Approximation Algorithms

Lecture 9: An Approximation Scheme for EuclideanTSP

Part I: TravelingSalesmanProblem

Joachim Spoerhase

Winter 2021/22

TravelingSalesmanProblem (TSP)

Question: What's the fastest way to deliver all parcels to their destination?

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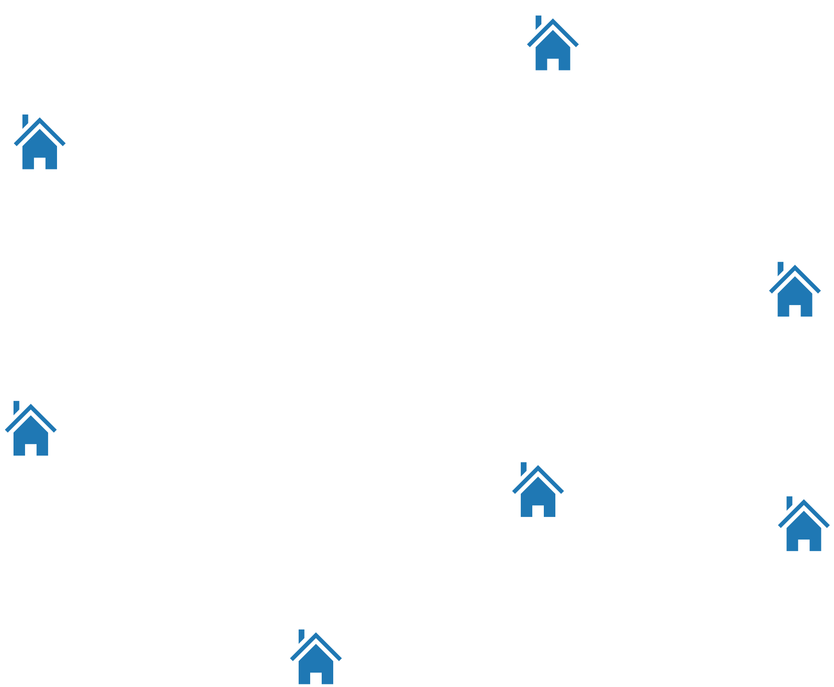
Given: A set of n houses (points) in $\mathbb{R}^2$.



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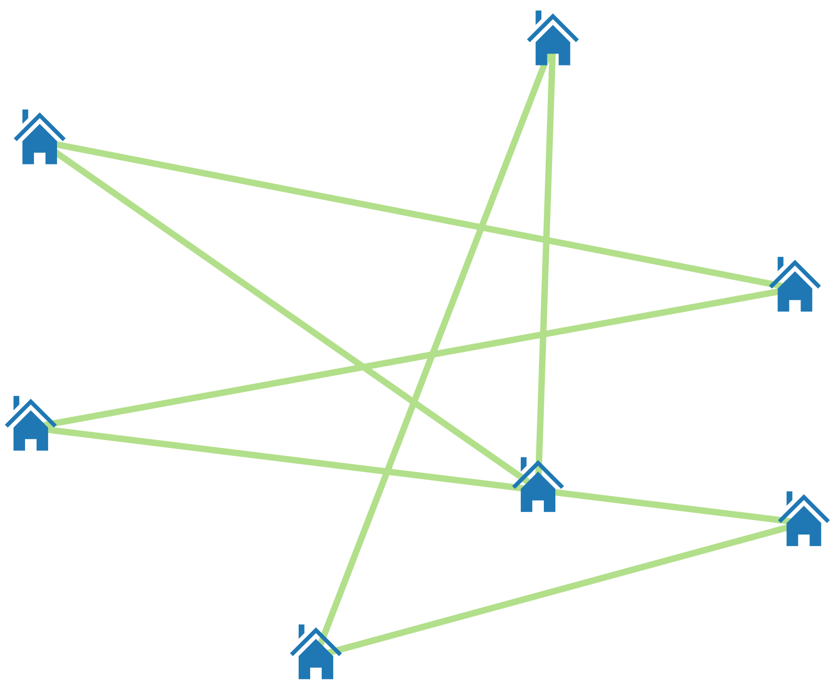

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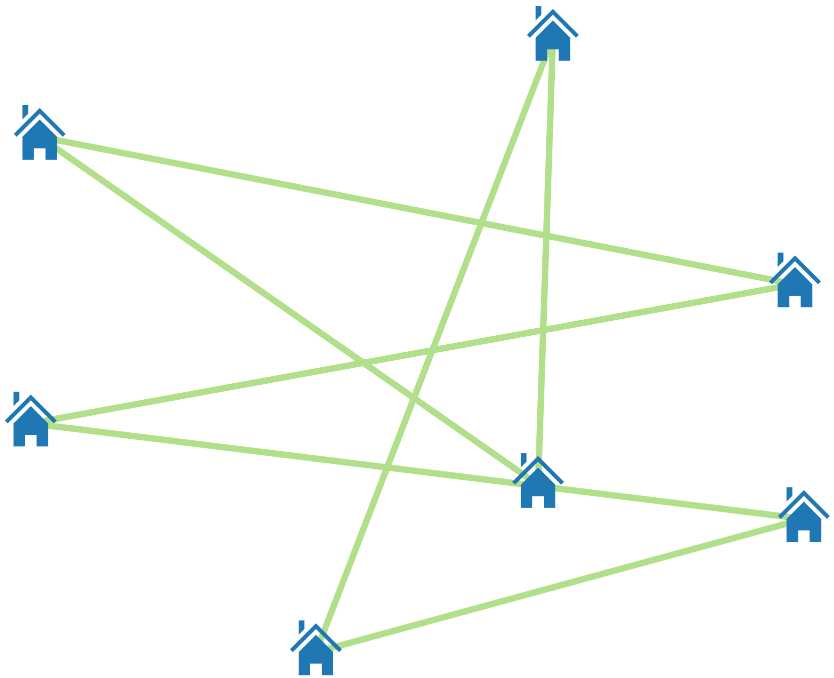

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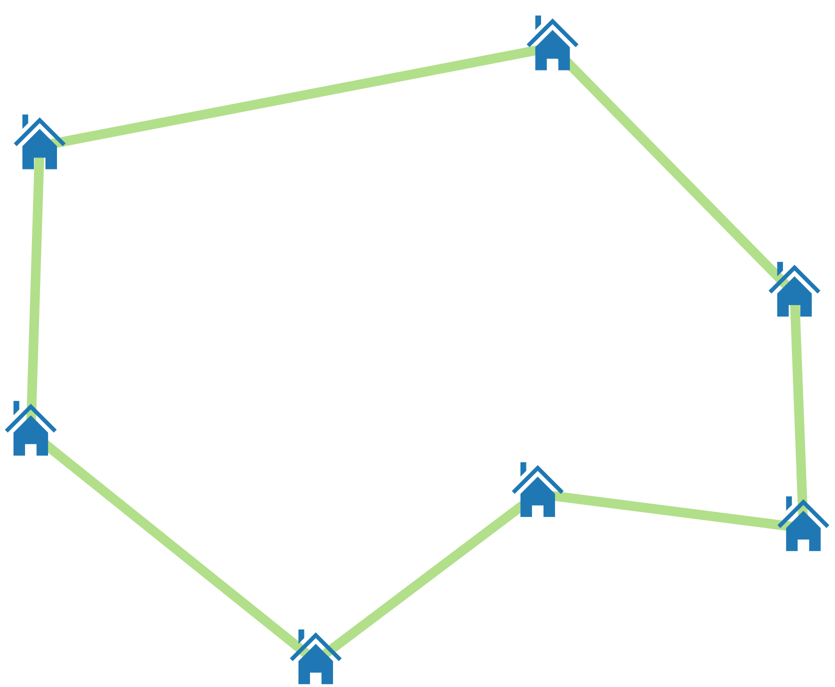

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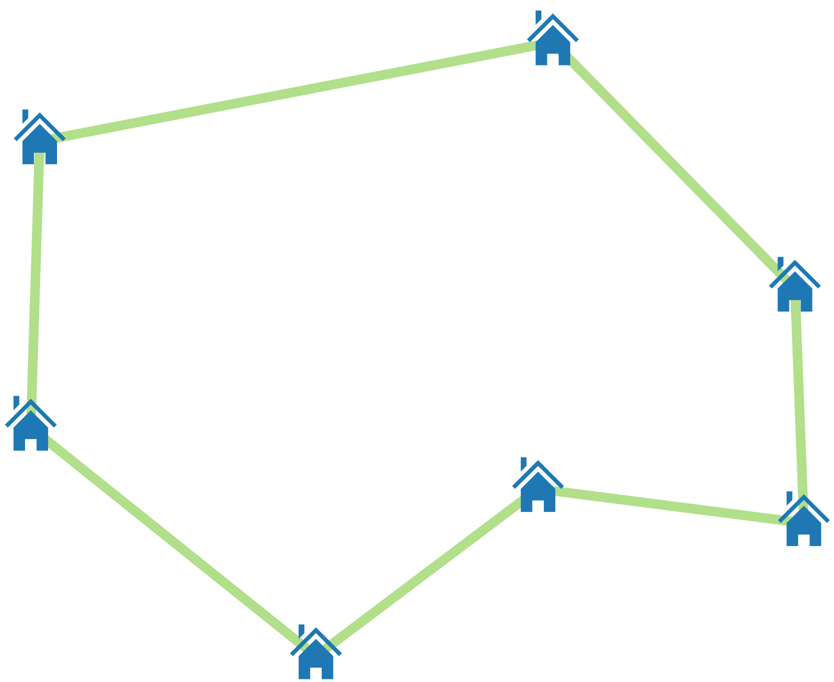

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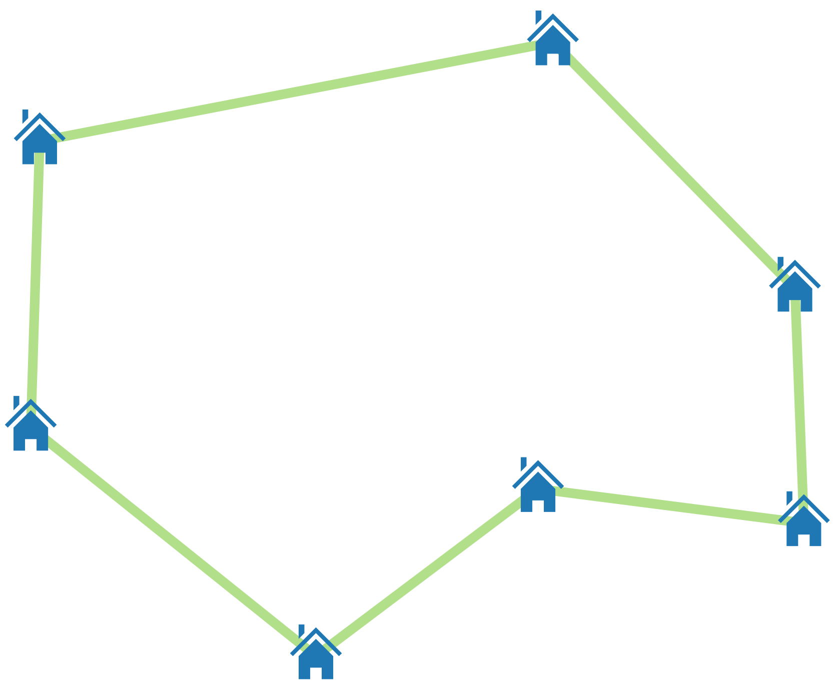

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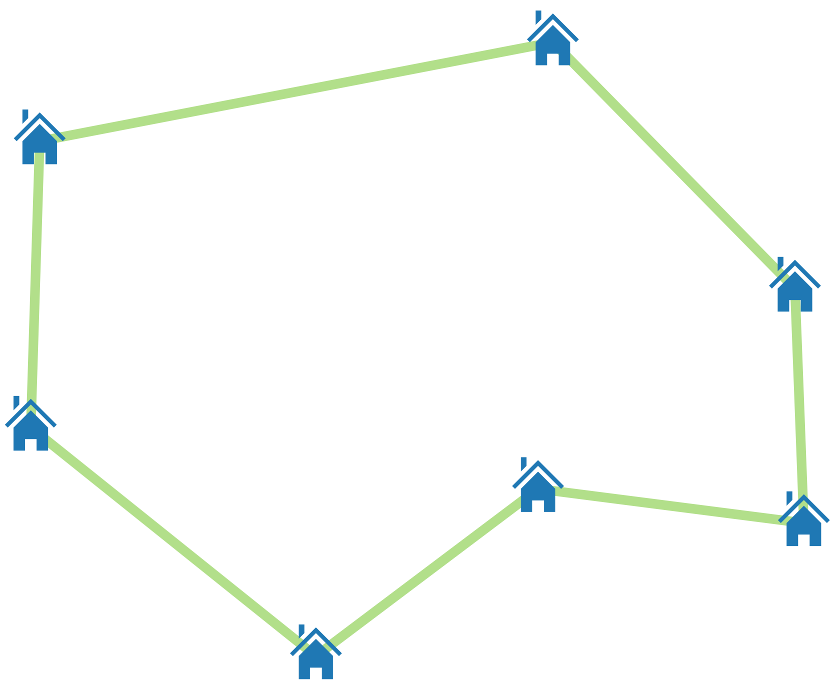
Distance between two points?



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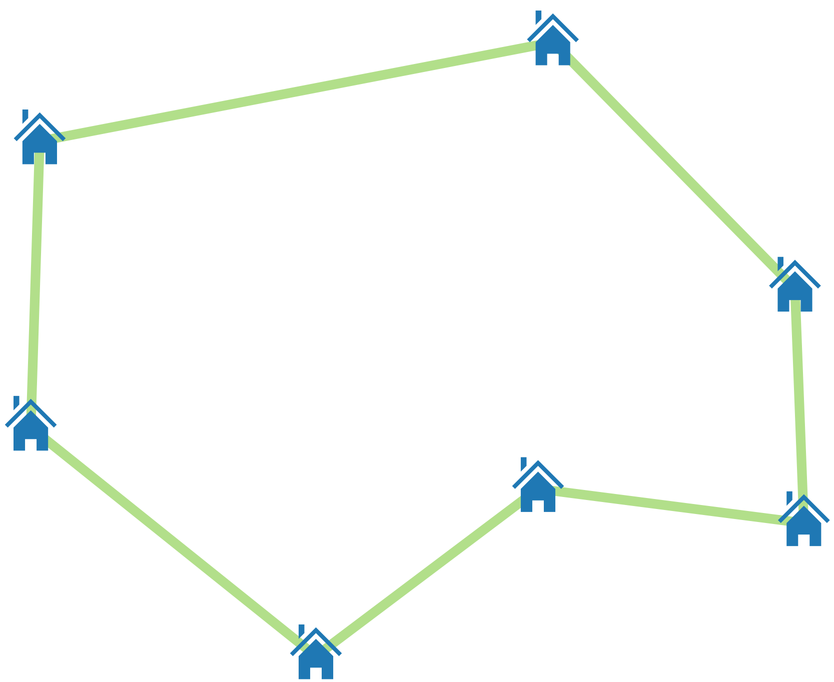

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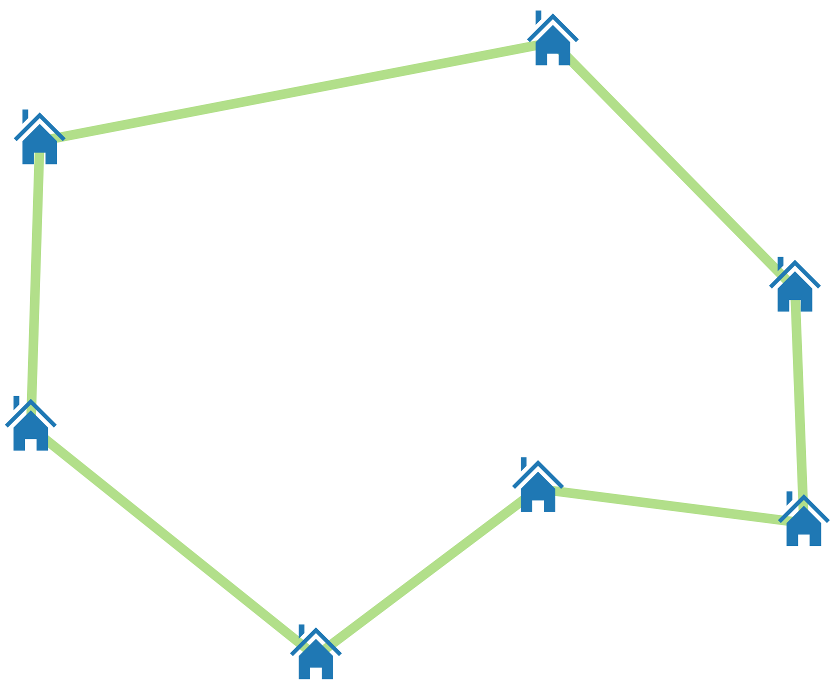

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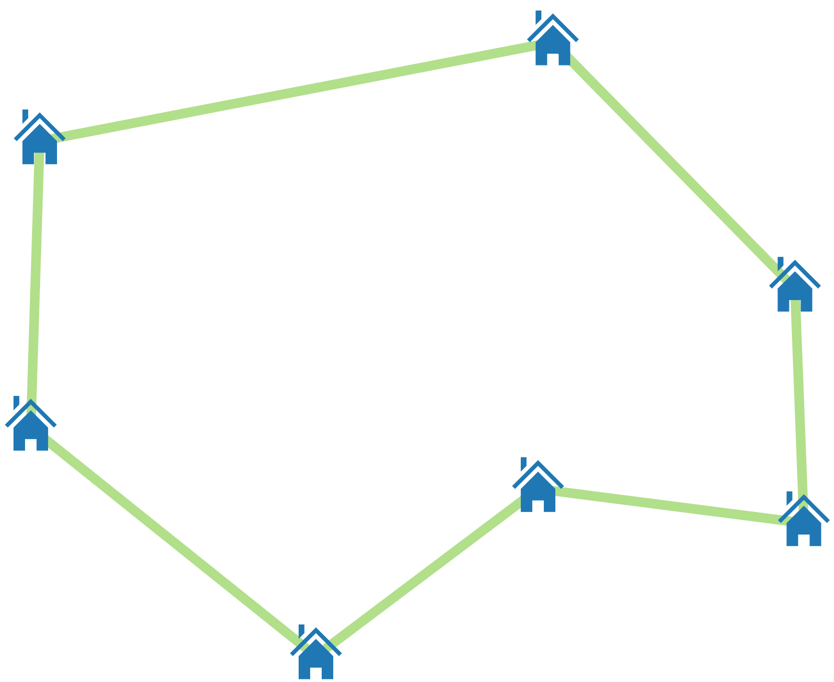
The Salesman can fly $\Rightarrow$ Euclidean distance.



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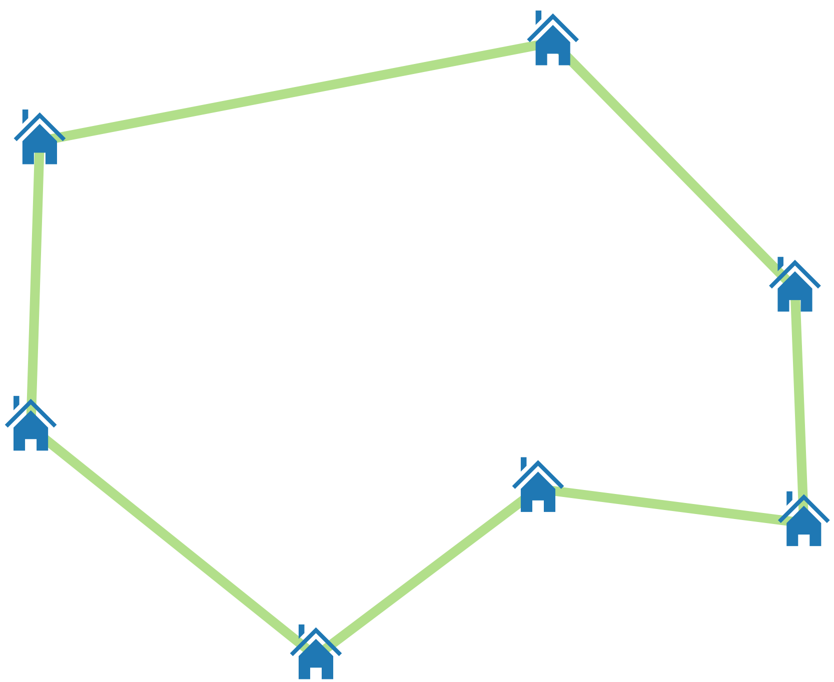

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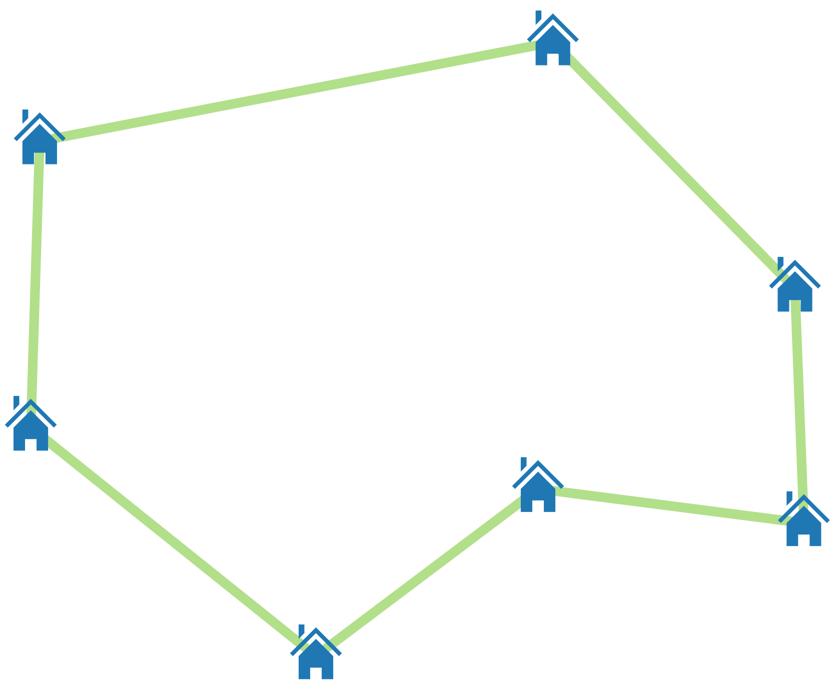

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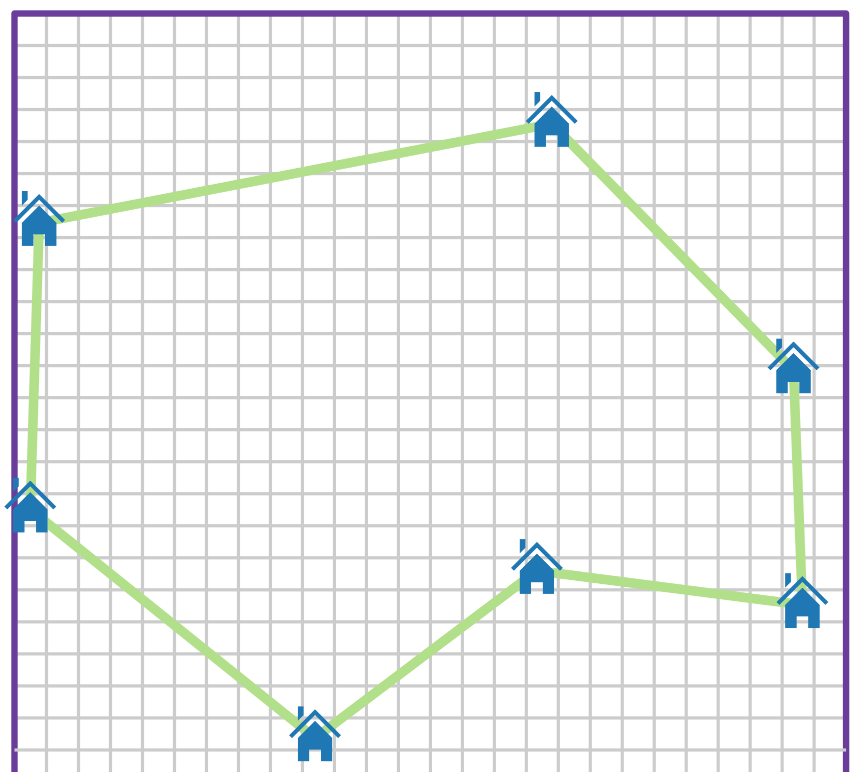

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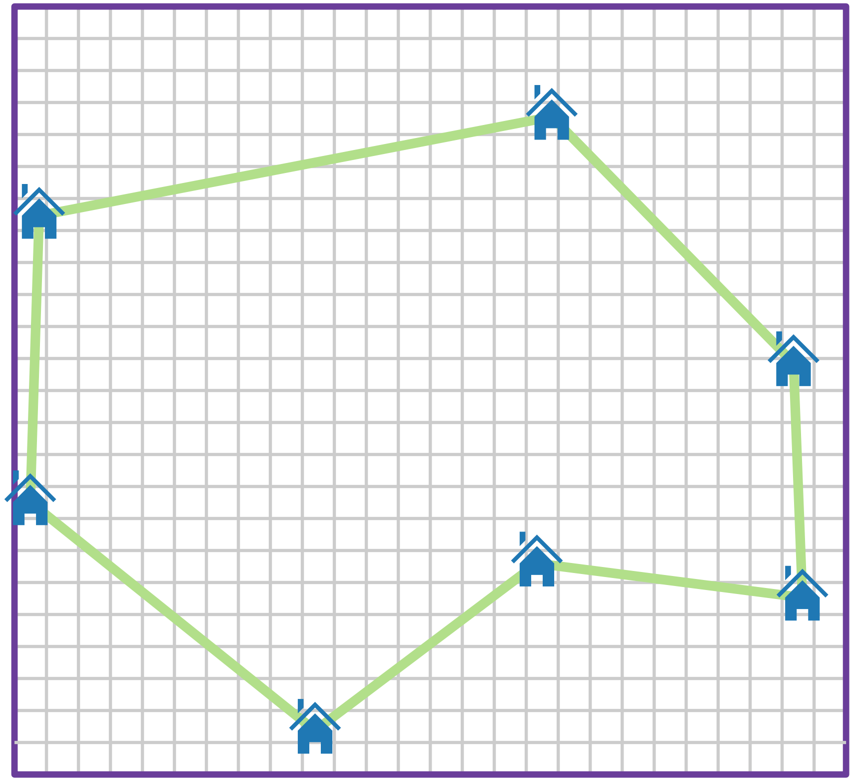

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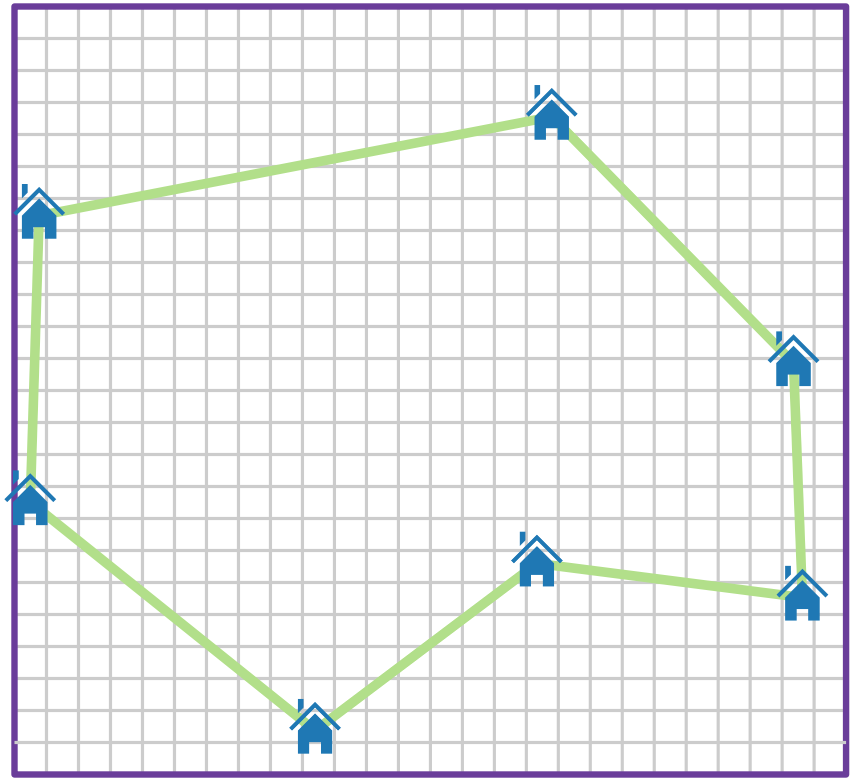

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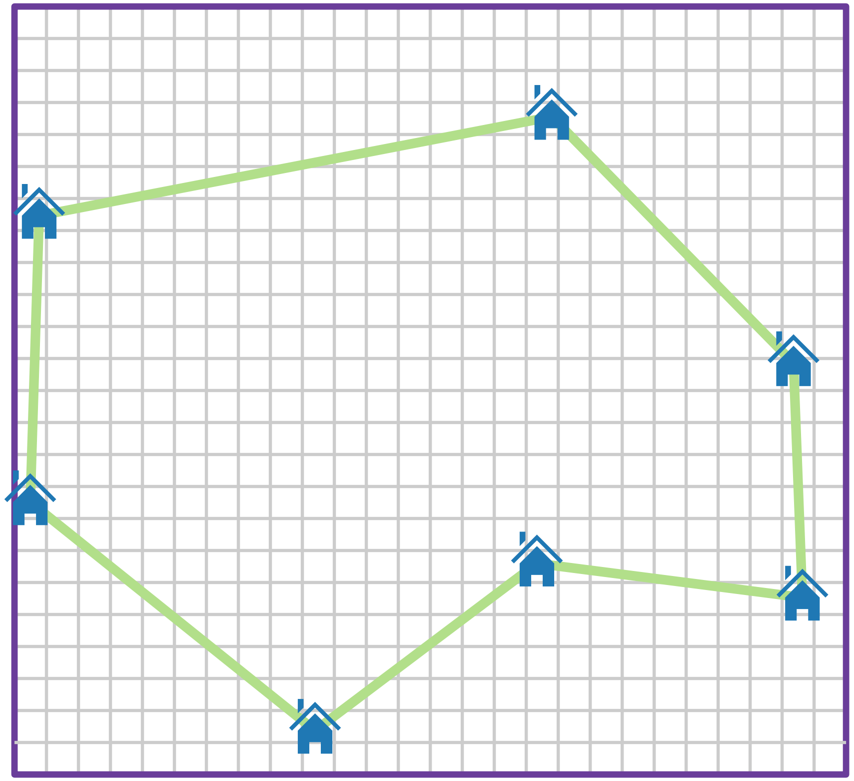

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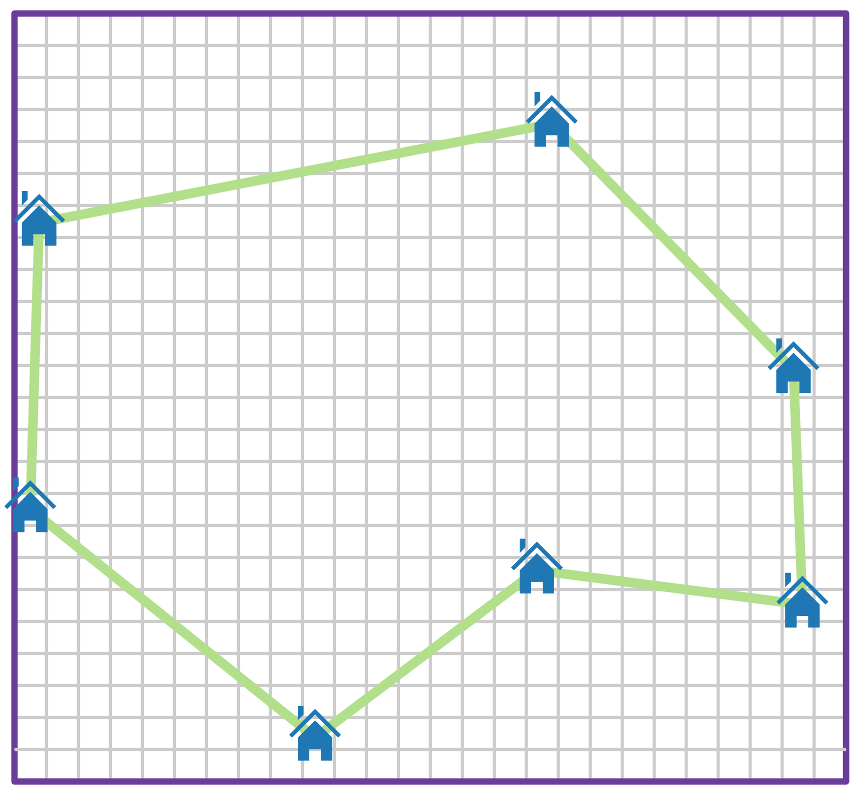

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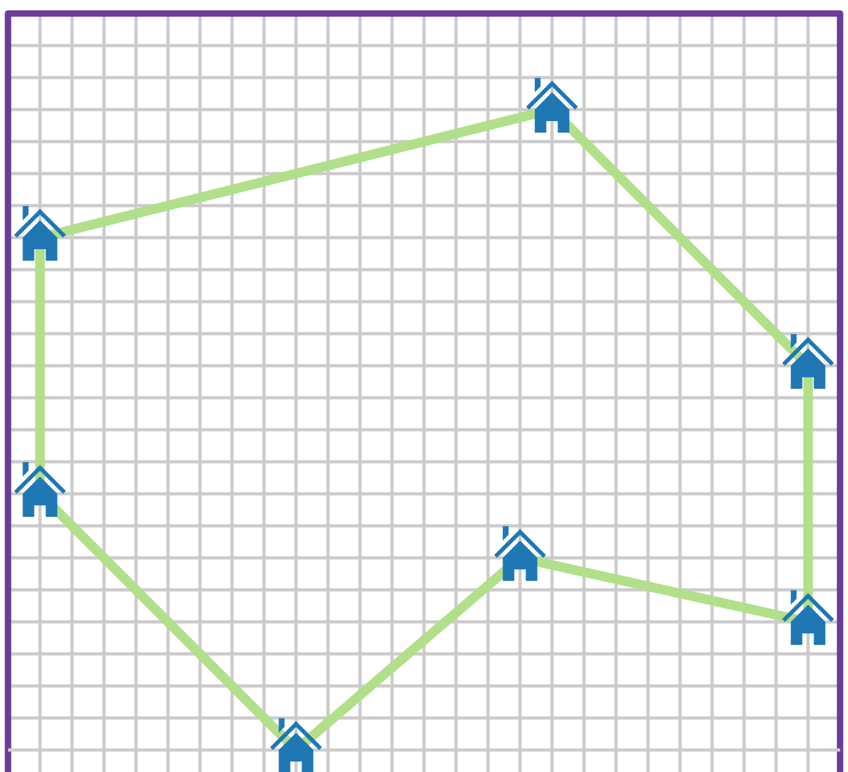

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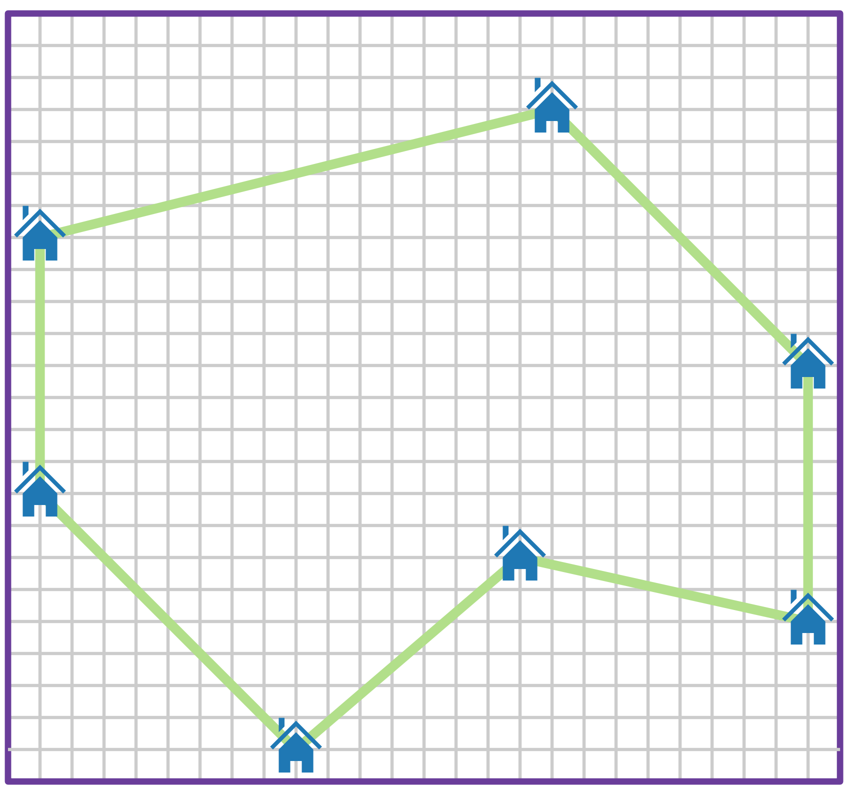

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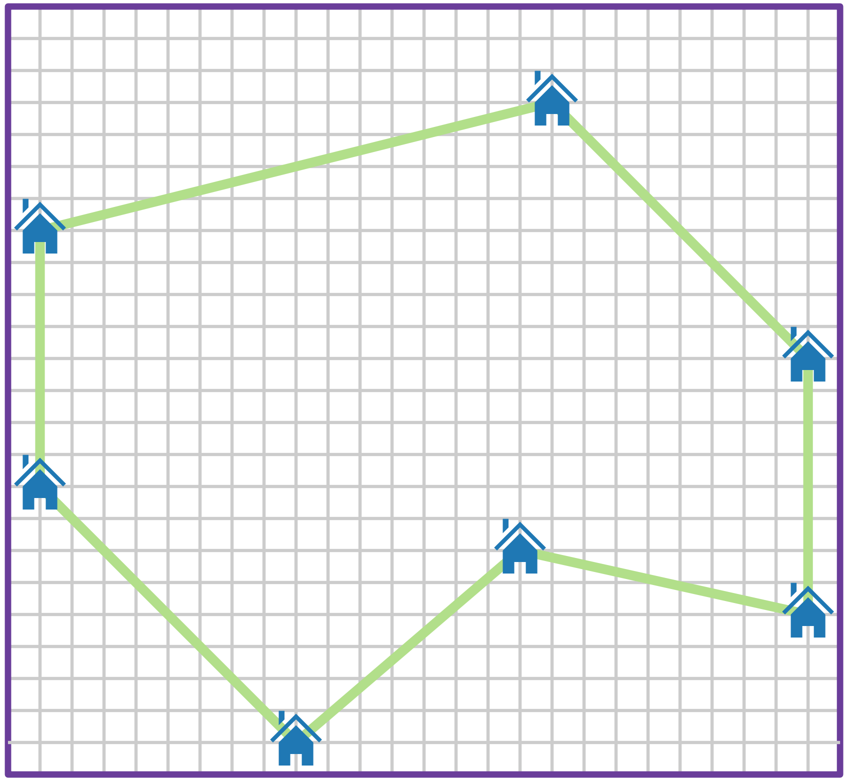

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Goal: $(1+\varepsilon)$-approximation!

Approximation Algorithms

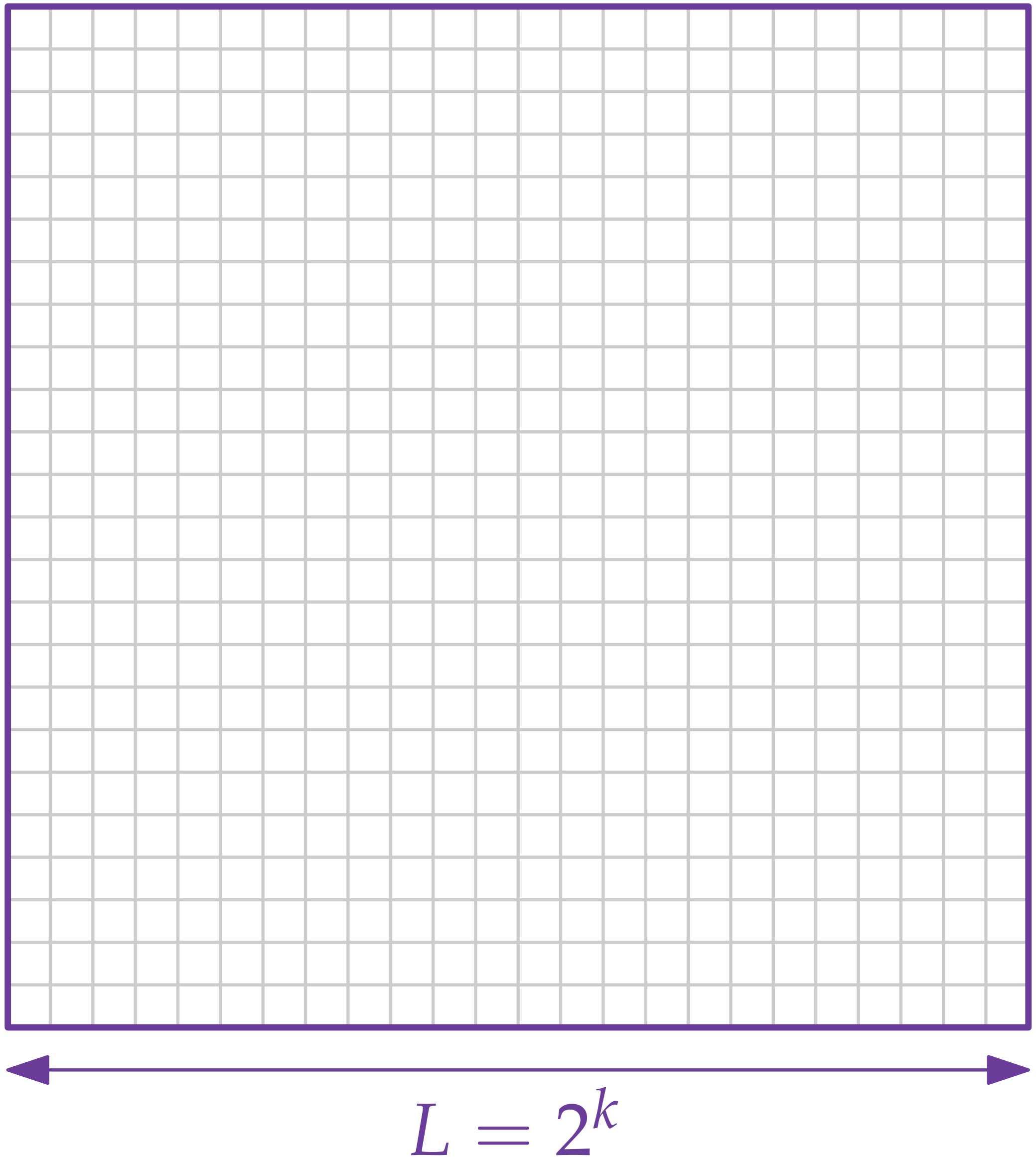
Lecture 9:
PTAS for EuclideanTSP

Part II:
Dissection

Joachim Spoerhase

Winter 2021/22

Basic Dissection



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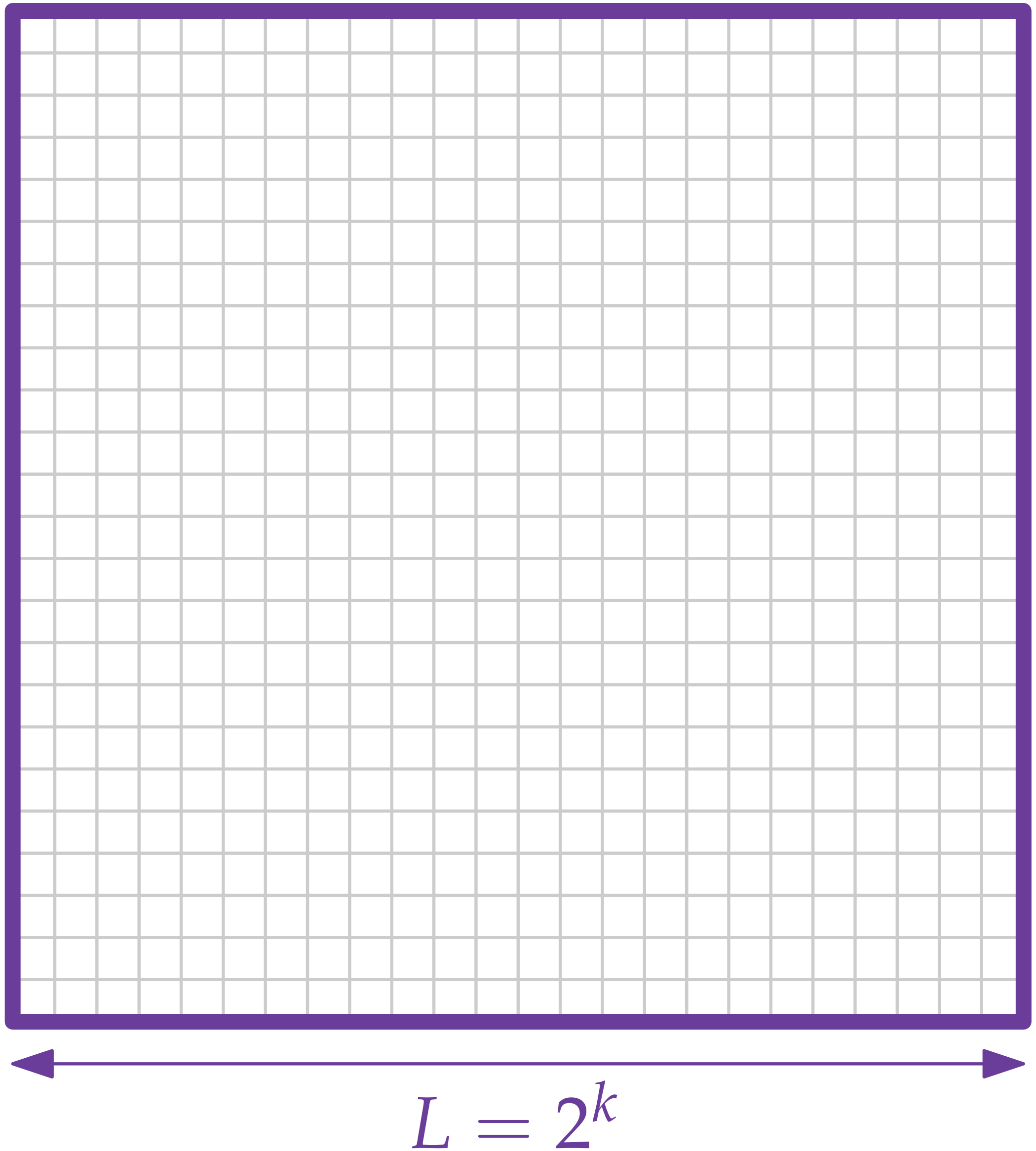

Level 0 □

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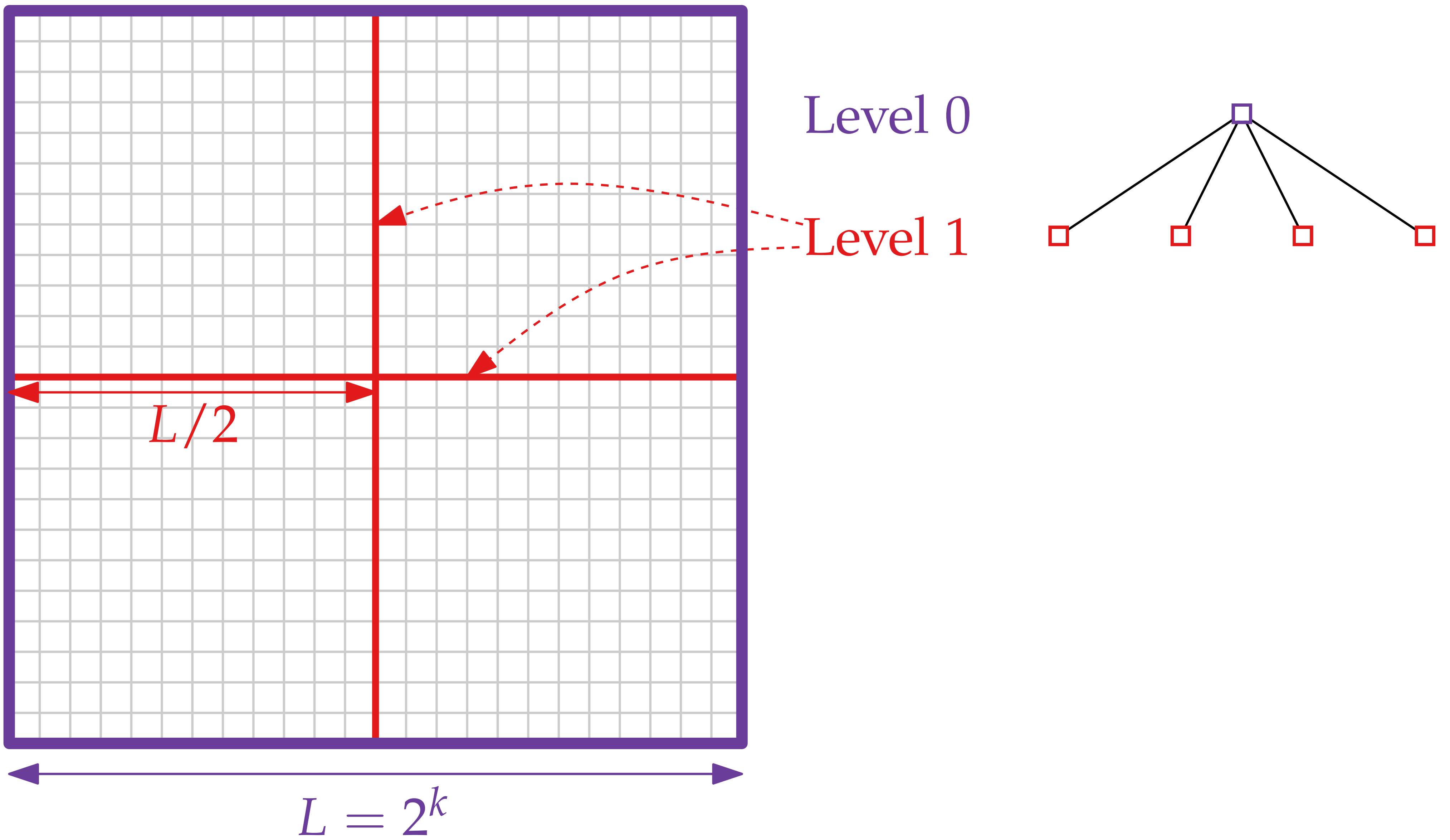

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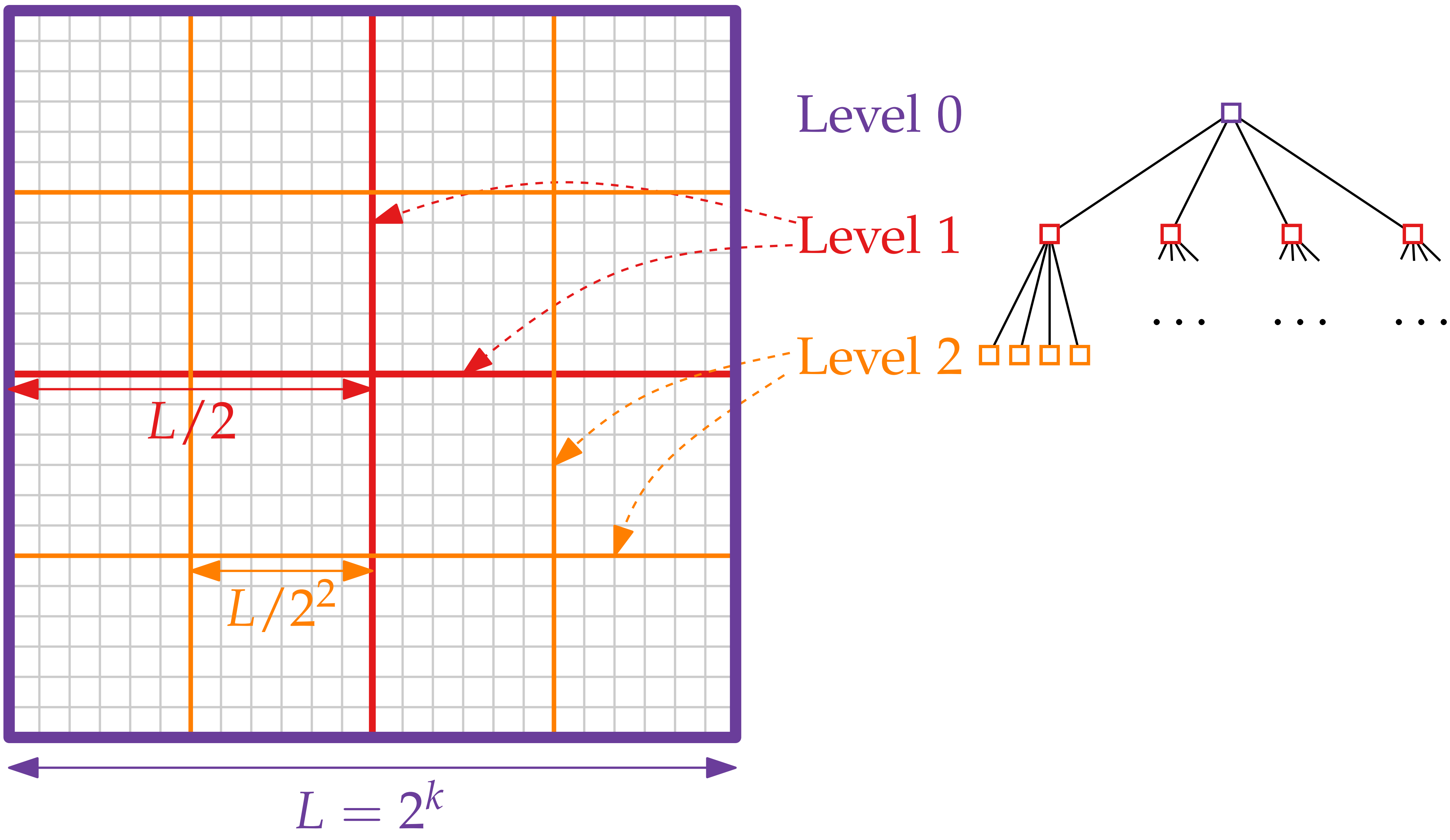

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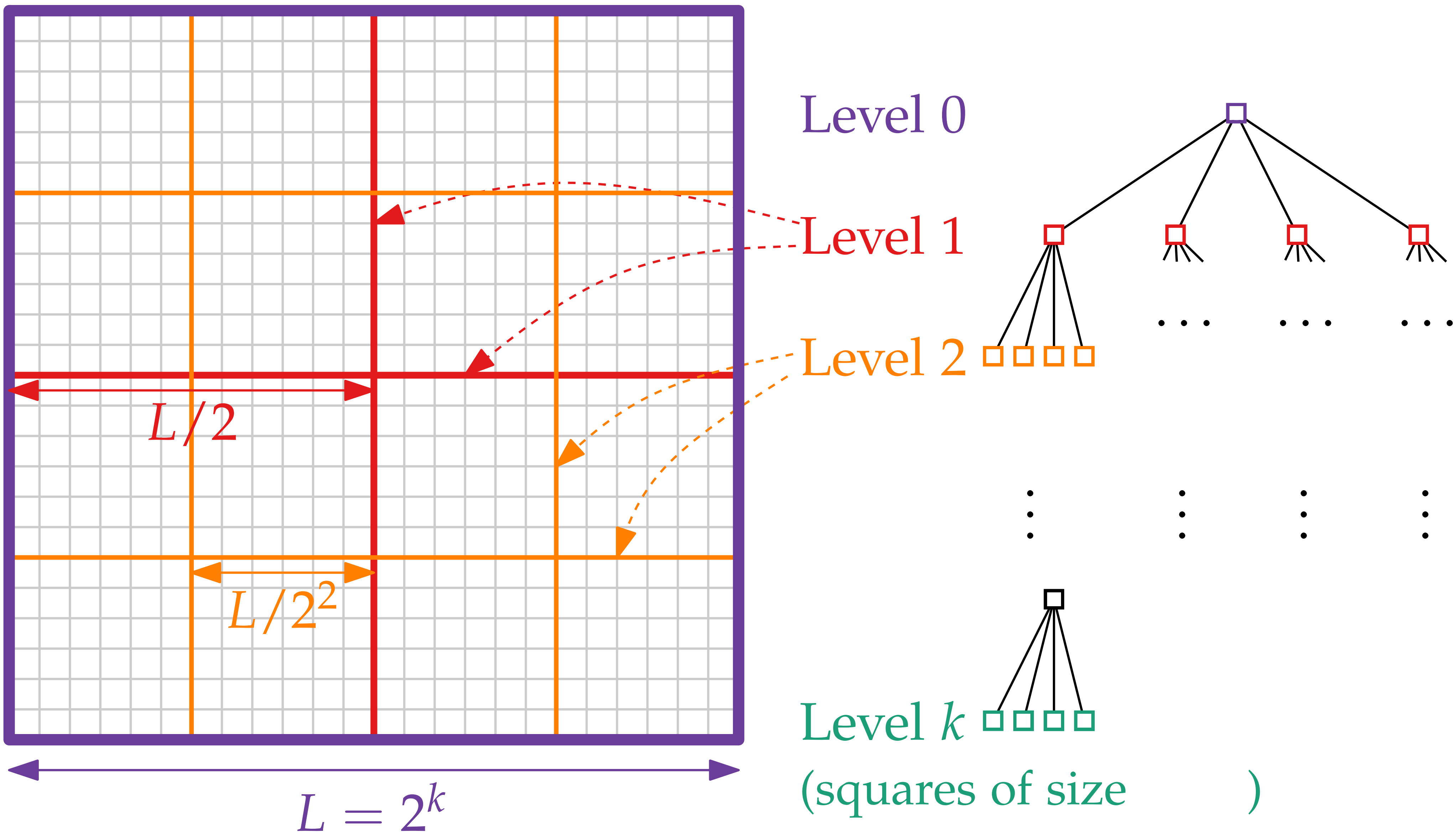

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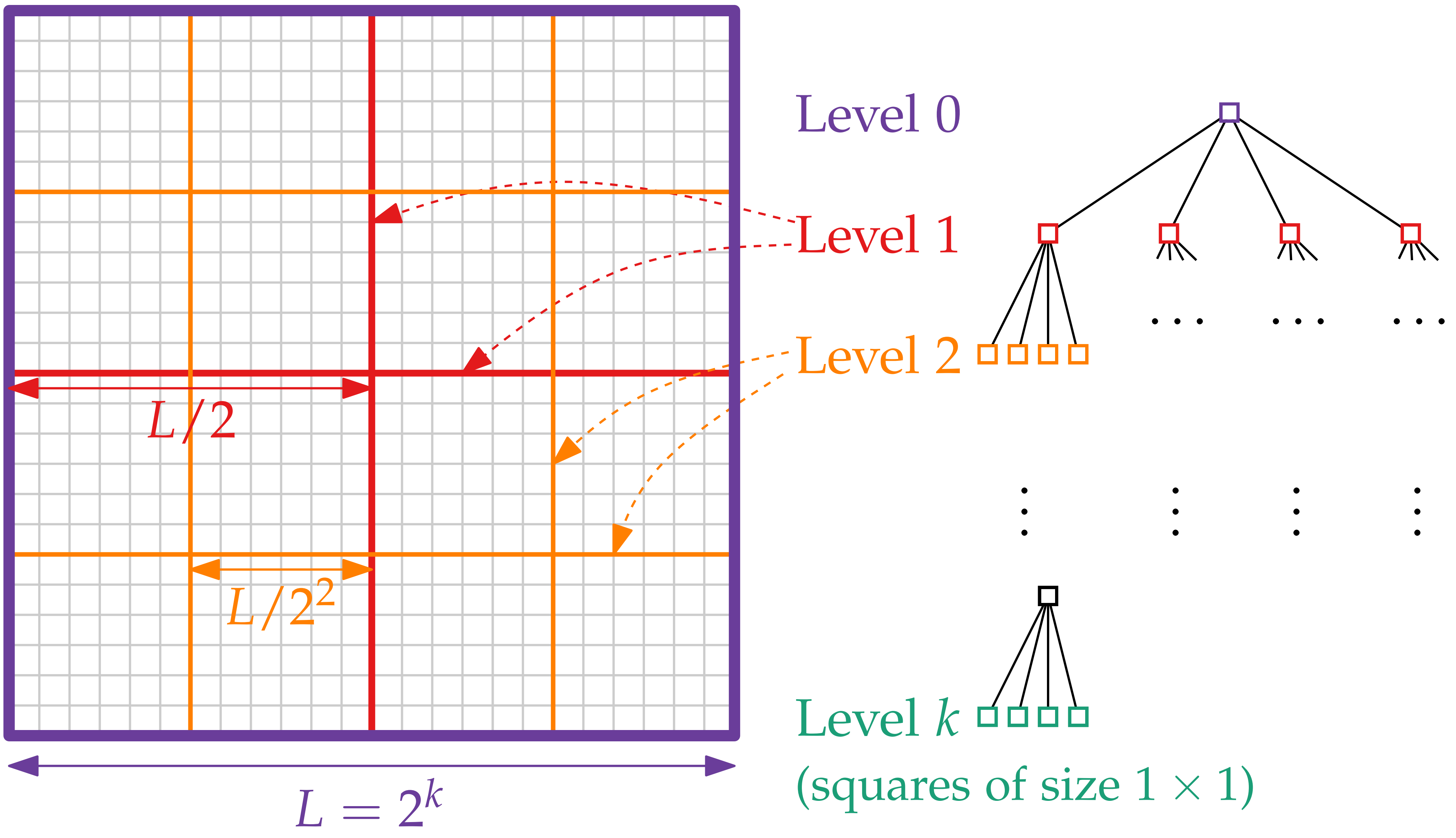
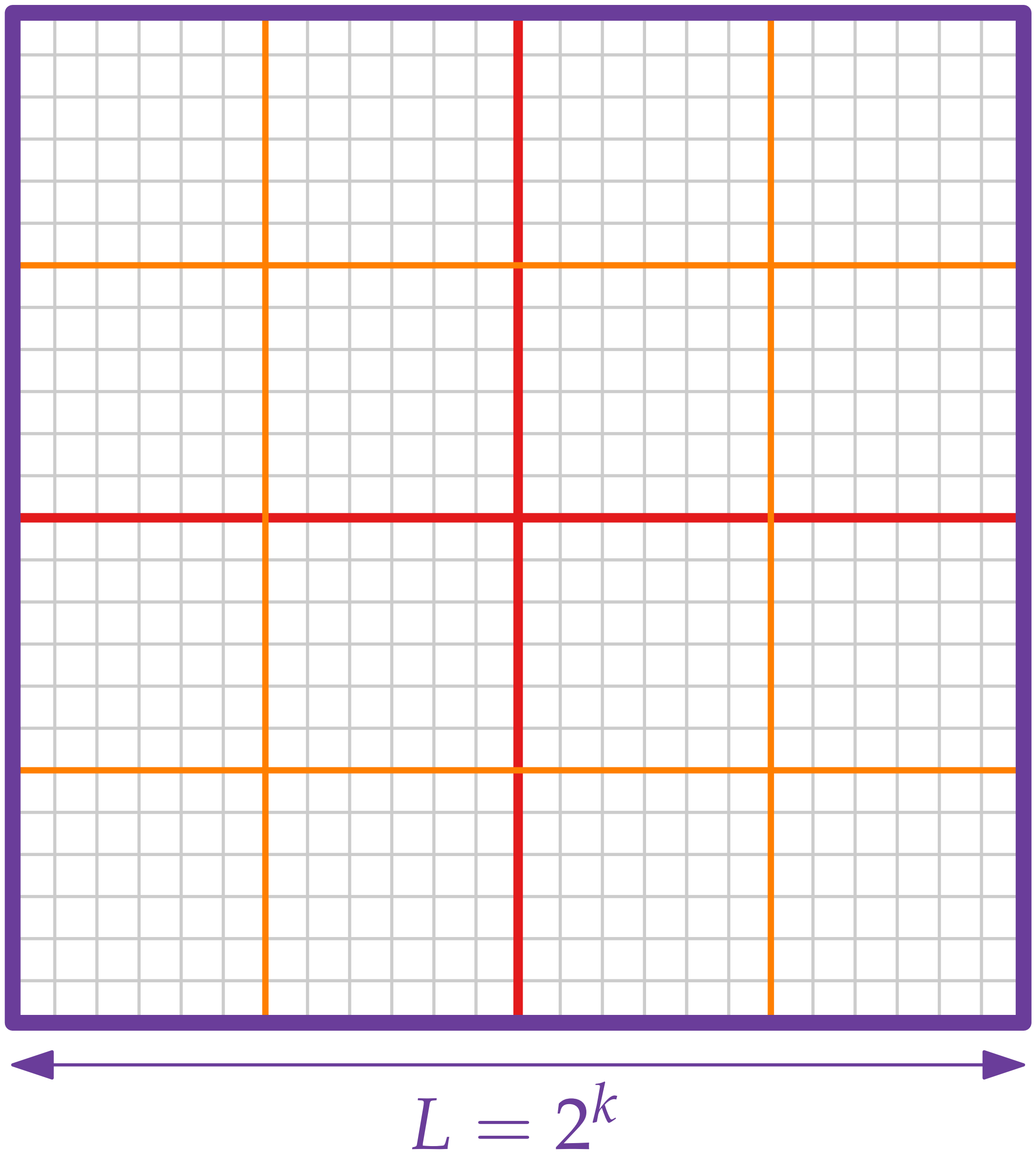
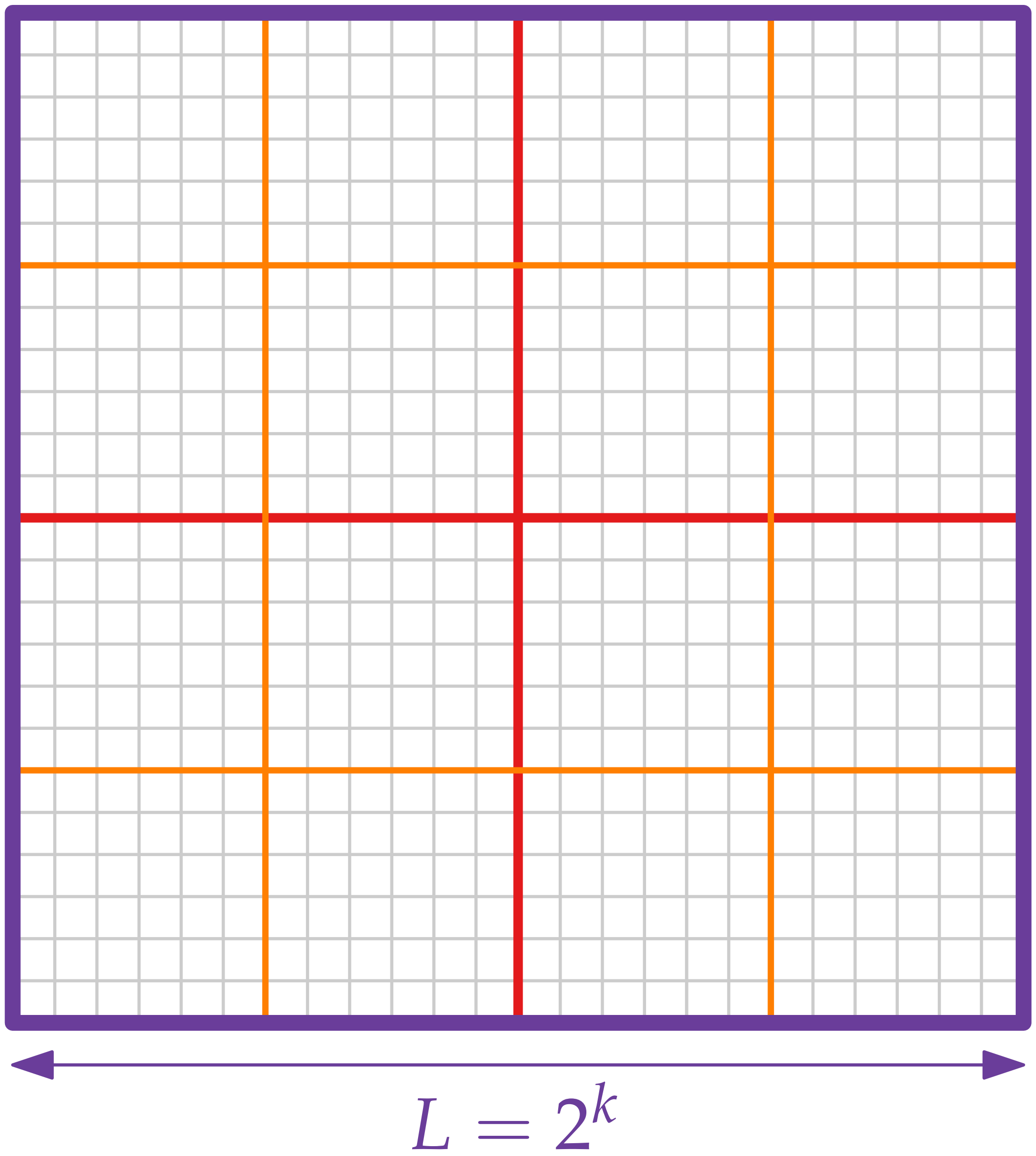


Portals



- m power of two in interval $[k/\varepsilon, 2k/\varepsilon]$

Portals



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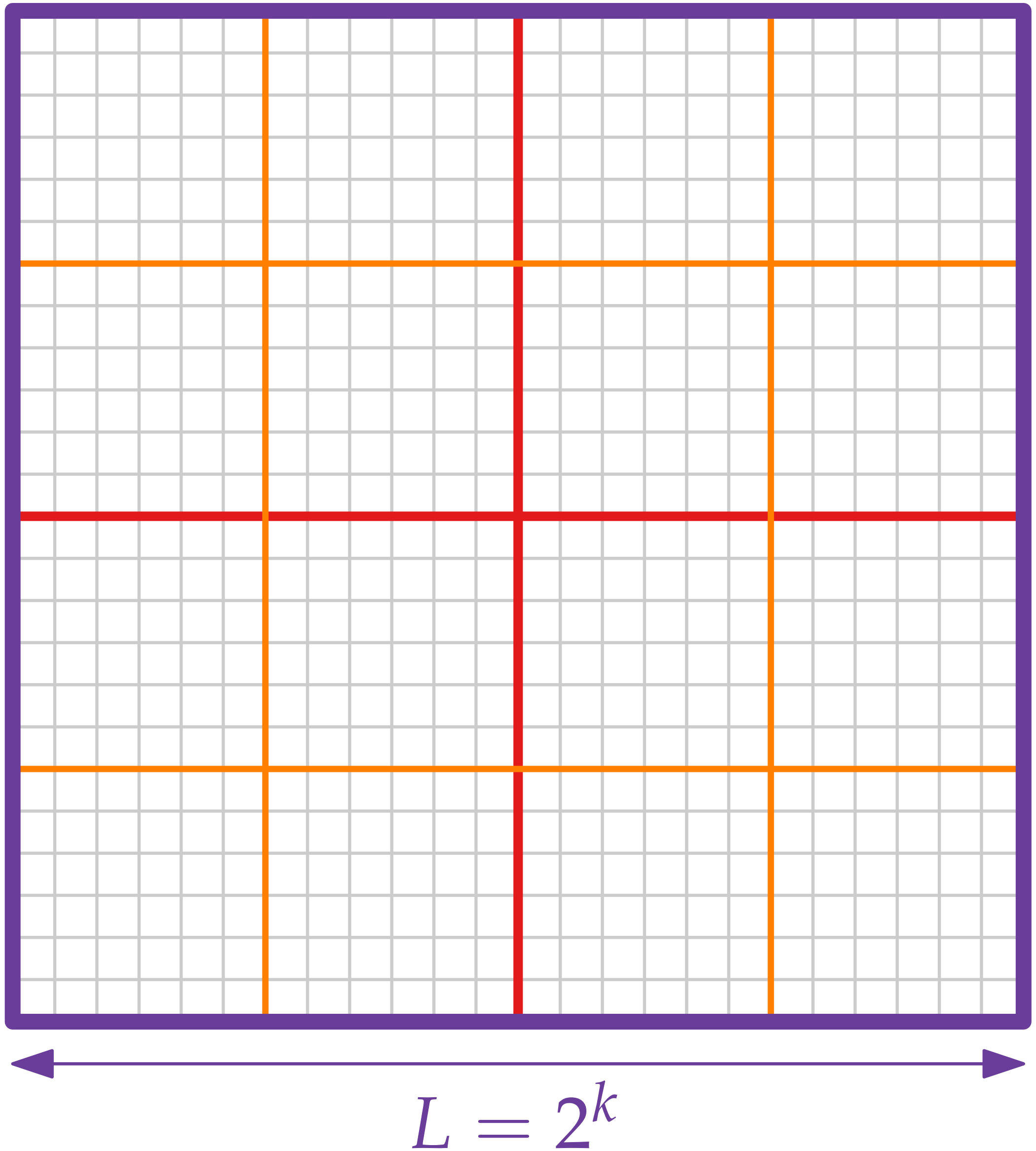

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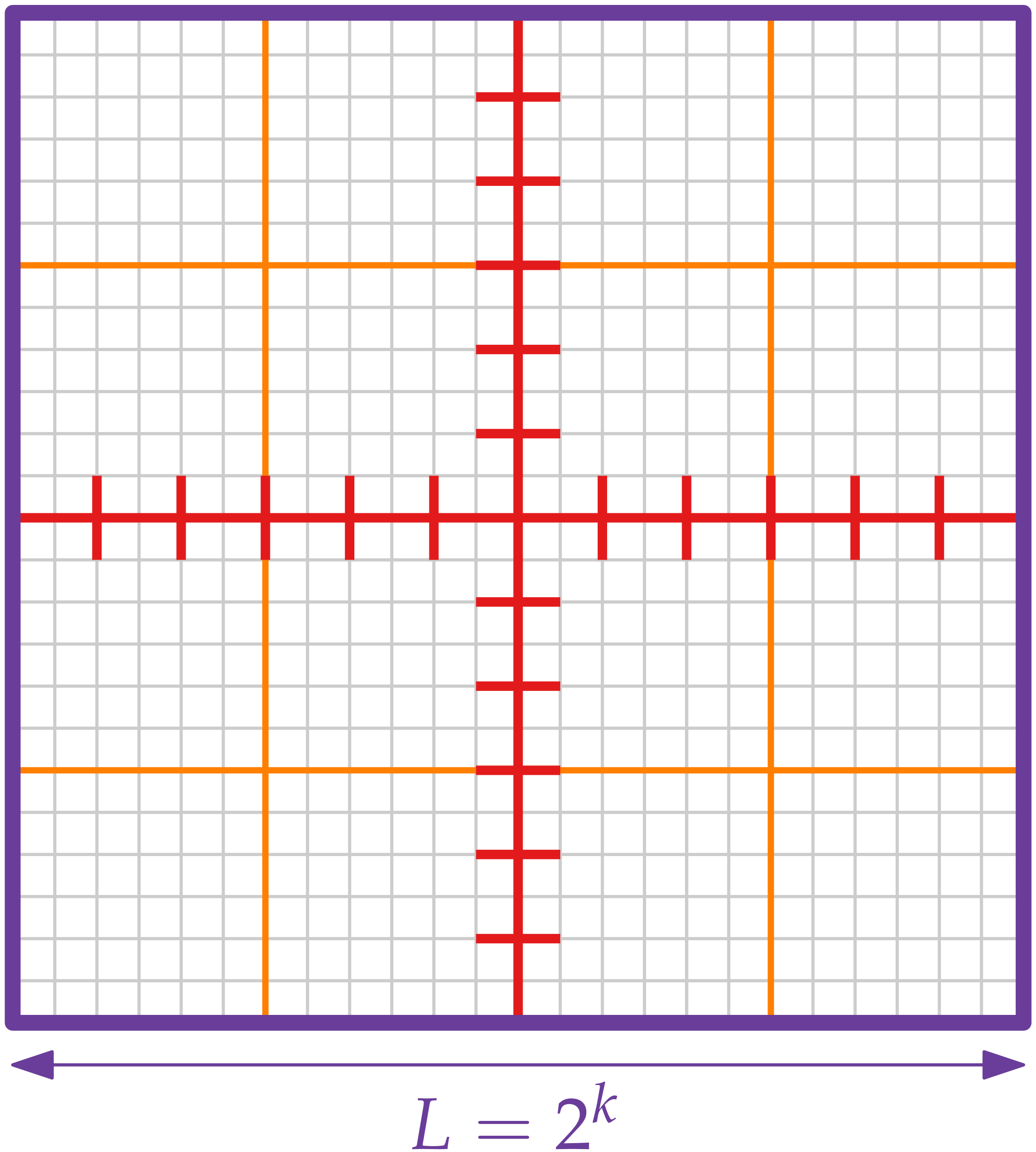

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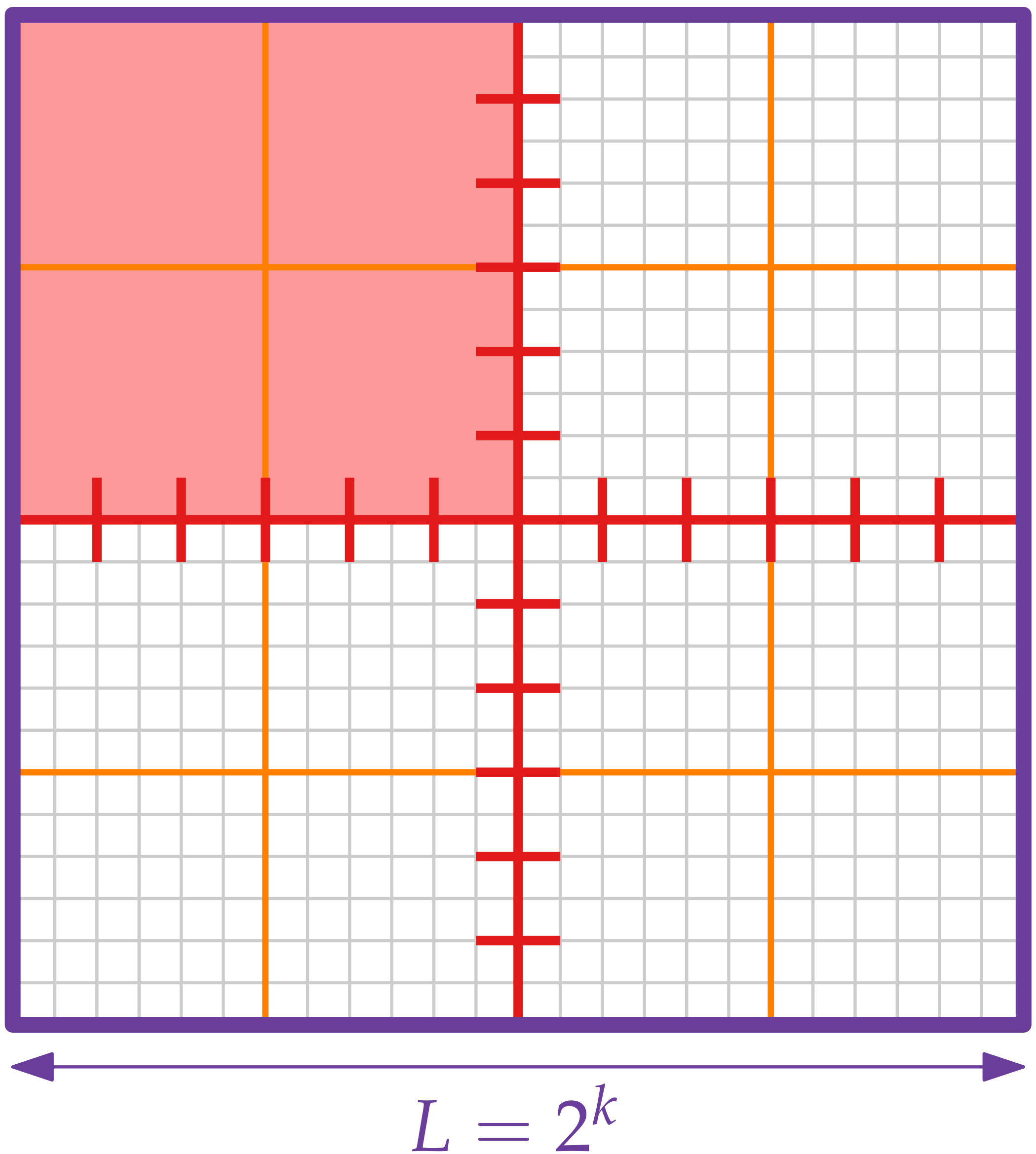

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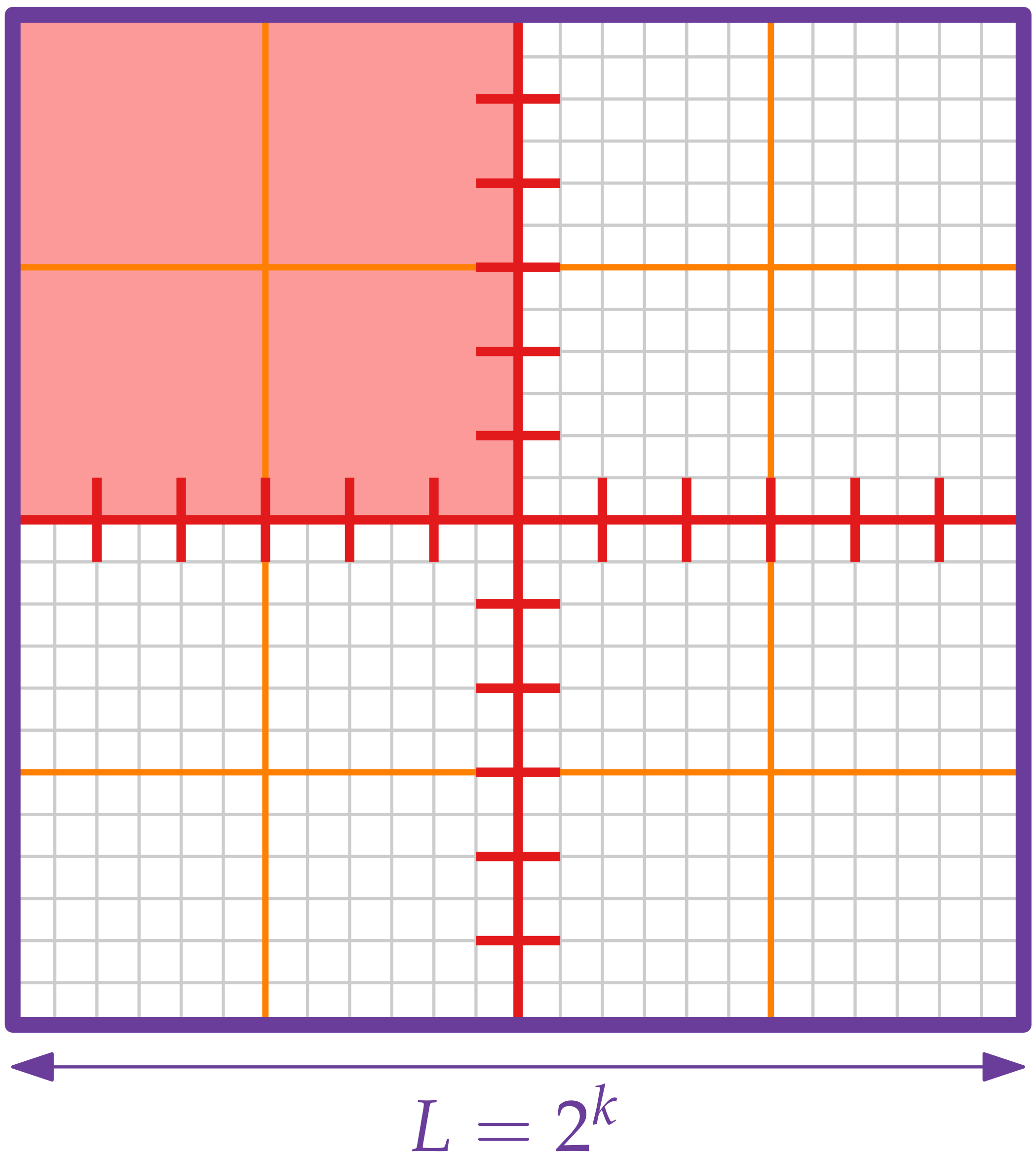
- Level-i-square: size

Portals



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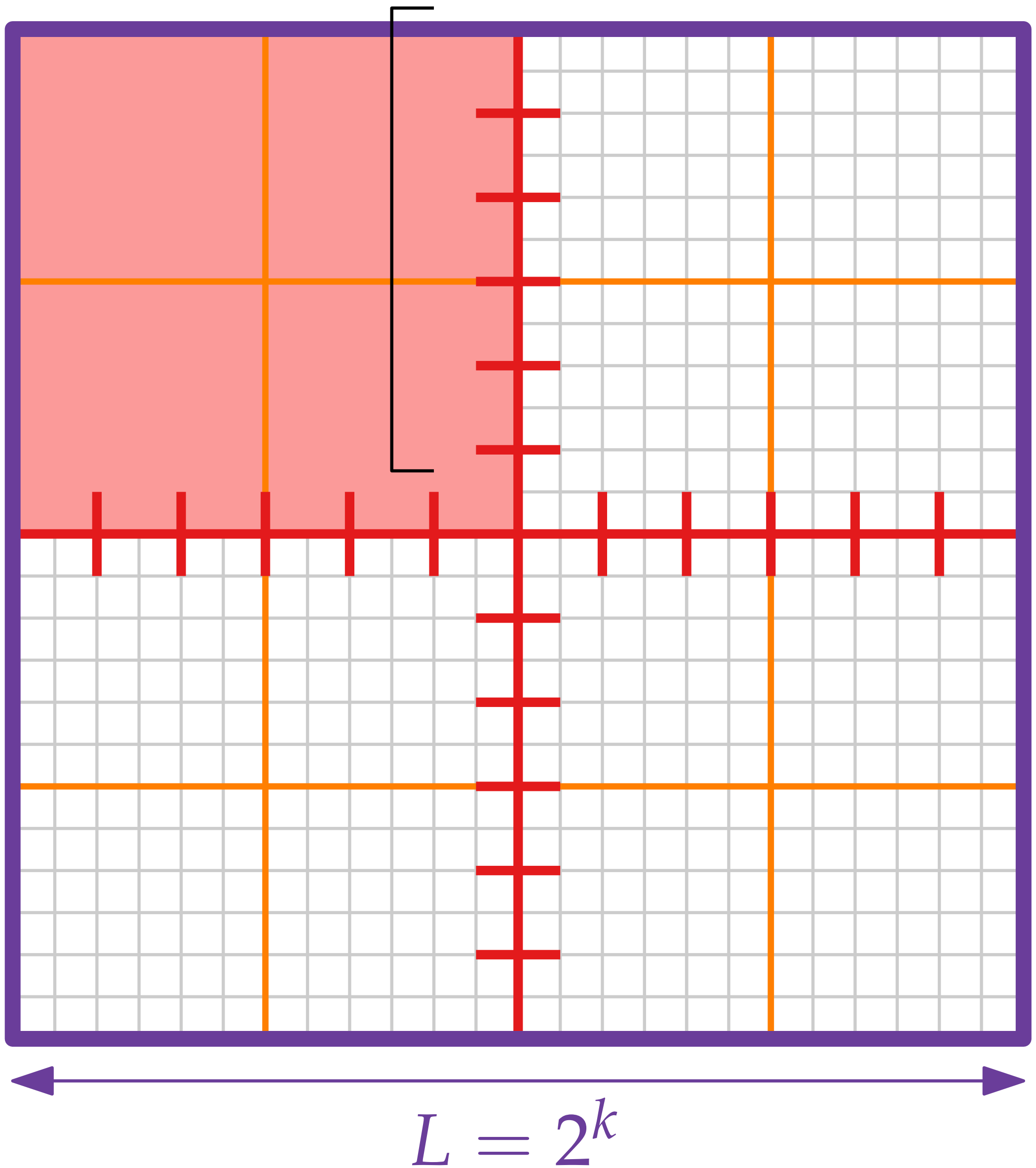

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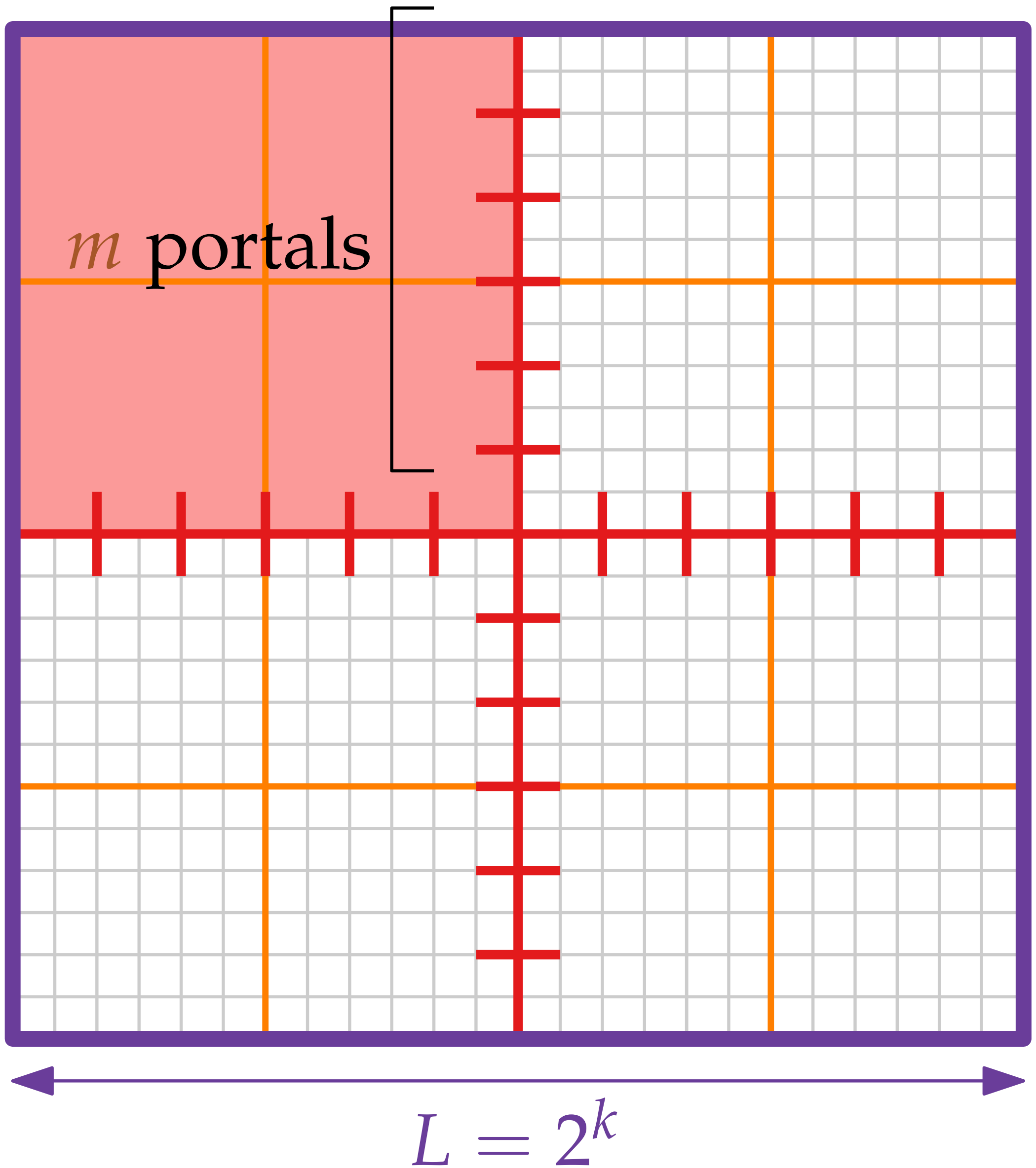

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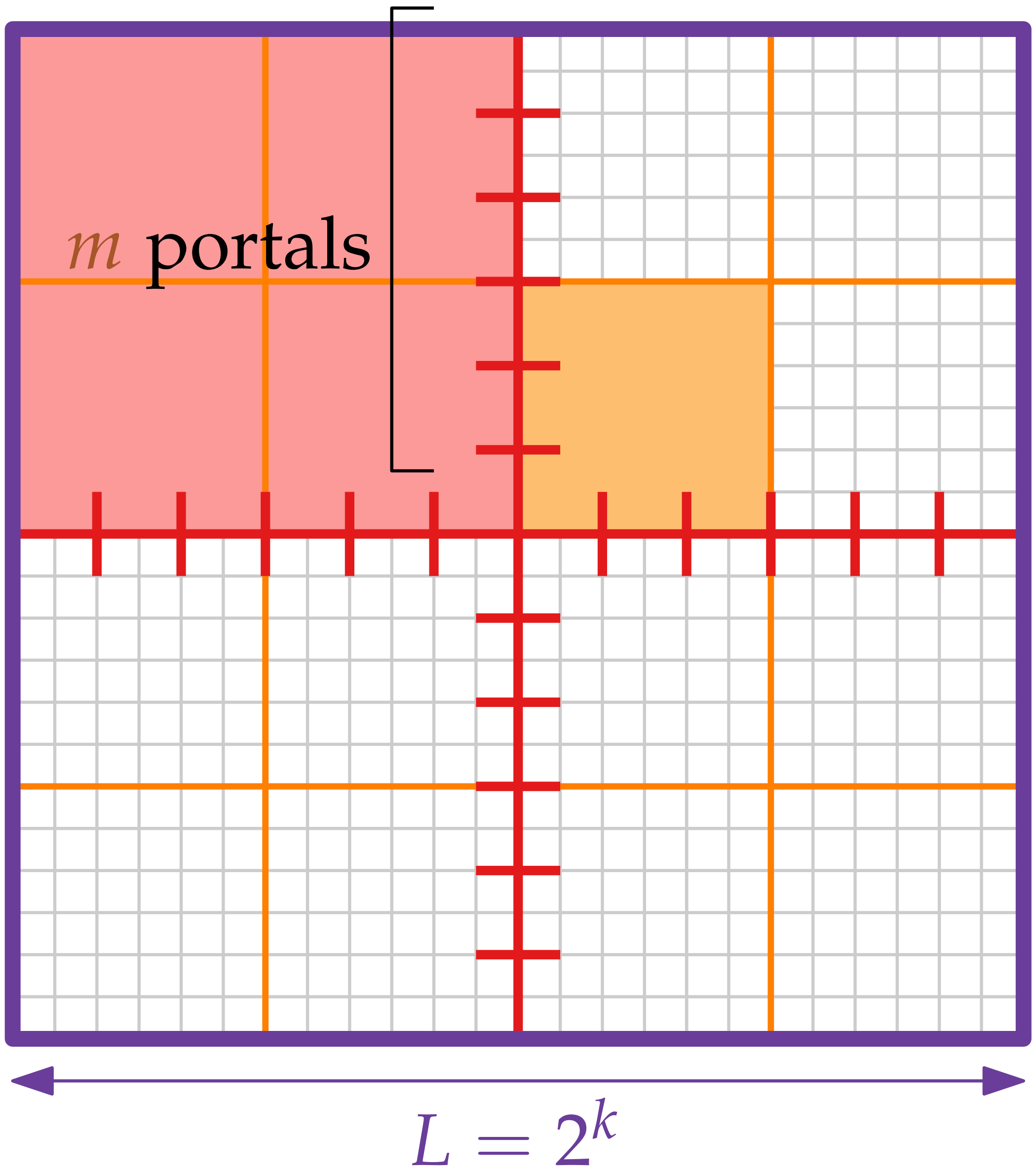

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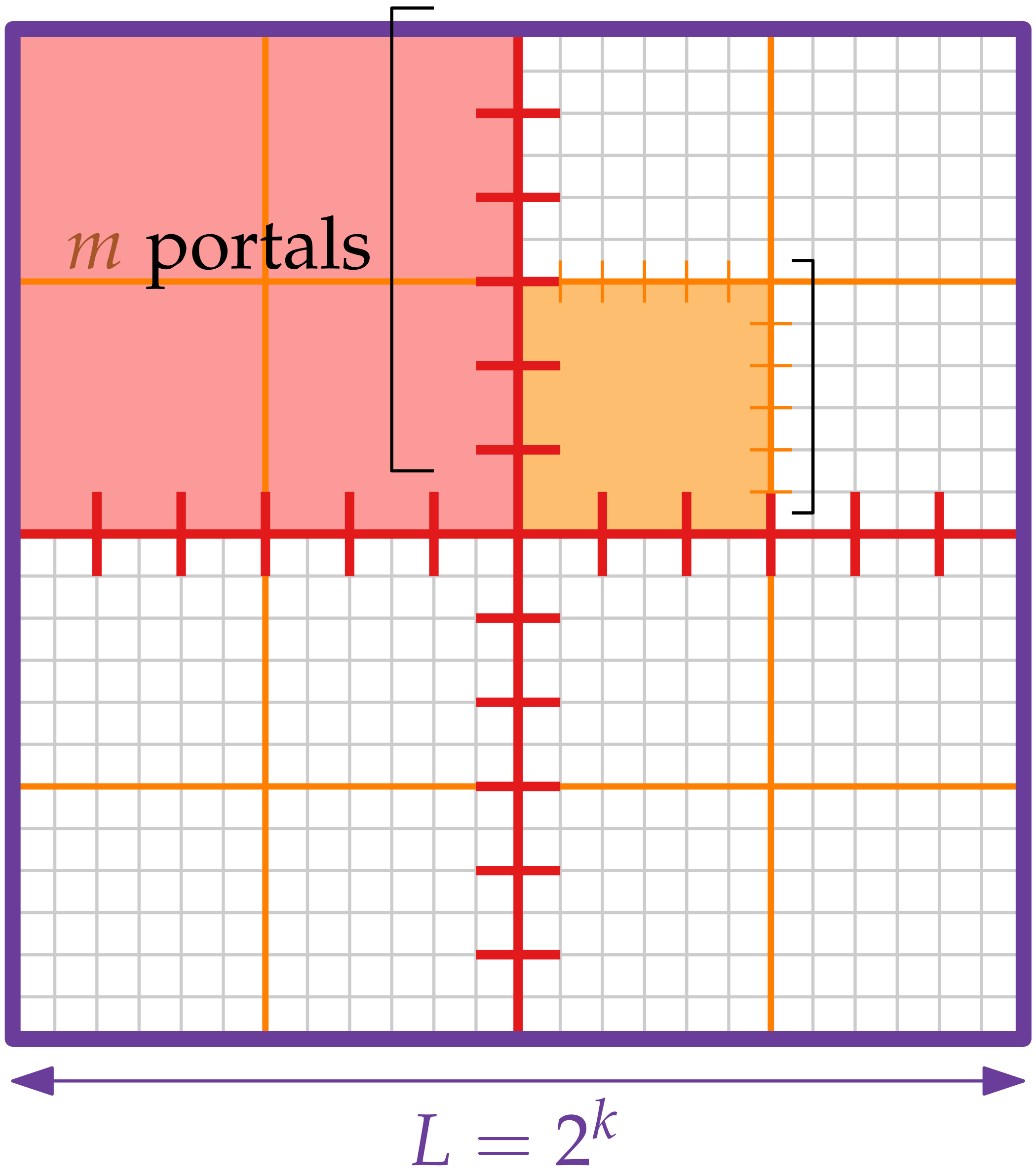

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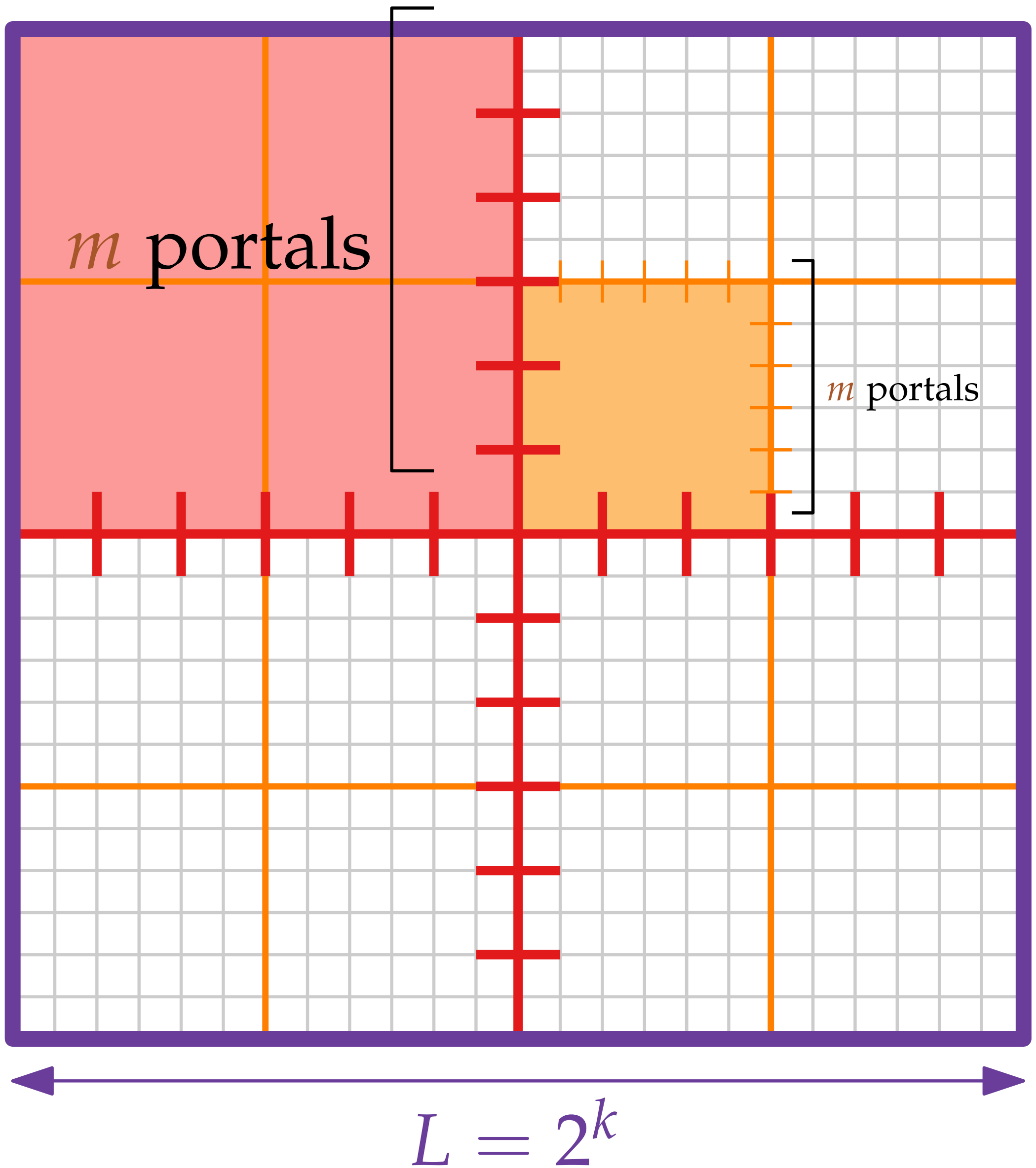

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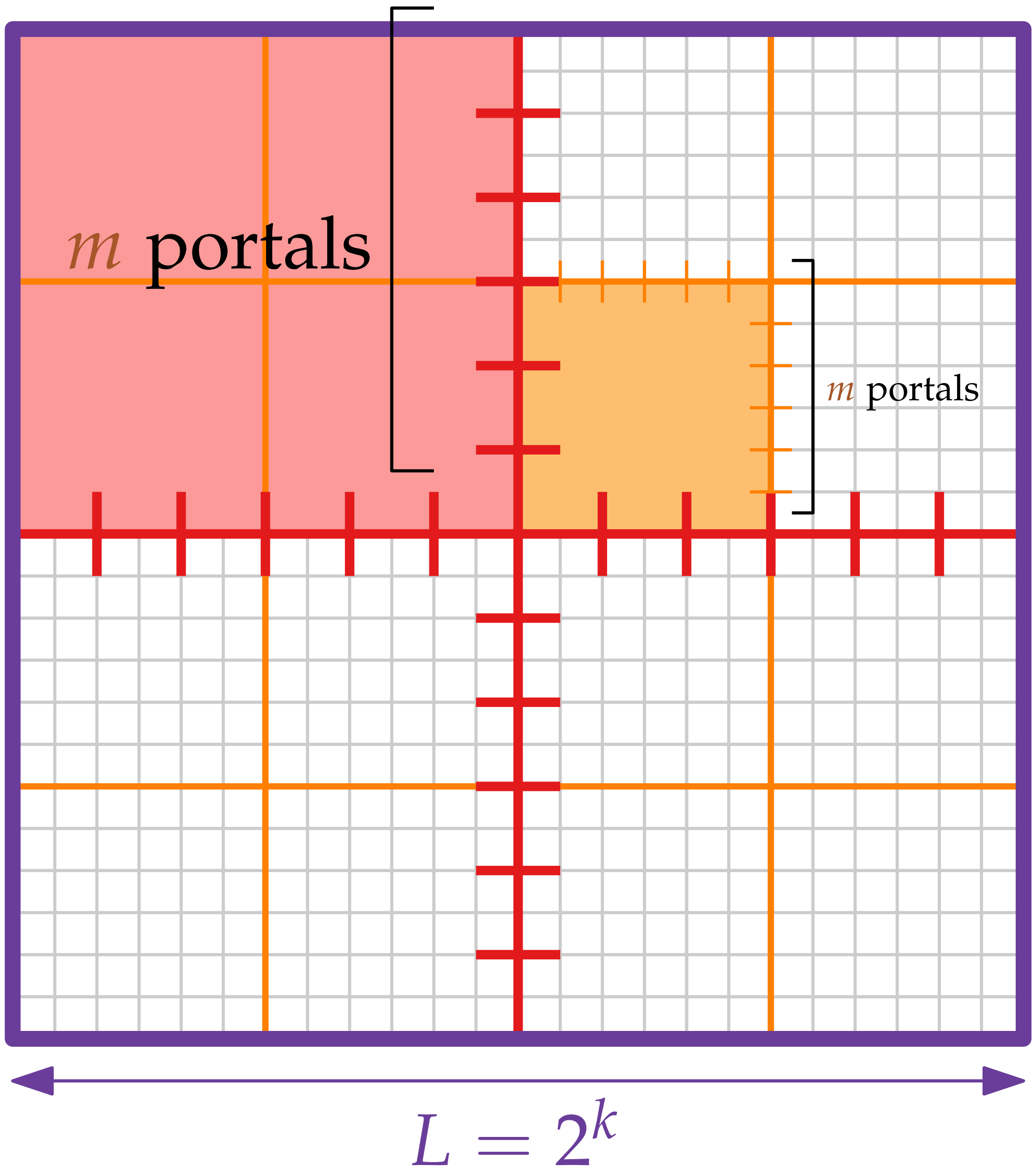

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- Level-i-square has at most $4m$ portals on its boundary.

Approximation Algorithms

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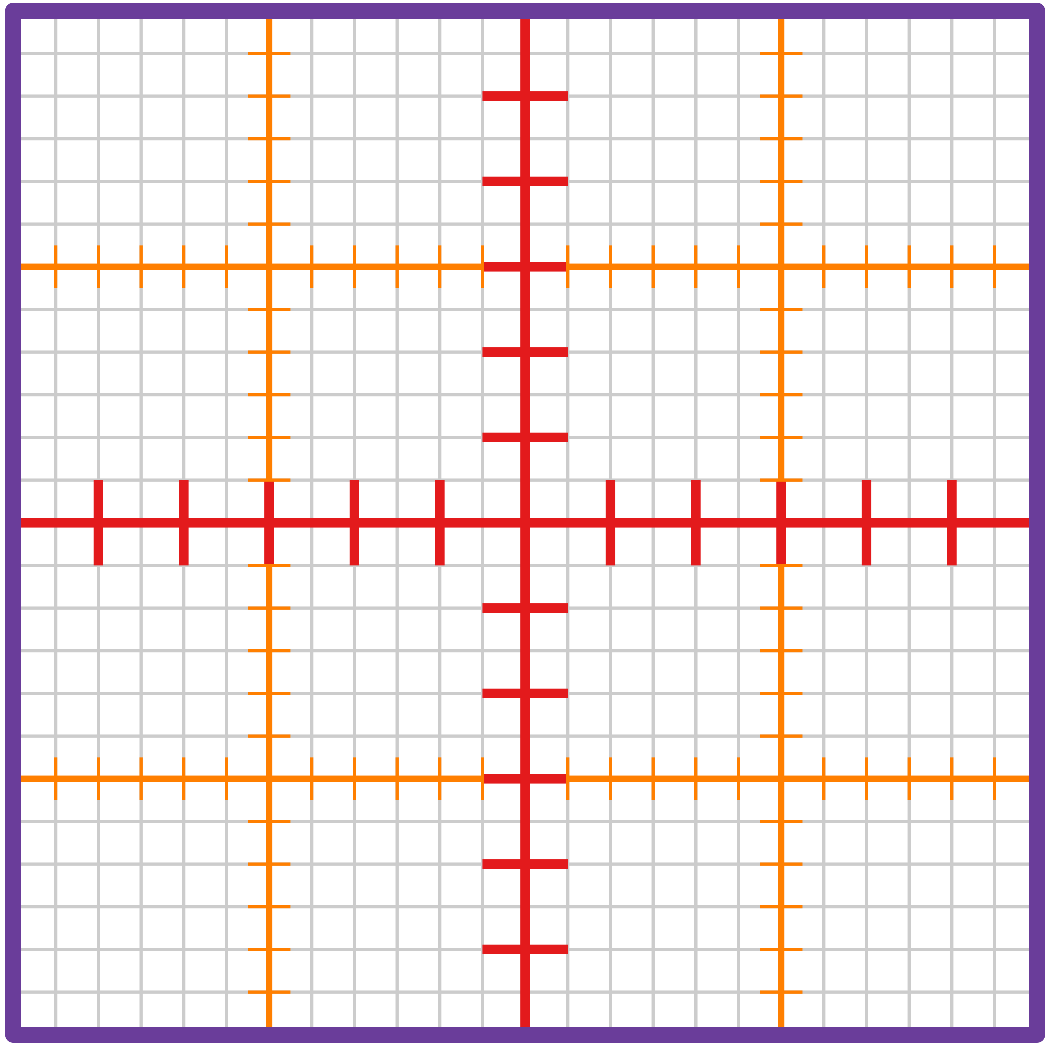
Part III:
Well Behaved Tours

Joachim Spoerhase

Winter 2021/22

Well Behaved Tours



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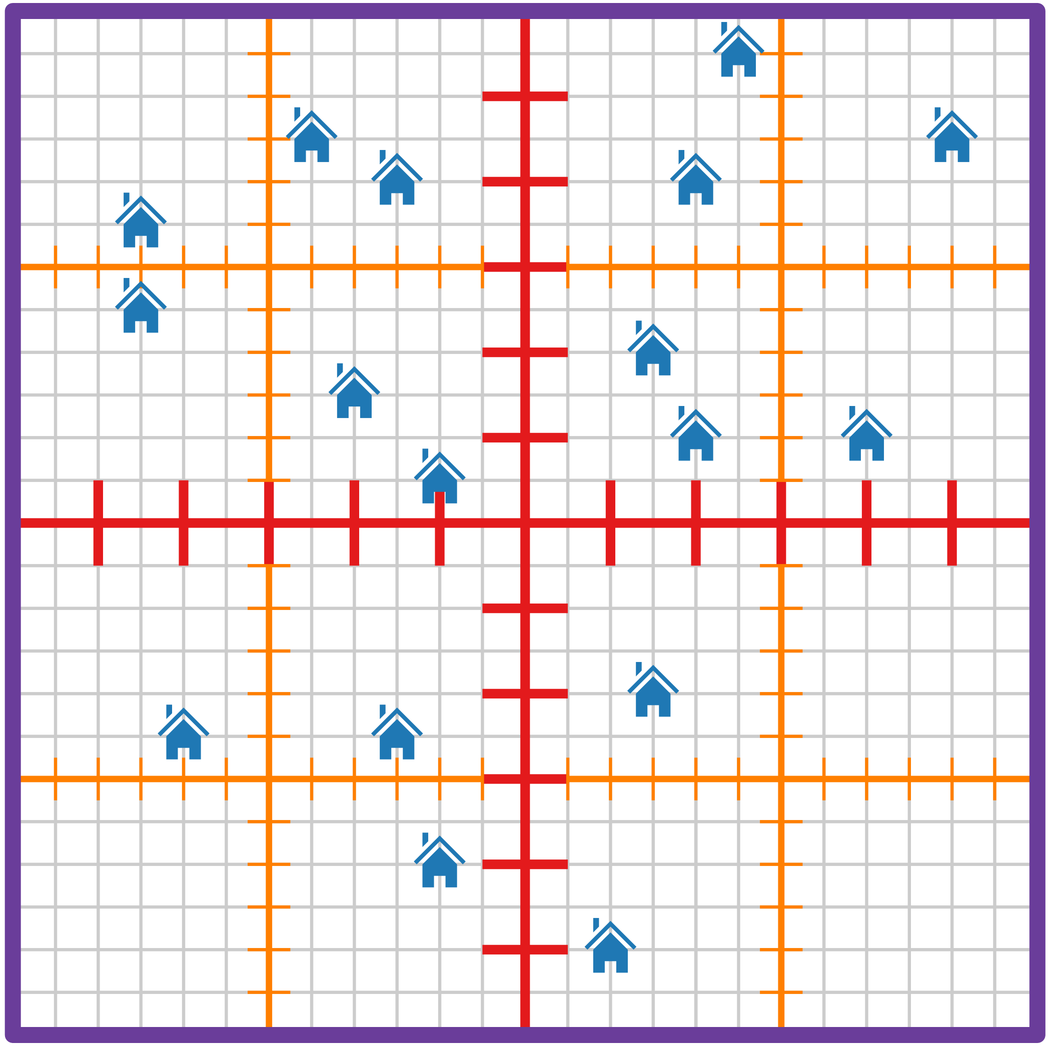

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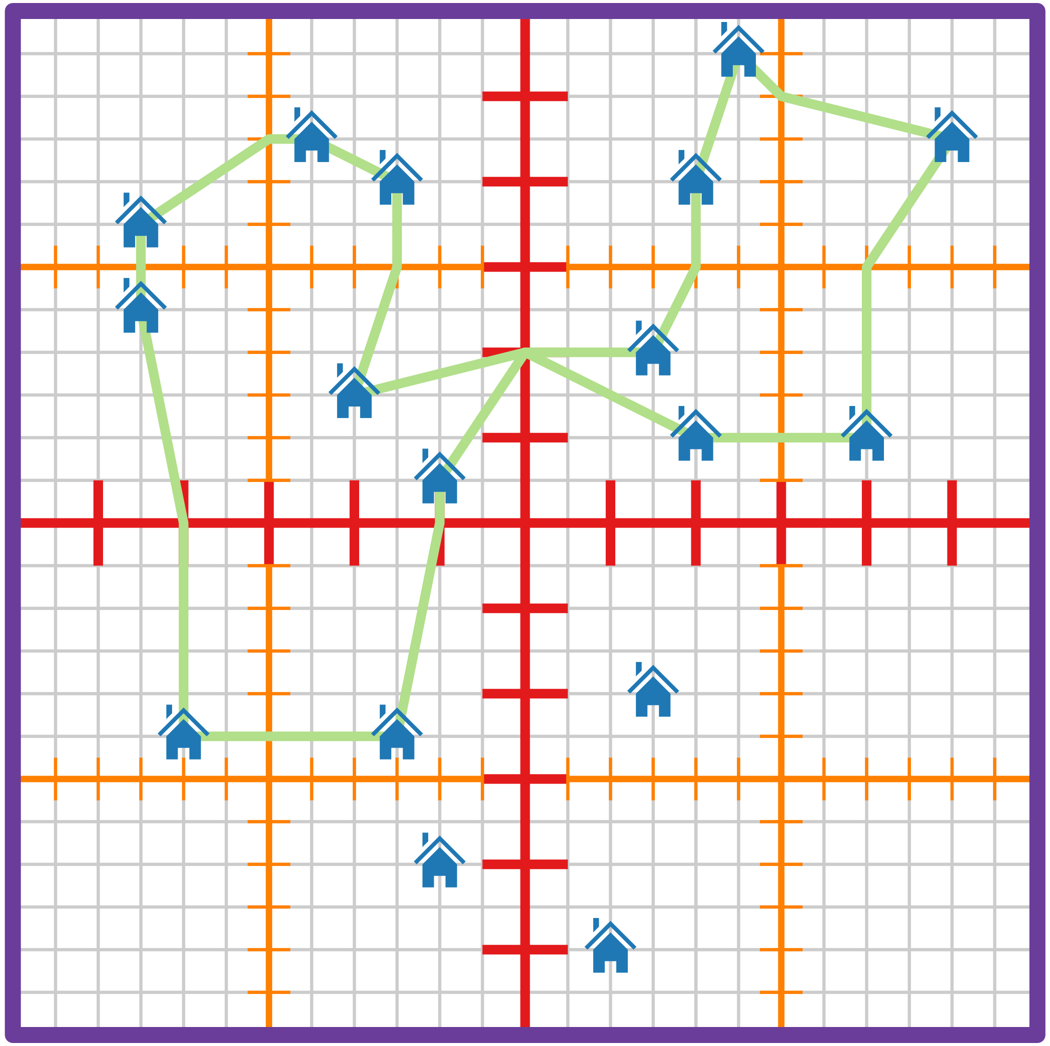

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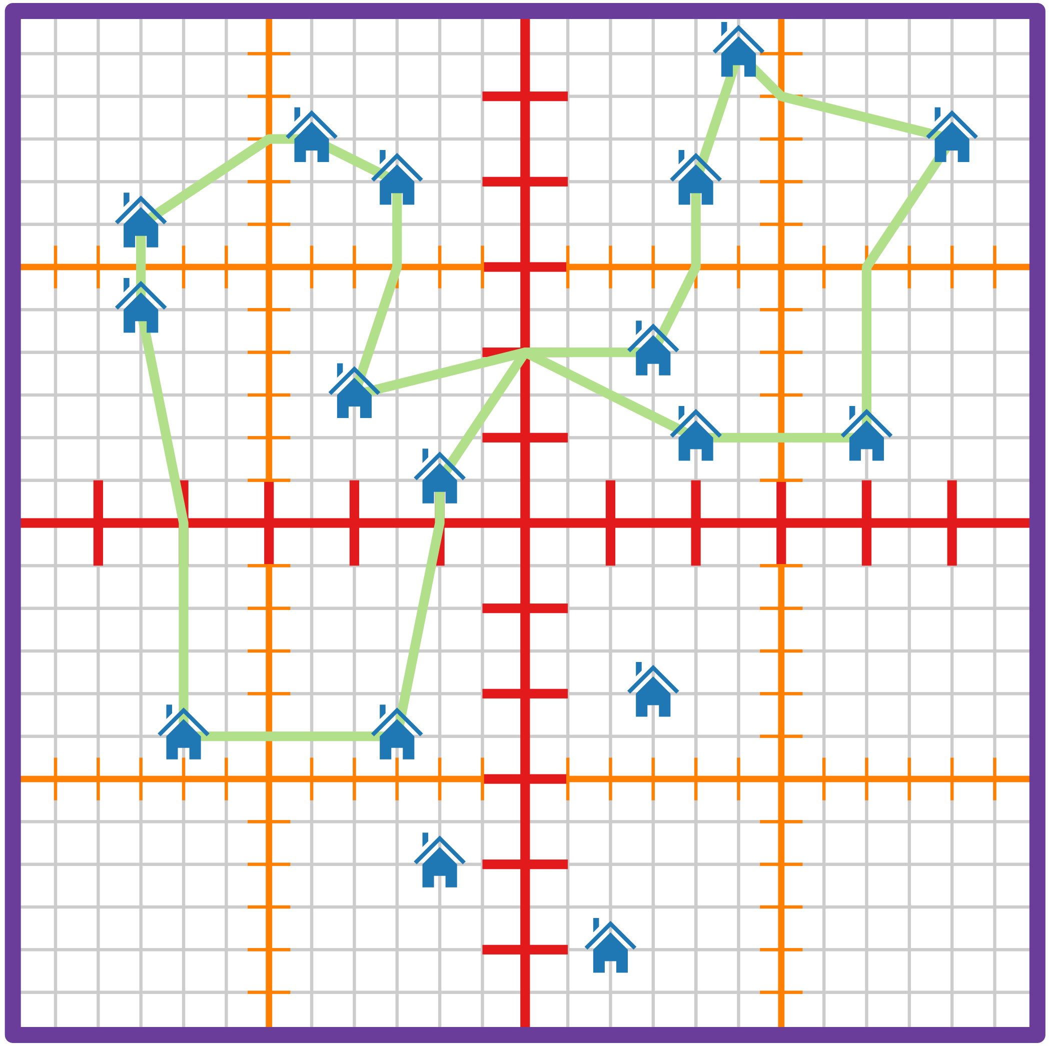

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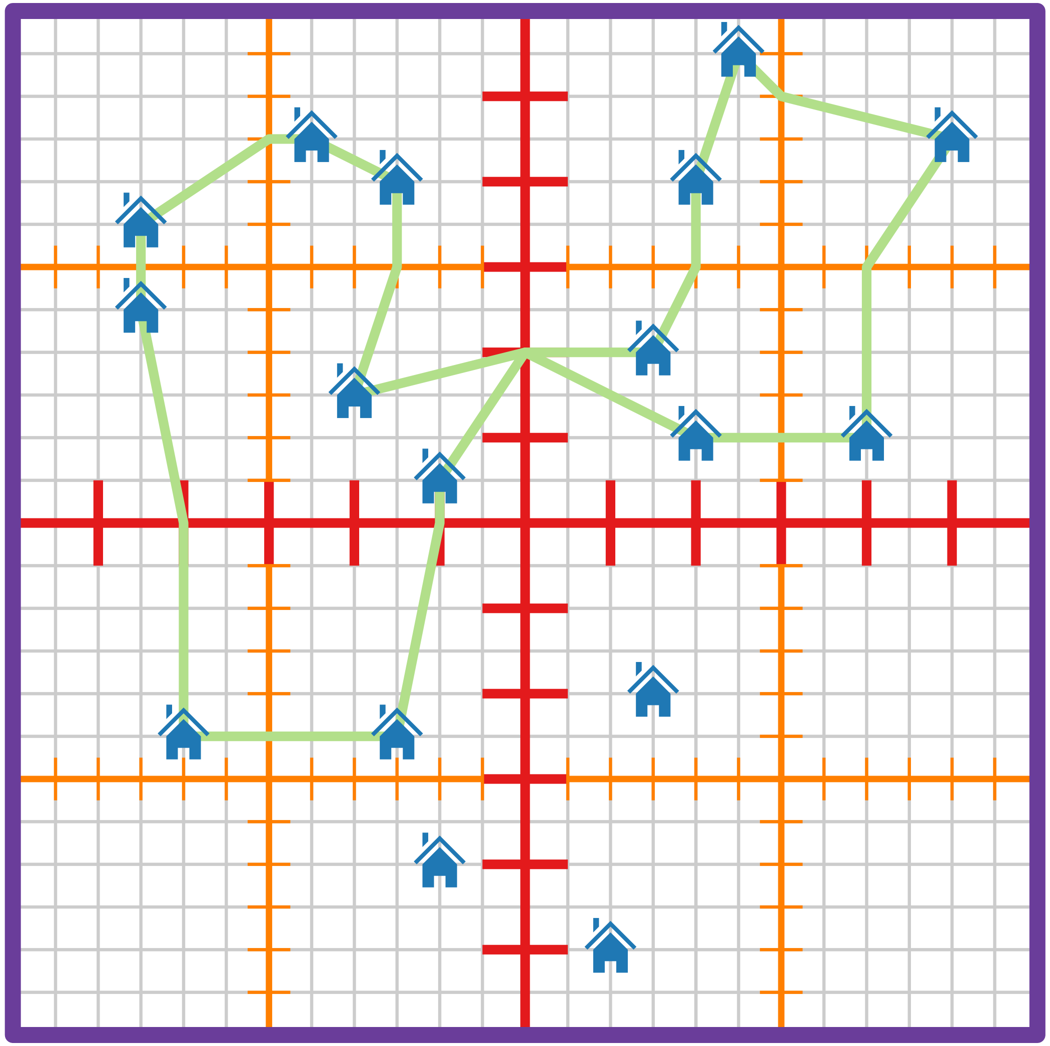
Well Behaved Tours



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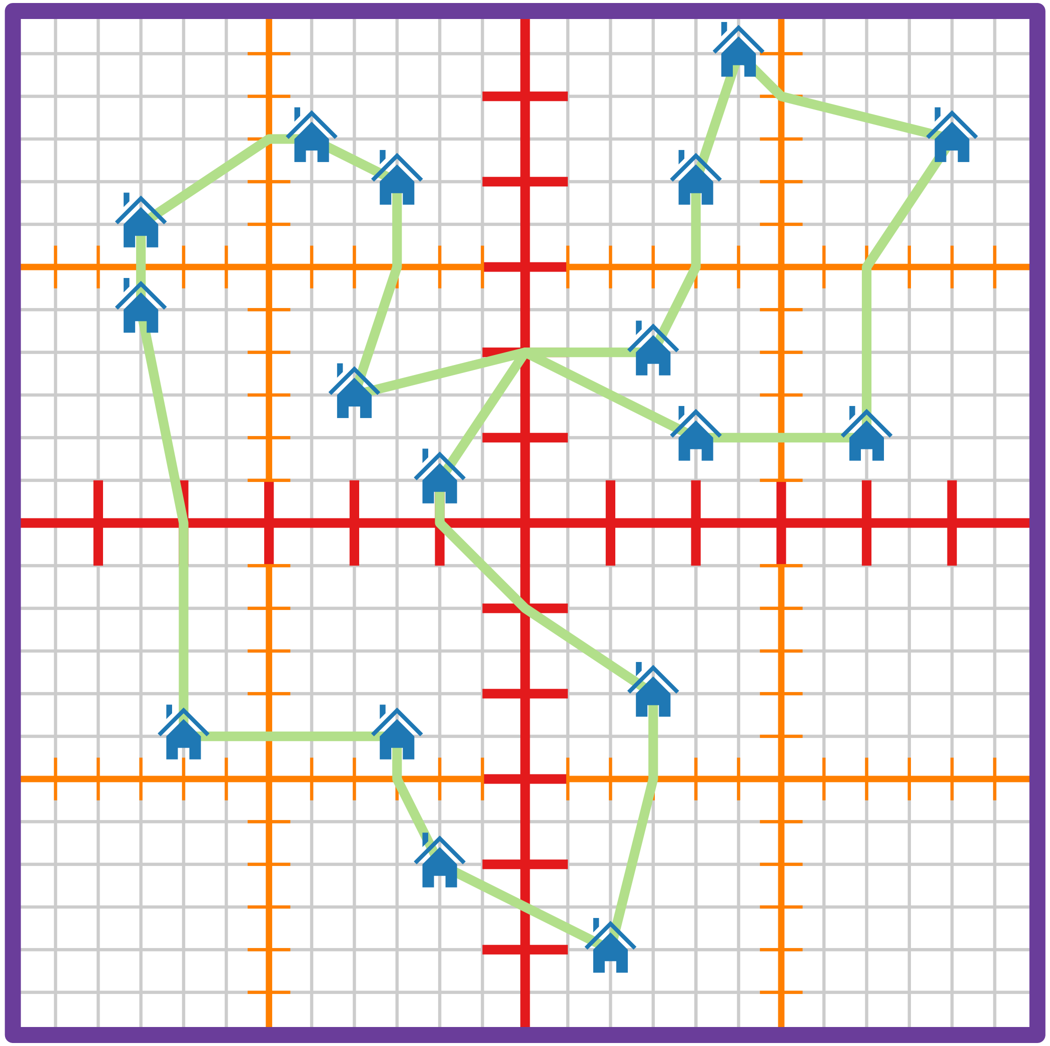

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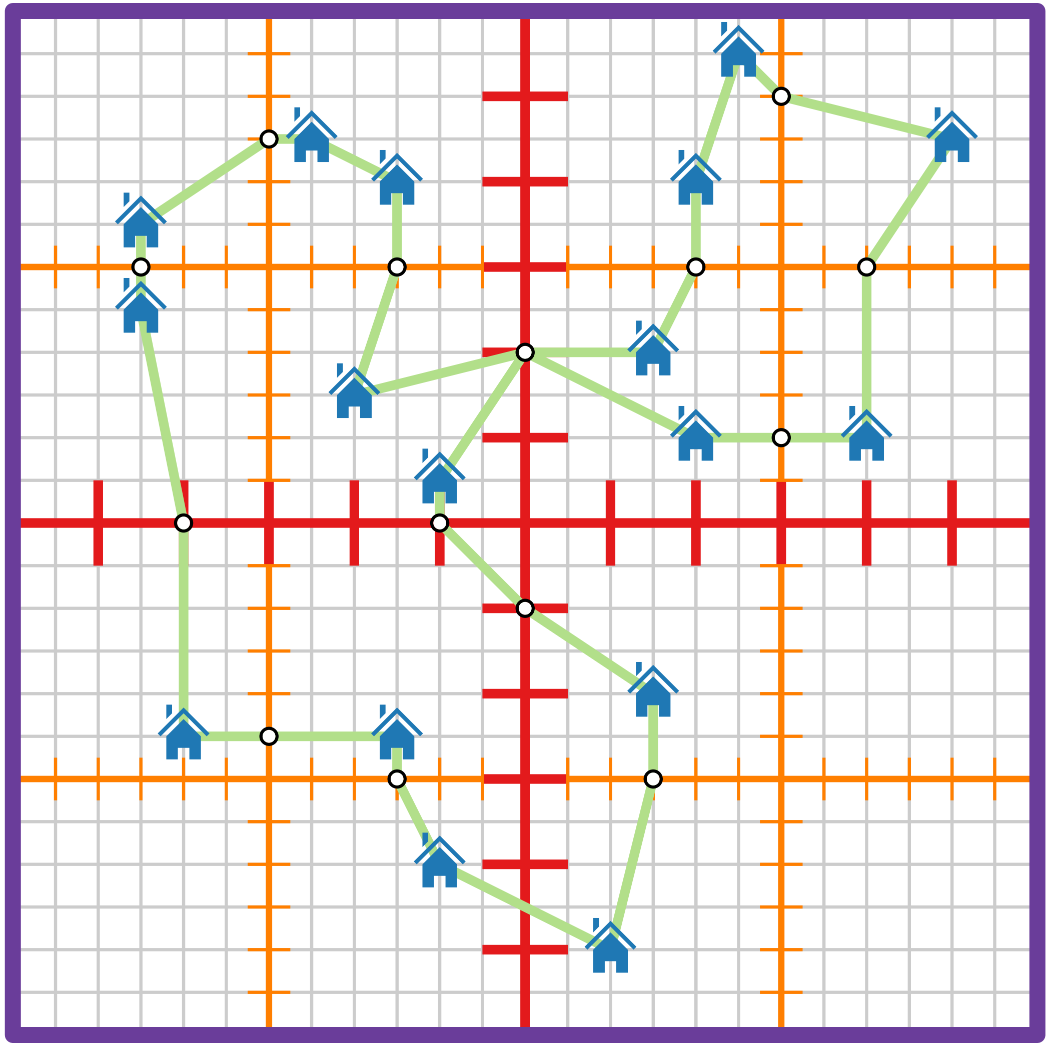
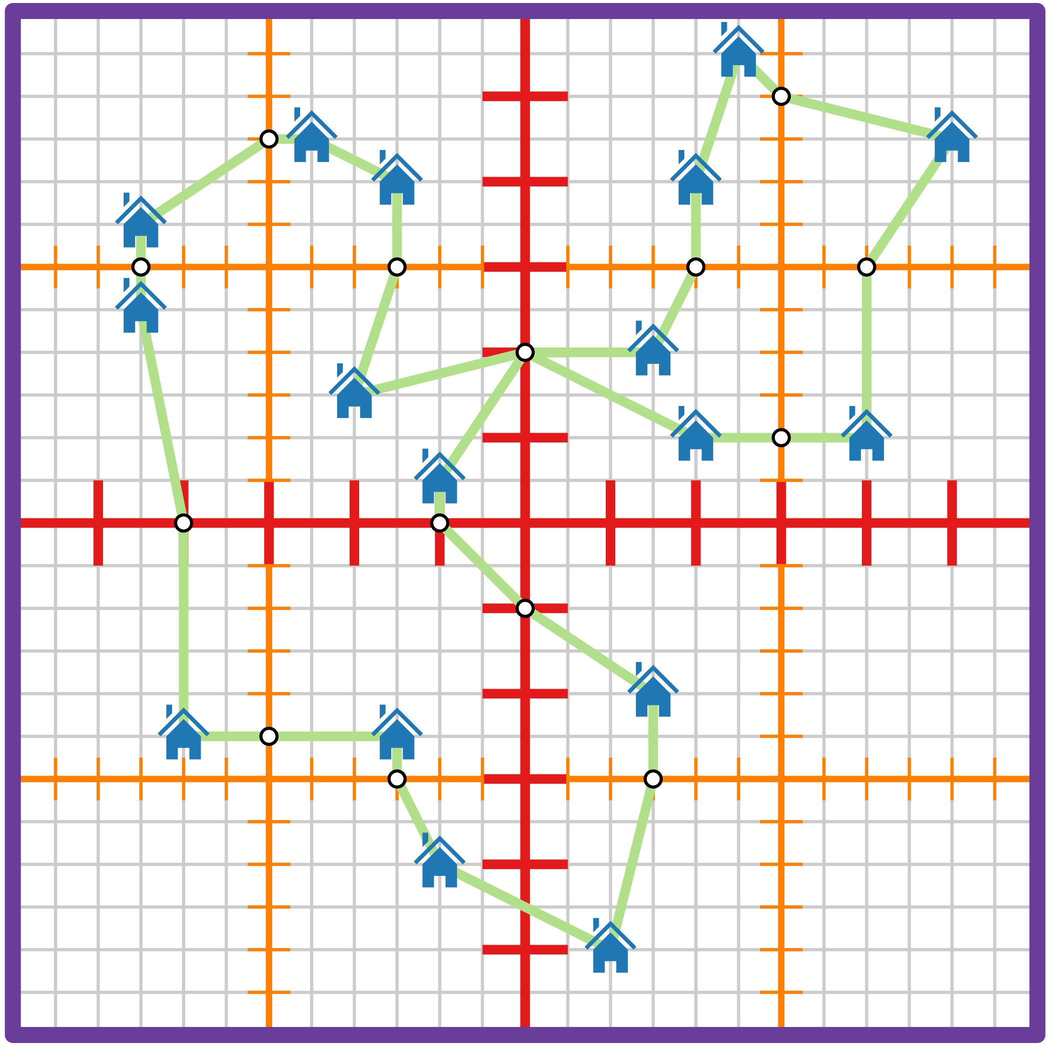


A tour is *well behaved* if

- it involves all houses and a subset of the portals,

Well Behaved Tours



A tour is *well behaved* if

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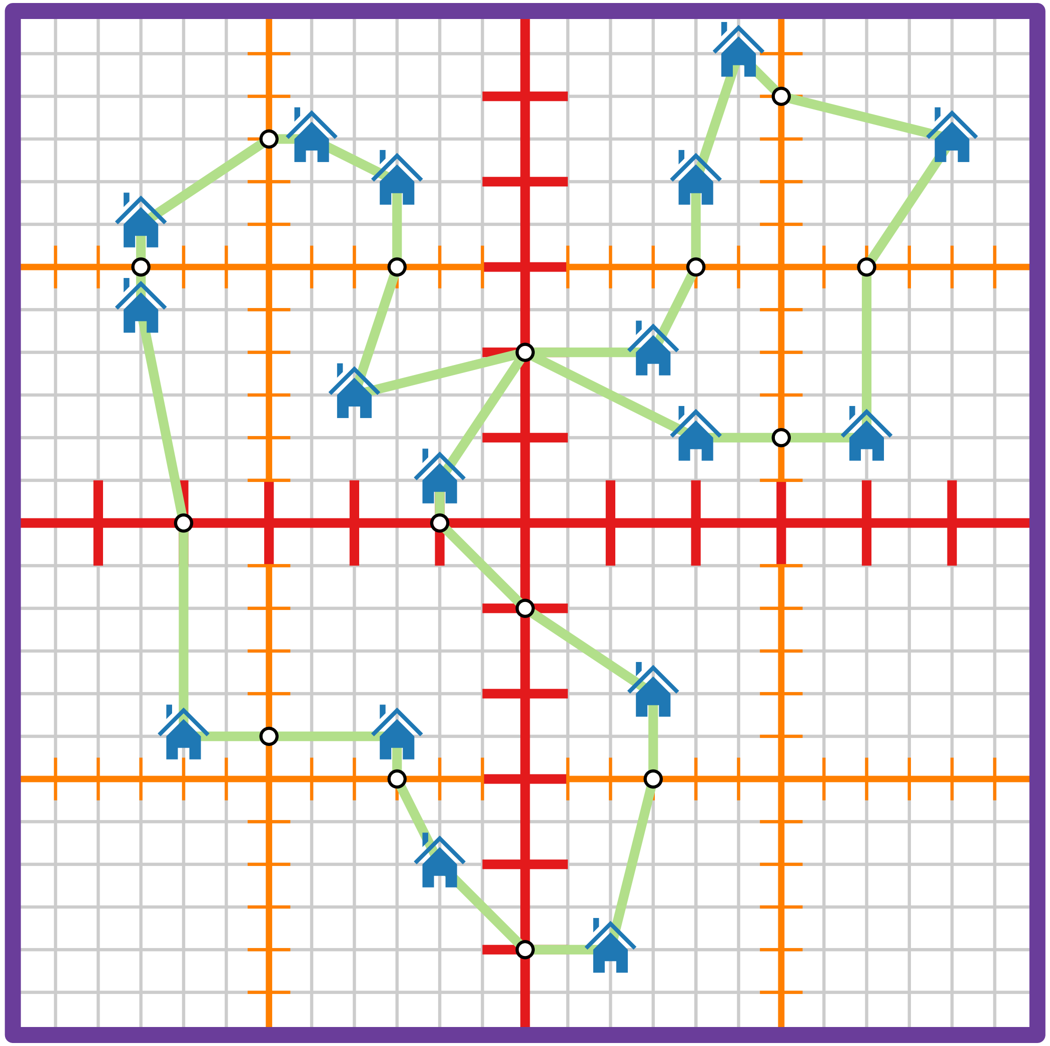
Well Behaved Tours



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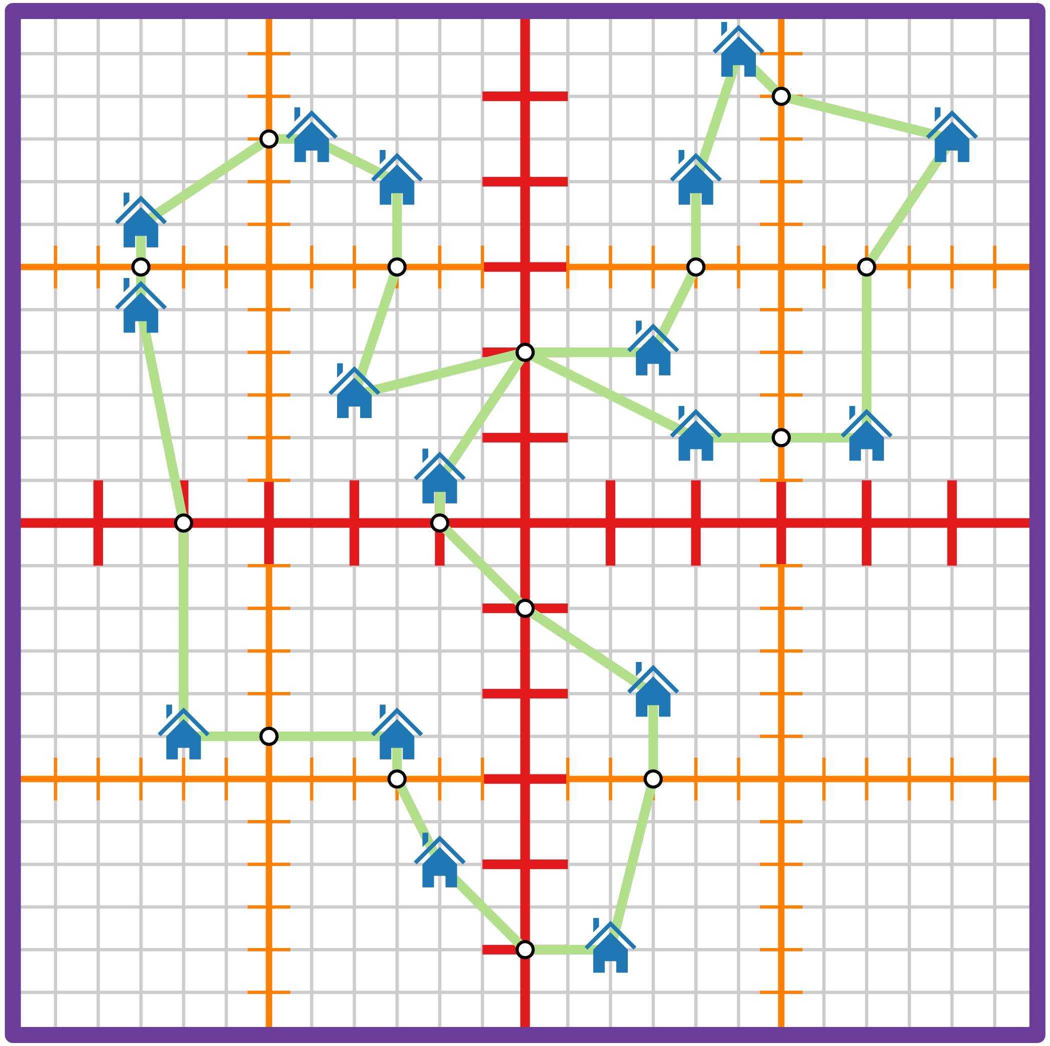

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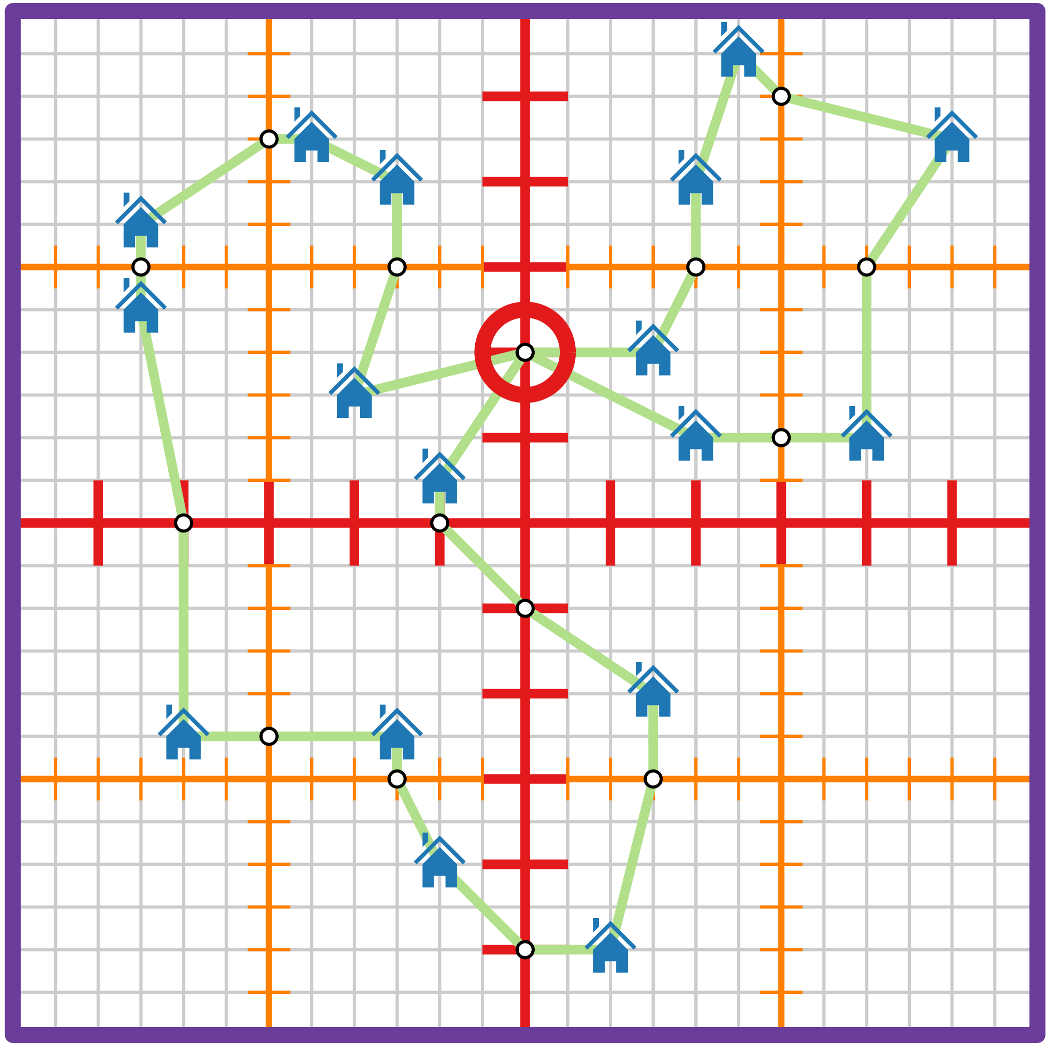

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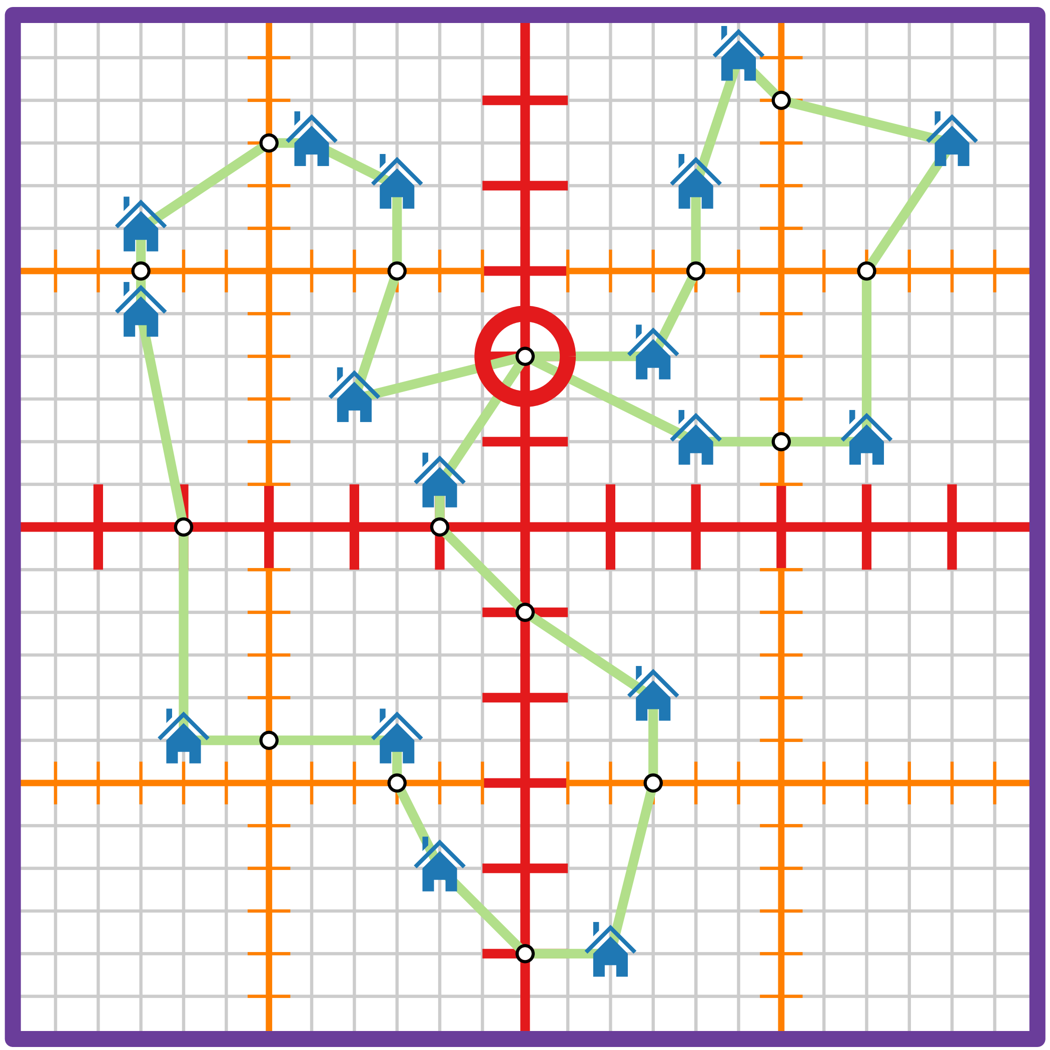

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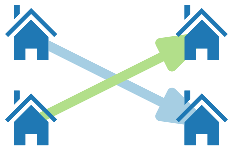
Crossing



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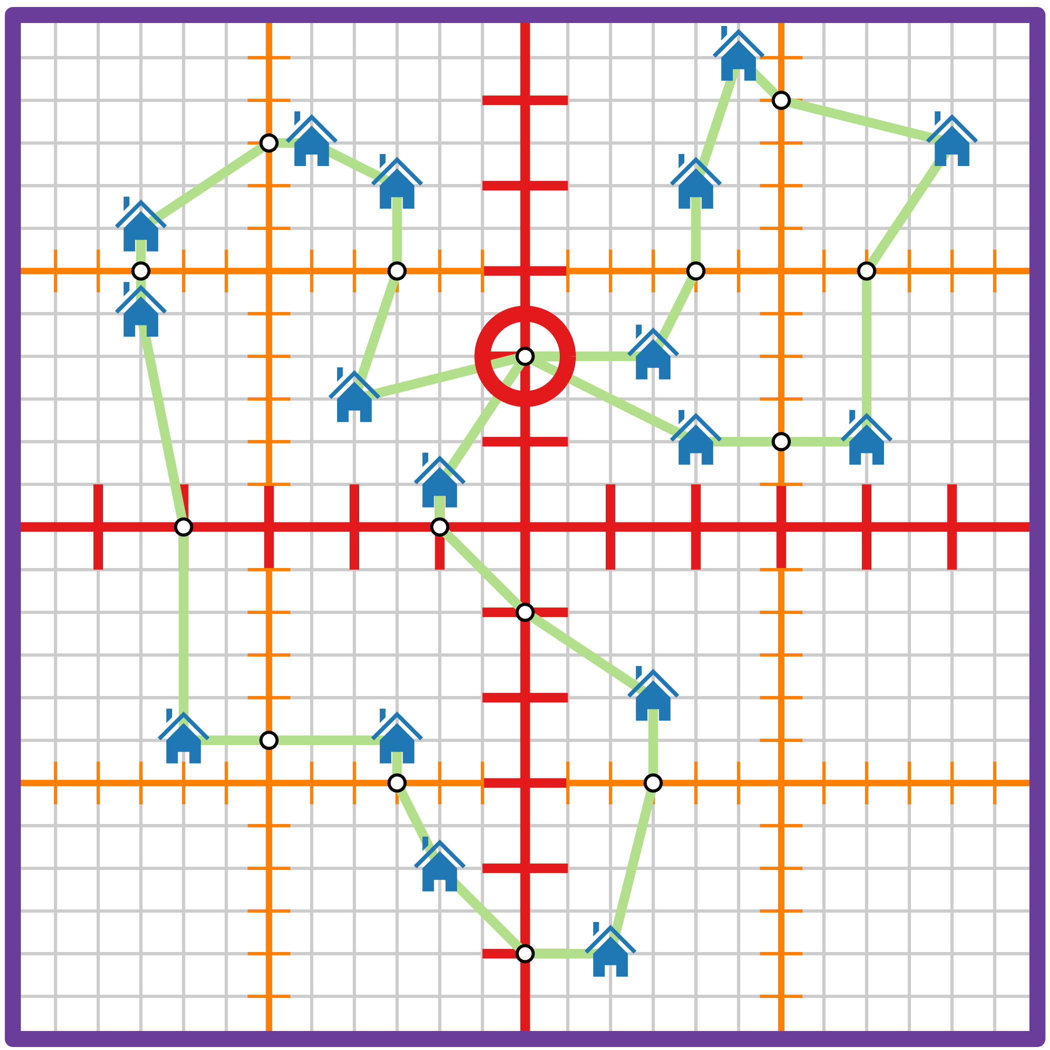

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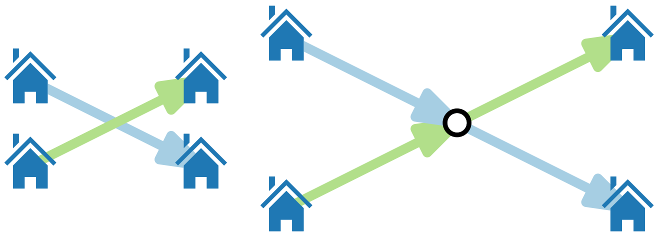

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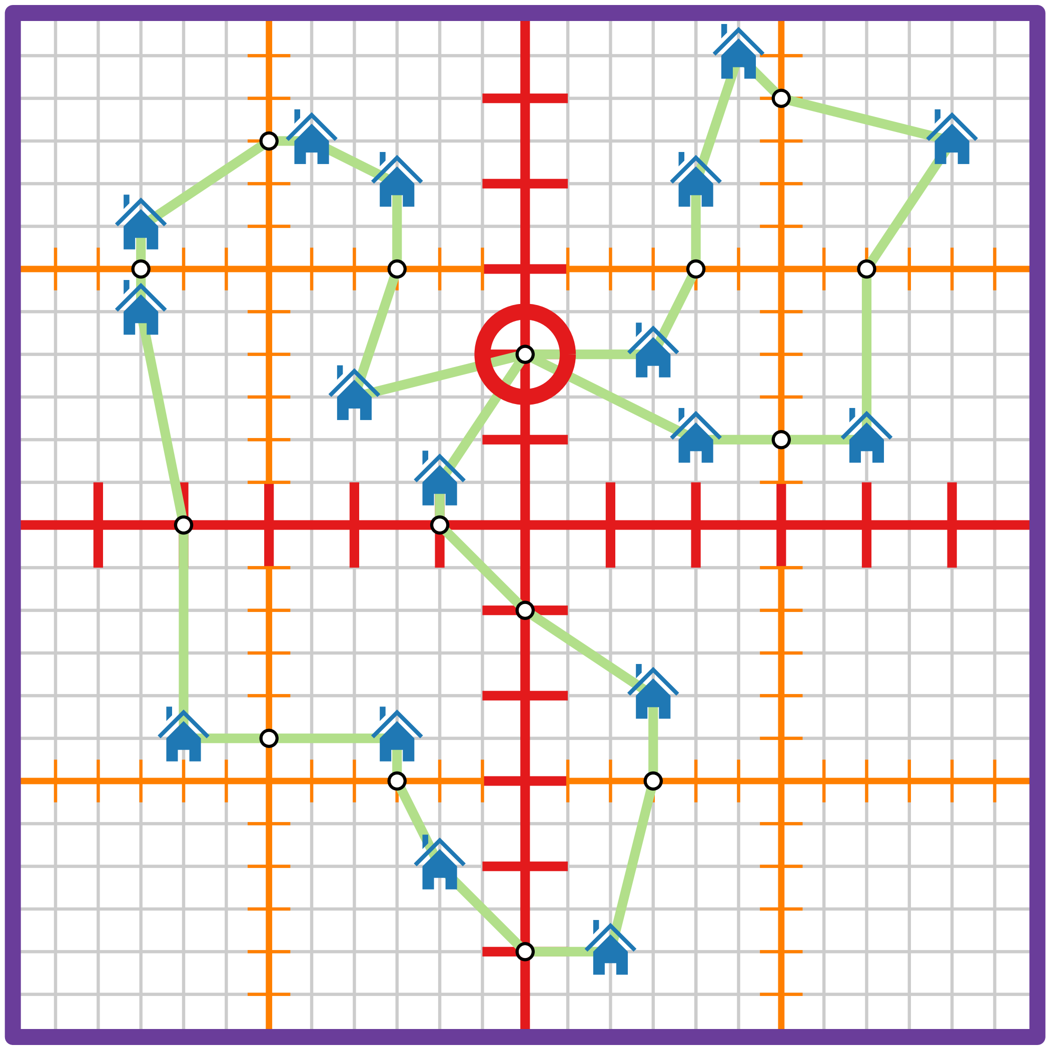

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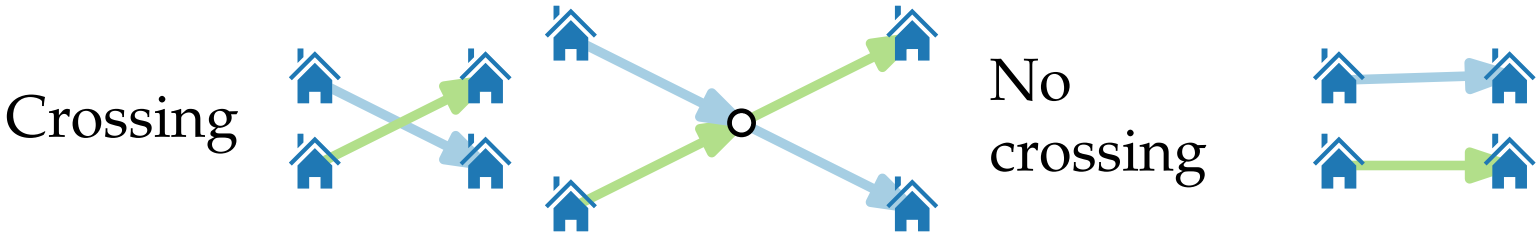
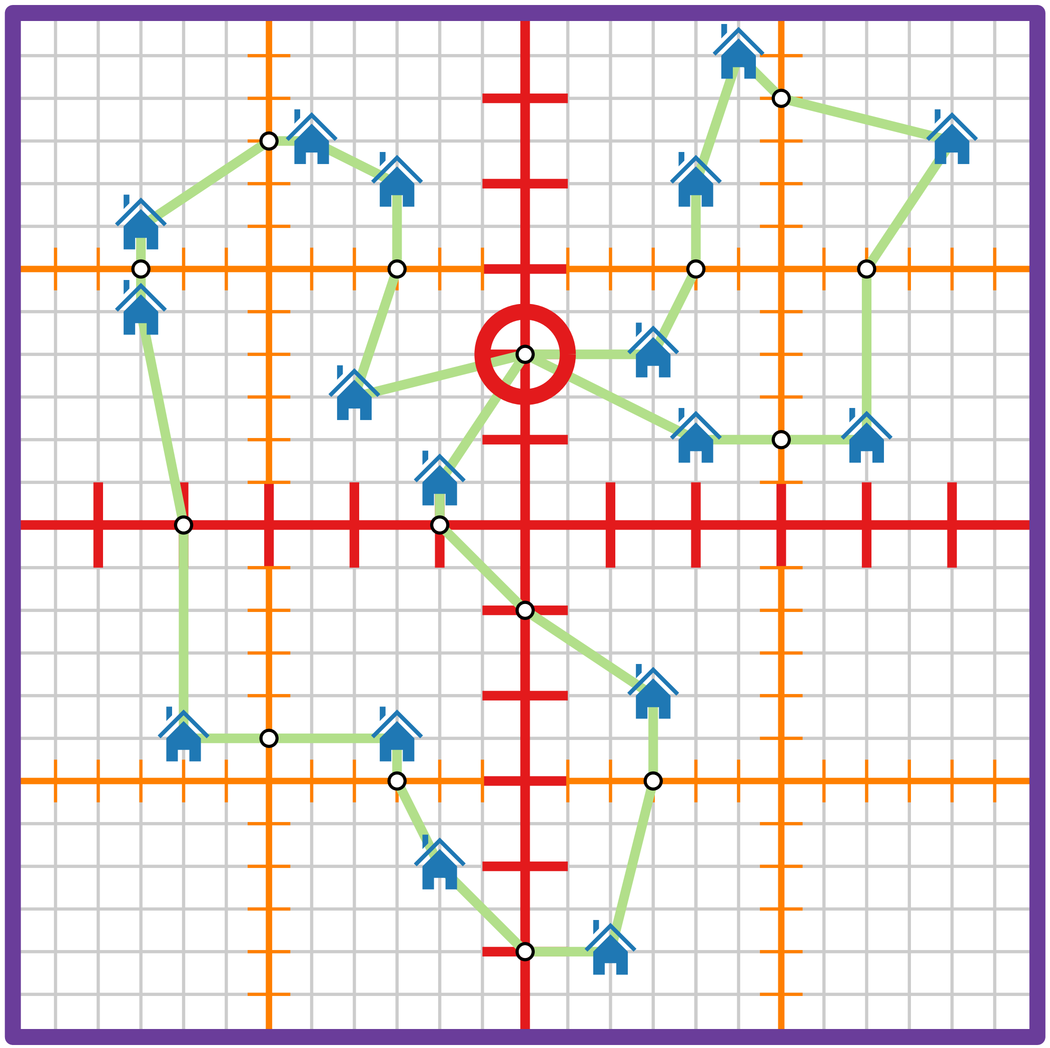


Well Behaved Tours



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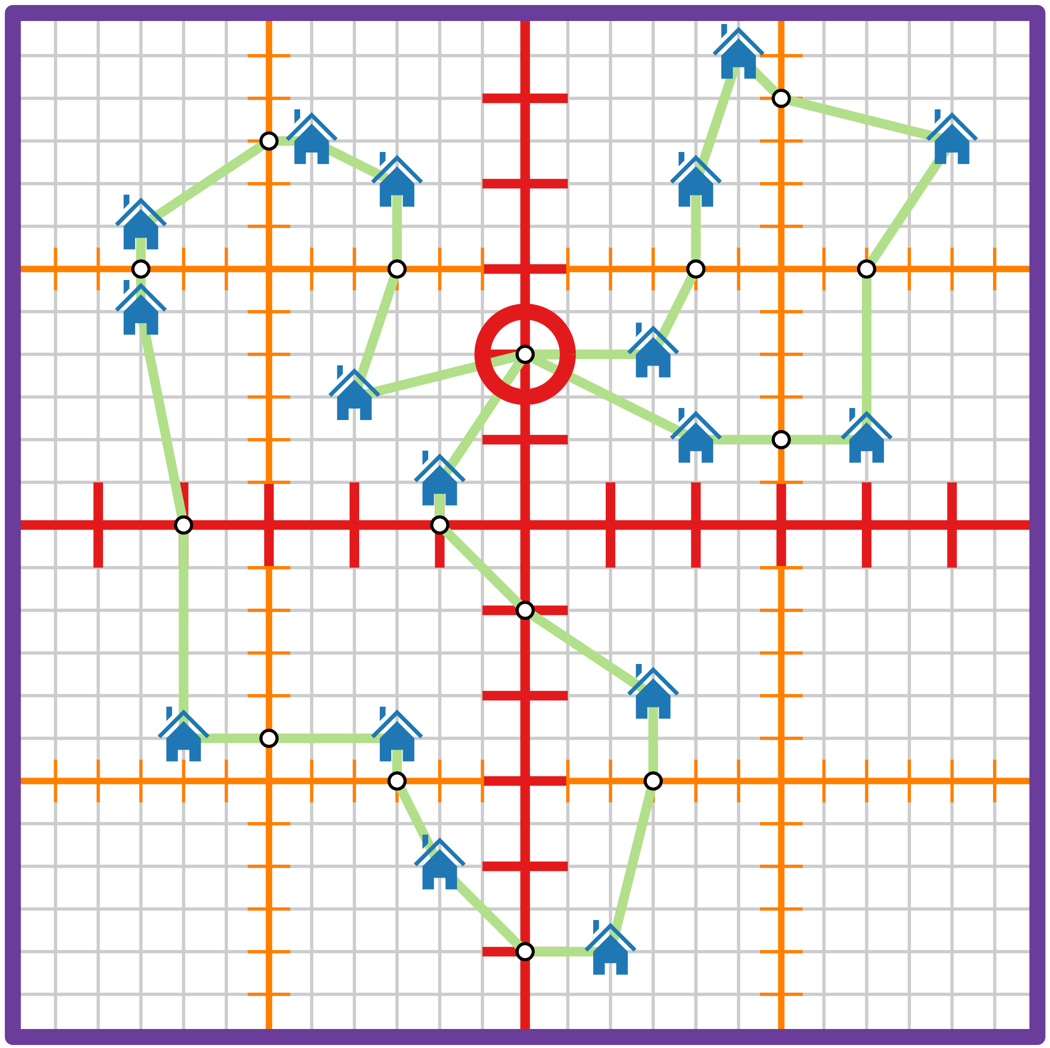

A tour is *well behaved* if

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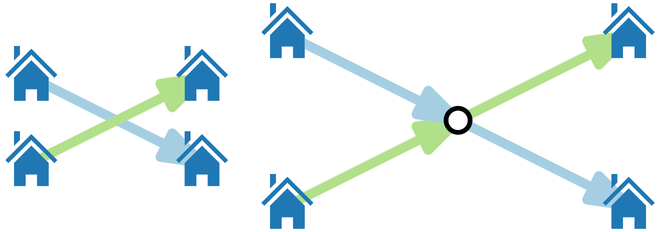
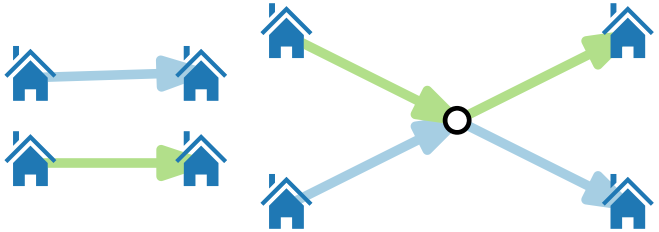
- it is crossing-free.

W.l.o.g. (homework):
No portal visited more than twice

Crossing



No crossing



Computing a Well Behaved Tour

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Lemma. An optimal well behaved tour can be computed in $2^{O(m)} = n^{O(1/\varepsilon)}$ time.

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Sketch.

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Computing a Well Behaved Tour

Lemma. An optimal well behaved tour can be computed in $2^{O(m)} = n^{O(1/\varepsilon)}$ time.

Sketch.

- Dynamic Programming!
- Compute sub-structure of an optimal tour for each square in the dissection tree.

Computing a Well Behaved Tour

Lemma. An optimal well behaved tour can be computed in $2^{O(m)} = n^{O(1/\varepsilon)}$ time.

Sketch.

- Dynamic Programming!
- Compute sub-structure of an optimal tour for each square in the dissection tree.
- These solutions can be efficiently propagated bottom-up through the dissection tree.

Approximation Algorithms

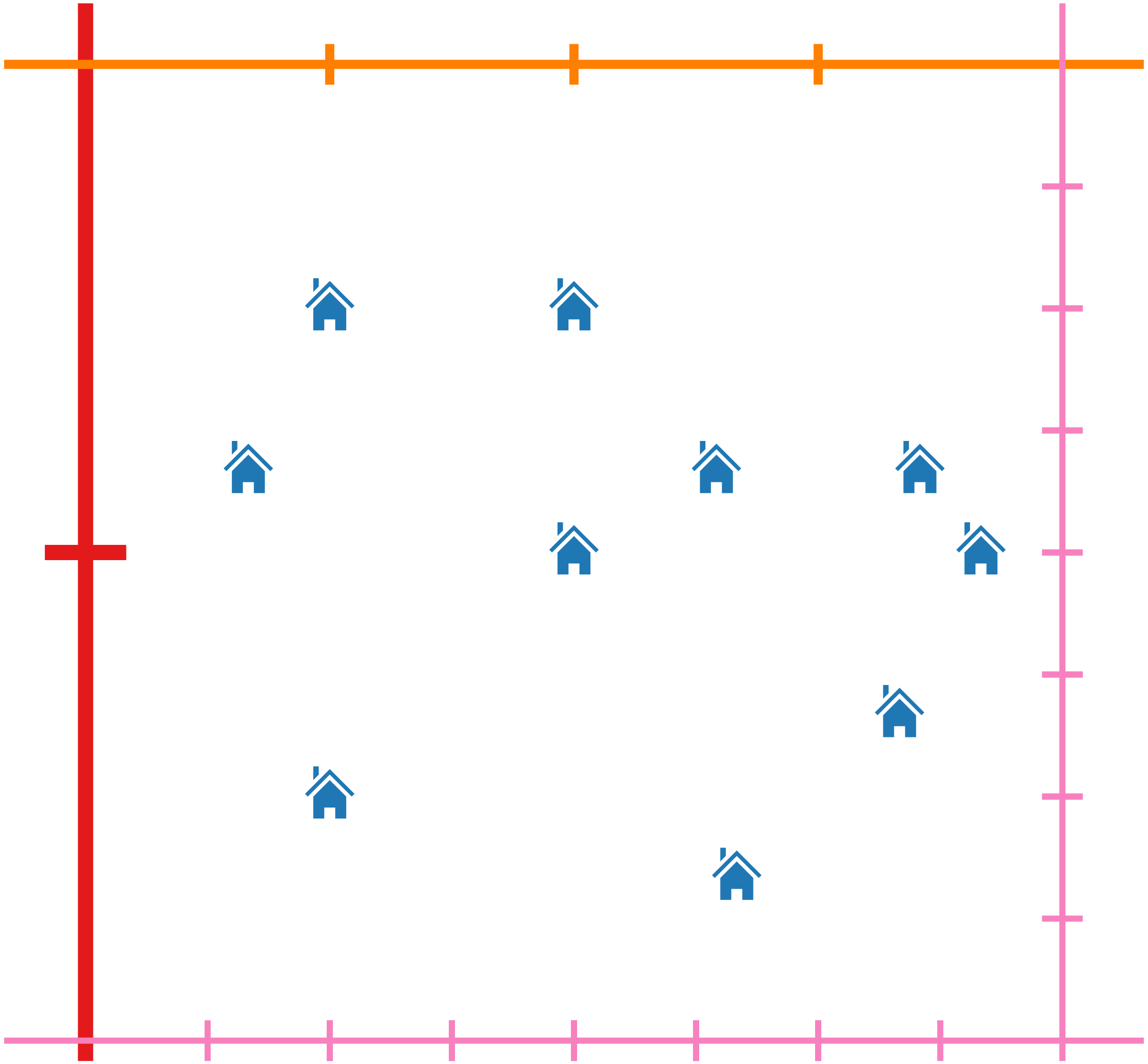
Lecture 9:
PTAS for EuclideanTSP

Part IV:
Dynamic Program

Joachim Spoerhase

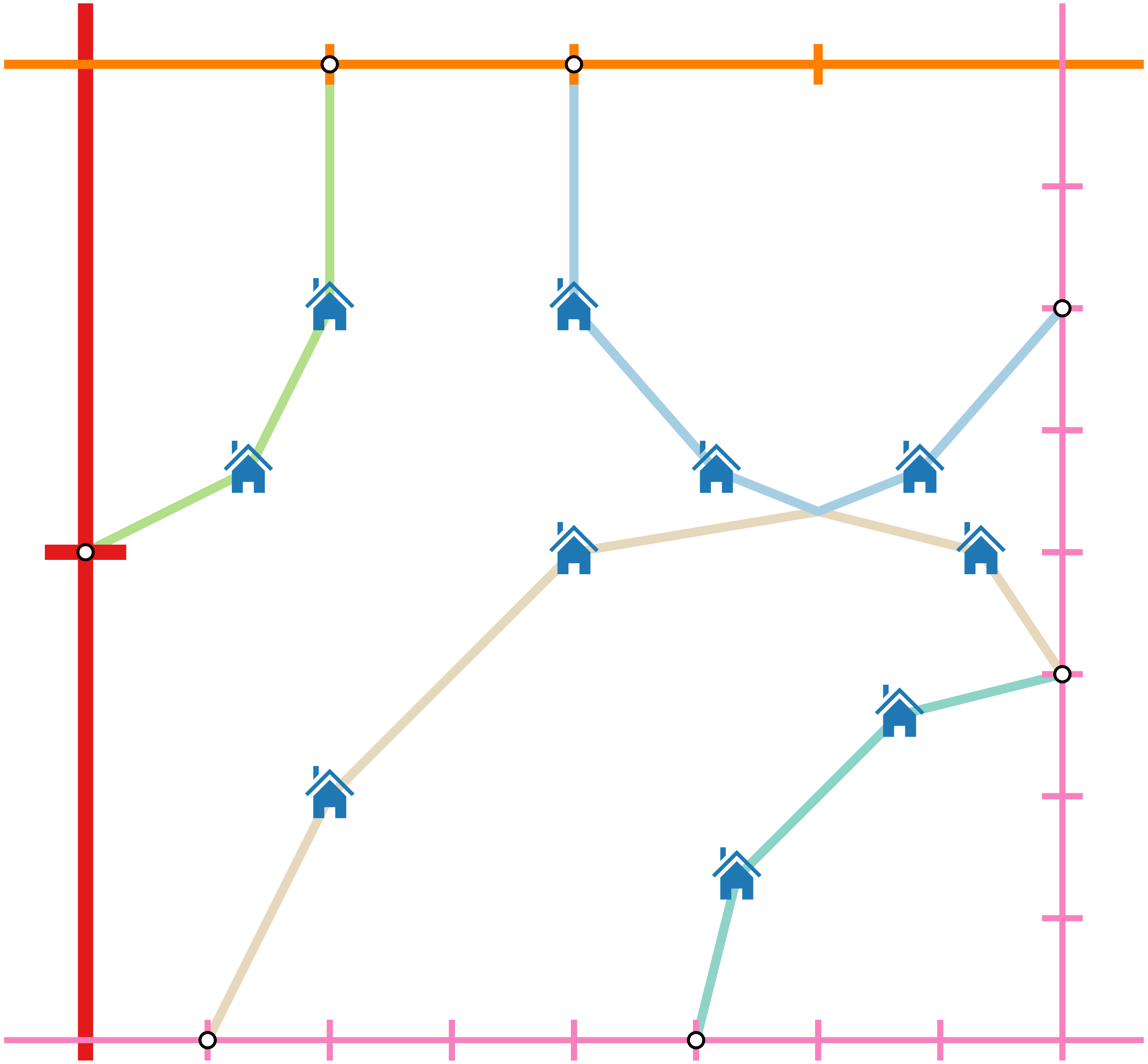
Winter 2021/22

Dynamic Program (I)



Each well behaved tour induces the following in each square Q of the dissection:

Dynamic Program (I)



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- A path cover of the houses in Q

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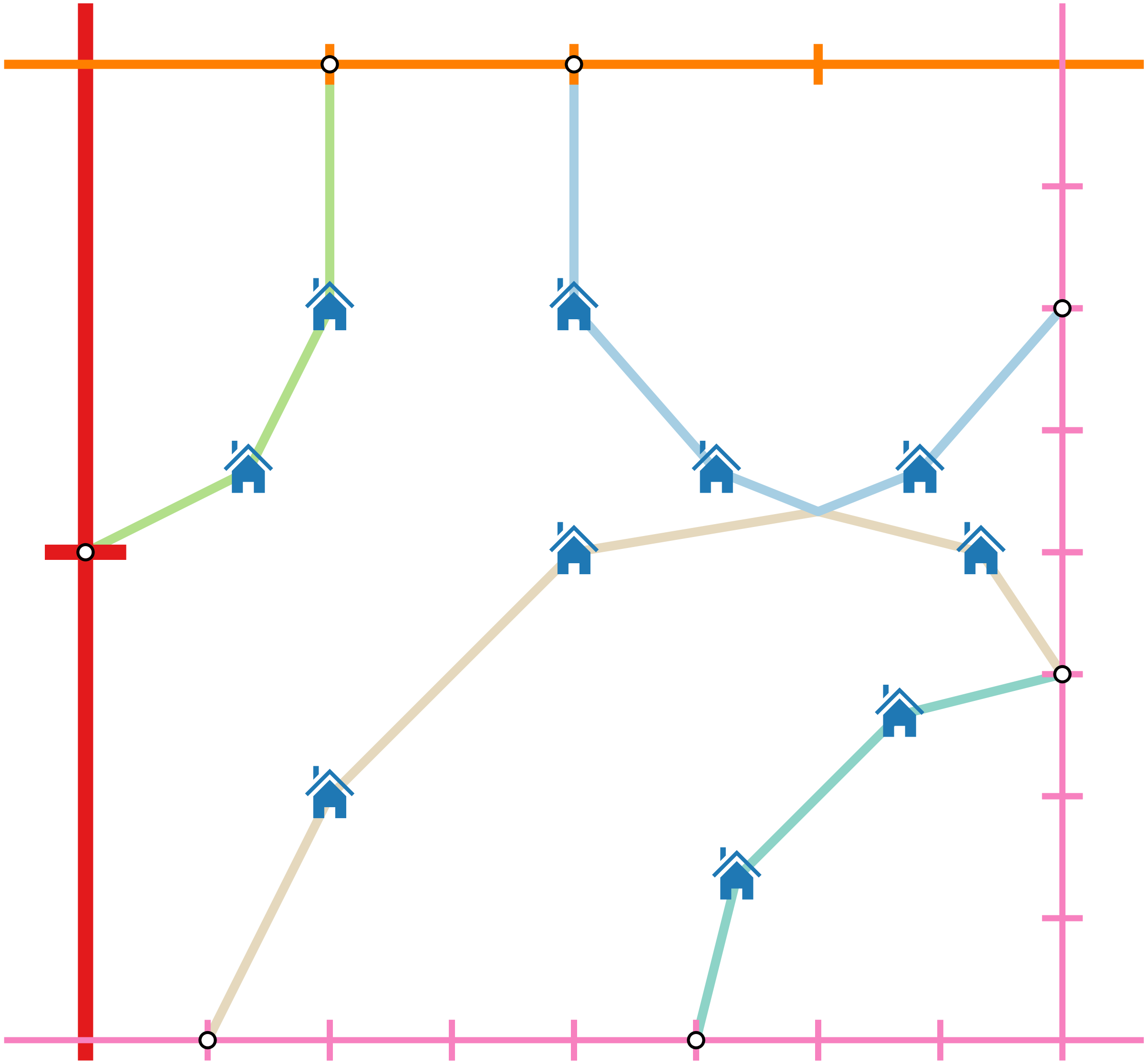

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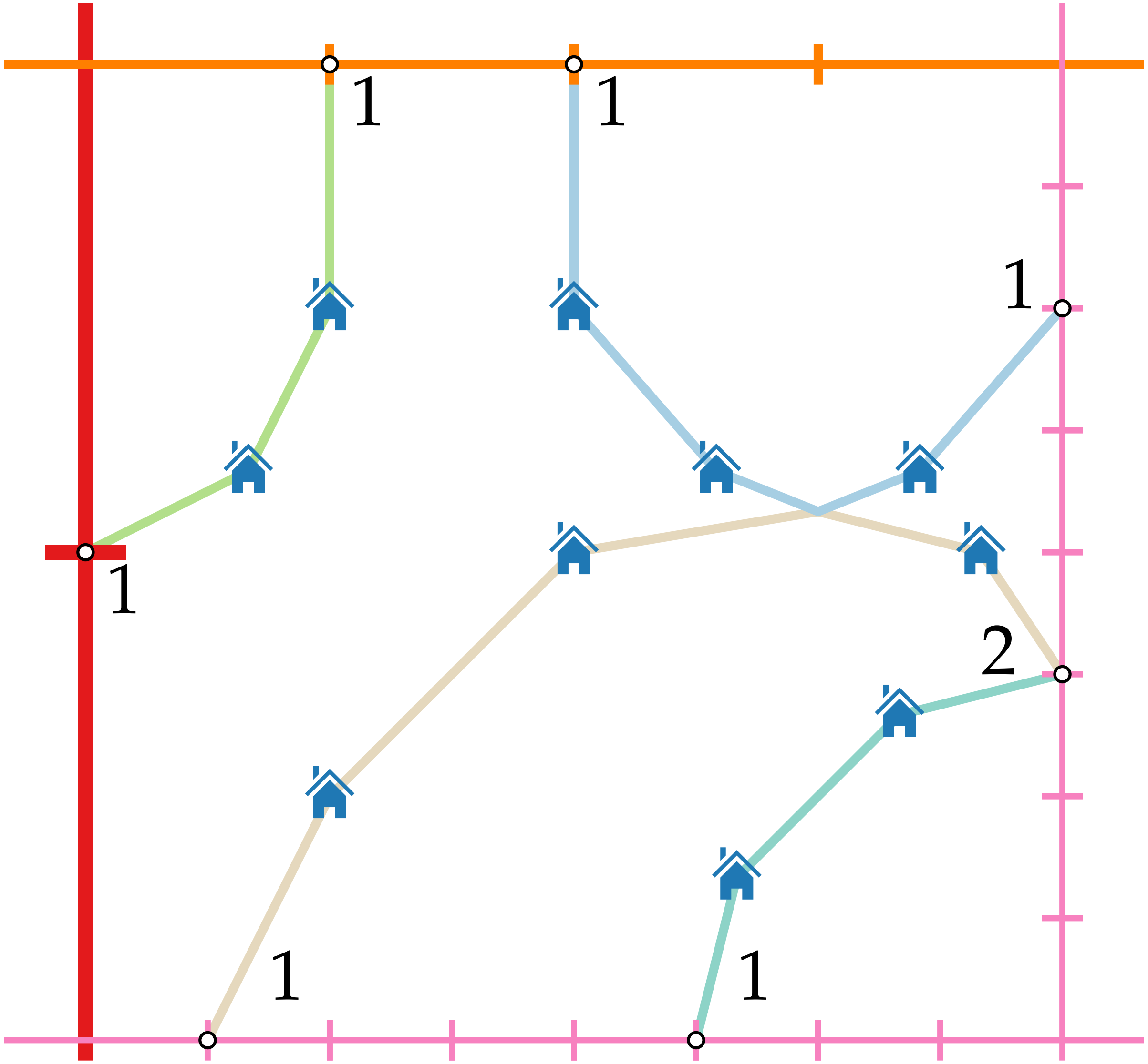

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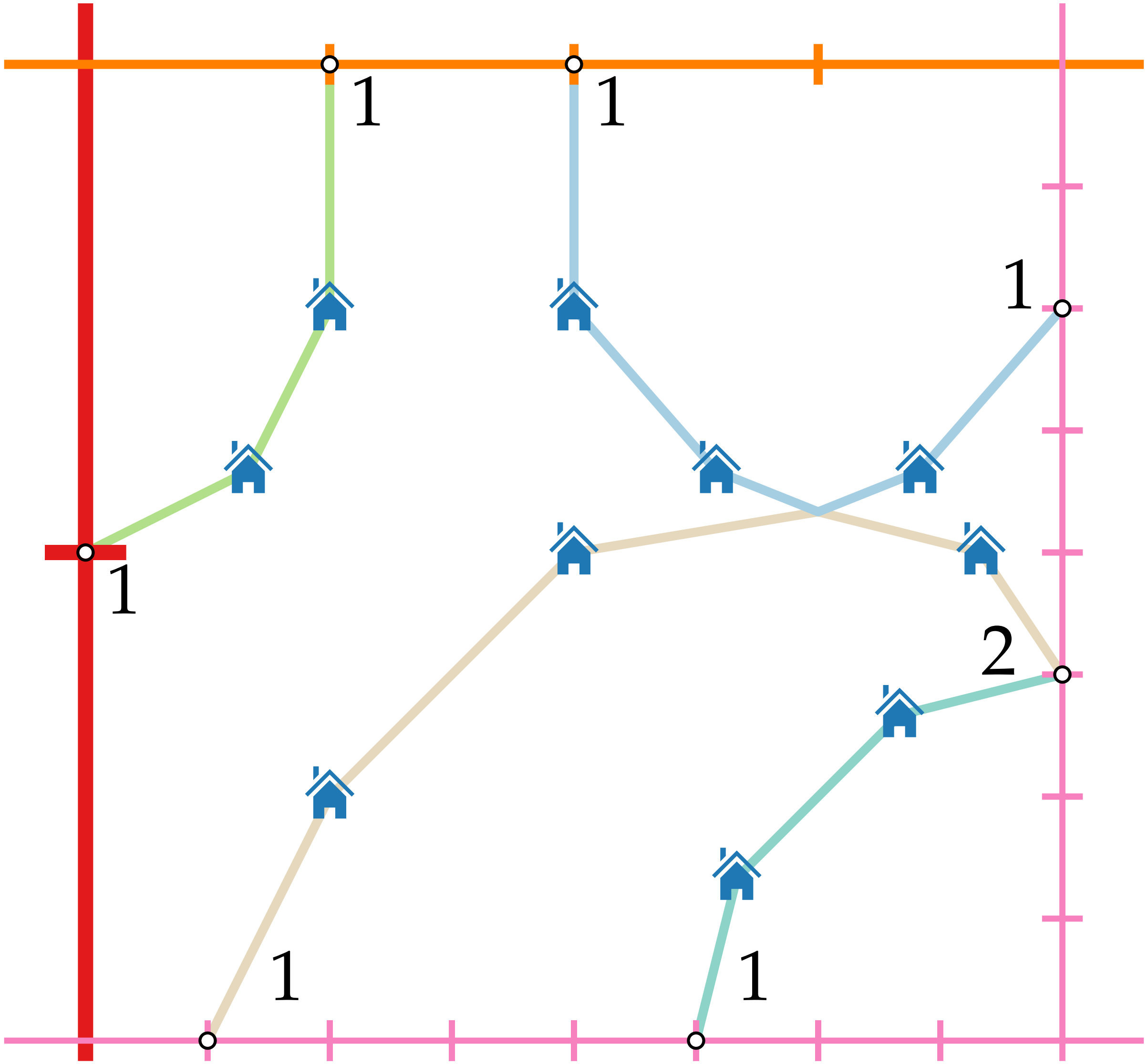

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$\Rightarrow$ max. possibilities

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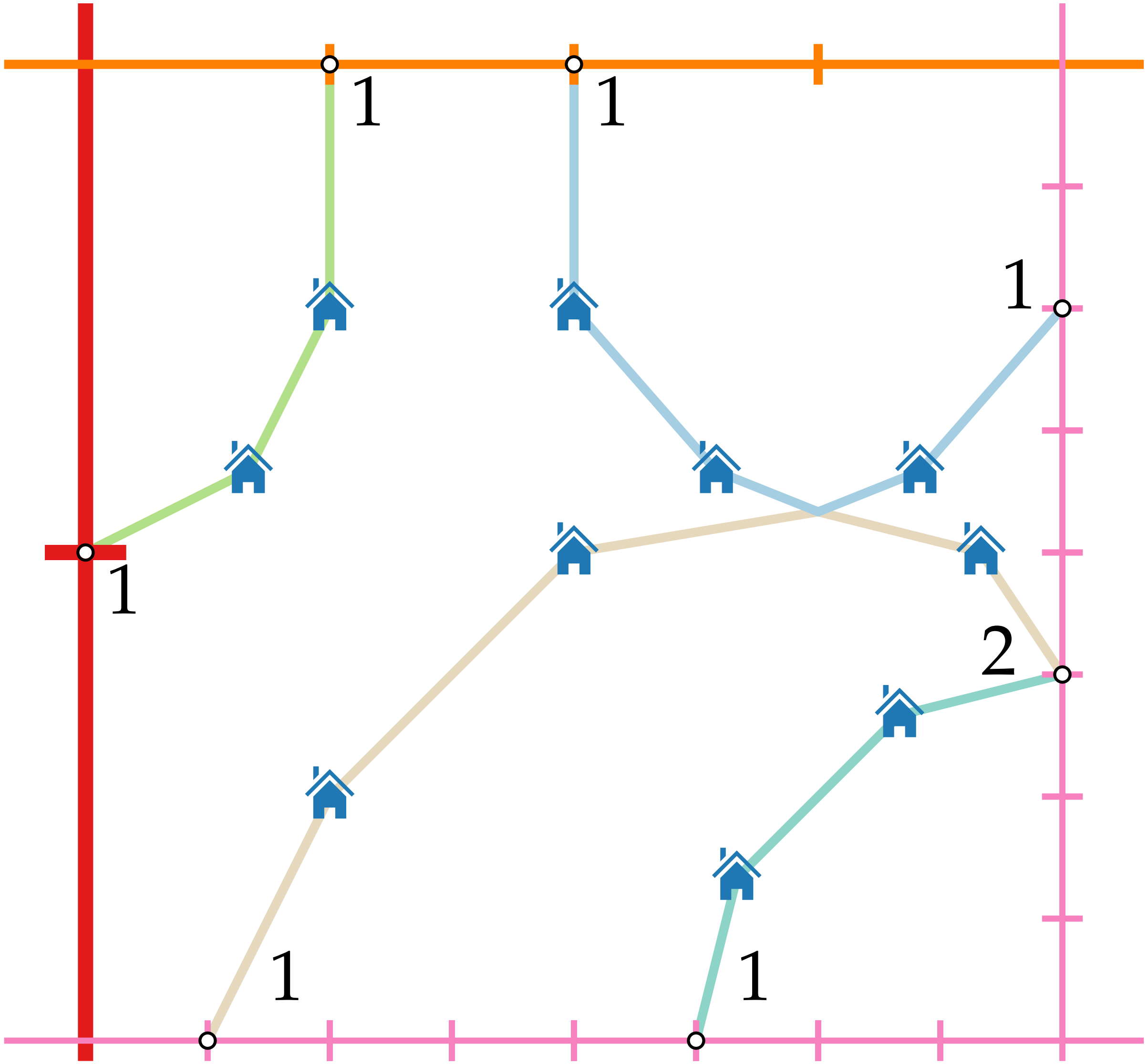

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$\Rightarrow$ max. $3^{4m} =$ possibilities

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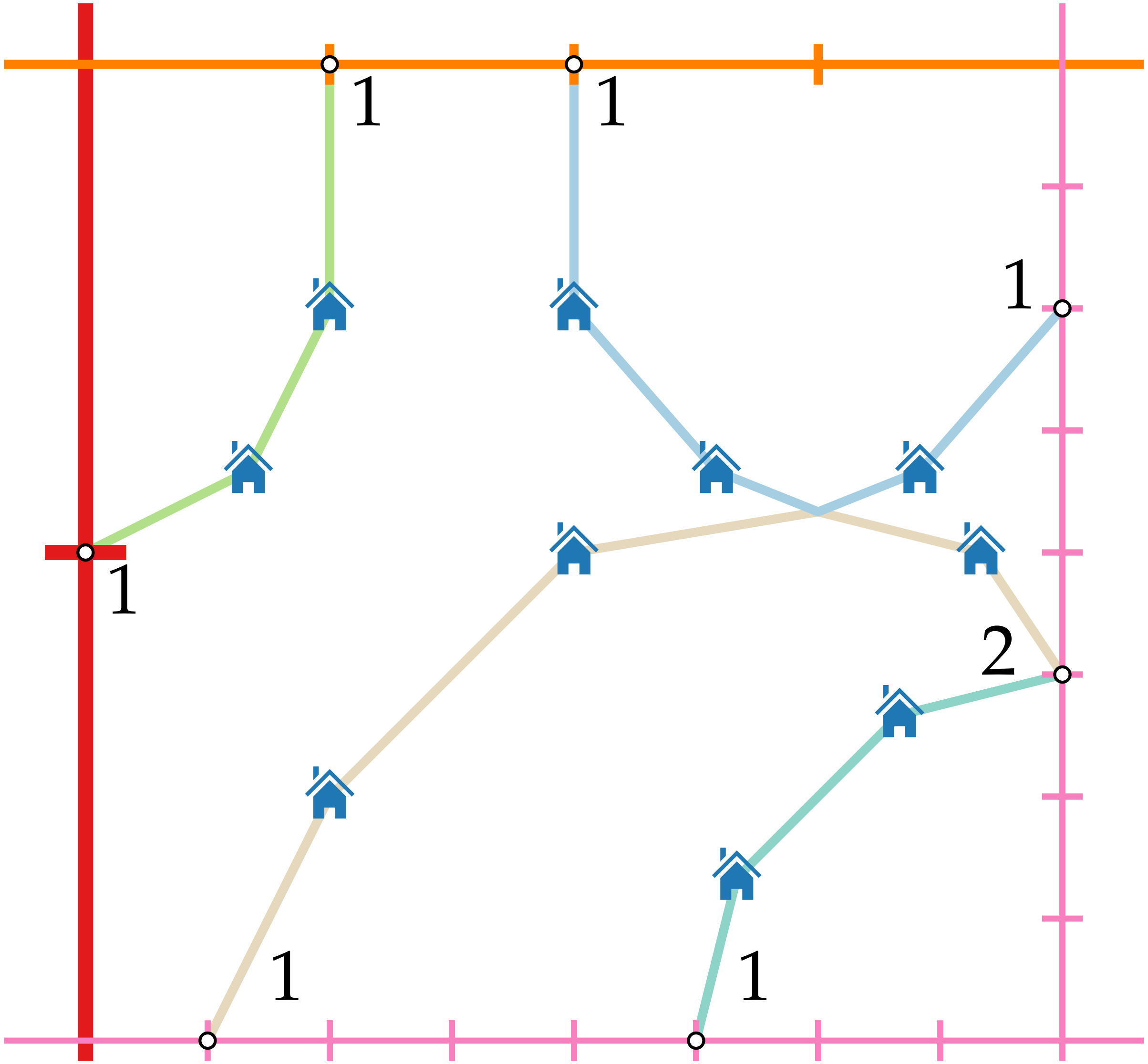

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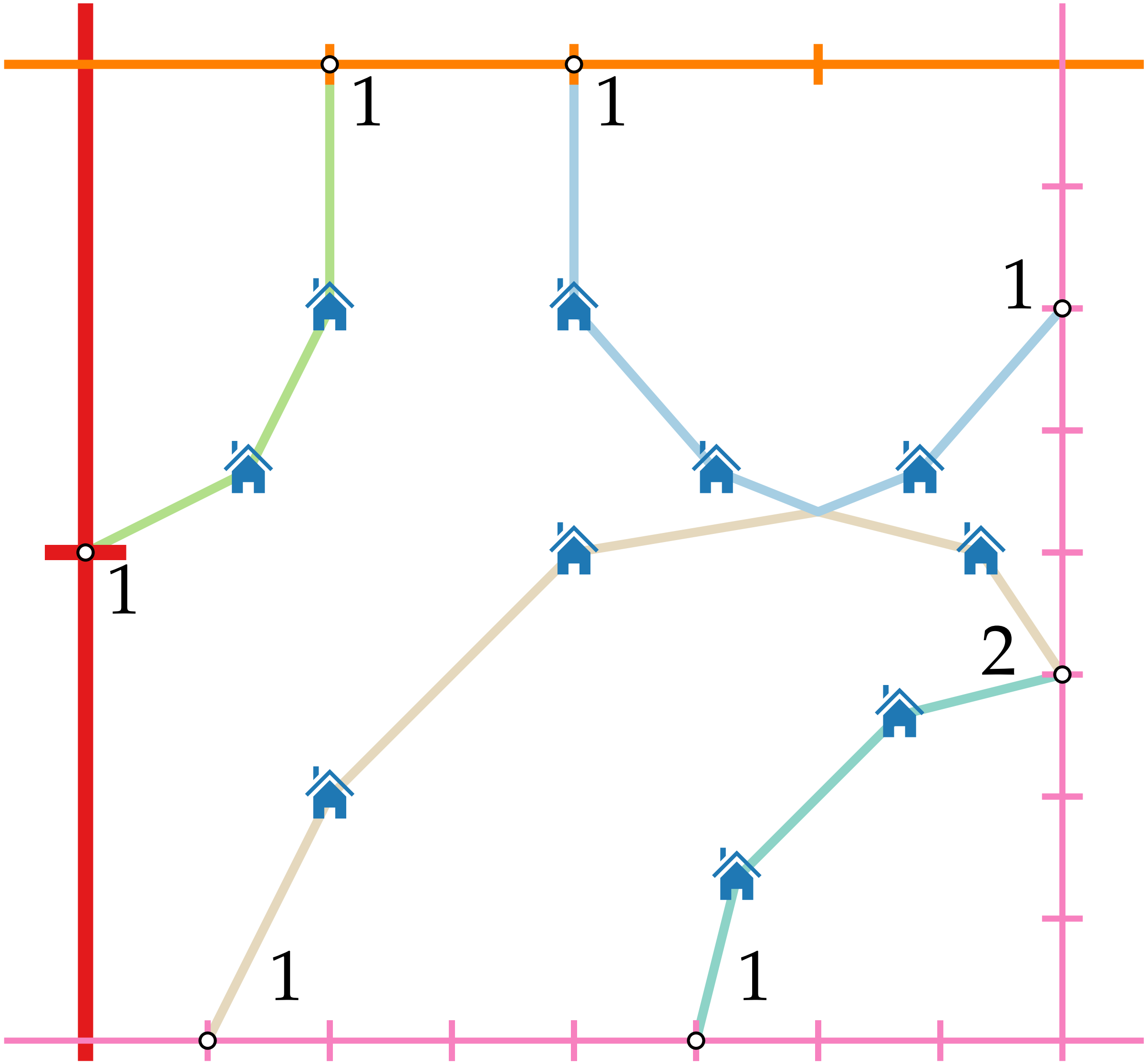

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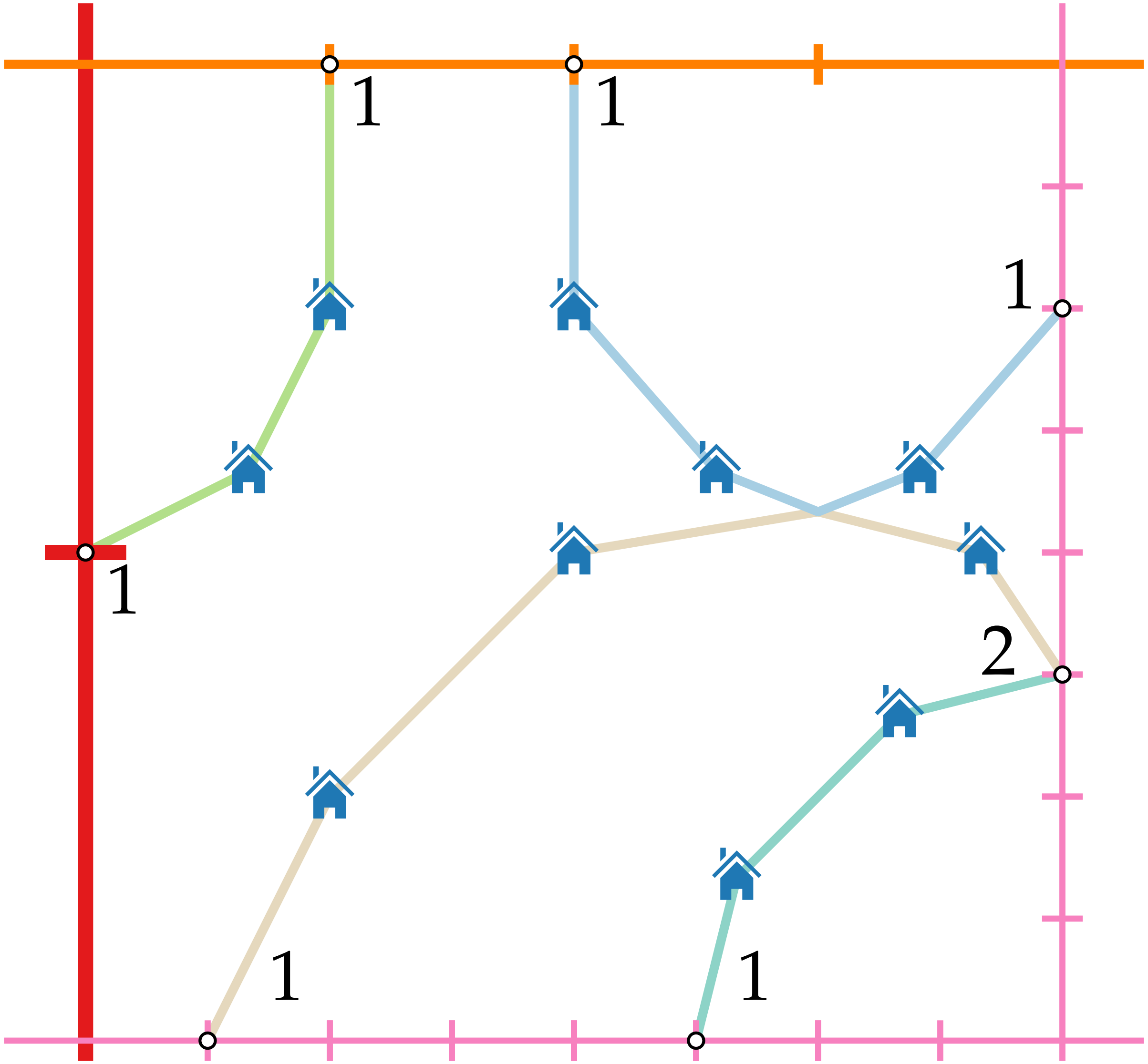

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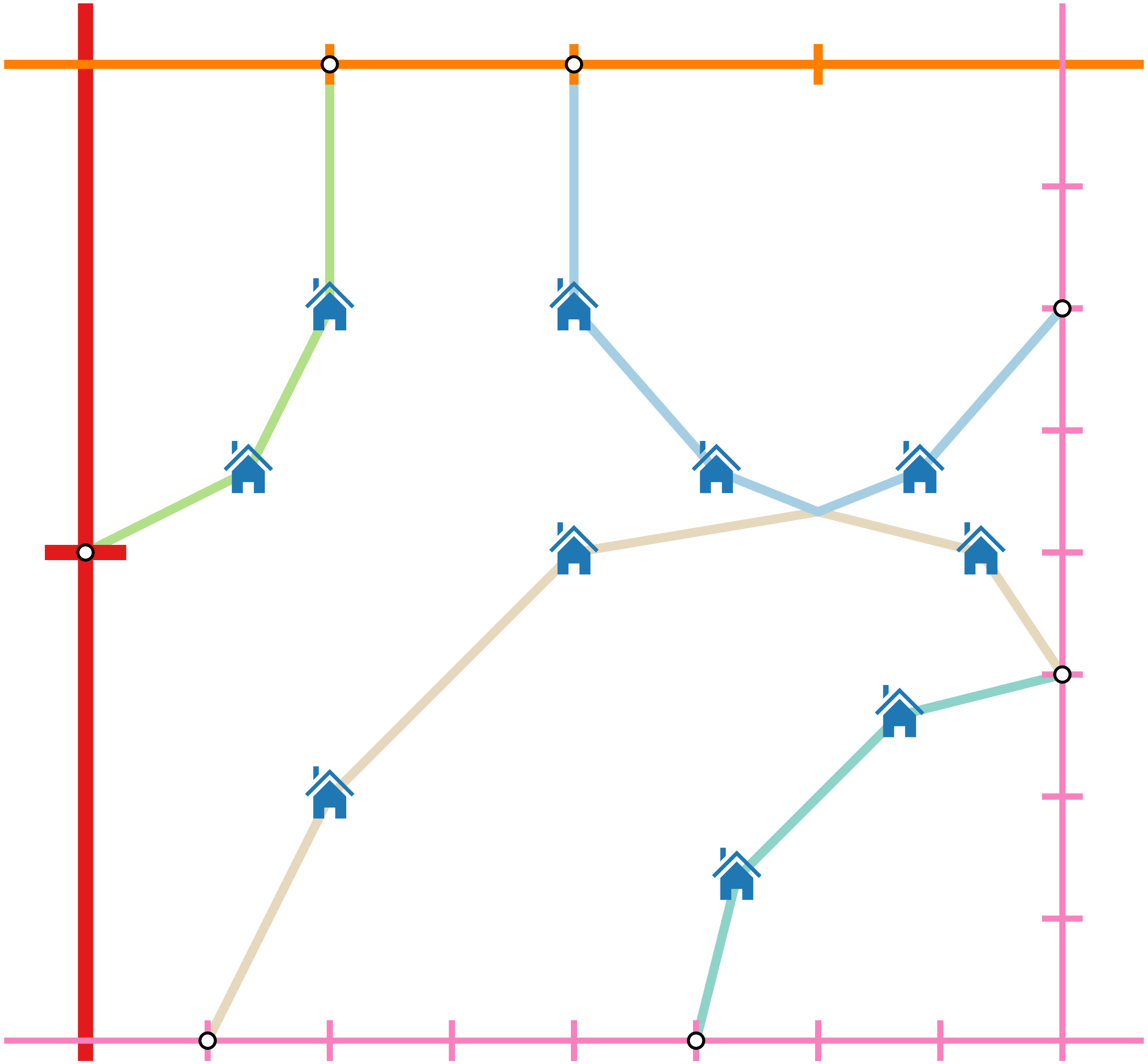

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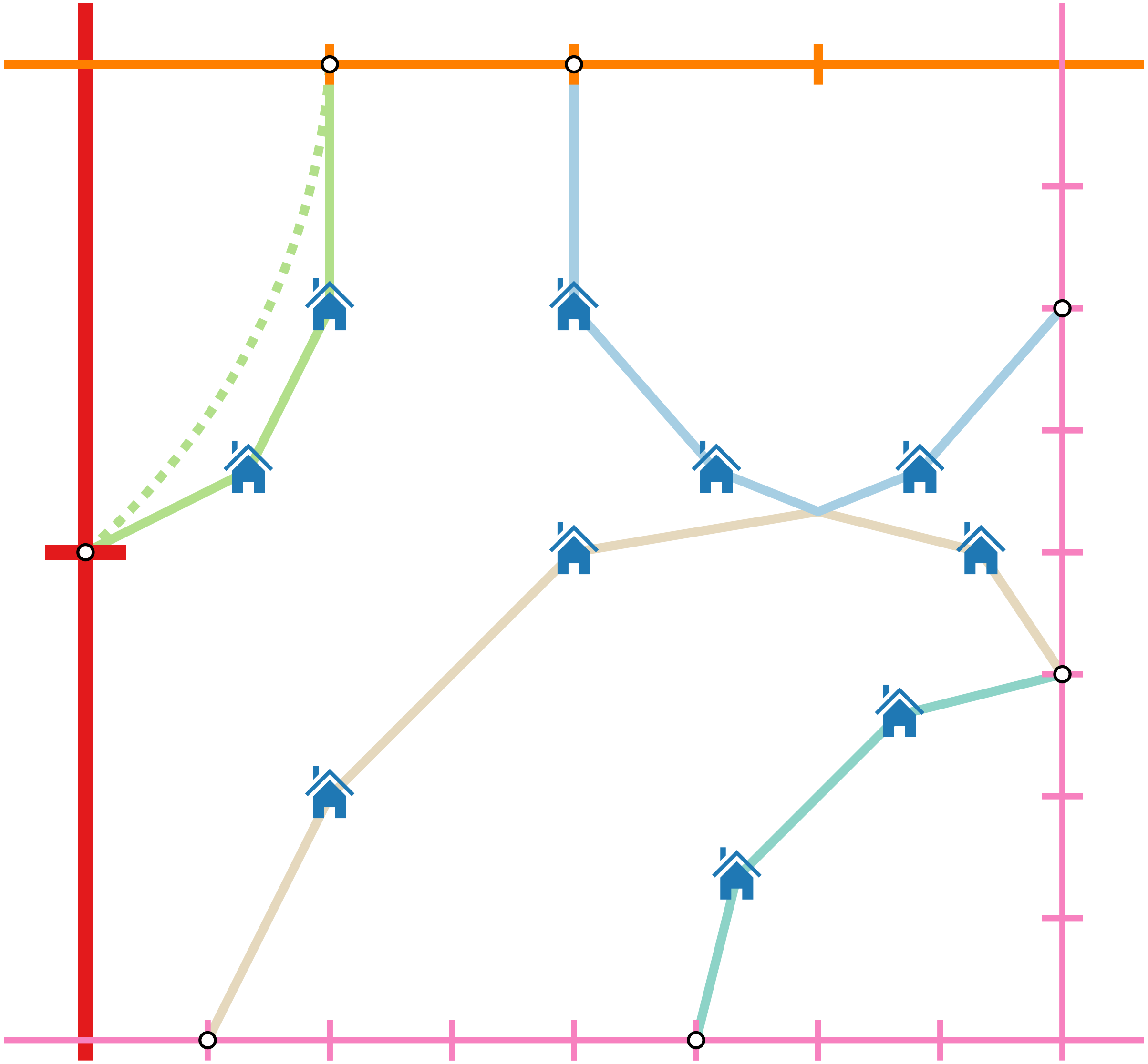

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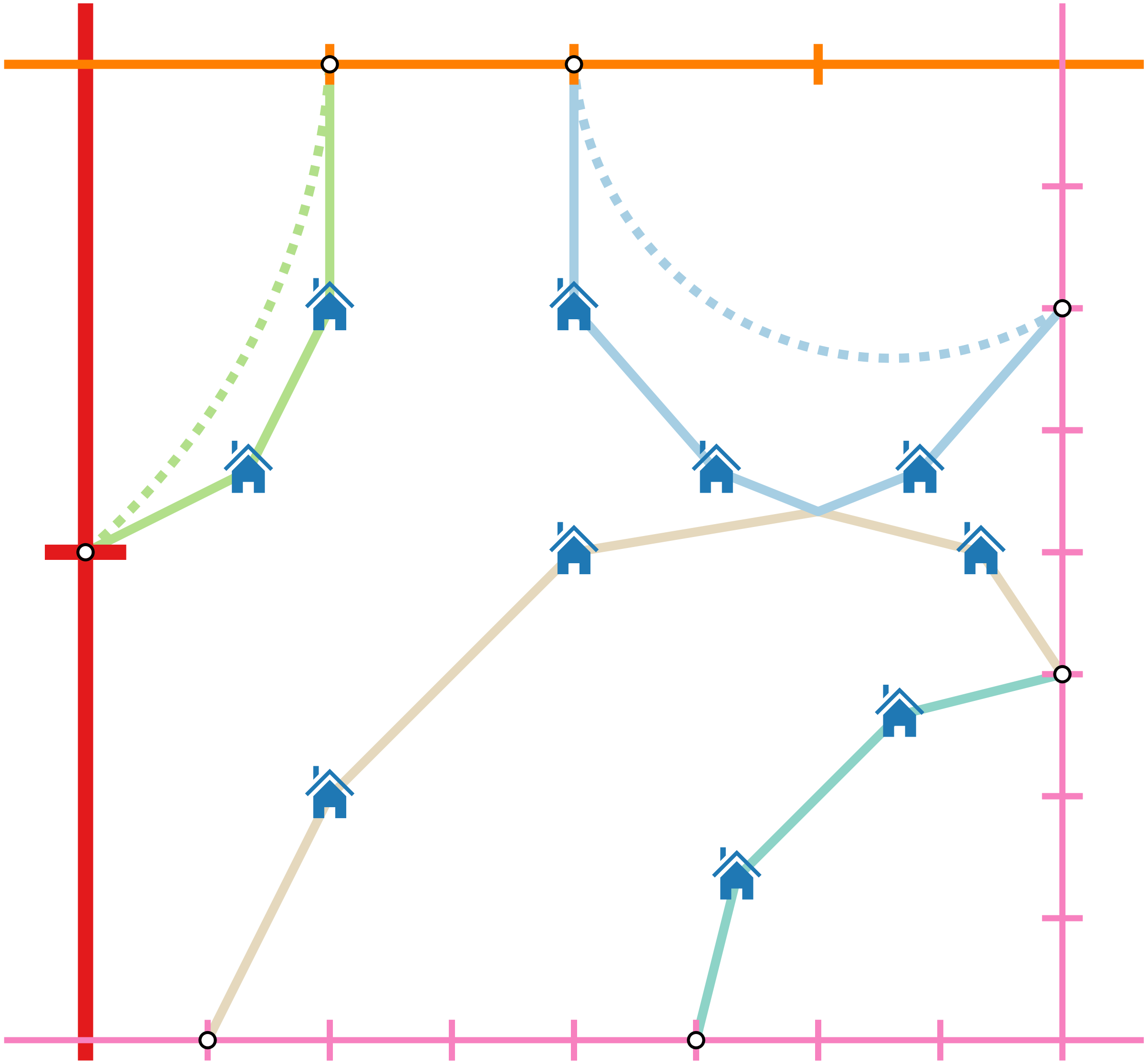

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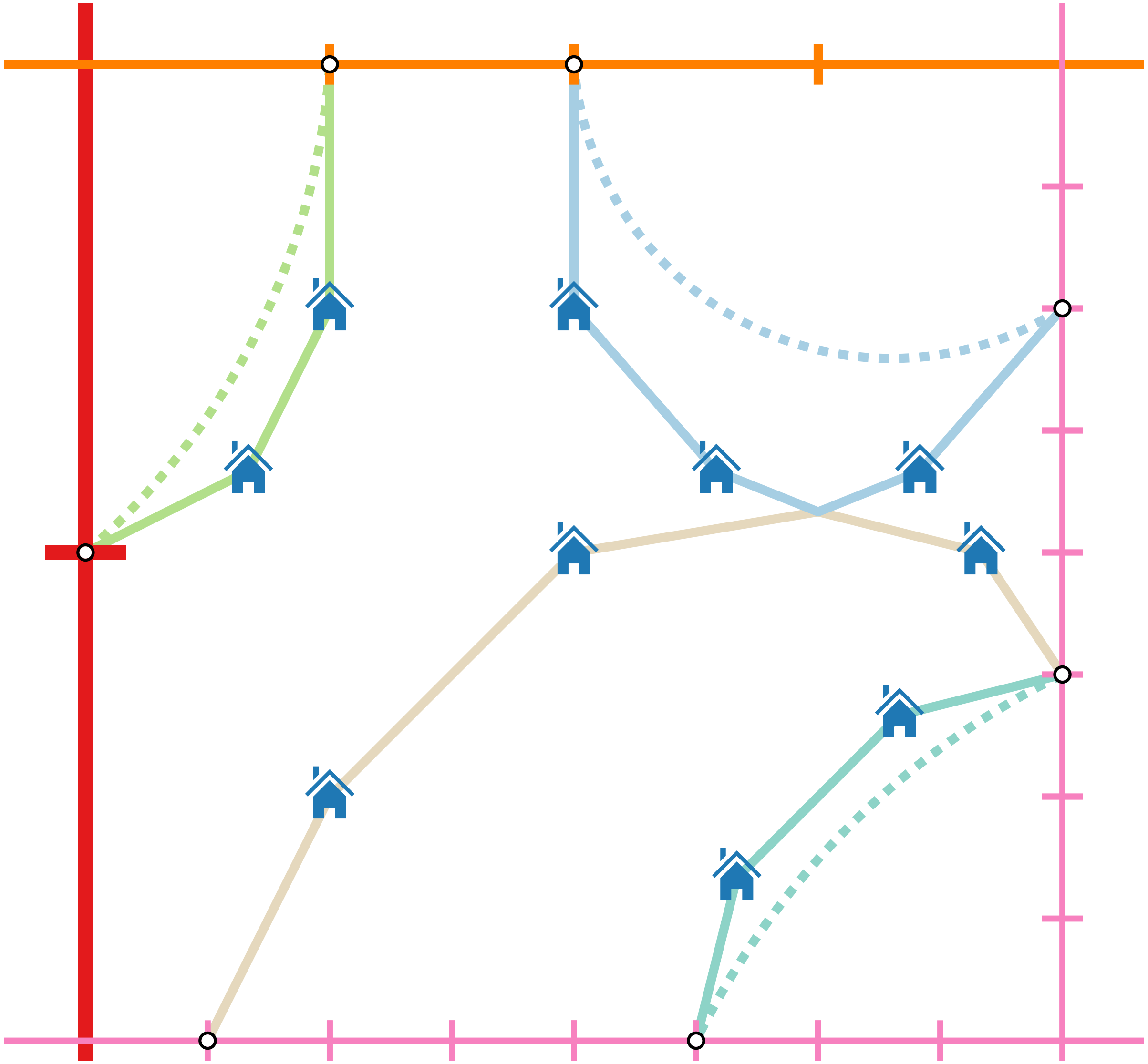

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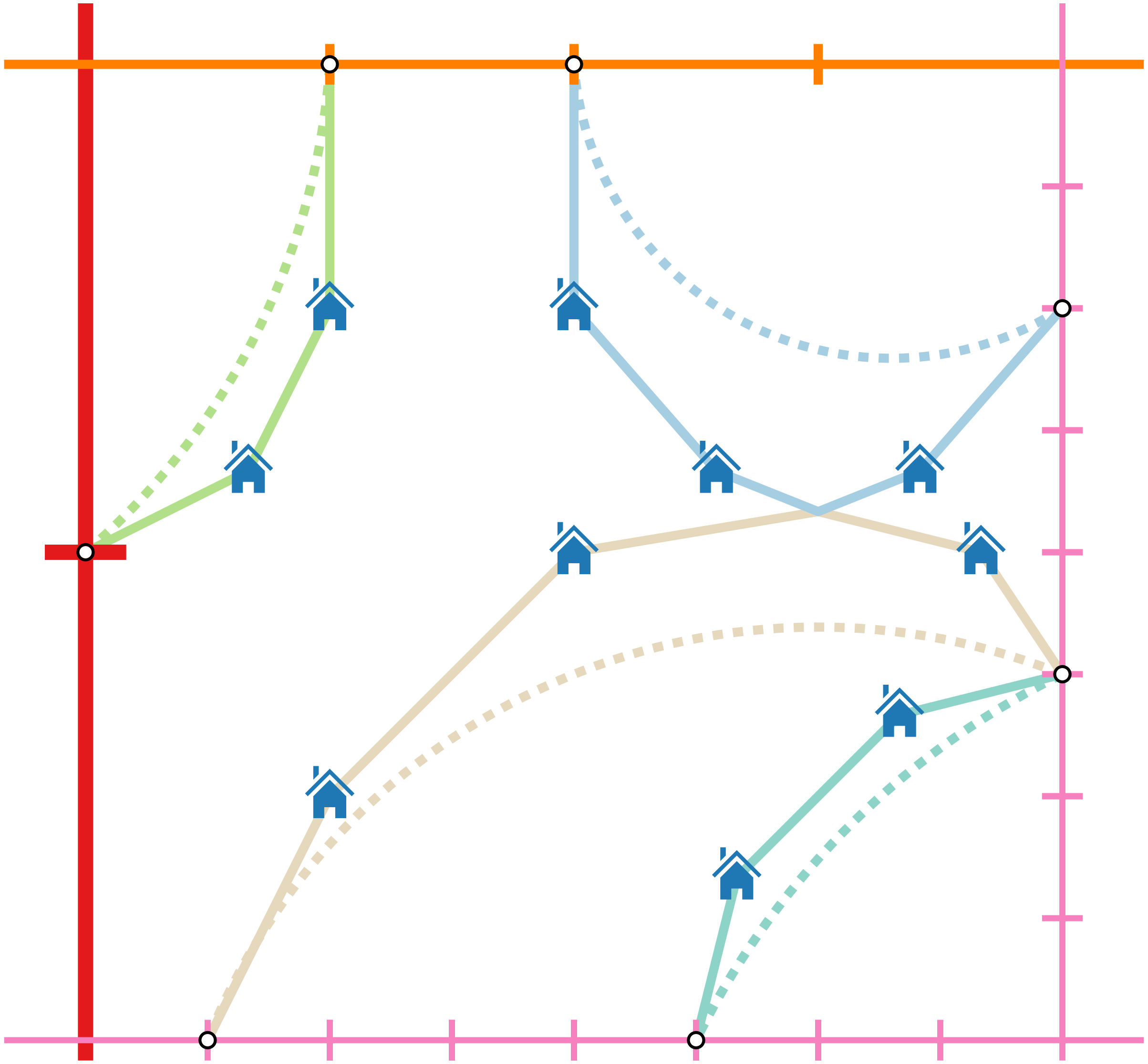

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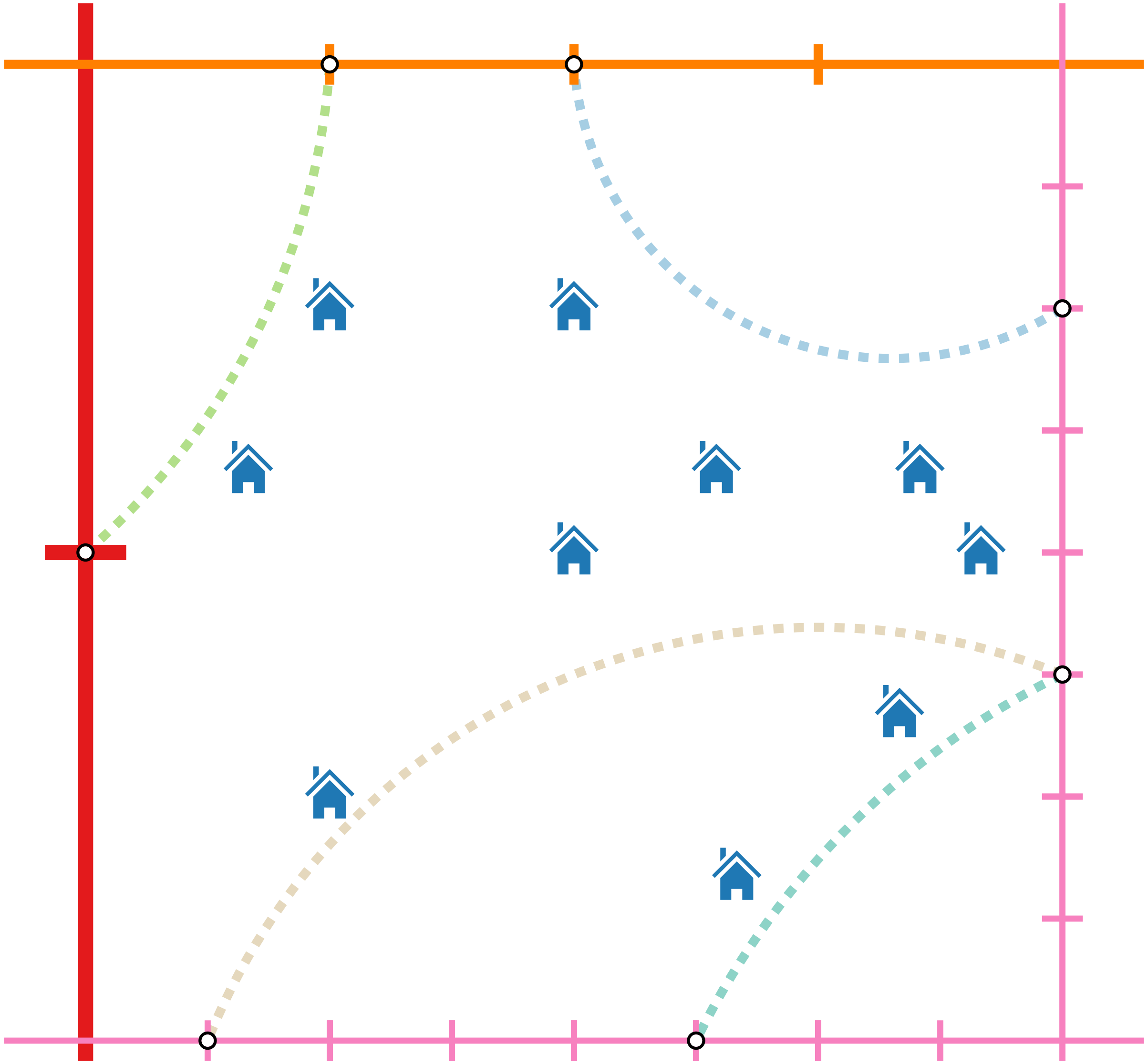

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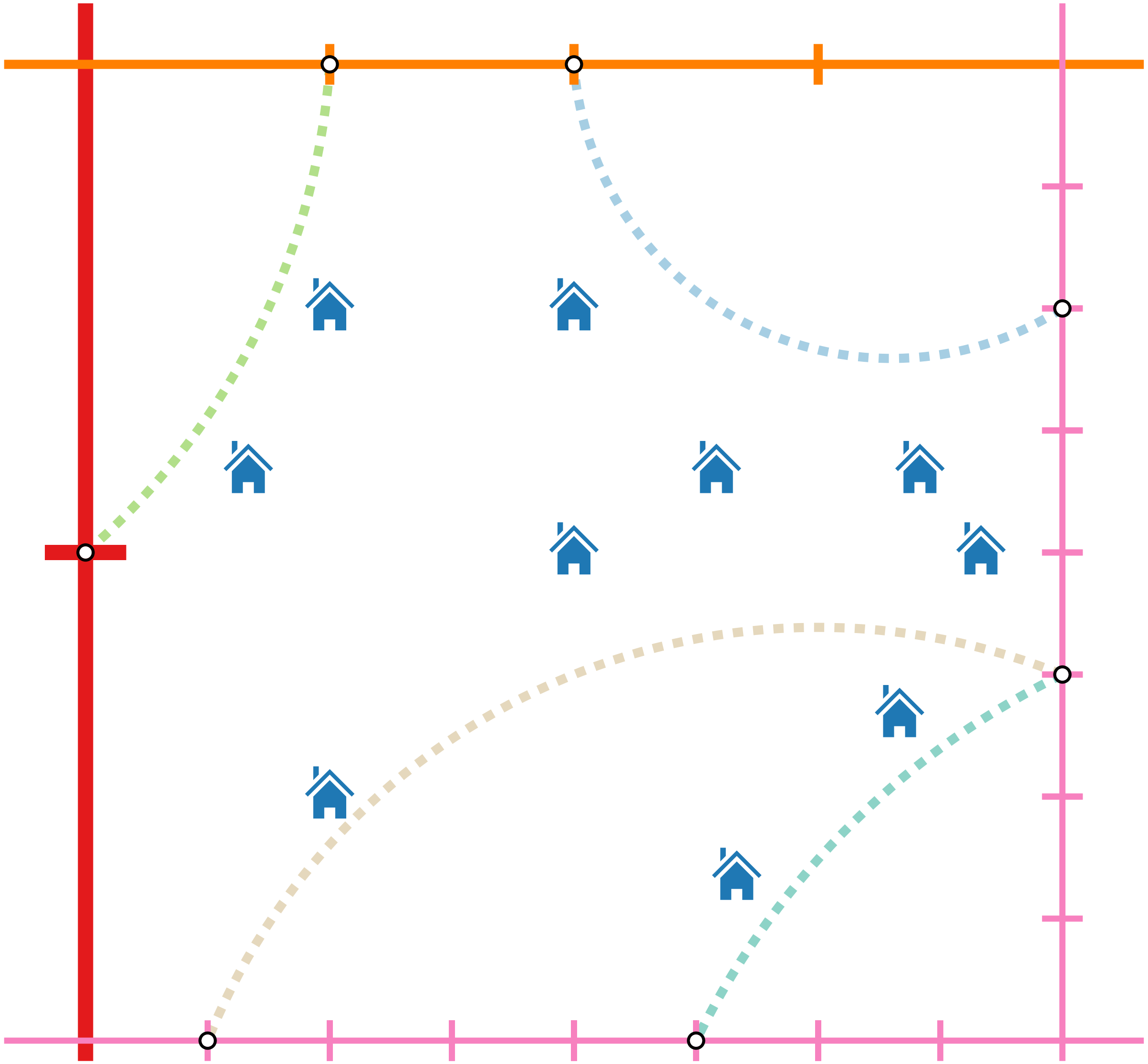

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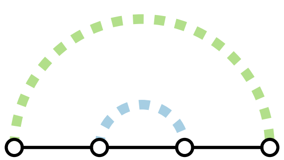

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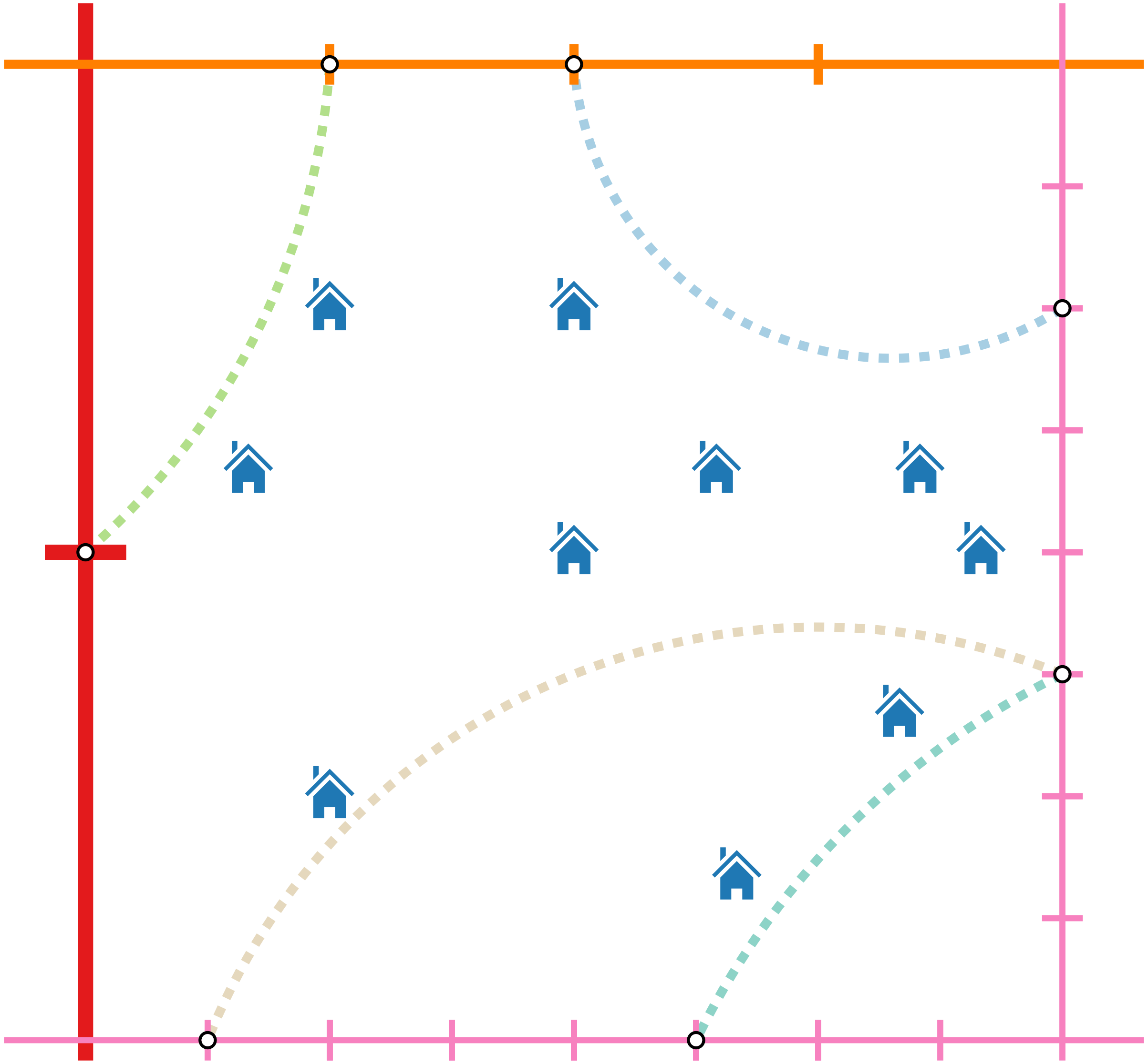

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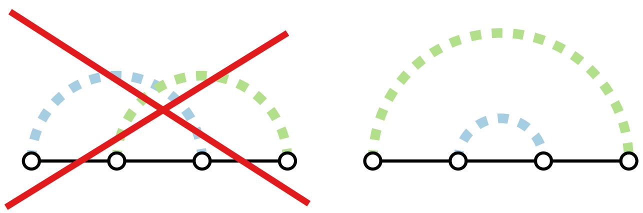

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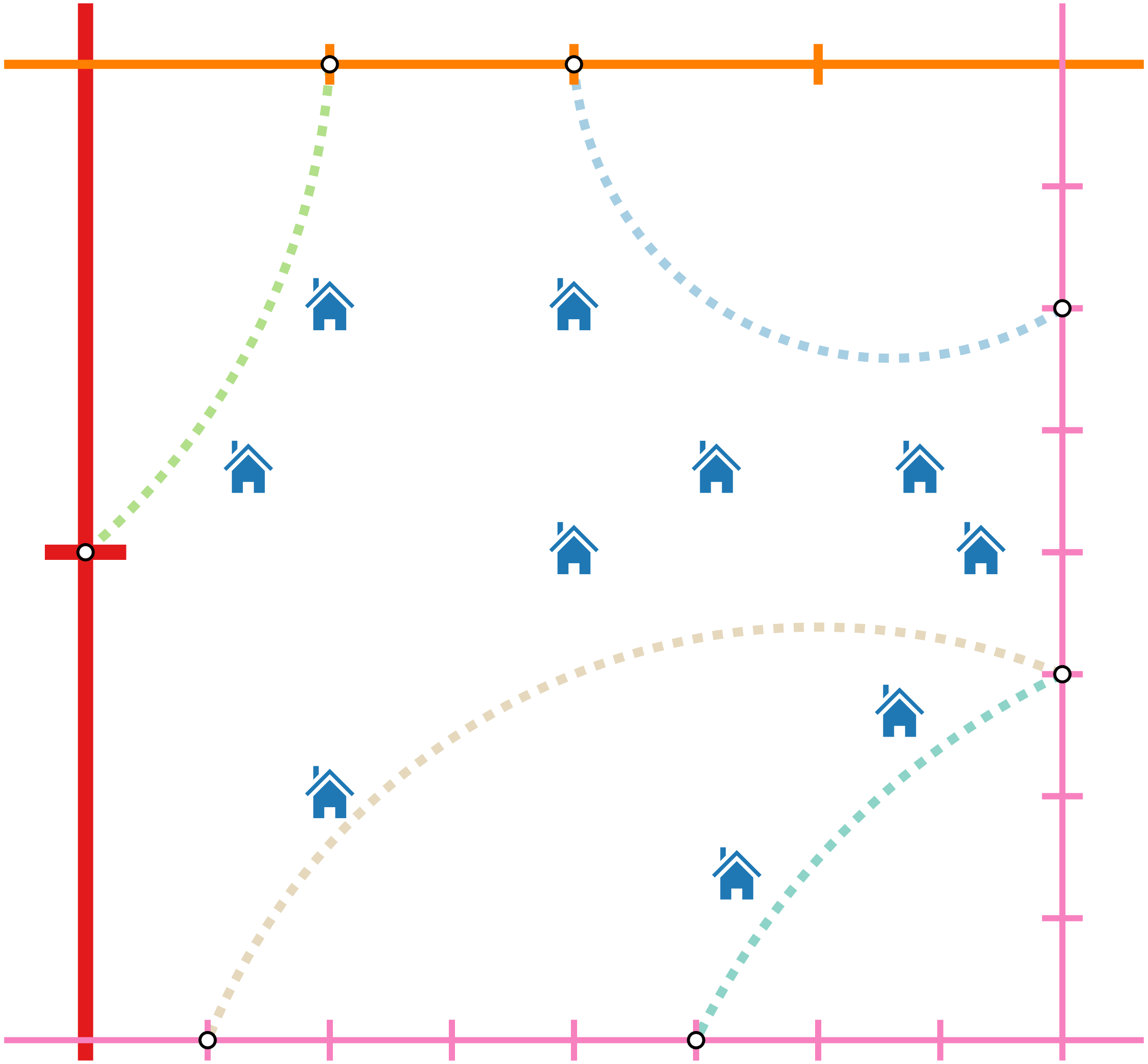

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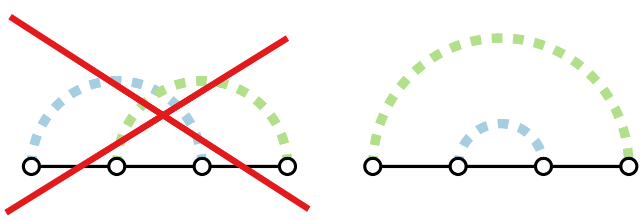
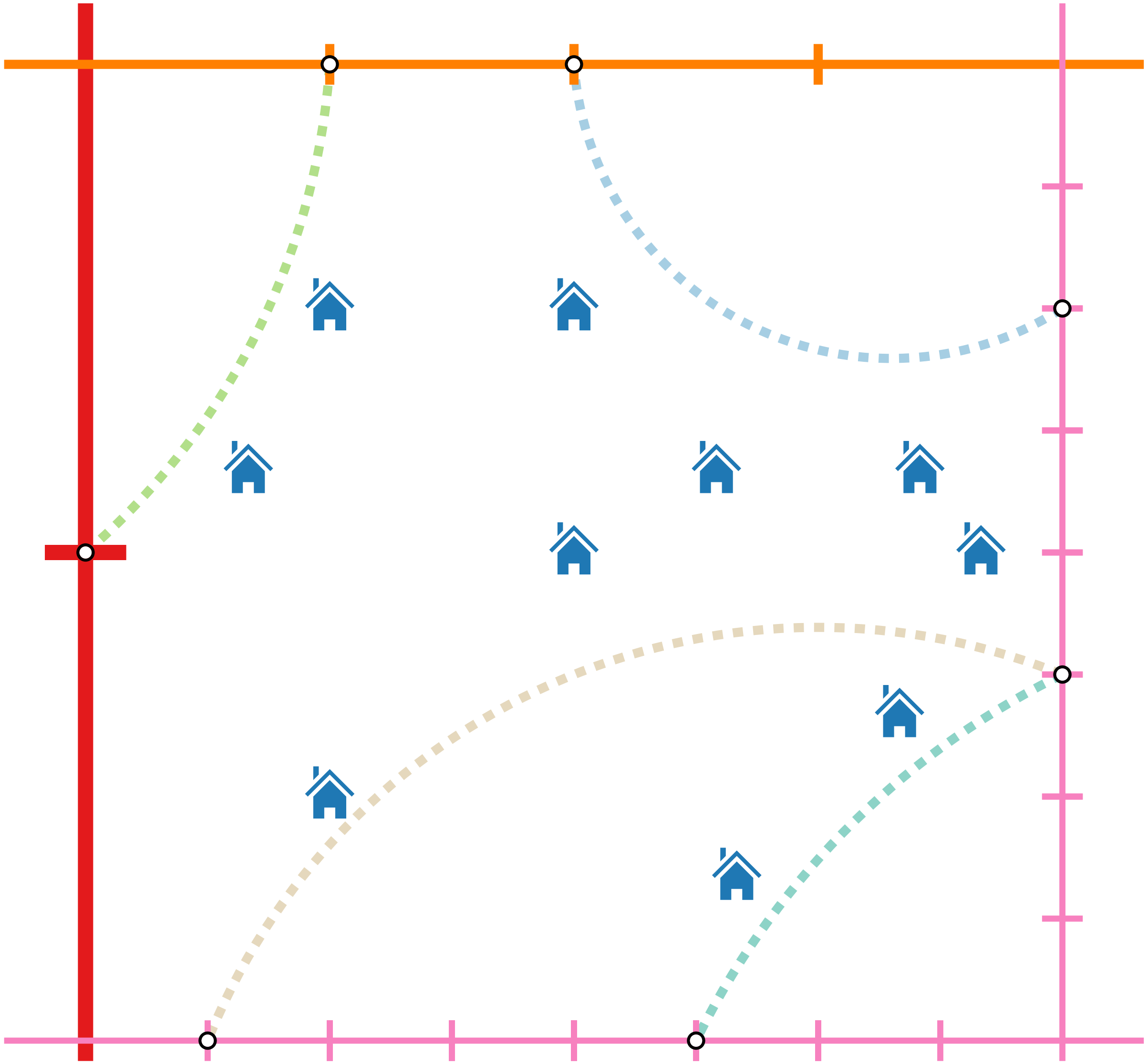


$\Rightarrow$ max.

Dynamic Program (I)



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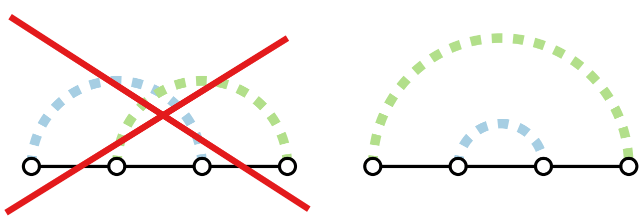
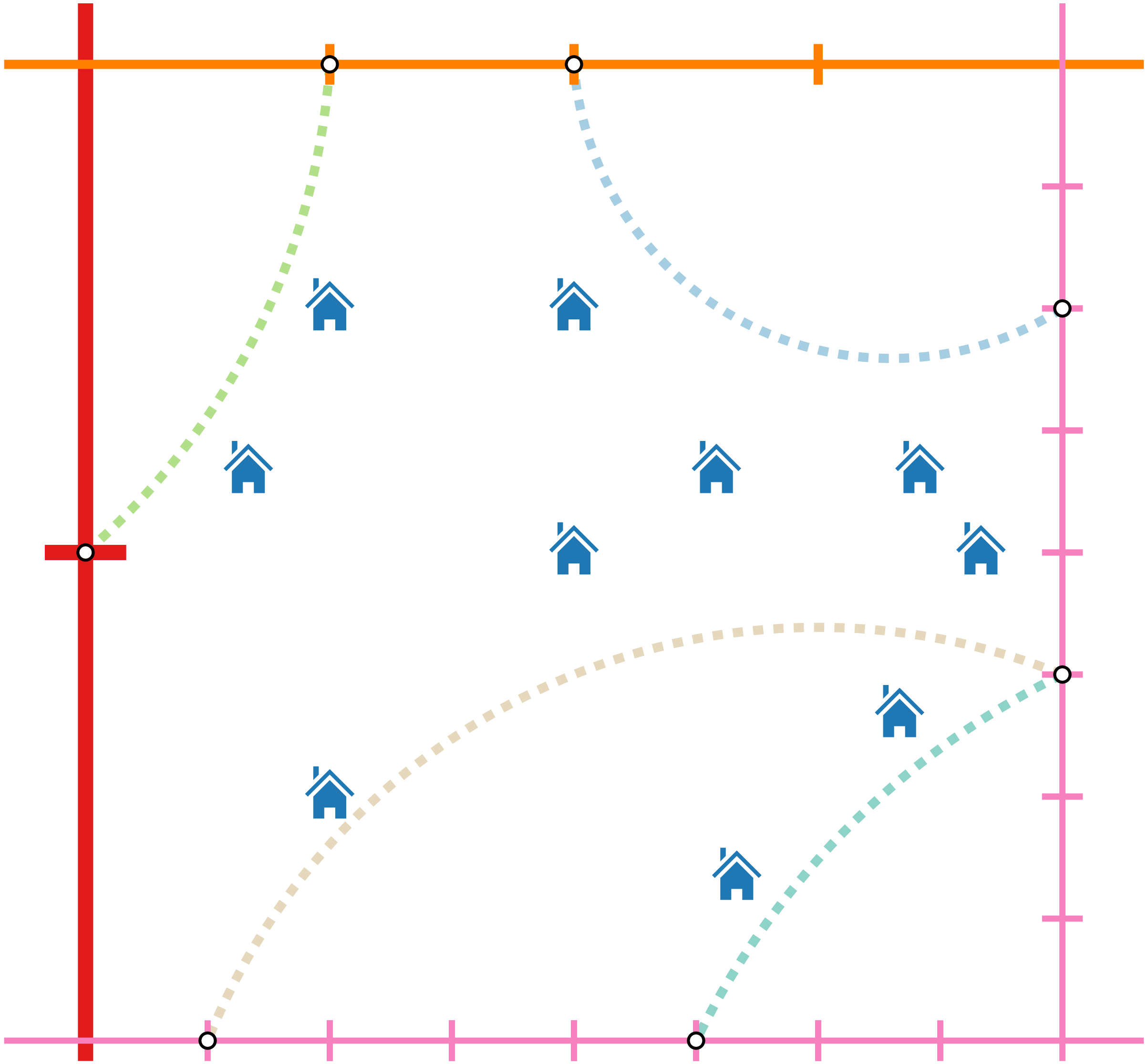


$\Rightarrow$ max. $\underbrace{}_{\text{\#visit vectors}}$

Dynamic Program (I)



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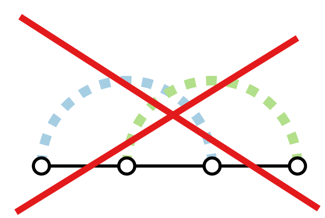
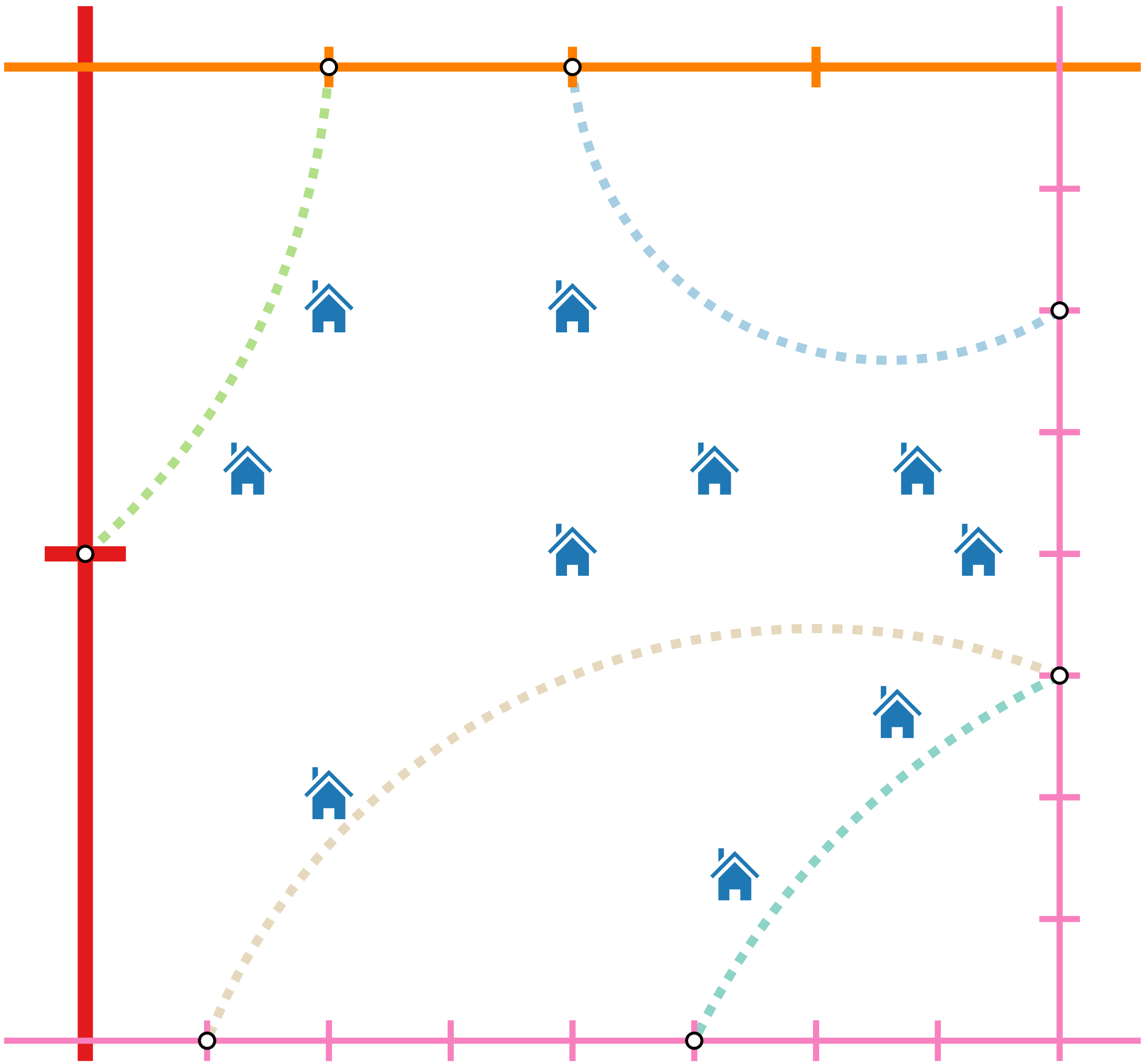


$$\Rightarrow \text{max.} \underbrace{\qquad\qquad}_{\text{\#visit vectors}} \times \underbrace{\qquad\qquad}_{\text{\#real. pairings}}$$

Dynamic Program (I)



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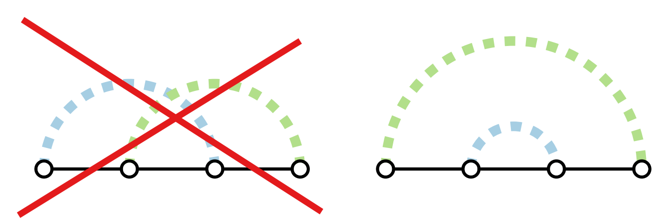
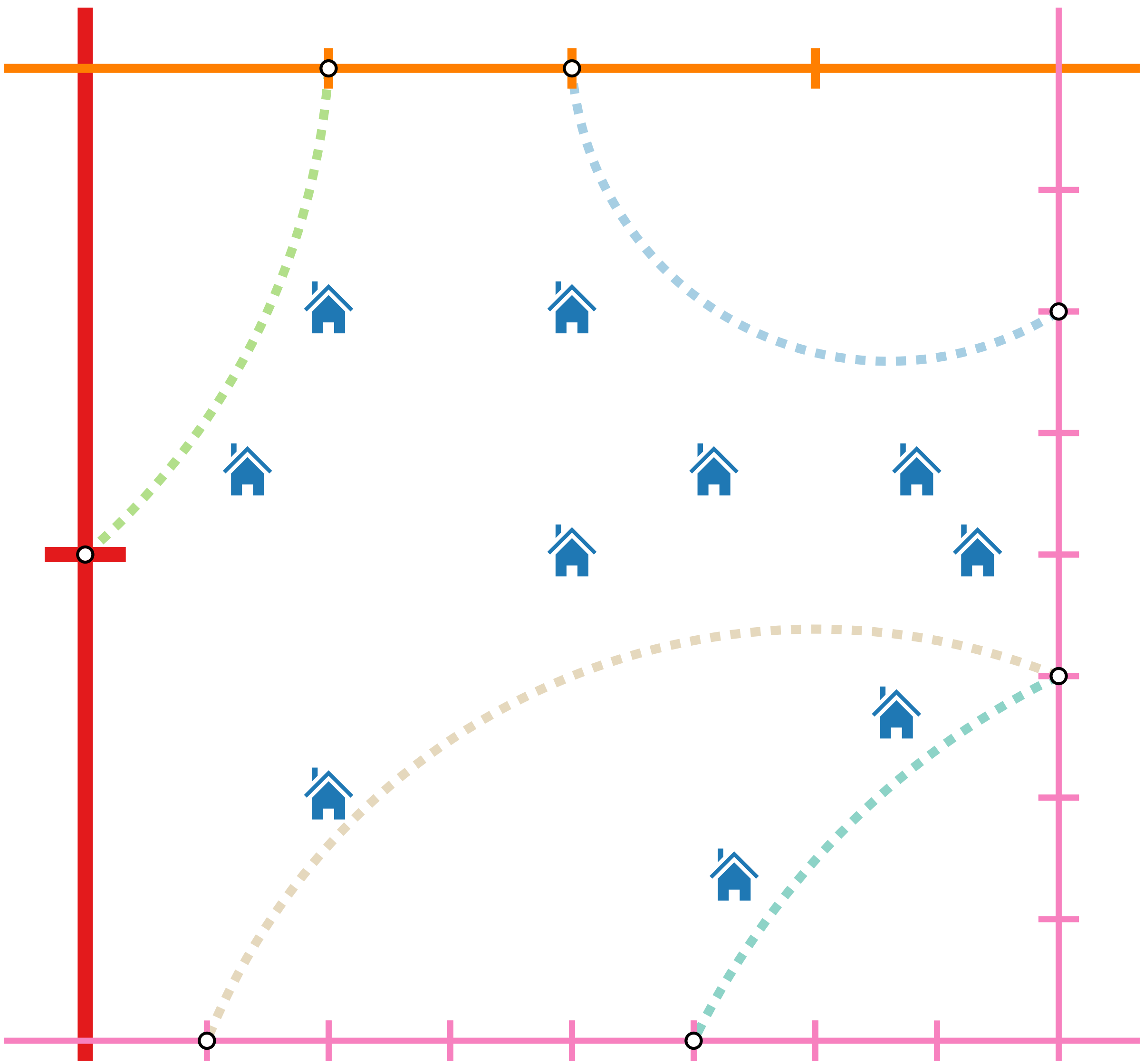


$$\Rightarrow \text{max. } \underbrace{n^{O(1/\varepsilon)}}_{\text{\#visit vectors}} \times \underbrace{\qquad}_{\text{\#real. pairings}}$$

Dynamic Program (I)



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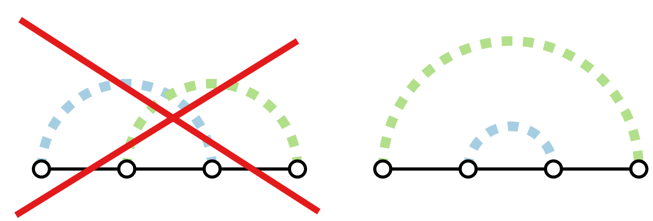
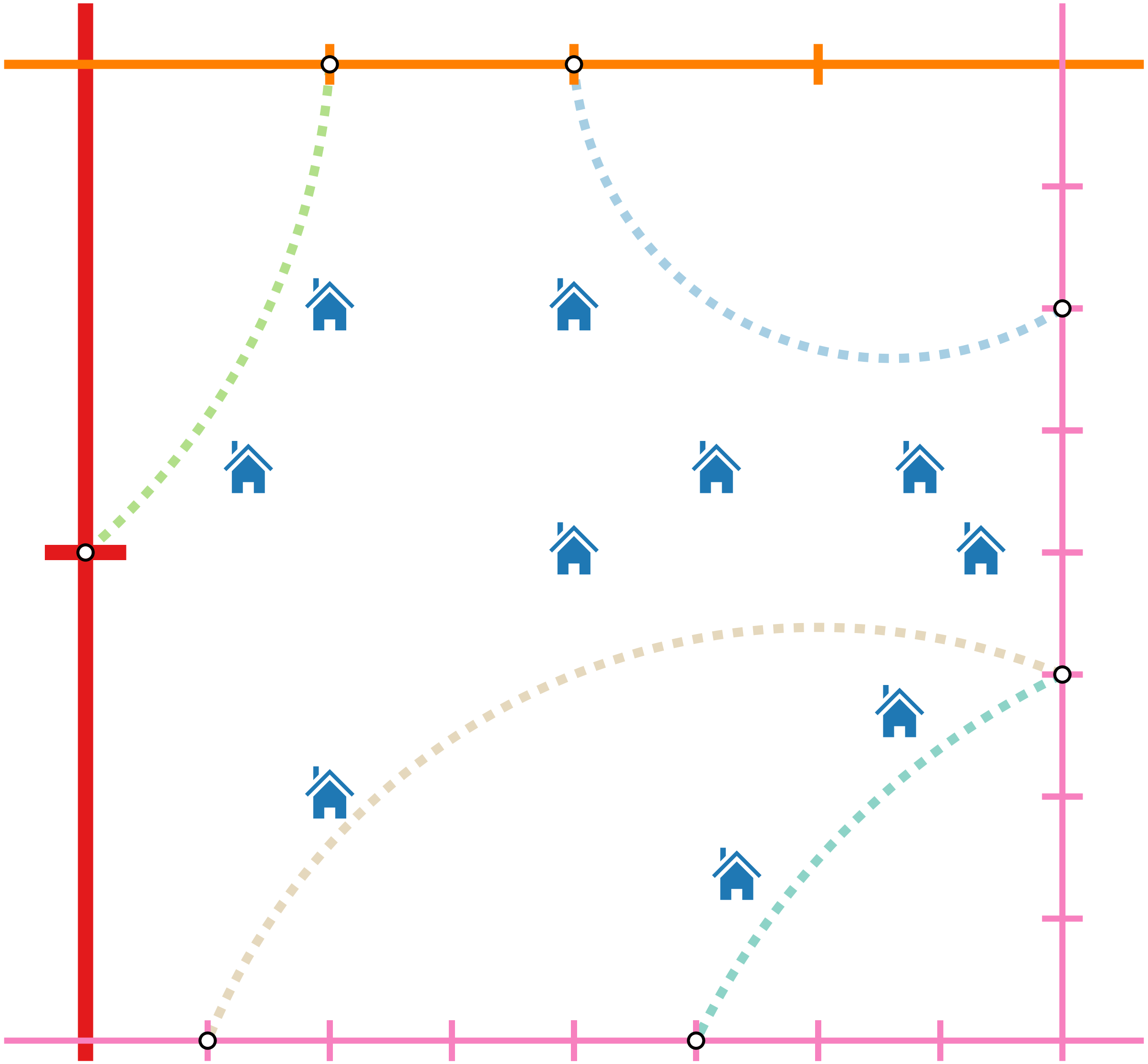


$$\Rightarrow \text{max.} \underbrace{n^{O(1/\varepsilon)}}_{\text{\#visit vectors}} \times \underbrace{2^{O(m)}}_{\text{\#real. pairings}}$$

Dynamic Program (I)



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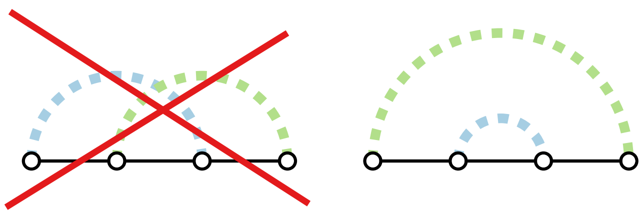
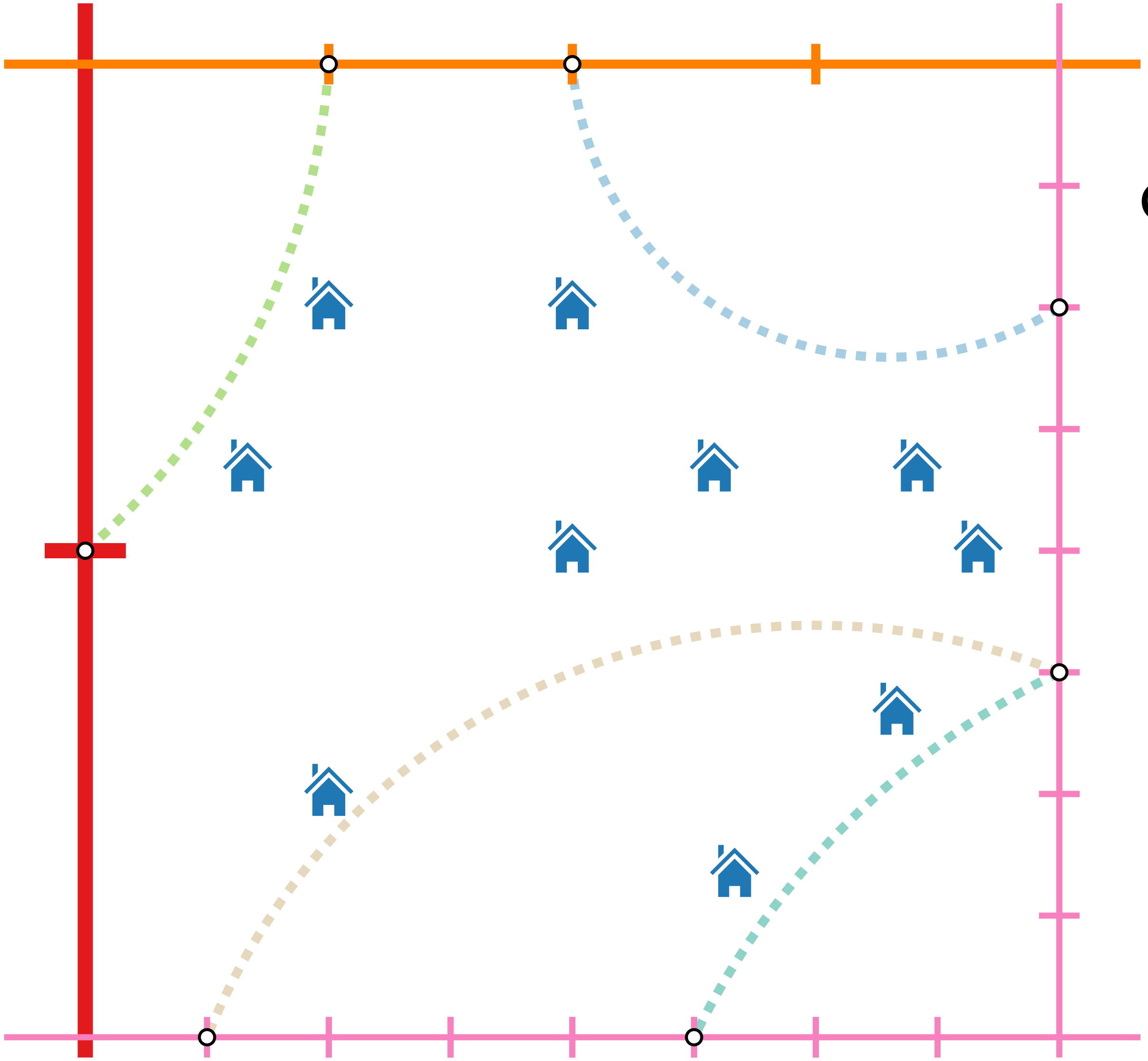


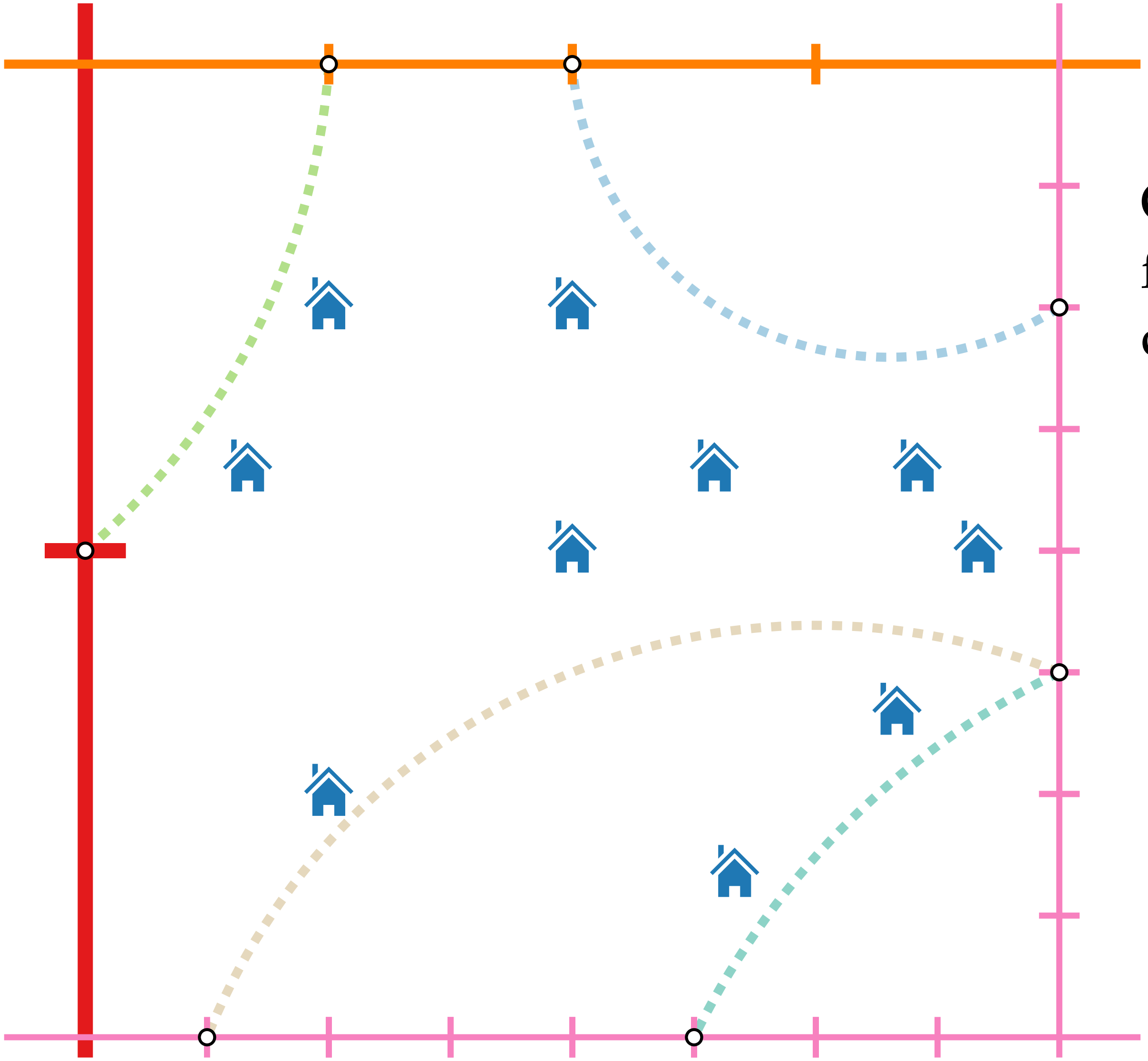
$$\Rightarrow \text{max. } \underbrace{n^{O(1/\varepsilon)}}_{\text{\#visit vectors}} \times \underbrace{2^{O(m)}}_{\text{\#real. pairings}} = n^{O(1/\varepsilon)} \text{ crossing-free pairings}$$

Dynamic Program (II)



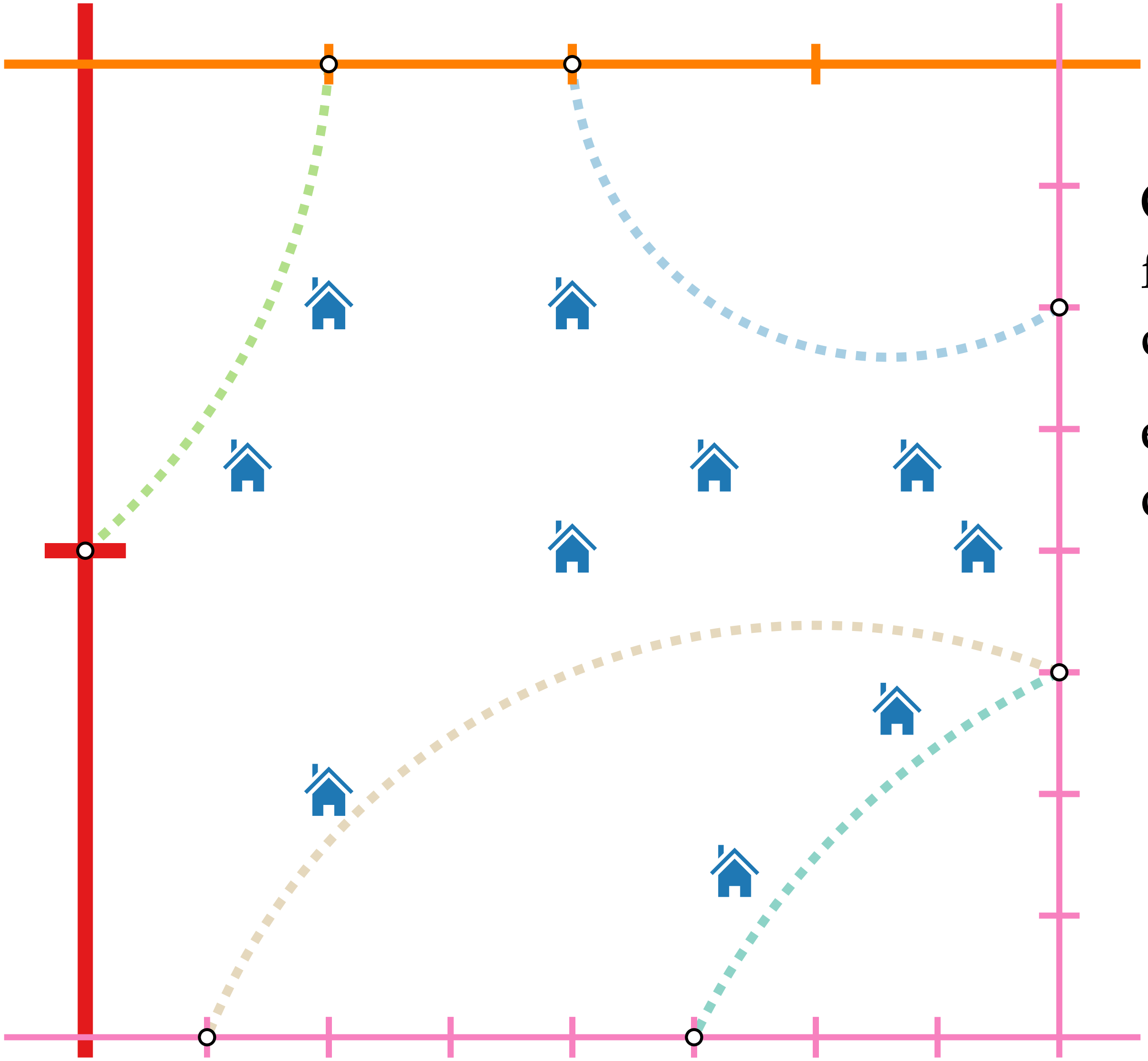
Compute:

Dynamic Program (II)



Compute:
for each square Q in the dissection and

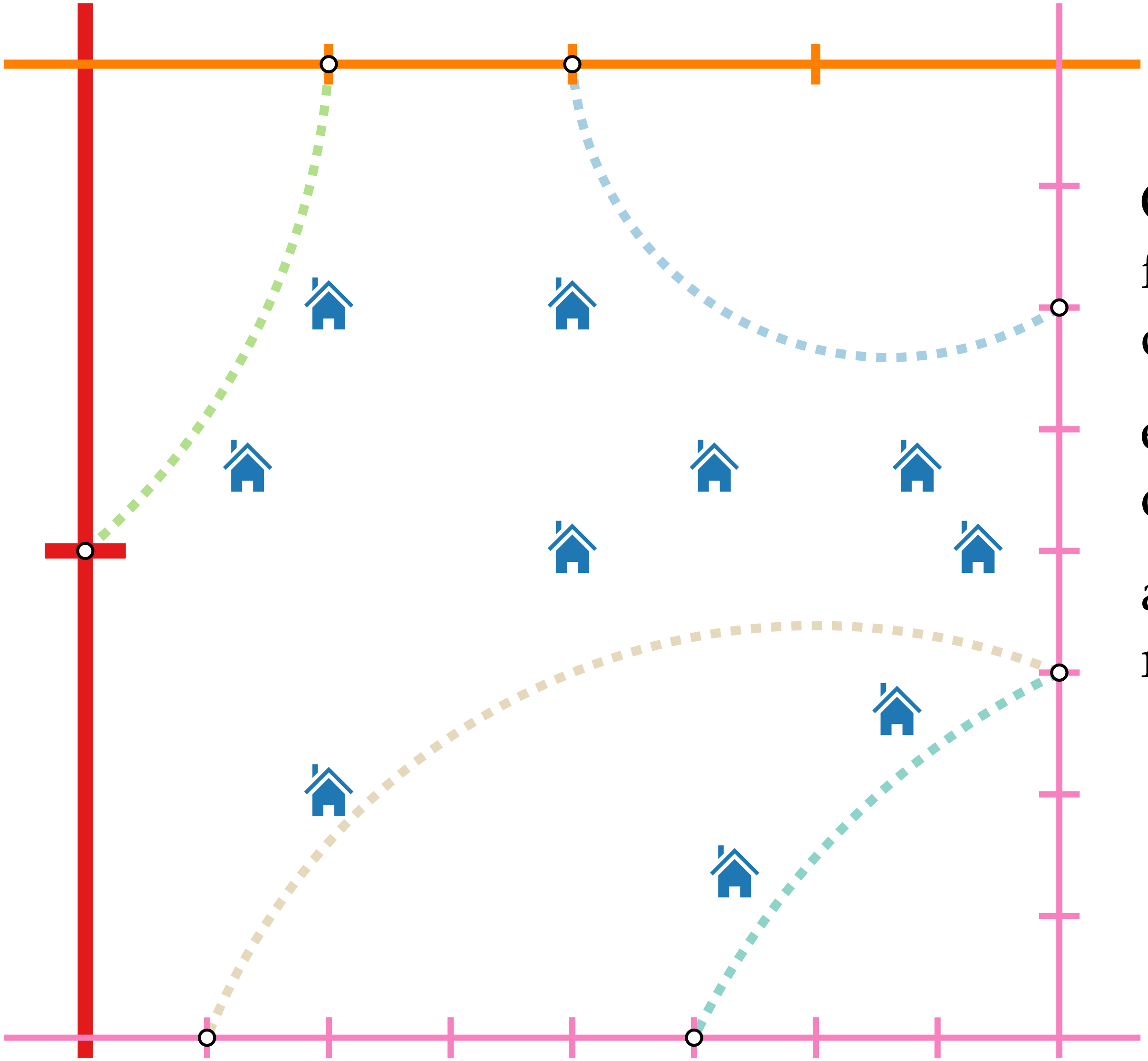
Dynamic Program (II)



Compute:
for each square Q in the dissection and

each crossing-free pairing P of Q

Dynamic Program (II)

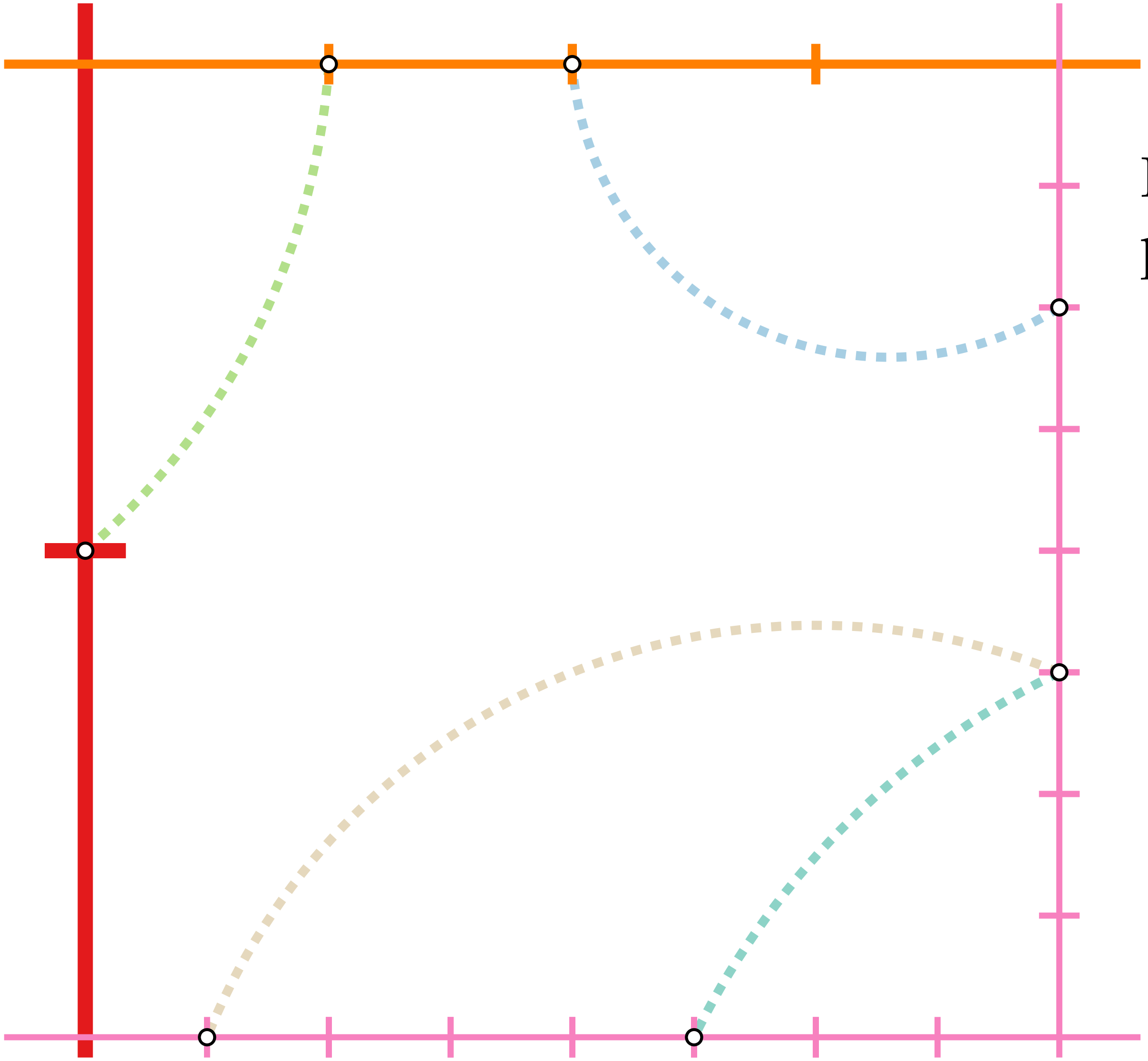


Compute:
for each square Q in the dissection and

each crossing-free pairing P of Q

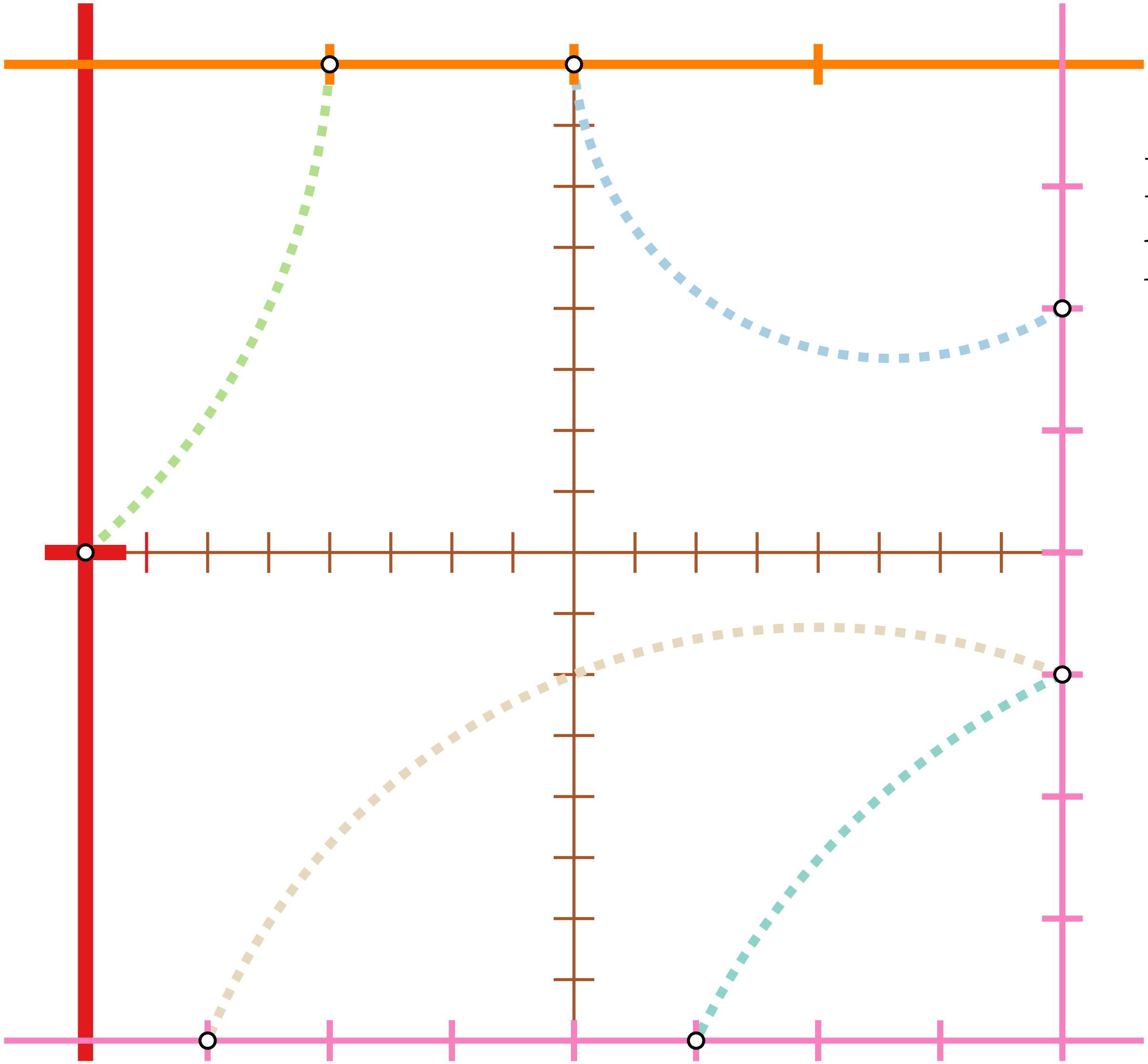
an optimal path cover that respects P.

Dynamic Program (III)



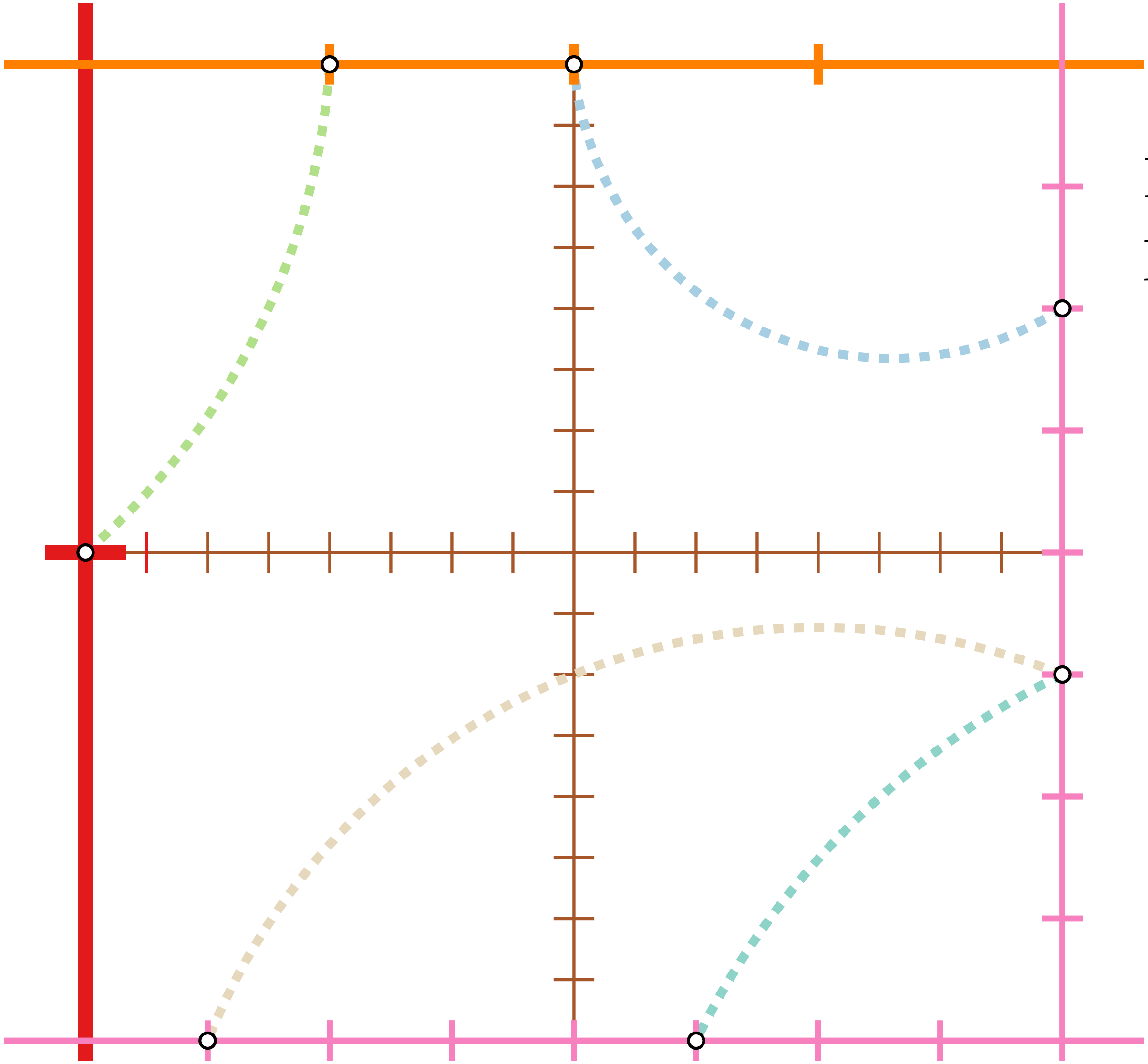
For a given square Q and pairing P:

Dynamic Program (III)



For a given square Q and pairing P:

Dynamic Program (III)

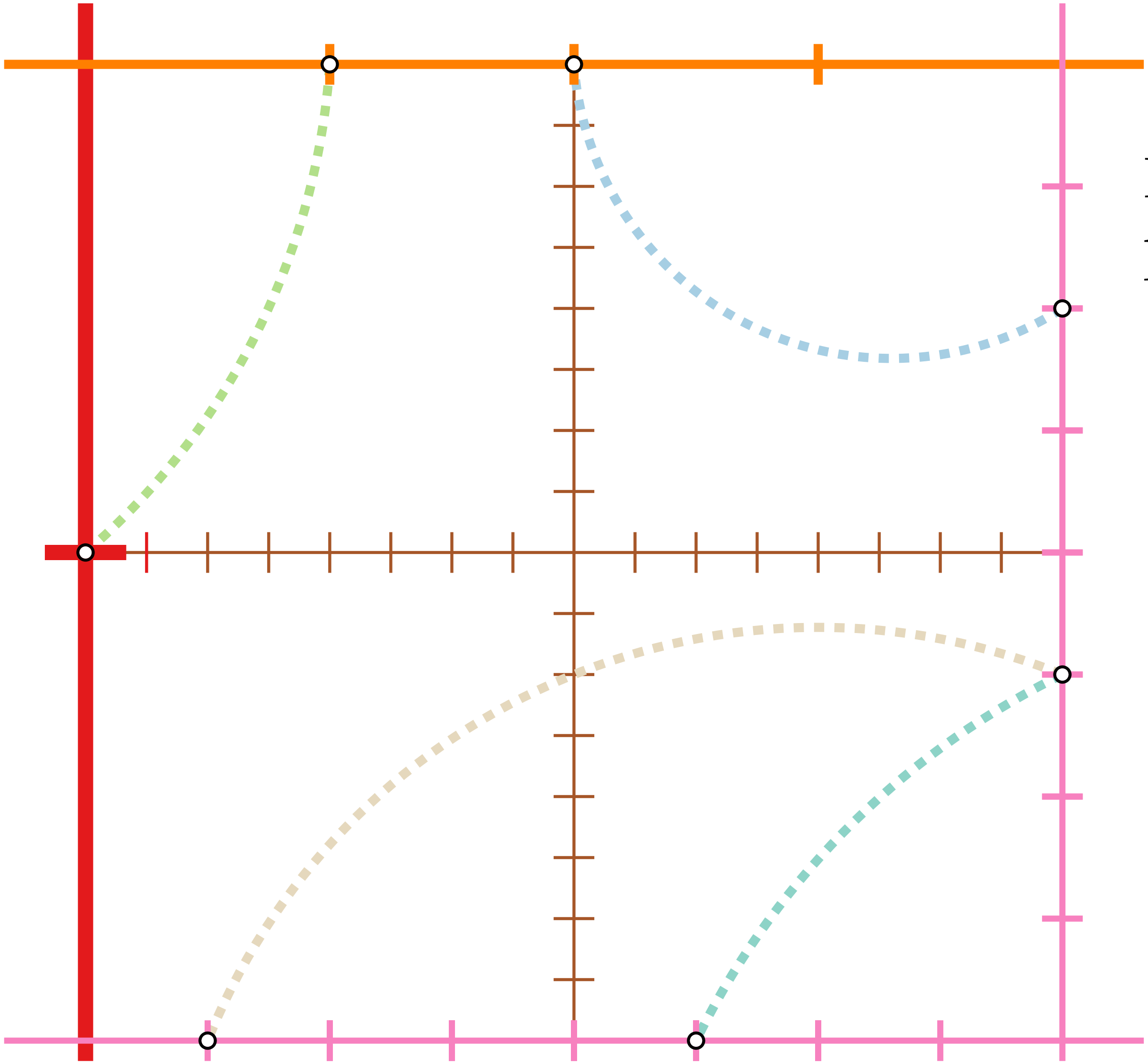


For a given square Q and pairing P:

- Iterate over all

  crossing-free pairings of the child-squares

Dynamic Program (III)



For a given square Q and pairing P:

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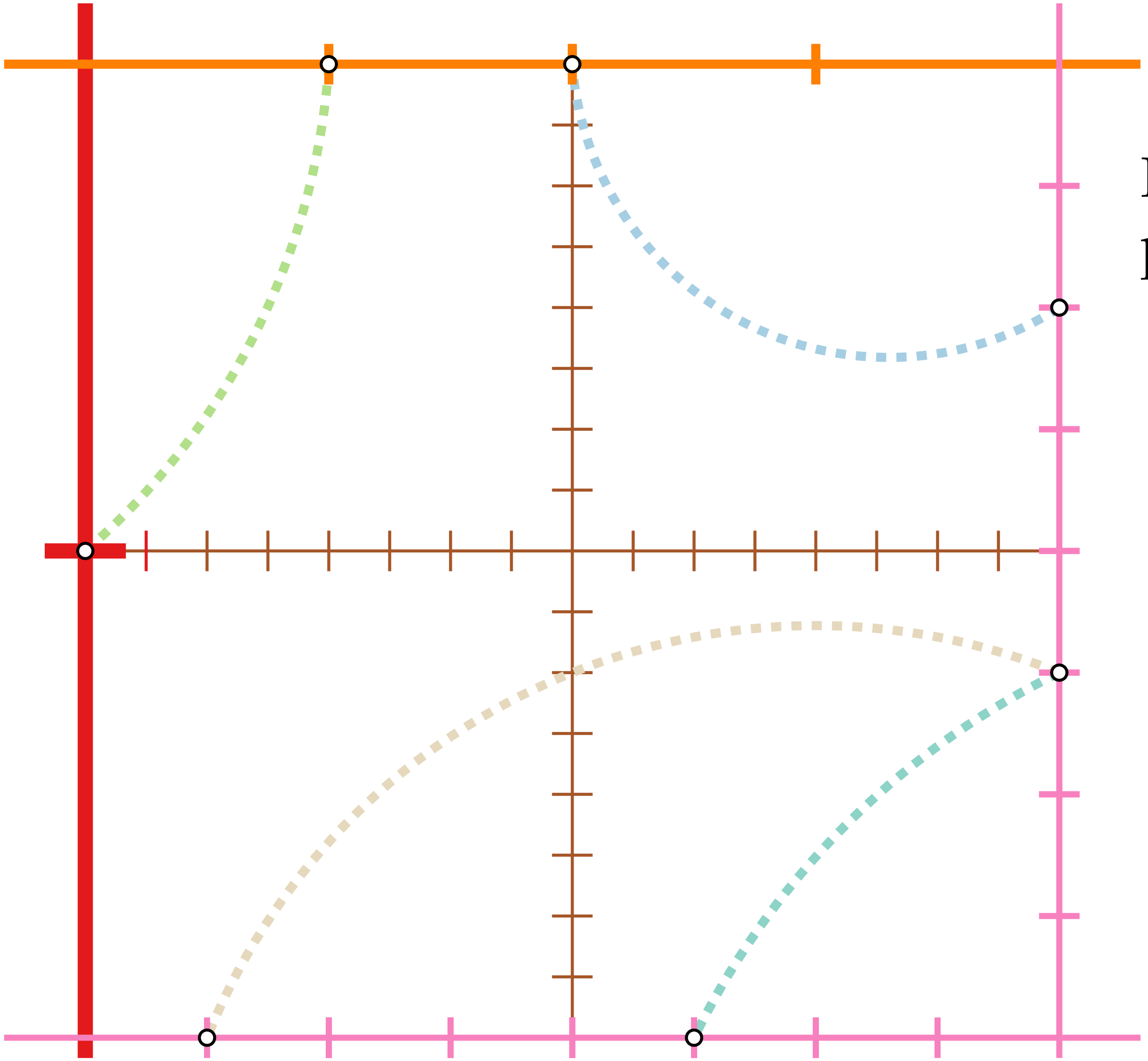
Dynamic Program (III)



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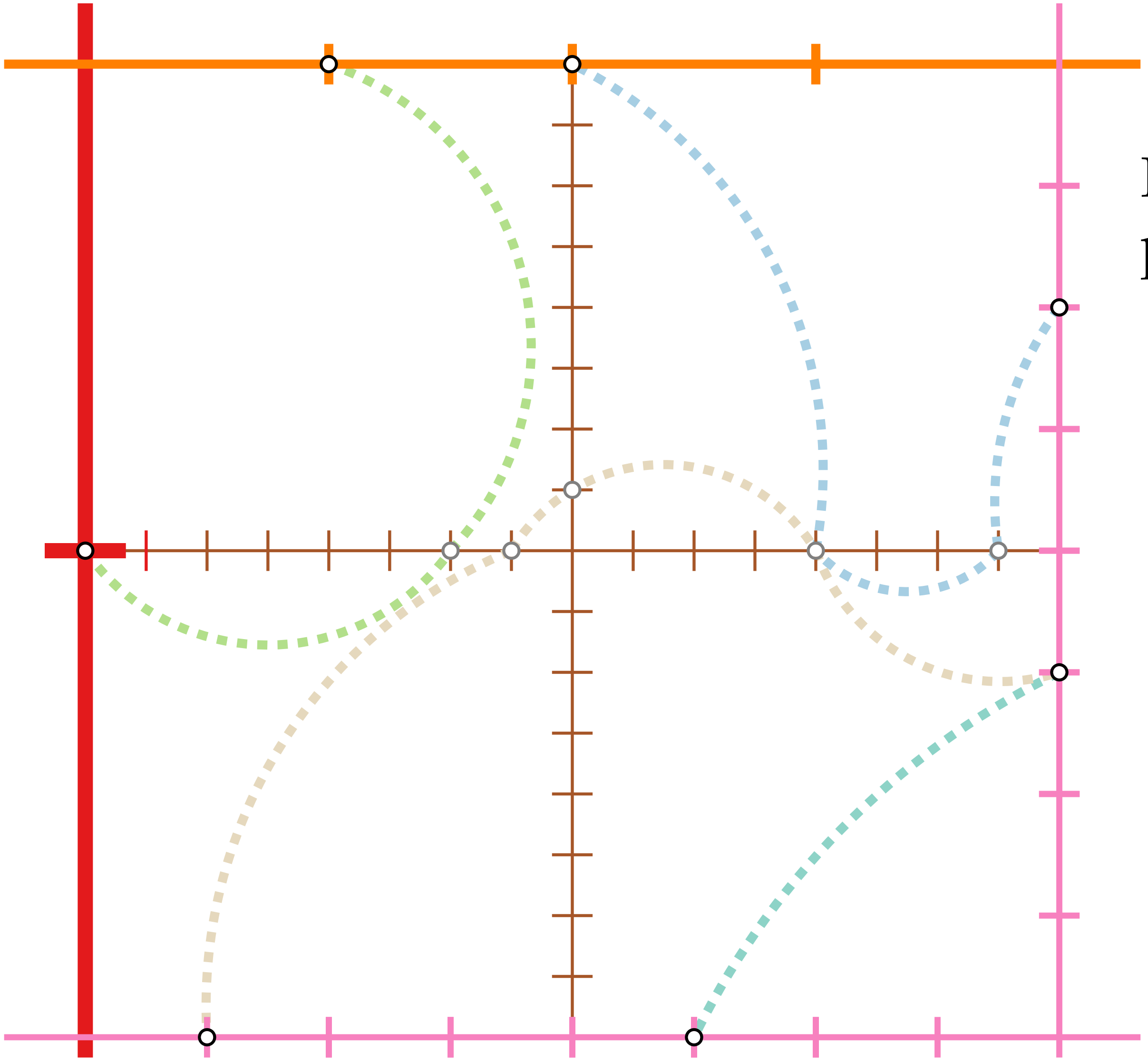
Dynamic Program (III)



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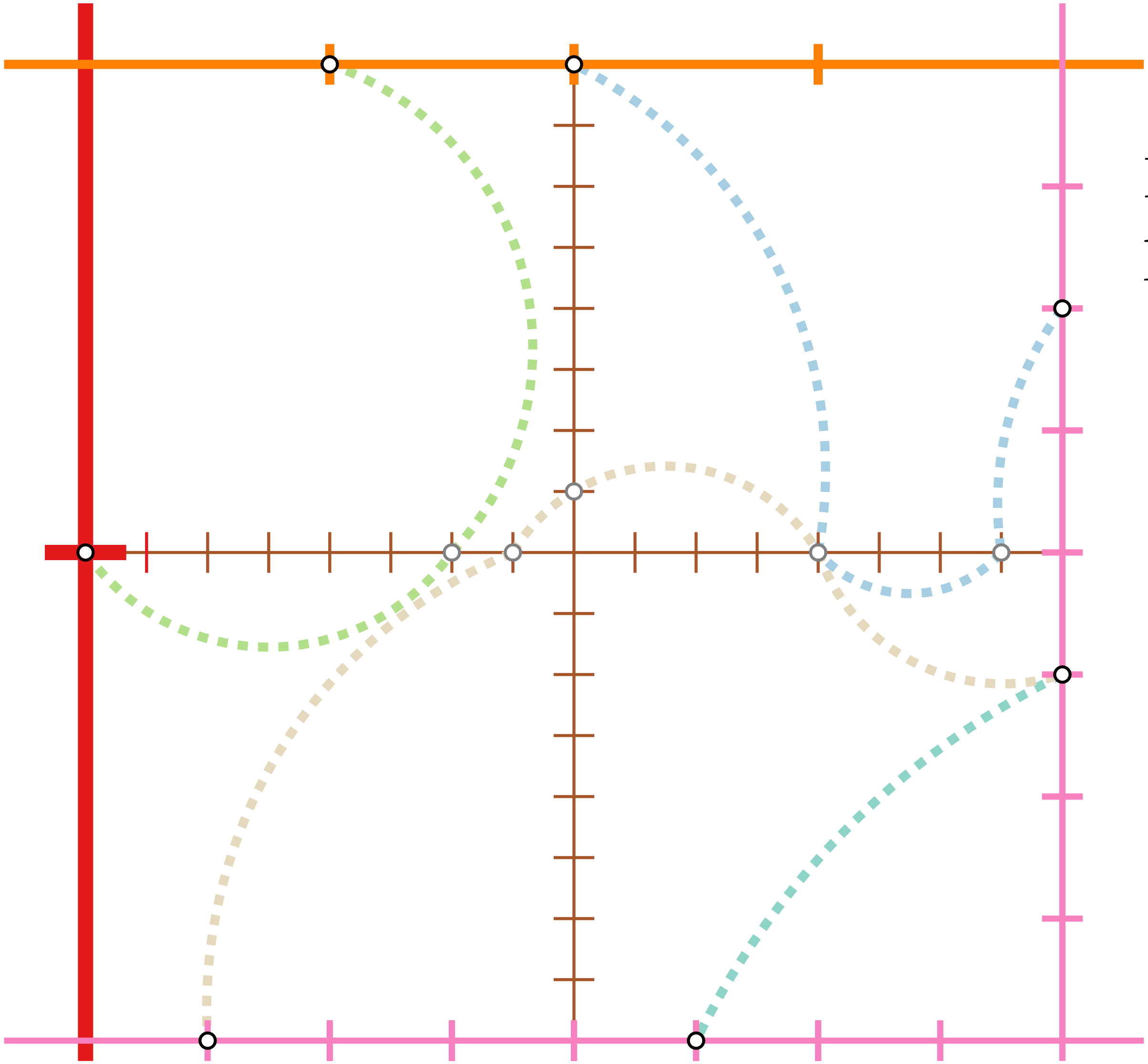

For a given square Q and pairing P:

- Iterate over all $(n^{O(1/\varepsilon)})^4 = n^{O(1/\varepsilon)}$ crossing-free pairings of the child-squares
- Minimize the cost over all such pairings that respect P

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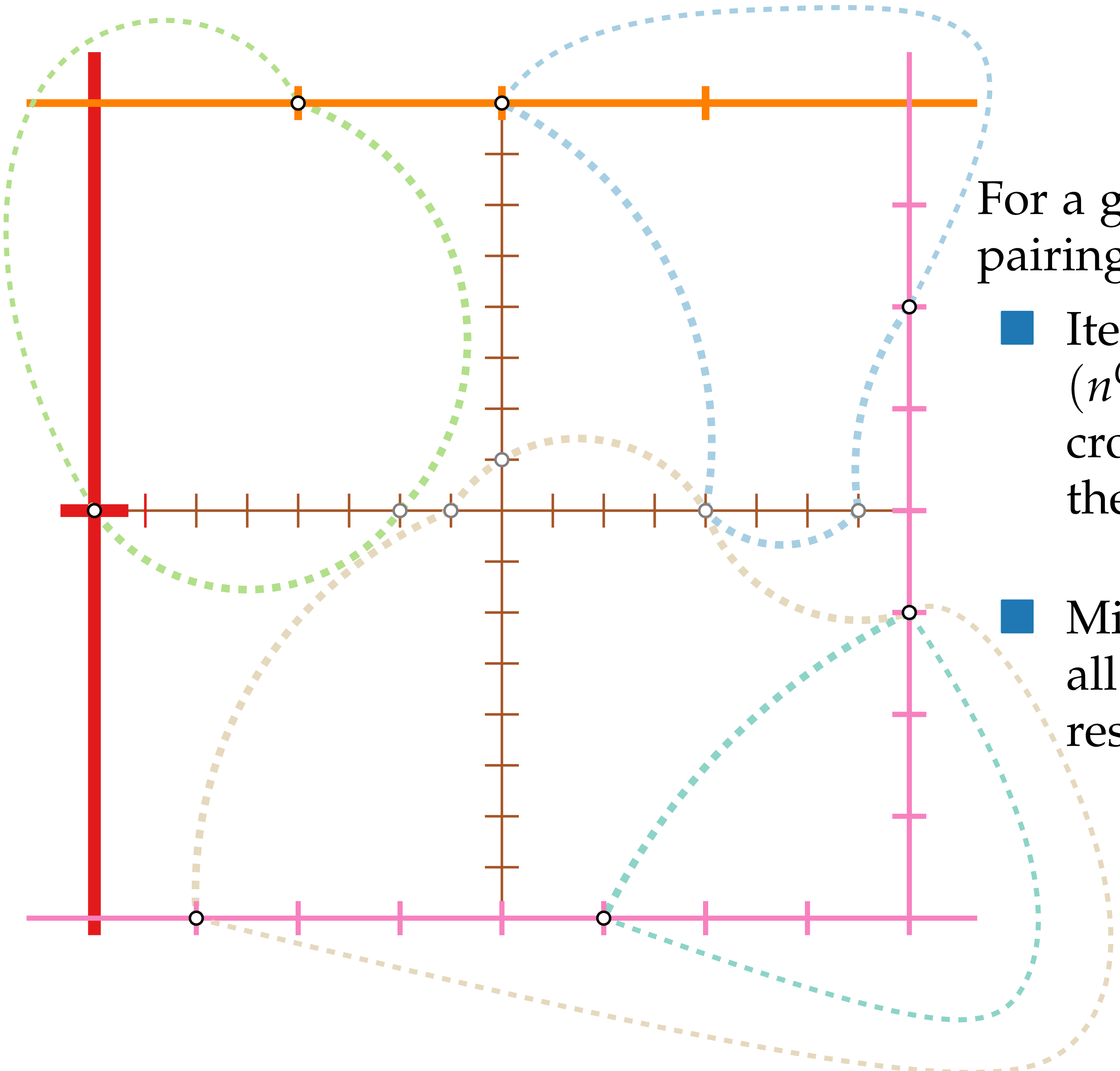

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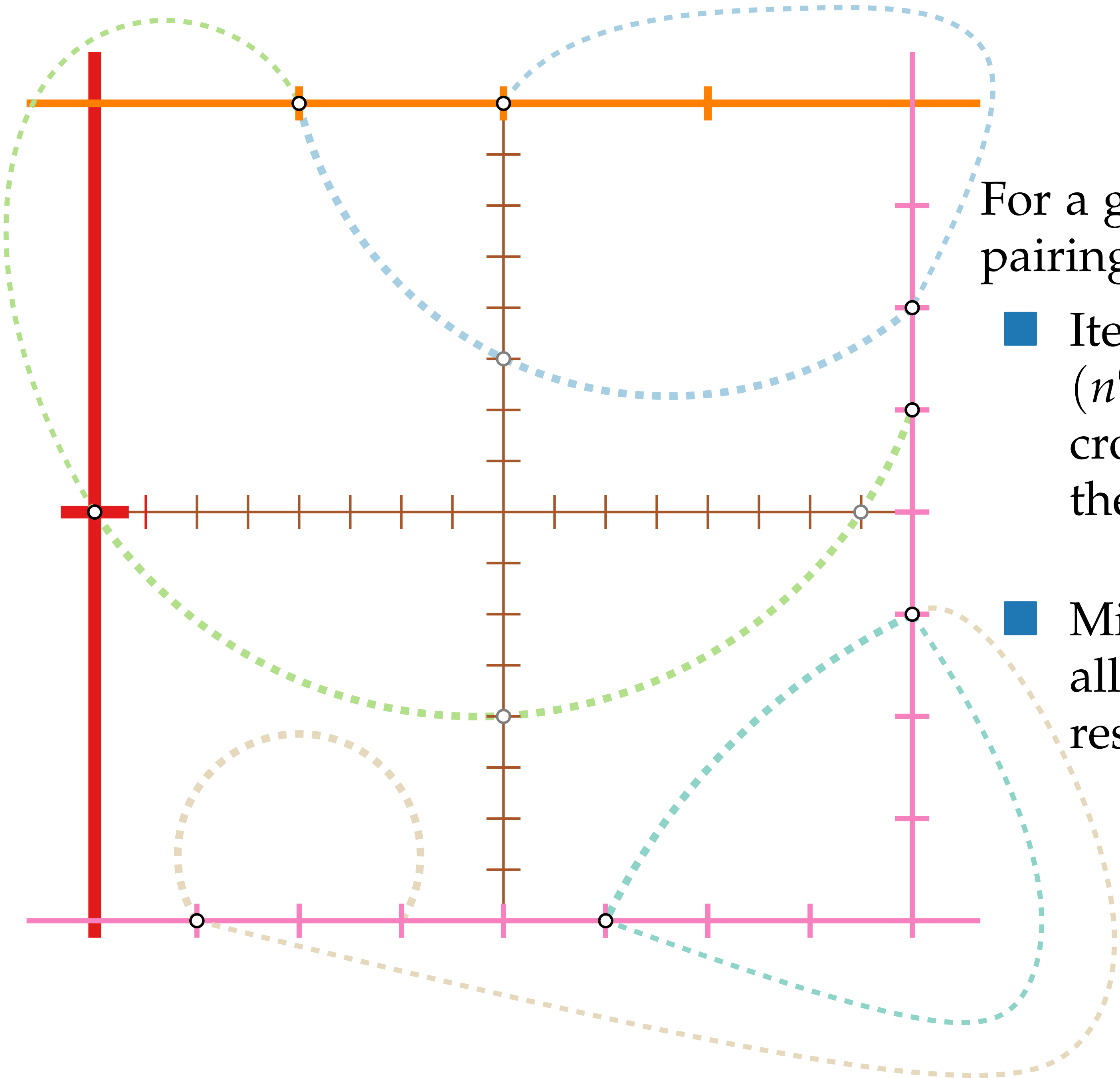

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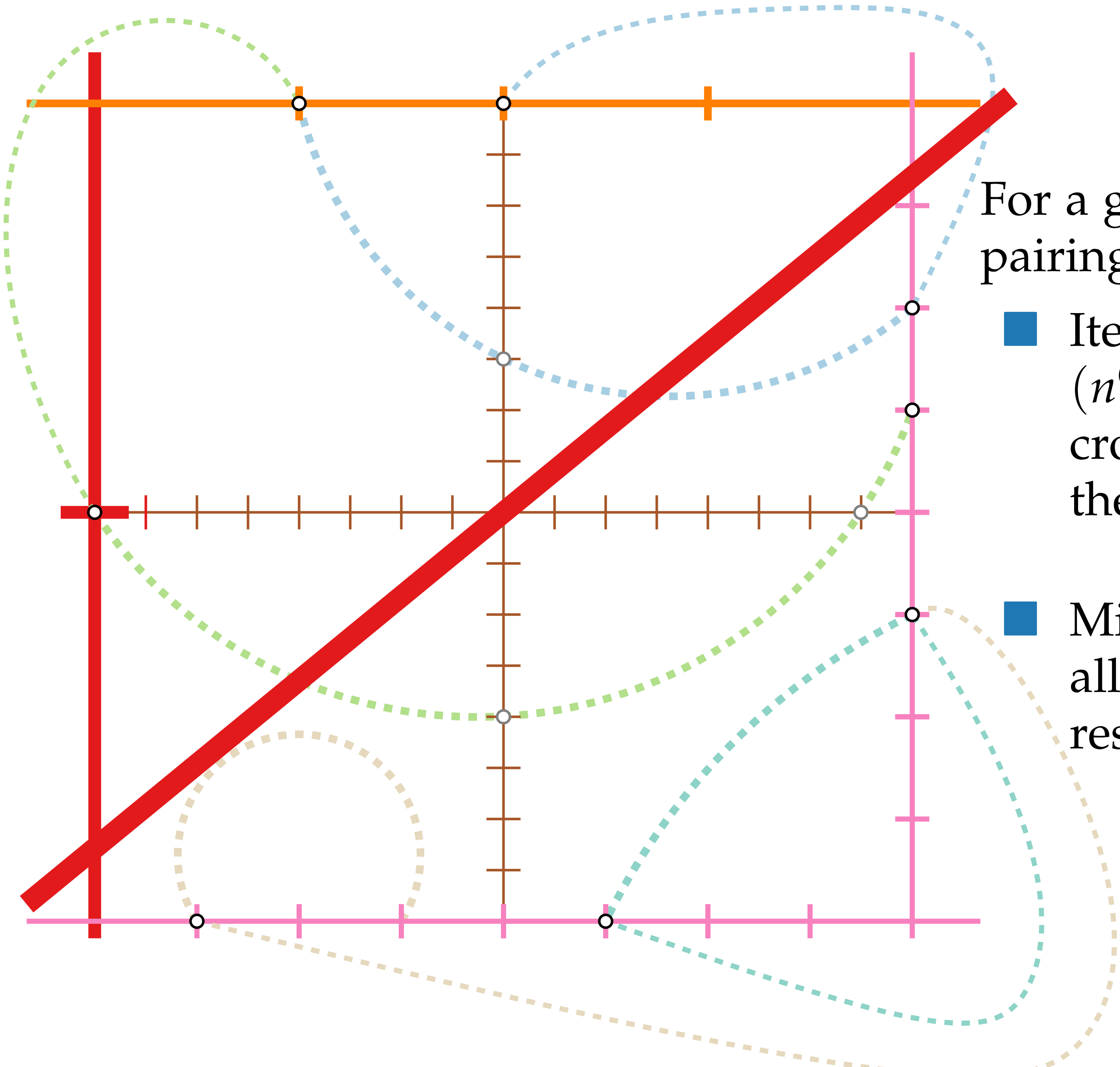

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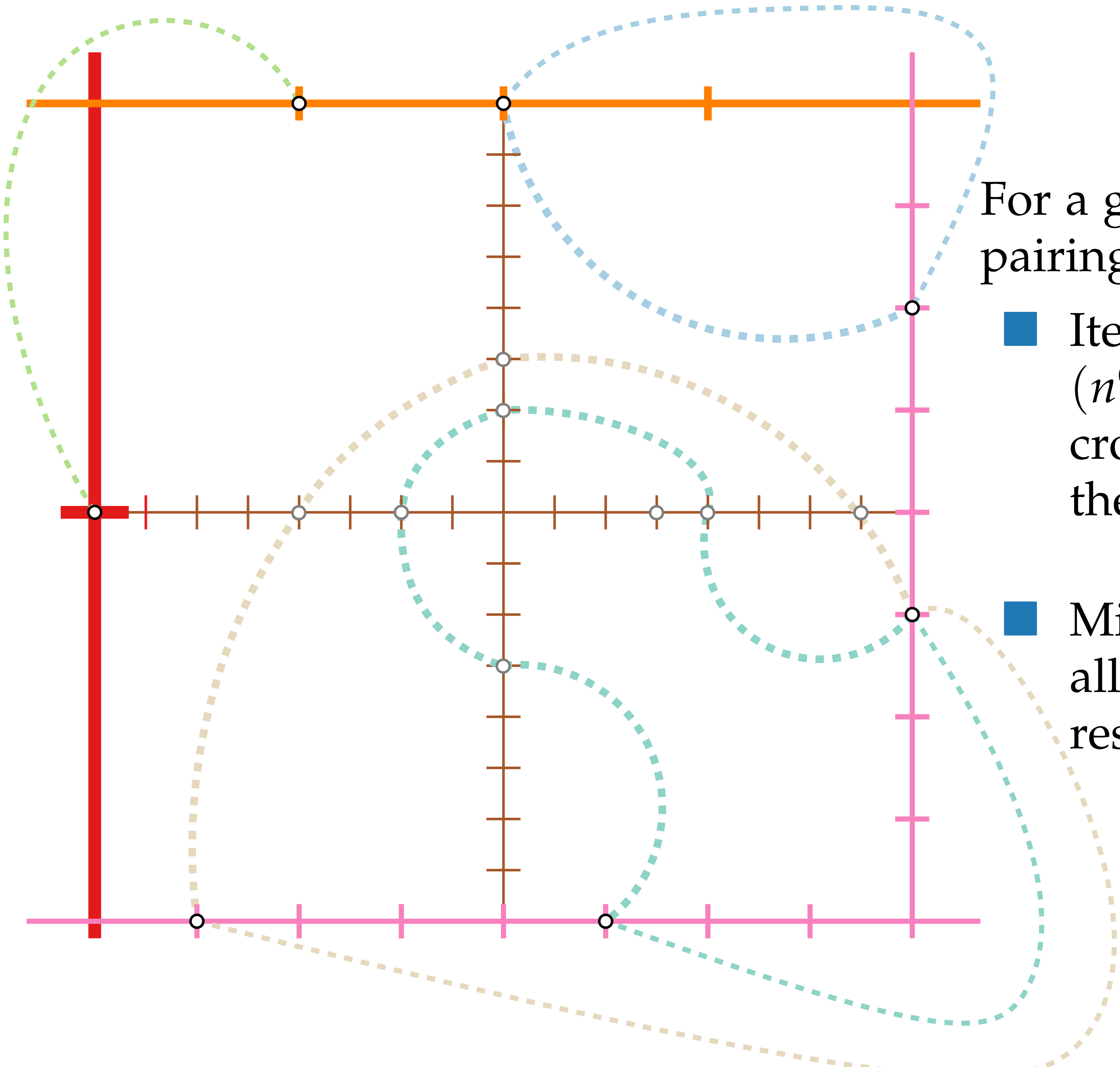

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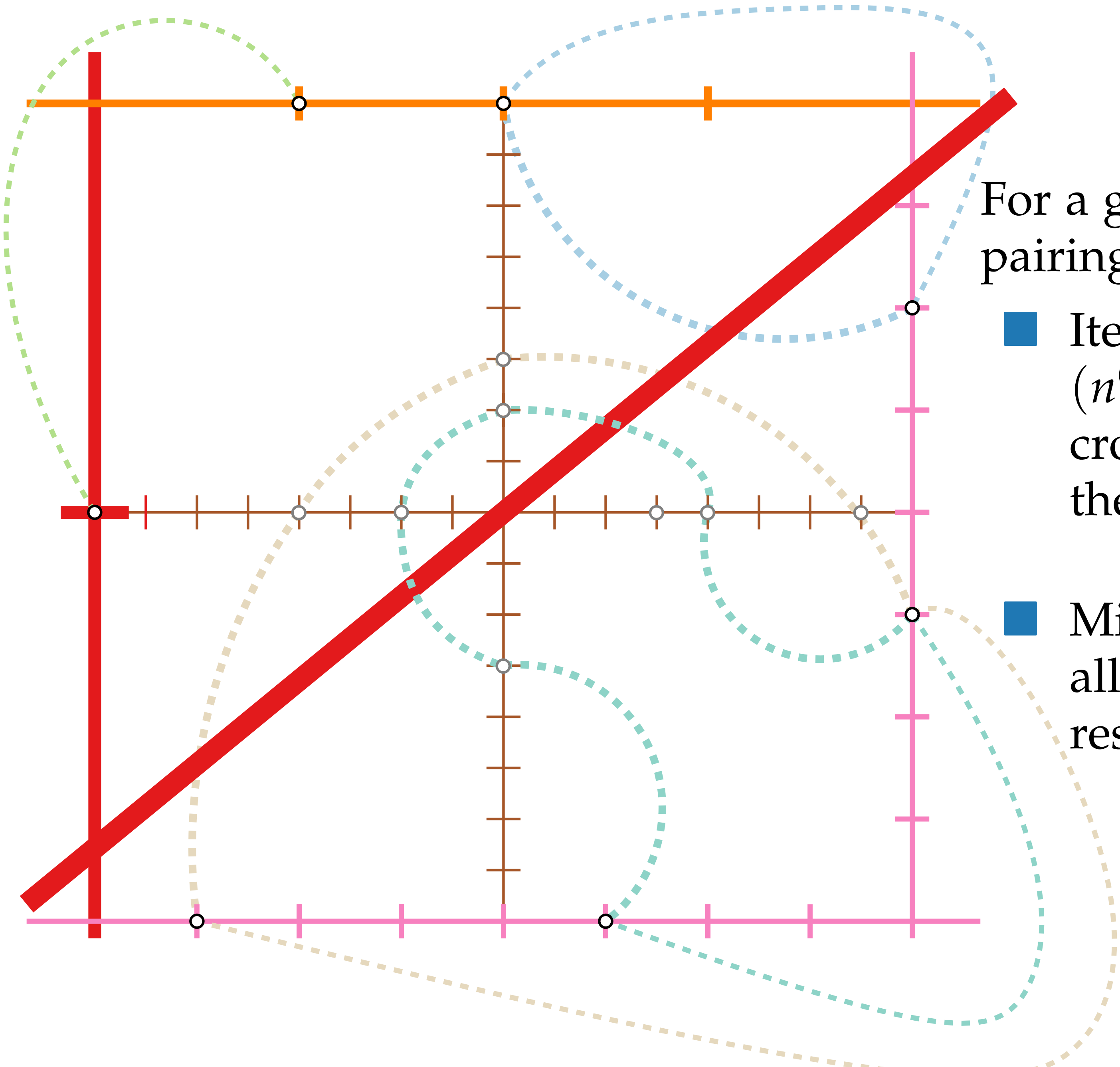

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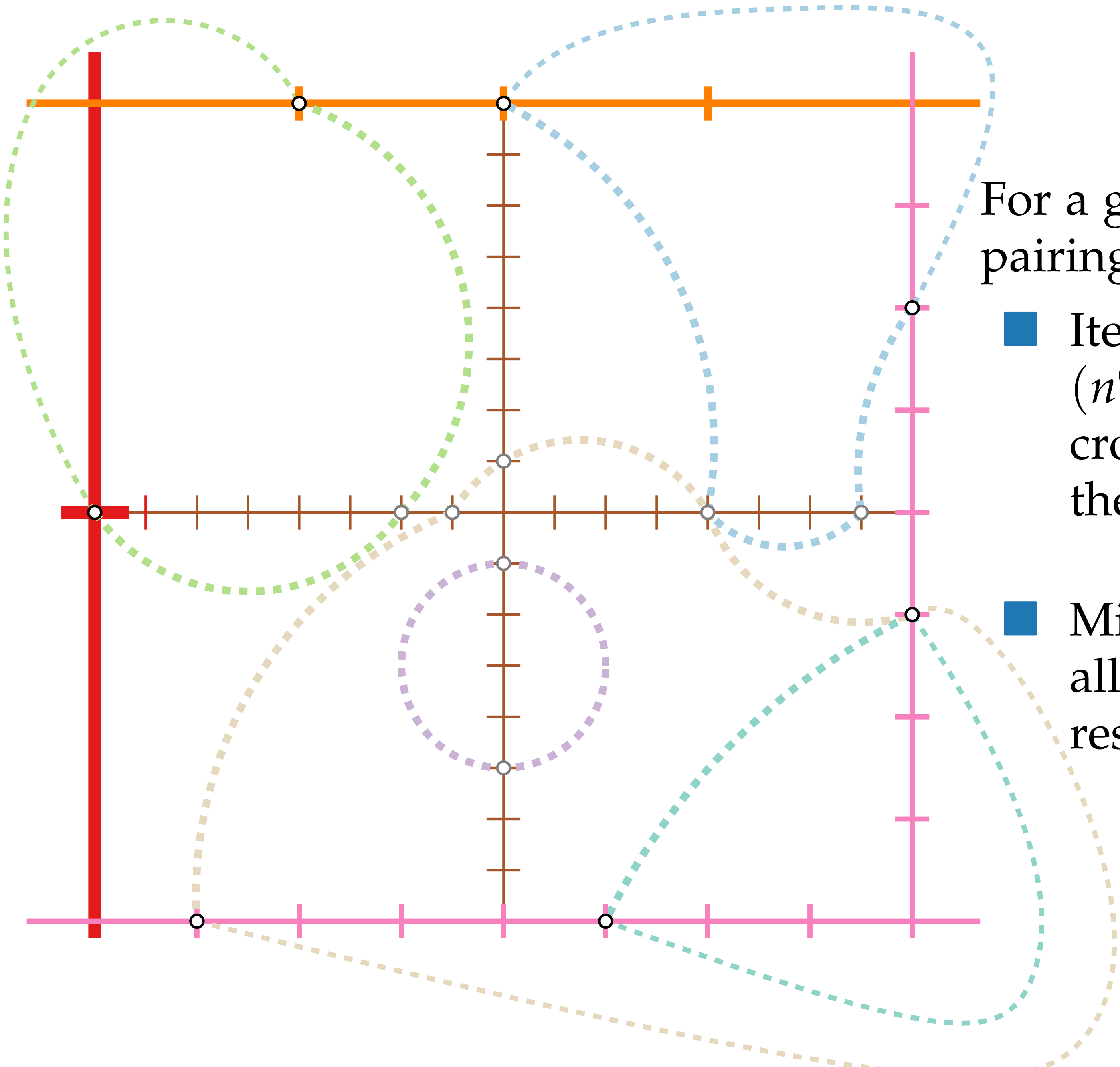

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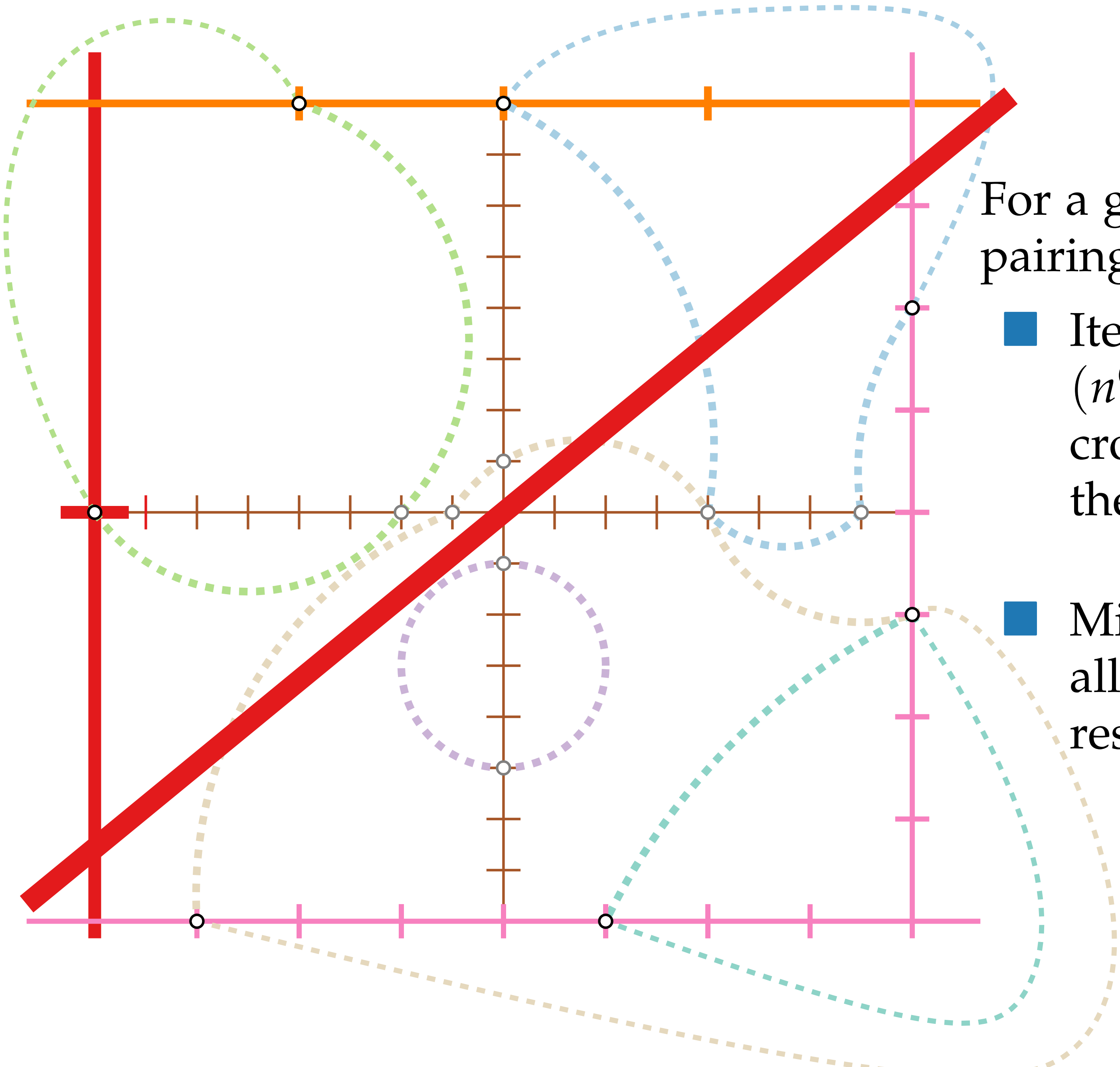

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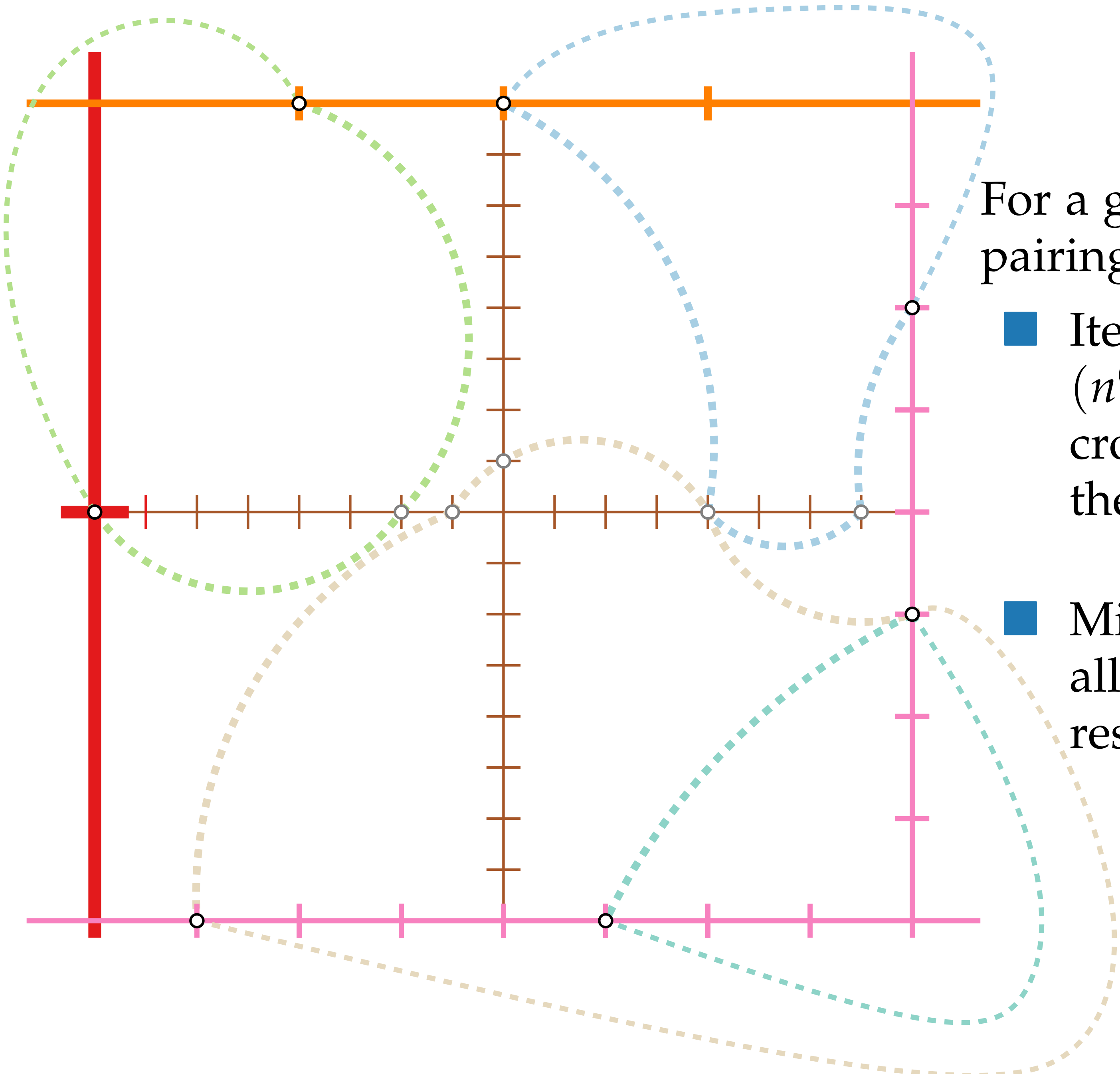

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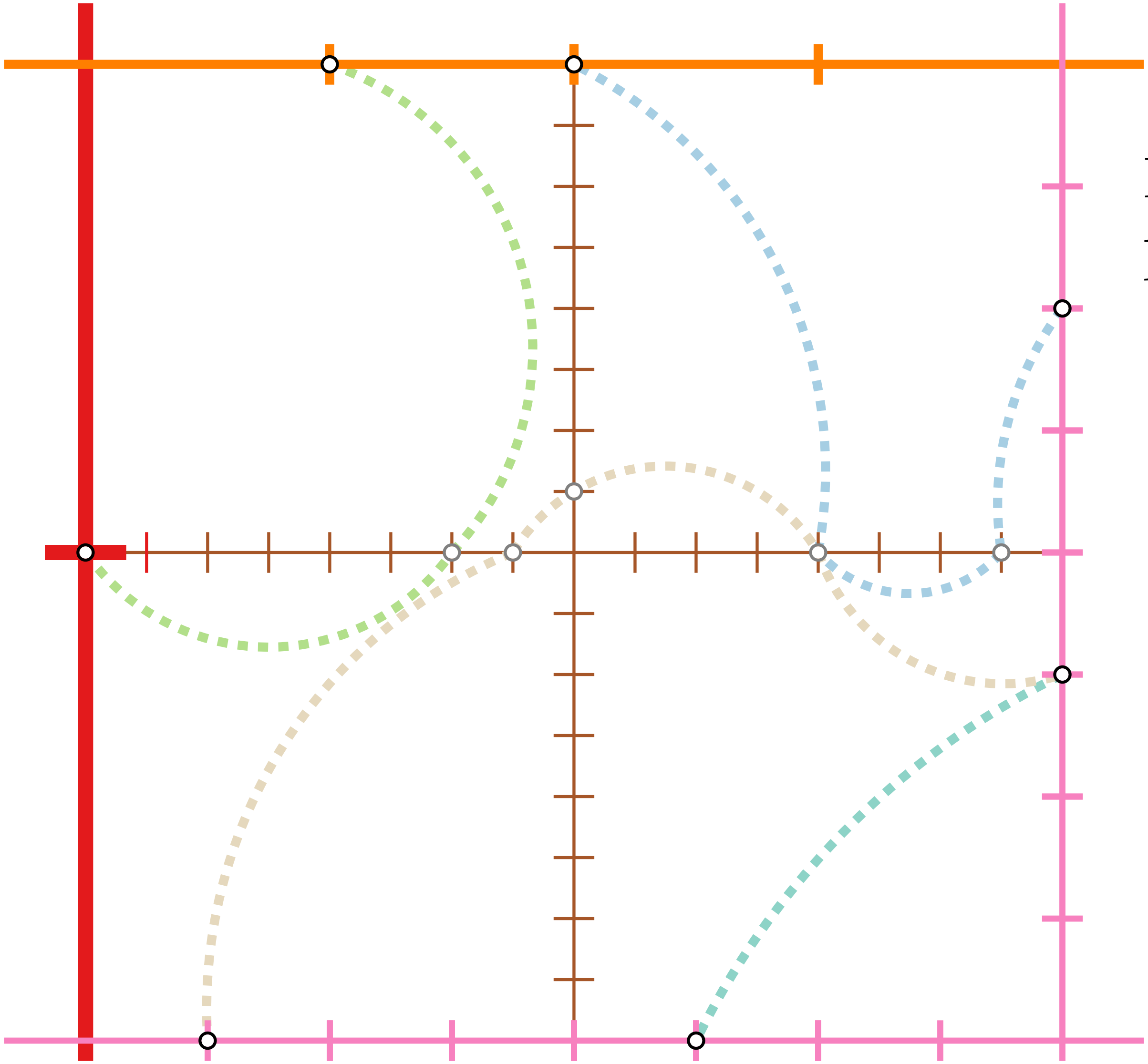

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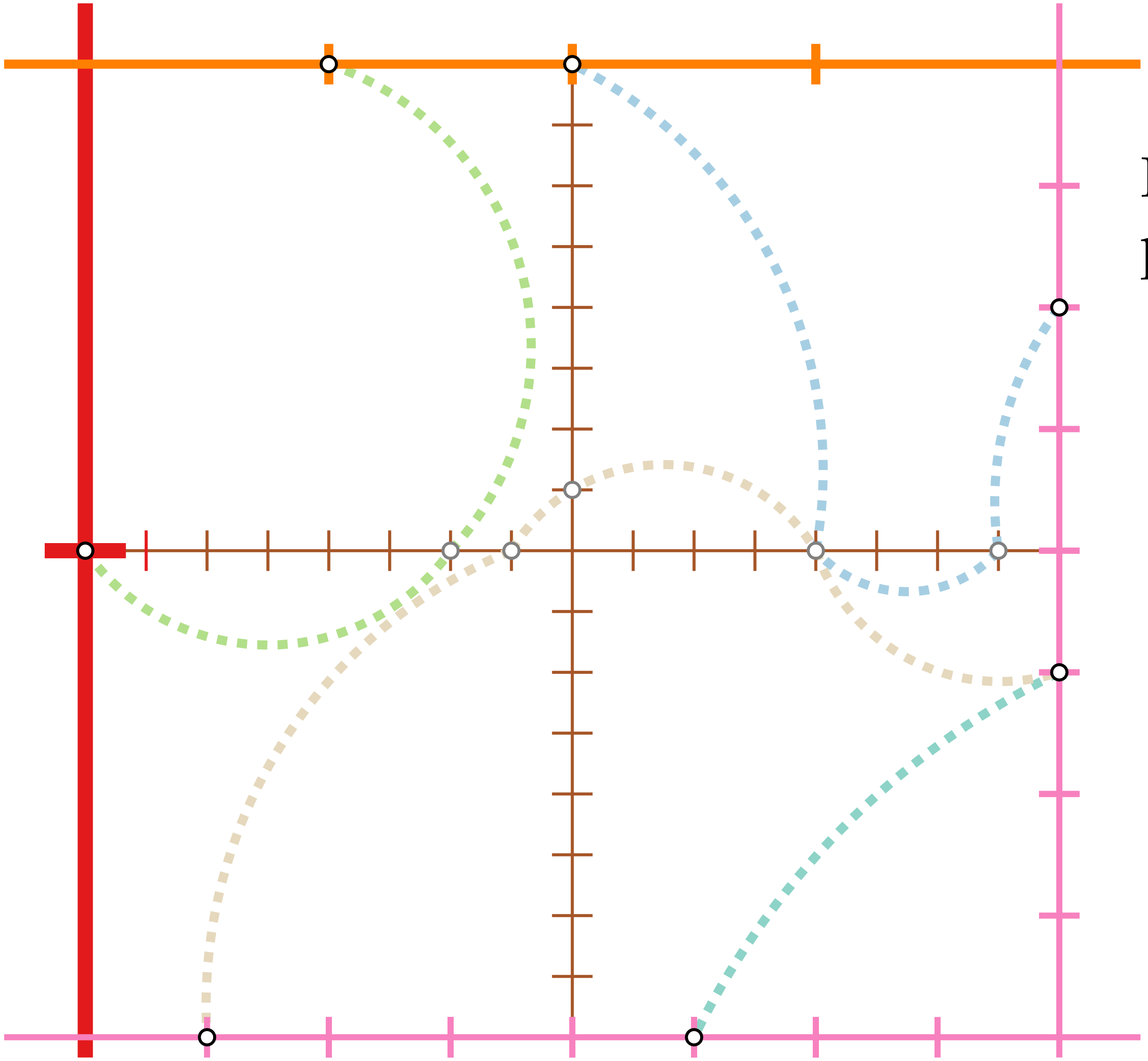

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- Correctness by induction

Lemma. An optimal well behaved tour can be computed in $2^{O(m)} = n^{O(1/\varepsilon)}$ time.

Approximation Algorithms

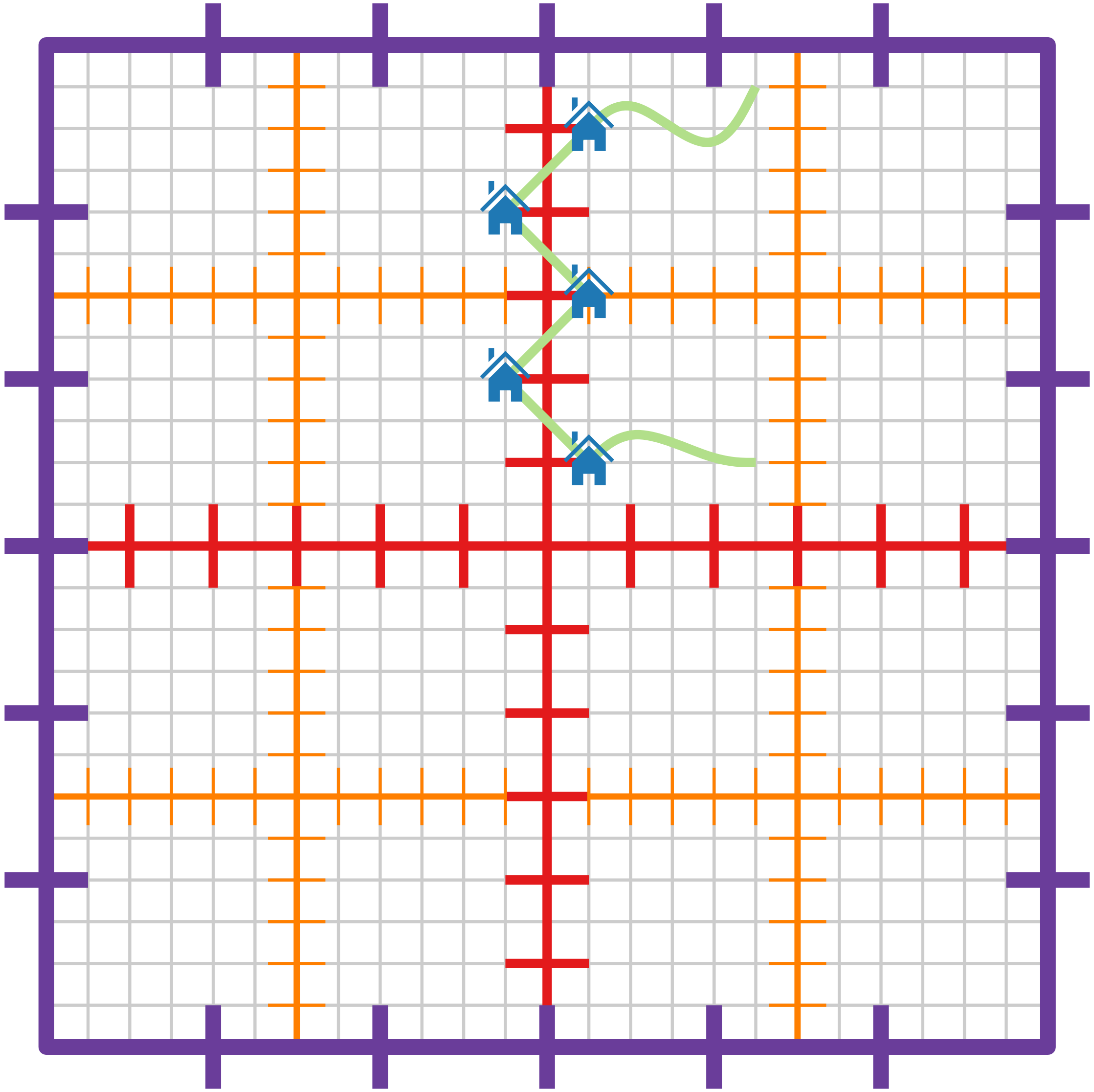
Lecture 9:
PTAS for EuclideanTSP

Part V:
Shifted Dissections

Joachim Spoerhase

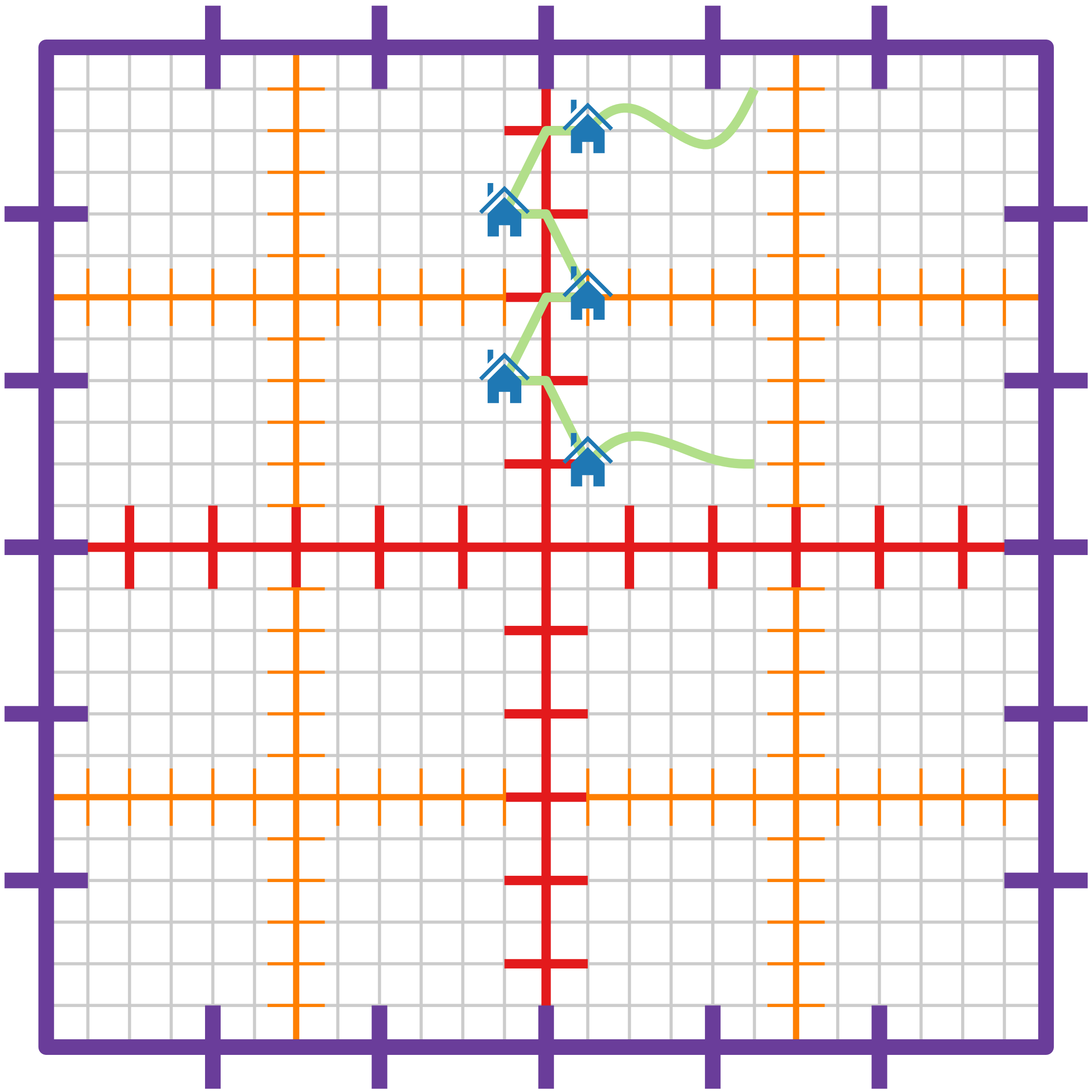
Winter 2021/22

Shifted Dissections



- The best well behaved tour can be a bad approximation.

Shifted Dissections



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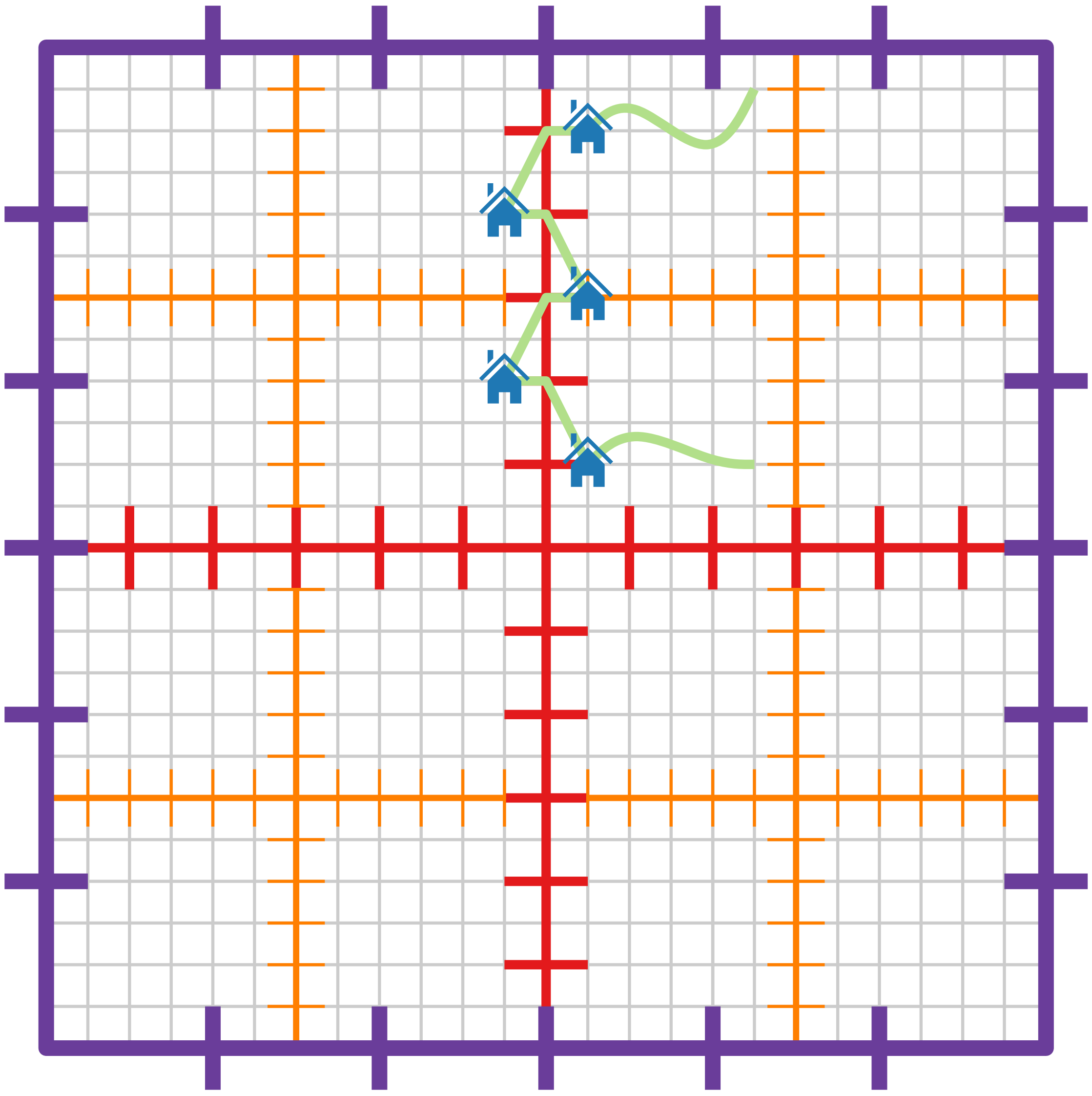
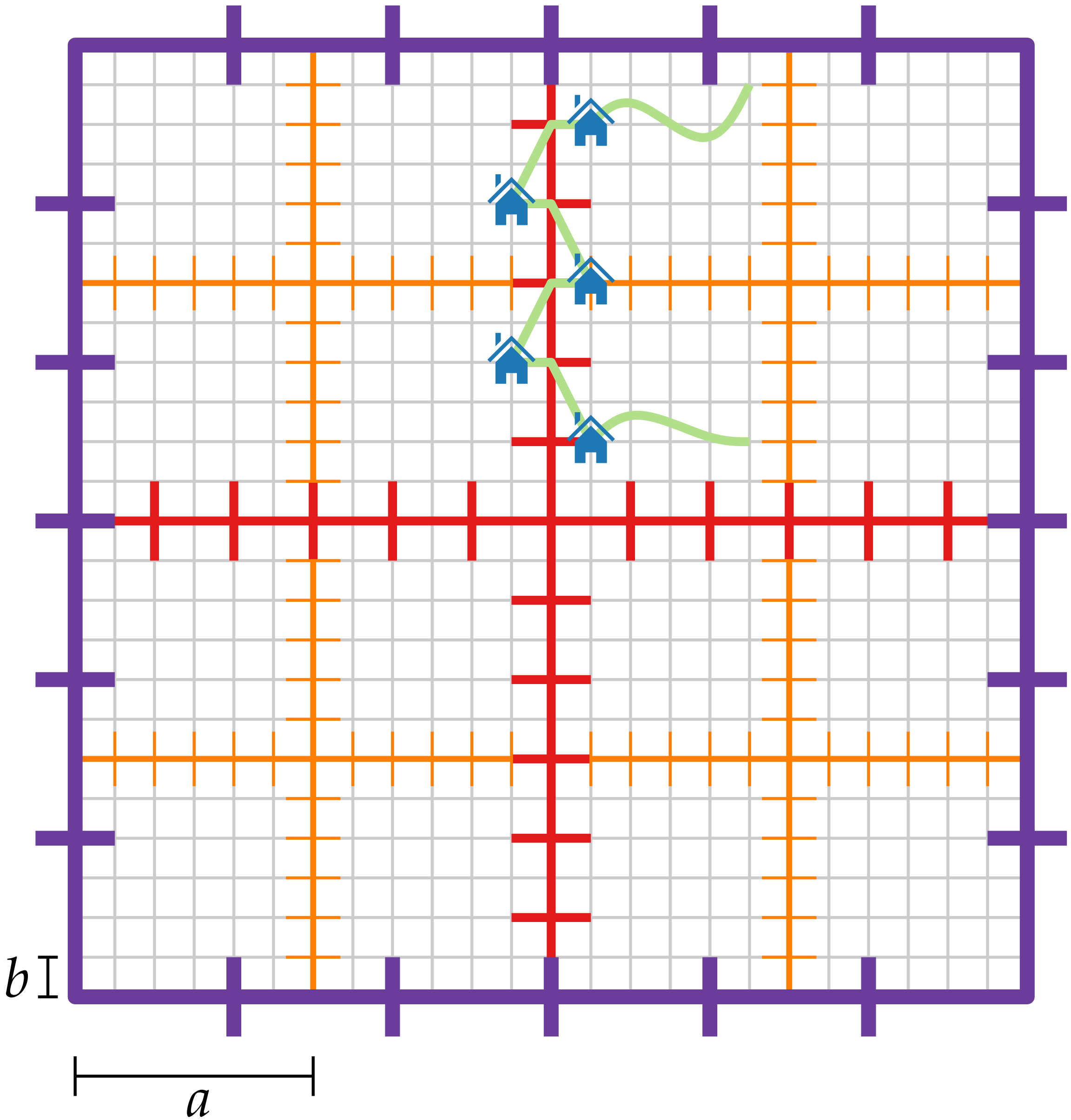


- The best well behaved tour can be a bad approximation.
- Consider an (a, b)-shifted dissection:

$$x \mapsto (x + a) \bmod L$$
$$y \mapsto (y + b) \bmod L$$

Shifted Dissections



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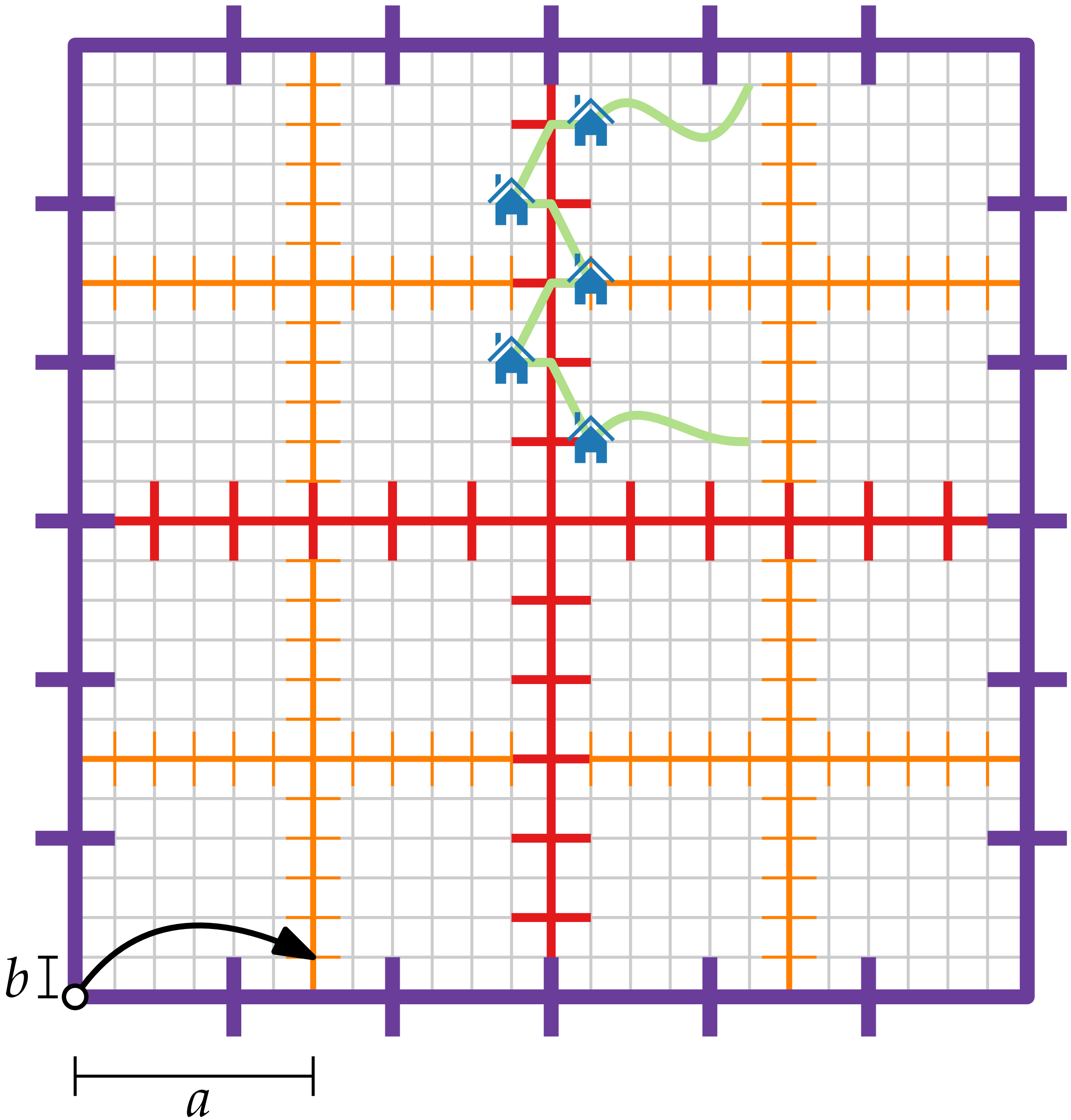

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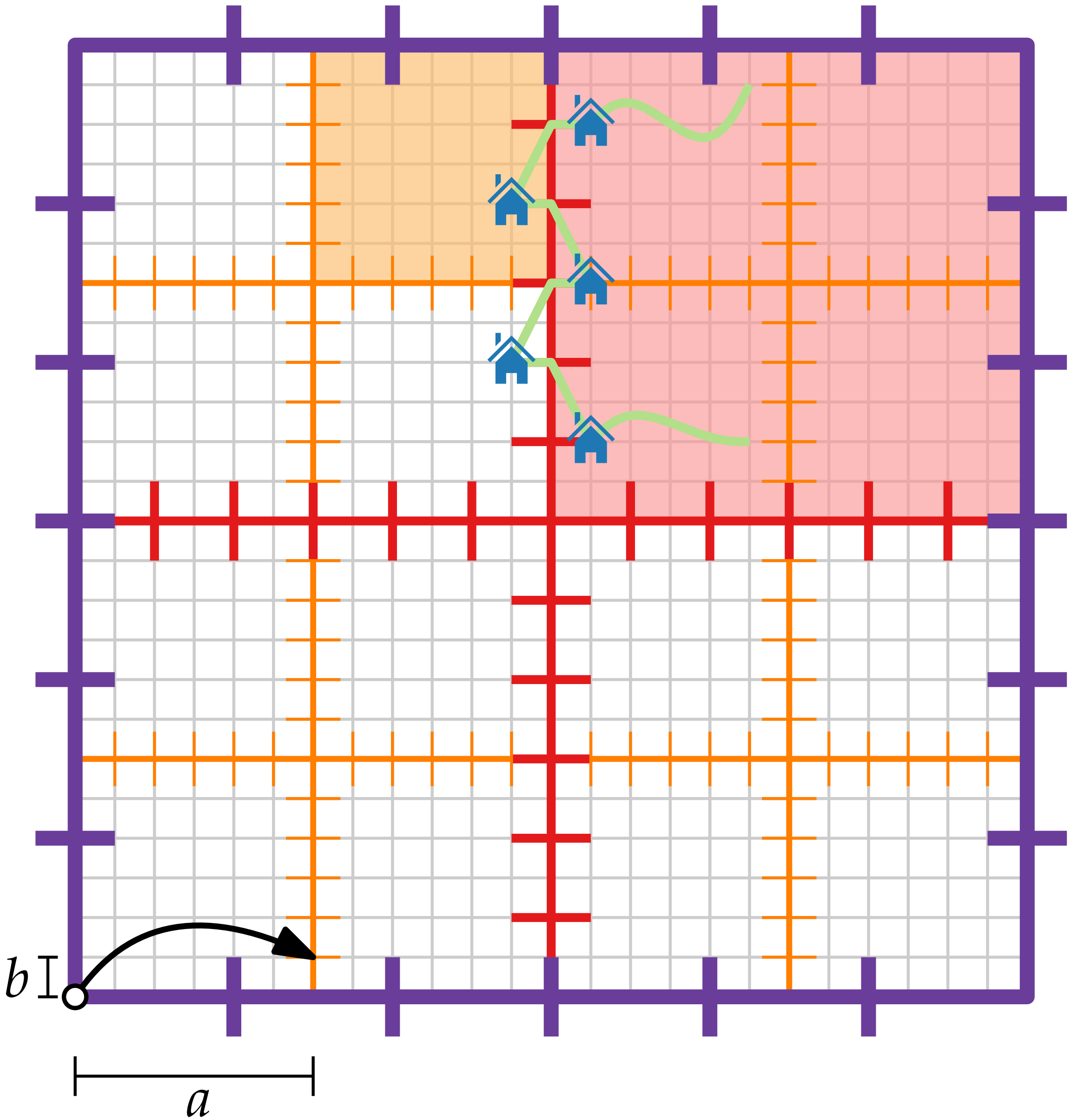

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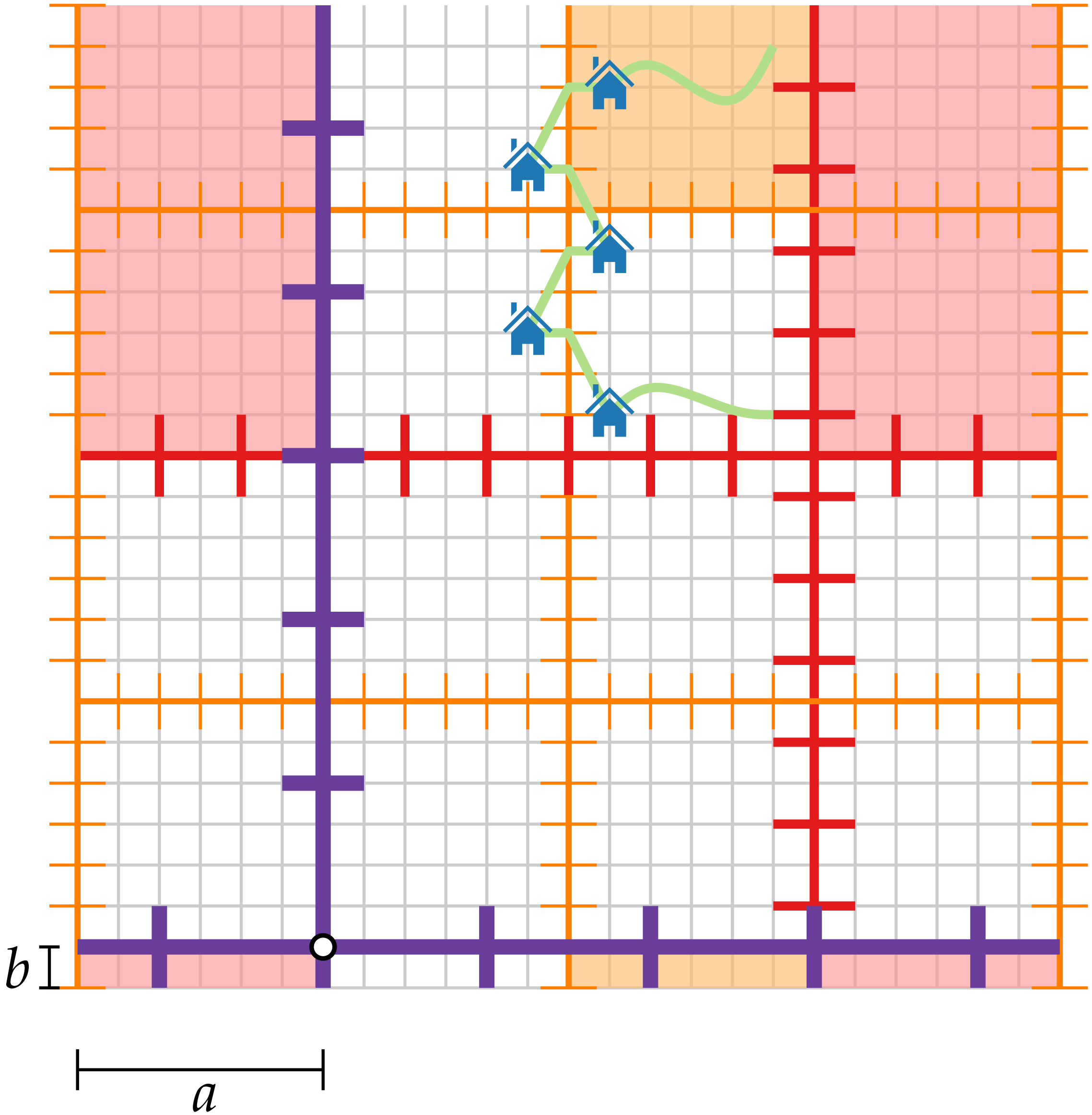

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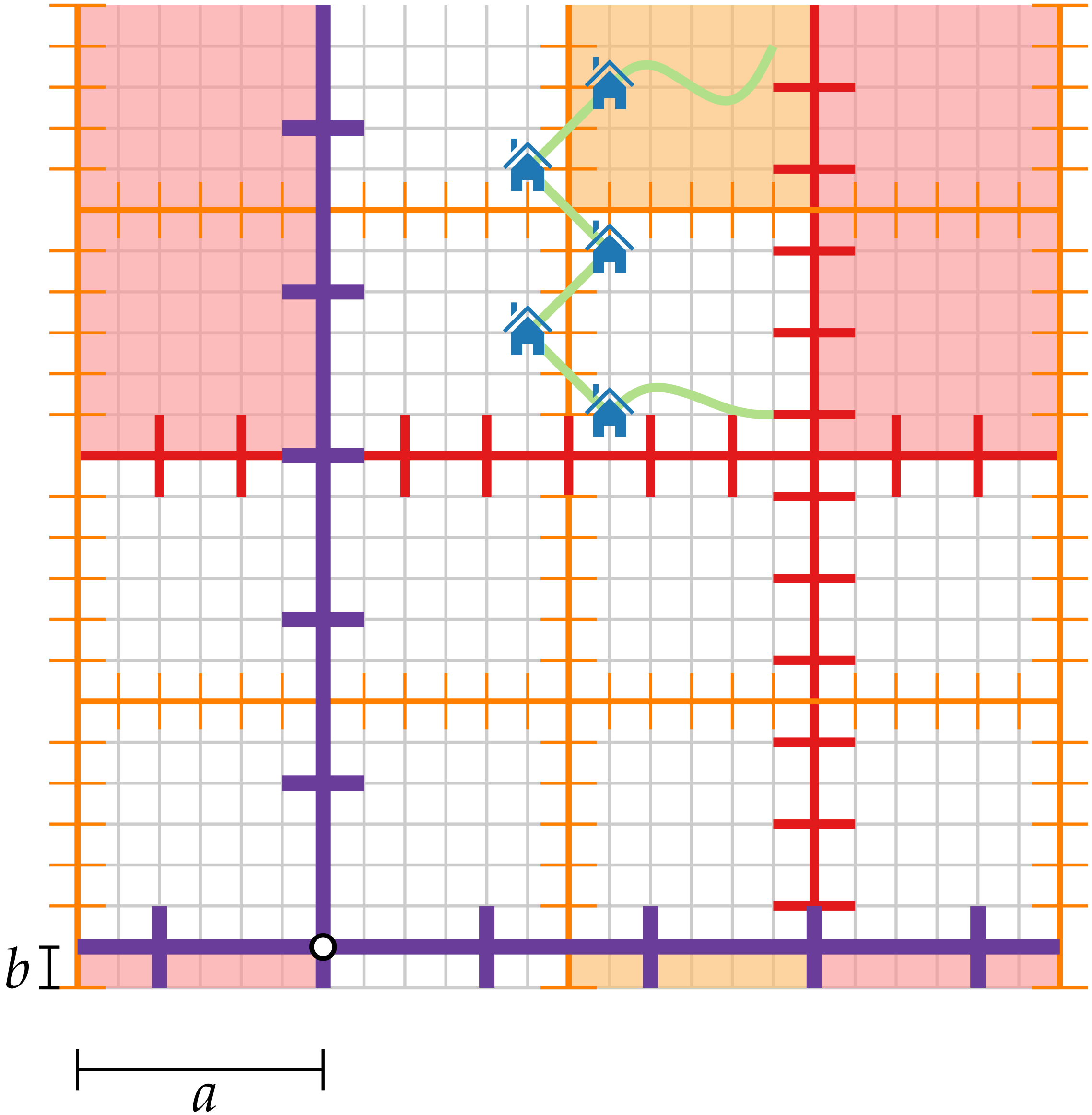

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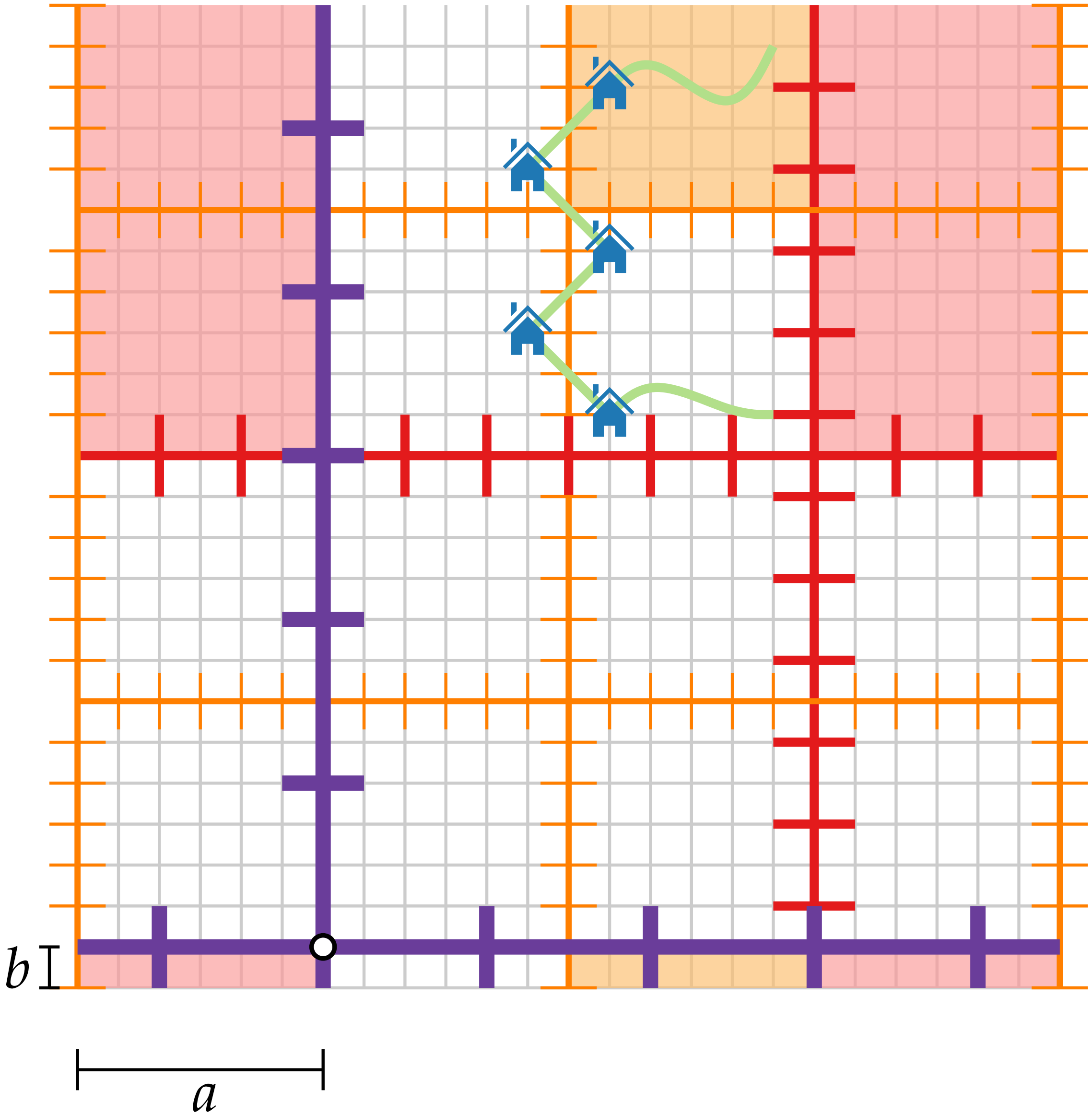

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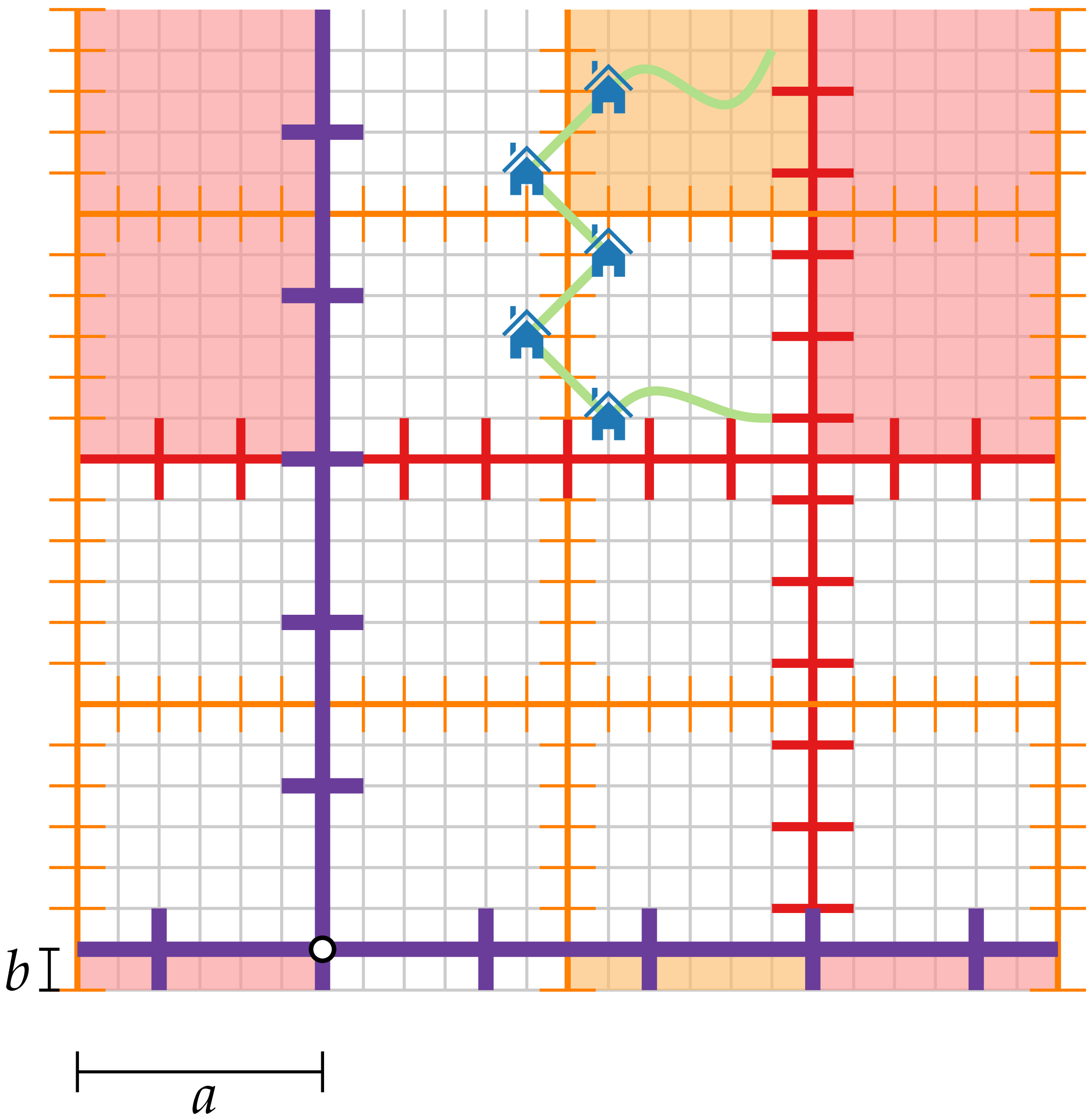

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- Squares in the dissection tree are "wrapped around".
- Dynamic program must be modified accordingly.

Shifted Dissections (II)

Lemma. Let π be an optimal tour and $N(\pi)$ be the number of crossings of π with the lines of the $(L \times L)$-grid.

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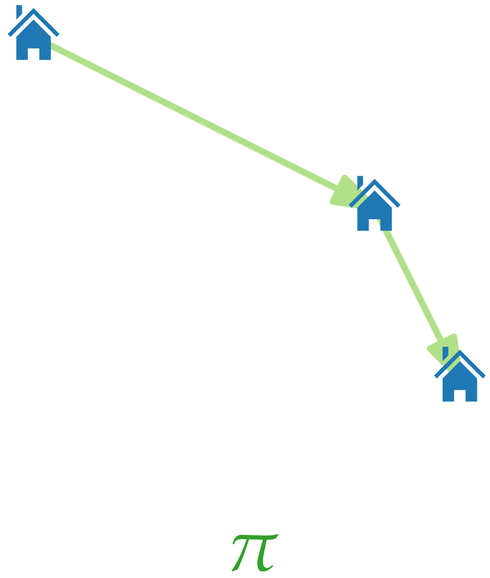

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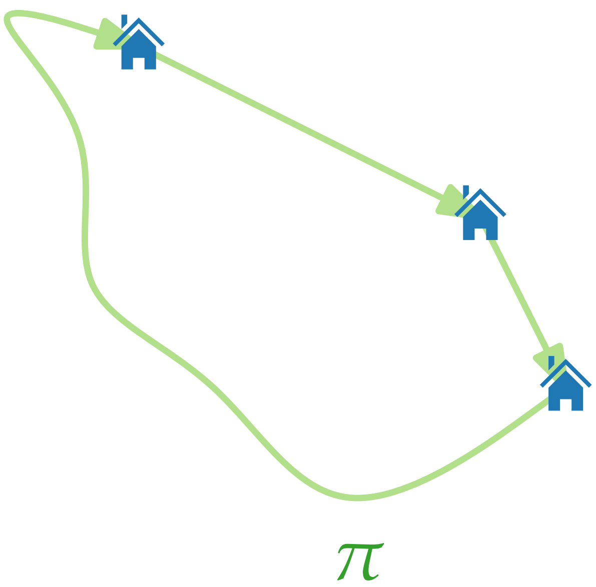

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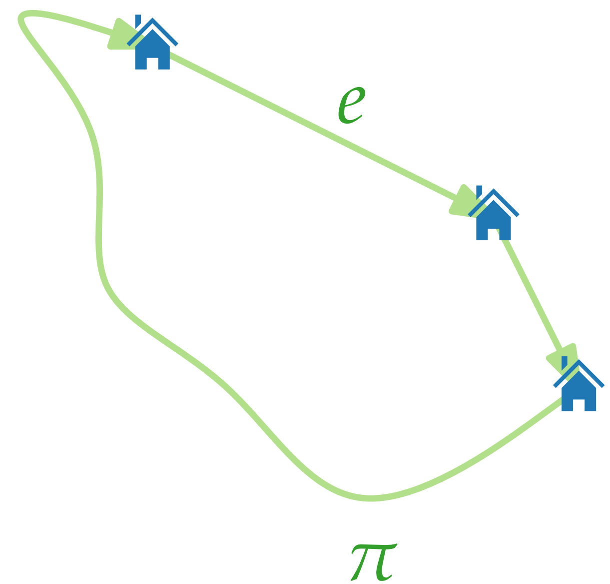

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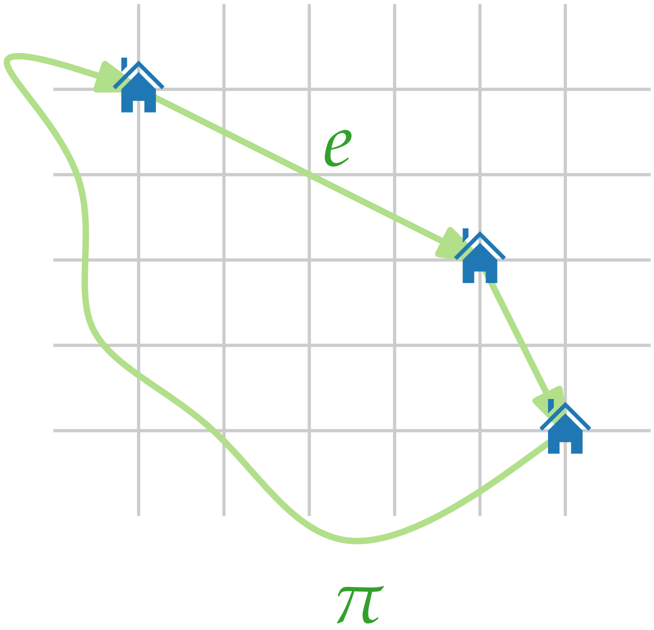

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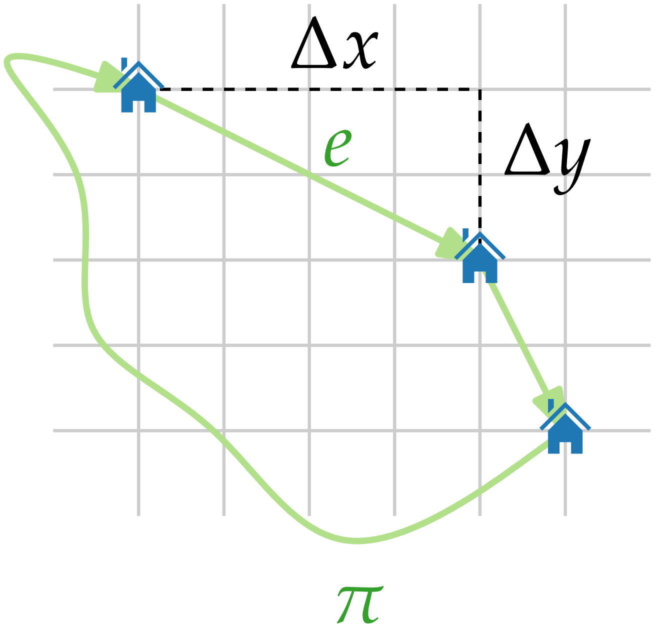

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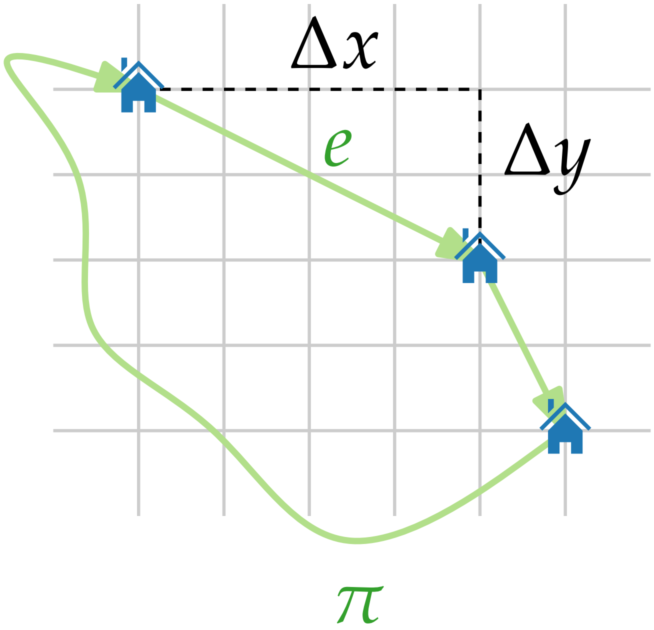

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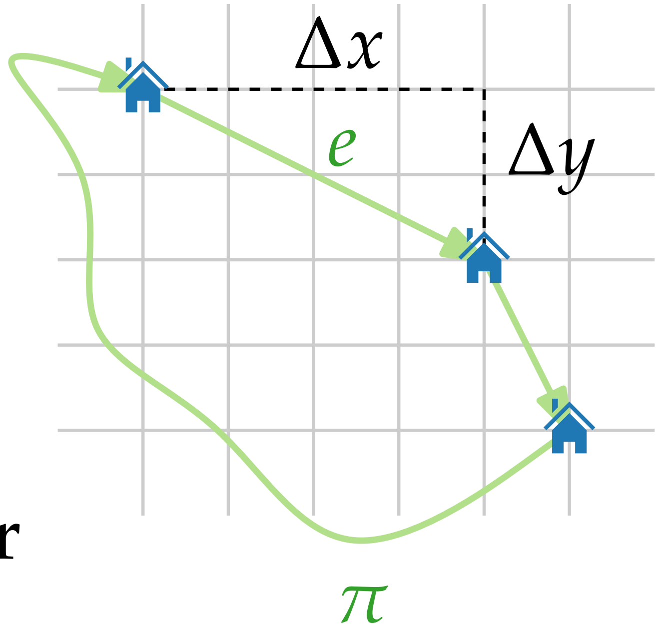

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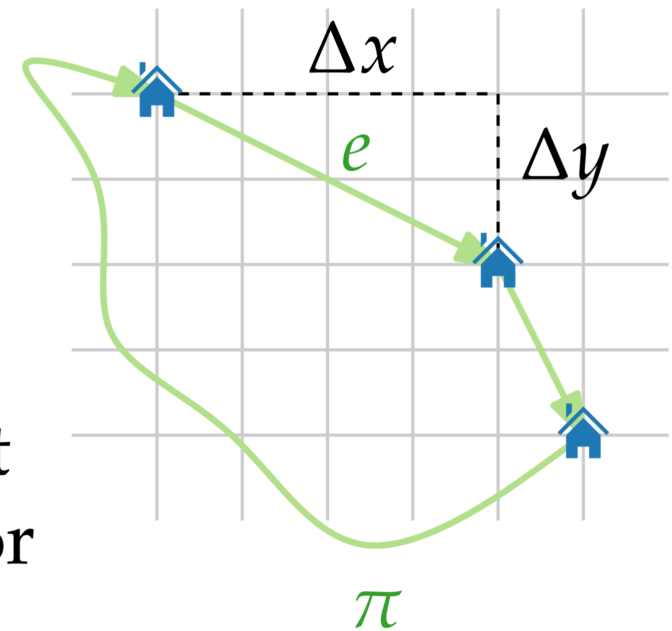

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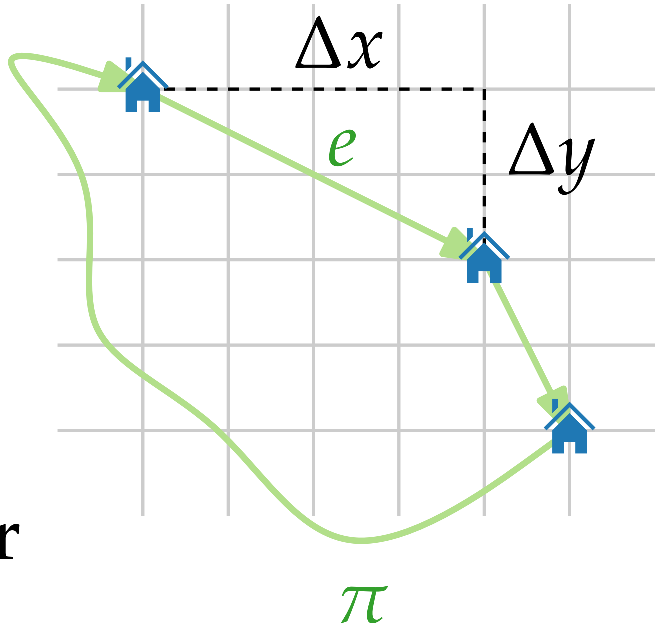

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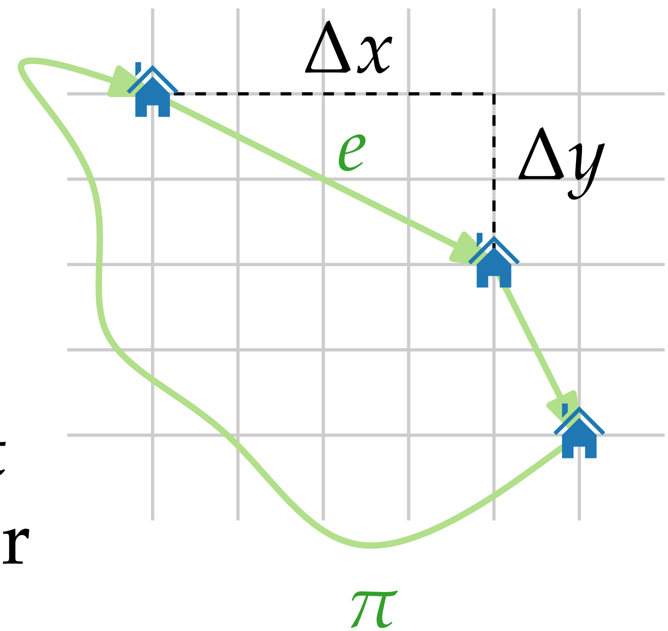

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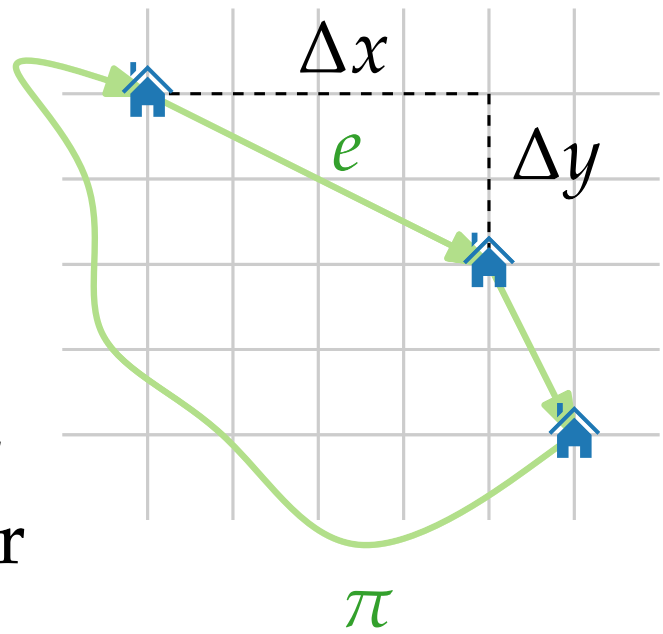

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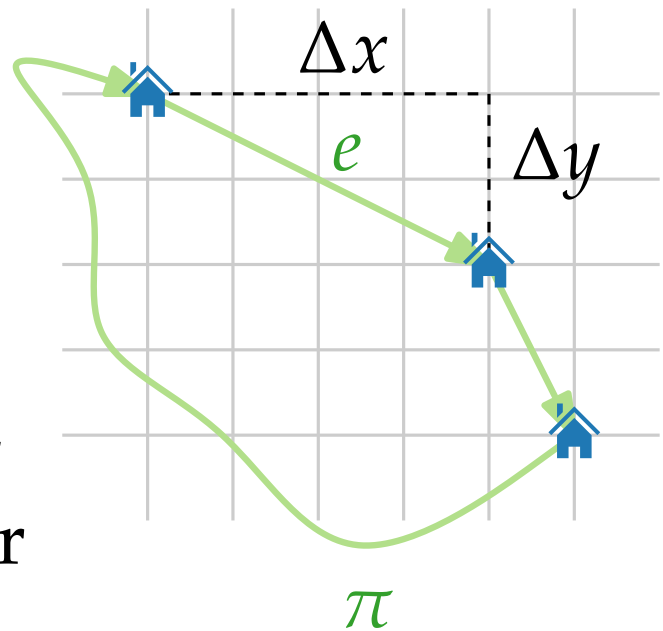

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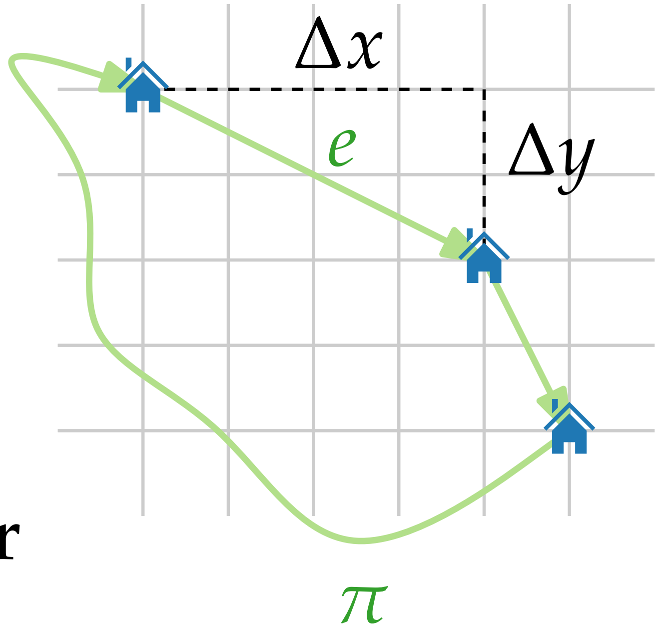

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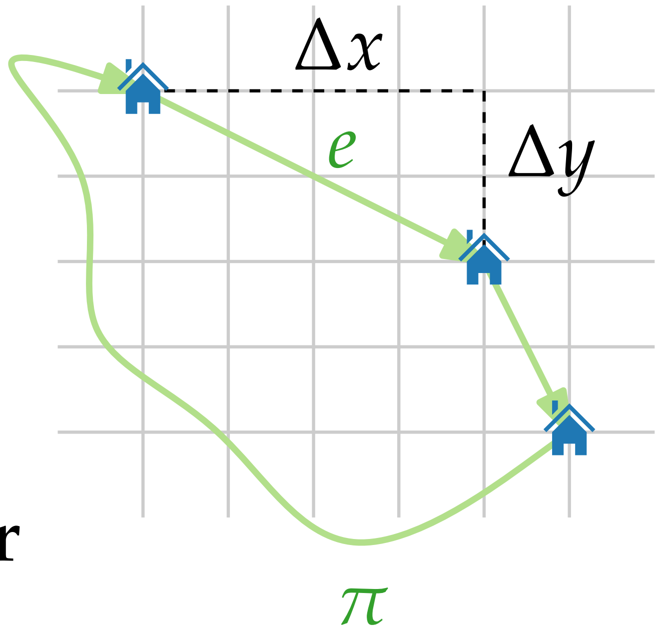

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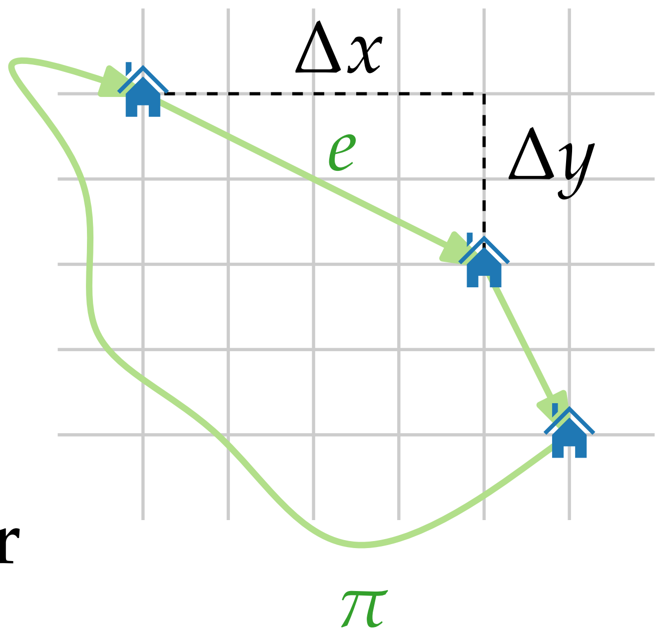

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Approximation Algorithms

Lecture 9:
PTAS for EuclideanTSP

Part VI:
Approximation Factor

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Shifted Dissections (III)

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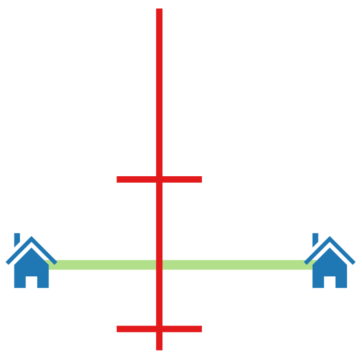

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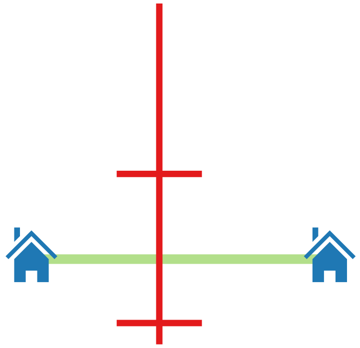

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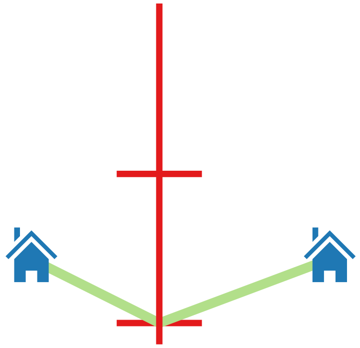

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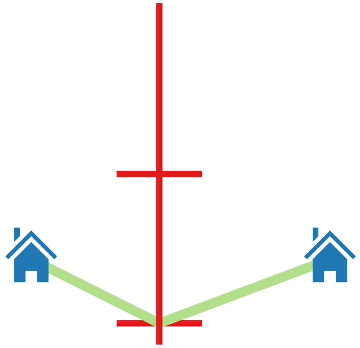

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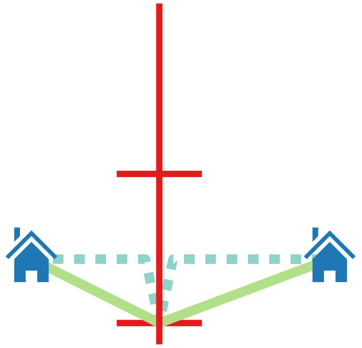

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- Summing over all $N(\pi) \leq \sqrt{2} \cdot \mathrm{OPT}$ intersection points, and applying linearity of expectation, provides the claim.

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Proof. Try all L^2 many (a, b)-shifted dissections. By the previous theorem, one of them is good enough. □