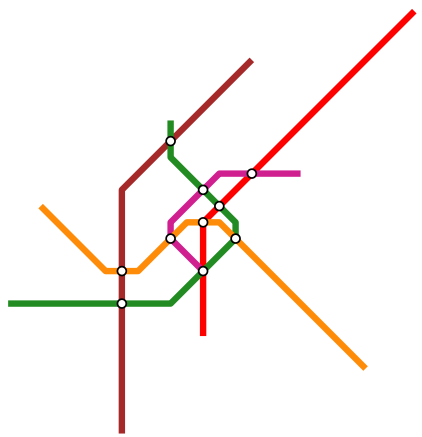
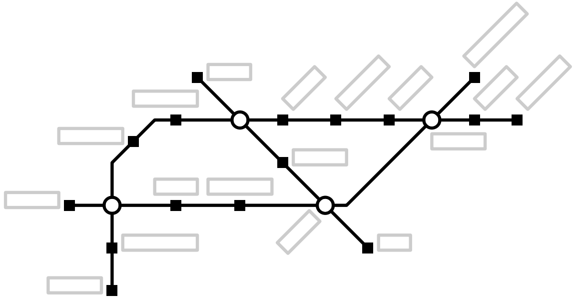
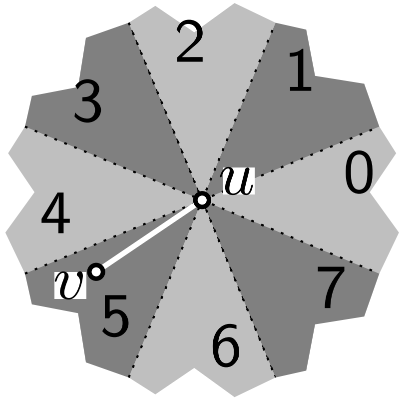
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Visualization of Graphs

Lecture 12:
Octilinear Graph Drawing
Metro Map Layout



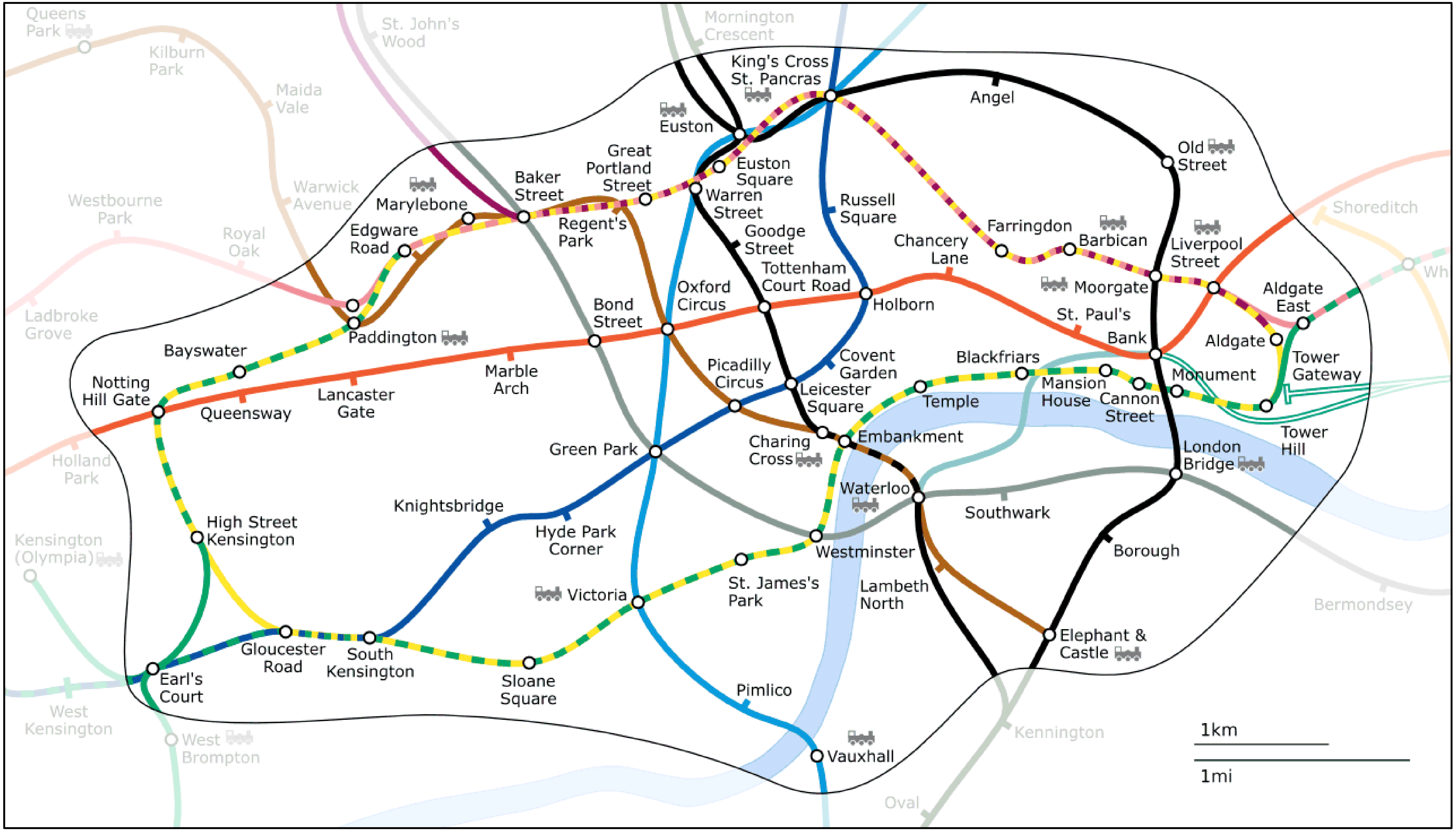




Part I:
Schematic Metro Maps

Jonathan Klawitter

What is a Schematic Metro Map?



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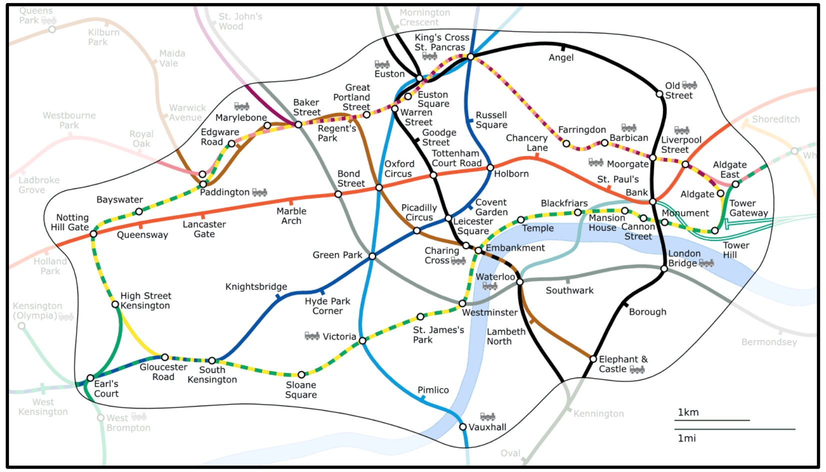

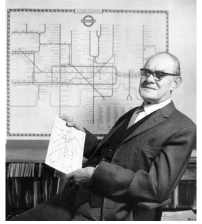

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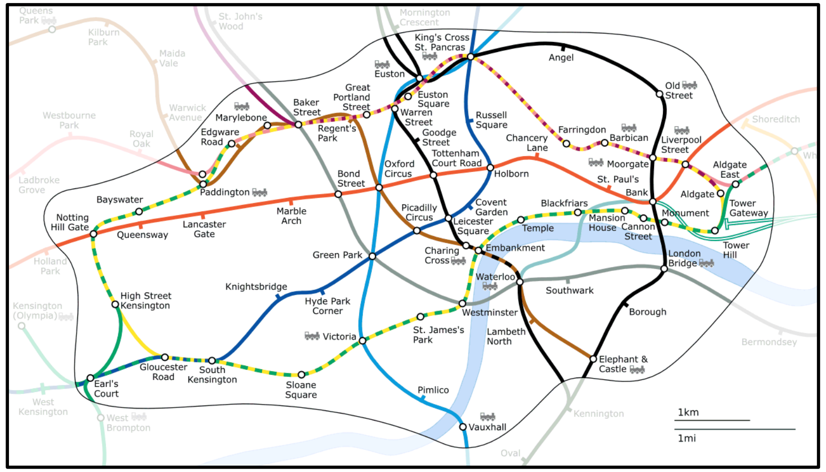

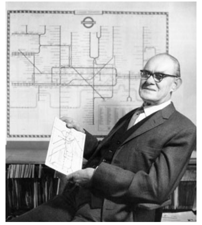

- map/diagram that shows stations connected by metro lines

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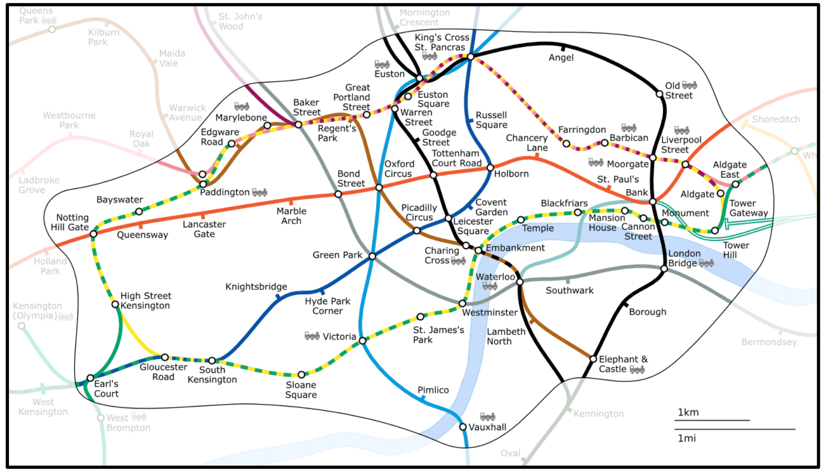

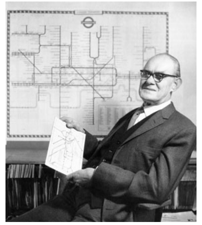

- map/diagram that shows stations connected by metro lines
- focus on topology rather than topography

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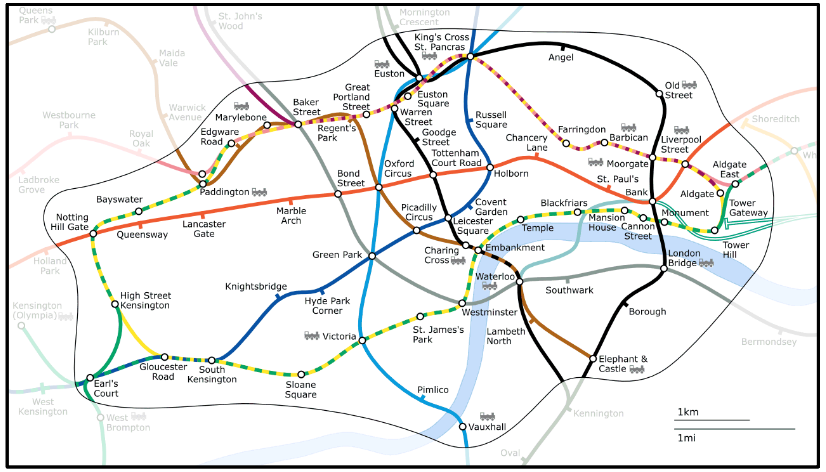

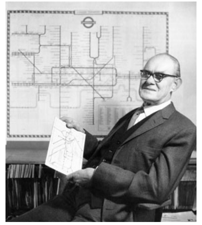

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- goal: easy-to-use visual navigation aid for passengers

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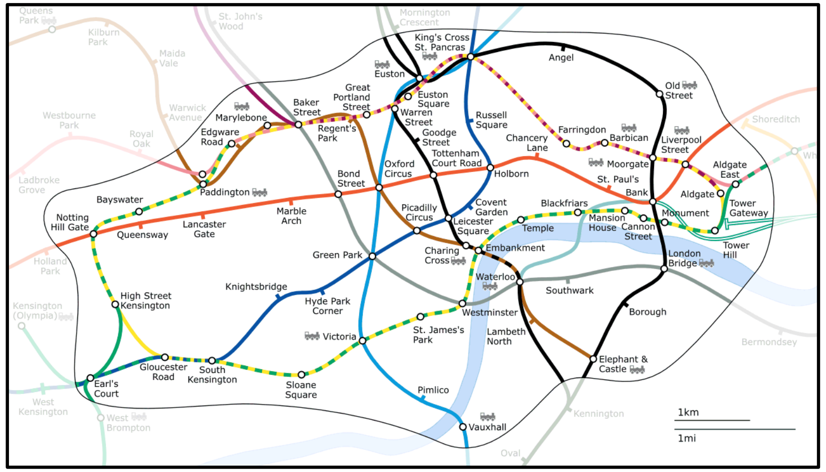

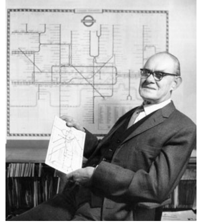

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  - “*How do I quickly get from A to B?*”

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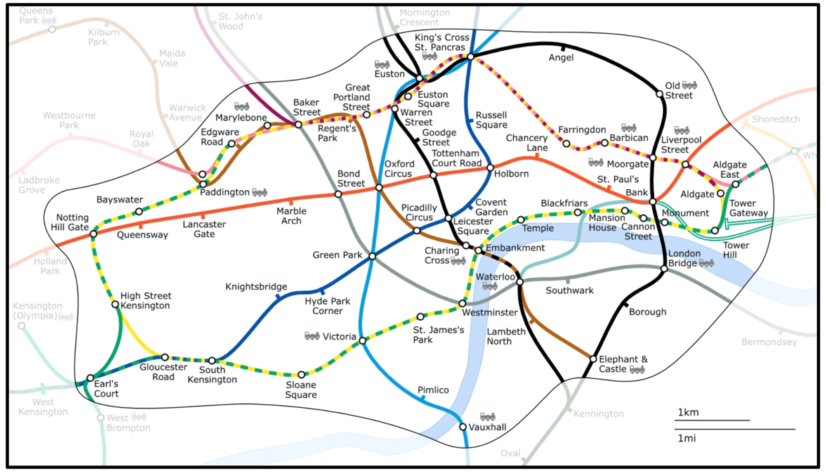

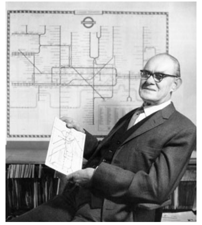

- map/diagram that shows stations connected by metro lines
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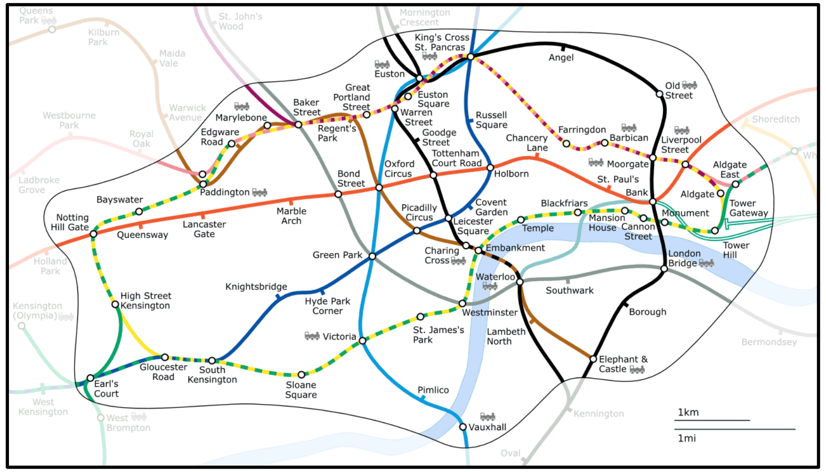

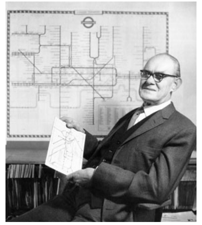

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- distorts scale and geometry

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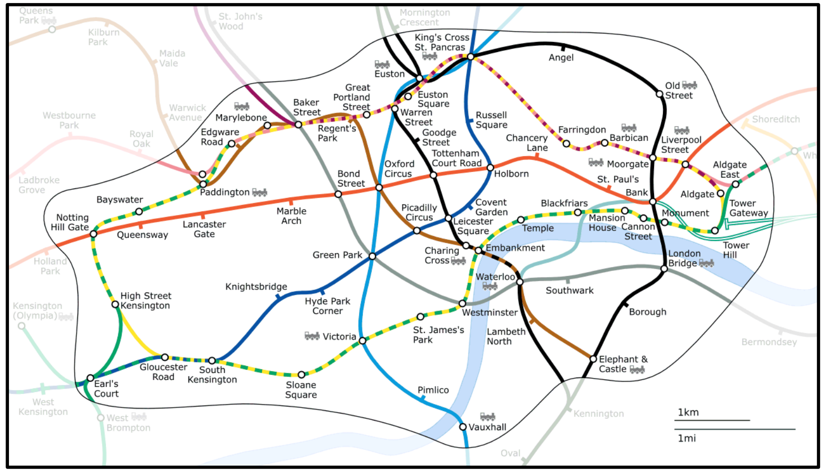

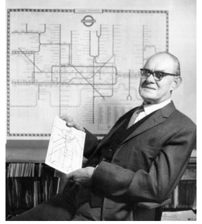

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- distorts scale and geometry
- metro map design still a largely manual process

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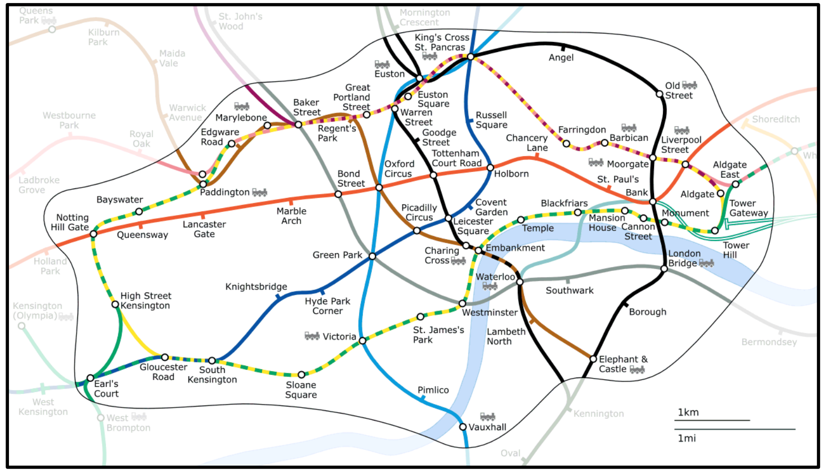

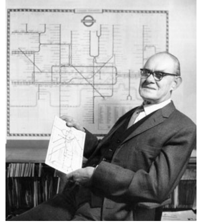

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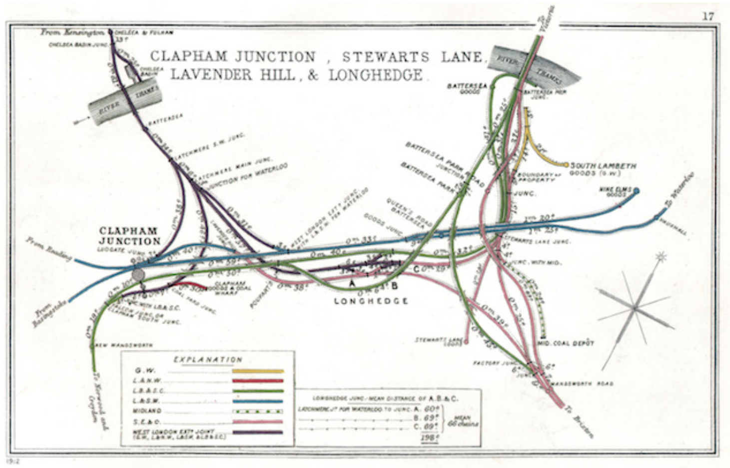
- goal: easy-to-use visual navigation aid for passengers
  - “*How do I quickly get from A to B?*”
  - “*Where do I need to change trains?*”
- distorts scale and geometry
- metro map design still a largely manual process
- optimizing network layout computationally challenging

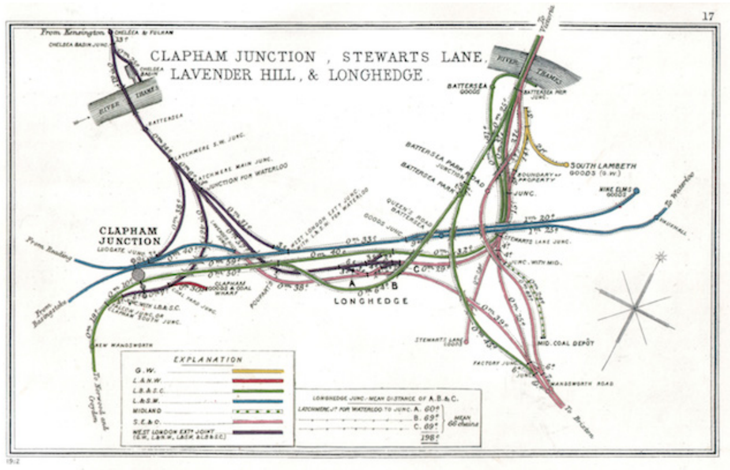
Subtasks in Metro Map Layout



Input.

- geographically embedded railway network G
- set of metro lines $\mathcal{L}$ serving G

Subtasks in Metro Map Layout



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Input.
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Output.
- **optimal** metro map layout (whatever it means)

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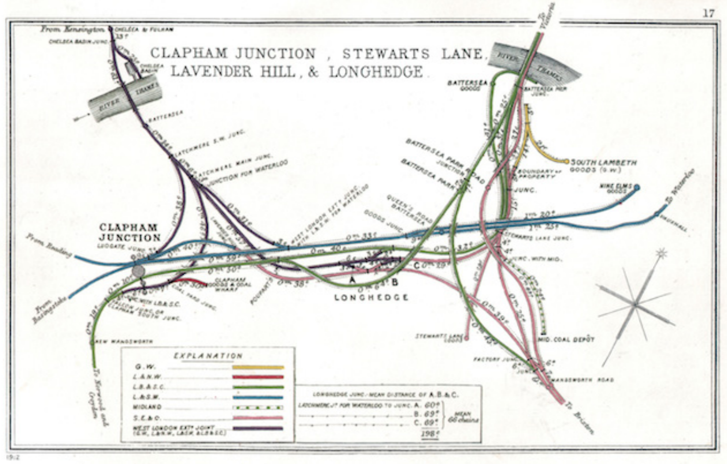

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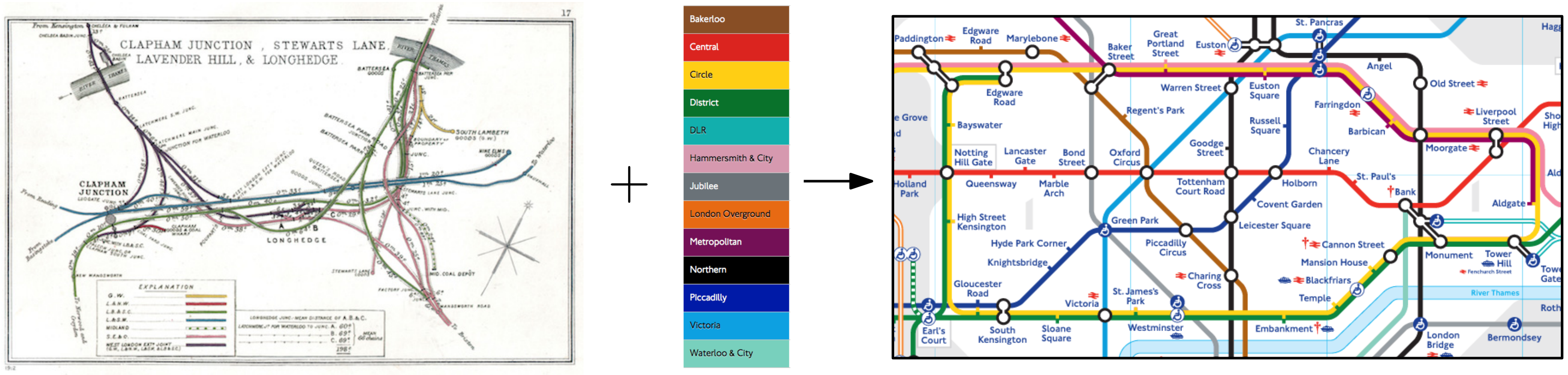

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Divide the task into several subtasks:

- rendering and design choices

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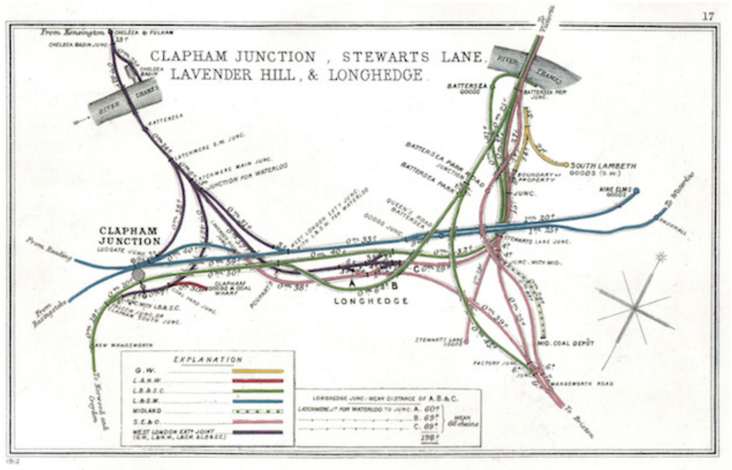

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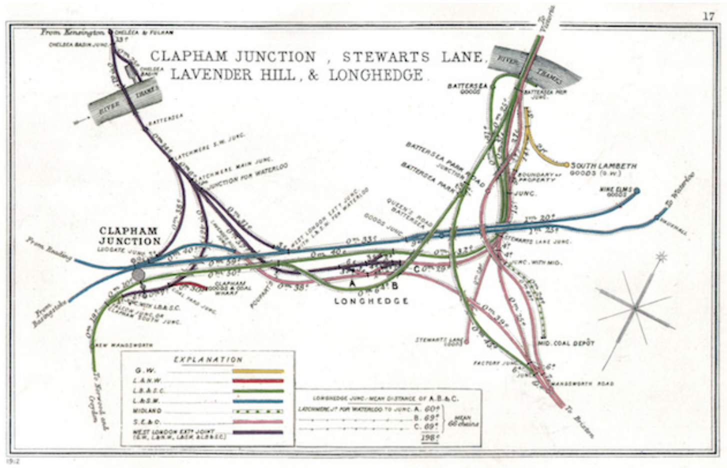

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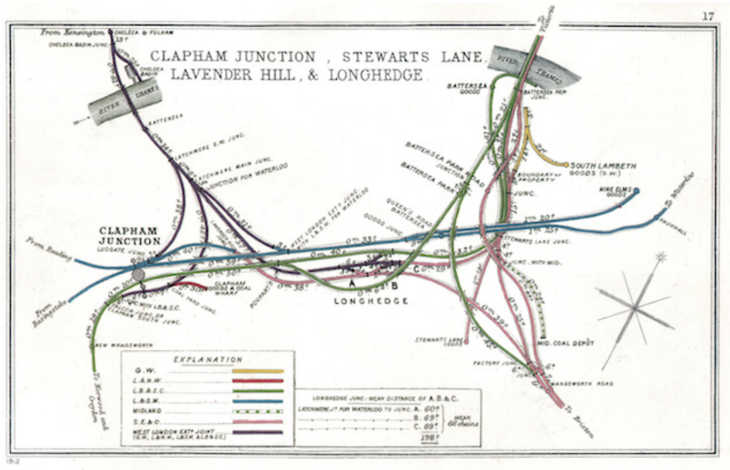

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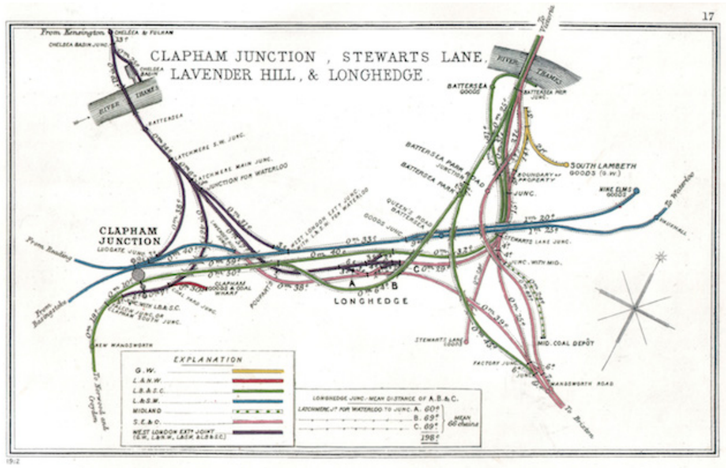

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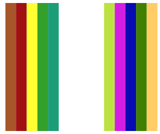

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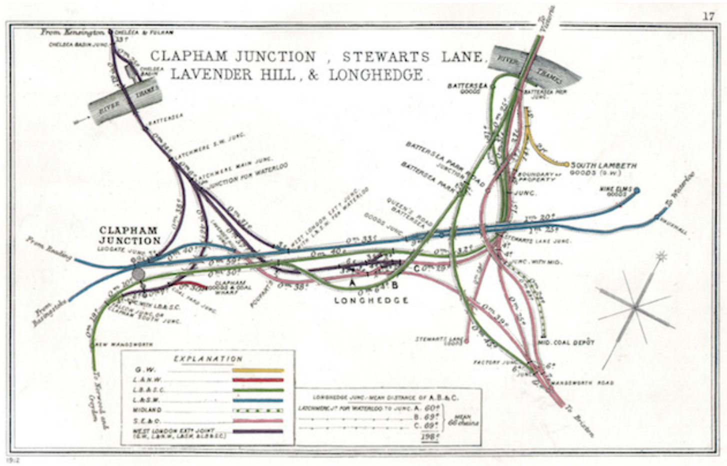

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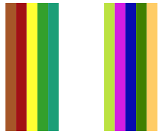
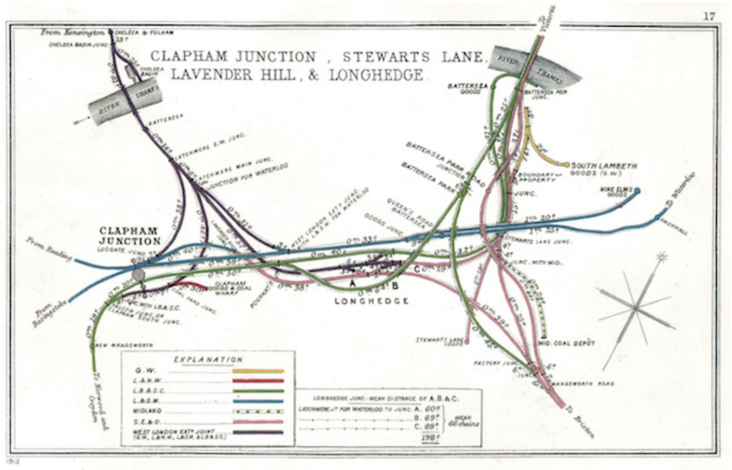


fonts

Paddington
PADDINGTON
Paddington

Subtasks in Metro Map Layout



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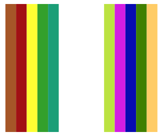

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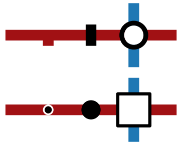
symbols



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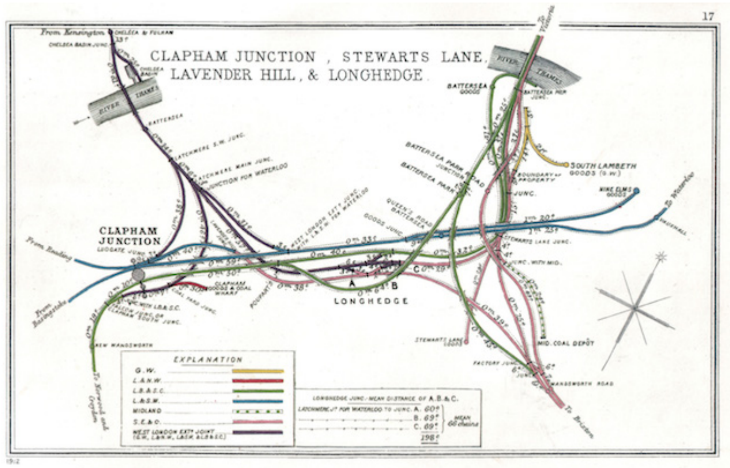

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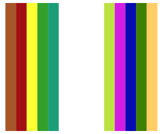

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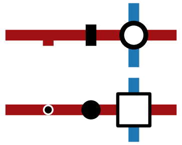
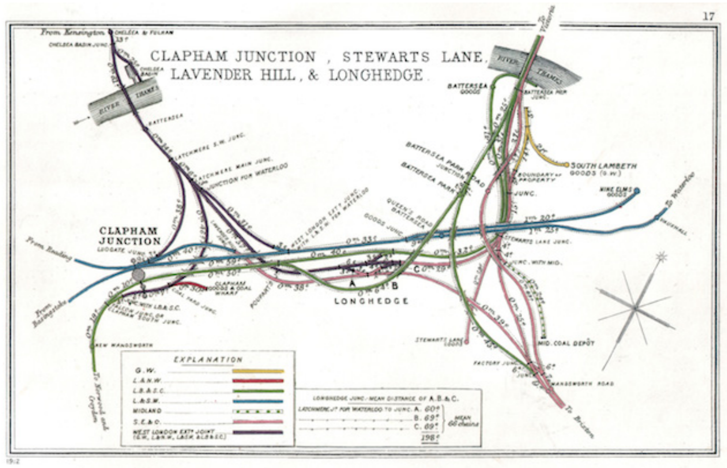


→ very salient,
but not a computational problem

Subtasks in Metro Map Layout



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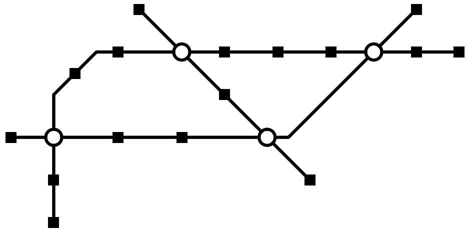
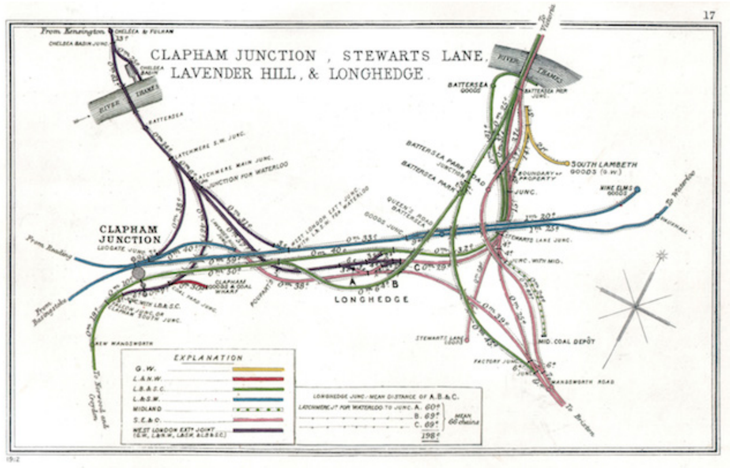


determine geometry of network layout

Subtasks in Metro Map Layout



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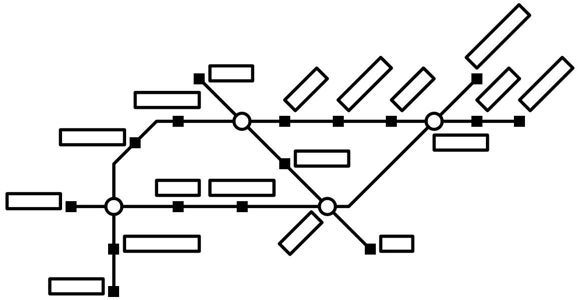
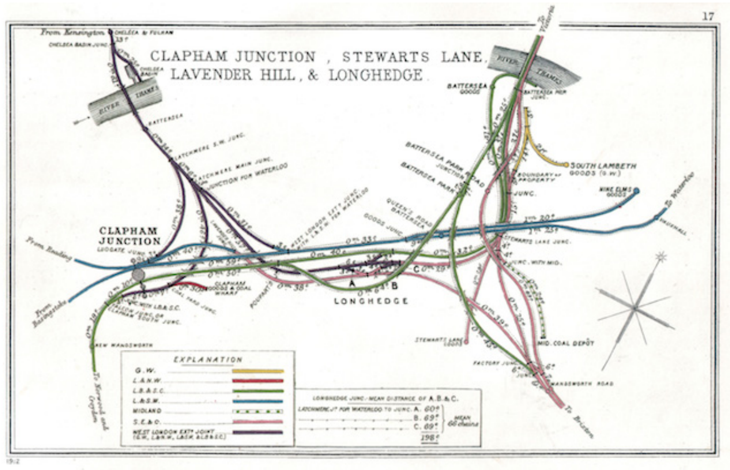


determine positions of station names

Subtasks in Metro Map Layout



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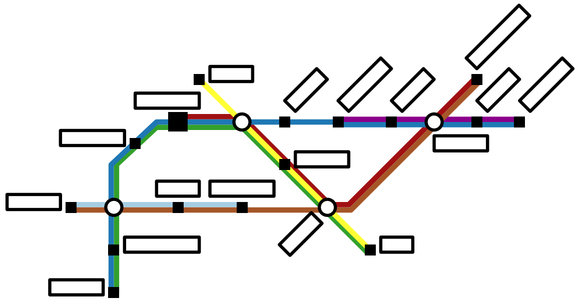
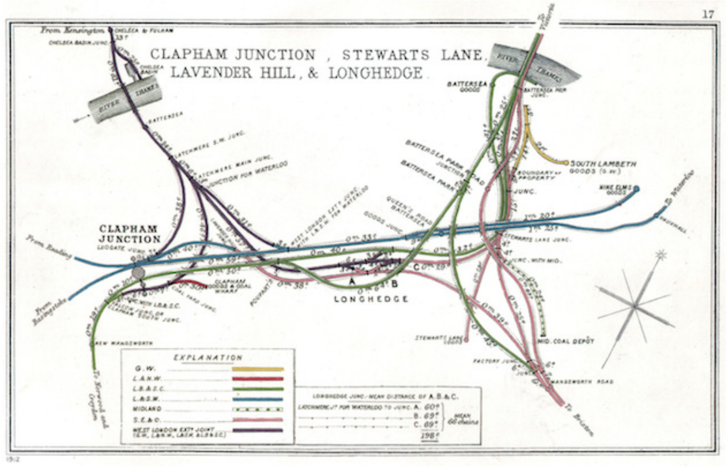


determine line routing and ordering of bundles

Subtasks in Metro Map Layout



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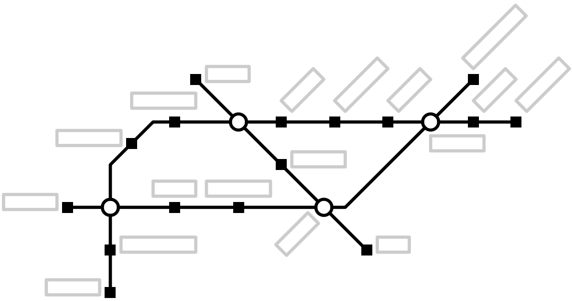
focus today



Formalizing the Network Layout Problem

Given.
- graph $G = (V, E)$ geometrically embedded in $\mathbb{R}^2$
- vertex set V (stations)
- edge set E (rail links)
- set of paths $\mathcal{L}$ (metro lines in G)

Goal. **schematic** layout of $(G, \mathcal{L})$ that
- satisfies a set of layout constraints
- optimizes a set of quality criteria

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Goal. **schematic** layout of $(G, \mathcal{L})$ that
- satisfies a set of layout constraints
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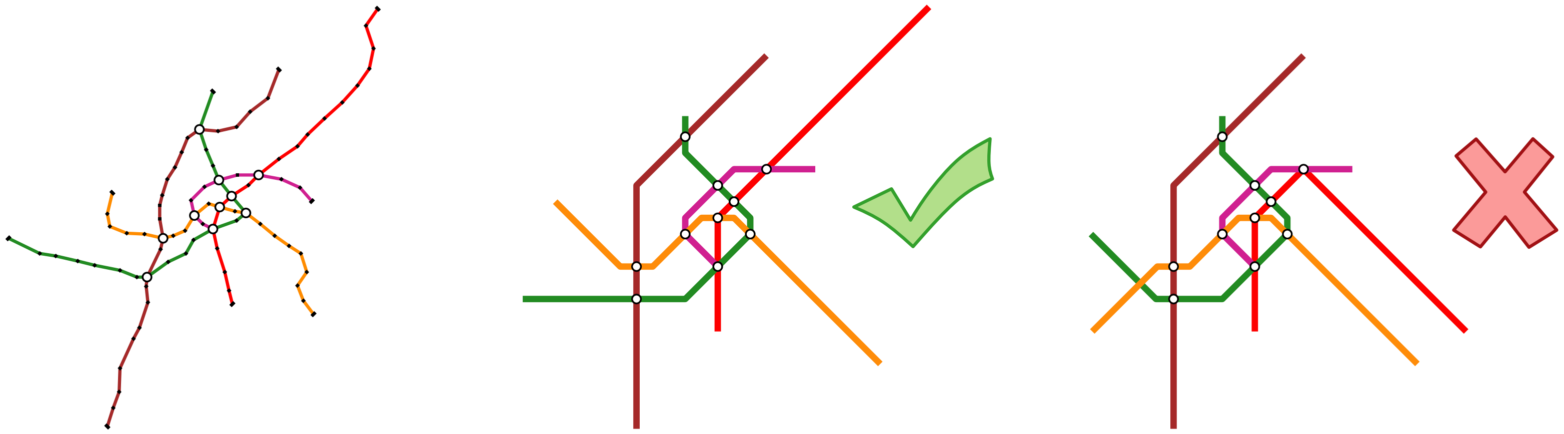
But what are the constraints and quality criteria?

$\rightarrow$ extract common principles of existing, manually designed metro maps

Design Rules

(R1) Do not change the network topology.

- no new crossings
- no changes in circular vertex orders

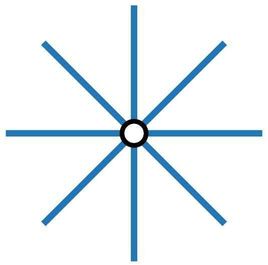


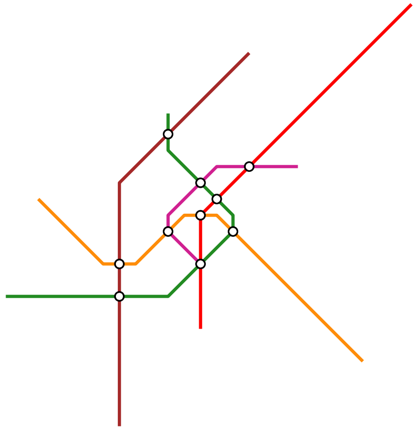
Design Rules

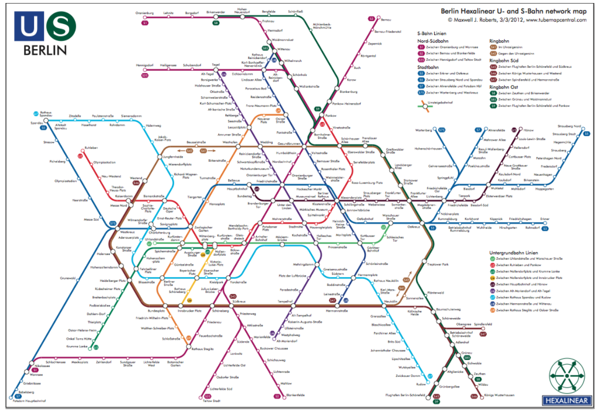
(R1) Do not change the network topology.

(R2) Restrict edge orientations.

- mostly octilinear (octolinear) orientation systems
- also curvilinear and other alternative orientation systems







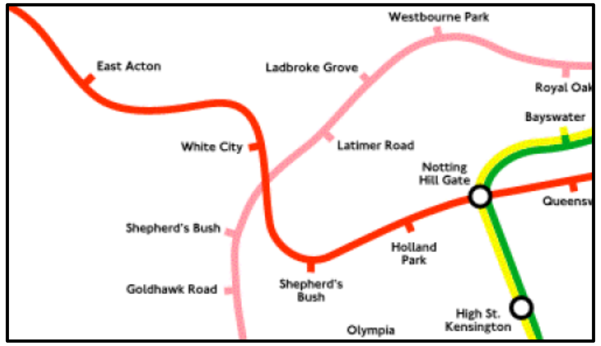
Design Rules

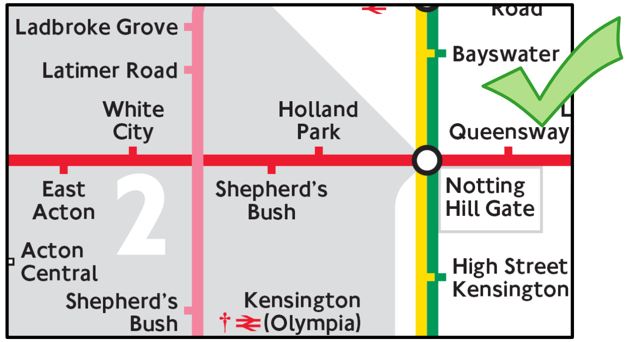
(R1) Do not change the network topology.

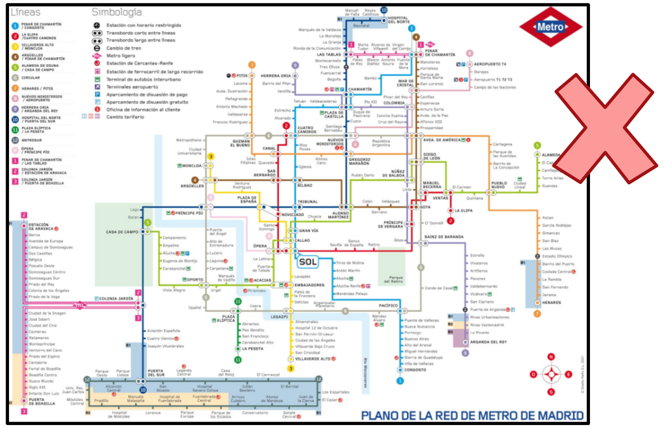
(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

- avoid bends
- prefer obtuse bend angles
- for curves: prefer uniform curvature, few inflection points







Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

- avoids visual ambiguities in complex stations

Design Rules

(R1) Do not change the network topology.

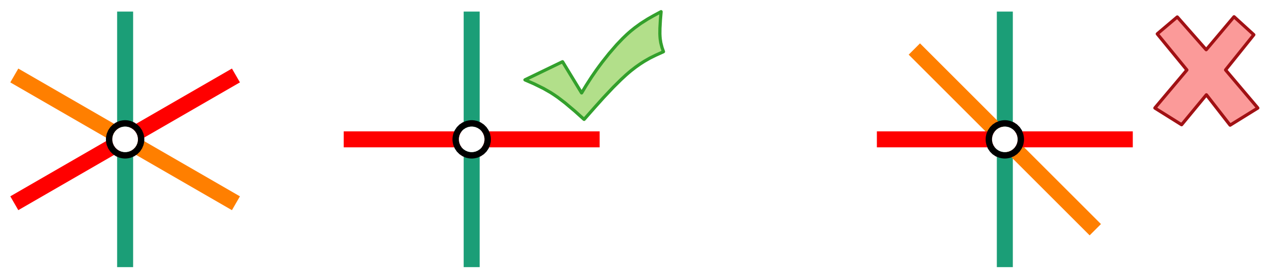
(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

(R5) Use large angular resolution in stations.

- distributes edges evenly for balanced appearance



Design Rules

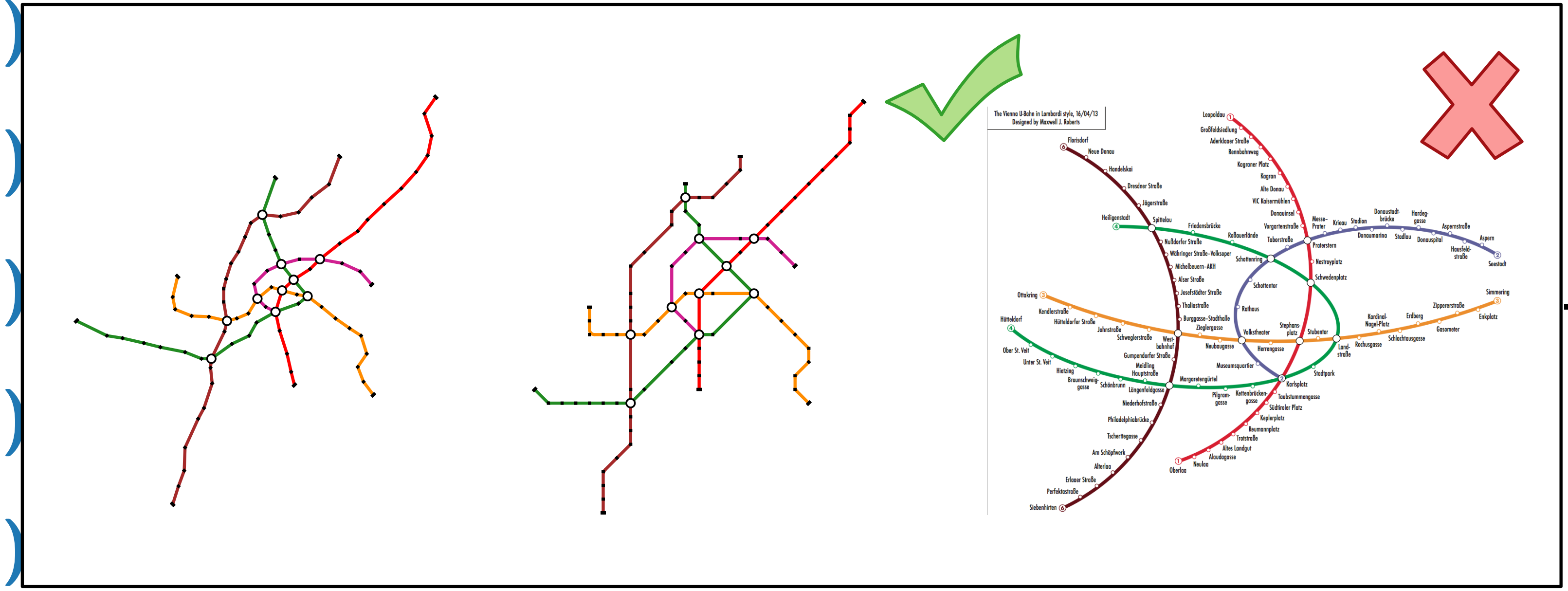
(R1)

(R2)

(R3)

(R4)

(R5)



(R6) Minimize geometric distortion and displacement.

- maintains resemblance to geography
- preserves user's mental map
- applicable locally or globally

topographicity

Design Rules

(R1) Do not change the network topology.

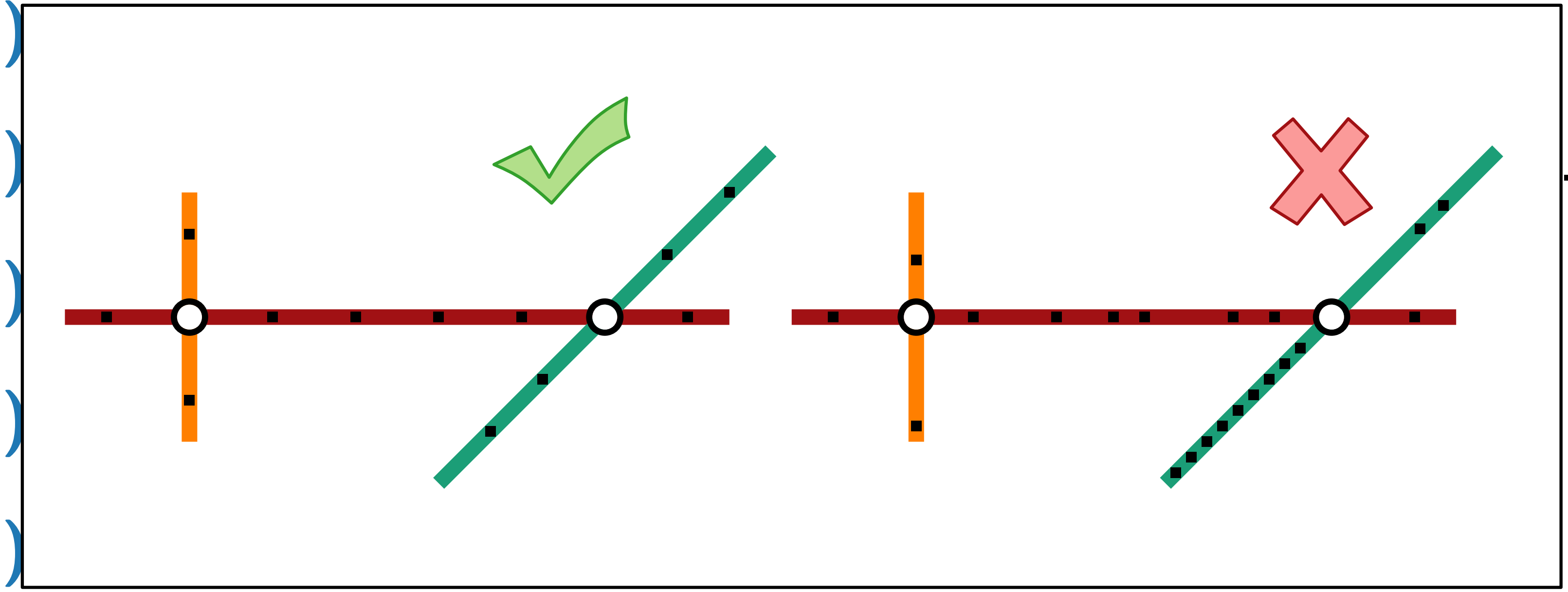
(R2)

(R3)

(R4)

(R5)

(R6)



(R7) Use uniform edge lengths.

- geographic distances less important
- network hop-distances more important
- balanced appearance

Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

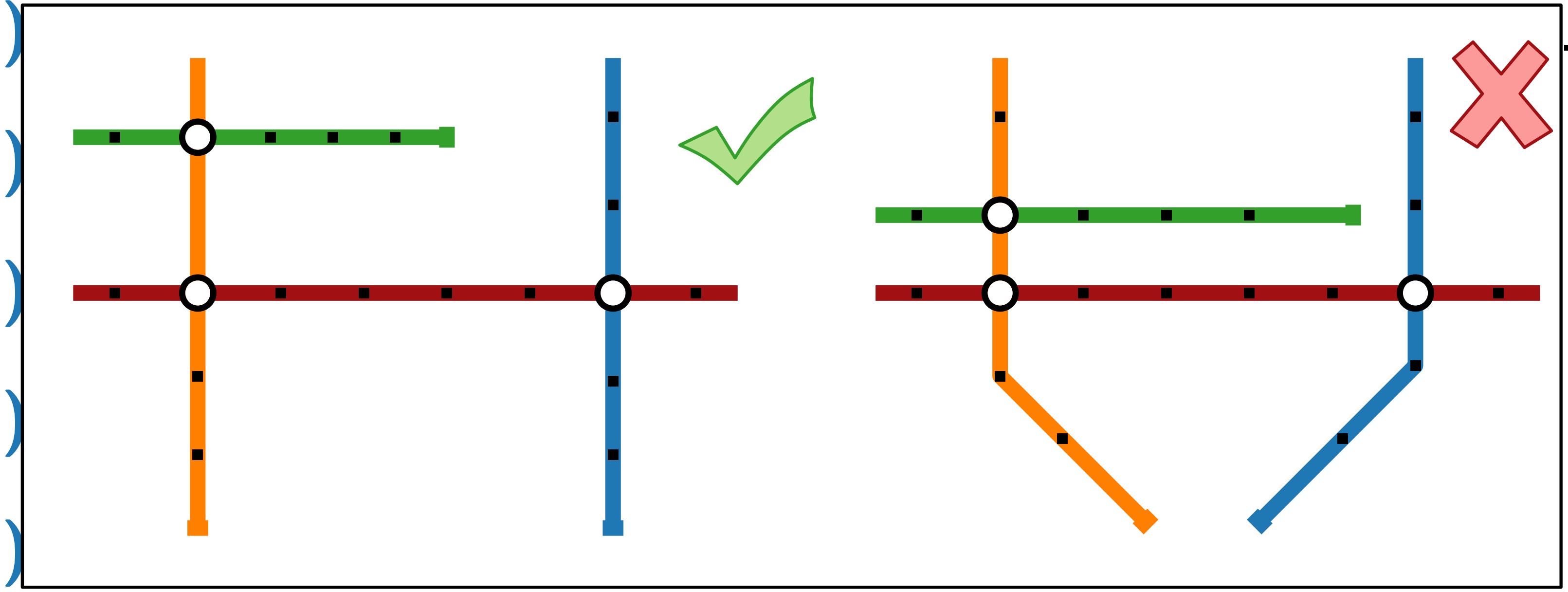
(R3)

(R4)

(R5)

(R6)

(R7)



(R8) Keep unrelated features apart.

- guarantees minimum clearance between features
- avoids ambiguities

Design Rules

(R1) Do not change the network topology.

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(R3)

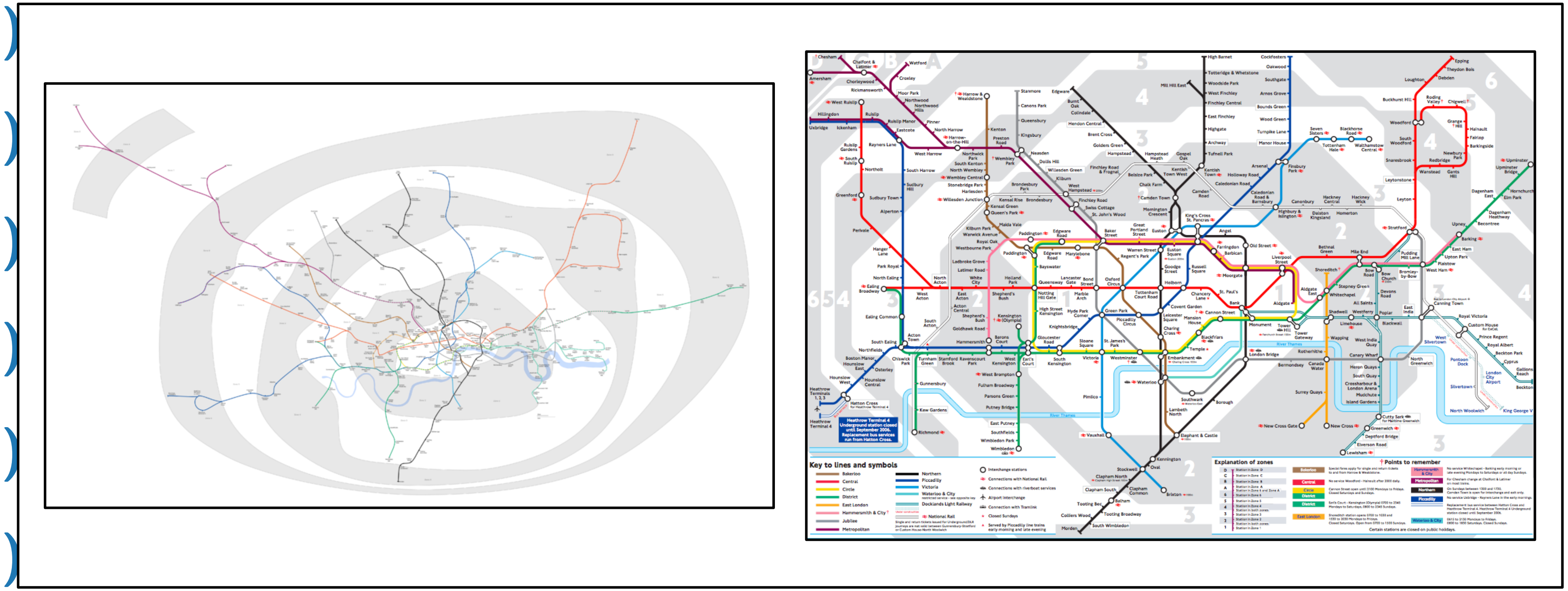
(R4)

(R5)

(R6)

(R7)

(R8)



(R9) Avoid large empty spaces.

- balances local feature density
- possibly fill gaps with legends

Design Rules

(R1) Do not change the network topology.

(R2) Restrict edge orientations.

(R3) Draw metro lines as simple and monotone as possible.

(R4) Let lines pass straight through interchanges.

(R5) Use large angular resolution in stations.

(R6) Minimize geometric distortion and displacement.

(R7) Use uniform edge lengths.

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(R9) Avoid large empty spaces.

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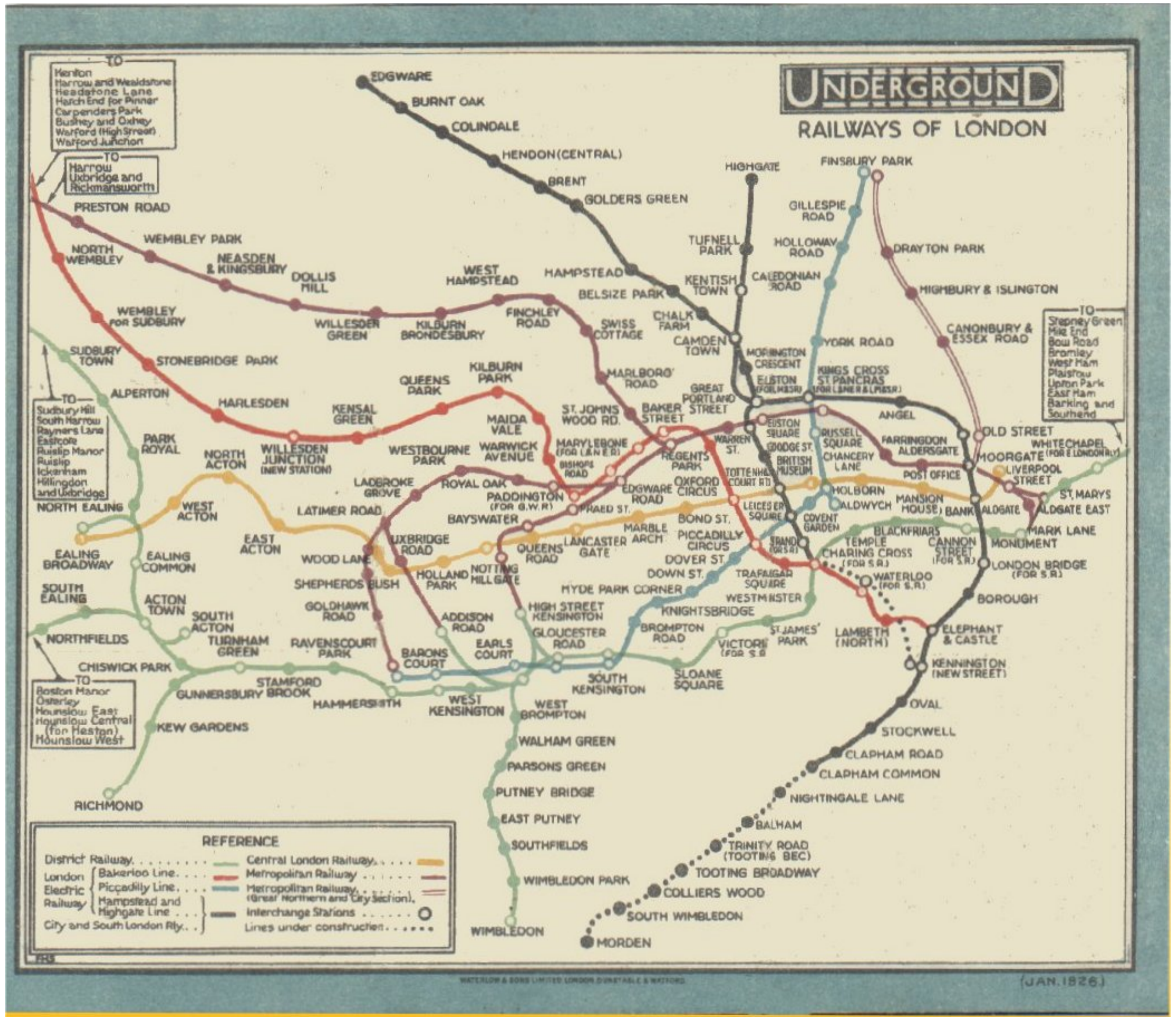
(R8) Keep unrelated features apart.

(R9) Avoid large empty spaces.

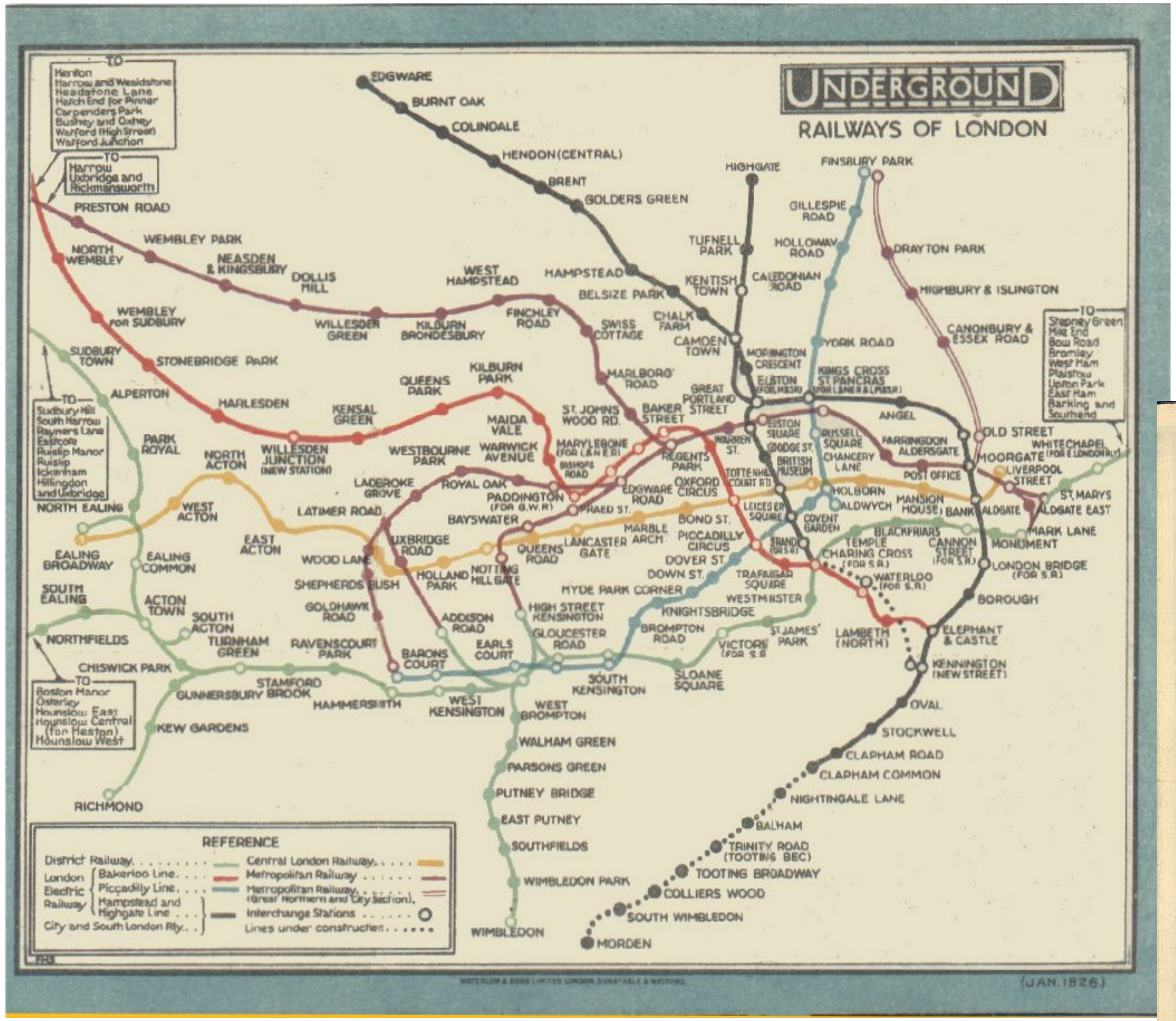
$\rightarrow$ **rules are potentially conflicting and need priorities**

A Bit of History



London 1927 (Fred H. Stingemore)

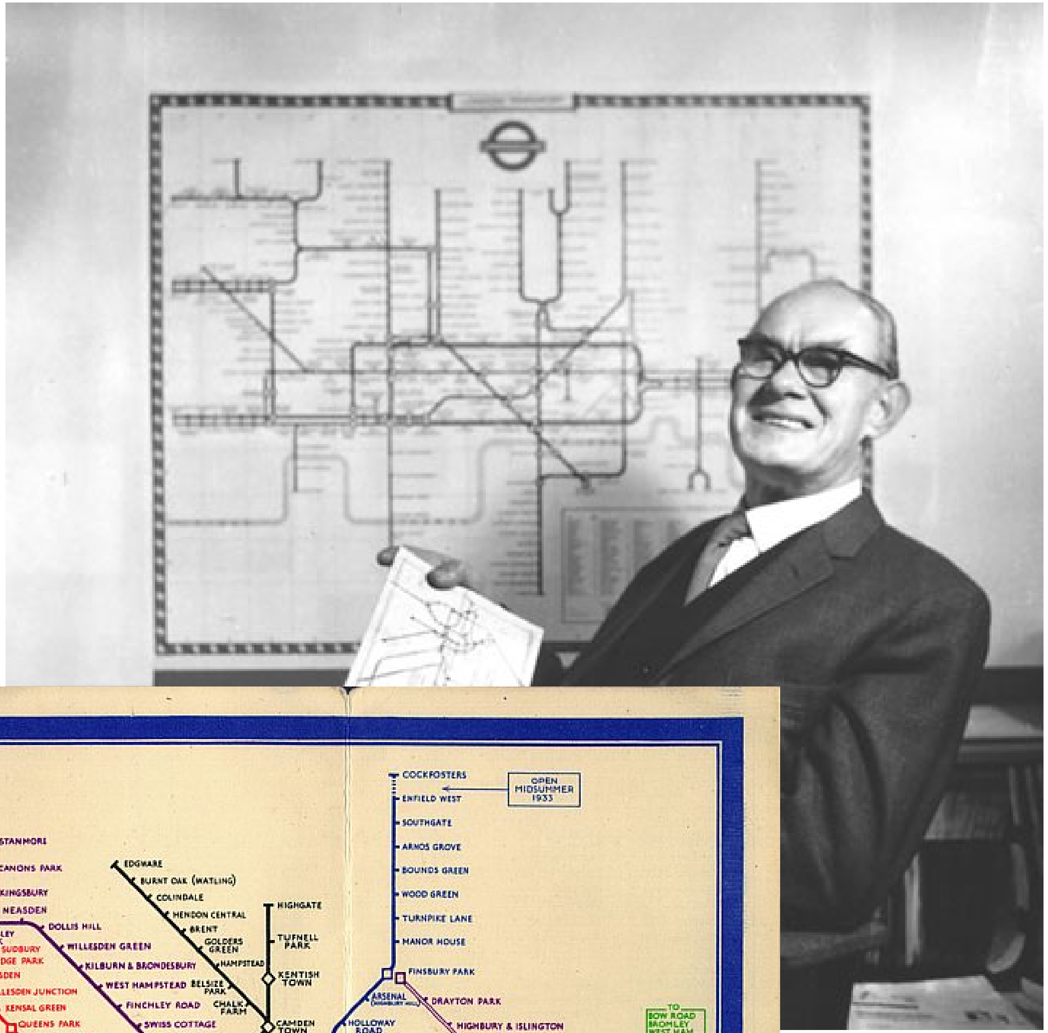
A Bit of History



London 1927 (Fred H. Stingemore)

(c) Mike Yashworth

Henry Beck
London 1933



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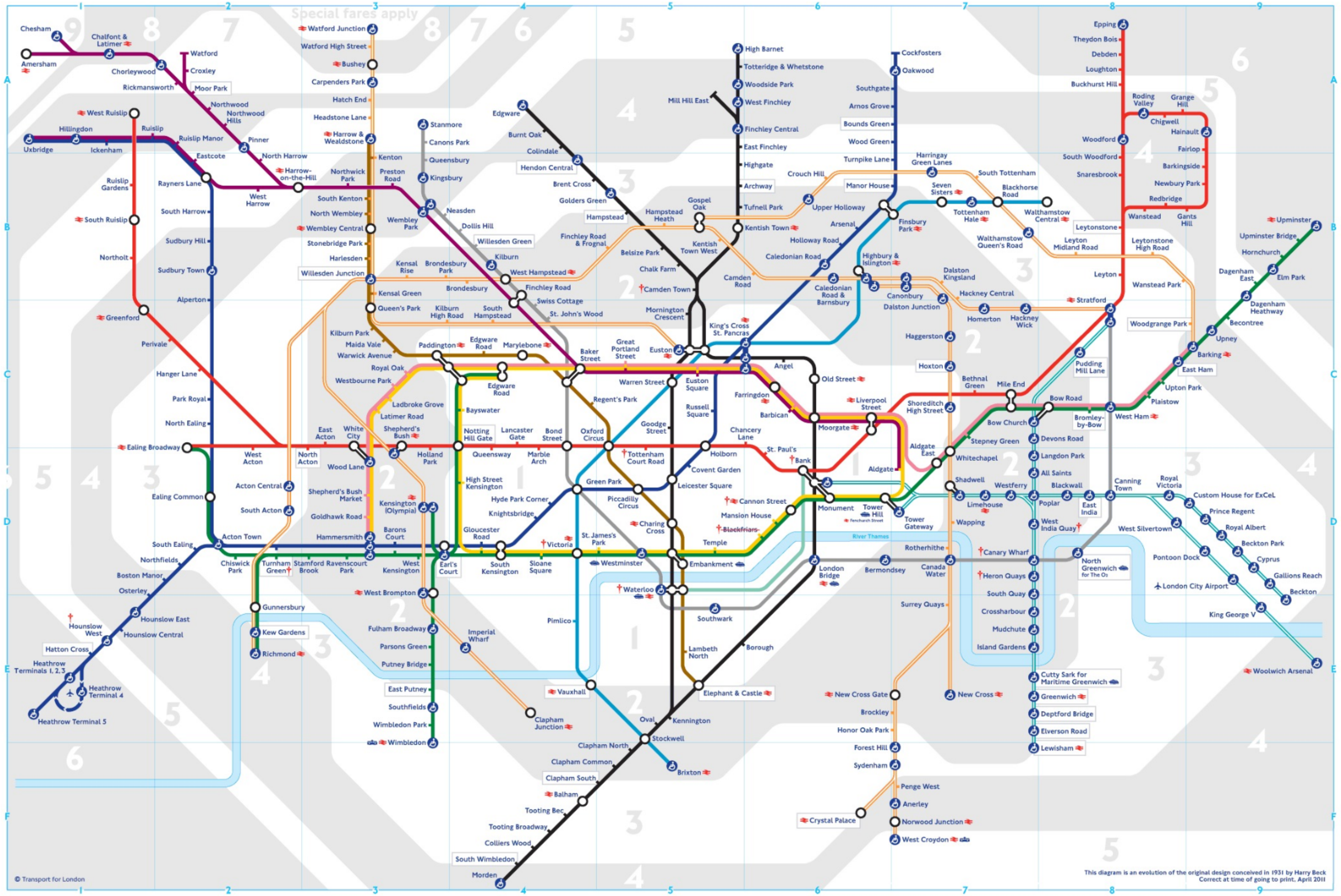
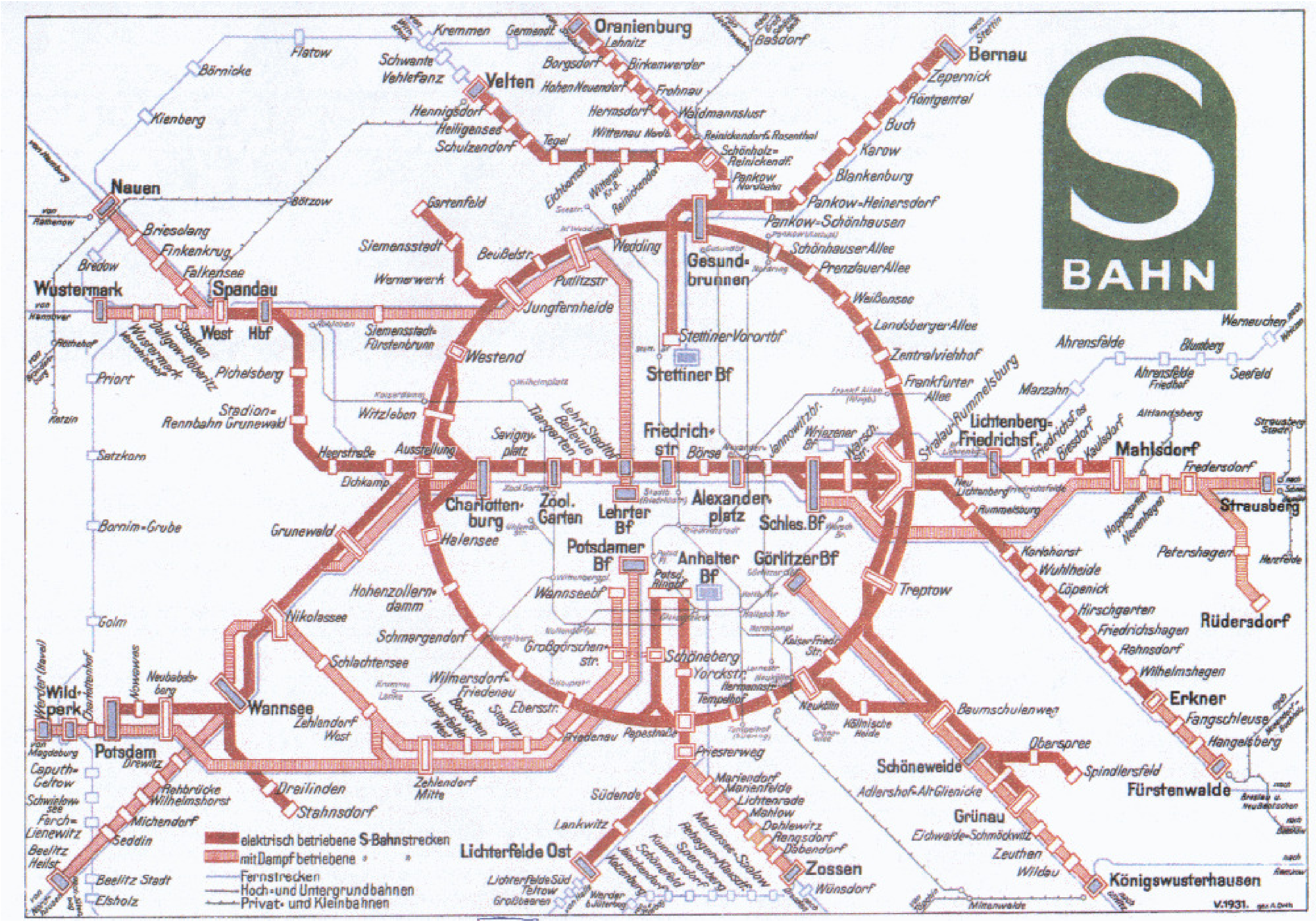


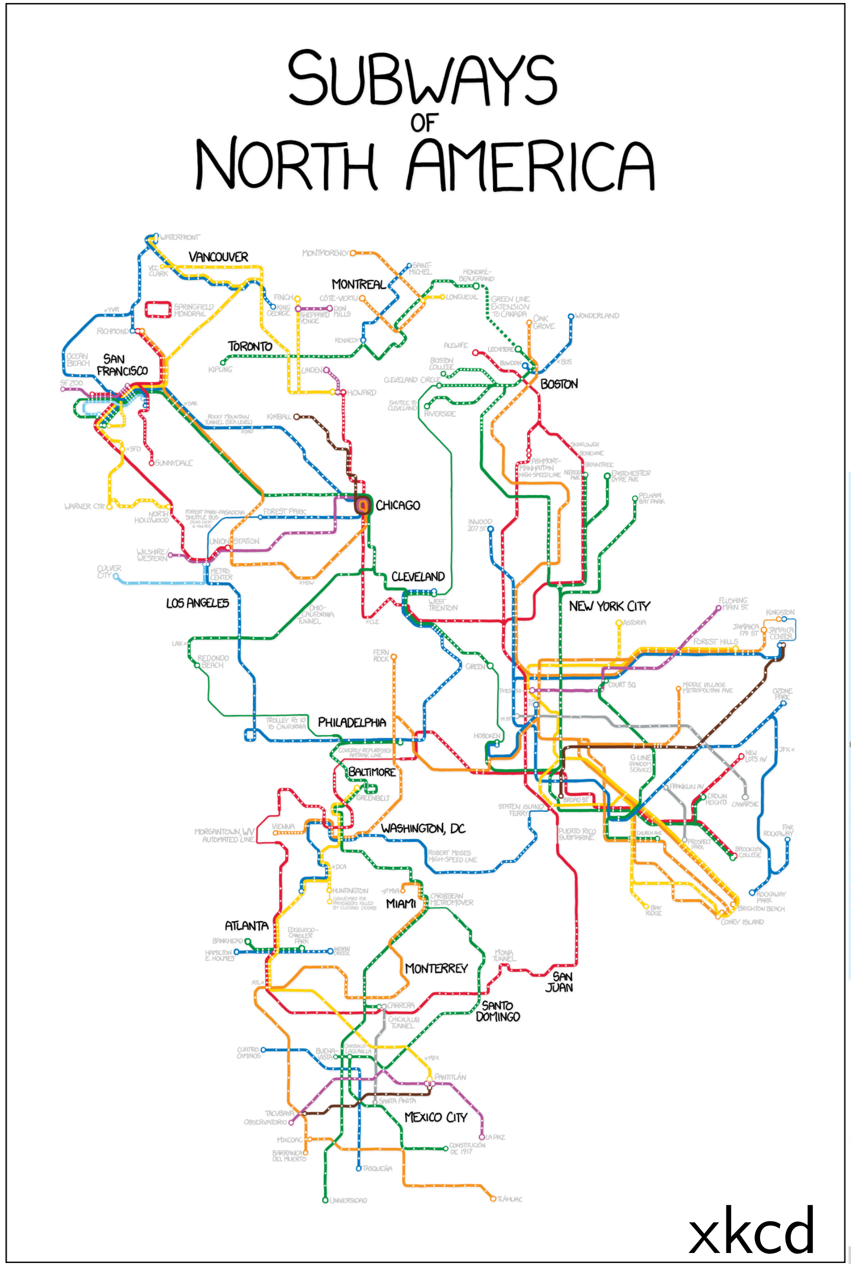
Tube Map
voted Design Icon 2006
(2nd after Concorde)

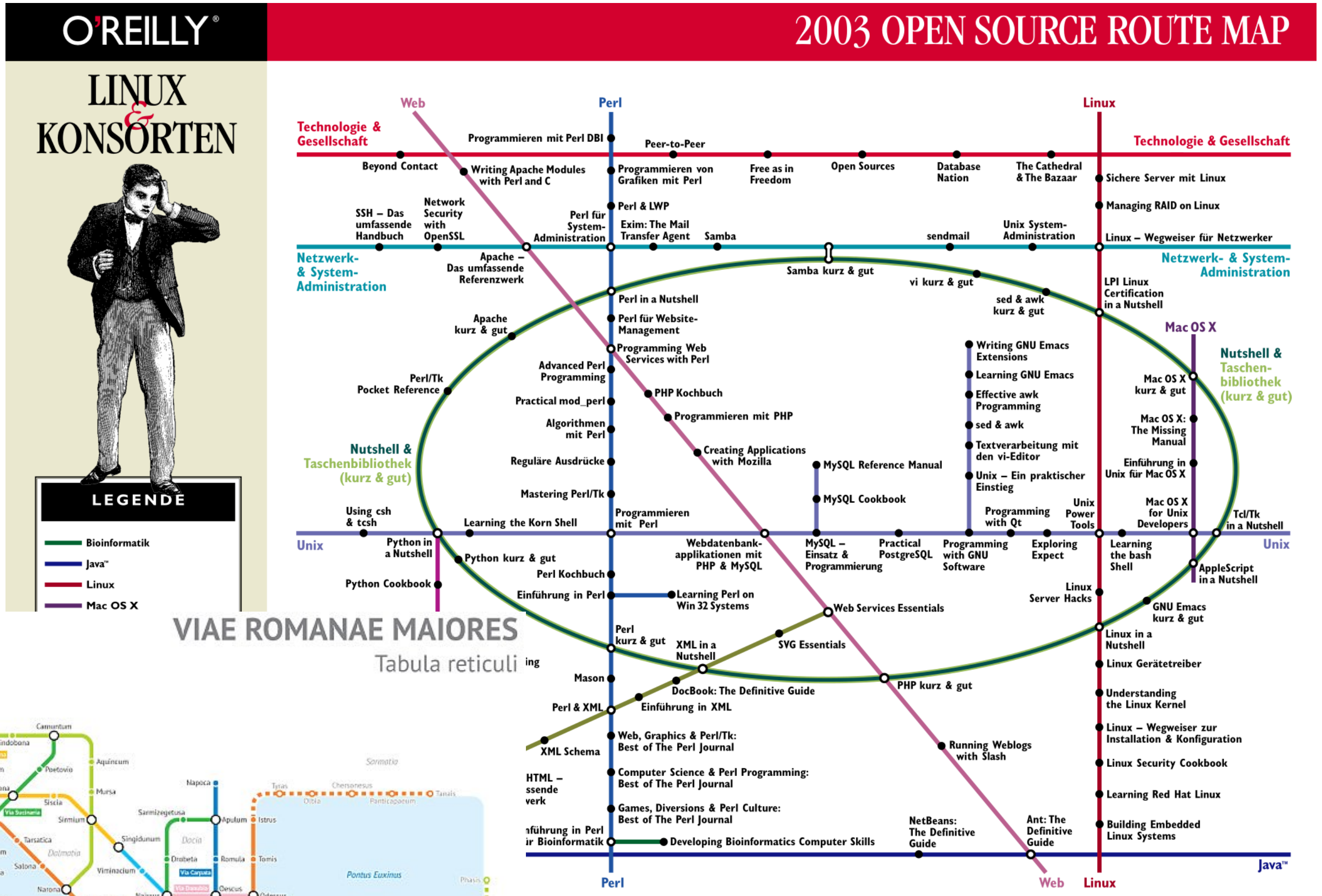
A Bit of History



Berlin 1931 (redrawn by Maxwell Roberts)

A Bit of History





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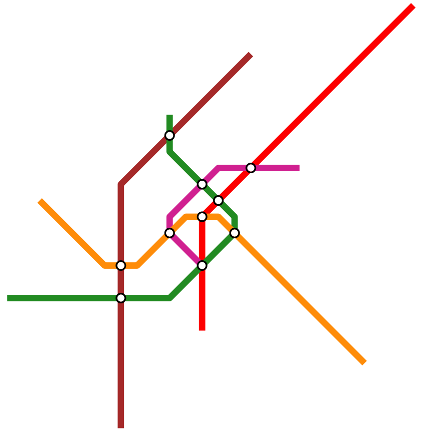
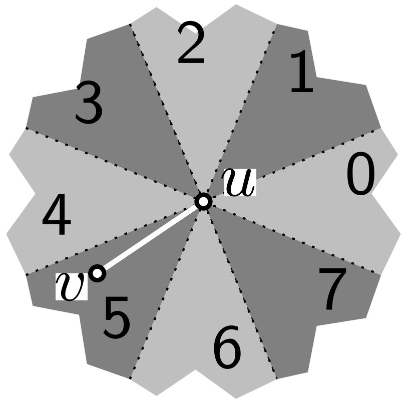
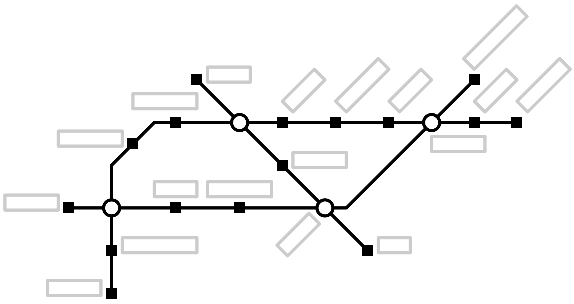
Visualization of Graphs



Lecture 12:
Octilinear Graph Drawing
Metro Map Layout



Part II:
Complexity and Path-Based Schematization



Jonathan Klawitter

Complexity

Theorem 1. [Nöllenburg 2005]
For an embedded graph G (vertex degrees ≤ 8) **bend minimization** (R3) is NP-hard if **preserving topology** (R1) and **octilinearity** (R2) are required.

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Sketch of proof.

Reduction from Boolean satisfiability problem PLANAR-3SAT using rigid "mechanical" gadgets

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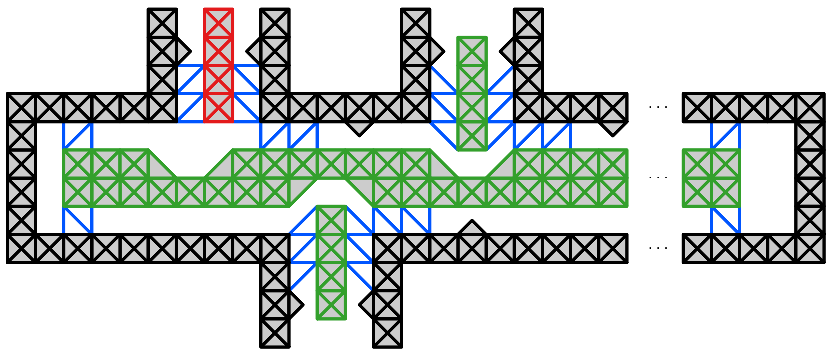

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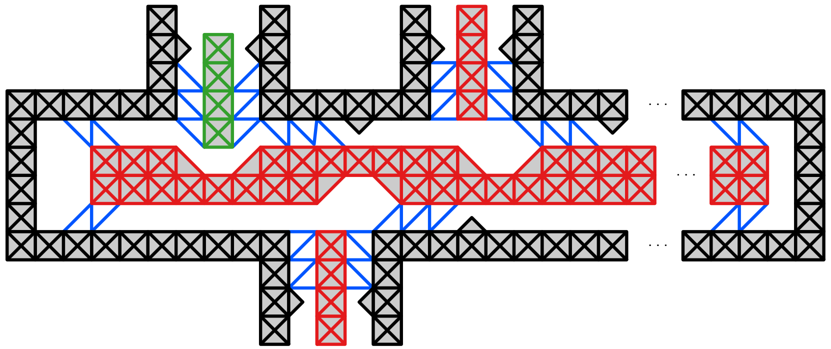

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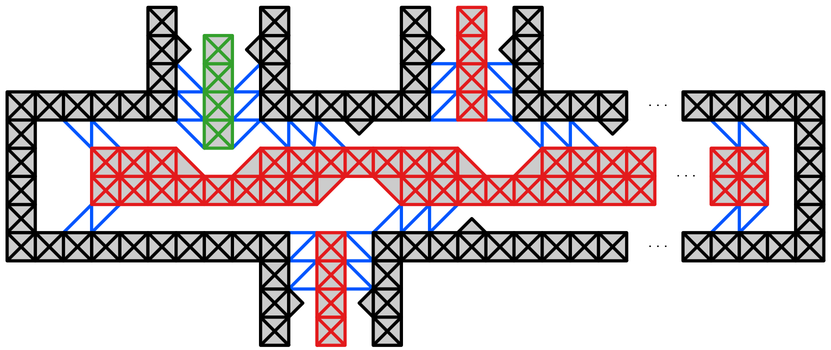

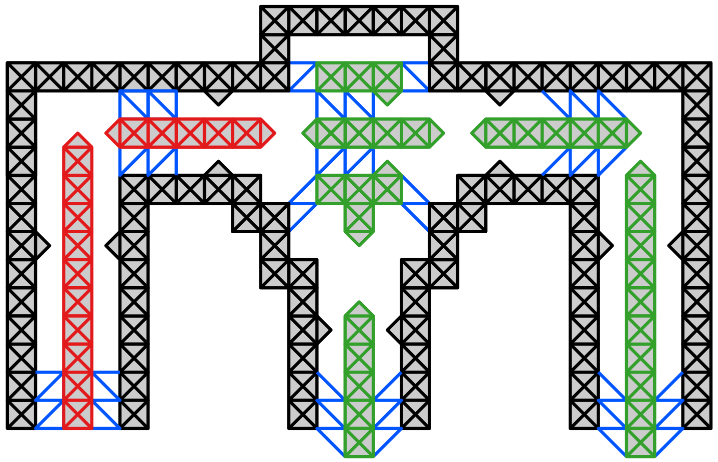

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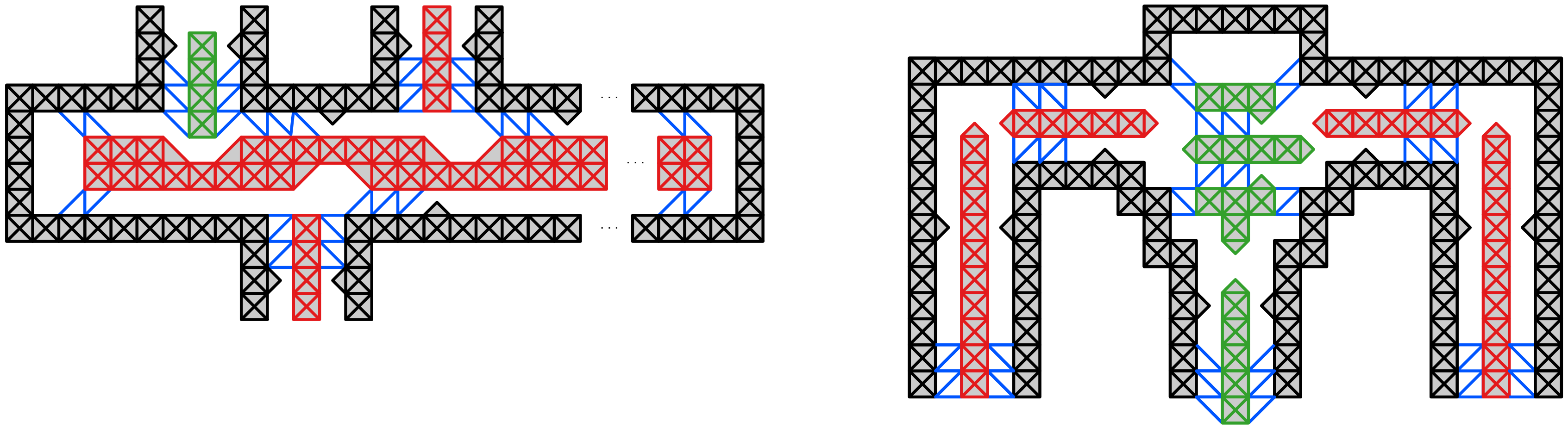

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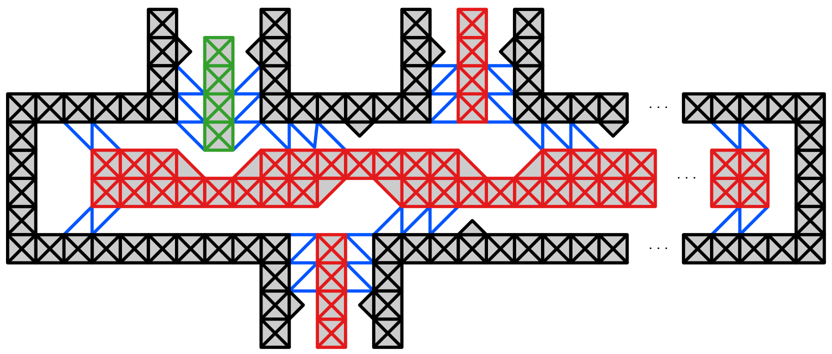

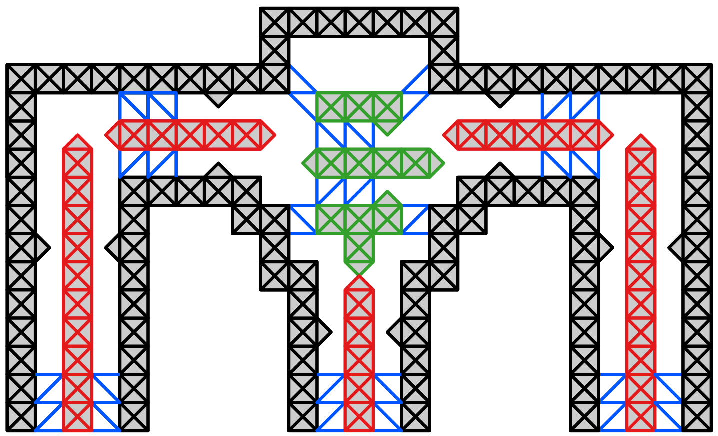

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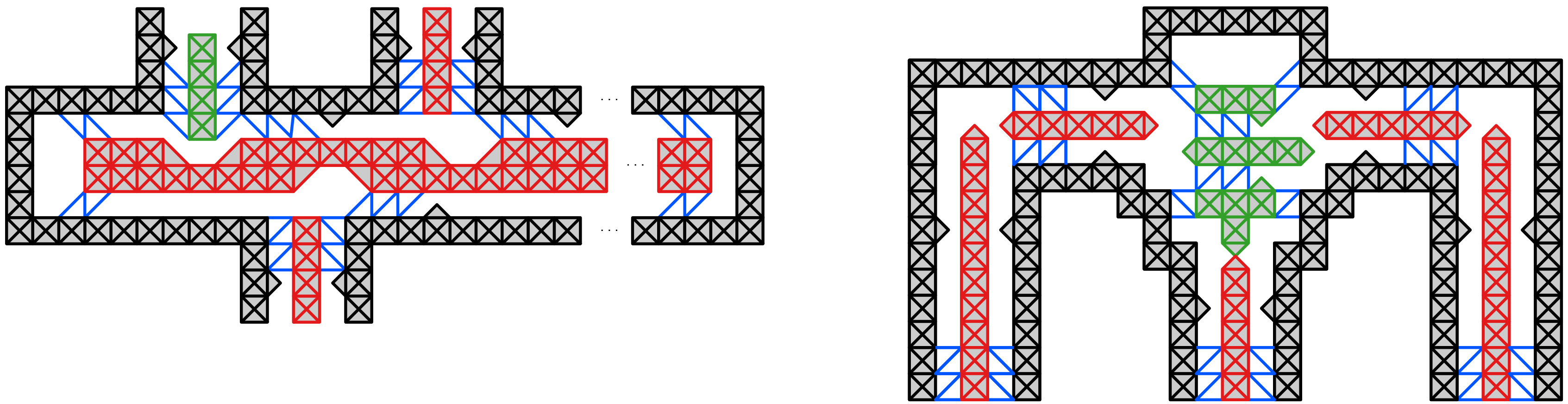
Sketch of proof.

Reduction from Boolean satisfiability problem $\textsc{planar-3Sat}$ using rigid "mechanical" gadgets



Remark.

- no efficient exact algorithms to expect
- same problem without diagonals (rectilinear) is efficiently solvable [Tamassia '87]

Path-Based Schematization

Goal. Solve restricted problem, where G is a path (or polyline)

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Constraints.
- $\mathcal{C}$-oriented edges (e.g. octilinear) **(R2)**
- bounded displacement **(R6)**

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- minimize number of links **(R3)**

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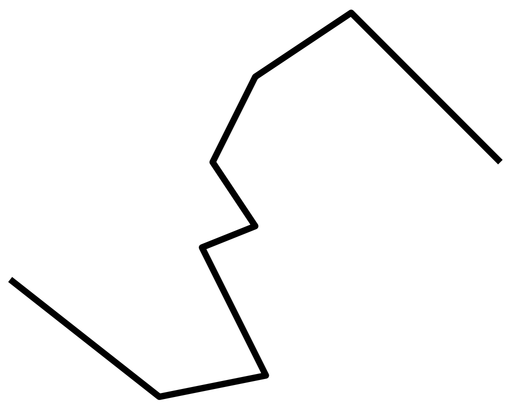

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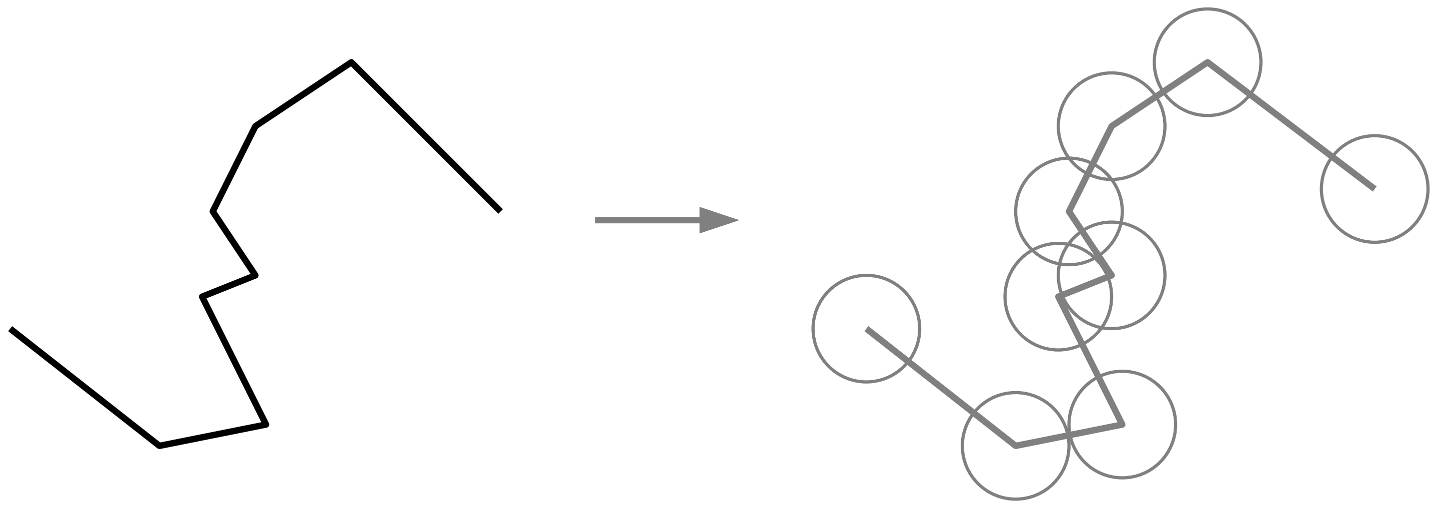

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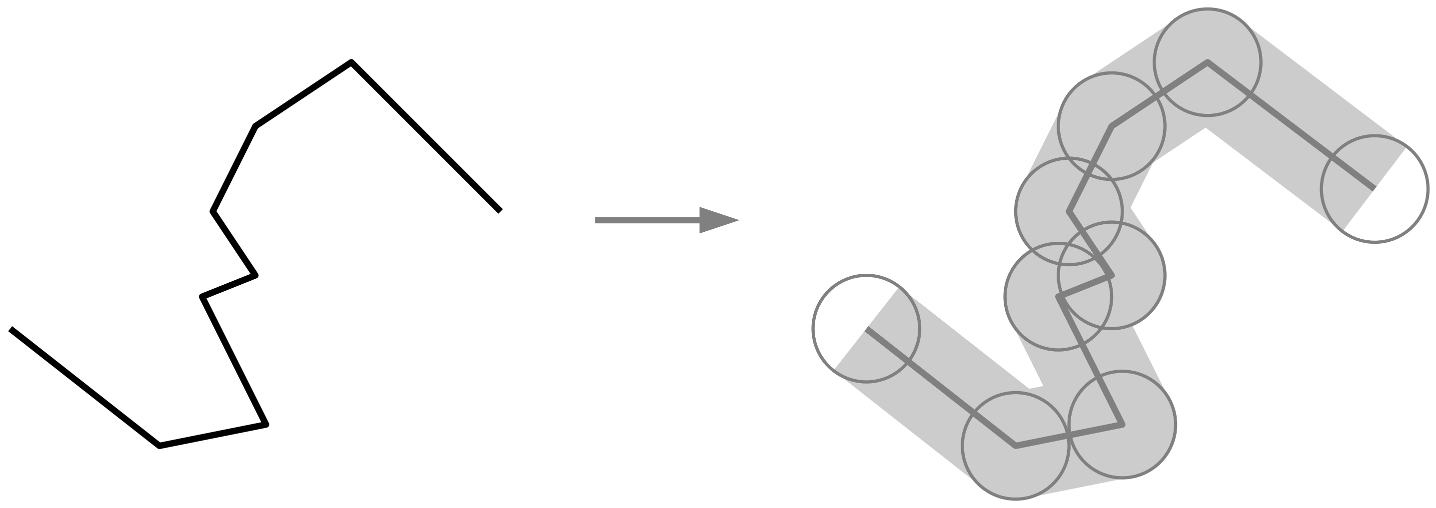

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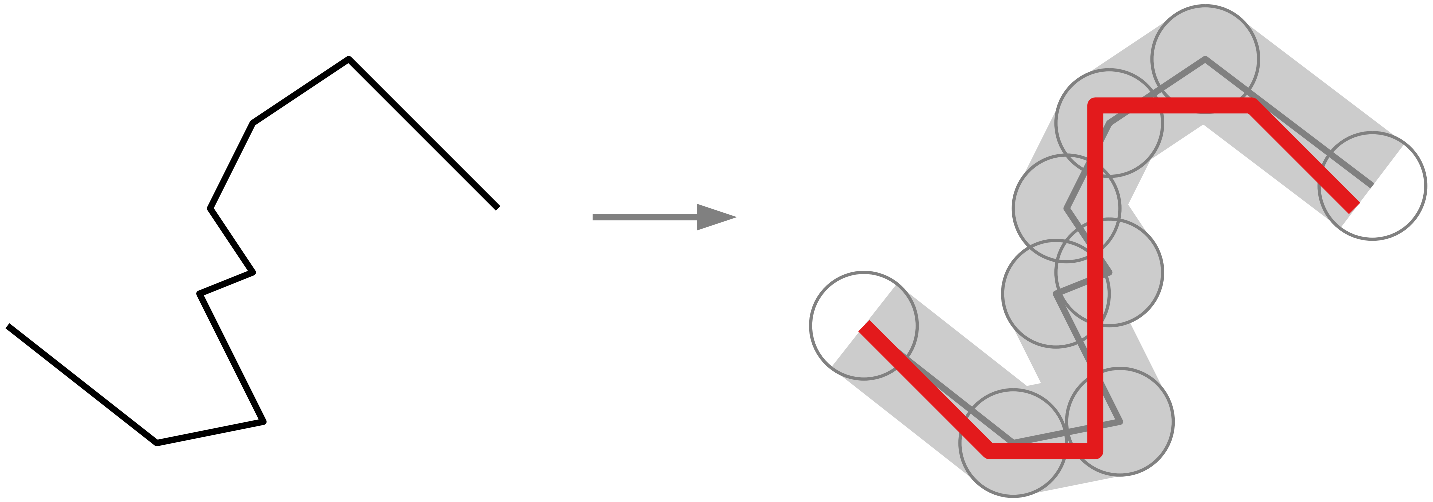
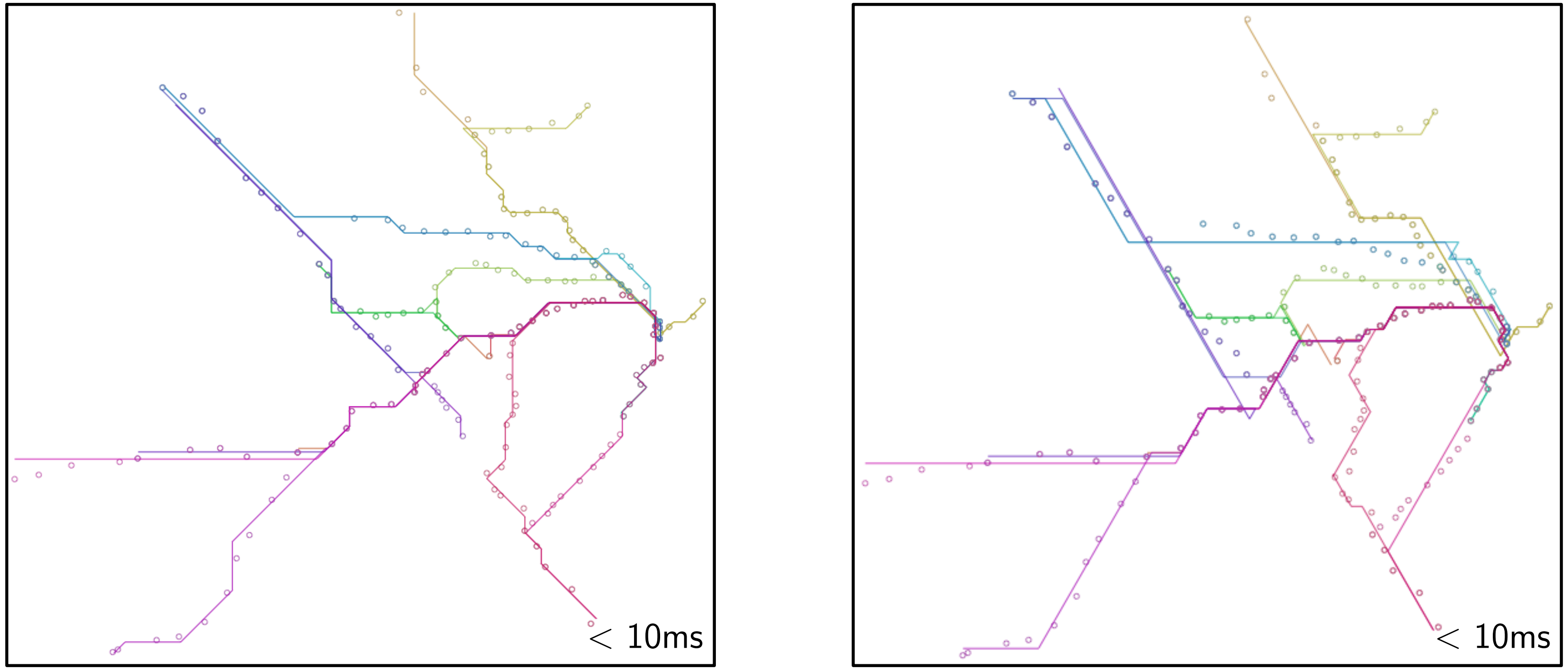


Path-based schematization – example

Theorem 2. [Dwyer, Hurst, Merrick '08]
For a path P of length n and orientation set $\mathcal{C}$ a $\mathcal{C}$-oriented schematized path can heuristically be fitted to the vertices in $O(|\mathcal{C}|n)$ time (or $O(|\mathcal{C}|n \log n)$) using least-squares regression.



$\mathcal{C}$-oriented Route Sketches

Theorem 3. [Delling et al. 2010]

Given a monotone path P and a set $\mathcal{C}$ of admissible edge slopes, we can compute, in $O(n^2)$+solve(LP) time, a $\mathcal{C}$-oriented schematization of P, which

- preserves the orthogonal order of P,
- has minimum slope deviation,
- has minimum total length.

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relative north-south-east-west relationship of all vertices

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Proof.
- dynamic programming for slope assignment
- LP for length assignment

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Proof.
- dynamic programming for slope assignment
- LP for length assignment

Theorem 4. [Brandes & Pampel 2009, Gemsa et al. 2011]
The d-regular (non-monotone) route sketch problem is NP-hard for any $d \geq 1$, where $\mathcal{C} = \{i \cdot 90°/d \mid i \in \mathbb{Z}\}$.

$\mathcal{C}$-oriented Route Sketches

Example. Bremen to Cuxhaven [Gemsa et al. 2011]

$\mathcal{C}$-oriented Route Sketches

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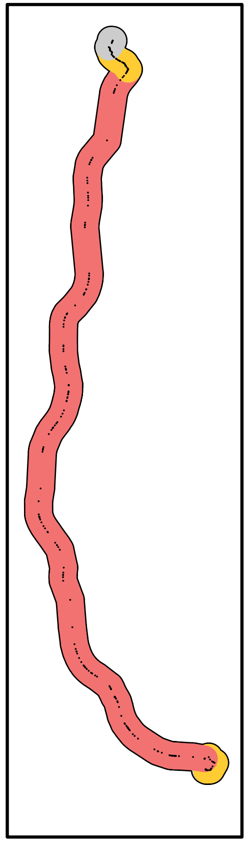
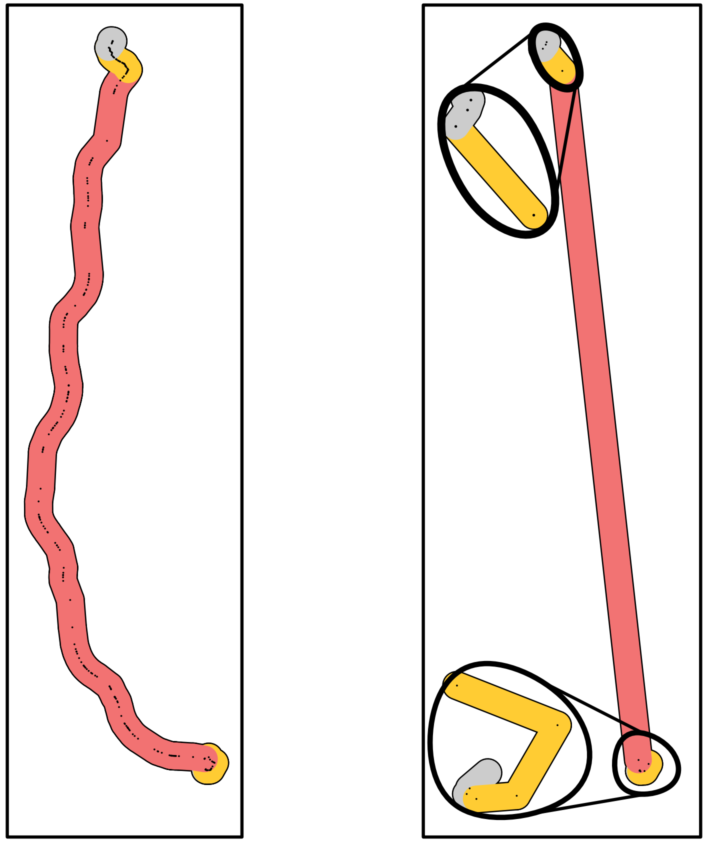


input

$\mathcal{C}$-oriented Route Sketches

Example. Bremen to Cuxhaven [Gemsa et al. 2011]



input simplified

$\mathcal{C}$-oriented Route Sketches

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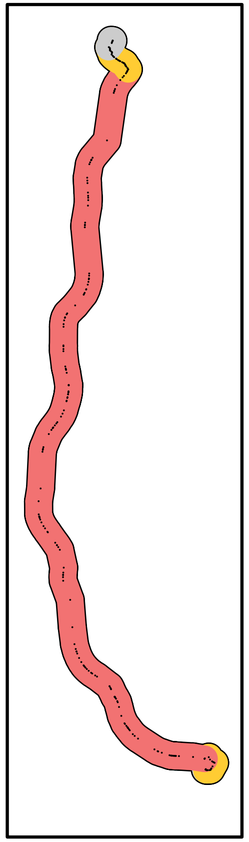

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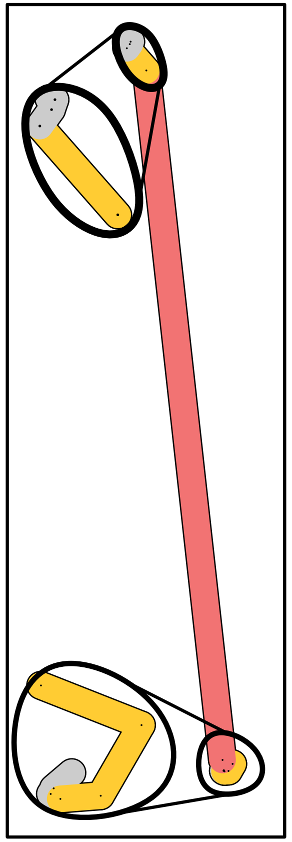

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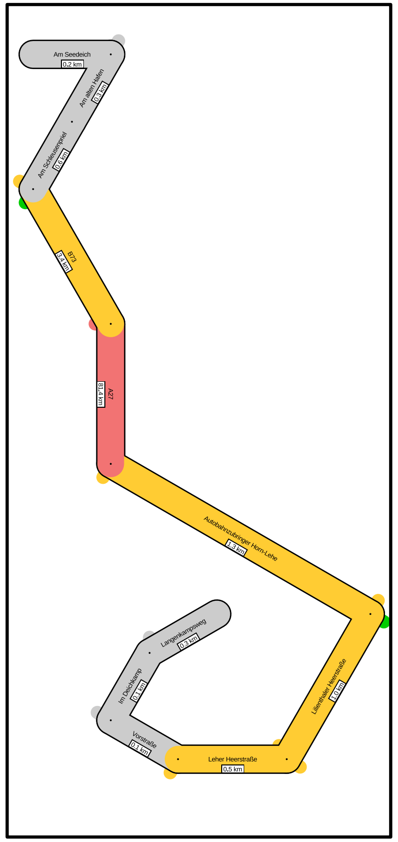

optimized by MIP

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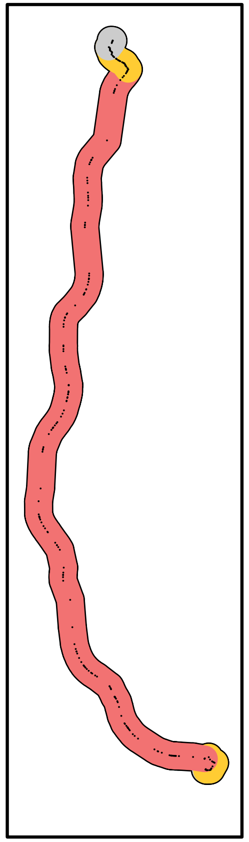

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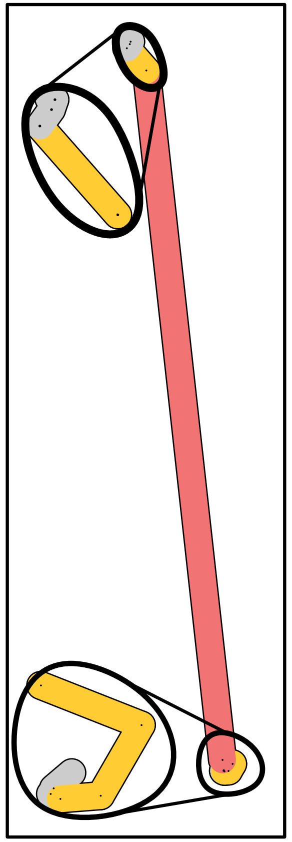

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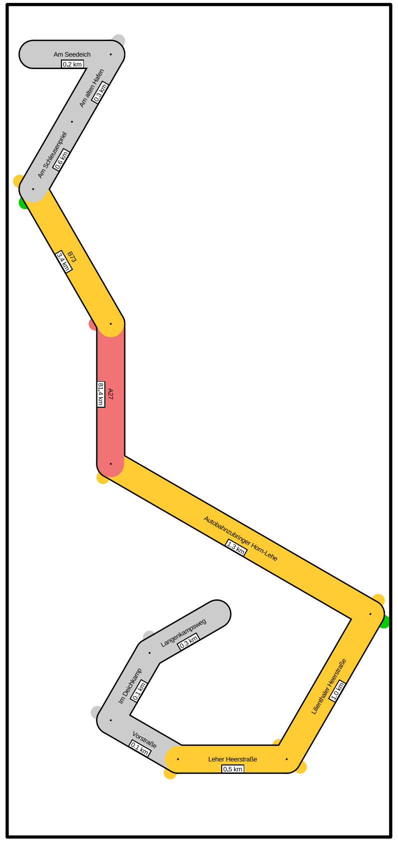

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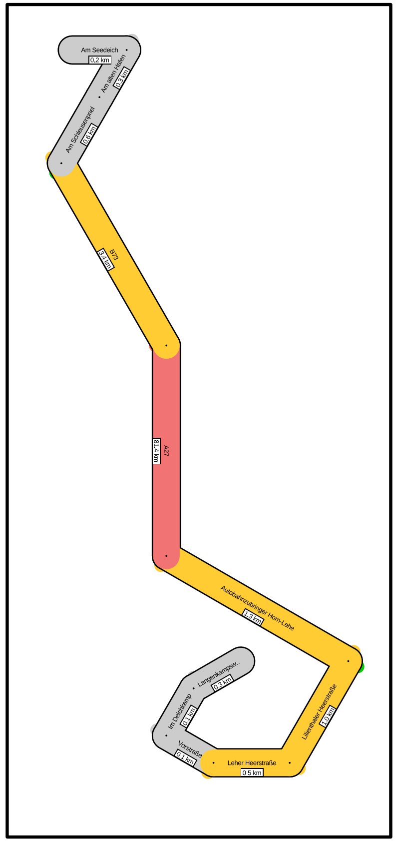

including length order constraint

Path-Based Schematization – Discussion

Pros.

Cons.

Path-Based Schematization – Discussion

Pros.

- polynomial running times

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Path-Based Schematization – Discussion

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- polynomial running times
- $\mathcal{C}$-orientation and bounded displacement guaranteed

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  - decompose metro network into paths

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Path-Based Schematization – Discussion

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  - schematize individual paths

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Path-Based Schematization – Discussion

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- extends to metro networks:
  - decompose metro network into paths
  - schematize individual paths
  - glue schematized paths together at interchanges

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- no guarantee on network topology **(R1)**

Path-Based Schematization – Discussion

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- polynomial running times
- $\mathcal{C}$-orientation and bounded displacement guaranteed
- bend minimization
- extends to metro networks:
  - decompose metro network into paths
  - schematize individual paths
  - glue schematized paths together at interchanges

Cons.

- no guarantee on network topology **(R1)**
- distortion/displacement too limited for metro maps

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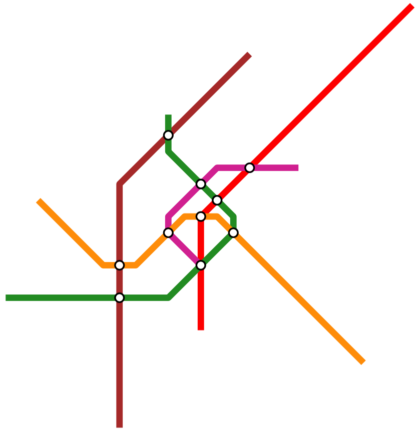
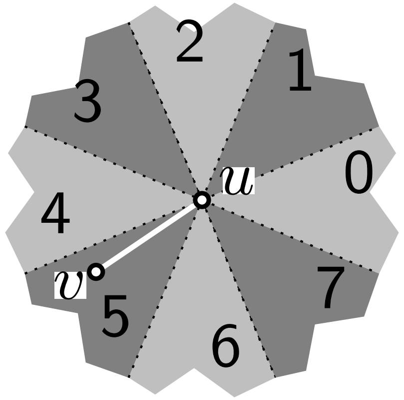
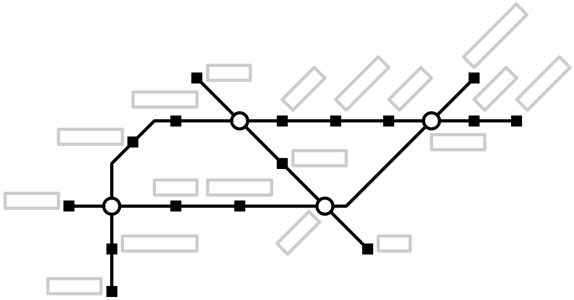
Visualization of Graphs



Lecture 12:
Octilinear Graph Drawing
Metro Map Layout



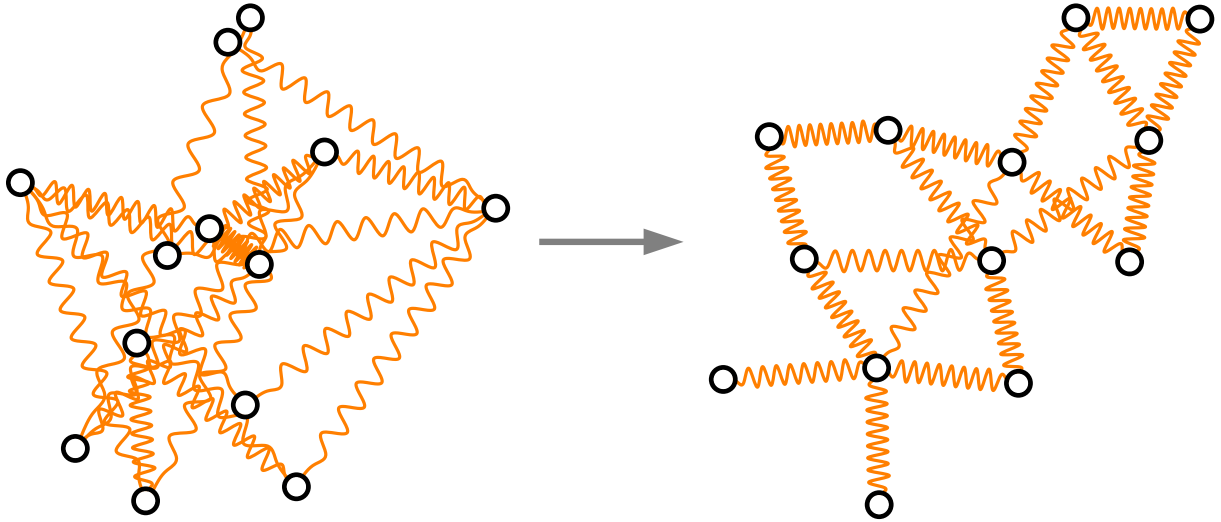
Part III:
Force-Based Schematization



Jonathan Klawitter

Force-Based Schematization

Idea. Apply well known force-based graph drawing approach



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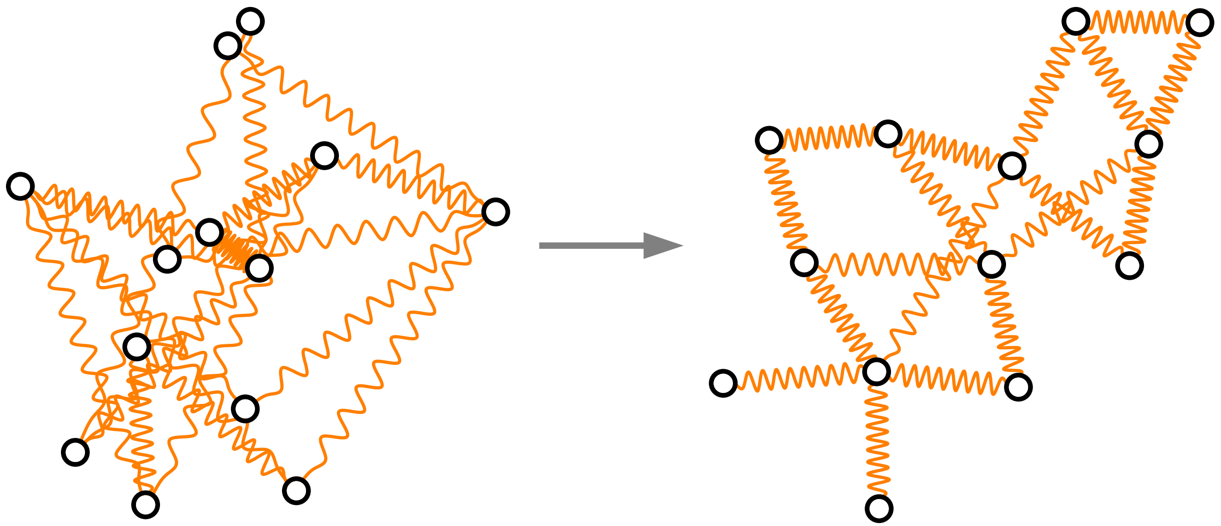
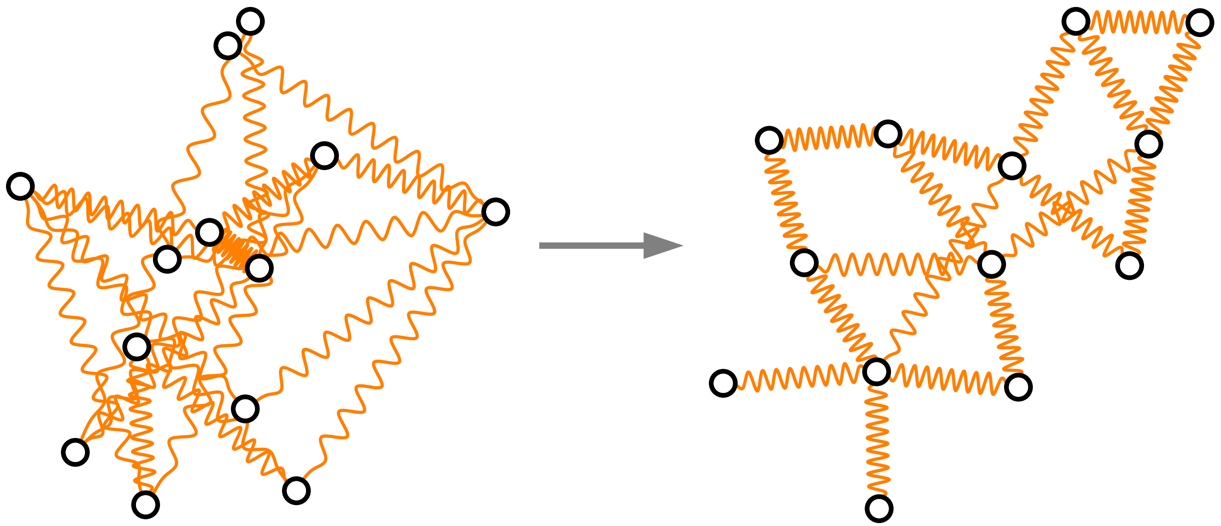


Recall.

- vertices are charged particles repelling each other

Force-Based Schematization

Idea. Apply well known force-based graph drawing approach

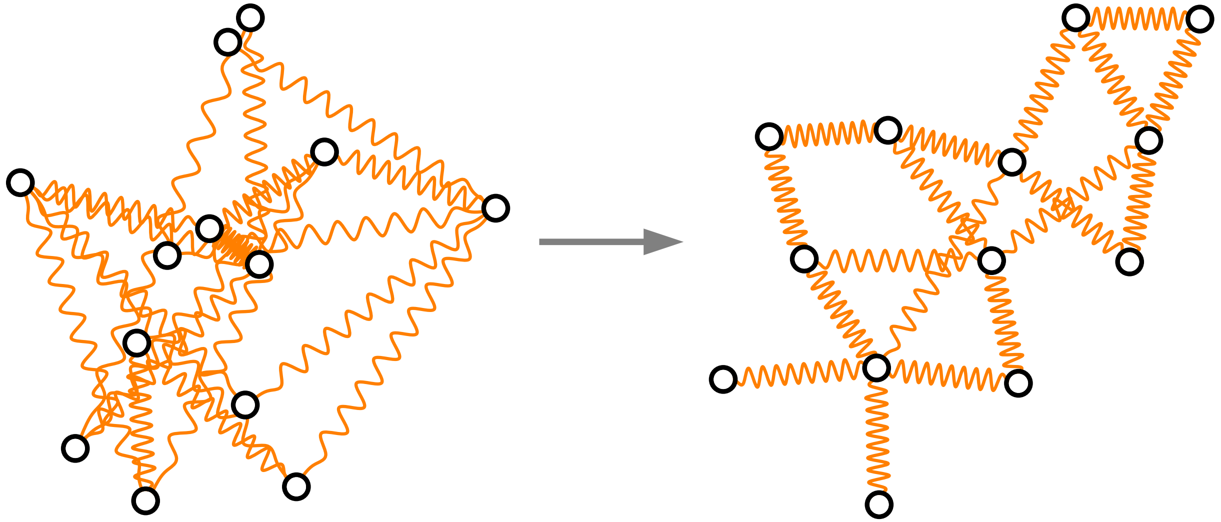


Recall.

- vertices are charged particles repelling each other
- edges are springs pulling edges into target length

Force-Based Schematization

Idea. Apply well known force-based graph drawing approach



Recall.

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- edges are springs pulling edges into target length
- iteratively calculate and apply forces until system stabilizes

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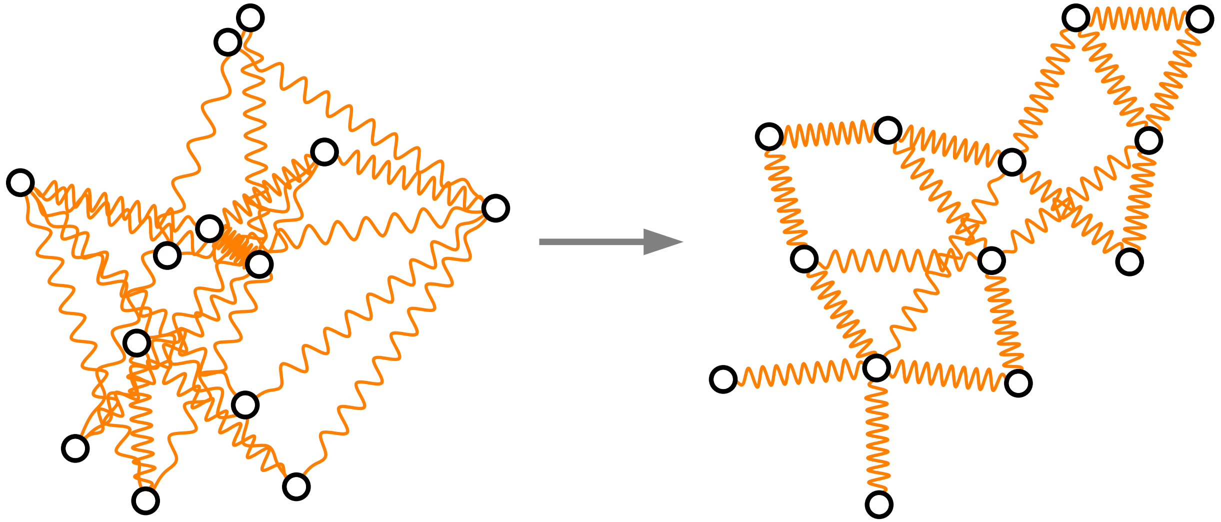

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$\rightarrow$ define additional forces to model
subset of metro map design rules

Force-Based Schematization – Octilinear

[Hong et al. 2006]

- contract degree-2 vertices into weighted edges **(R3)**

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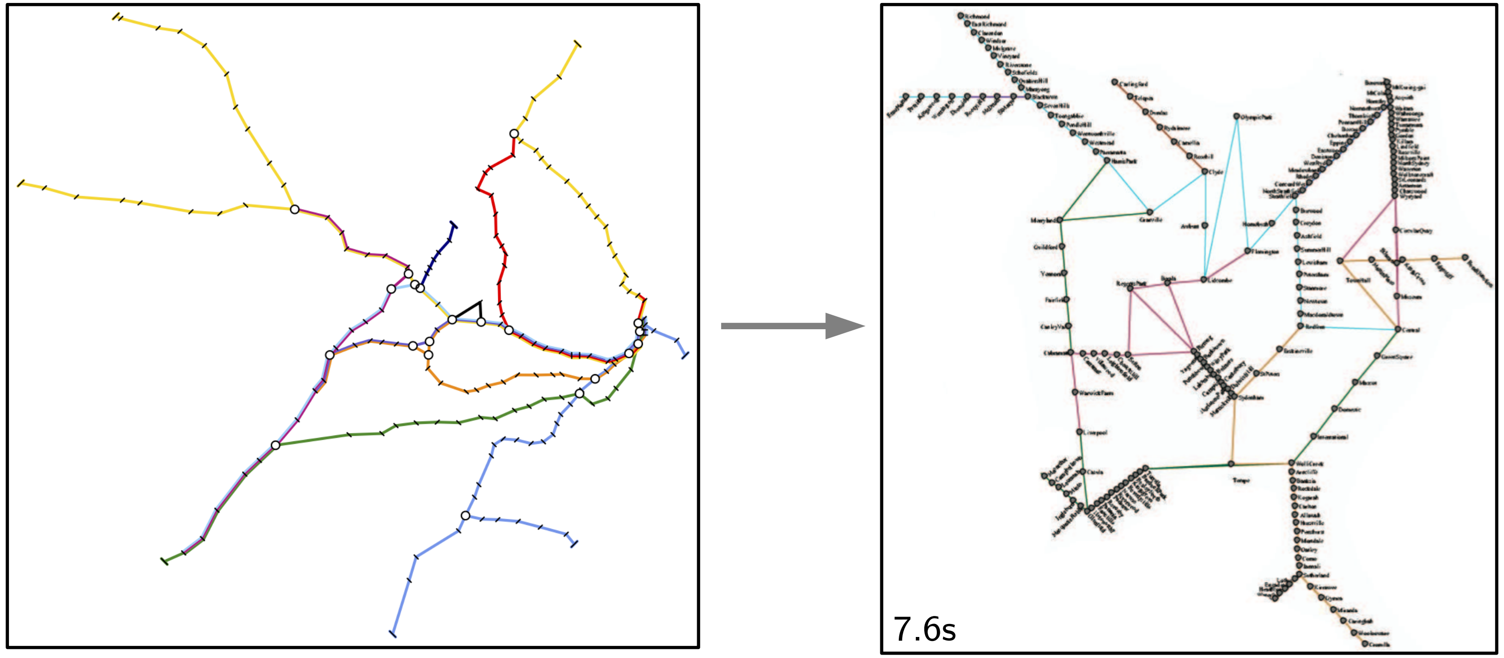

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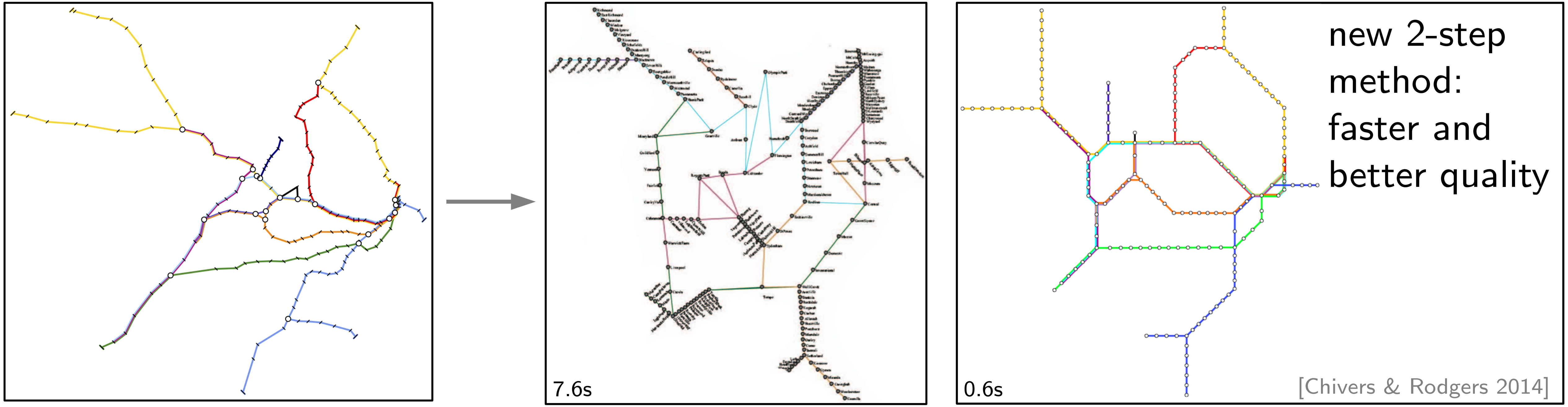

Force-Based Schematization – Bézier

[Fink et al. 2013]

- convert (octilinear) input layout into Bézier curves
  $\rightarrow$ vertices and control points, both subject to forces

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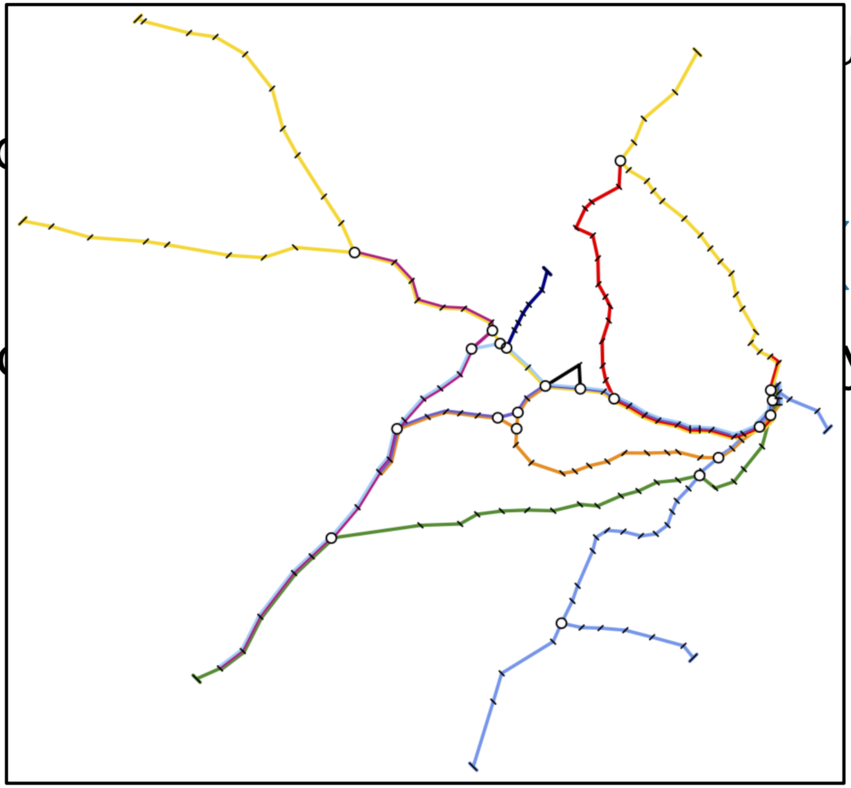

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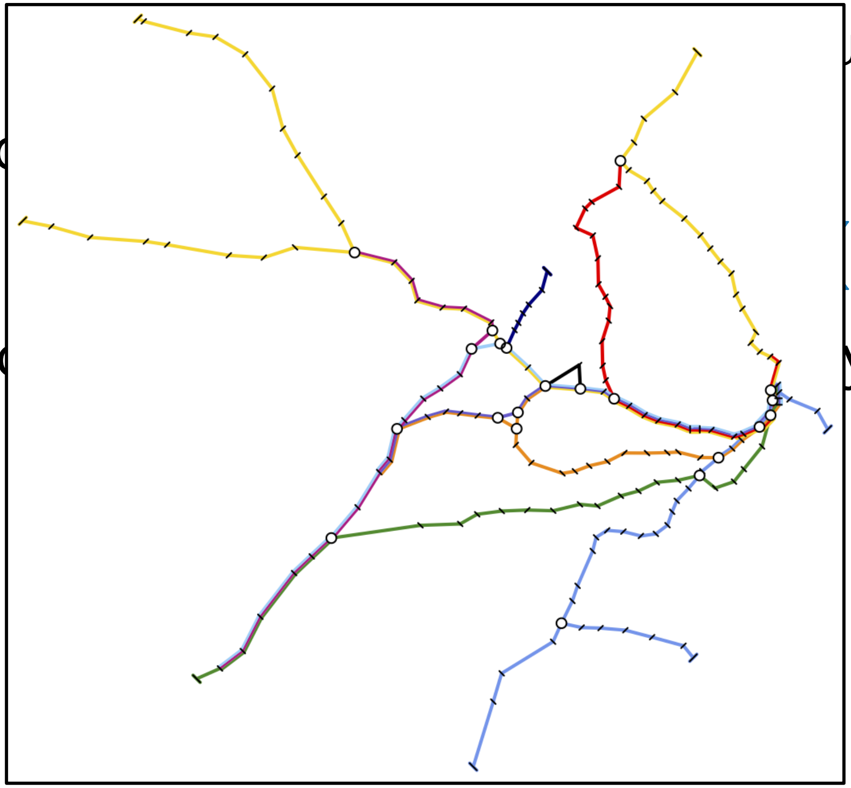

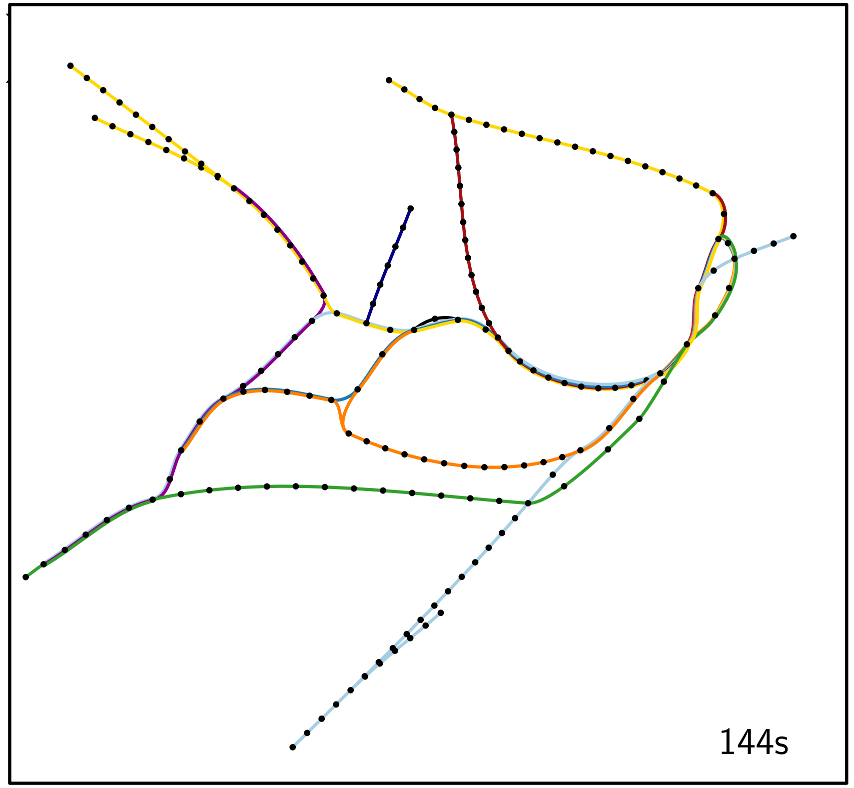

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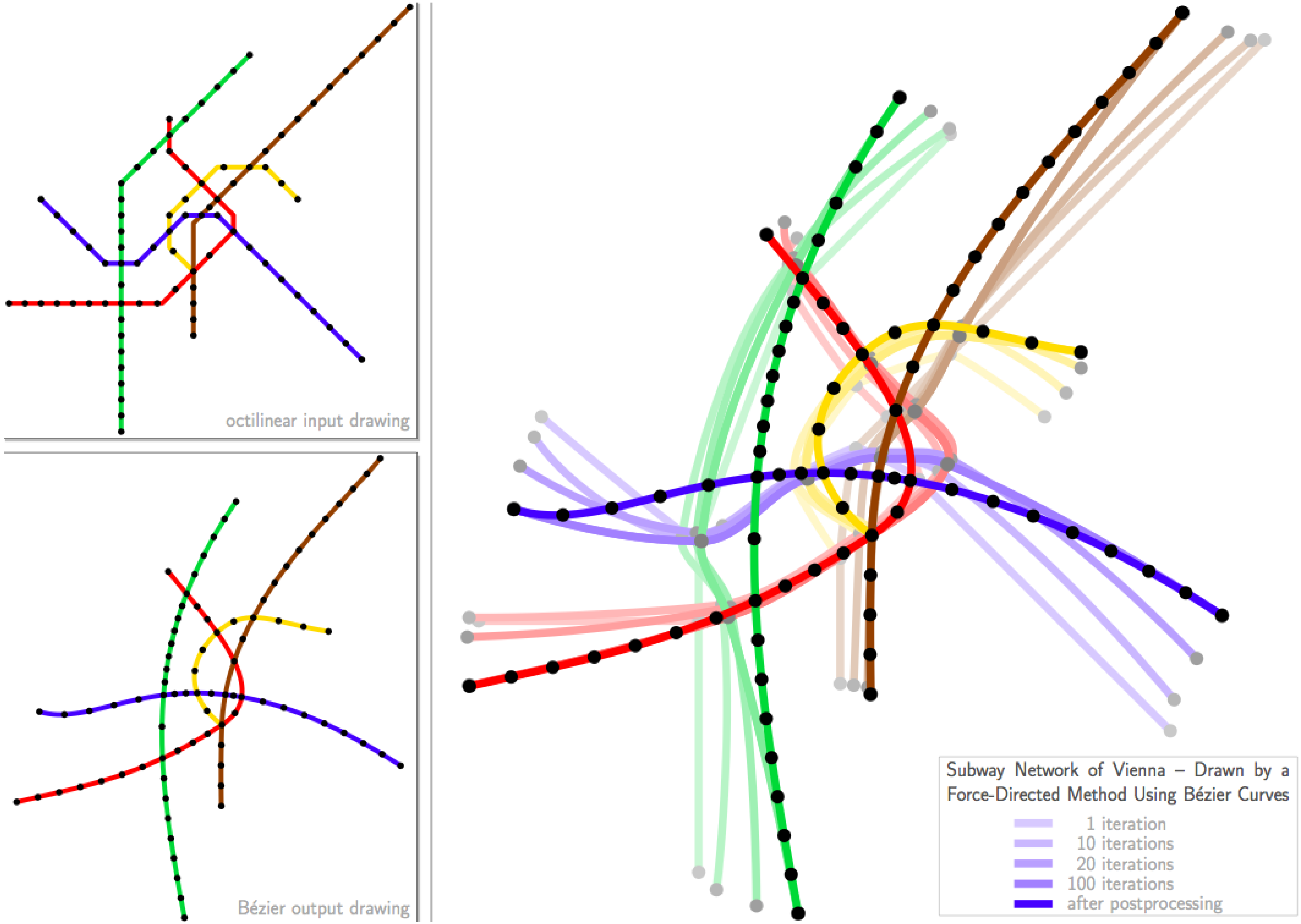
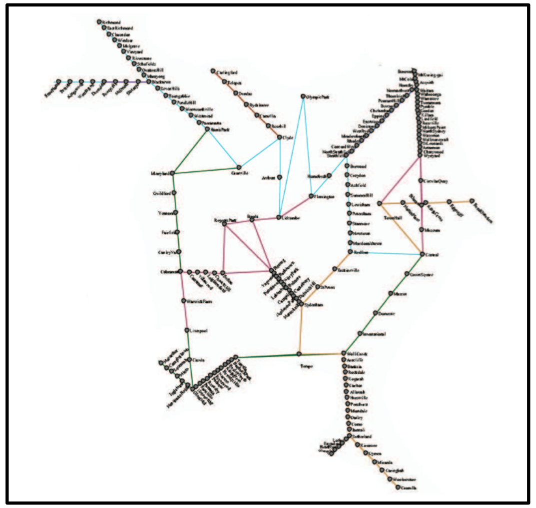


Force-Based Schematization – Discussion

Octilinear.



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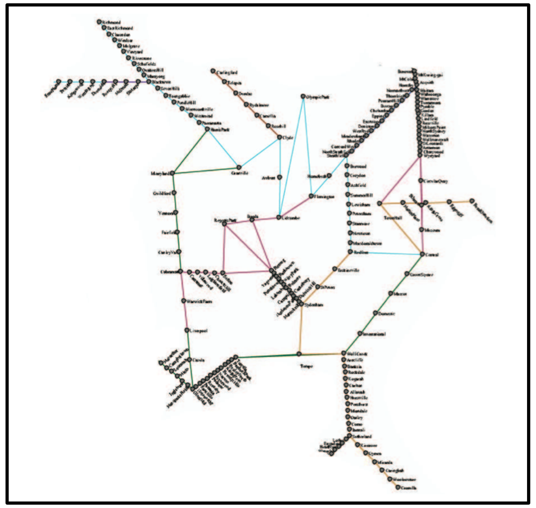

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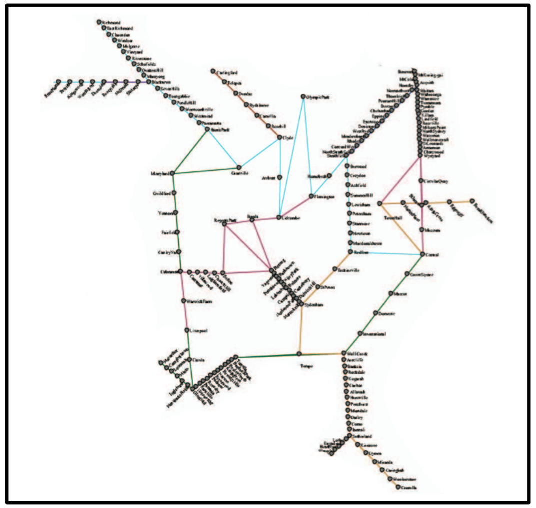
- guarantees topology **(R1)**
- slower than path-based algorithms, but still fast enough



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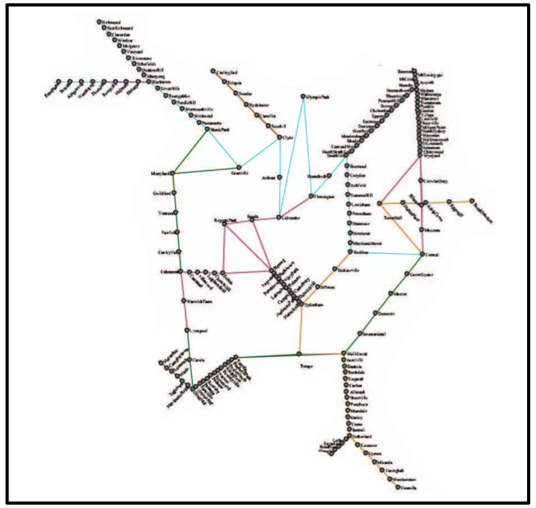

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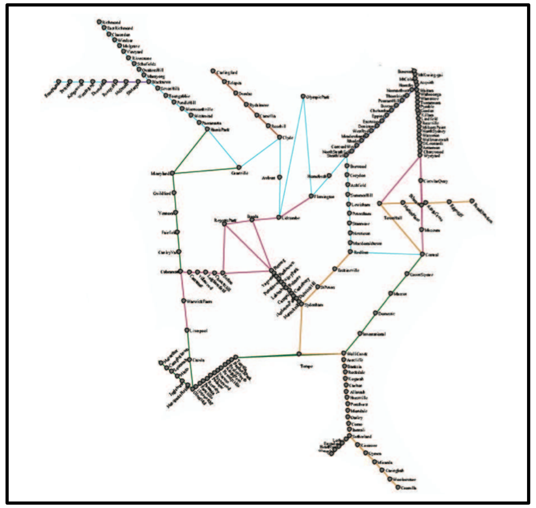

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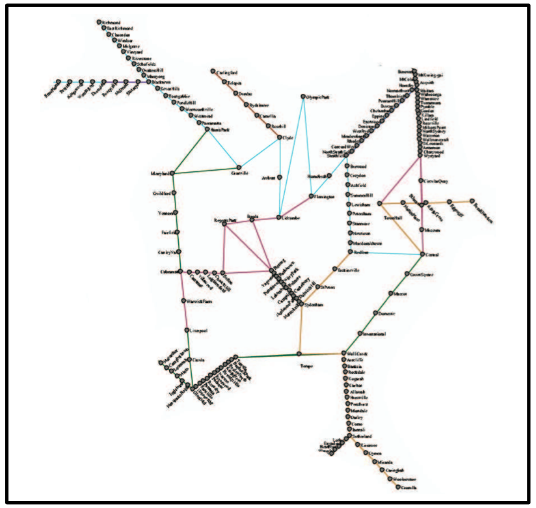

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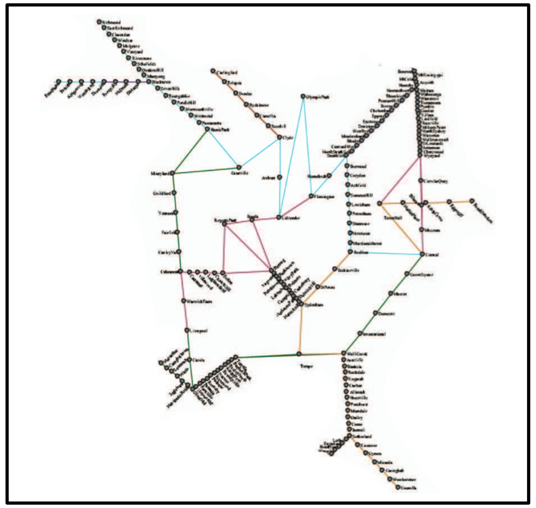

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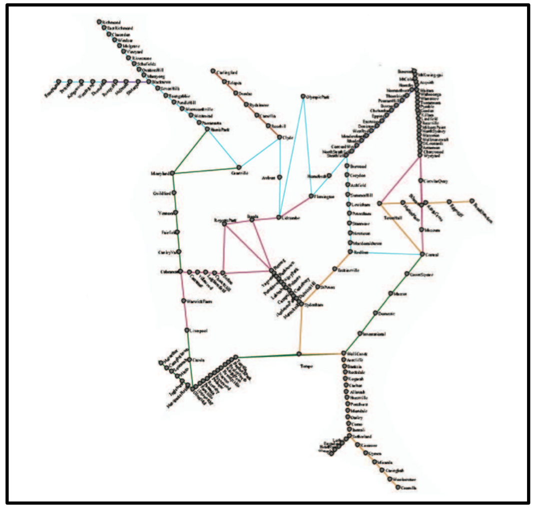

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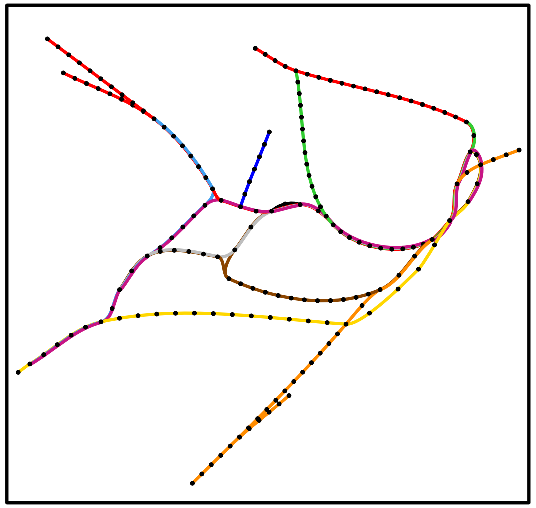

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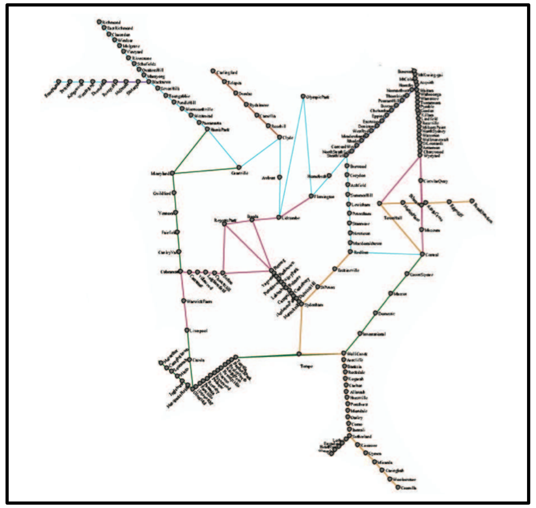

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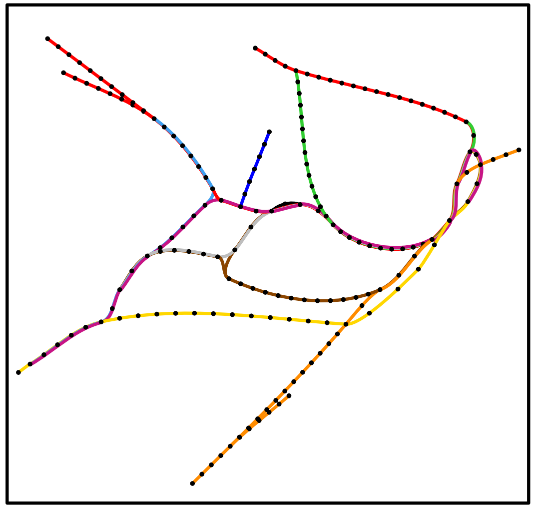

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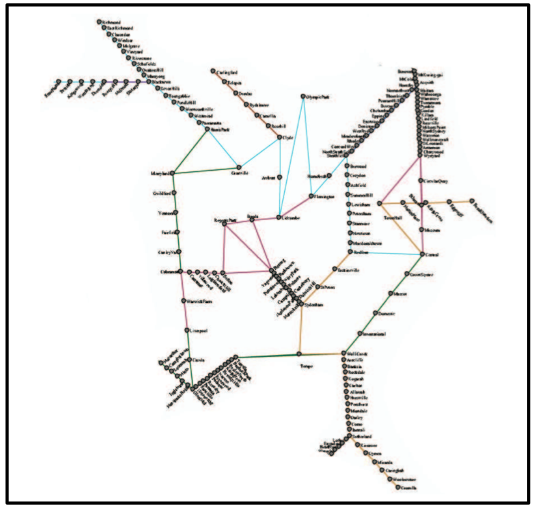
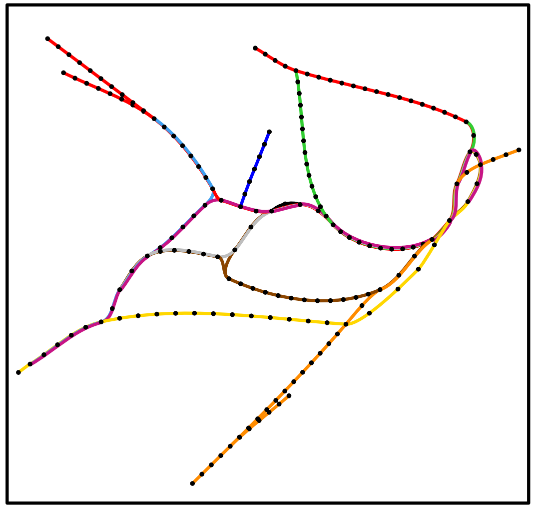


Bézier.

- guarantees topology **(R1)**
- takes almost all design rules into account



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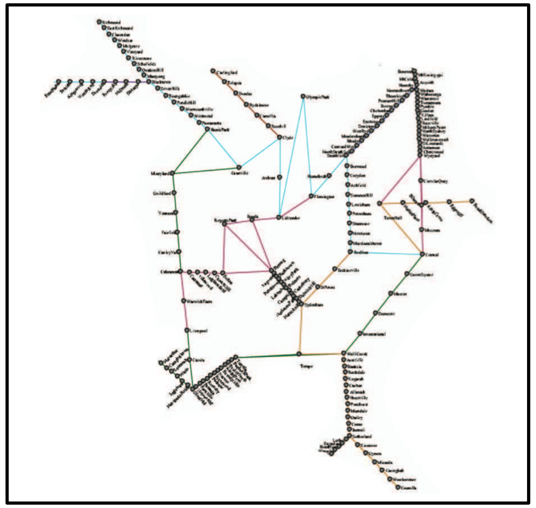

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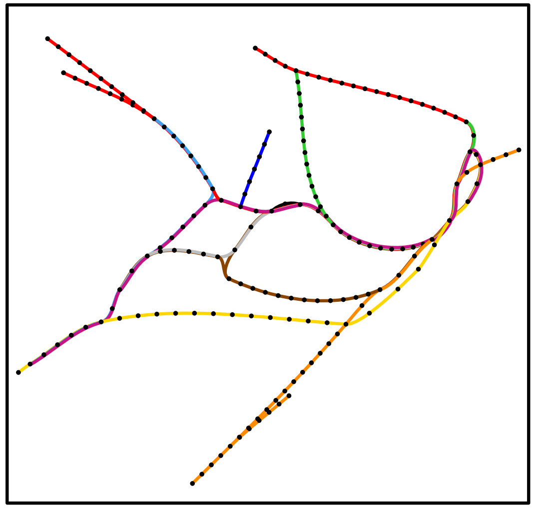

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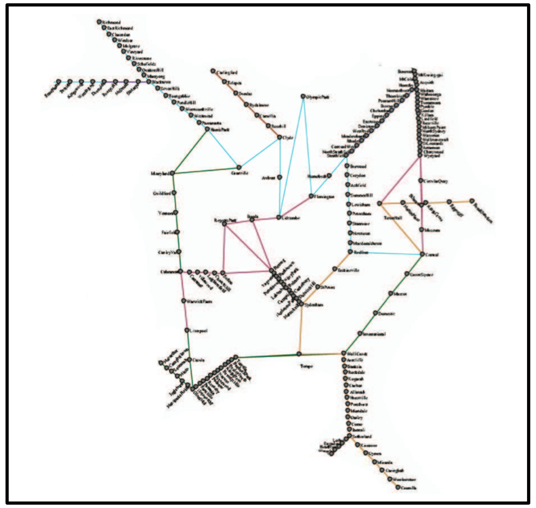

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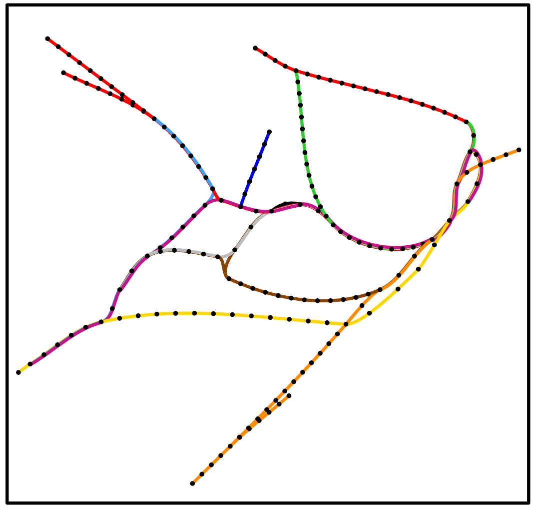
- works well on small and medium instances



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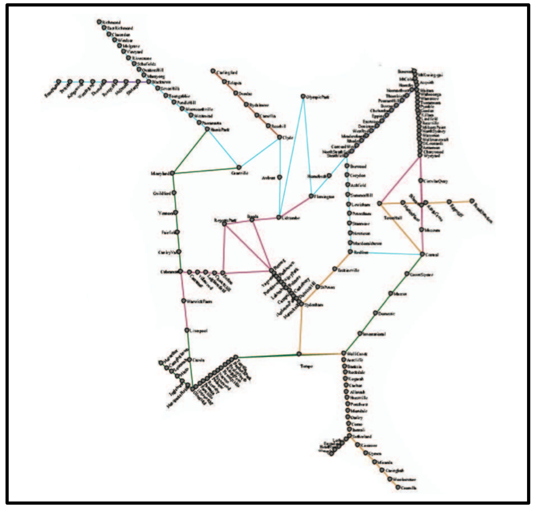
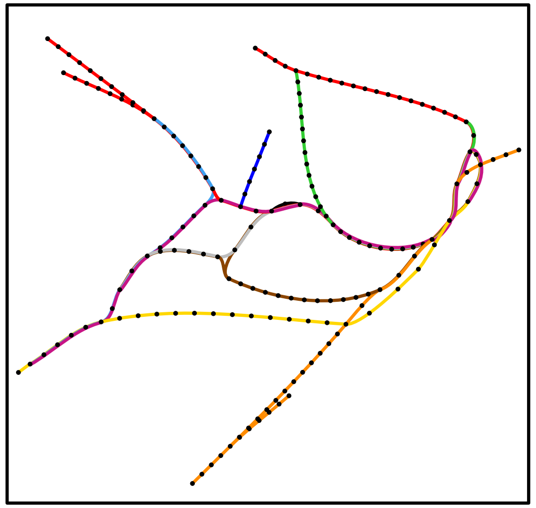


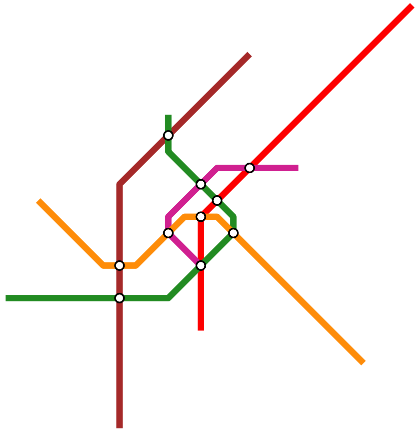
Bézier.

- guarantees topology **(R1)**
- takes almost all design rules into account
- first curvilinear metro map algorithm
- works well on small and medium instances
- difficulties with more complex networks

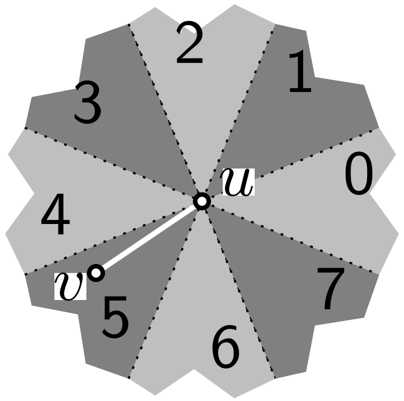


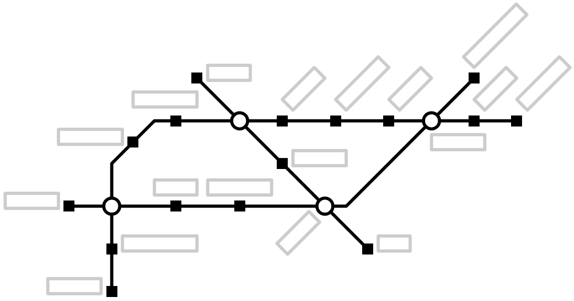
Visualization of Graphs

Lecture 12:
Octilinear Graph Drawing
Metro Map Layout



Part IV:
Local Schematization





Jonathan Klawitter

Local Schematization

Idea.

- define a (multi-criteria) layout quality function

es

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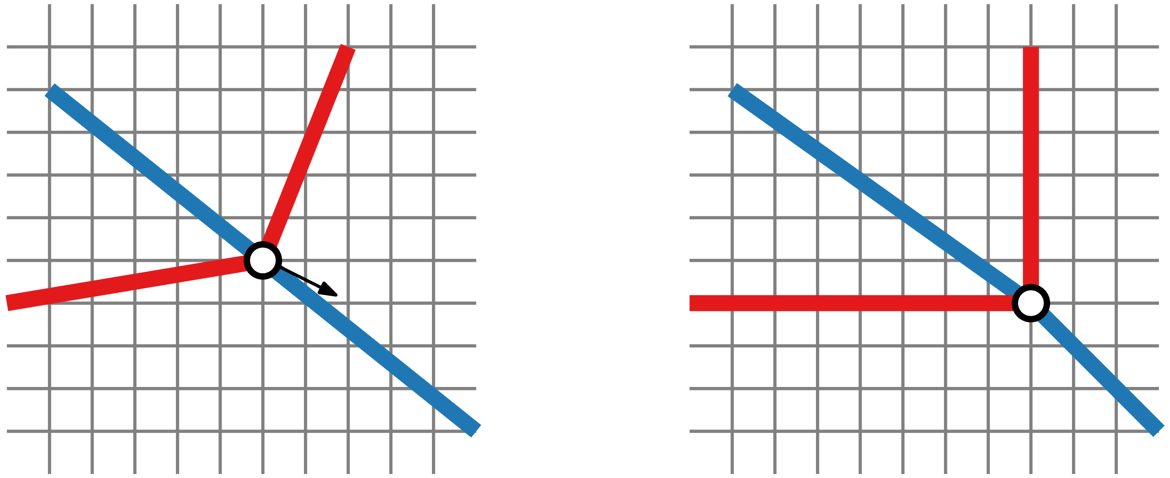

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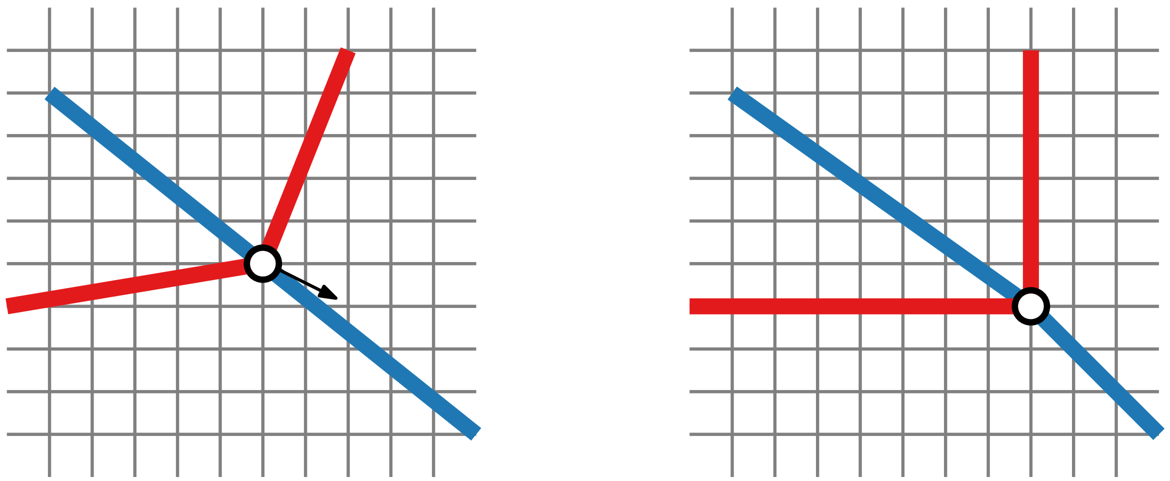

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[Avelar & Müller 2000]

- calculate best vertex position in each criterion (octilinearity **(R2)**, min. separation **(R8)**)

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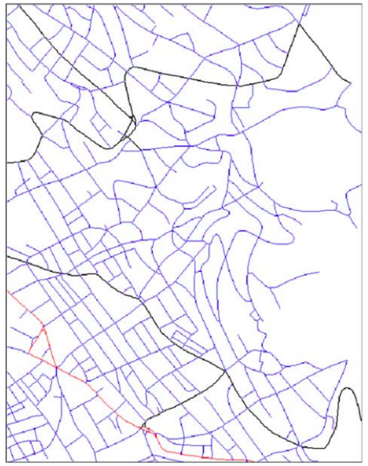

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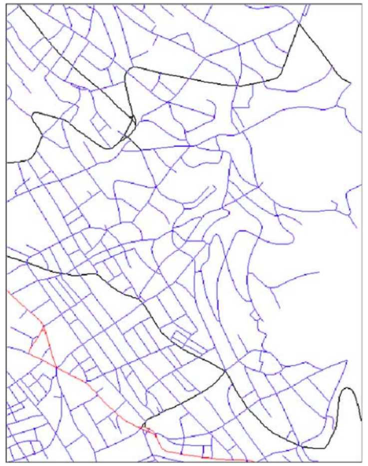

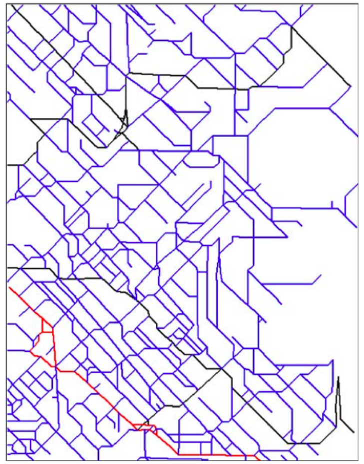

100

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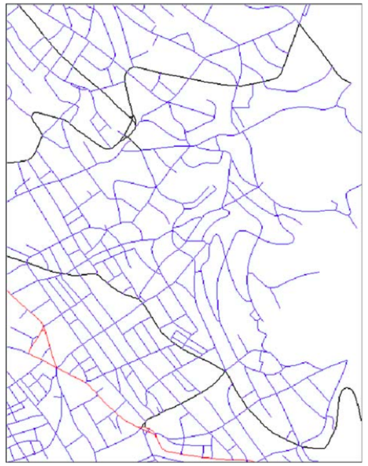

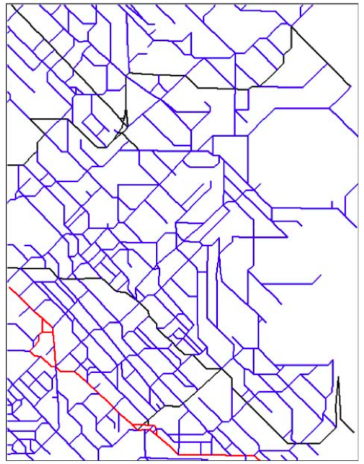

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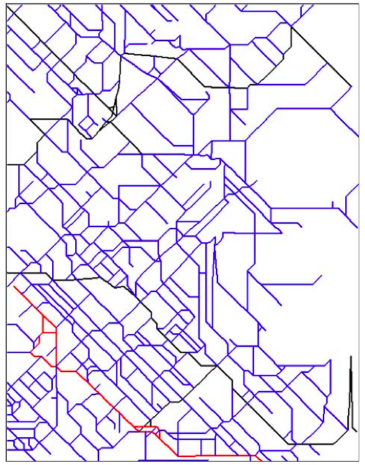

1,000

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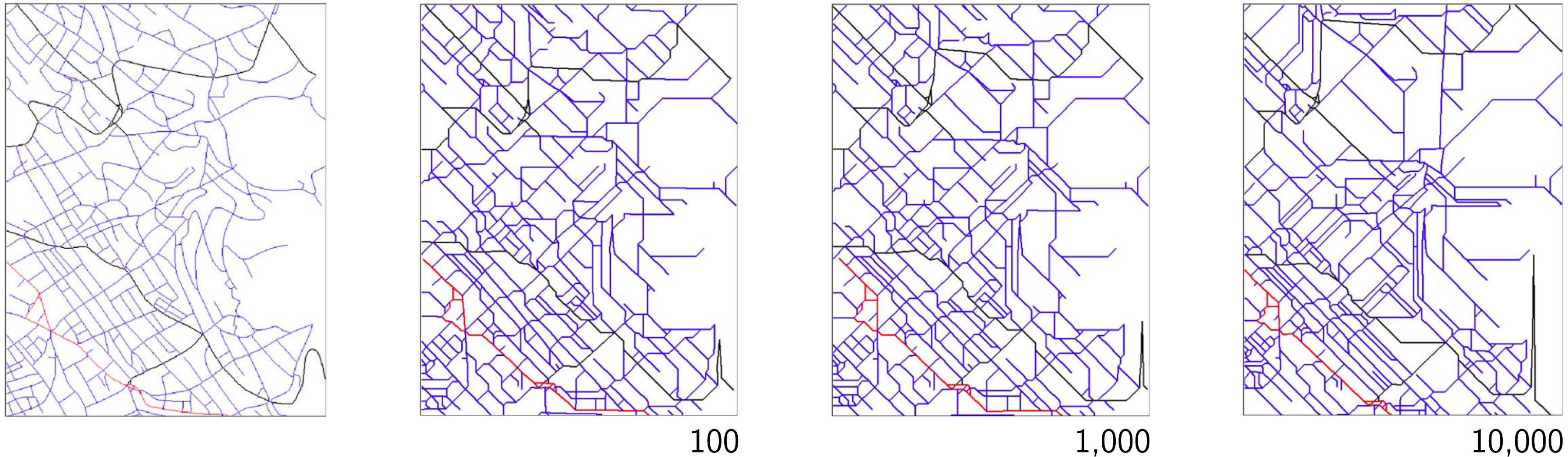

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[Ware et al. 2006, Ware & Richards 2013]

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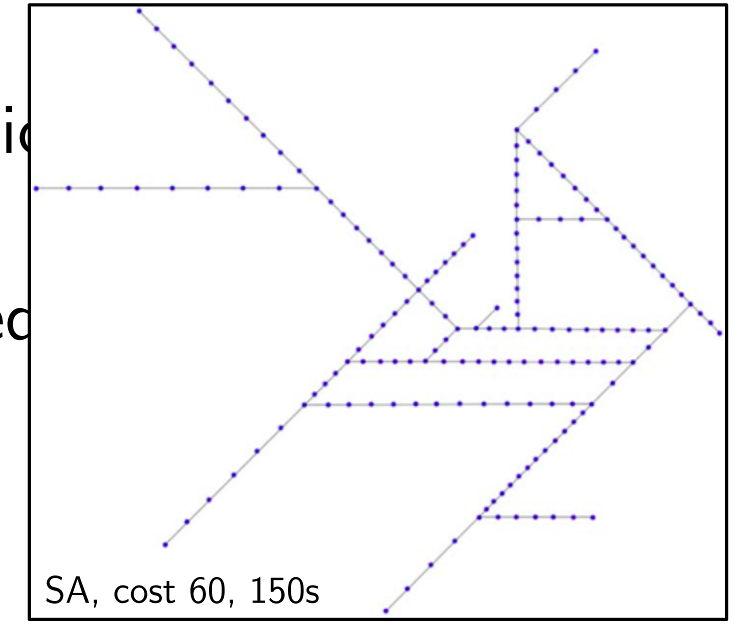
Local Schematization

Idea.

- define a (multi-criteria) layout quality function
- improve layout quality step-by-step by locally moving verti[ces] to better nearby (grid) positions
- applicable optimization techniques: hill climbing, simulated annealing, ant colony optimization, . . .


SA, cost 60, 150s

[Ware et al. 2006, Ware & Richards 2013]

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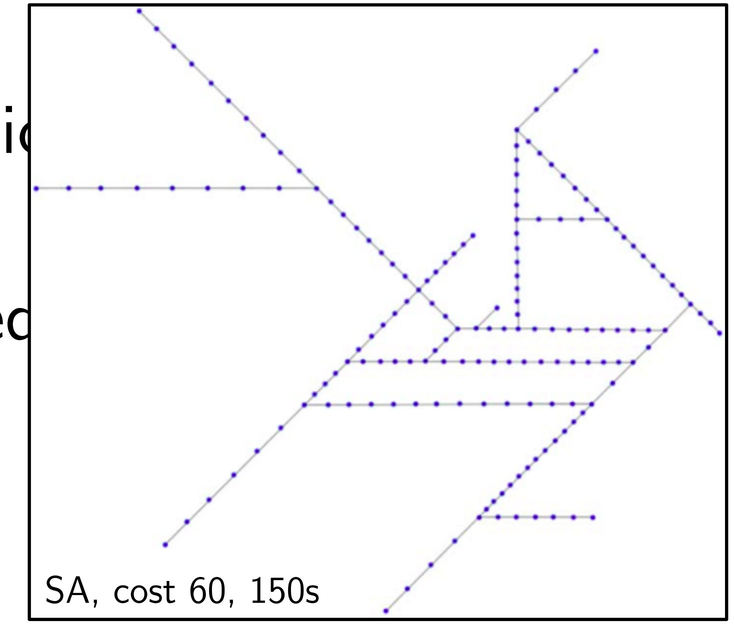

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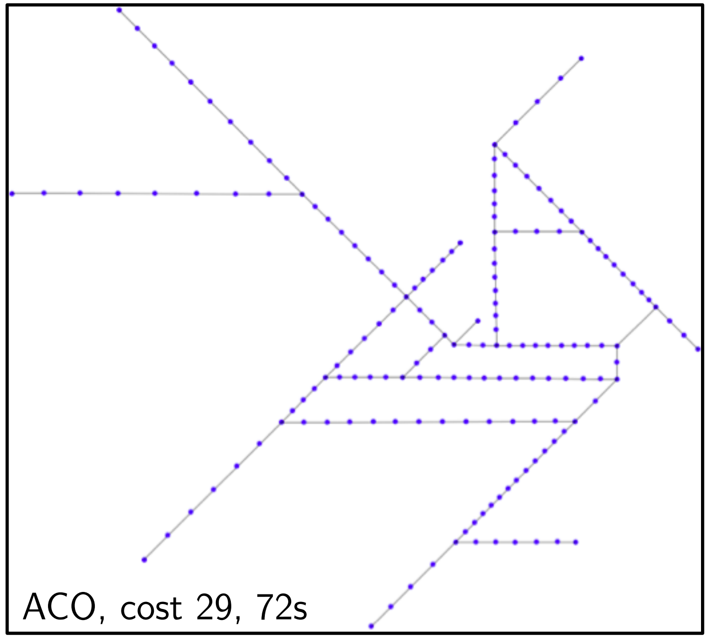

ACO, cost 29, 72s

Local Schematization

[Stott et al. 2011]

Idea.

- design rules as before

Local Schematization

[Stott et al. 2011]

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- additionally include metro map specific criteria (bend minimization **(R3)**, interchange straightness **(R4)**, angular resolution **(R5)**, relative positions **(R6)**)

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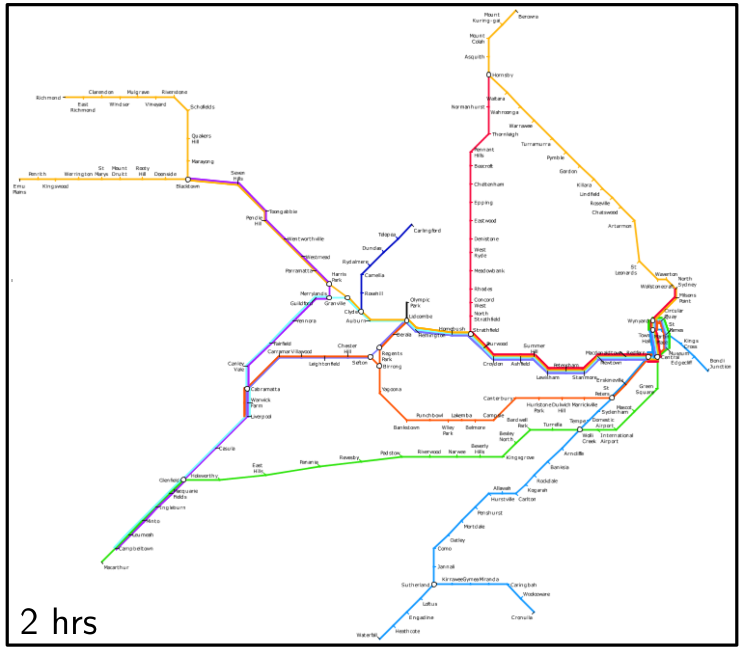

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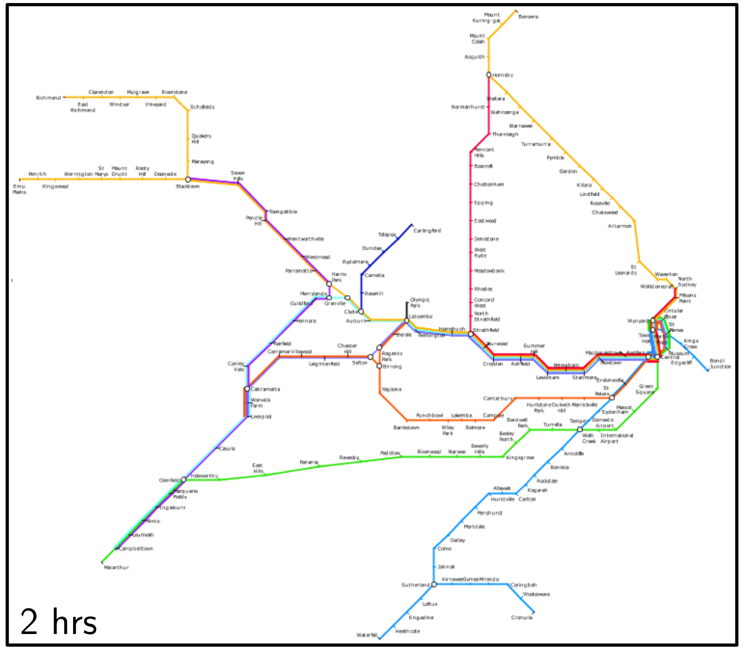

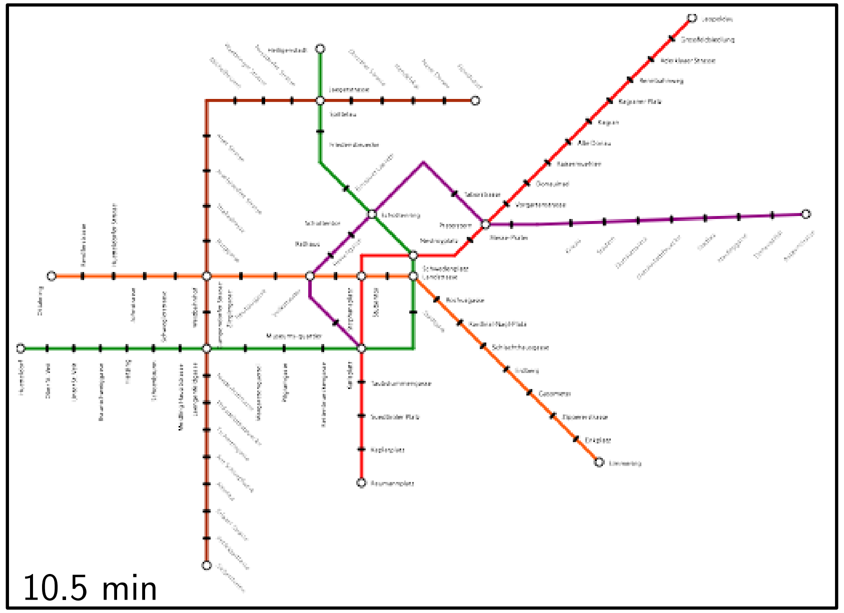

Local Schematization – Discussion

Pros.

Cons.

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Local Schematization – Discussion

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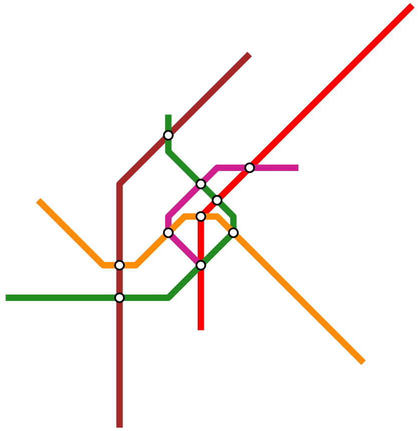
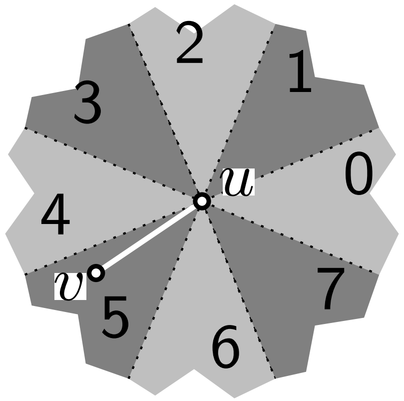
- susceptible to local minima
- long running times

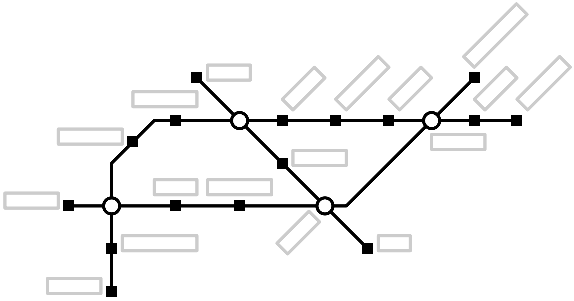
Visualization of Graphs



Lecture 12:
Octilinear Graph Drawing
Metro Map Layout



Part V:
Mixed-Integer Programming



Jonathan Klawitter

Mixed-Integer Programming

[Nöllenburg & Wolff 2011]

- find exact optimum solution using combinatorial optimization

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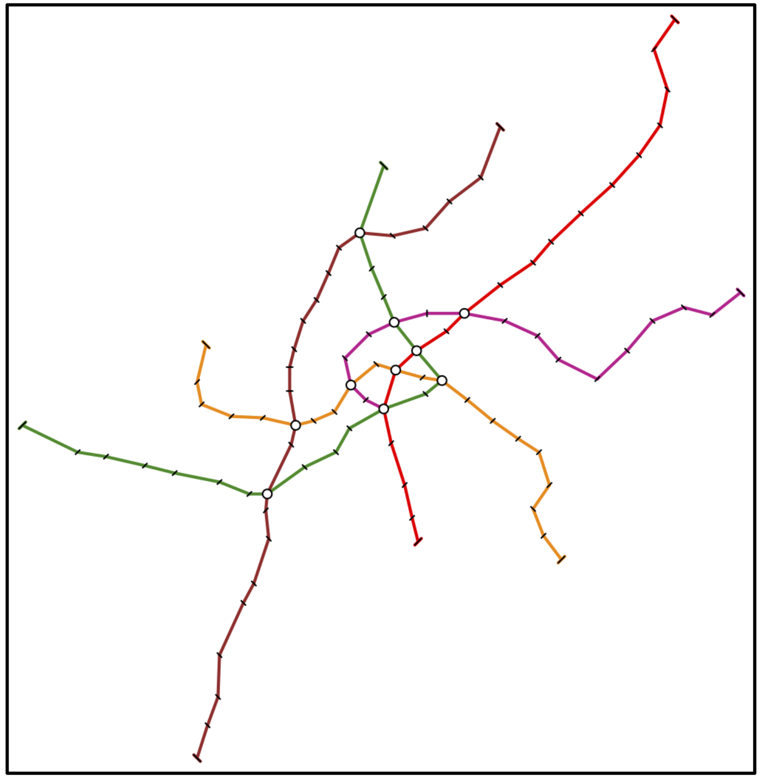
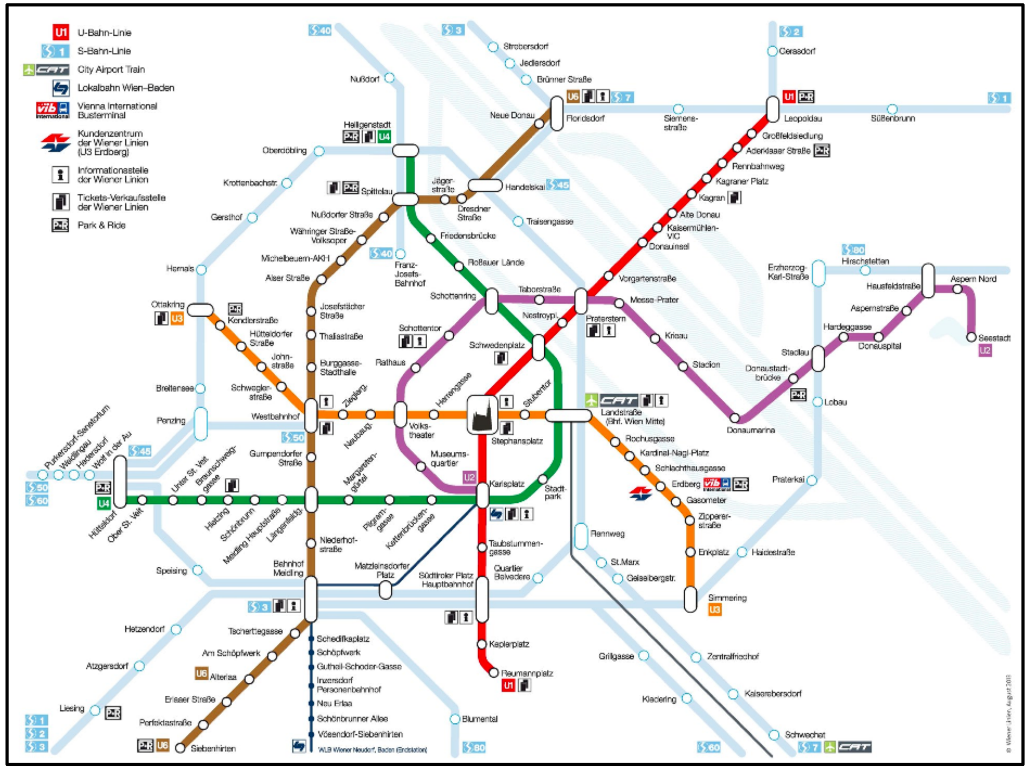
- model constraints as linear (in)equalities and linear objective function $\rightarrow$ mixed-integer programming **MIP**
- integrate overlap-free station labeling in same model
- can use sophisticated optimization tools as black box solvers (e.g., CPLEX, Gurobi)

Hard Constraints

(R1) preserve embedding/**topology** and **planarity**

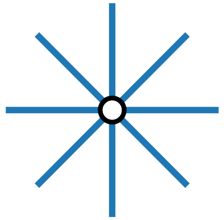


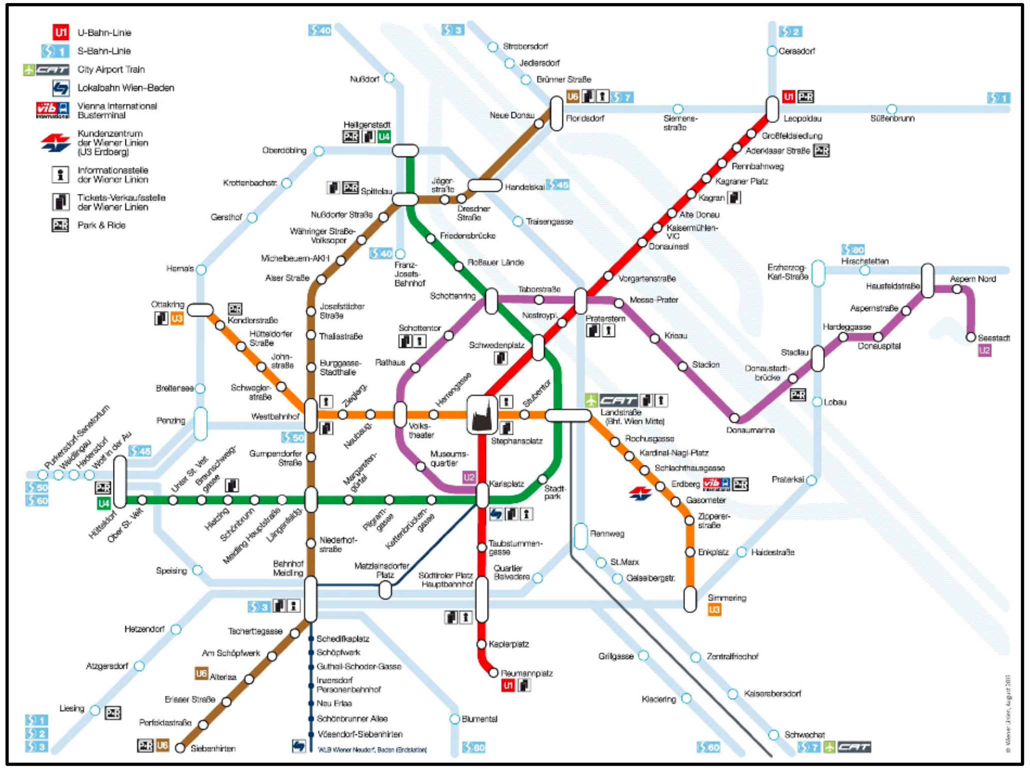


Hard Constraints

(R1) preserve embedding/**topology** and **planarity**

(R2) draw all edges **octilinearly**





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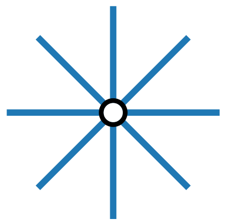
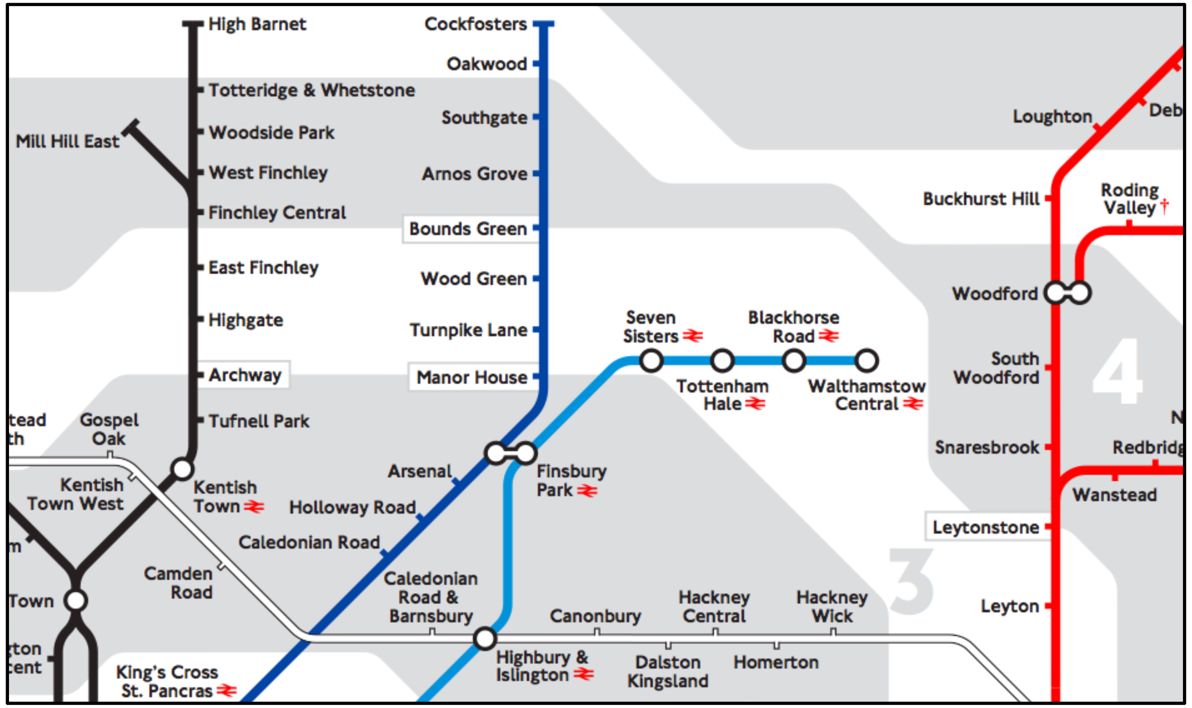
(R7) enforce **minimum edge length** $\ell_{\min}$





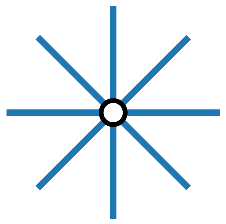
Hard Constraints

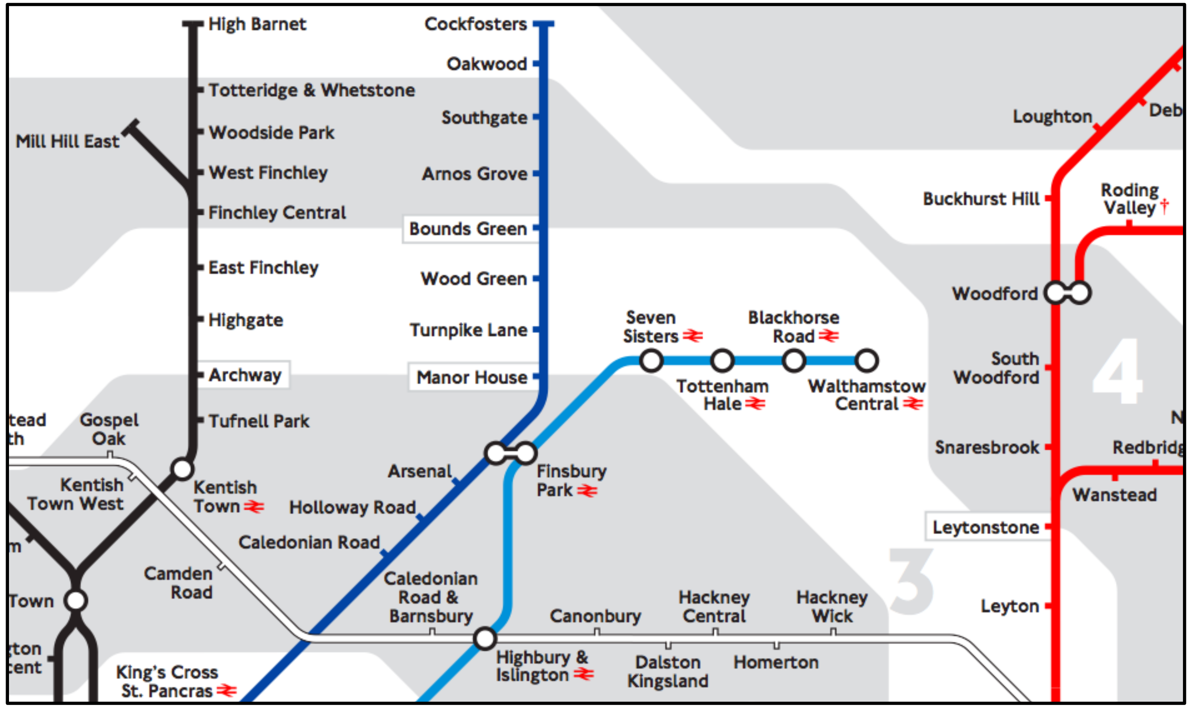
(R1) preserve embedding/**topology** and **planarity**

(R2) draw all edges **octilinearly**

(R7) enforce **minimum edge length** ℓ_{min}

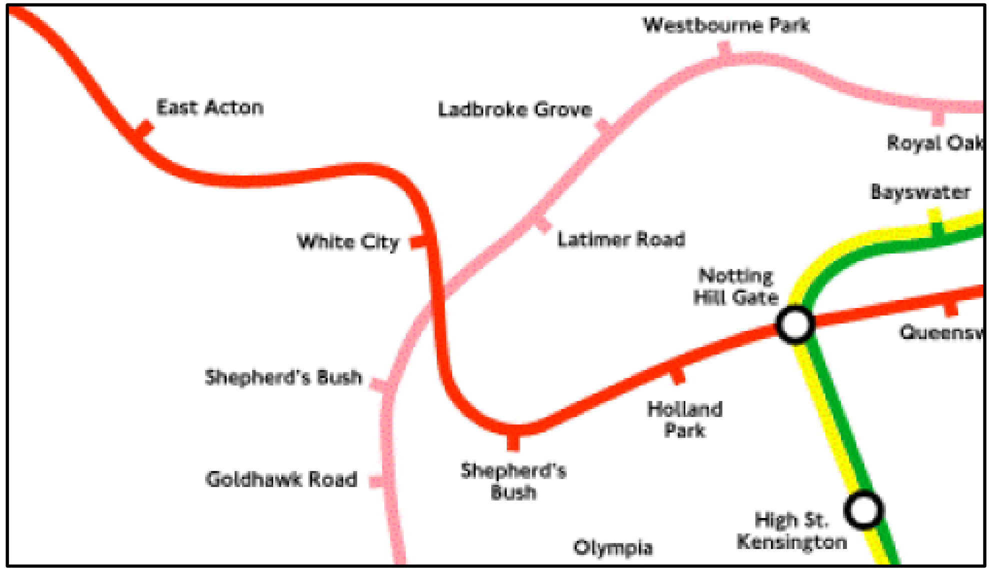
(R8) enforce **minimum feature separation** d_{min}

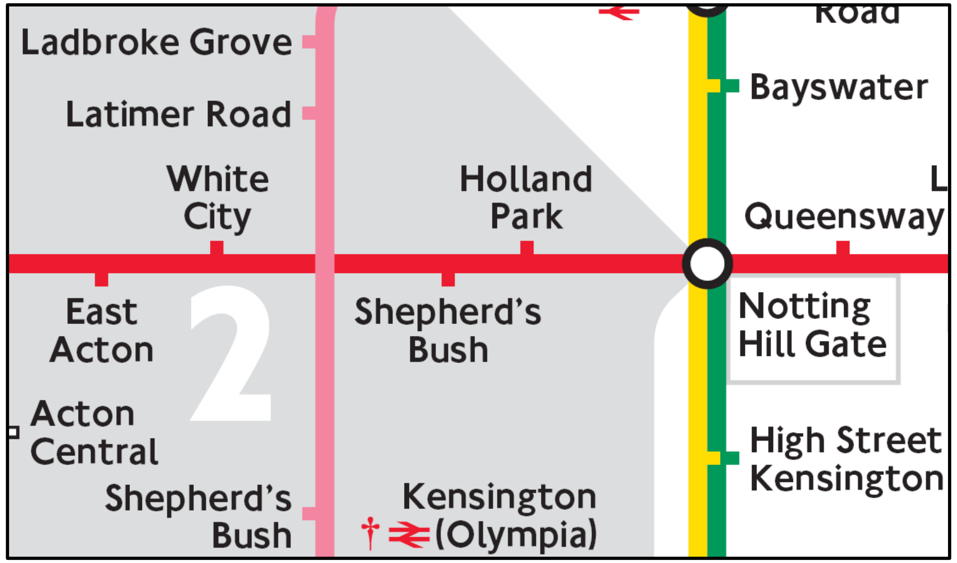




Soft Constraints

(R3+R4) draw lines in $\mathcal{L}$ with **few bends**





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(R3+R4) draw lines in $\mathcal{L}$ with **few bends**

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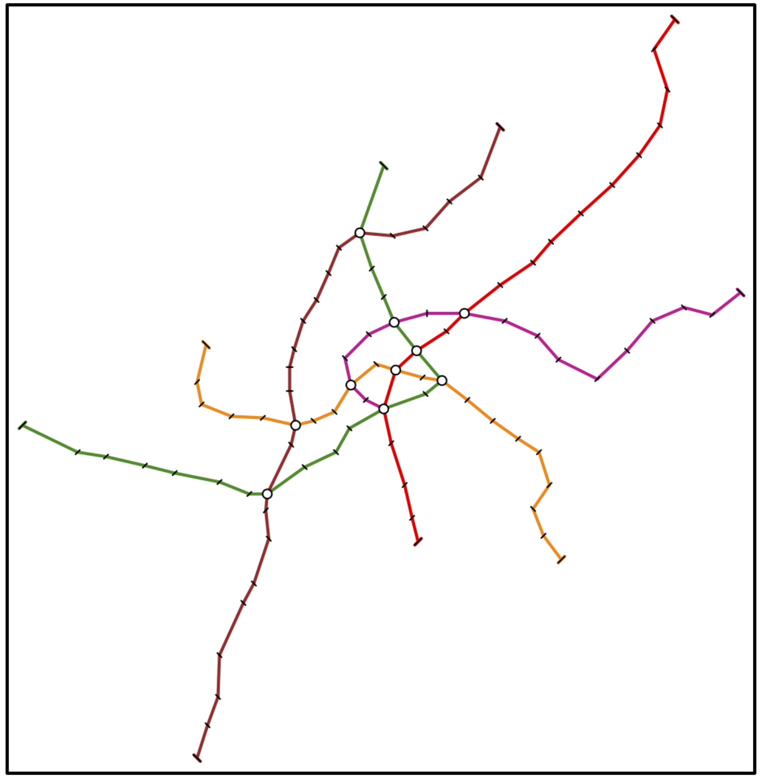

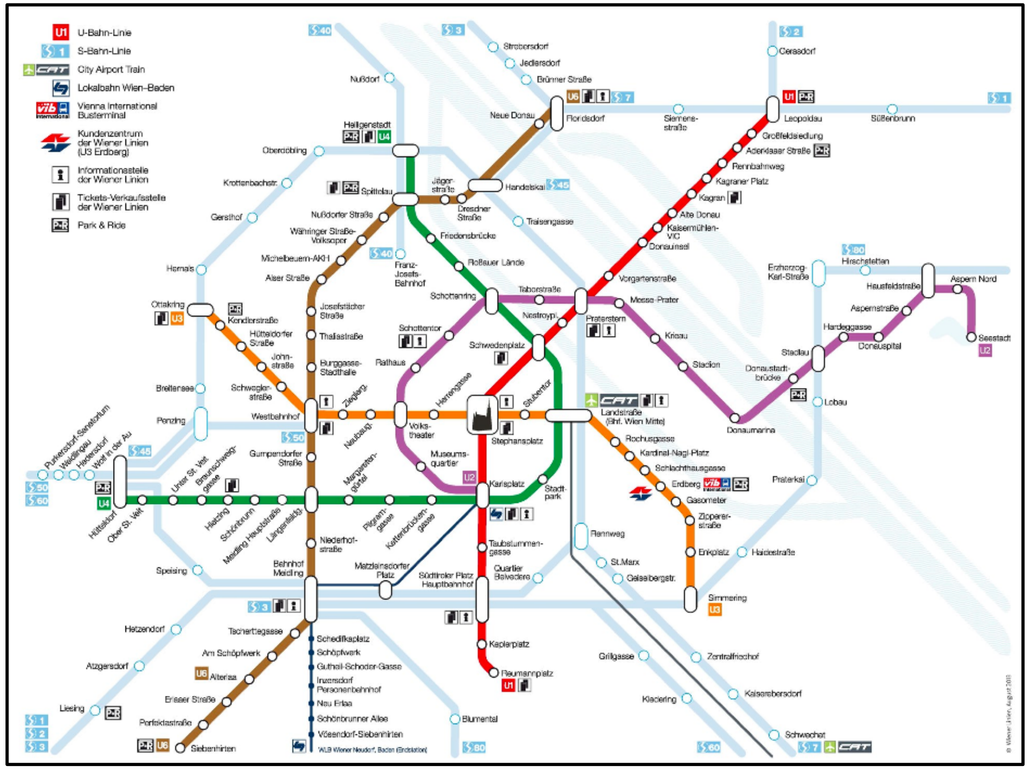

Soft Constraints

(R3+R4) draw lines in $\mathcal{L}$ with **few bends**

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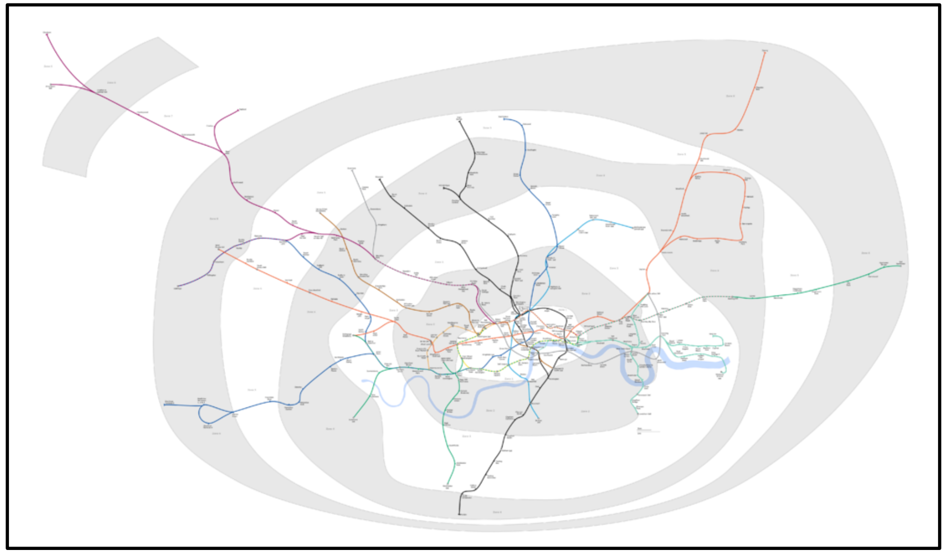
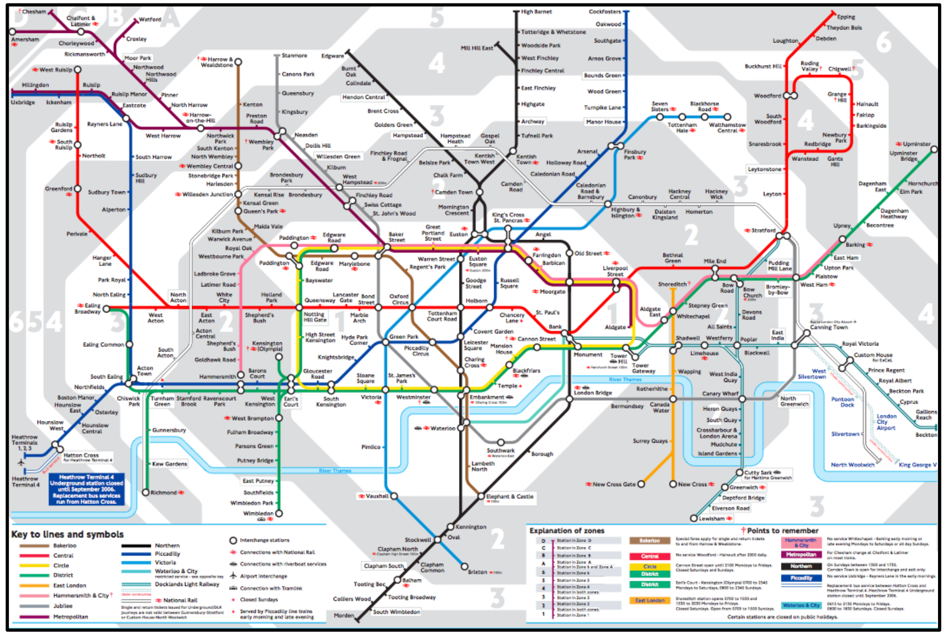
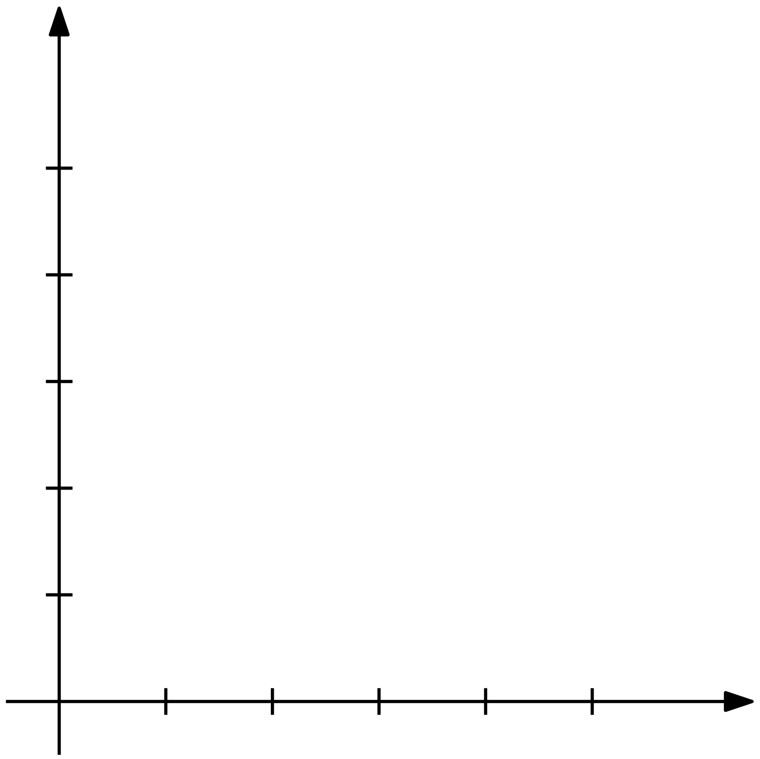




Linear Programming

Linear Programming (LP) is an efficient optimization method for



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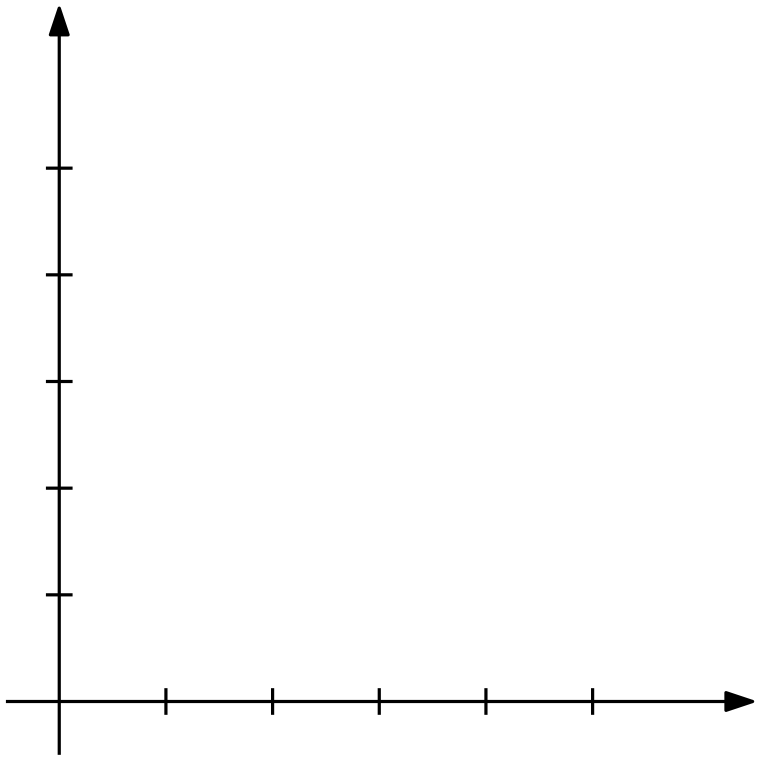

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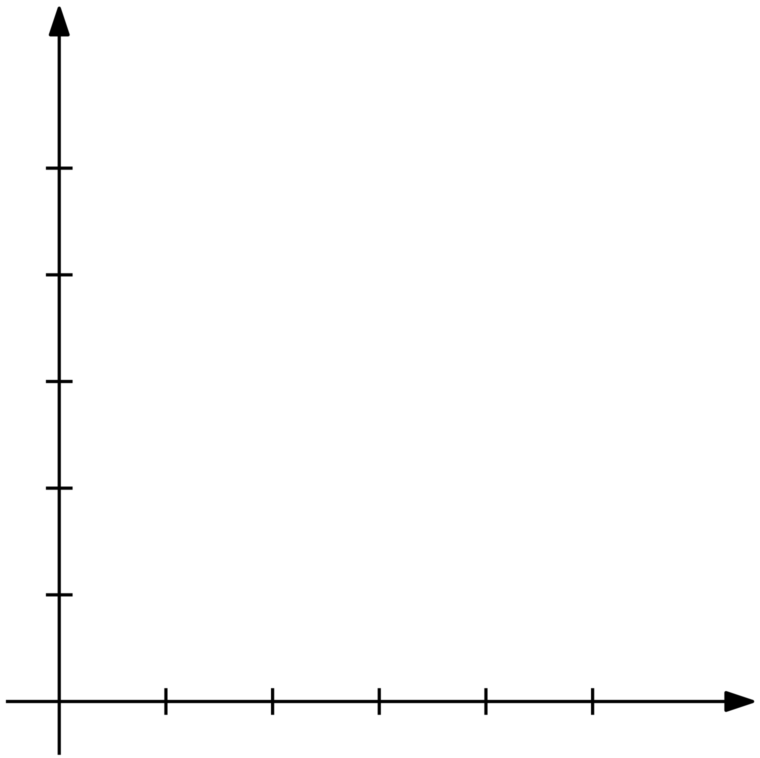

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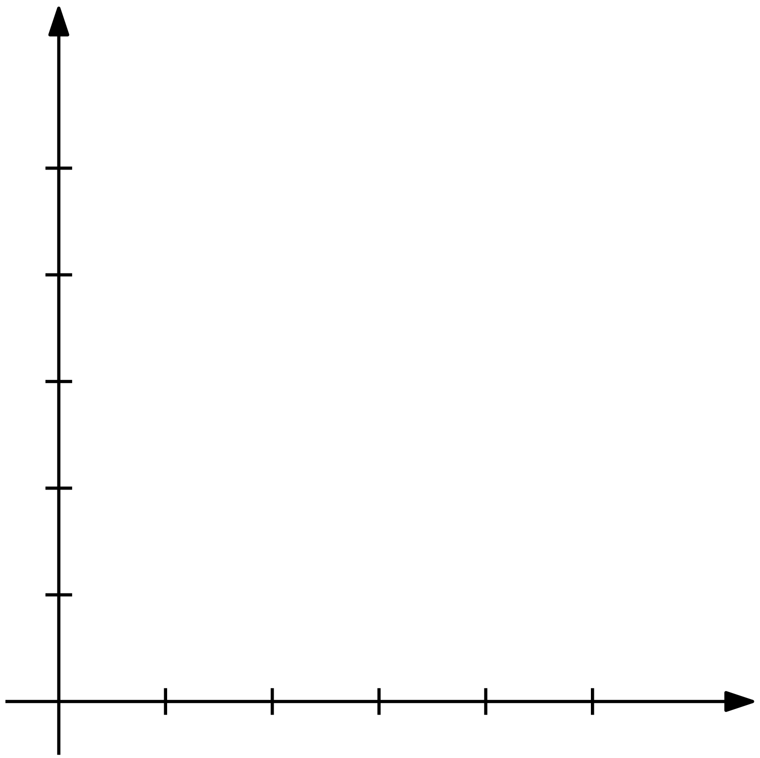

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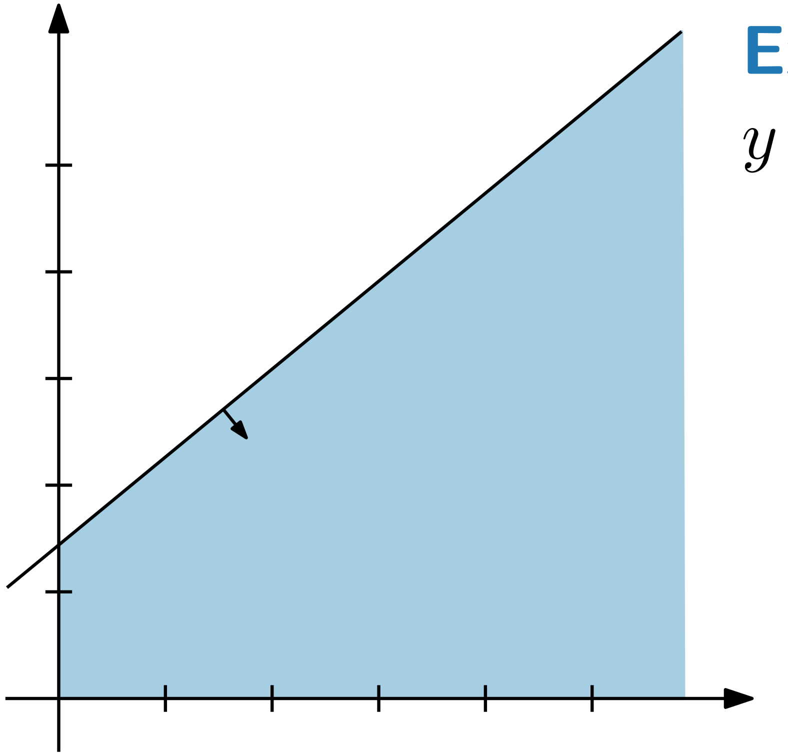

Example.
$y \leq 0.9x + 1.5$

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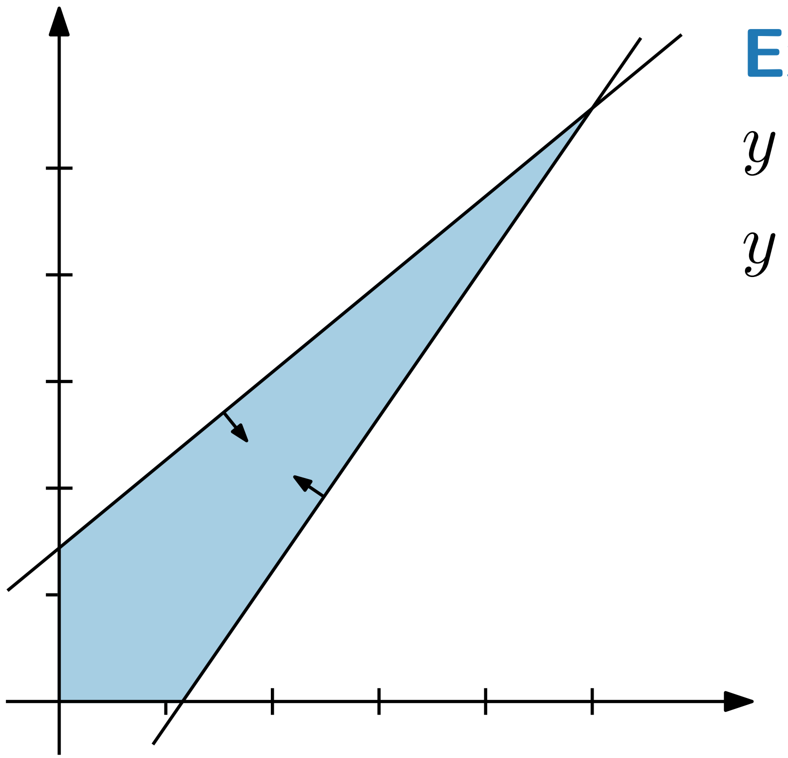

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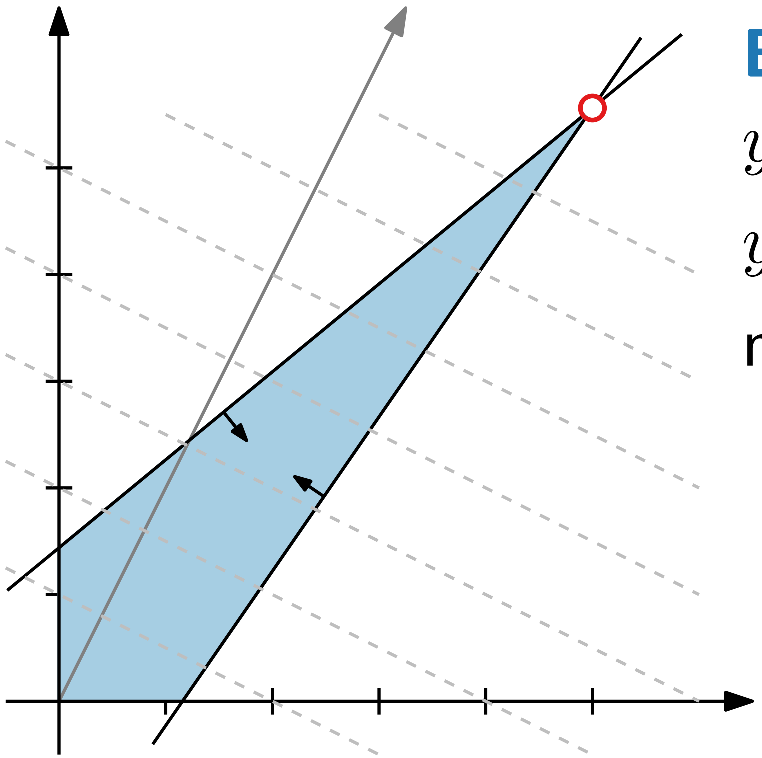

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maximize $x + 2y$

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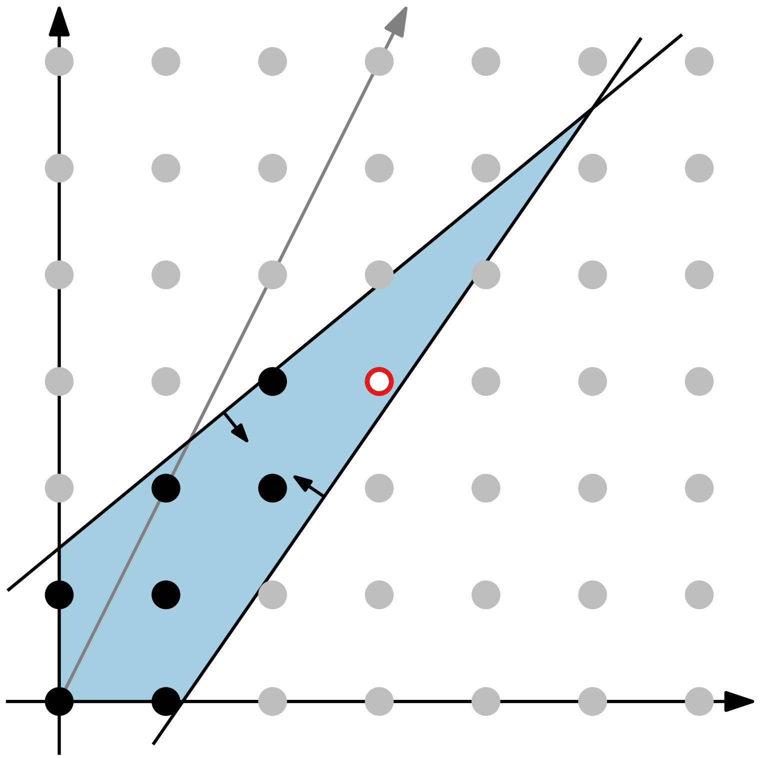
Mixed Integer Programming (MIP)

- in addition binary and integer variables



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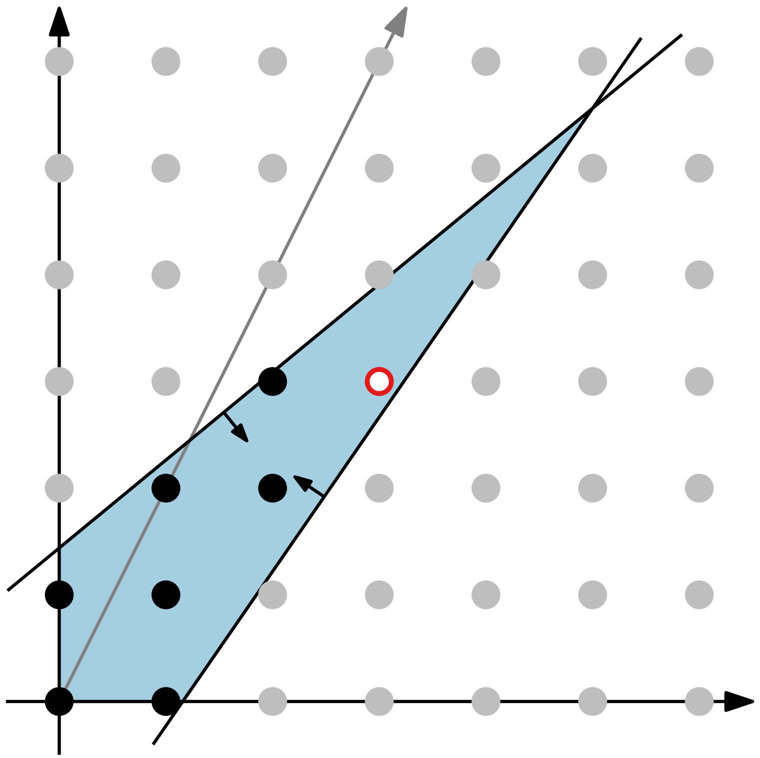
- NP-hard optimization problem



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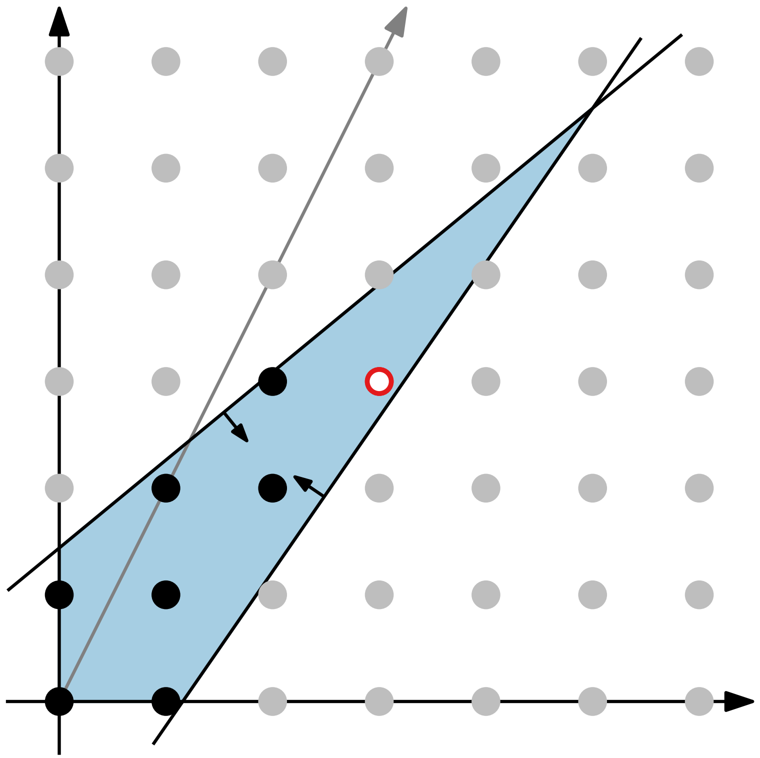

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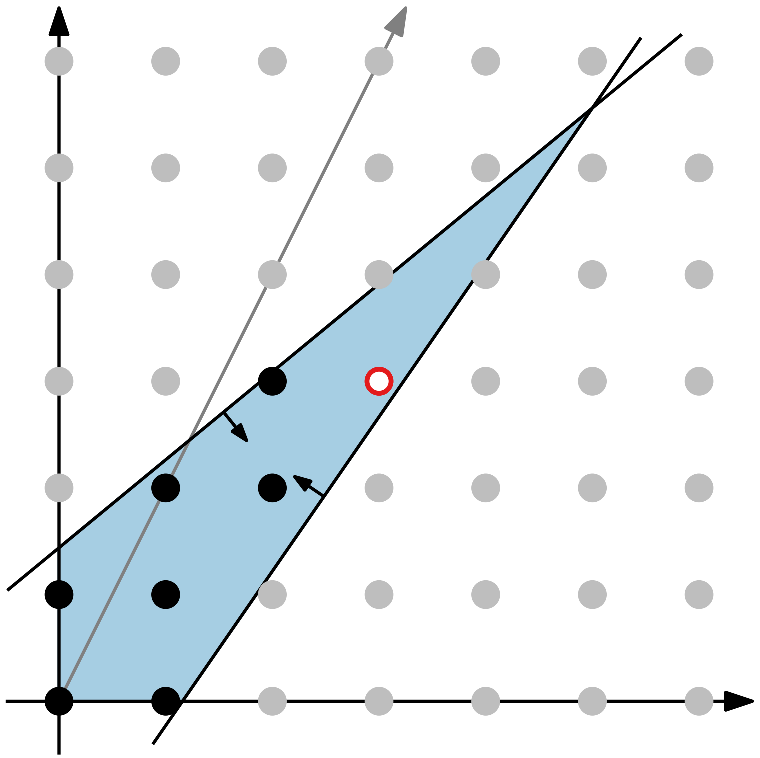
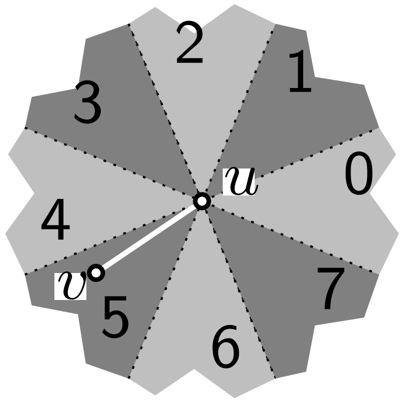


Theorem 5.
Metro map layout can be modeled as MIP such that

- hard constraints $\rightarrow$ linear constraints
- soft constraints $\rightarrow$ linear objective function

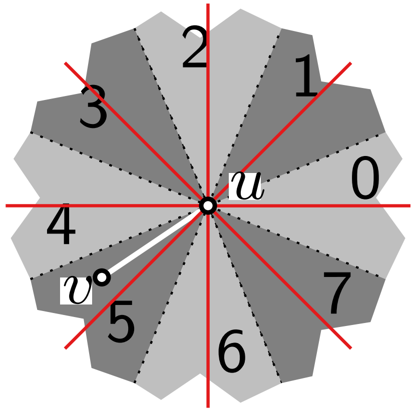
Sectors and Coordinates



Sectors.

- for each vertex u partition the plane into eight sectors numbered 0–7
  here: $\text{sec}(u, v) = 5$ in the input

Sectors and Coordinates



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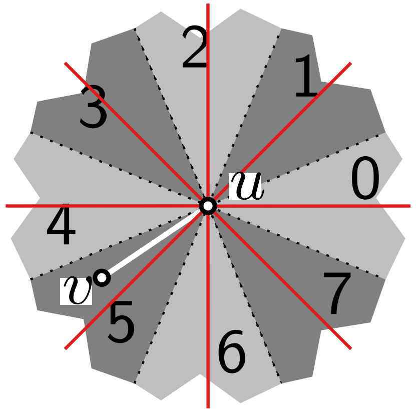

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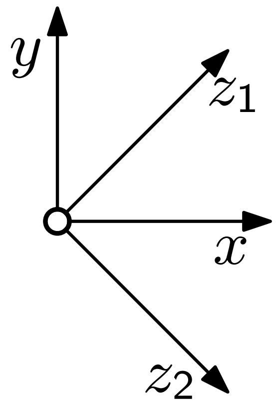
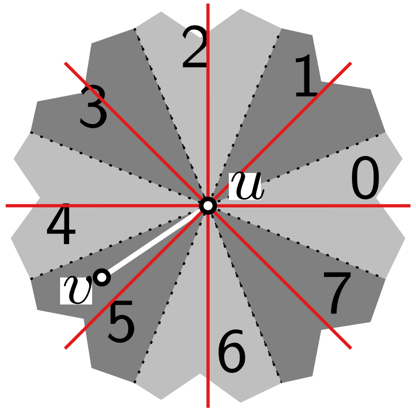


Coordinates.

- assign (redundant) z_1- and z_2-coordinates to each vertex v
  - $z_1(v) = \frac{1}{2} \cdot (x(v) + y(v))$
  - $z_2(v) = \frac{1}{2} \cdot (x(v) - y(v))$

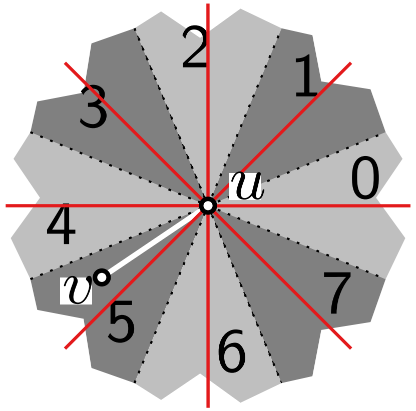
Octilinearity and Relative Position



Goal.

Draw the edge uv

Octilinearity and Relative Position

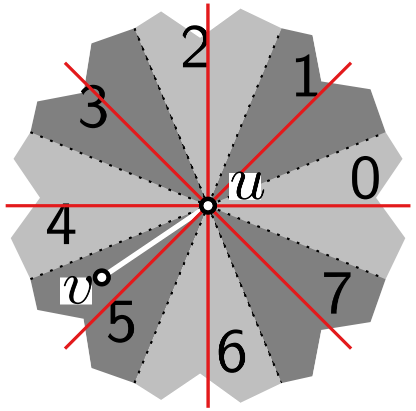


Goal.

Draw the edge uv

- octilinearly **(R2)**

Octilinearity and Relative Position

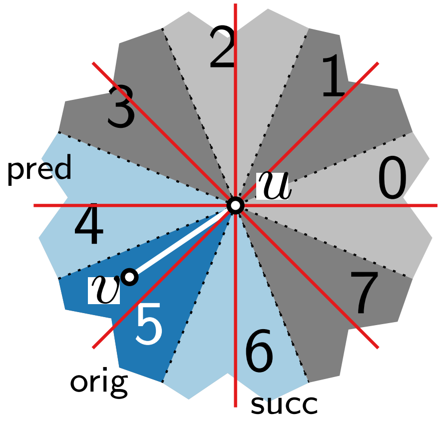


Goal.

Draw the edge uv

- octilinearly **(R2)**
- with minimum length $\ell = \ell_{uv}$ **(R7)**

Octilinearity and Relative Position



Goal.
Draw the edge uv

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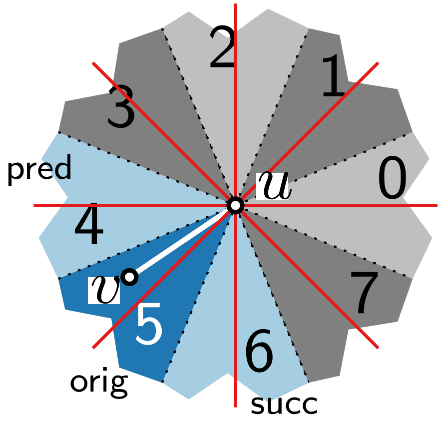
- with minimum length $\ell = \ell_{uv}$ **(R7)**
- restricted to the three best directions **(R6)**

Octilinearity and Relative Position



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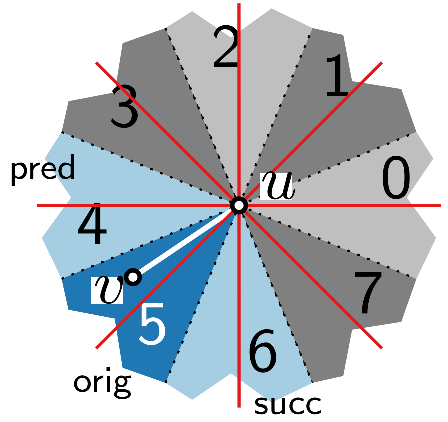
How to model this using linear constraints in a MIP?

Octilinearity and Relative Position



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Draw the edge uv

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- with minimum length $\ell = \ell_{uv}$ **(R7)**
- restricted to the three best directions **(R6)**

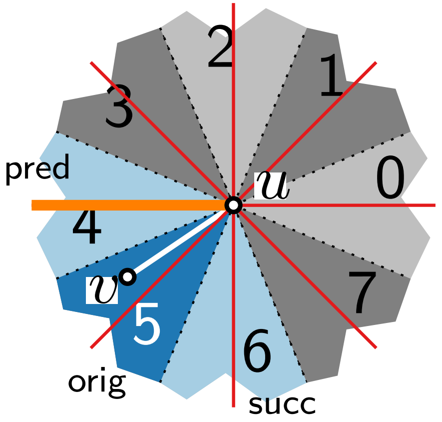
How to model this using linear constraints in a MIP?

Introduce **binary variables**

$$\alpha_{\mathsf{pred}}(u, v) + \alpha_{\mathsf{orig}}(u, v) + \alpha_{\mathsf{succ}}(u, v) = 1$$

to select exactly one of the three sectors.

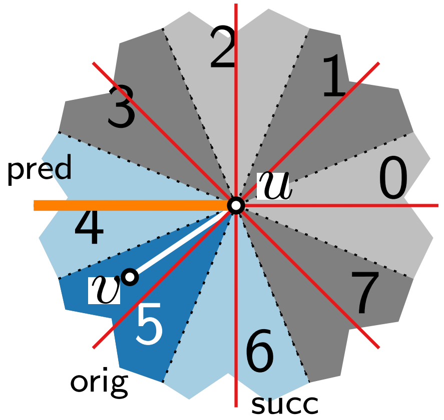
Octilinearity and Relative Position



Predecessor sector.

$$\begin{aligned} y(u) - y(v) &\leq M(1 - \alpha_{\text{pred}}(u,v)) \\ -y(u) + y(v) &\leq M(1 - \alpha_{\text{pred}}(u,v)) \\ x(u) - x(v) &\geq -M(1 - \alpha_{\text{pred}}(u,v)) + \ell \end{aligned}$$

Octilinearity and Relative Position



very large constant

Predecessor sector.

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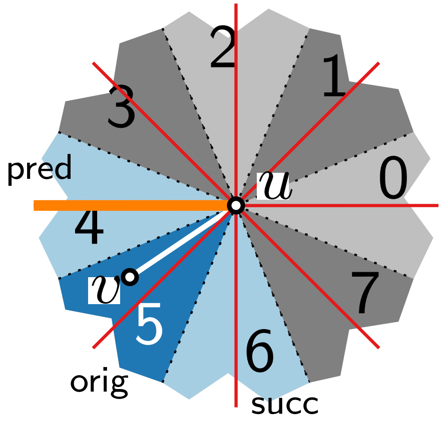
Octilinearity and Relative Position



Predecessor sector.

very large constant

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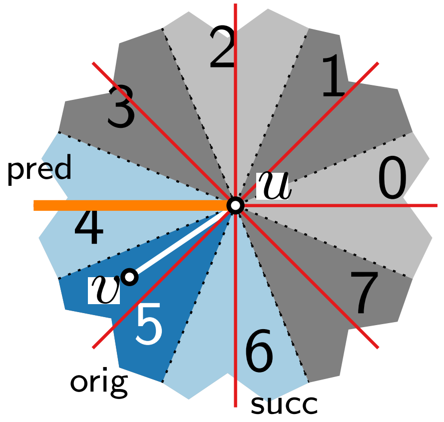
How does this work?

Octilinearity and Relative Position



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\begin{aligned}
y(u) - y(v) &\le M(1 - \alpha_{\mathsf{pred}}(u,v)) \\
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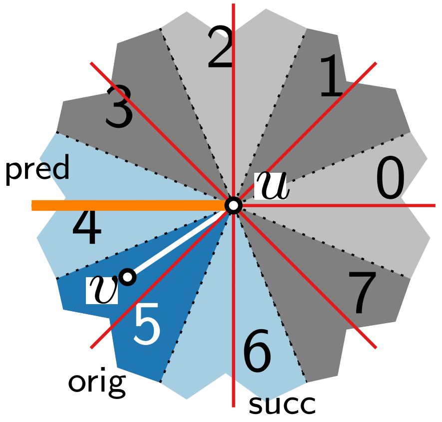
\end{aligned}
$$

How does this work?

Case 1: $\alpha_{\mathsf{pred}}(u,v) = 1$

Octilinearity and Relative Position



Predecessor sector.

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y(u) - y(v) &\leq M(1 - \alpha_{\text{pred}}(u,v)) \\
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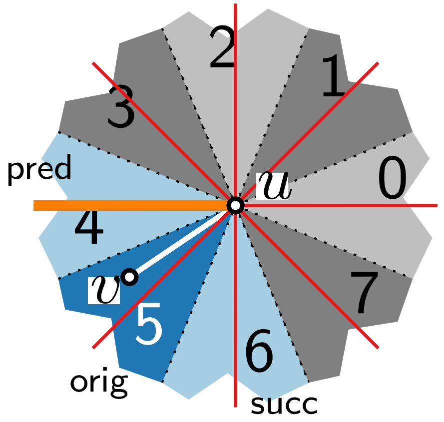
\end{aligned}
$$

How does this work?

Case 1: $\alpha_{\text{pred}}(u,v) = 1$

$$
\begin{aligned}
y(u) - y(v) &\leq 0 \\
-y(u) + y(v) &\leq 0 \\
x(u) - x(v) &\geq \ell
\end{aligned}
$$

Octilinearity and Relative Position



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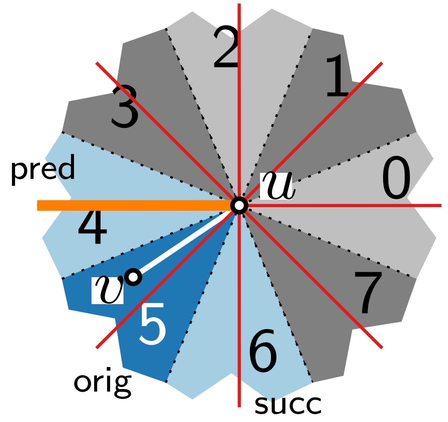
Case 2: $\alpha_{\mathsf{pred}}(u,v) = 0$

Octilinearity and Relative Position



Predecessor sector.

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y(u) - y(v) &\leq M(1 - \alpha_{\text{pred}}(u,v)) \\
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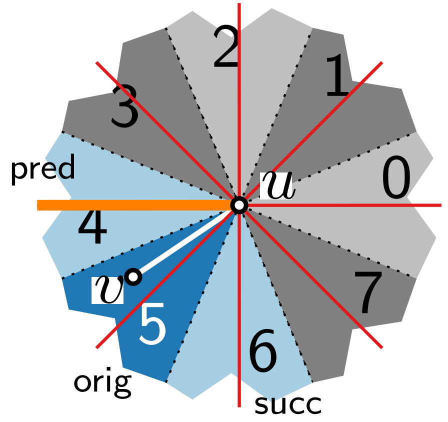
x(u) - x(v) &\geq \ell
\end{aligned}
$$

Case 2: $\alpha_{\text{pred}}(u,v) = 0$

$$
\begin{aligned}
y(u) - y(v) &\leq M \\
-y(u) + y(v) &\leq M \\
x(u) - x(v) &\geq -M + \ell
\end{aligned}
$$

Octilinearity and Relative Position



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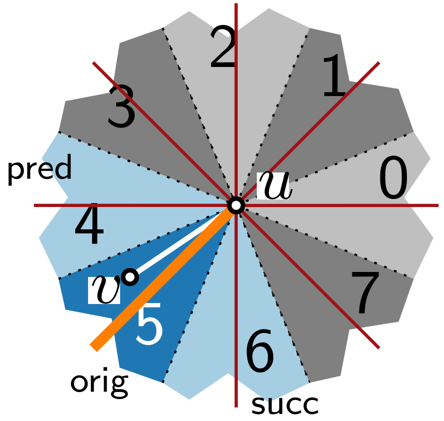
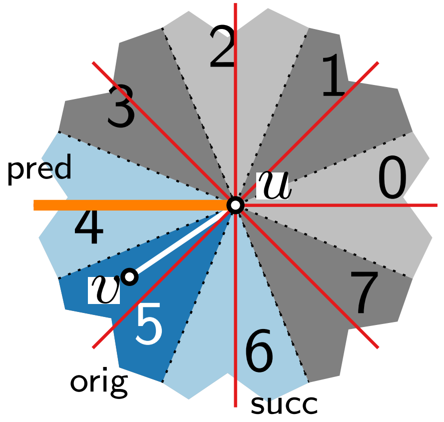
Original sector.

$$\begin{aligned} z_2(u) - z_2(v) &\leq M(1 - \alpha_{\mathsf{orig}}(u,v)) \\ -z_2(u) + z_2(v) &\leq M(1 - \alpha_{\mathsf{orig}}(u,v)) \\ z_1(u) - z_1(v) &\geq -M(1 - \alpha_{\mathsf{orig}}(u,v)) + \ell \end{aligned}$$



Octilinearity and Relative Position



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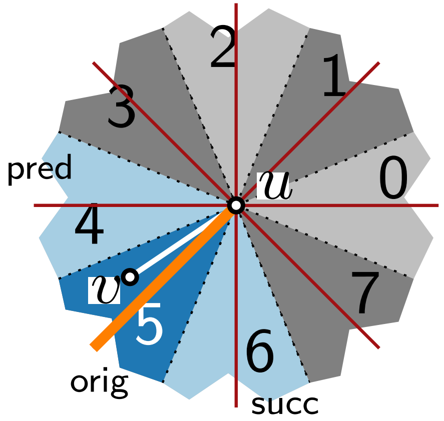

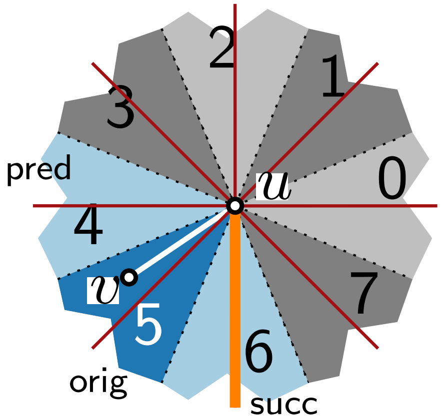

Successor sector.

$$\begin{aligned} x(u) - x(v) &\leq M(1 - \alpha_{\text{succ}}(u,v)) \\ -x(u) + x(v) &\leq M(1 - \alpha_{\text{succ}}(u,v)) \\ y(u) - y(v) &\geq -M(1 - \alpha_{\text{succ}}(u,v)) + \ell \end{aligned}$$

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- models the three soft constraints

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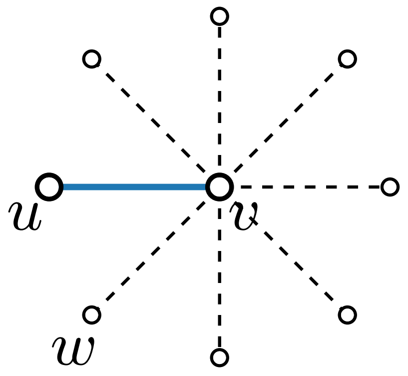
Example: line bends (R3/R4)



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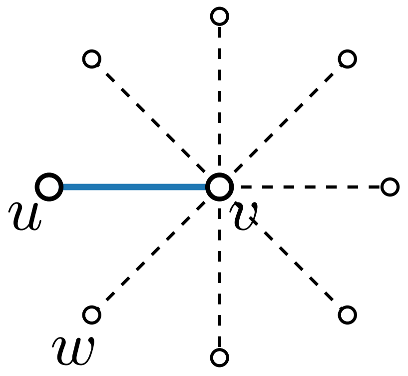
Edges uv and vw on a line $L \in \mathcal{L}$



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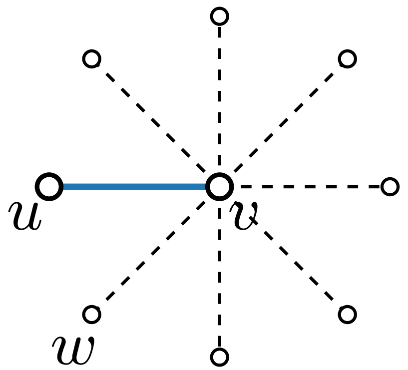

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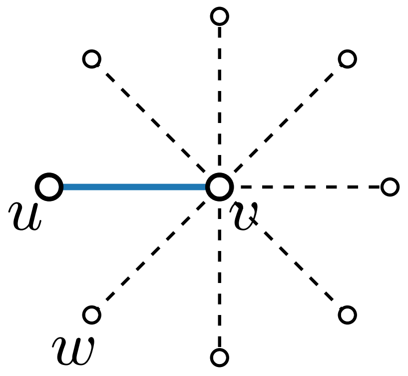

Edges uv and vw on a line $L \in \mathcal{L}$

- draw as straight as possible
- increasing cost bend(u, v, w) for increasing acuteness of $\angle(\overline{uv}, \overline{vw})$

$$\text{cost}_{\text{bends}} = \sum_{L \in \mathcal{L}} \sum_{uv, vw \in L} \text{bend}(u, v, w)$$

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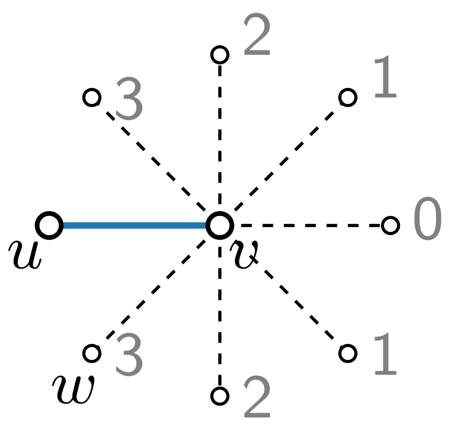

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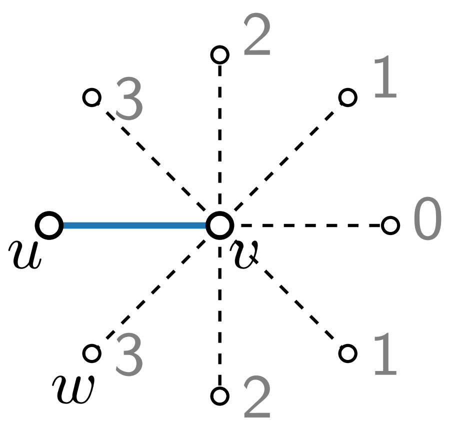

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To assign bend(u, v, w) correctly, we need to define some linear constraints based on the direction variables dir(u, v) and dir(v, w).

Overview MIP model

Constraints.

- linearization of all hard constraints
- $O(n^2)$ variables and constraints (due to planarity)

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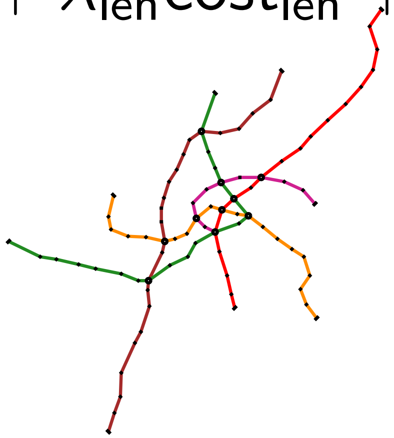

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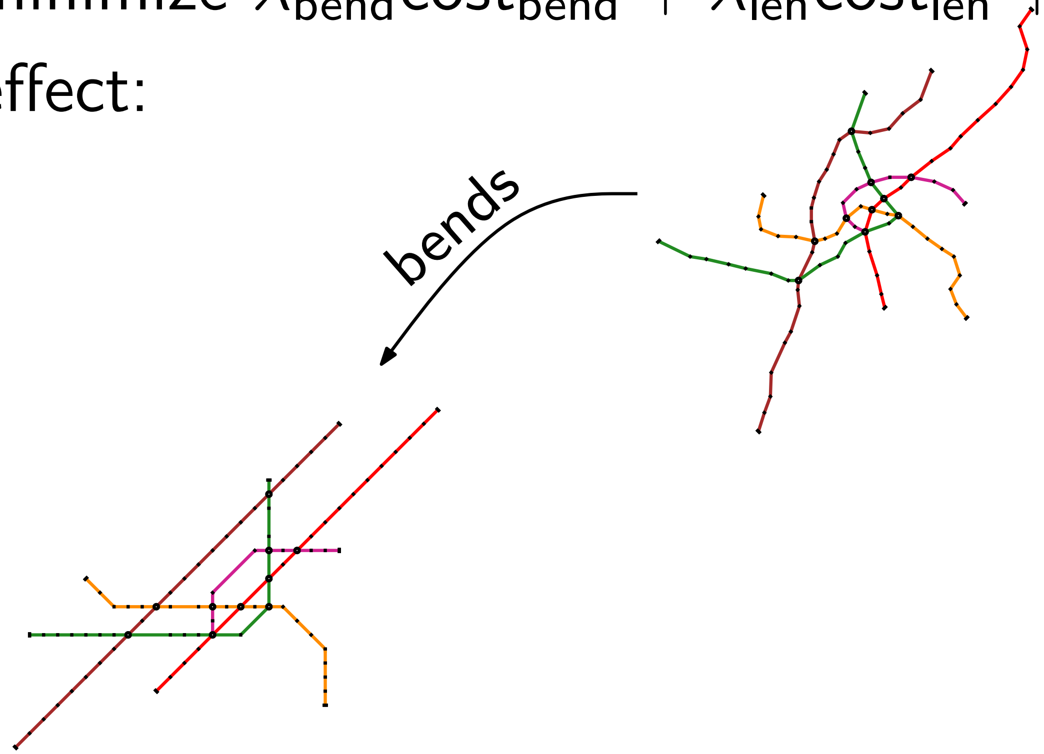

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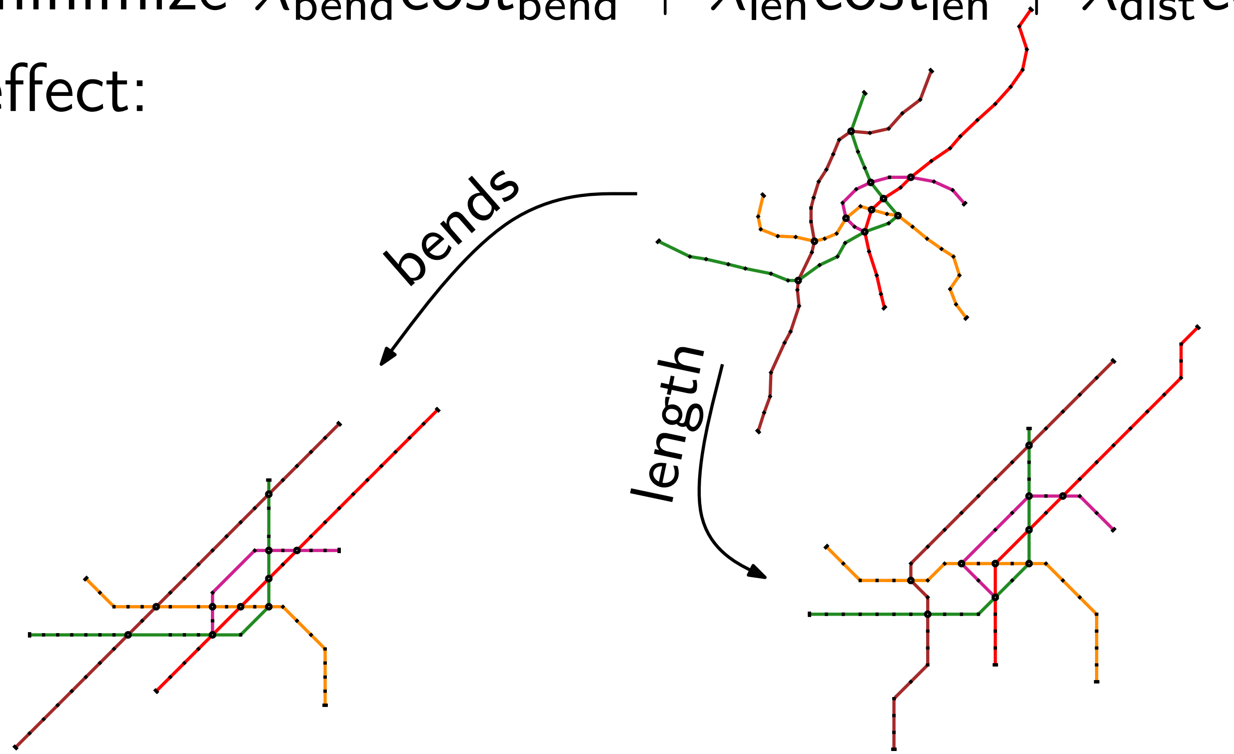

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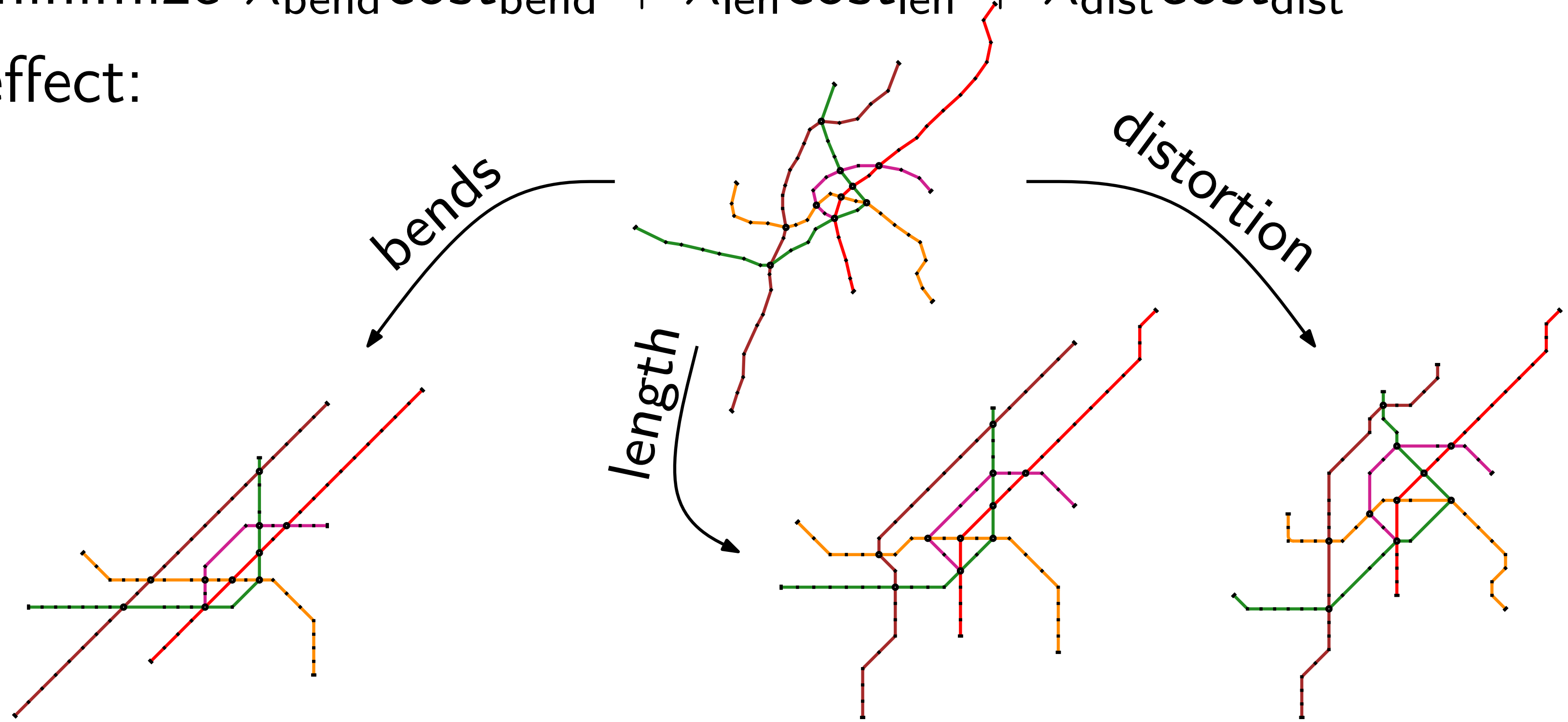

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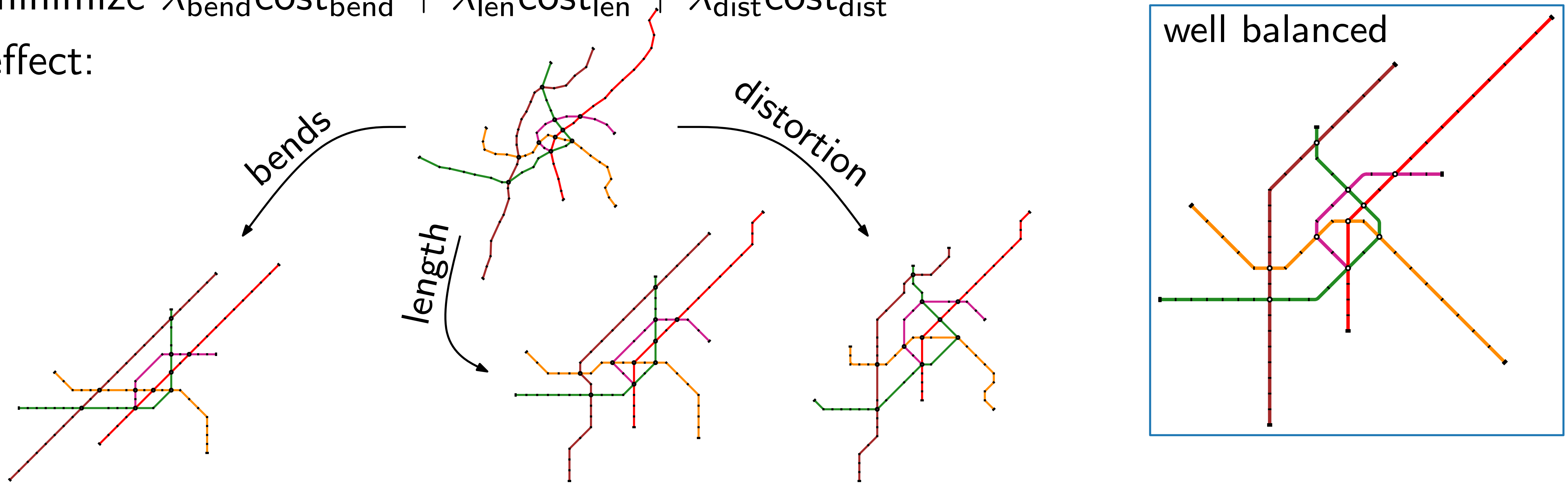
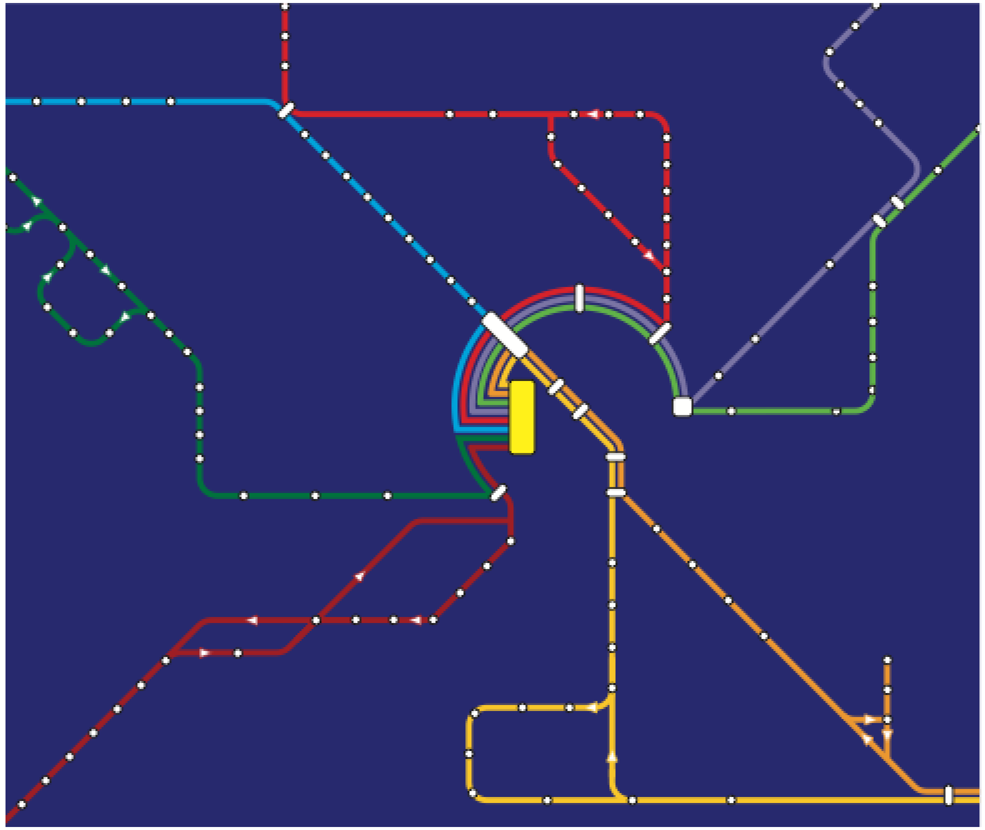


Station Labeling

- unlabeled map mostly useless



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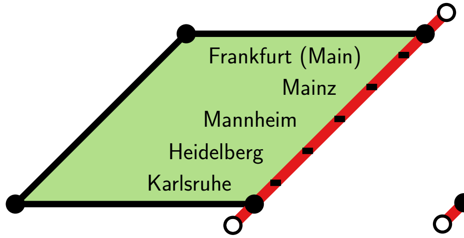
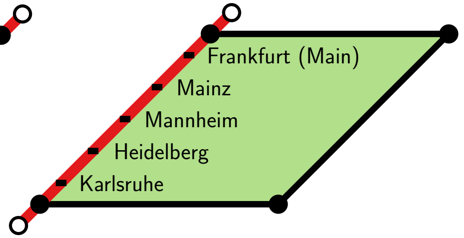
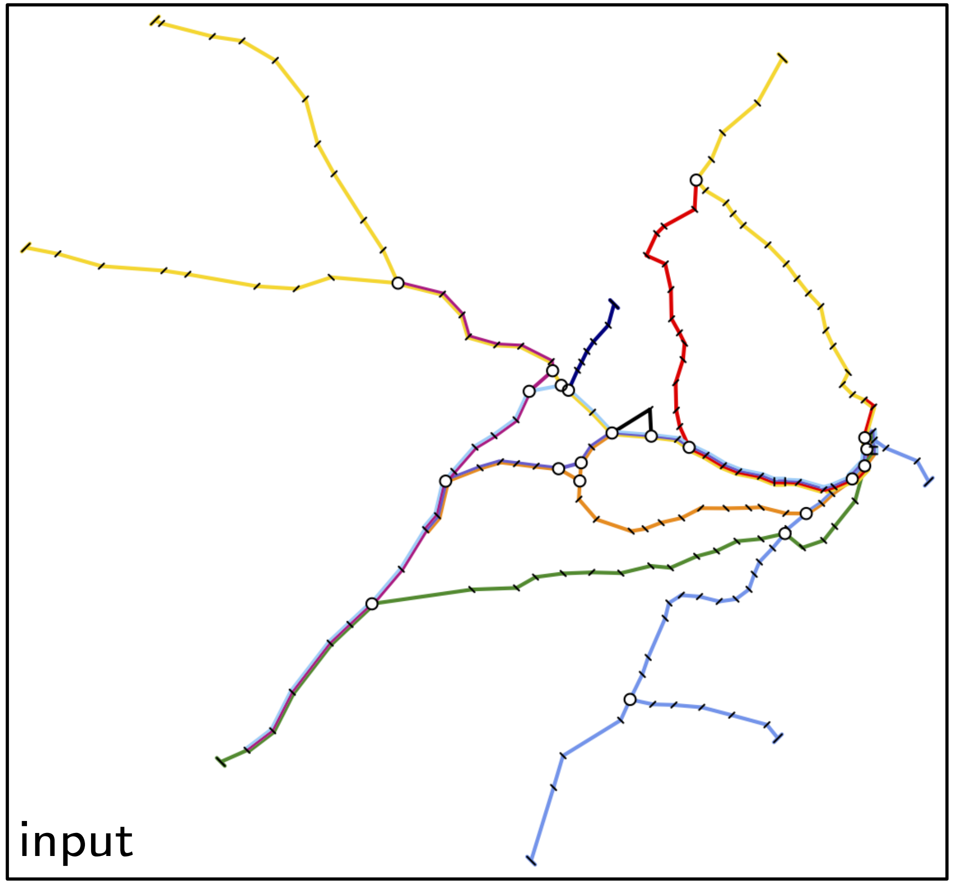
→ combine layout & labeling for optimal results!





- parallelogram as special metro line
- switching sides allowed

Example: Sydney &



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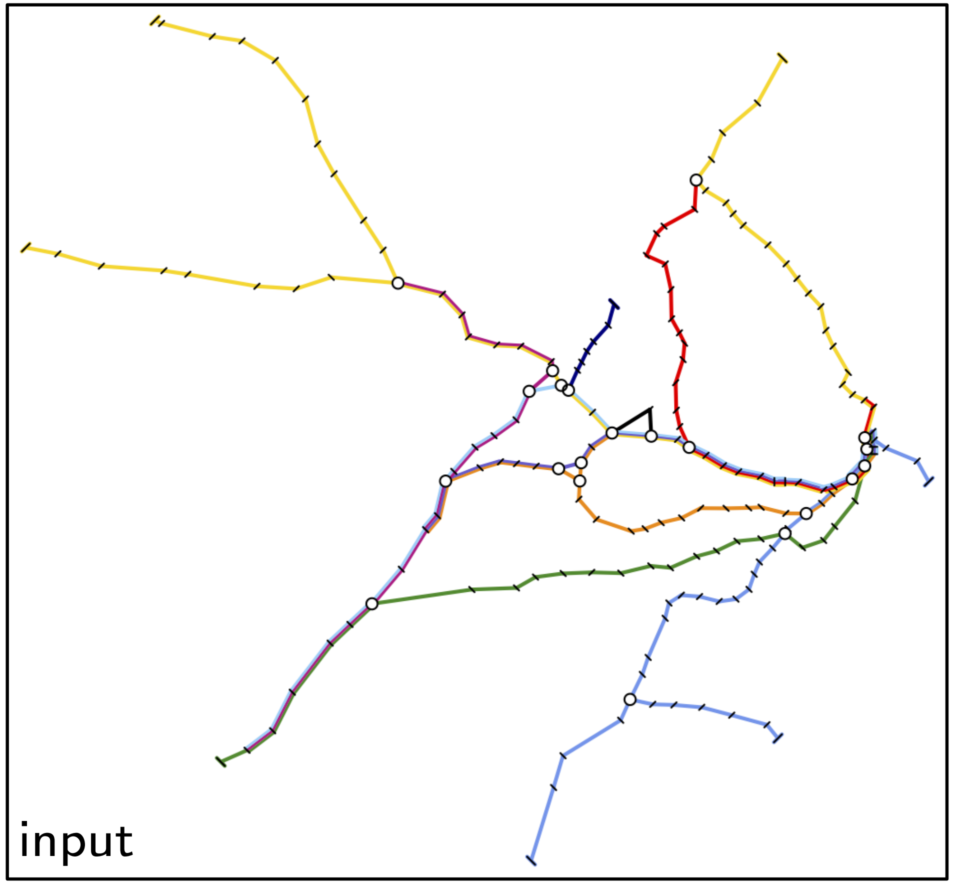

Input	$\lvert V \rvert$	$\lvert E \rvert$	fcs.	$\lvert \mathcal{L} \rvert$
full	174	183	11	10
reduced	88	97		
labeled	242	270	30	

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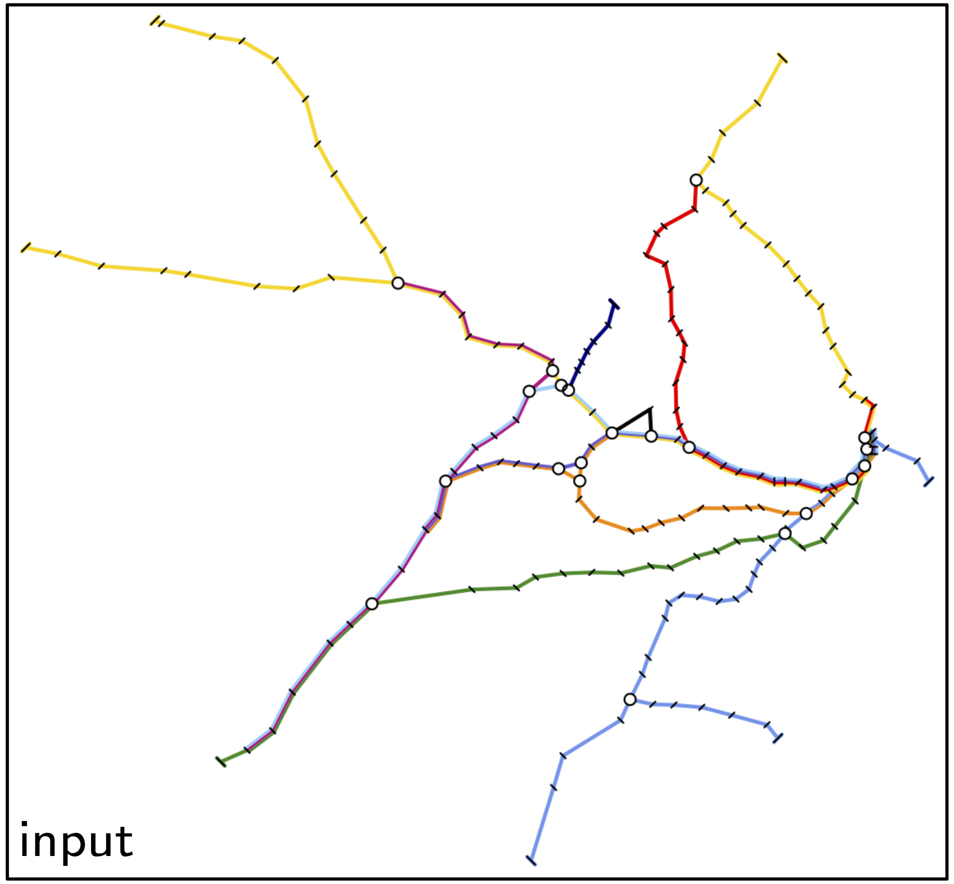

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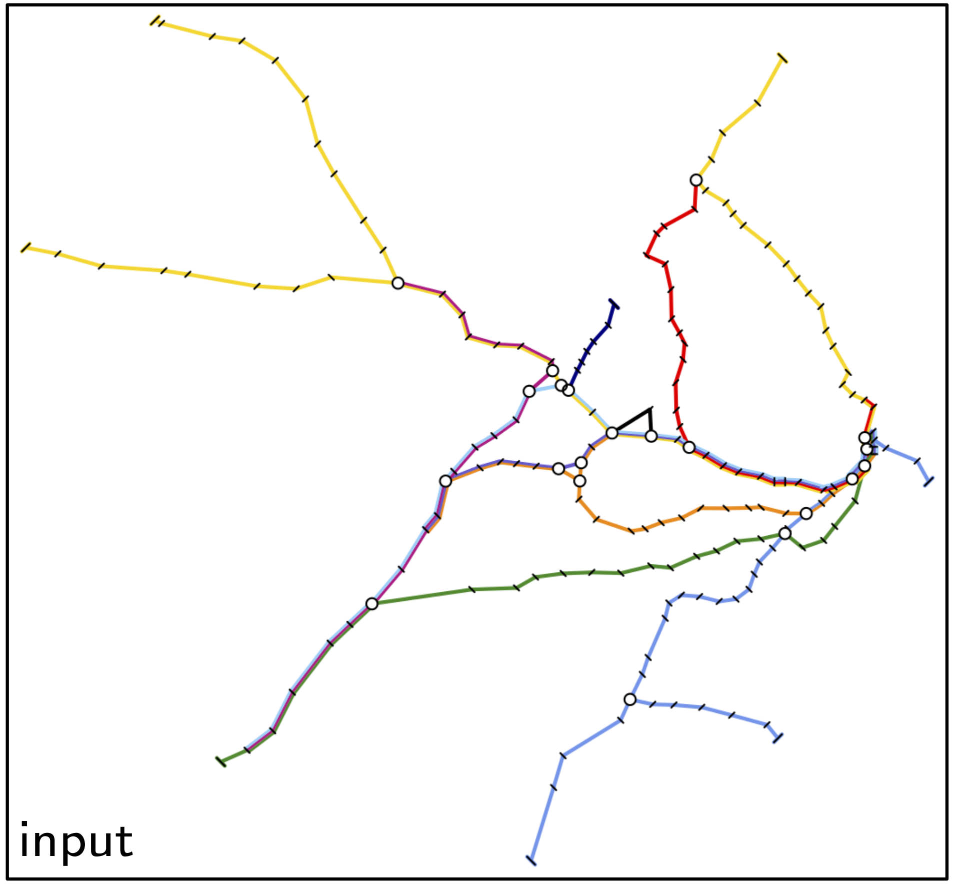

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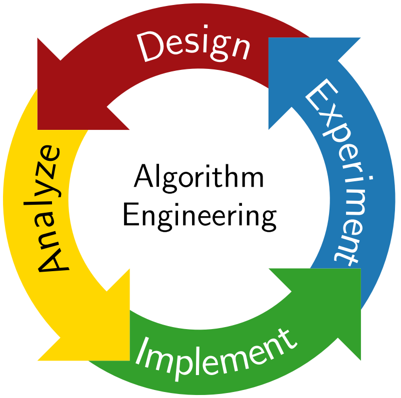

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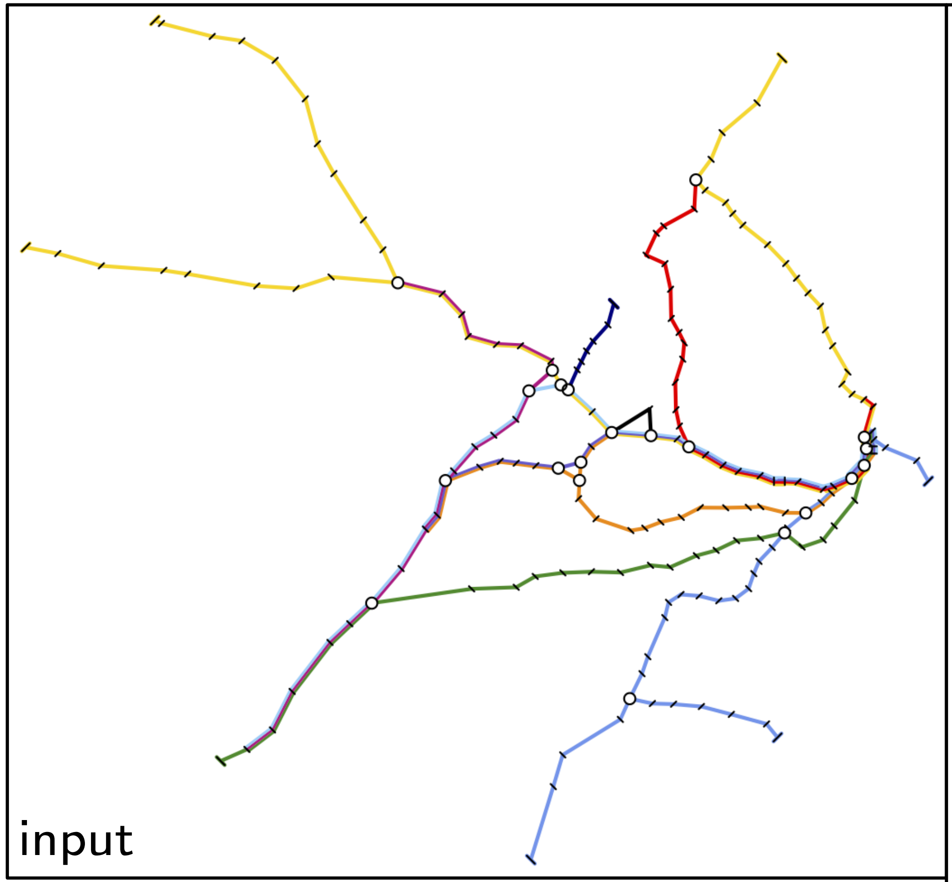

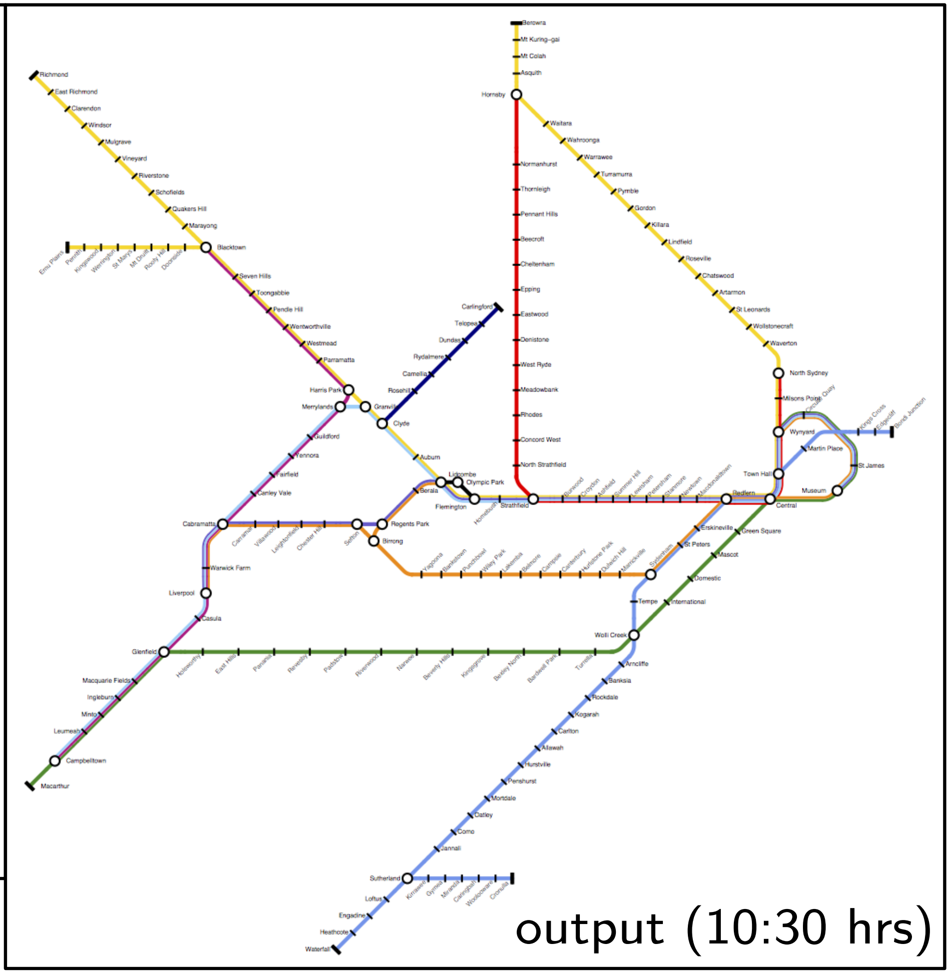

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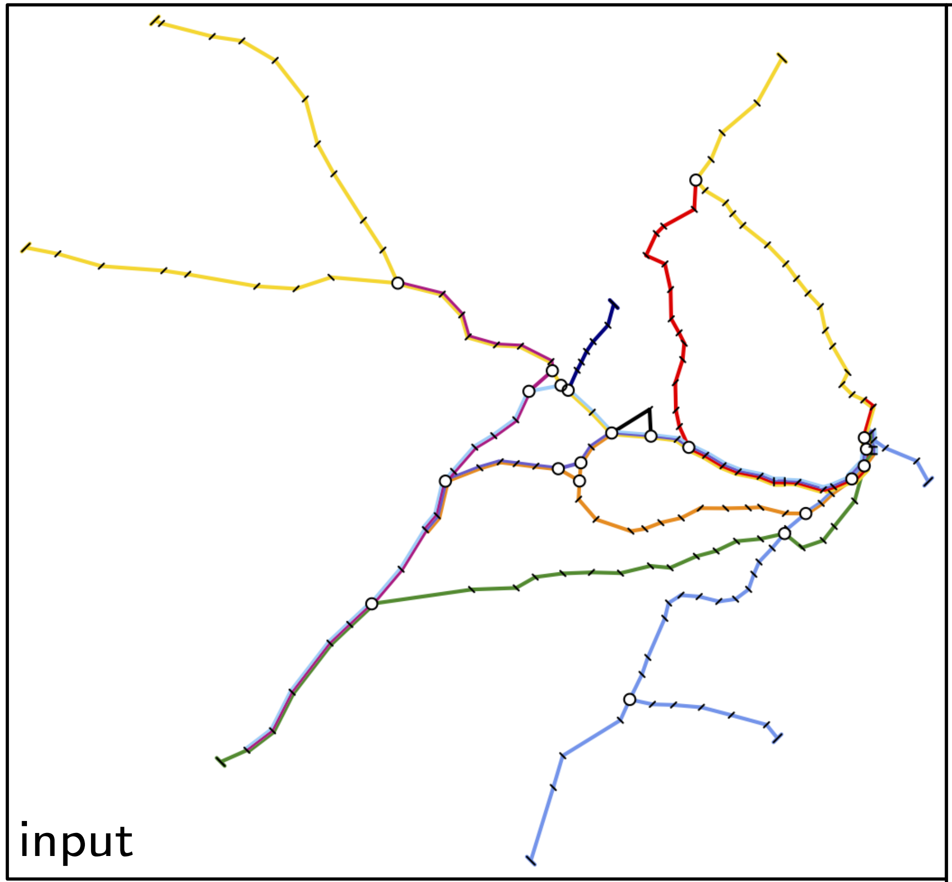
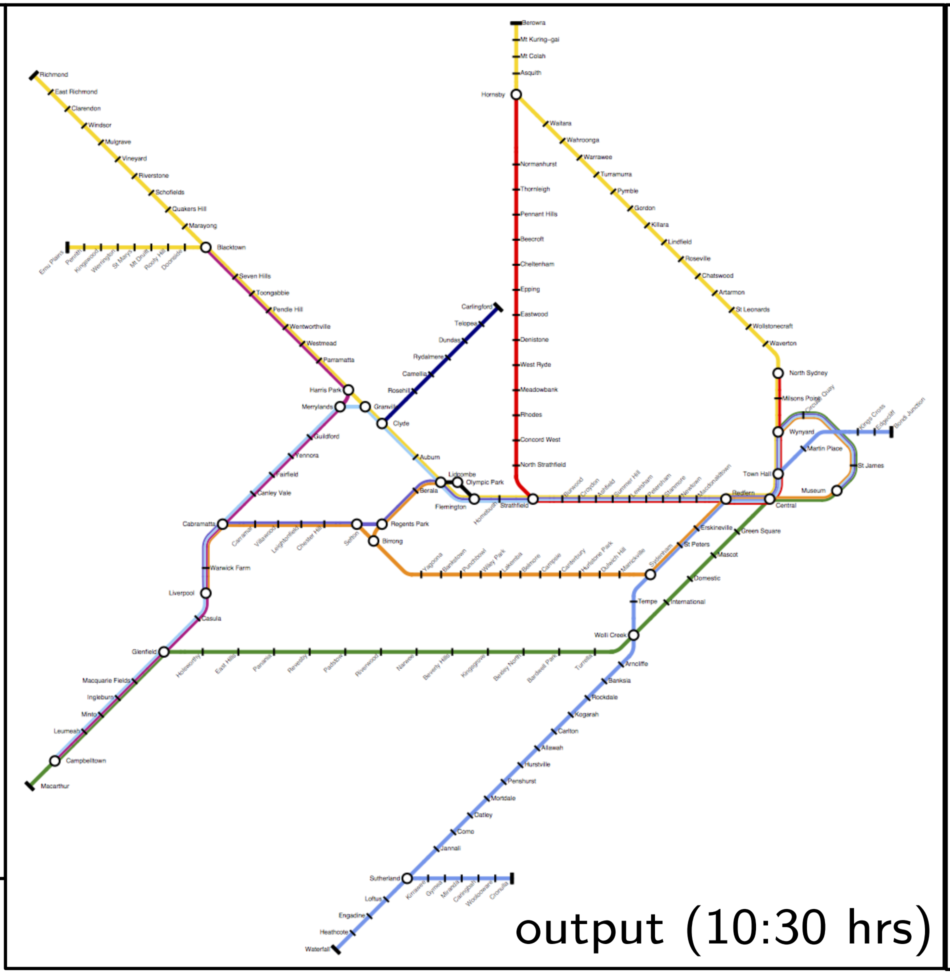
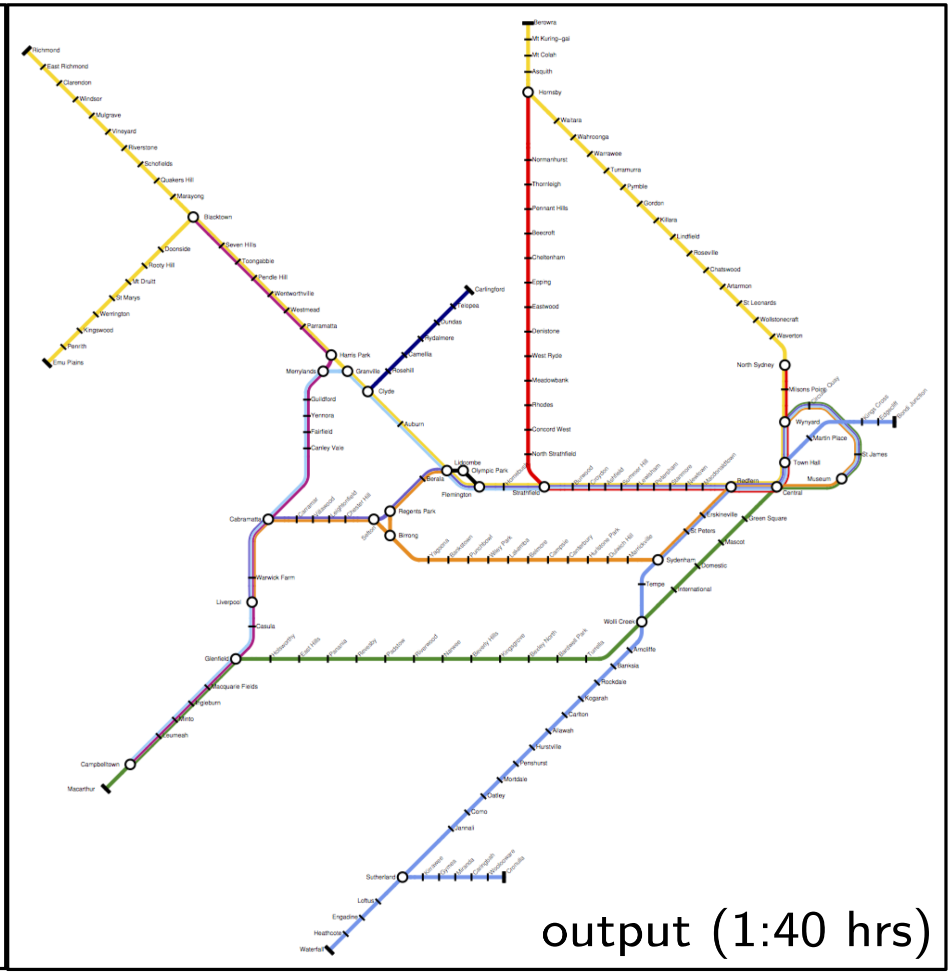


input

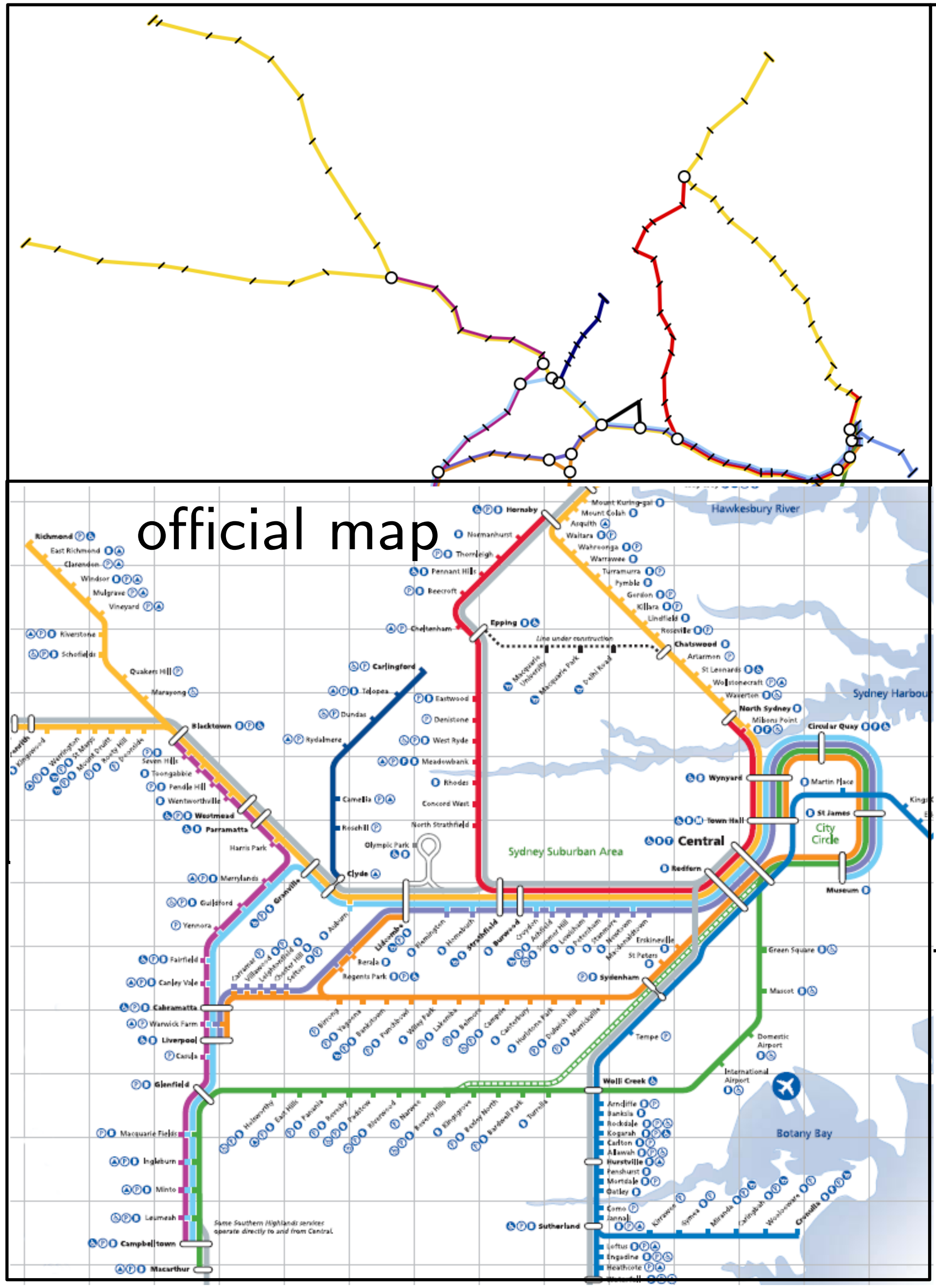


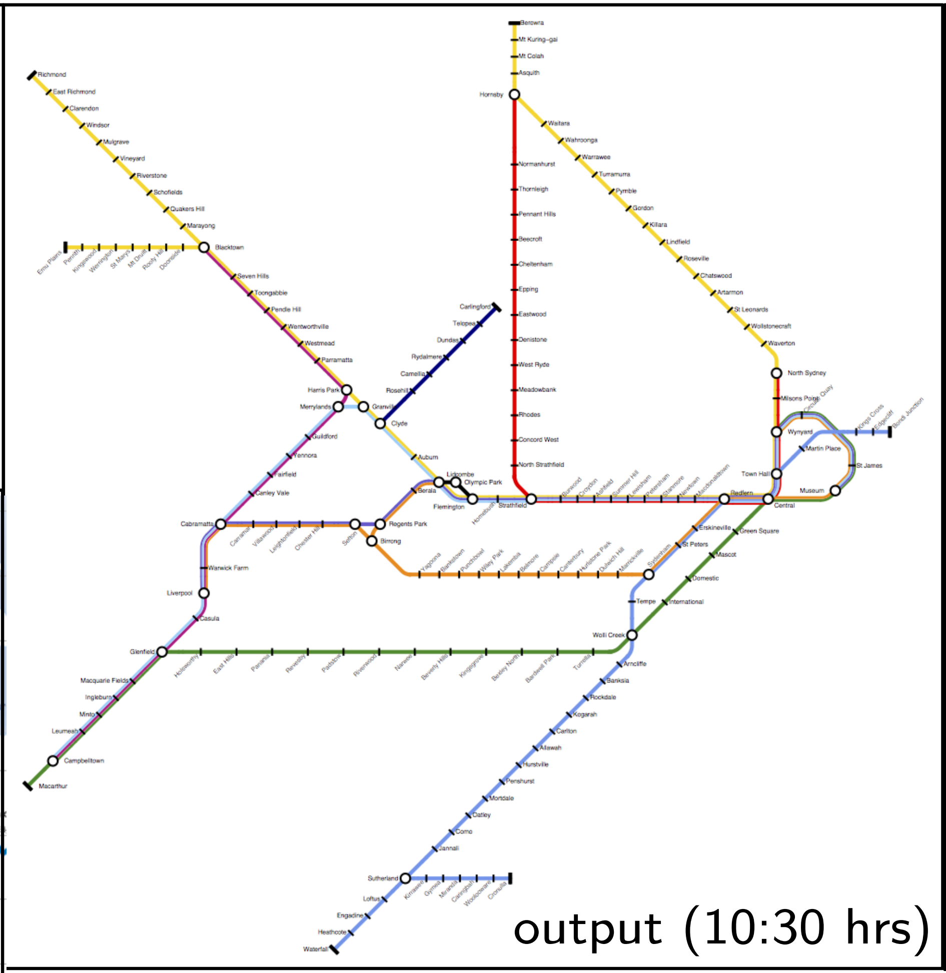
output (10:30 hrs)

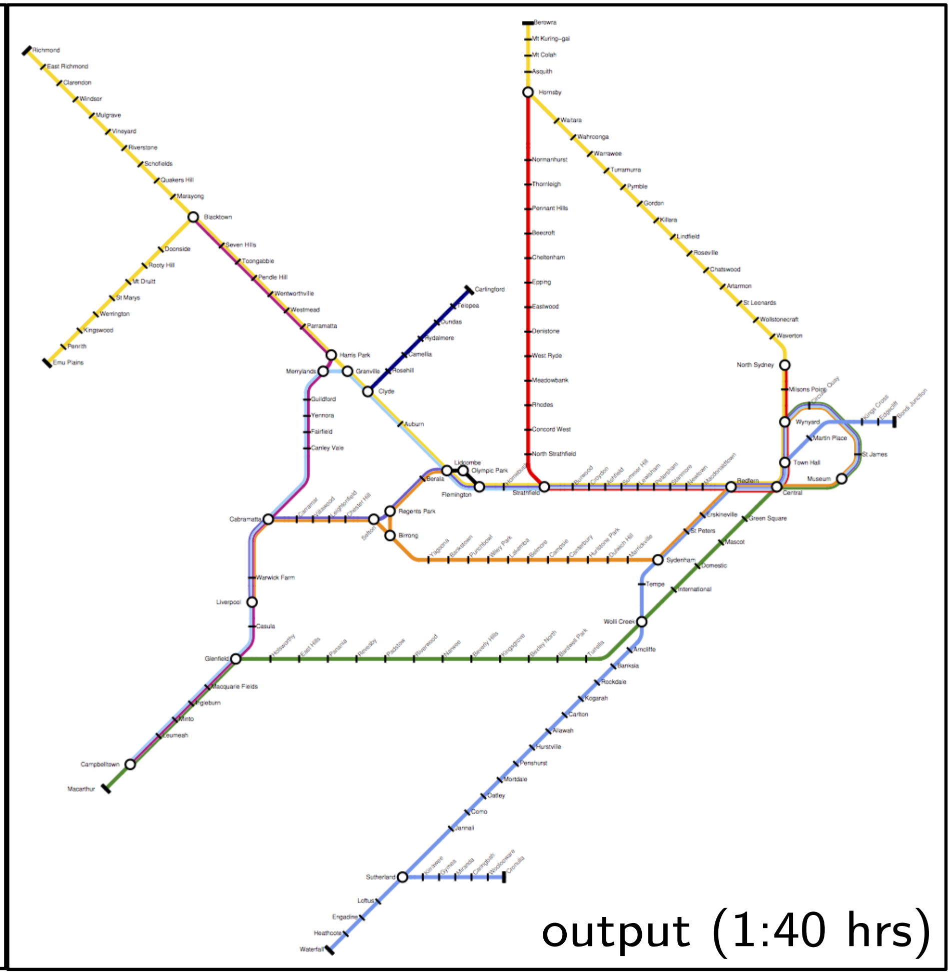


output (1:40 hrs)

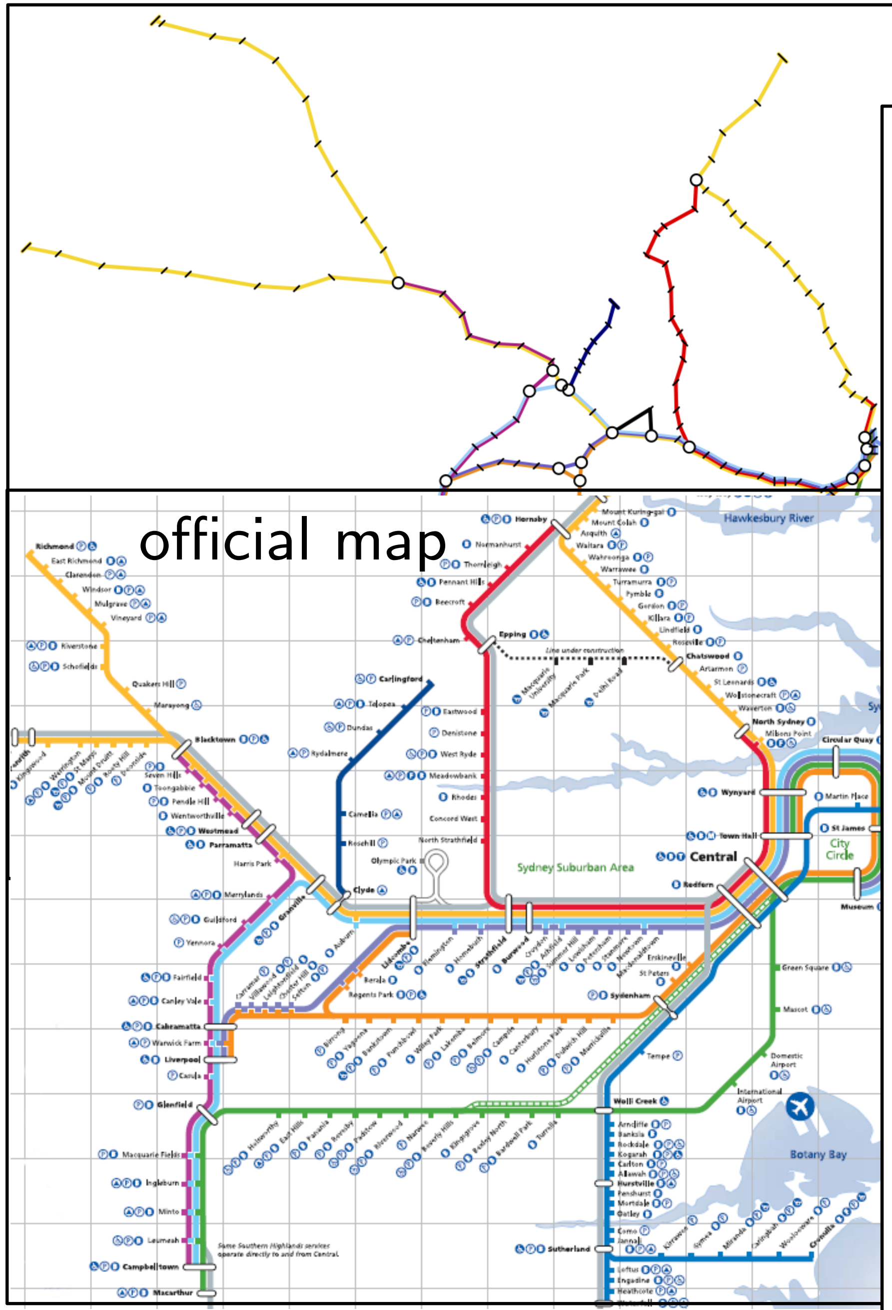
Example: Sydney &

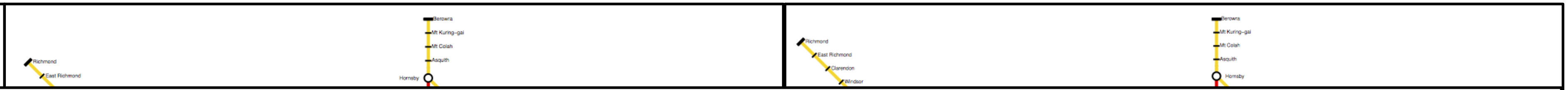

official map


output (10:30 hrs)

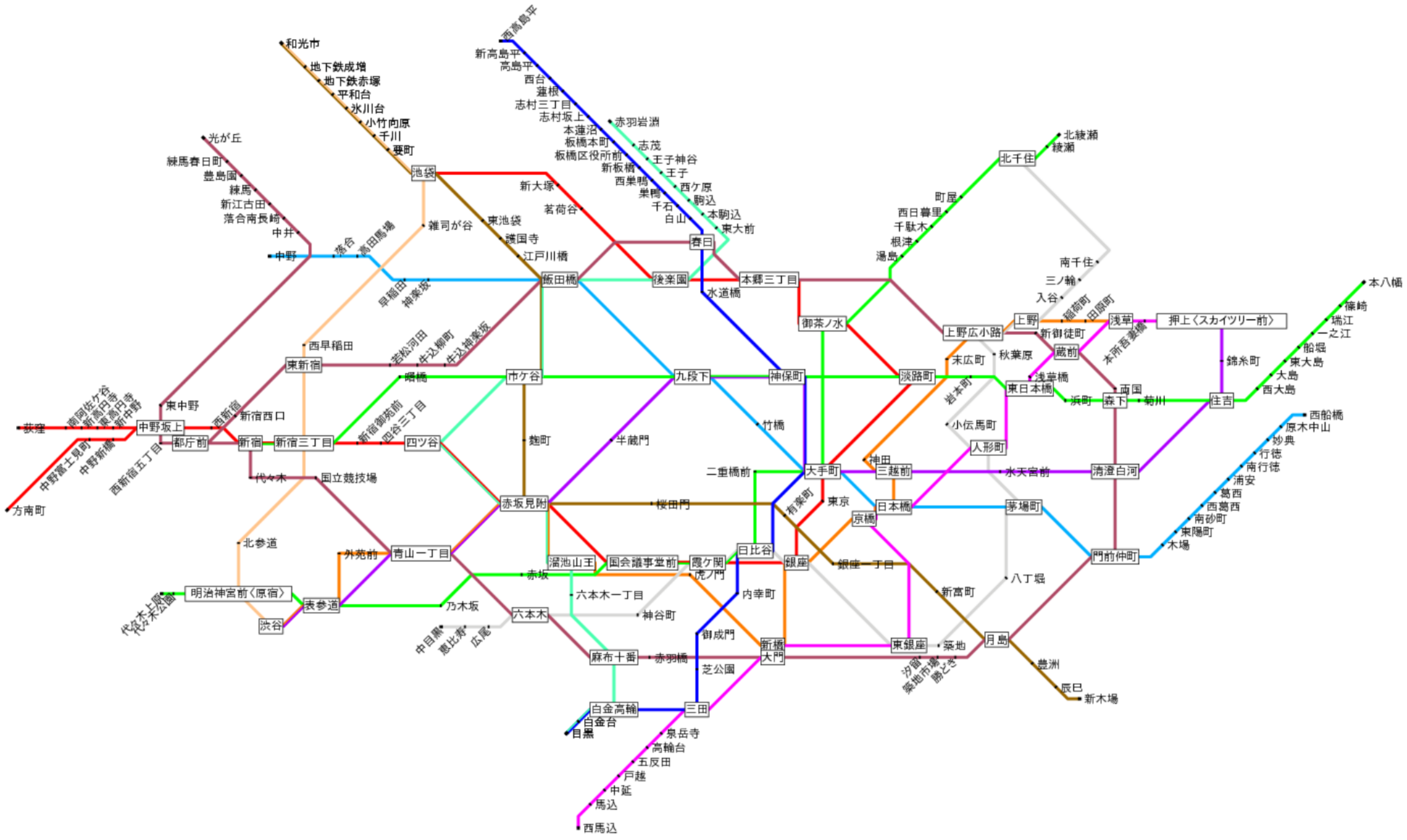

output (1:40 hrs)

Example: Sydney & Tokyo





Tokyo



- multi-phase MIP model [Onda et al. 2018]

5 hrs

Mixed-Integer Programming – Discussion

Pros.

Cons.

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- solutions only as good as the model specification

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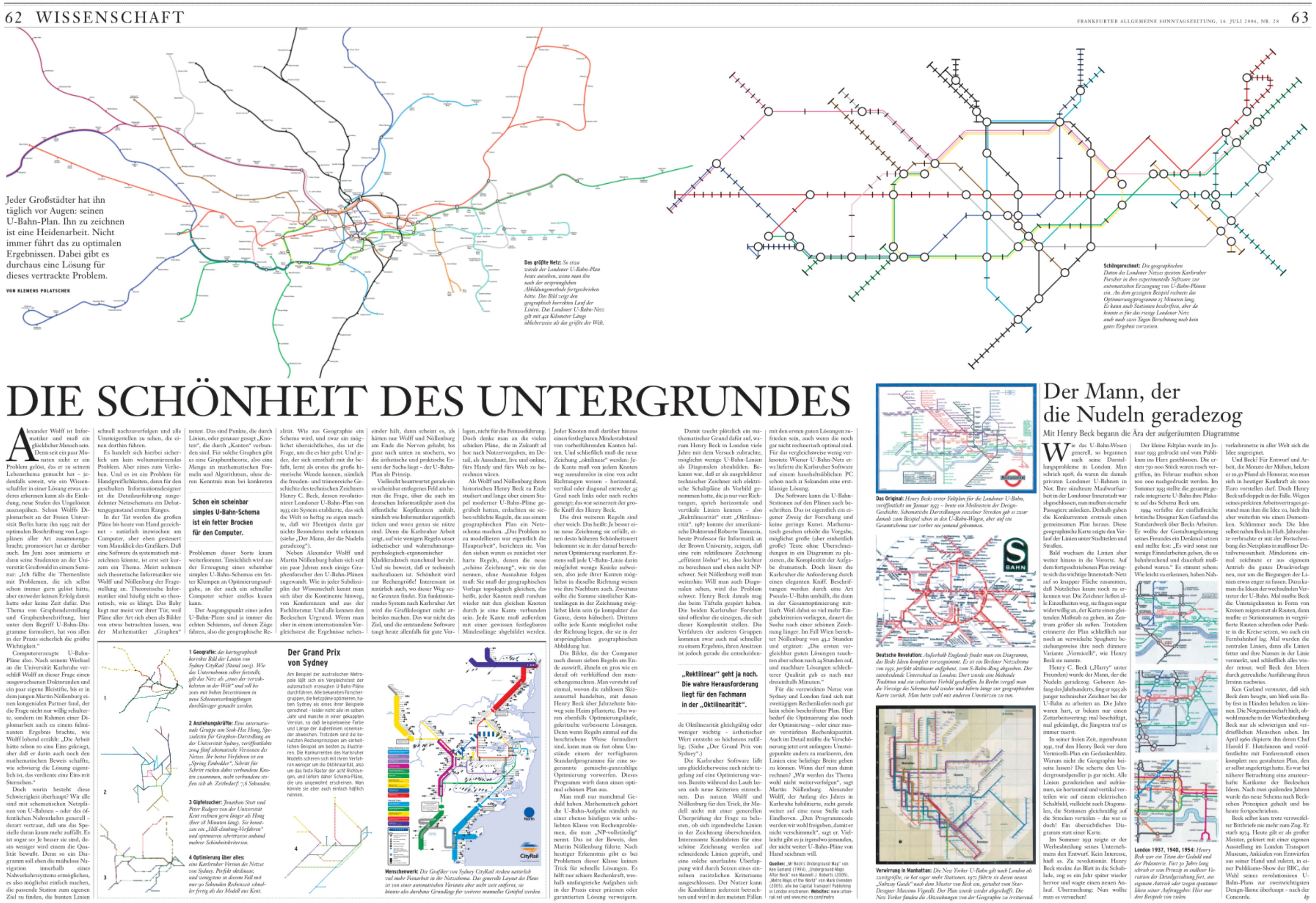
62 WISSENSCHAFT

FRANKFURTER ALLGEMEINE SONNTAGSZEITUNG, 16. JULI 2006, NR. 28 63

Jeder Großstädter hat ihn täglich vor Augen: seinen U-Bahn-Plan. Ihn zu zeichnen ist eine Heidenarbeit. Nicht immer führt das zu optimalen Ergebnissen. Dabei gibt es durchaus eine Lösung für dieses vertrackte Problem.

VON KLEMENS POLATSCHEK

DIE SCHÖNHEIT DES UNTERGRUNDES

Schon ein scheinbar simples U-Bahn-Schema ist ein fetter Brocken für den Computer.

Der Grand Prix von Sydney

„Rektilinear" geht ja noch. Die wahre Herausforderung liegt für den Fachmann in der „Oktilinearität".

Der Mann, der die Nudeln geradezog

Mit Henry Beck begann die Ära der aufgeräumten Diagramme

Least-Squares Schematization

[Wang, Chi '11]

Idea.

- model layout problem as minimization of a set of (squared) energy terms
- variables for vertex positions and edge slopes
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  - compute topologically correct non-octilinear layout optimizing angular resolution **(R5)**, uniform edge lengths **(R7)**, displacement **(R6)**
  - octilinearly discretize edge orientations **(R2)** by extra energy term
  - optimize label placement by energy minimization in fixed layout
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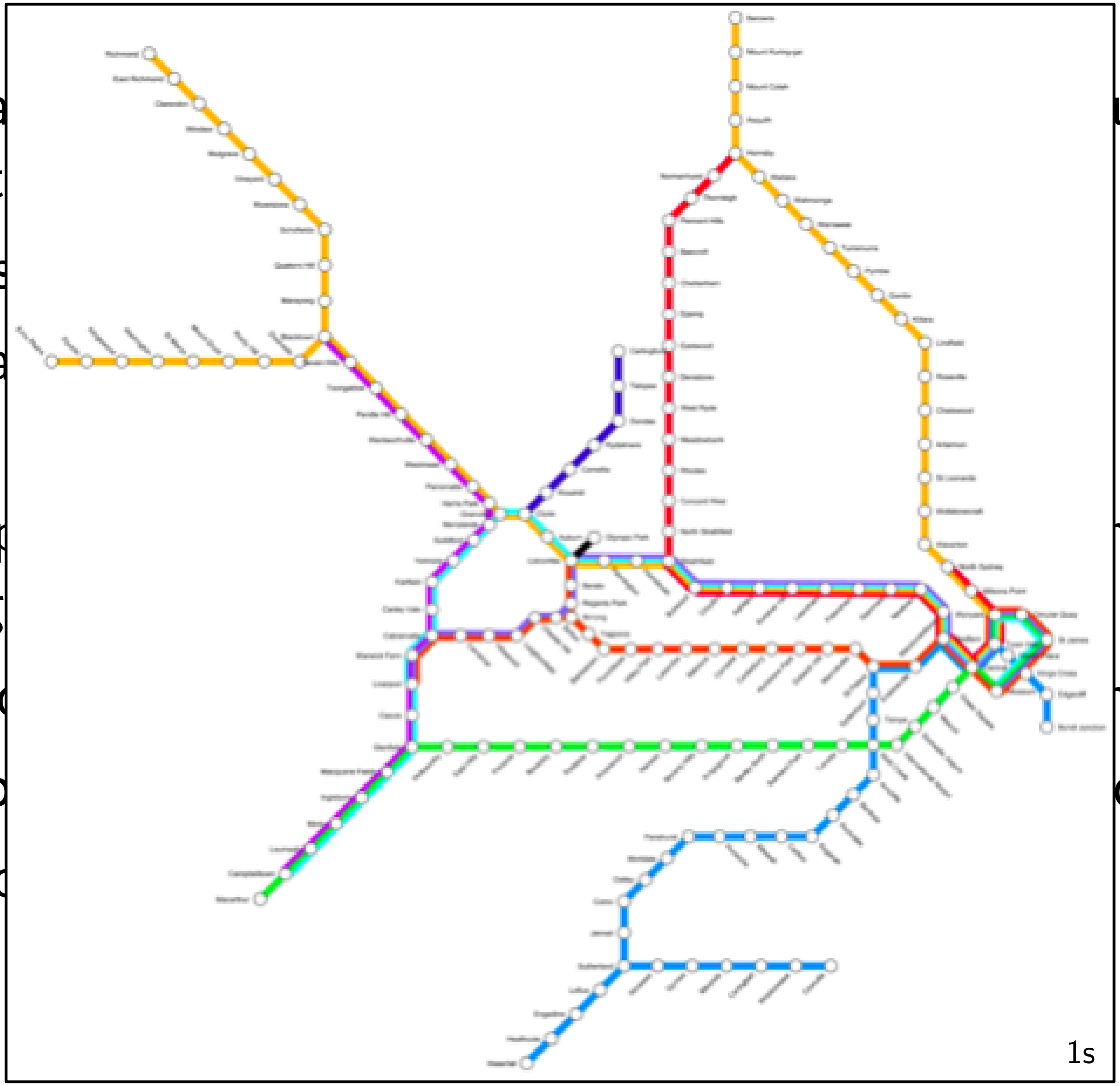

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Discussion.
- very fast method for good quality layouts
- no guarantee on constraints unless final energy is zero

Summary

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- edge bundles and large vertices

Literature

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