Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part I:
SetCover

Alexander Wolff

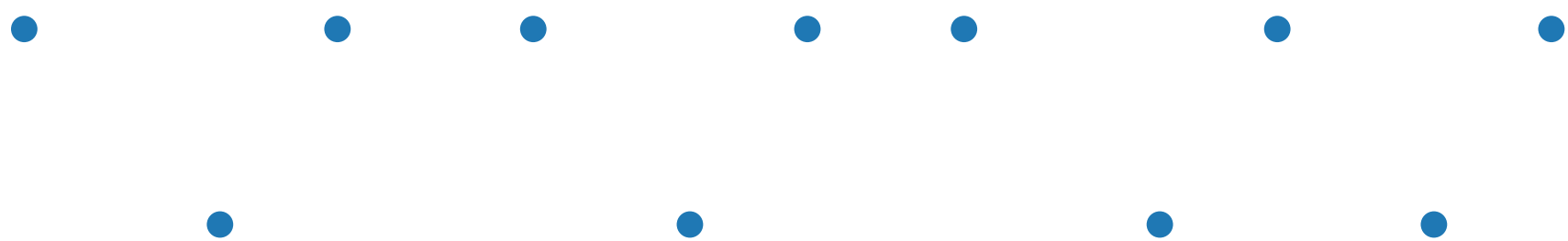
Winter 2022/23

SetCover (card.)

Let U be some **ground set** (universe),

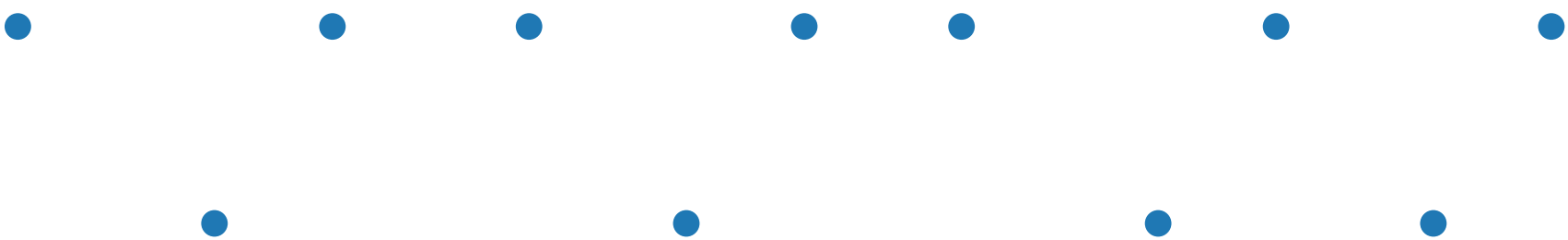
SetCover (card.)

Let U be some **ground set** (universe),



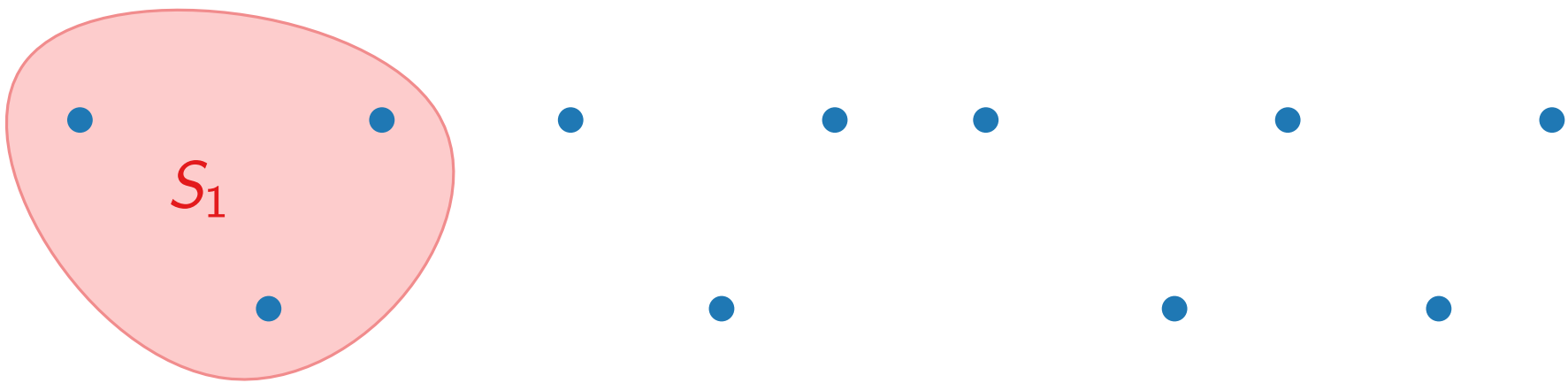
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



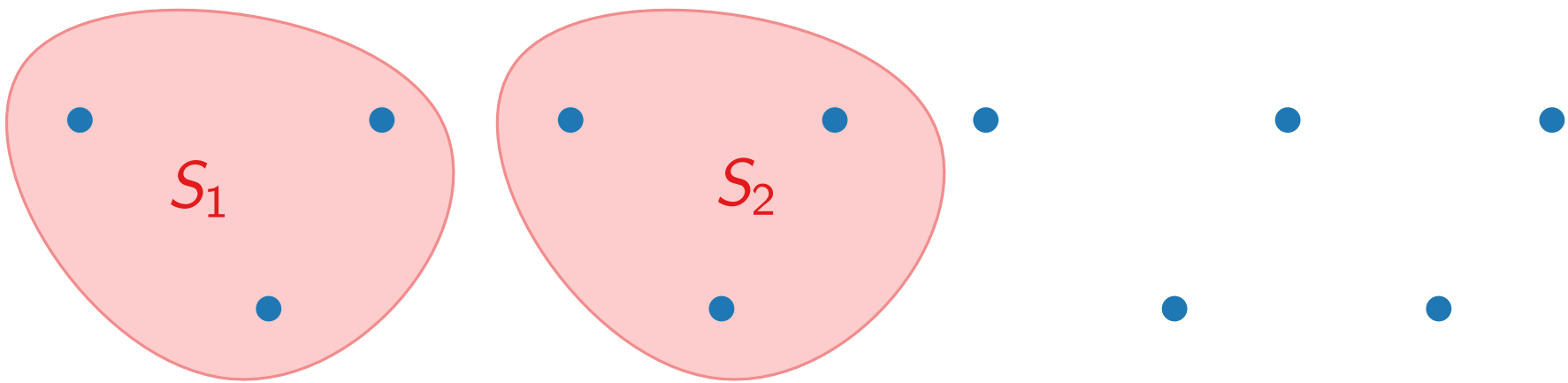
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



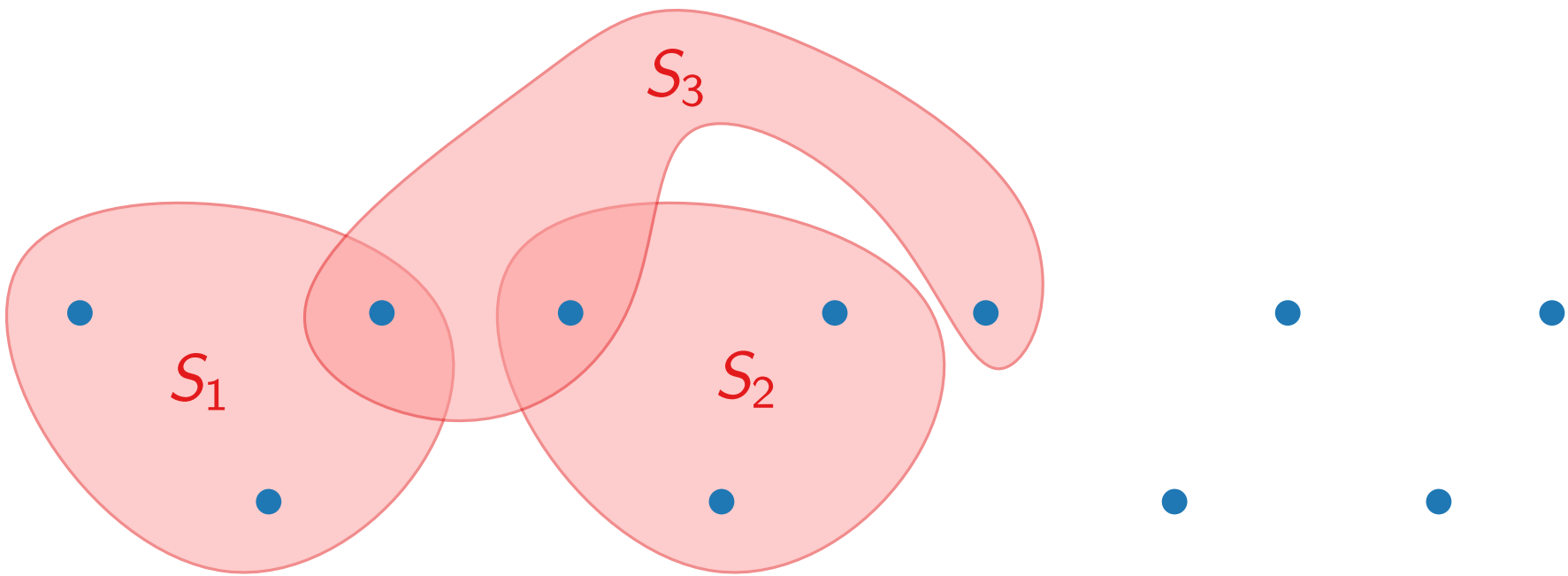
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



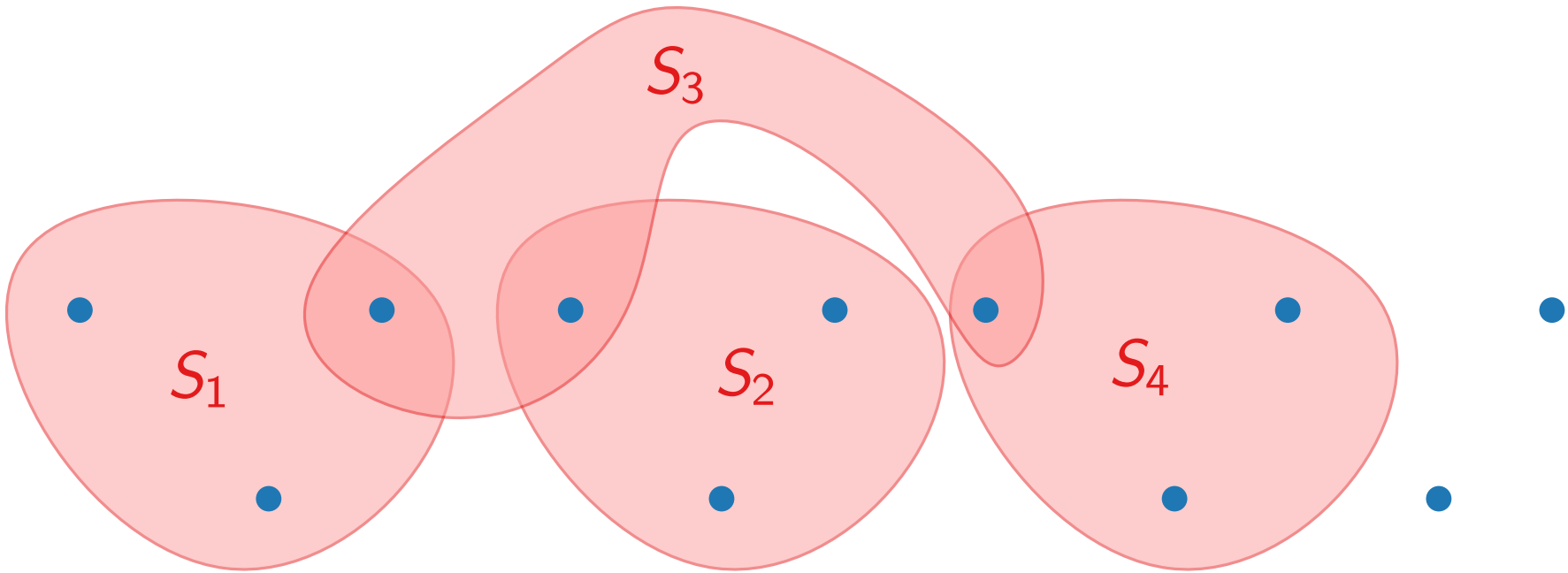
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



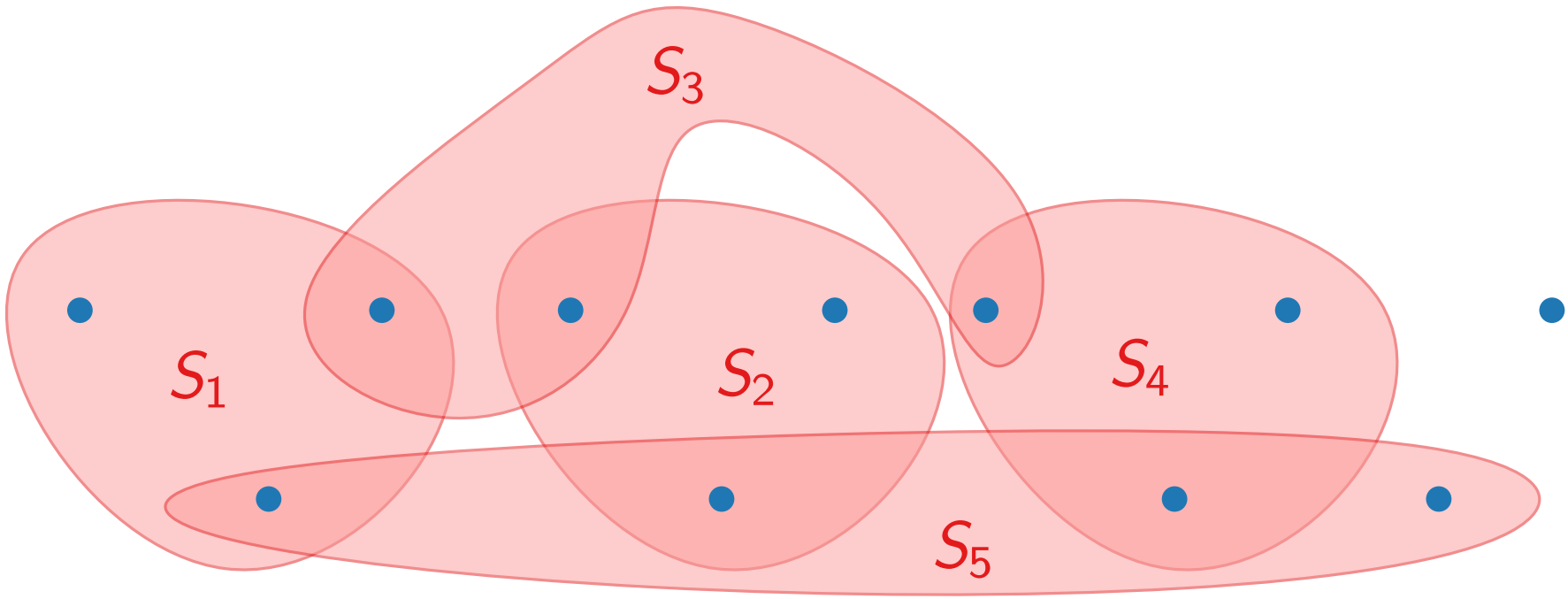
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



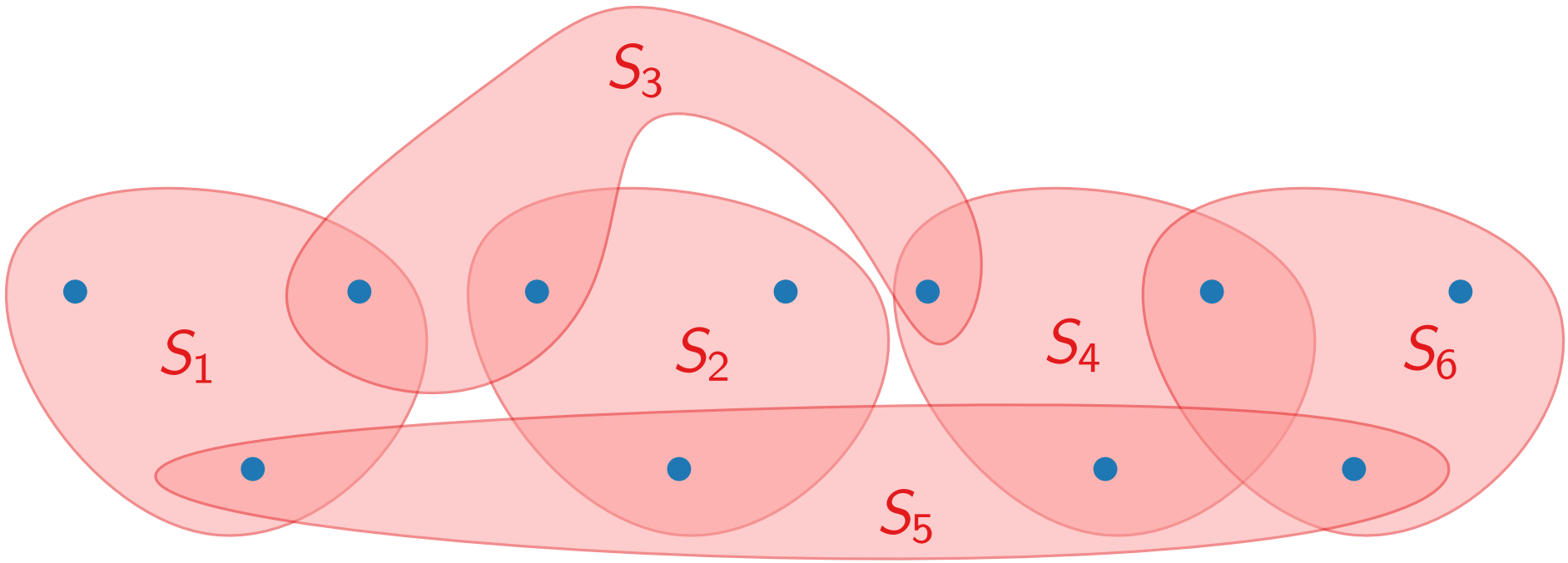
SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



SetCover (card.)

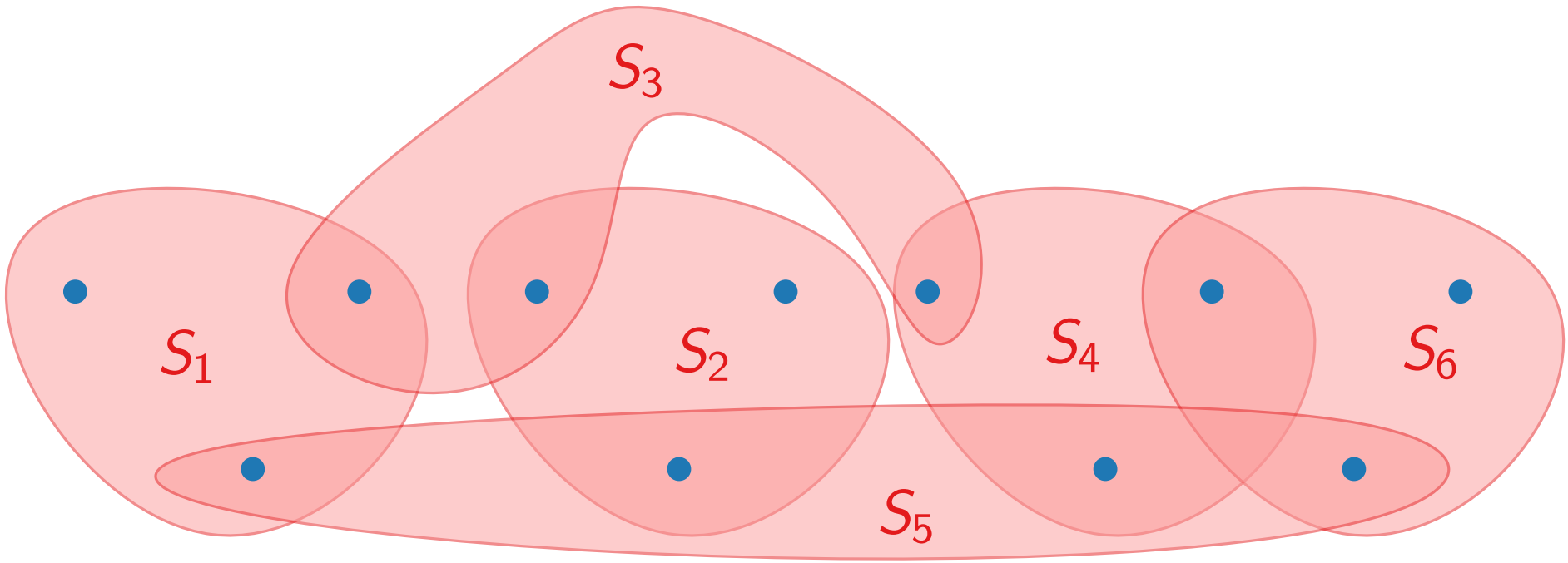
Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.



SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

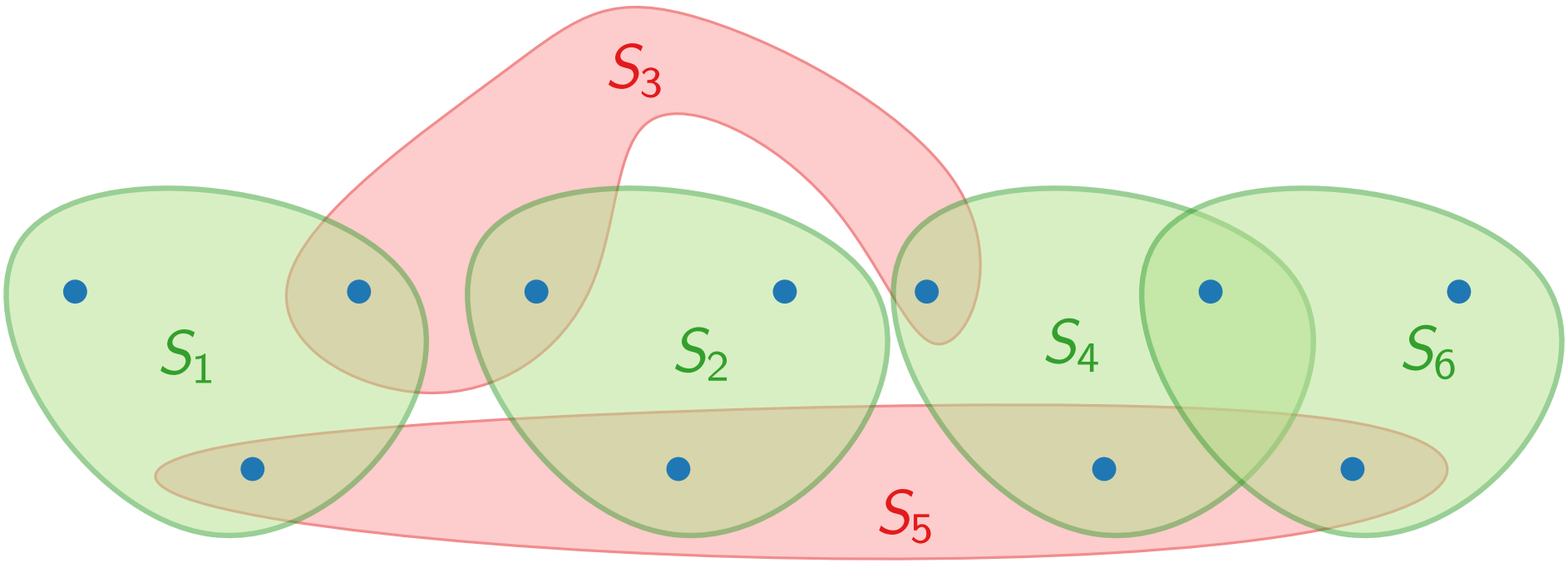
Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum cardinality.



SetCover (card.)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

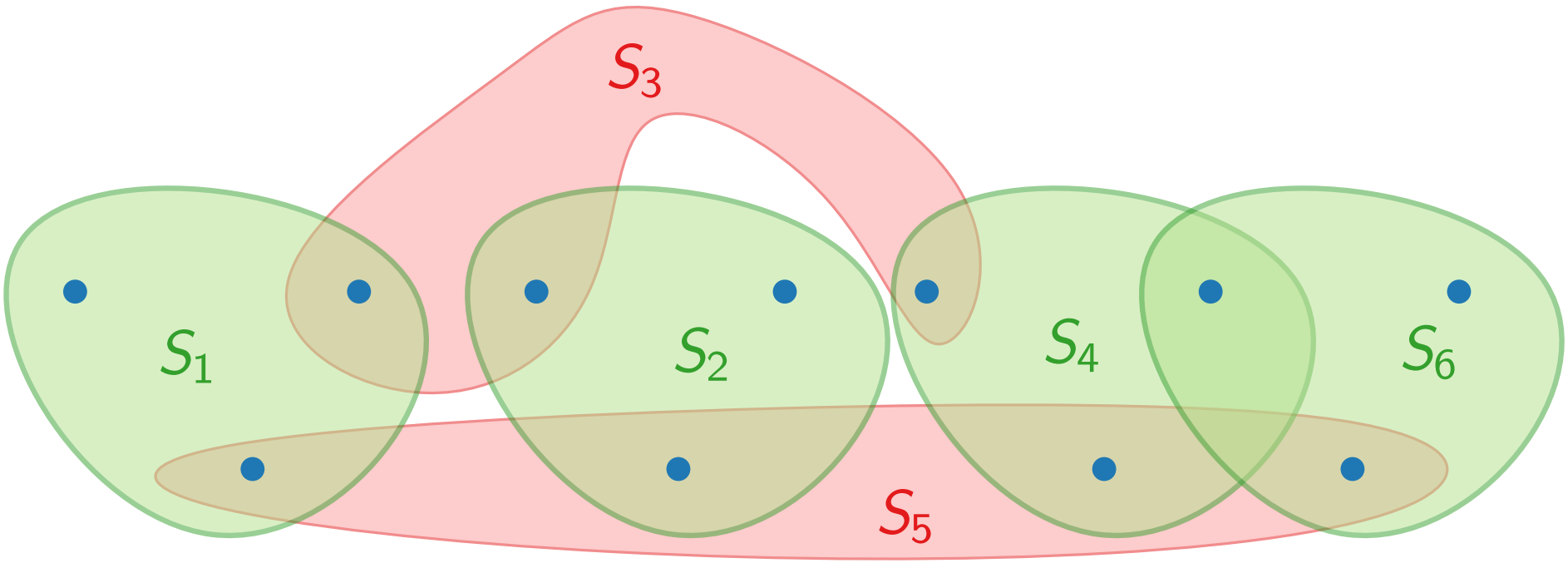
Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum cardinality.



SetCover (general)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum cardinality.

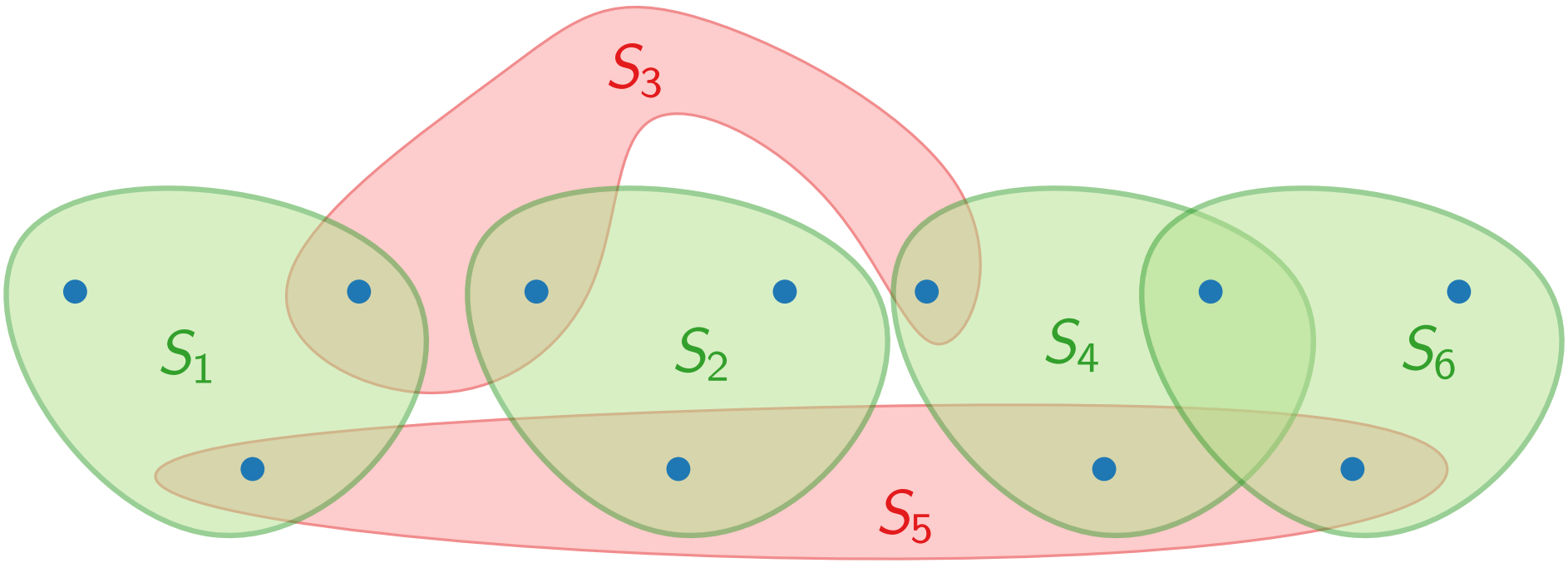


SetCover (general)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

Each $S \in \mathcal{S}$ has cost $c(S) > 0$.

Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum cardinality.

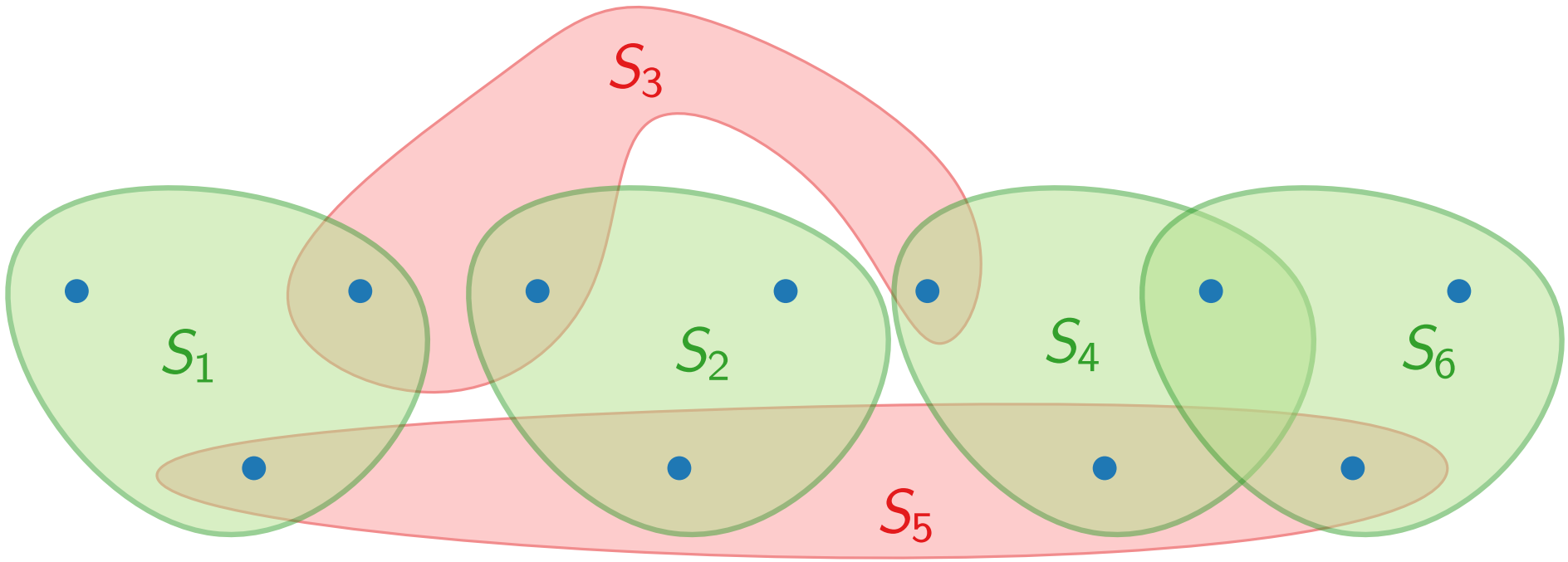


SetCover (general)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

Each $S \in \mathcal{S}$ has cost $c(S) > 0$.

Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum ~~cardinality.~~ total cost $c(\mathcal{S}') := \sum_{S \in \mathcal{S}'} c(S)$.

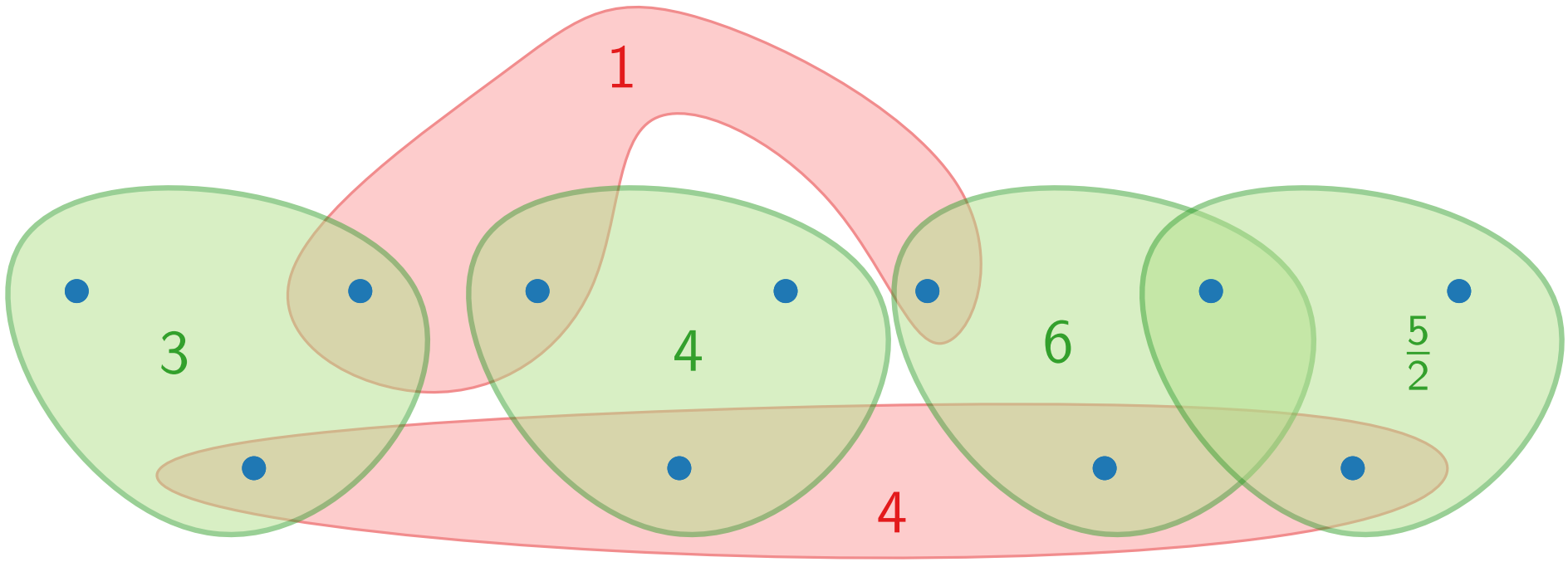


SetCover (general)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

Each $S \in \mathcal{S}$ has cost $c(S) > 0$.

Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum ~~cardinality.~~ total cost $c(\mathcal{S}') := \sum_{S \in \mathcal{S}'} c(S)$.

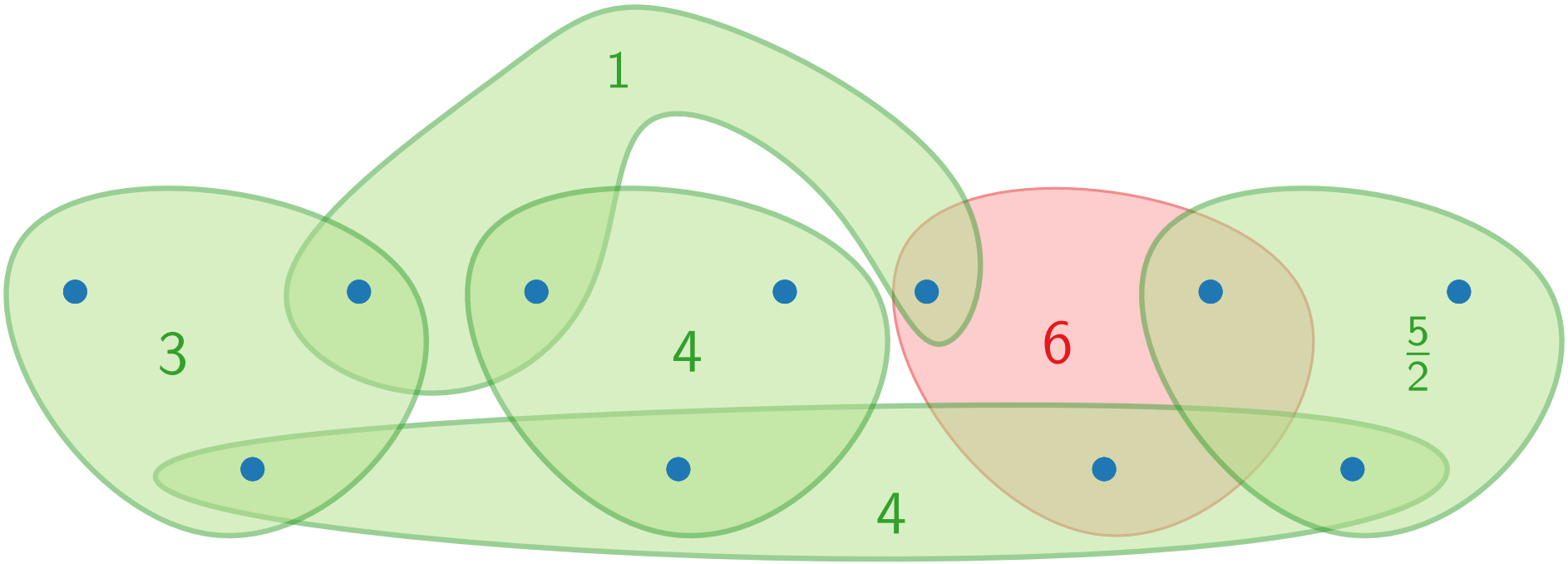


SetCover (general)

Let U be some **ground set** (universe),
and let $\mathcal{S}$ be a family of **subsets** of U with $\bigcup \mathcal{S} = U$.

Each $S \in \mathcal{S}$ has cost $c(S) > 0$.

Find a **cover** $\mathcal{S}' \subseteq \mathcal{S}$ of U (i.e., with $\bigcup \mathcal{S}' = U$) of minimum ~~cardinality.~~ total cost $c(\mathcal{S}') := \sum_{S \in \mathcal{S}'} c(S)$.



Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

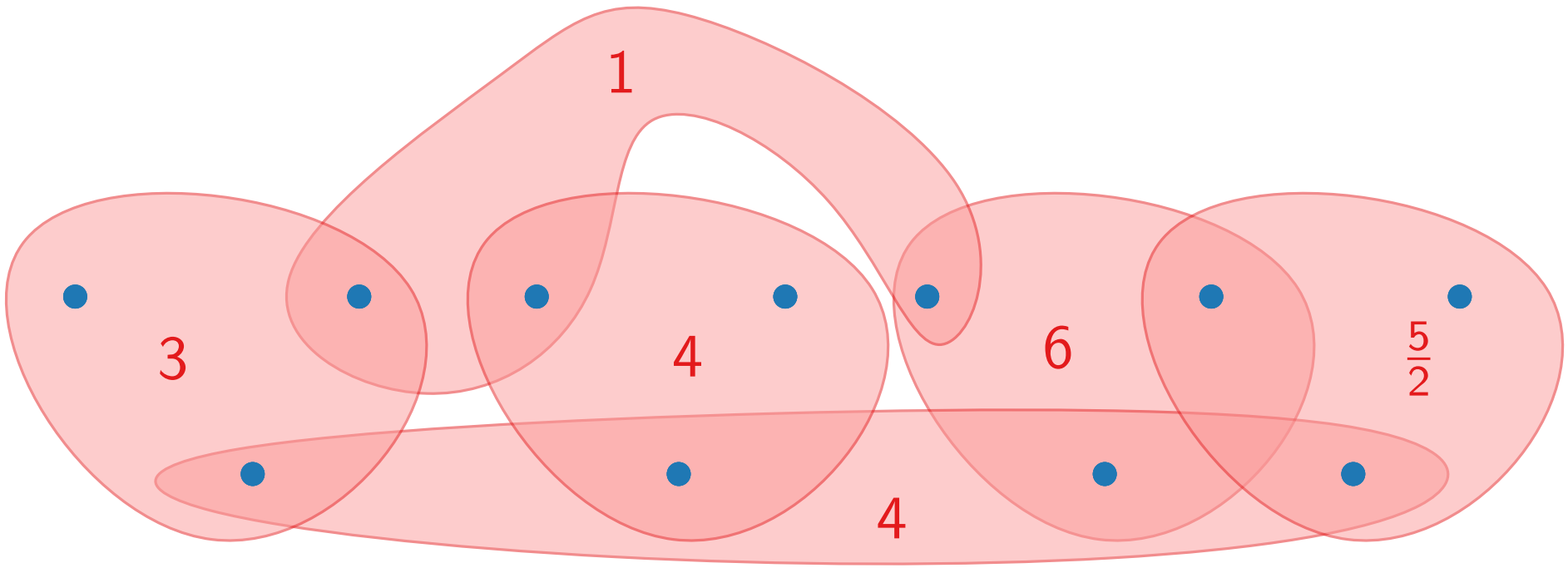
Part II:
Greedy for SetCover

Alexander Wolff

Winter 2022/23

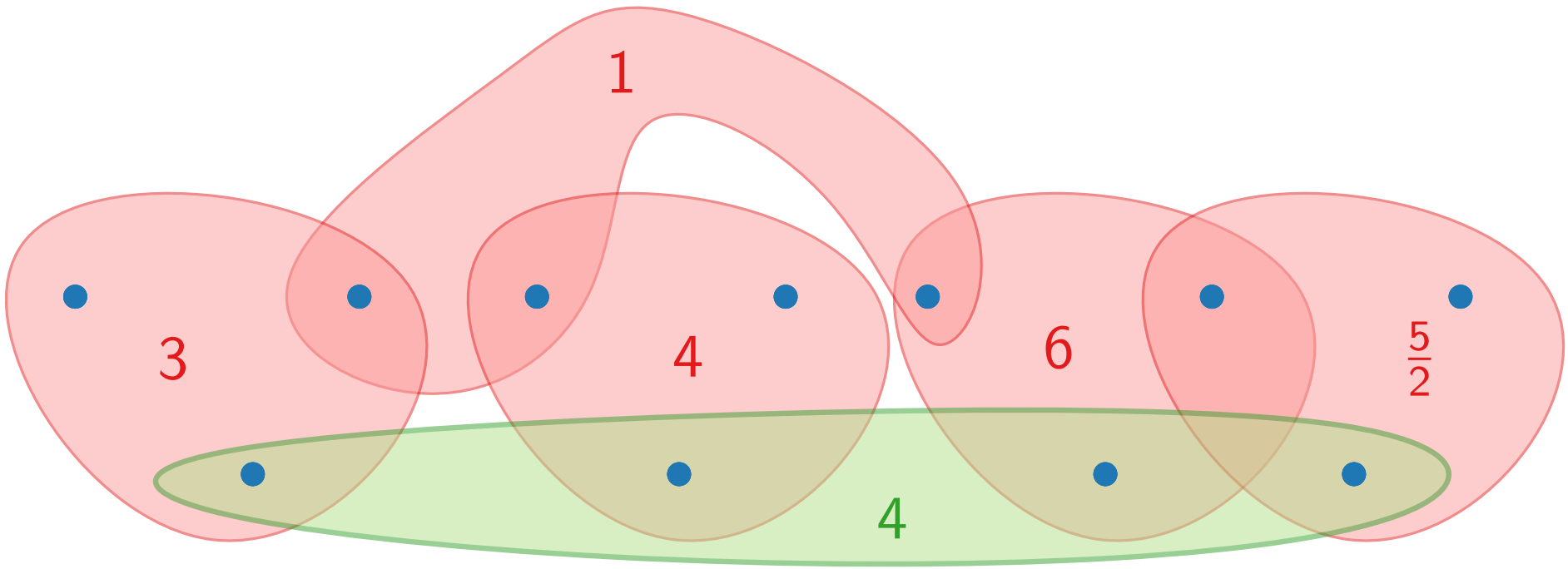
Iterative "Buying" of Elements

What is the real cost of picking a set?



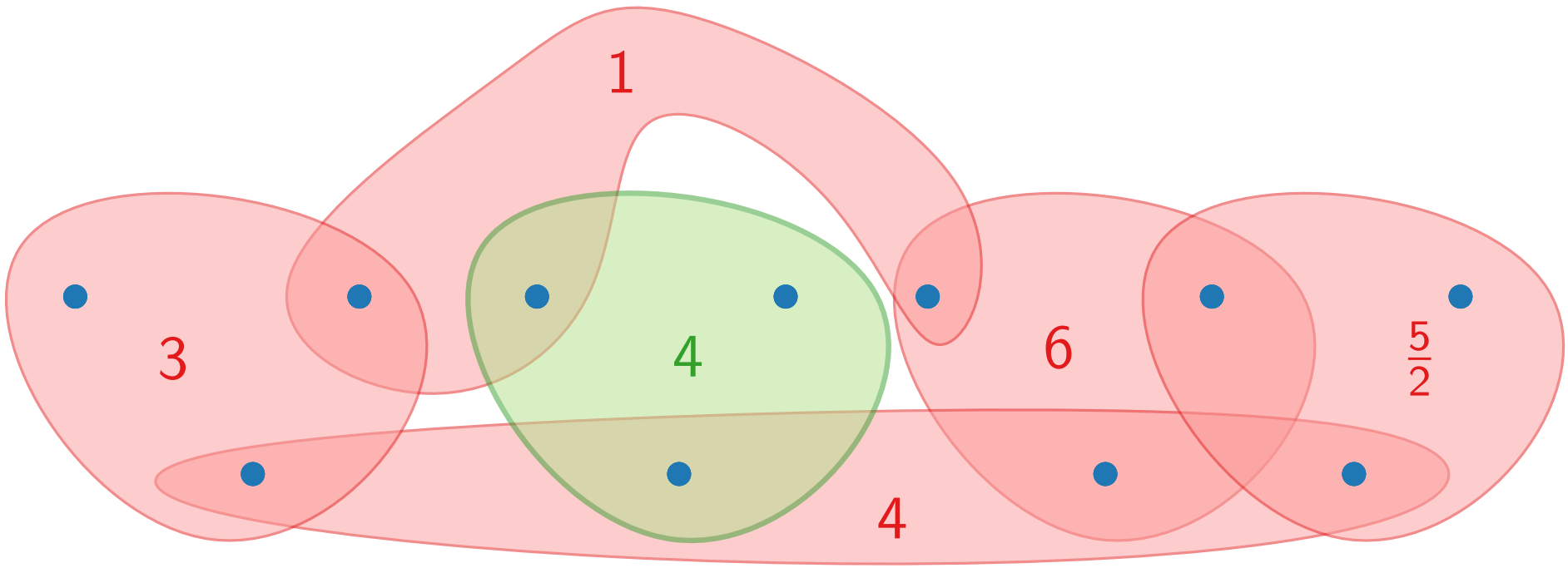
Iterative "Buying" of Elements

What is the real cost of picking a set?



Iterative "Buying" of Elements

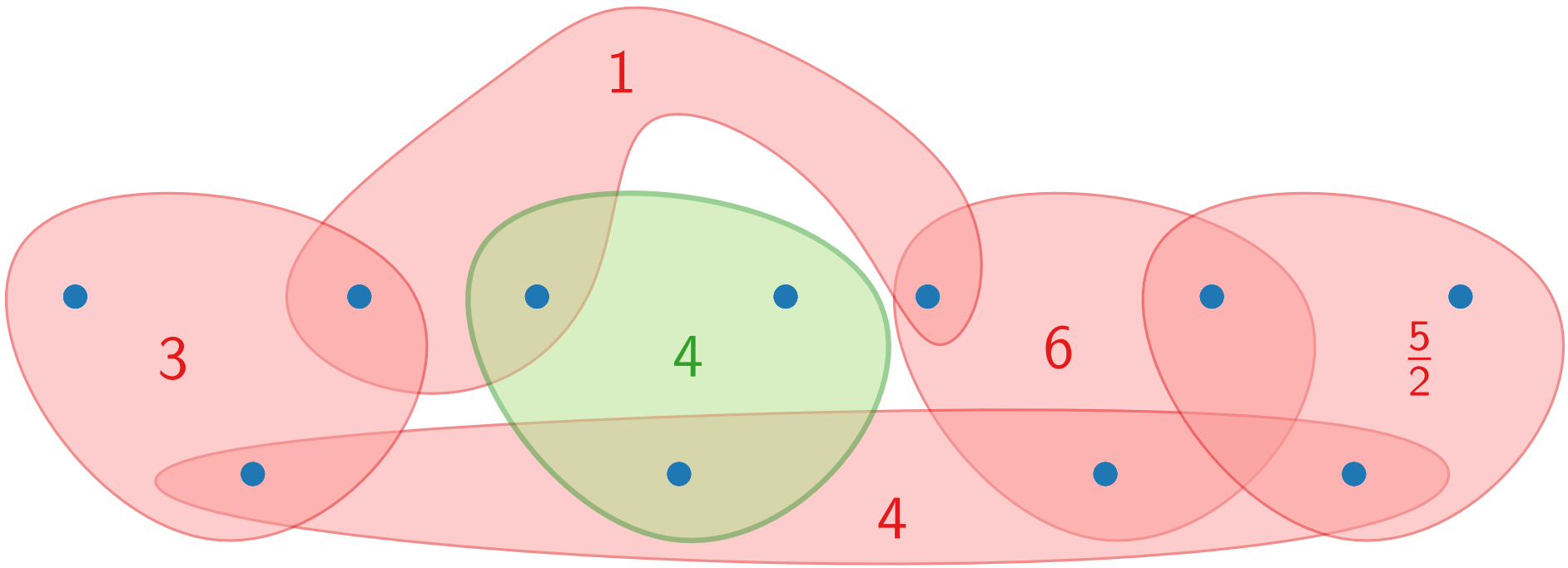
What is the real cost of picking a set?



Iterative "Buying" of Elements

What is the real cost of picking a set?

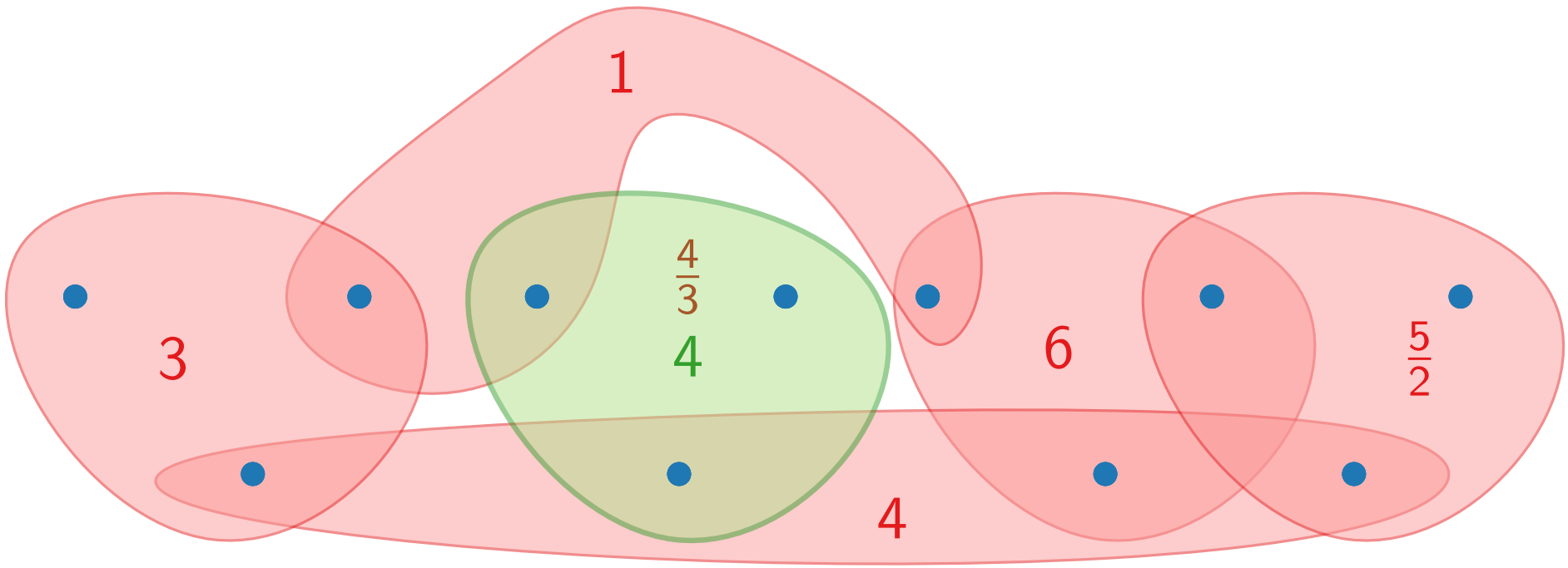
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

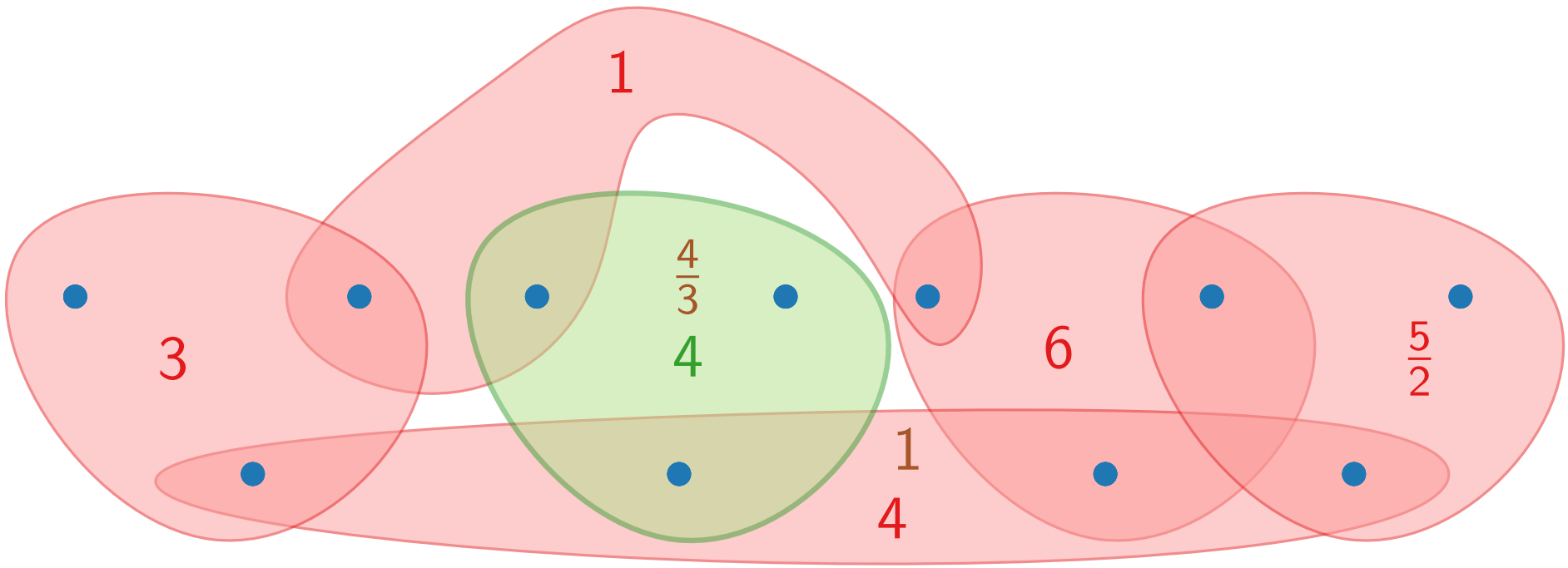
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

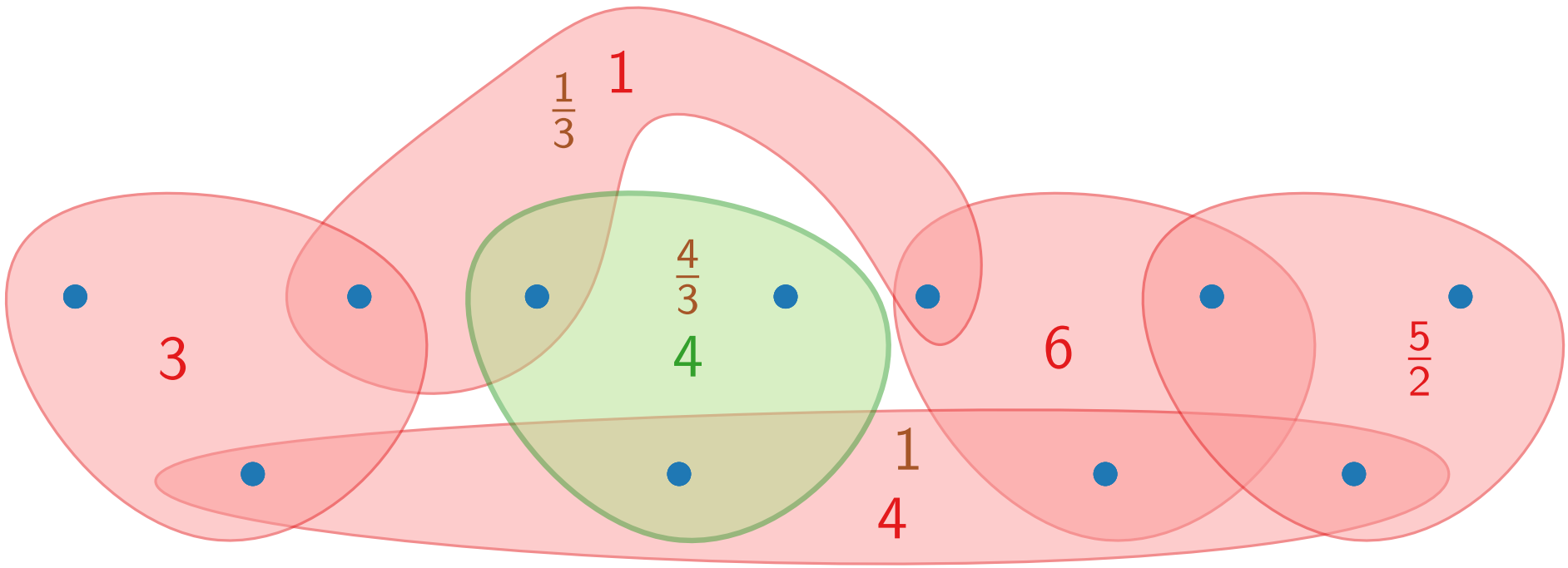
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

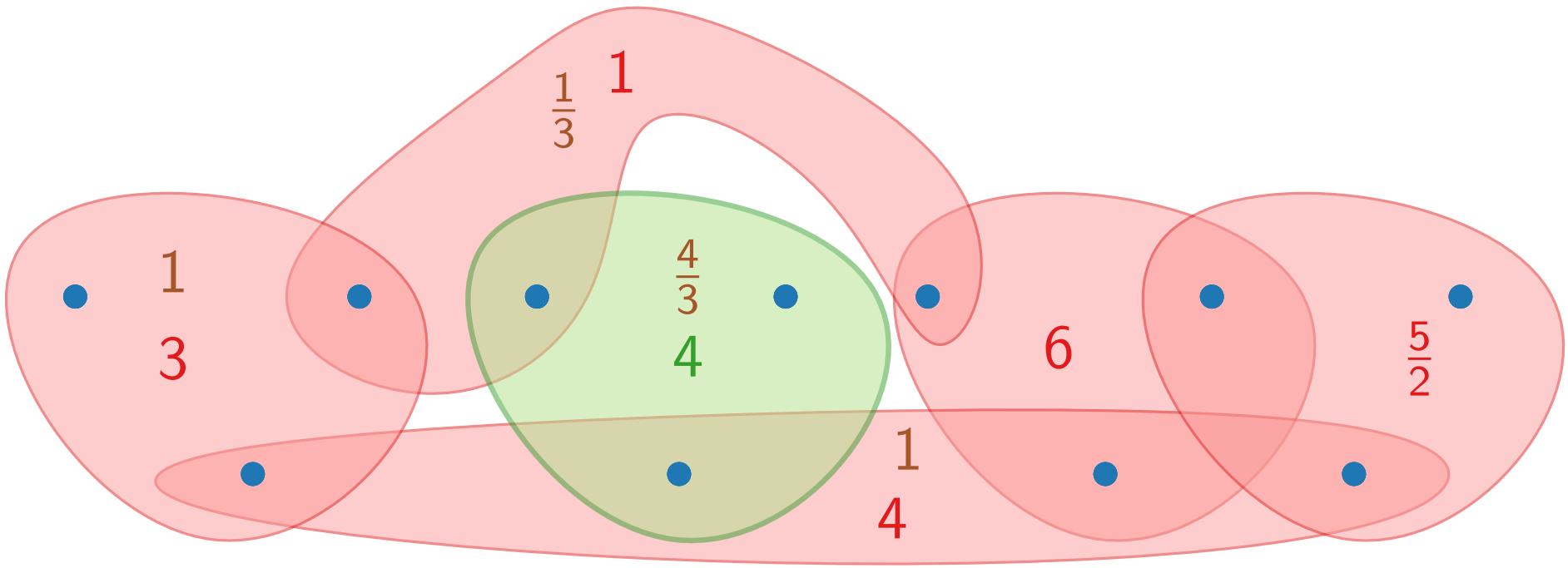
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

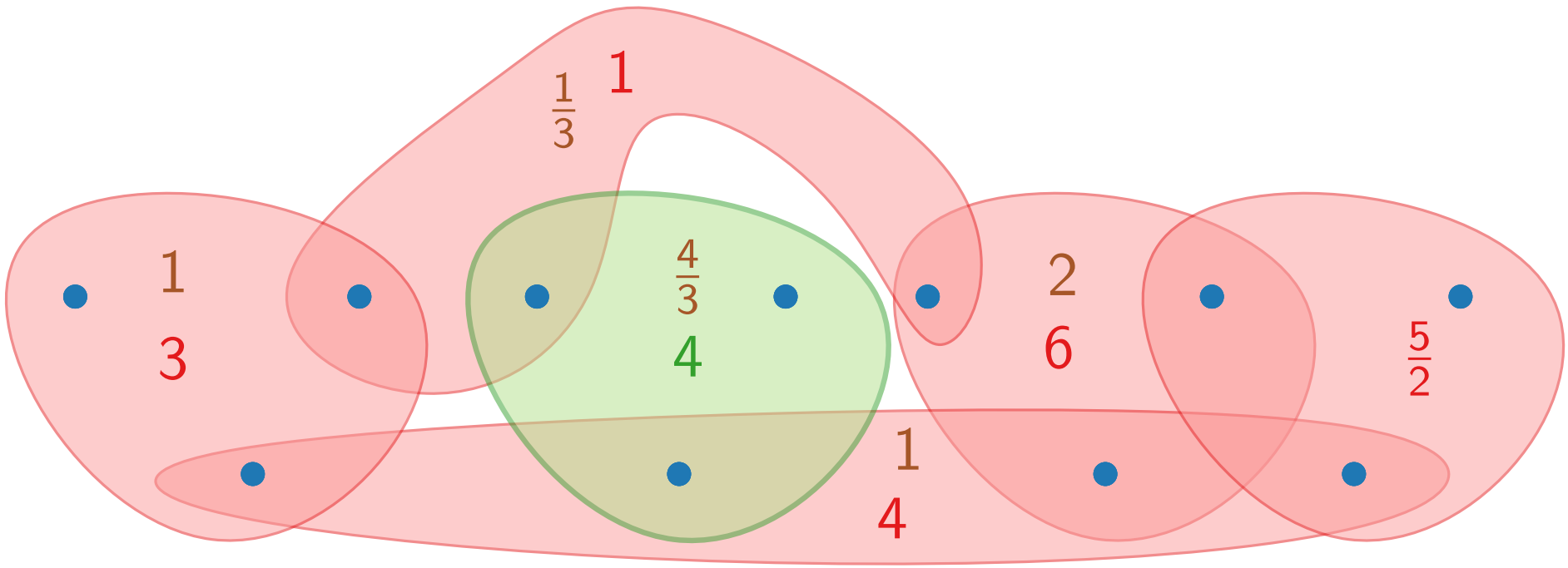
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

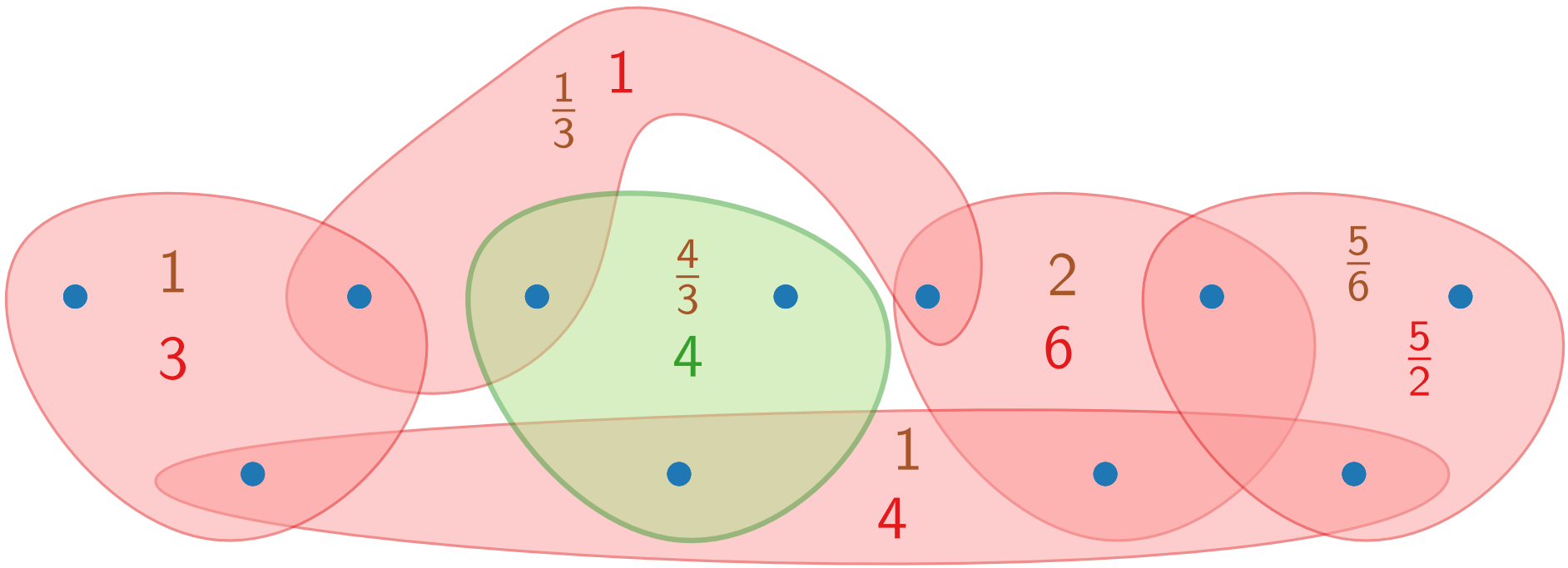
Set with k elements and cost c has per-element cost c/k.



Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

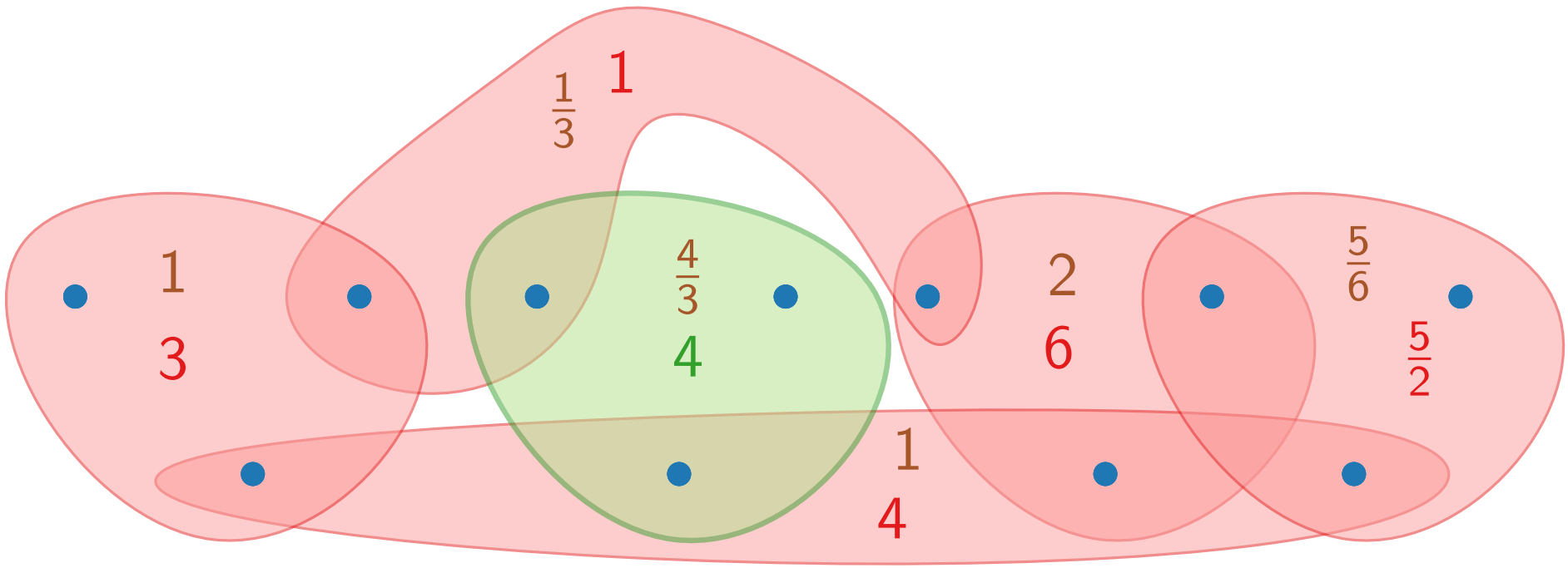


Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?



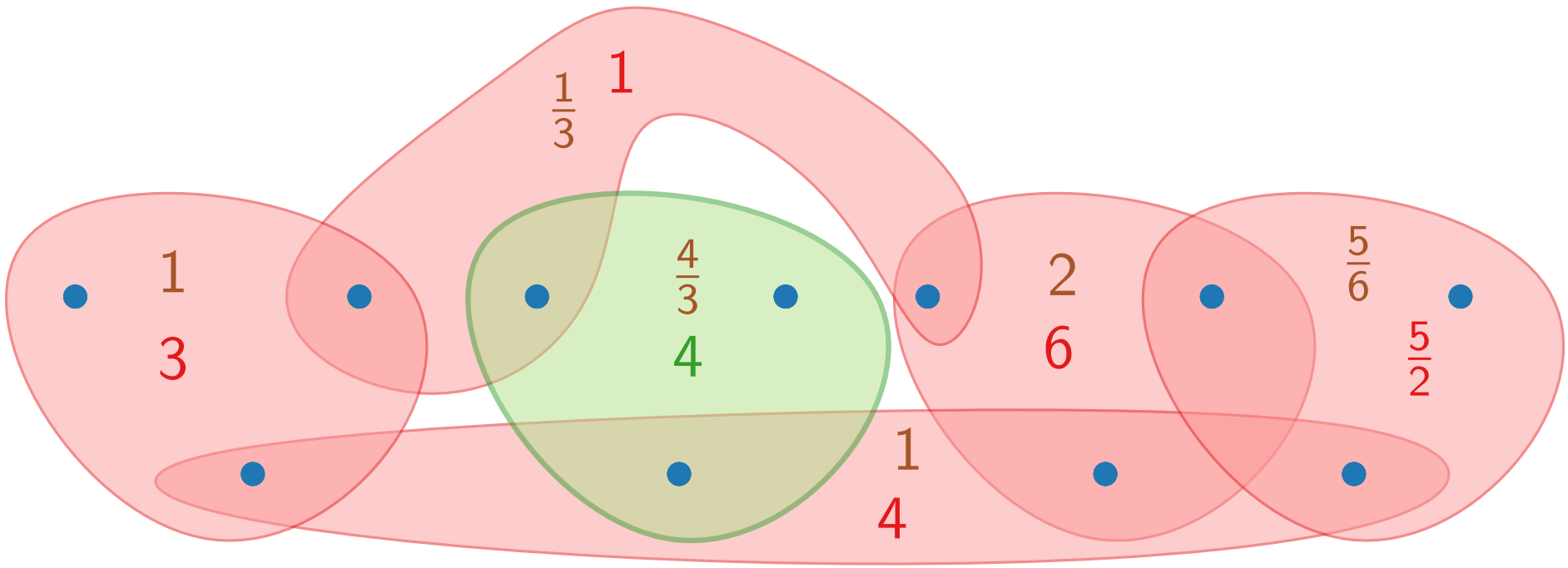
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



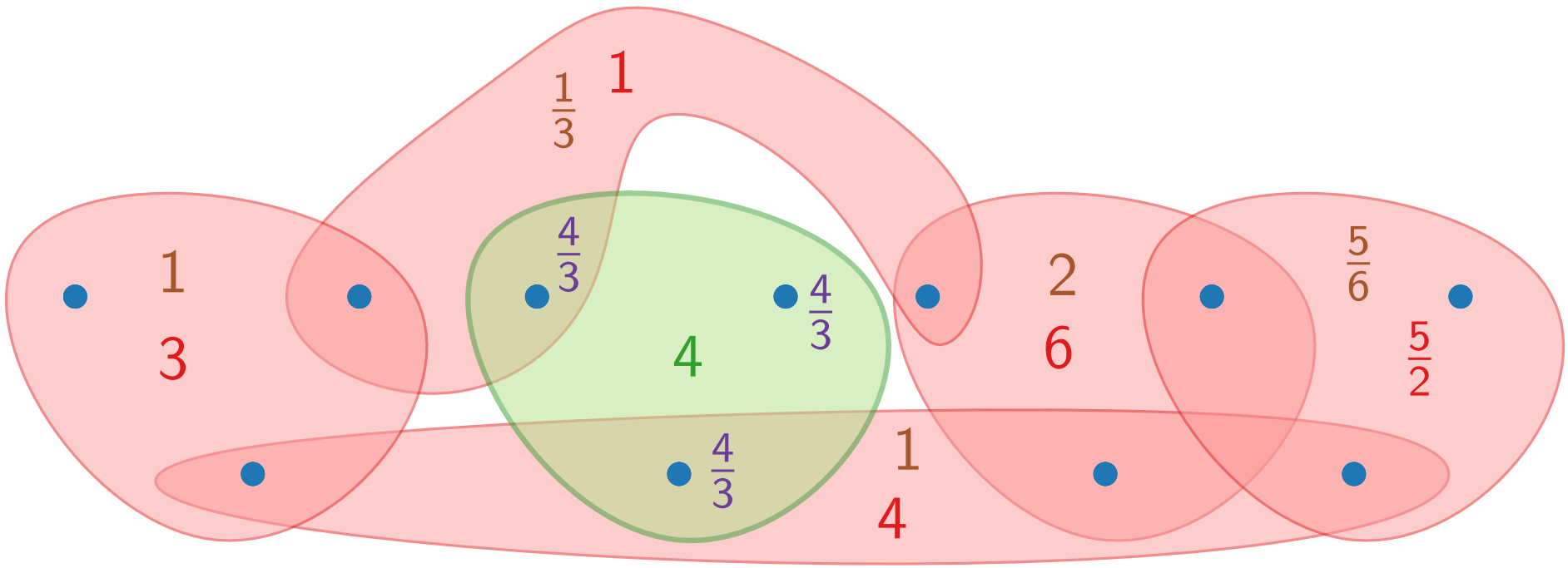
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



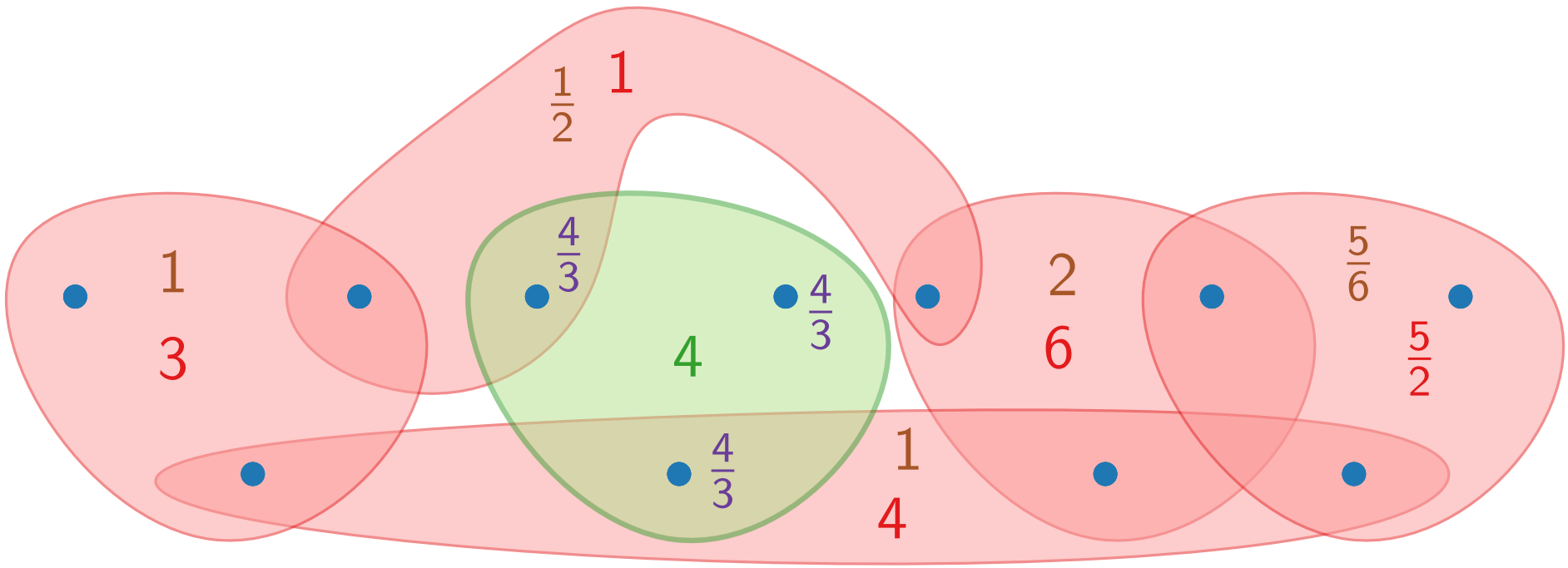
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



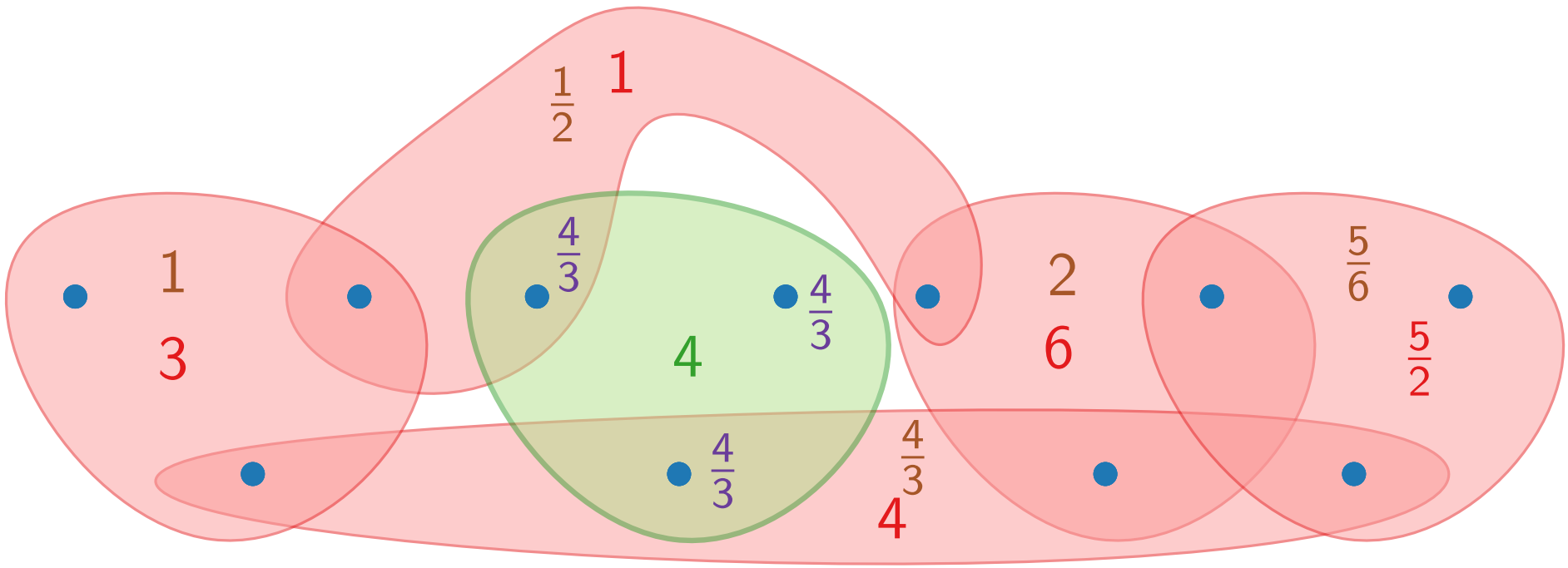
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



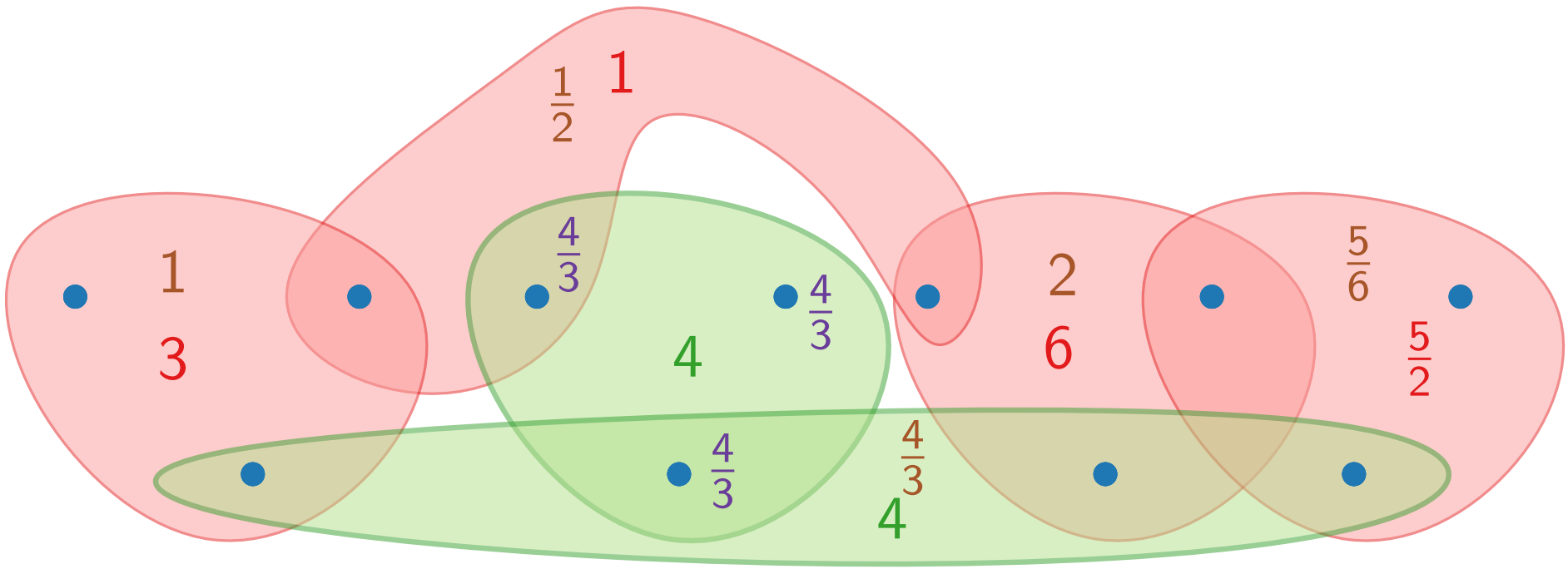
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



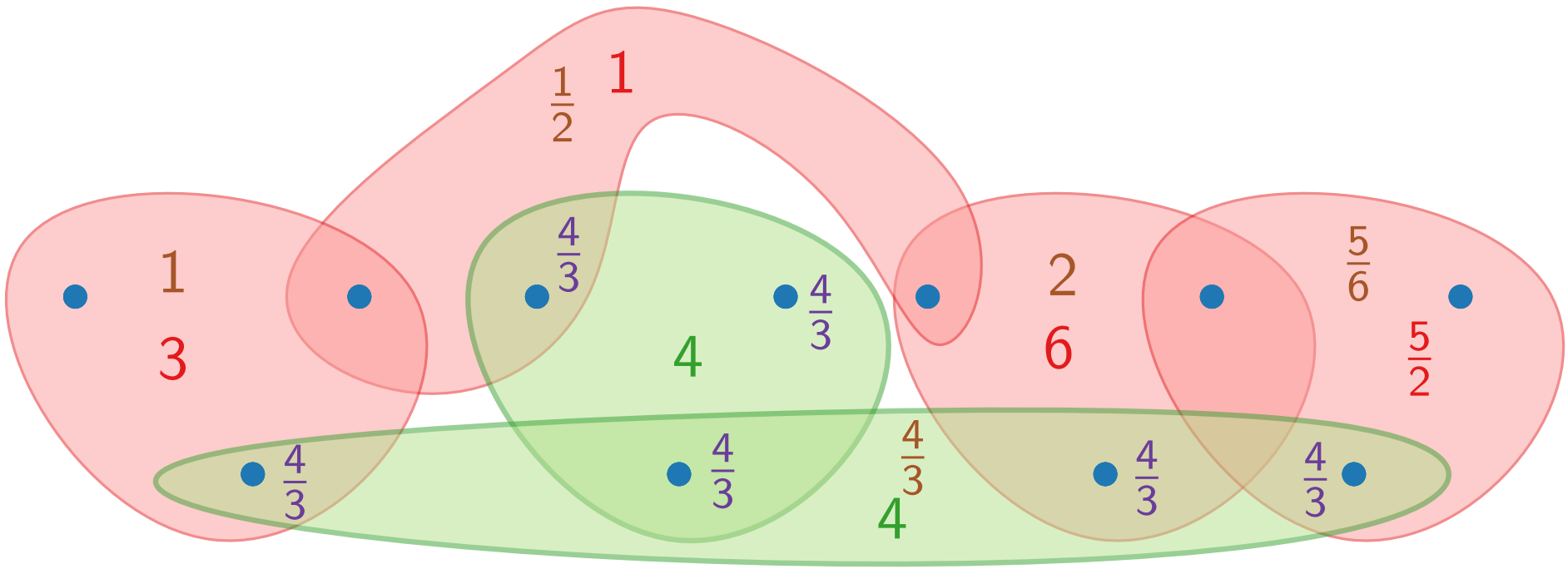
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

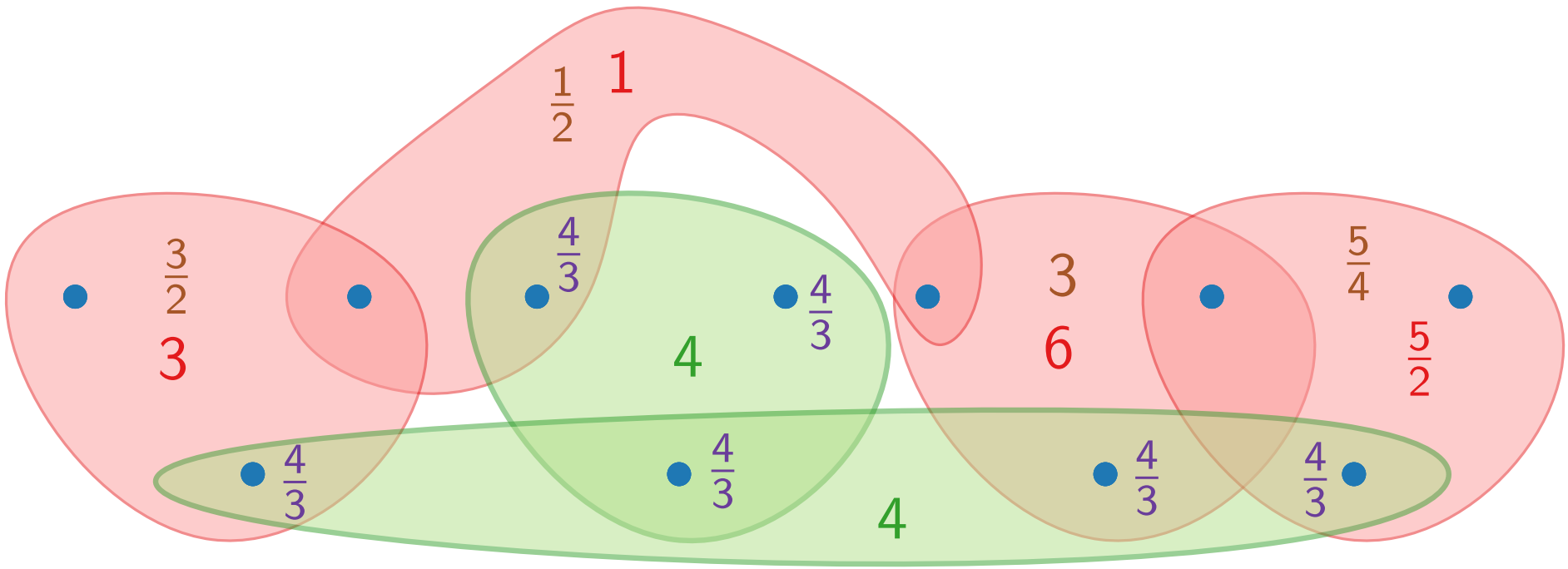
What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



Iterative "Buying" of Elements

What is the real cost of picking a set?
Set with k elements and cost c has per-element cost c/k.
What happens if we "buy" a set?
Fix price of elements bought and recompute per-element cost.



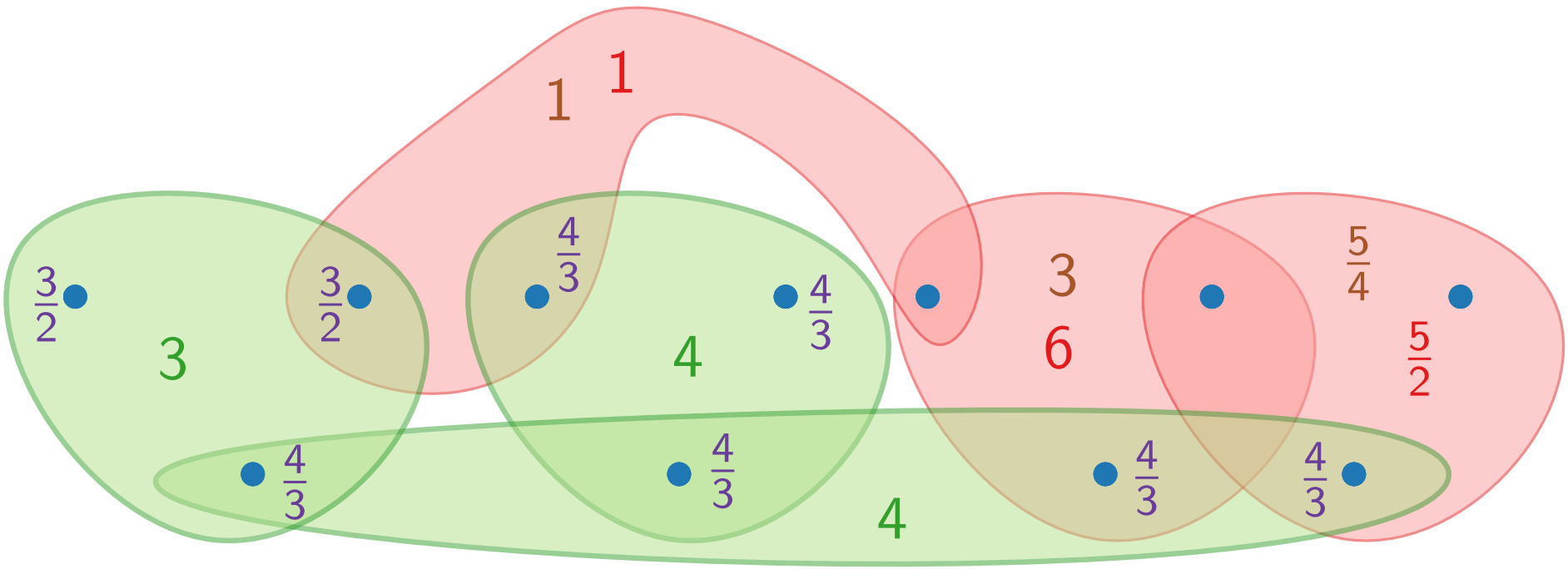
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



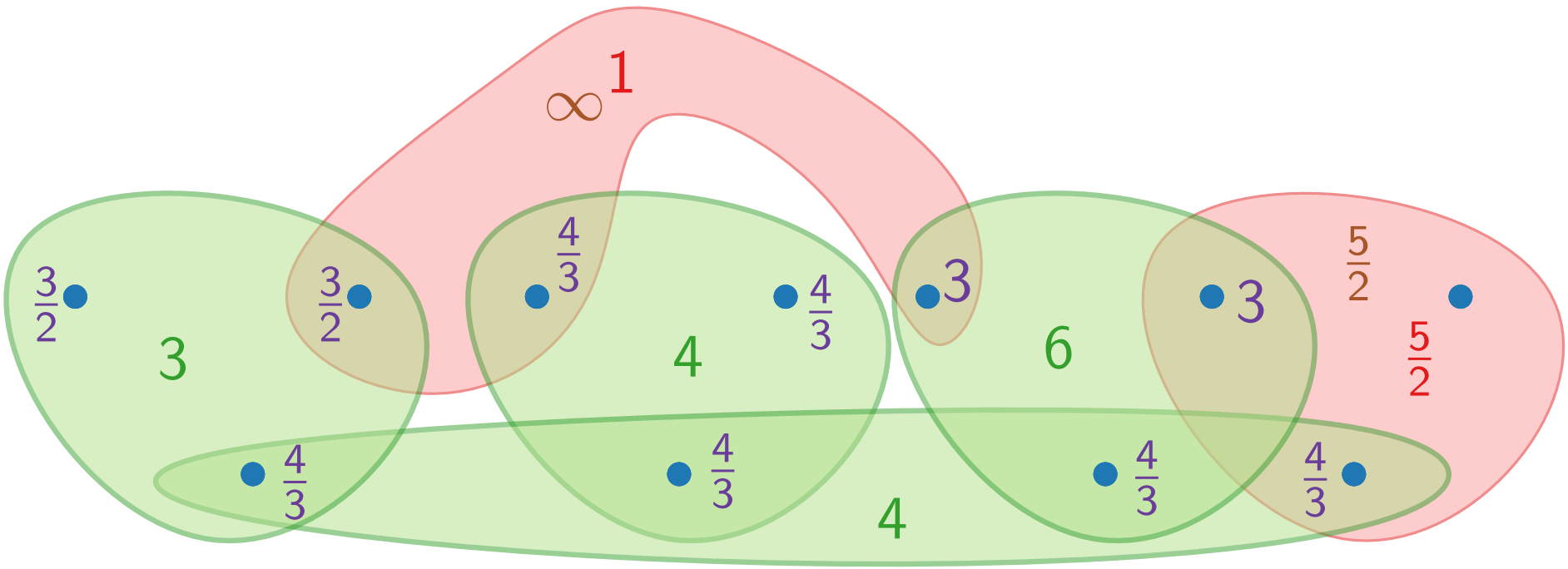
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



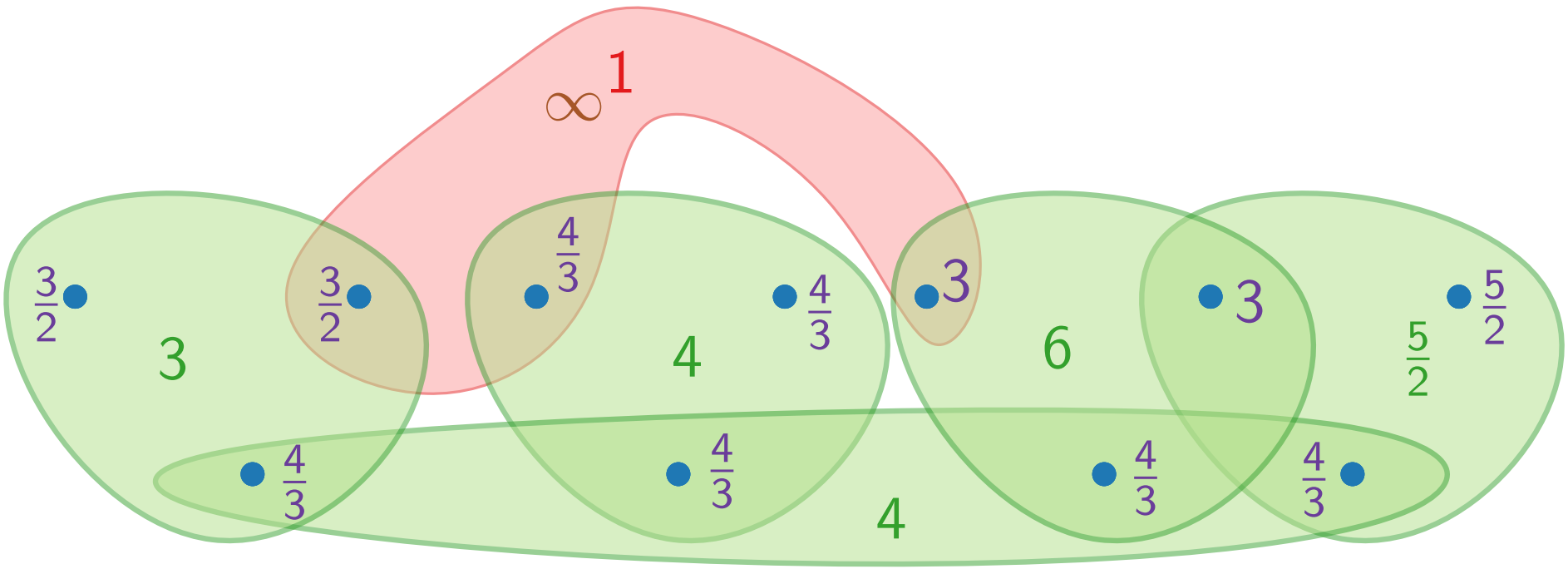
Iterative "Buying" of Elements

What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.



Iterative "Buying" of Elements

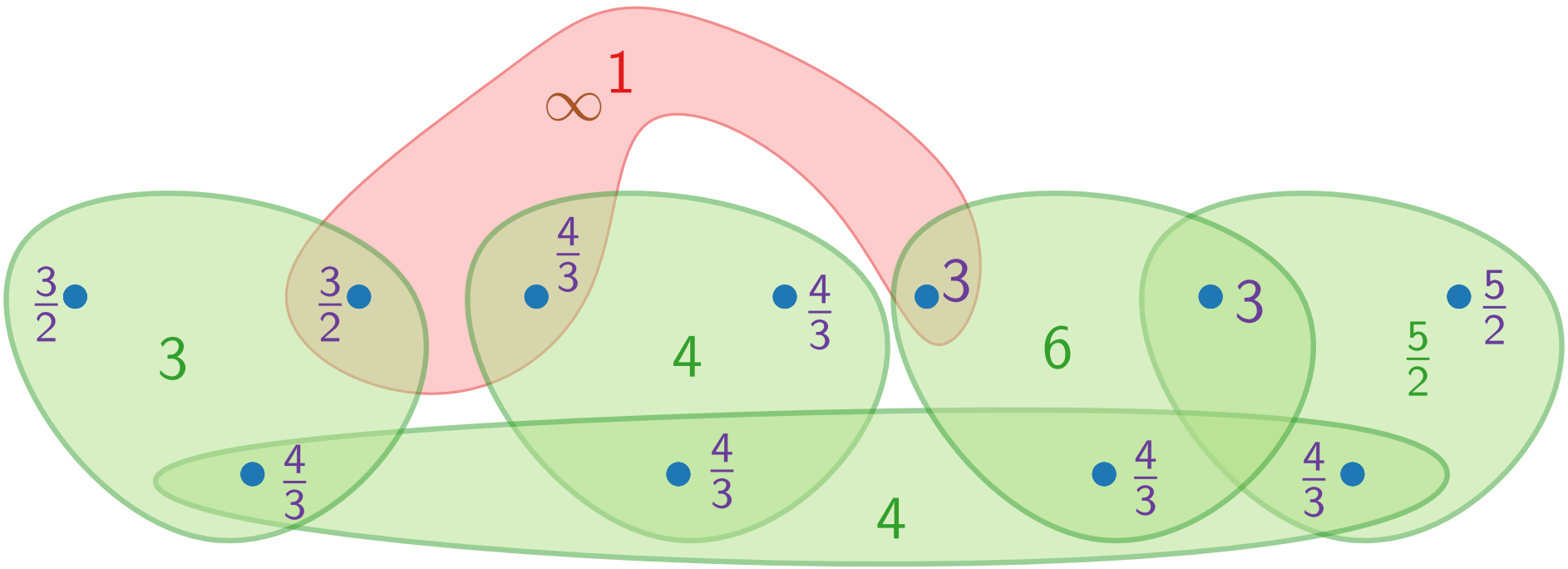
What is the real cost of picking a set?

Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.

total cost: $\sum_{u \in U} \text{price}(u)$



Iterative "Buying" of Elements

What is the real cost of picking a set?

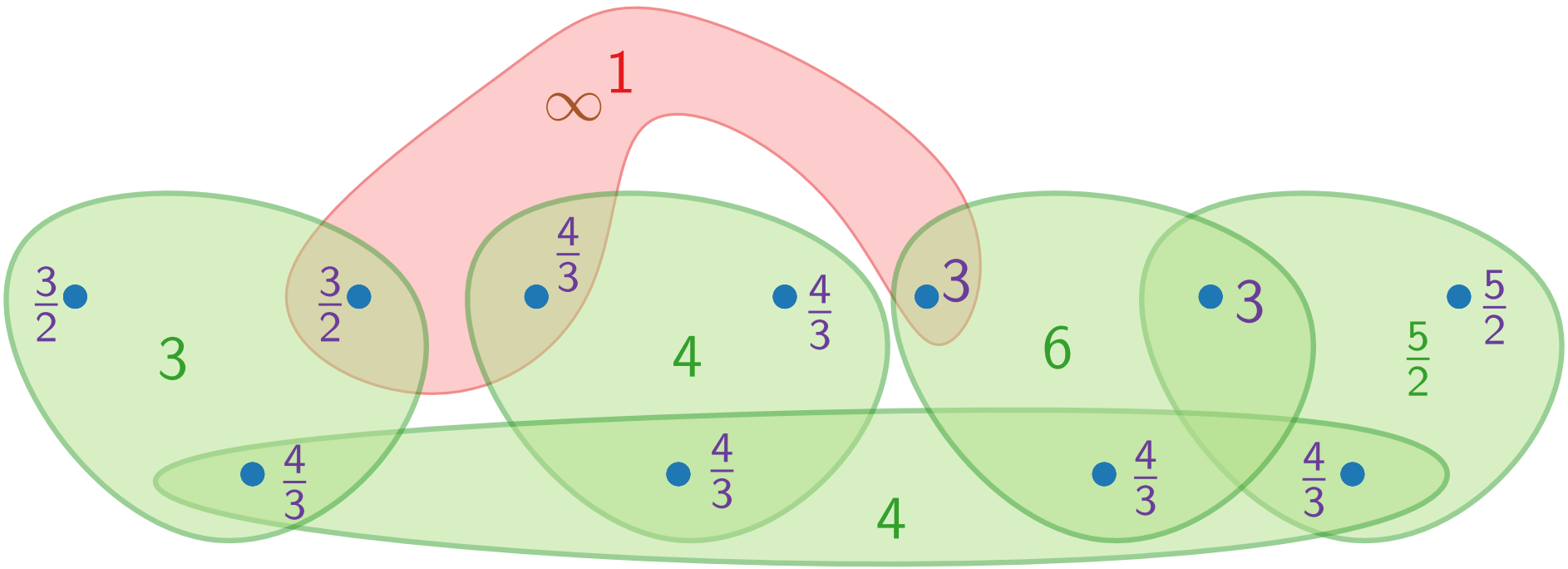
Set with k elements and cost c has per-element cost c/k.

What happens if we "buy" a set?

Fix price of elements bought and recompute per-element cost.

total cost: $\sum_{u \in U} \text{price}(u)$

Greedy: Always choose the set with minimum per-element cost.



Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

  foreach $u \in S \setminus C$ **do**

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

  foreach $u \in S \setminus C$ **do**

    price(u) $\leftarrow \frac{c(S)}{|S \setminus C|}$

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

  foreach $u \in S \setminus C$ **do**

    price(u) $\leftarrow \frac{c(S)}{|S \setminus C|}$

  $C \leftarrow C \cup S$

return $\mathcal{S}'$ // Cover of U

Greedy for SetCover

GreedySetCover($U, \mathcal{S}, c$)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

while $C \neq U$ **do**

  $S \leftarrow$ set in $\mathcal{S}$ that minimizes $\frac{c(S)}{|S \setminus C|}$

  foreach $u \in S \setminus C$ **do**

    price(u) $\leftarrow \frac{c(S)}{|S \setminus C|}$

  $C \leftarrow C \cup S$

  $\mathcal{S}' \leftarrow \mathcal{S}' \cup \{S\}$

return $\mathcal{S}'$ // Cover of U

Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part III:
Analysis

Alexander Wolff

Winter 2022/23

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

- $\leq j - 1$ elements of S already bought

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

- $\leq j - 1$ elements of S already bought
- $\geq \ell - j + 1$ elements of S not yet bought

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

- $\leq j - 1$ elements of S already bought
- $\geq \ell - j + 1$ elements of S not yet bought
- per-element cost for S:

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

- $\leq j - 1$ elements of S already bought
- $\geq \ell - j + 1$ elements of S not yet bought
- per-element cost for S: $\leq c(S)/(\ell - j + 1)$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Proof. Iteration at which alg. buys $u_j \Rightarrow$

- $\leq j - 1$ elements of S already bought
- $\geq \ell - j + 1$ elements of S not yet bought
- per-element cost for S: $\leq c(S)/(\ell - j + 1)$
- price by alg. no larger due to greedy choice

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol.

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

$\text{price}(U) =$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

$\text{price}(U) = \sum_{u \in U} \text{price}(u) \leq$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

$$\text{price}(U) = \sum_{u \in U} \text{price}(u) \leq \sum_{i=1}^{m} \text{price}(S_i)$$

$$\leq$$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

$$\text{price}(U) = \sum_{u \in U} \text{price}(u) \leq \sum_{i=1}^{m} \text{price}(S_i)$$
$$\leq \sum_{i=1}^{m} c(S_i) \cdot \mathcal{H}_k =$$

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

Lemma. Let $S \in \mathcal{S}$, and let $u_1, \ldots, u_\ell$ be the elements of S in the order in which they are covered ("bought") by GreedySetCover. Then $\text{price}(u_j) \leq c(S)/(\ell - j + 1)$.

Lemma. $\text{price}(S) := \sum_{i=1}^{\ell} \text{price}(u_i) \leq c(S) \cdot \mathcal{H}_\ell$.

Proof. Let $\{S_1, \ldots, S_m\}$ be opt. sol. $\text{OPT} = \sum_{i=1}^{m} c(S_i)$

$$\begin{aligned} \text{price}(U) &= \sum_{u \in U} \text{price}(u) \leq \sum_{i=1}^{m} \text{price}(S_i) \\ &\leq \sum_{i=1}^{m} c(S_i) \cdot \mathcal{H}_k = \text{OPT} \cdot \mathcal{H}_k \end{aligned}$$

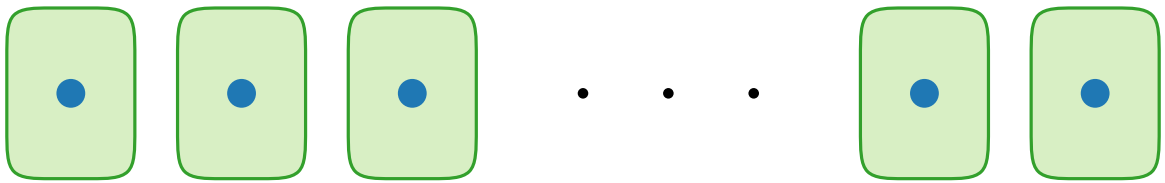
□

Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

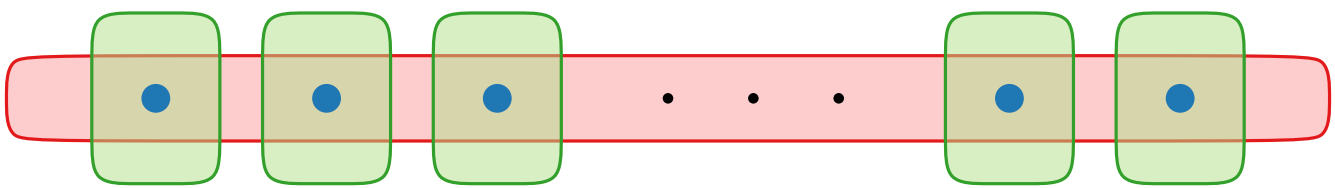
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



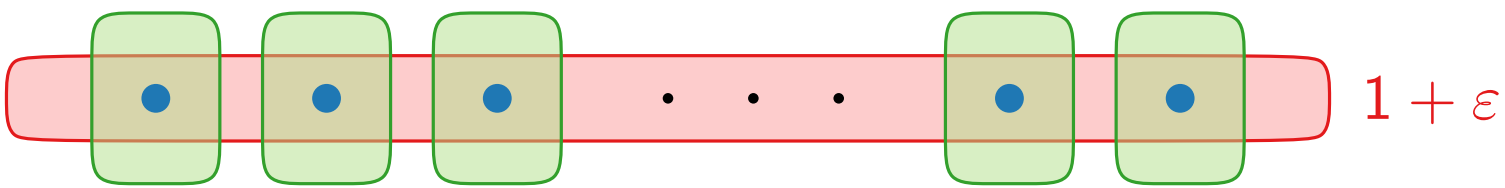
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



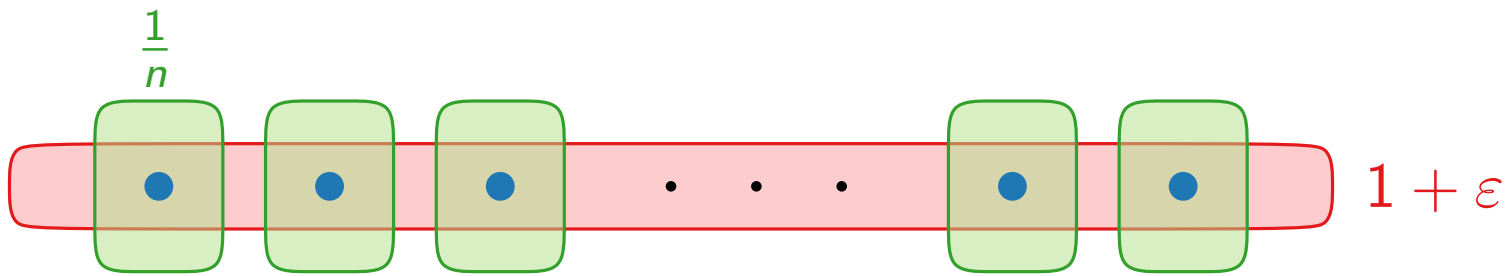
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



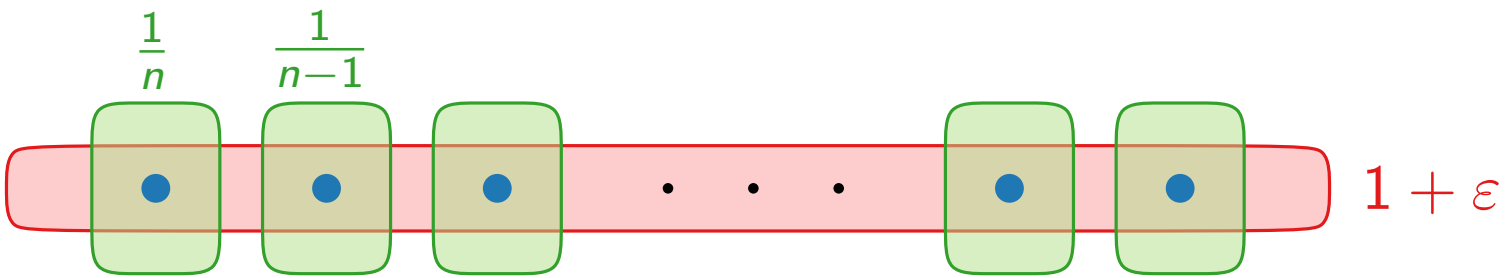
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



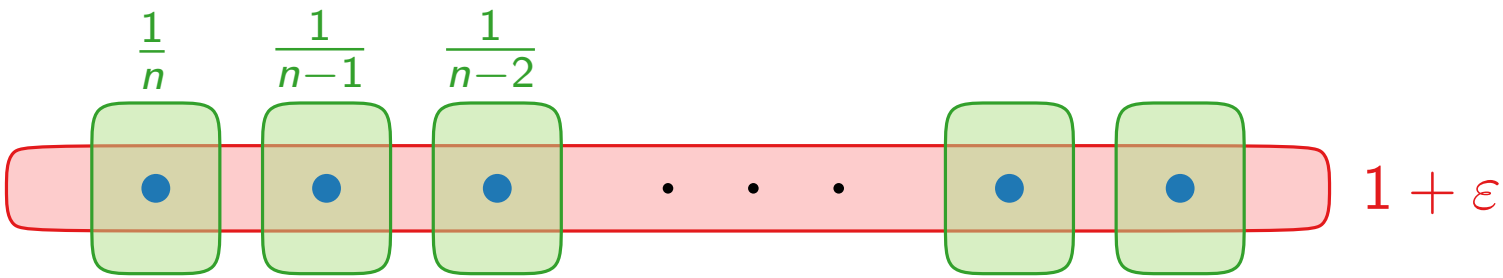
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



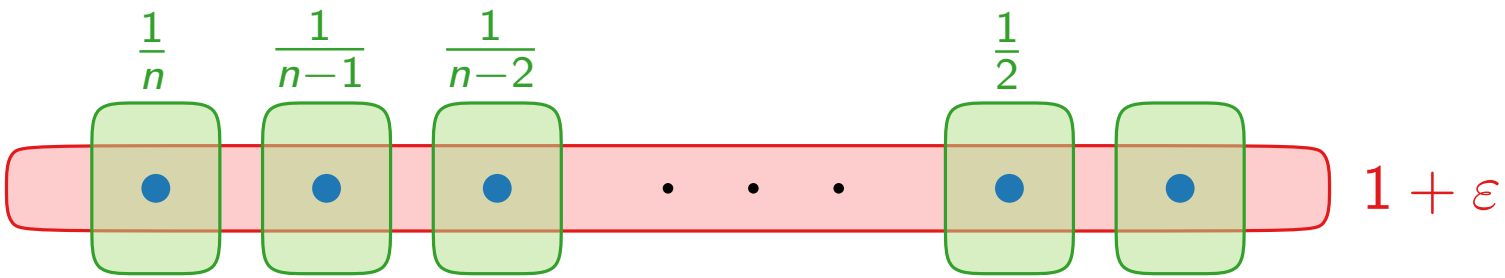
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



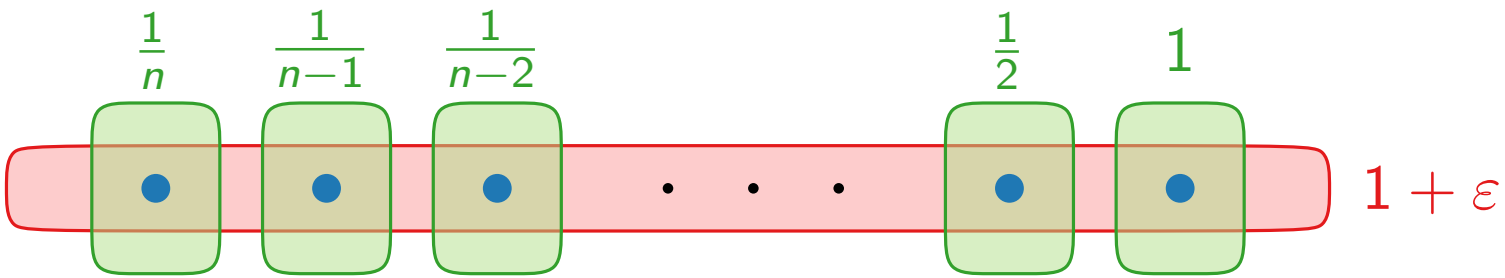
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



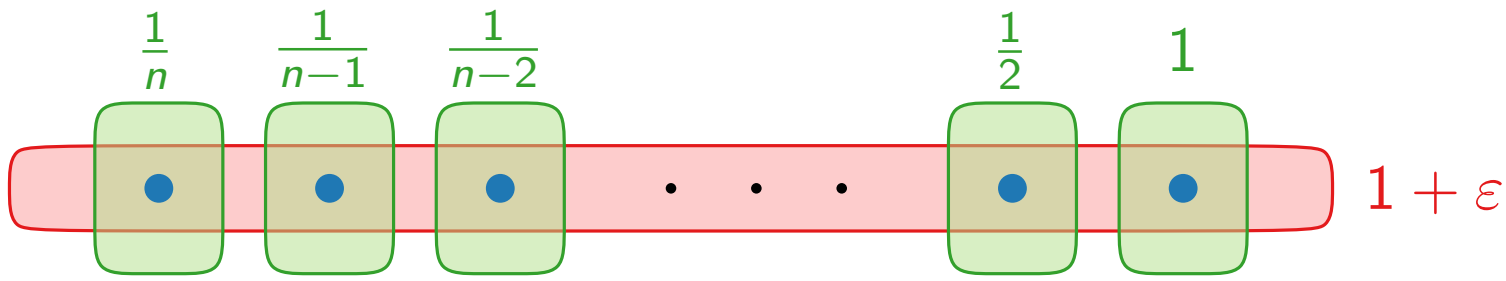
Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



Analysis tight?

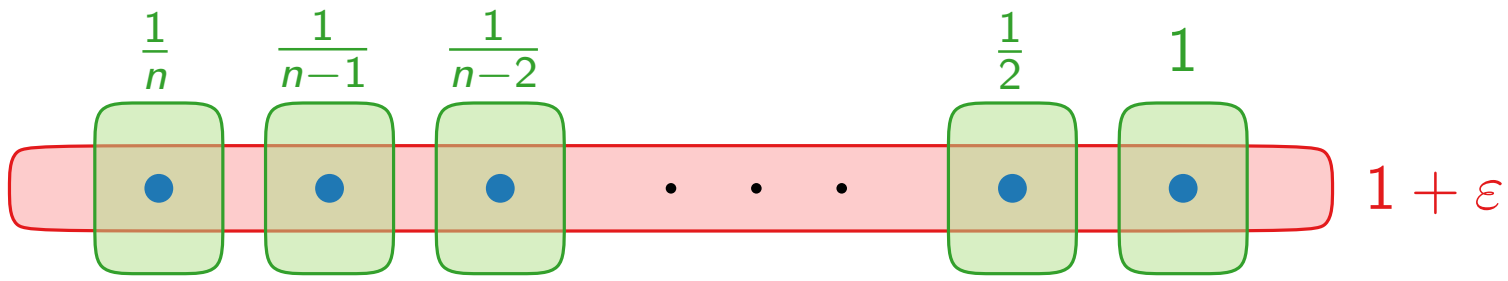
Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



price$(U) = \mathcal{H}_n$ OPT $= 1 + \varepsilon$

Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

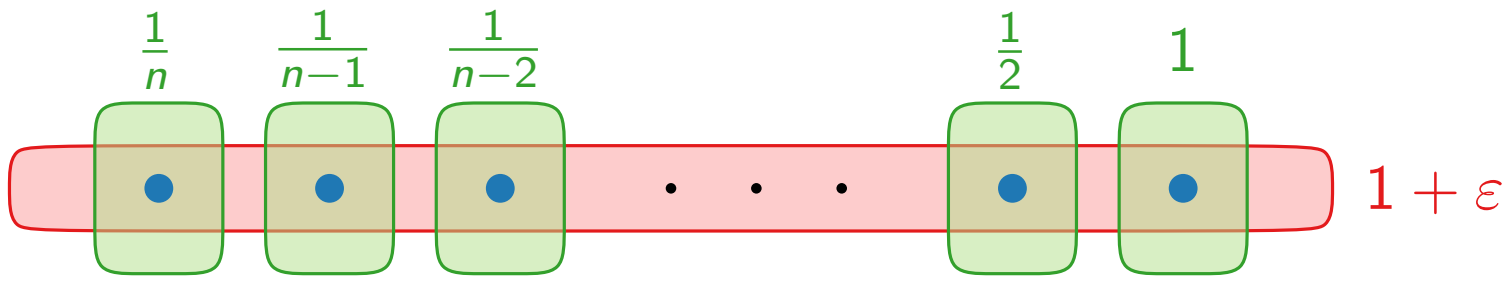


$\text{price}(U) = \mathcal{H}_n$ $\qquad$ $\text{OPT} = 1 + \varepsilon$

Can we do better?

Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.



price$(U) = \mathcal{H}_n$ OPT $= 1 + \varepsilon$

Can we do better?

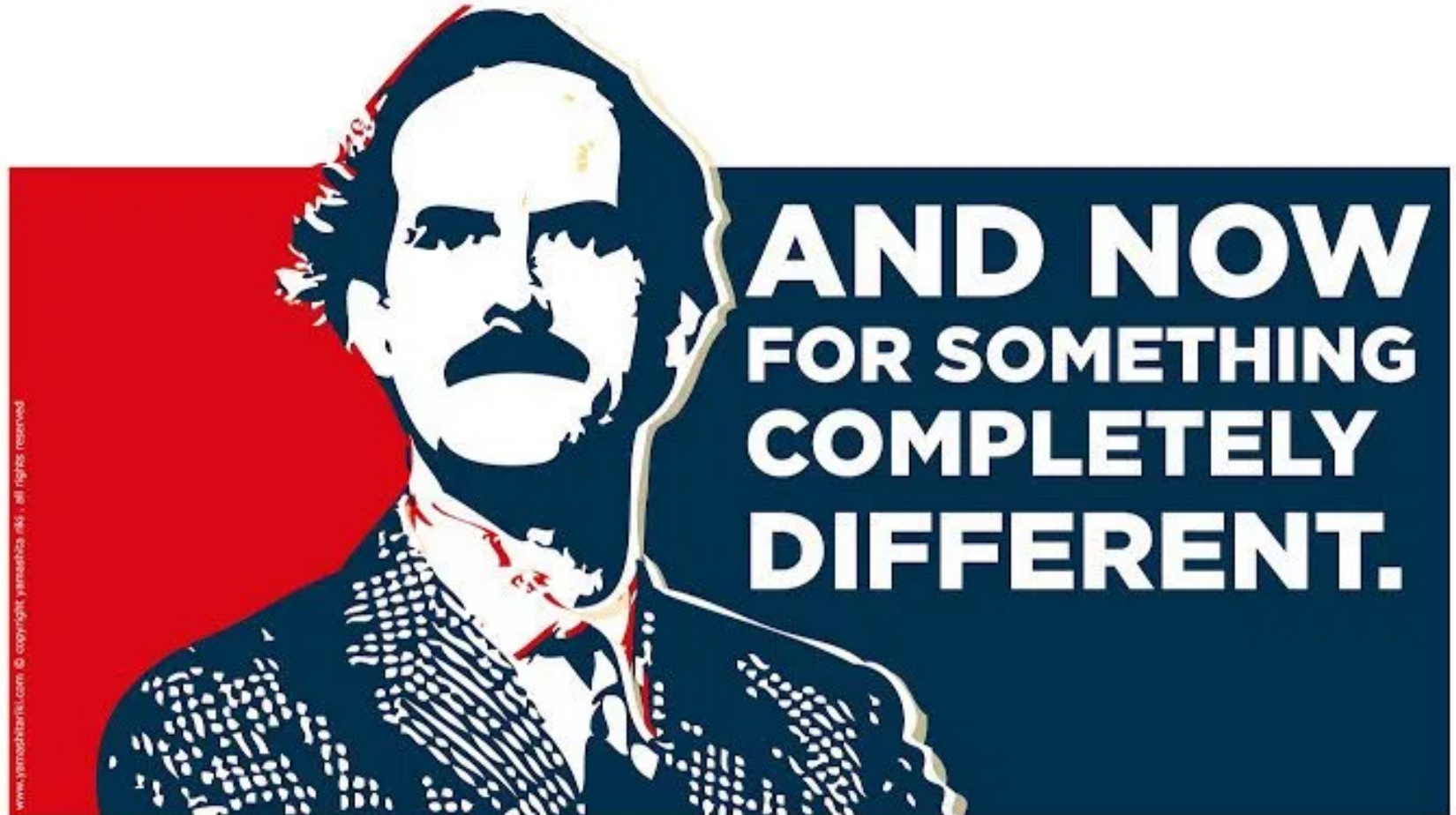
No – SetCover cannot be approximated within factor $(1 - o(1)) \cdot \ln n$ (unless P = NP). [Feige, JACM 1998]

Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

Can we do b

No – SetCo
$(1 - o(1)) \cdot l$

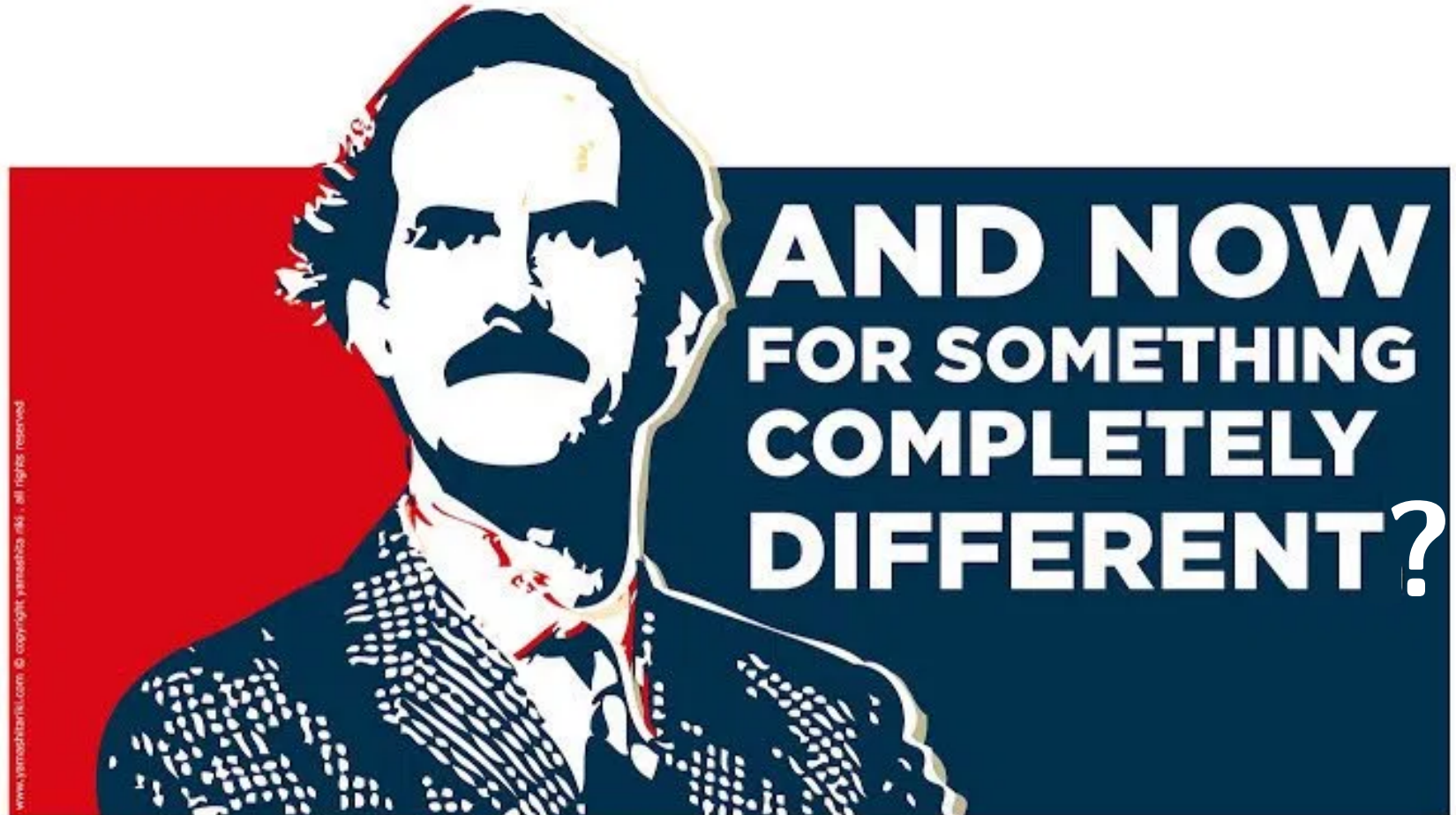


Analysis tight?

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

Can we do b

No – SetCo
$(1 - o(1)) \cdot l$



Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part IV:
ShortestSuperString

Alexander Wolff

Winter 2022/23

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\}$

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

abc

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

abc
$\quad bcb$

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

$$\begin{array}{l} abc \\ \quad bcb \\ \quad\quad cbaa \end{array}$$

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

$abcbaa$

abc
$\quad bcb$
$\quad\quad cbaa$

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb$?

$abcbaa$ "covers" all strings in U

abc
bcb
$cbaa$

ShortestSuperString (SSS)

Given a set $\{s_1, \ldots, s_n\} \subseteq \Sigma^+$ of strings over a finite alphabet Σ.

Find a **shortest string** s (*superstring*) such that,
for each $i \in \{1, \ldots, n\}$, the string s_i is a *substring* of s.

Example. $U := \{cbaa, abc, bcb\} \quad \rightarrow cbaabcb\ ?$

W.l.o.g.: No string s_i is a substring of any other string s_j.

$abcbaa$ "covers" all strings in U

abc
$\ \ bcb$
$\ \ \ \ cbaa$

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

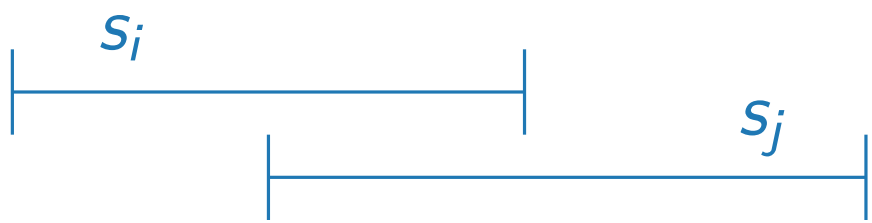
Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

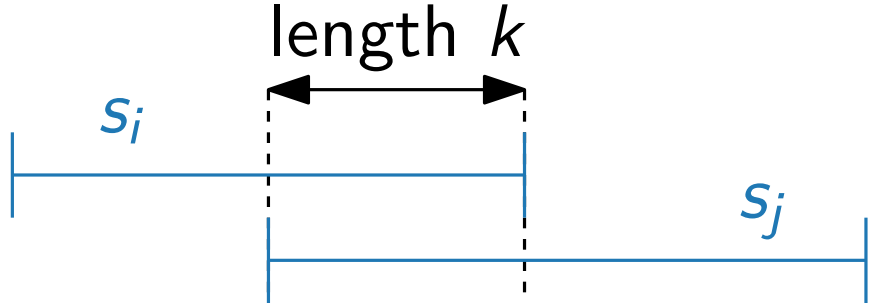


SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

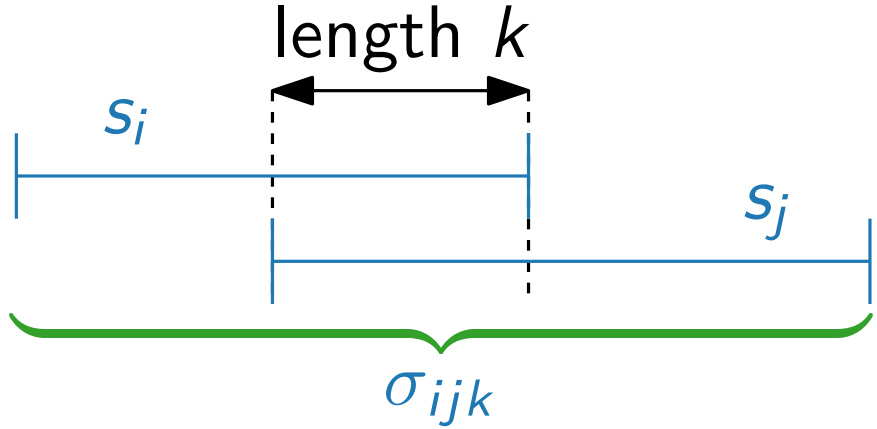


SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)



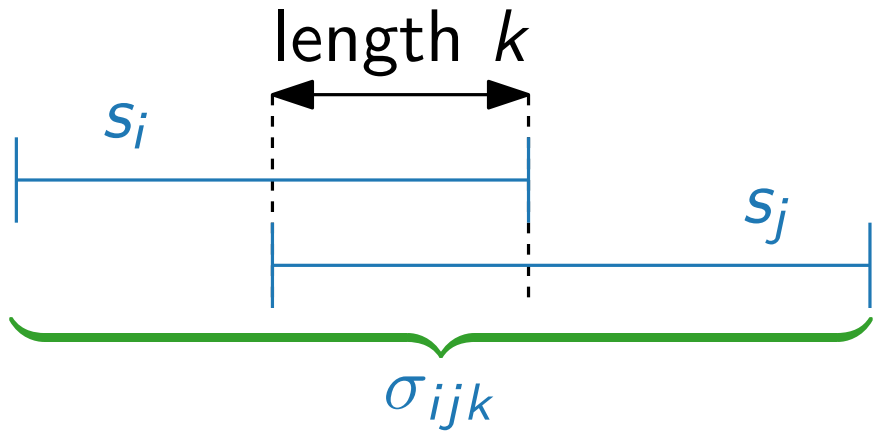
SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc



SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

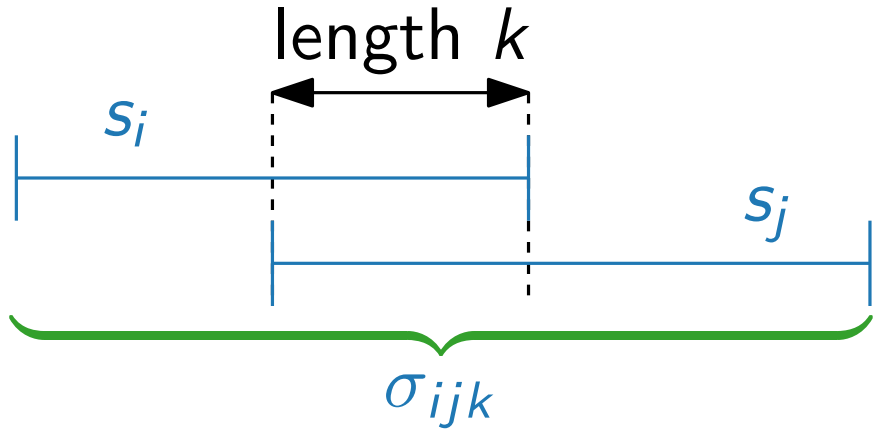
Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab

ababc



SSS as a SetCover Problem

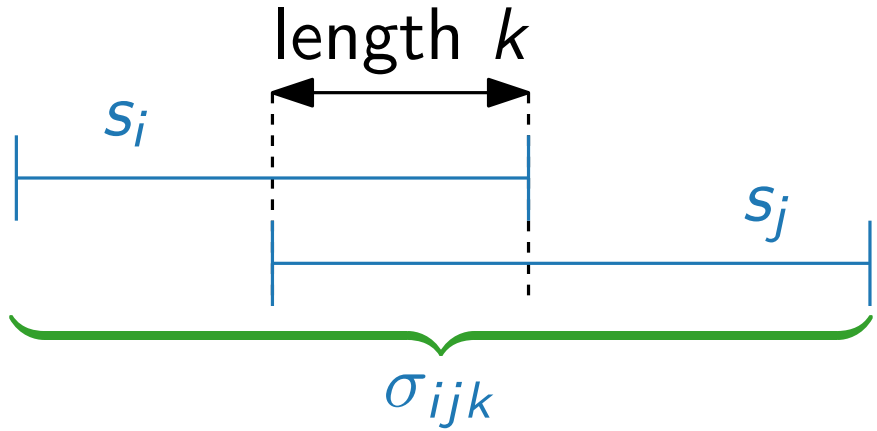
SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cababc



SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

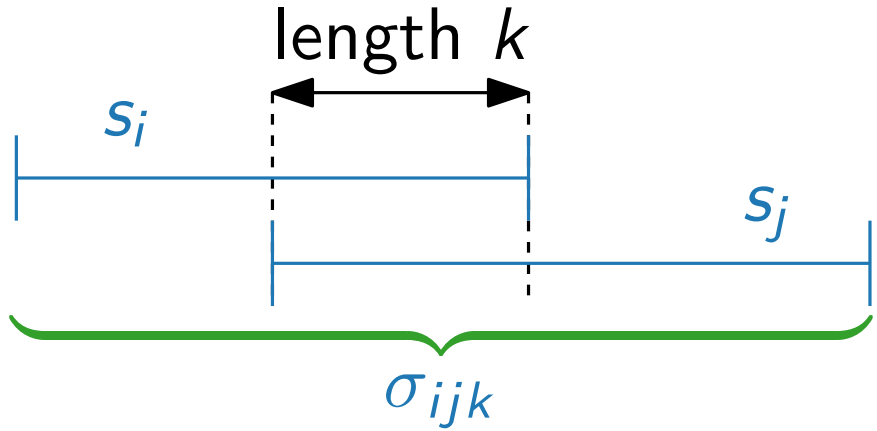
Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab

ababc

σ_{ij2}: cababababc



SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

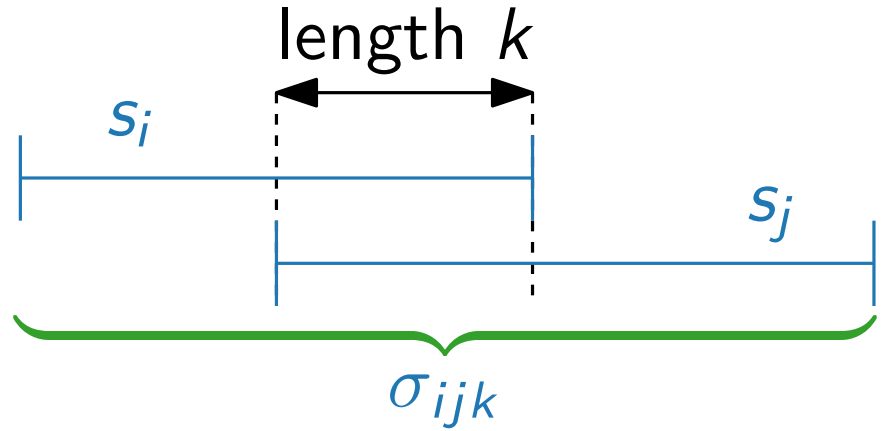
Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab cabab

ababc ababc

σ_{ij2}: cababababc



SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

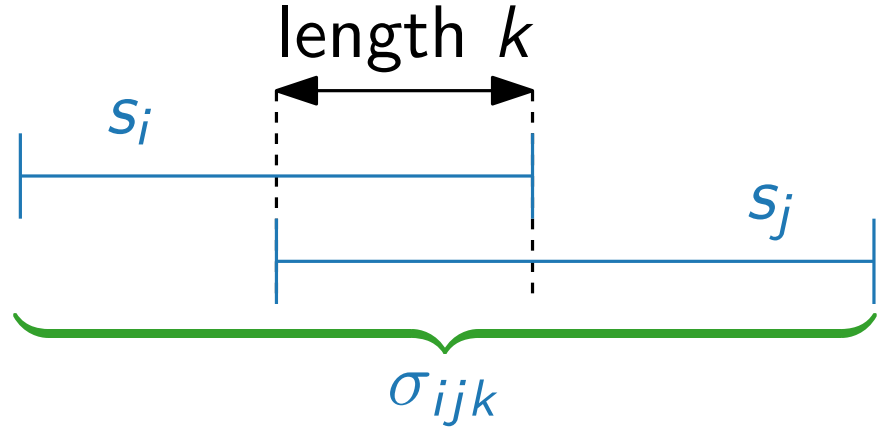
Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab cabab

ababc ababc

σ_{ij2}: cababababc σ_{ij4}: cababc



SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

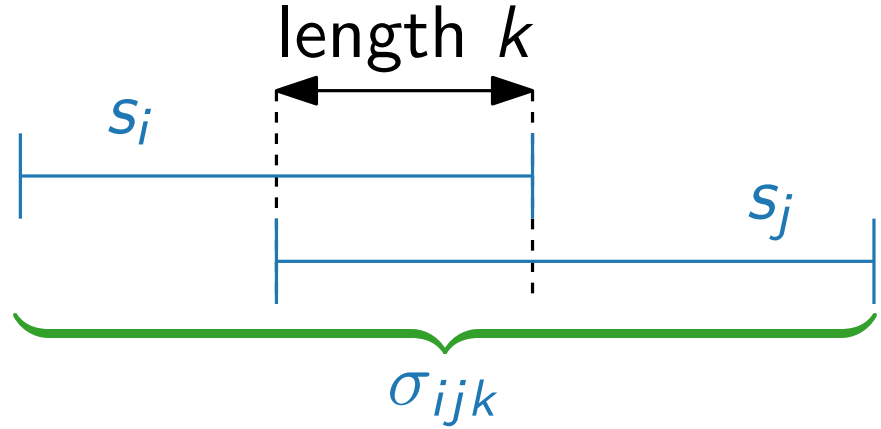
Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab
ababc
σ_{ij2}: cababababc

cabab
ababc
σ_{ij4}: cababc



$S(\sigma_{ijk}) = \{s \in U \mid s \text{ substring of } \sigma_{ijk}\}$ contains the elements of the ground set covered by σ_{ijk}.

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

Ground set $U := \{s_1, \ldots, s_n\}$.

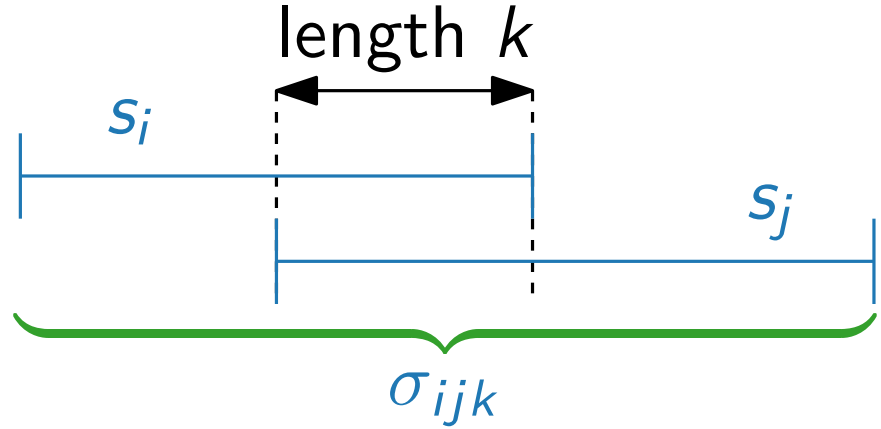
Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab cabab

ababc ababc

σ_{ij2}: cababababc σ_{ij4}: cababc



$S(\sigma_{ijk}) = \{s \in U \mid s \text{ substring of } \sigma_{ijk}\}$ contains the elements of the ground set covered by σ_{ijk}.

$c(S(\sigma_{ijk})) = |\sigma_{ijk}|$ (number of characters in σ_{ijk})

SSS as a SetCover Problem

SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

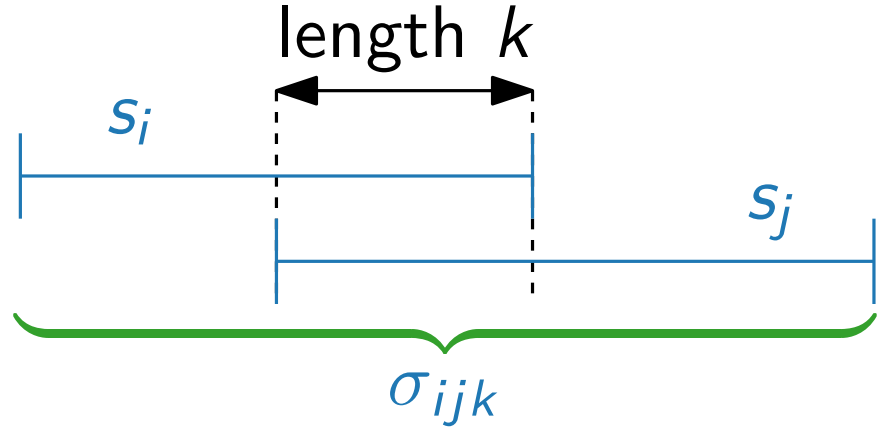
Ground set $U := \{s_1, \ldots, s_n\}$.

Let be σ_{ijk} be the unique string with prefix s_i and suffix s_j where s_i and s_j *overlap* on k characters (for suitable i, j, k)

s_i: cabab s_j: ababc

cabab
ababc
σ_{ij2}: cababababc

cabab
ababc
σ_{ij4}: cababc



$S(\sigma_{ijk}) = \{s \in U \mid s \text{ substring of } \sigma_{ijk}\}$ contains the elements of the ground set covered by σ_{ijk}.

$c(S(\sigma_{ijk})) = |\sigma_{ijk}|$ (number of characters in σ_{ijk})

$\mathcal{S} = \{S(\sigma_{ijk}) \mid k > 0\}$ (possibly $i = j$)

Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part V:
Solving ShortestSuperString via SetCover

Alexander Wolff

Winter 2022/23

Relating SSS and SetCover

Lemma. Let $\mathsf{OPT}_{\mathsf{SSS}}$ be the length of a shortest superstring of U, and let $\mathsf{OPT}_{\mathsf{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

$$\mathsf{OPT}_{\mathsf{SSS}} \leq \mathsf{OPT}_{\mathsf{SC}}.$$

Relating SSS and SetCover

Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U, and let $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

$$\mathrm{OPT}_{\mathrm{SSS}} \leq \mathrm{OPT}_{\mathrm{SC}}.$$

Proof.

Consider an optimal set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ of U.

Relating SSS and SetCover

Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U, and let $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

$$\mathrm{OPT}_{\mathrm{SSS}} \leq \mathrm{OPT}_{\mathrm{SC}}.$$

Proof.

Consider an optimal set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ of U.

Then $s := \pi_1 \circ \ldots \circ \pi_k$ is a superstring of U of length

Relating SSS and SetCover

Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U, and let $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

$$\mathrm{OPT}_{\mathrm{SSS}} \leq \mathrm{OPT}_{\mathrm{SC}}.$$

Proof.

Consider an optimal set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ of U.

Then $s := \pi_1 \circ \ldots \circ \pi_k$ is a superstring of U of length $\sum_{i=1}^{k} |\pi_i| = \sum_{i=1}^{k} c(S(\pi_i)) = \mathrm{OPT}_{\mathrm{SC}}$.

Relating SSS and SetCover

Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U, and let $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SetCover instance. Then

$$\mathrm{OPT}_{\mathrm{SSS}} \leq \mathrm{OPT}_{\mathrm{SC}}.$$

Proof.

Consider an optimal set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ of U.

Then $s := \pi_1 \circ \ldots \circ \pi_k$ is a superstring of U of length $\sum_{i=1}^{k} |\pi_i| = \sum_{i=1}^{k} c(S(\pi_i)) = \mathrm{OPT}_{\mathrm{SC}}$.

Thus, $\mathrm{OPT}_{\mathrm{SSS}} \leq |s| = \mathrm{OPT}_{\mathrm{SC}}$.

Relating SSS and SetCover

Lemma. $\mathsf{OPT}_{\mathsf{SC}} \leq 2 \cdot \mathsf{OPT}_{\mathsf{SSS}}$.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Relating SSS and SetCover

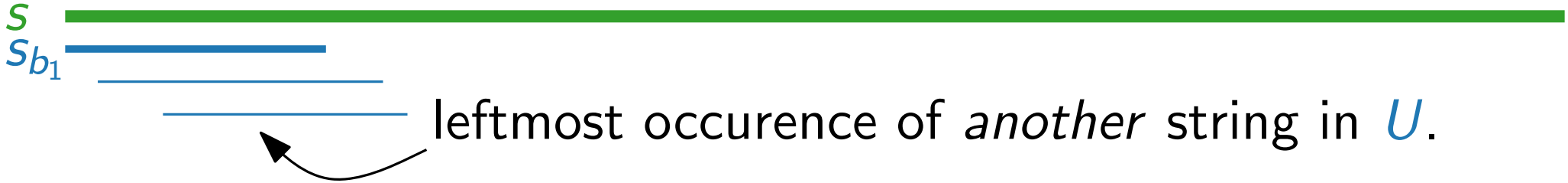
Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

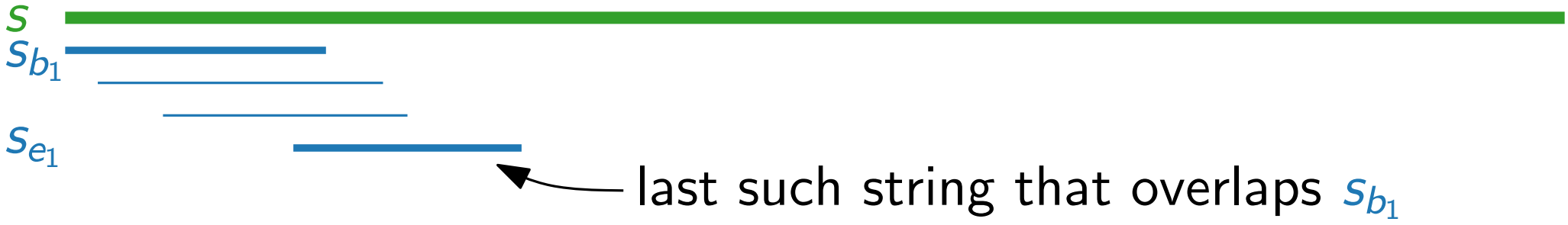
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

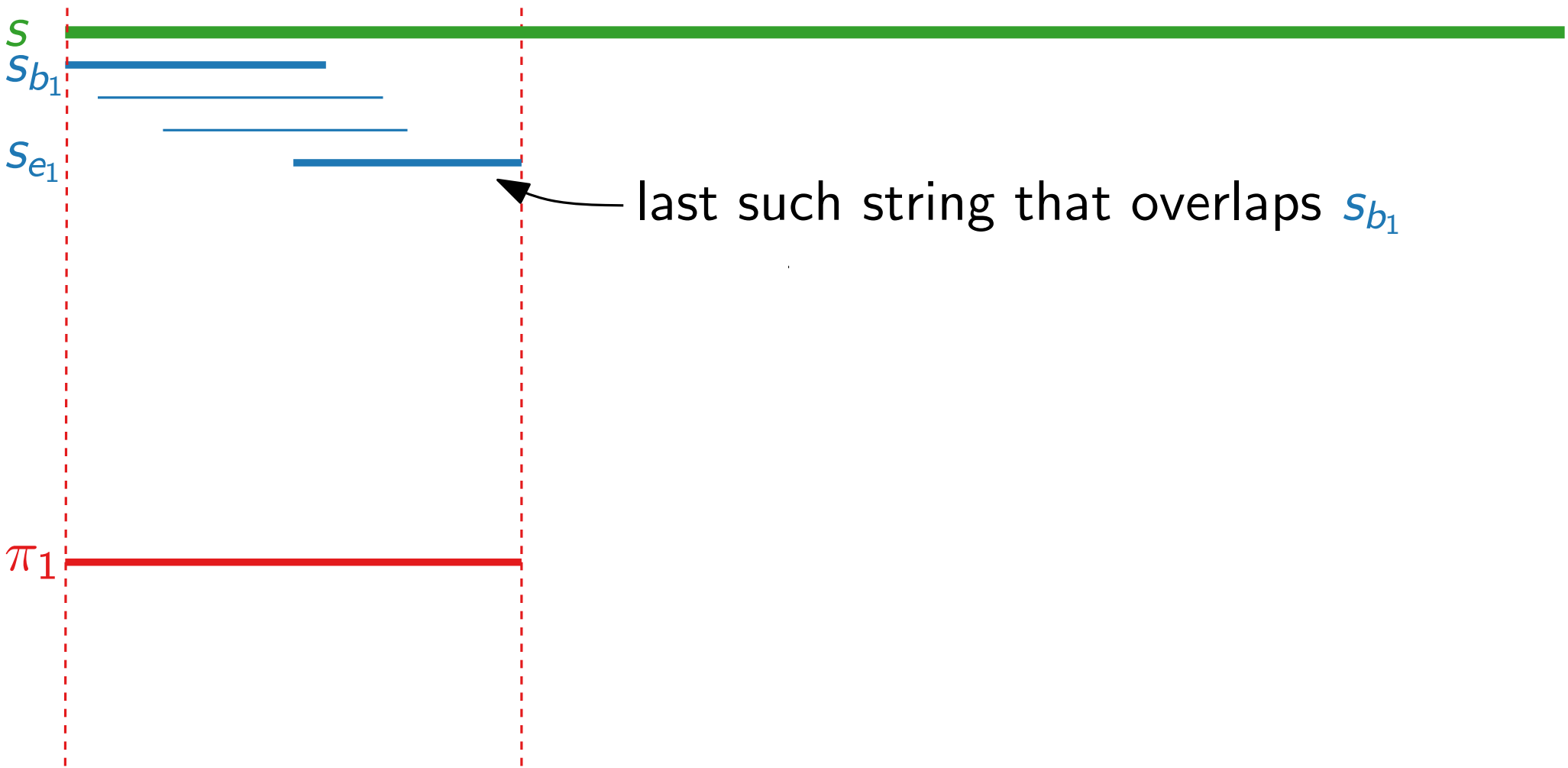
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

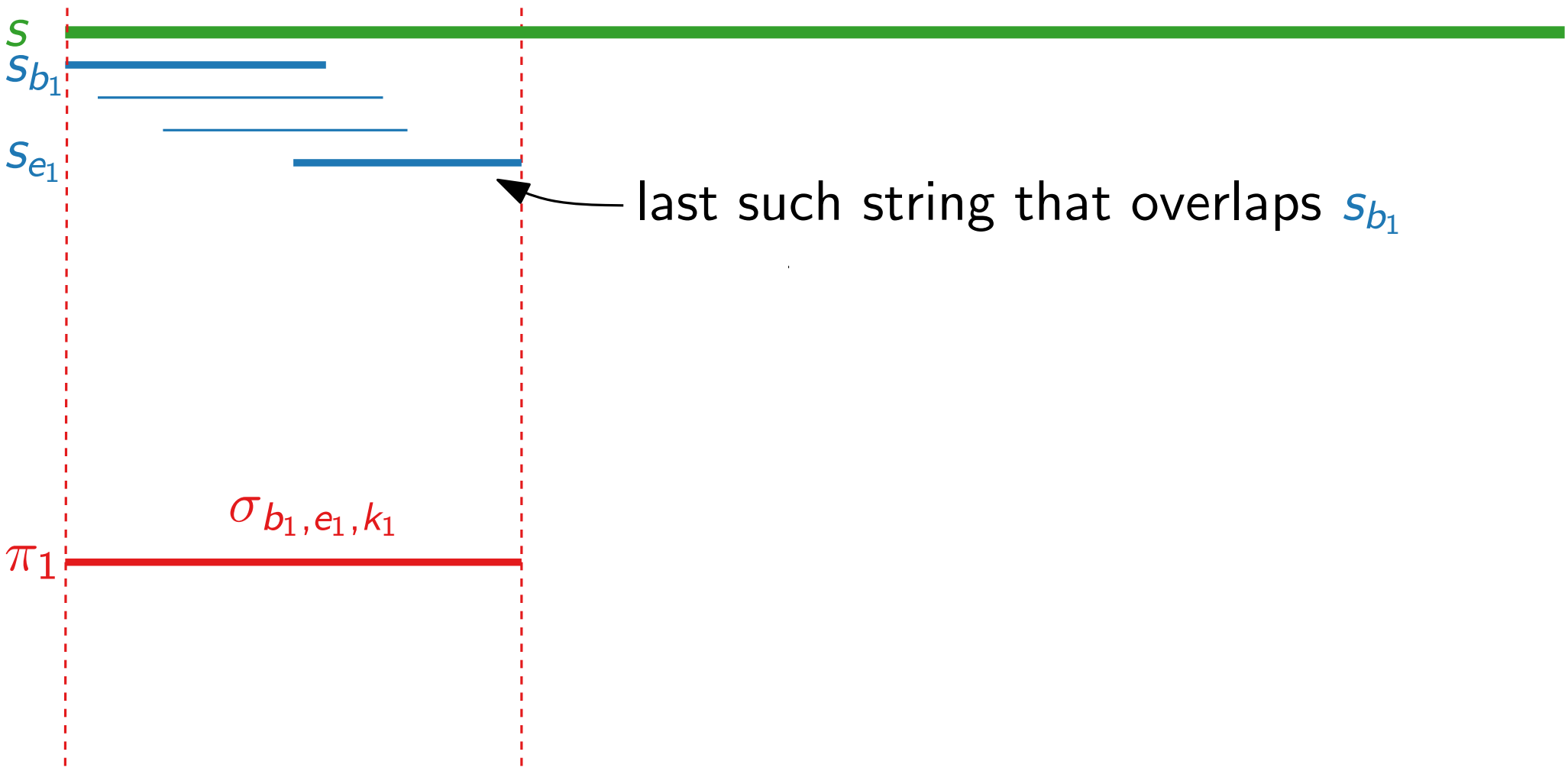
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

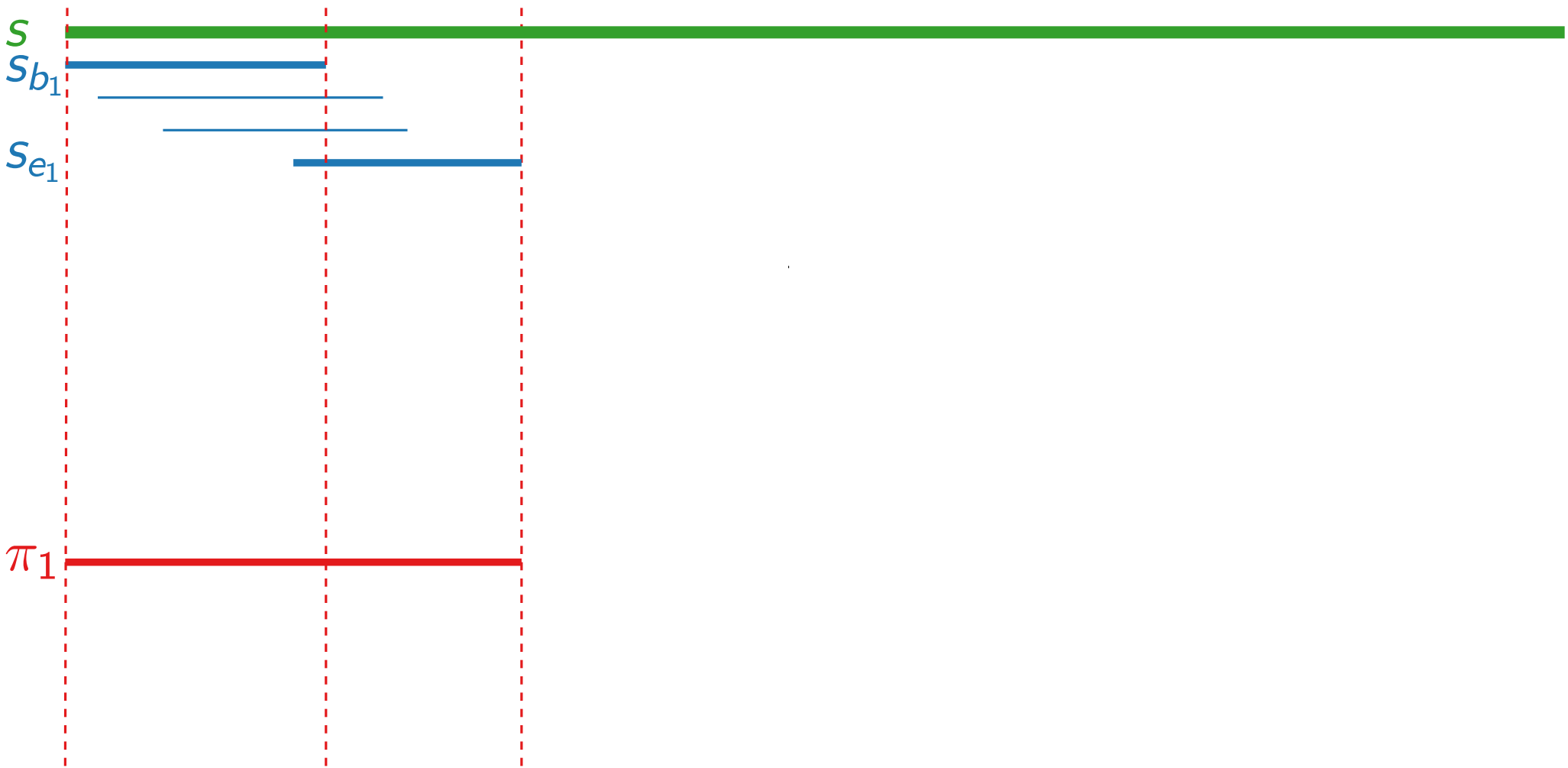
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

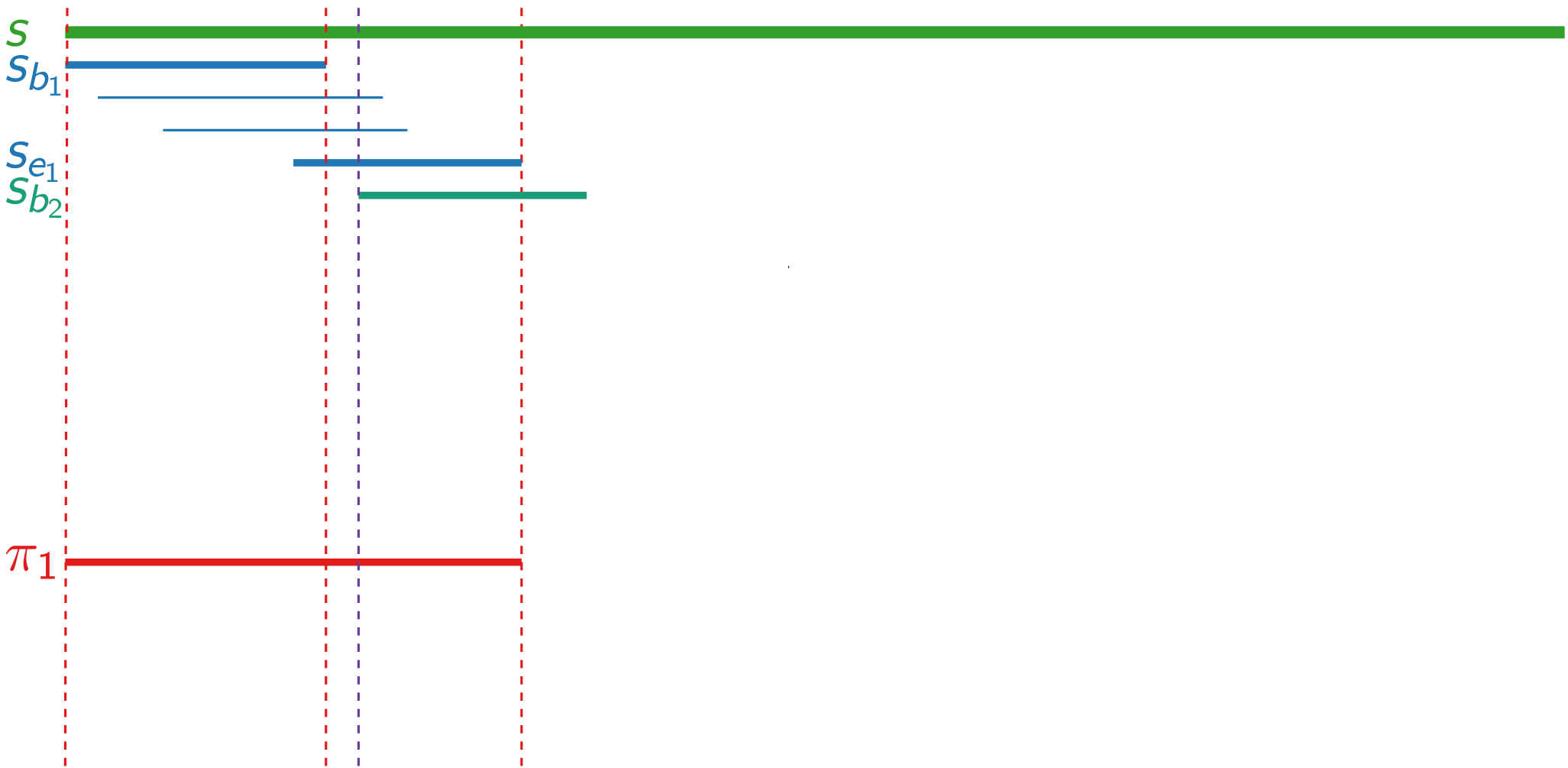
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

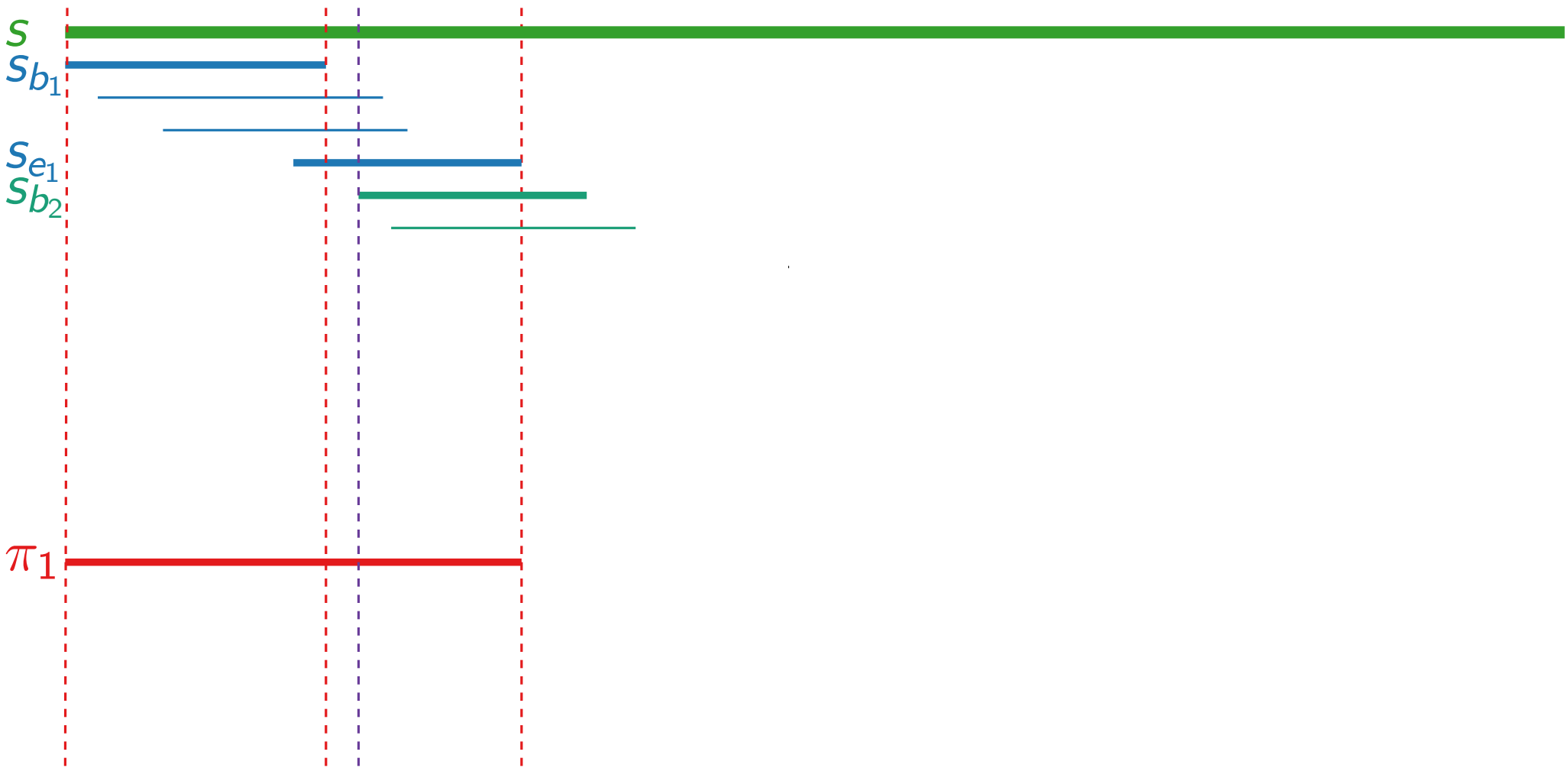
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

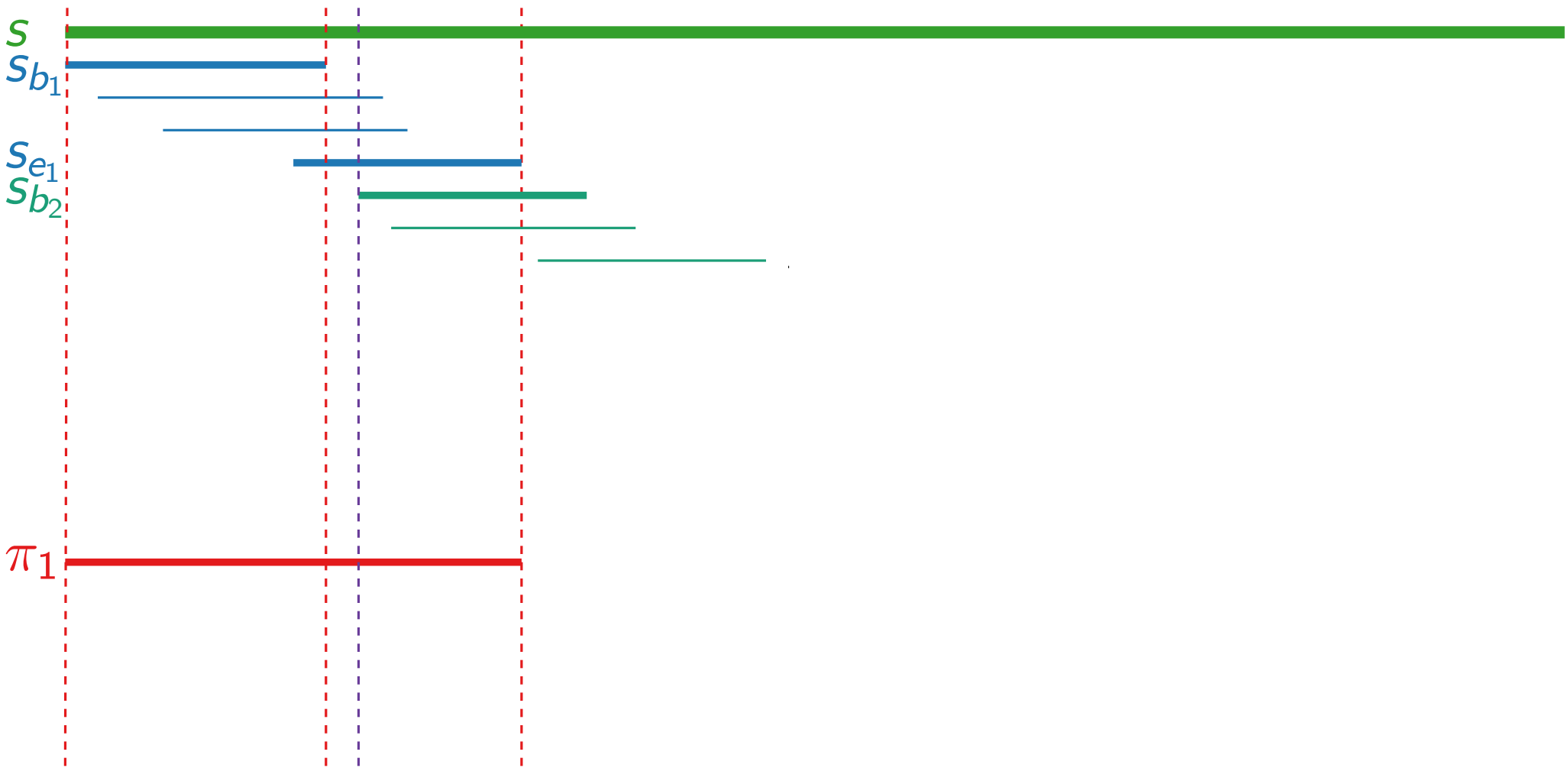
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

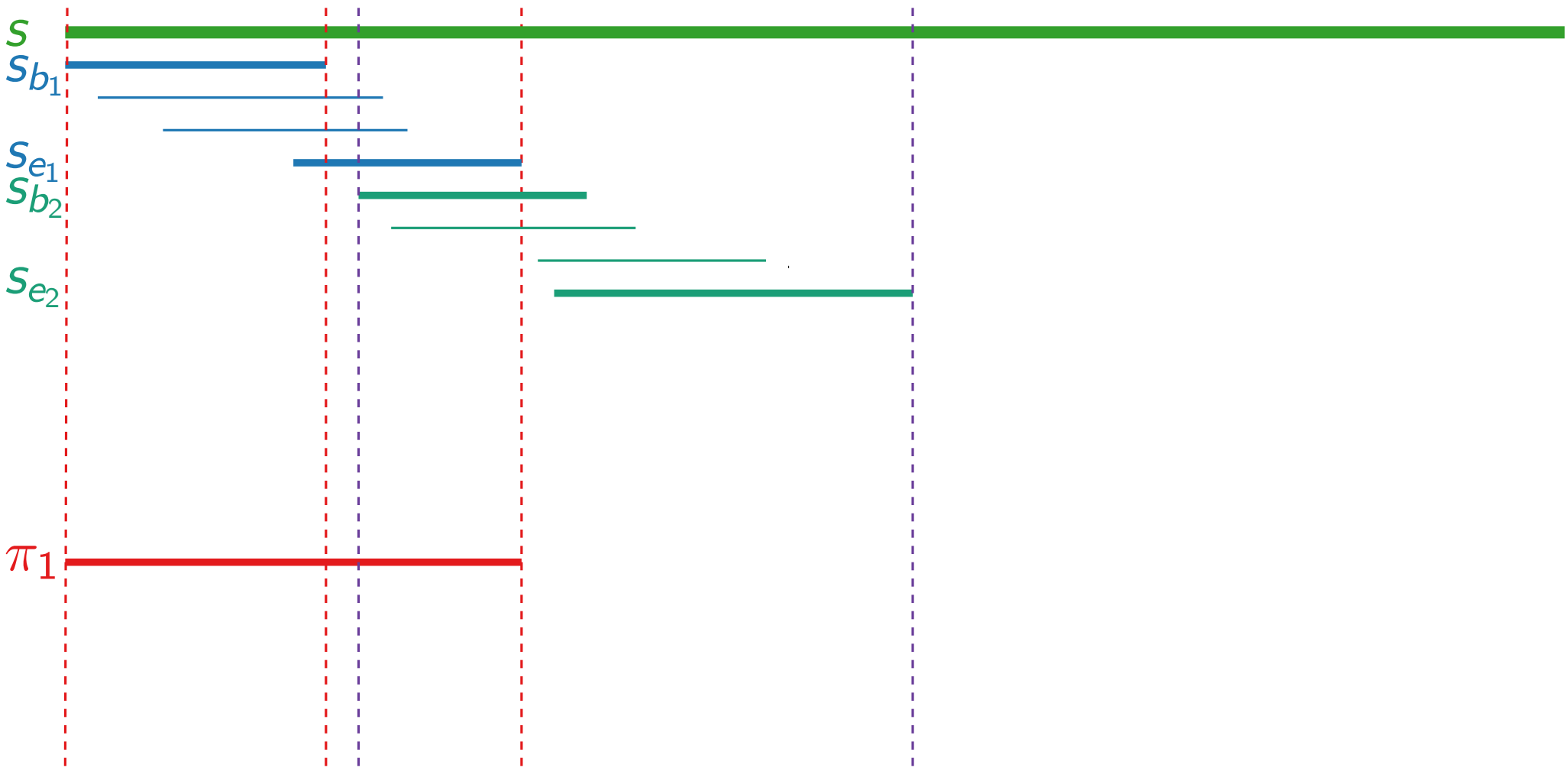
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

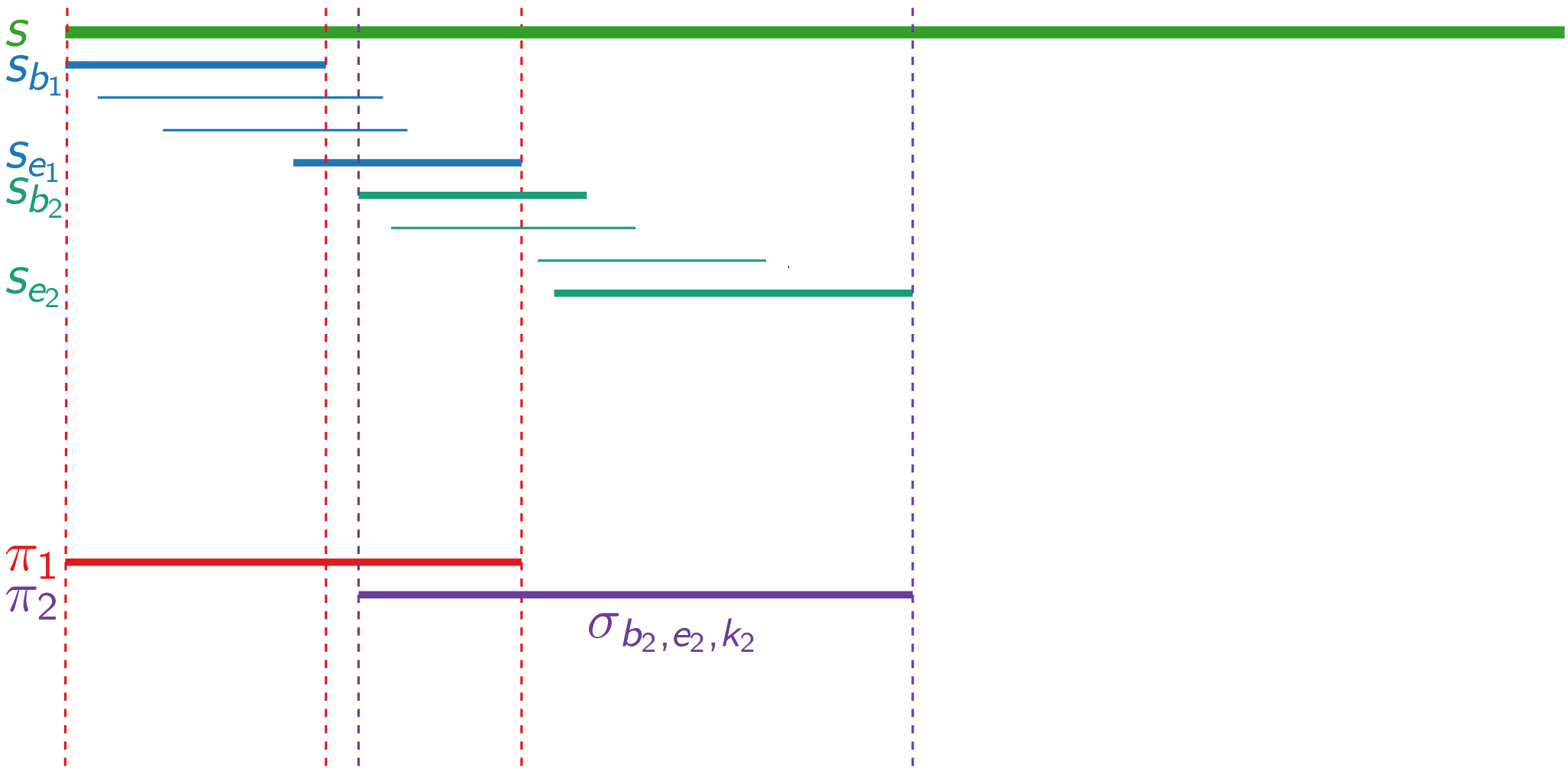
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

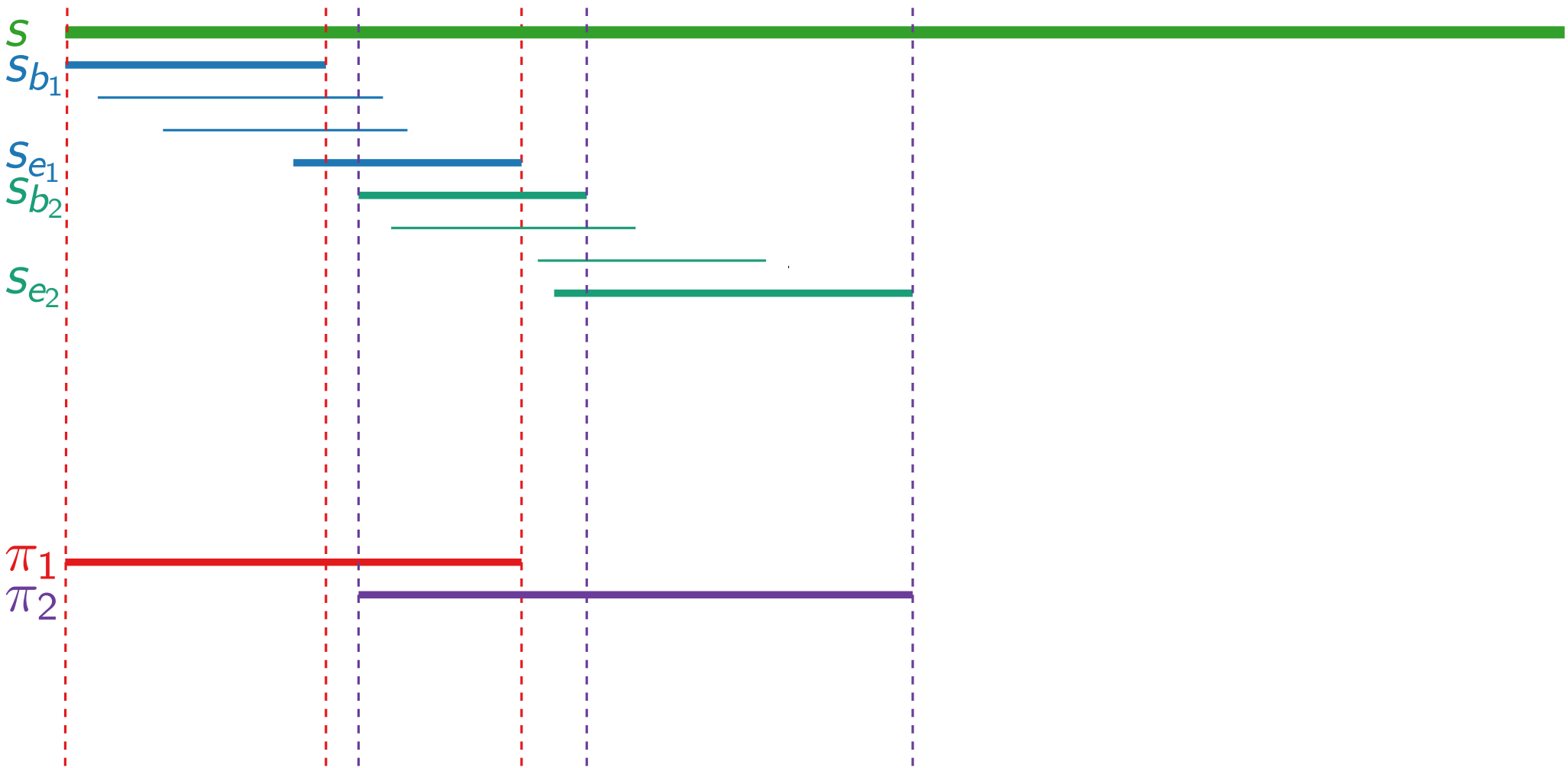
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

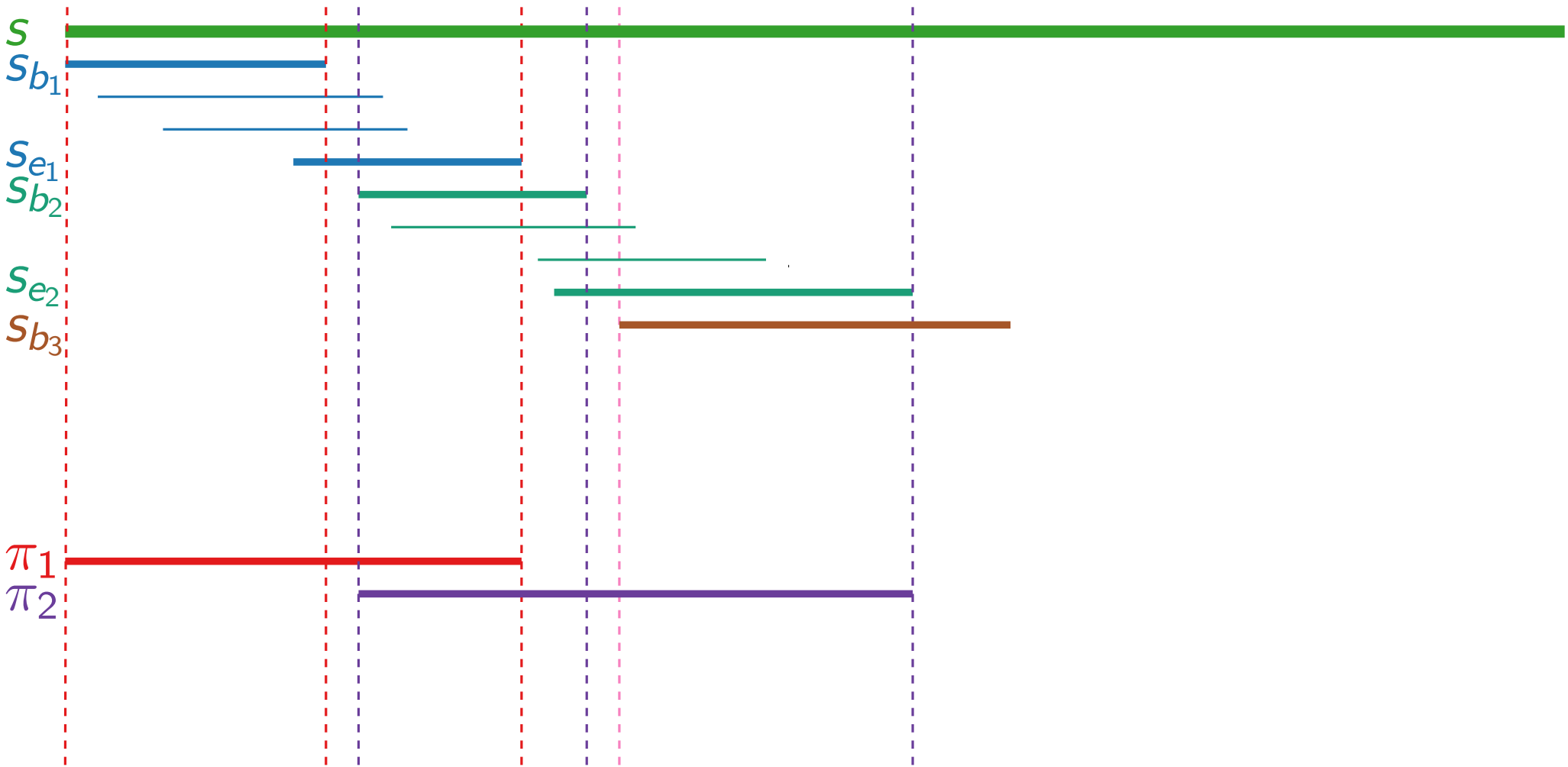
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

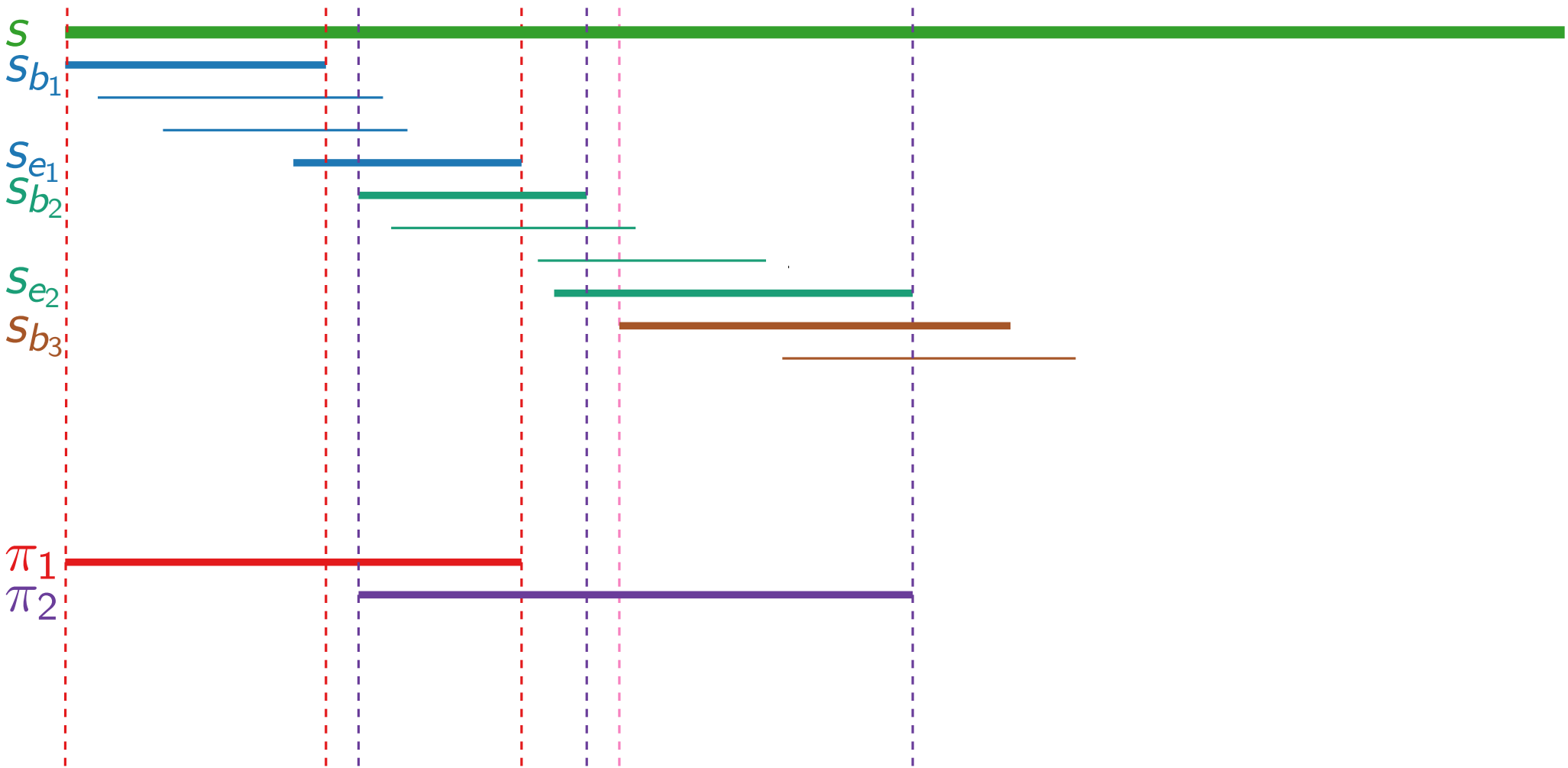
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

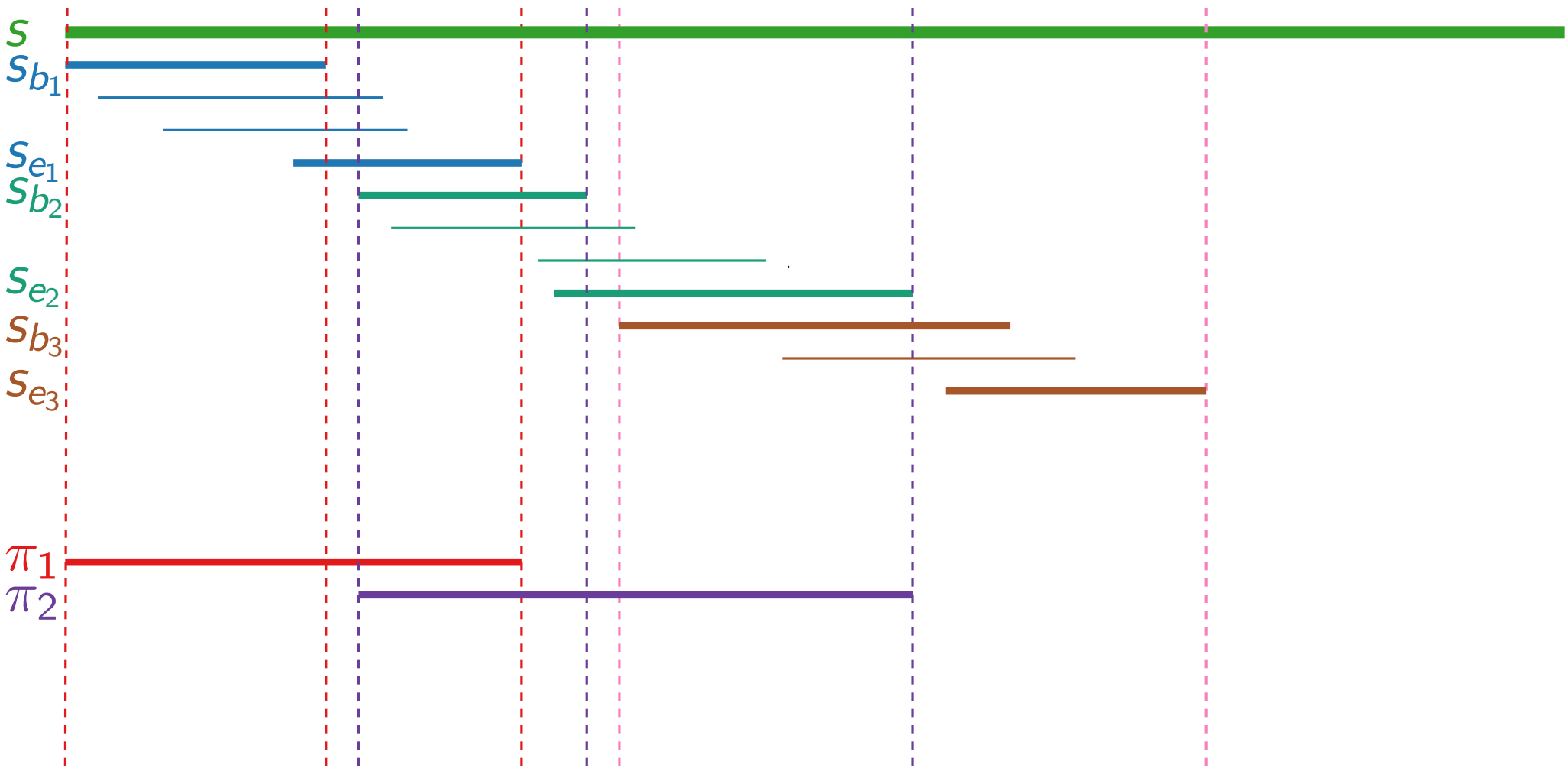
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

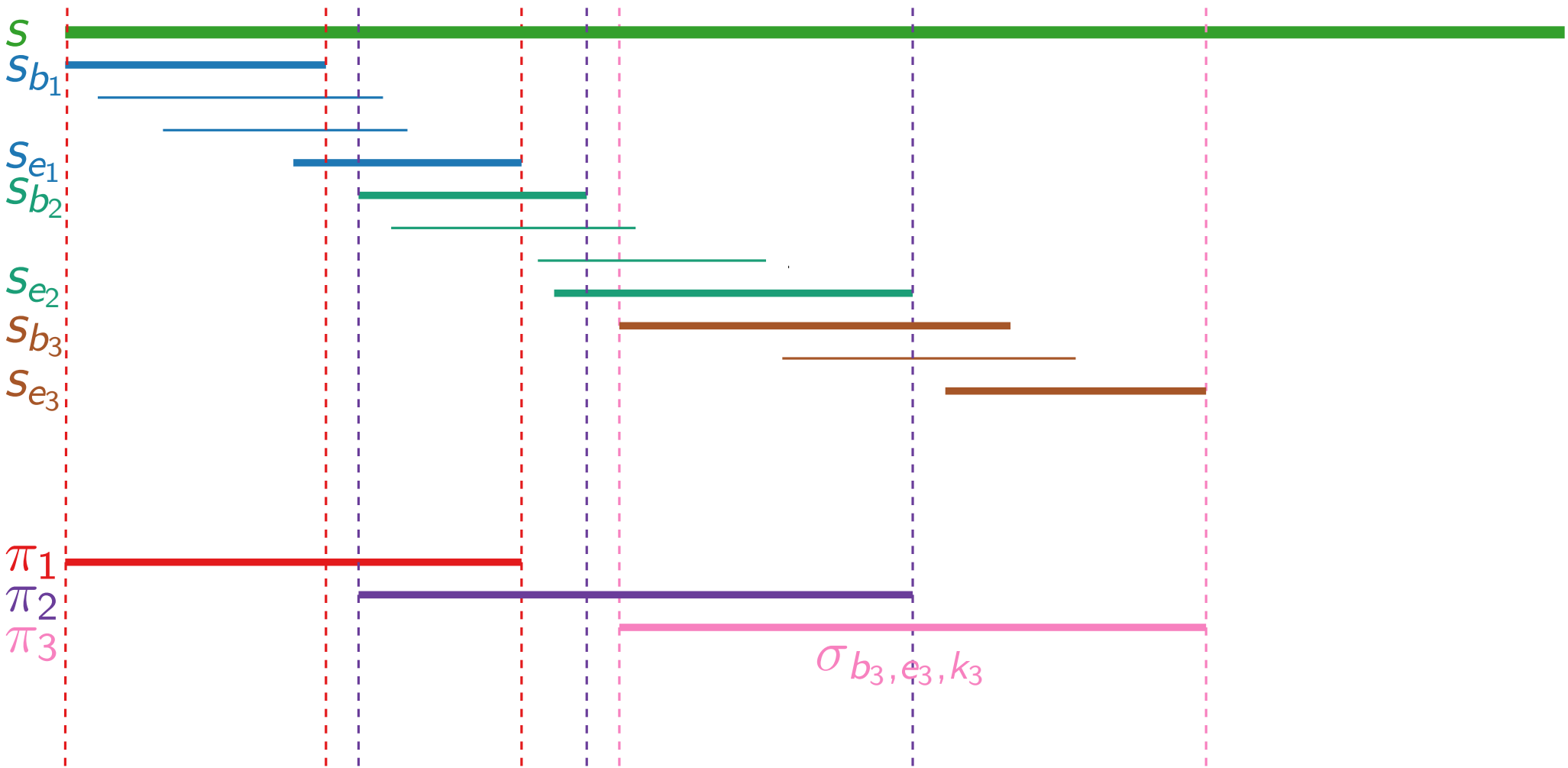
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

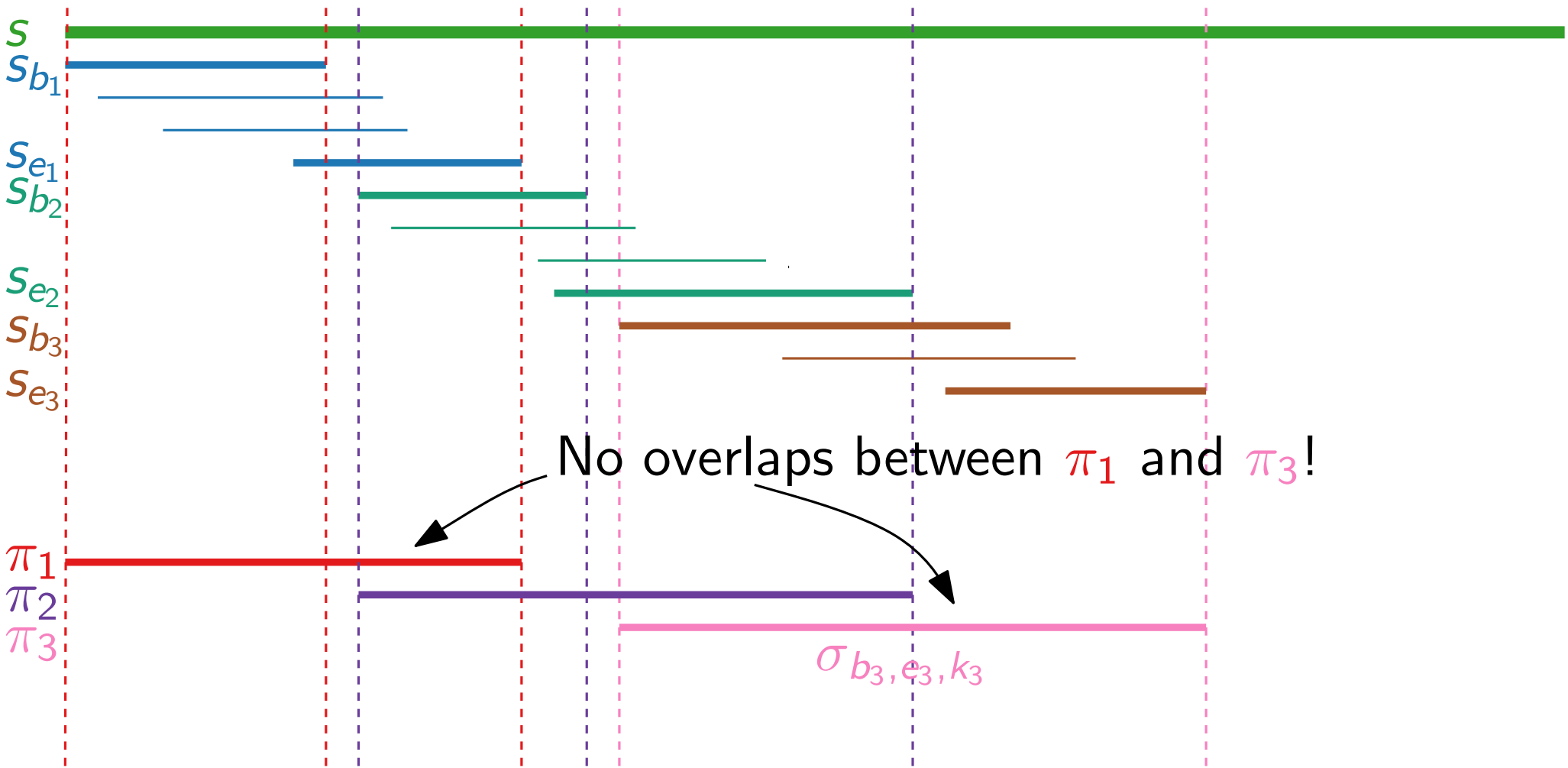
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

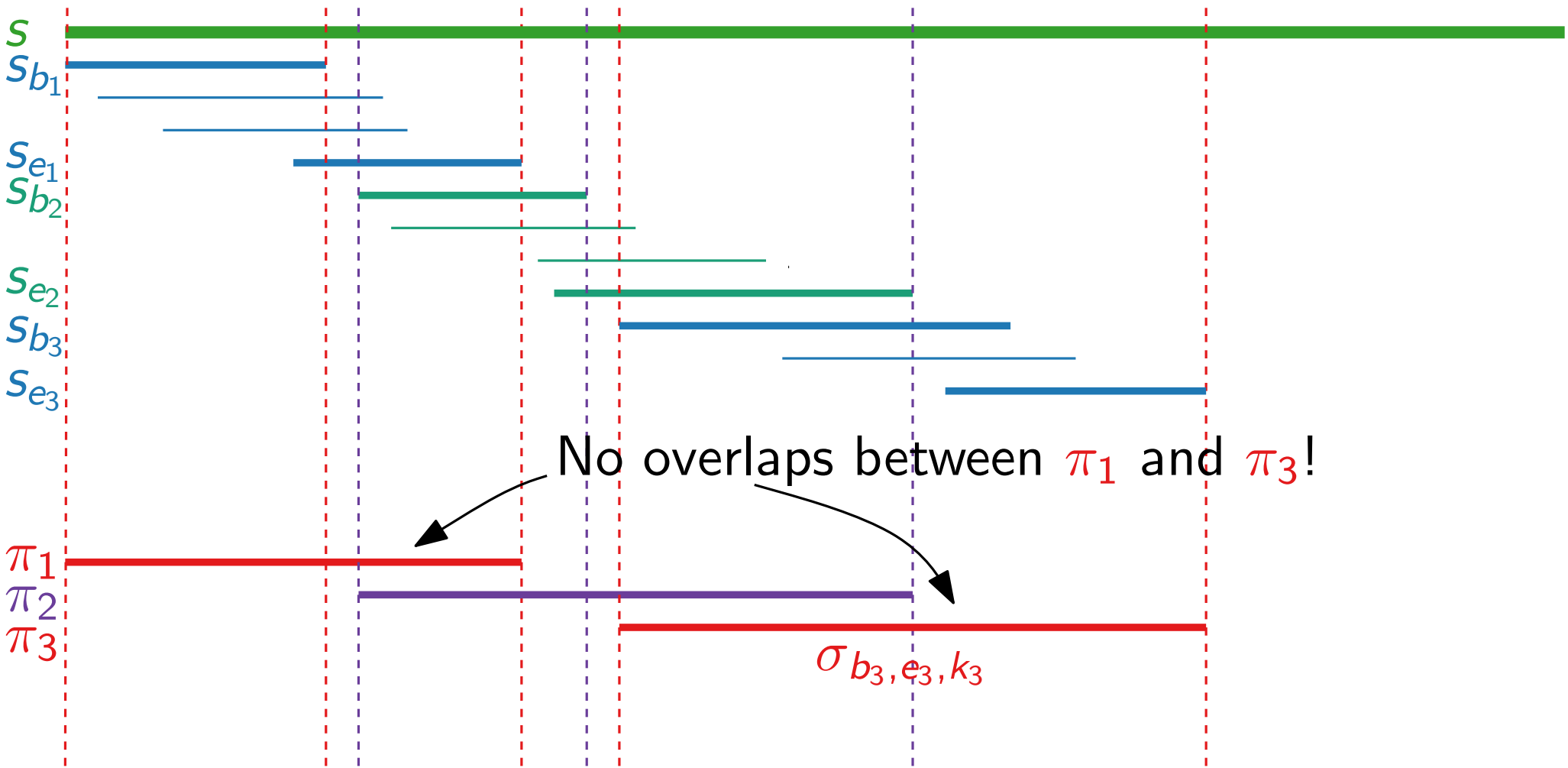
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

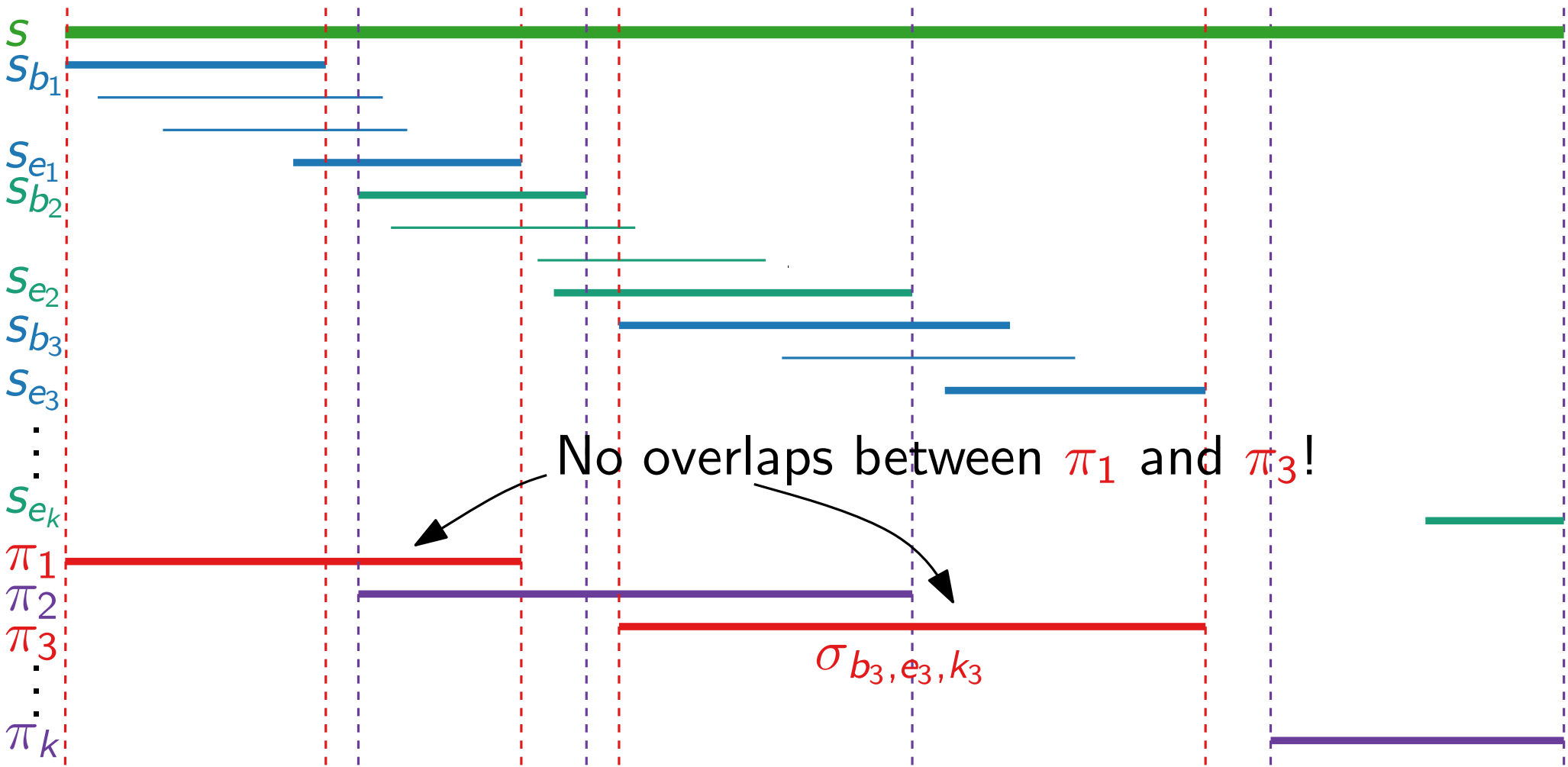
Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof. Consider an optimal superstring s.
Construct a set cover of cost $\leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.



Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof.

Each string $s_i \in U$ is a substring of some π_j.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof.

Each string $s_i \in U$ is a substring of some π_j.

$\{S(\pi_1), \ldots, S(\pi_k)\}$ is a solution for the SetCover instance with cost $\sum_i |\pi_i|$.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof.

Each string $s_i \in U$ is a substring of some π_j.

$\{S(\pi_1), \ldots, S(\pi_k)\}$ is a solution for the SetCover instance with cost $\sum_i |\pi_i|$.

Substrings π_j, π_{j+2} do not overlap.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof.

Each string $s_i \in U$ is a substring of some π_j.

$\{S(\pi_1), \ldots, S(\pi_k)\}$ is a solution for the SetCover instance with cost $\sum_i |\pi_i|$.

Substrings π_j, π_{j+2} do not overlap.

Each character in s lies in at most **two** (subsequent) substrings, namely π_j and π_{j+1}.

Relating SSS and SetCover

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Proof.

Each string $s_i \in U$ is a substring of some π_j.

$\{S(\pi_1), \ldots, S(\pi_k)\}$ is a solution for the SetCover instance with cost $\sum_i |\pi_i|$.

Substrings π_j, π_{j+2} do not overlap.

Each character in s lies in at most **two** (subsequent) substrings, namely π_j and π_{j+1}.

$\sum_i |\pi_i| \leq 2|s| = 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with the algorithm GreedySetCover.

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with the algorithm GreedySetCover.
3. Return $\pi_1 \circ \ldots \circ \pi_k$ as the superstring.

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with the algorithm GreedySetCover.
3. Return $\pi_1 \circ \ldots \circ \pi_k$ as the superstring.

Theorem. This algorithm is a factor-$2\mathcal{H}_n$ approximation algorithm for ShortestSuperString.

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with the algorithm GreedySetCover.
3. Return $\pi_1 \circ \ldots \circ \pi_k$ as the superstring.

Theorem. This algorithm is a factor-$2\mathcal{H}_n$ approximation algorithm for ShortestSuperString.

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Algorithm for SSS

1. Construct SetCover instance $\langle U, \mathcal{S}, c \rangle$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with the algorithm GreedySetCover.
3. Return $\pi_1 \circ \ldots \circ \pi_k$ as the superstring.

Theorem. This algorithm is a factor-$2\mathcal{H}_n$ approximation algorithm for ShortestSuperString.

Lemma. $\mathrm{OPT}_{\mathrm{SC}} \leq 2 \cdot \mathrm{OPT}_{\mathrm{SSS}}$.

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$ approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \leq 1 + \ln k$.

Can we do better?

Can we do better?

- The best known approximation factor for ShortestSuperString is $\frac{71}{30} \approx 2.367$.

Can we do better?

- The best known approximation factor for ShortestSuperString is $\frac{71}{30} \approx 2.367$.
- ShortestSuperString cannot be approximated within factor $\frac{333}{332} \approx 1.003$ (unless $\mathsf{P} = \mathsf{NP}$).