Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part I:
SetCover

Joachim Spoerhase

Winter 2021/22

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Given a **ground set** U

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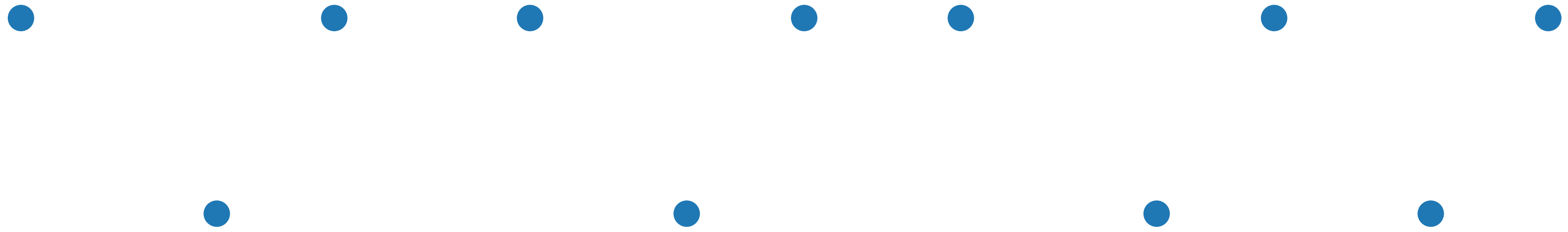

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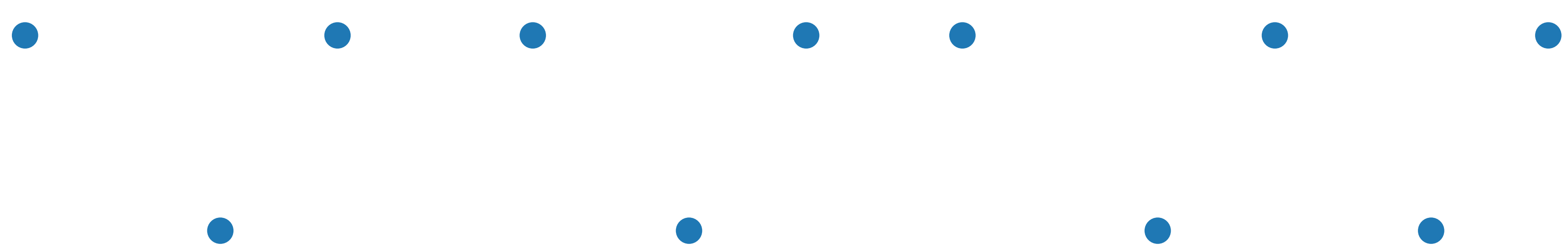

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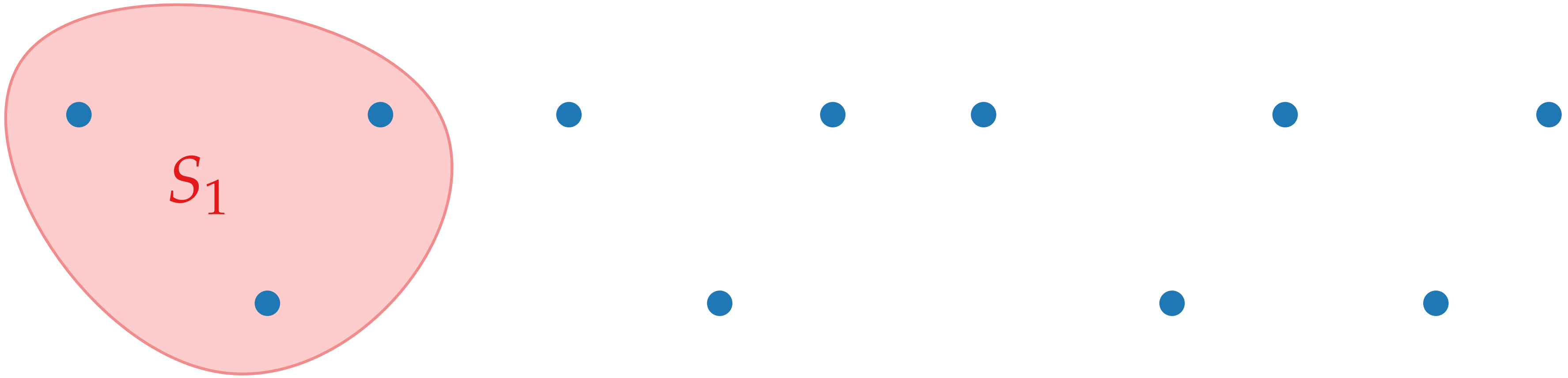

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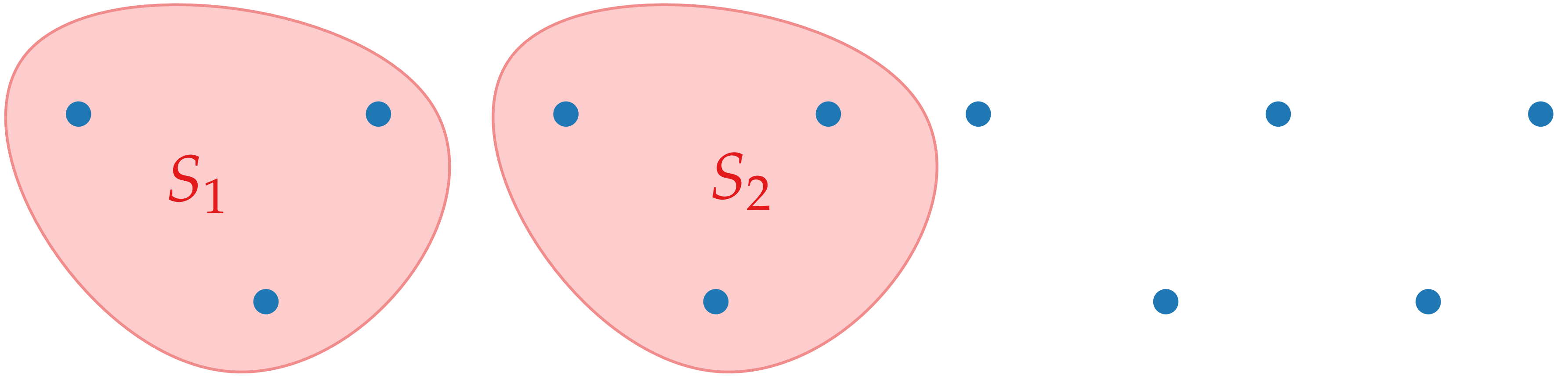

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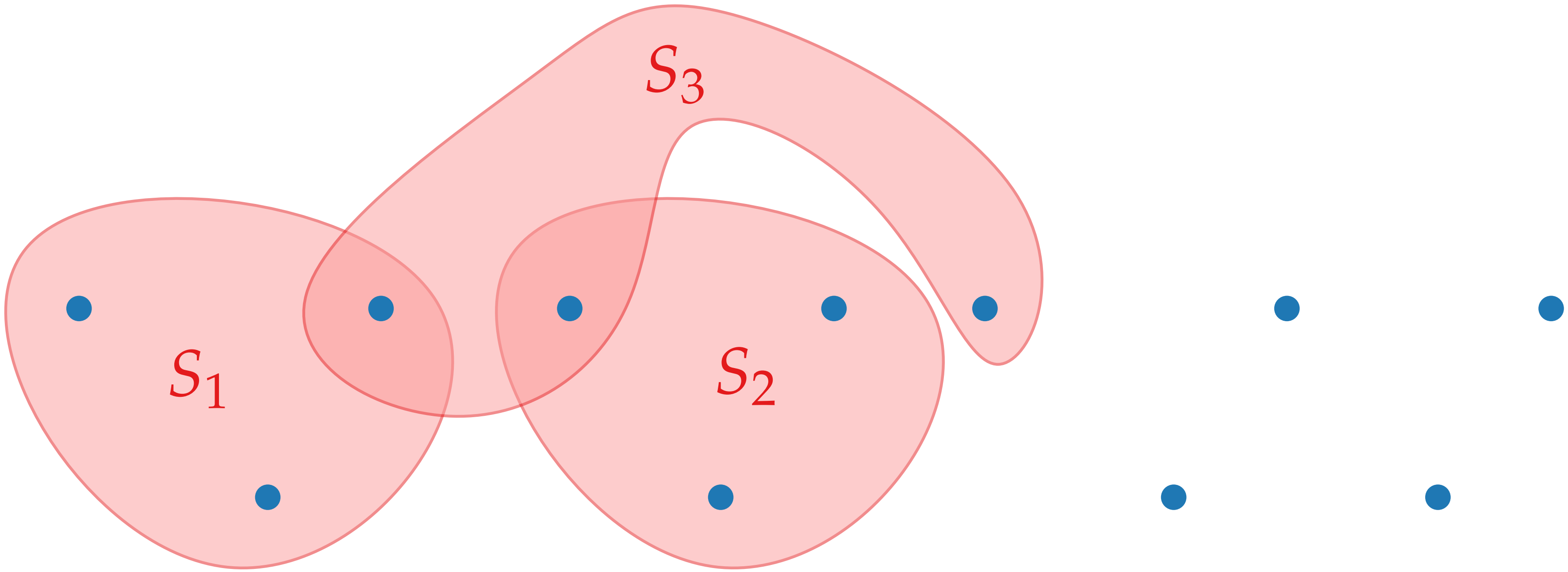

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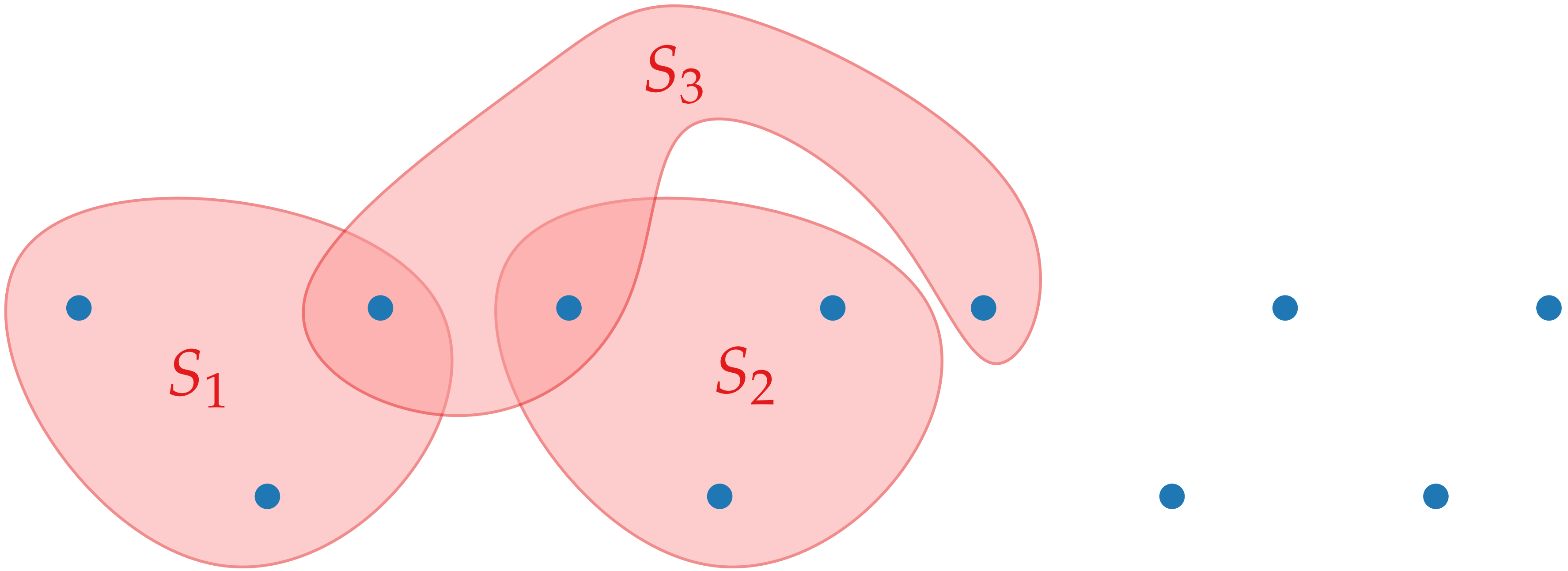

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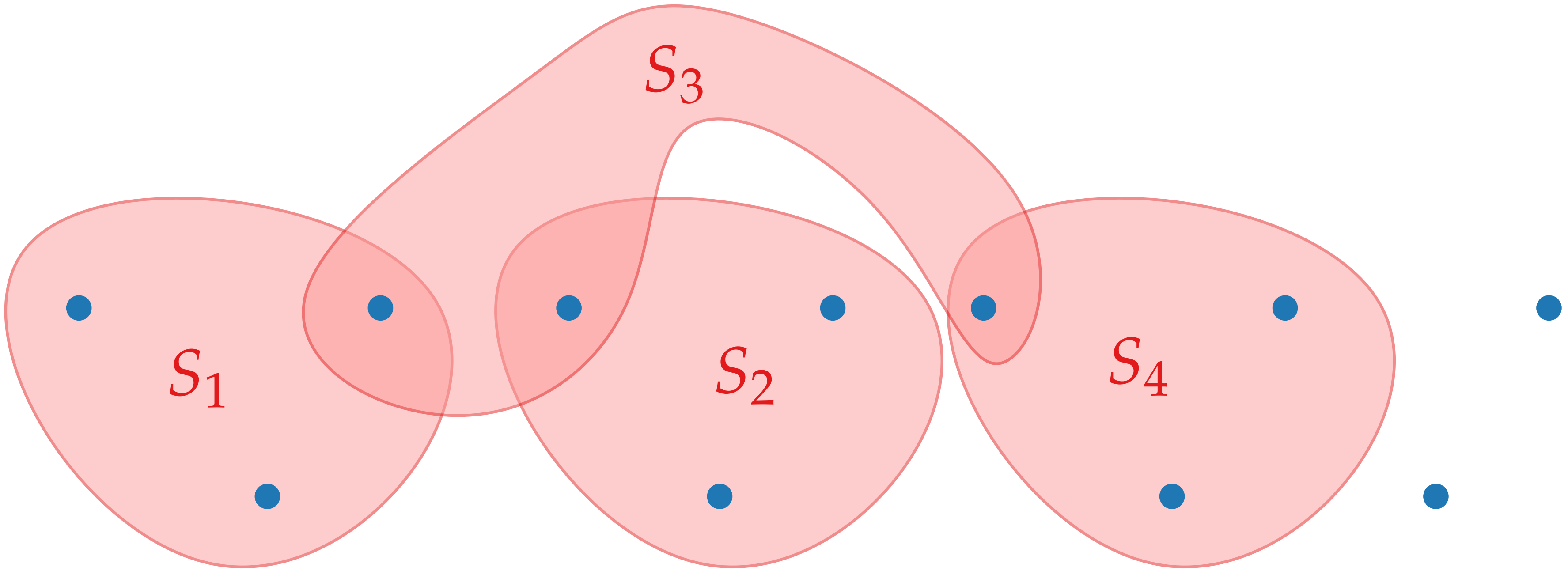

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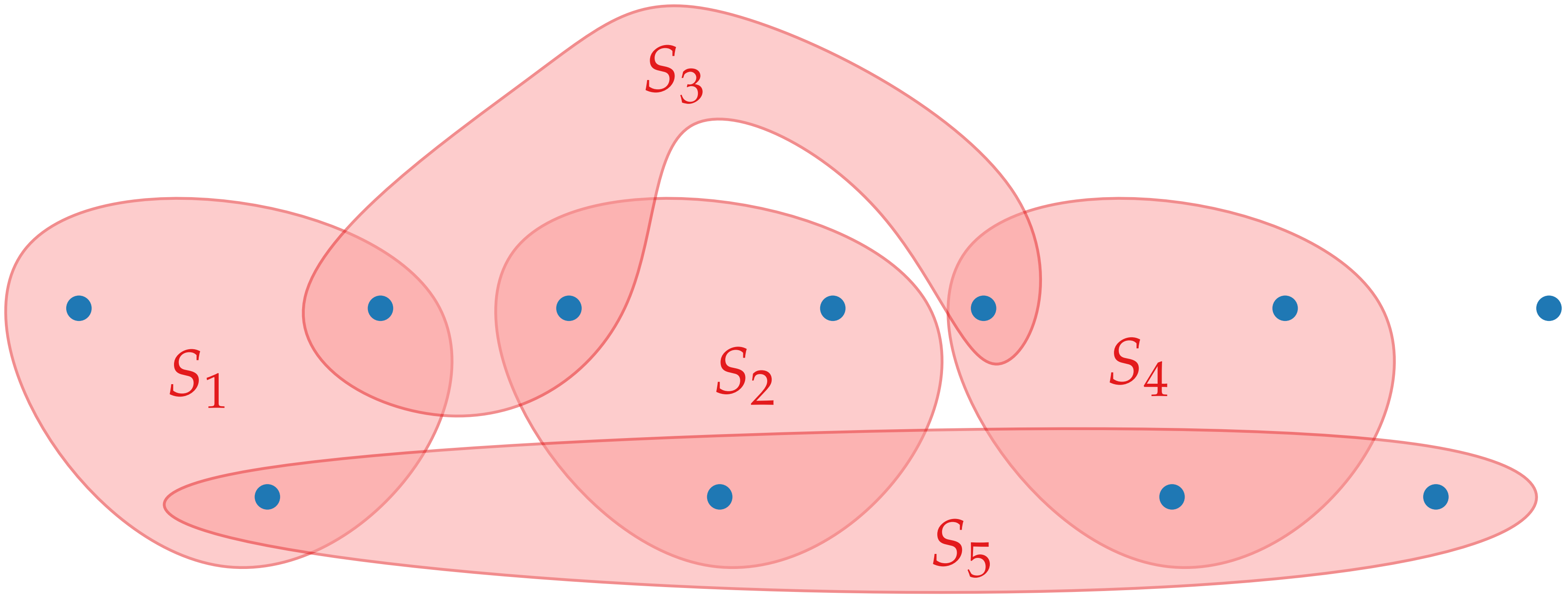

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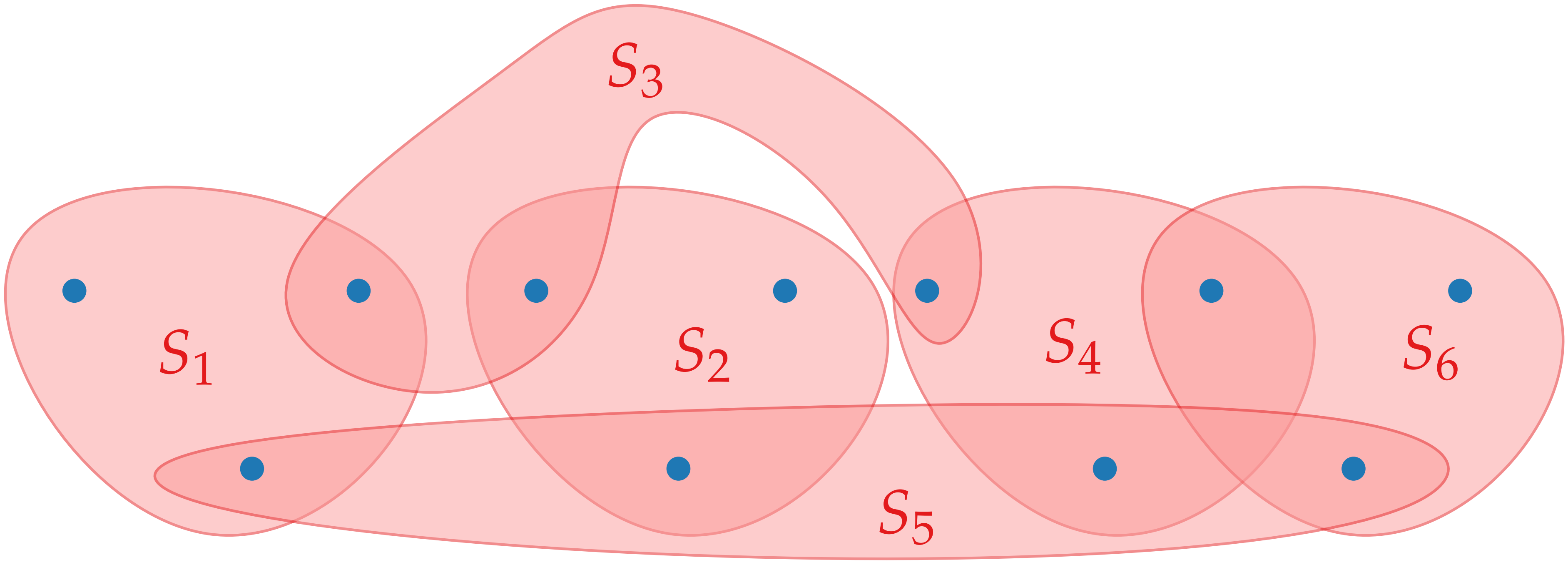

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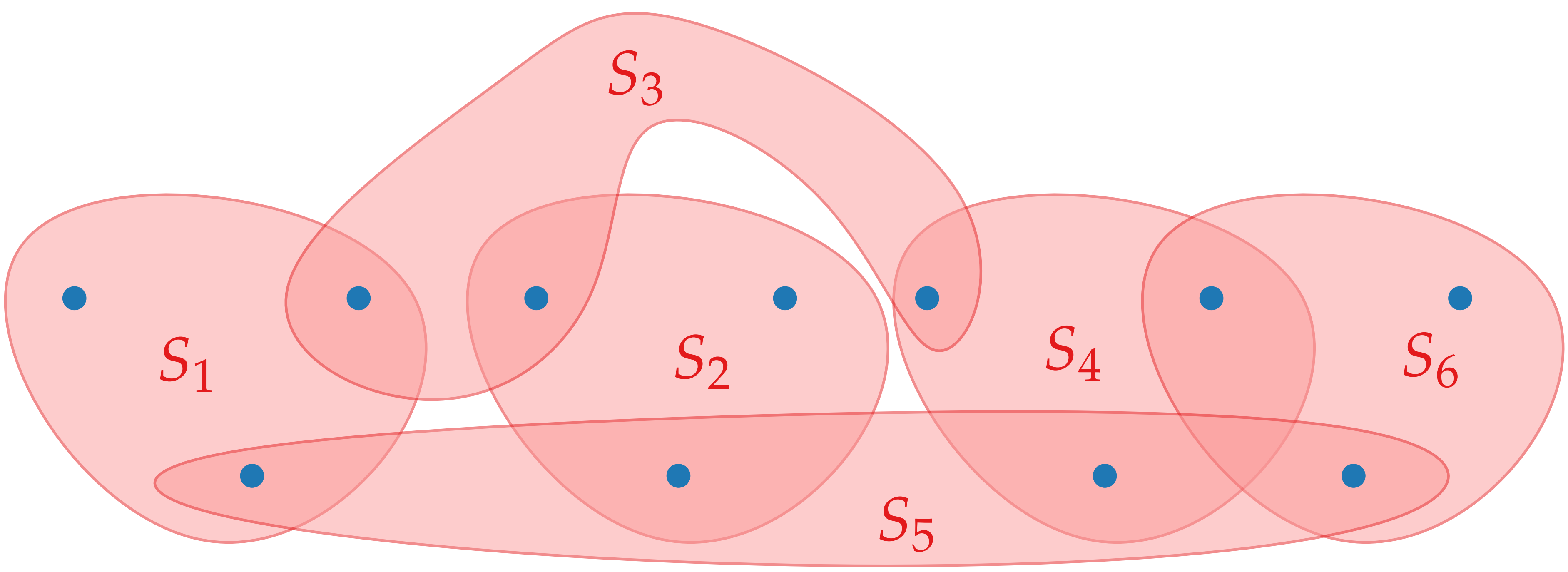

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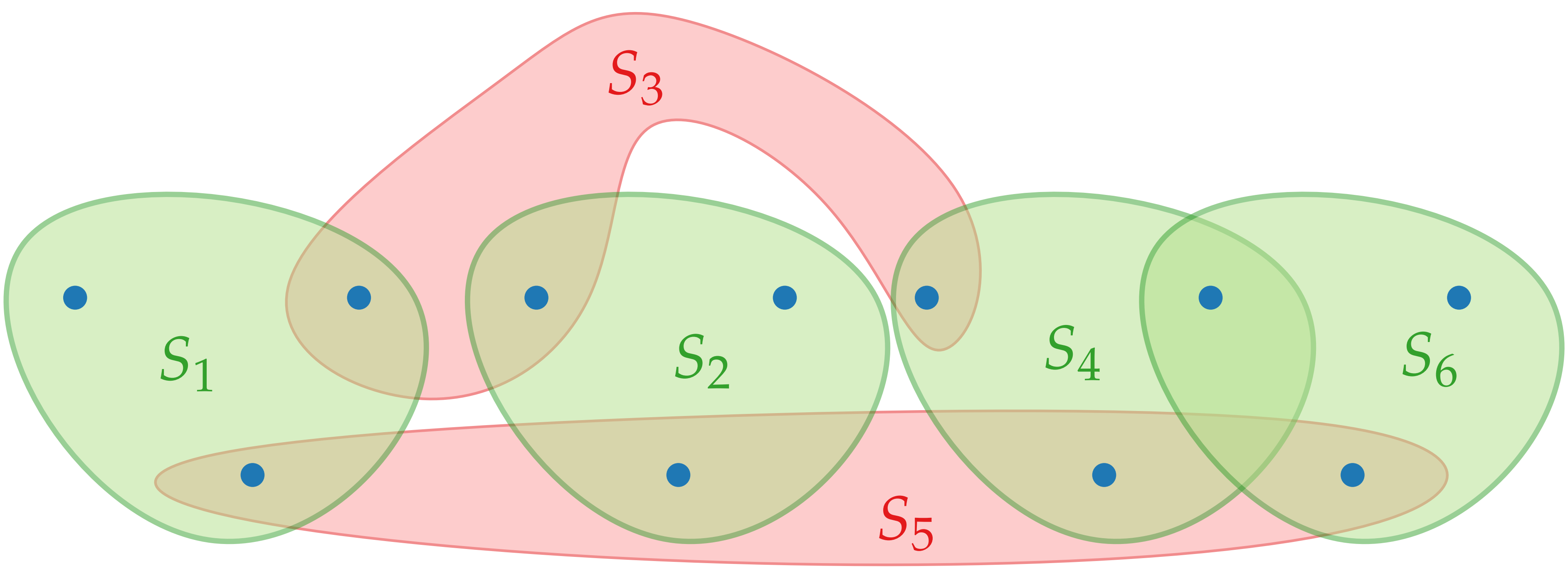

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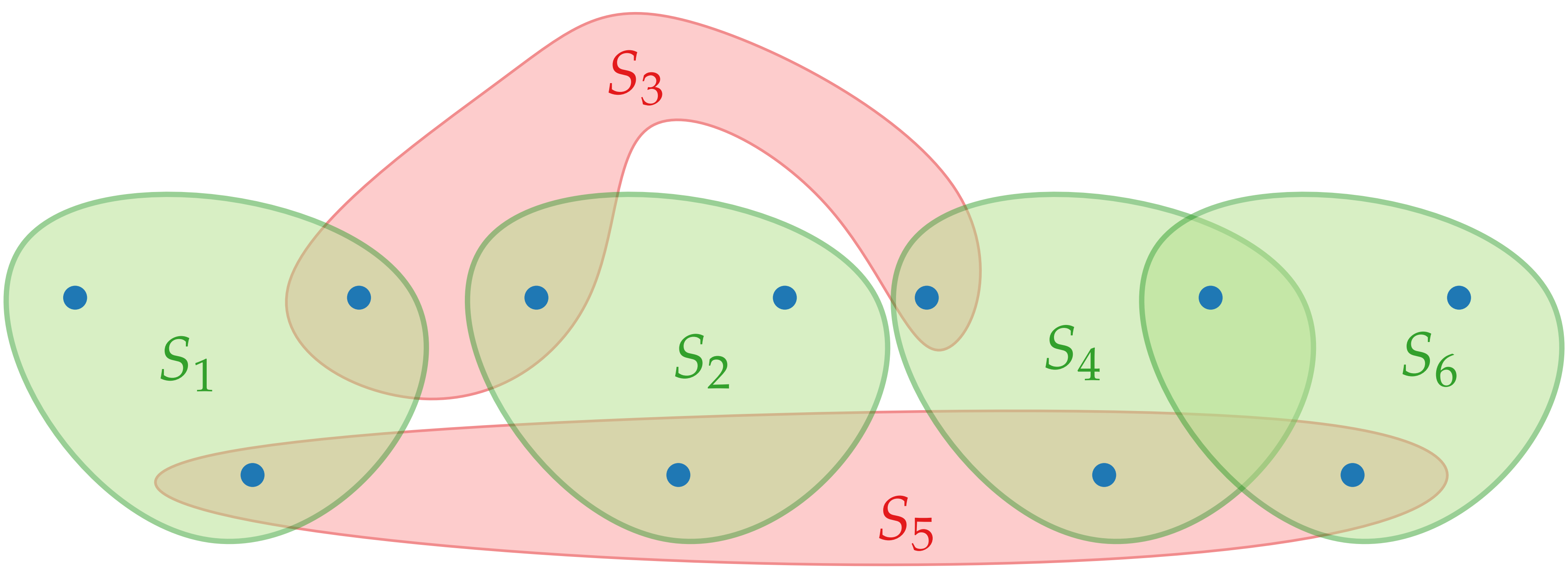

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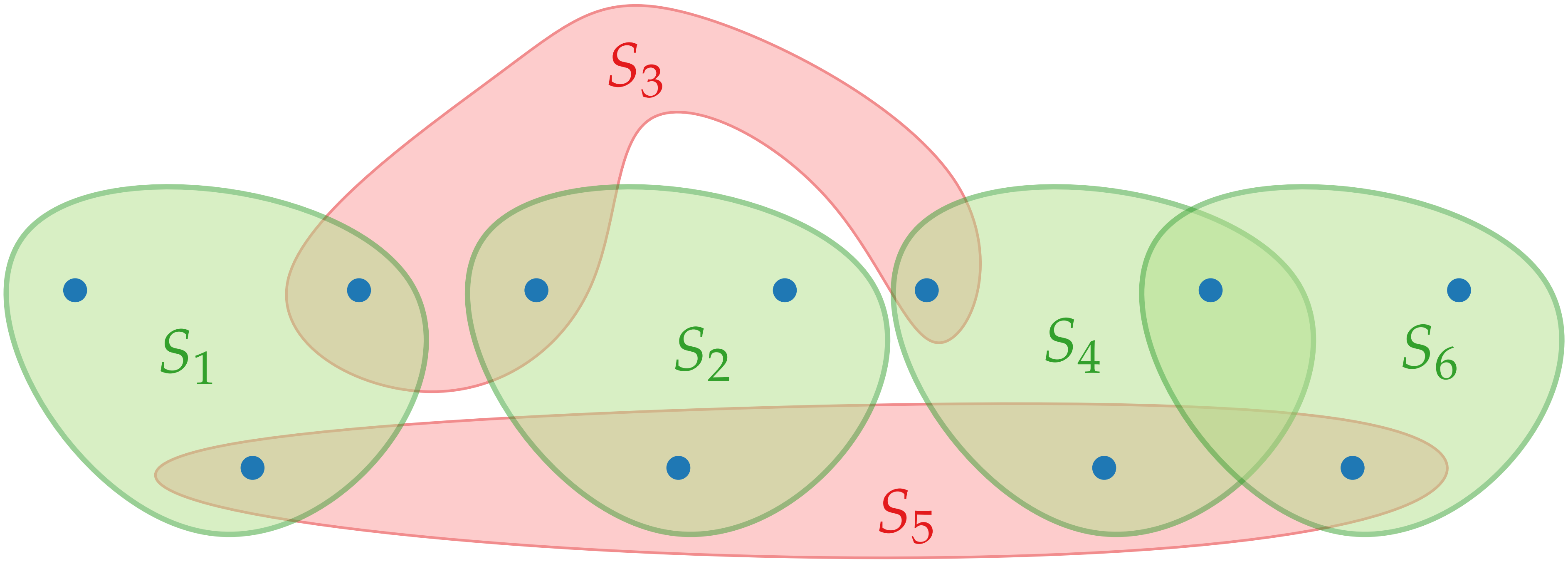

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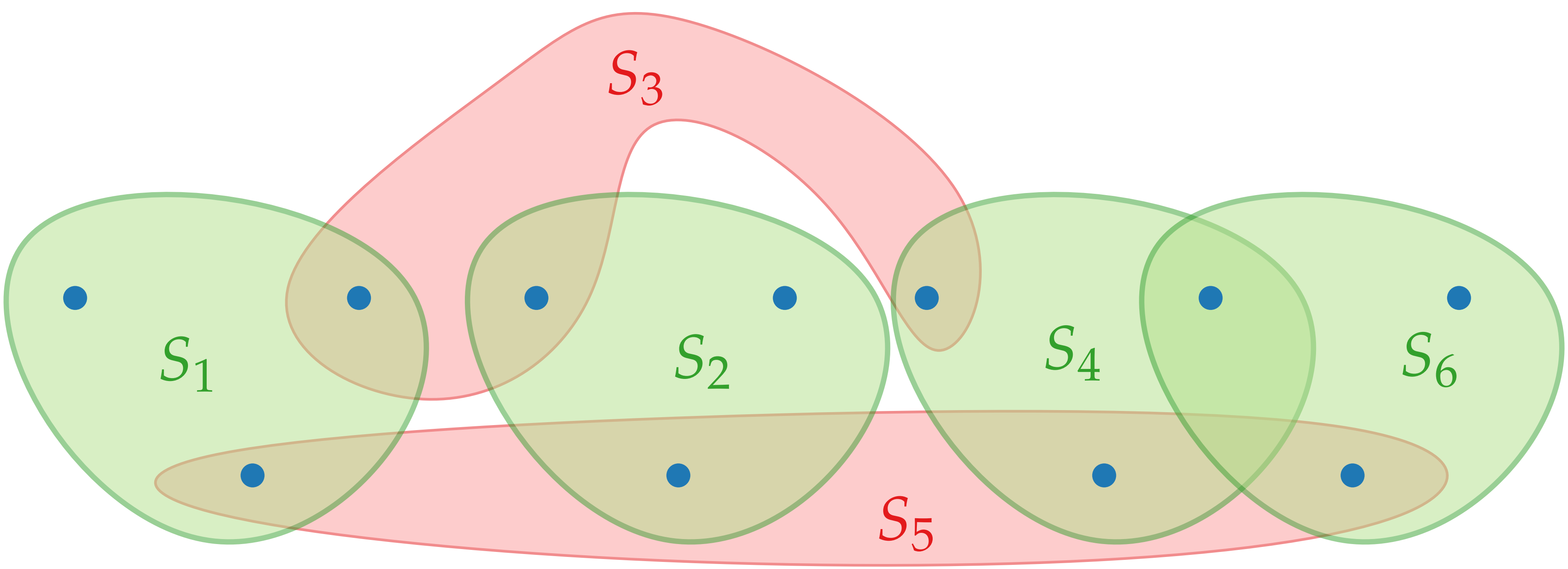

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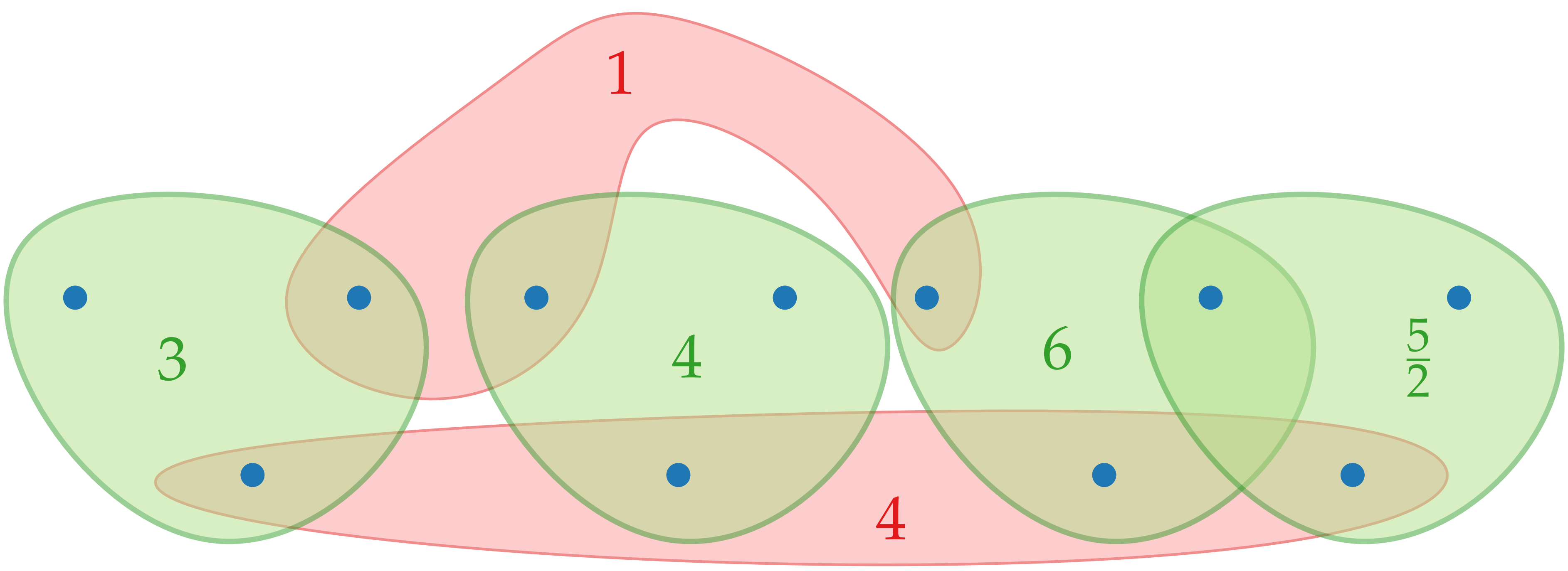

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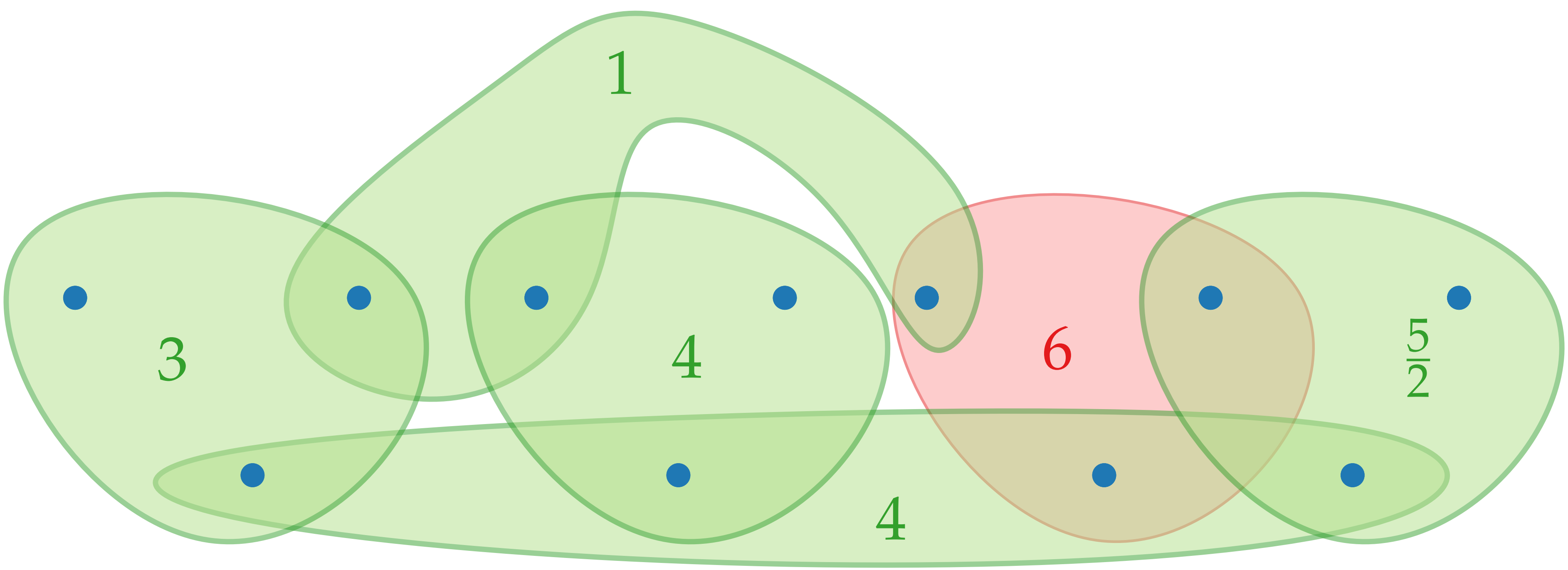

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SetCover and ShortestSuperString

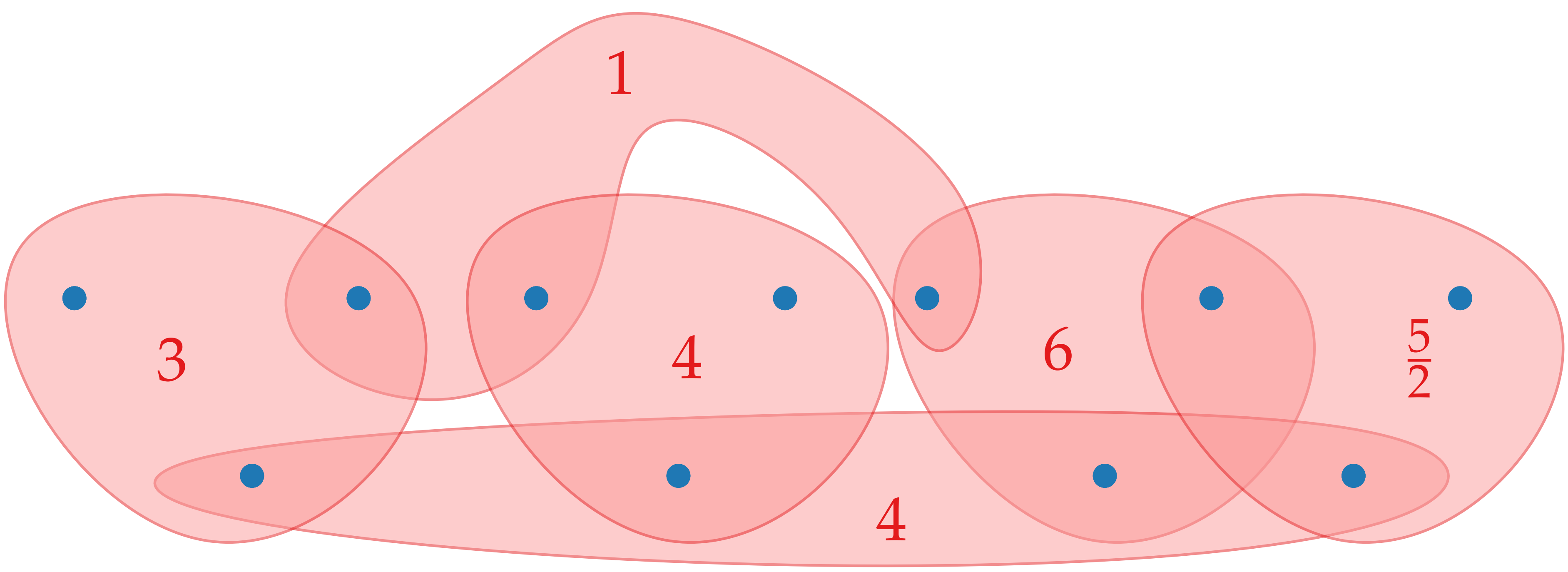
Part II:
Greedy for SetCover

Joachim Spoerhase

Winter 2020/21

Iterative "Buying" of Elements

What is the real cost of picking a set?



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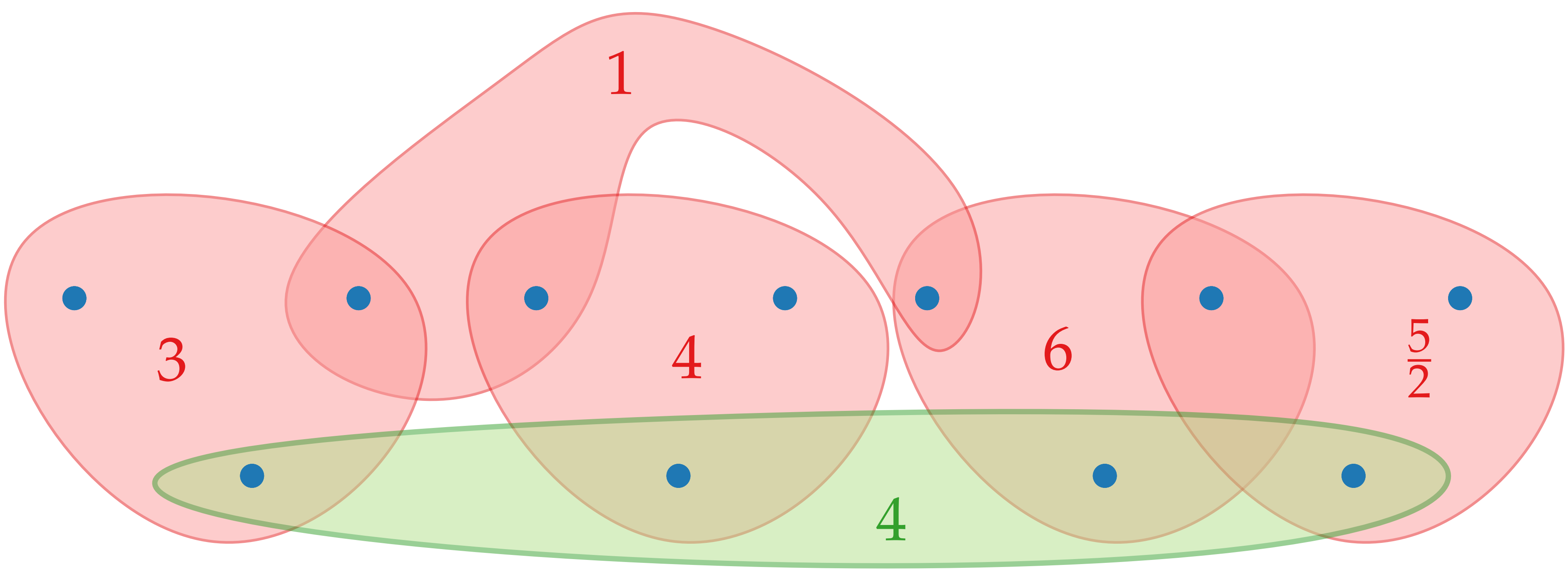

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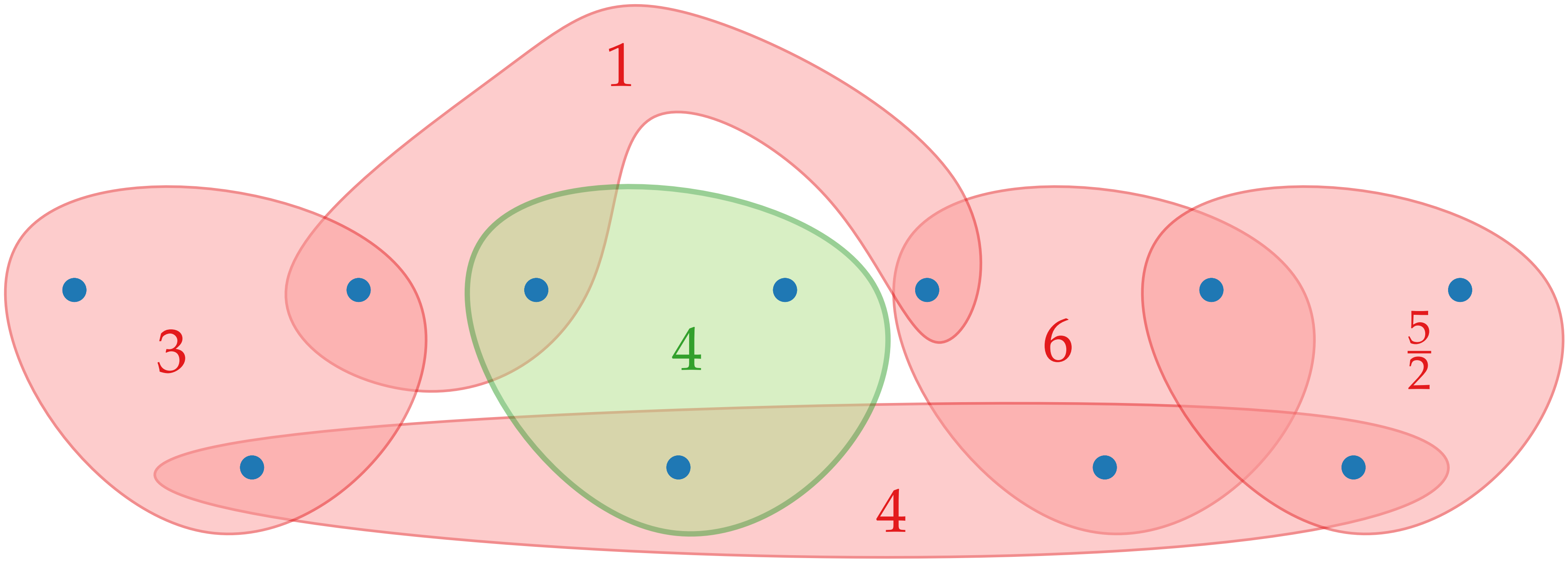

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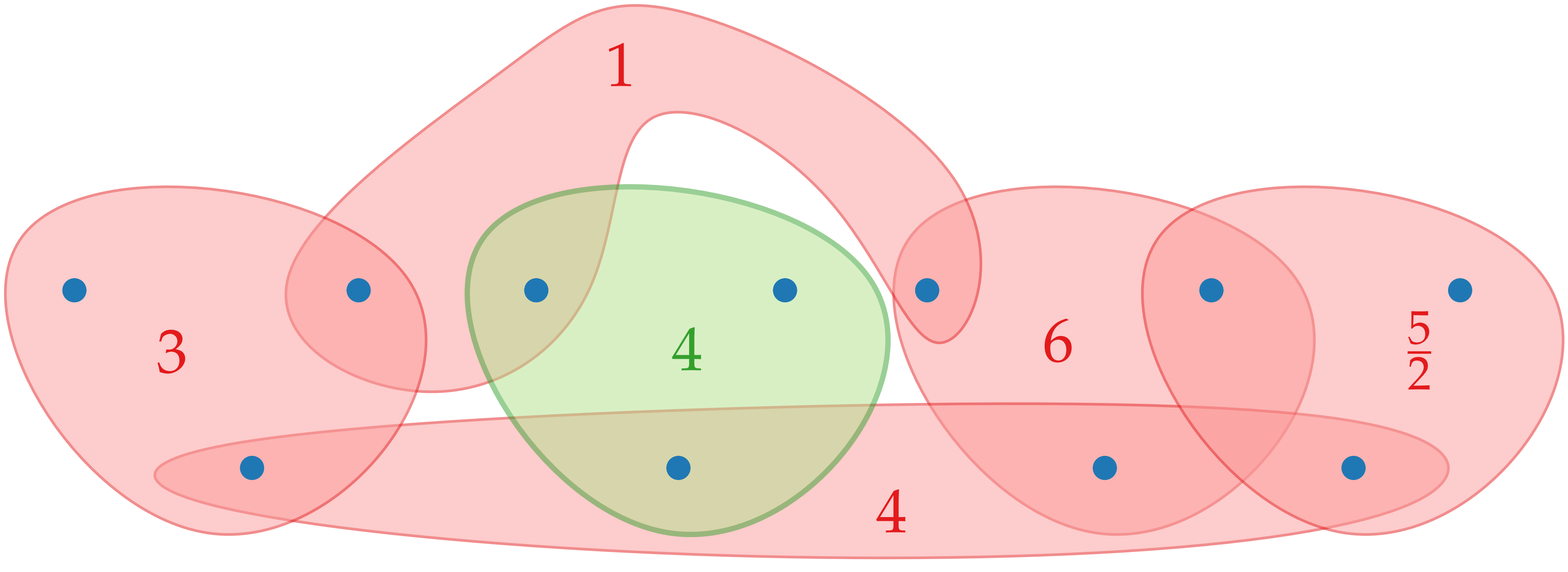

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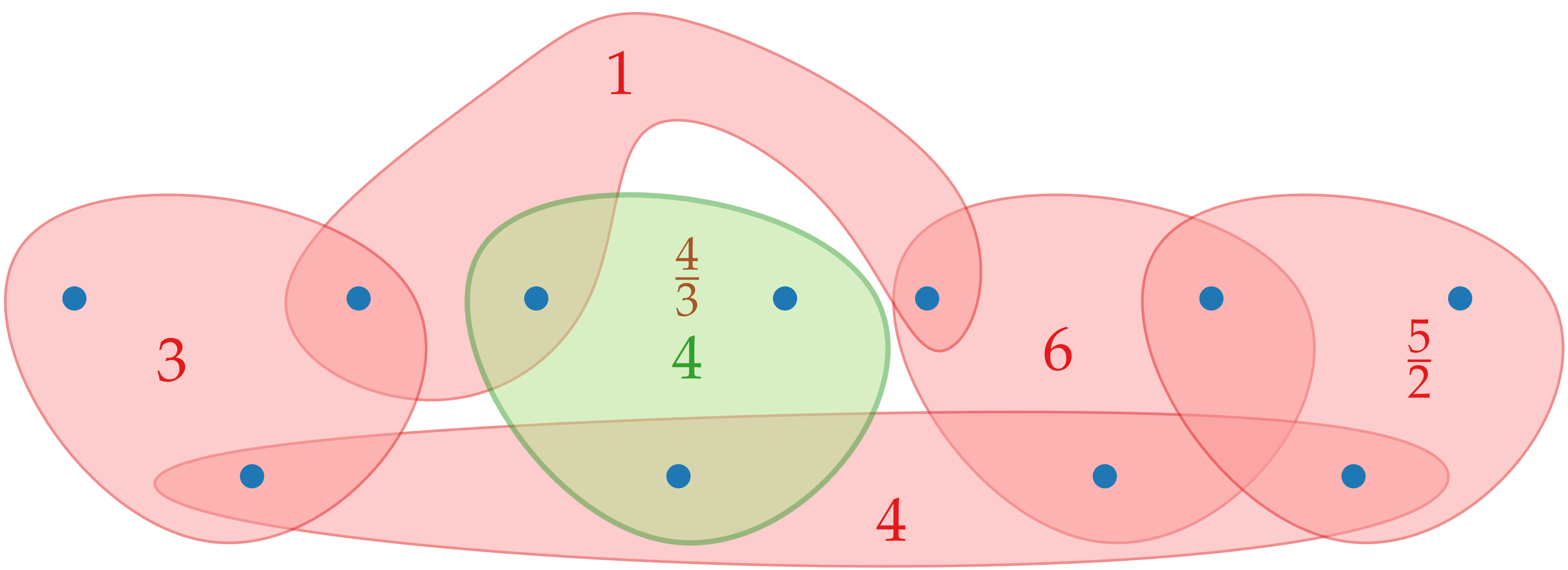

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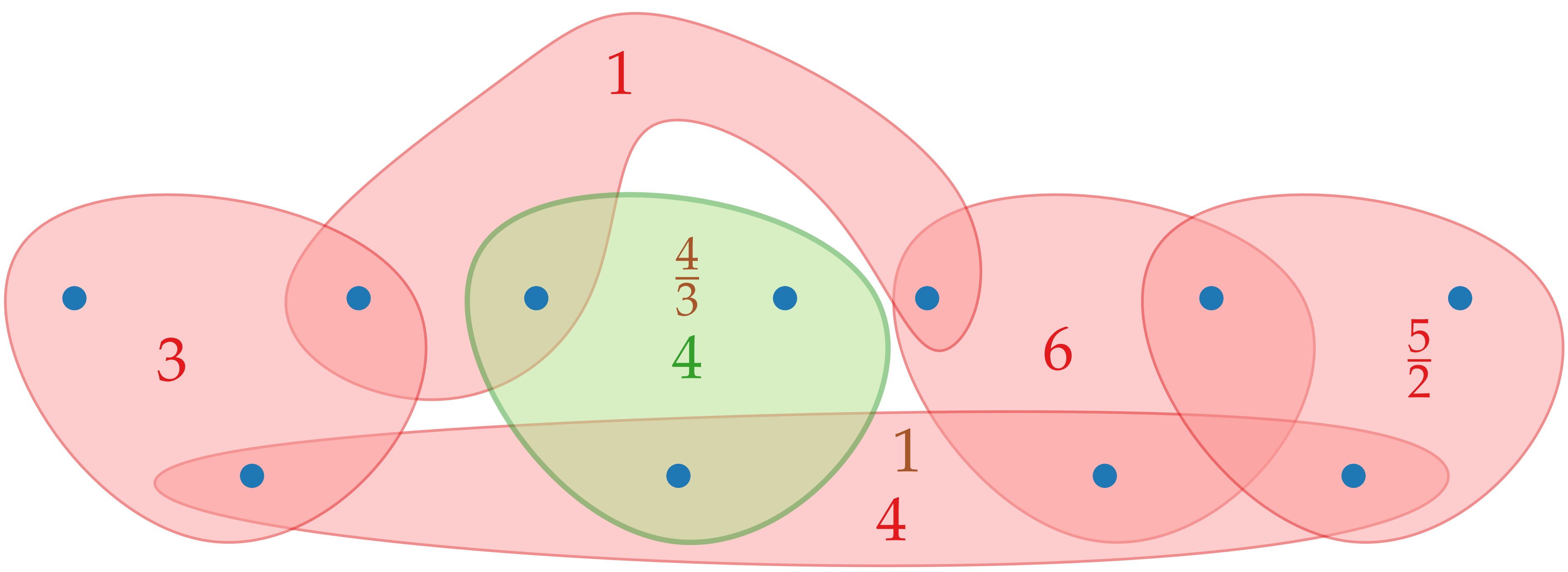

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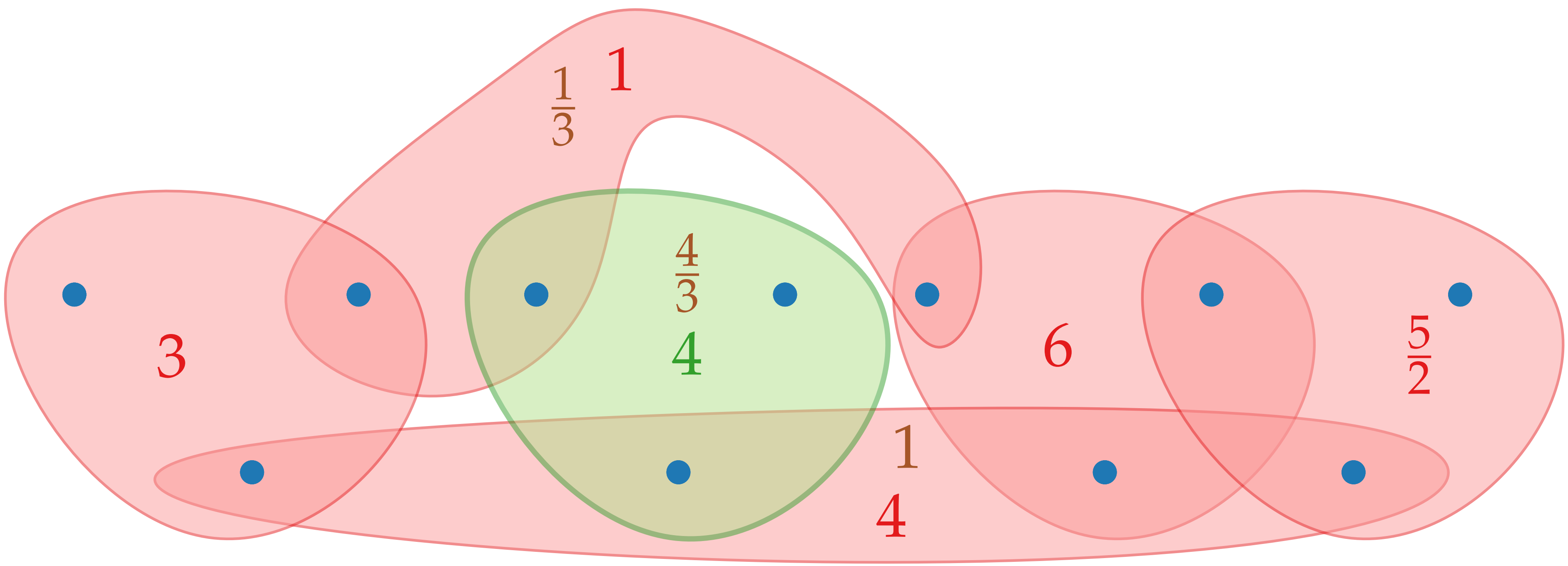

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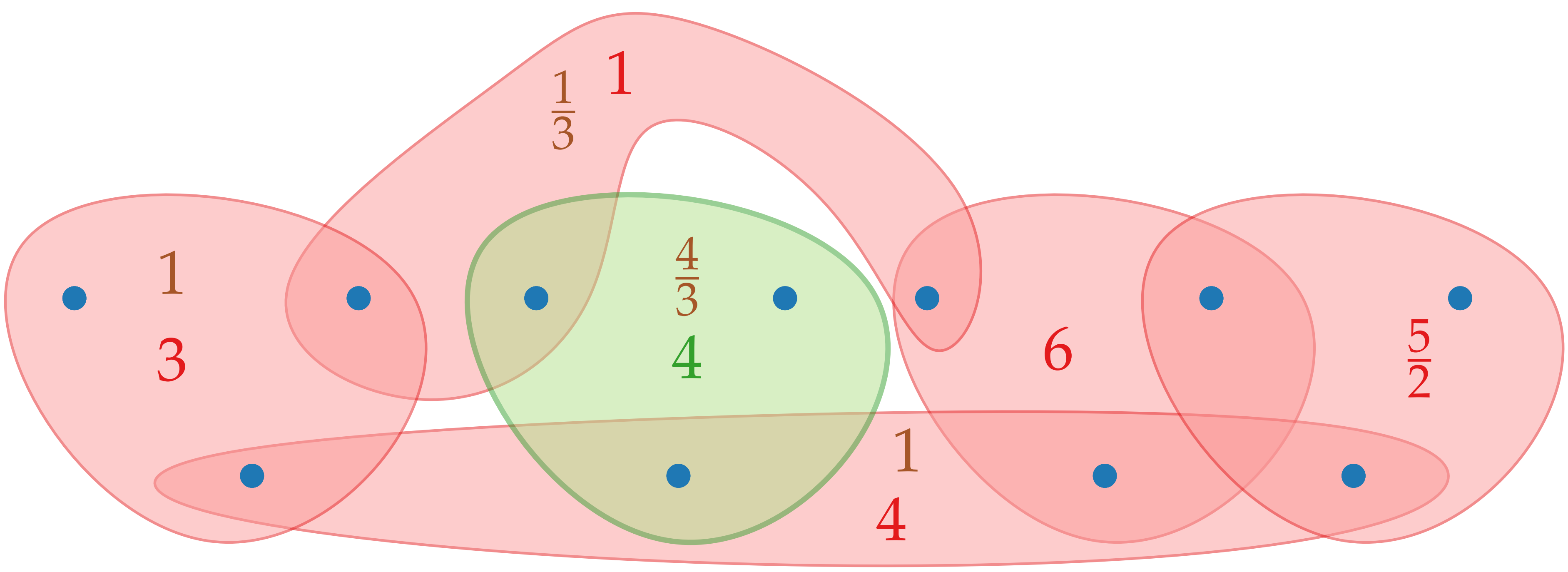

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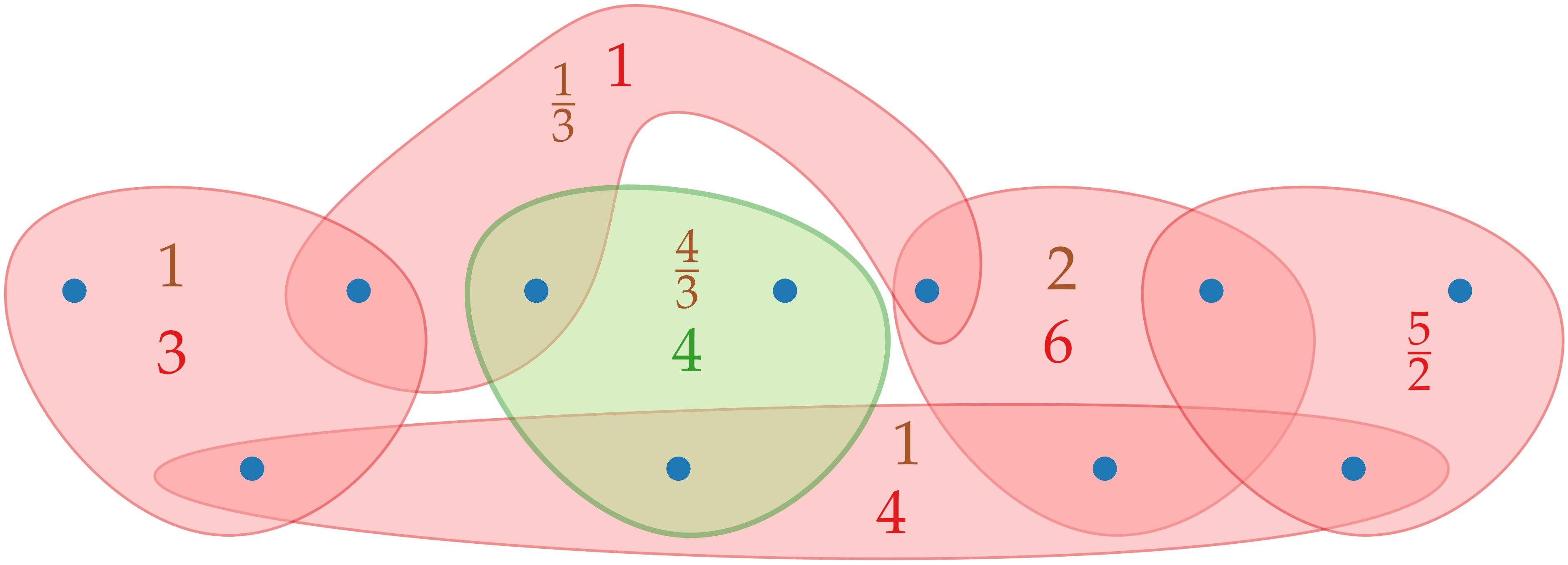

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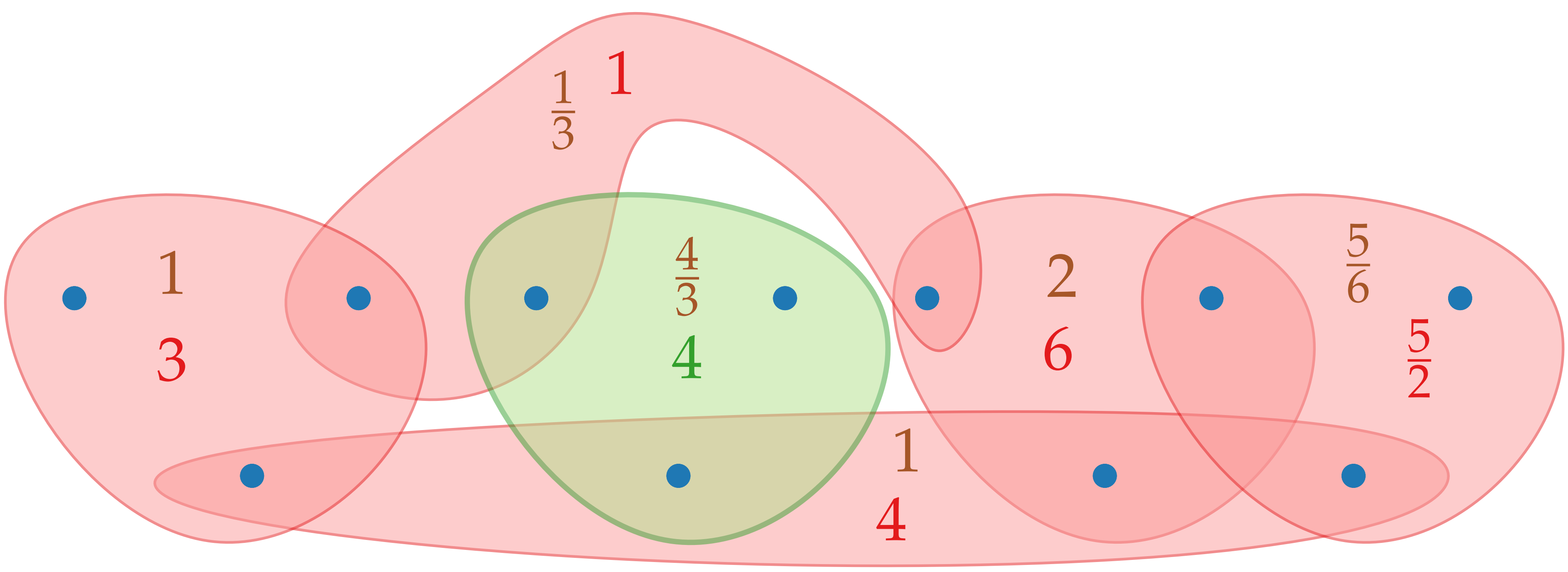

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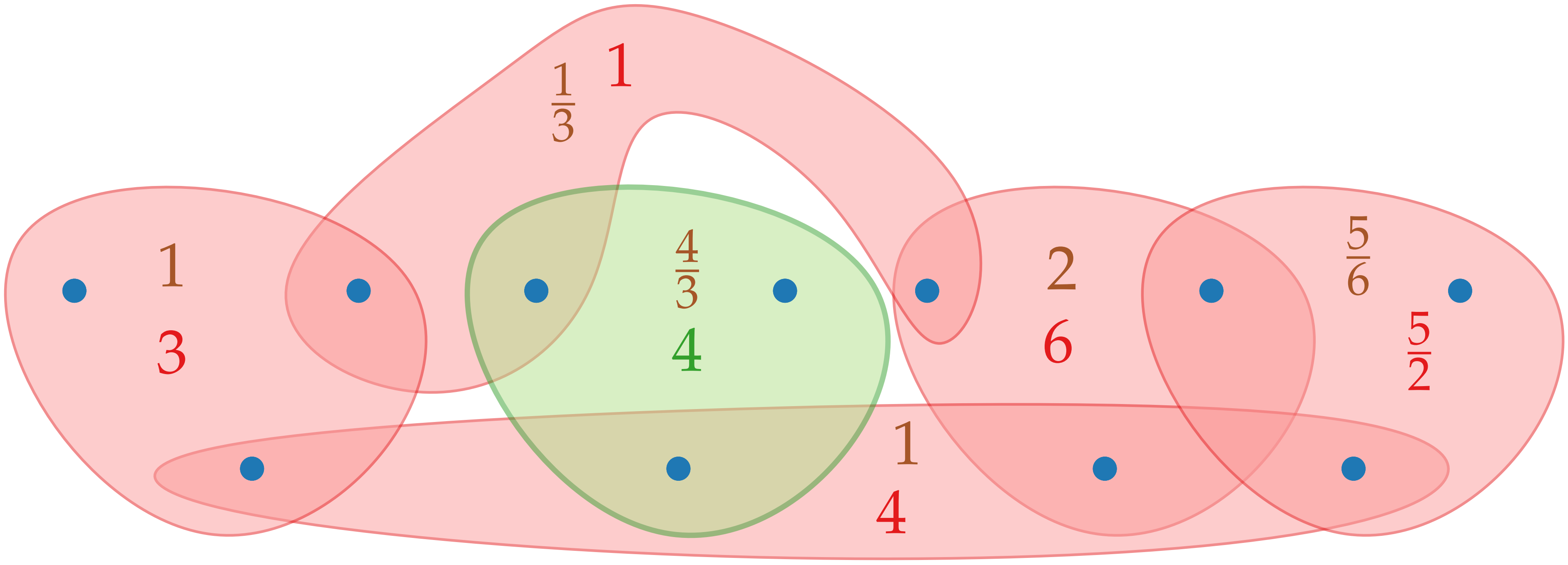

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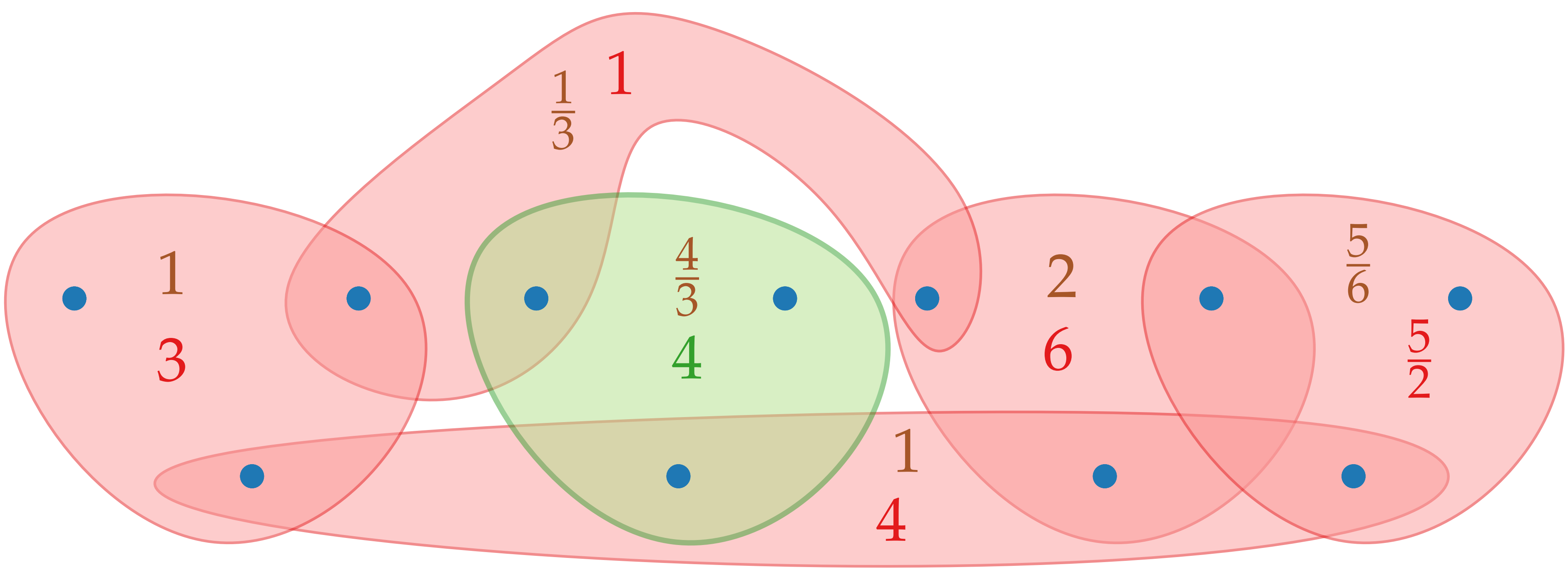

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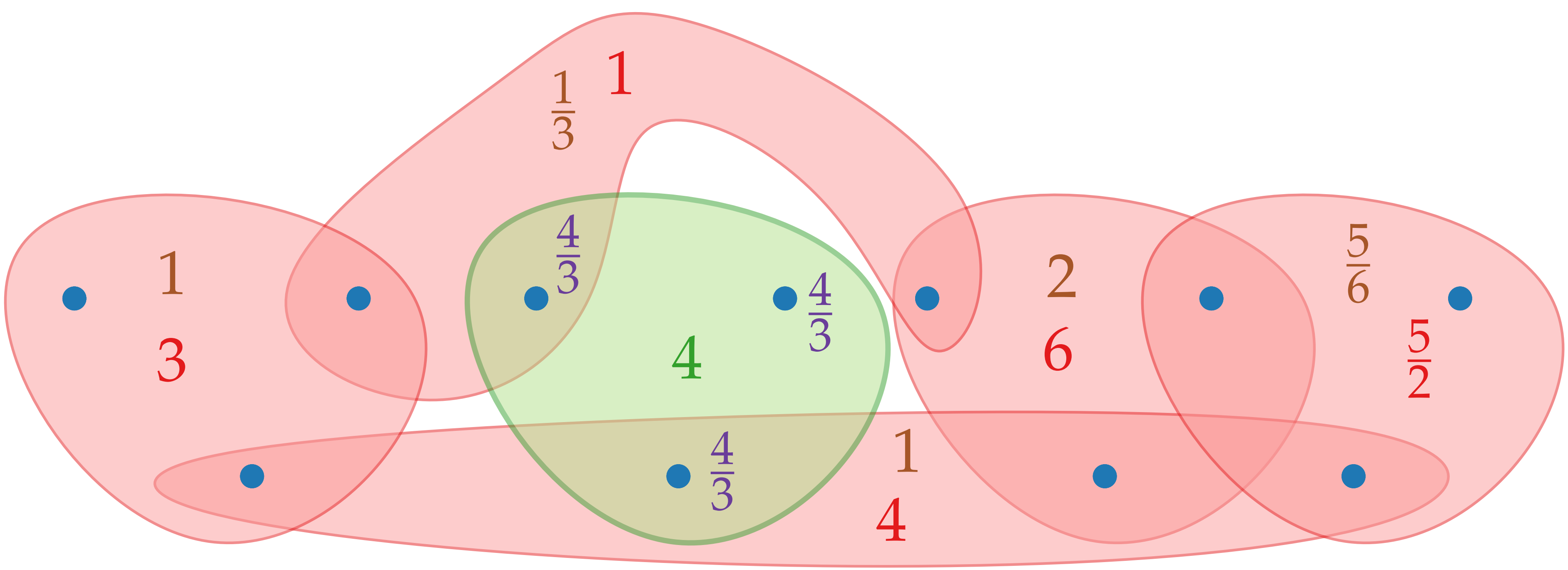

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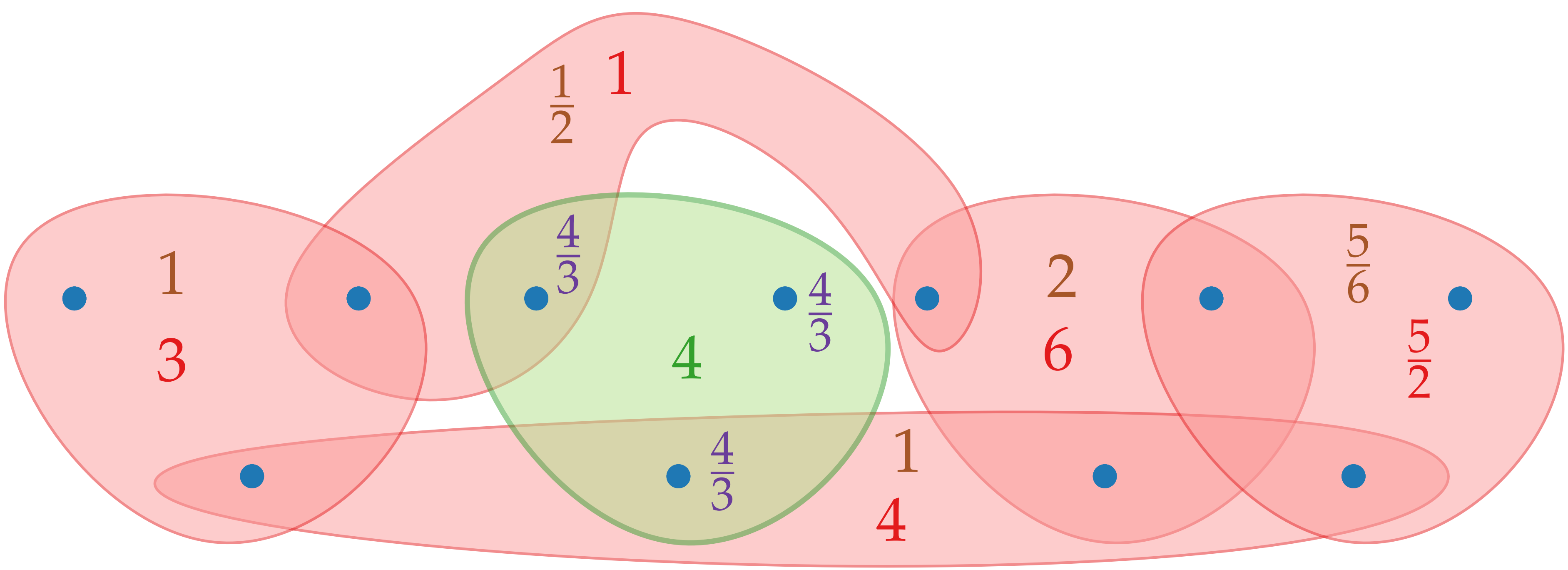

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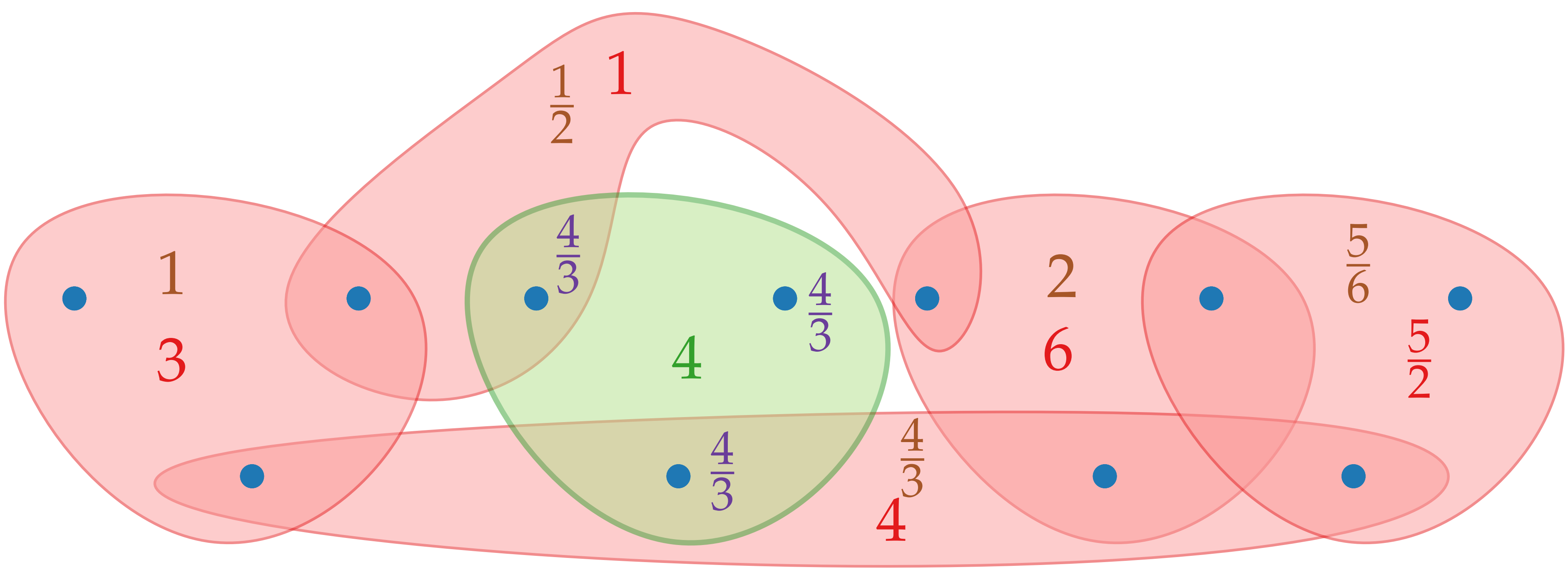

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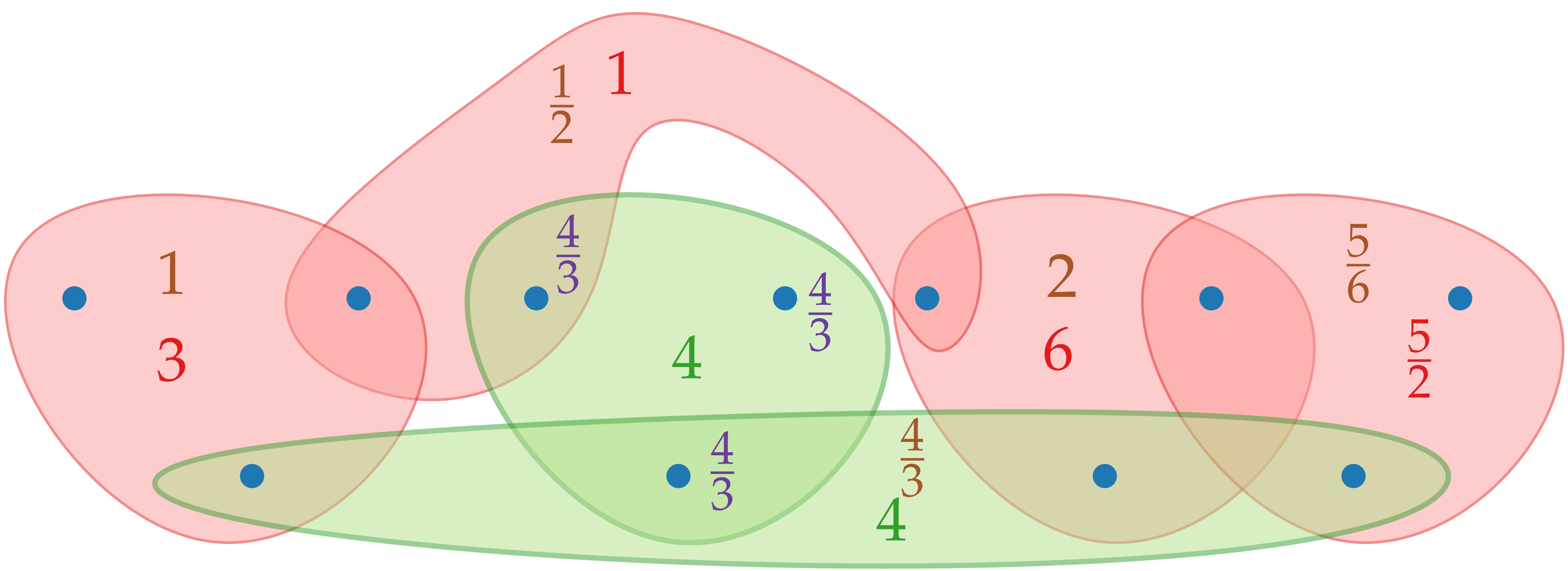

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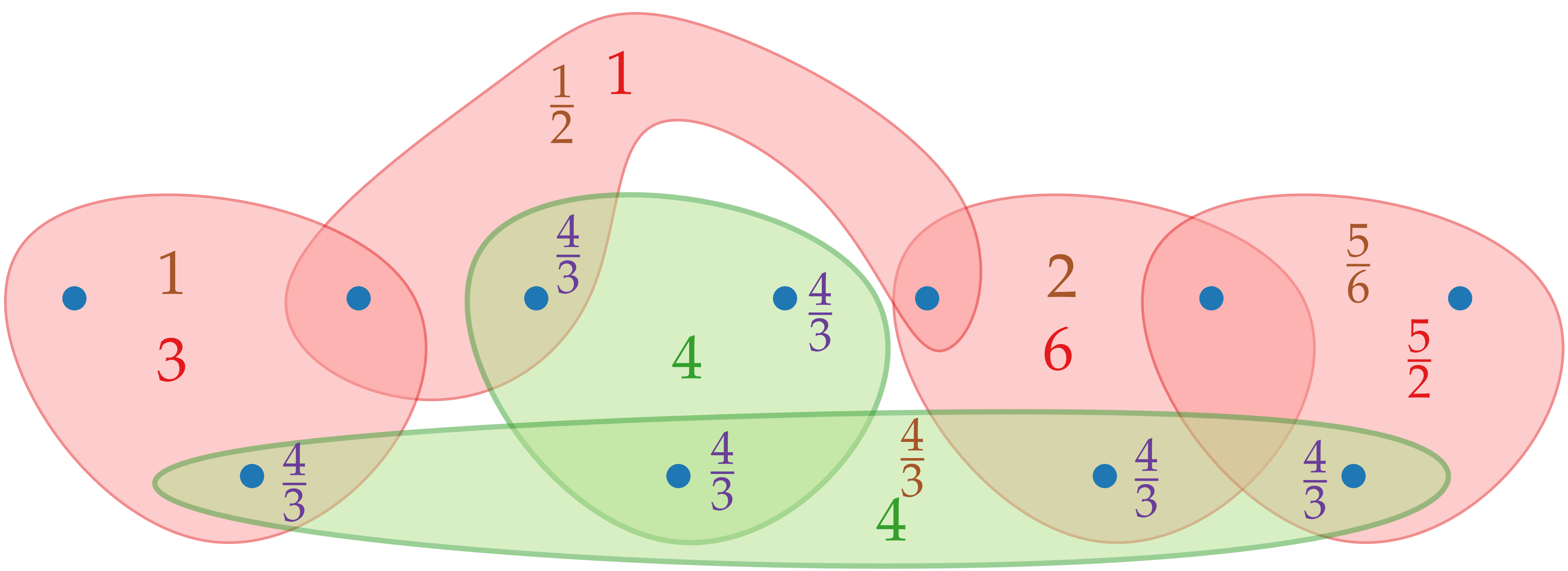

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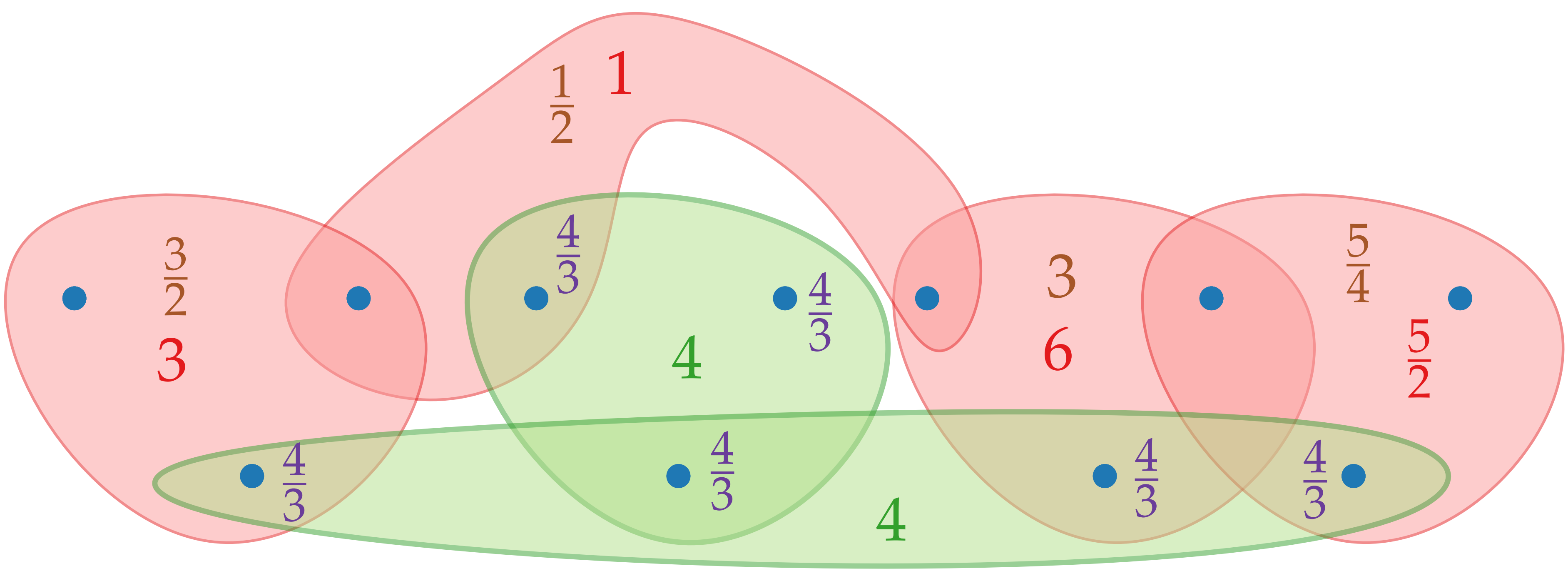

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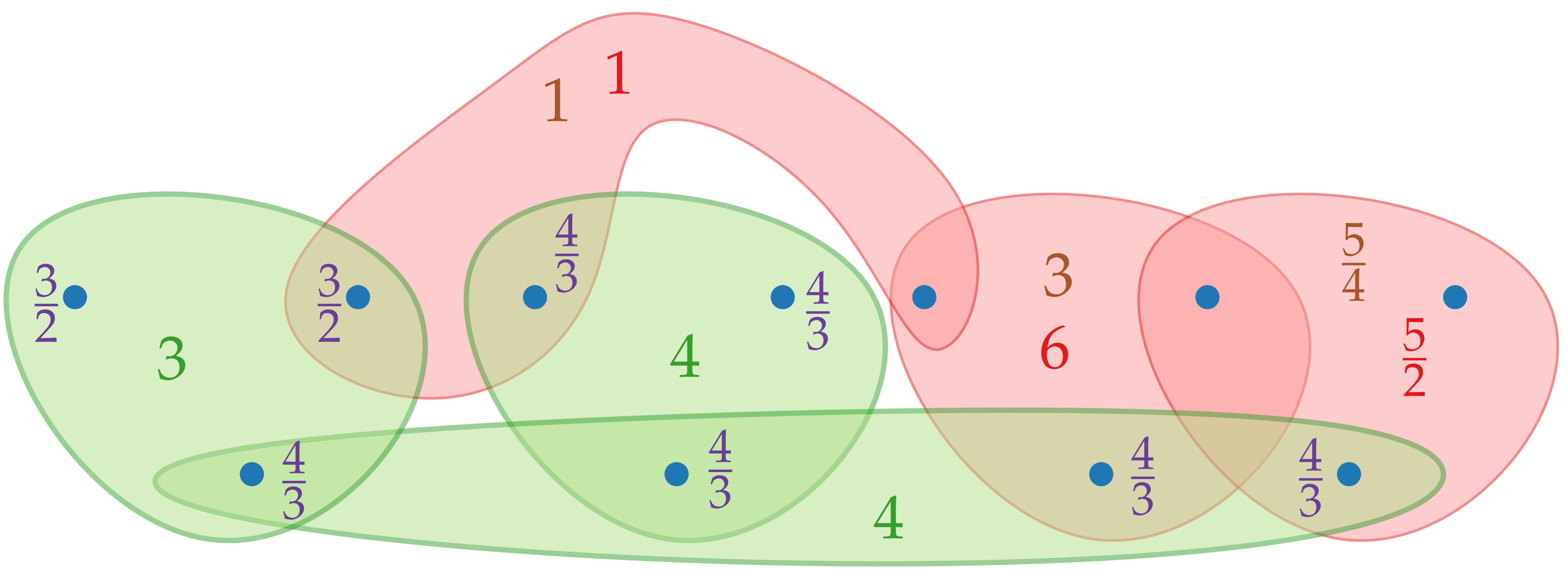

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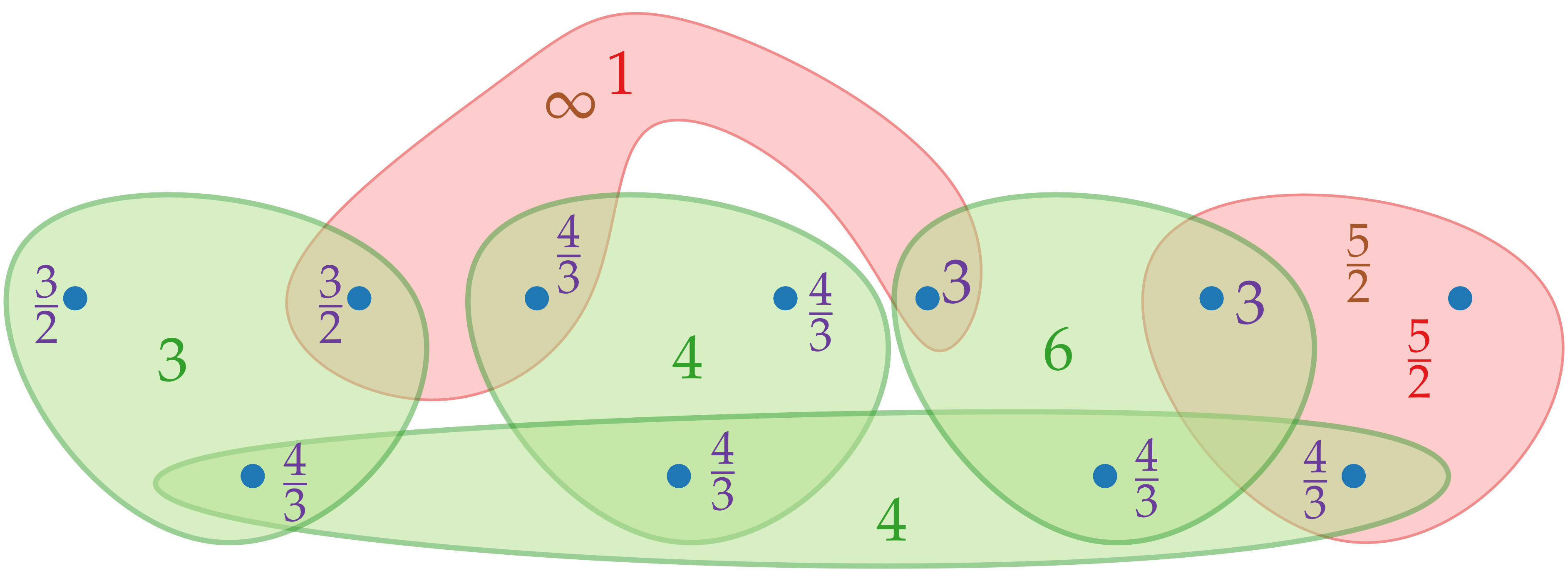

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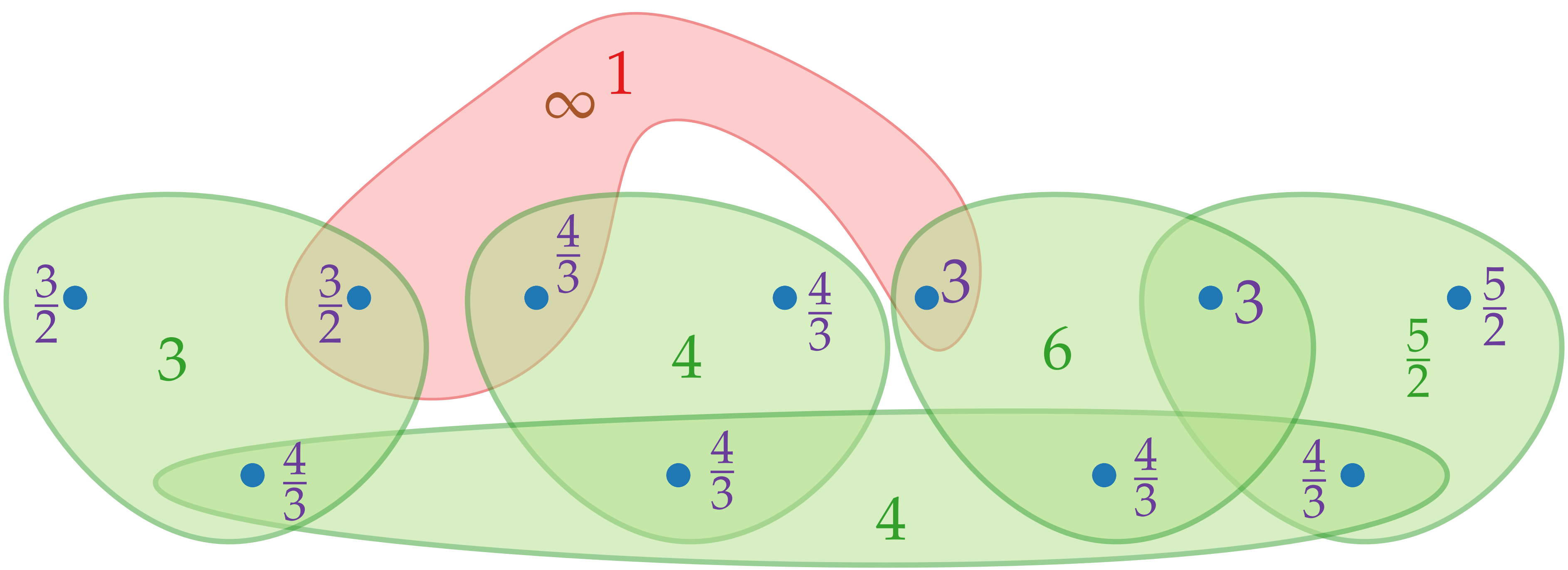

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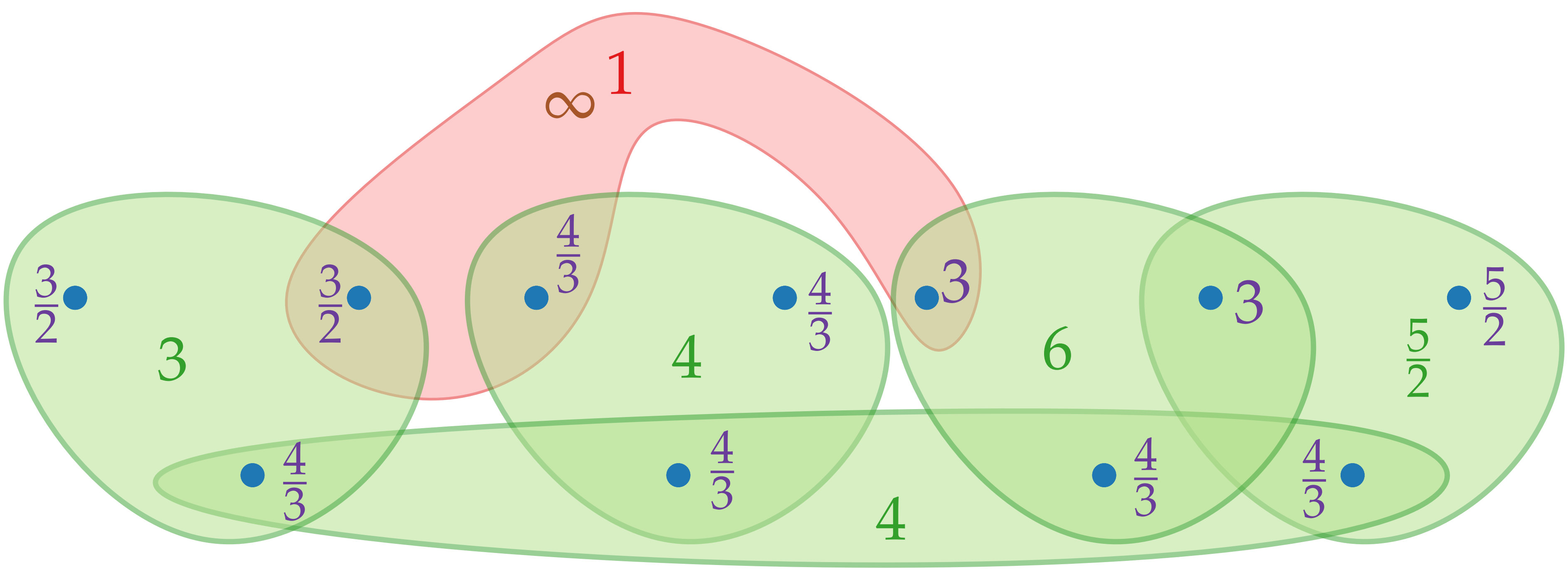

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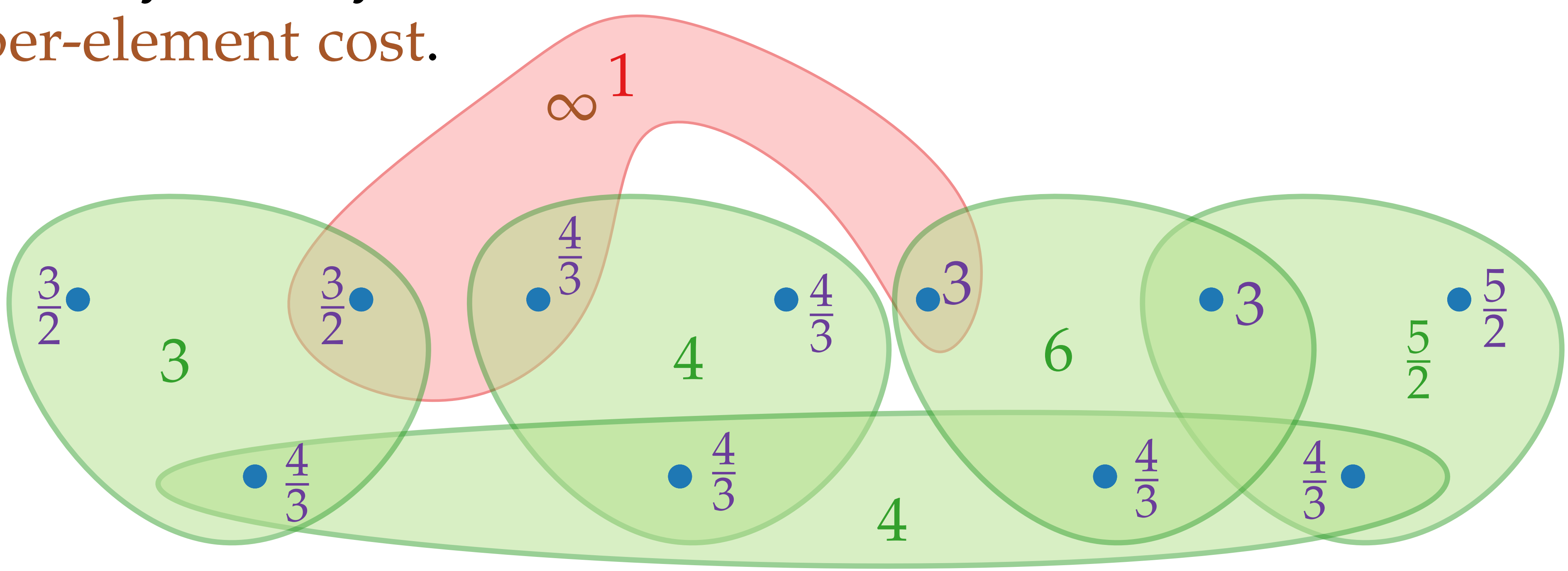
Greedy: Always choose the set with the minimum per-element cost.



Greedy for SetCover

GreedySetCover(U, $\mathcal{S}$, c)

$C \leftarrow \emptyset$

$\mathcal{S}' \leftarrow \emptyset$

return $\mathcal{S}'$ `// Cover of` U

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Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part III:
Analysis

Joachim Spoerhase

Winter 2021/22

Analysis

Theorem. GreedySetCover is a factor-$\mathcal{H}_k$-approximation algorithm for SetCover, where k is the cardinality of the largest set in $\mathcal{S}$ and $\mathcal{H}_k := 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k} \to 0.5 + \ln k$.

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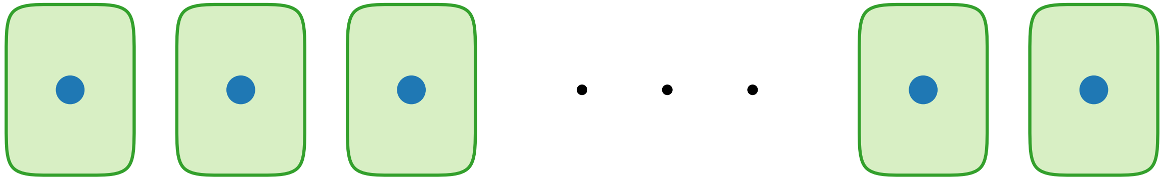

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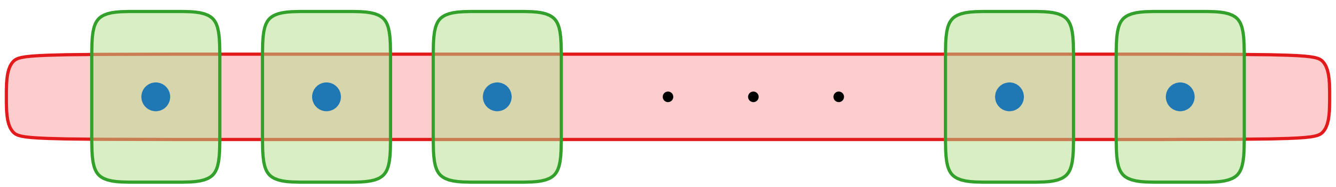

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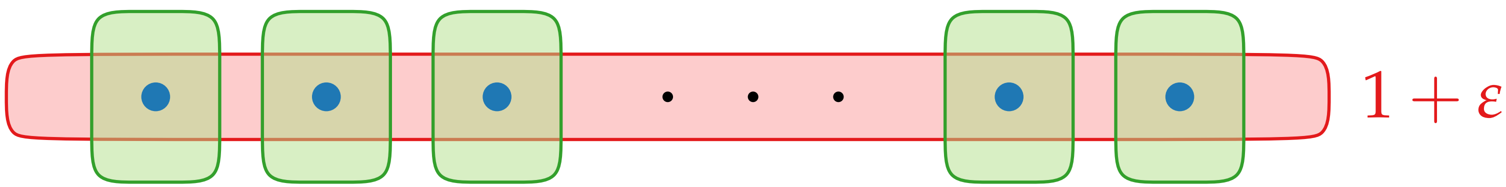

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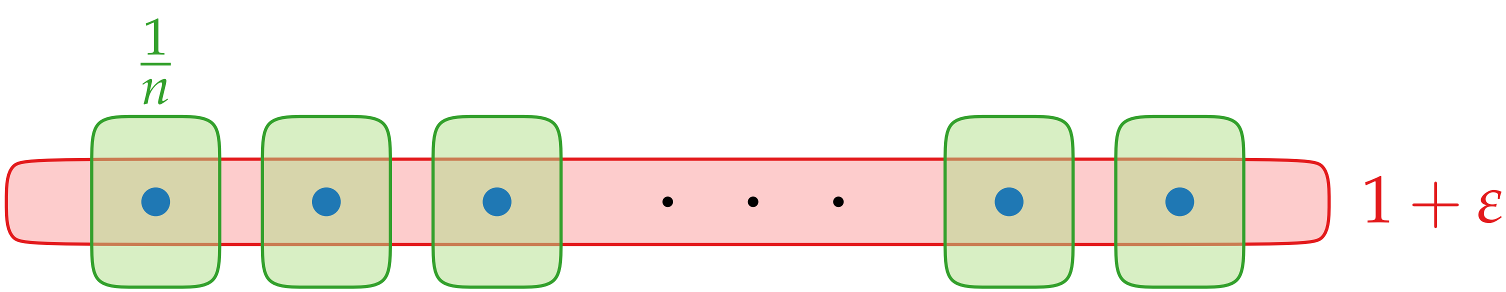

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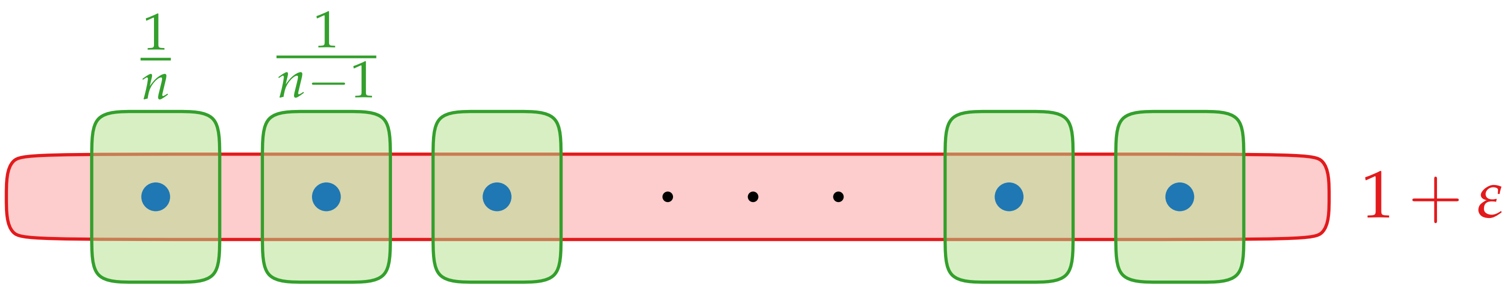

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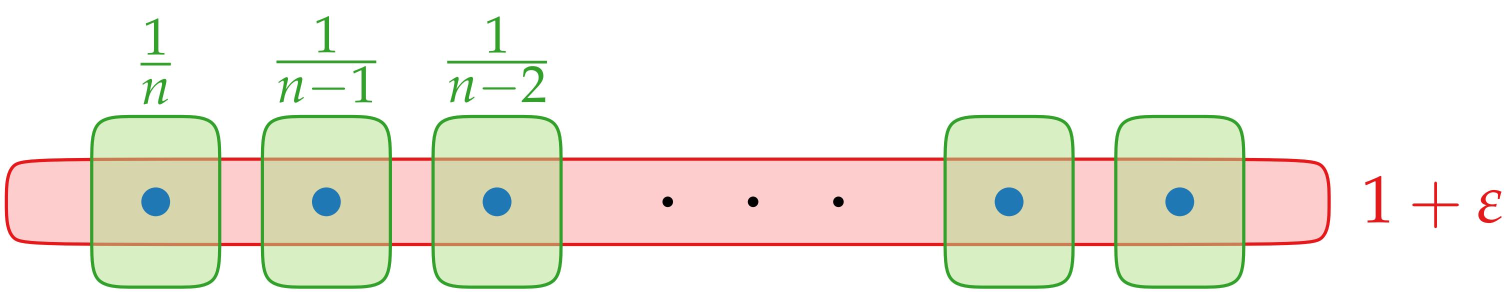

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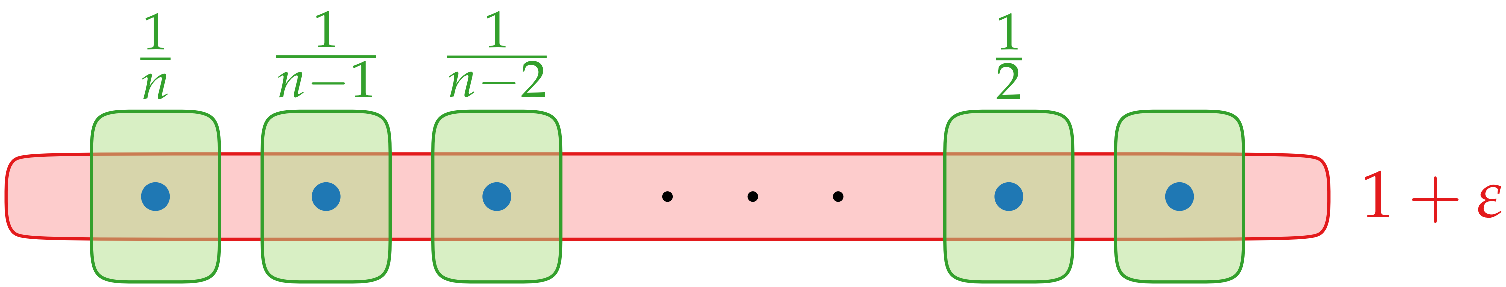

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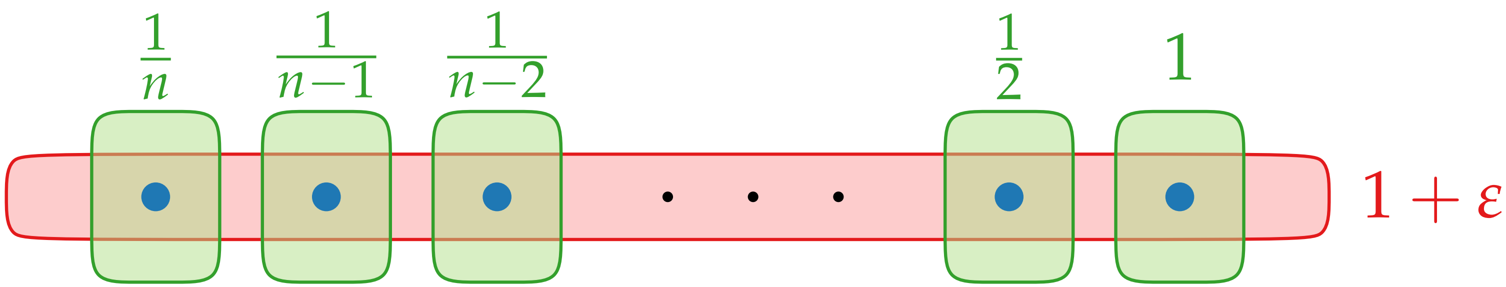

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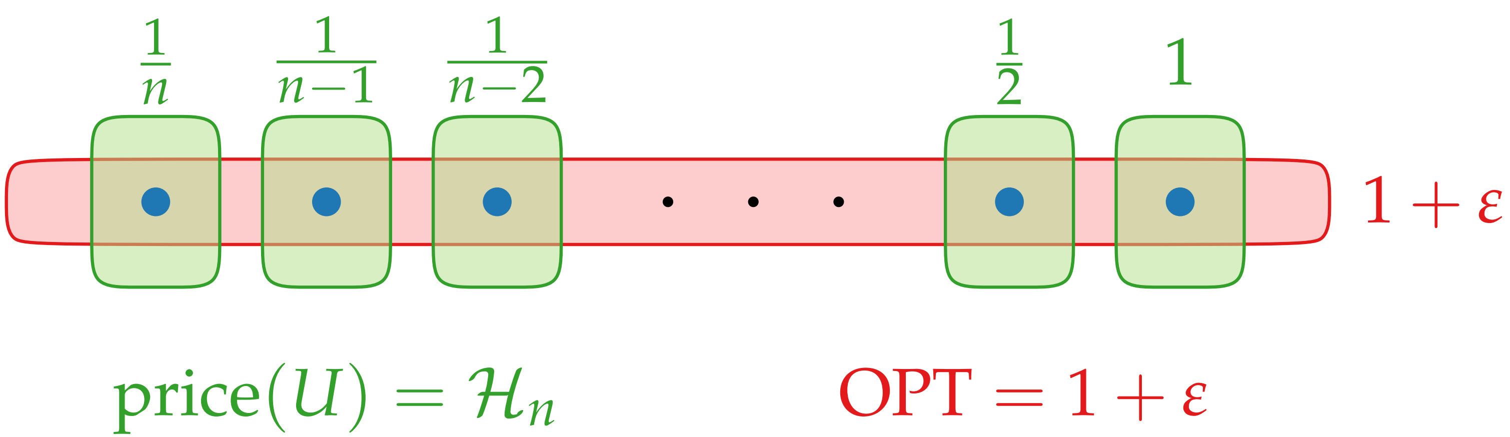

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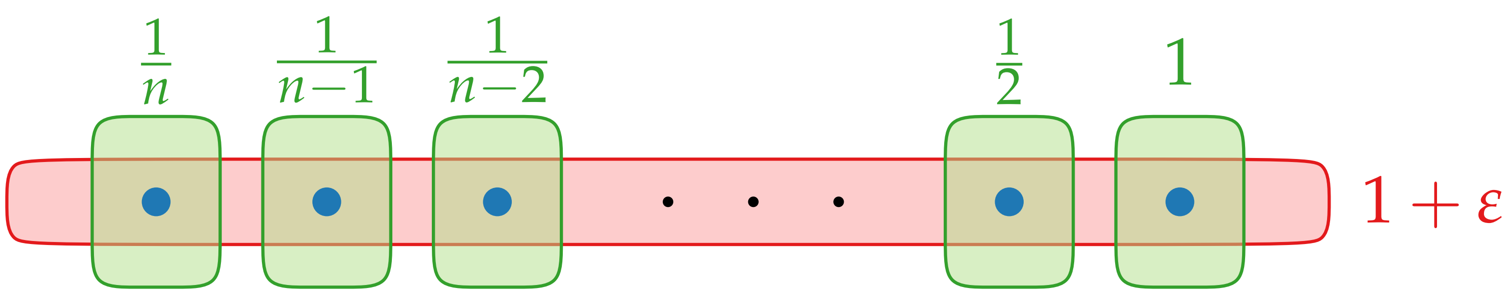

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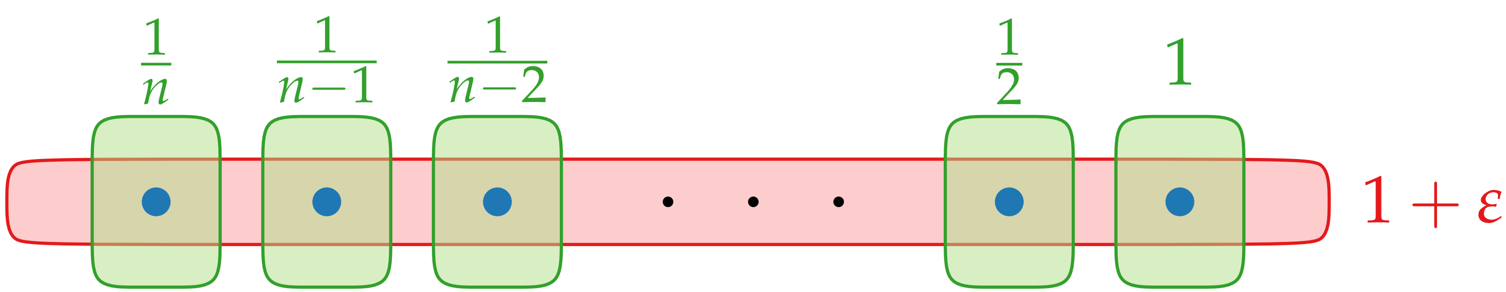

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SetCover cannot be approximated within factor $(1 - o(1)) \cdot \ln(n)$ (unless P=NP)

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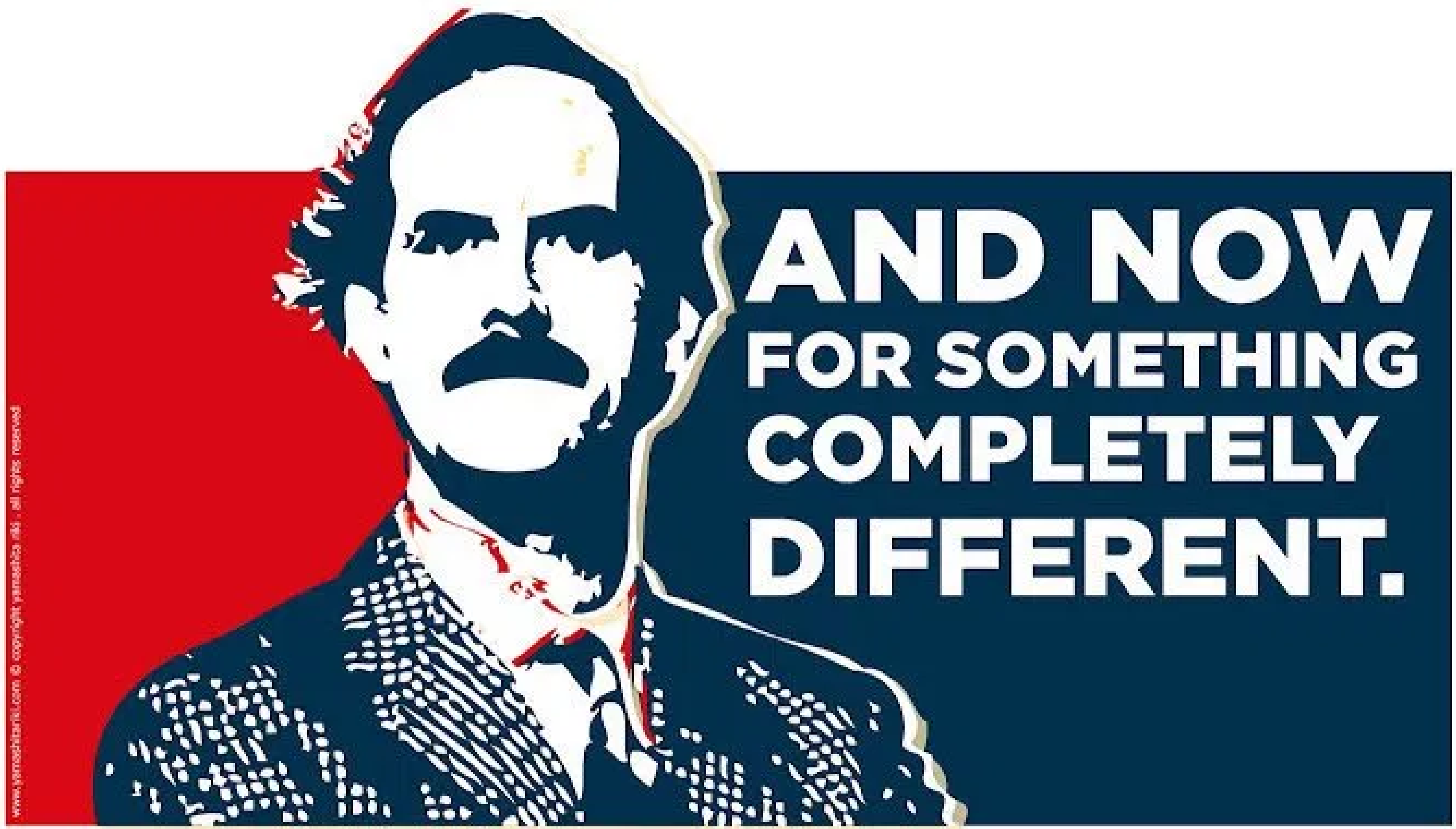
bette

SetC

$(1 -$



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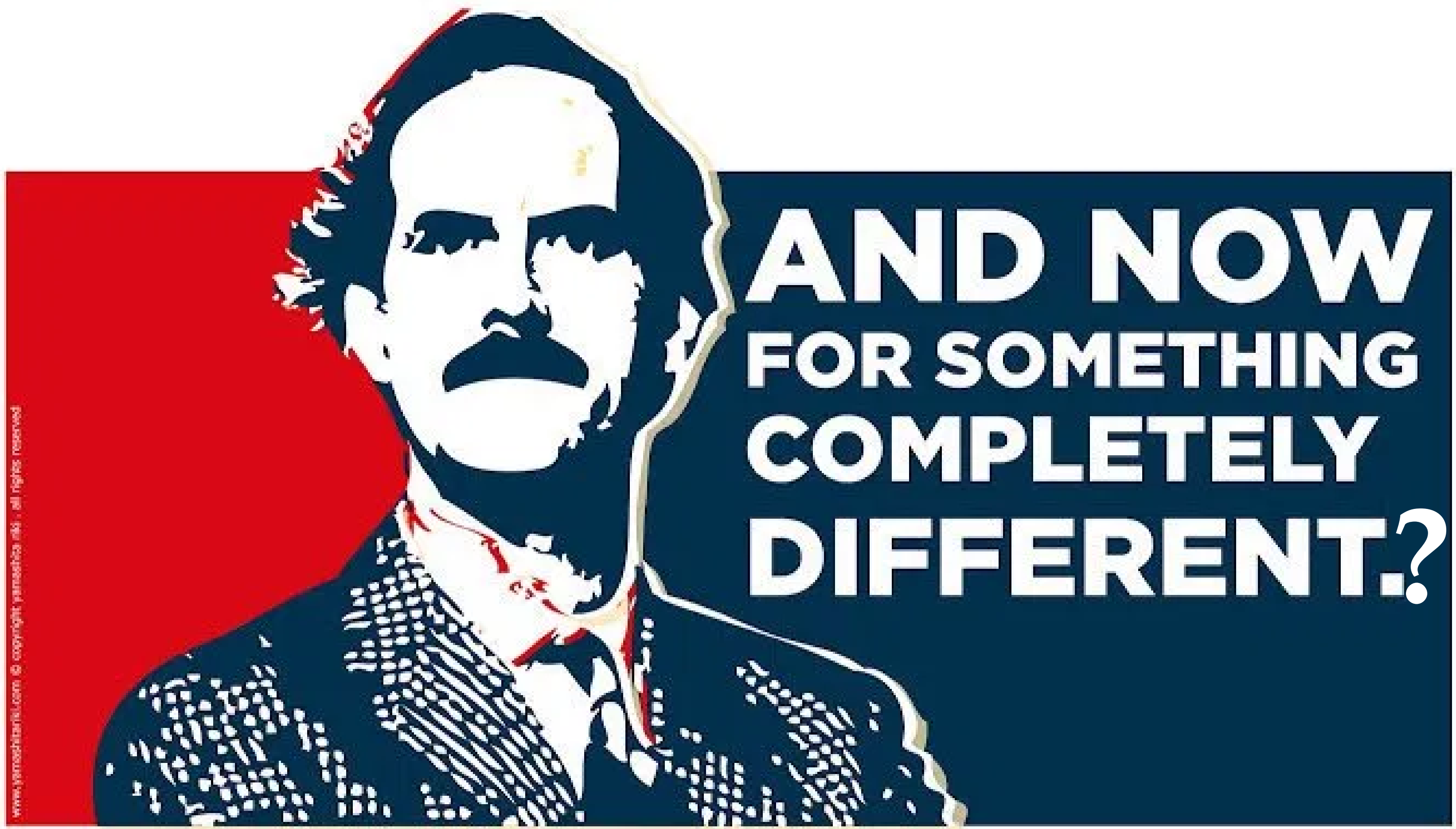

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Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part IV:
ShortestSuperString

Joachim Spoerhase

Winter 2021/22

ShortestSuperString (SSS)

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W.l.o.g.: No string s_i is a substring of any other string s_j.

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SetCover Instance: ground set U, set family $\mathcal{S}$, costs c.

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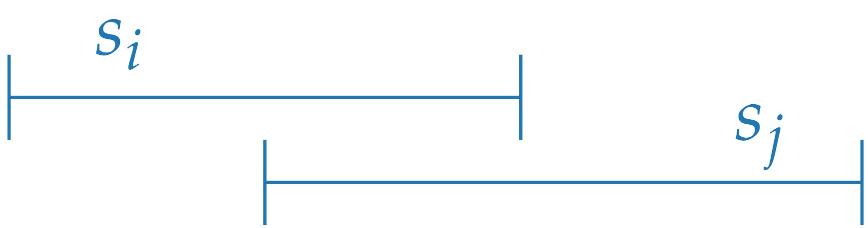

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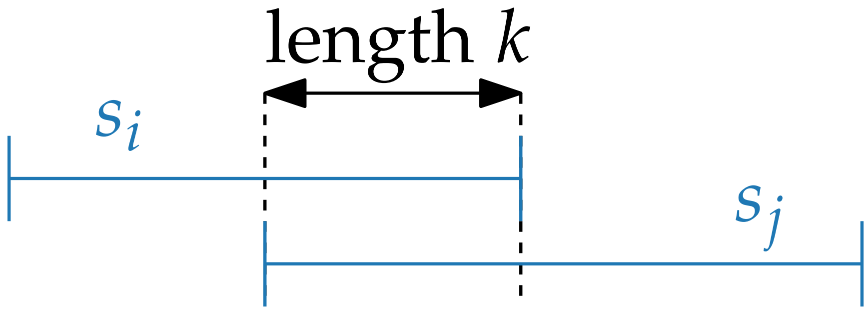

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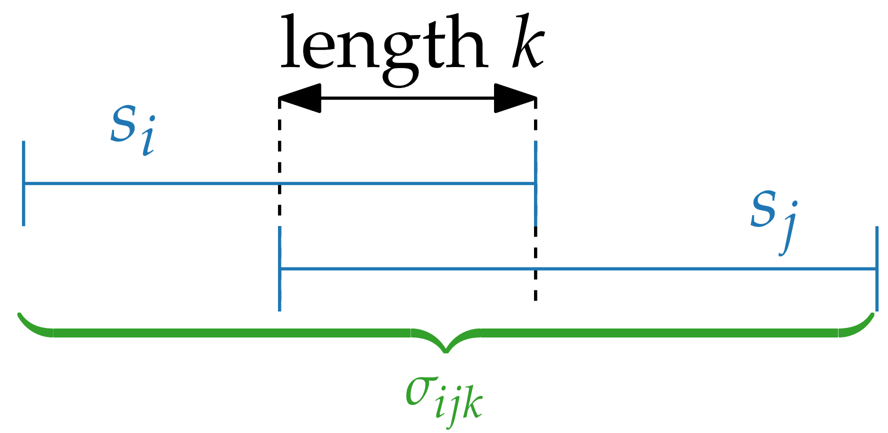

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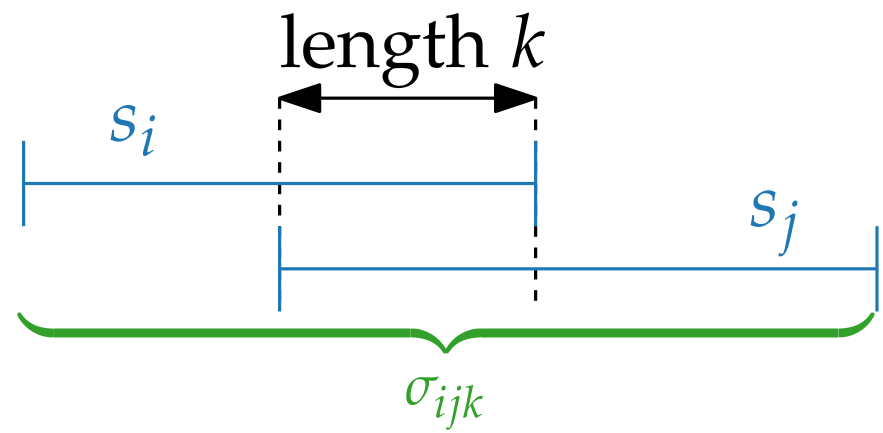

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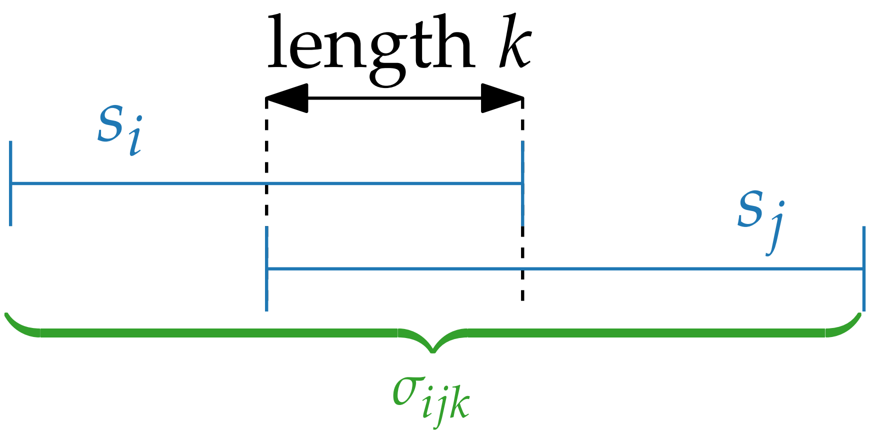

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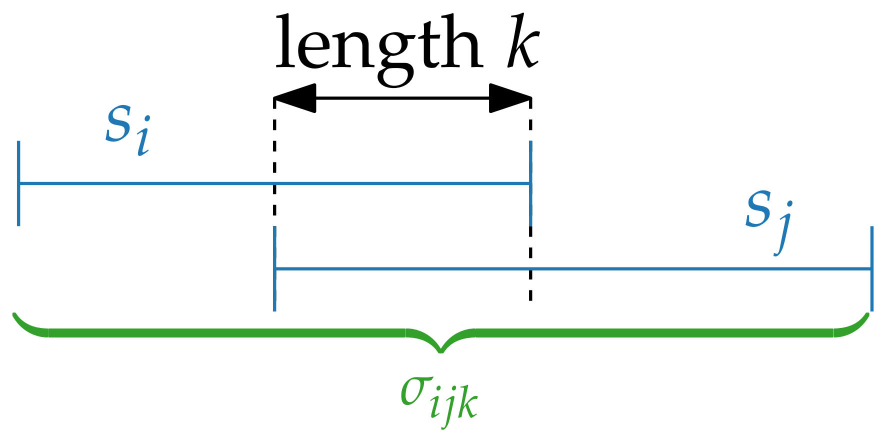

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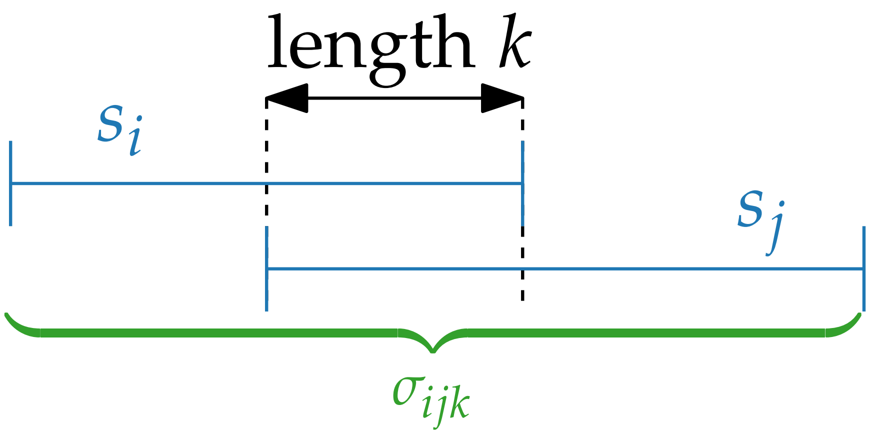

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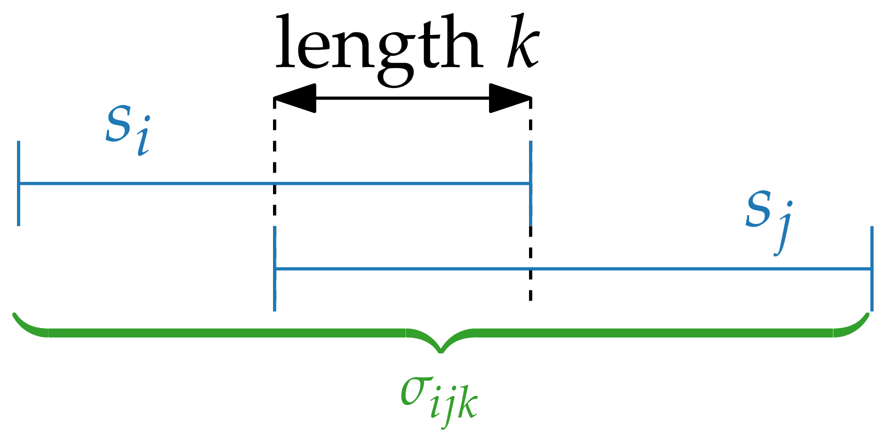

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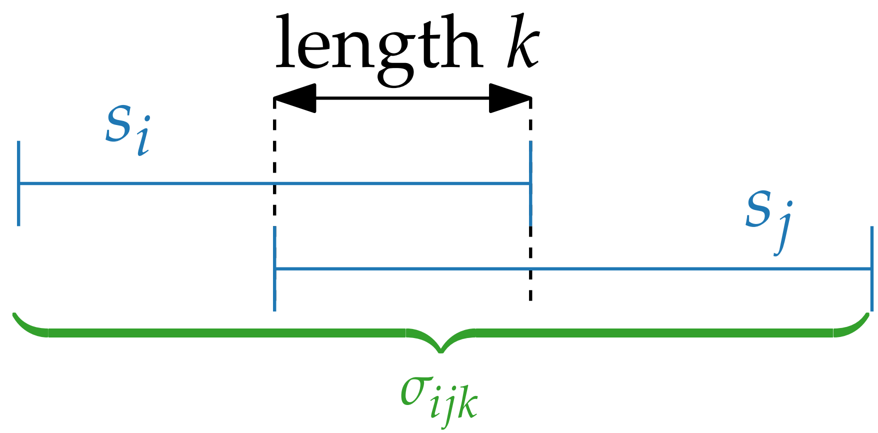

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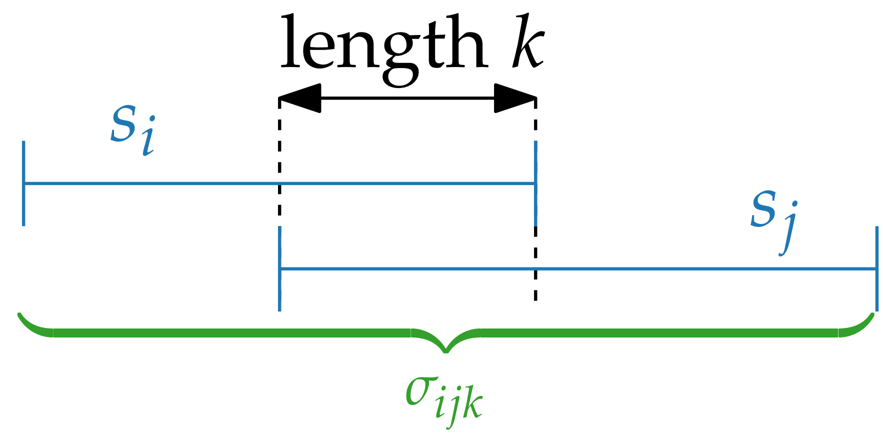

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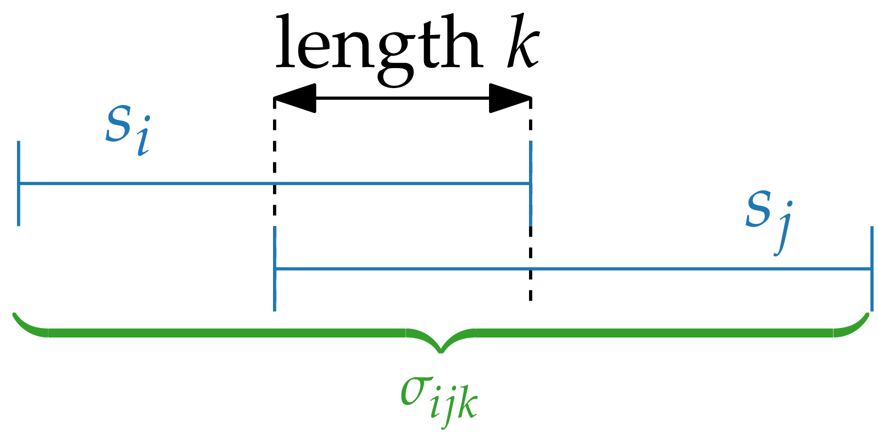

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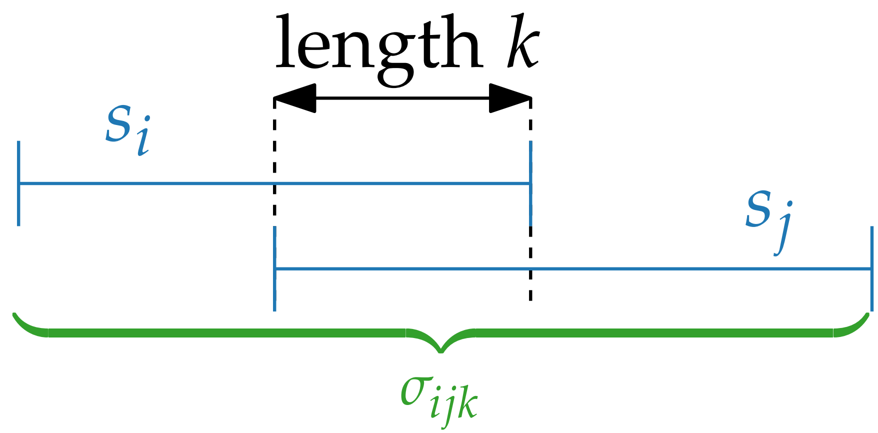

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$\mathcal{S} = \{S(\sigma_{ijk}) \mid k > 0\}$ (possibly $i = j$)

Approximation Algorithms

Lecture 2:
SetCover and ShortestSuperString

Part V:
Solving ShortestSuperString via SetCover

Joachim Spoerhase

Winter 2021/22

Relating SSS and SETCOVER

Lemma. Let $\mathrm{OPT}_{\mathrm{SSS}}$ be the length of a shortest superstring of U and $\mathrm{OPT}_{\mathrm{SC}}$ be the minimum cost of the corresponding SETCOVER instance. Then:

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Thus, $\mathrm{OPT}_{\mathrm{SSS}} \leq |s| = \mathrm{OPT}_{\mathrm{SC}}$.

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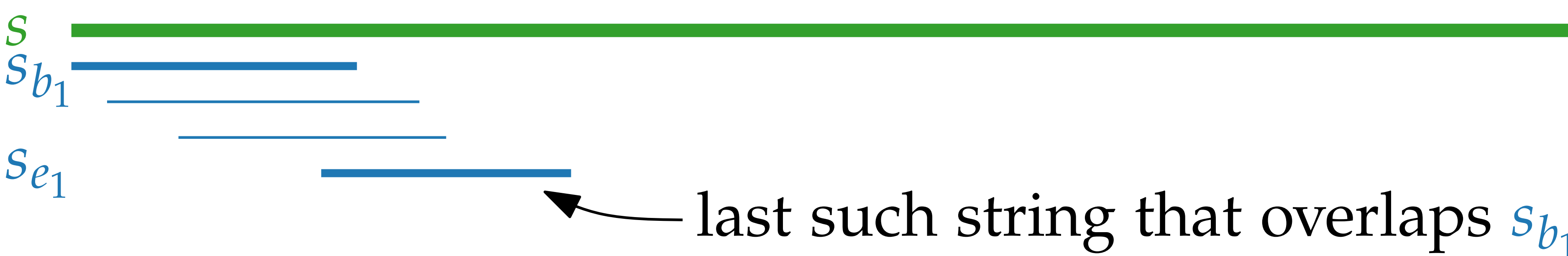

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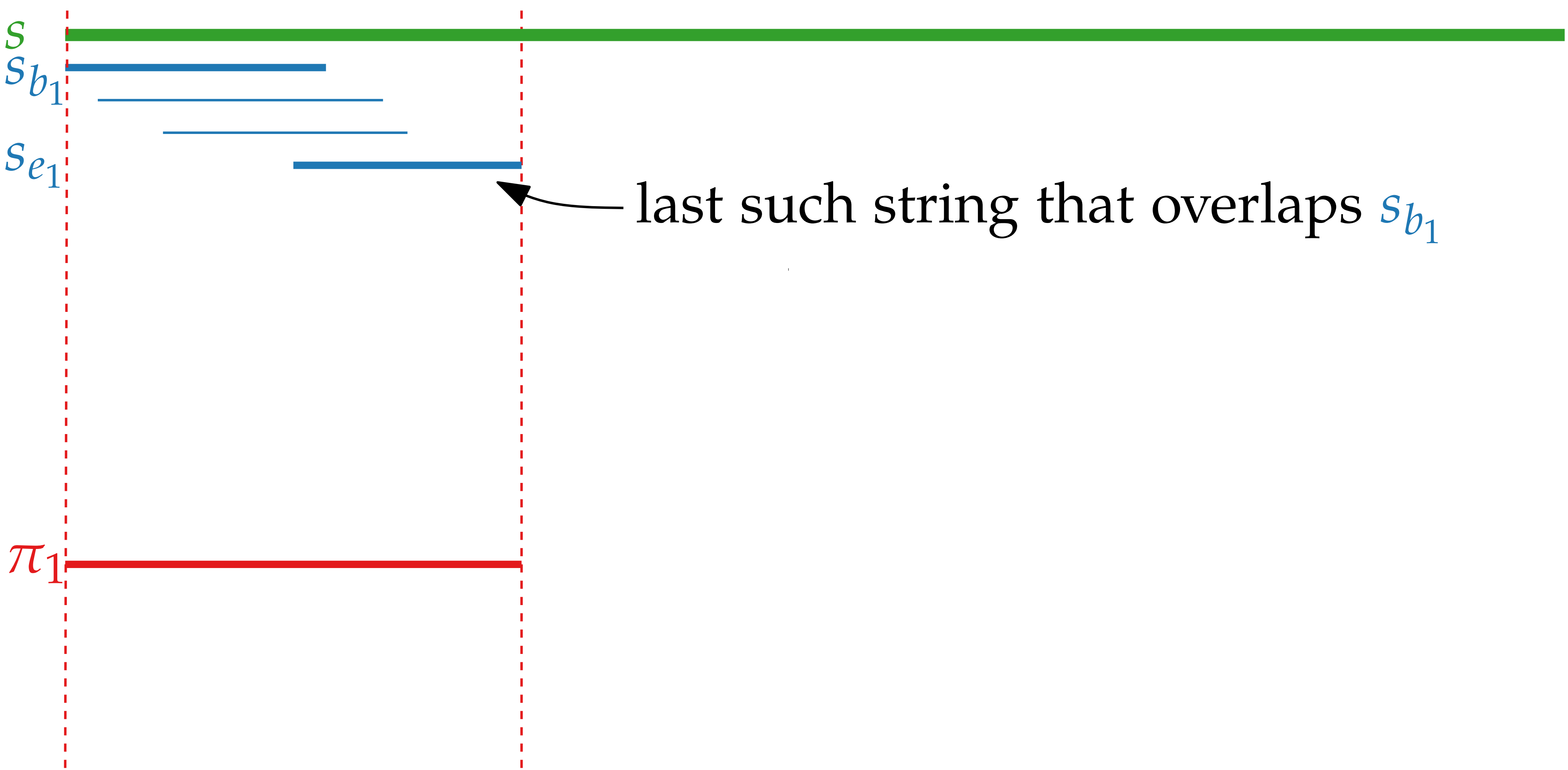

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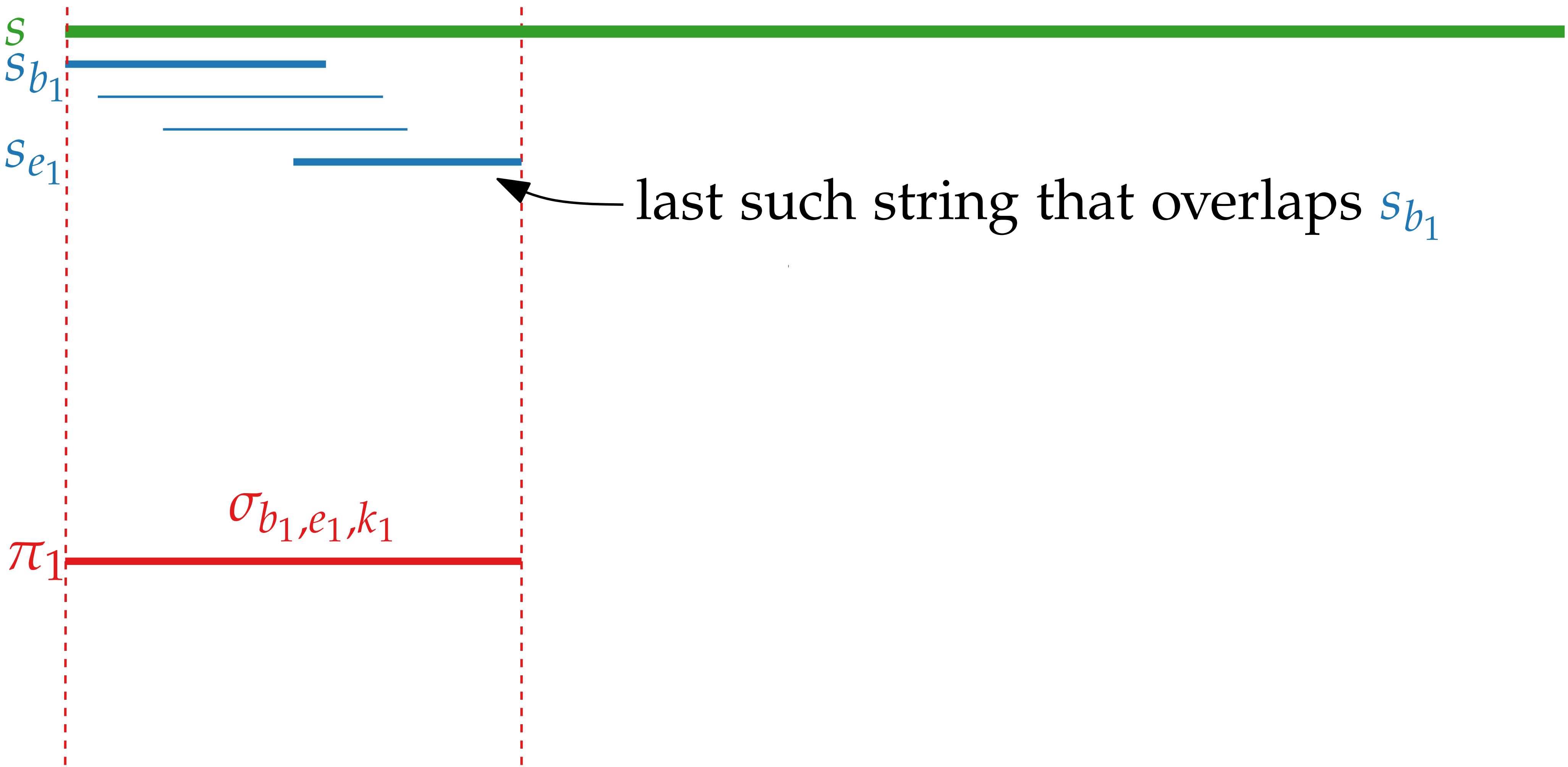

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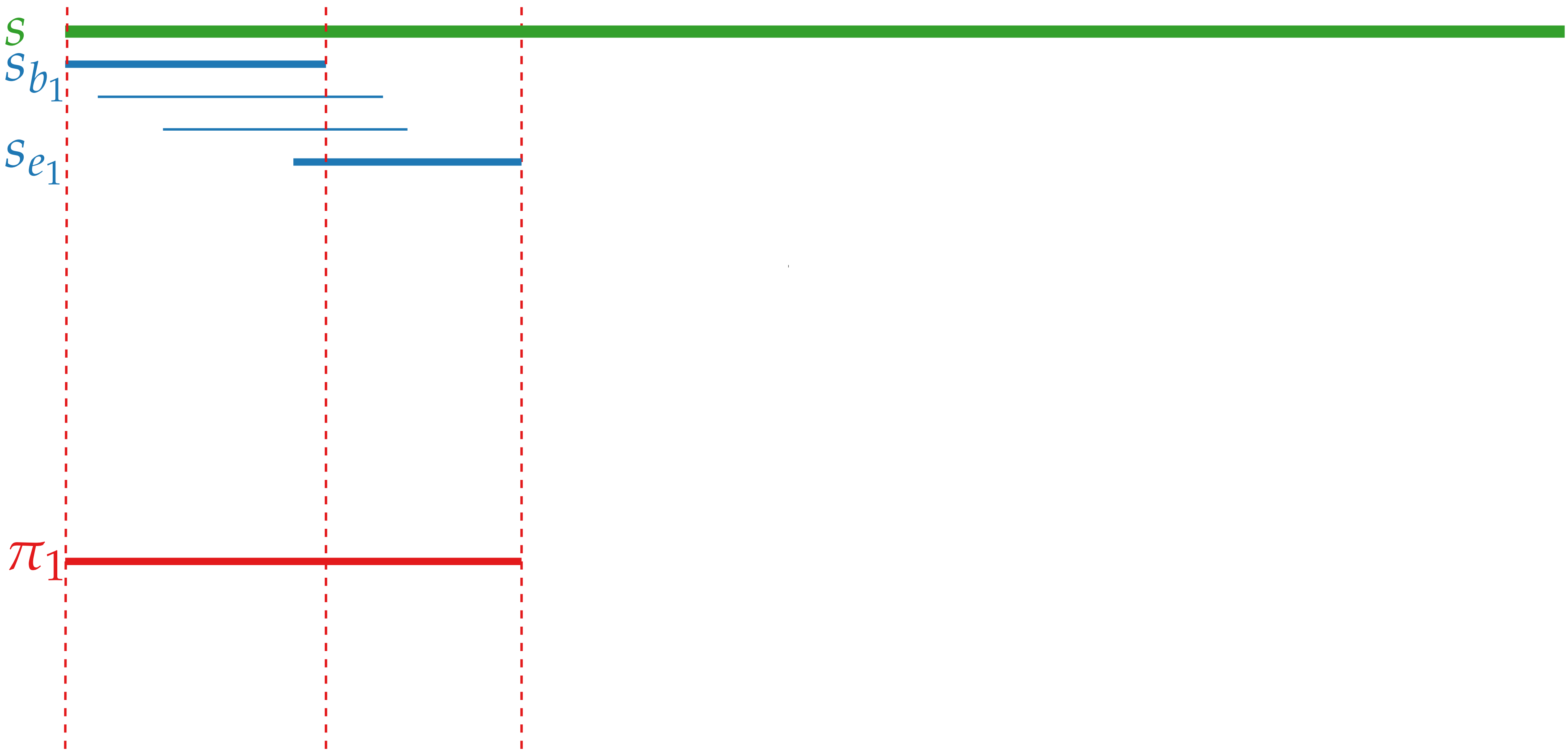

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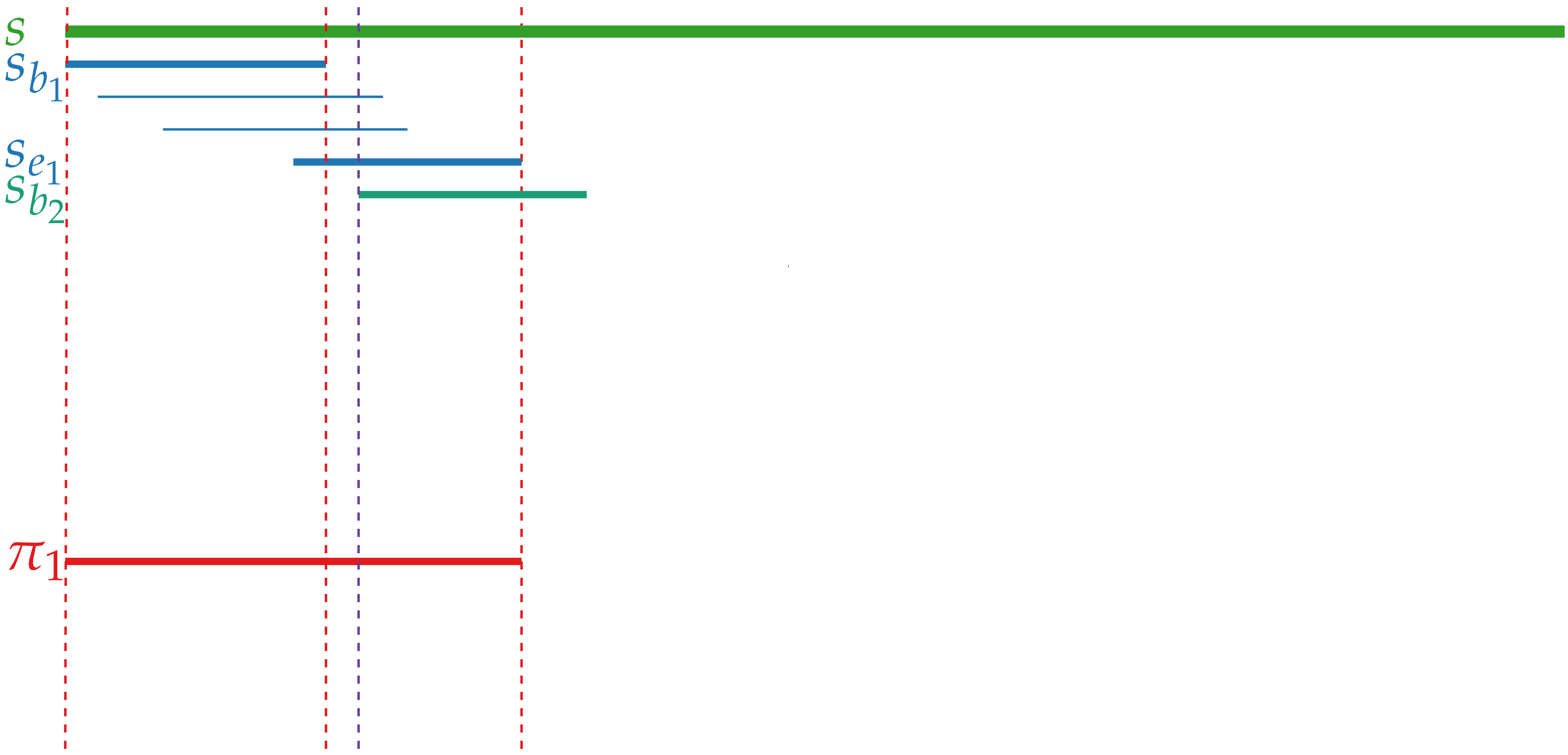

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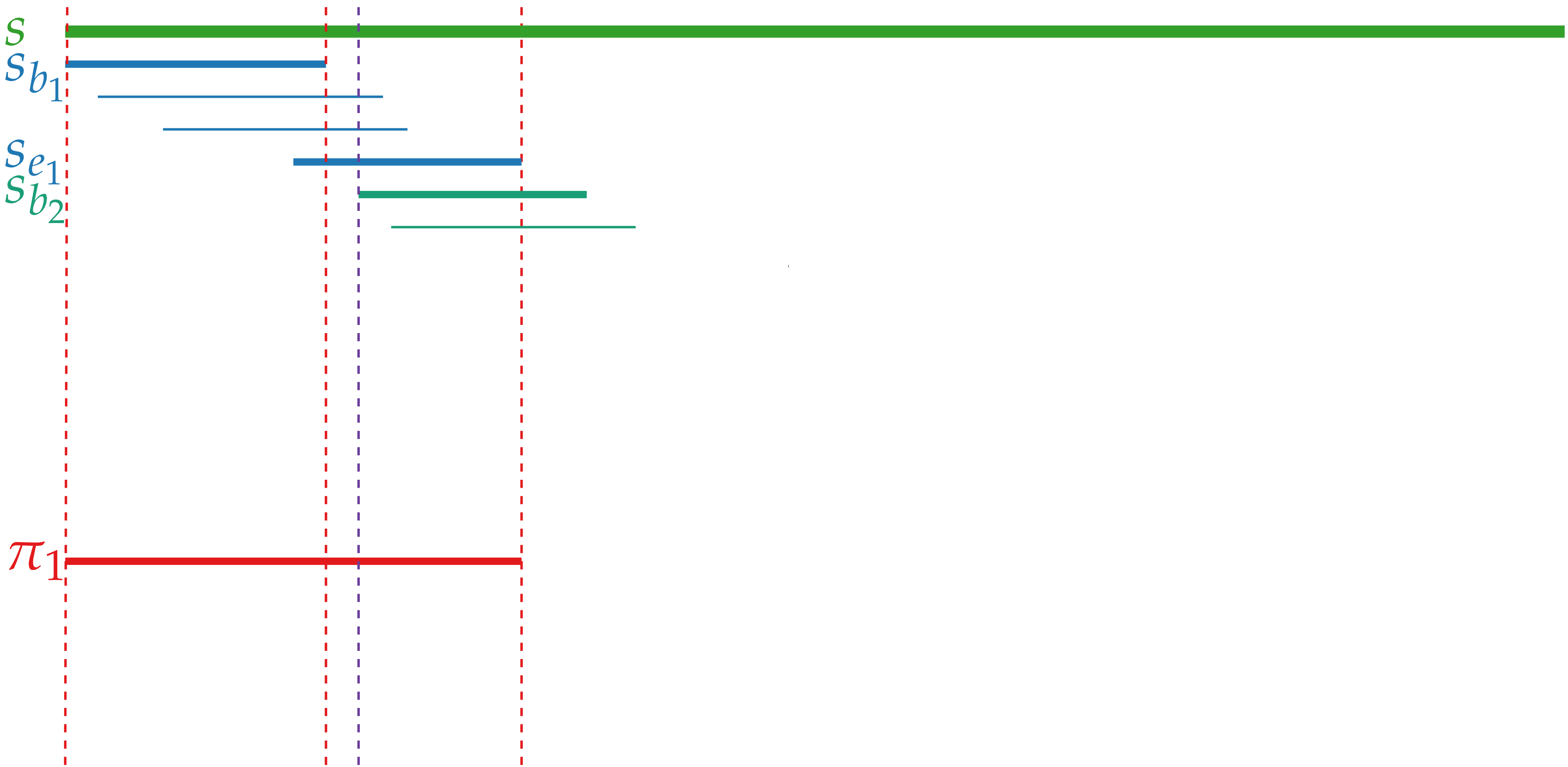

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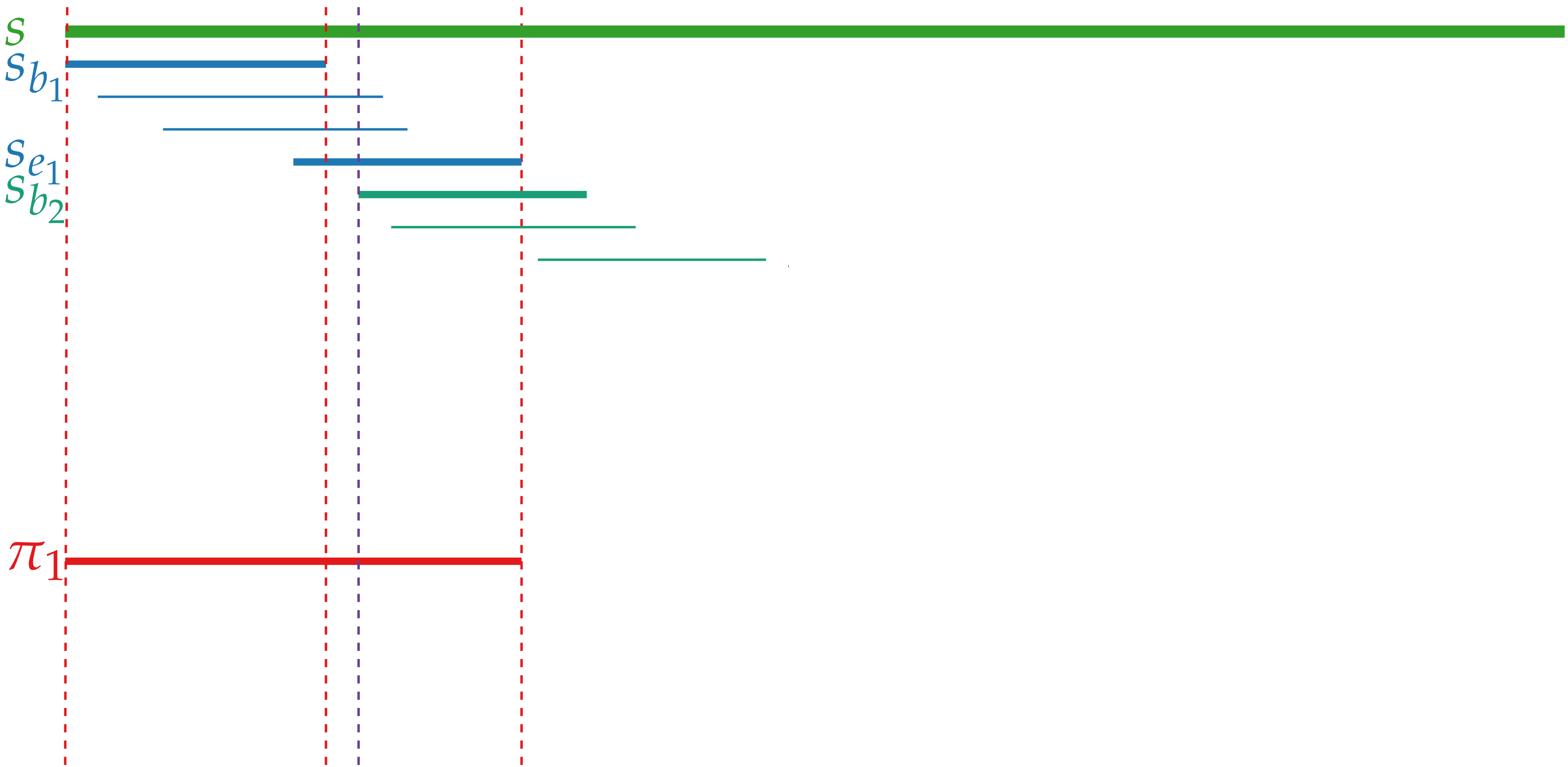

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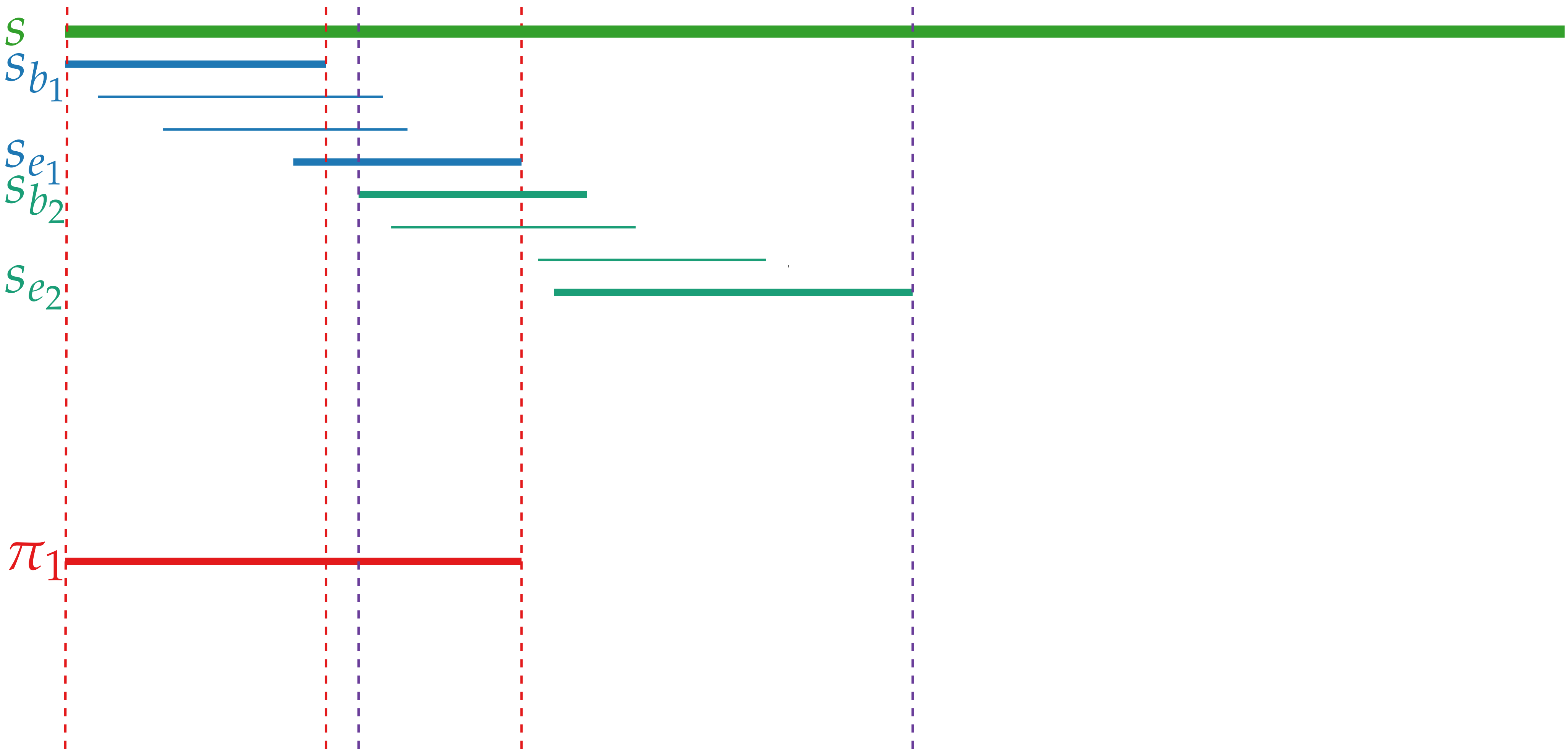

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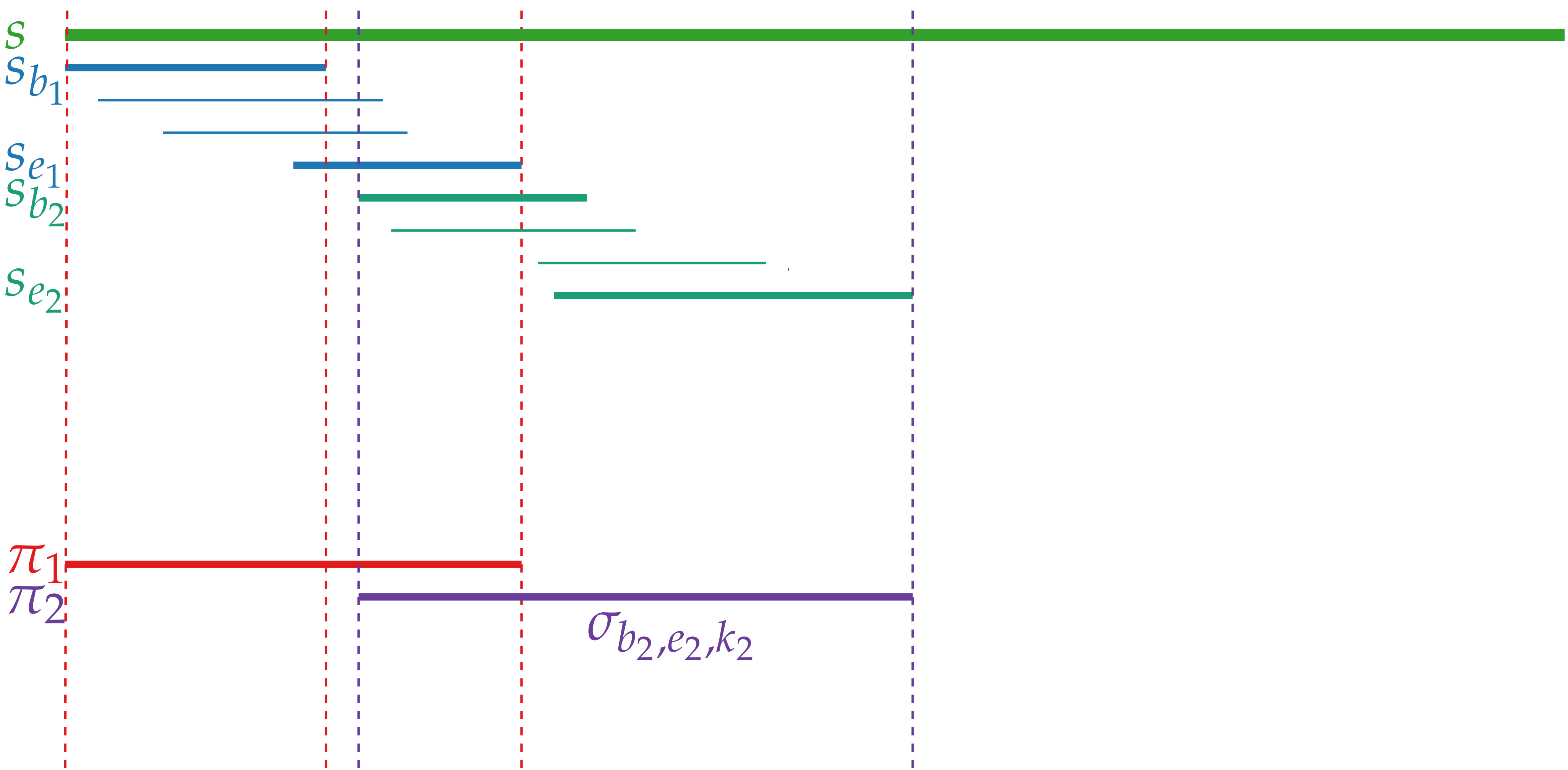

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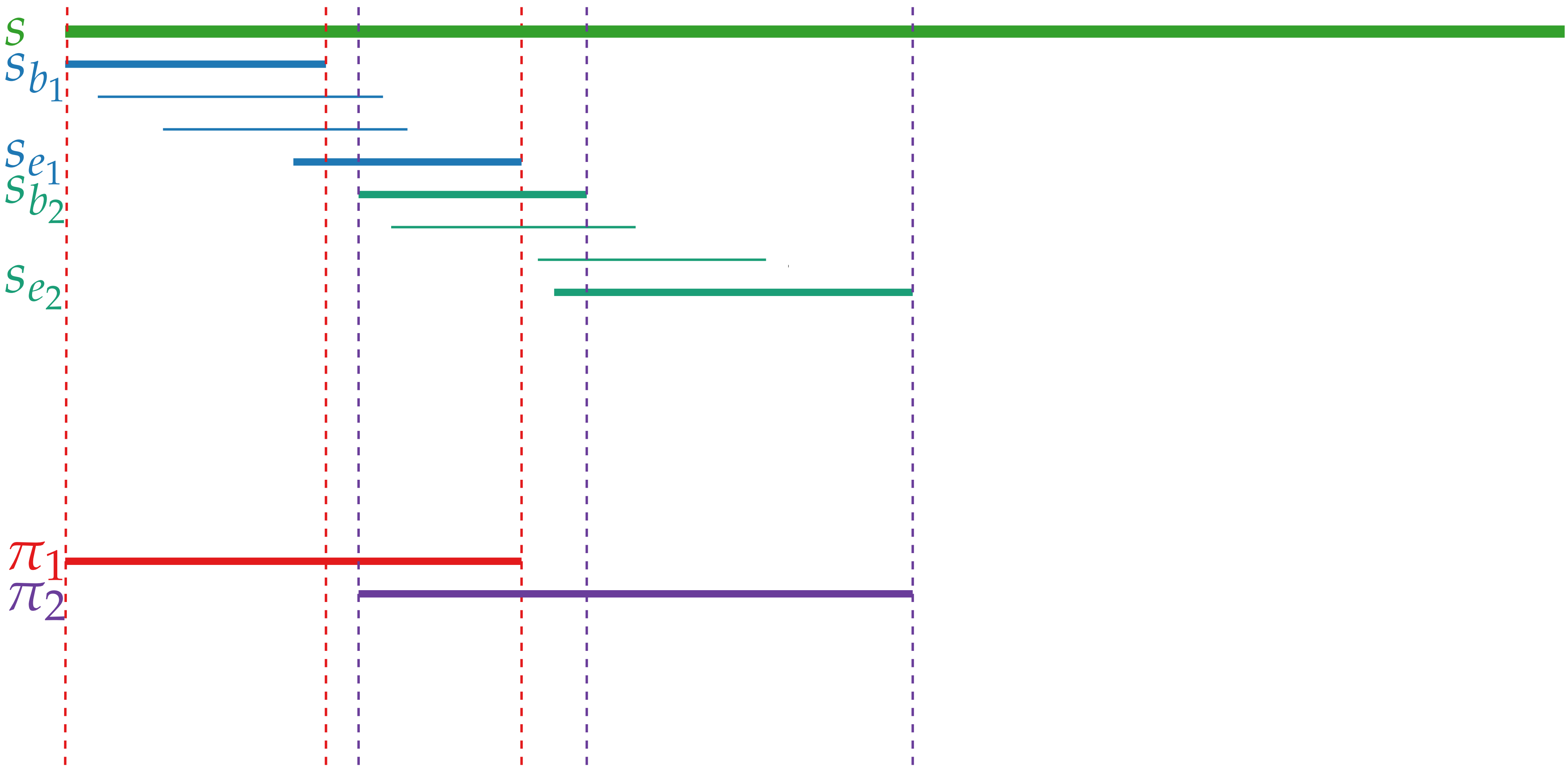

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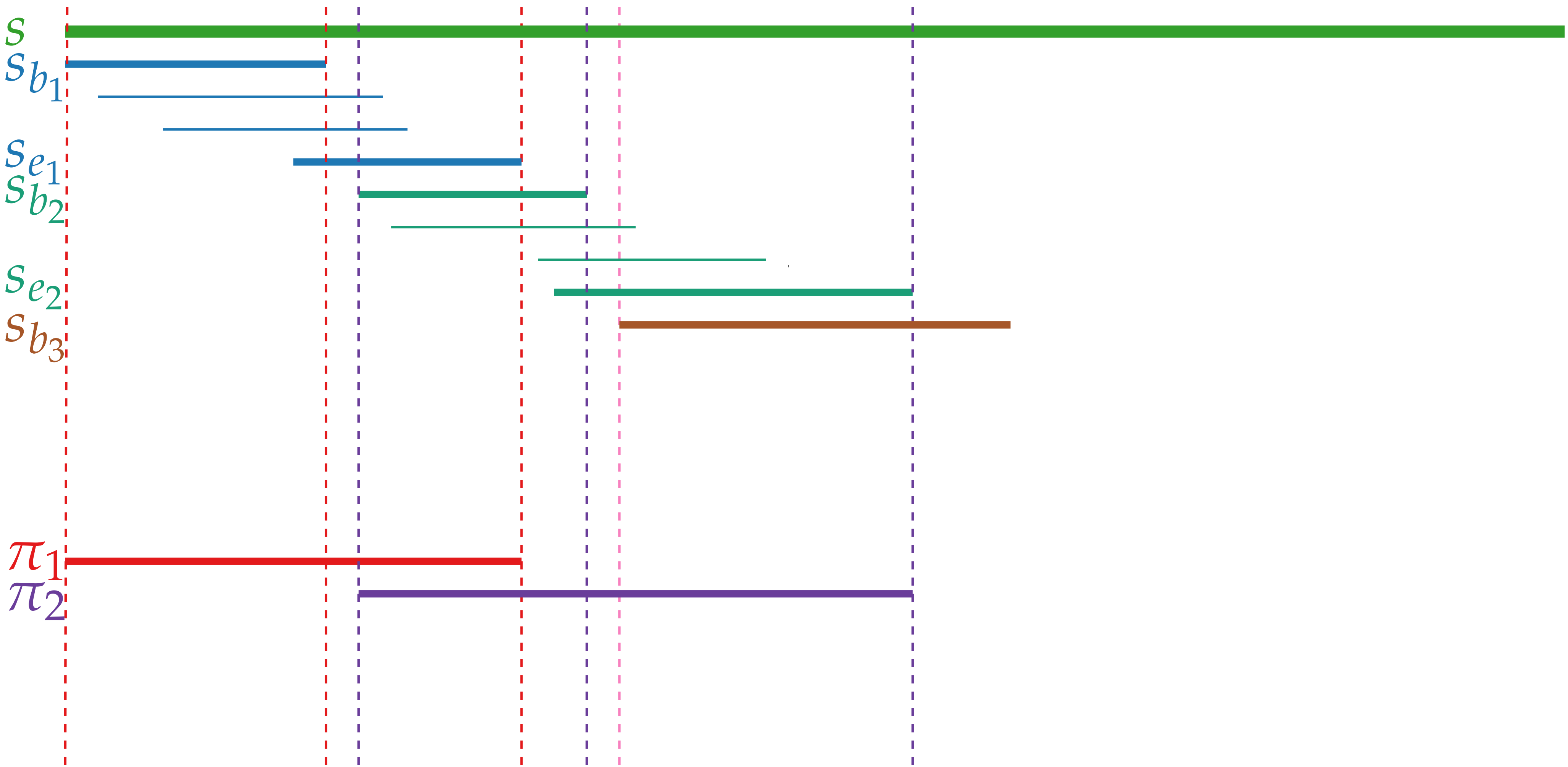

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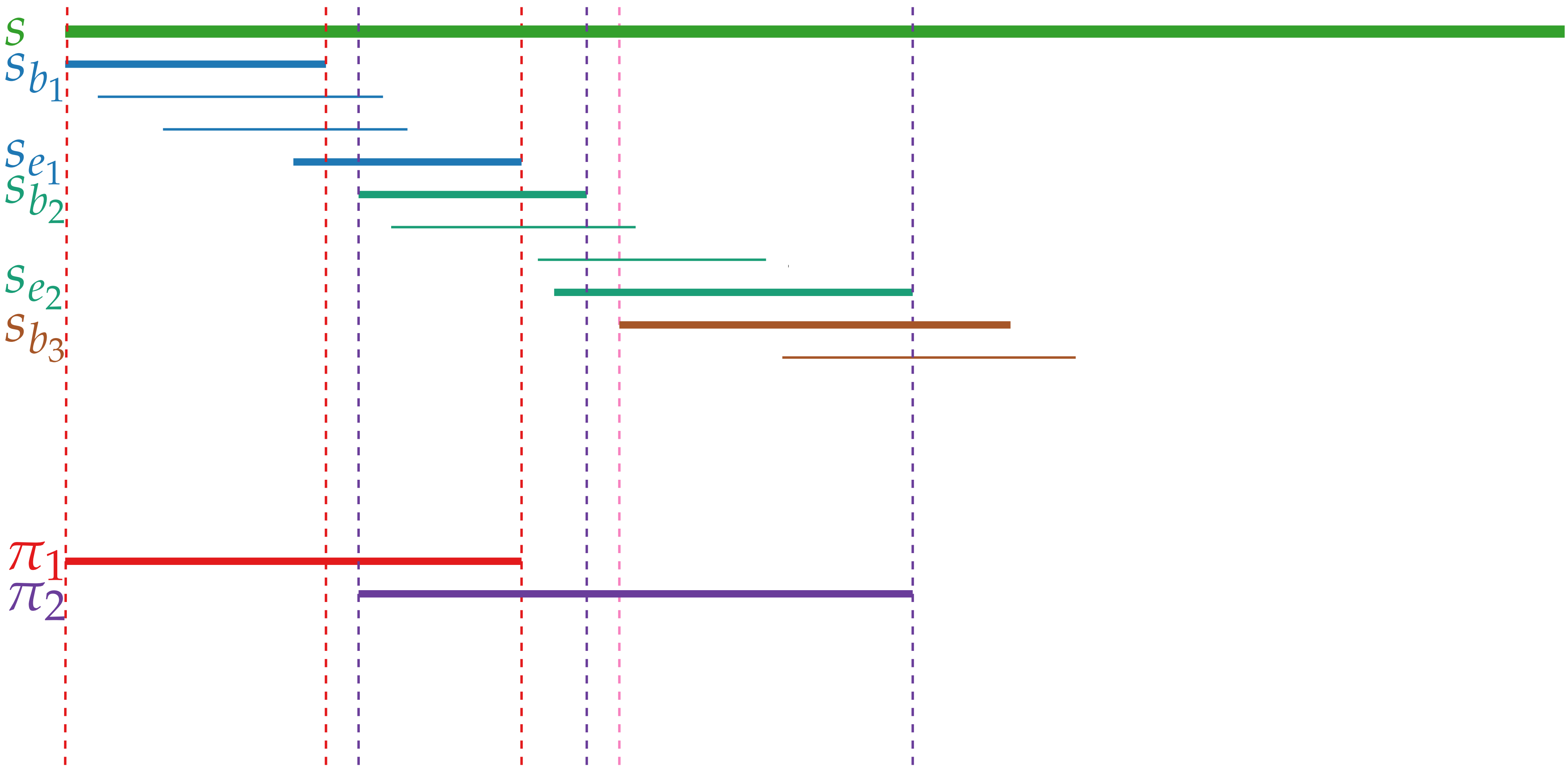

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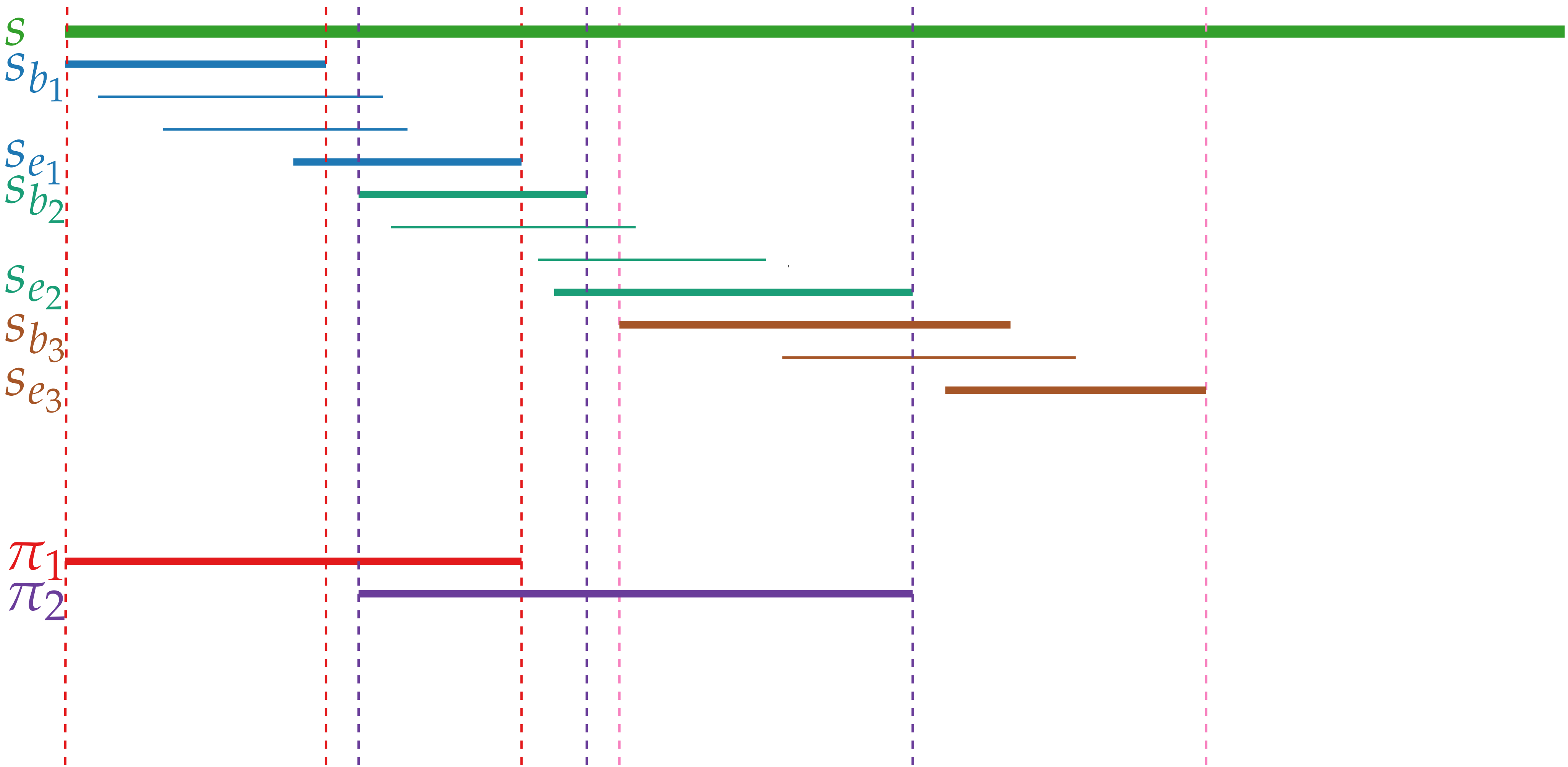

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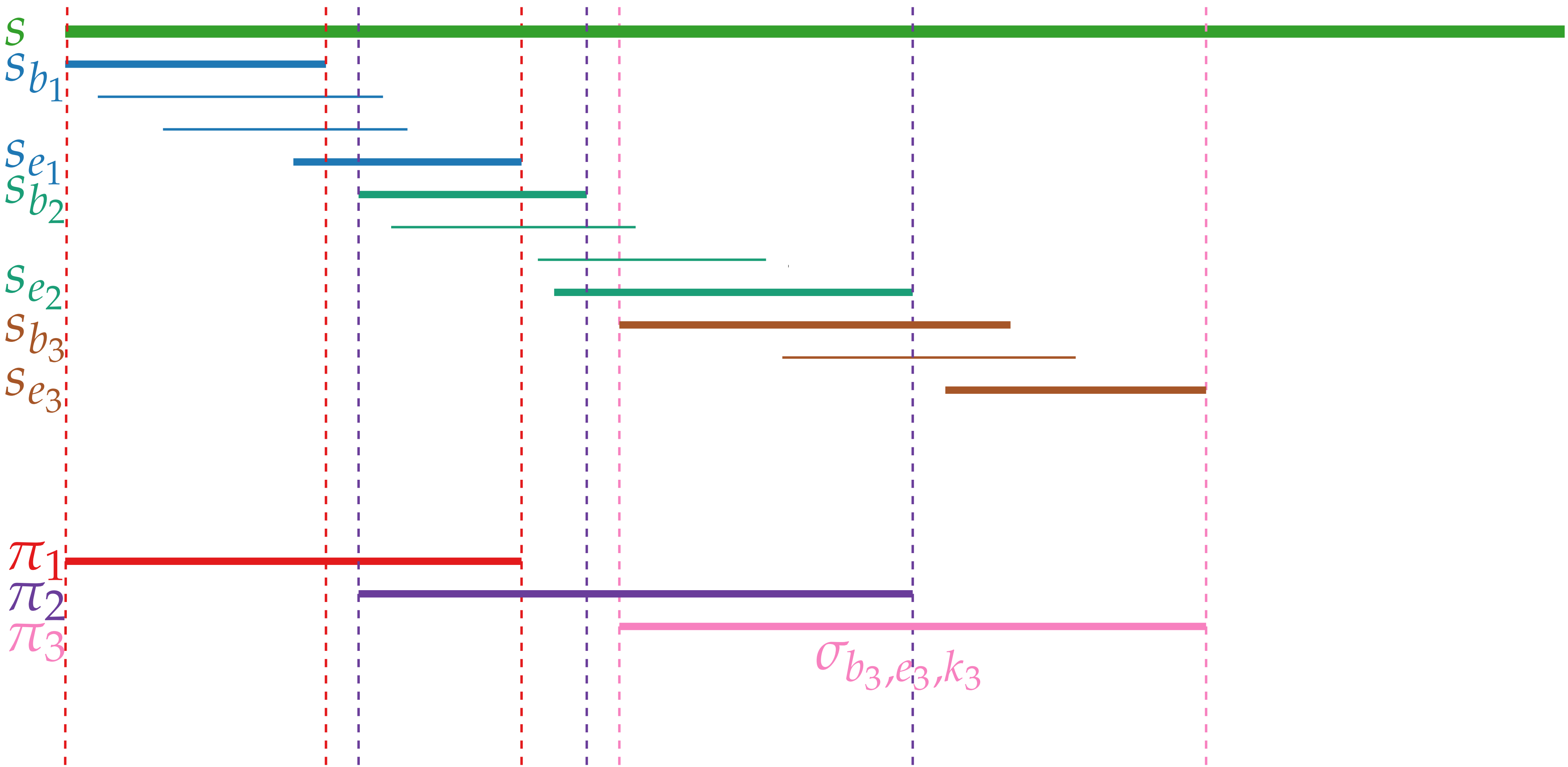

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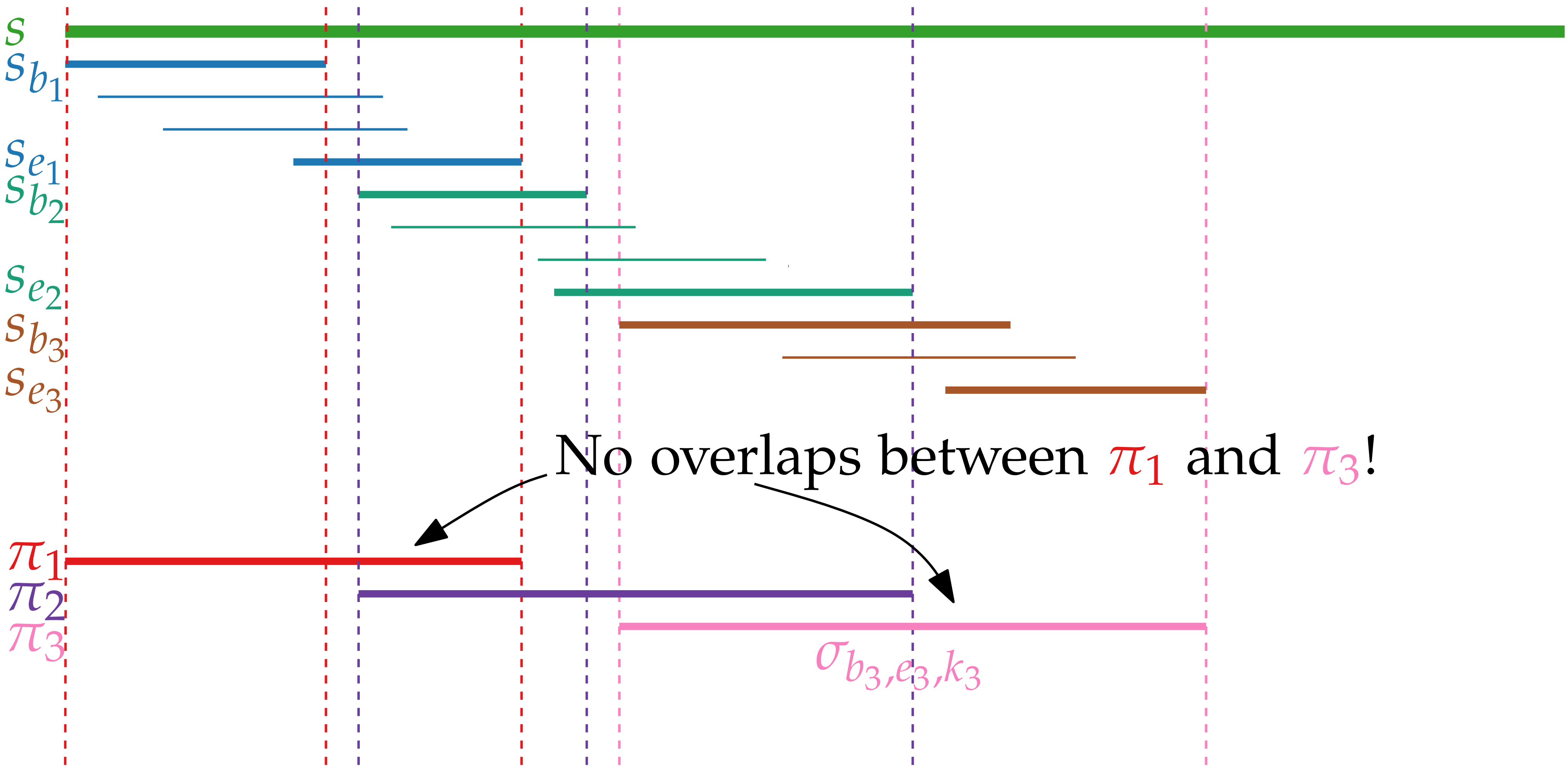

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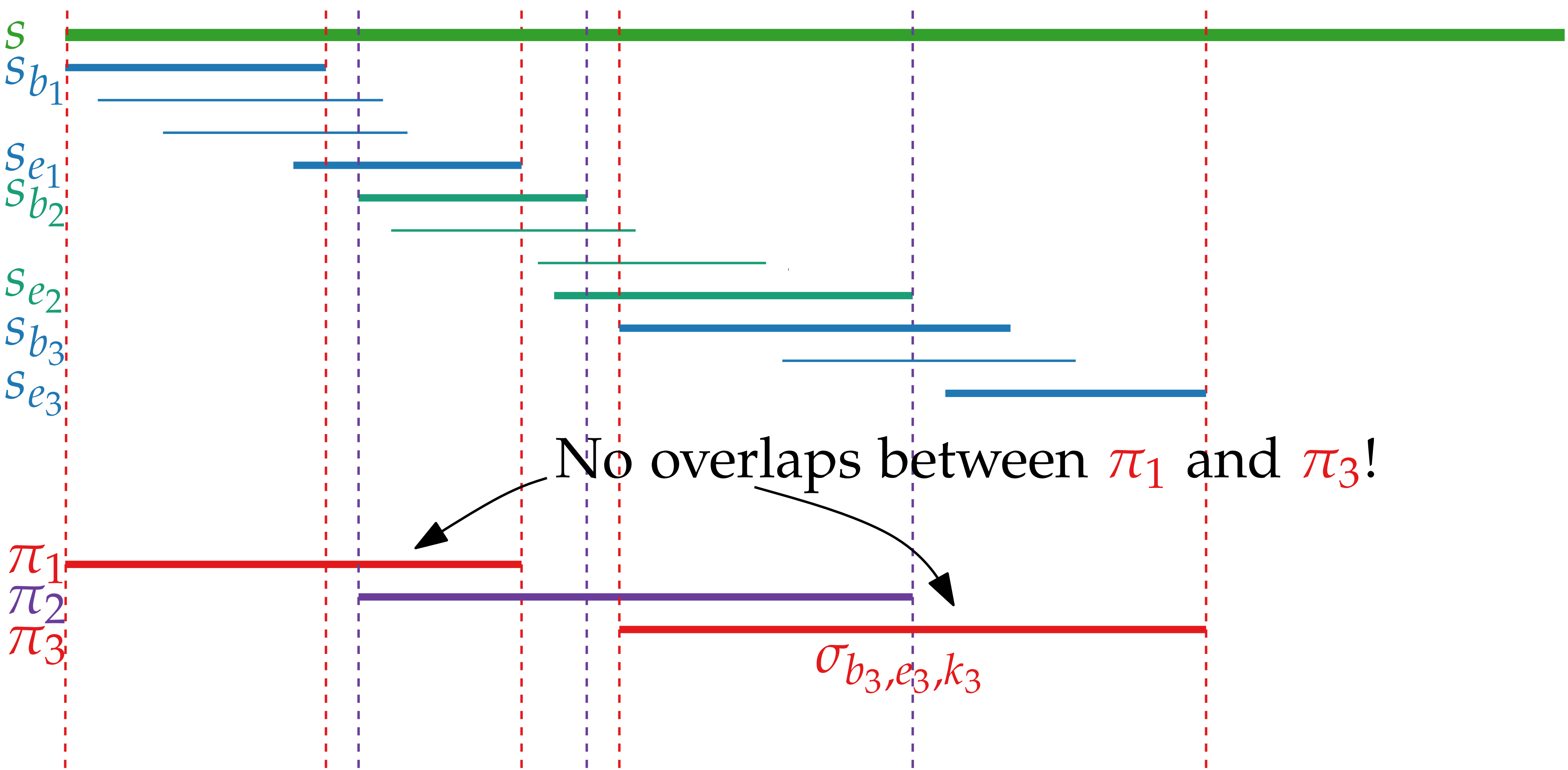

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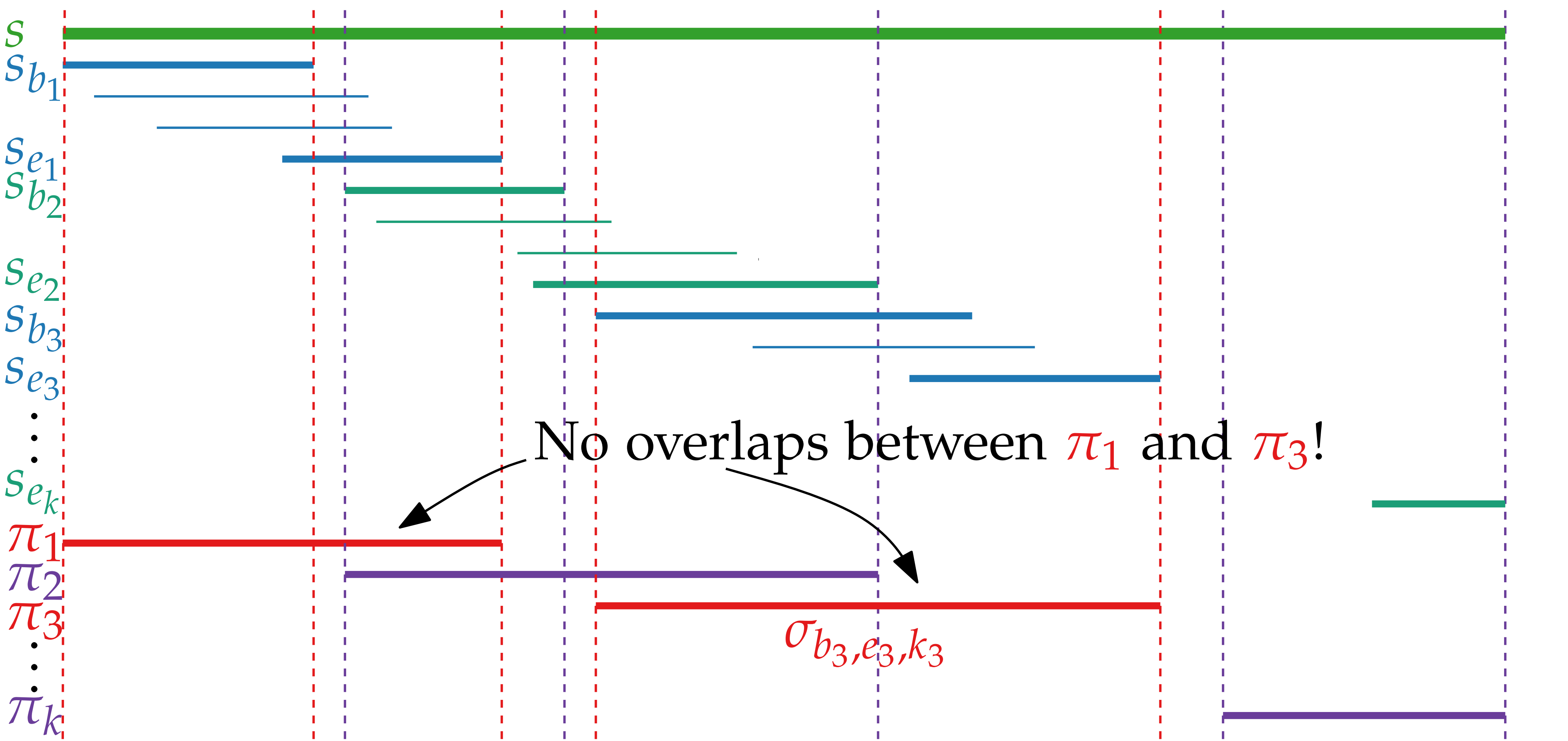

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Theorem. This algorithms is a factor-$2\mathcal{H}_n$-approximation algorithm for ShortestSuperString.

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1. Construct SetCover instance $U, \mathcal{S}, c$.
2. Compute a set cover $\{S(\pi_1), \ldots, S(\pi_k)\}$ with algorithm GreedySetCover.
3. Return $\pi_1 \circ \cdots \circ \pi_k$ as the superstring.

Theorem. This algorithms is a factor-$2\mathcal{H}_n$-approximation algorithm for ShortestSuperString.

better?

The best-known approximation factor for ShortestSuperString is $\frac{71}{30} \approx 2.367$.

ShortestSuperString cannot be approximation within factor $\frac{333}{332} \approx 1.003$ (unless P=NP).