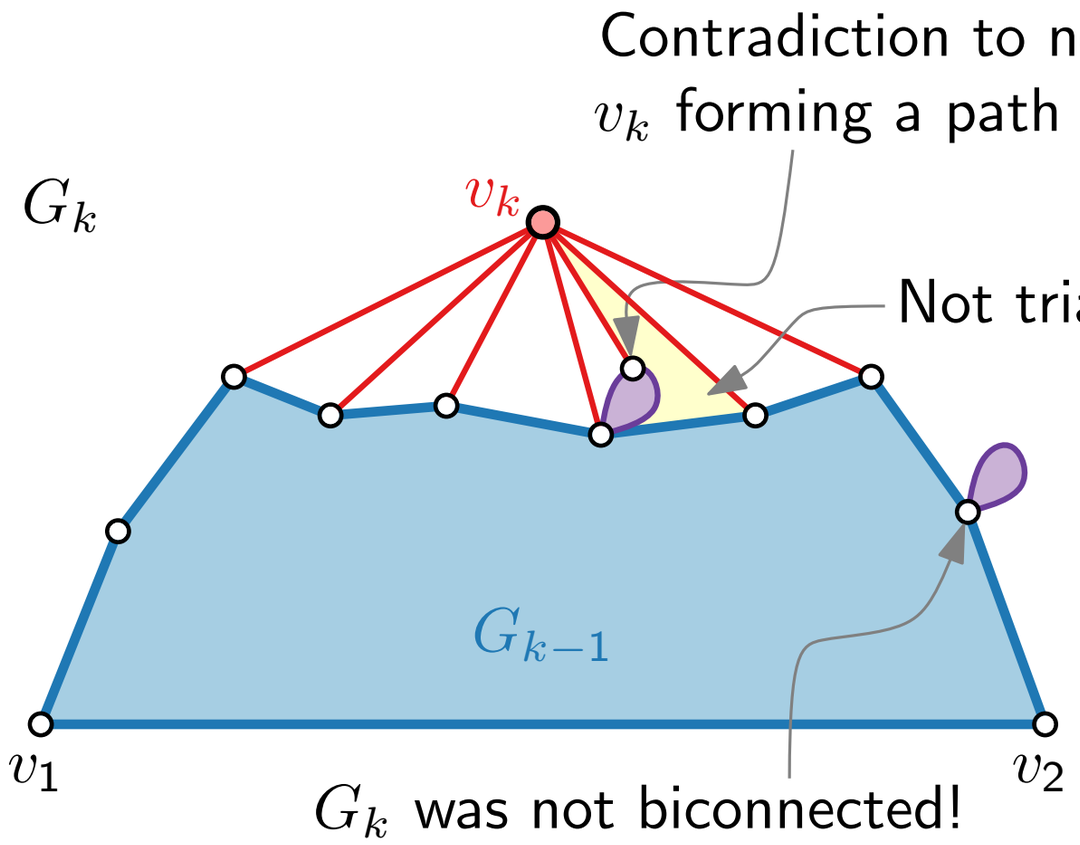

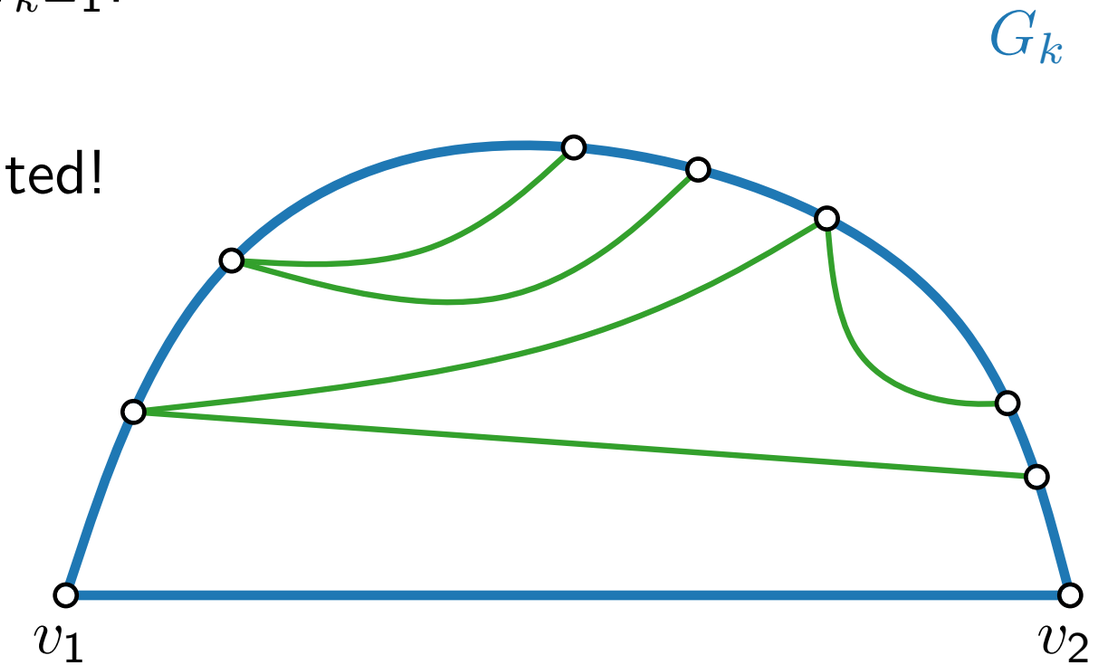

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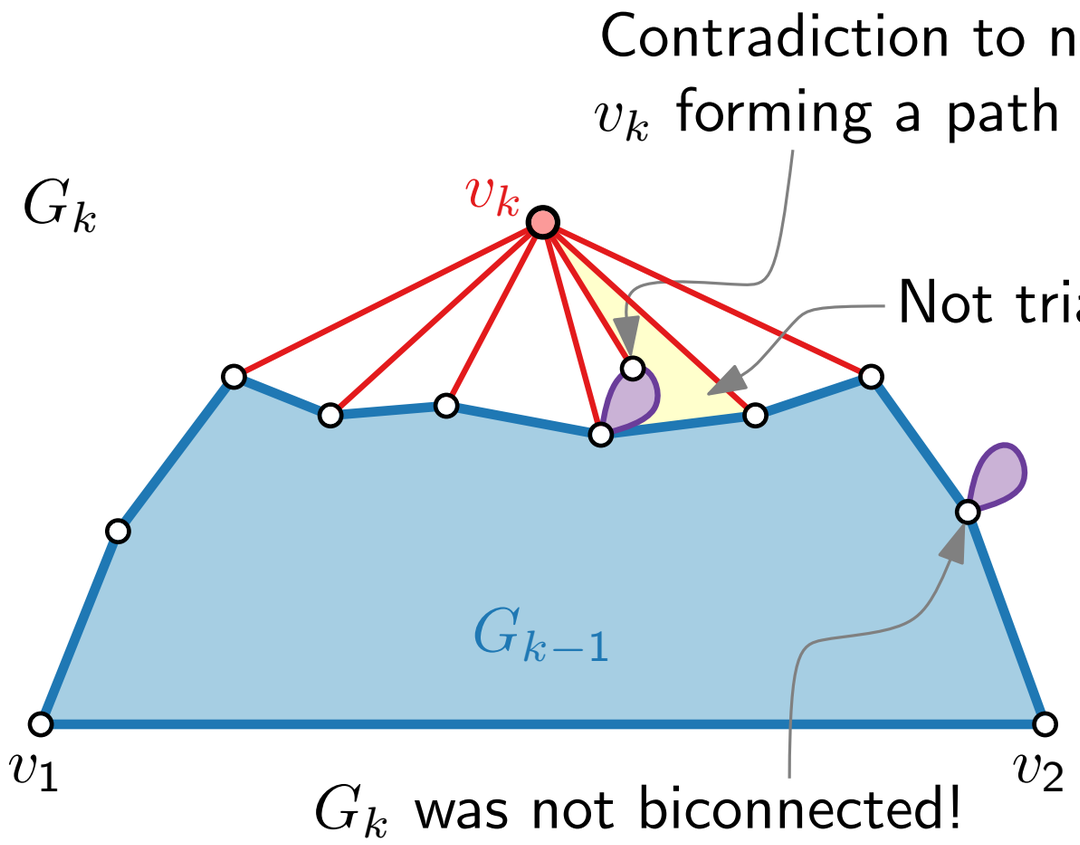

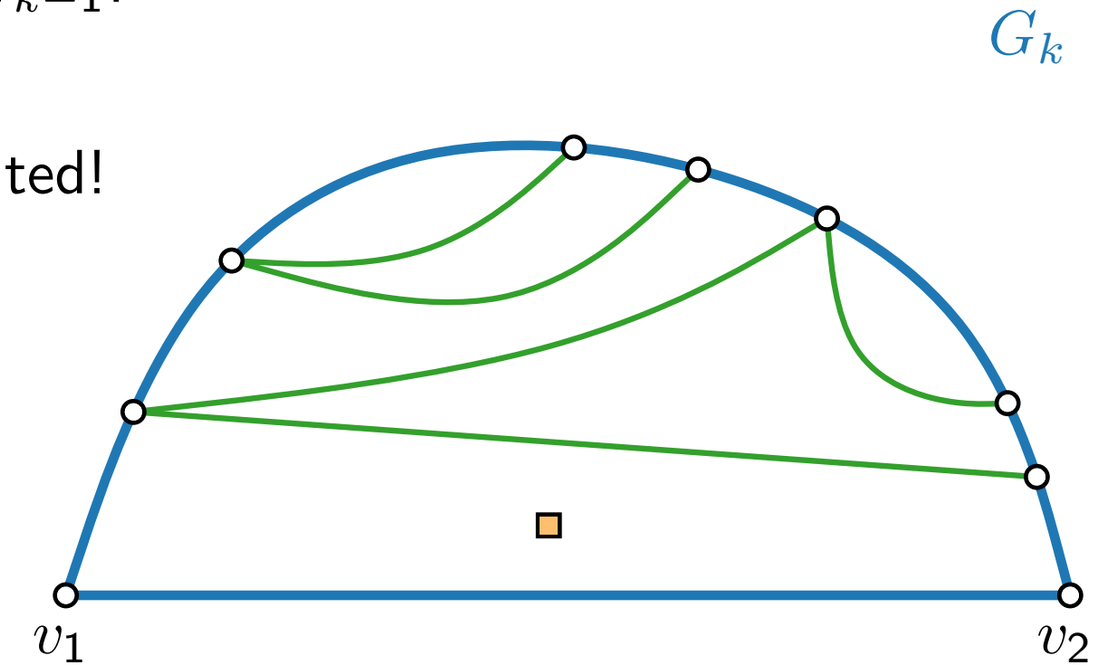

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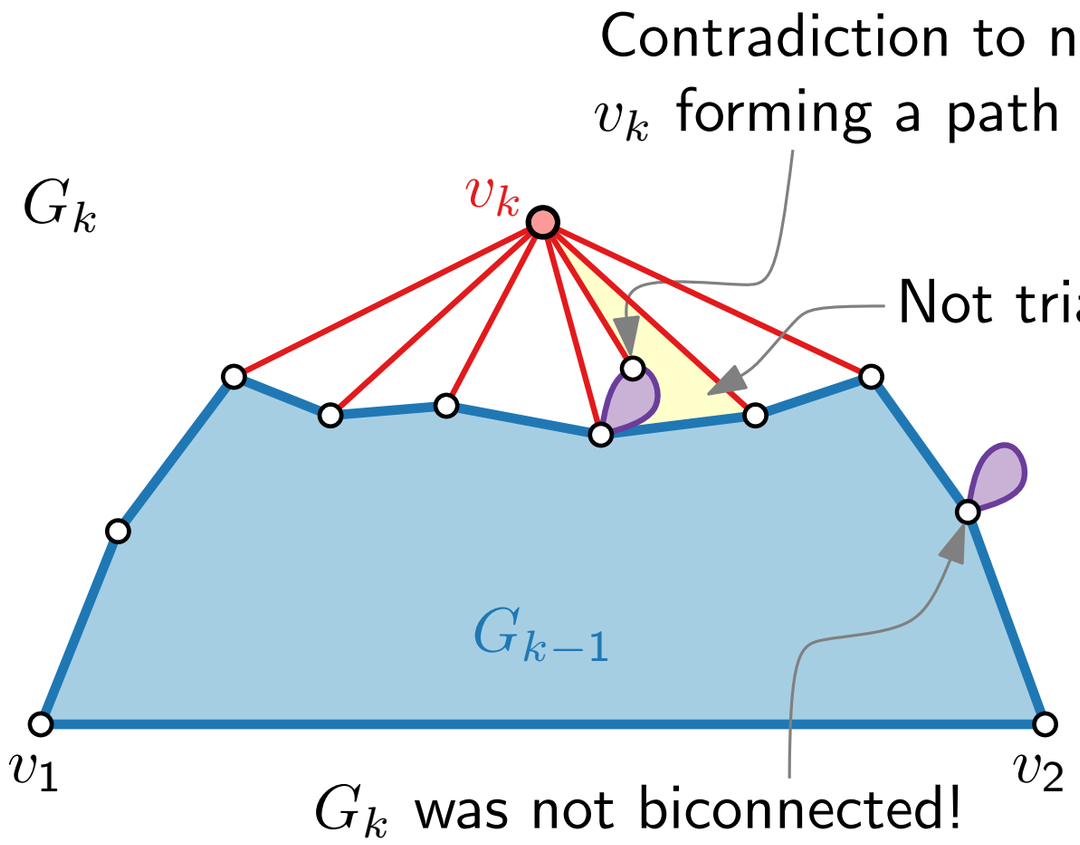

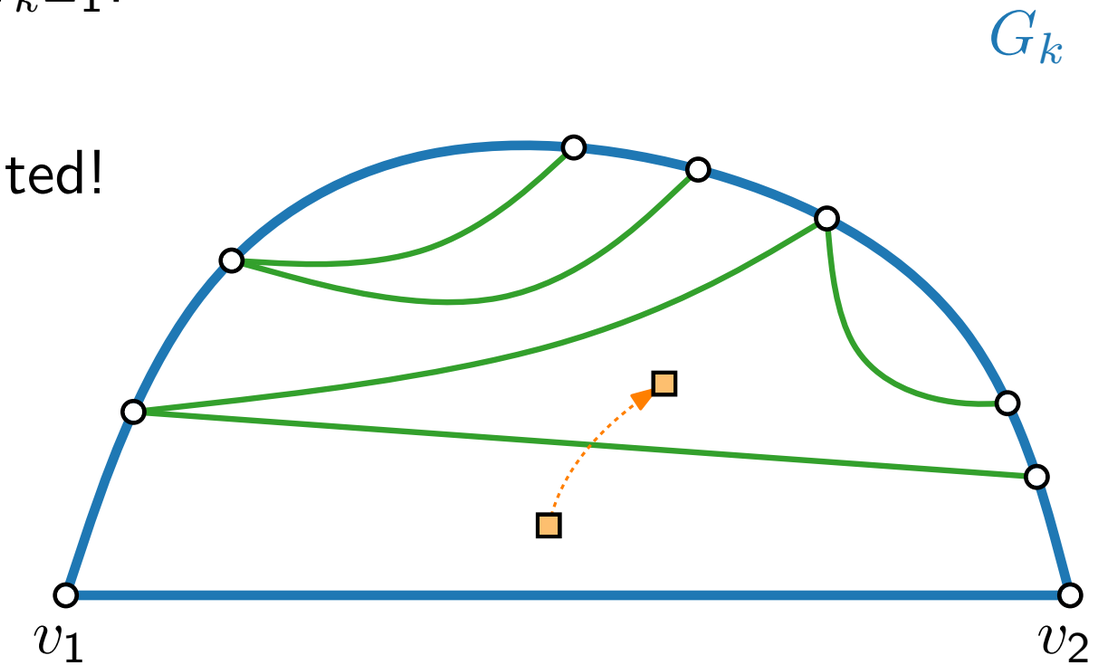

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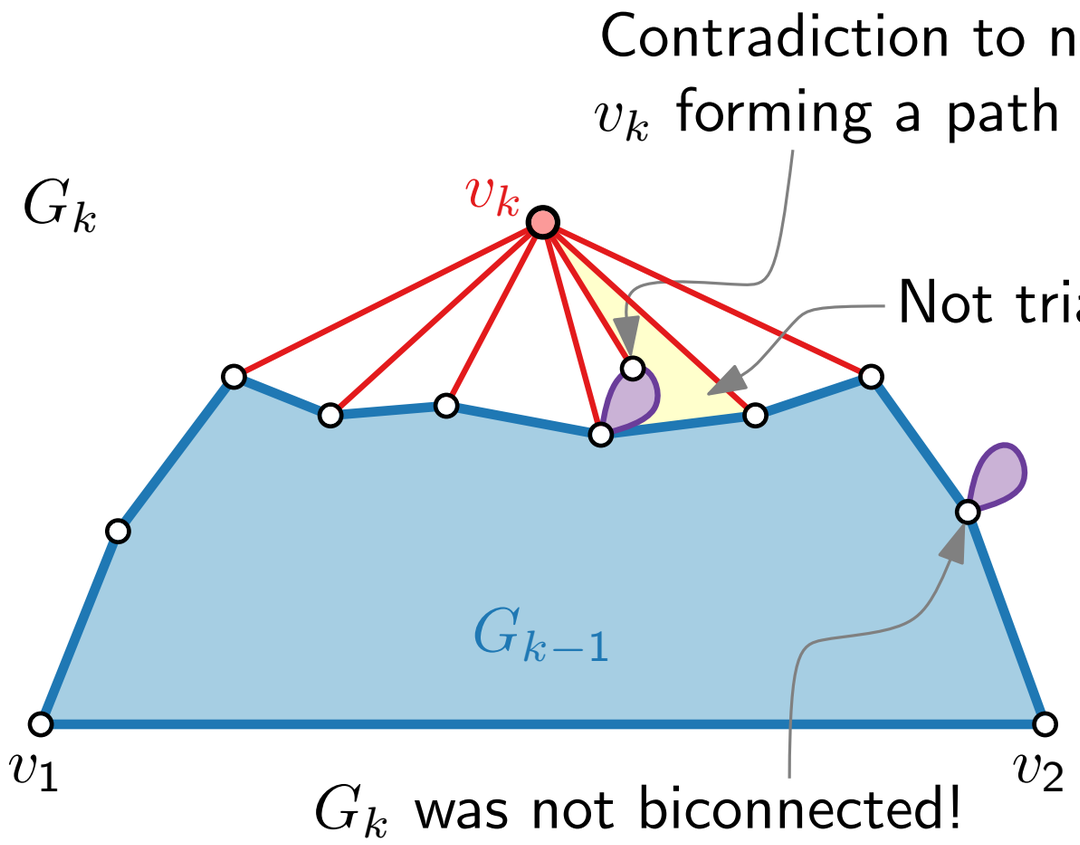

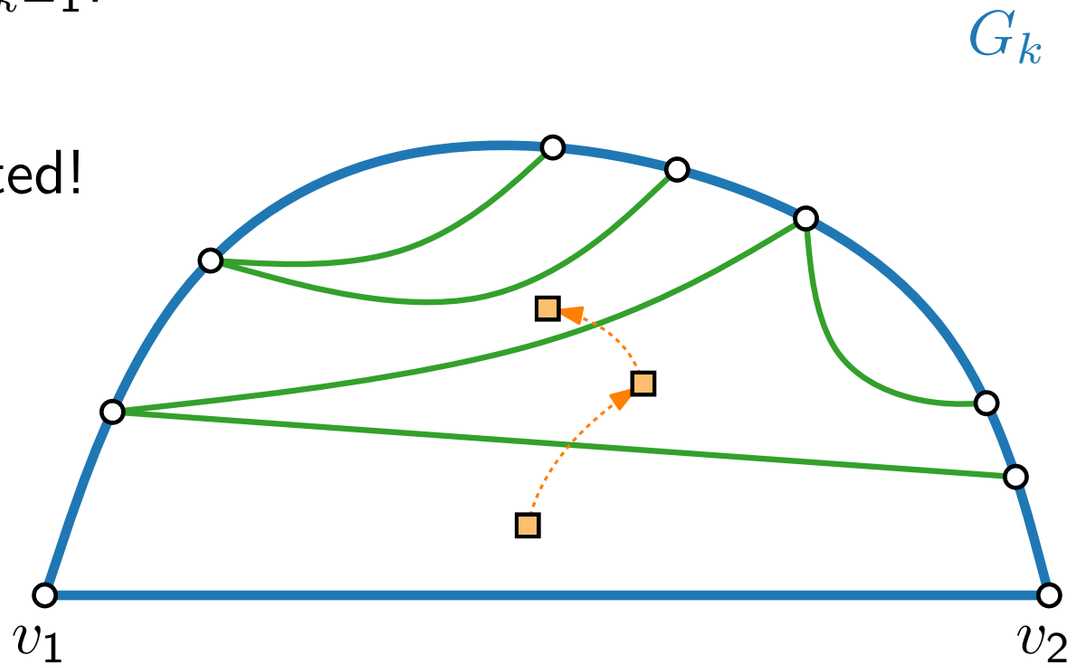

Canonical Order – Existence

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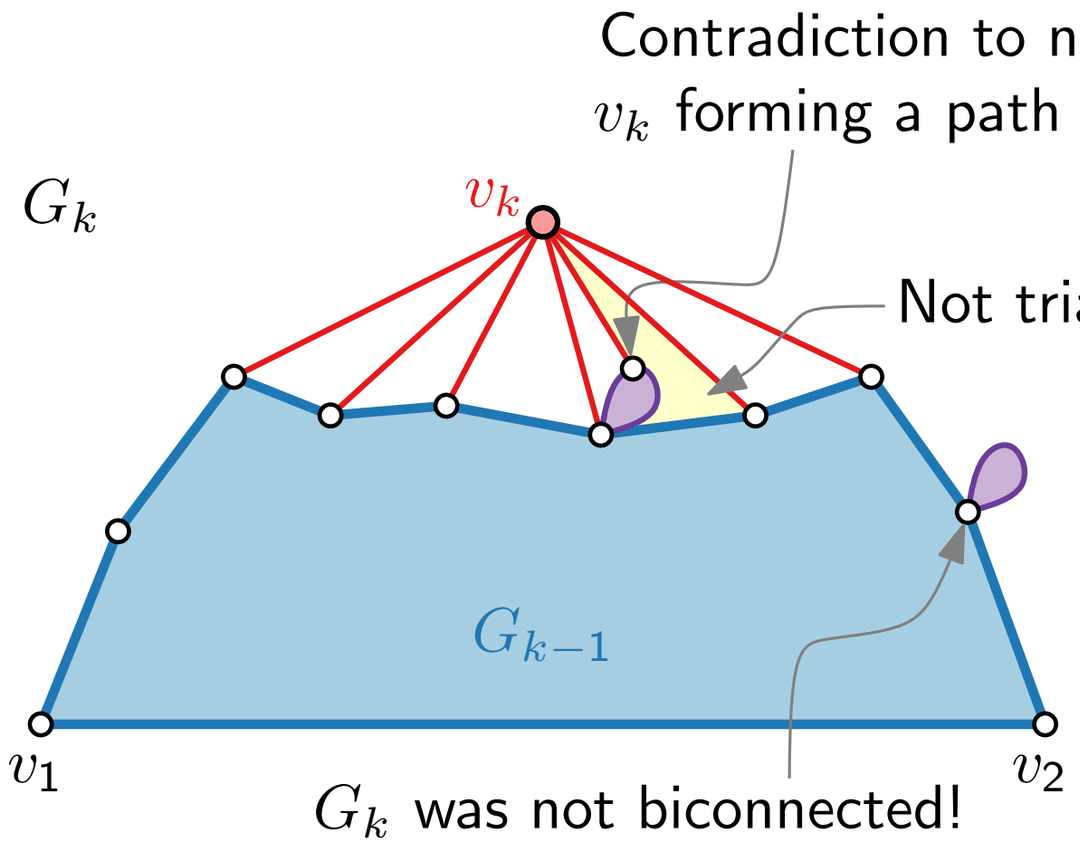

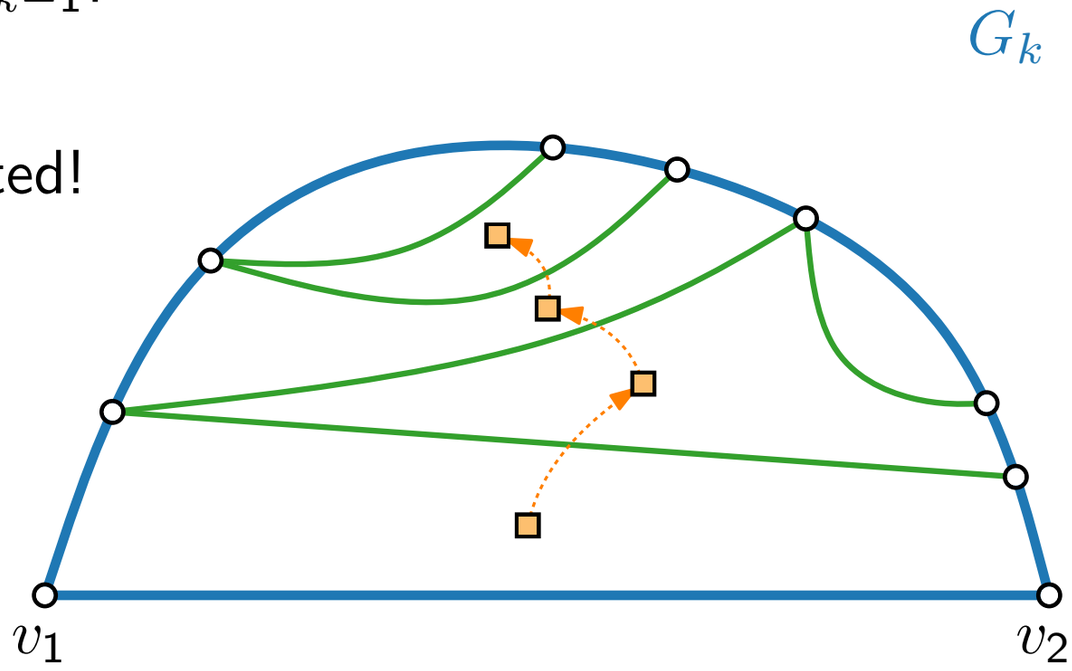

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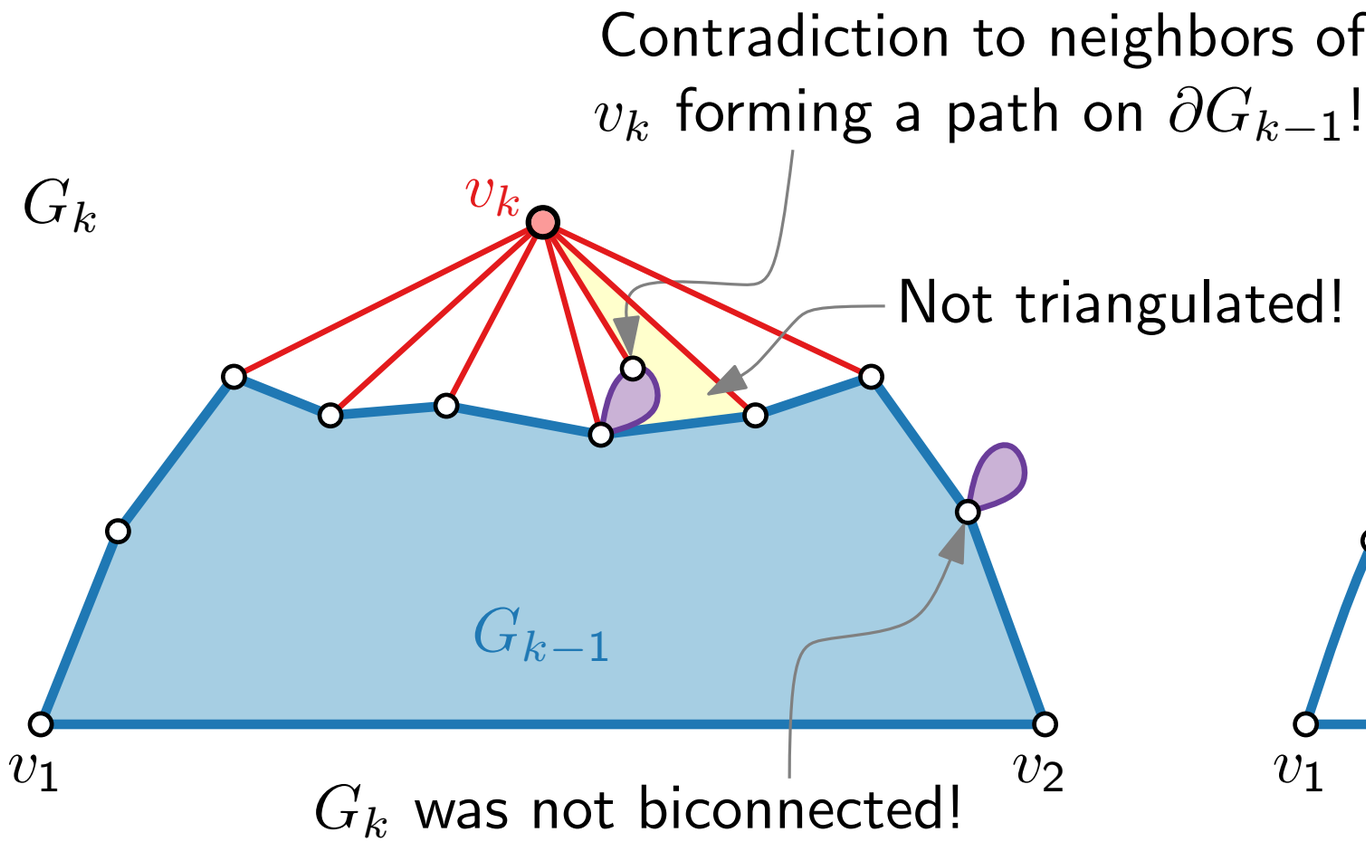

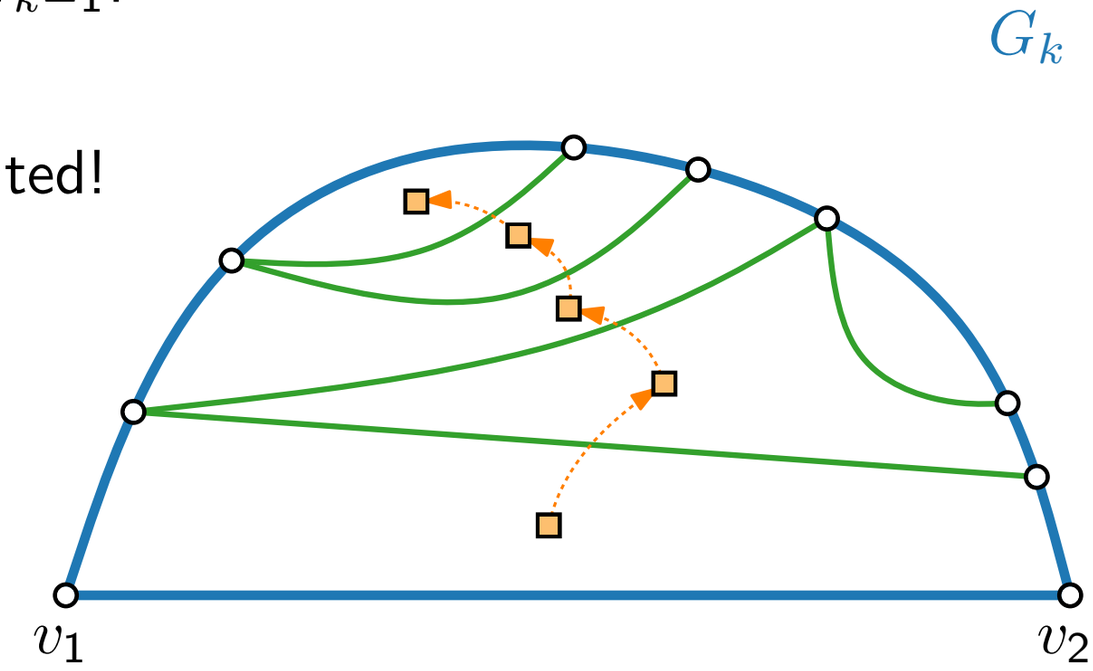

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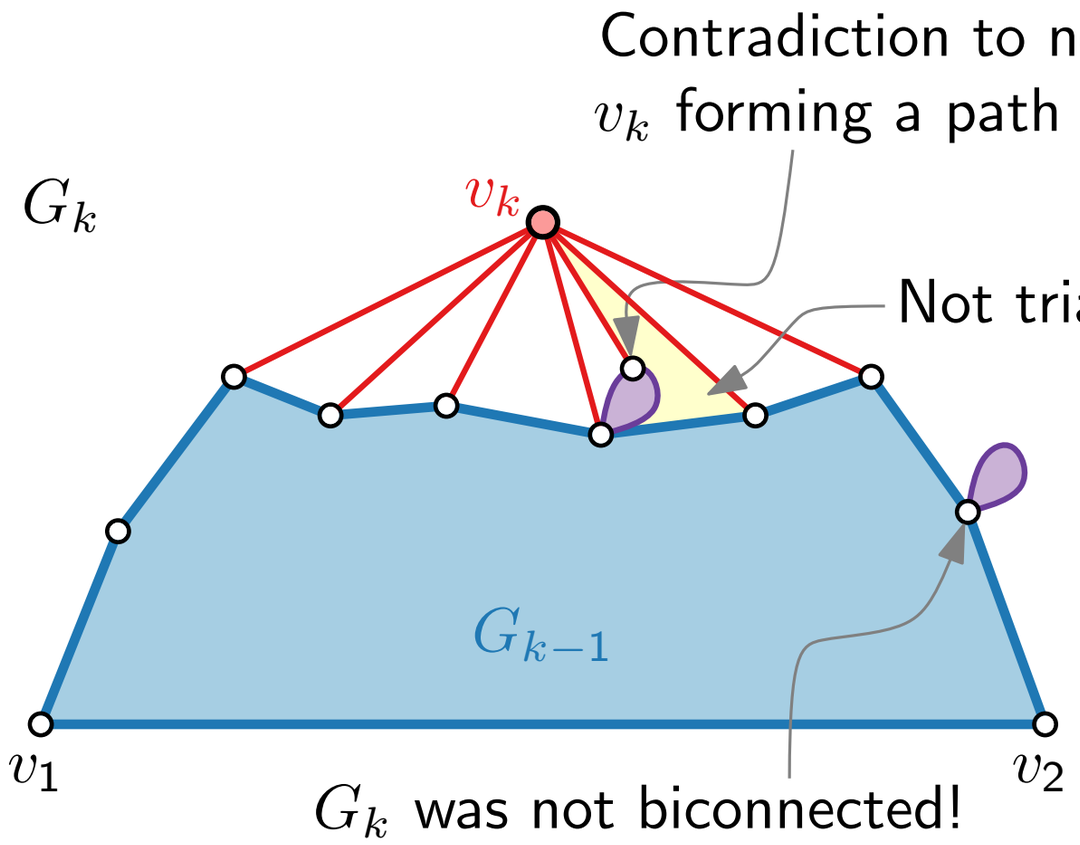

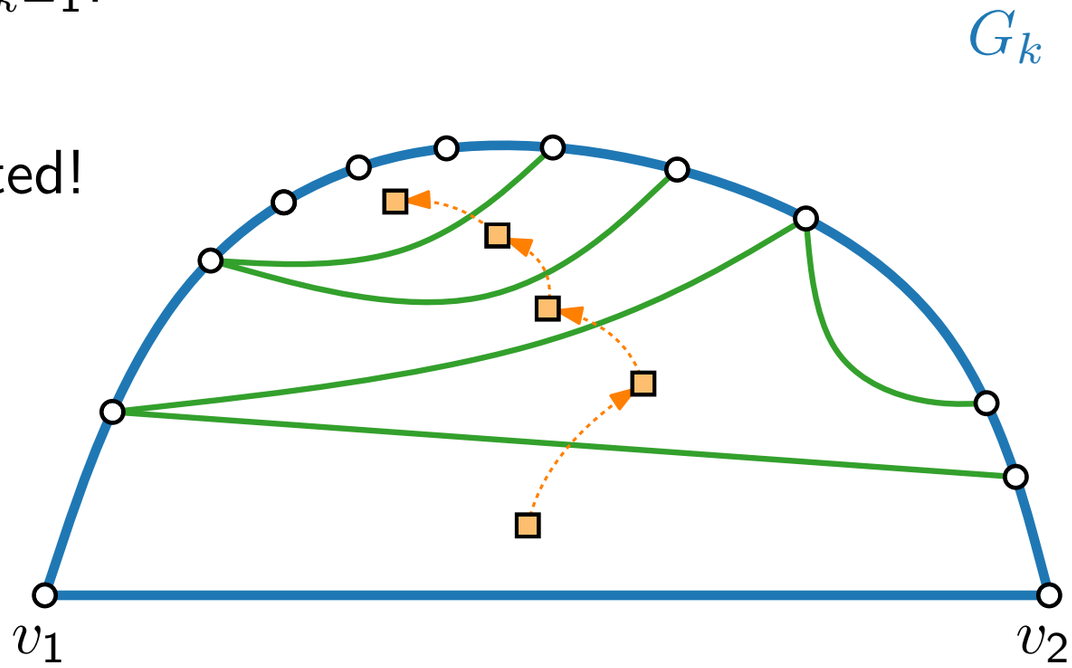

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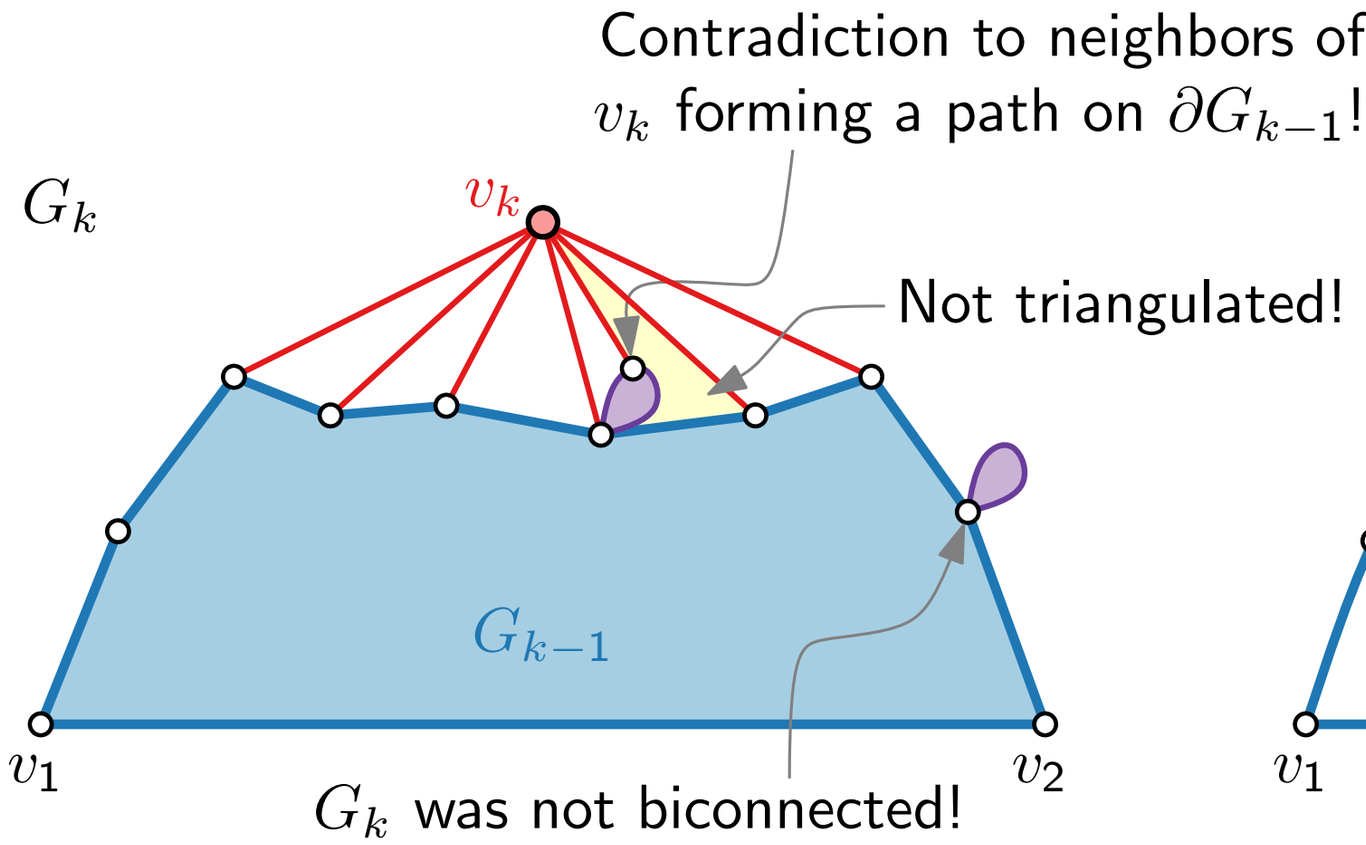

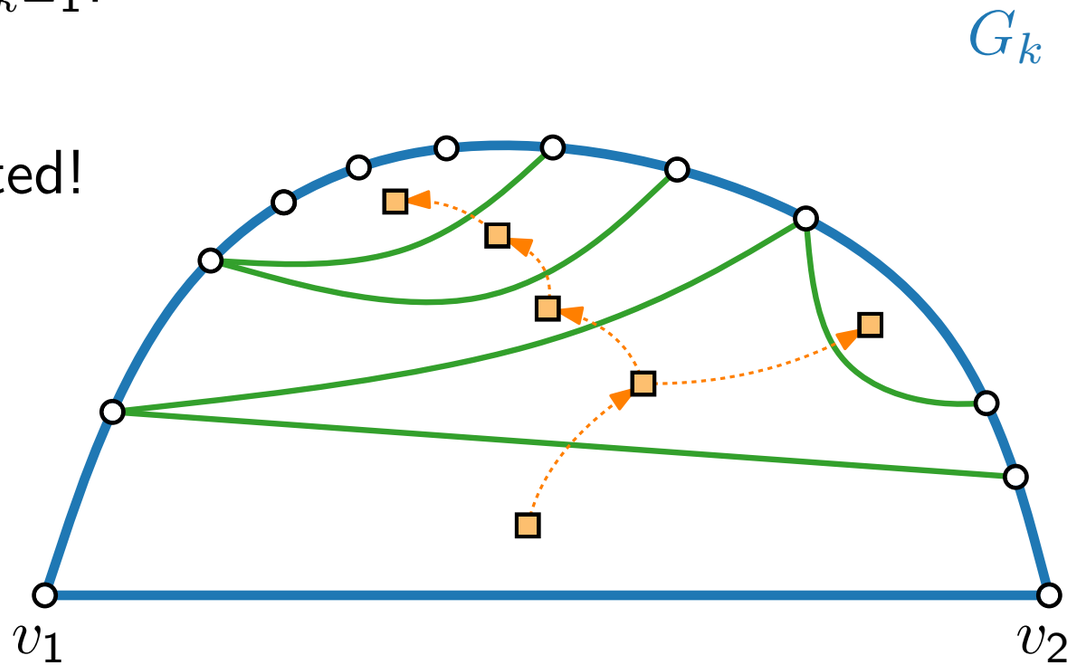

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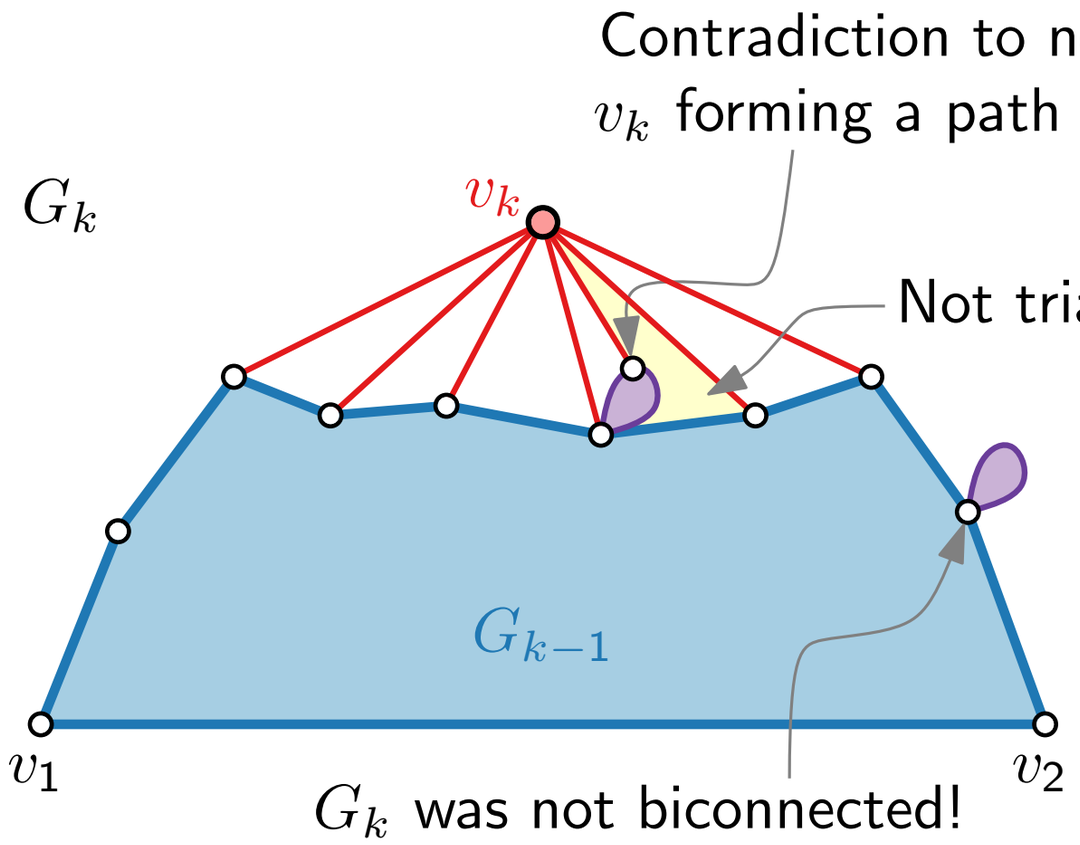

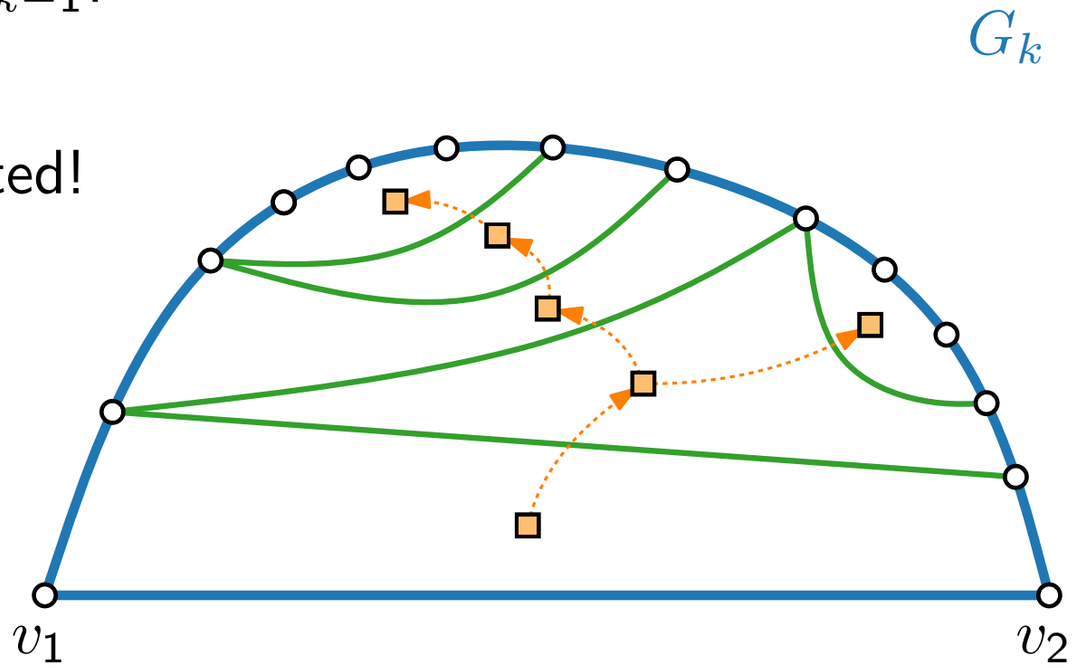

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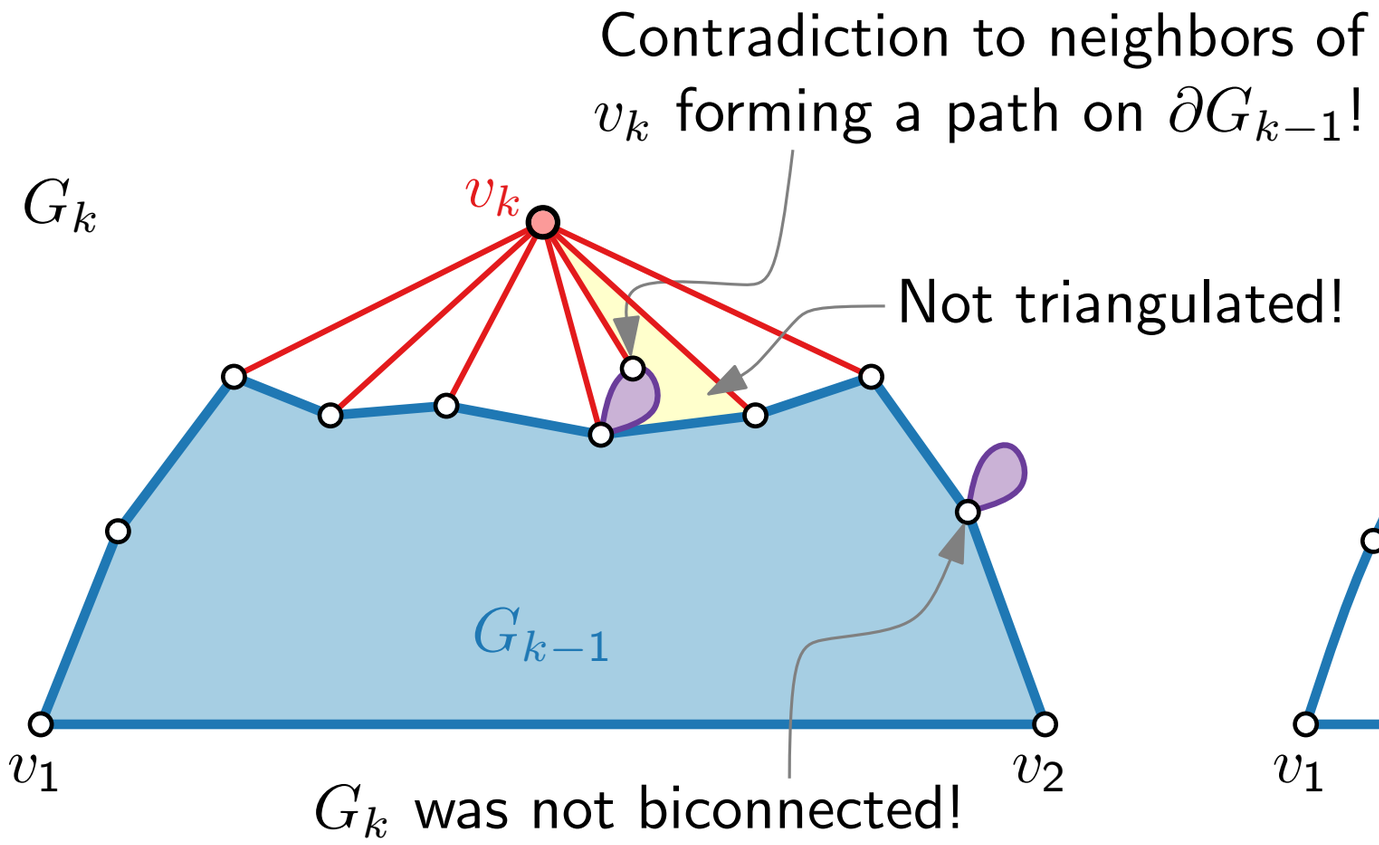

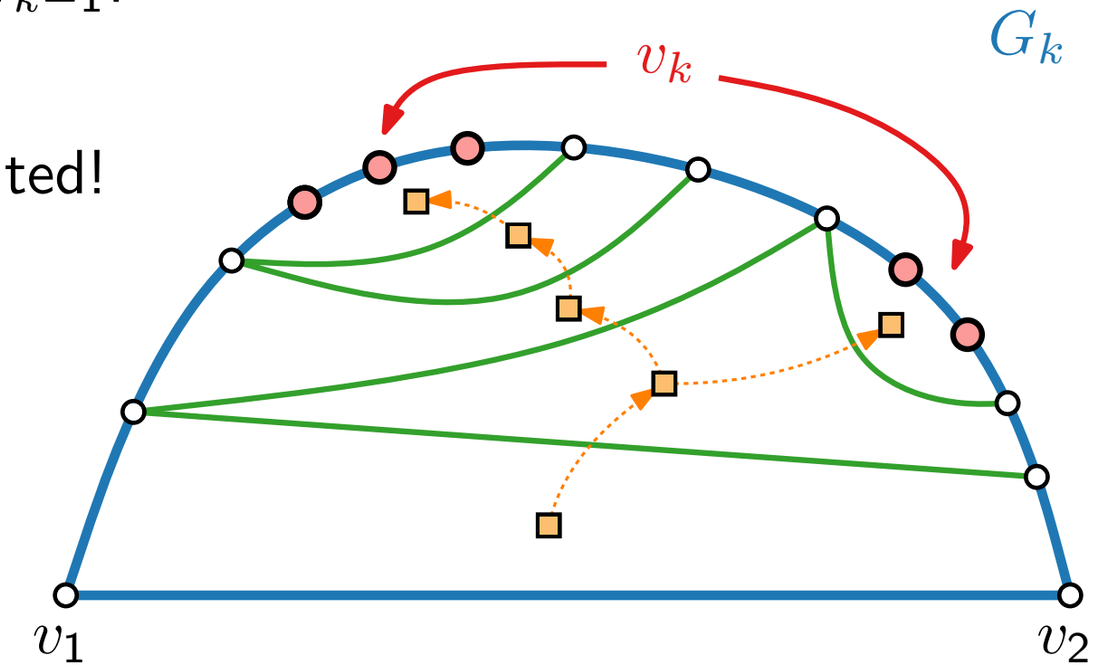

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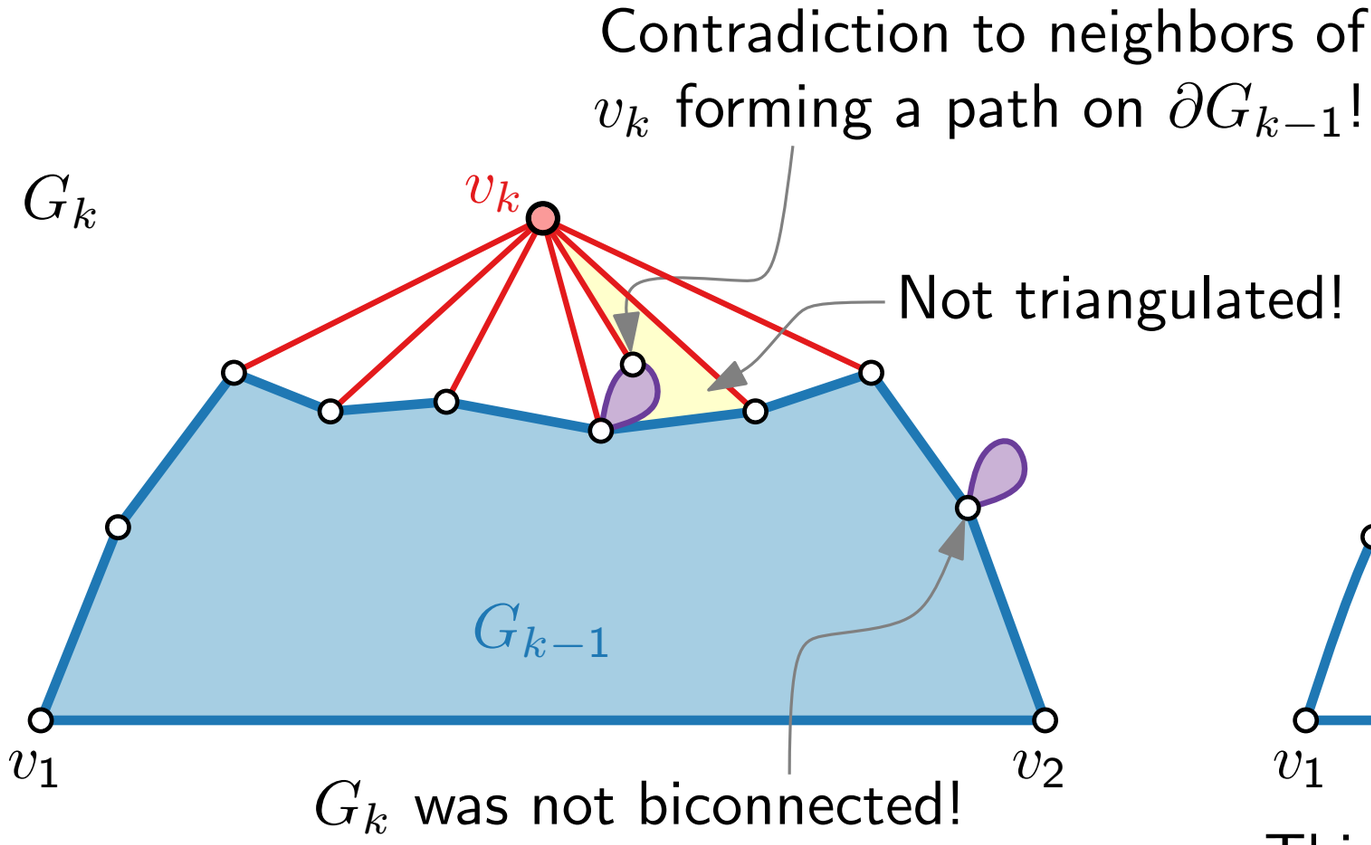

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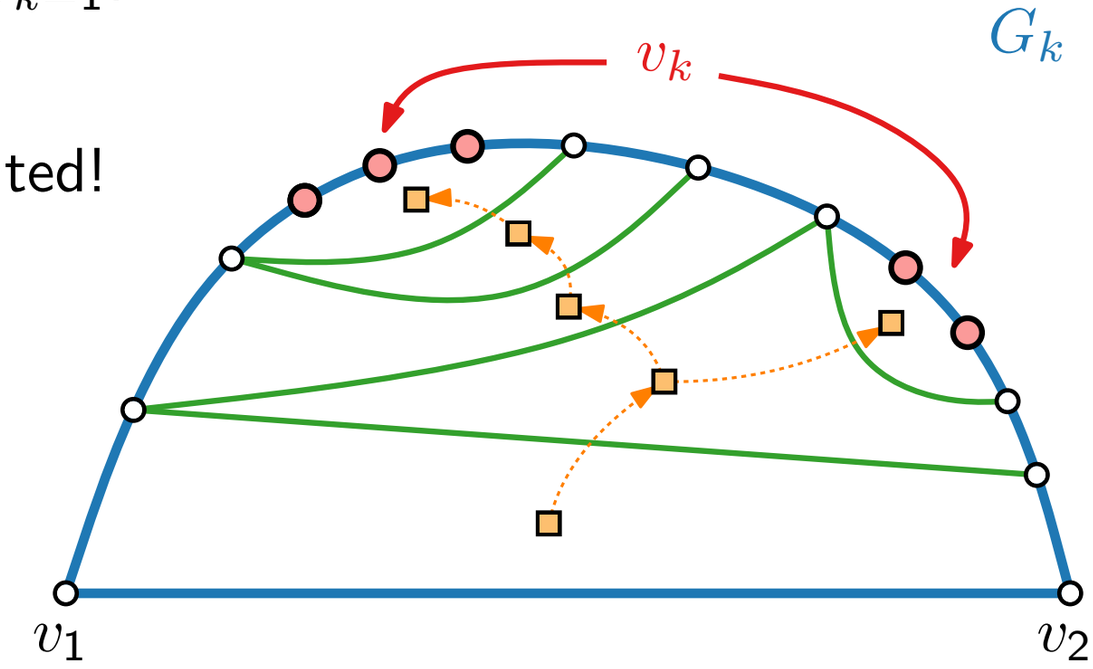

This completes the proof of the lemma. □

Canonical Order – Implementation

CanonicalOrder($G = (V, E), (v_1, v_2, v_n)$)

Canonical Order – Implementation

outer face

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Canonical Order – Implementation

outer face

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forall $v \in V$ **do**
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- out(v) = true iff v is on the current outer face

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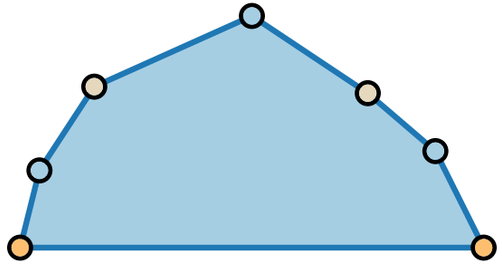

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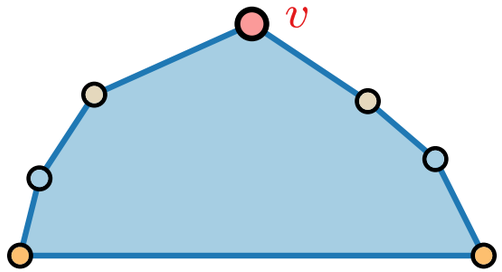

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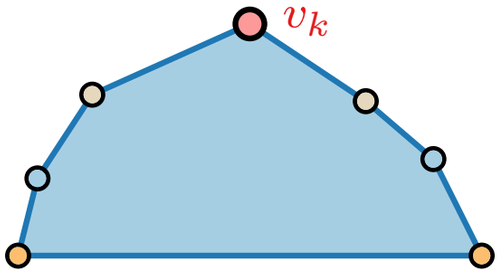

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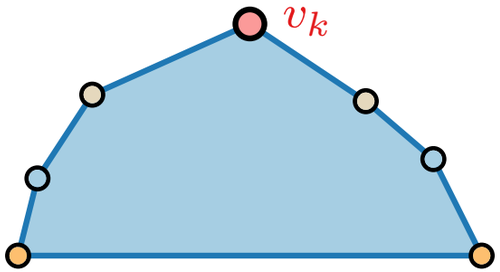

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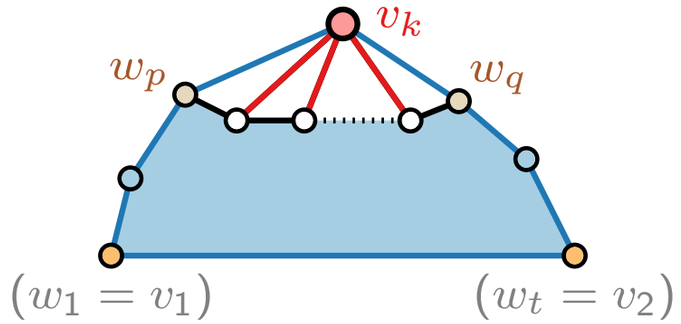

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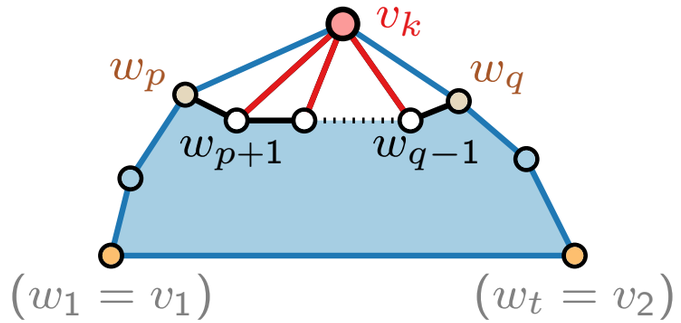

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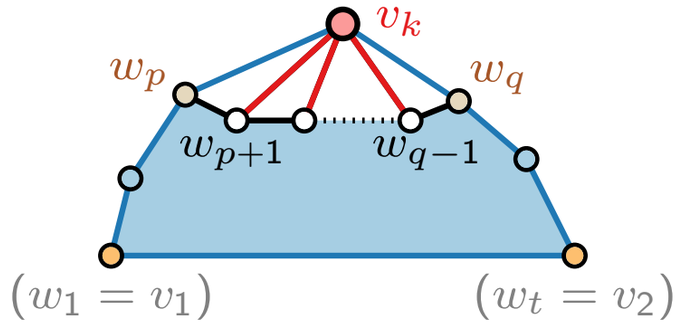

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  for $i = p + 1$ **to** $q - 1$ **do**
    out(w_i) $\leftarrow$ true
    foreach $u \in$ Adj[w_i] $\setminus \{w_{i-1}, w_{i+1}\}$ **do**
      if out(u) **then** chords(w_i)++, chords(u)++
  if $p + 1 = q$ **then** chords(w_p)−−, chords(w_q)−−

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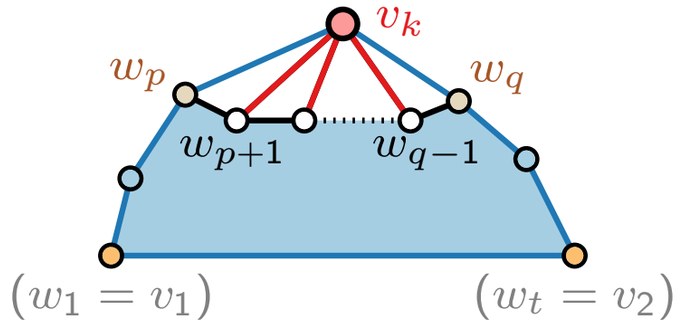

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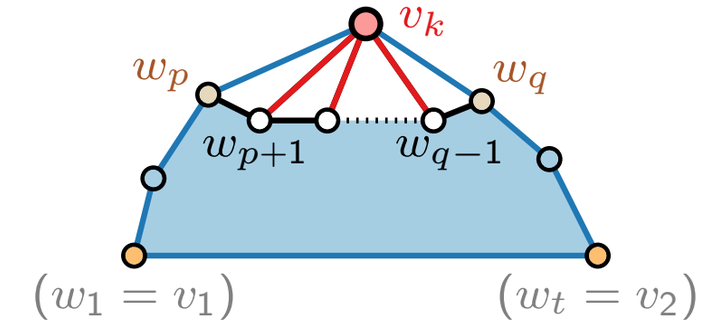

Lemma.
Algorithm CanonicalOrder computes a canonical order of a plane graph in $\mathcal{O}(n)$ time.

Canonical Order – Implementation

outer face

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forall $v \in V$ **do**
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for $k = n$ **downto** 3 **do**
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  $v_k \leftarrow v$; mark(v) $\leftarrow$ true; out(v) $\leftarrow$ false
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  for $i = p + 1$ **to** $q - 1$ **do**
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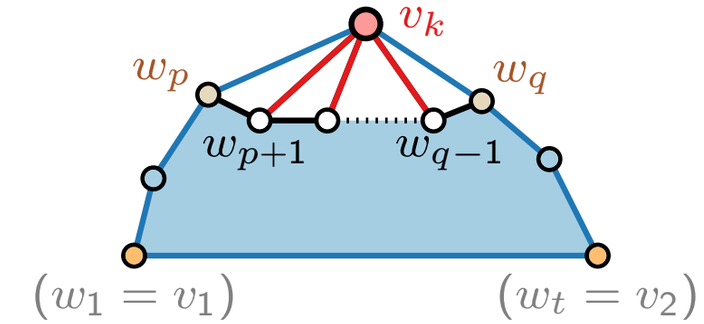

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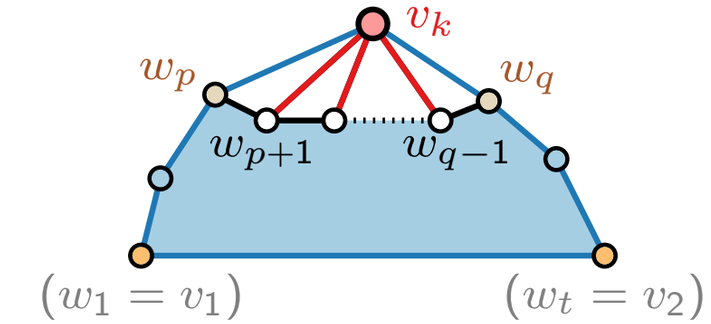

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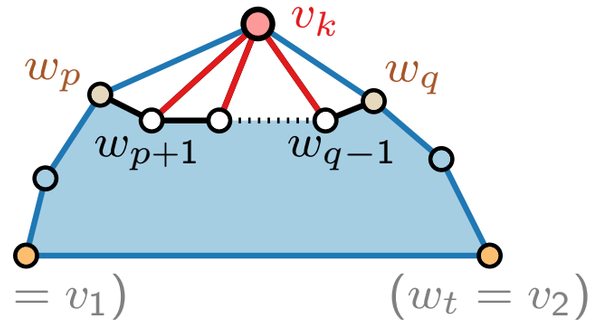

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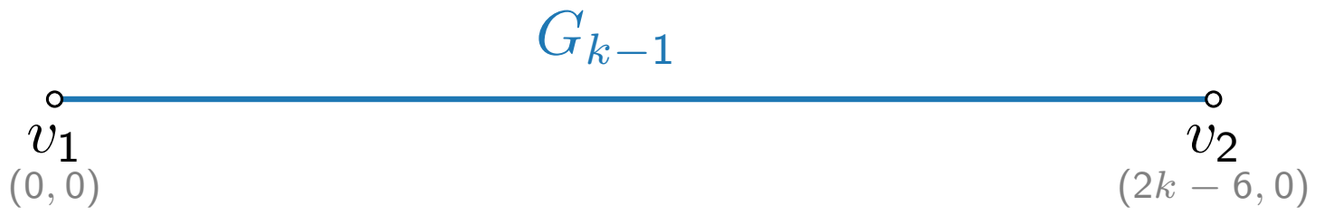

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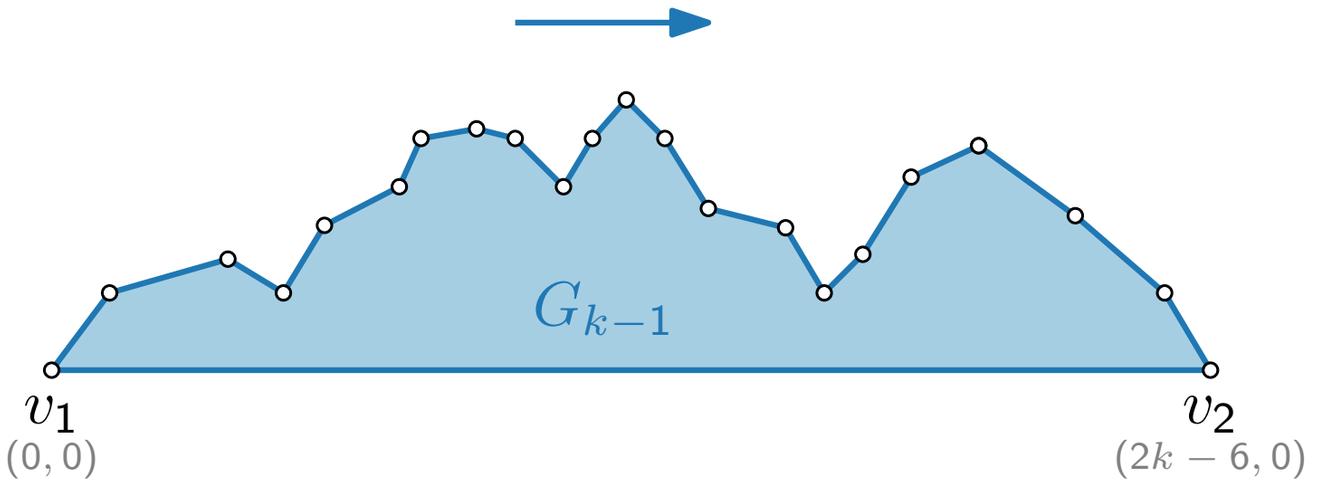

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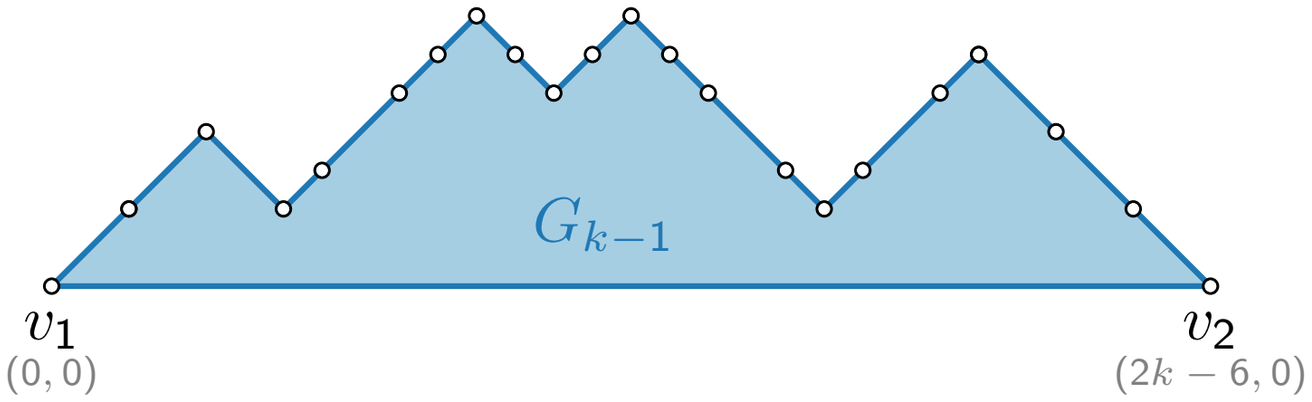

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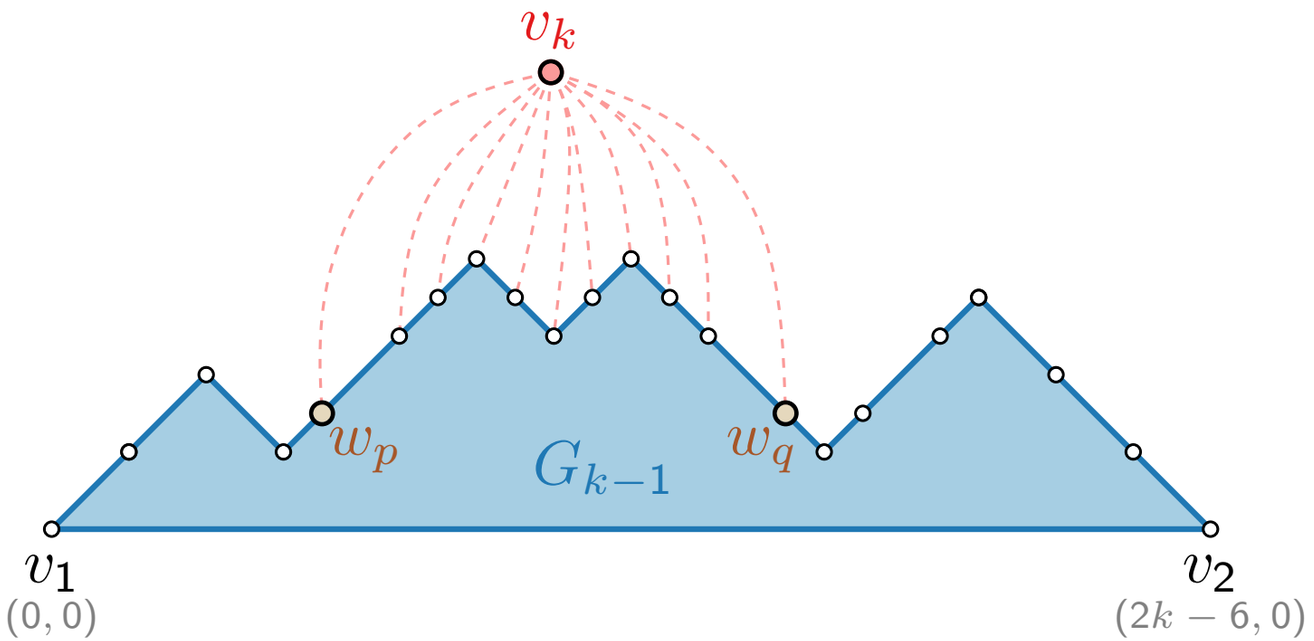

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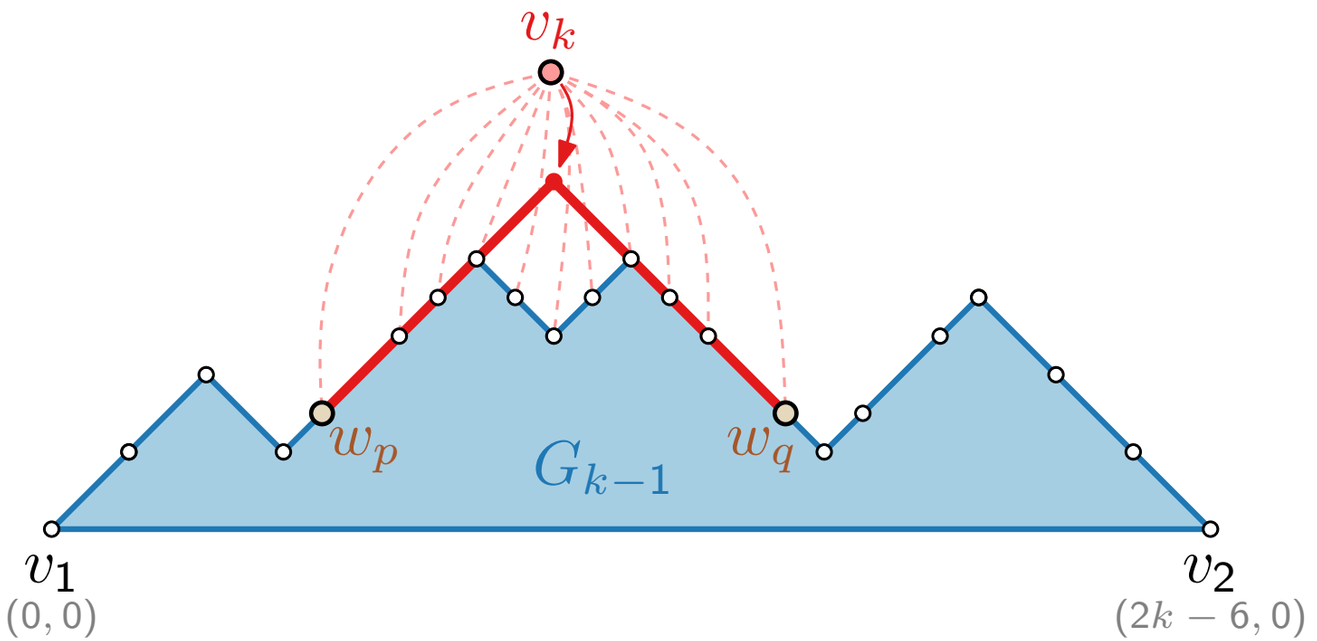

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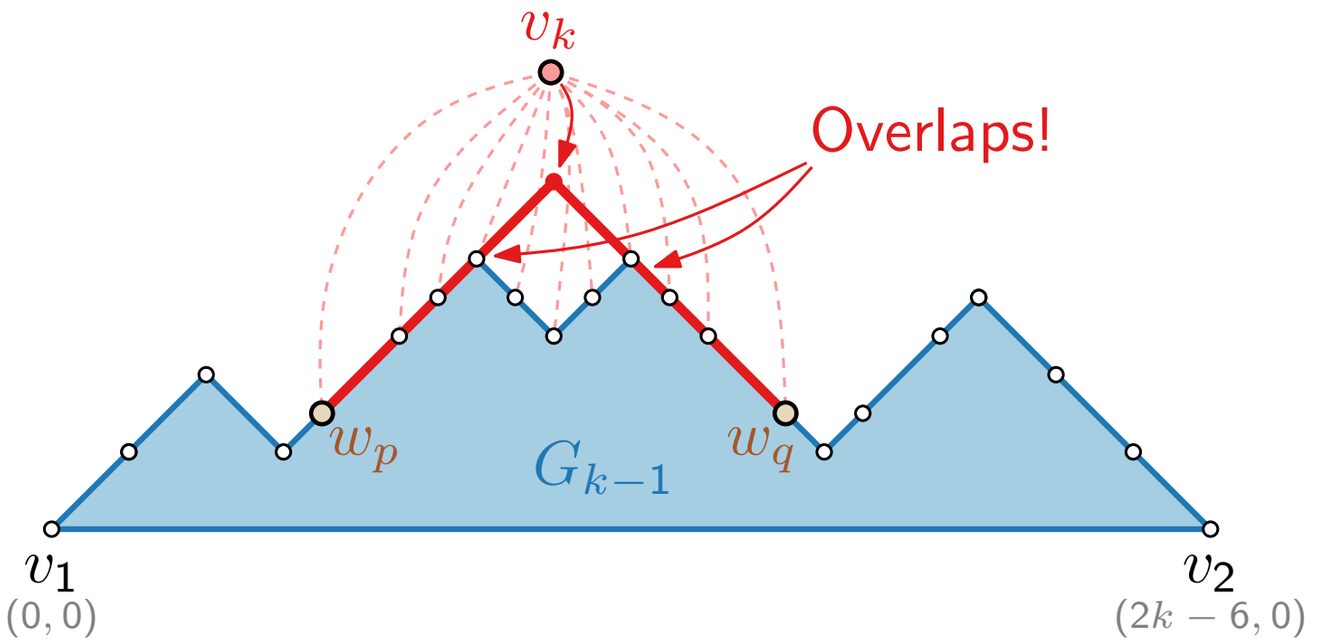

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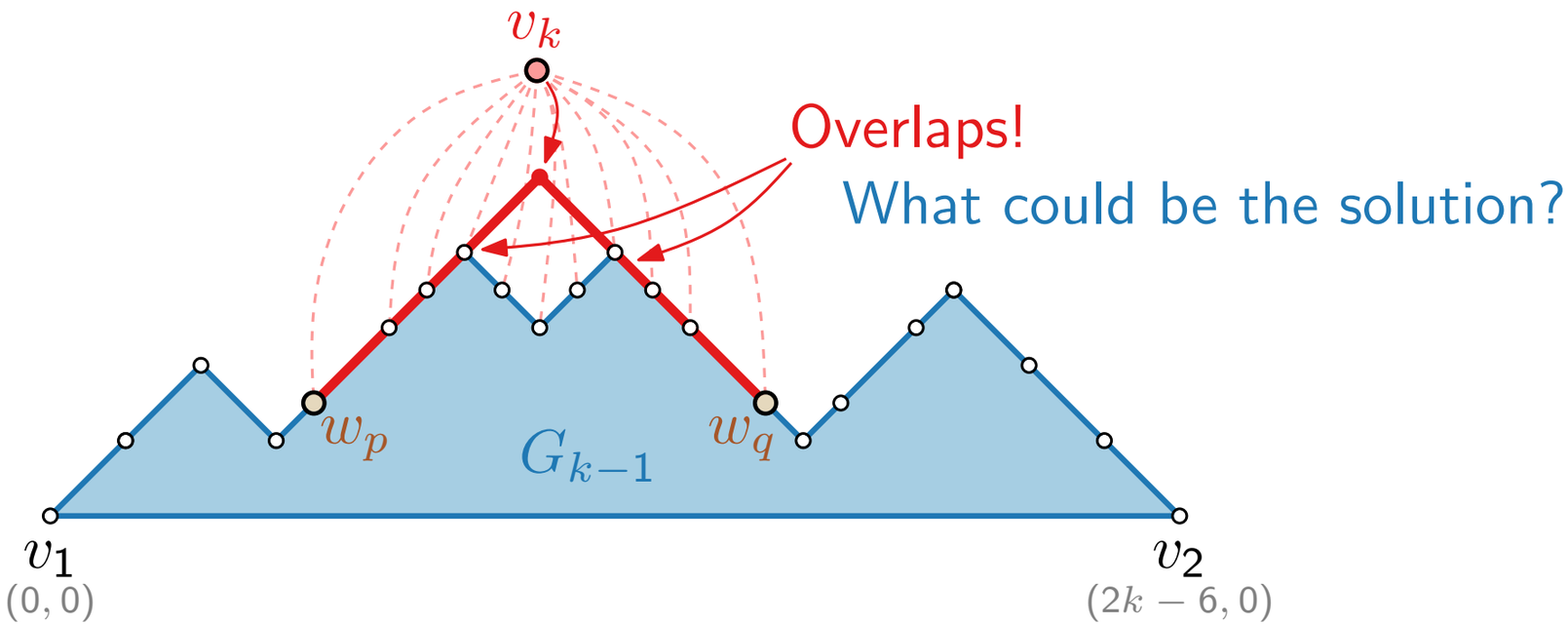

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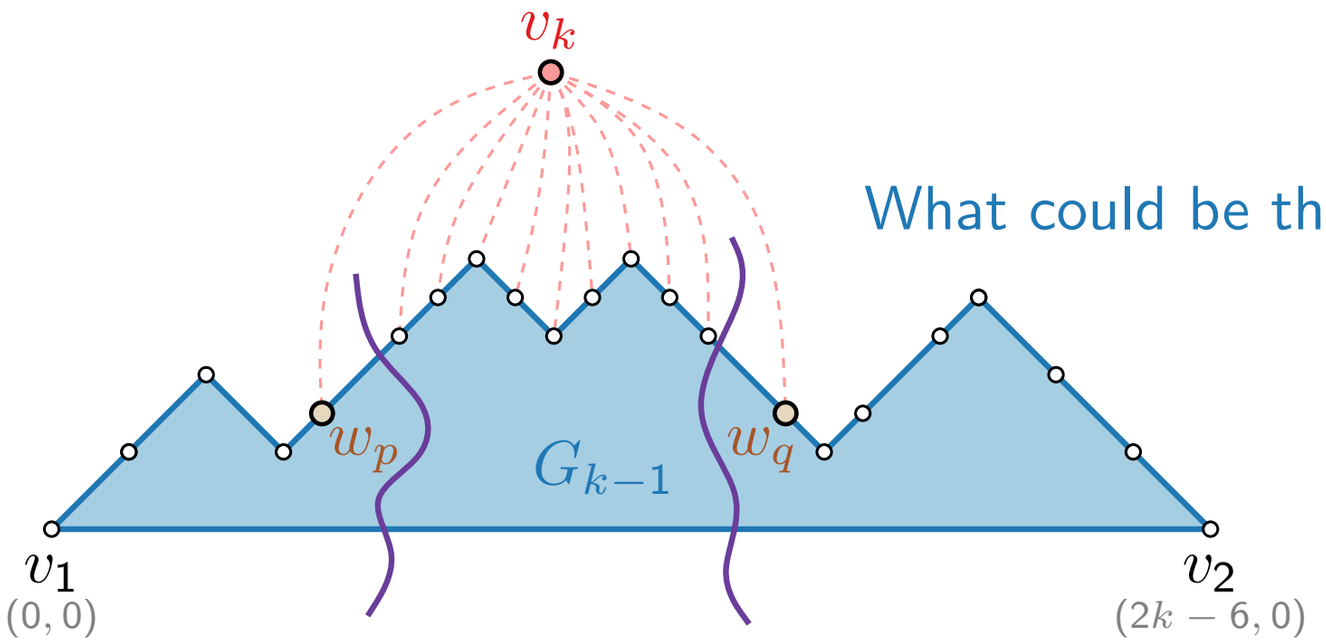

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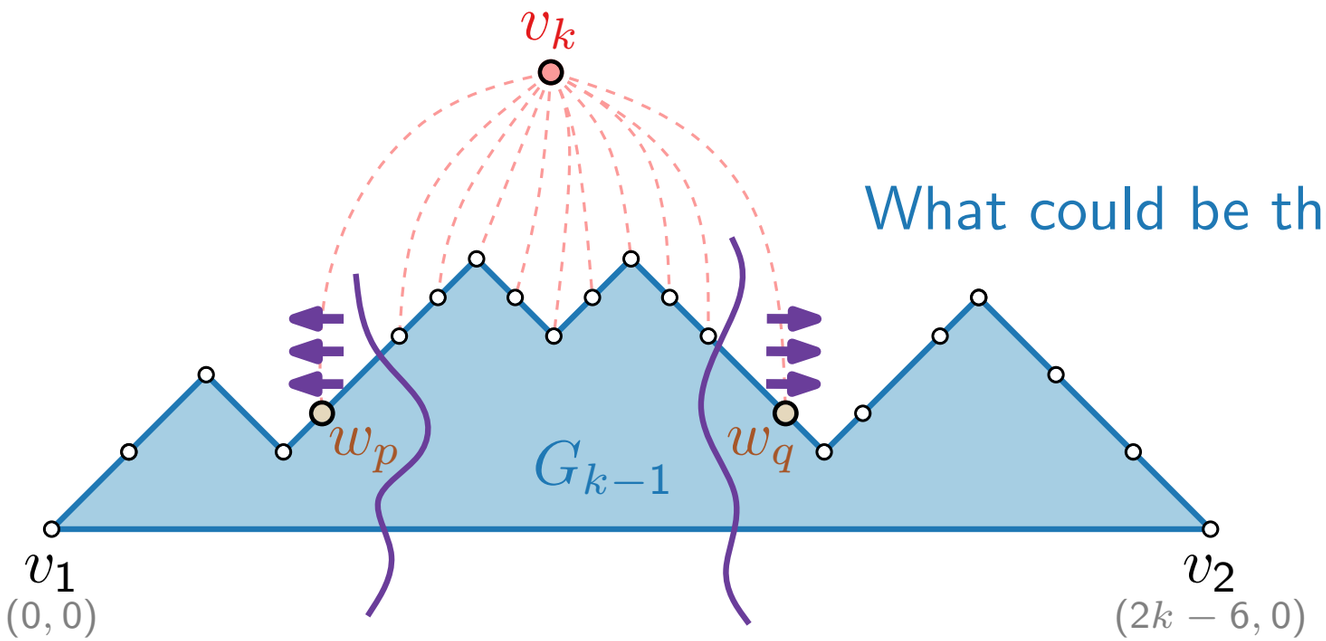

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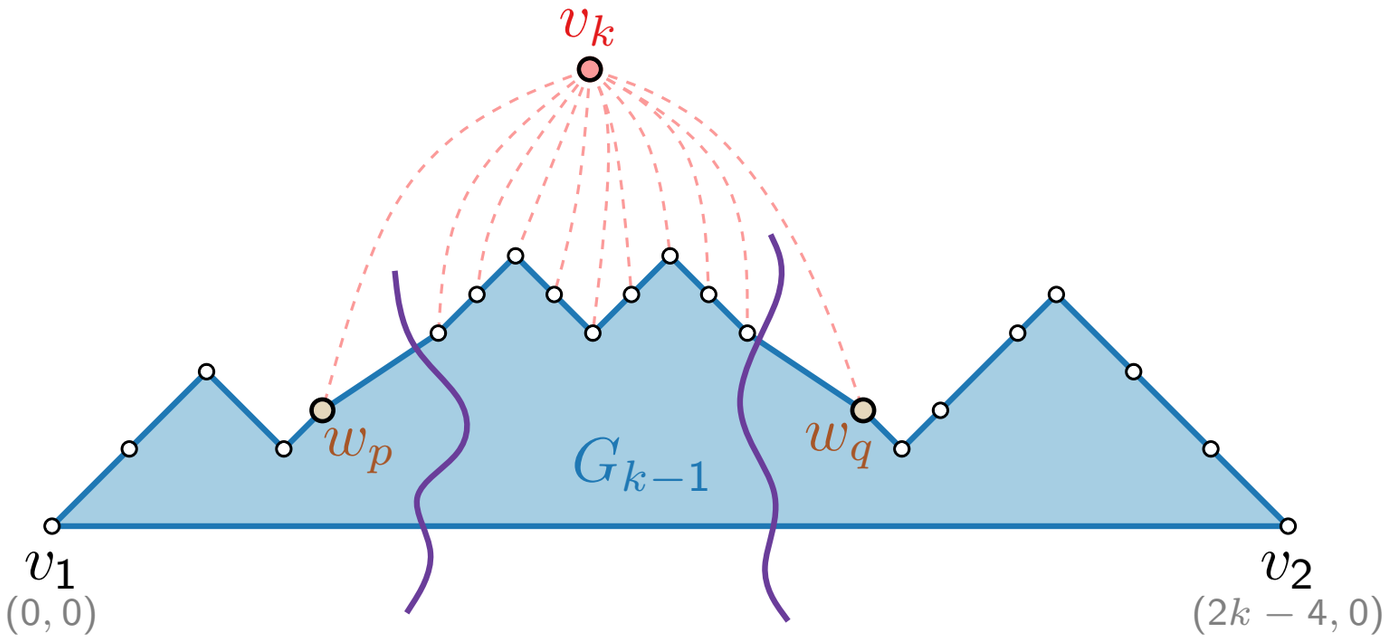

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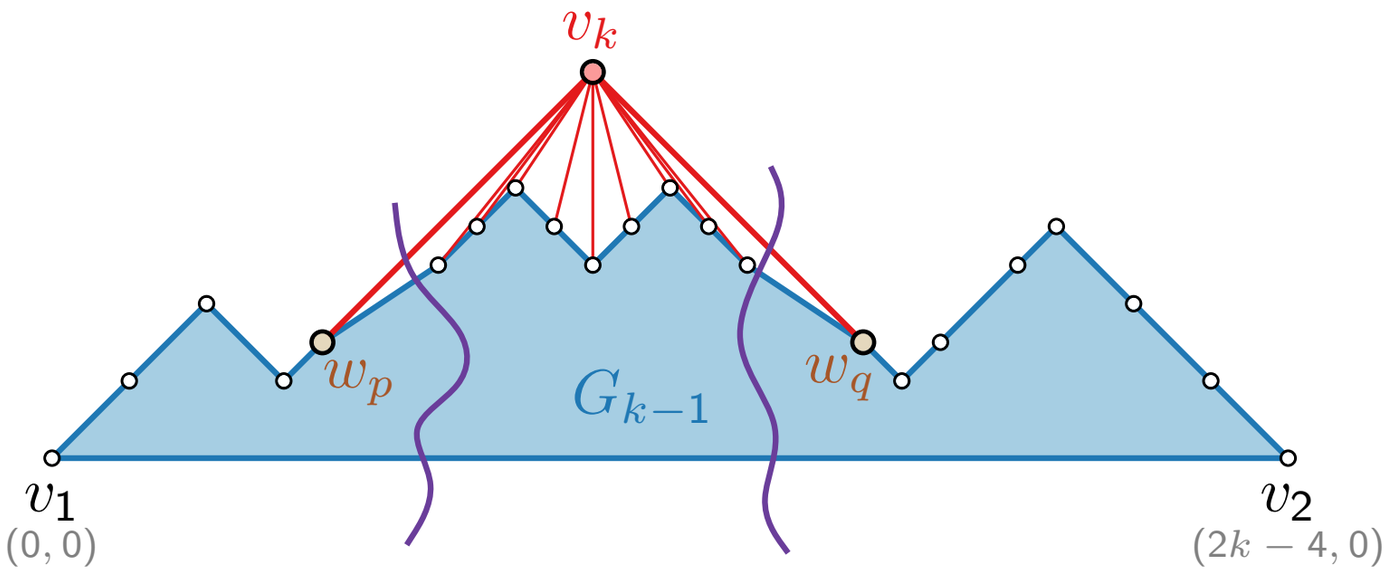

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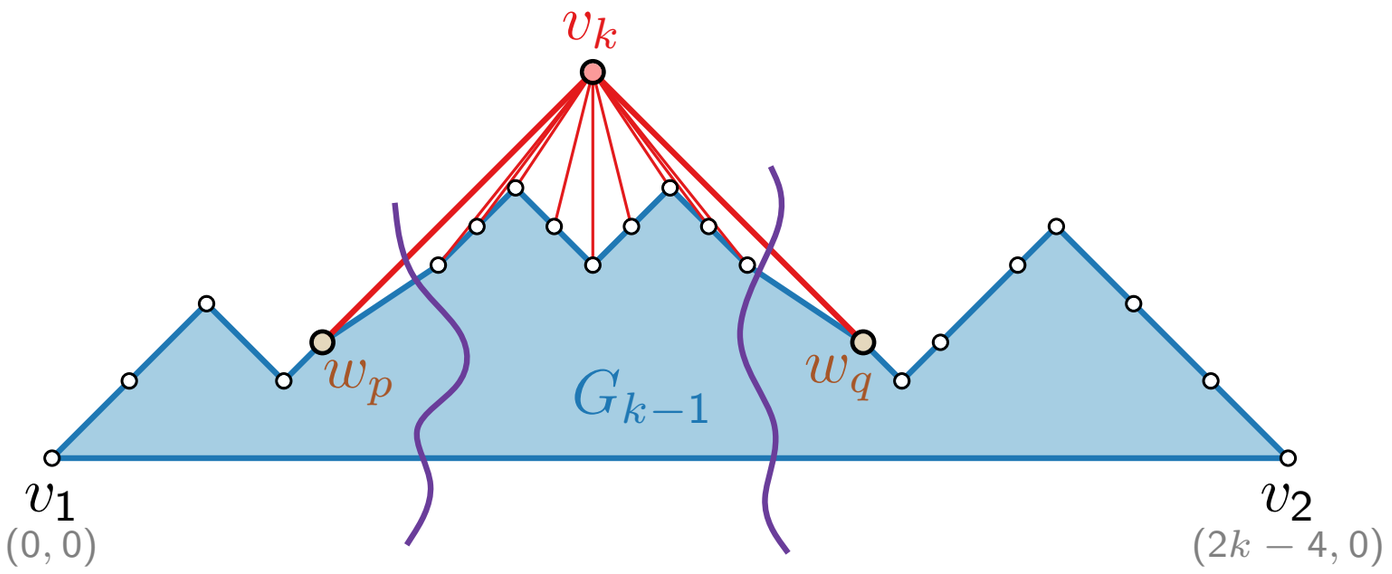

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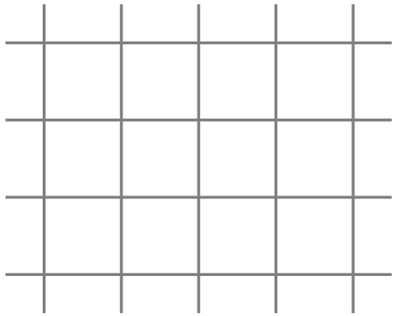

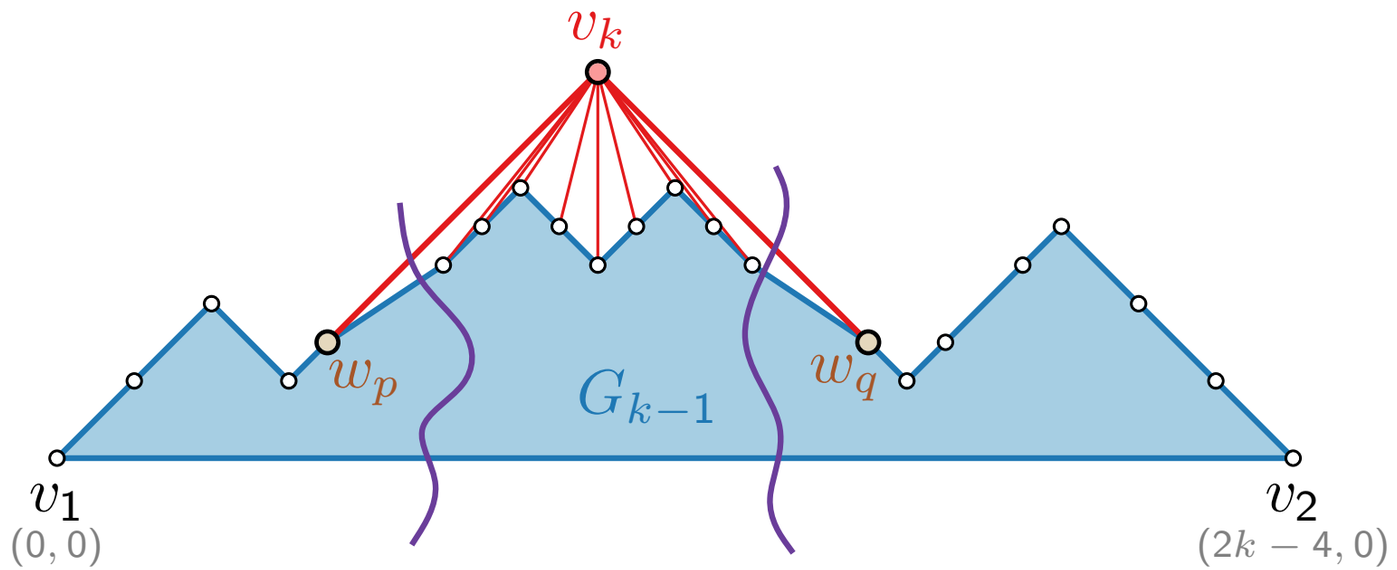

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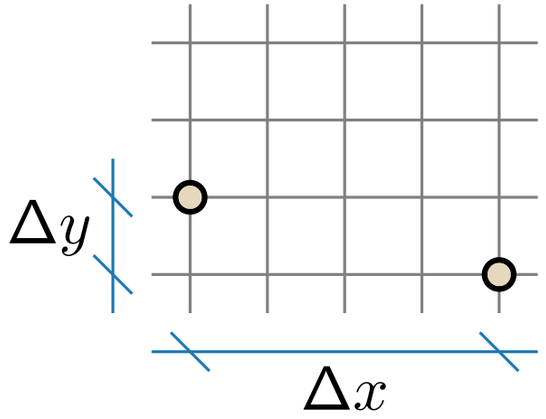

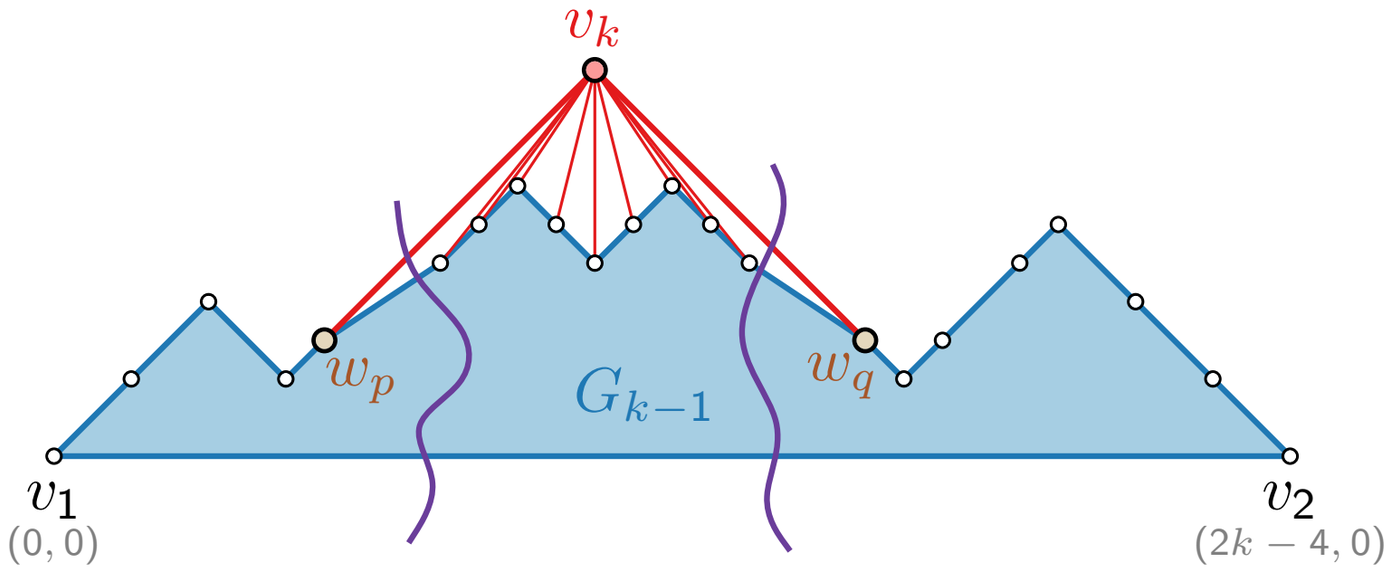

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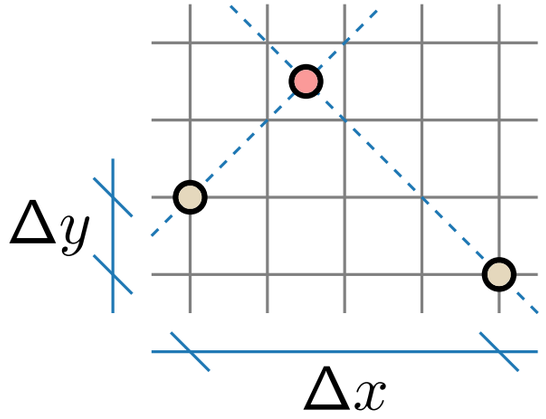

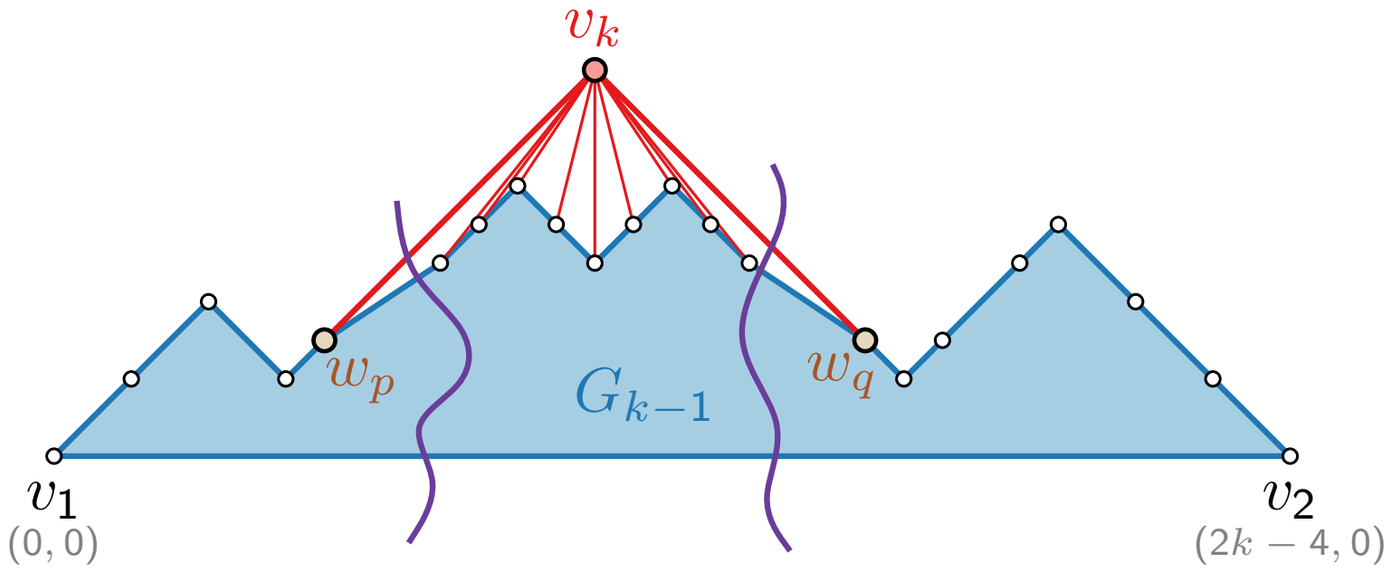

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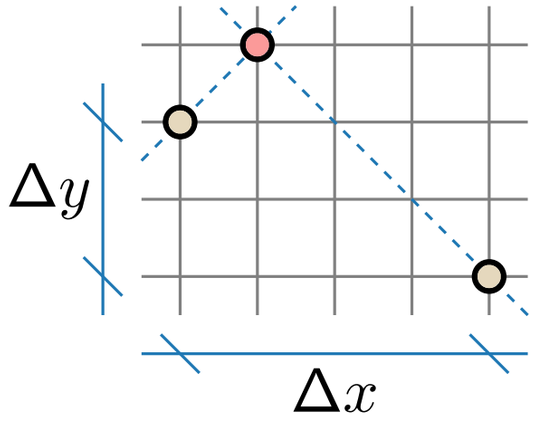

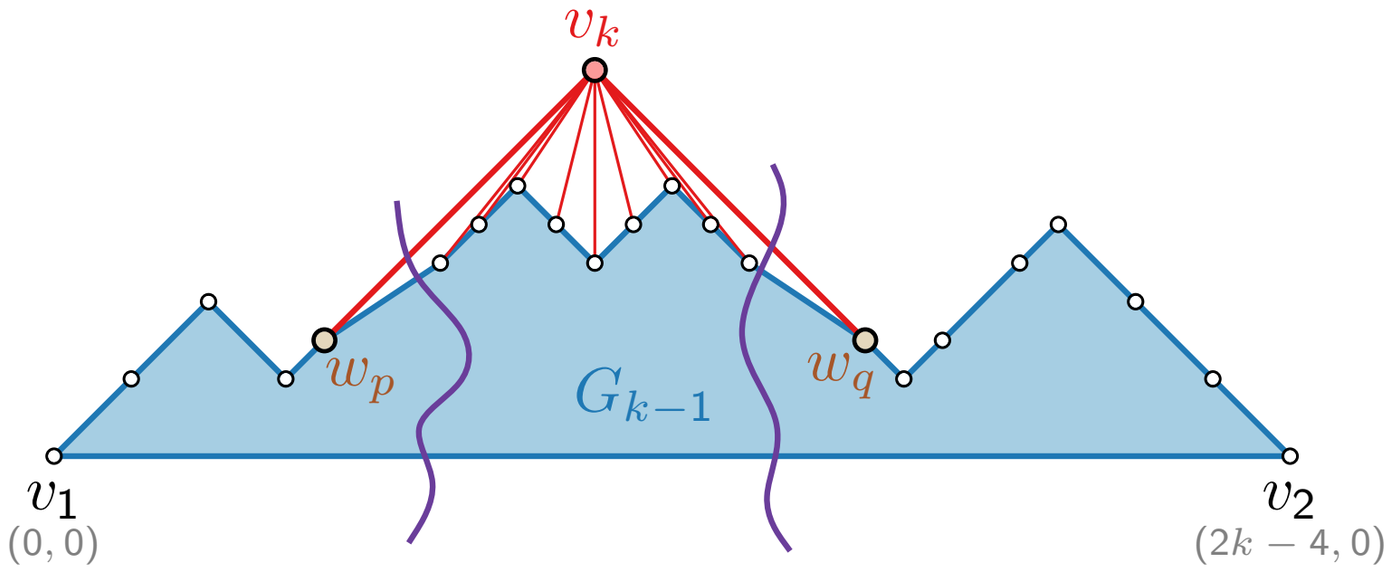

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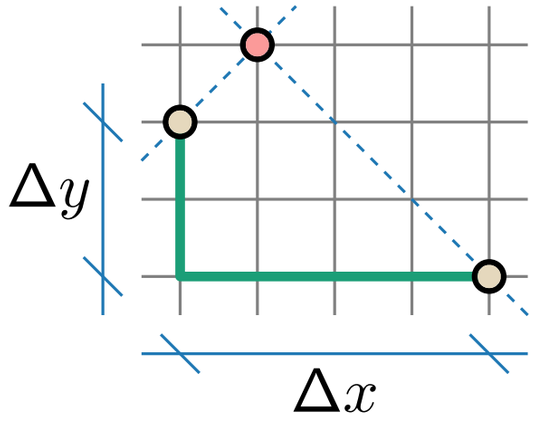

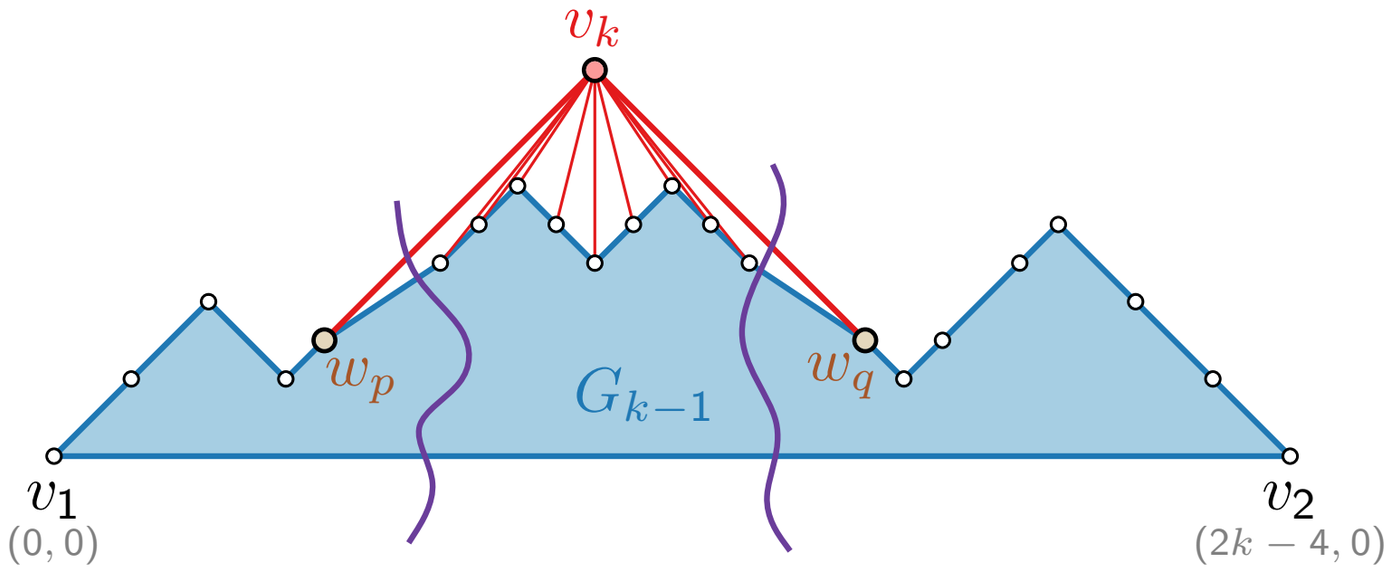

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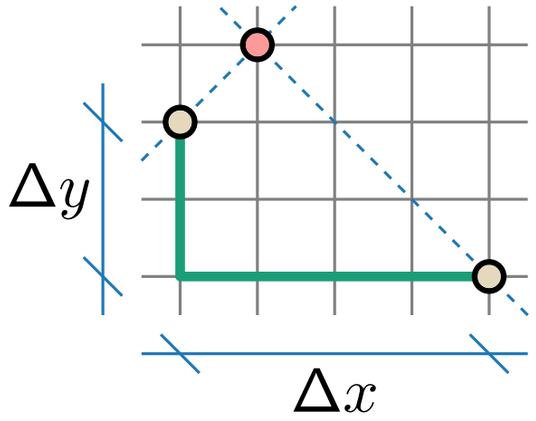
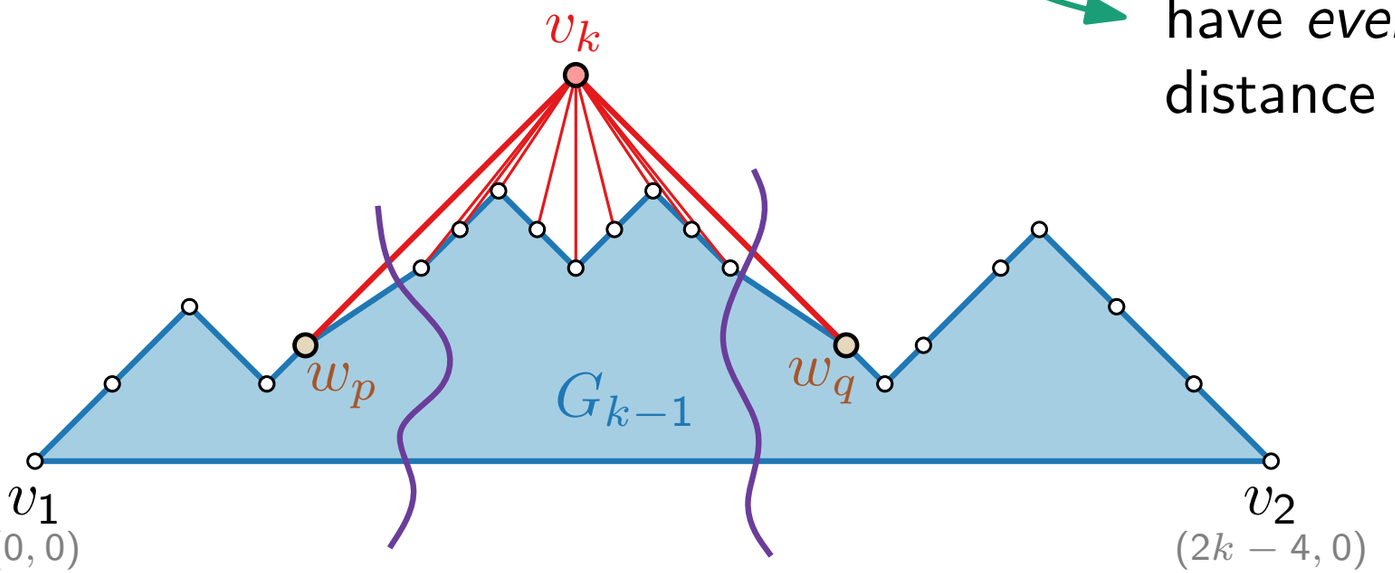


Yes, because w_p and w_q have *even* Manhattan distance $\Delta x + \Delta y$.



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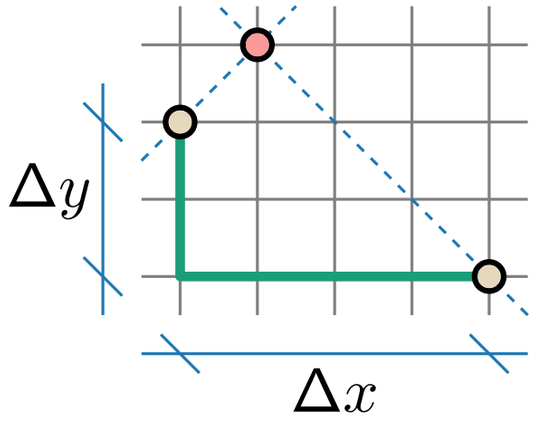
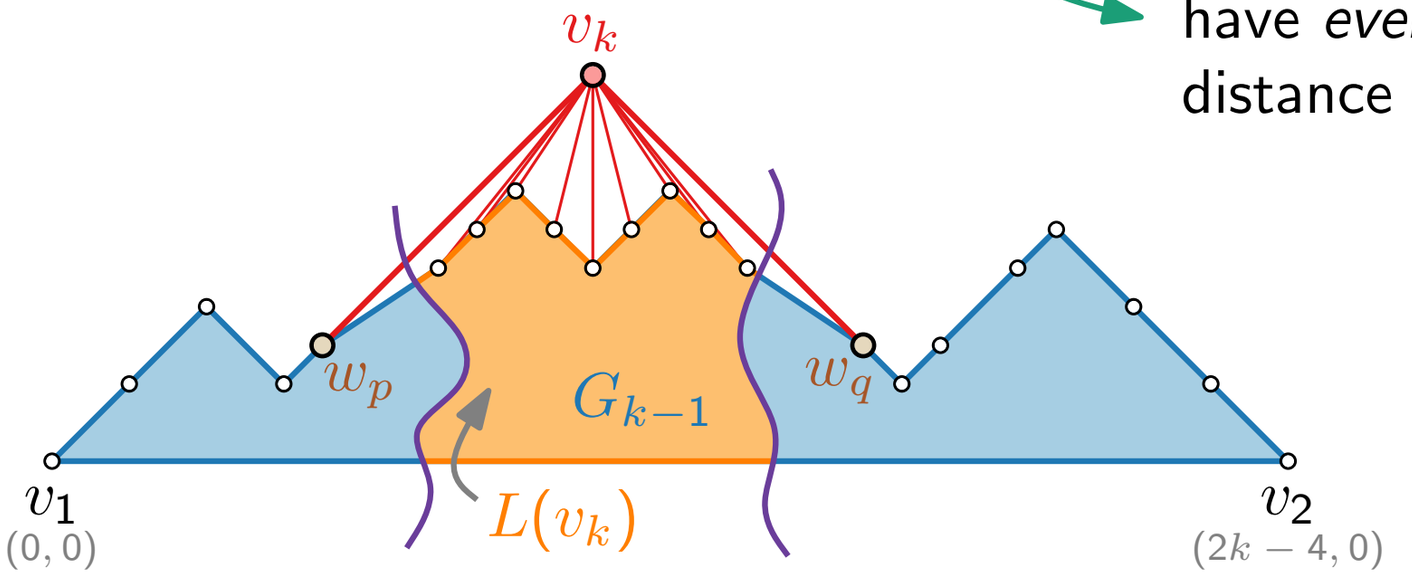
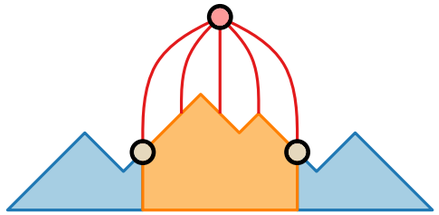
Will v_k lie on the grid?



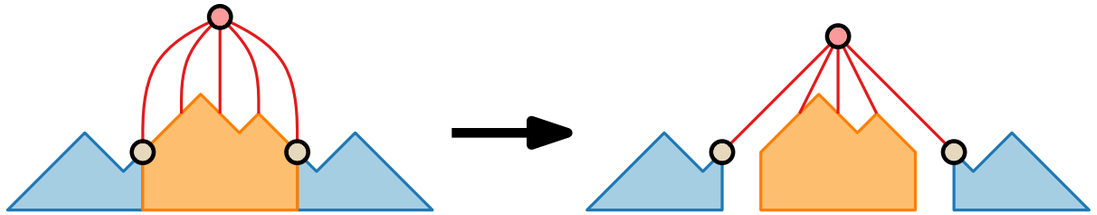
Yes, because w_p and w_q have *even* Manhattan distance $\Delta x + \Delta y$.



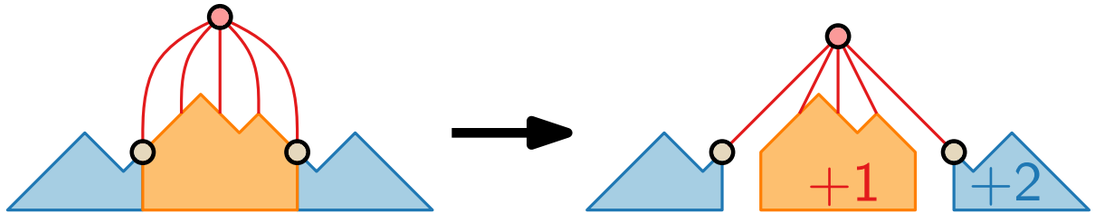
Shift Method – Example



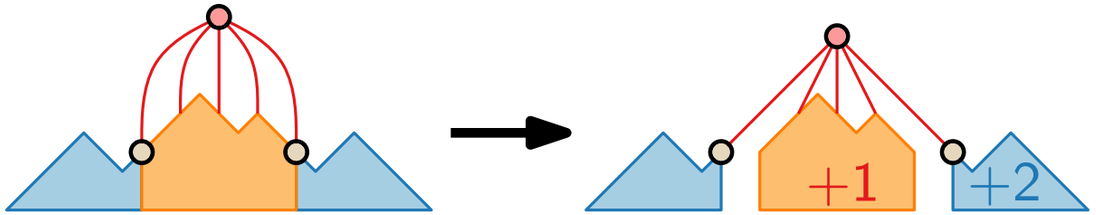
Shift Method – Example

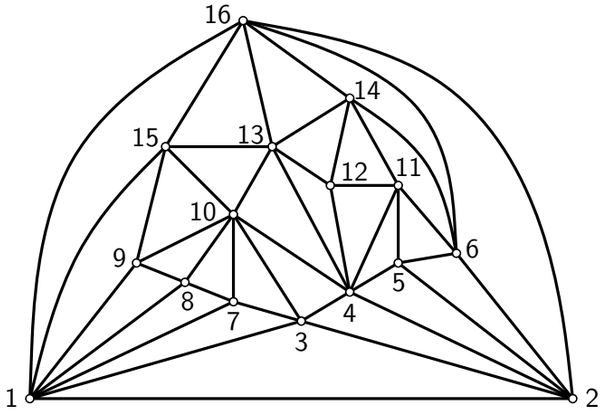


Shift Method – Example

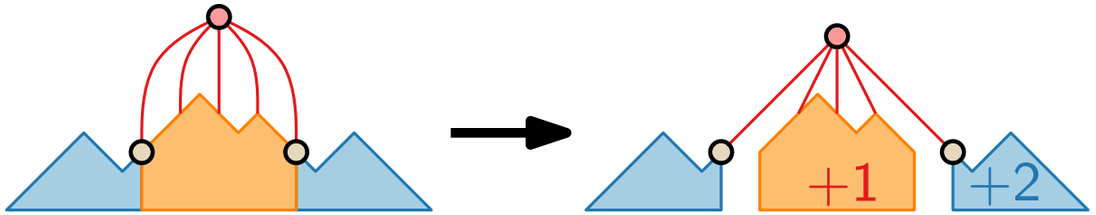


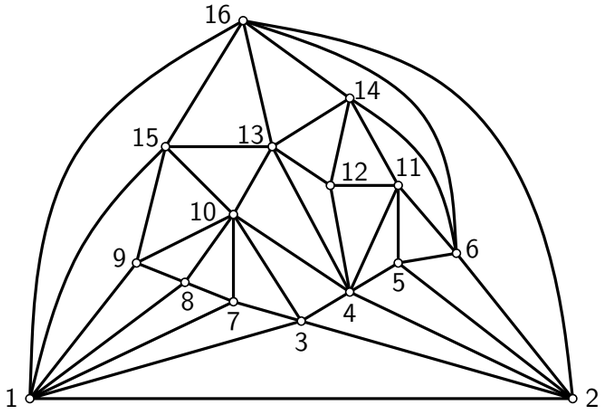
Shift Method – Example

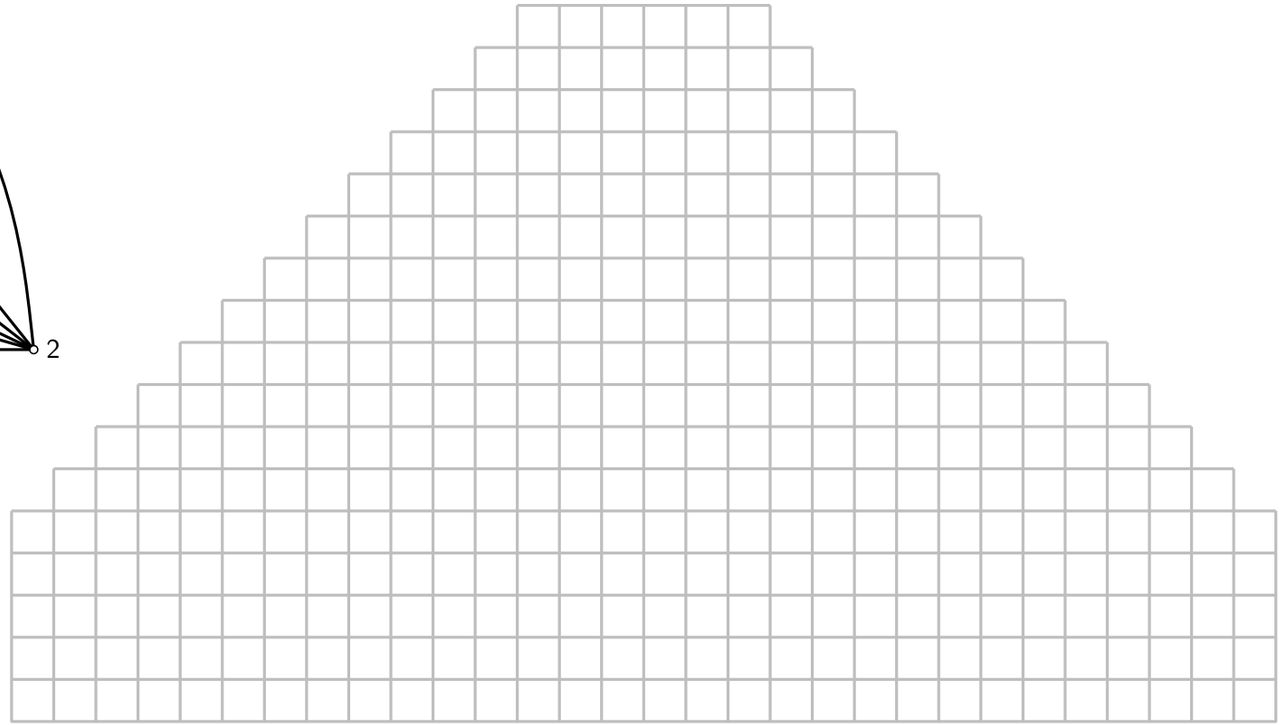




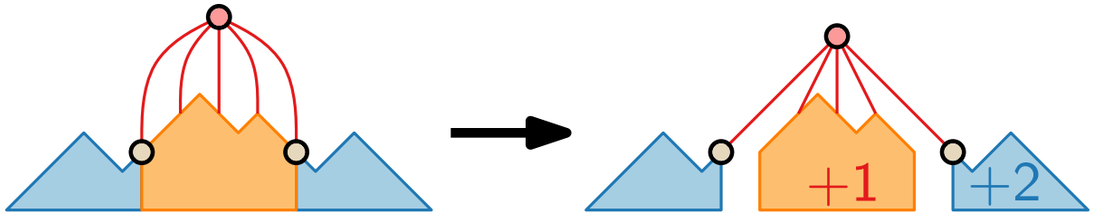
Shift Method – Example

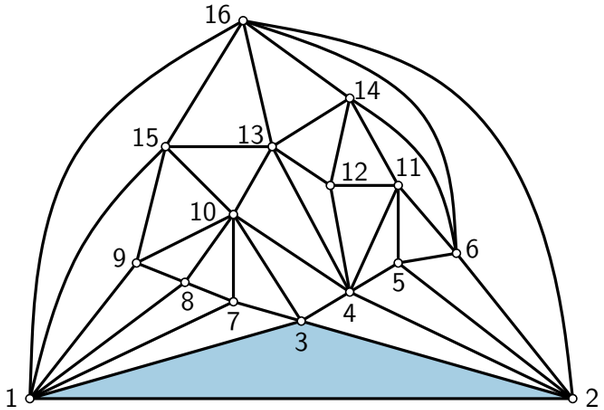


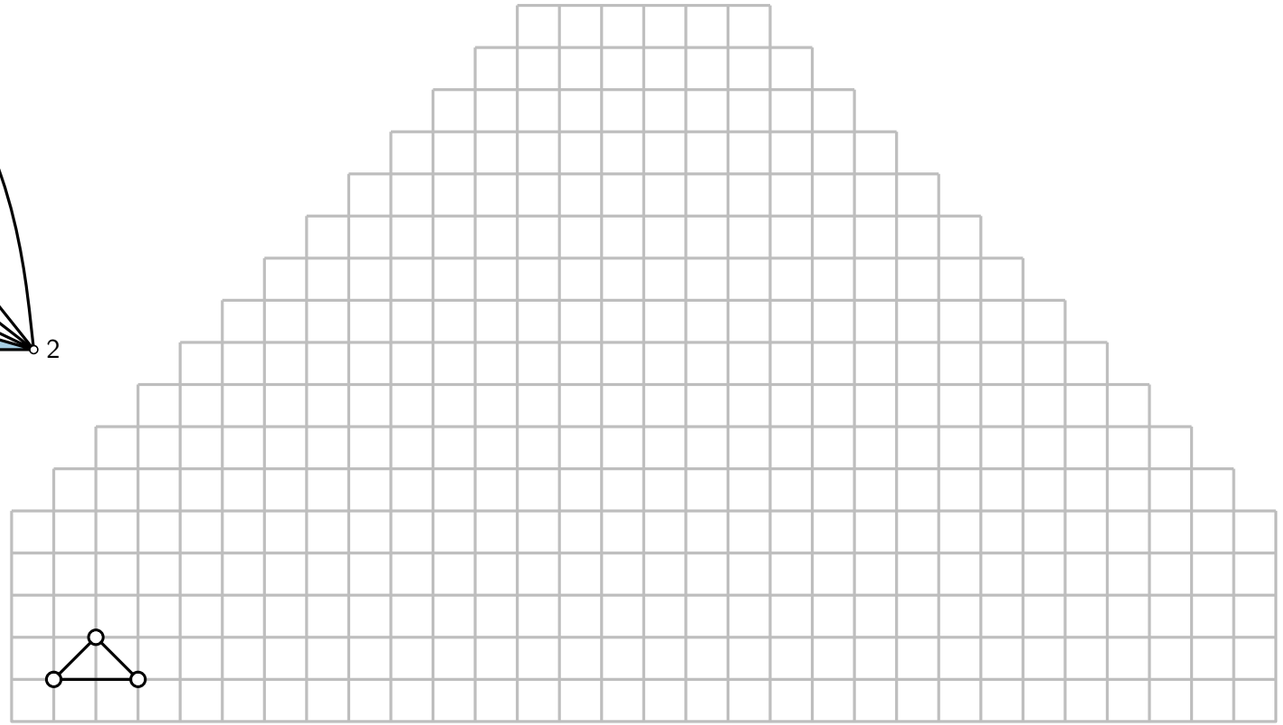




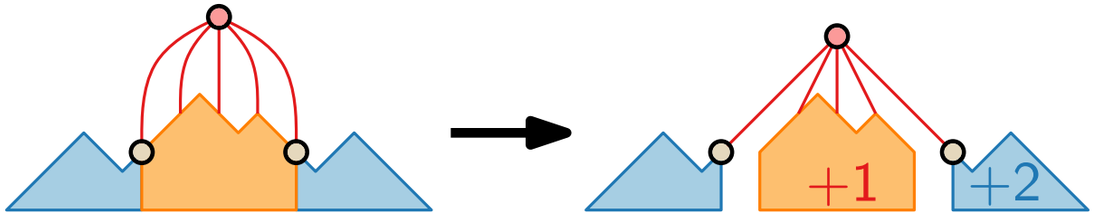
Shift Method – Example

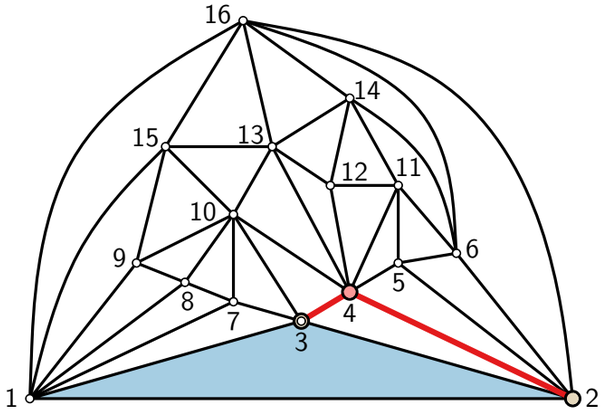


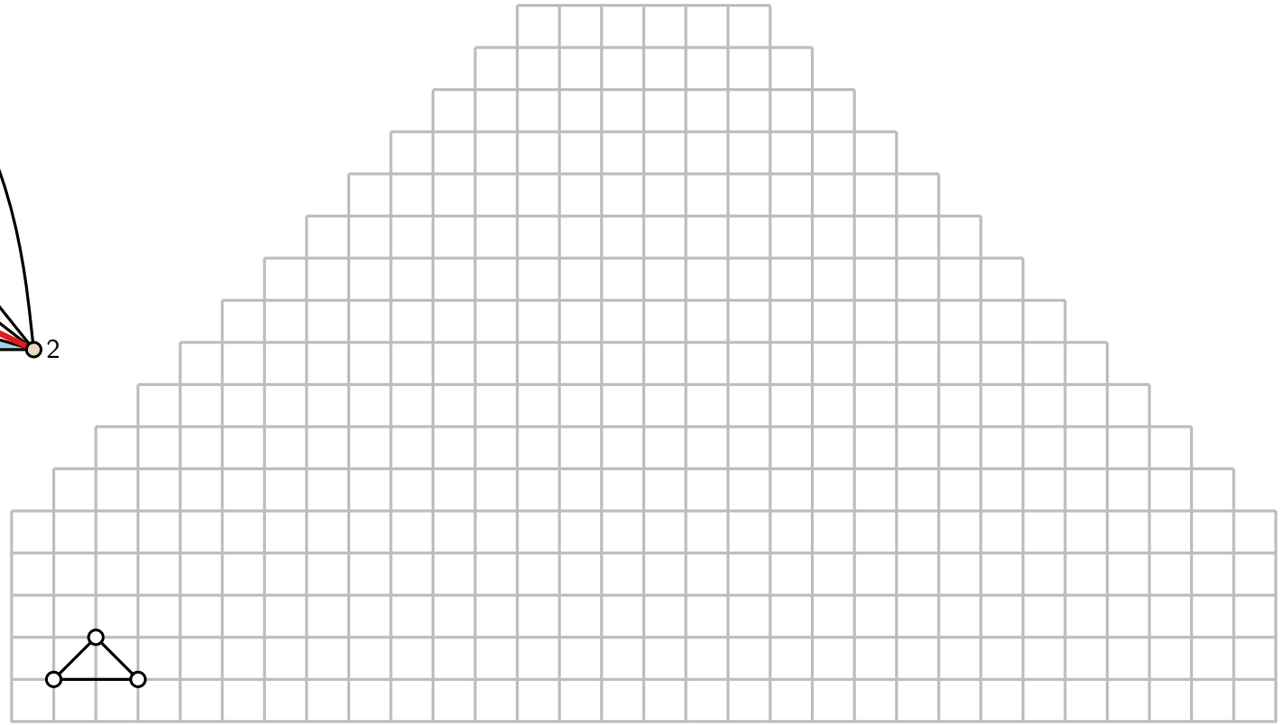




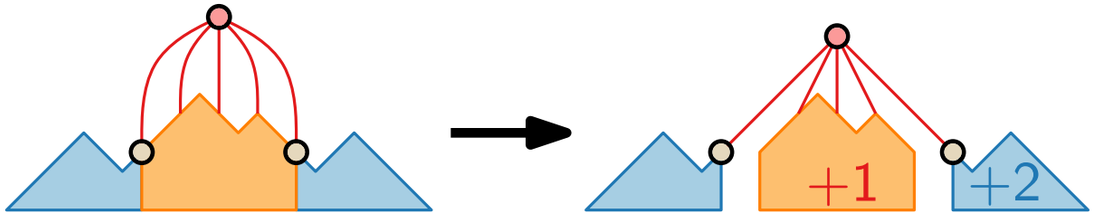
Shift Method – Example

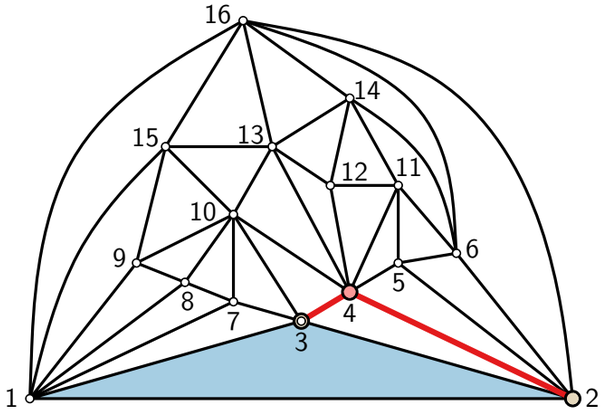


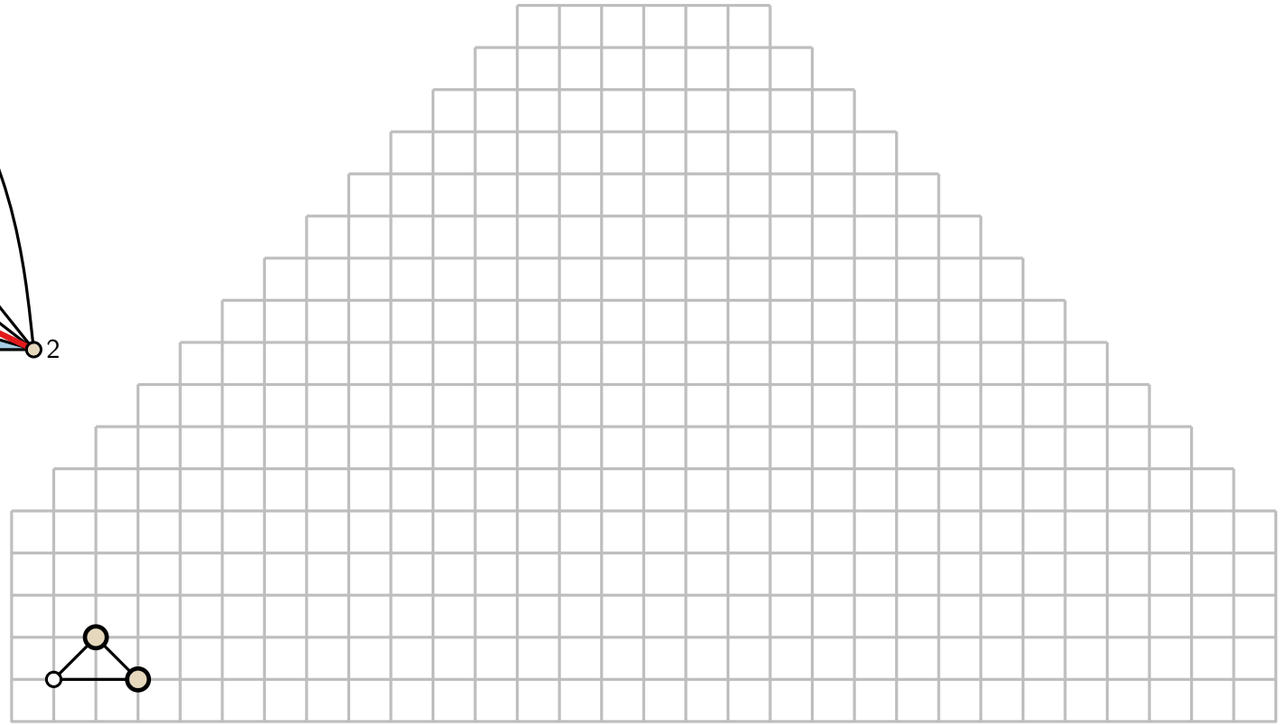




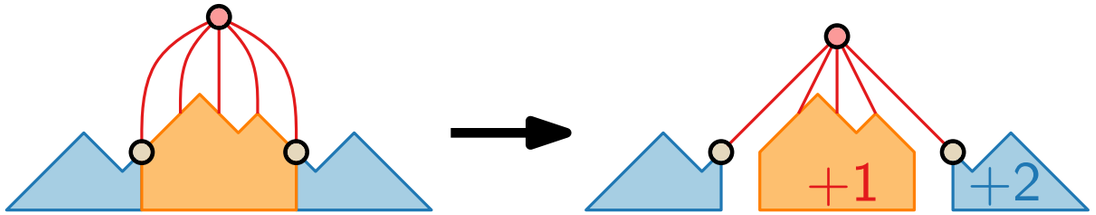
Shift Method – Example

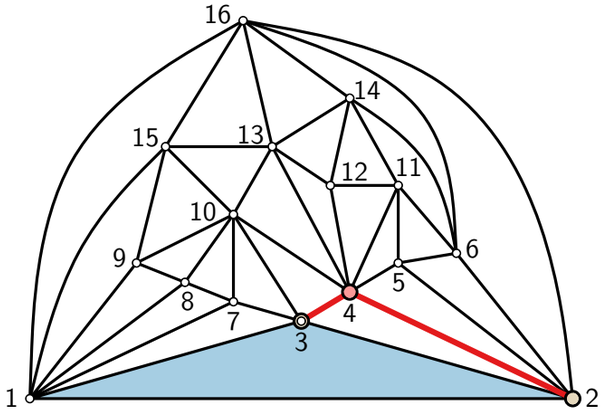


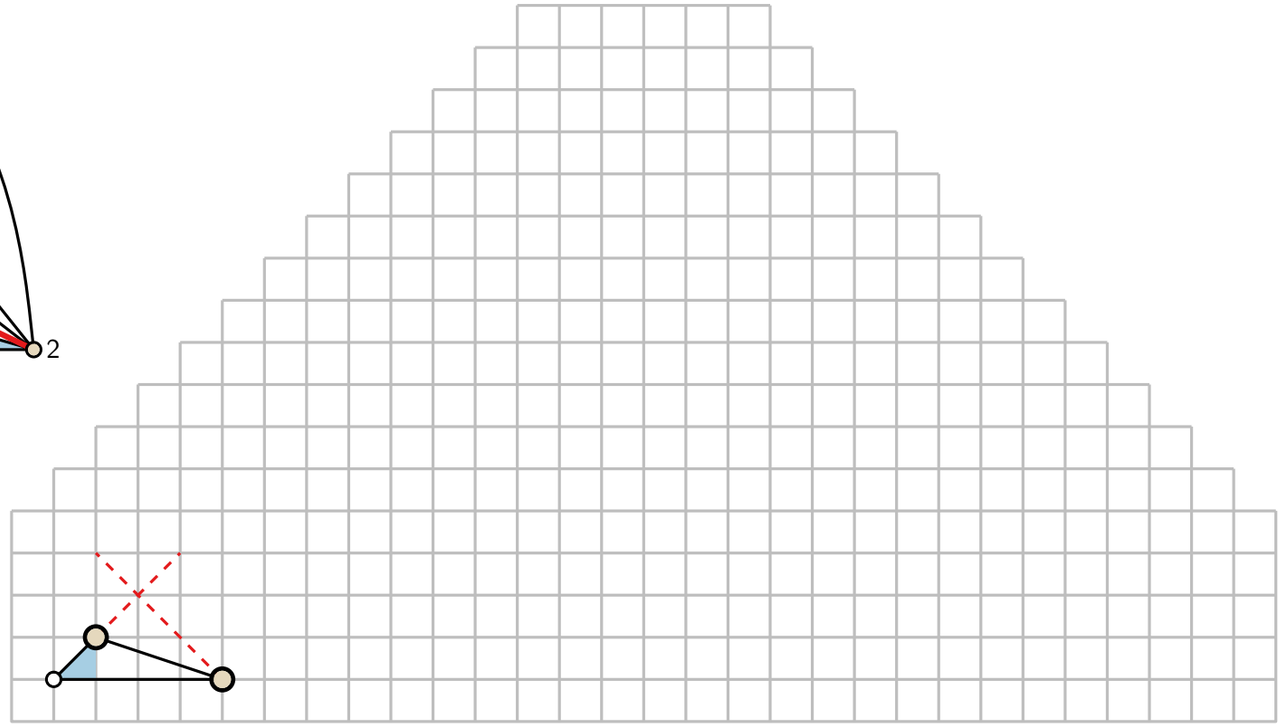




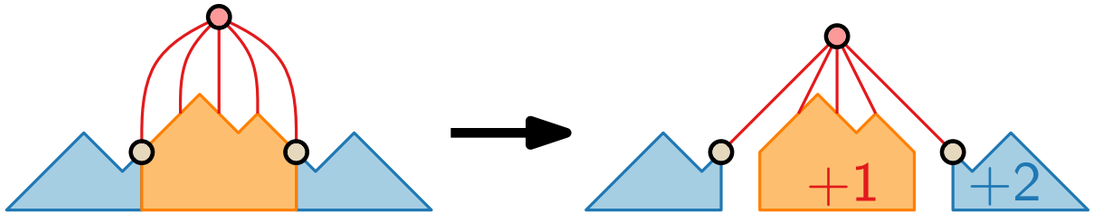
Shift Method – Example

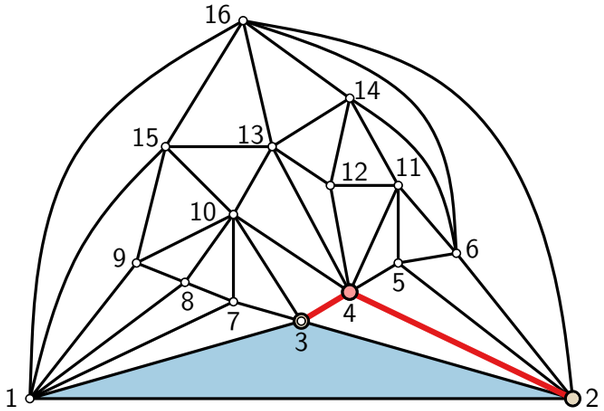


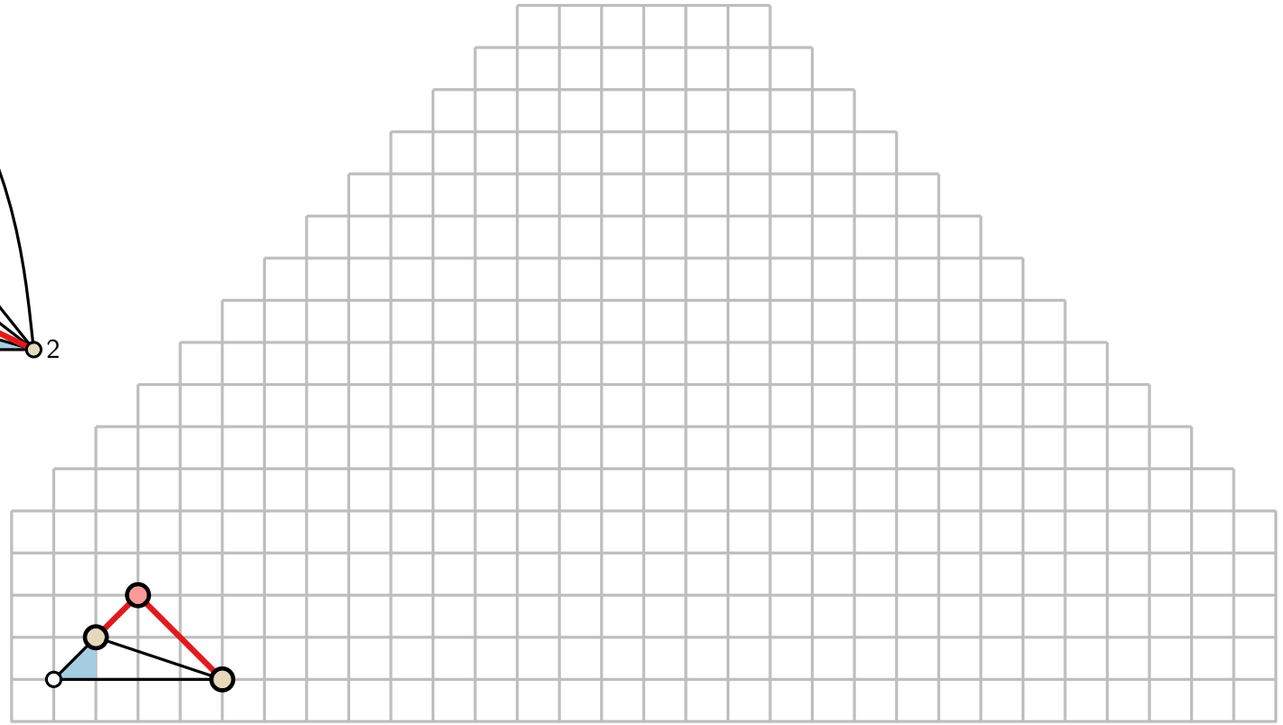




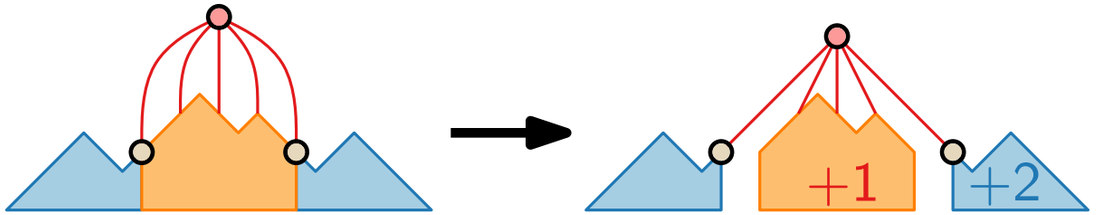
Shift Method – Example

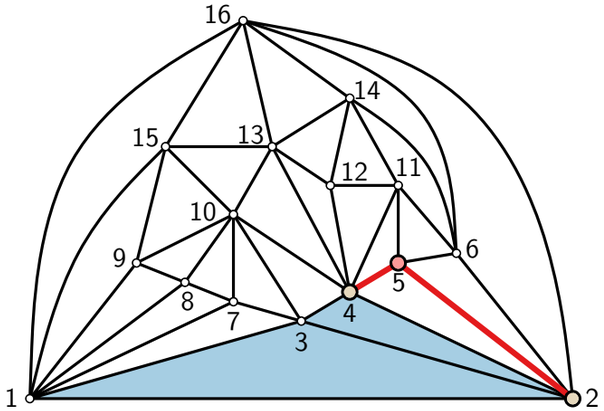


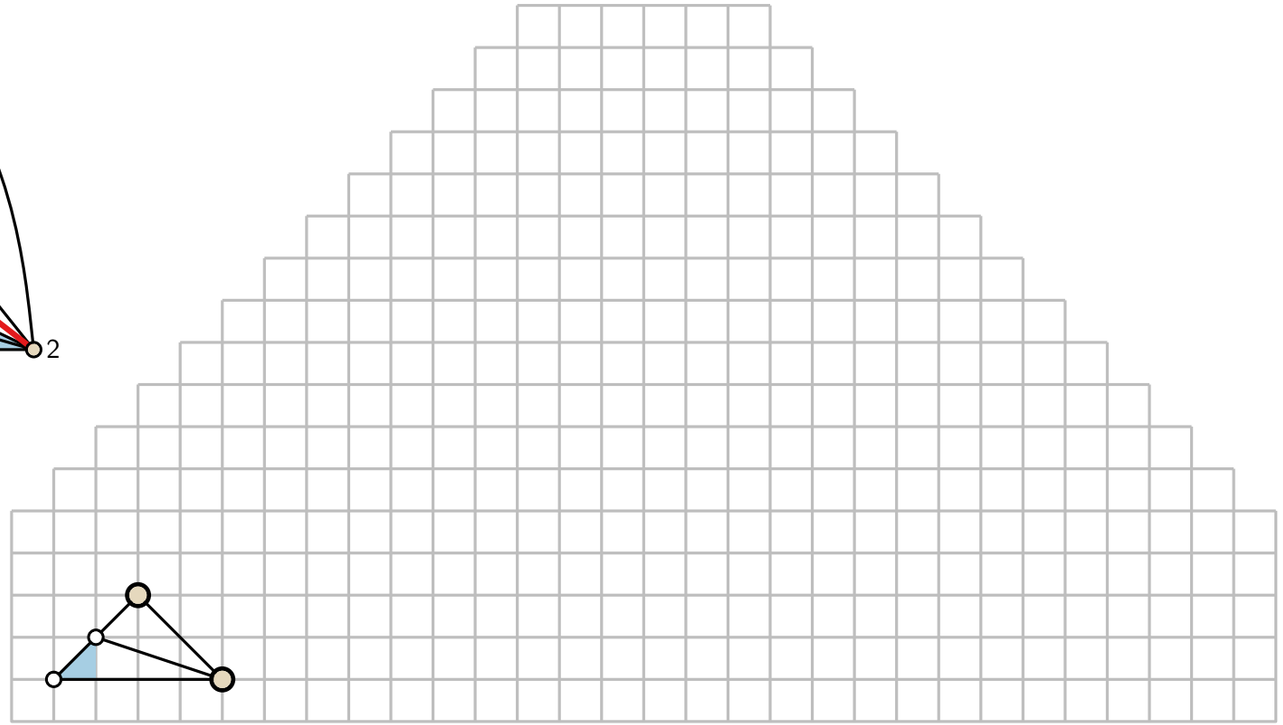




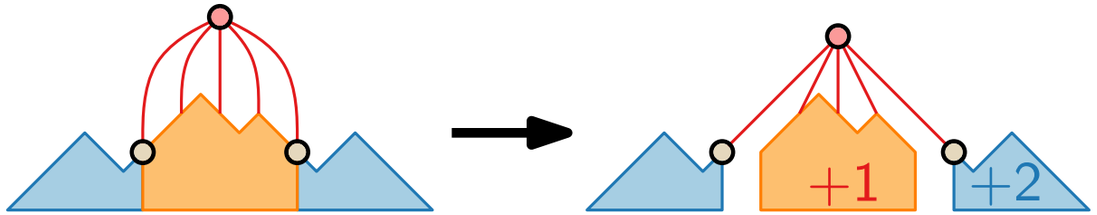
Shift Method – Example

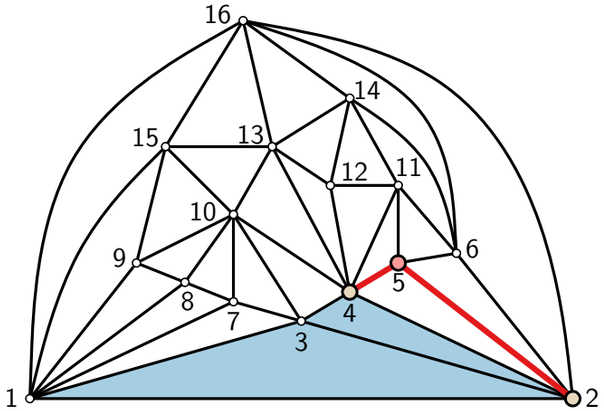


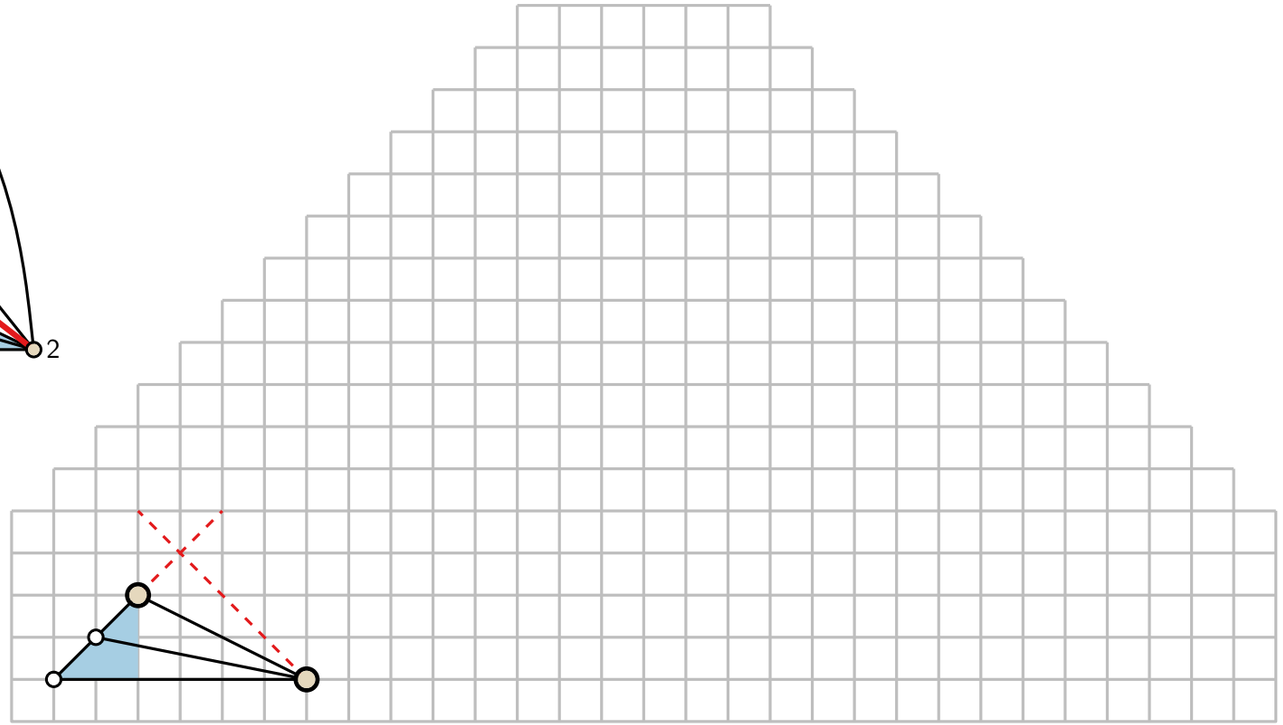




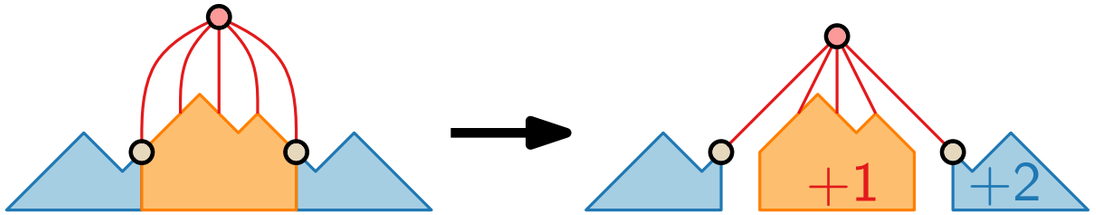
Shift Method – Example

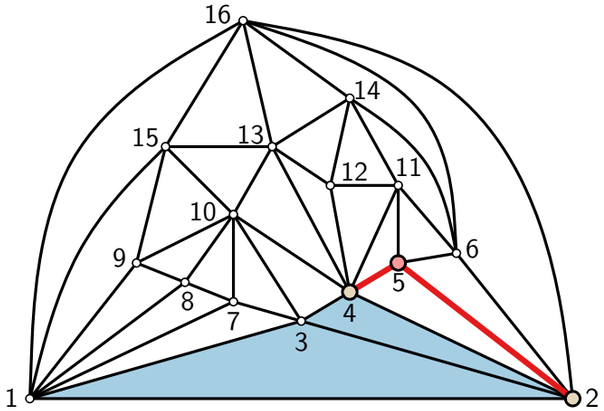


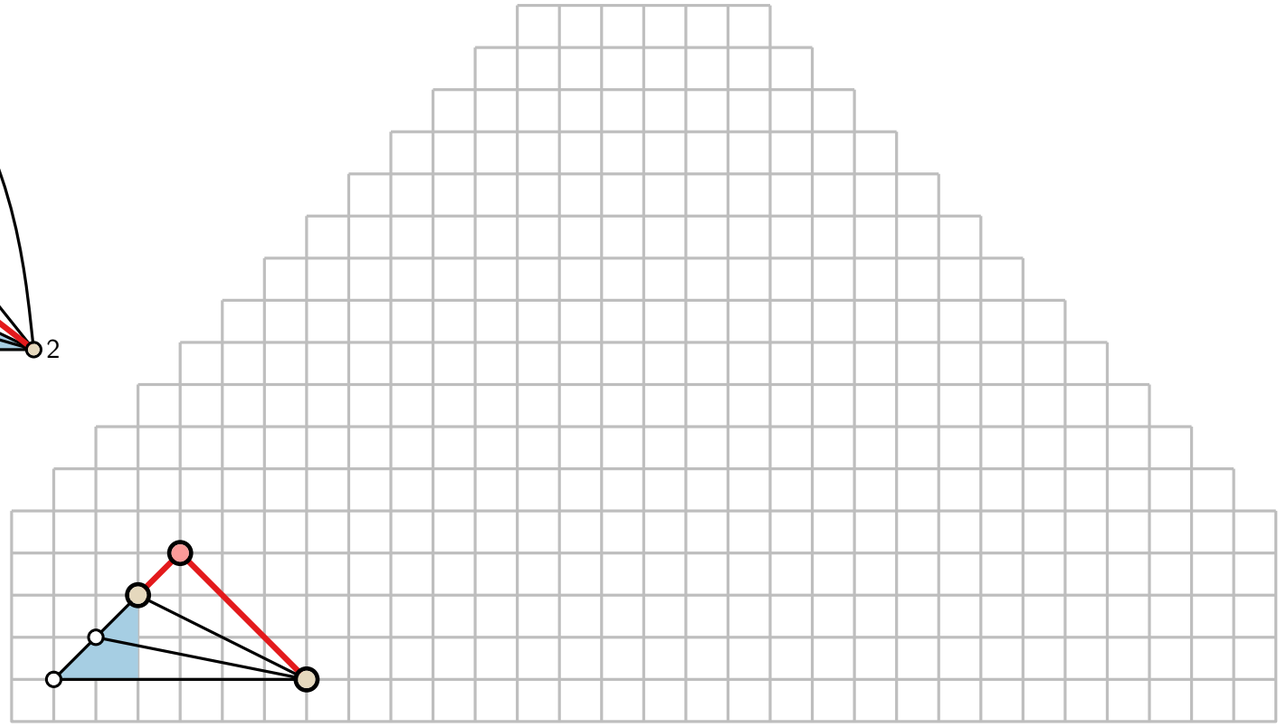




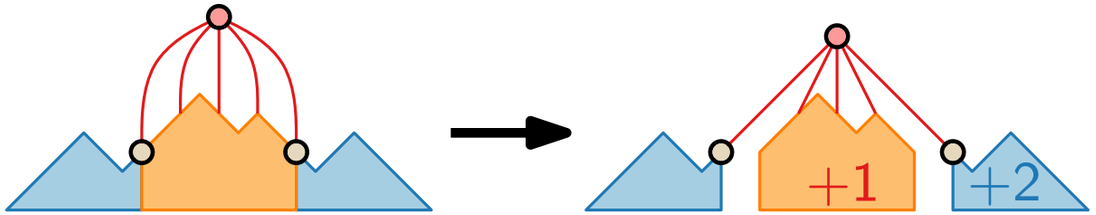
Shift Method – Example

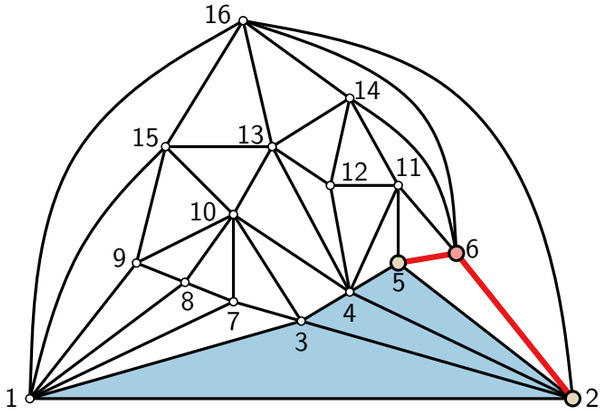


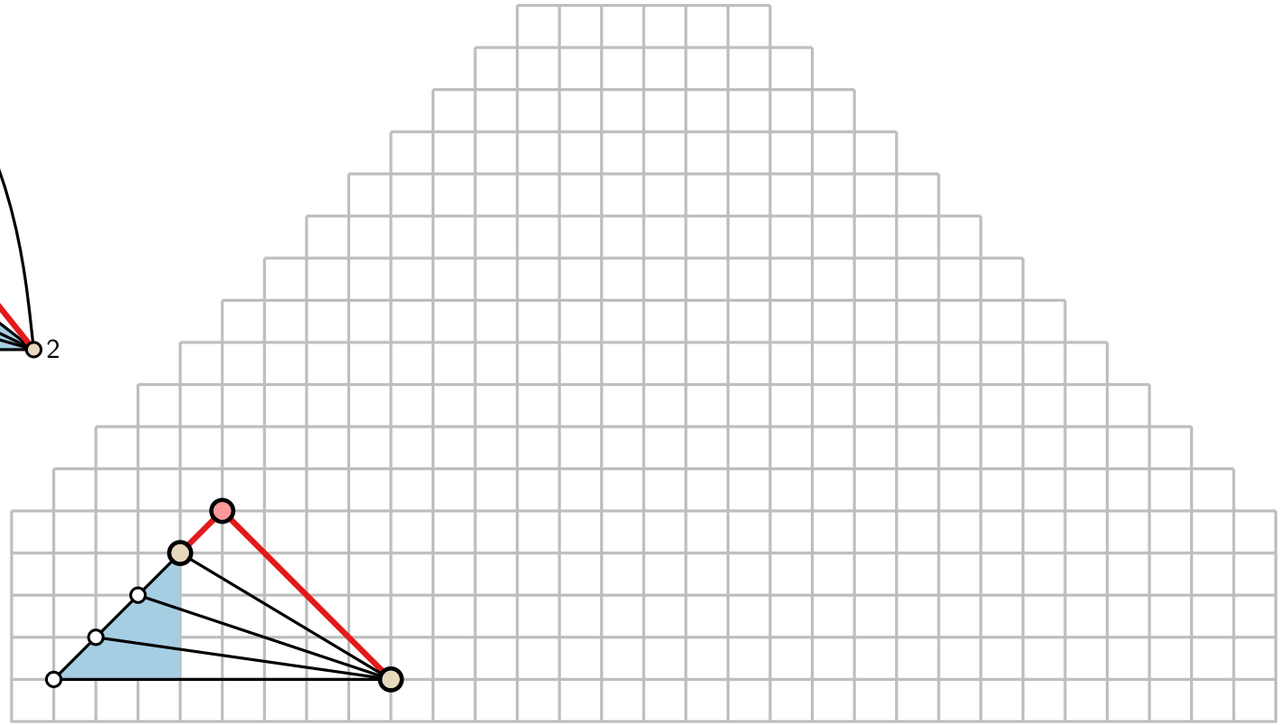




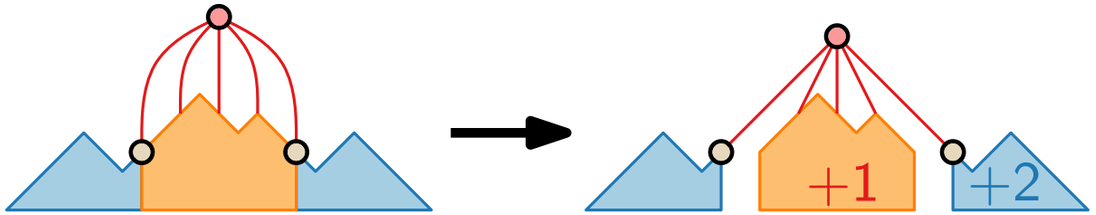
Shift Method – Example

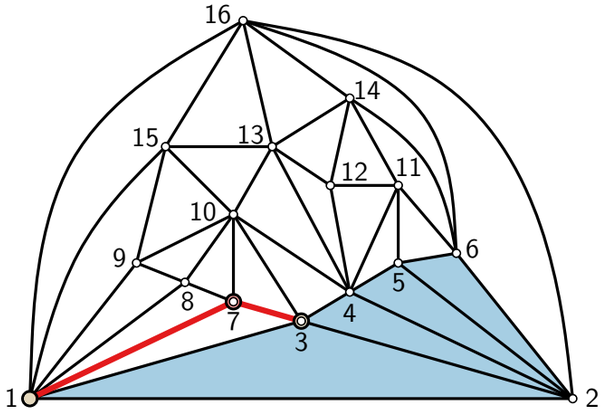


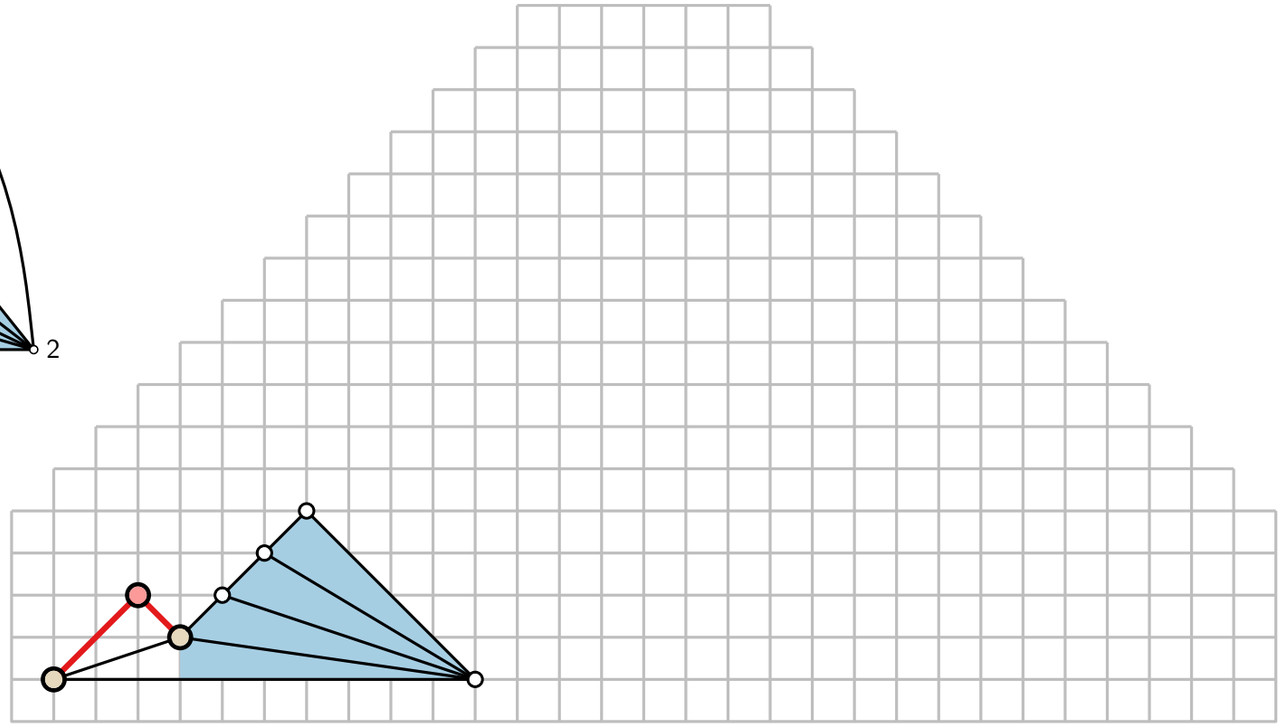




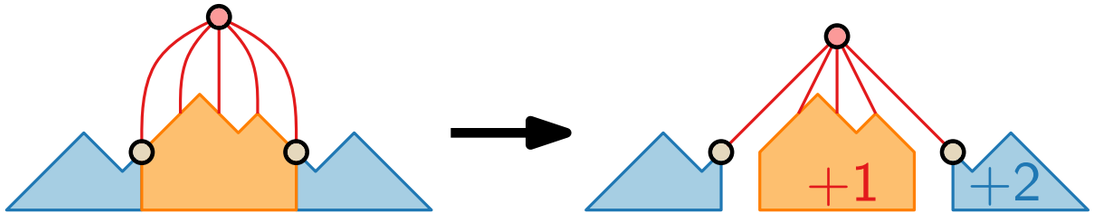
Shift Method – Example

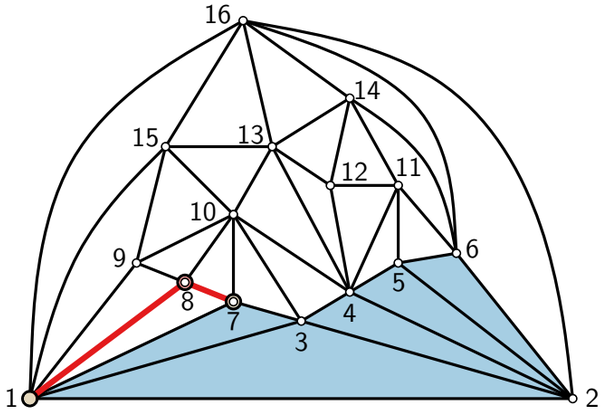


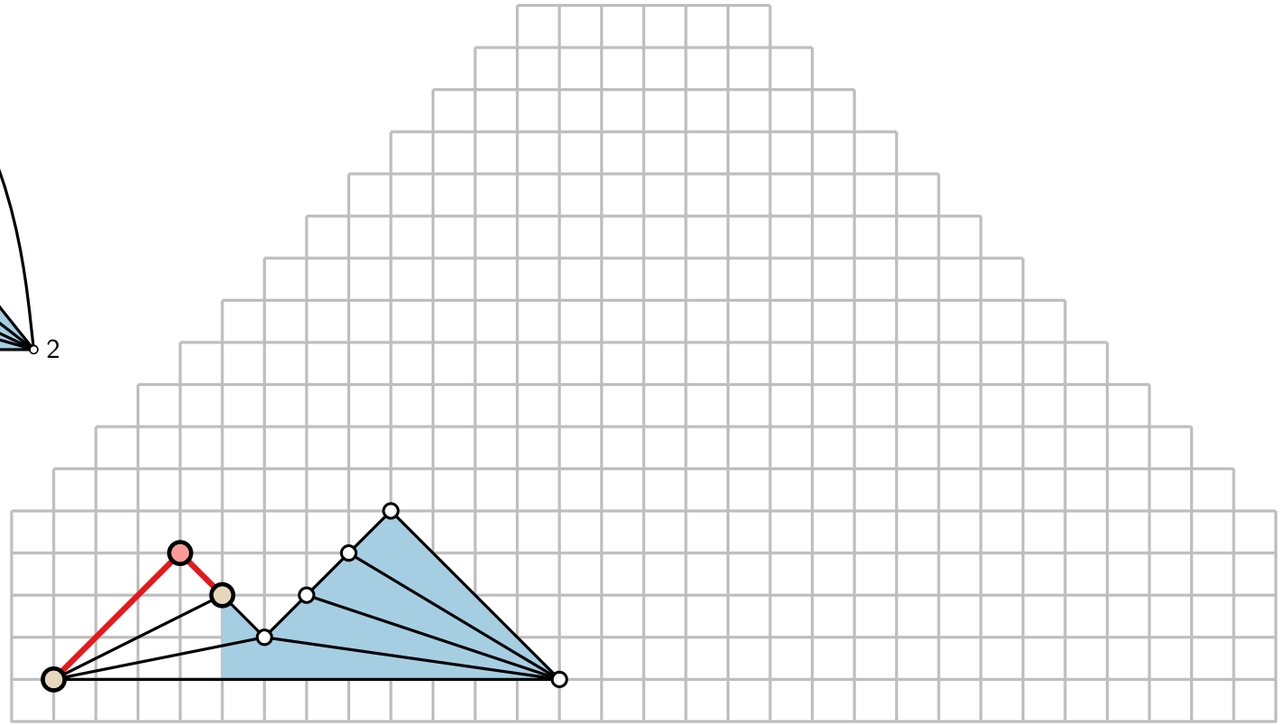




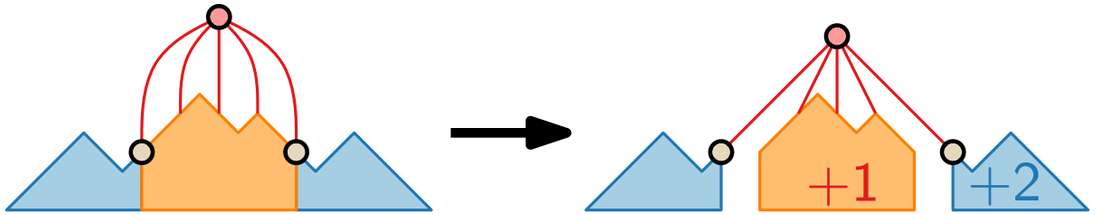
Shift Method – Example

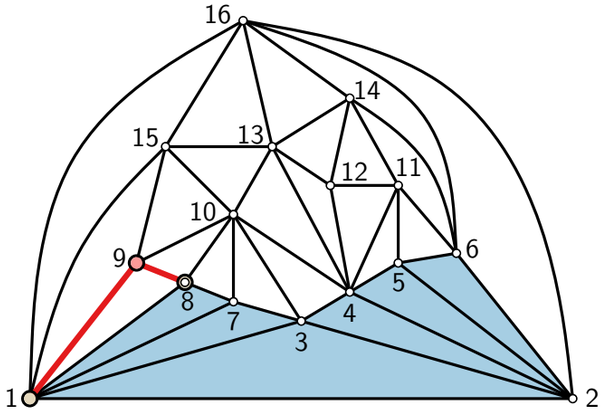


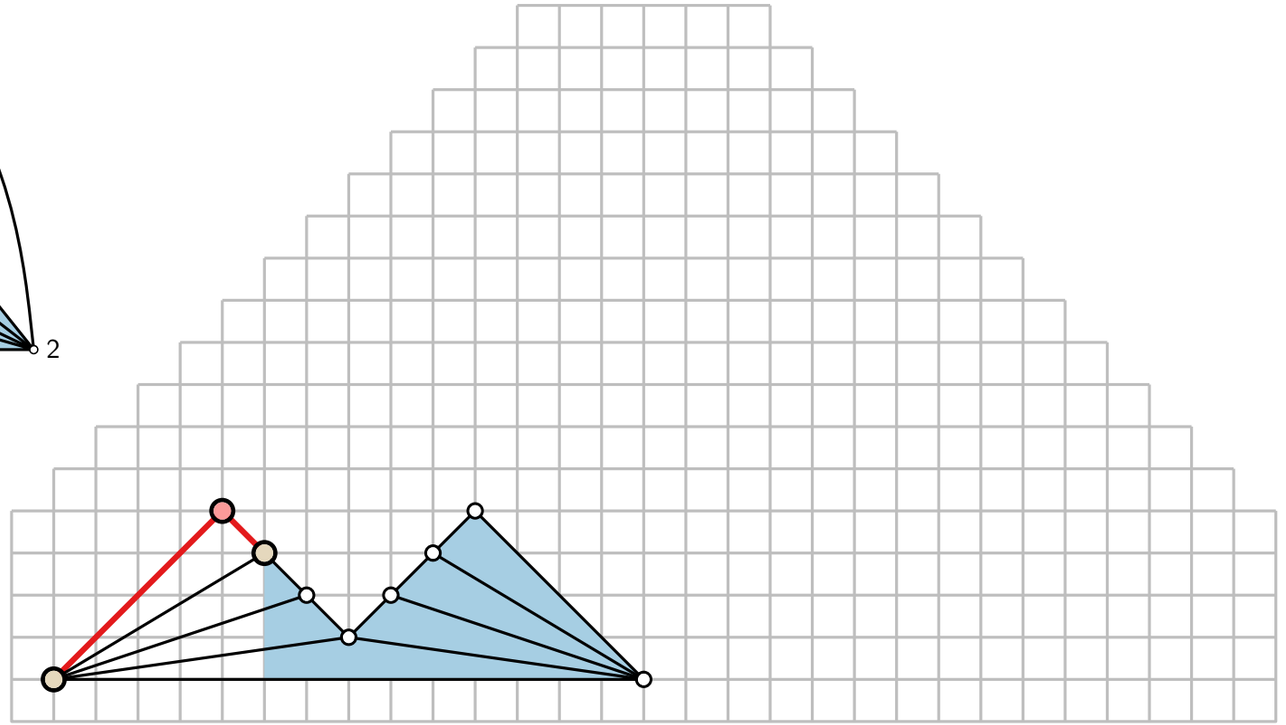




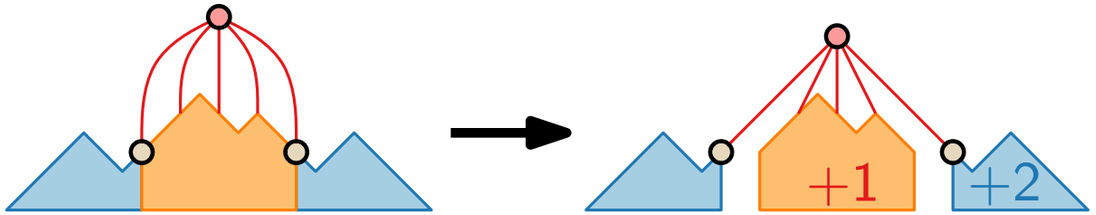
Shift Method – Example

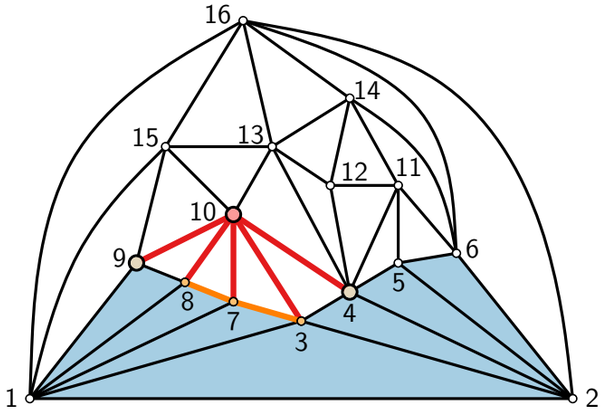


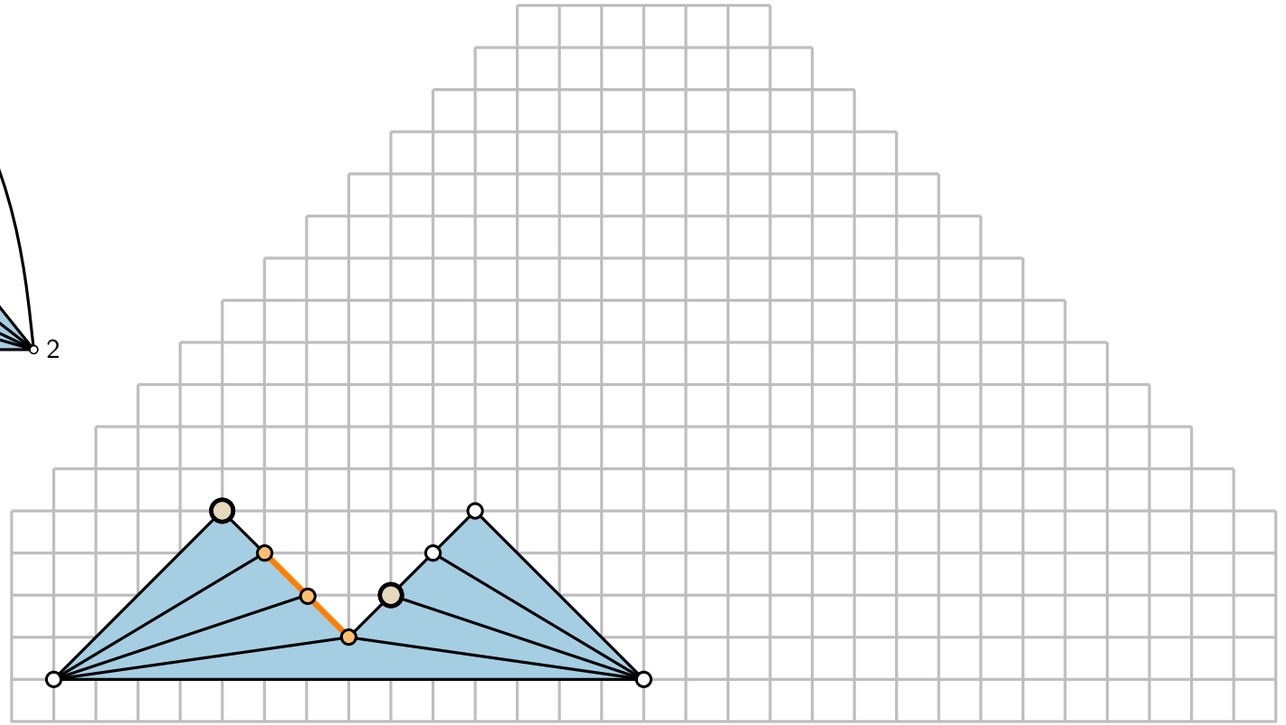




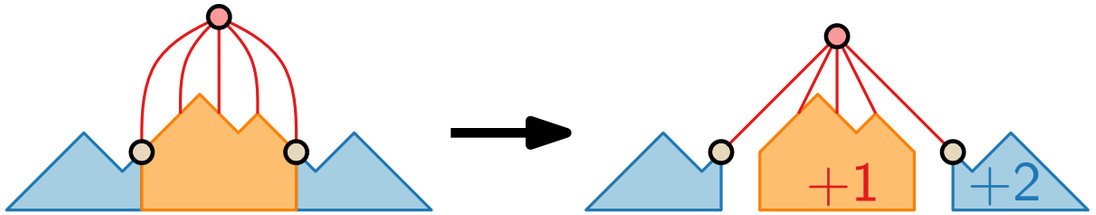
Shift Method – Example

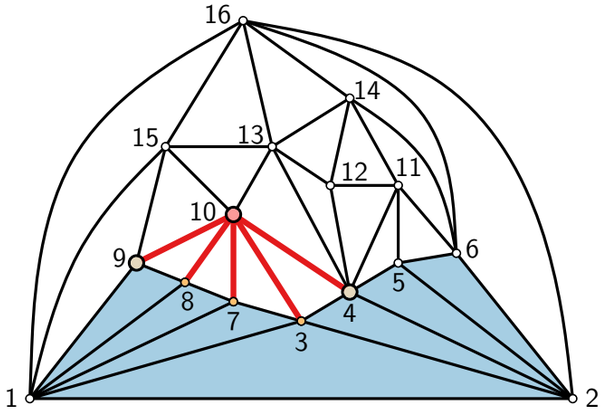


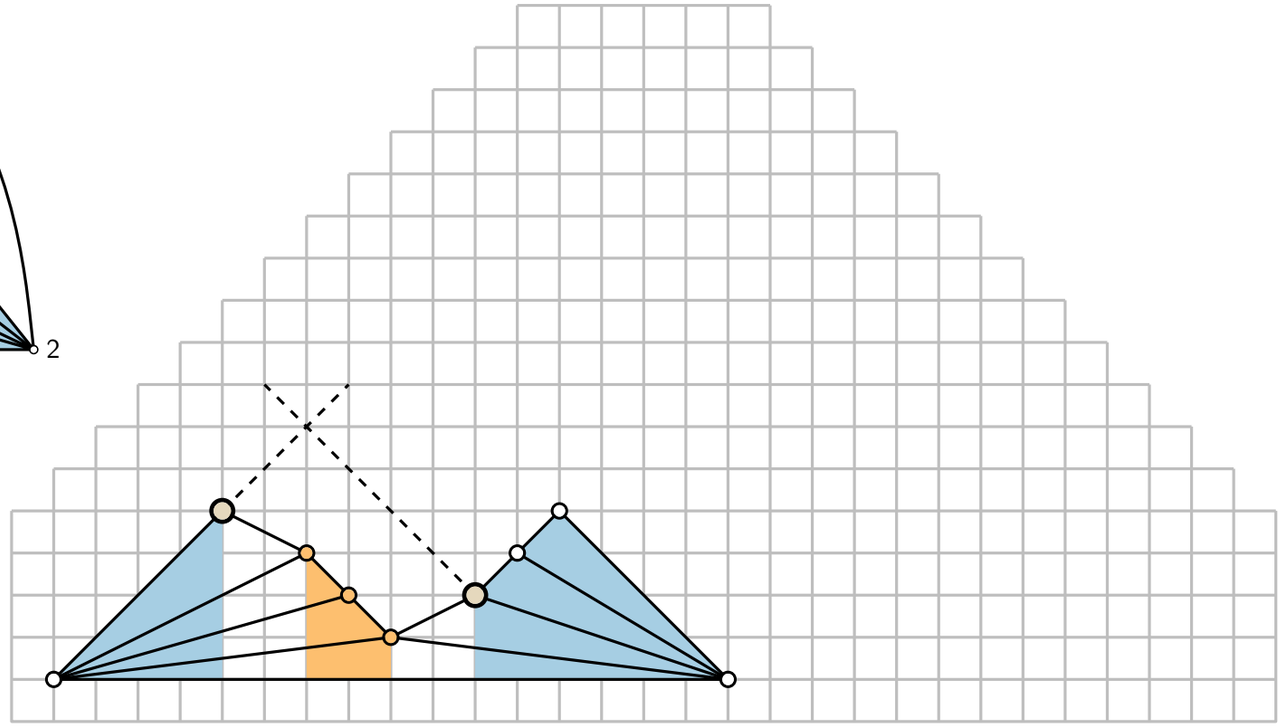




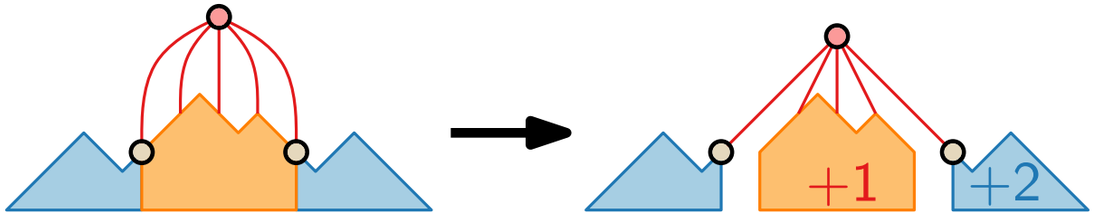
Shift Method – Example

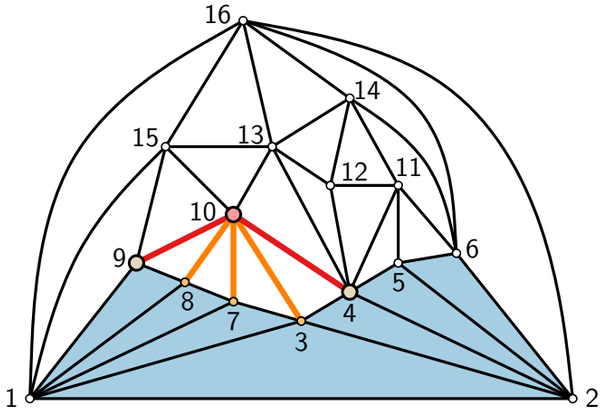


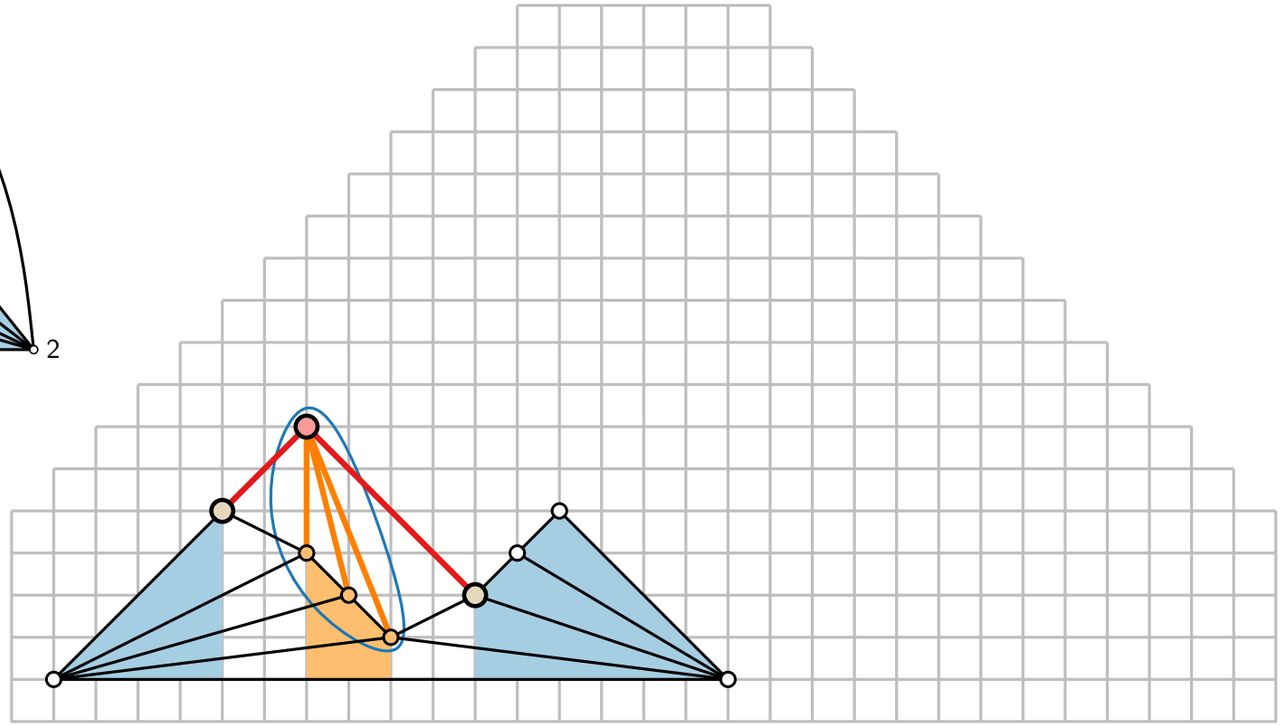




Shift Method – Example

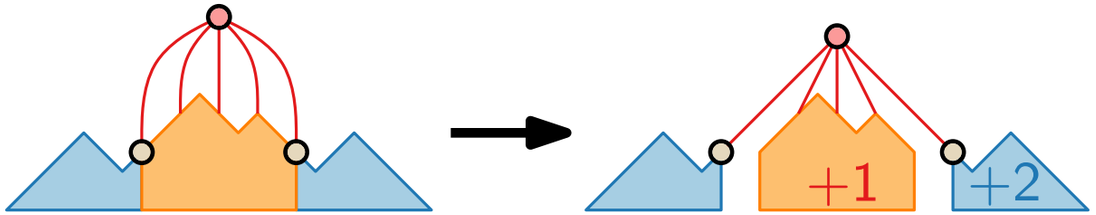


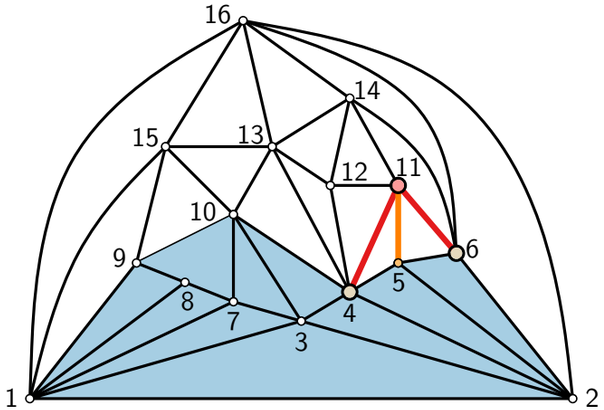


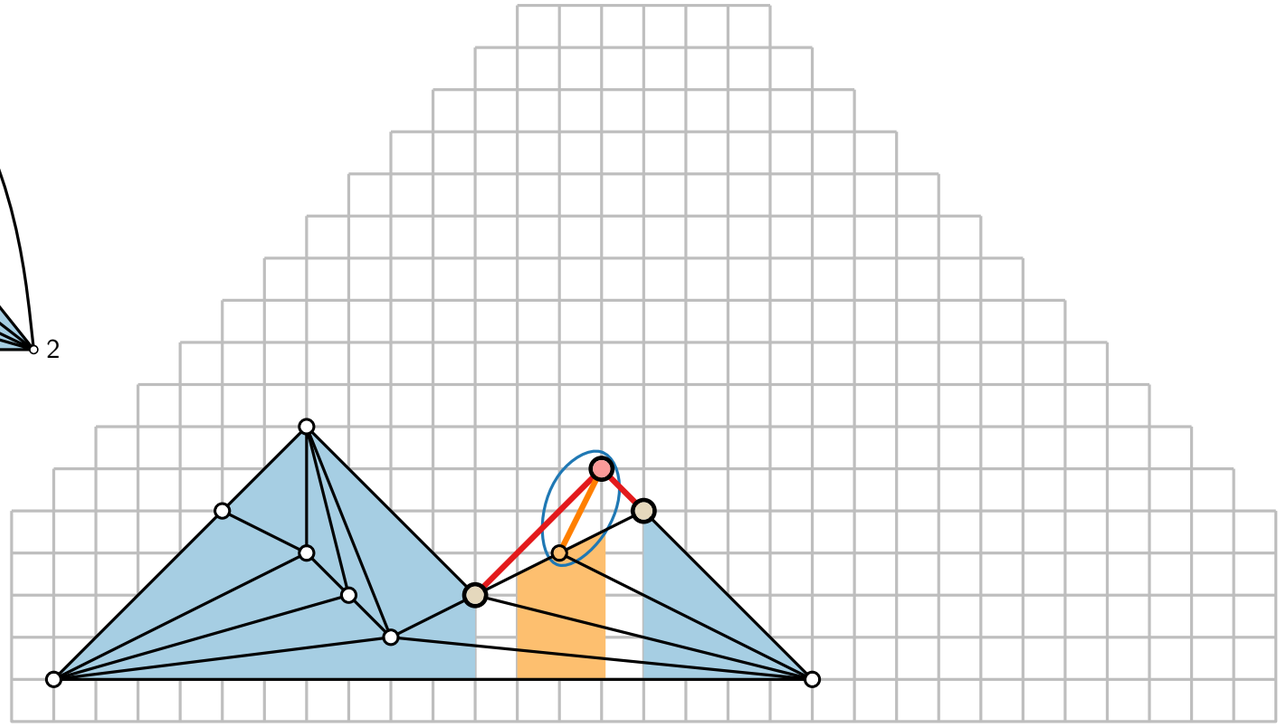


$L(10)$

Shift Method – Example

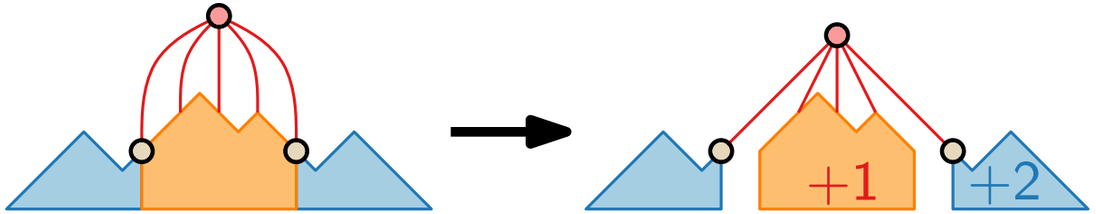


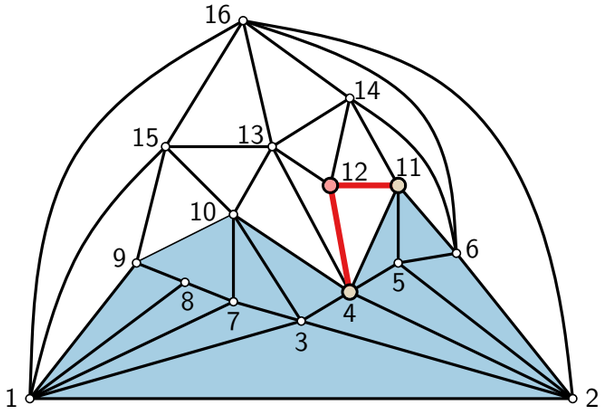


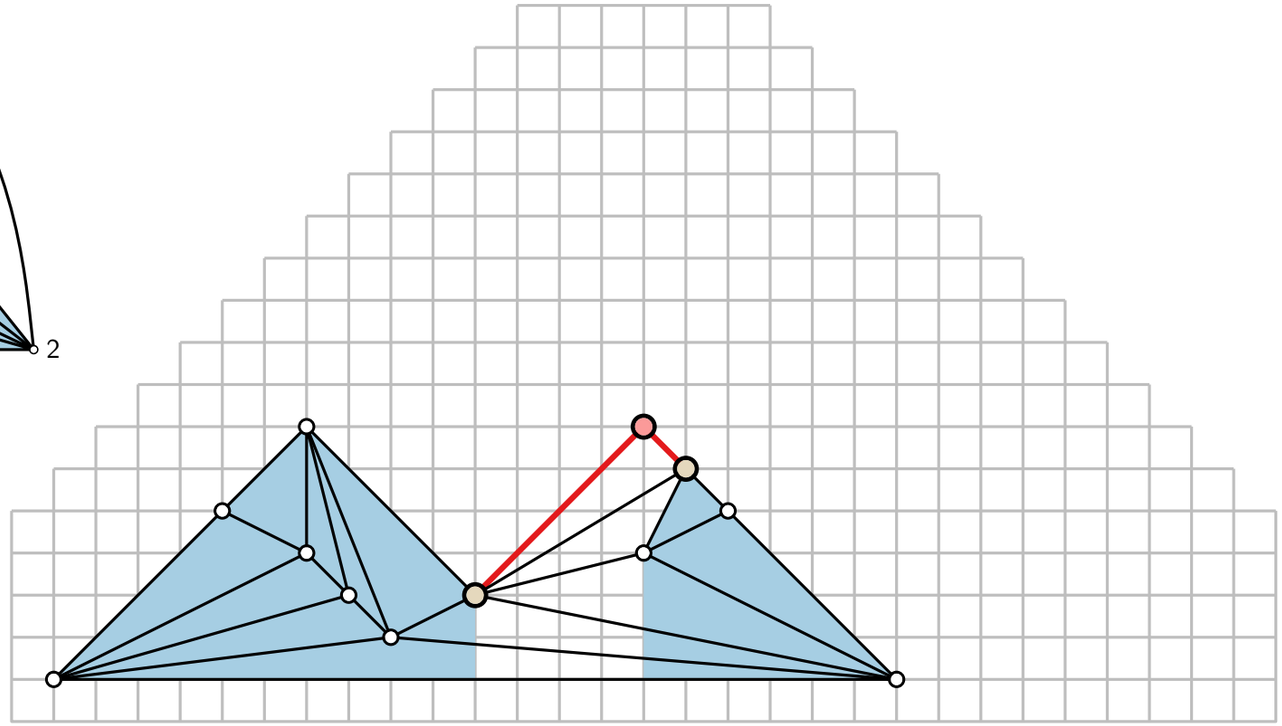


$L(11)$

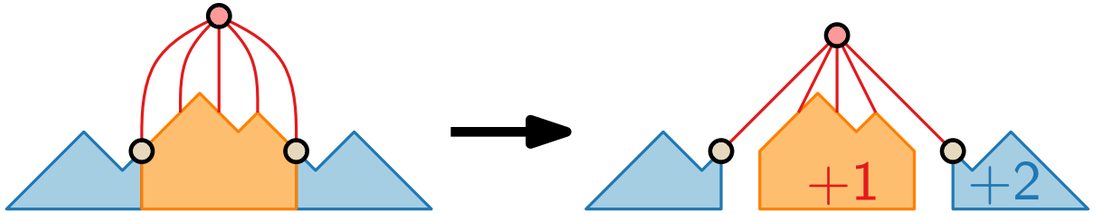
Shift Method – Example

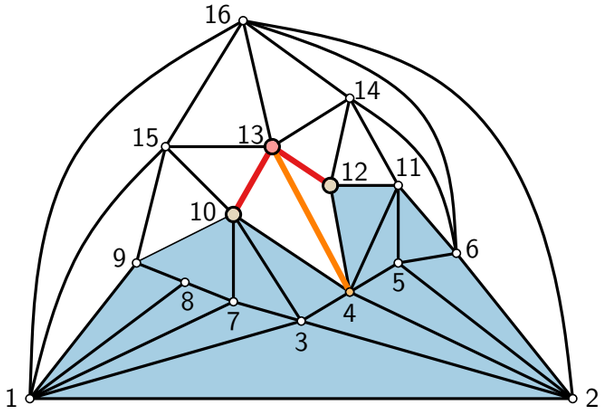


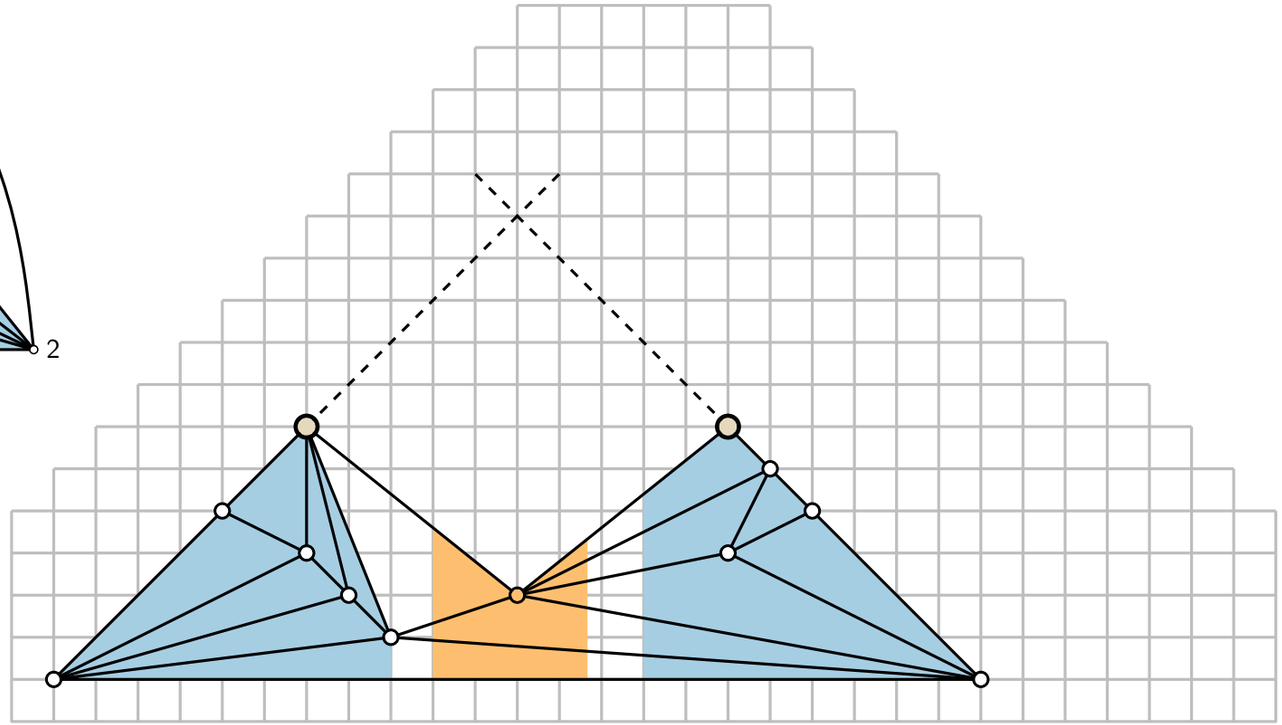




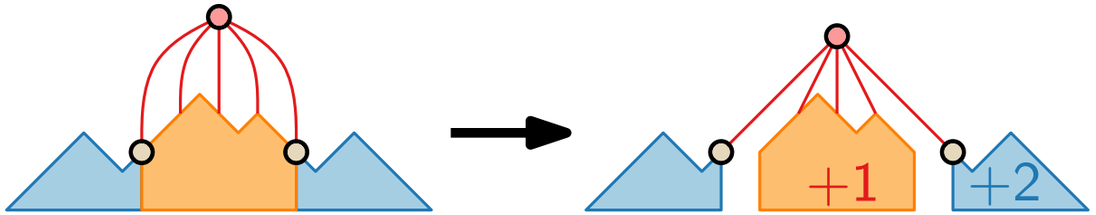
Shift Method – Example

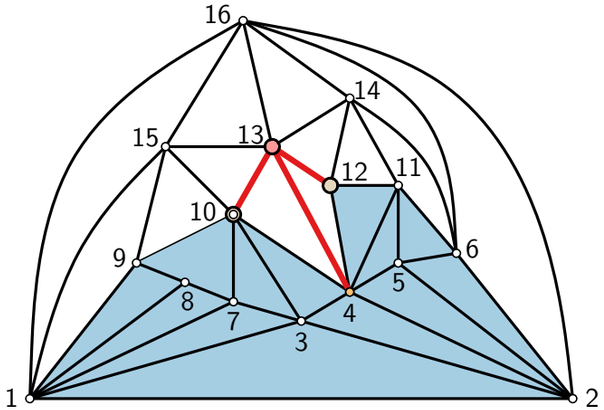


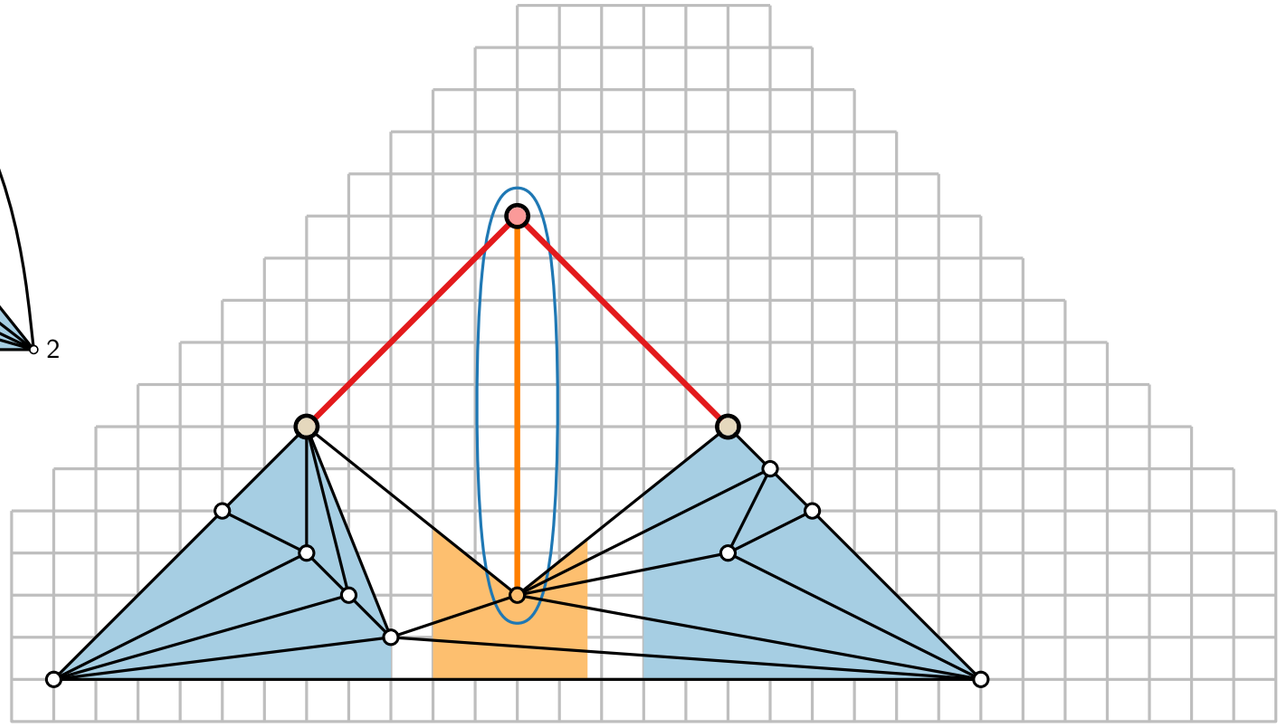




Shift Method – Example

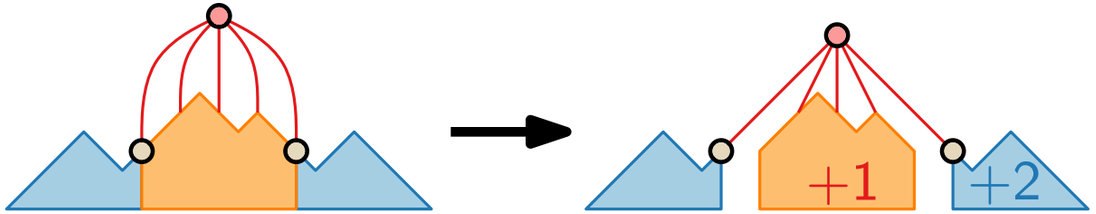


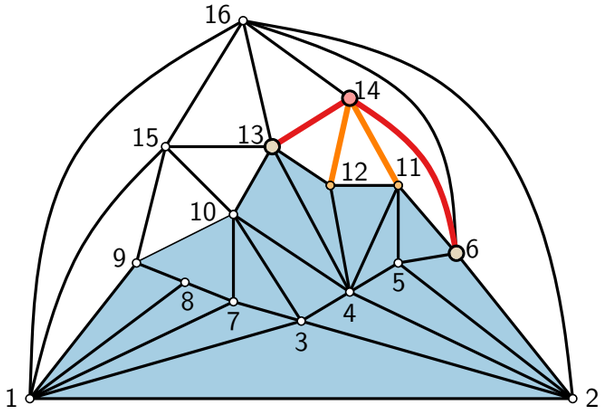


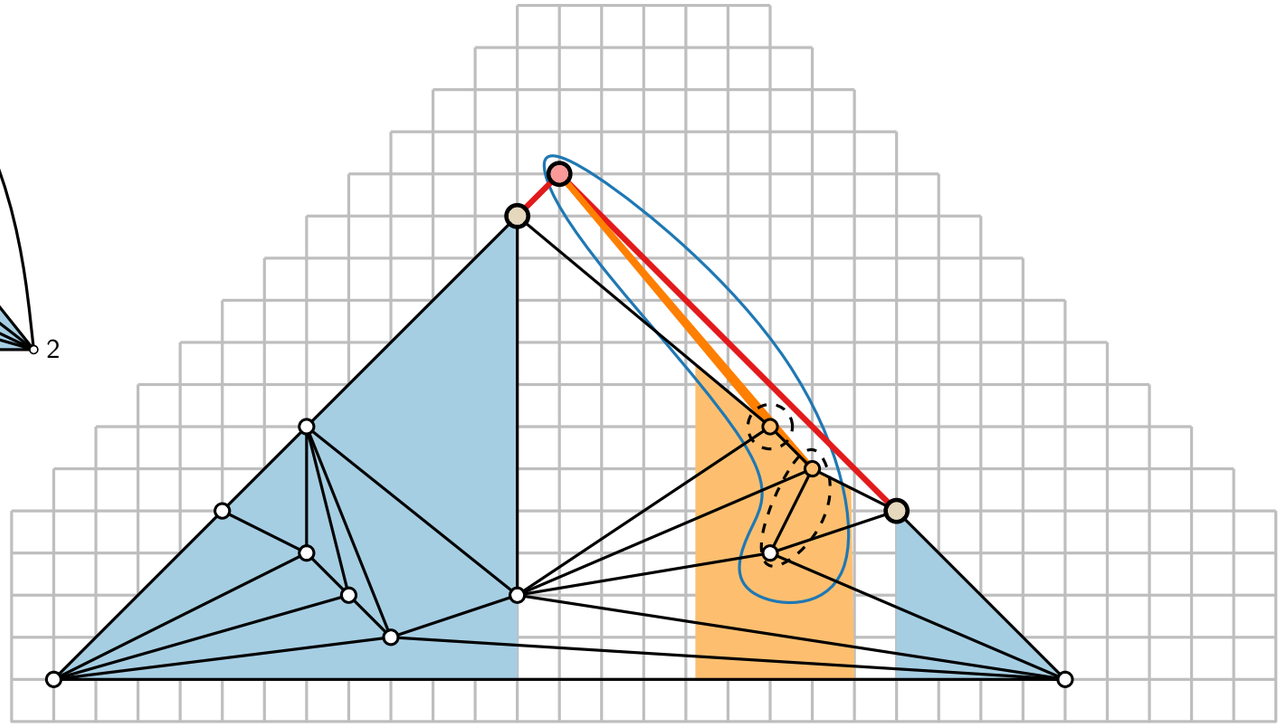


$L(13)$

Shift Method – Example

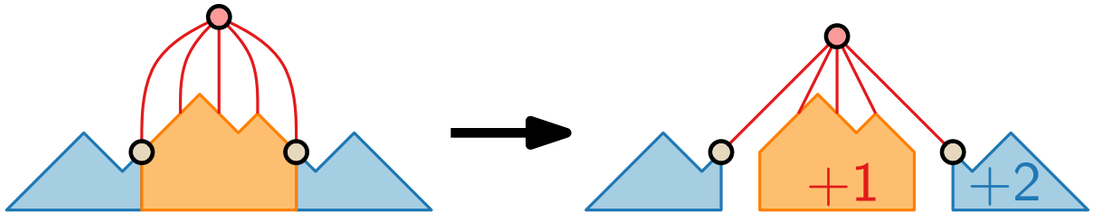


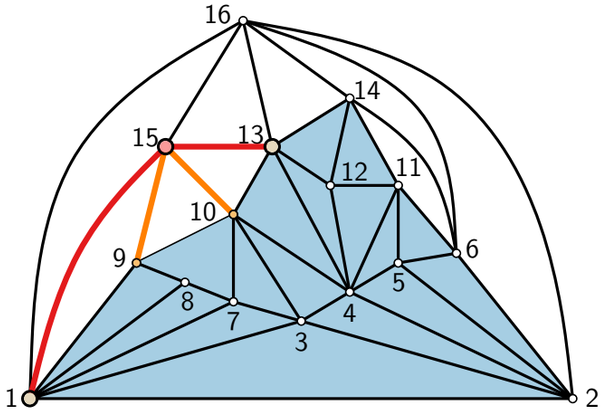


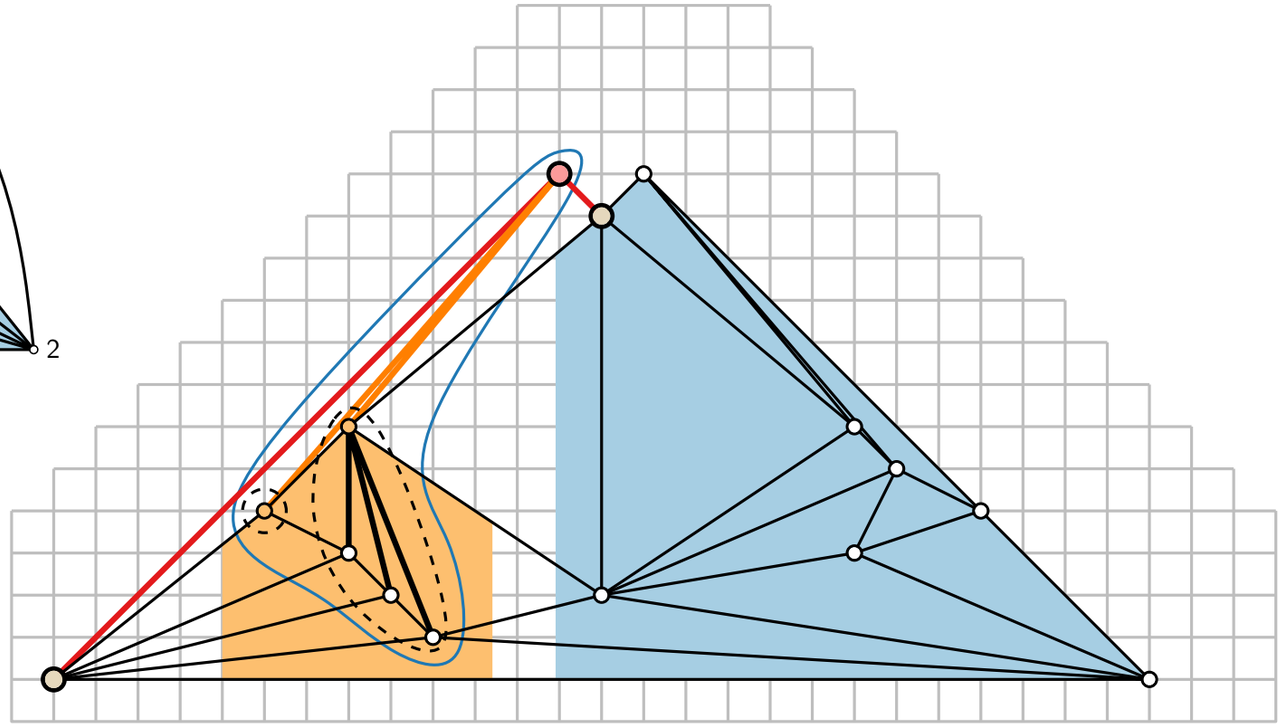


$L(14)$

Shift Method – Example

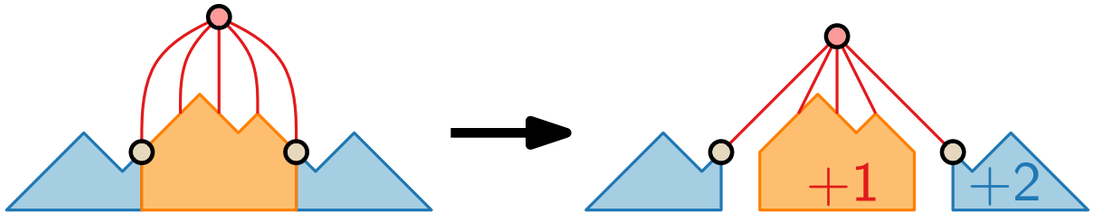


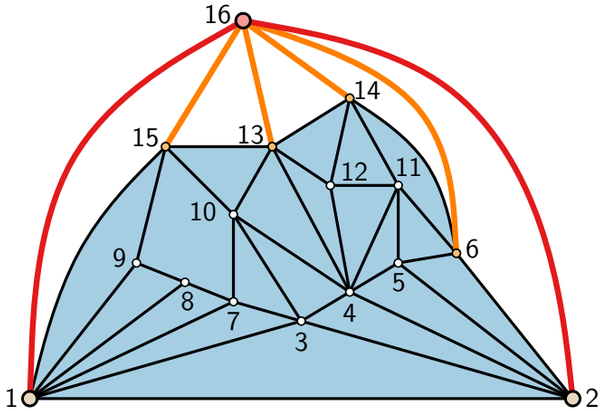


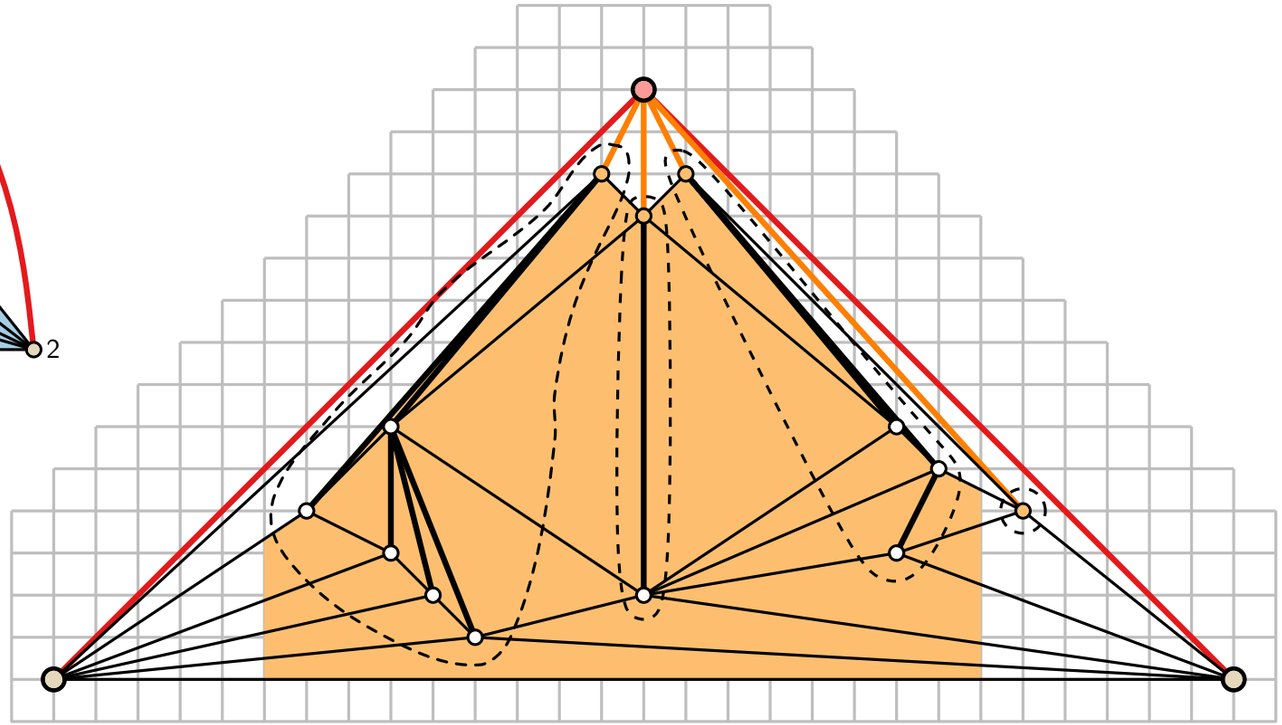


$L(15)$

Shift Method – Example

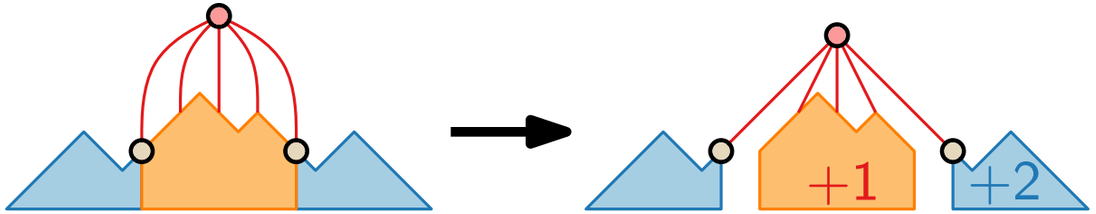


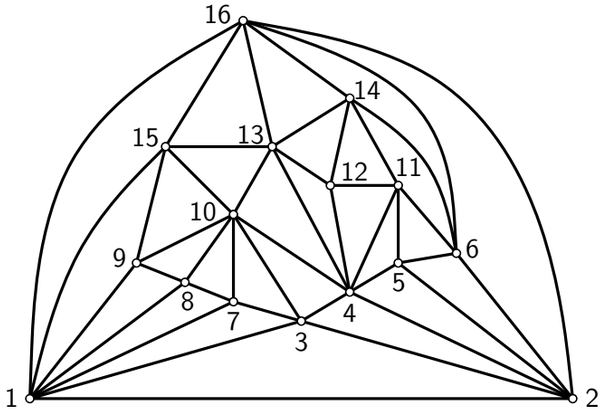


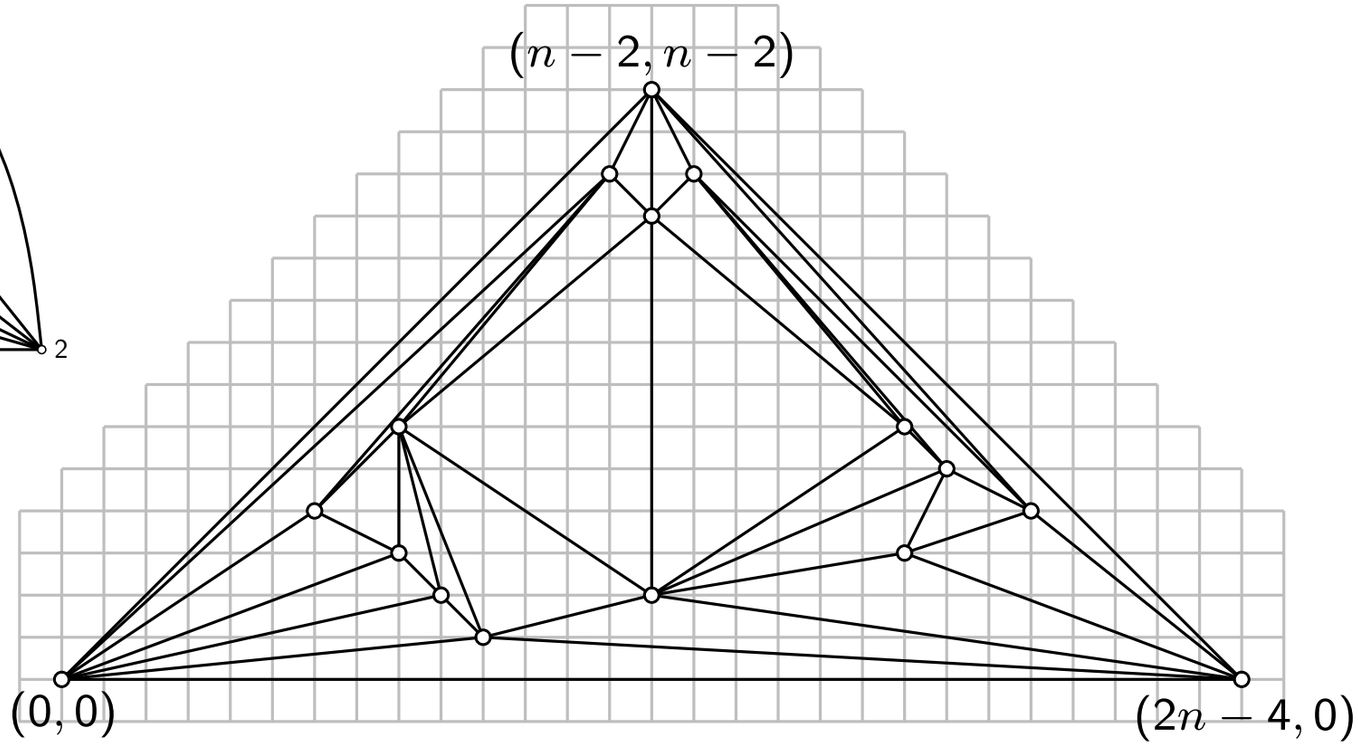


$L(16)$

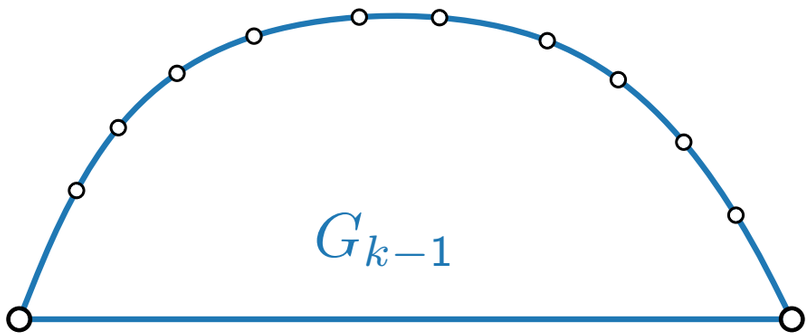
Shift Method – Example



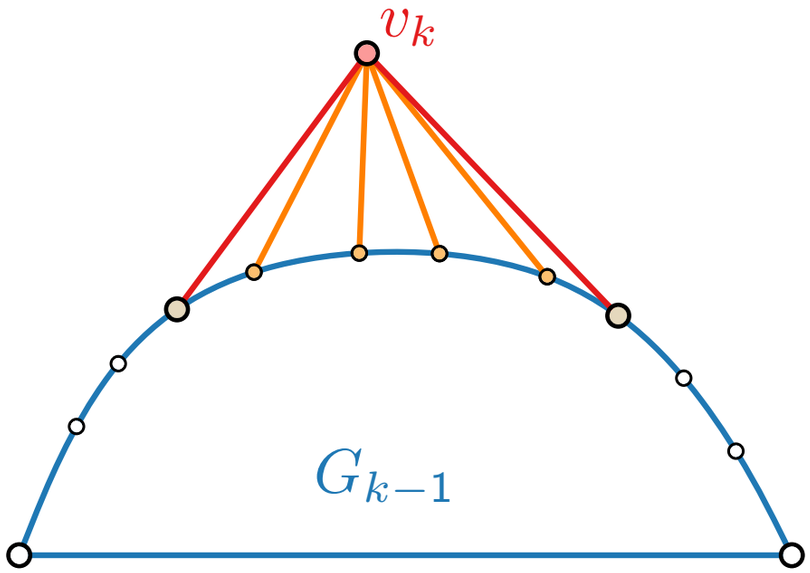




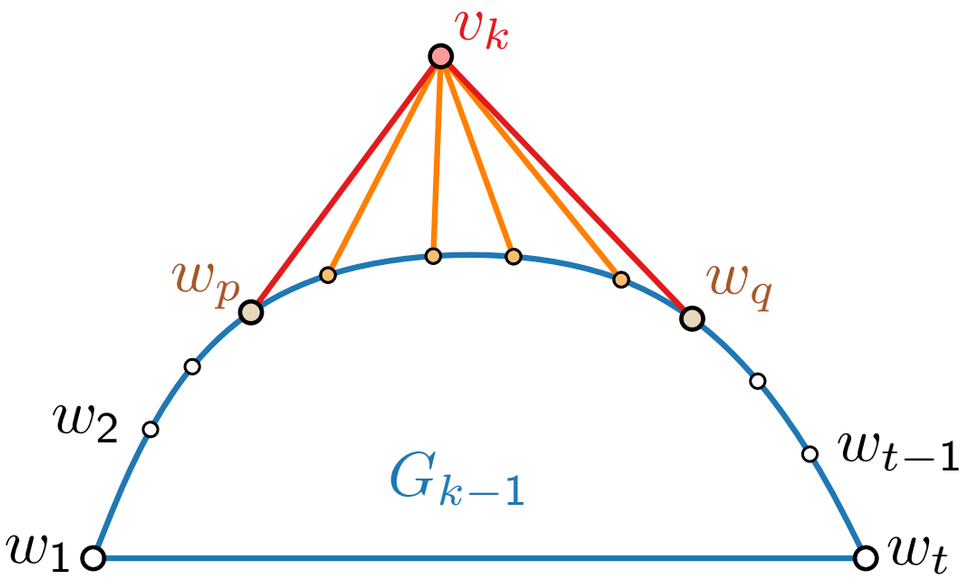
Shift Method – Planarity



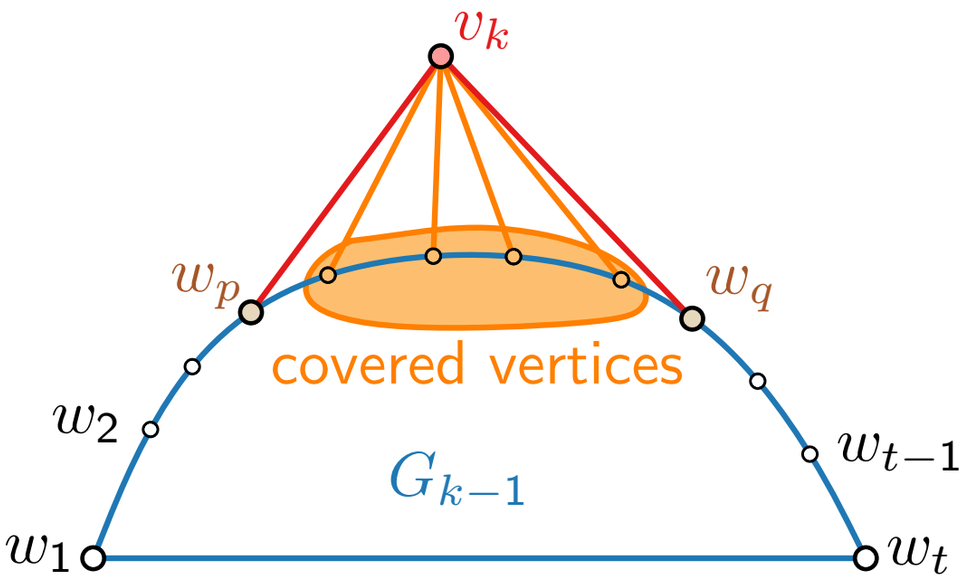
Shift Method – Planarity



Shift Method – Planarity



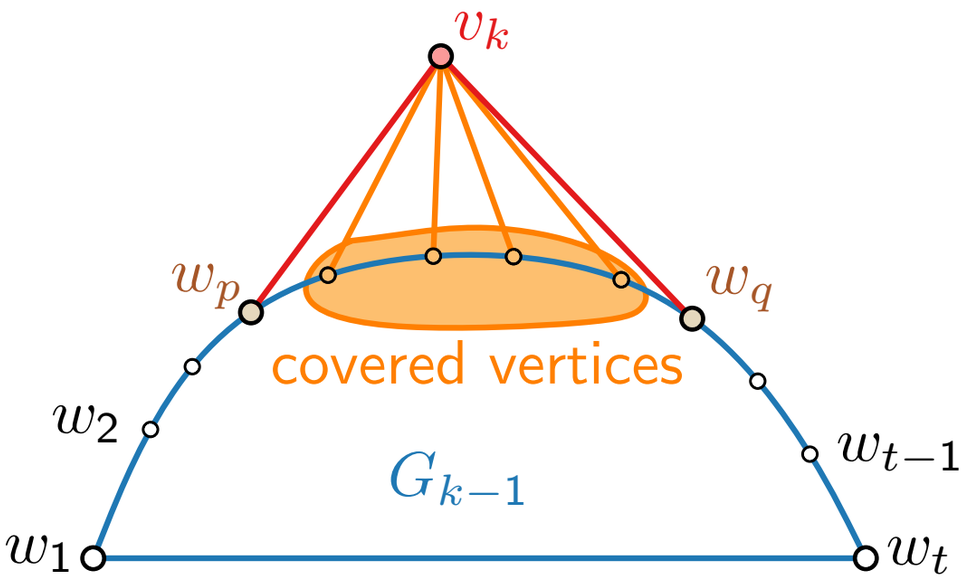
Shift Method – Planarity



Shift Method – Planarity

Observations.

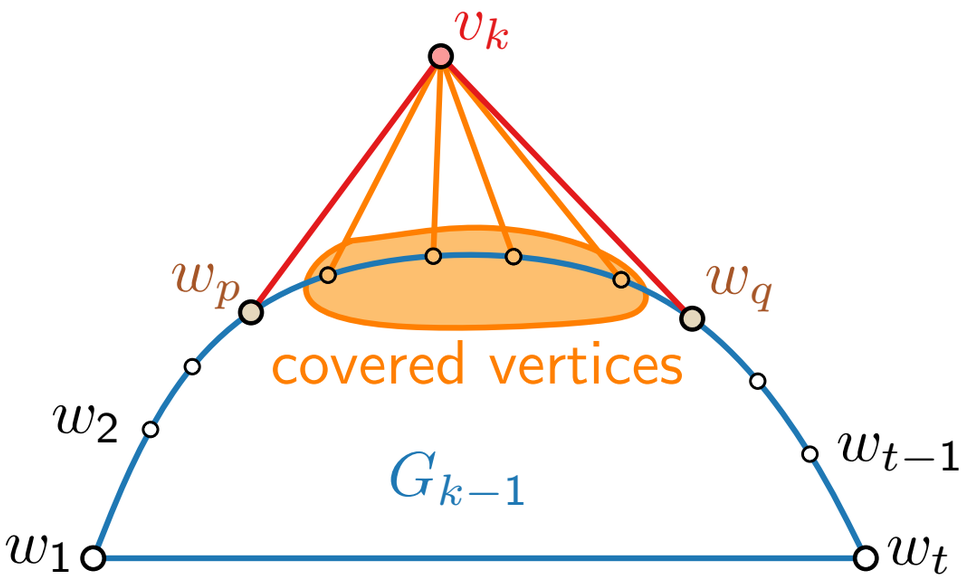
- Each internal vertex is covered exactly once.



Shift Method – Planarity

Observations.

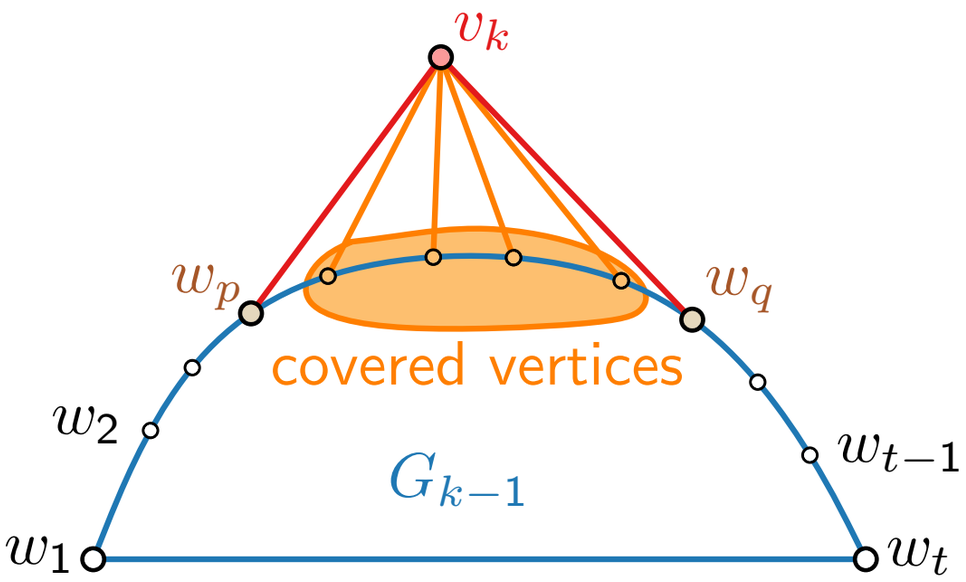
- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G



Shift Method – Planarity

Observations.

- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G
- and a forest in G_i, $1 \leq i \leq n-1$.



Shift Method – Planarity

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- Covering relation defines a tree in G
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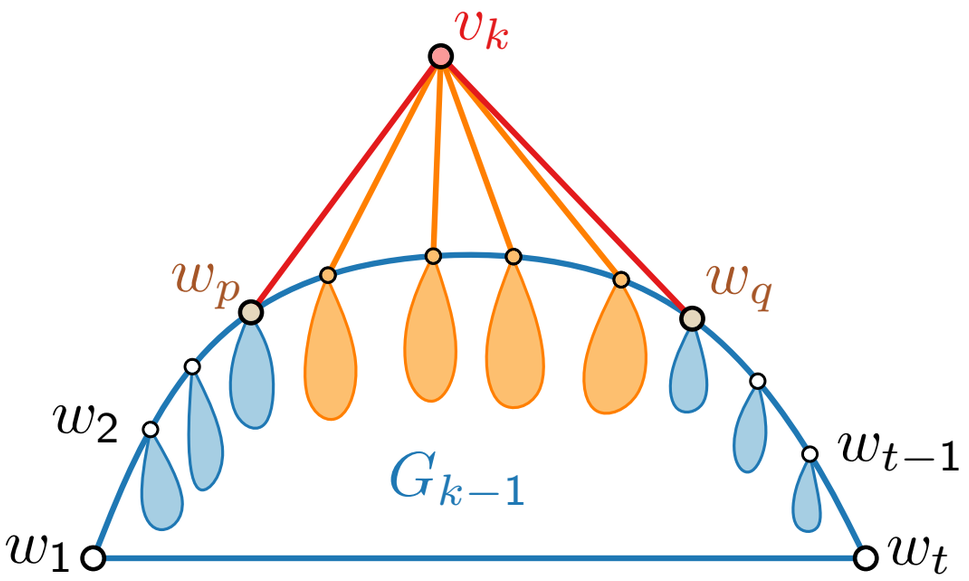

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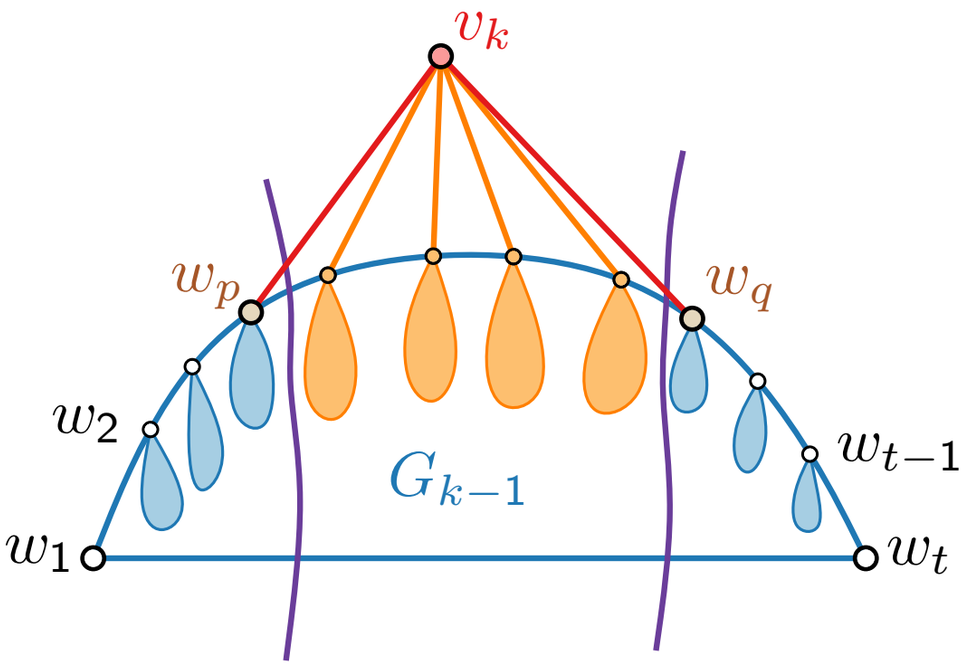

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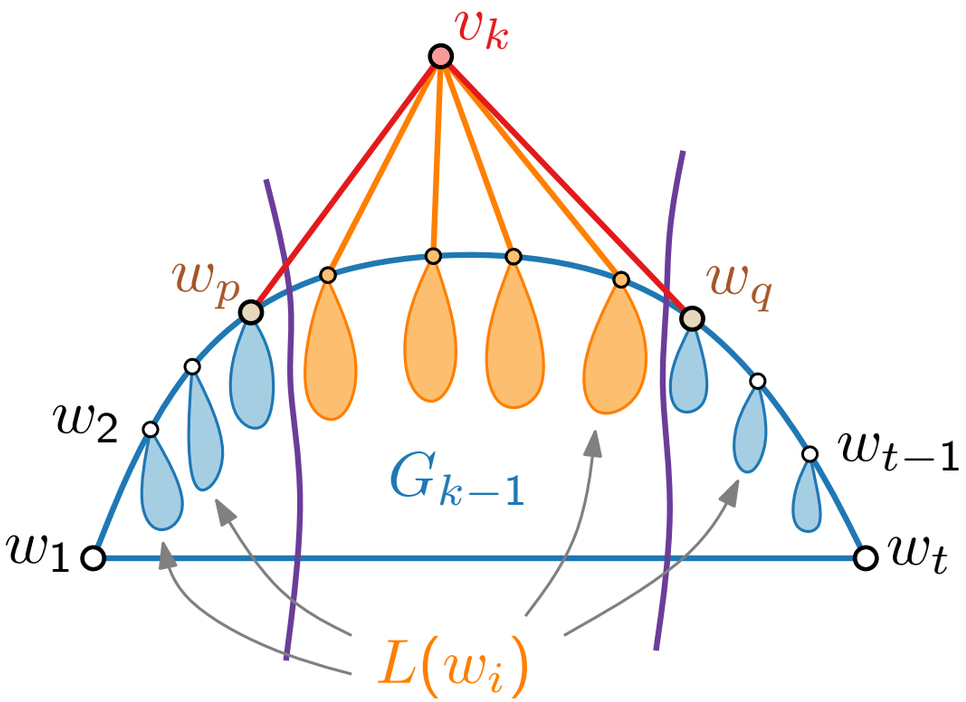
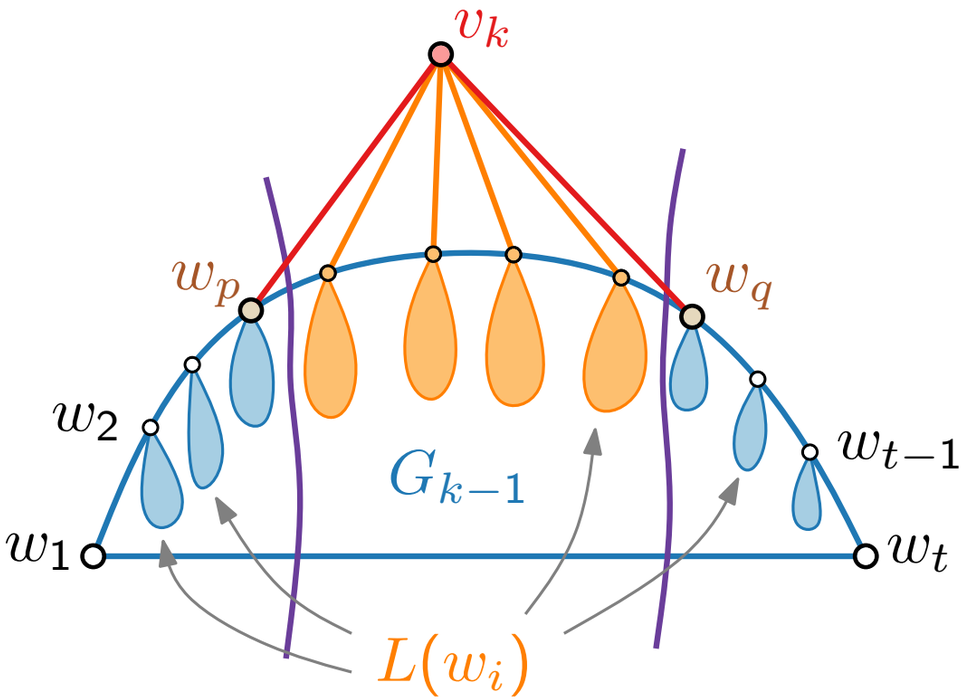


Shift Method – Planarity

Observations.

- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G
- and a forest in G_i, $1 \leq i \leq n-1$.

Lemma.
Let $0 \leq \delta_1 \leq \delta_2 \leq \cdots \leq \delta_t \in \mathbb{N}$, s.t. $\delta_{p+1} - \delta_p \geq 1$, $\delta_q - \delta_{q-1} \geq 1$, $\delta_q - \delta_p \geq 2$ and even.



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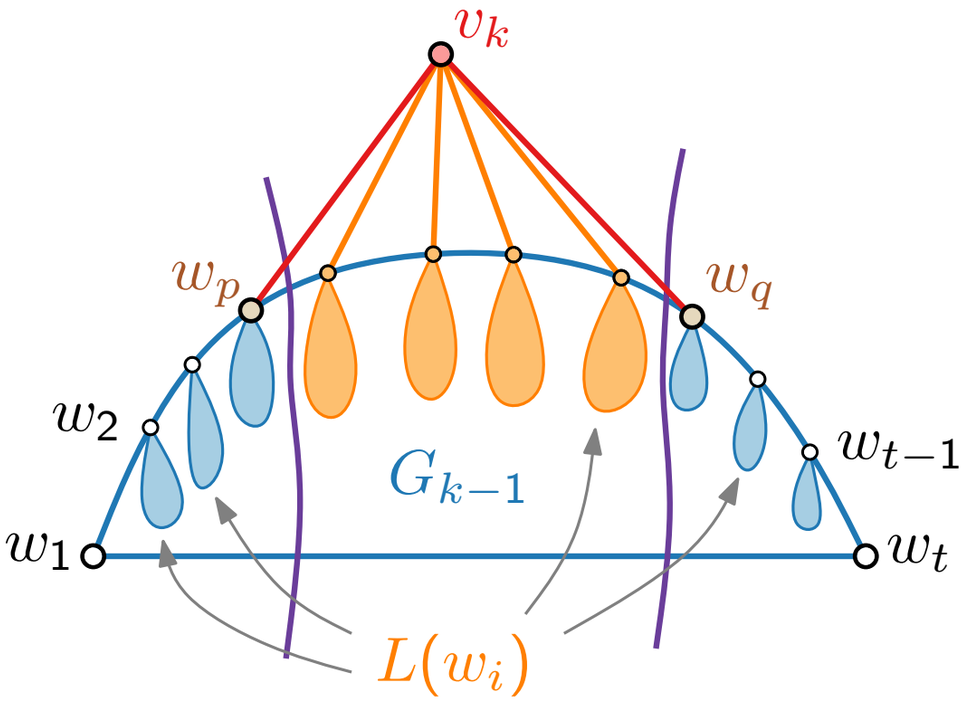

Lemma.
Let $0 \leq \delta_1 \leq \delta_2 \leq \cdots \leq \delta_t \in \mathbb{N}$, s.t. $\delta_{p+1} - \delta_p \geq 1$, $\delta_q - \delta_{q-1} \geq 1$, $\delta_q - \delta_p \geq 2$ and even. If we shift $L(w_i)$ by δ_i to the right, then we get a planar straight-line drawing.

Shift Method – Planarity

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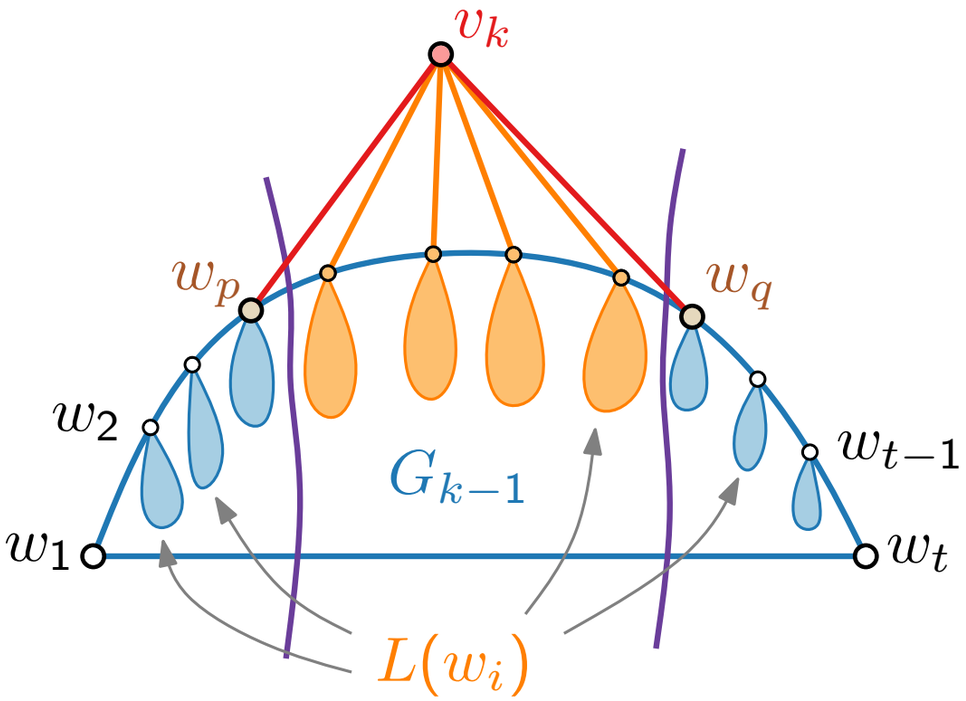

Proof by induction:
If G_{k-1} is drawn planar and straight-line, then so is G_k.

Shift Method – Planarity

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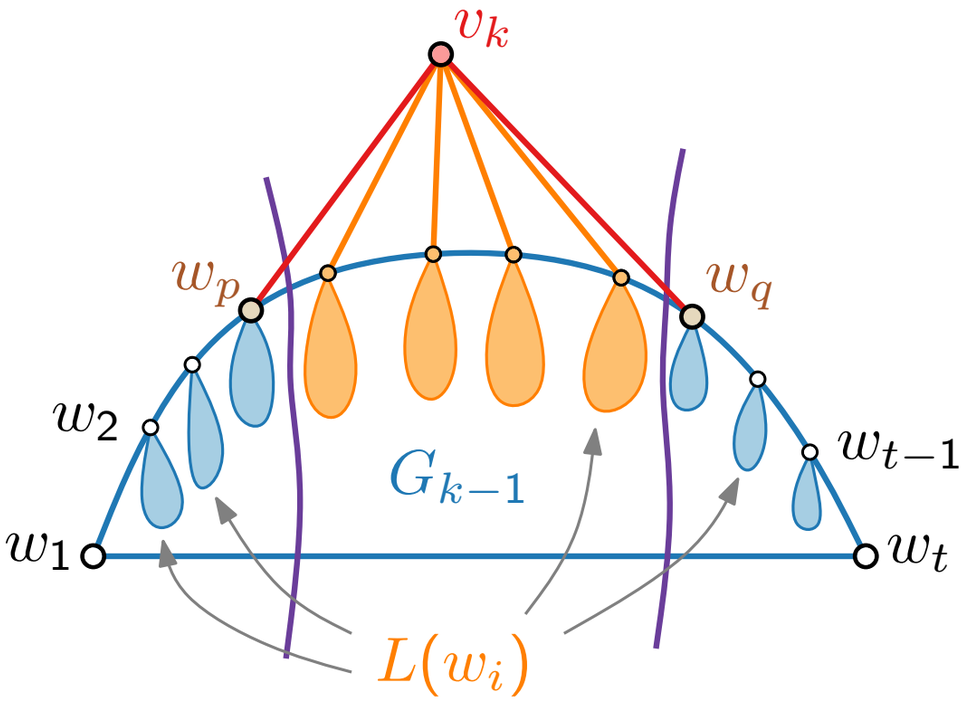


Proof by induction:
If G_{k-1} is drawn planar and straight-line, then so is G_k.

Ideas:

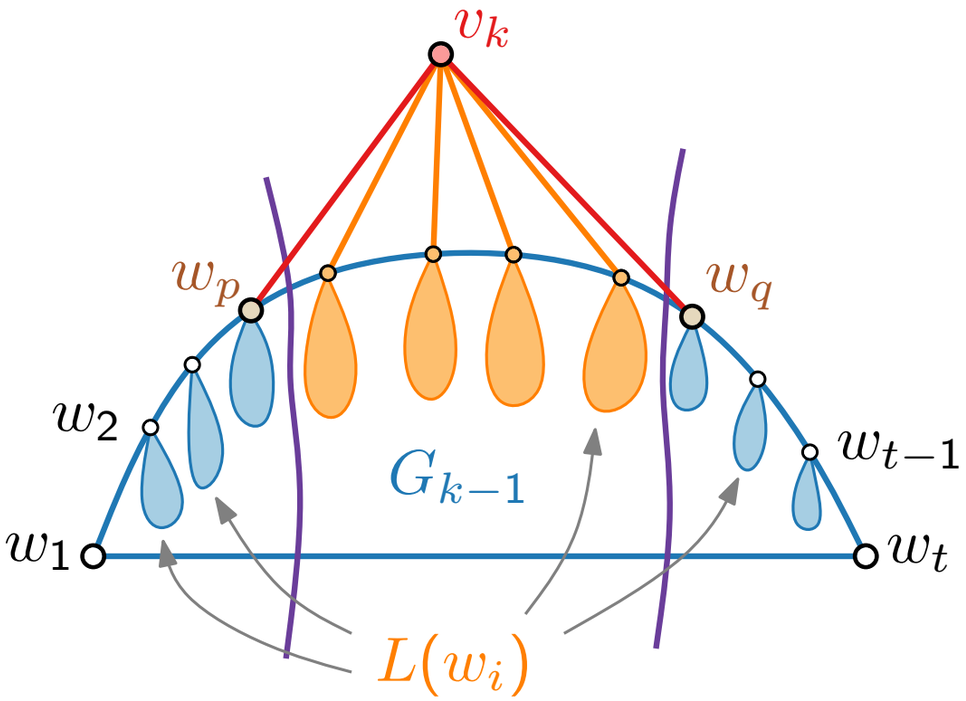
- New edges don't intersect other edges ($\rightarrow$ invariants).
- Edges within each $L(w_i)$ do not change.
- Other edges lie within triangles that only become flatter without causing new intersections.

Shift Method – Planarity

Observations.

- Each internal vertex is covered exactly once.
- Covering relation defines a tree in G
- and a forest in G_i, $1 \leq i \leq n-1$.

Lemma.
Let $0 \leq \delta_1 \leq \delta_2 \leq \cdots \leq \delta_t \in \mathbb{N}$, s.t. $\delta_{p+1} - \delta_p \geq 1$, $\delta_q - \delta_{q-1} \geq 1$, $\delta_q - \delta_p \geq 2$ and even. If we shift $L(w_i)$ by δ_i to the right, then we get a planar straight-line drawing.



Proof by induction:
If G_{k-1} is drawn planar and straight-line, then so is G_k.

Ideas:

- New edges don't intersect other edges ($\rightarrow$ invariants).
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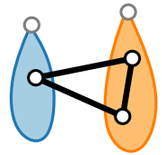

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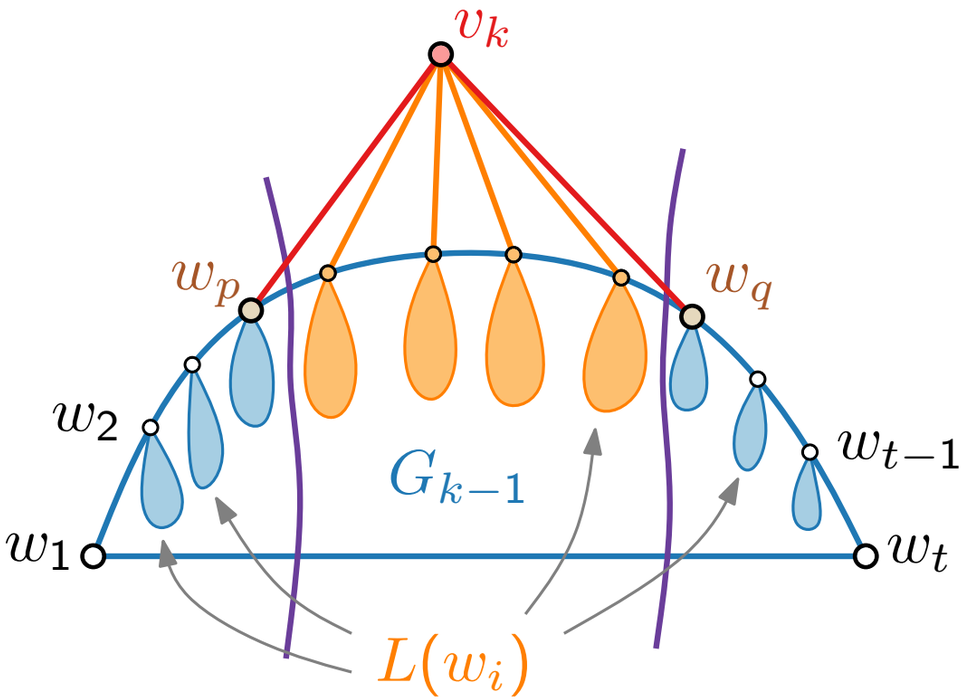

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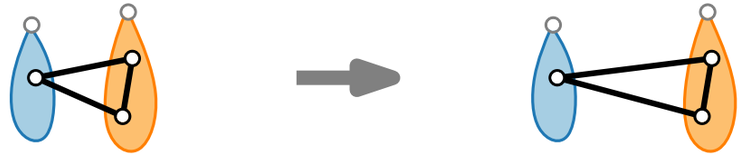

Shift Method – Pseudocode

canonical order of V

ShiftMethod($G = (V, E), (v_1, v_2, \ldots, v_n)$)

for $k = 1$ to 3 **do**

for $k = 4$ to n **do**

Shift Method – Pseudocode

canonical order of V

ShiftMethod$(G = (V, E), (v_1, v_2, \ldots, v_n))$

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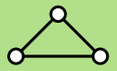

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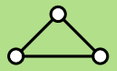
$P(v_1) \leftarrow (0, 0); P(v_2) \leftarrow (2, 0), P(v_3) \leftarrow (1, 1)$ 

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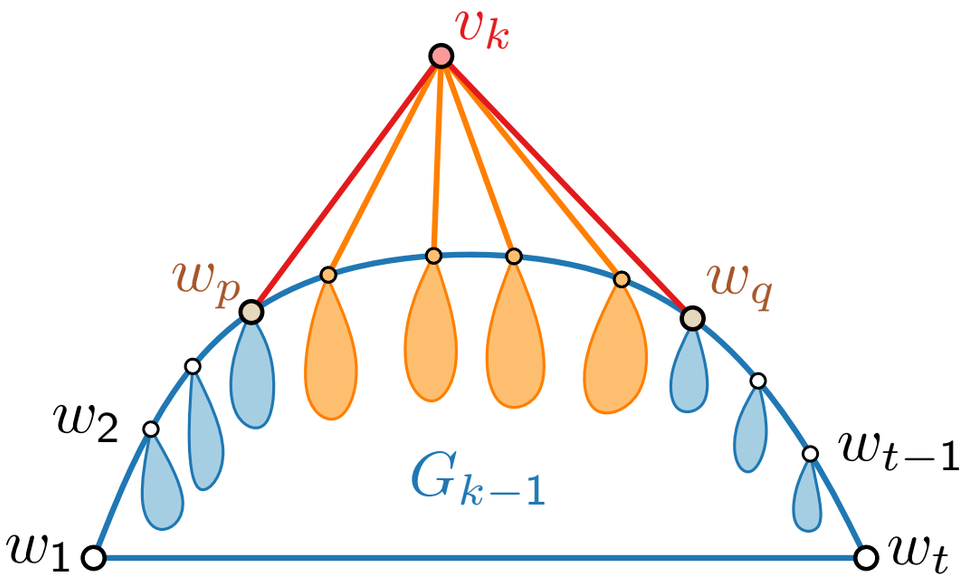

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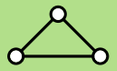

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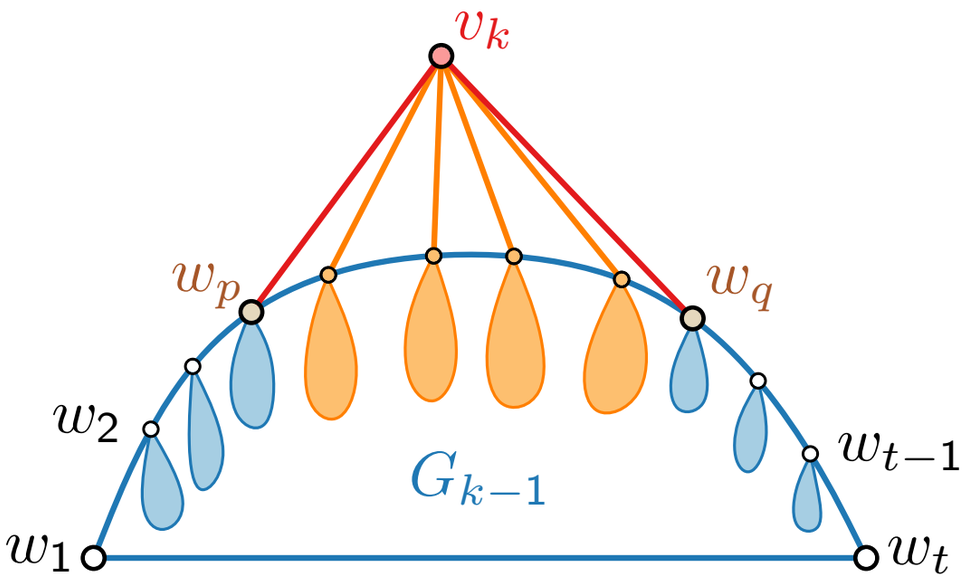

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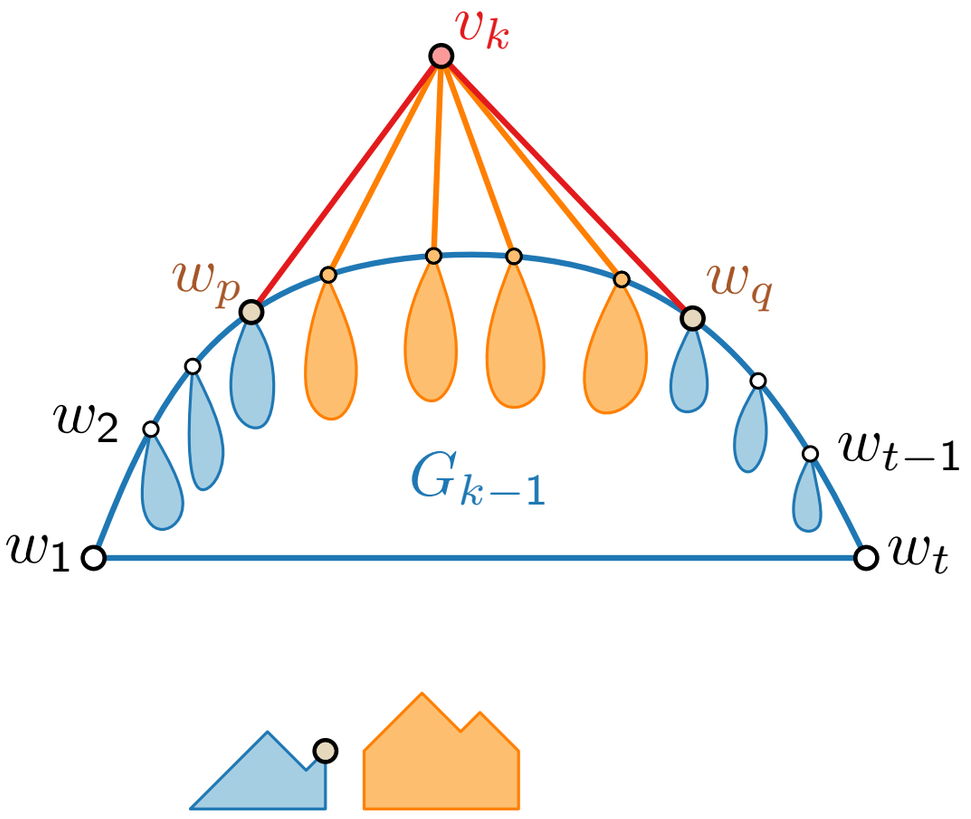

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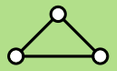

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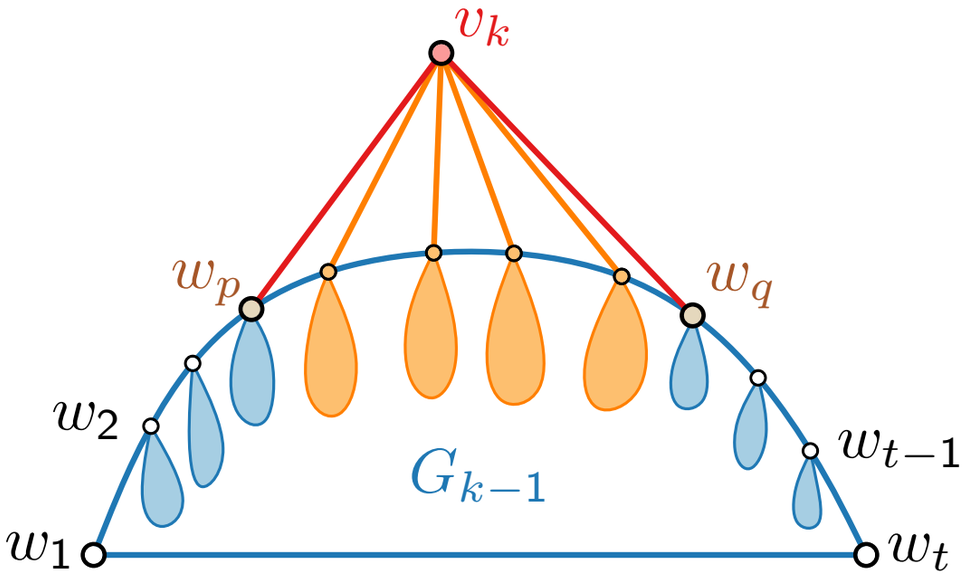

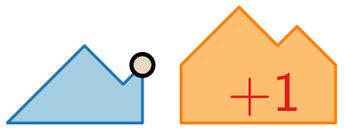

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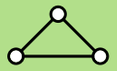

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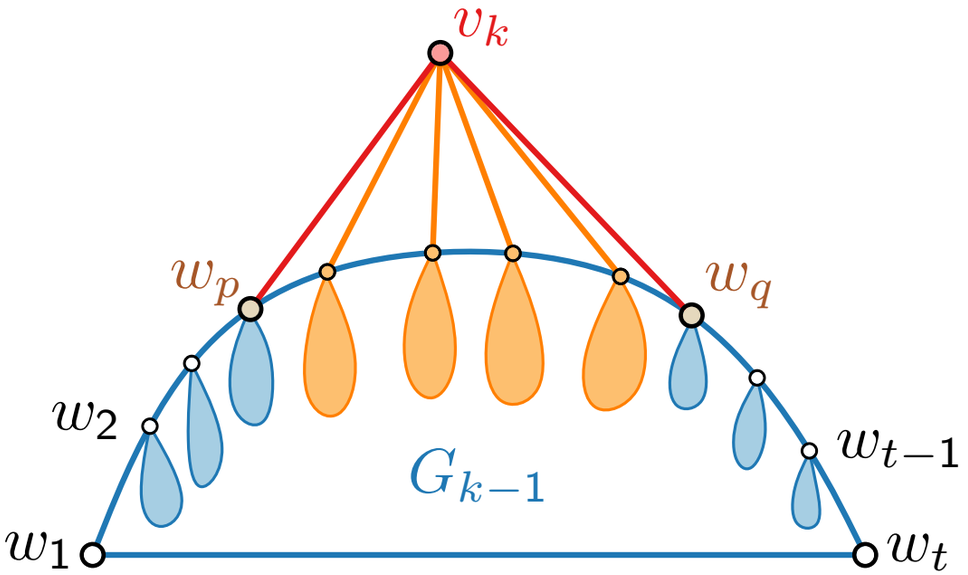

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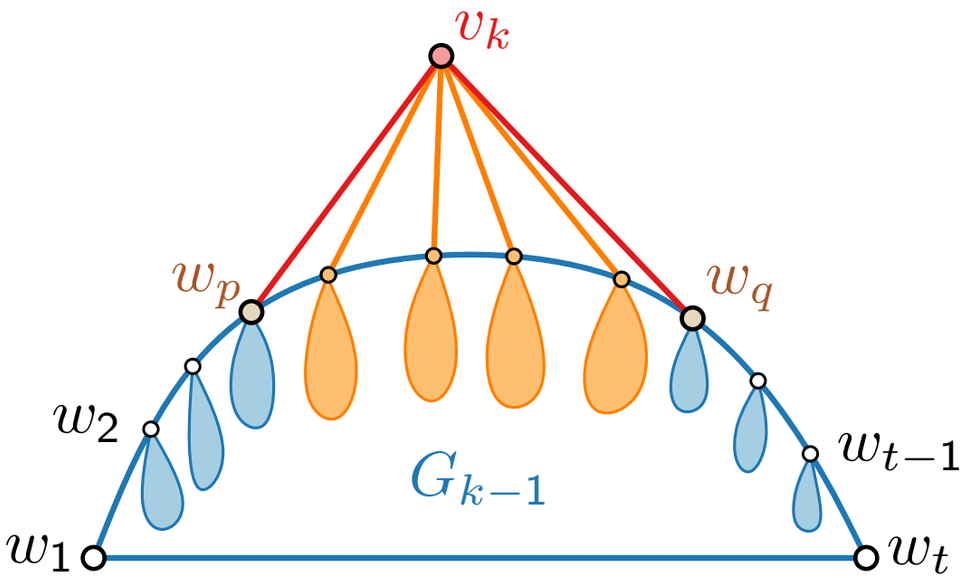

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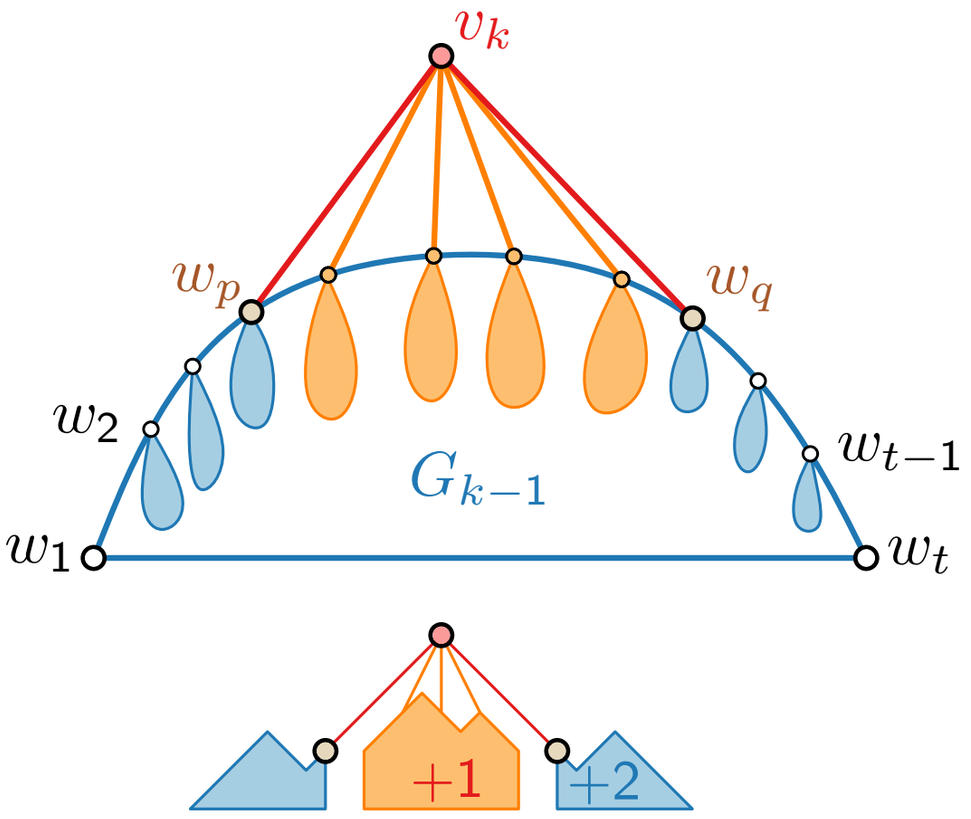

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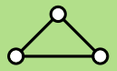

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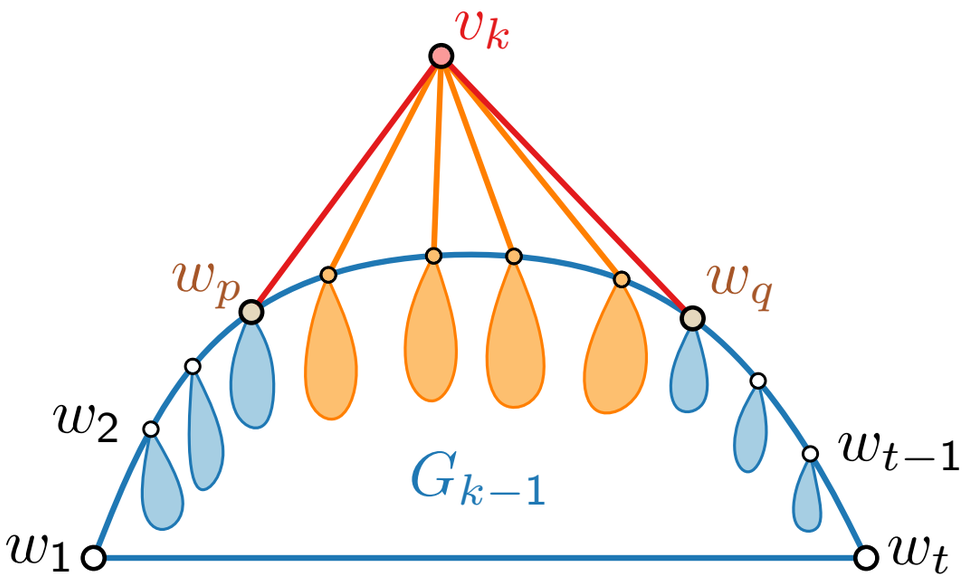

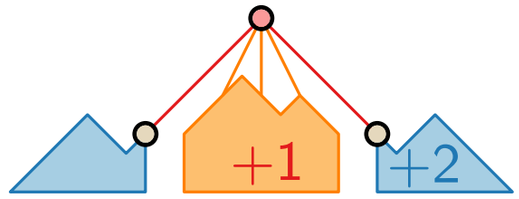

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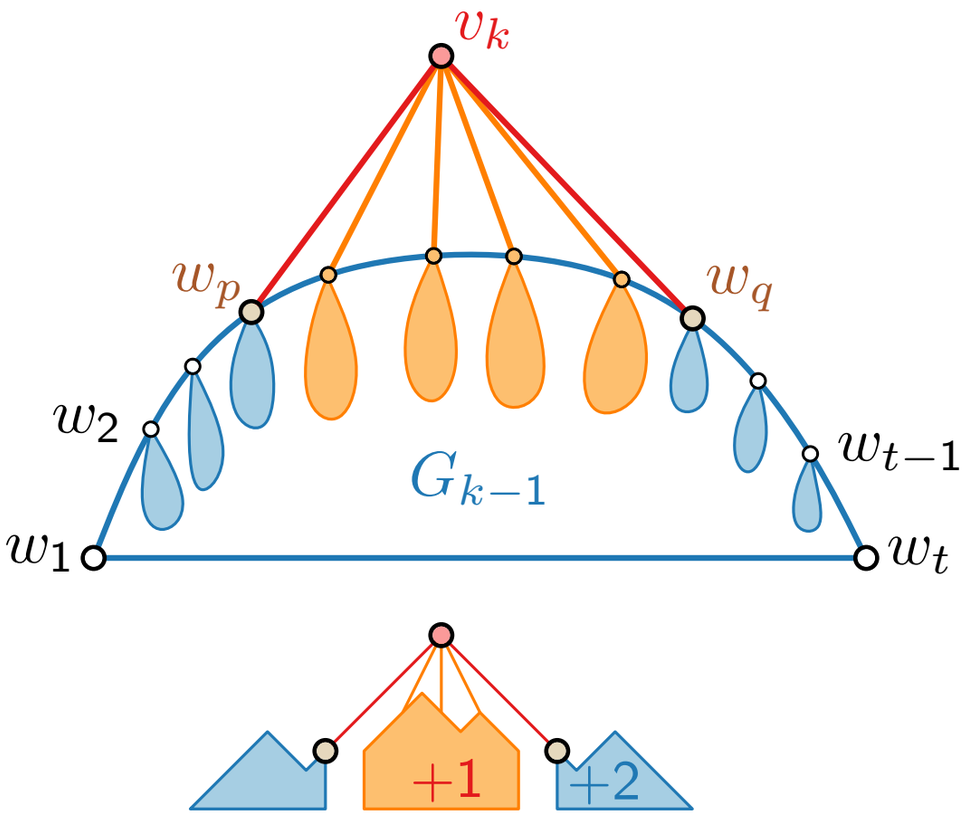

Running Time?

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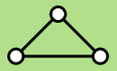
$P(v_1) \leftarrow (0, 0)$; $P(v_2) \leftarrow (2, 0)$, $P(v_3) \leftarrow (1, 1)$ 

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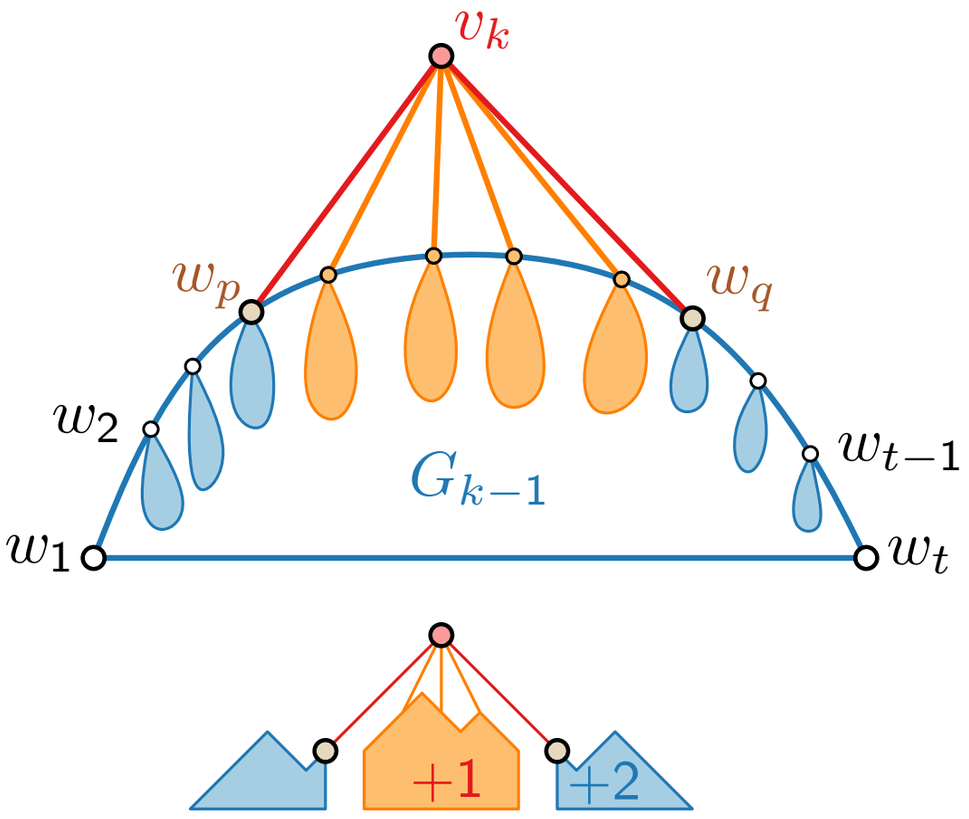
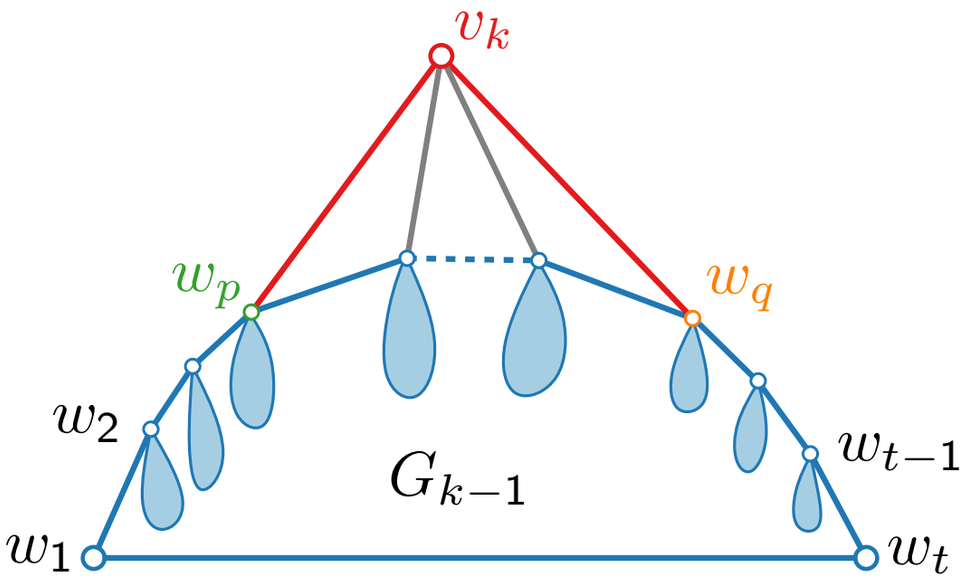


Running Time?

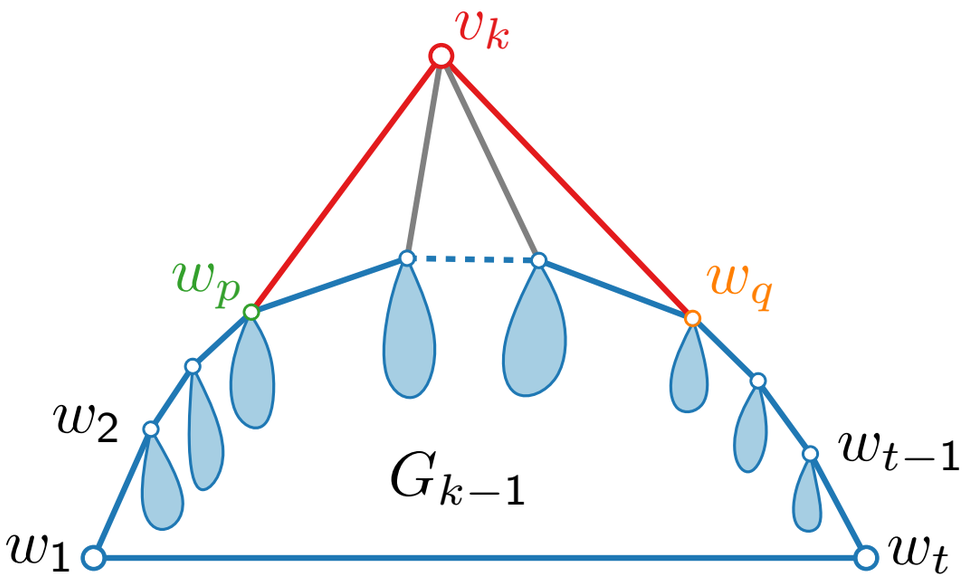
Shift Method – Linear-Time Implementation



Shift Method – Linear-Time Implementation

Idea 1.

To compute $x(v_k)$ & $y(v_k)$,
we only need $y(w_p)$, $y(w_q)$, and $x(w_q) - x(w_p)$



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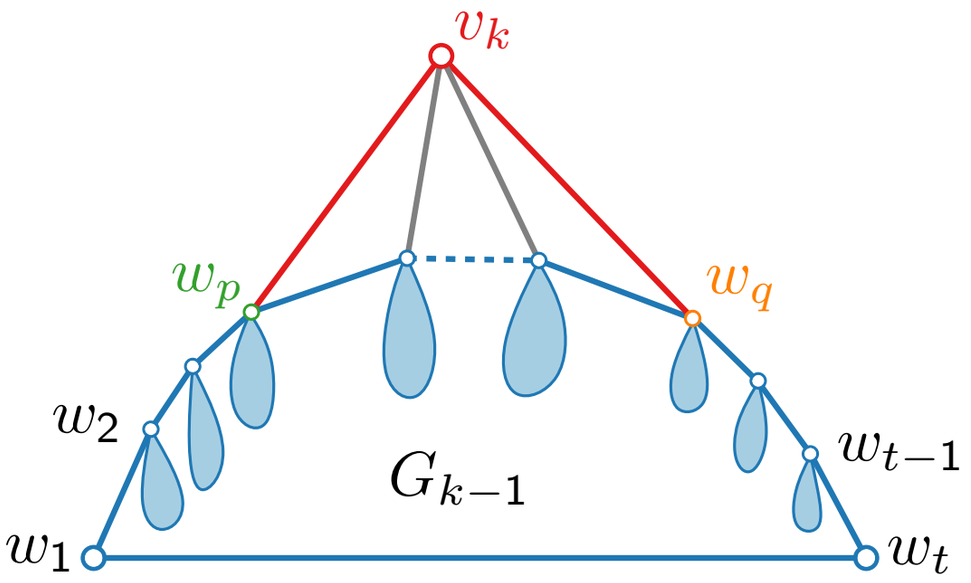

(1) $x(v_k) = \frac{1}{2}(x(w_q) + x(w_p) + y(w_q) - y(w_p))$

Shift Method – Linear-Time Implementation

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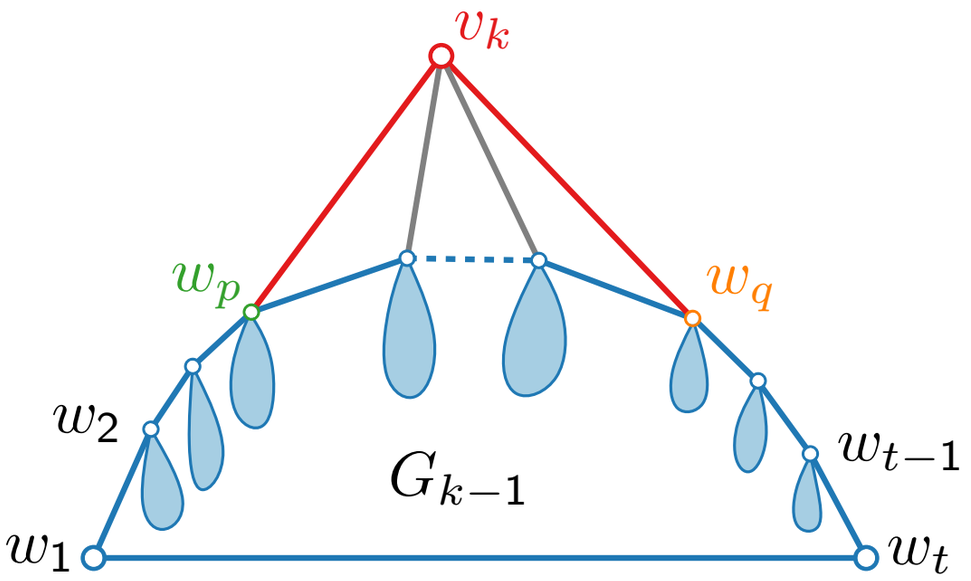

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Shift Method – Linear-Time Implementation

Idea 1.
To compute $x(v_k)$ & $y(v_k)$,
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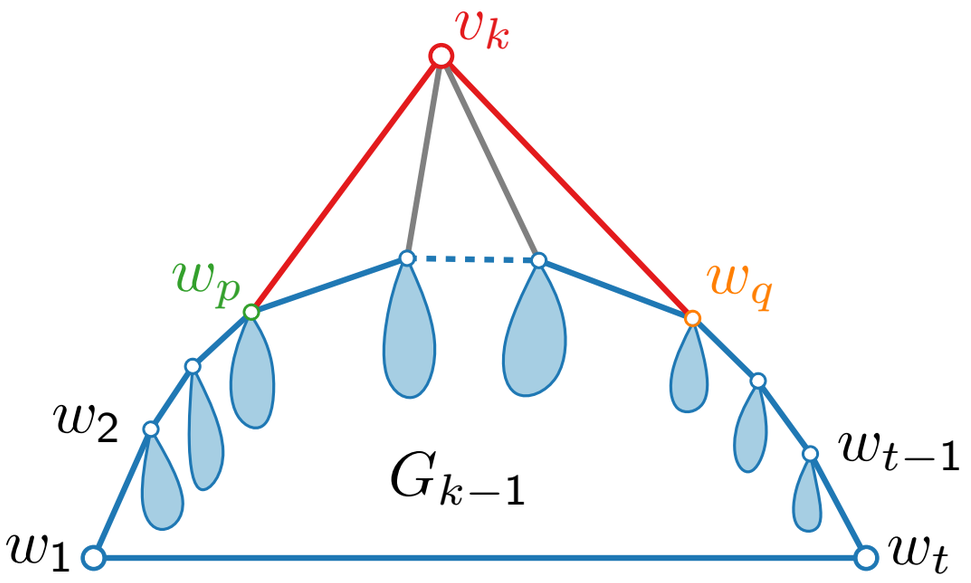
Idea 2.
Instead of storing explicit x-coordinates,
we store x-distances.



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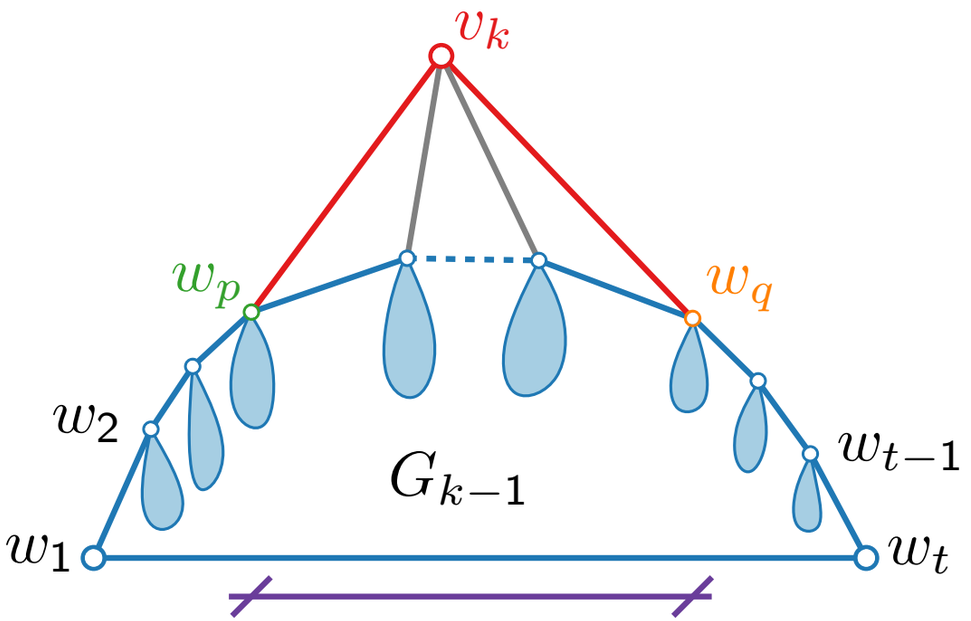

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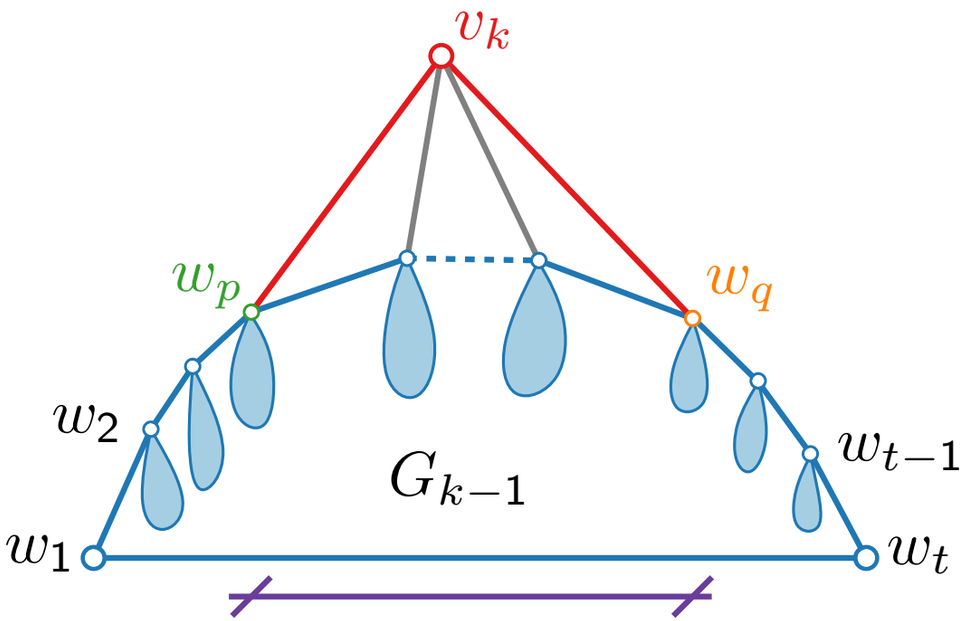
Idea 2.

Instead of storing explicit x-coordinates,
we store x-distances.

After an x-distance is computed for each v_k,
use preorder traversal to compute all x-coordinates.



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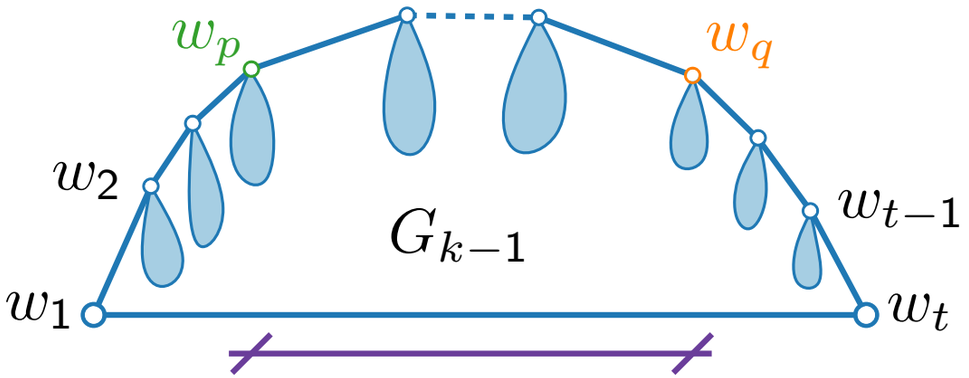
Shift Method – Linear-Time Implementation

Relative x-distance tree.

For each vertex v store

- x-offset $\Delta_x(v)$ from parent
- y-coordinate $y(v)$



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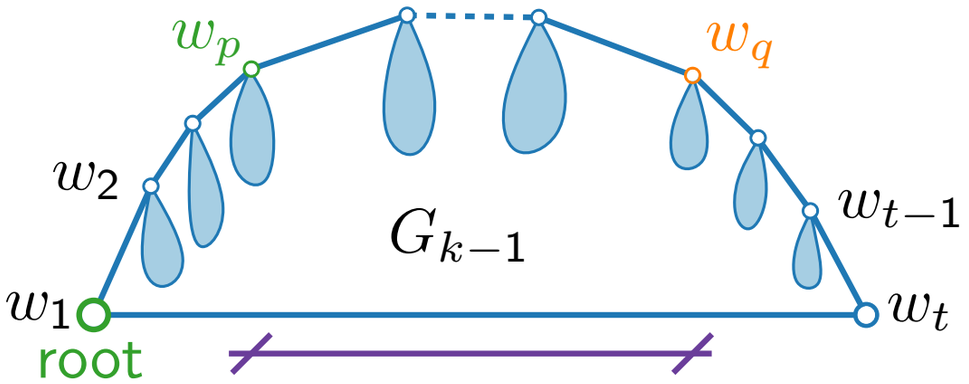

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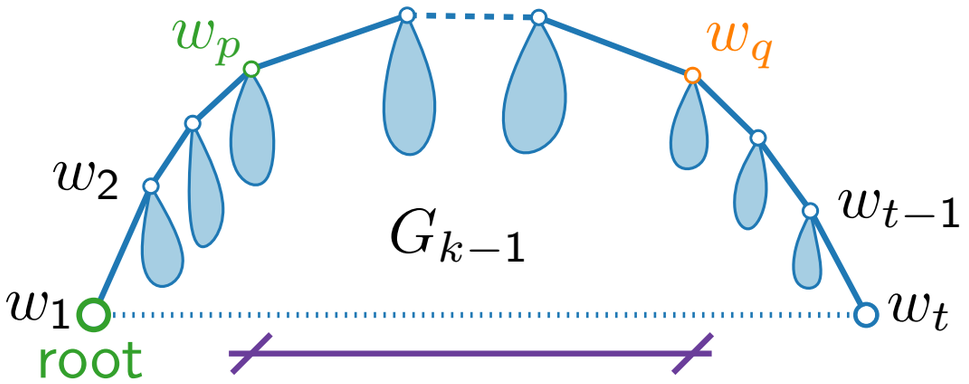

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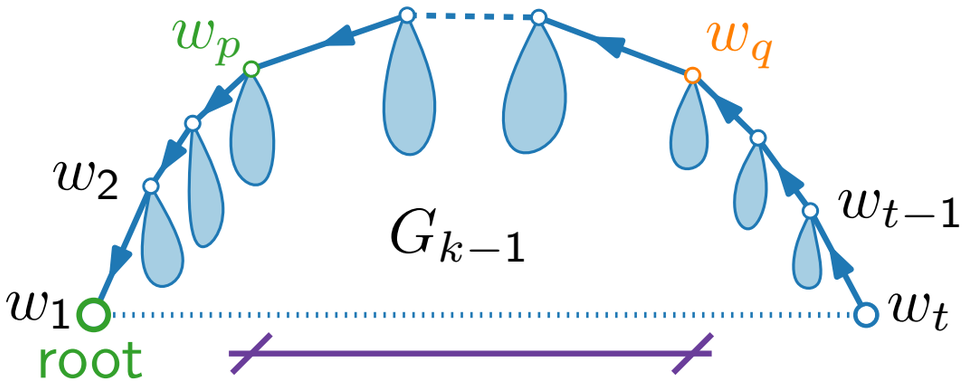

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Shift Method – Linear-Time Implementation

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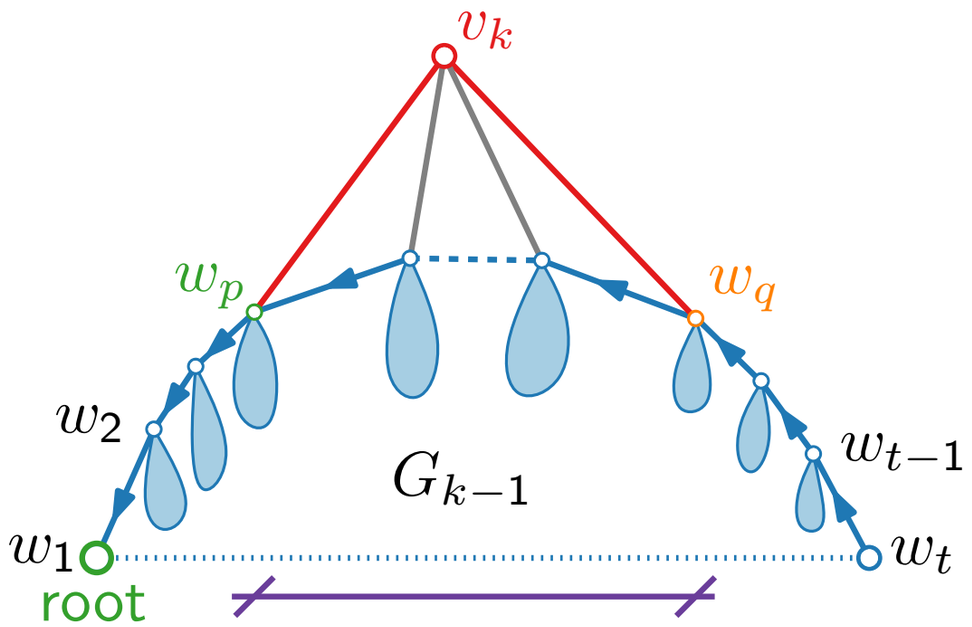

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Shift Method – Linear-Time Implementation

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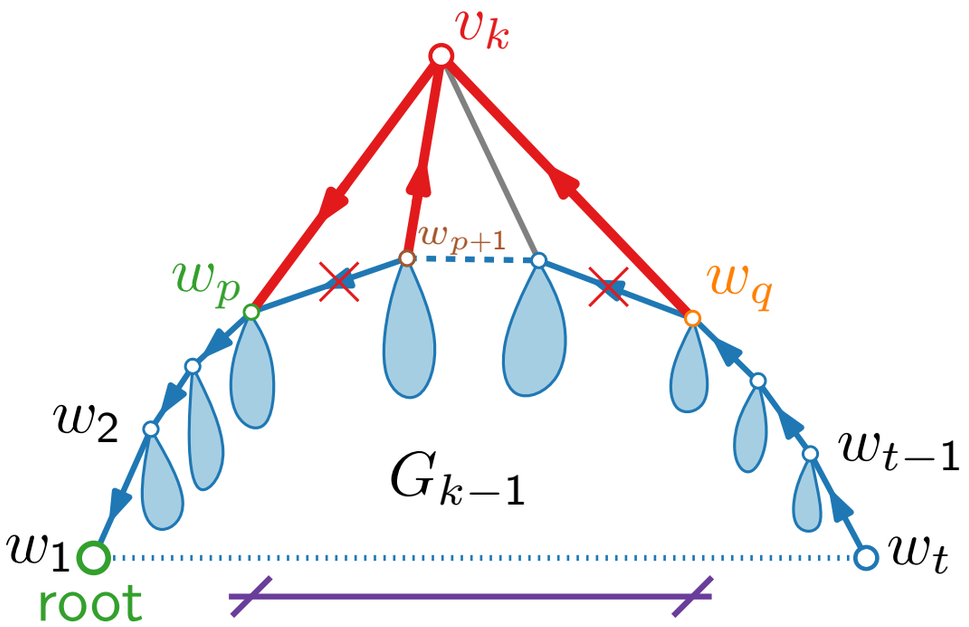

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Shift Method – Linear-Time Implementation

Relative x-distance tree.

For each vertex v store

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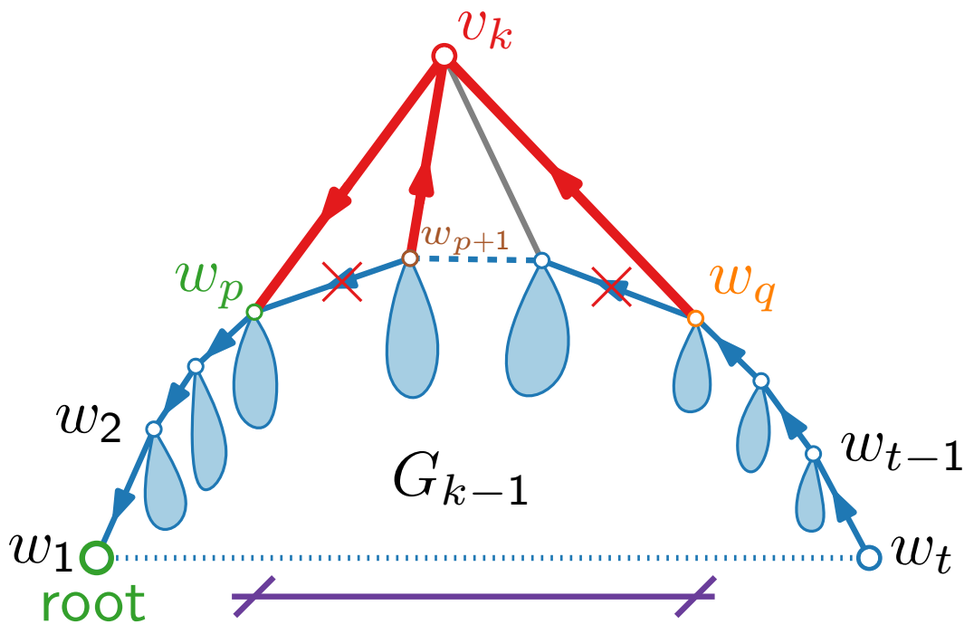
Calculations.

- $\Delta_x(w_{p+1})$++, $\Delta_x(w_q)$++



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Shift Method – Linear-Time Implementation

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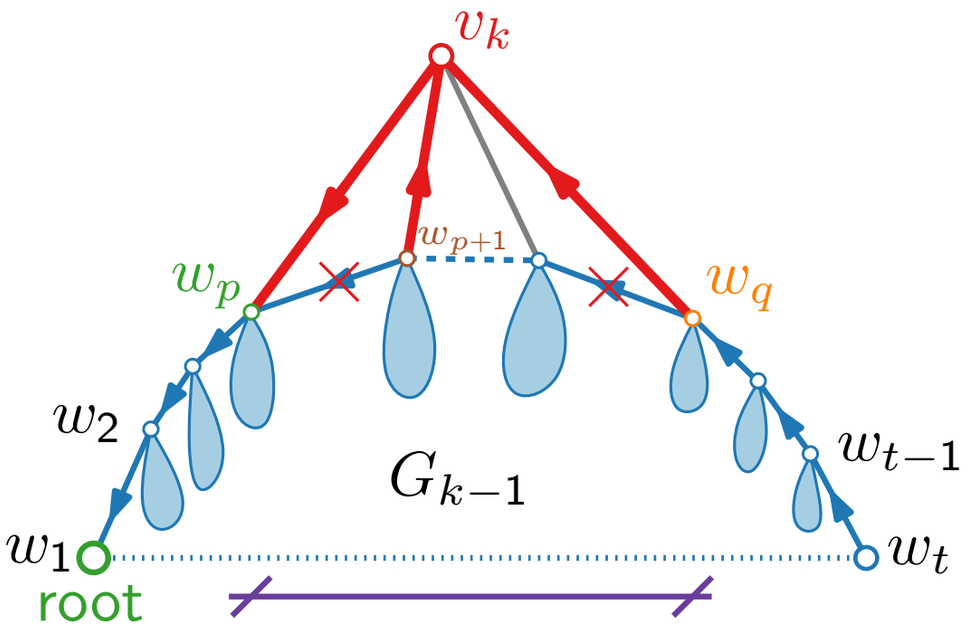

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Shift Method – Linear-Time Implementation

Relative x-distance tree.

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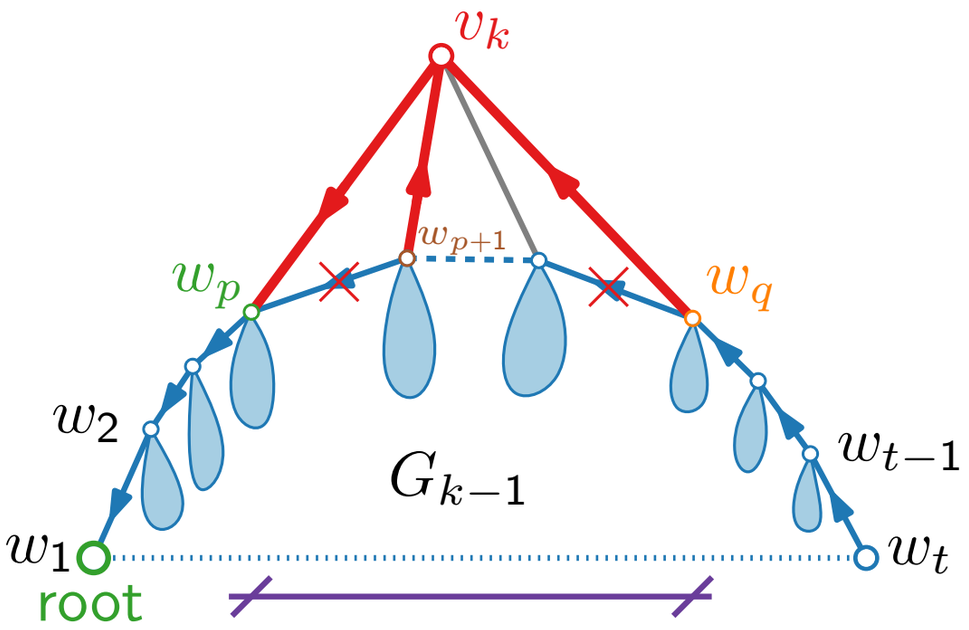

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Shift Method – Linear-Time Implementation

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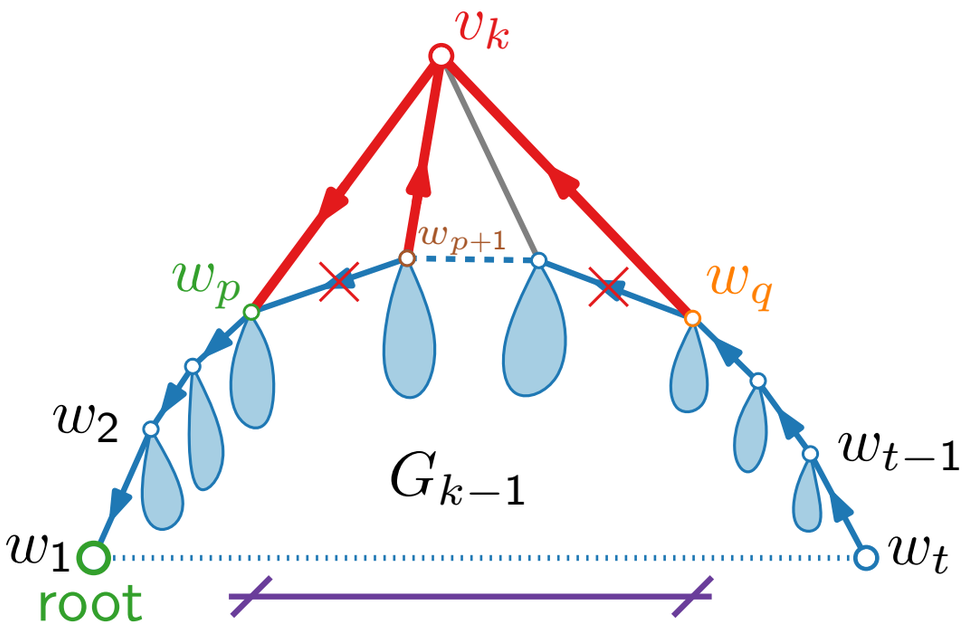

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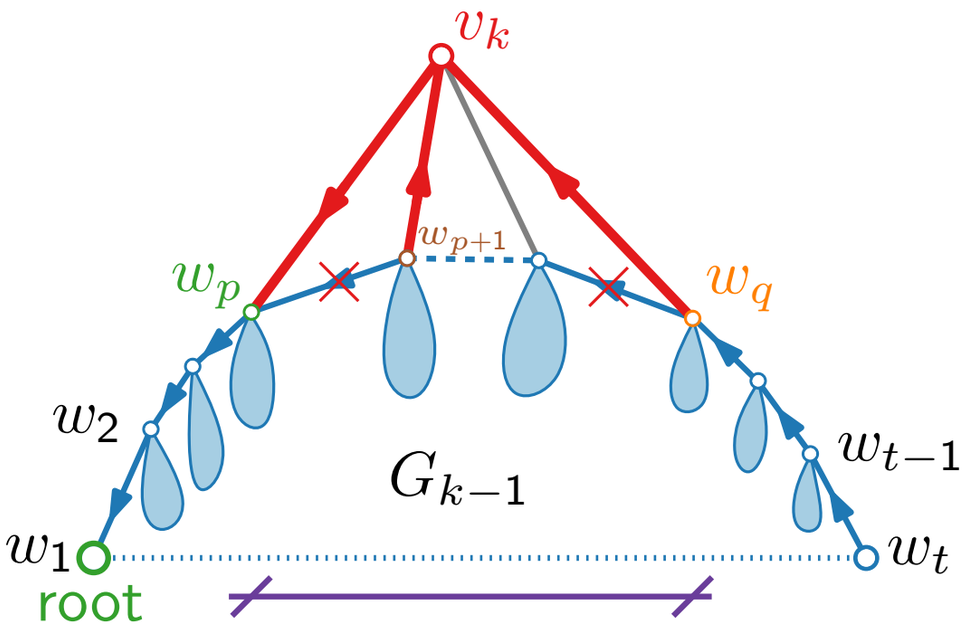

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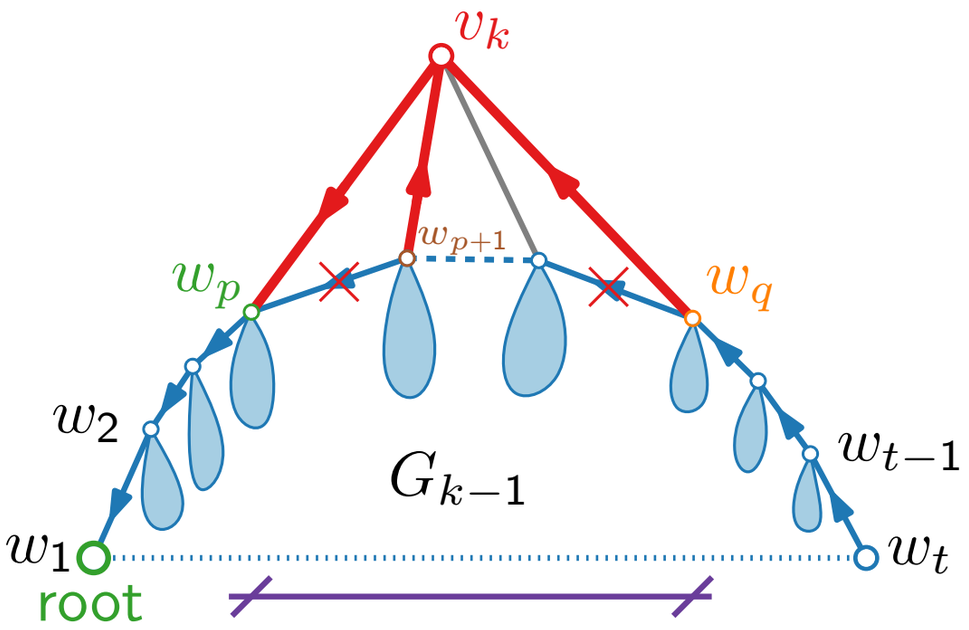

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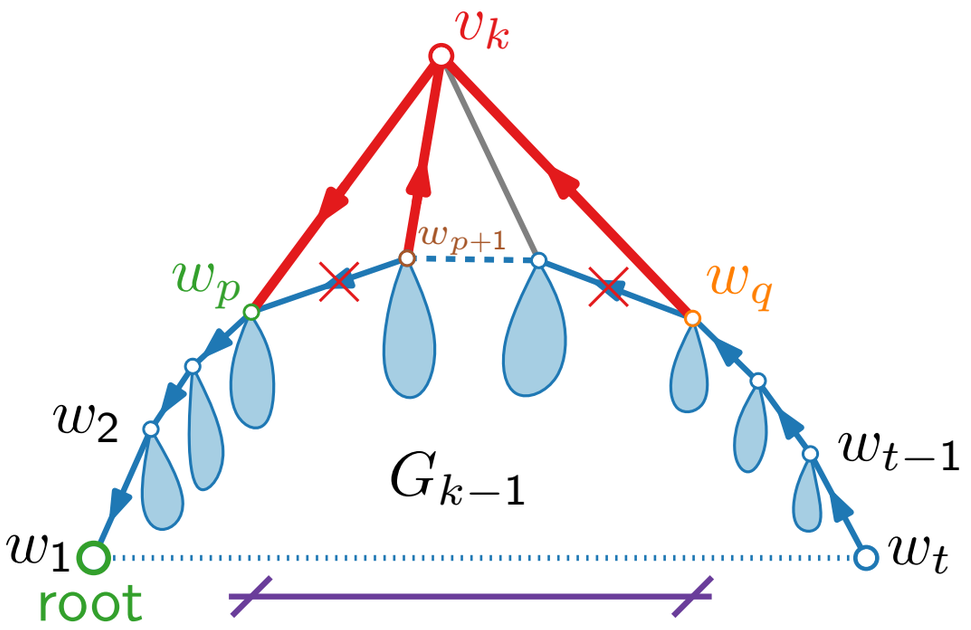
$\mathcal{O}(n)$ in total



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Literature

- [PGD Ch. 4.2] for detailed explanation of shift method
- [de Fraysseix, Pach, Pollack 1990] “How to draw a planar graph on a grid” – original paper on the shift method