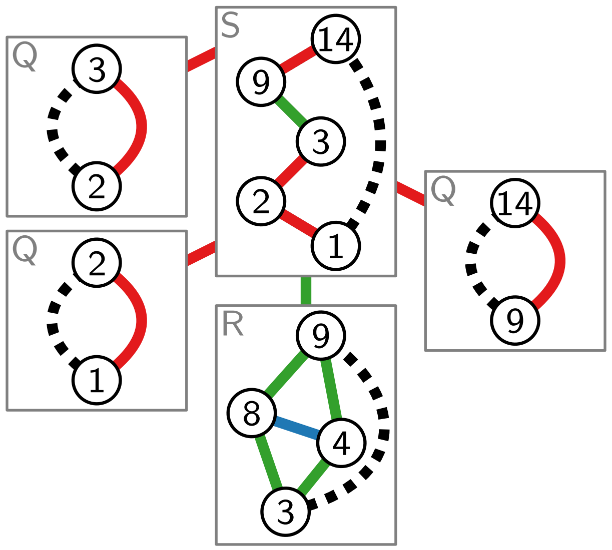
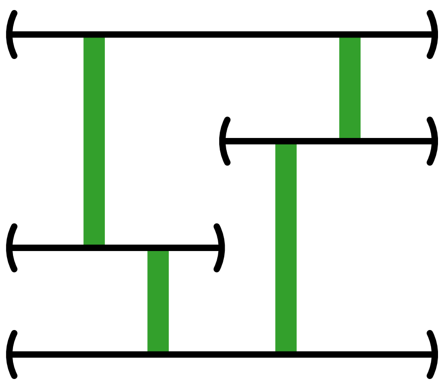
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



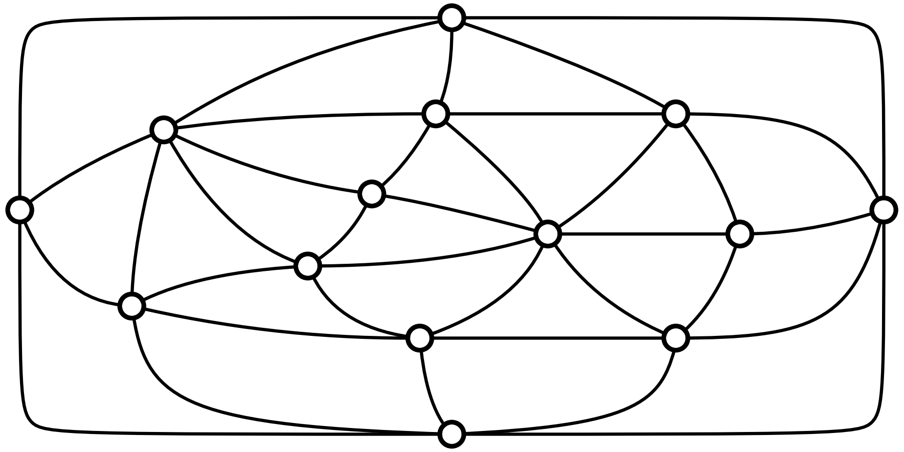
Part I:
Problem Definition

Jonathan Klawitter



Partial Representation Extension Problem

Let $G = (V, E)$ be a graph.



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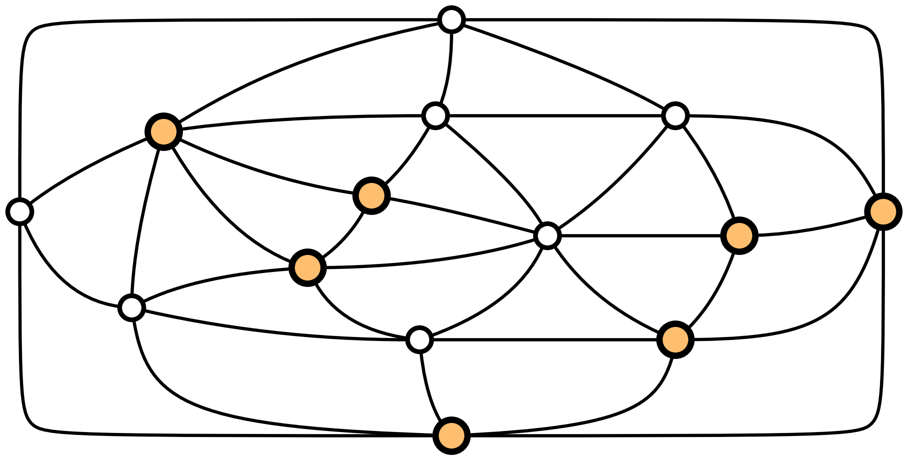

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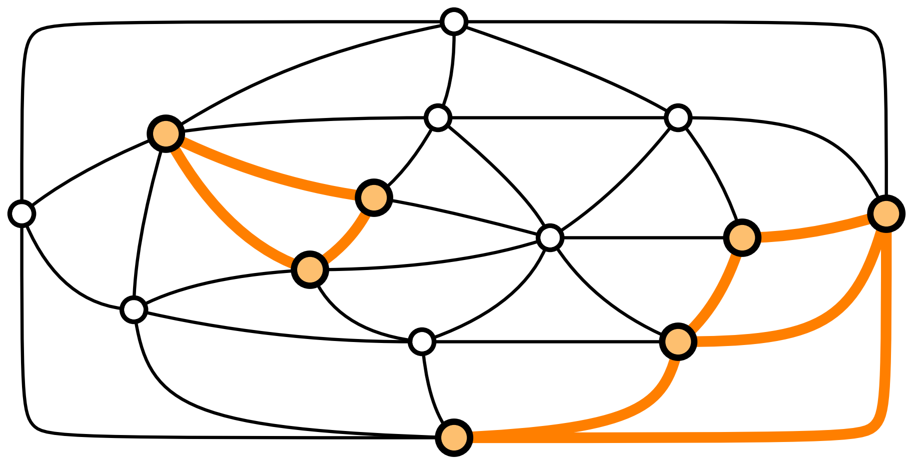

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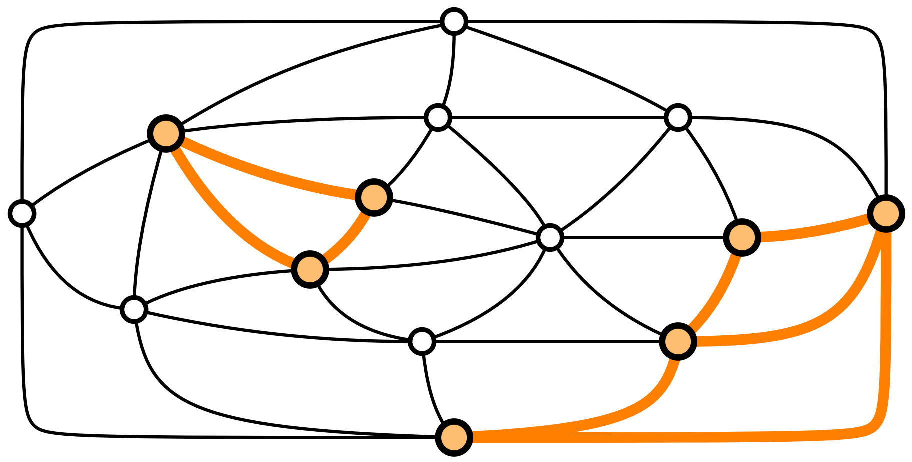

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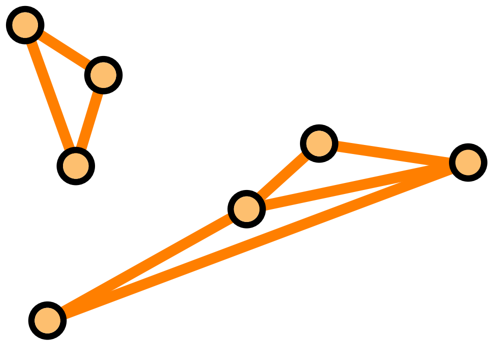

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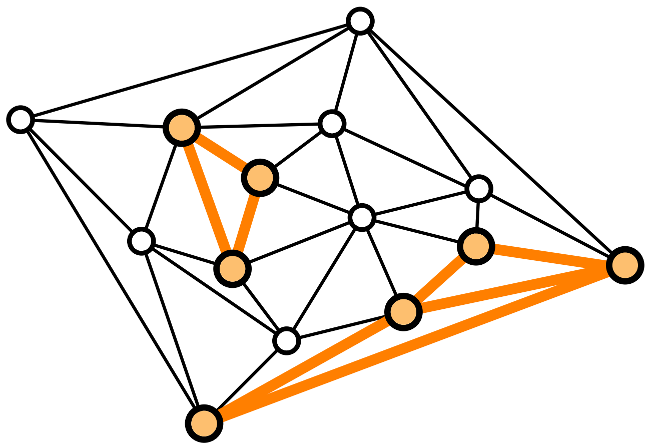

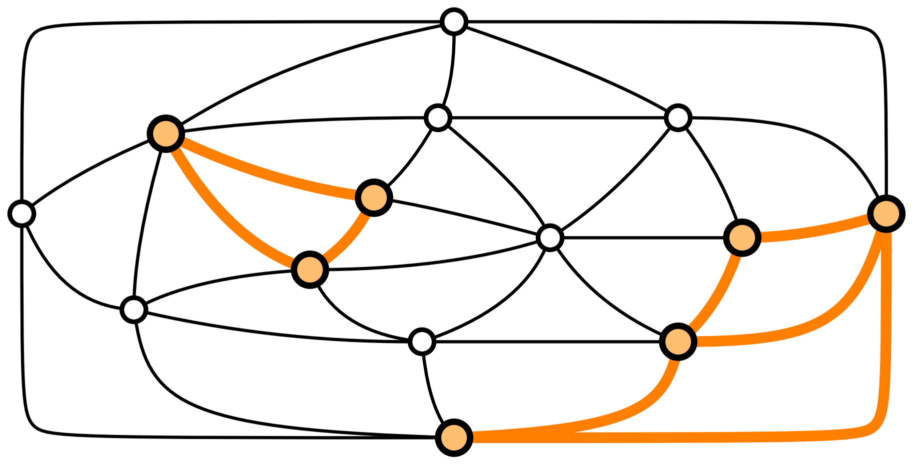

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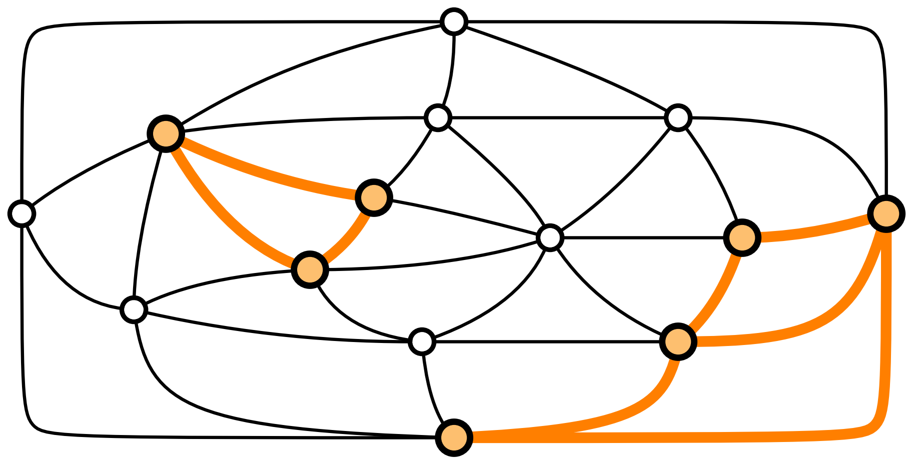

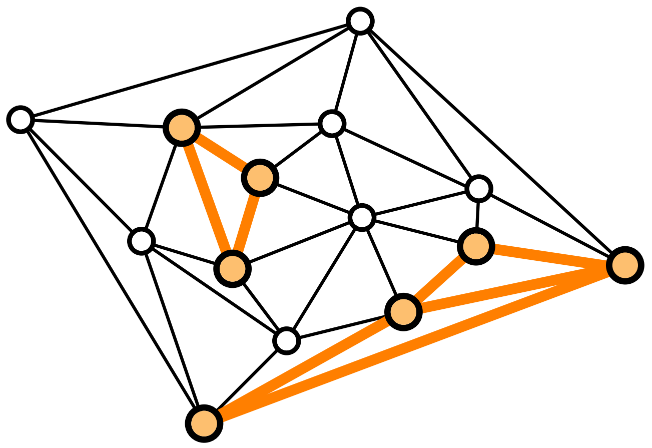

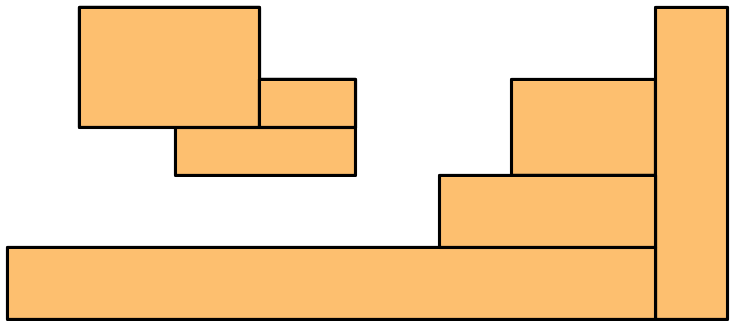

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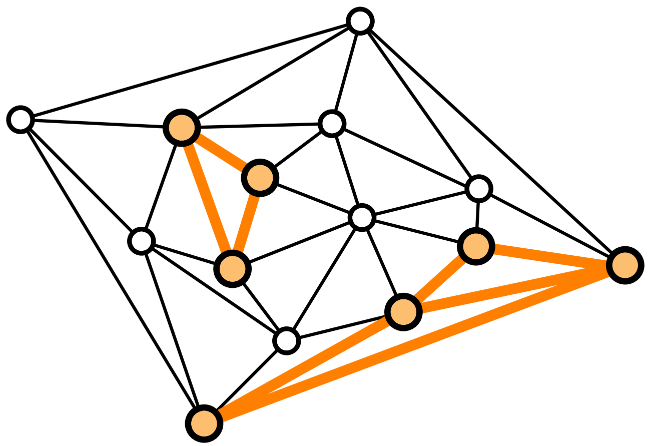

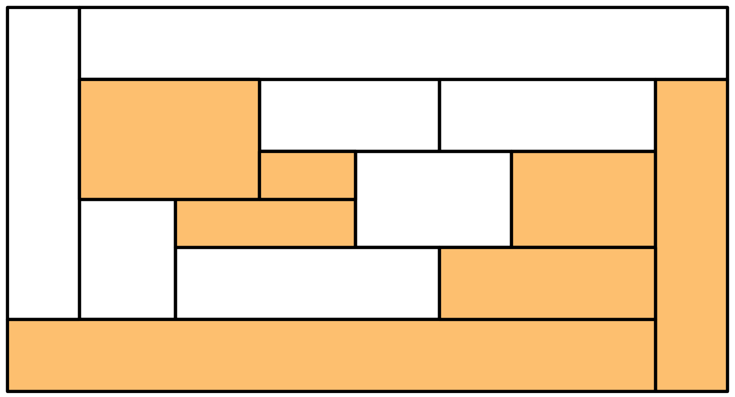

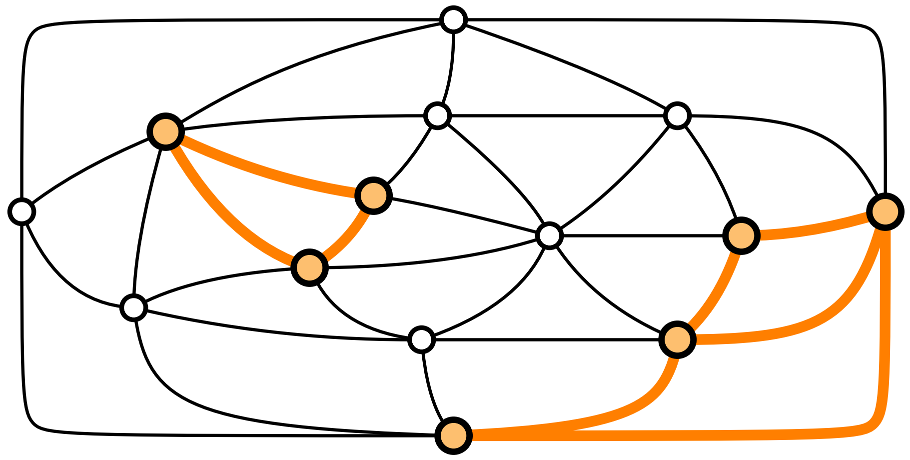

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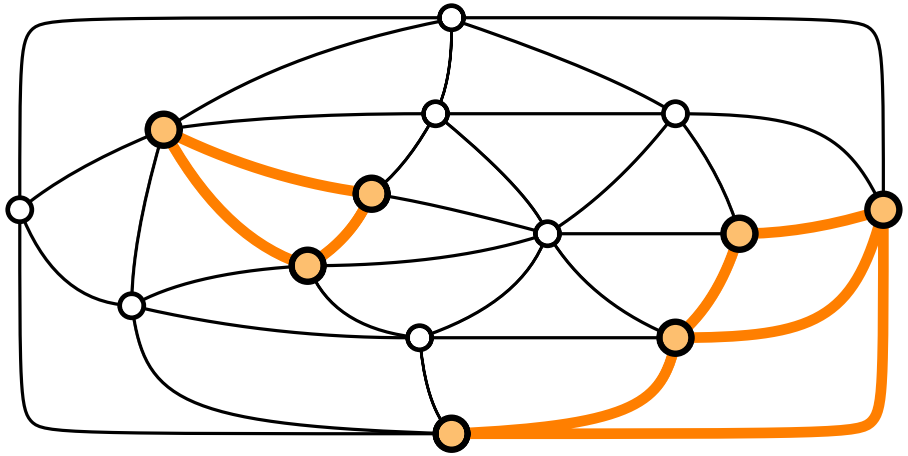

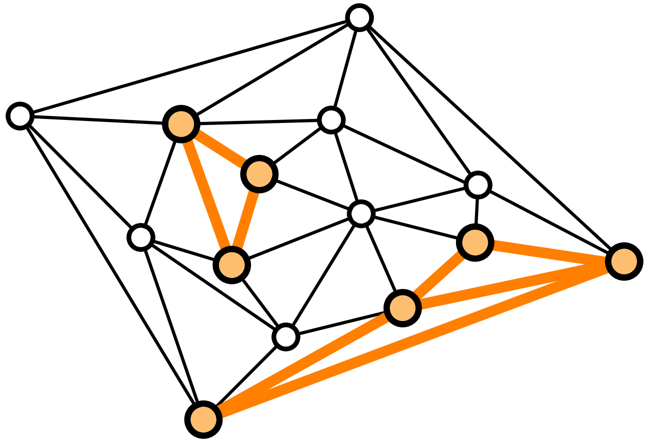

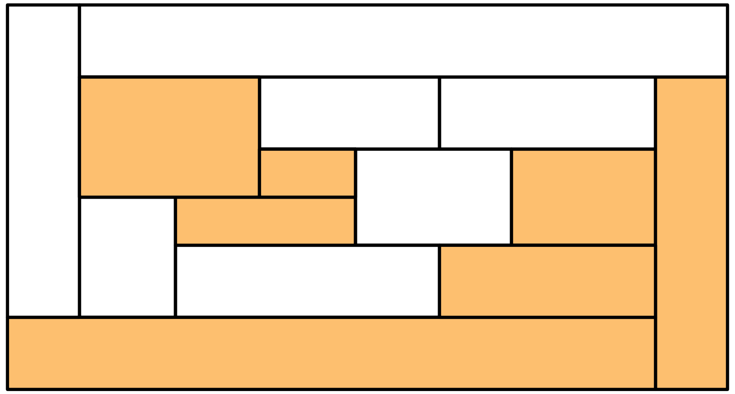

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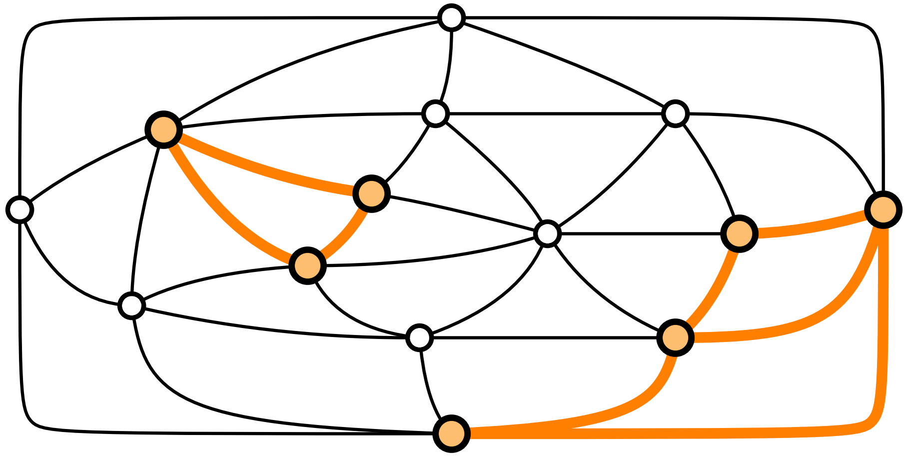

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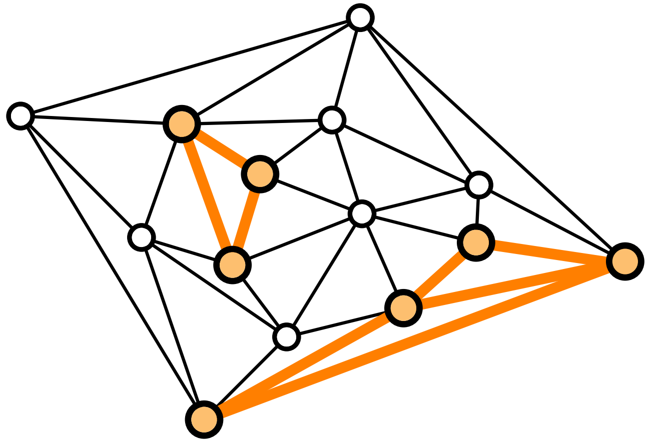
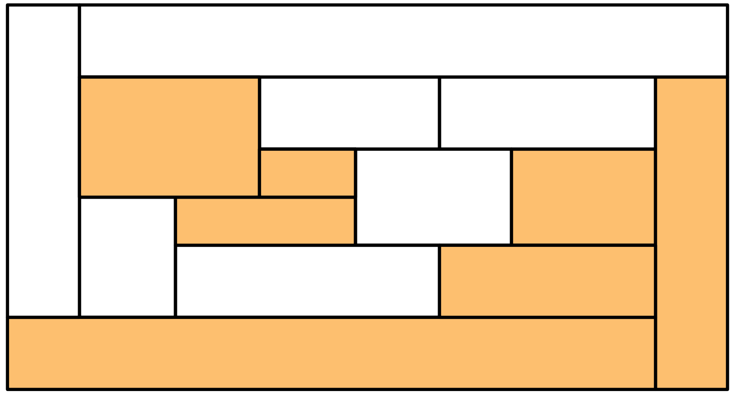
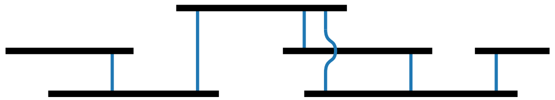




Polytime for:

- (unit) interval graphs



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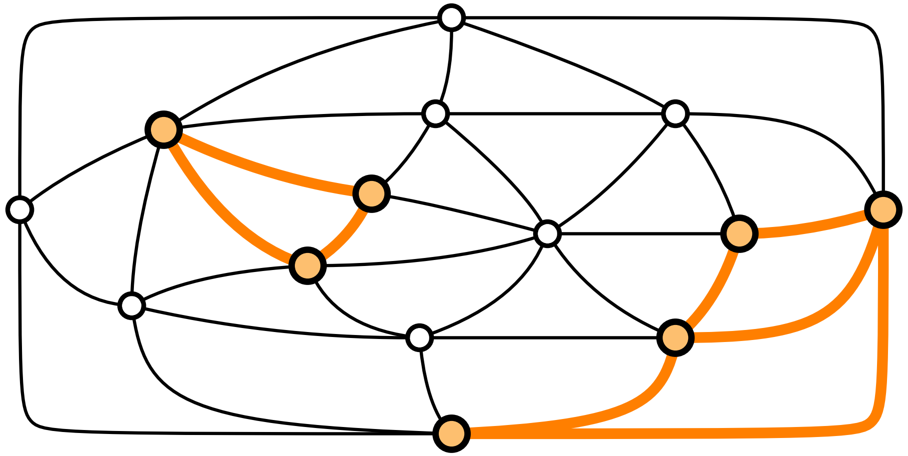

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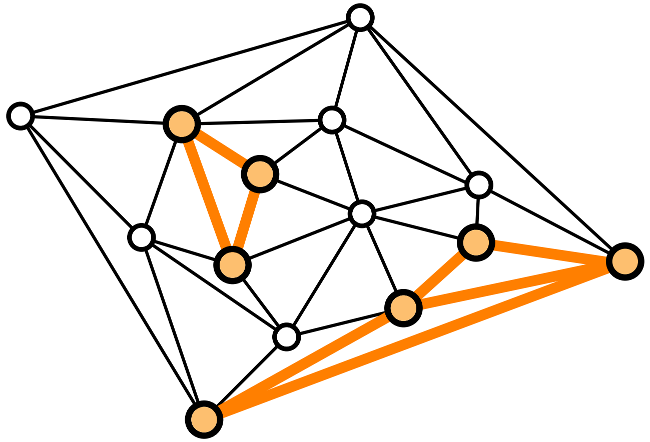

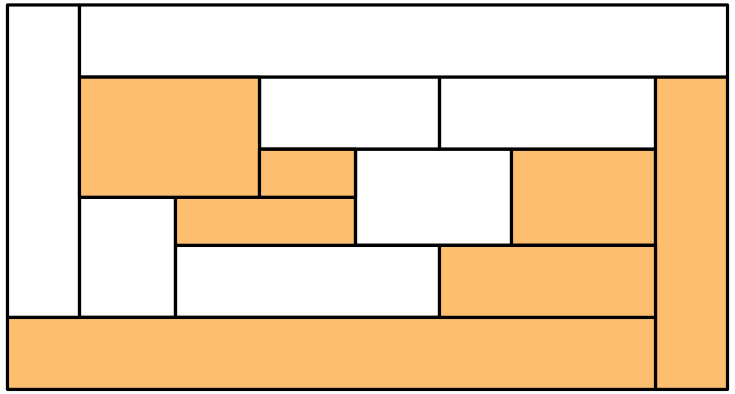

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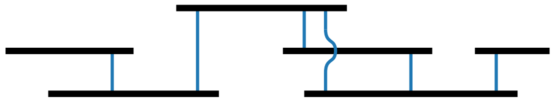

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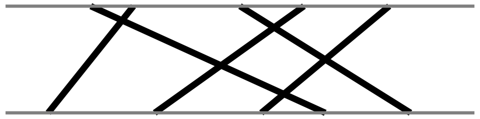

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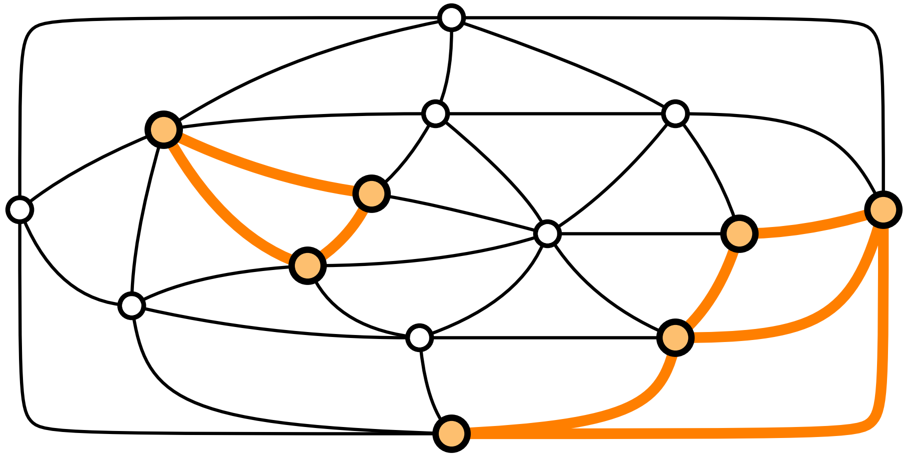

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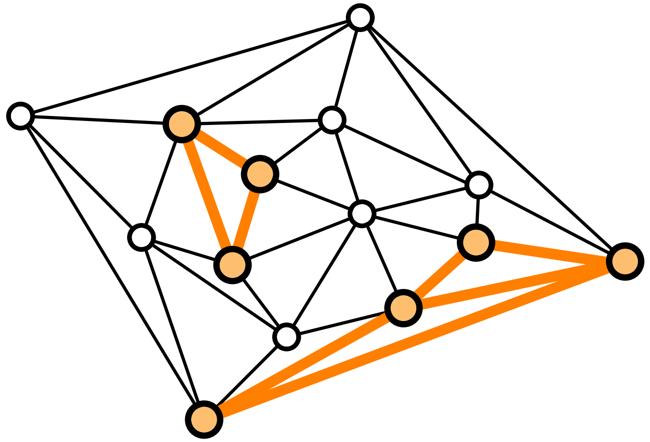

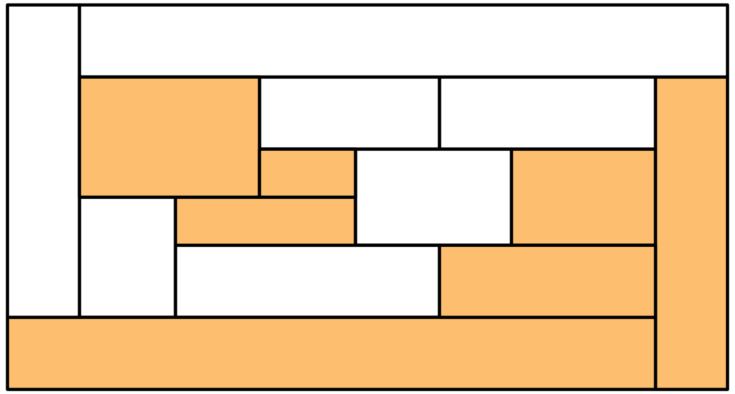

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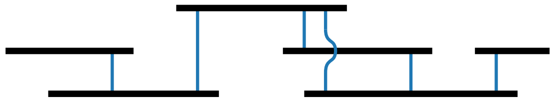

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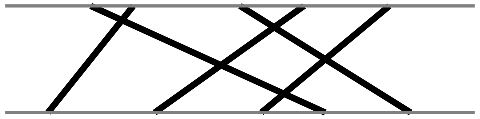

- circle graphs

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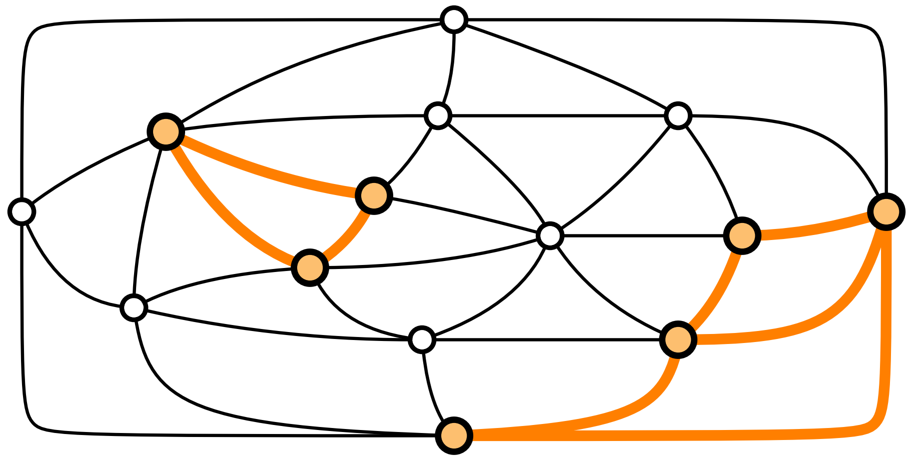

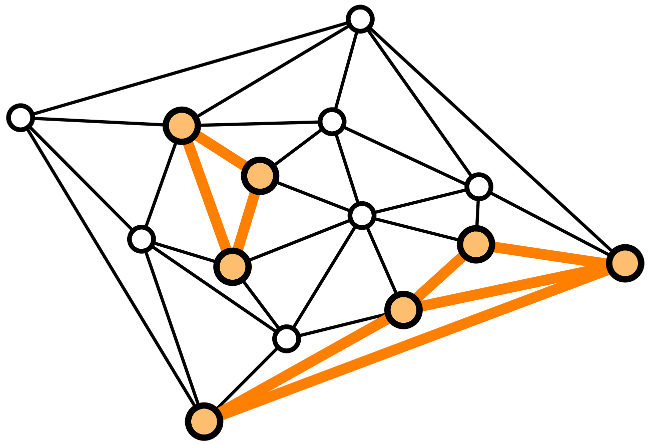

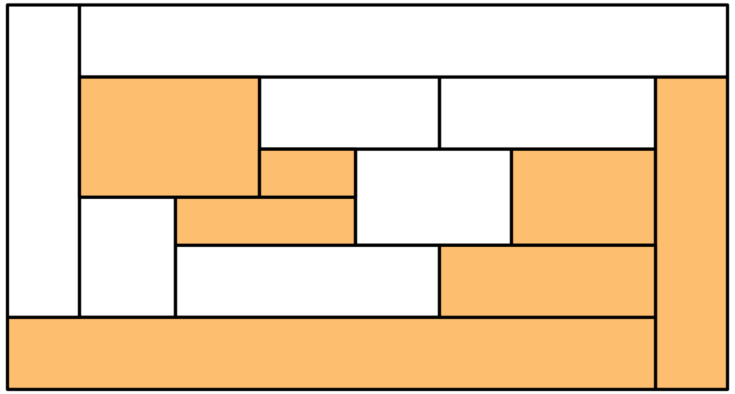

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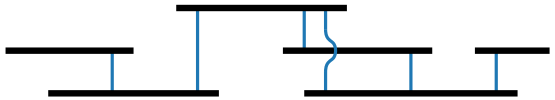

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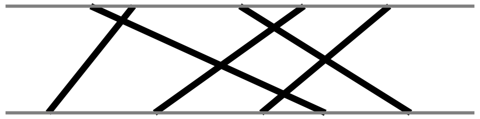

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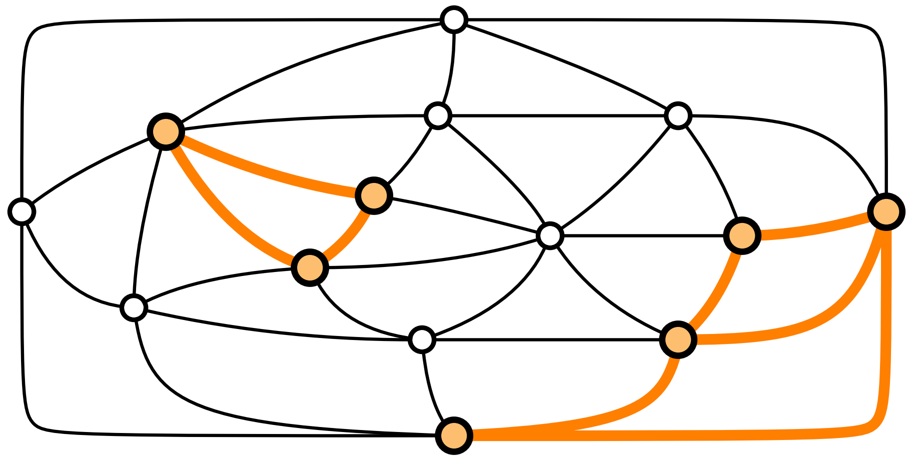

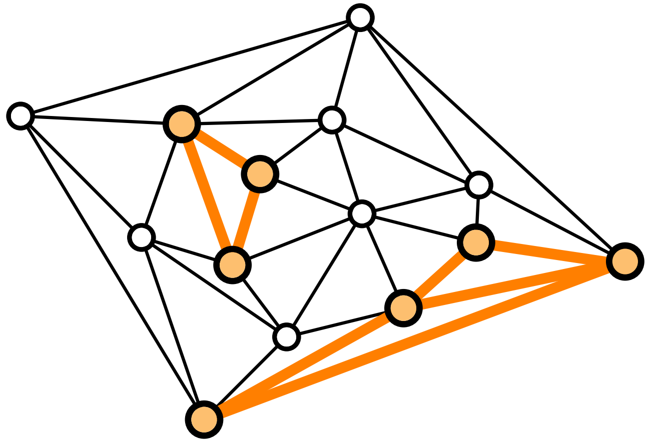

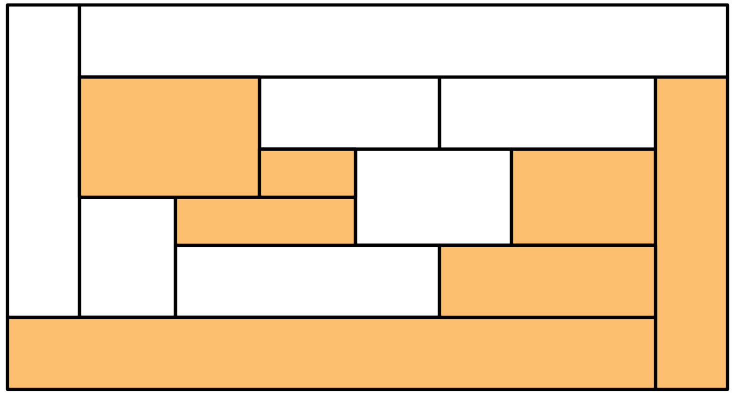

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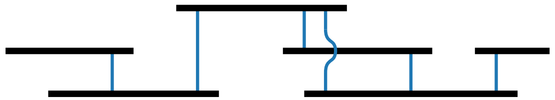

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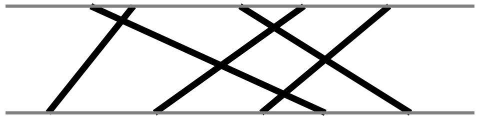

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NP-hard for:

- planar straight-line drawings

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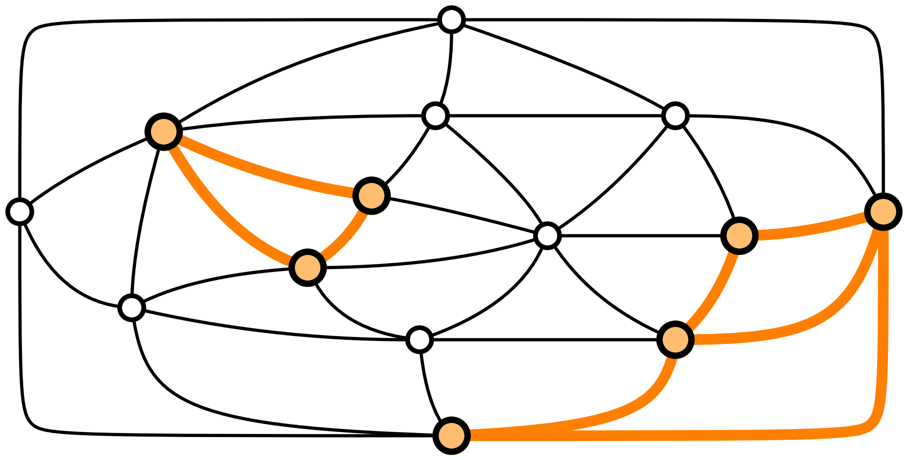

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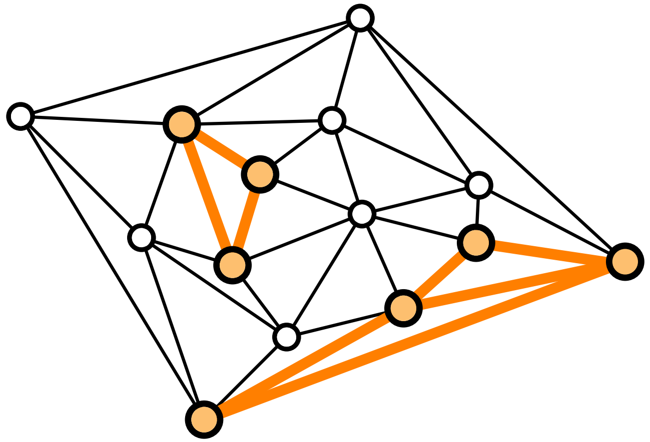

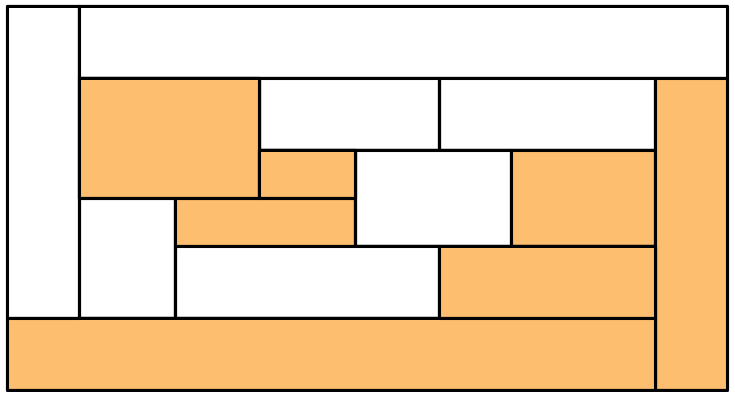

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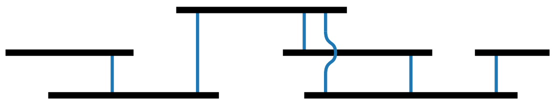

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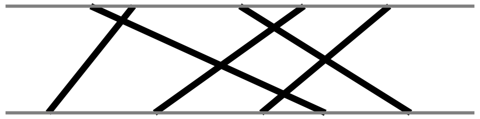

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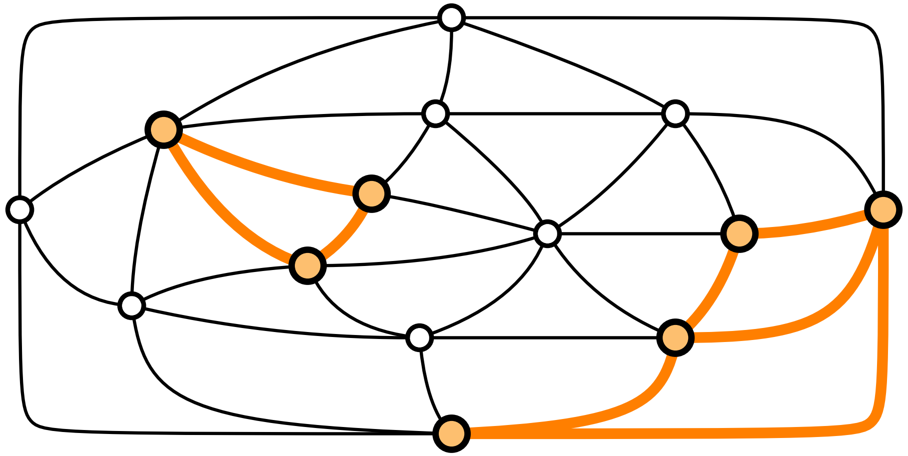

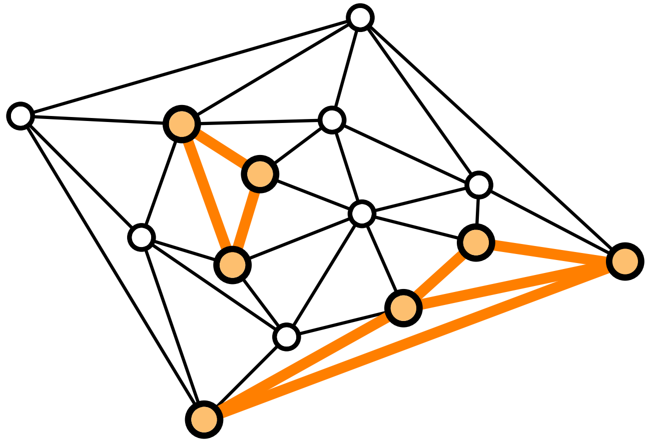

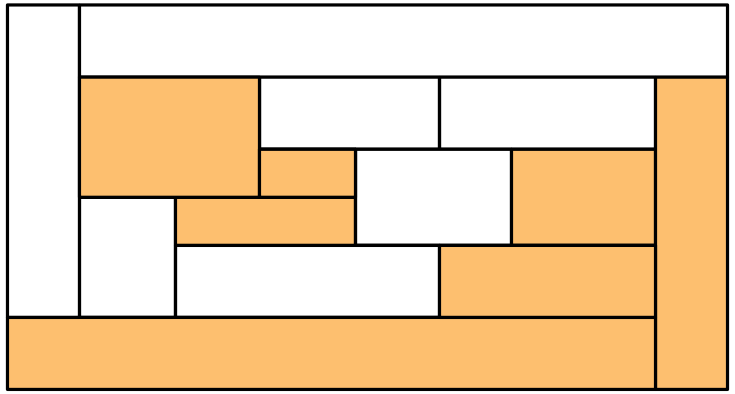

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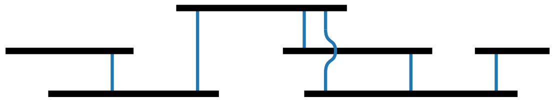

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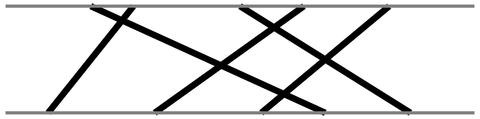

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NP-hard for:

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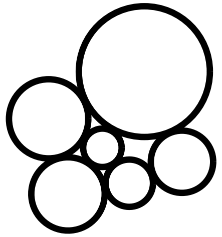

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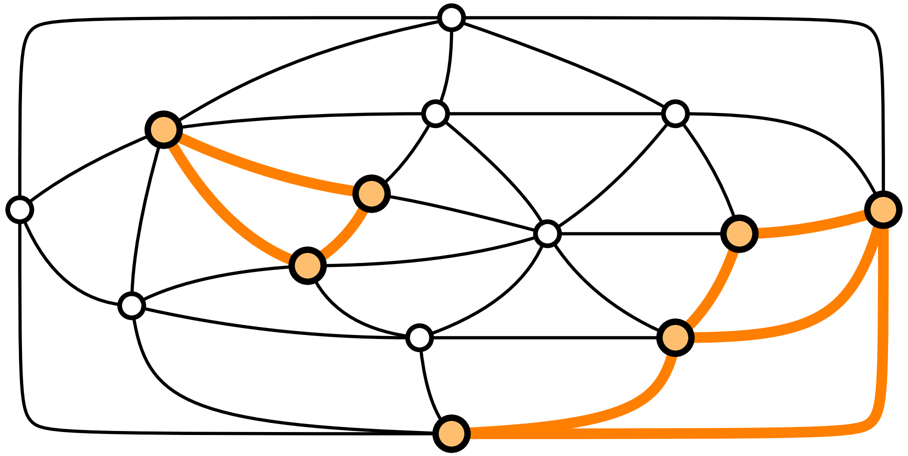

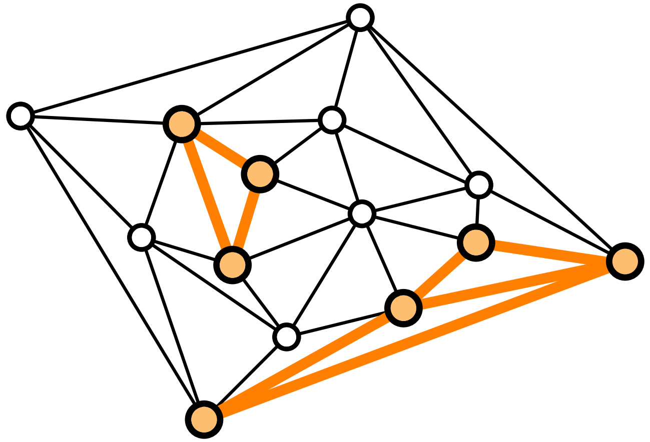

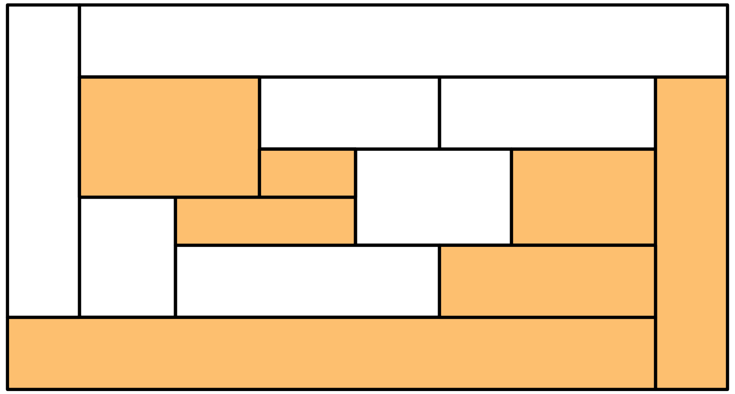

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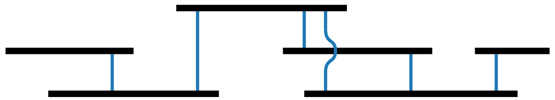

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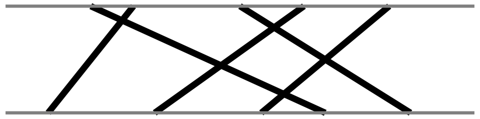

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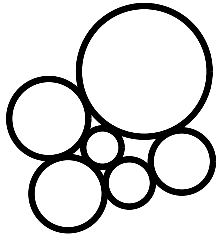

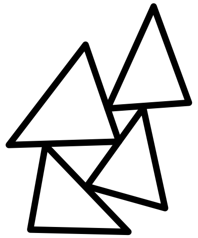

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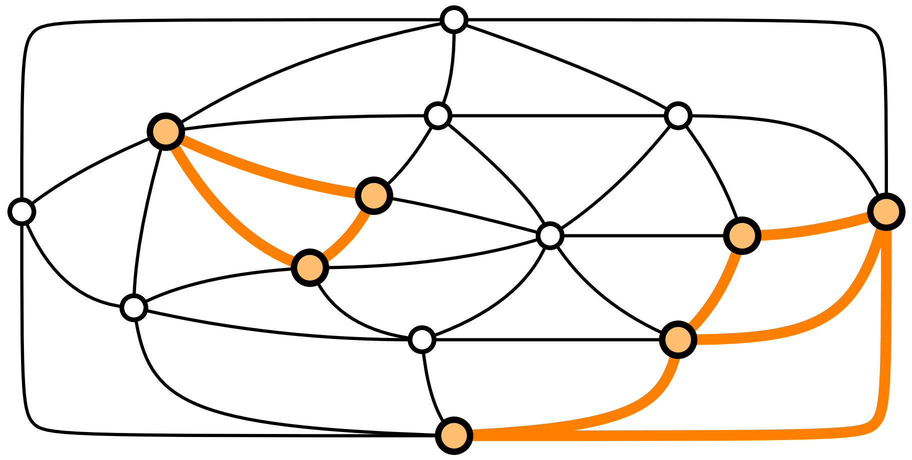

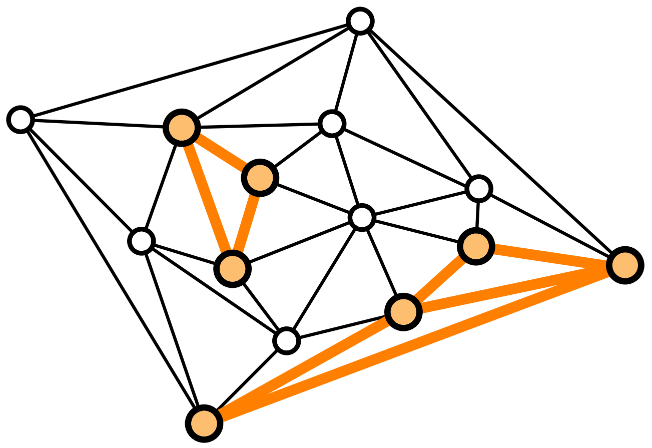

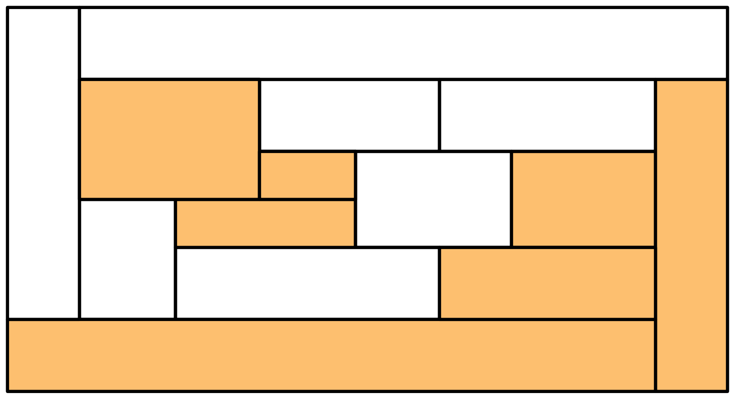

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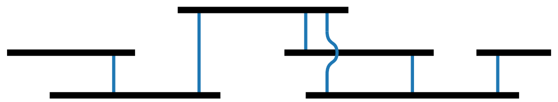

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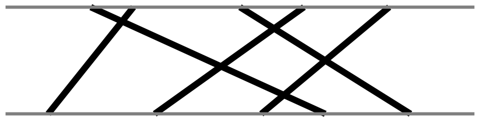
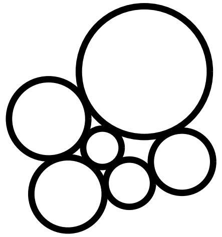


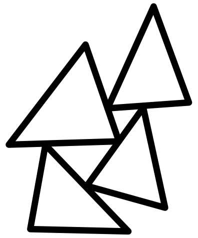
- circle graphs

NP-hard for:

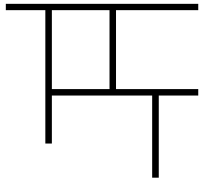
- planar straight-line drawings
- contacts of
  - disks



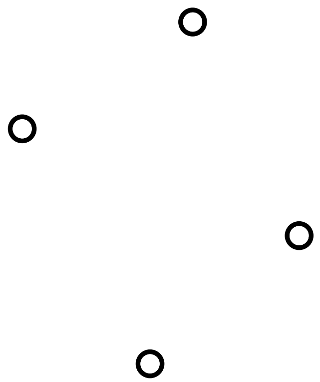
  - triangles



  - orthogonal segments



Bar Visibility Representation



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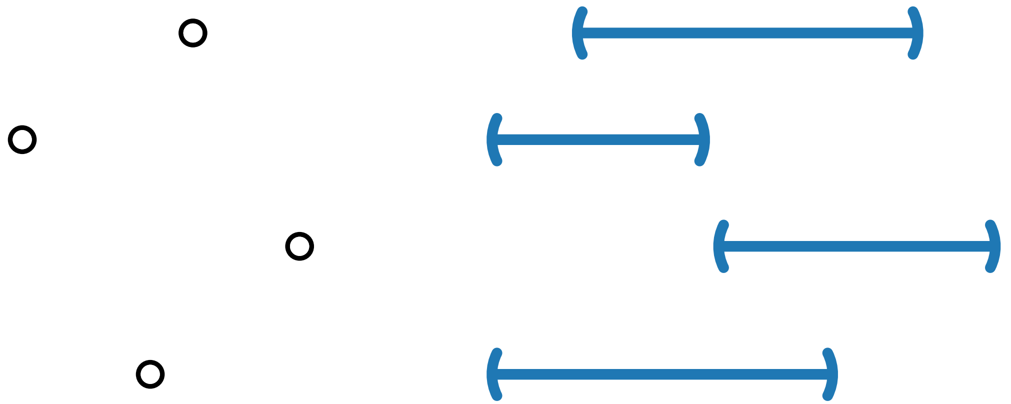

Bar Visibility Representation

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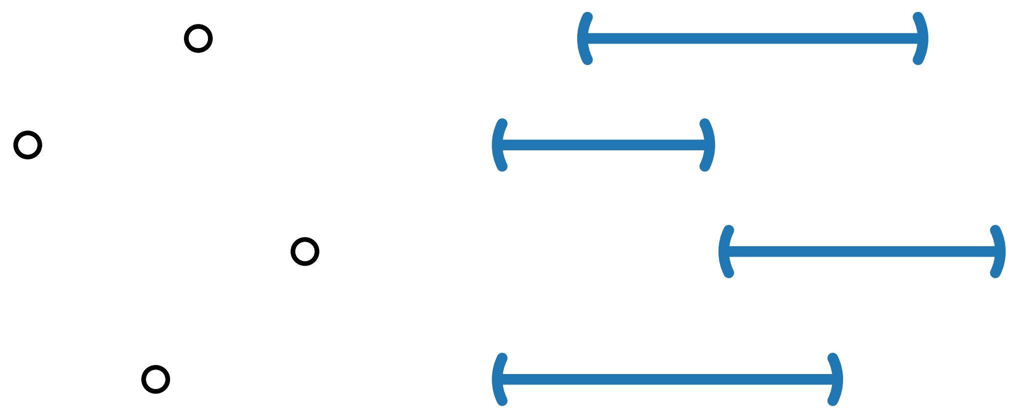

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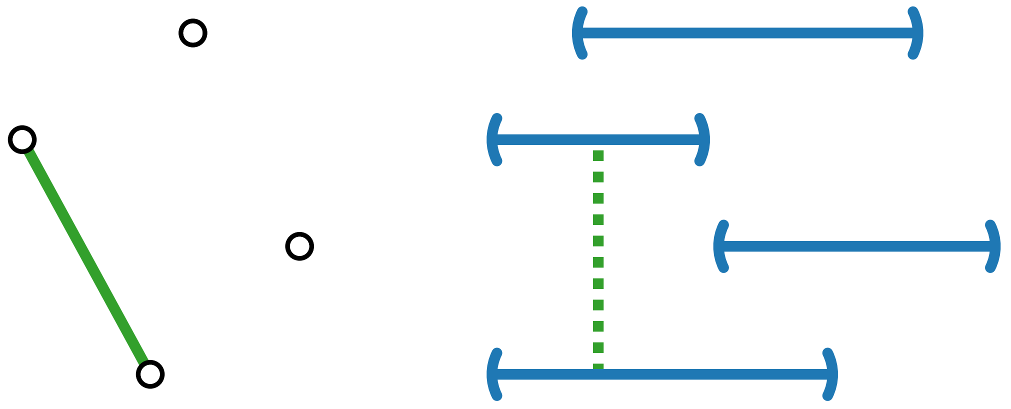

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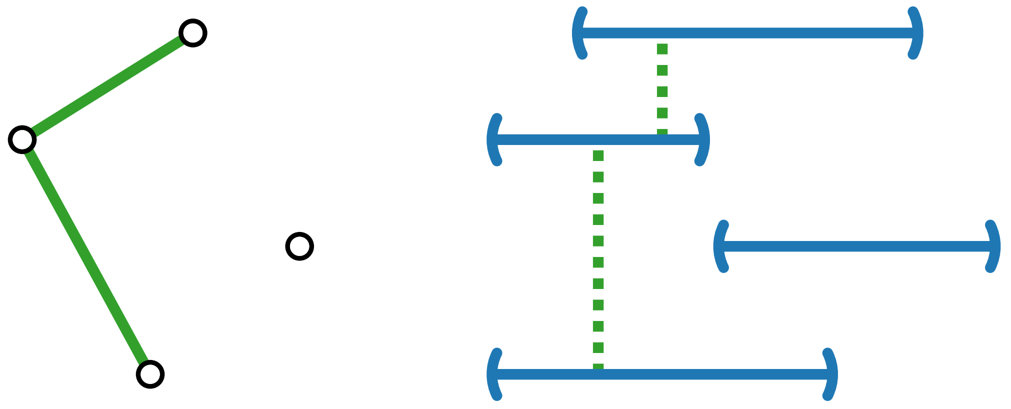

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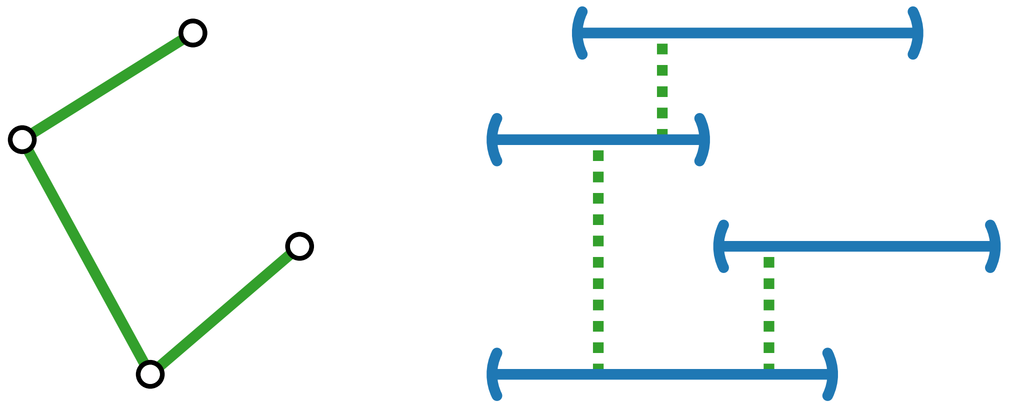

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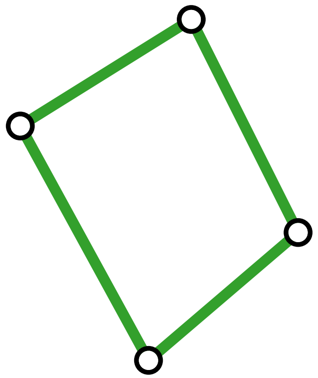

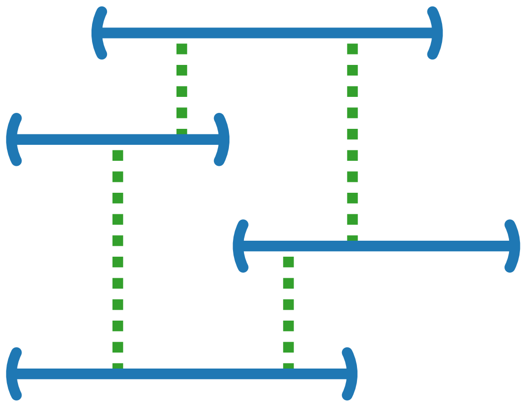

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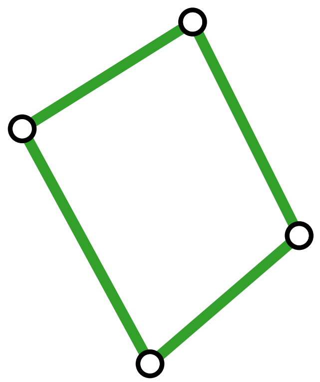

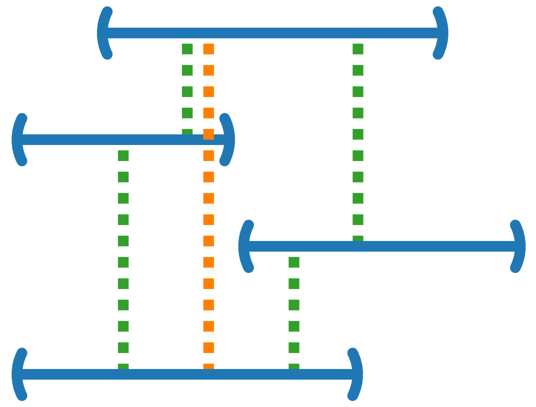

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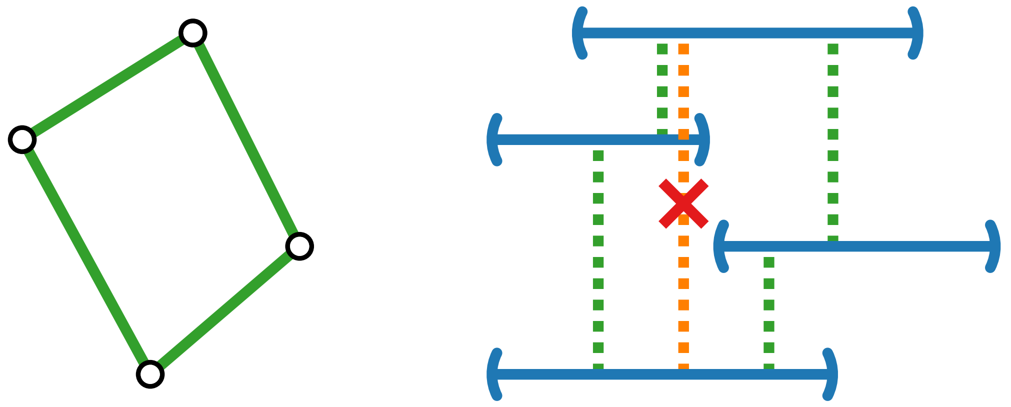

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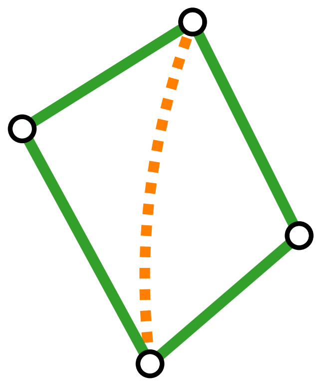

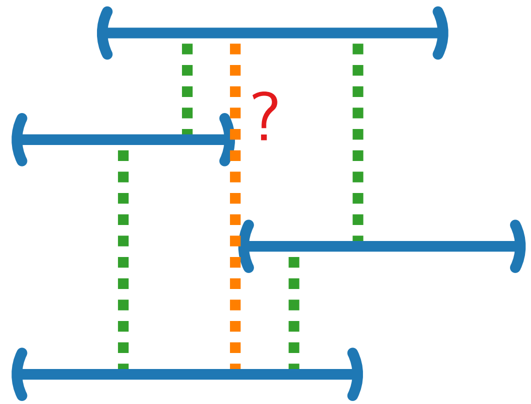

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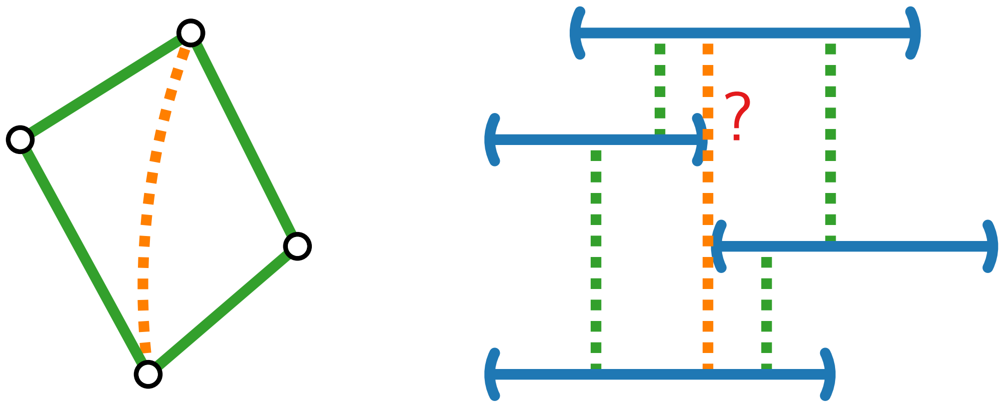

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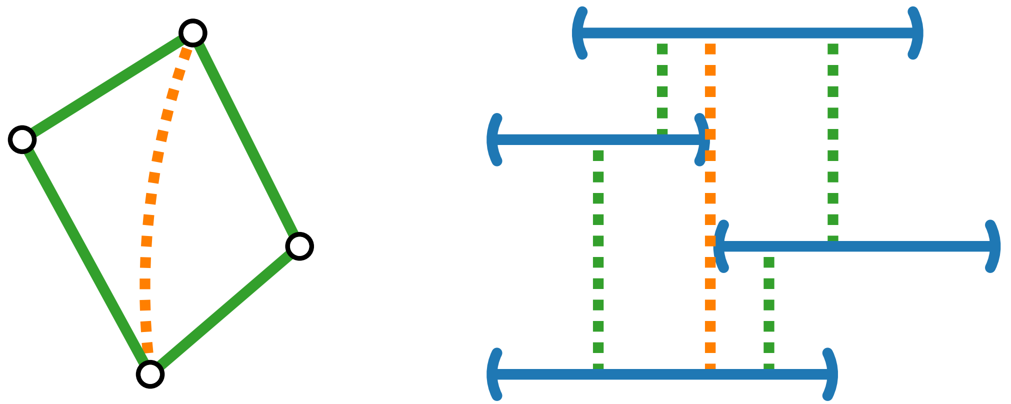

Models.

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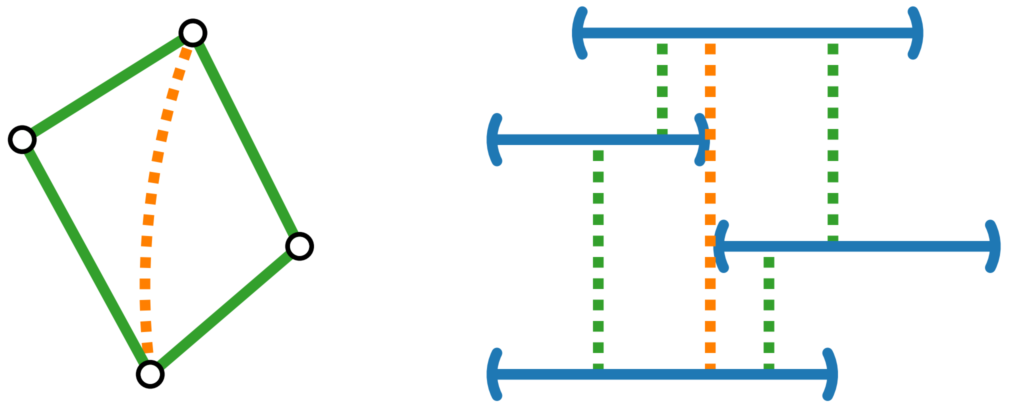

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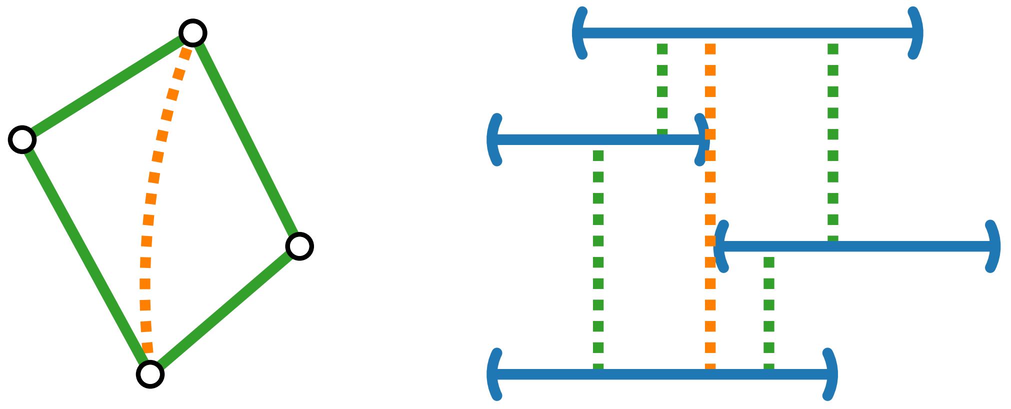

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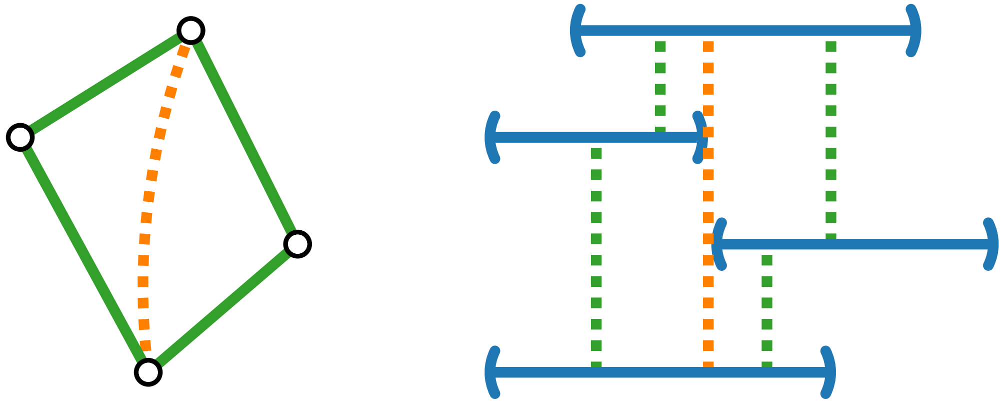

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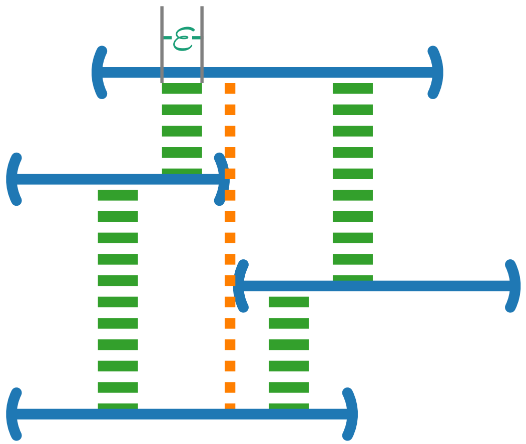

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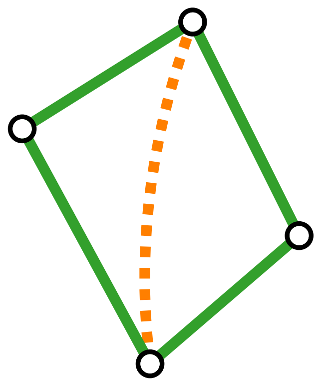

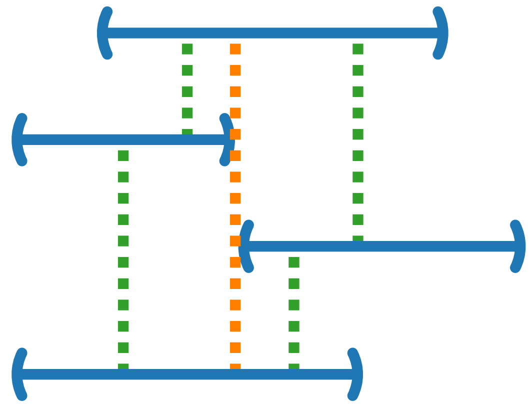

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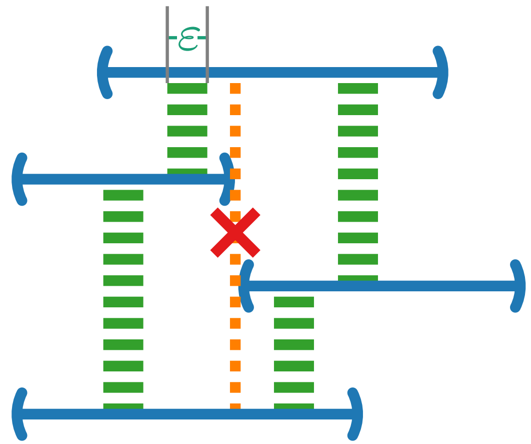

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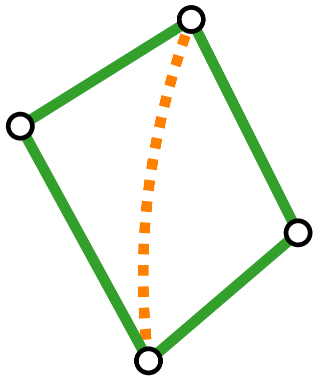

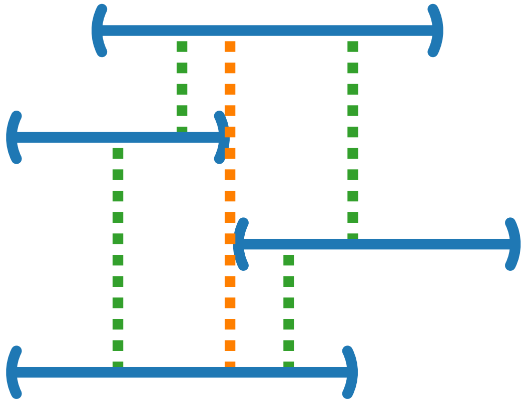

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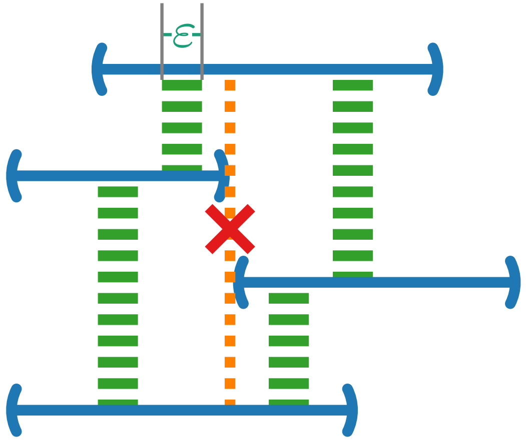

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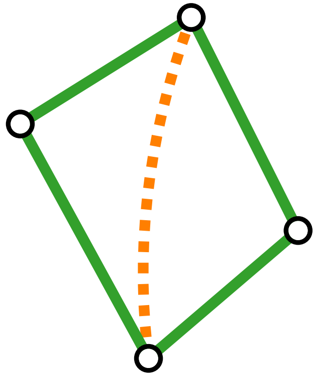

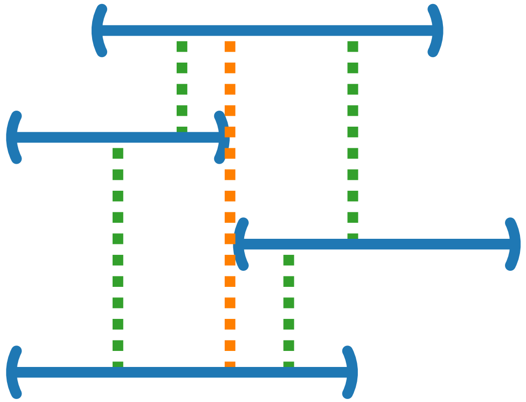

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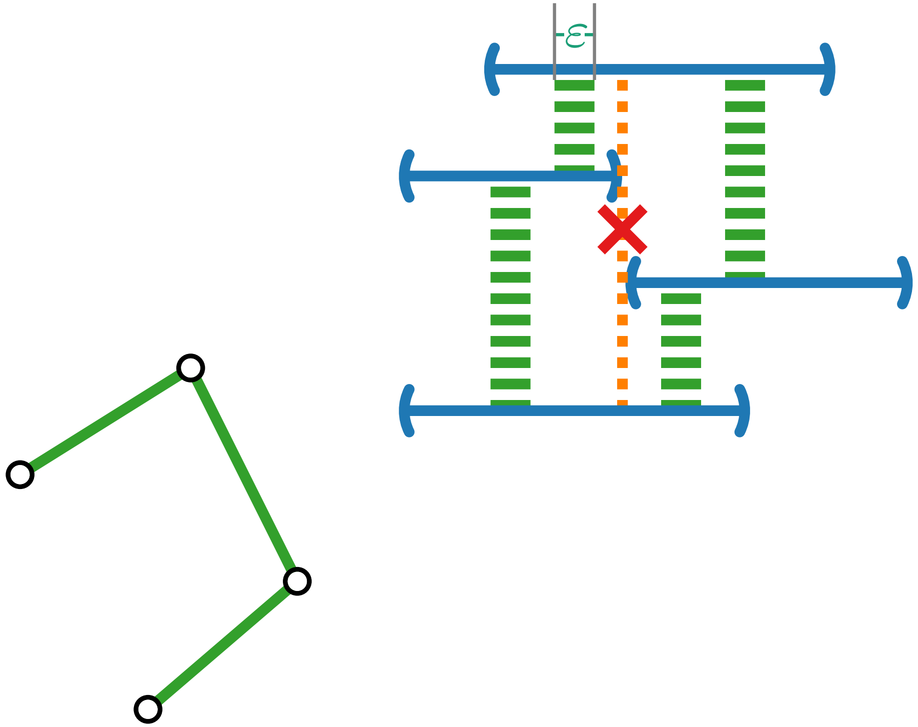
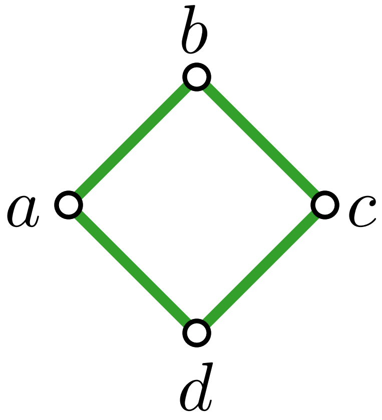
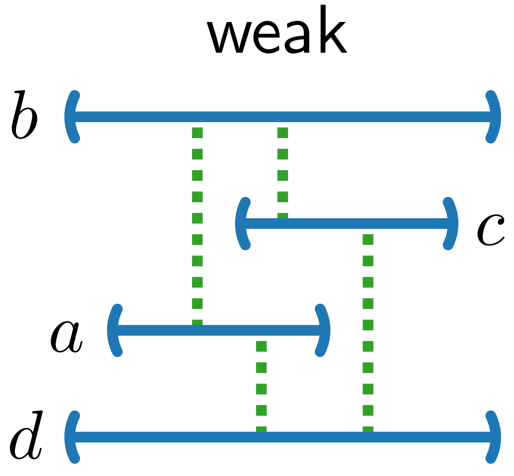


Problems





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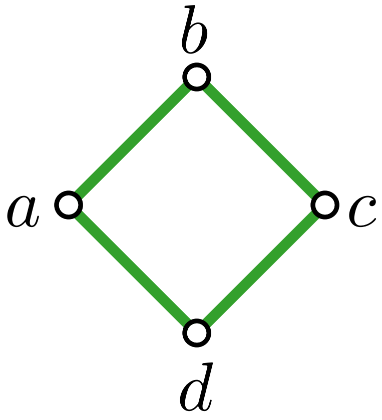

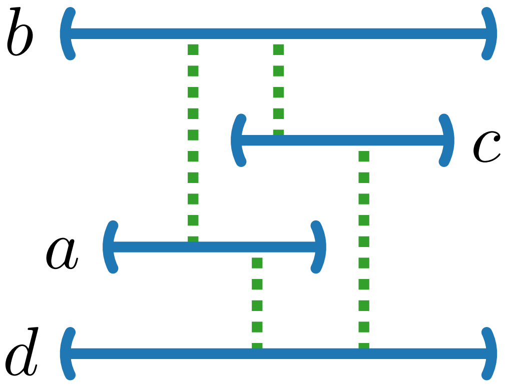

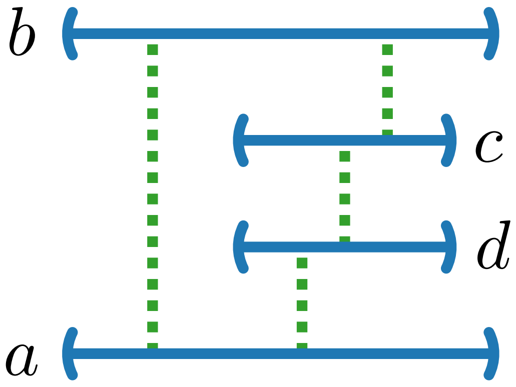

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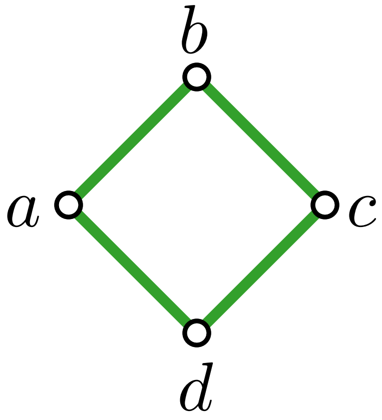

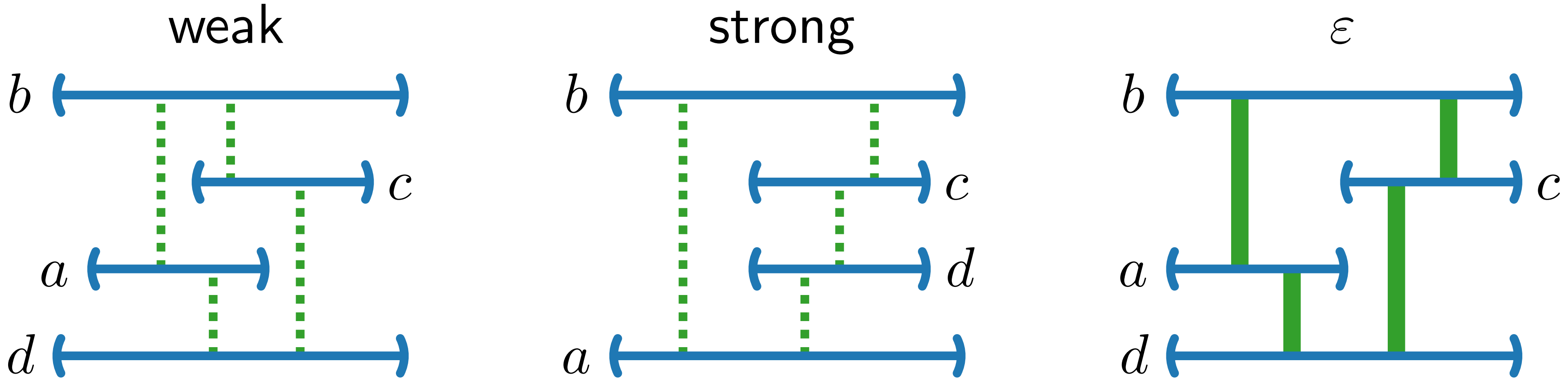

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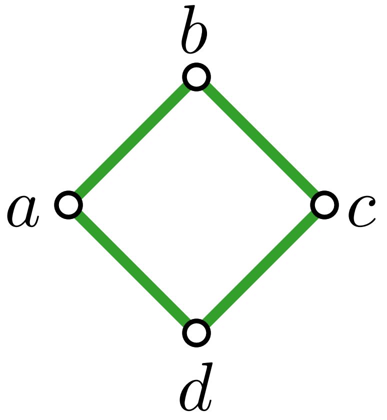

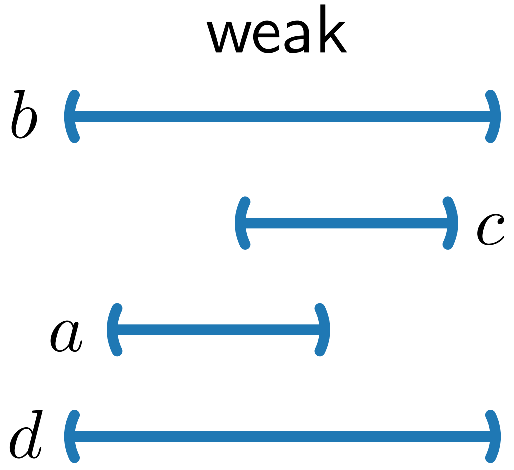

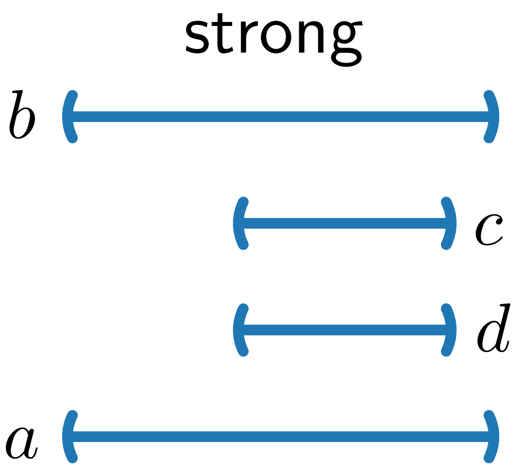

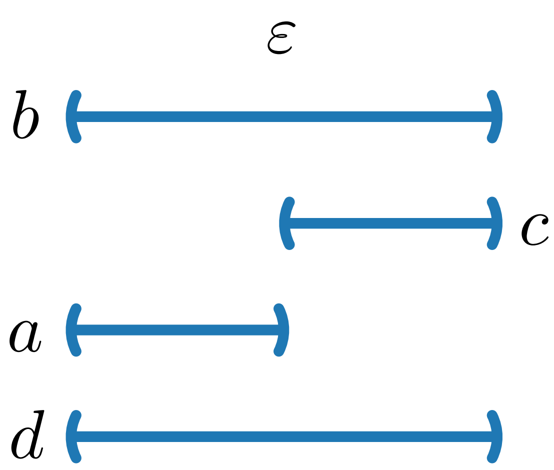

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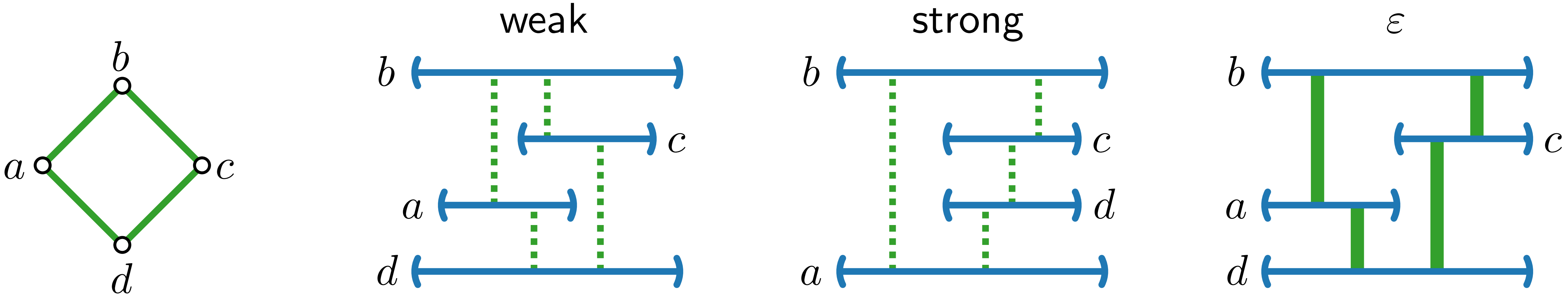

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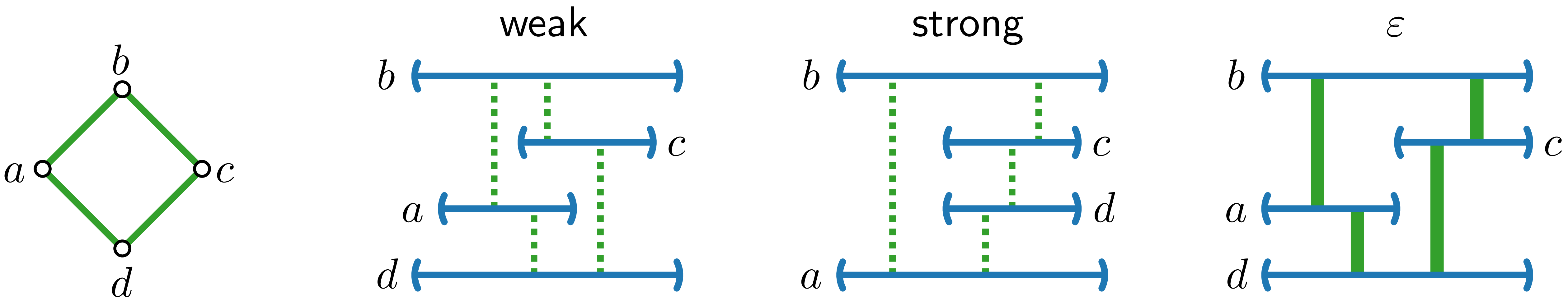
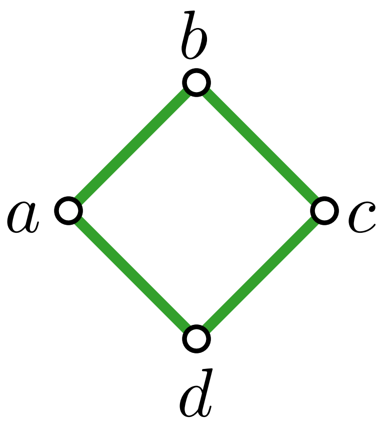
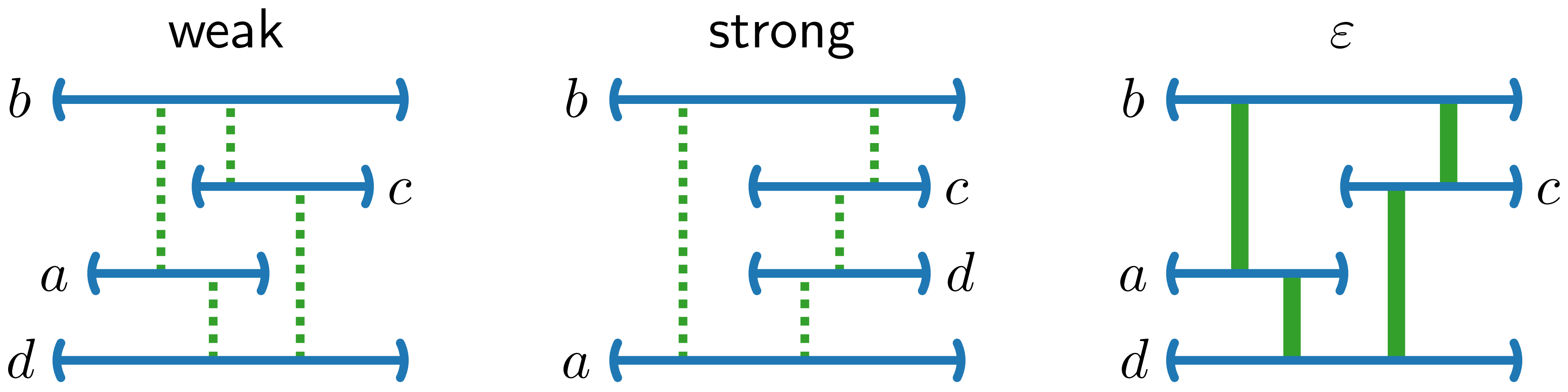


Recognition Problem.
Given a graph G, **decide** if there exists a weak/strong/ε bar visibility representation ψ of G.

Problems





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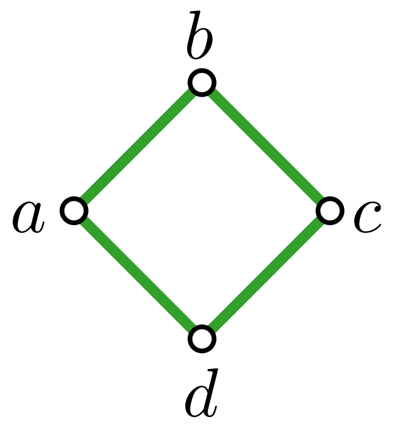

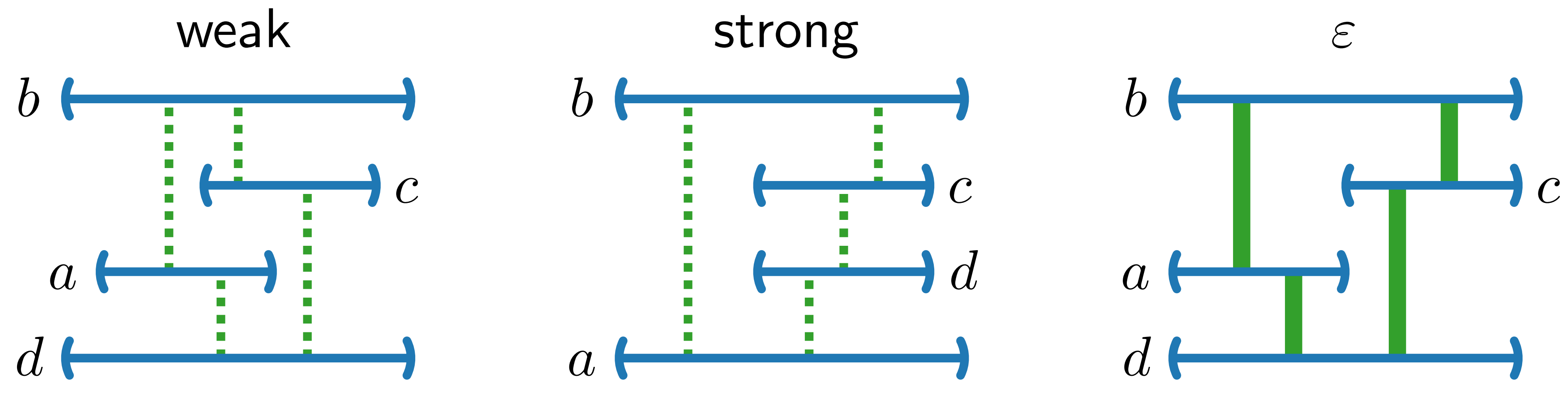

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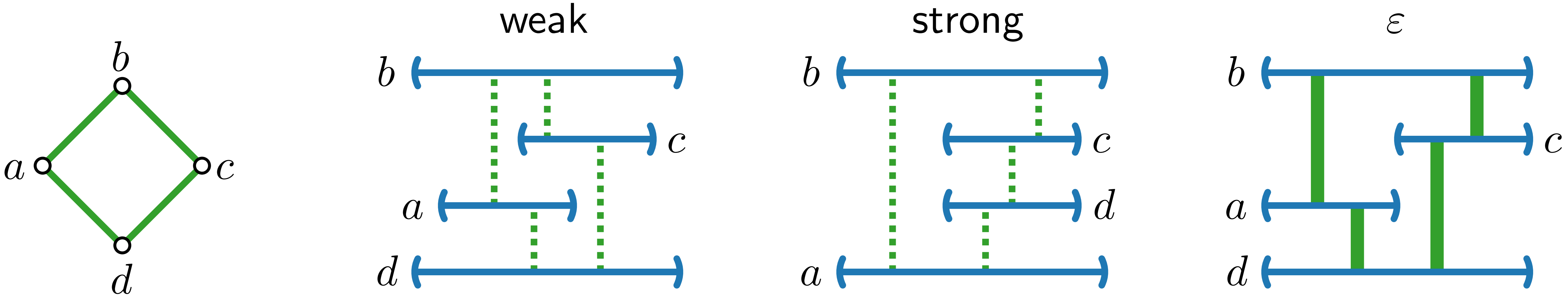
Partial Representation Extension Problem.
Given a graph G and a **set of bars ψ' of $V' \subset V(G)$**, **decide** if there exists a weak/strong/ε bar visibility representation ψ of G **where $\psi|_{V'} = \psi'$** (and **construct** ψ when it exists).

Background



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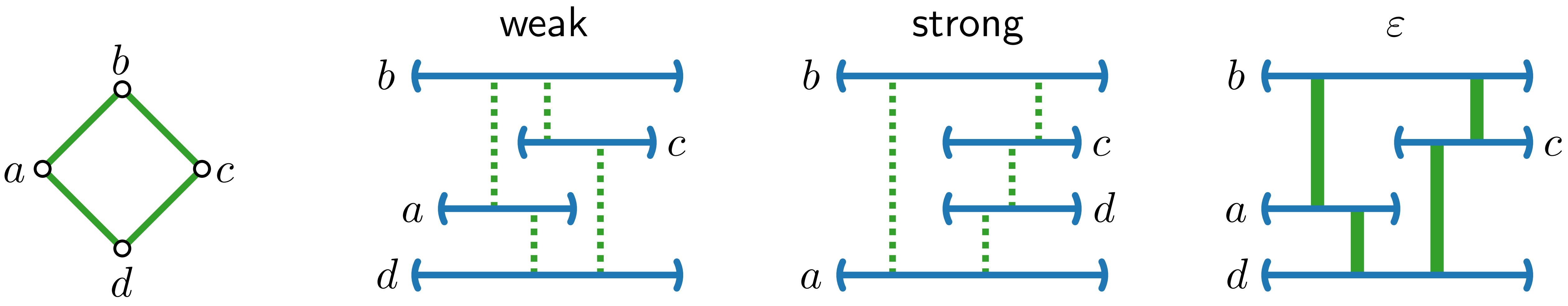
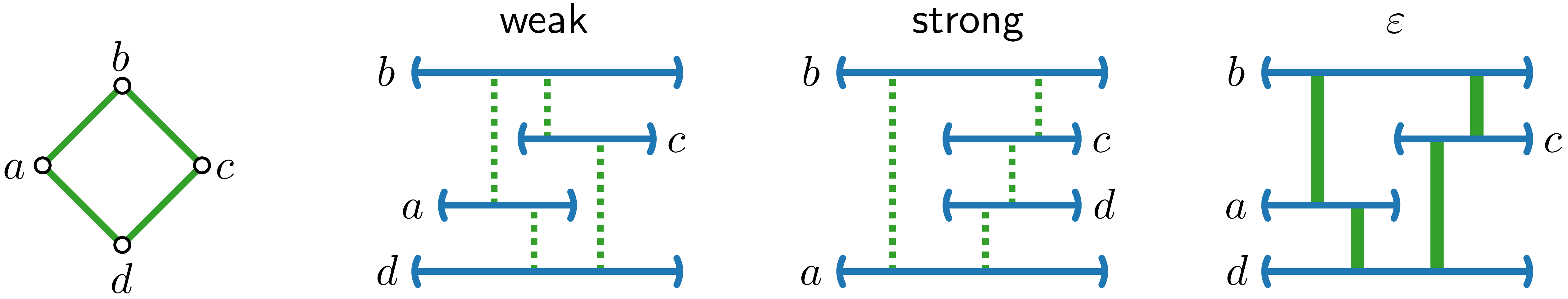


Weak Bar Visibility.

Background



Weak Bar Visibility.

- All planar graphs. [Tamassia & Tollis '86; Wismath '85]

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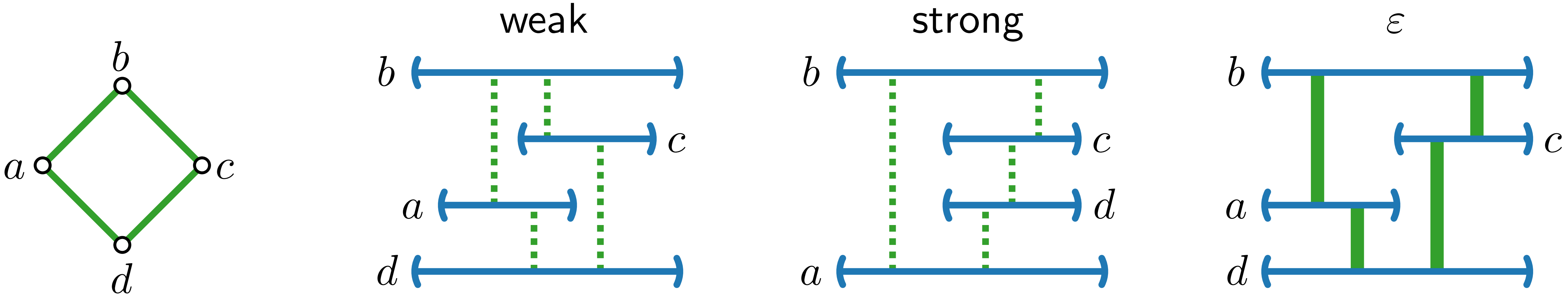

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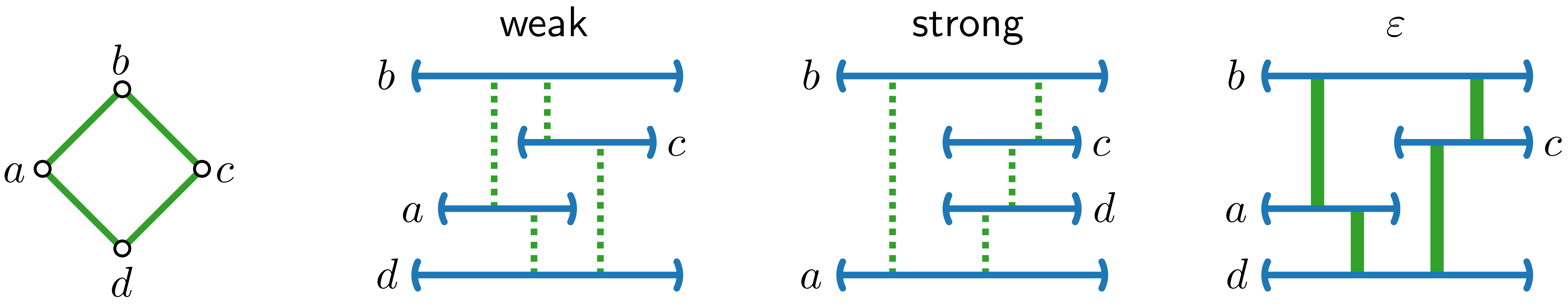

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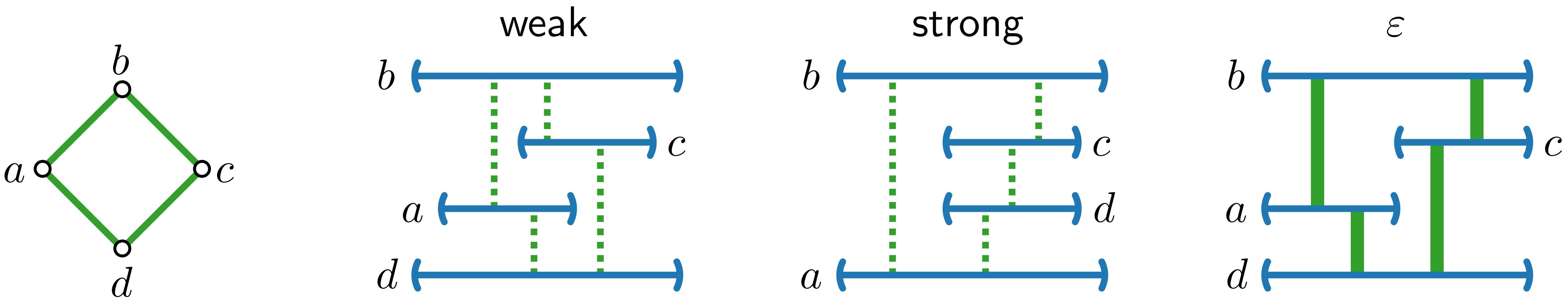

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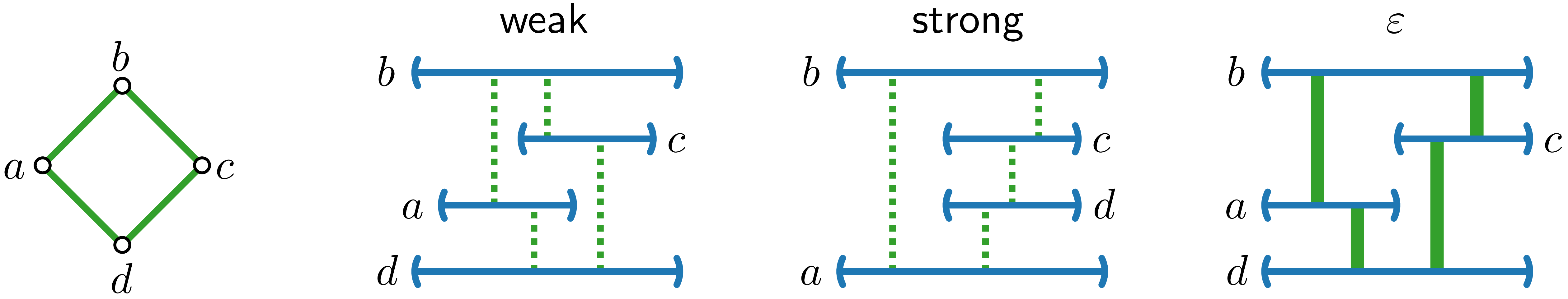

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Strong Bar Visibility.

- NP-complete to recognize [Andreae '92]

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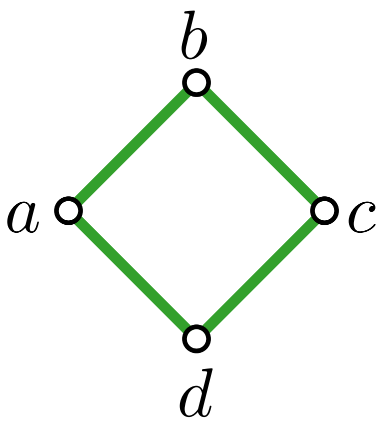
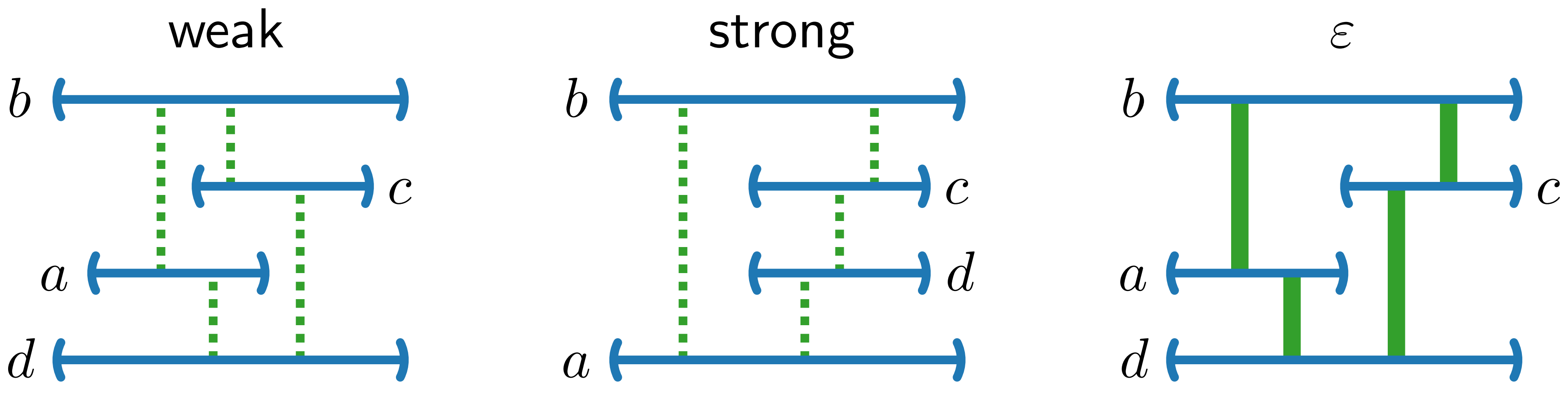
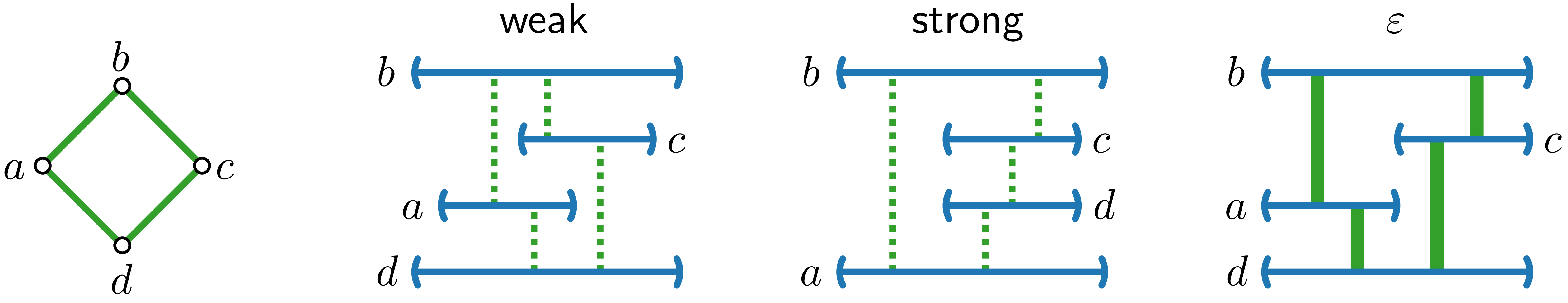




ε-Bar Visibility.

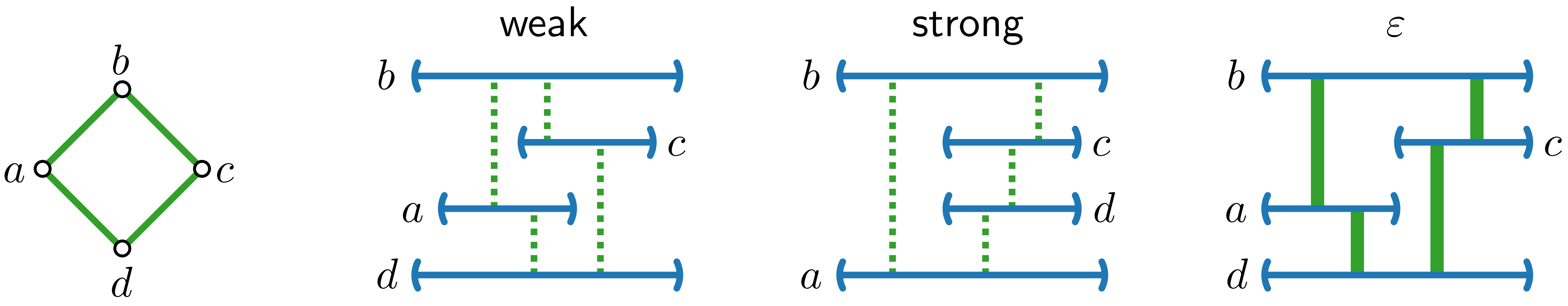
Background



ε-Bar Visibility.

- Planar graphs that can be embedded with all **cut vertices** on the outerface. [T&T '86, Wismath '85]

Background



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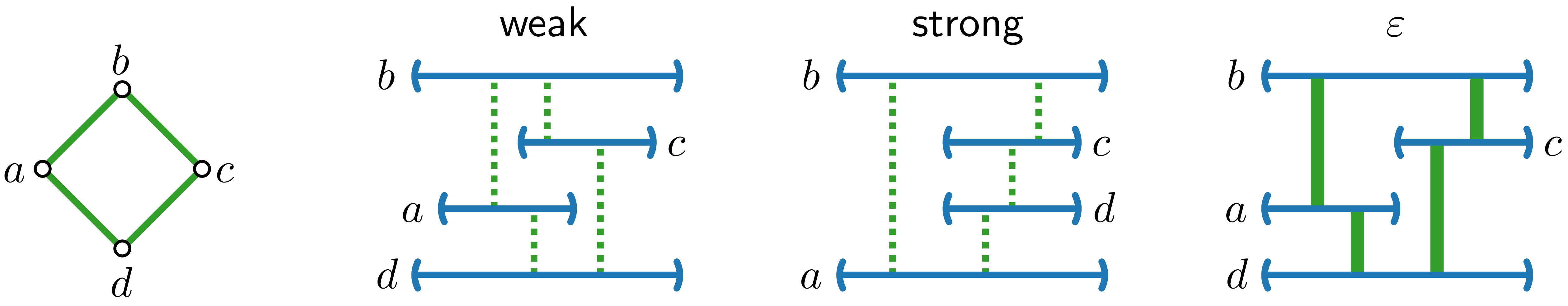

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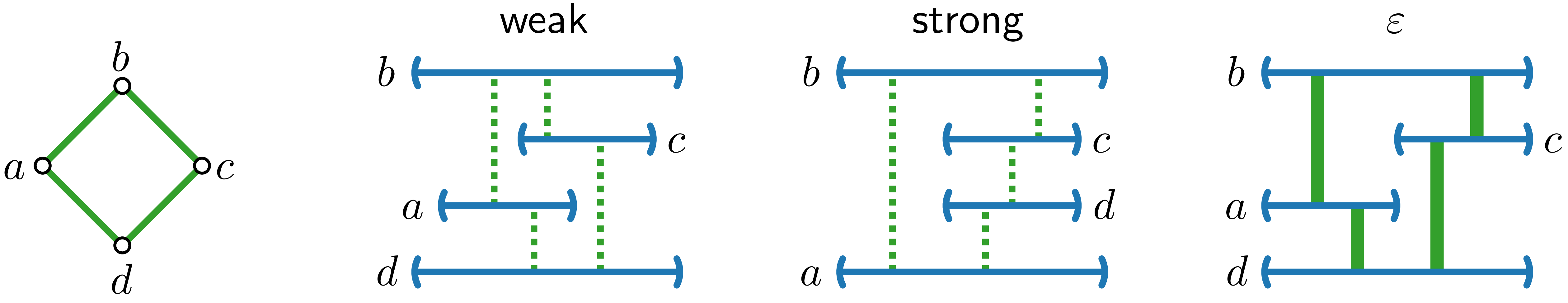

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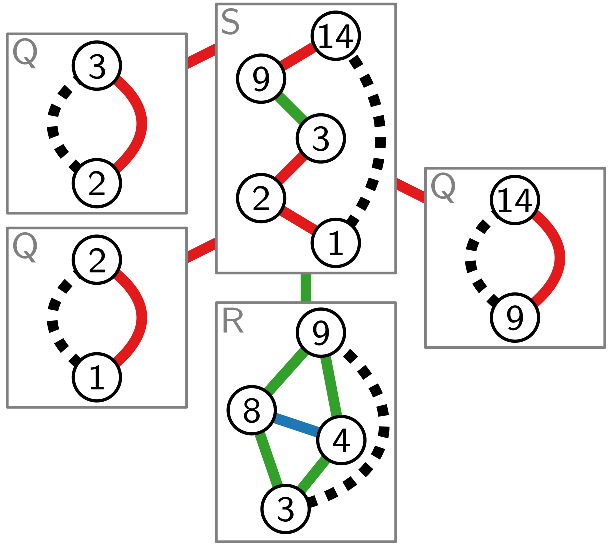
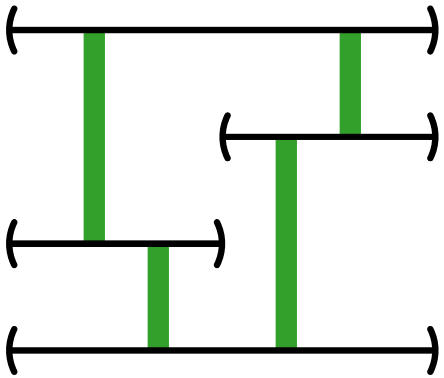
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part II:
Recognition & Construction

Jonathan Klawitter



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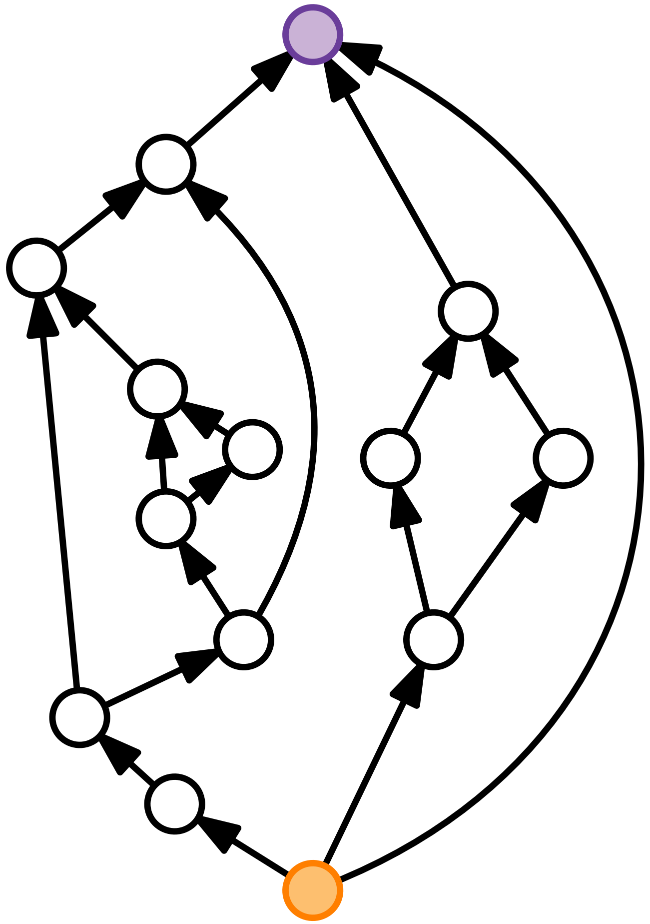

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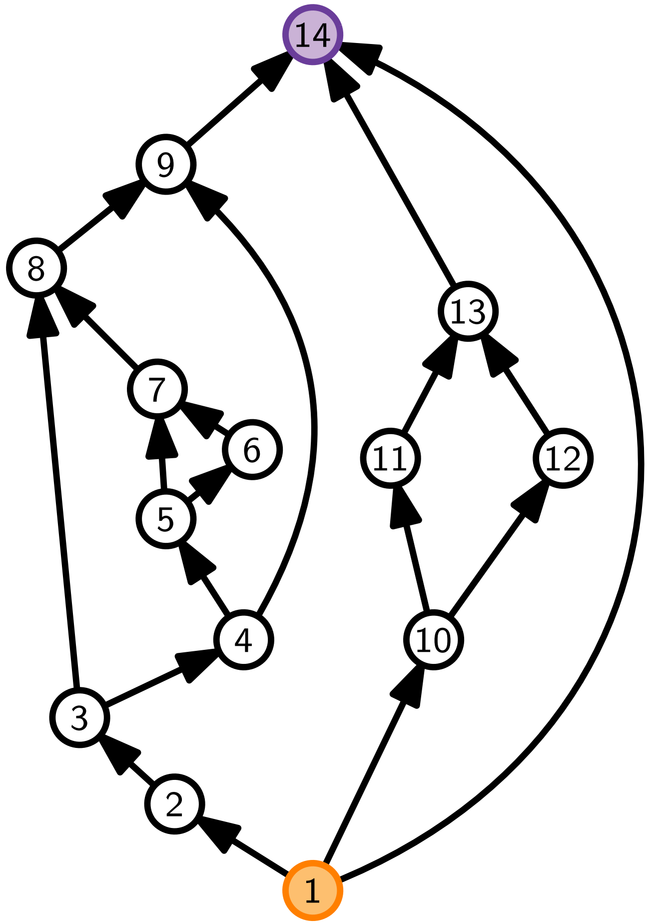

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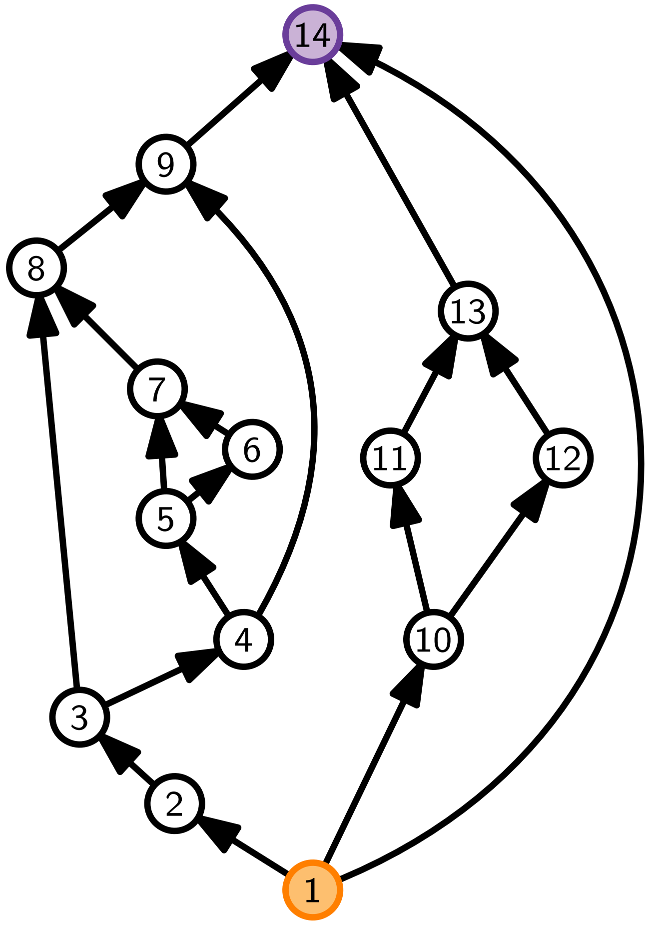

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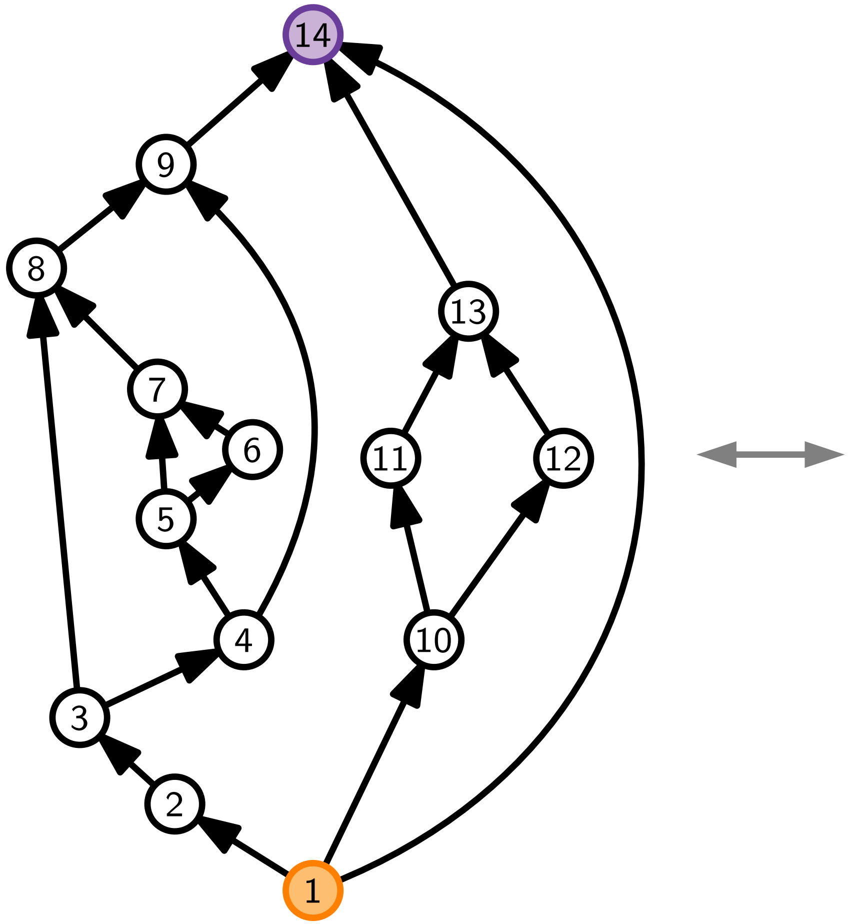

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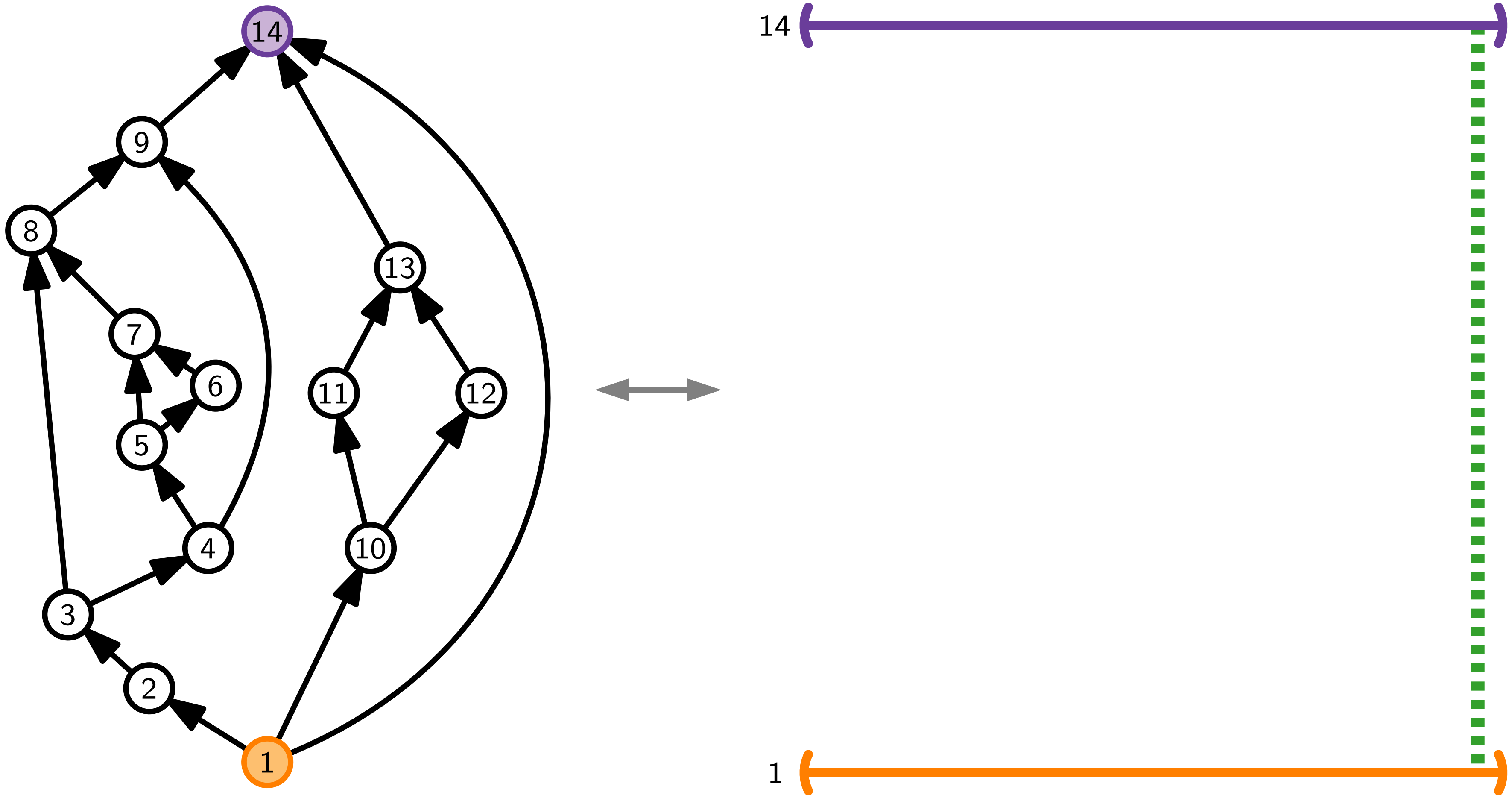

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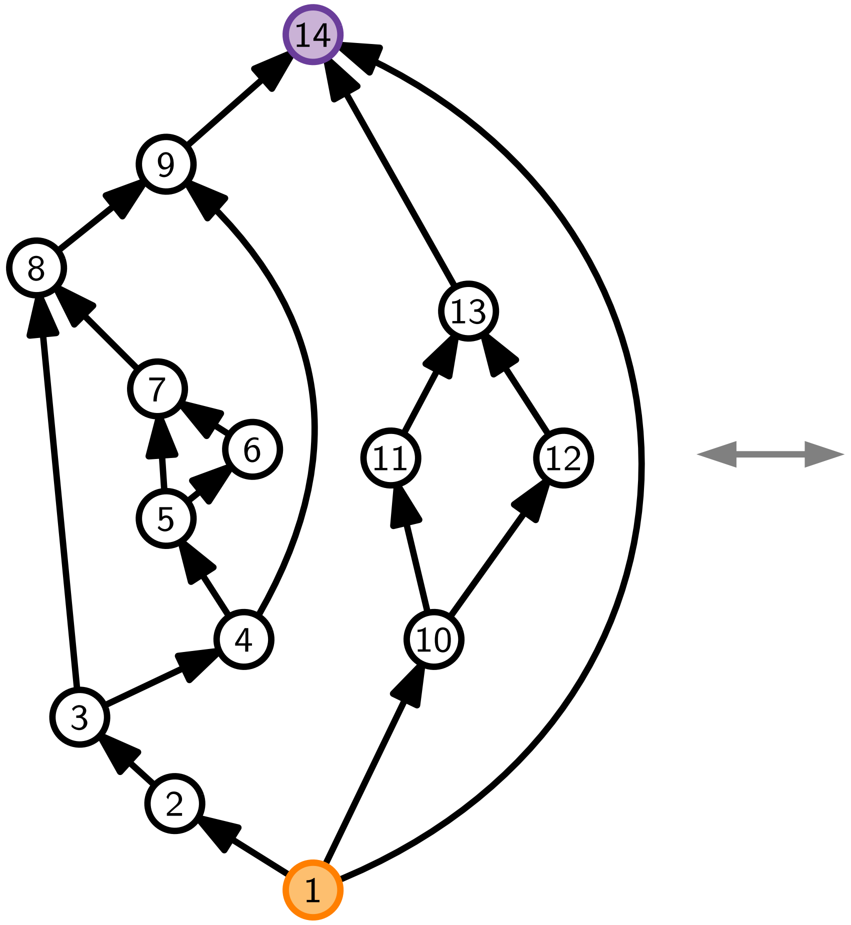

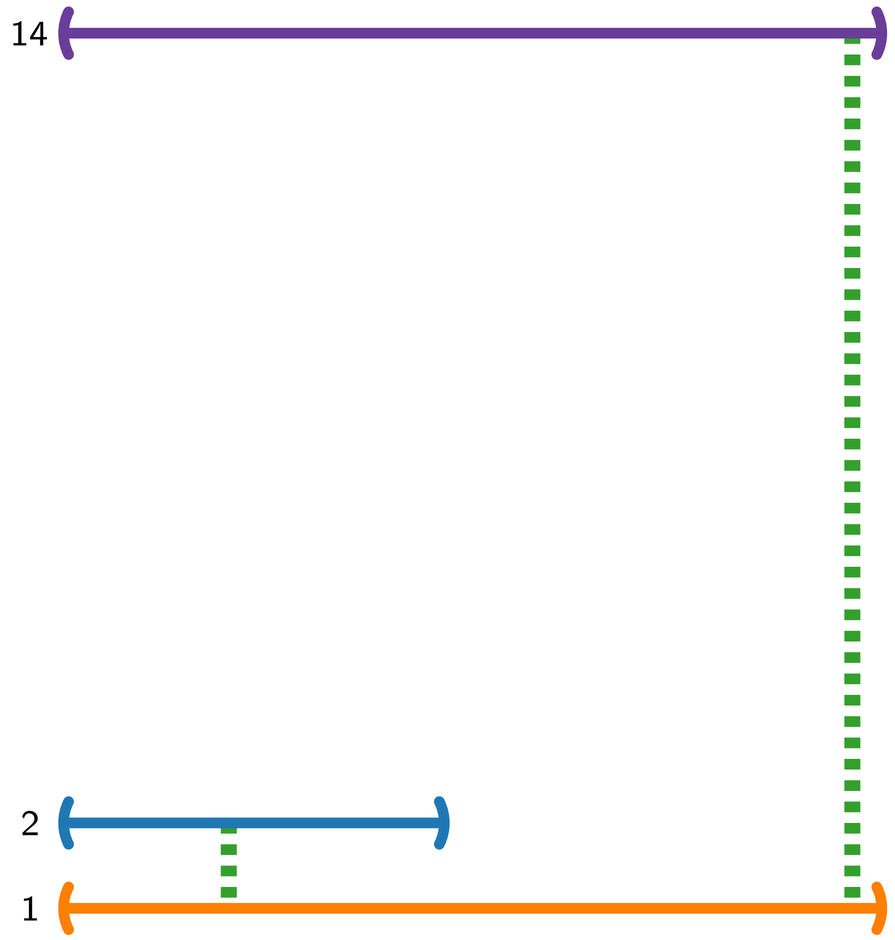

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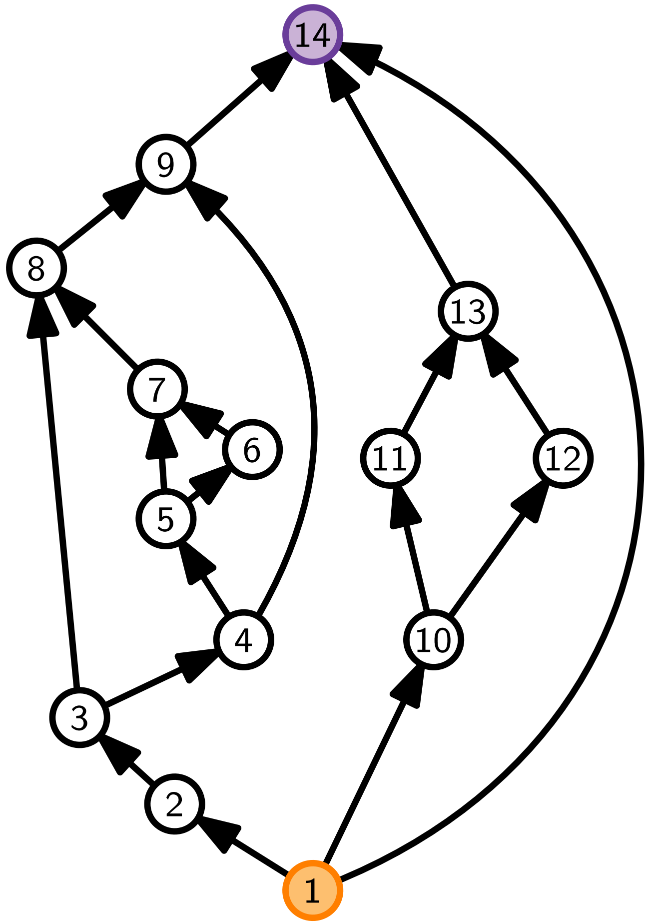

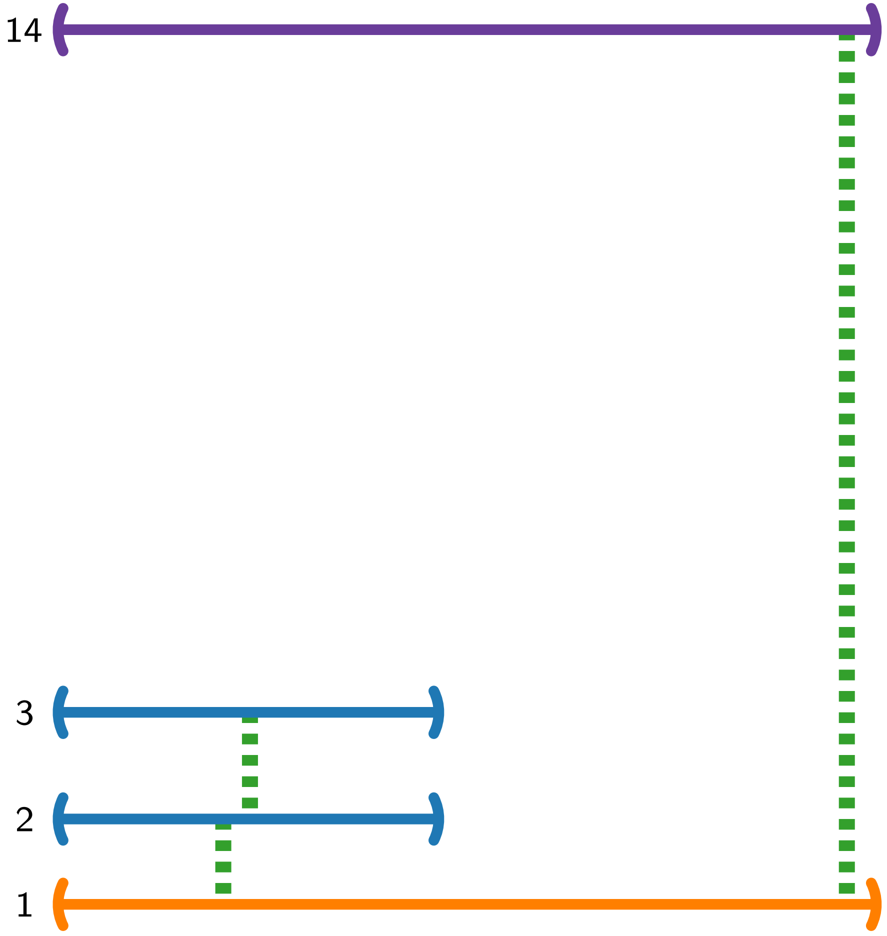

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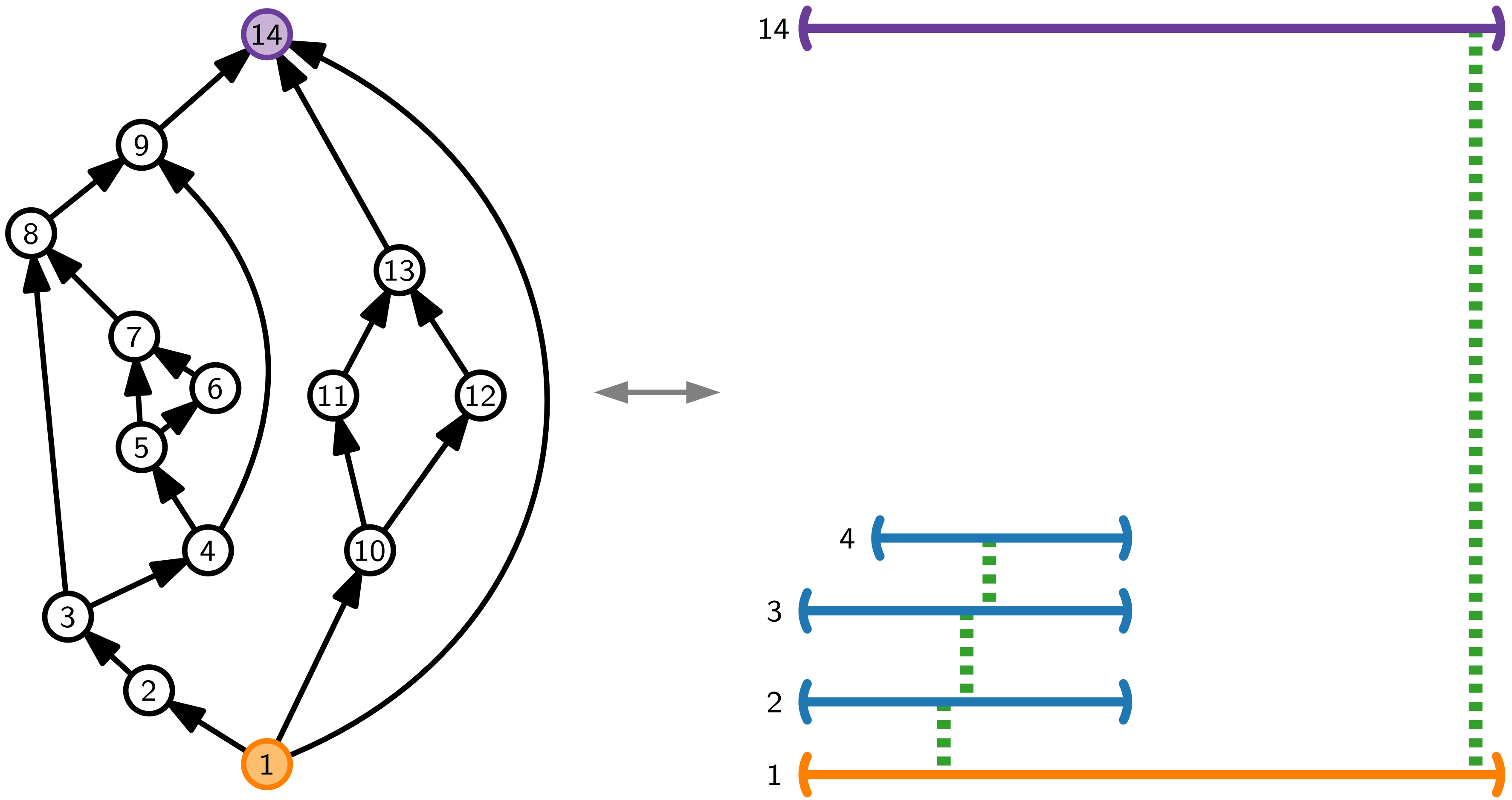

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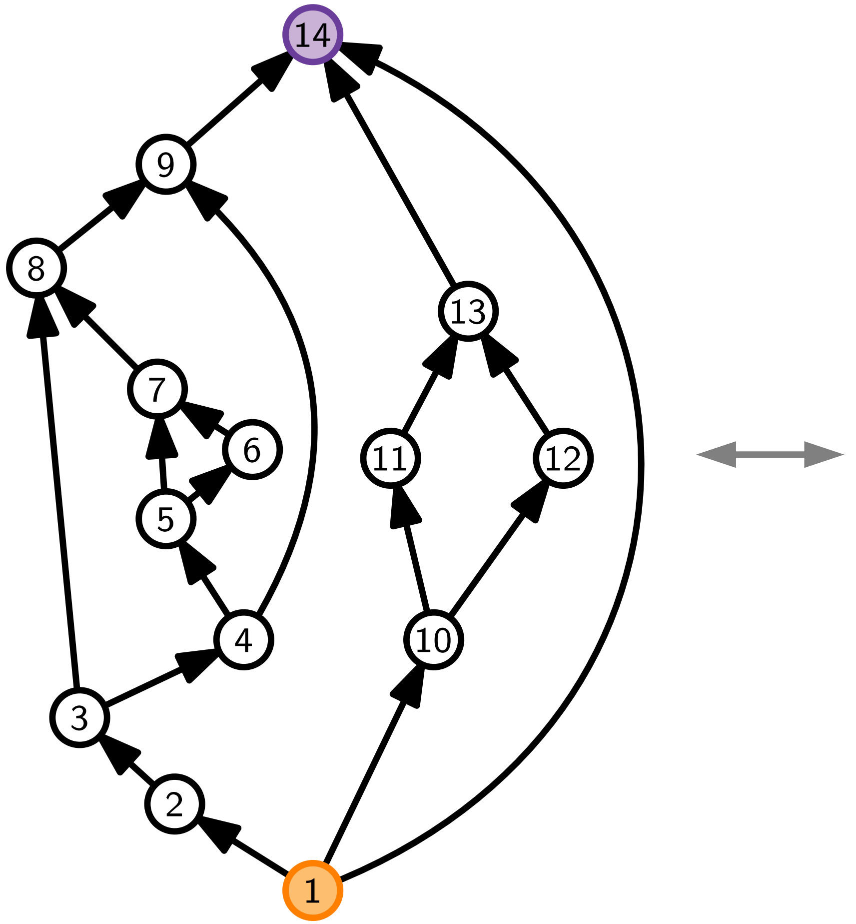

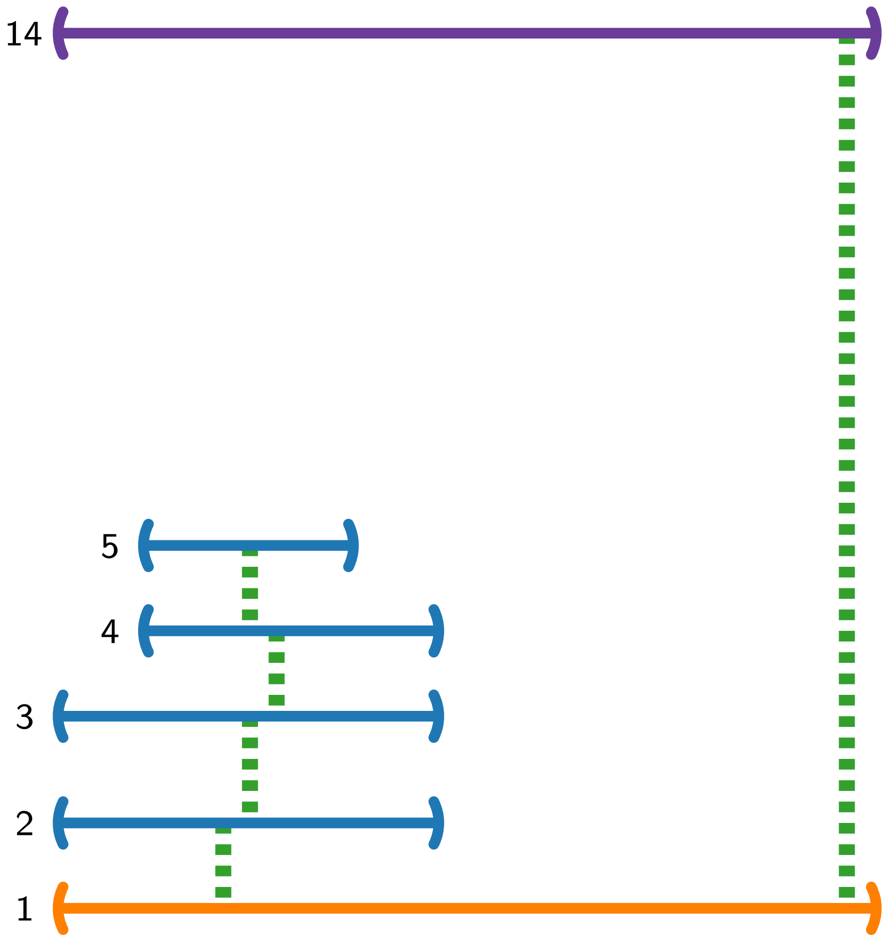

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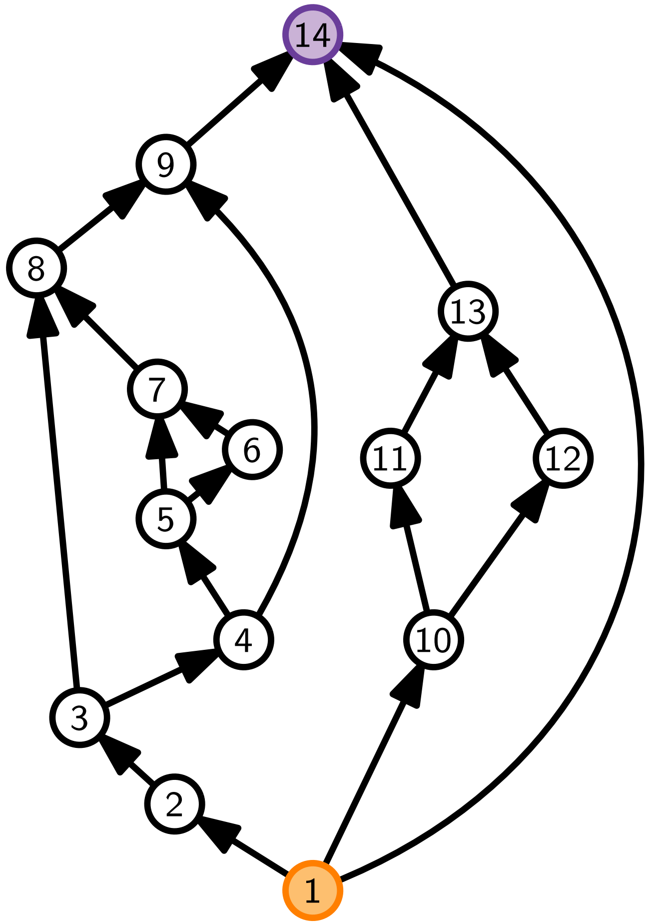

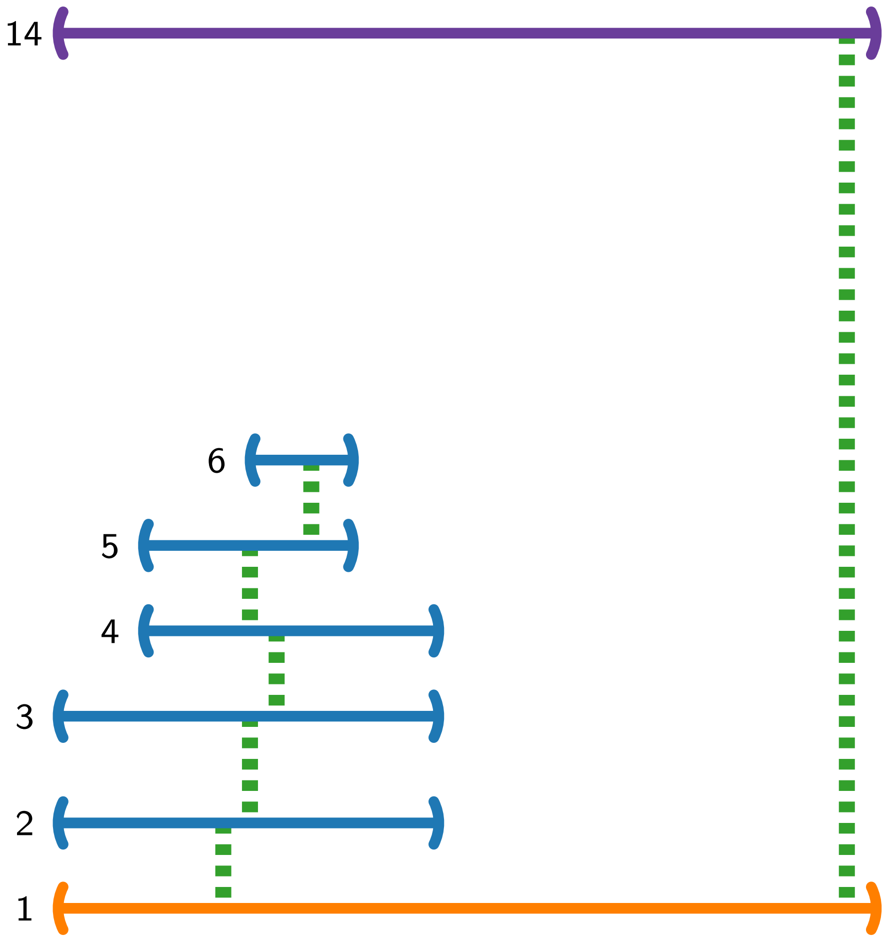

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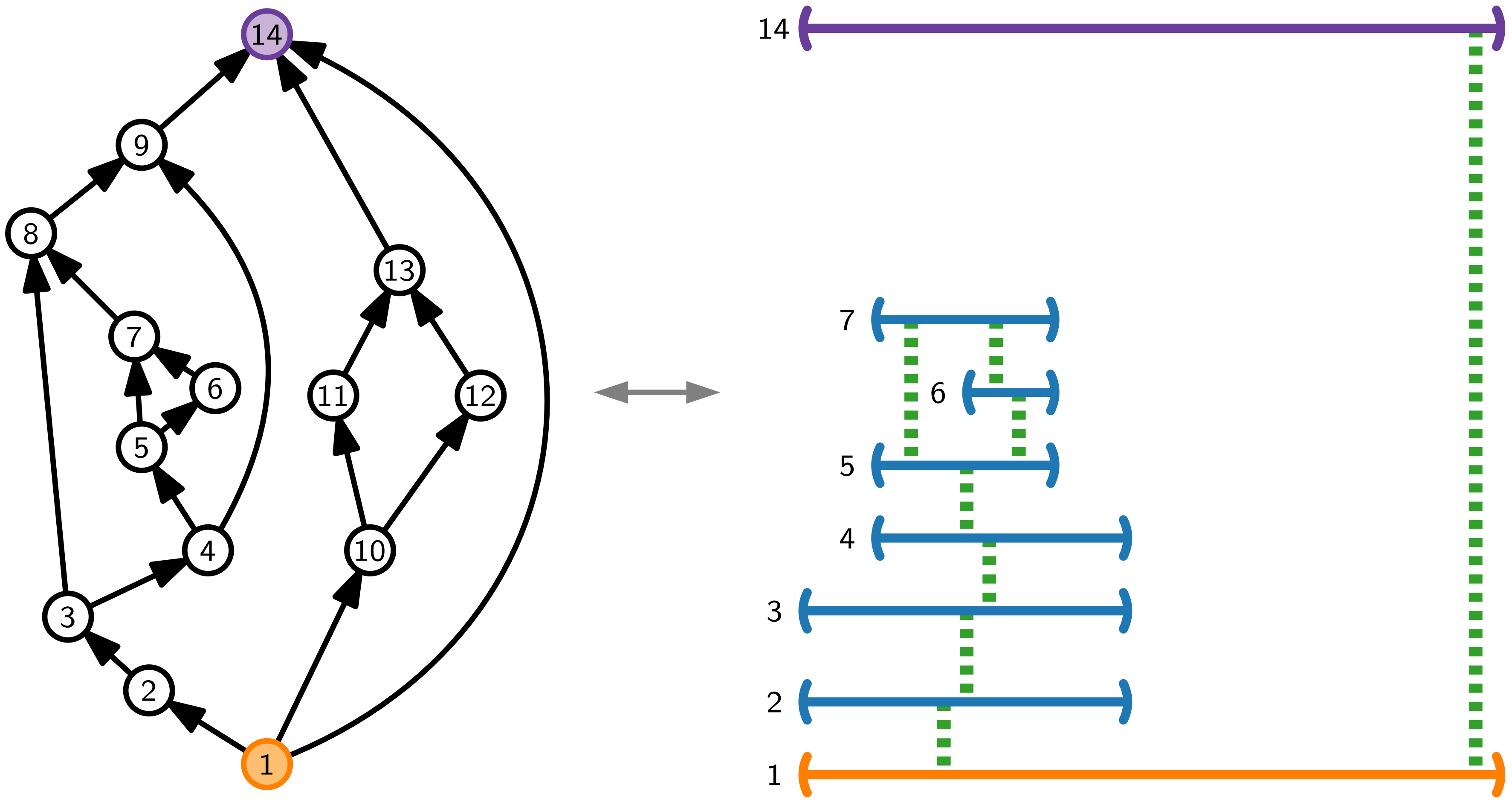

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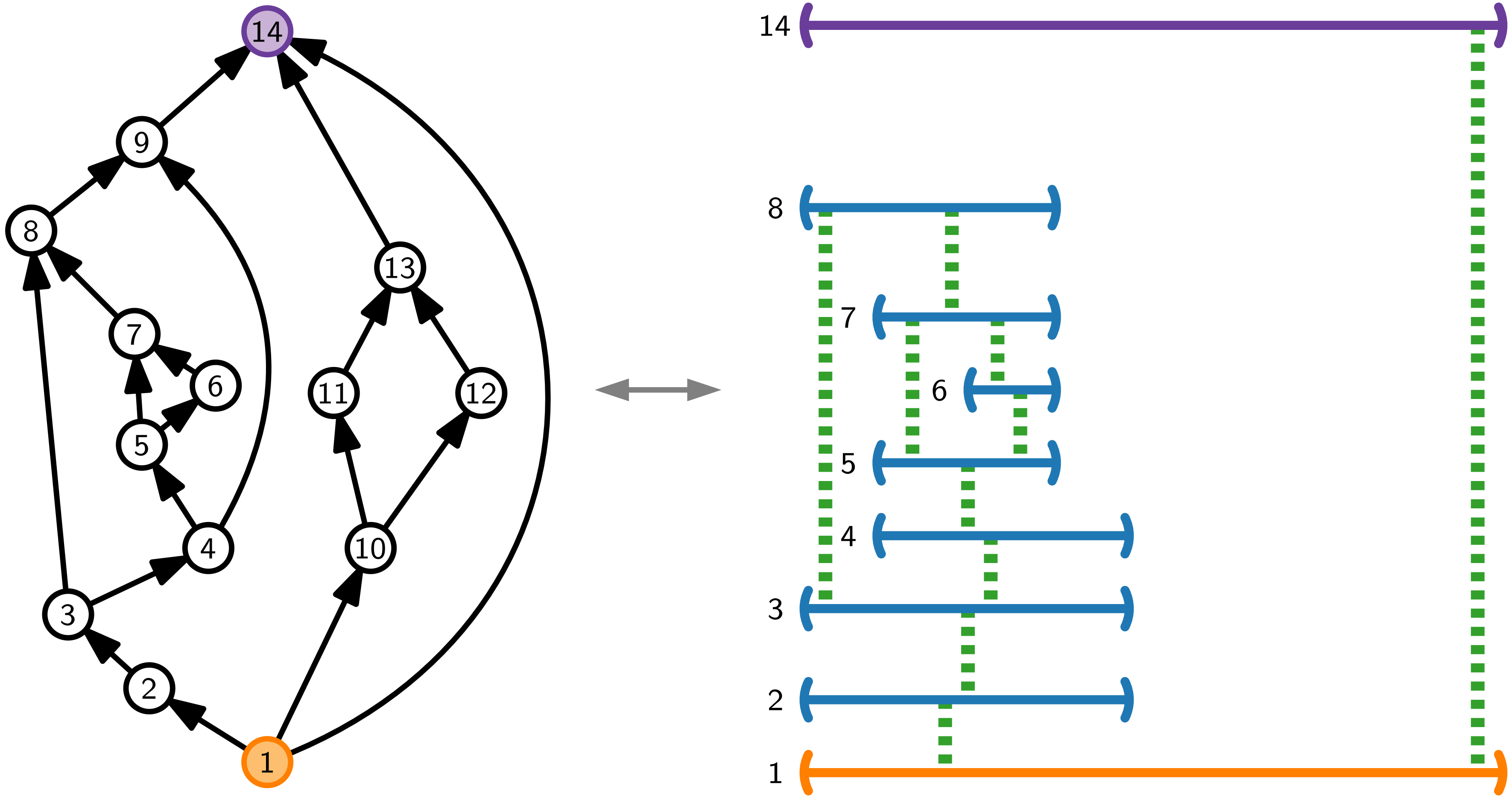

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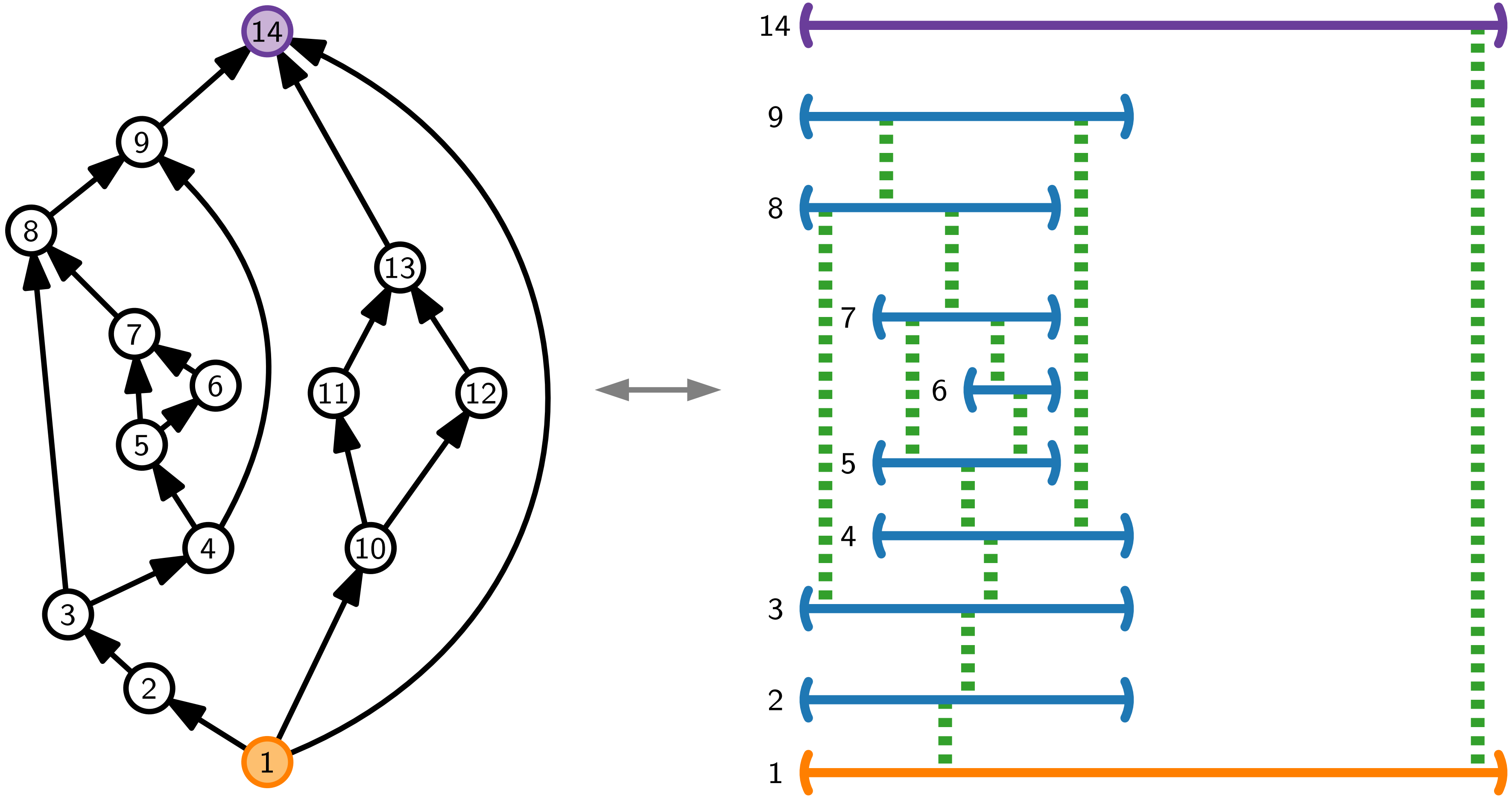

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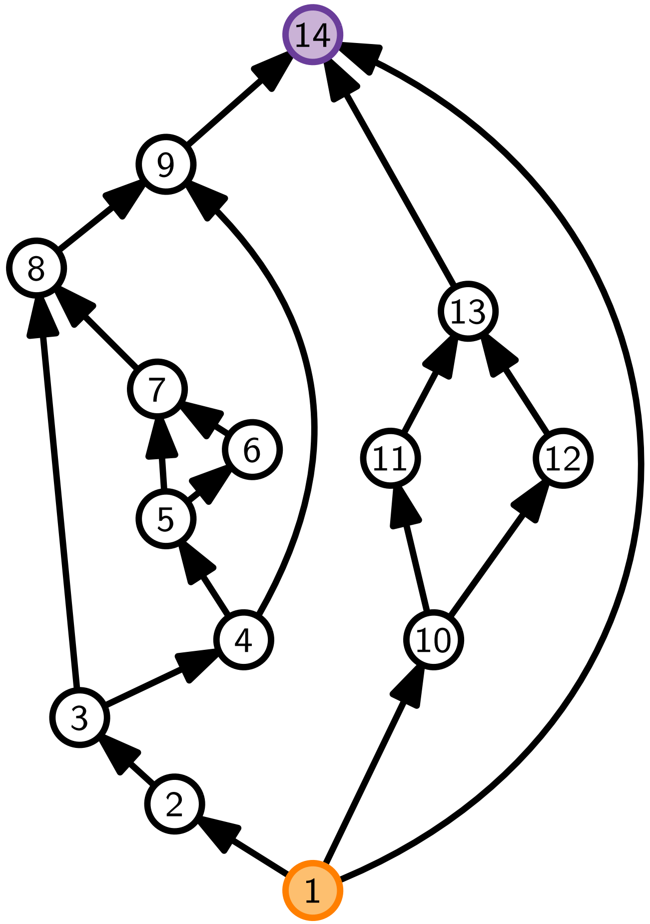

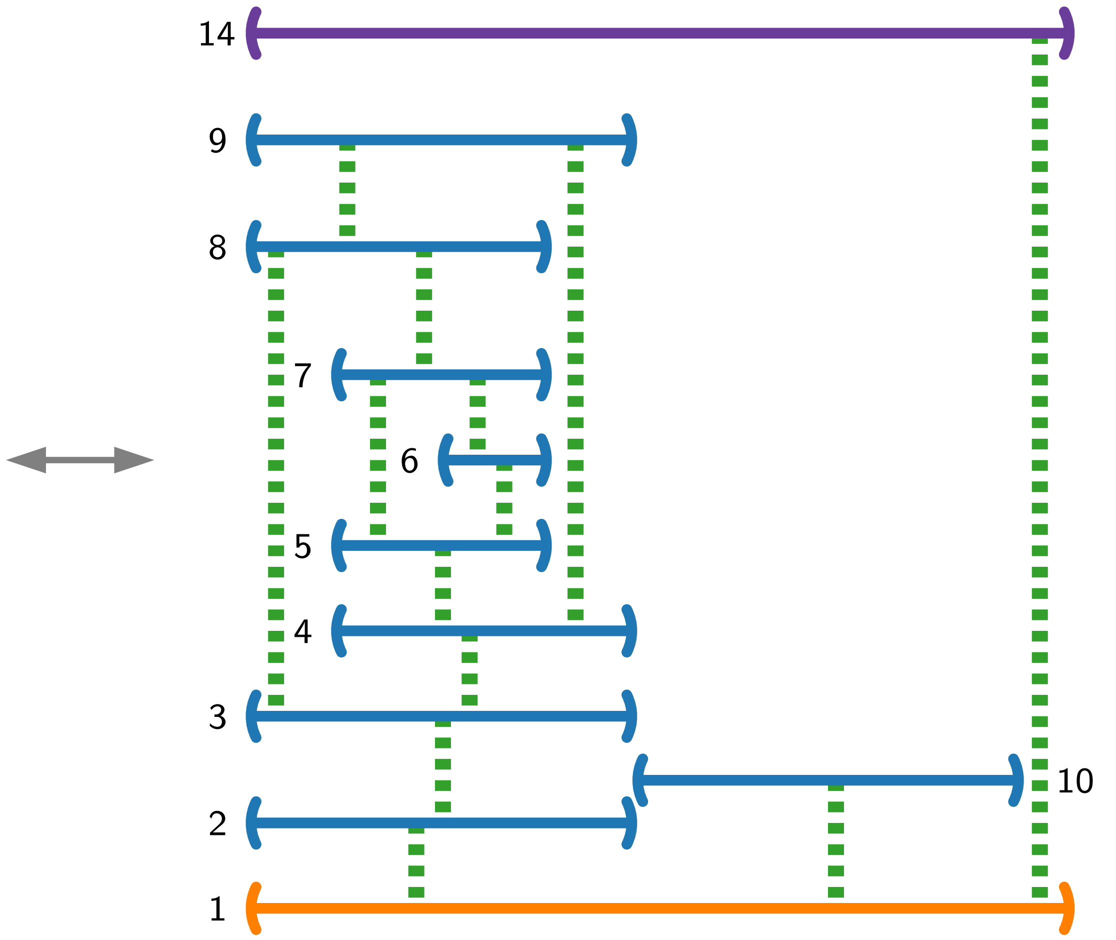

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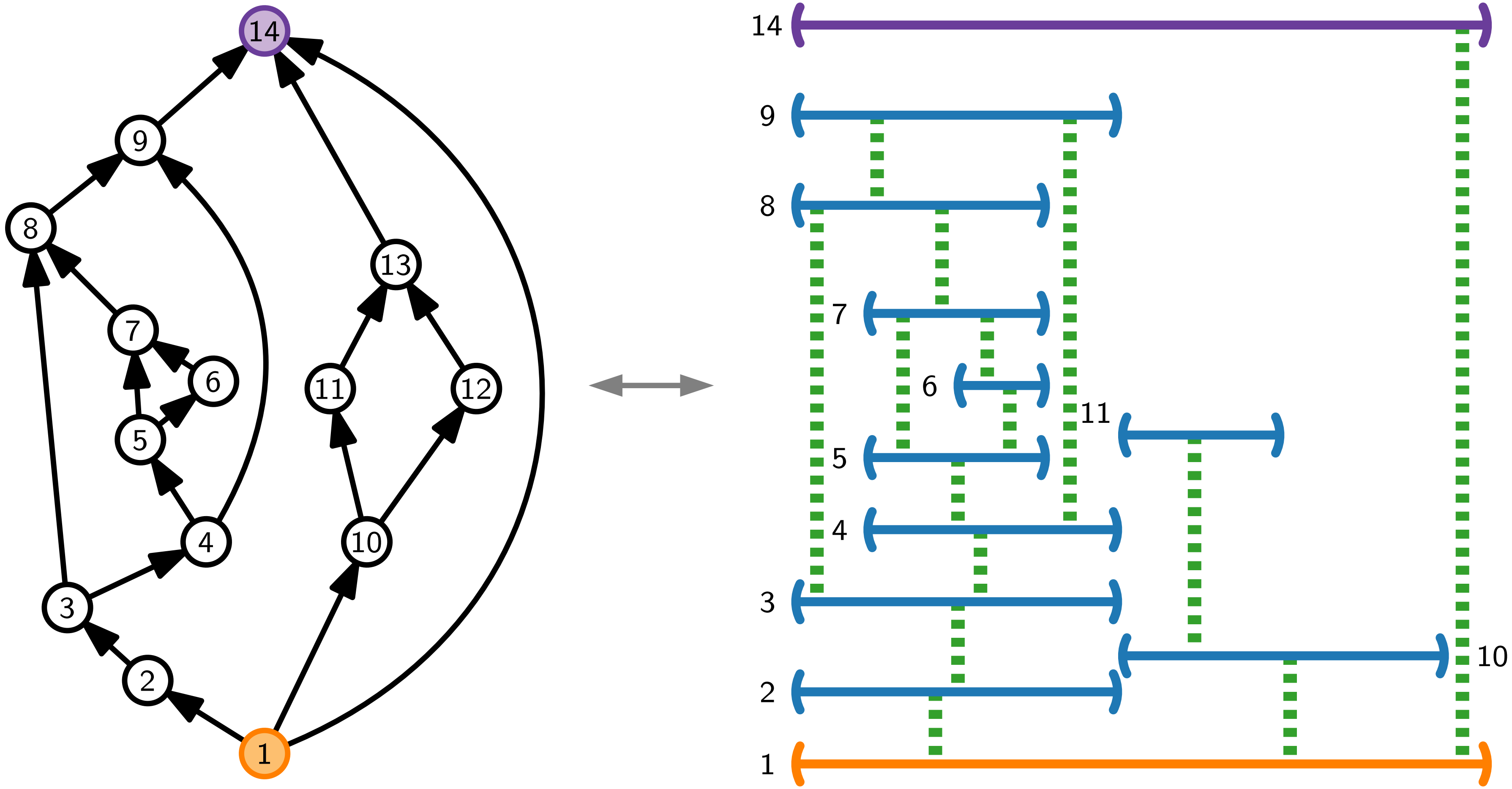

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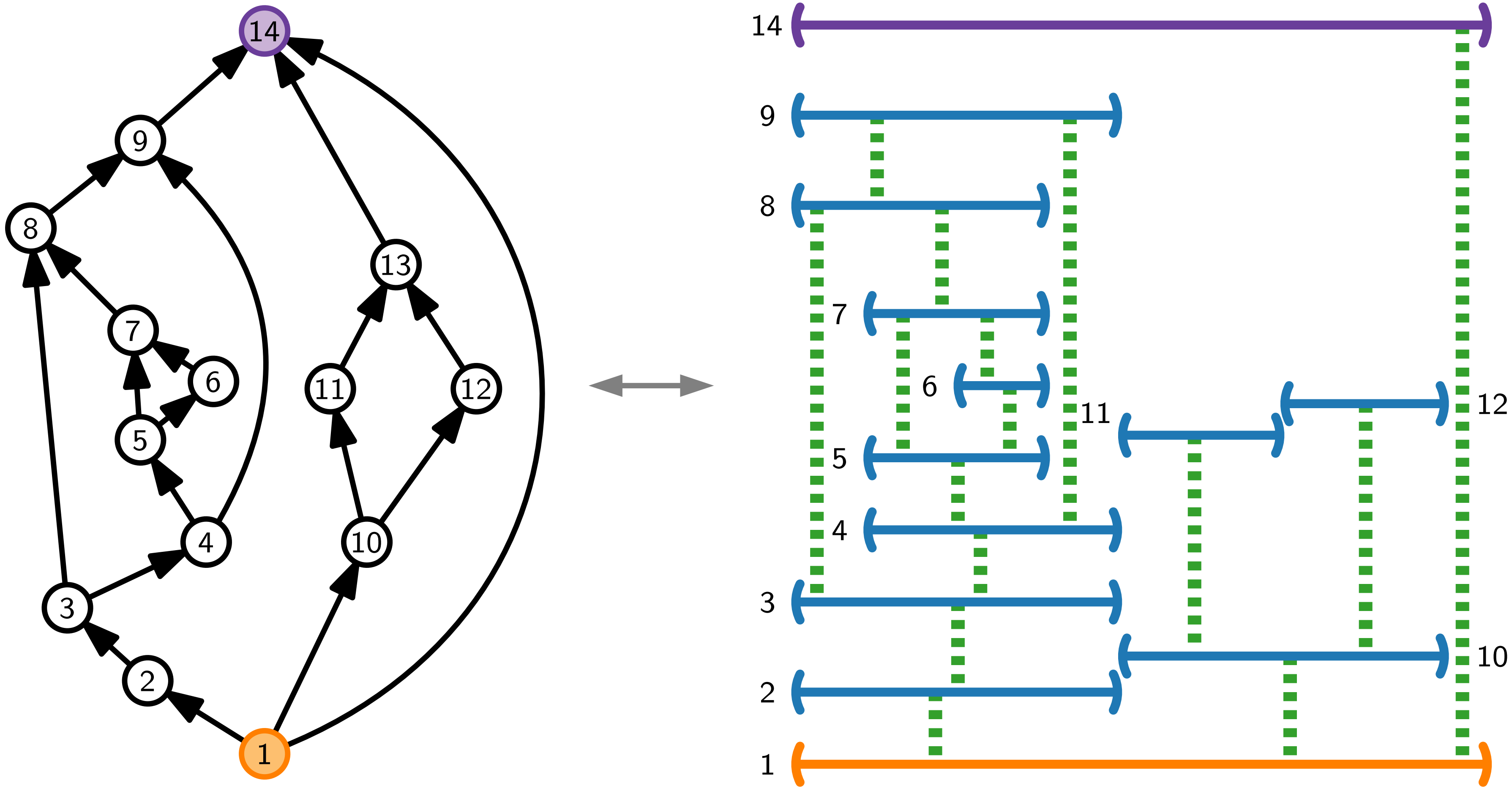

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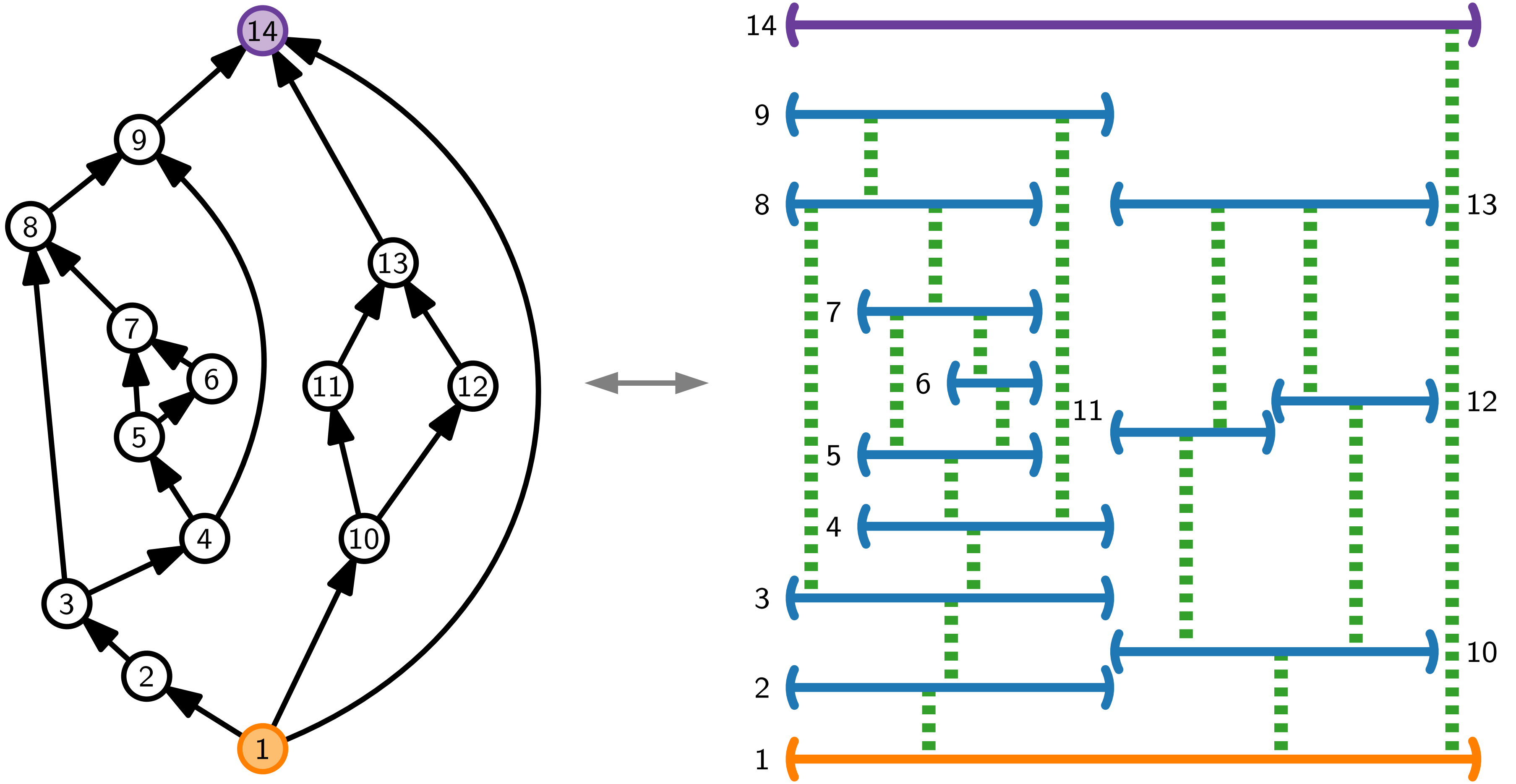

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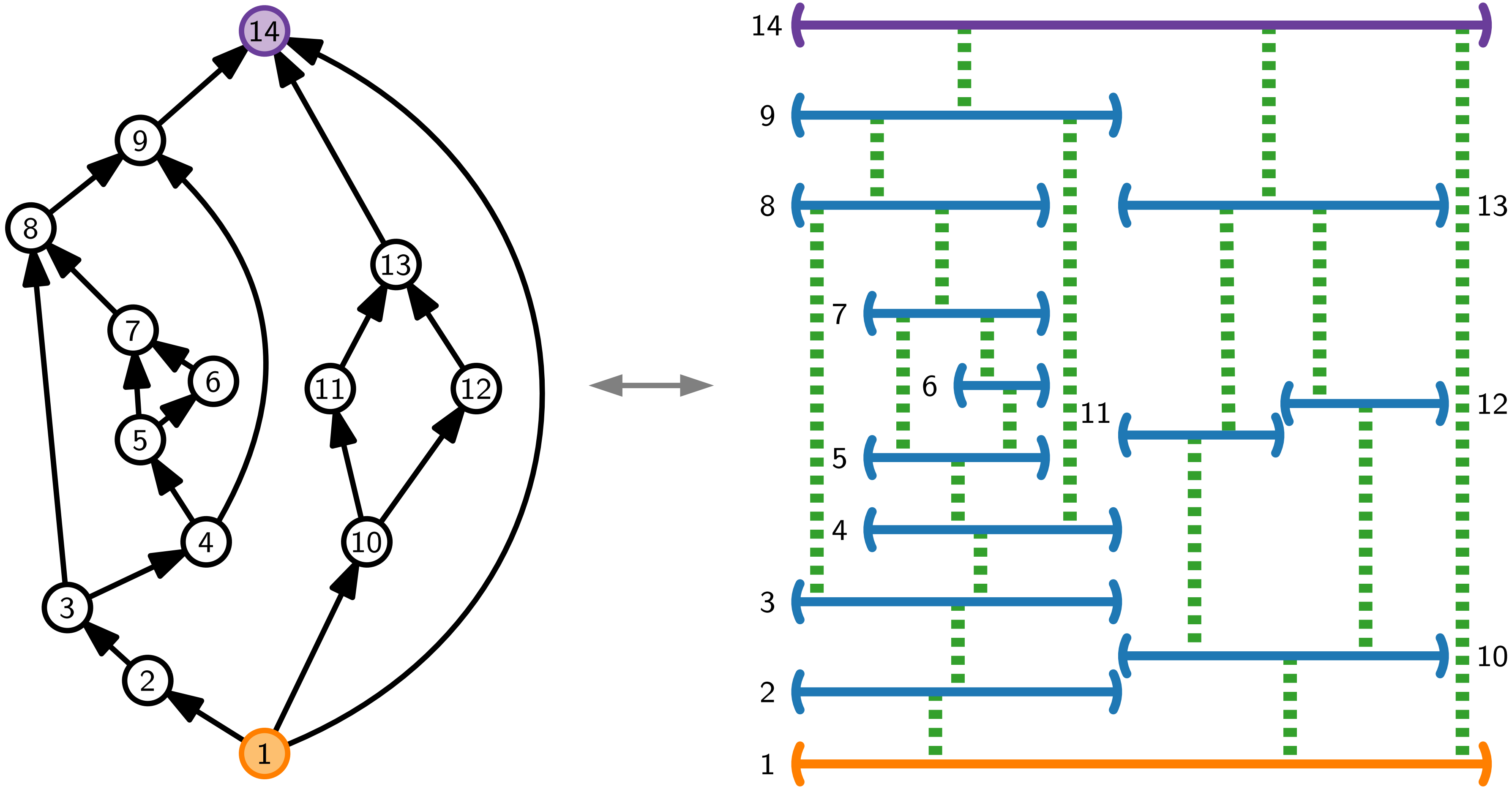

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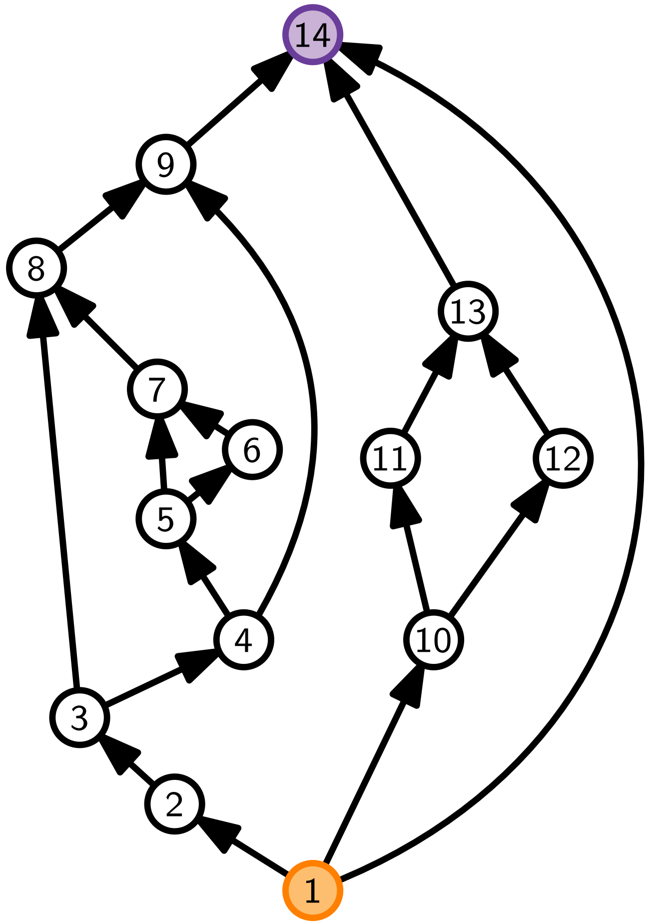

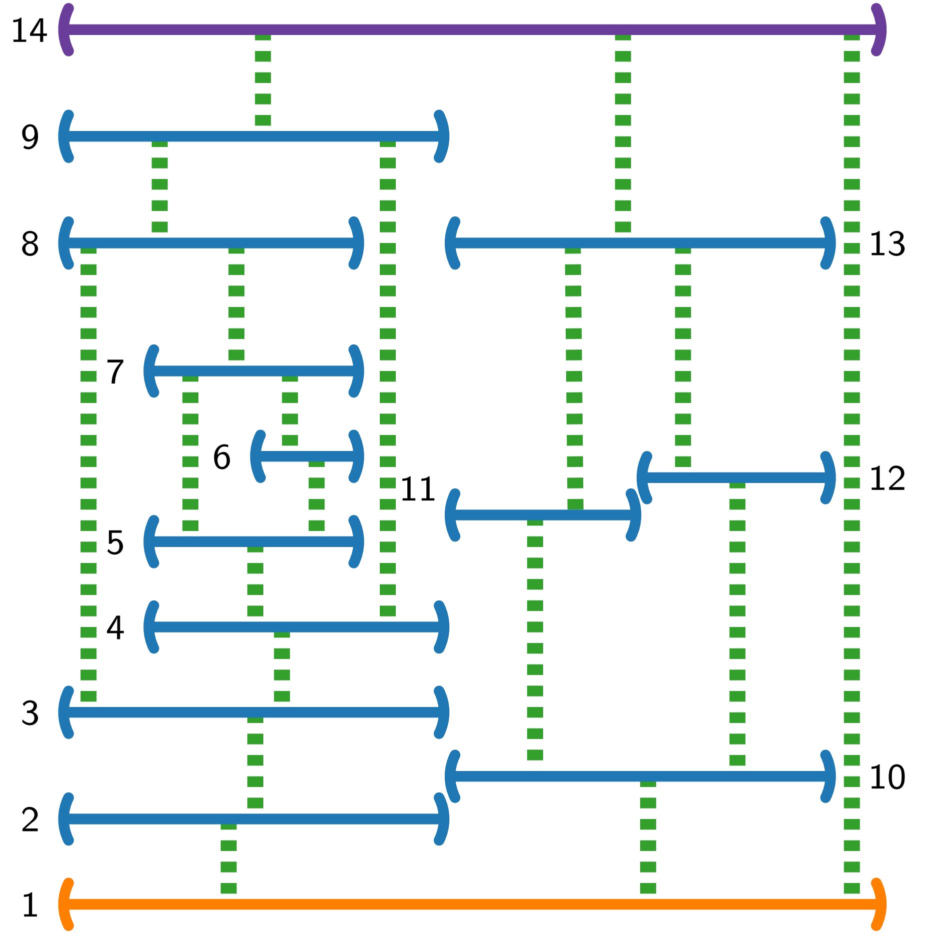

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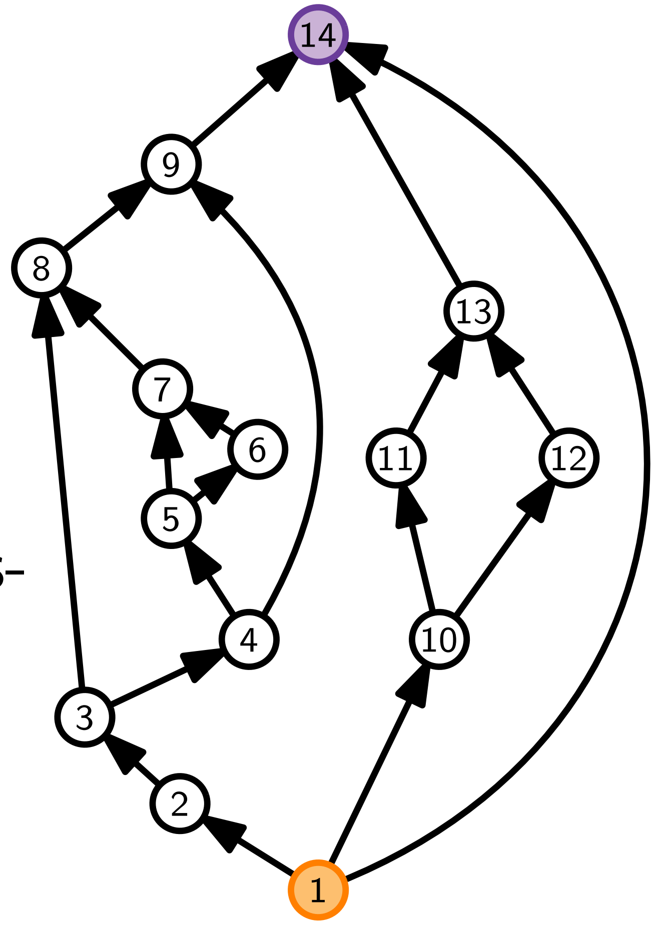
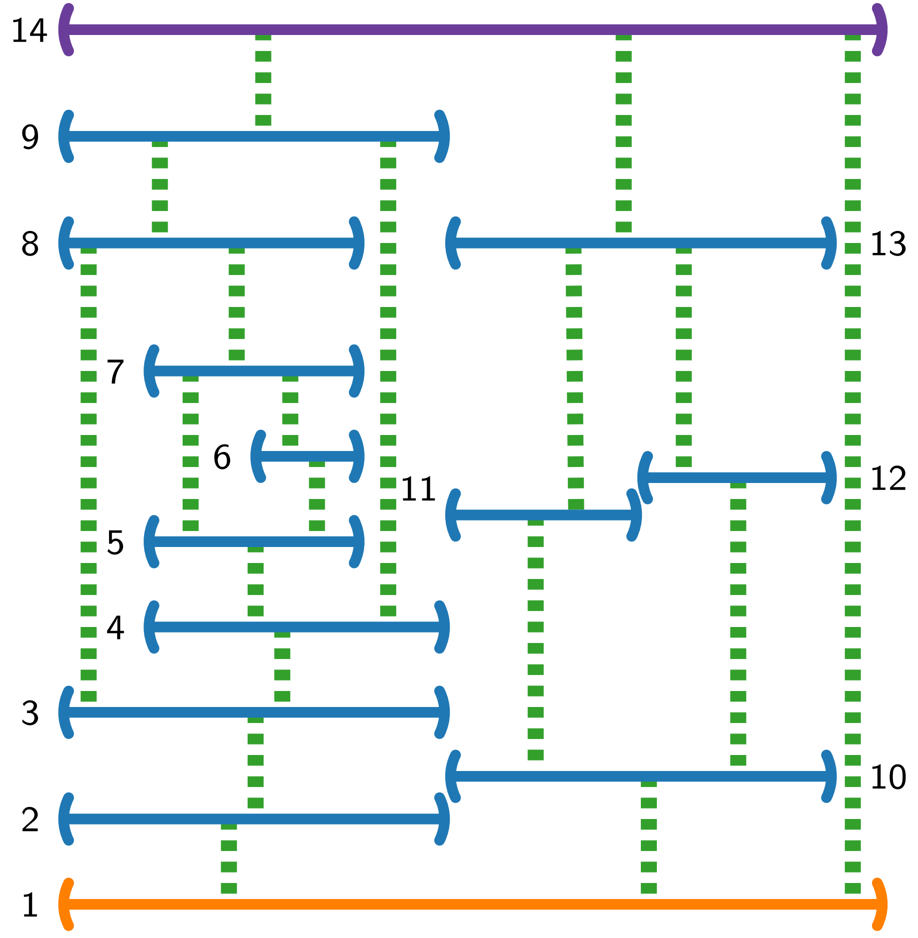
- This is upward planarity testing!
  [Garg & Tamassia '01]





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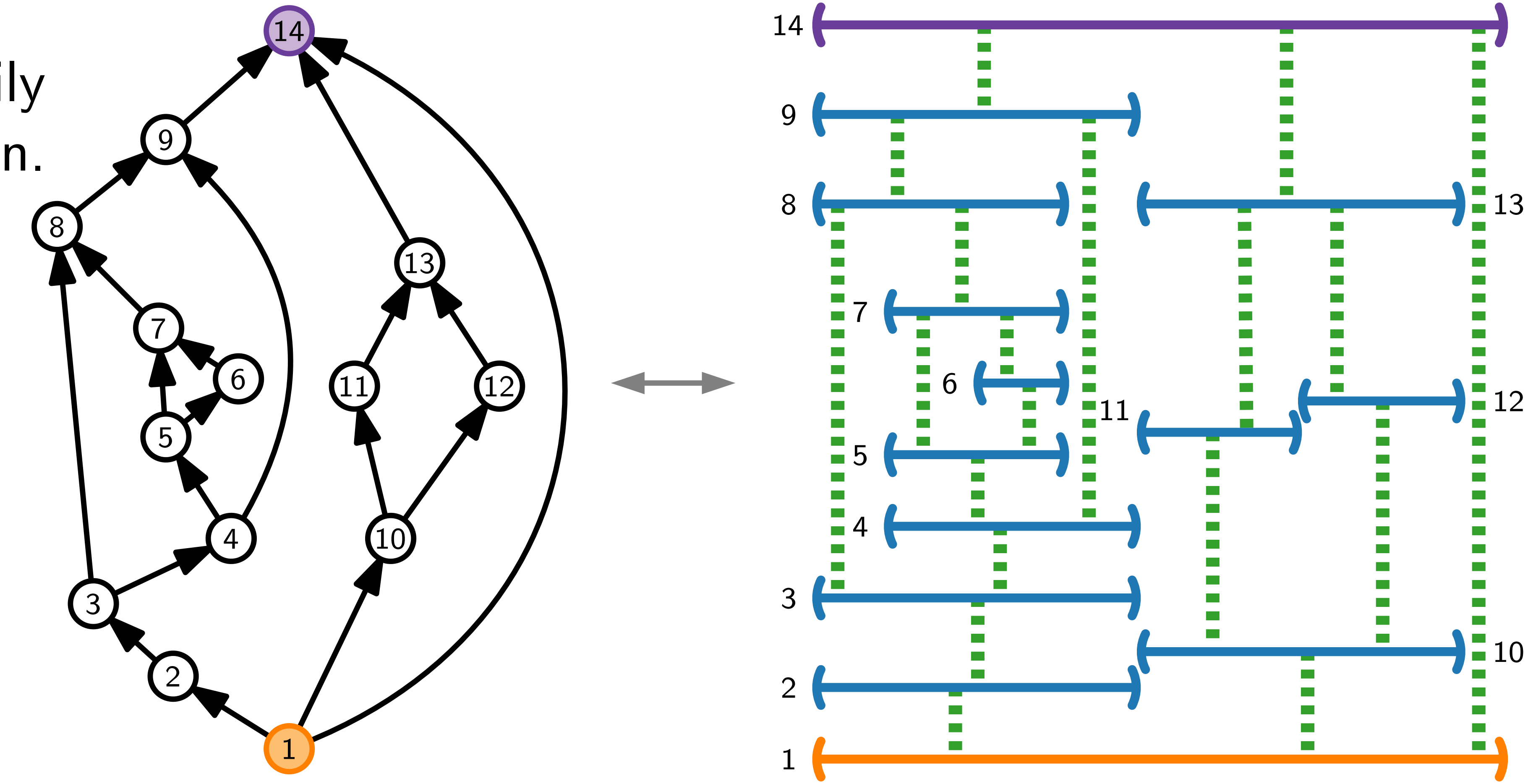

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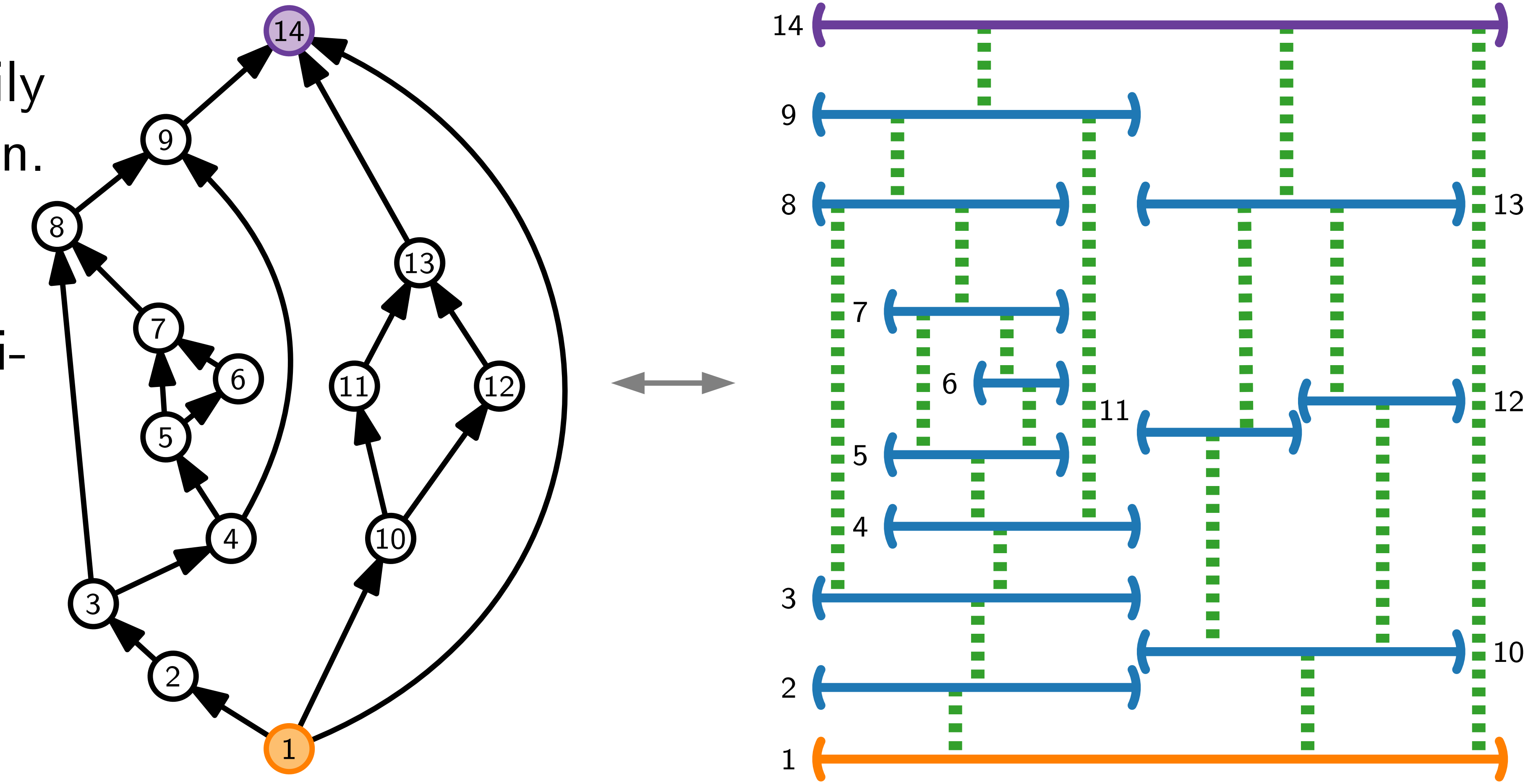

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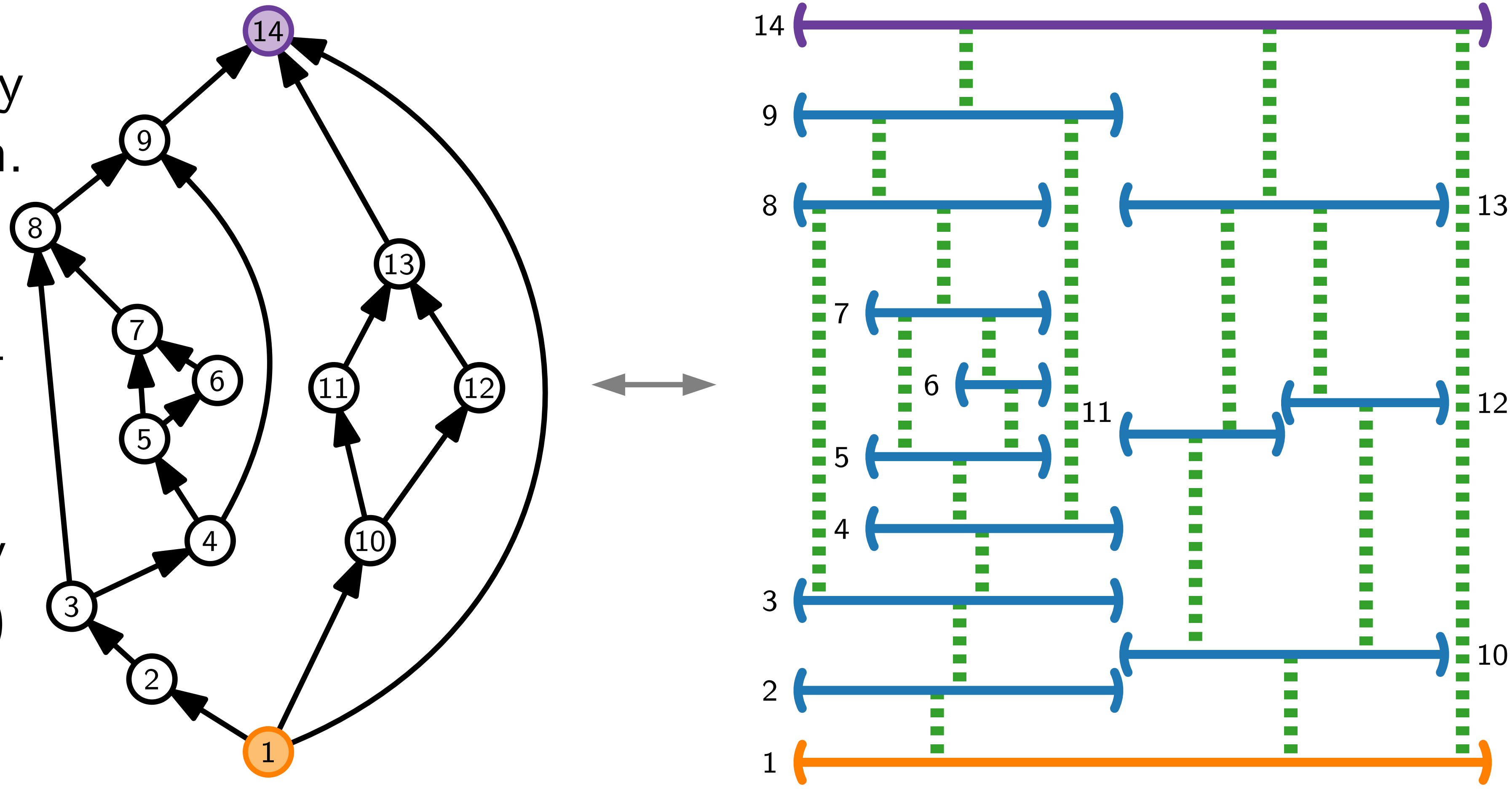

Results and Outline

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Lecture 9:
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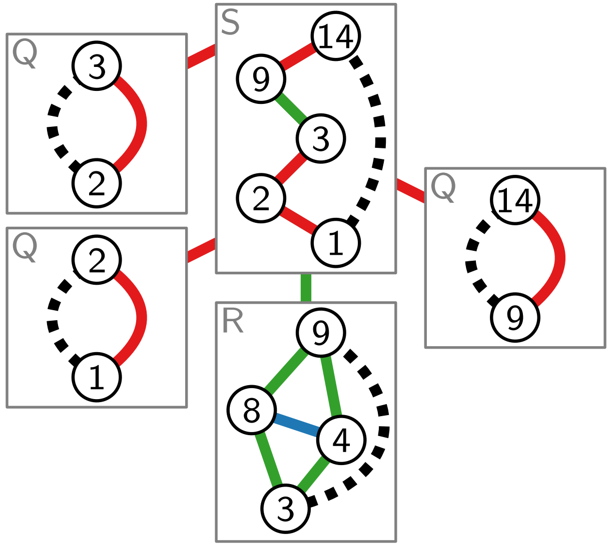
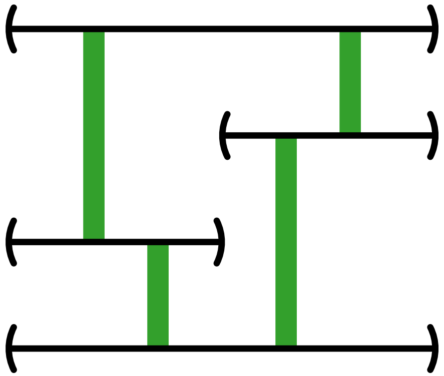


Part III:
SPQR-Trees



Jonathan Klawitter

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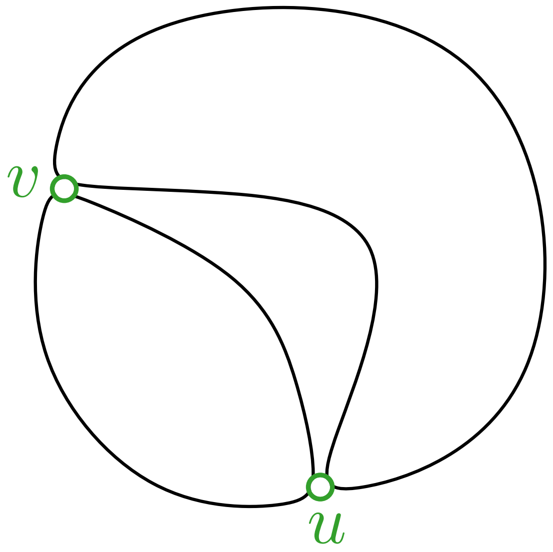

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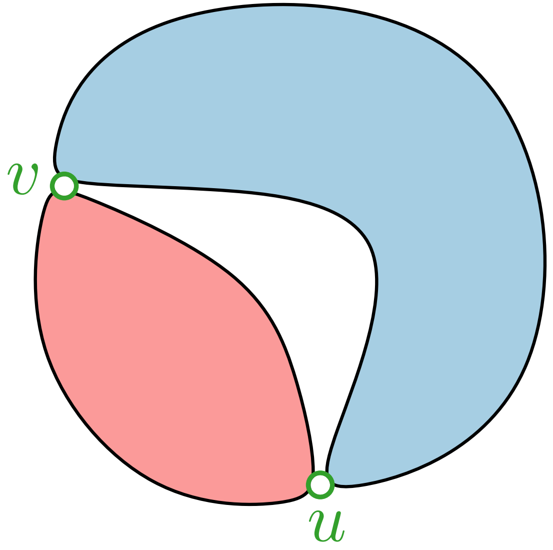

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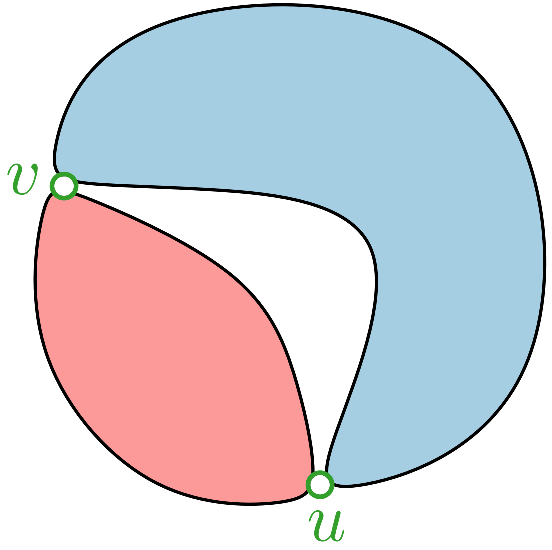

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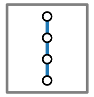

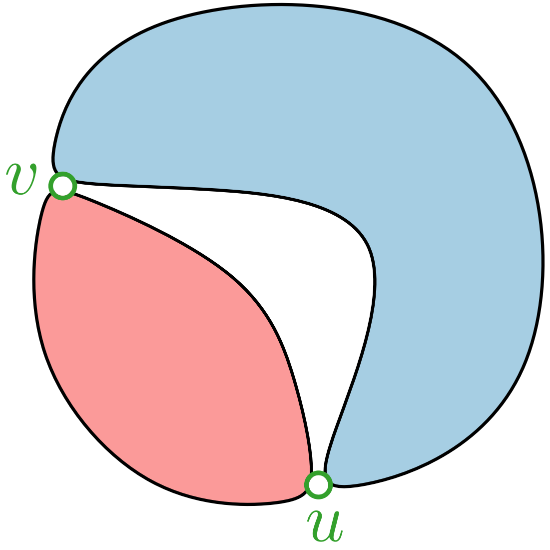

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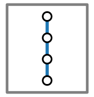
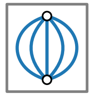
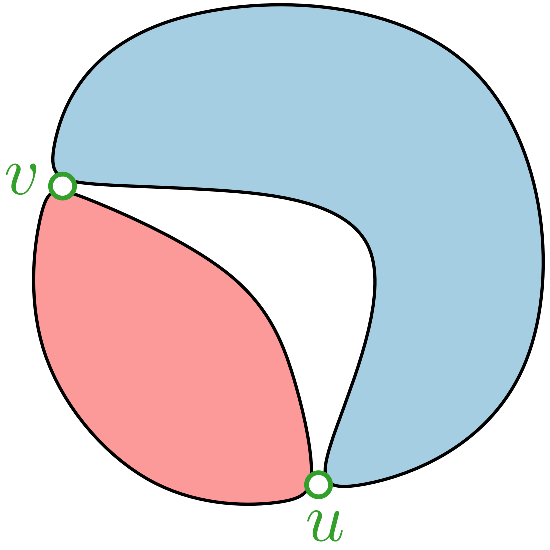
  - **S** nodes represent a series composition 
  - **P** nodes represent a parallel composition 



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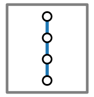
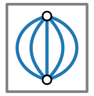
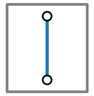
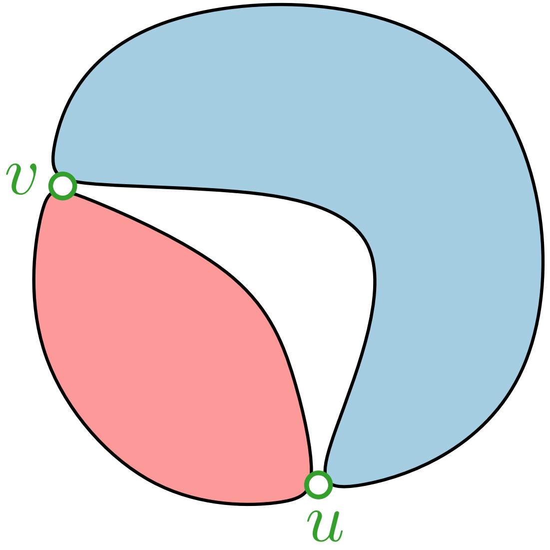
  - **S** nodes represent a series composition 
  - **P** nodes represent a parallel composition 
  - **Q** nodes represent a single edge 



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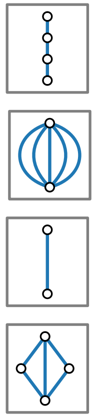

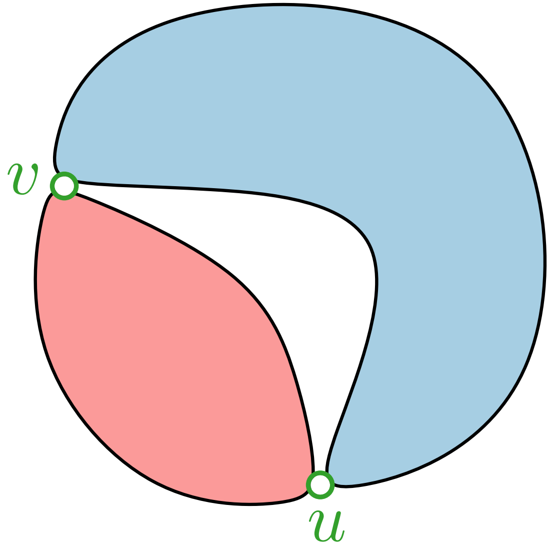

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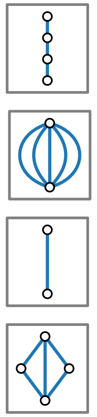

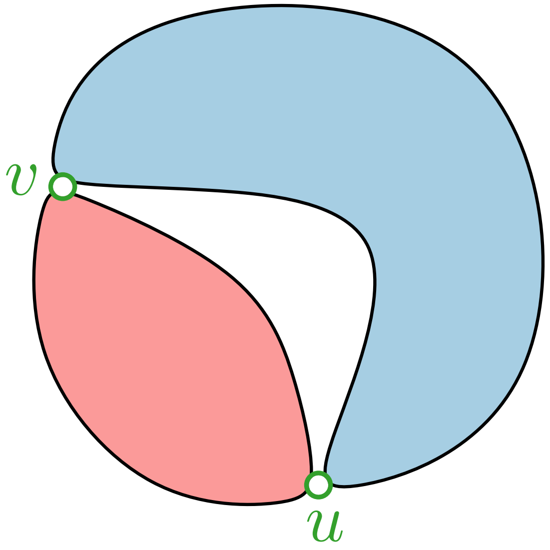

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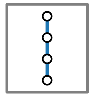
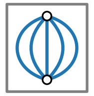
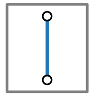
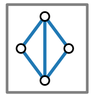
  - **S** nodes represent a series composition 
  - **P** nodes represent a parallel composition 
  - **Q** nodes represent a single edge 
  - **R** nodes represent 3-connected (*rigid*) subgraphs 
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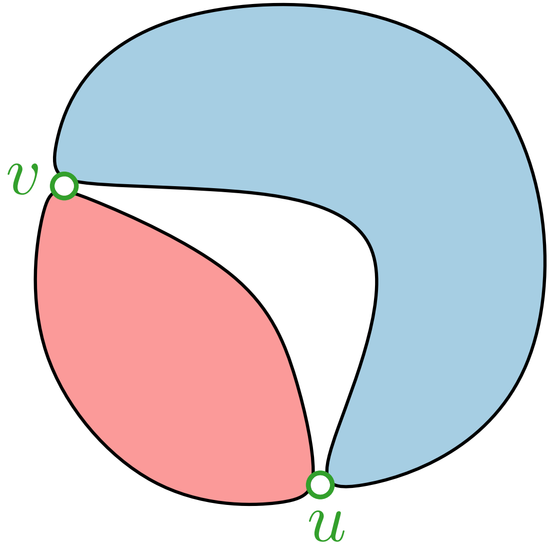

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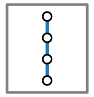
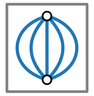
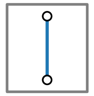
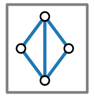
  - **S** nodes represent a series composition 
  - **P** nodes represent a parallel composition 
  - **Q** nodes represent a single edge 
  - **R** nodes represent 3-connected (*rigid*) subgraphs 
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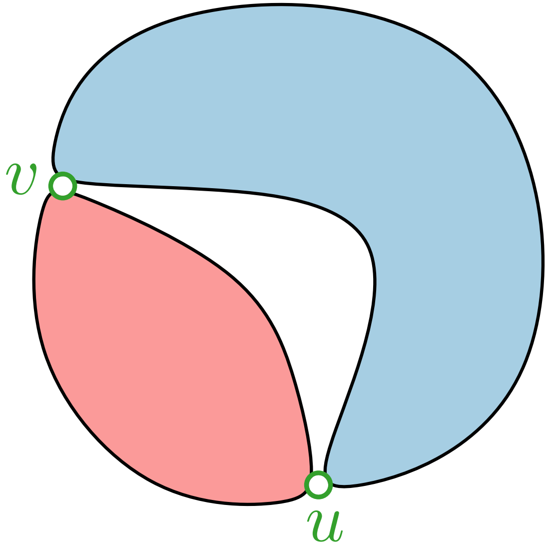

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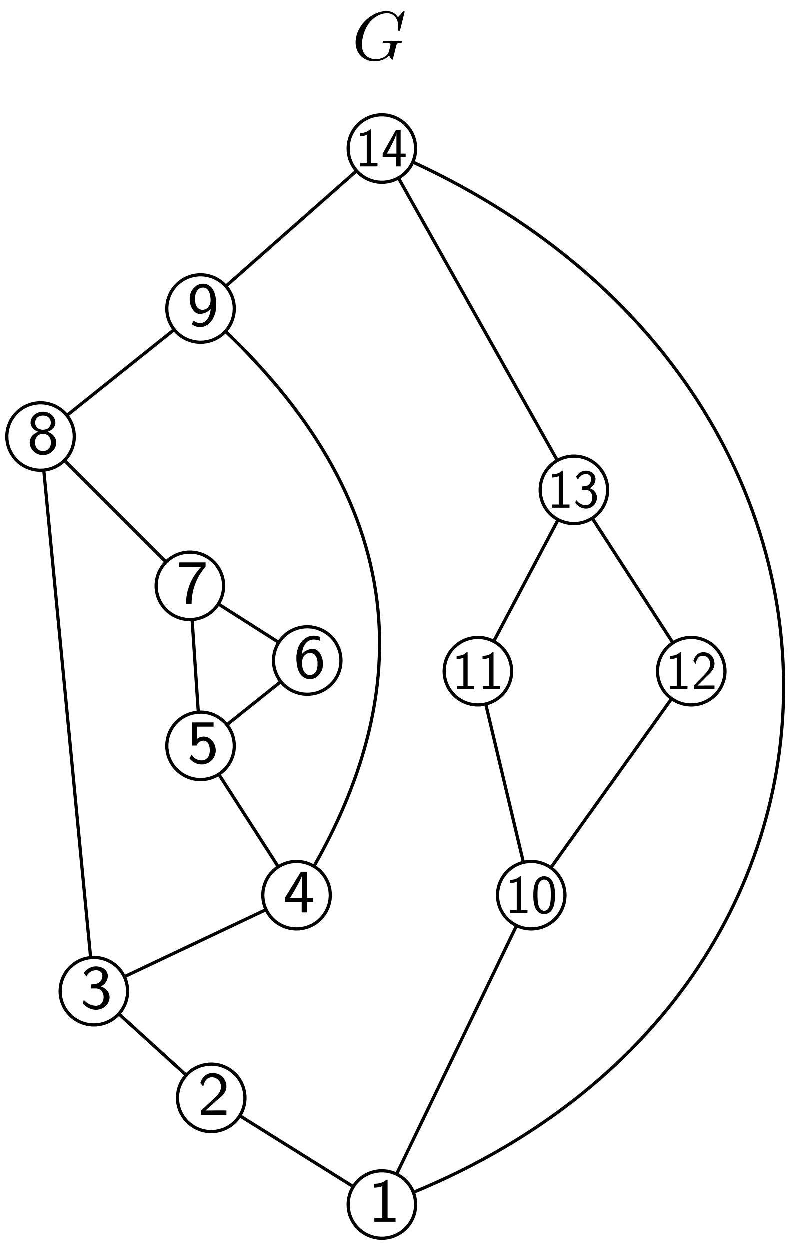

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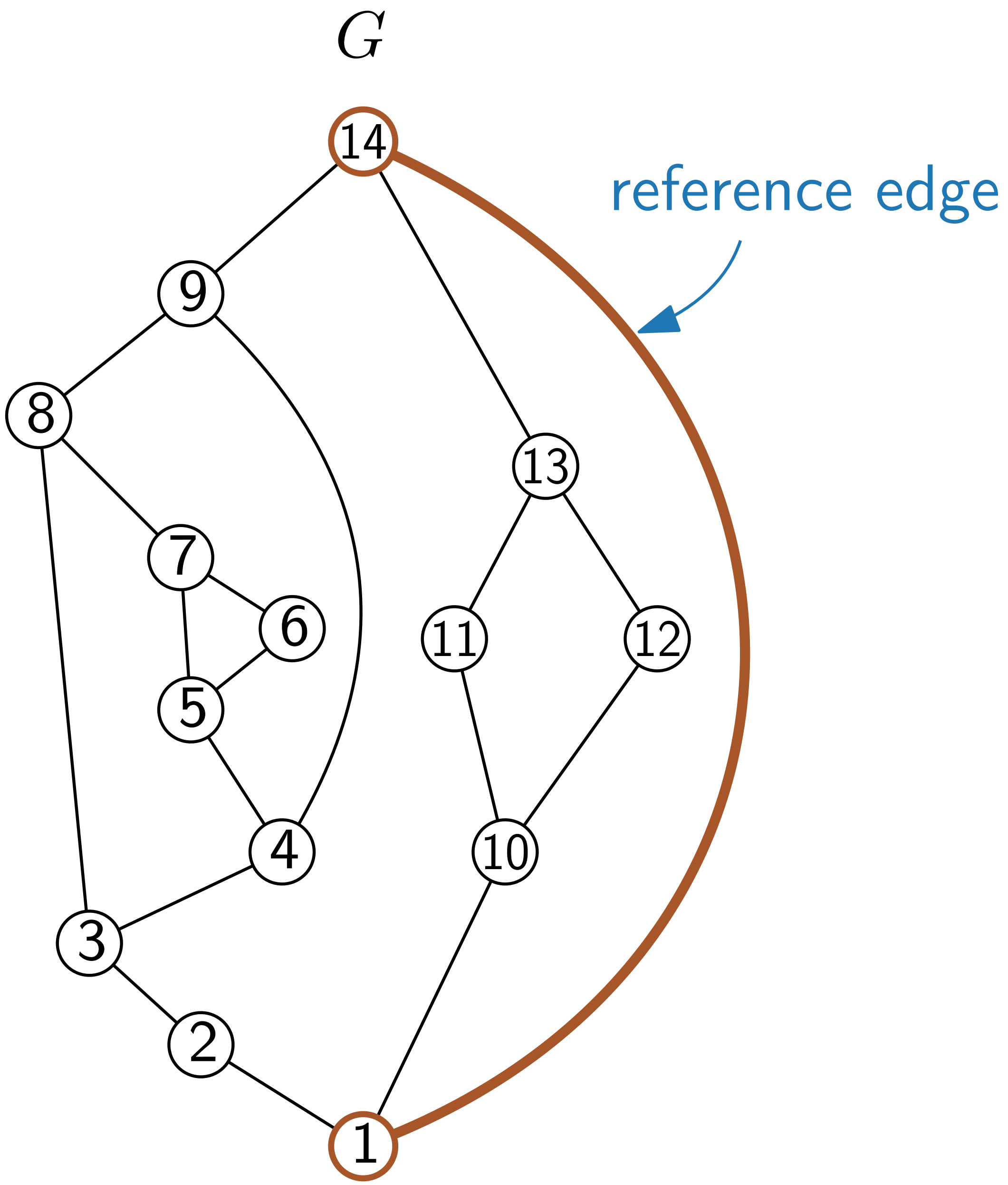

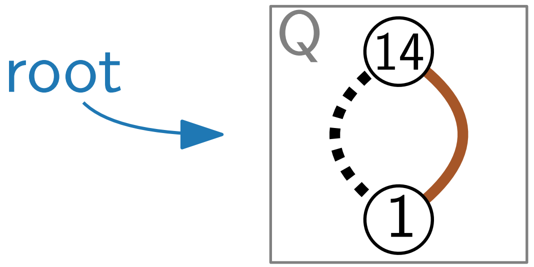

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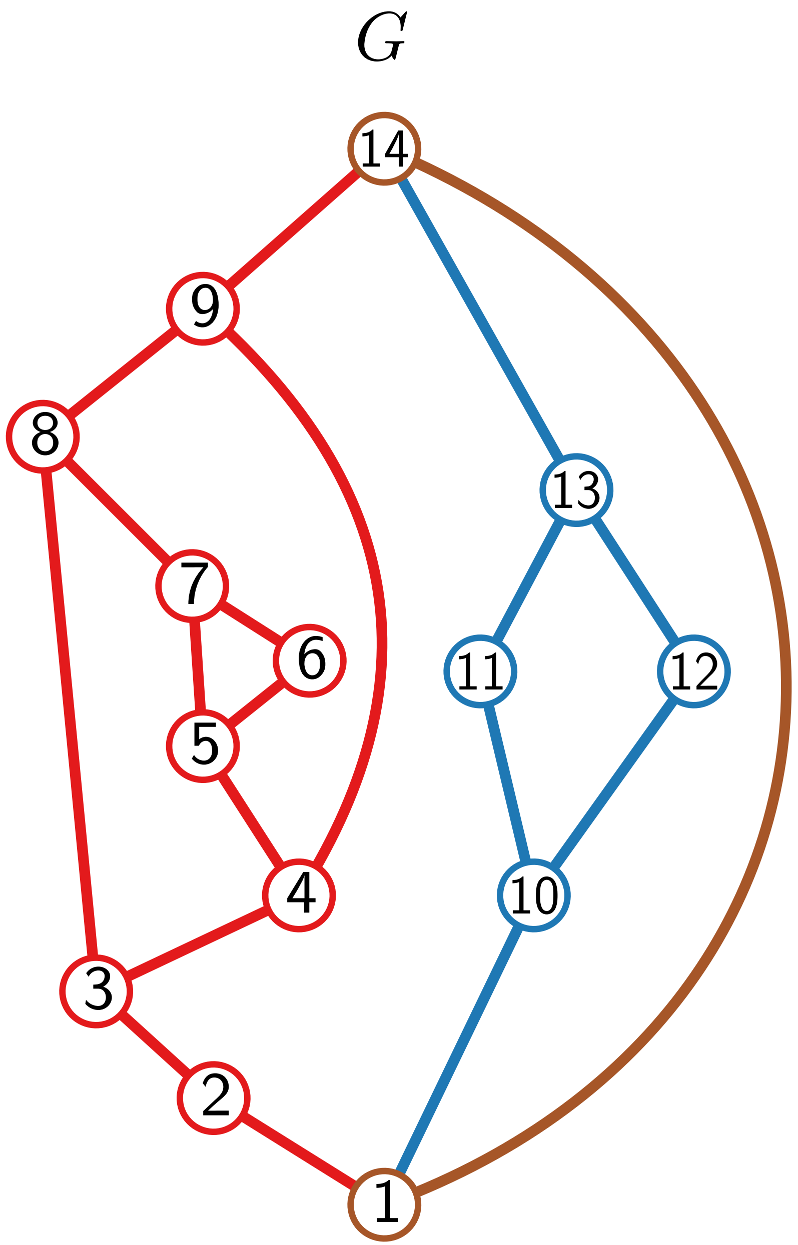

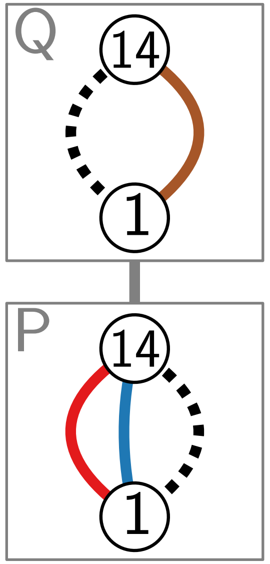

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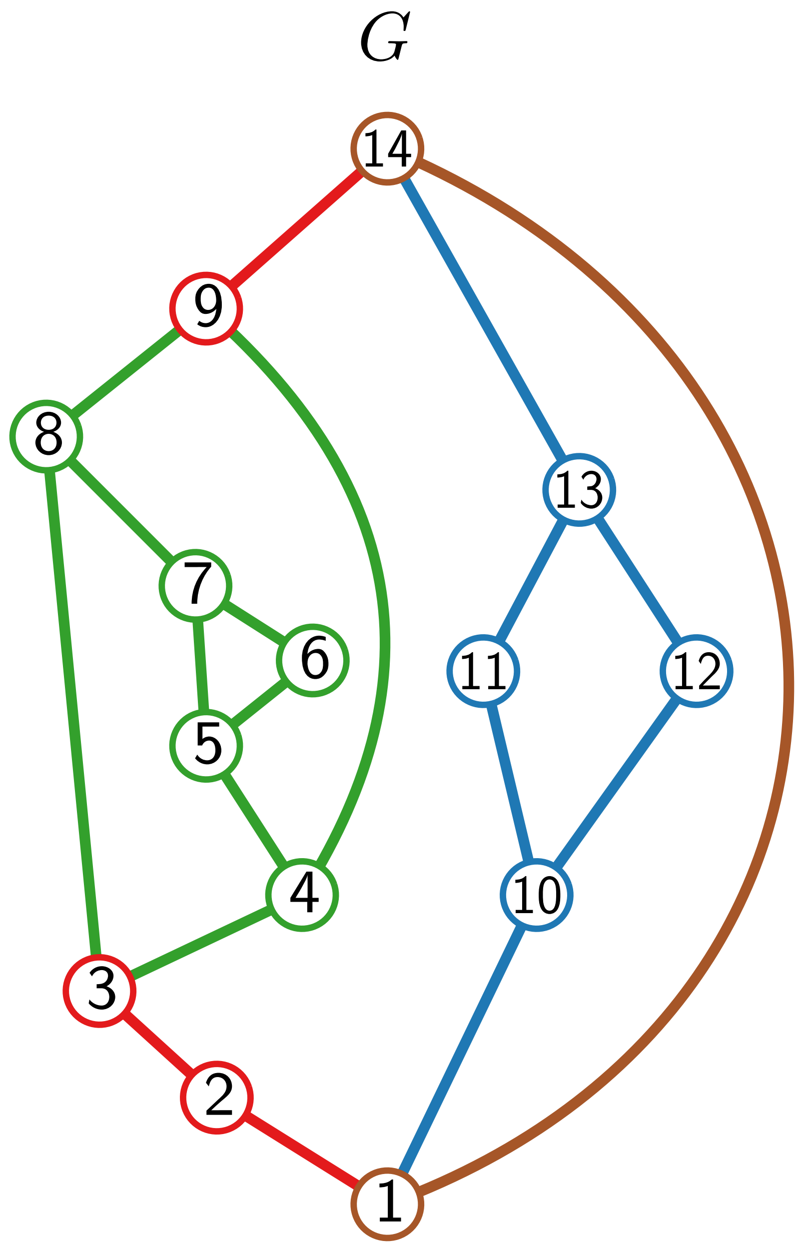

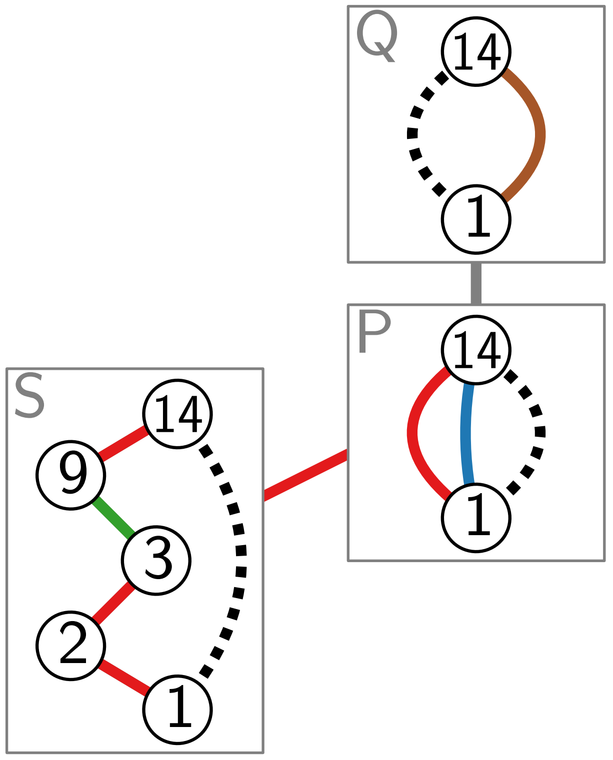

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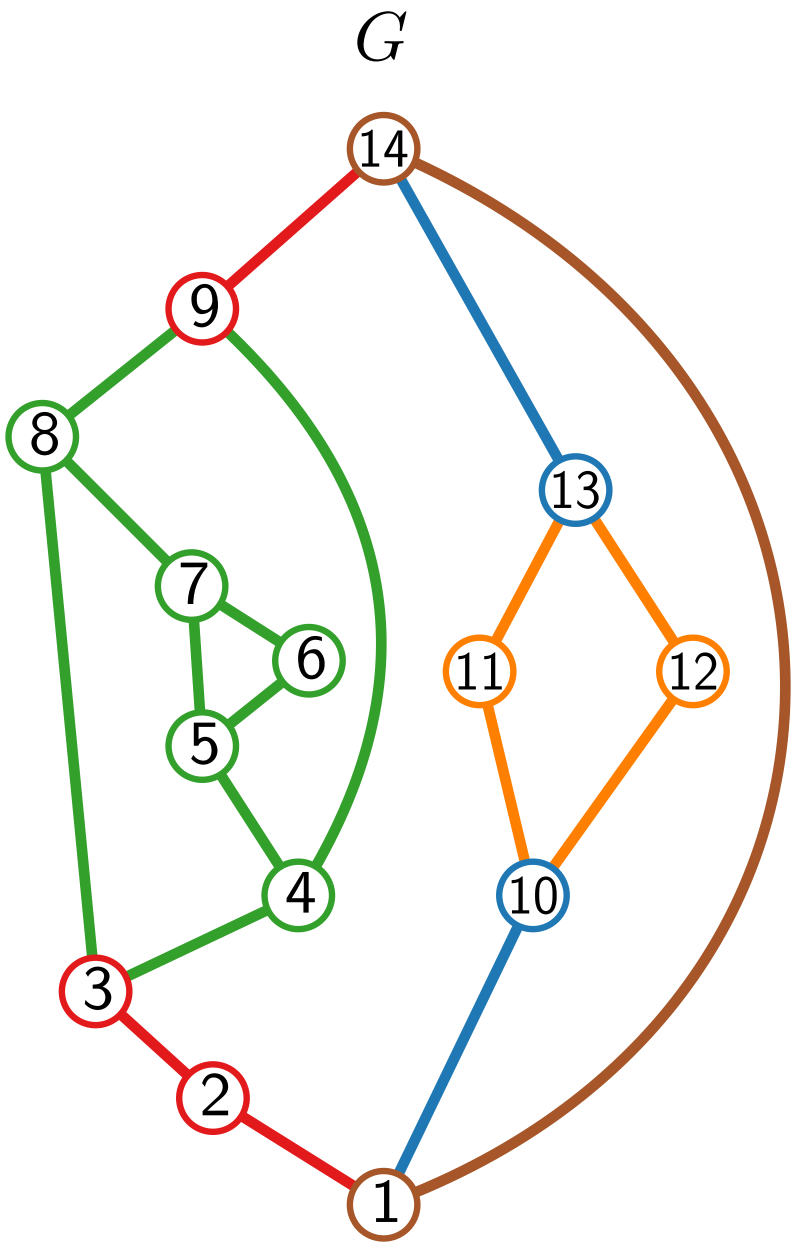

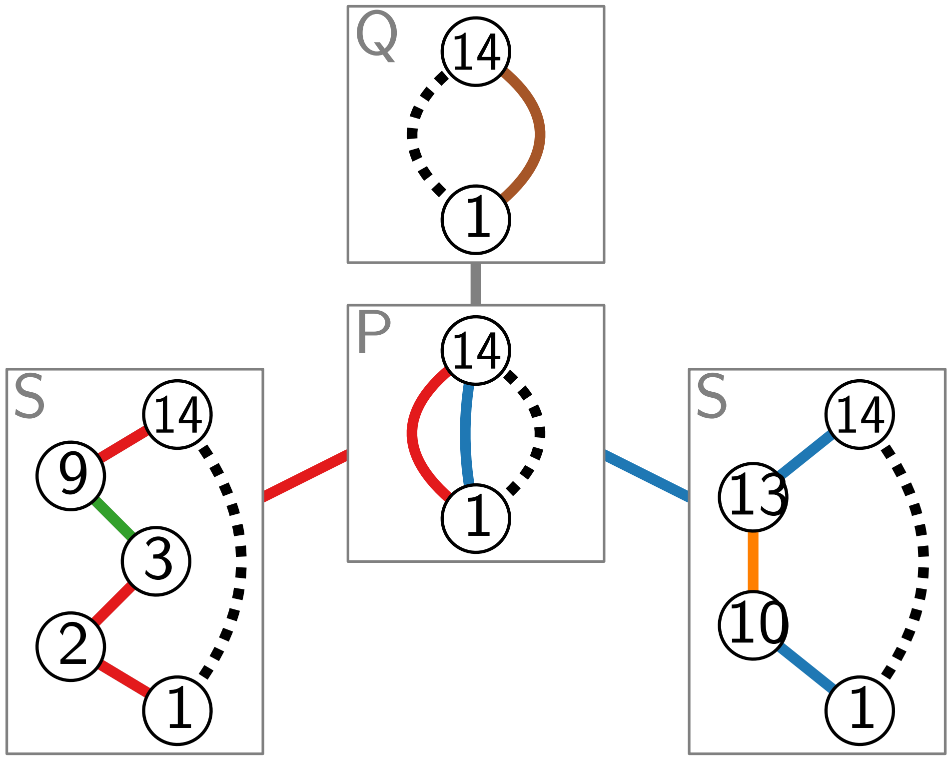

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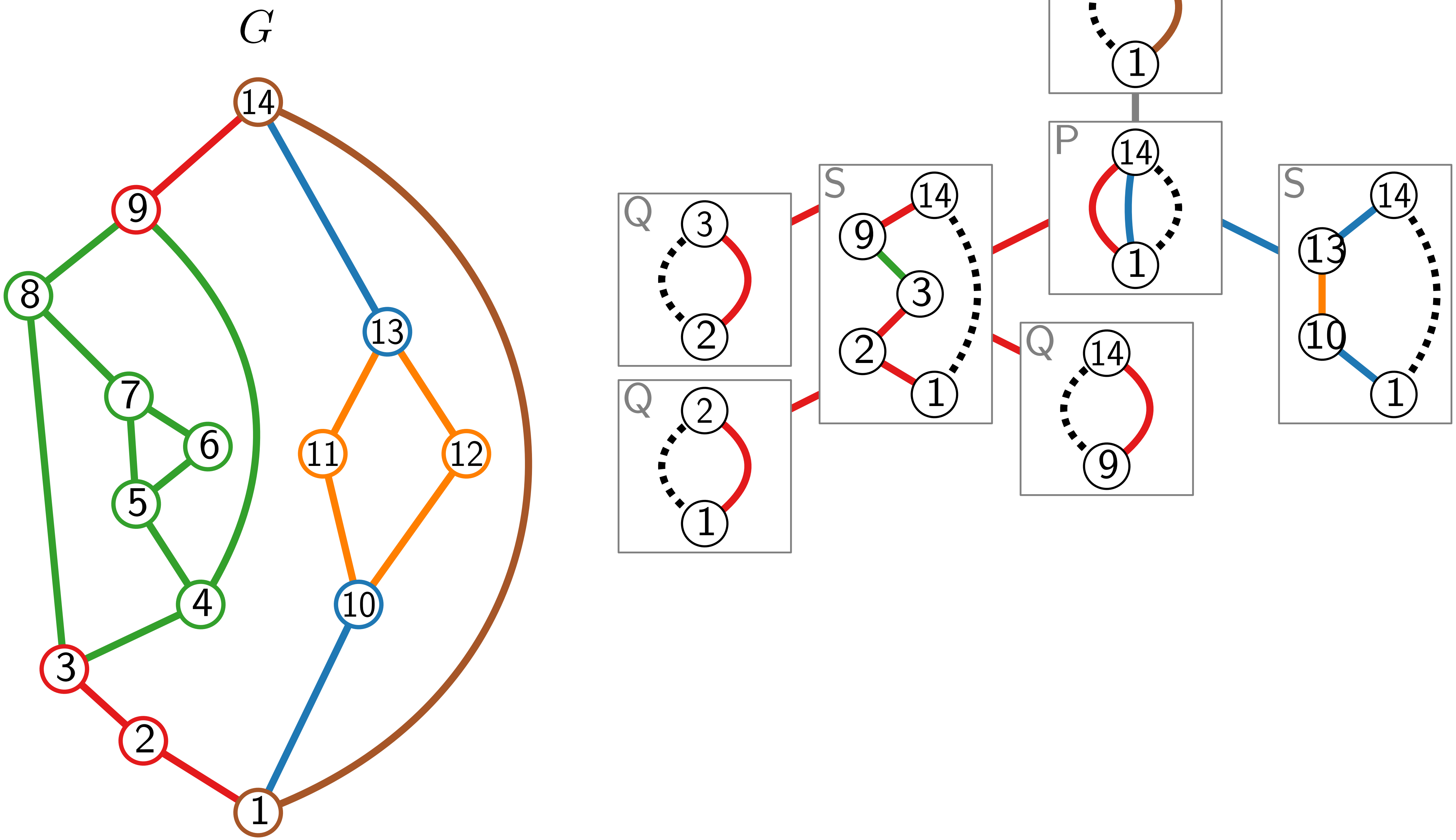

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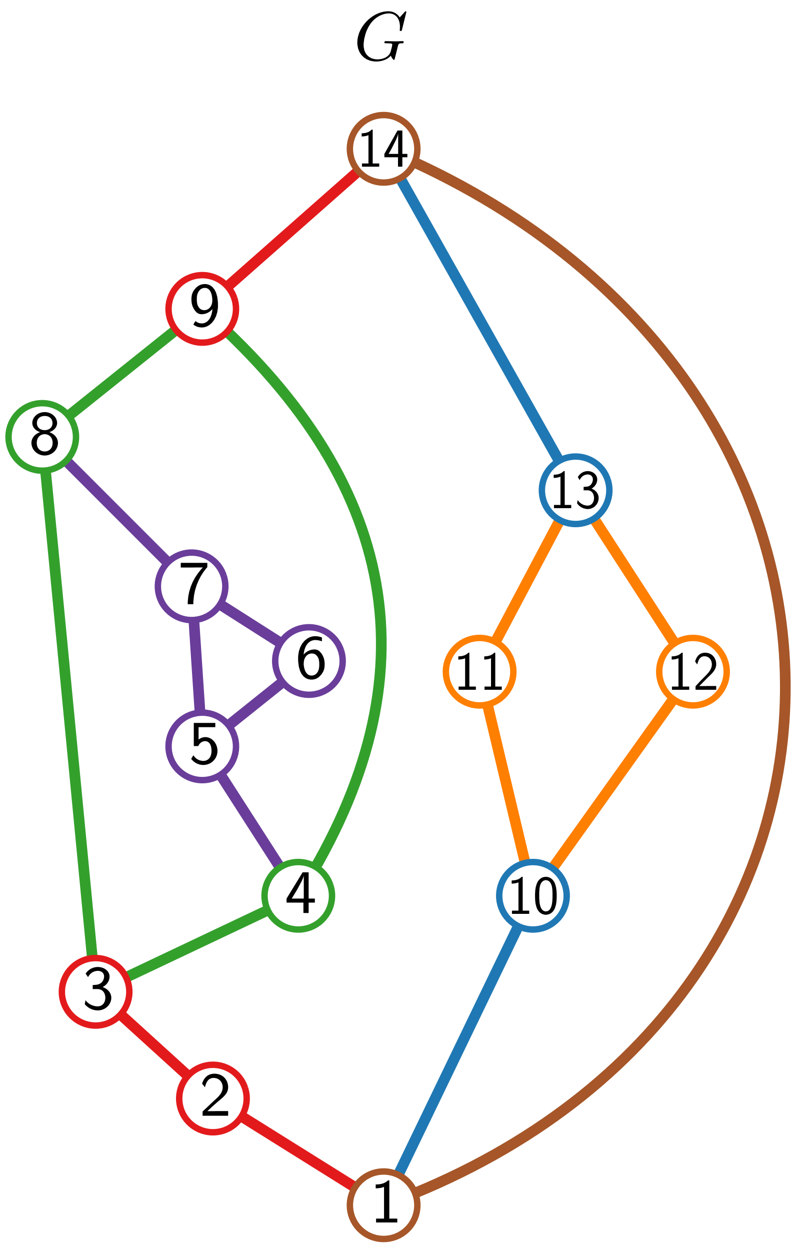

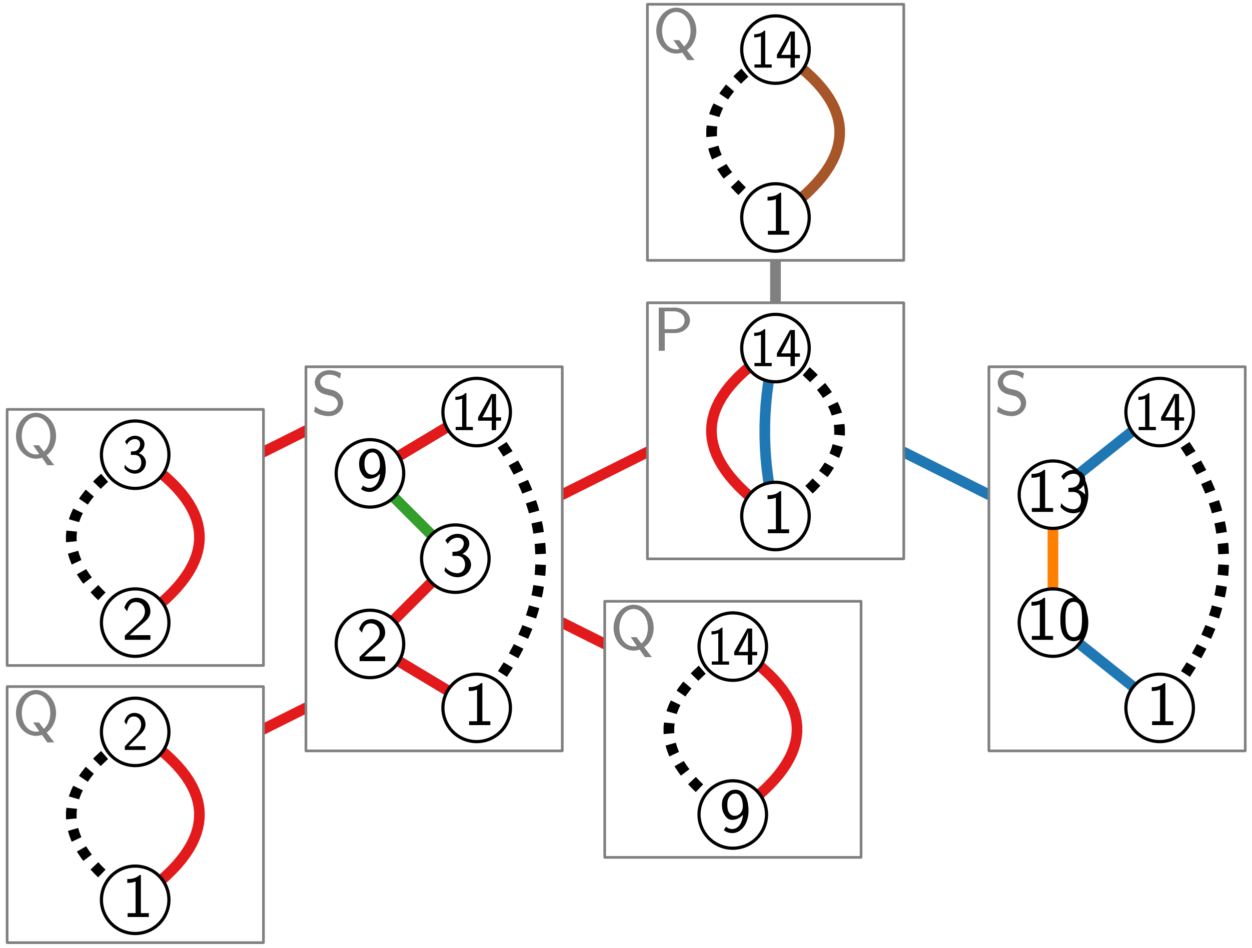

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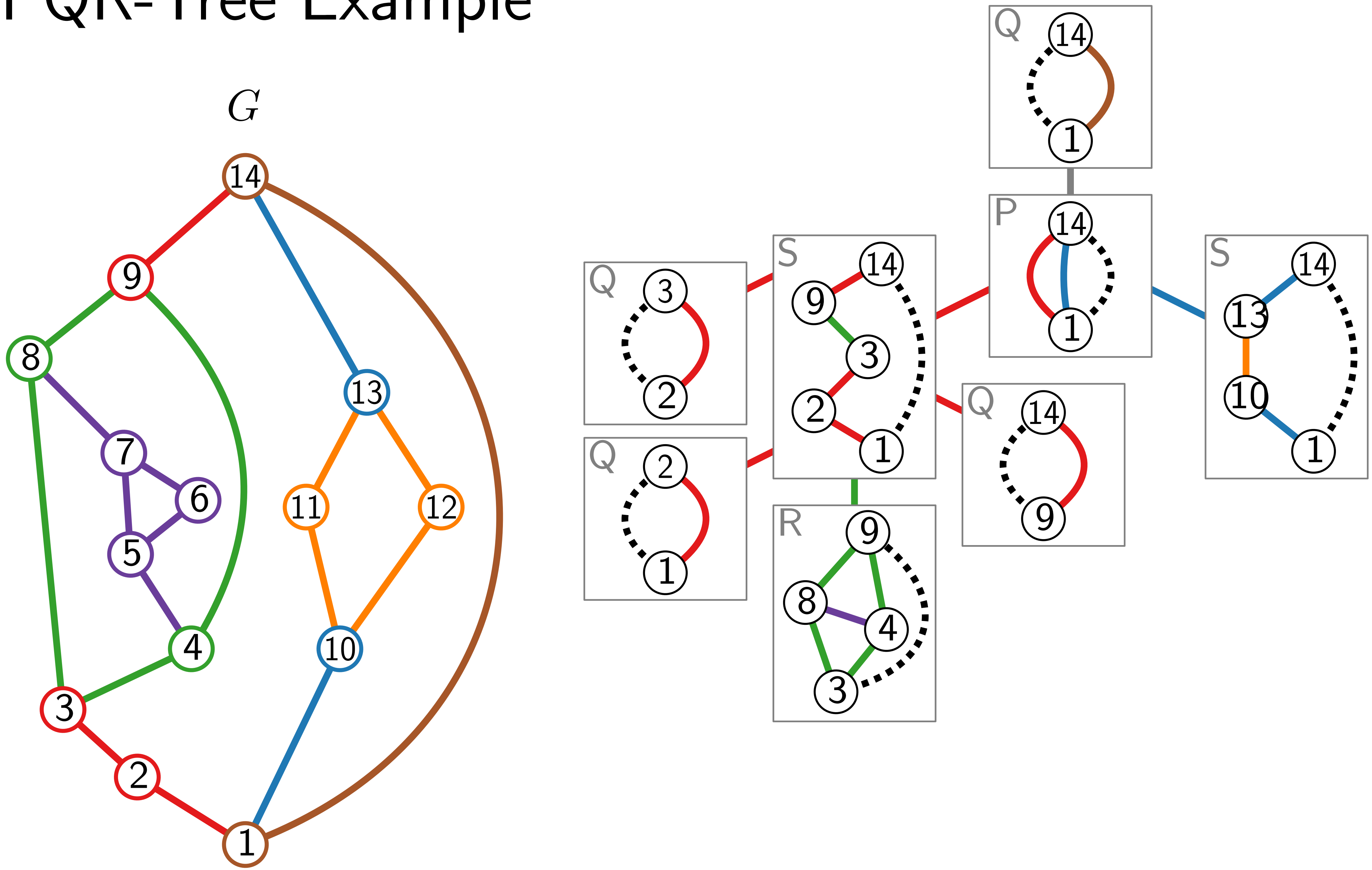

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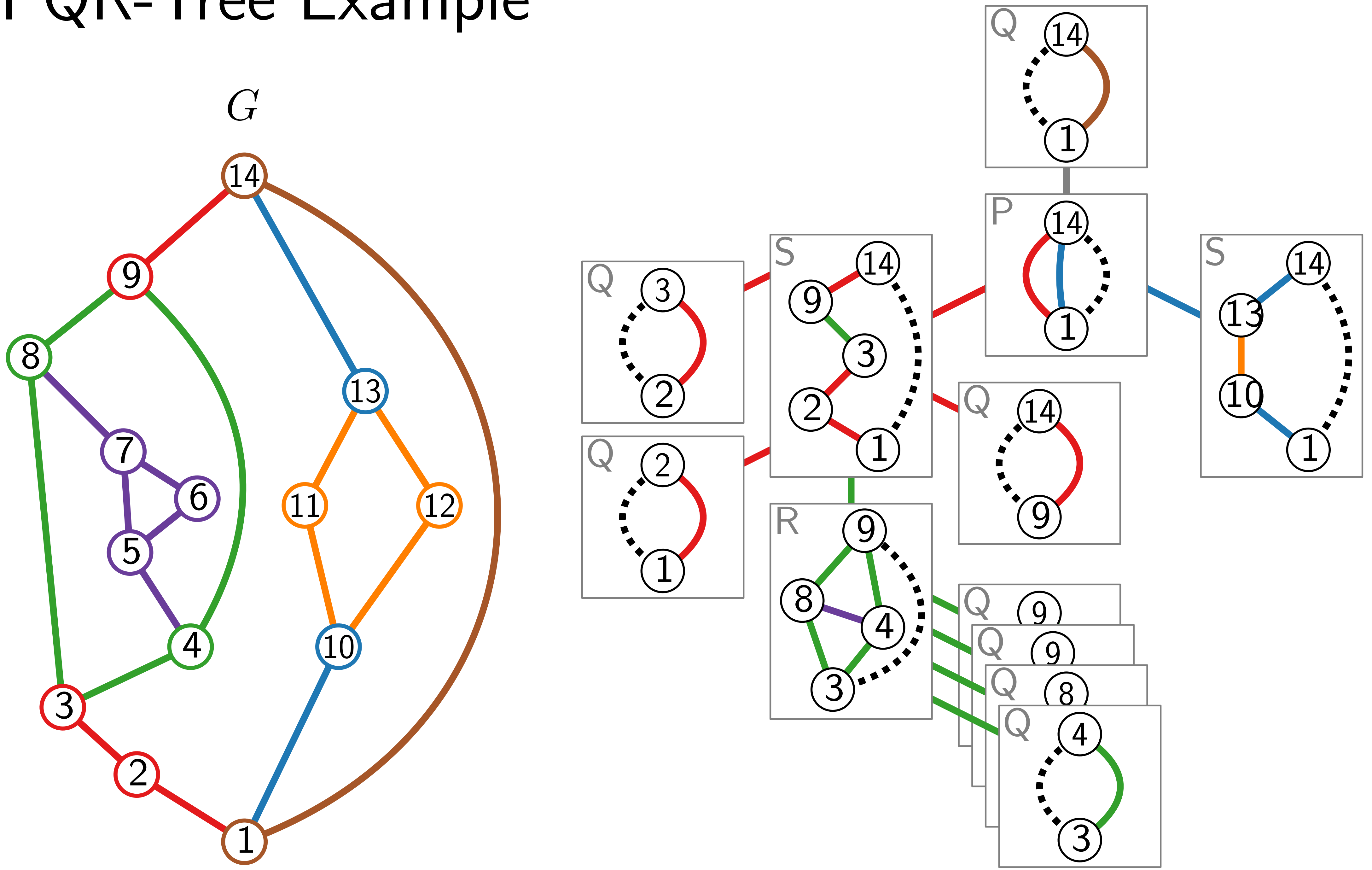

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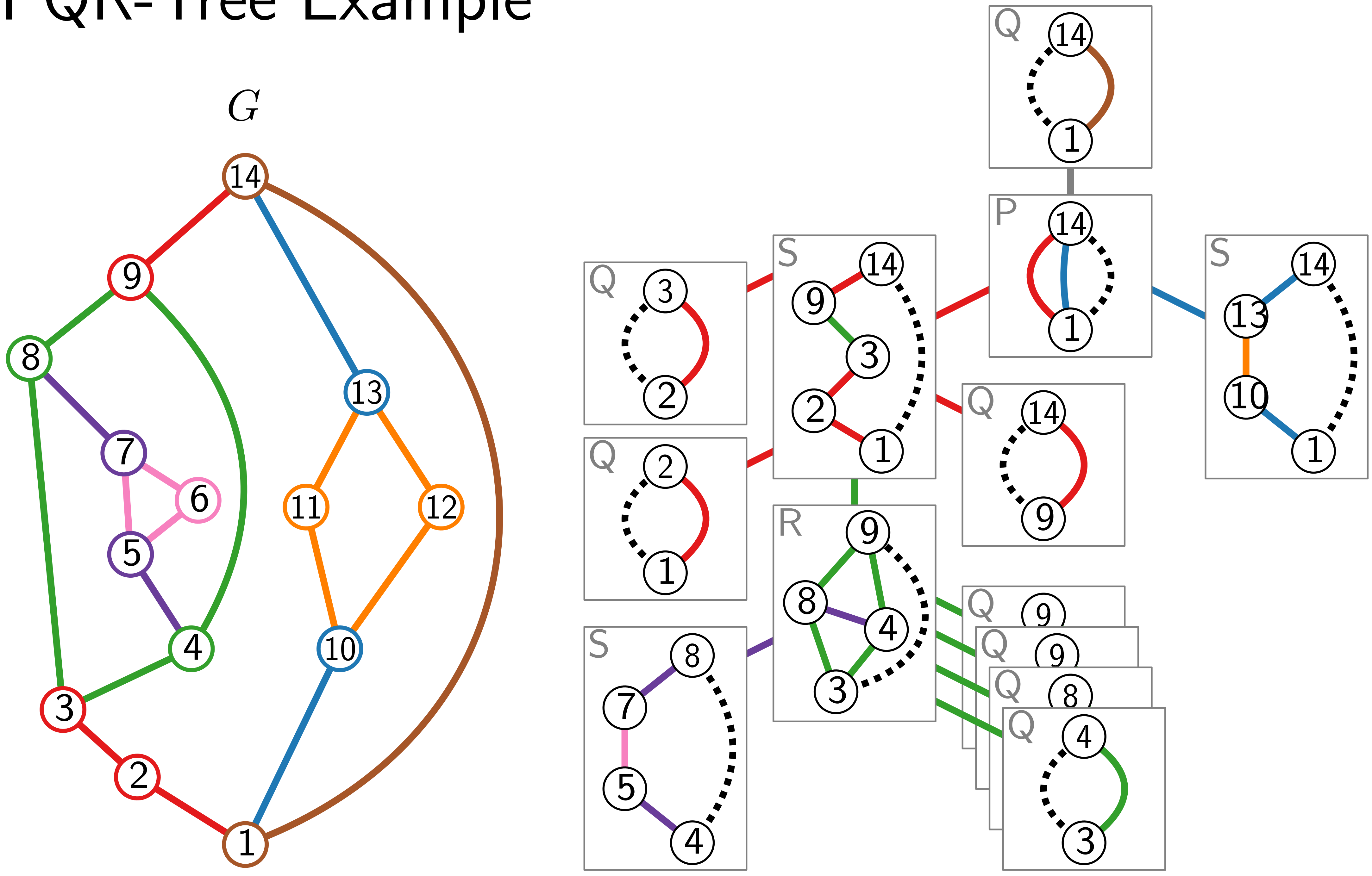

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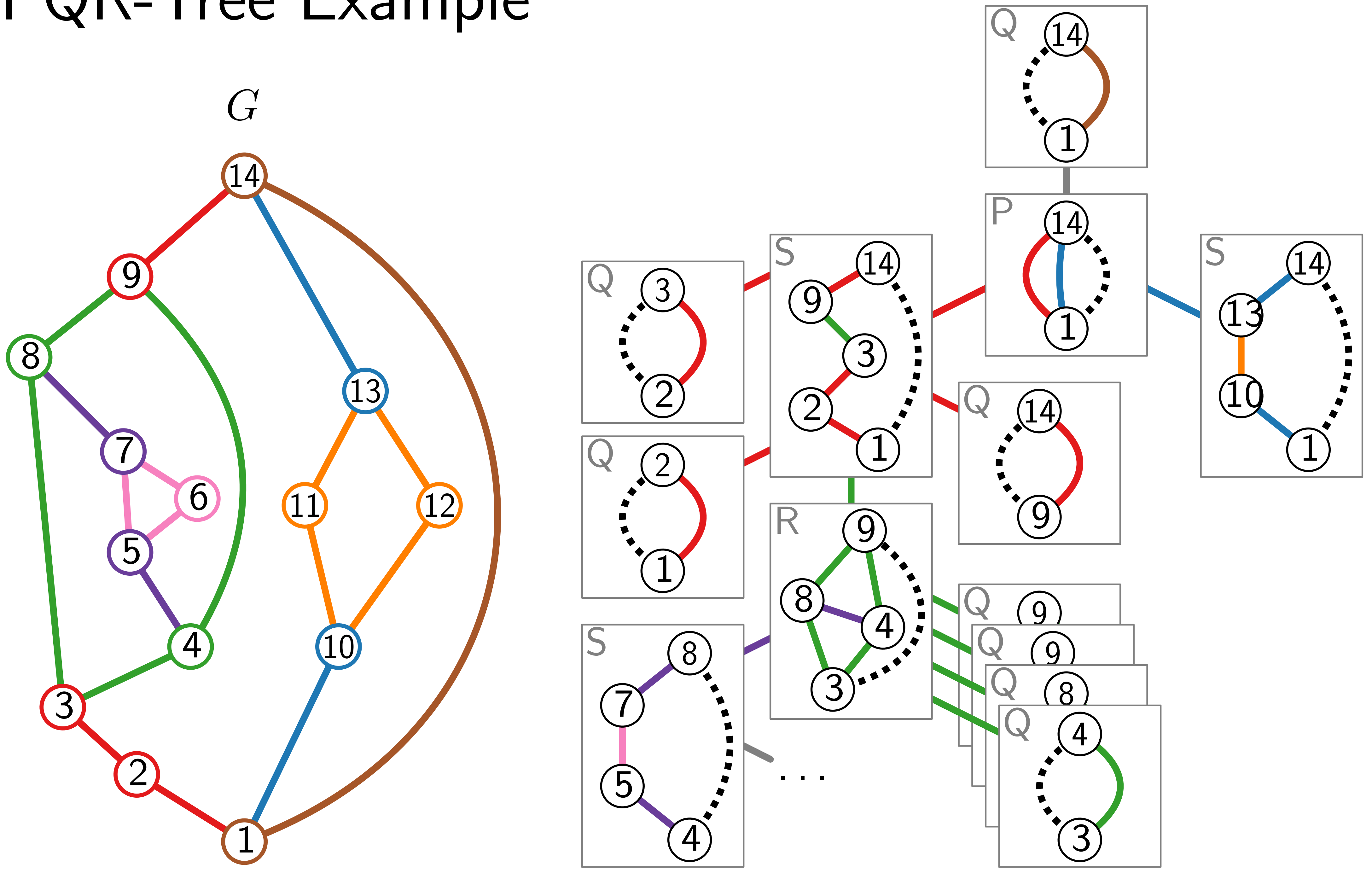

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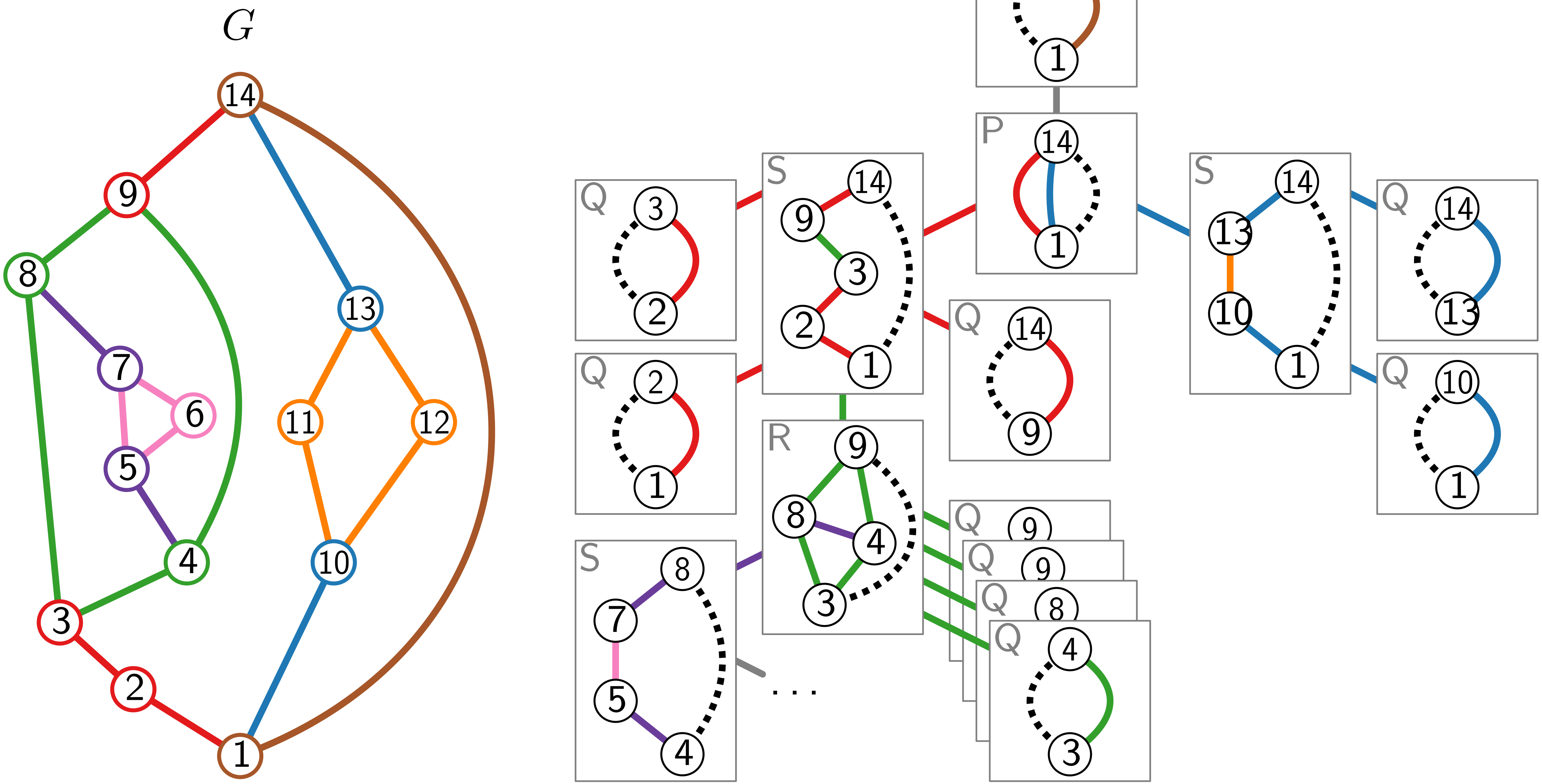

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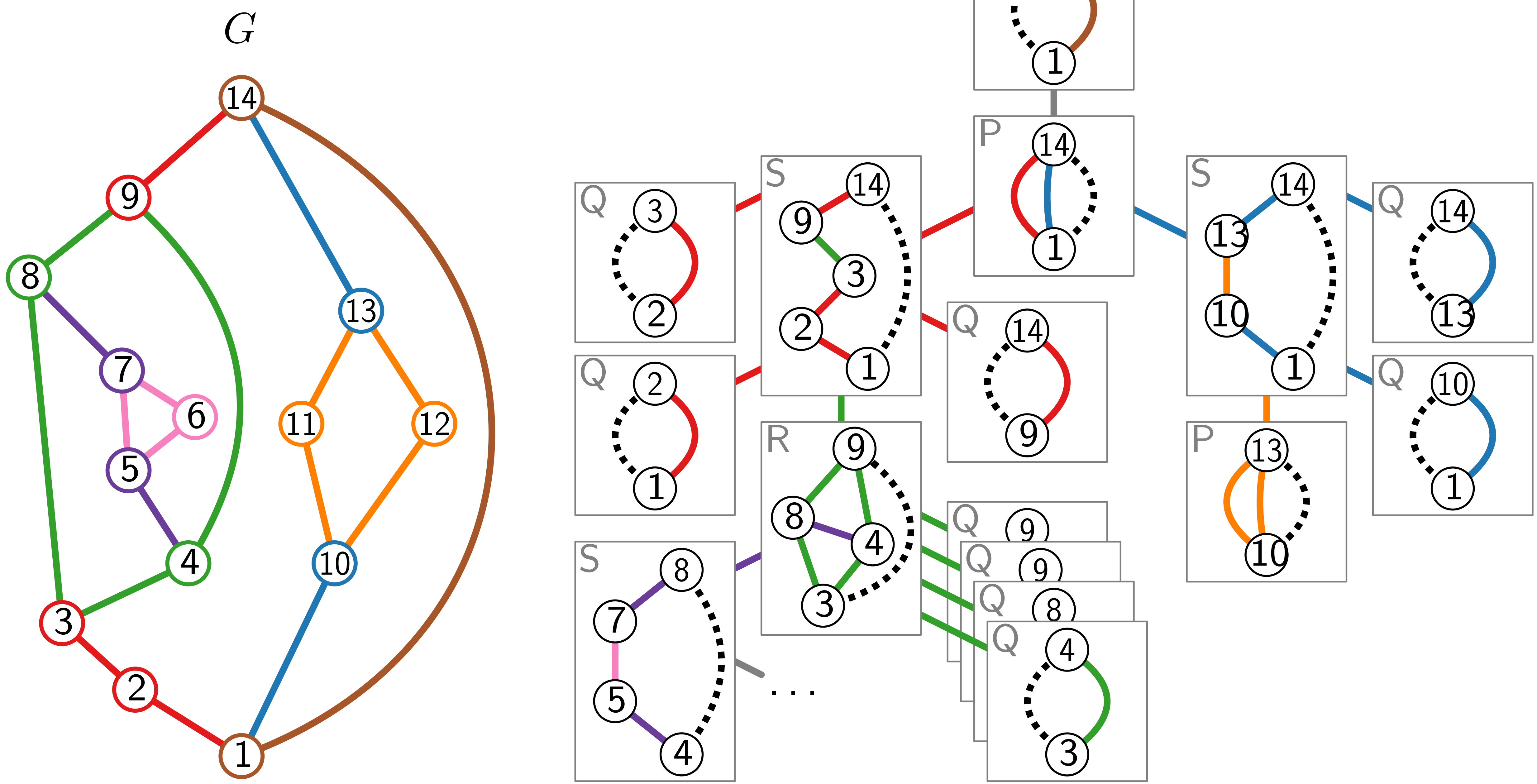

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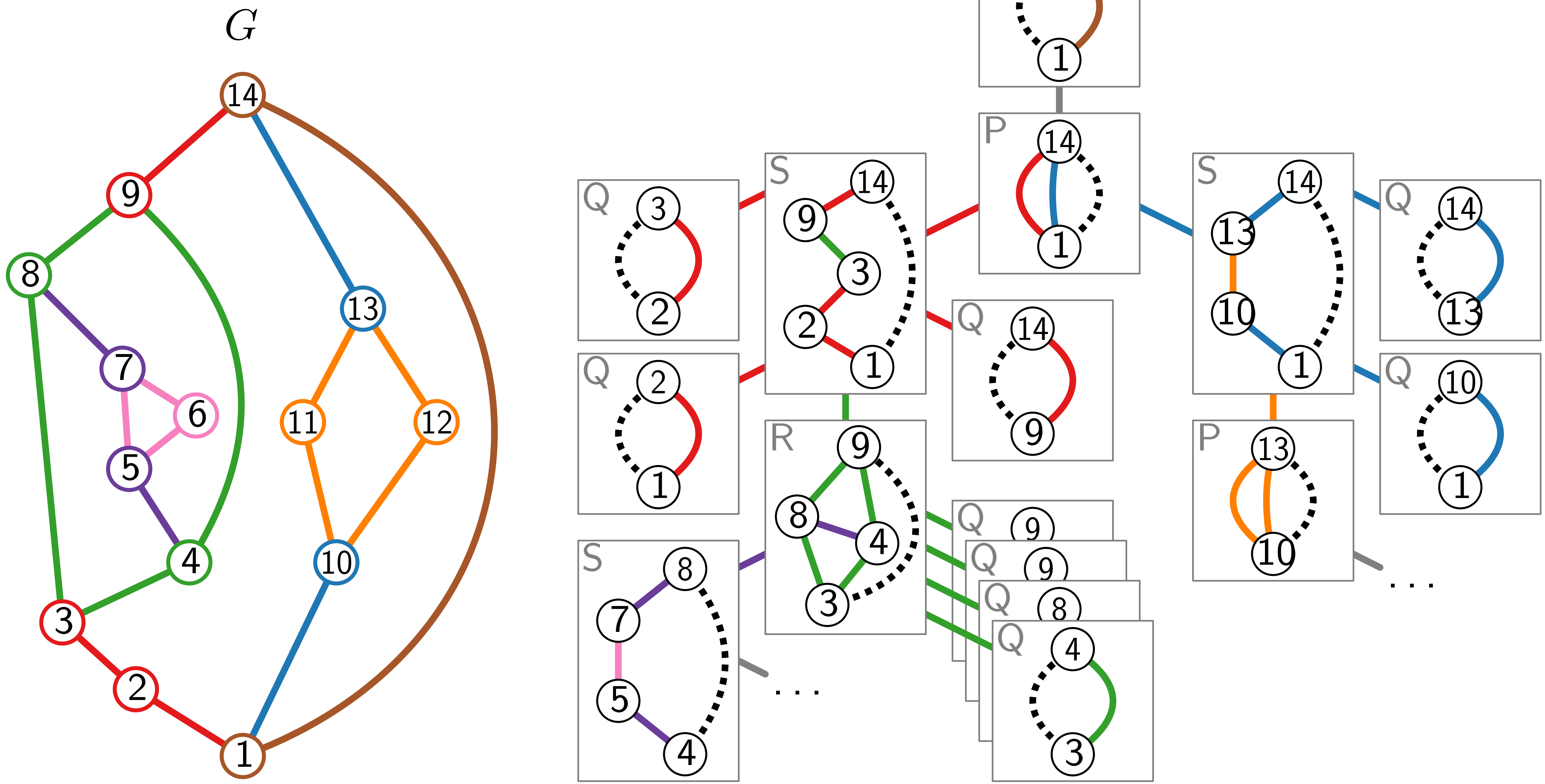


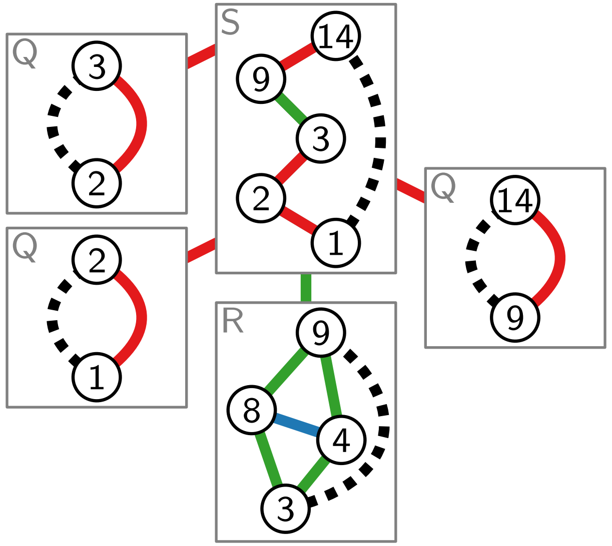
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

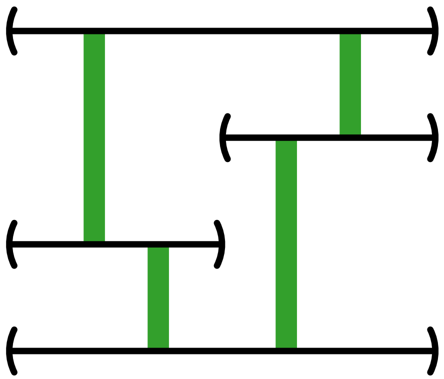
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension

Part IV:
Rectangular
Representation Extension

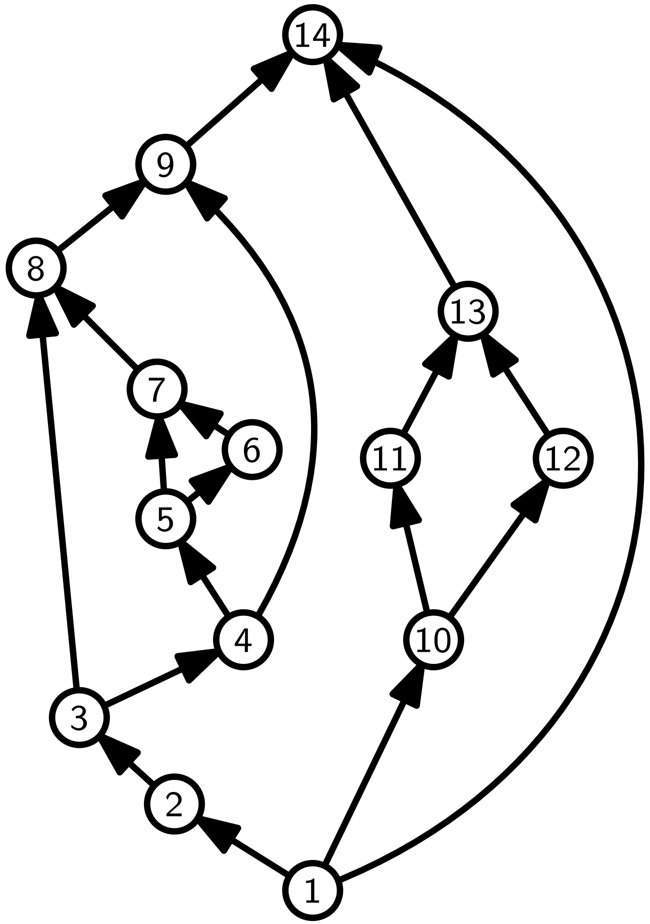
Jonathan Klawitter





Representation Extension for st-Graphs

Theorem 1'.
Rectangular ε-Bar Visibility Representation Extension can be solved in $\mathcal{O}(n^2)$ time for st-graphs.



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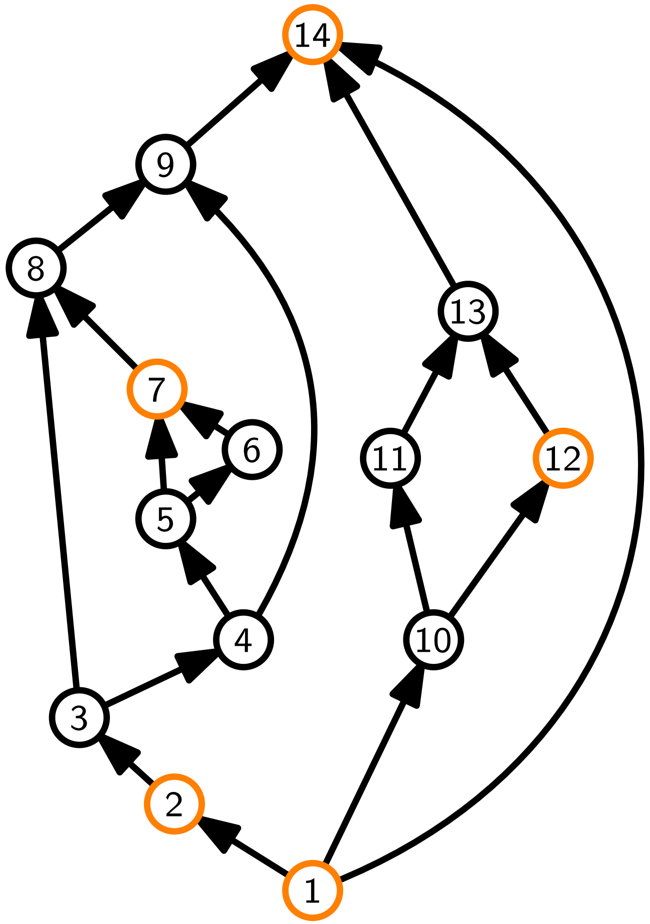

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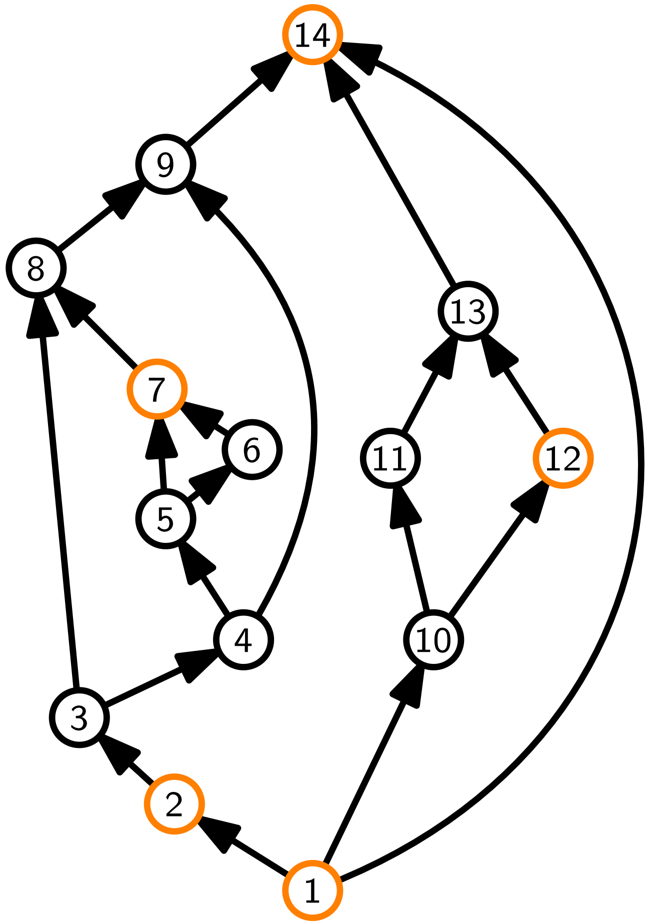

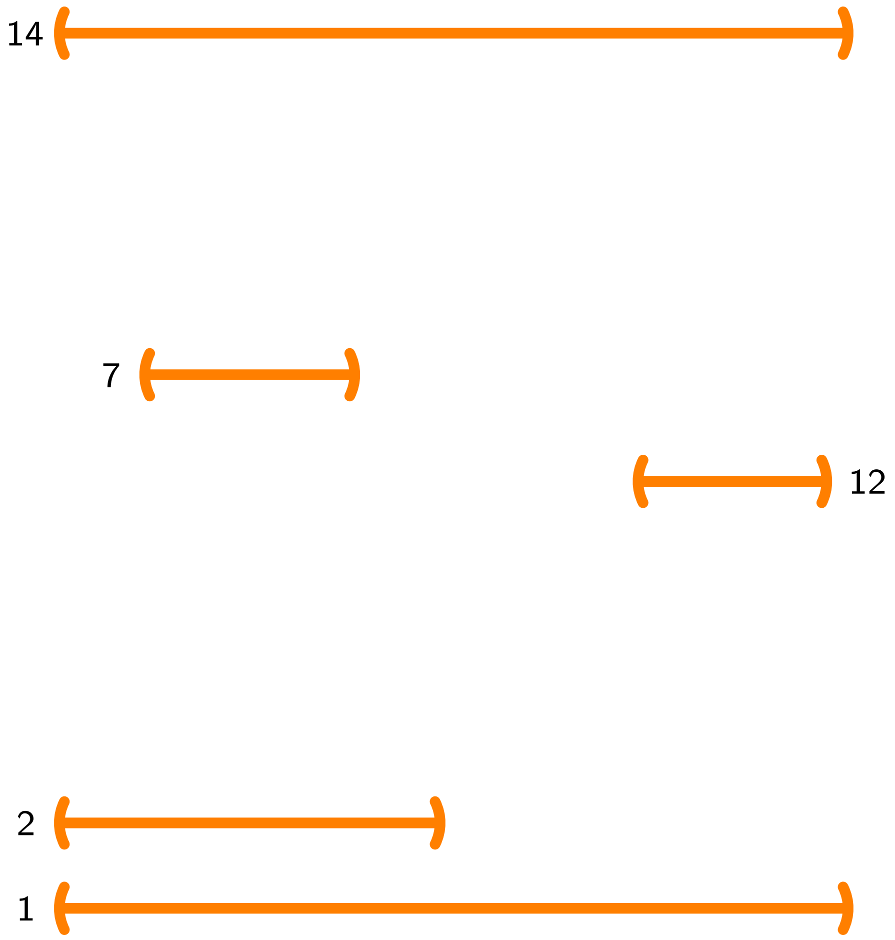

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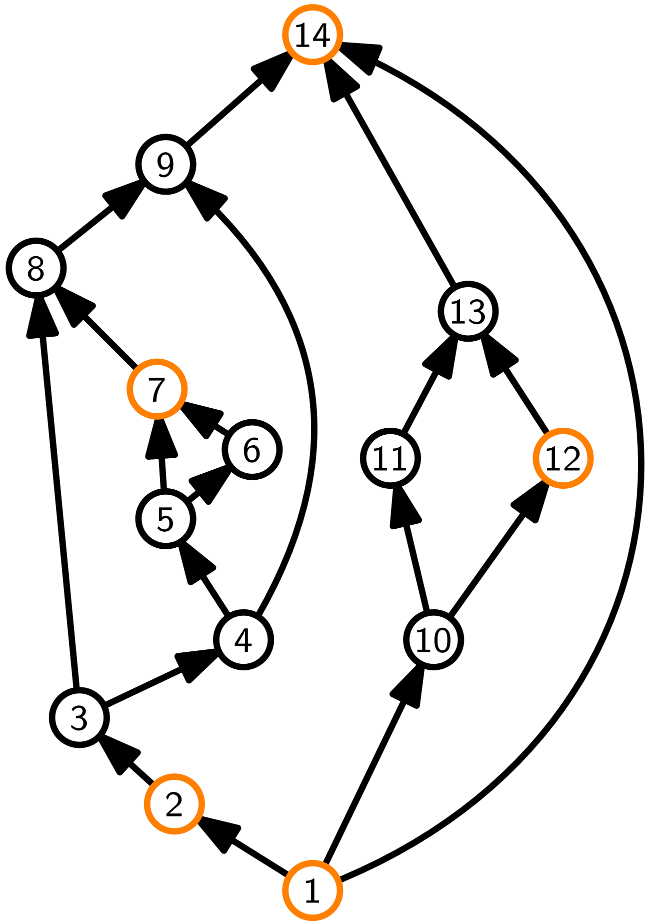

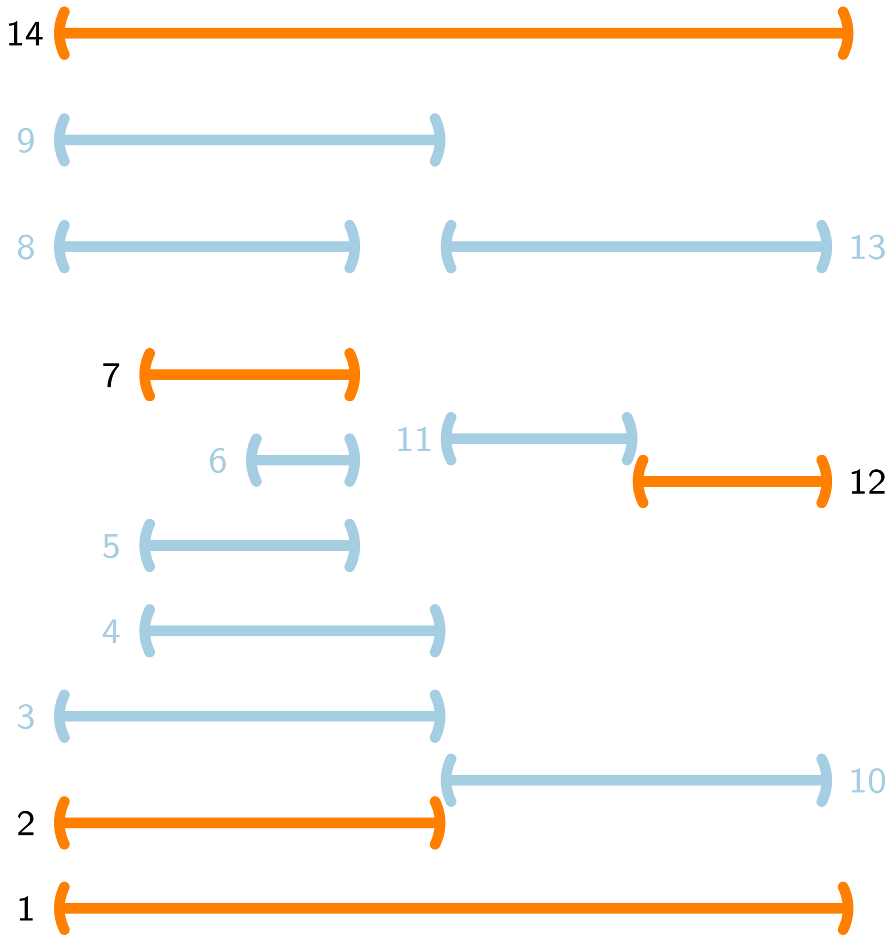

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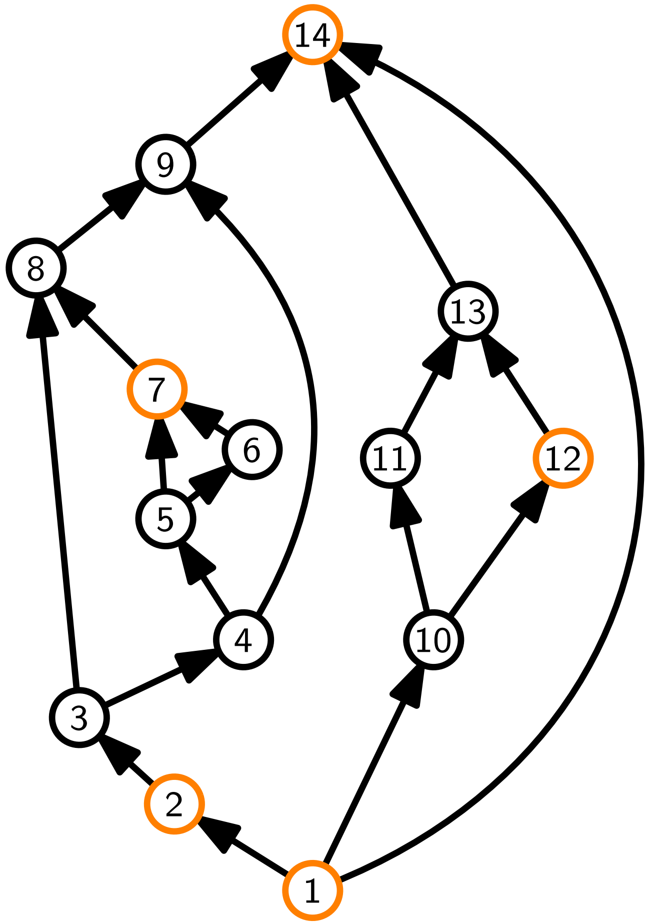

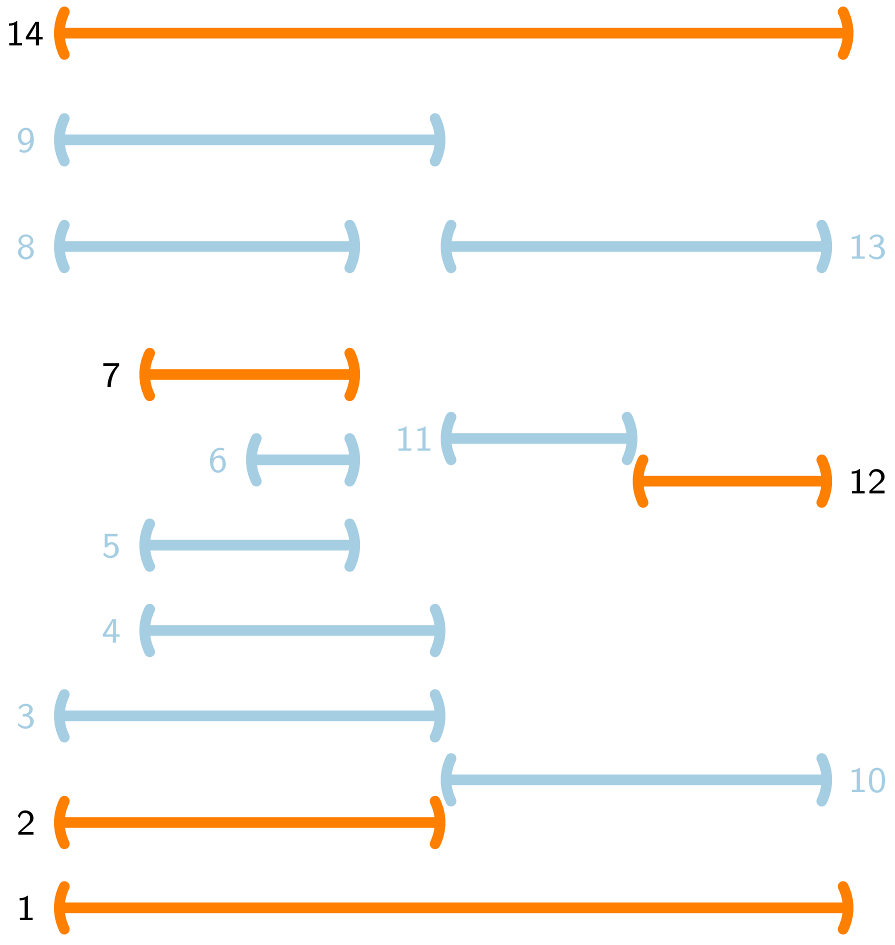

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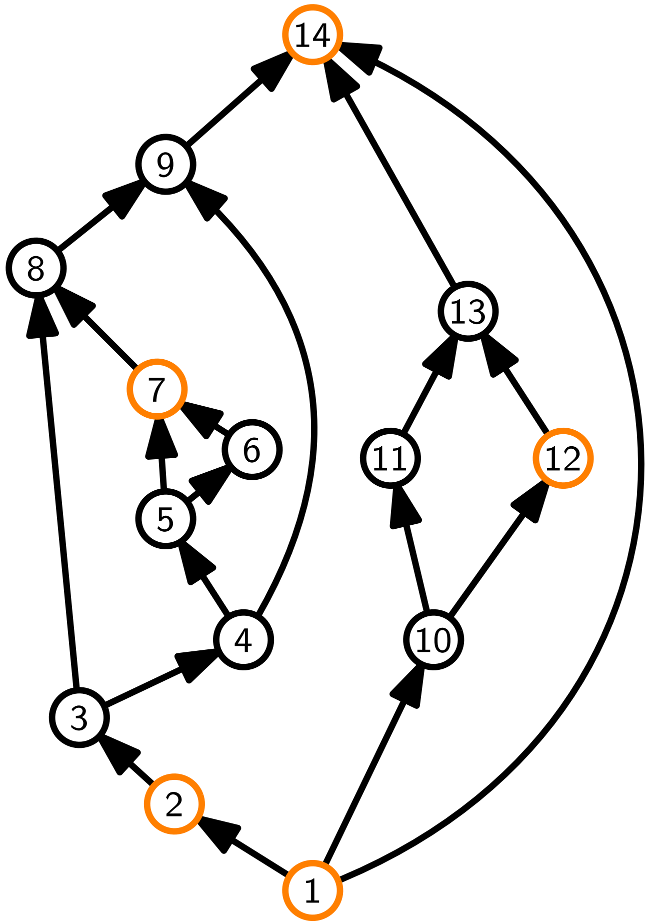

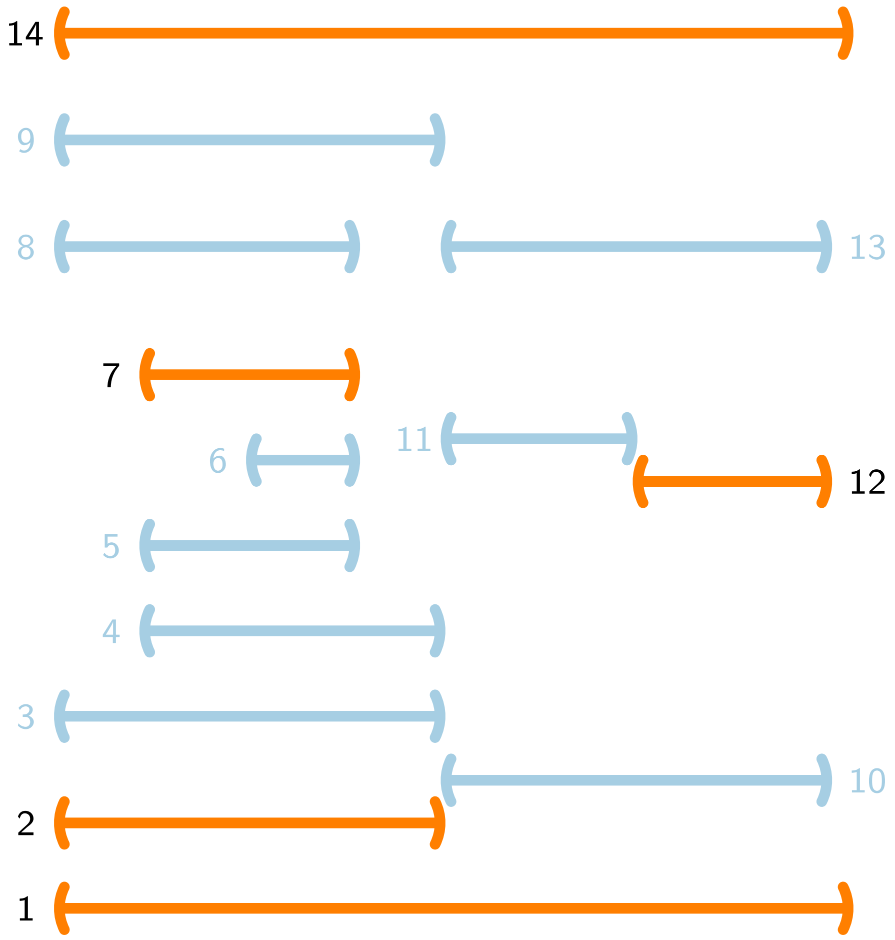

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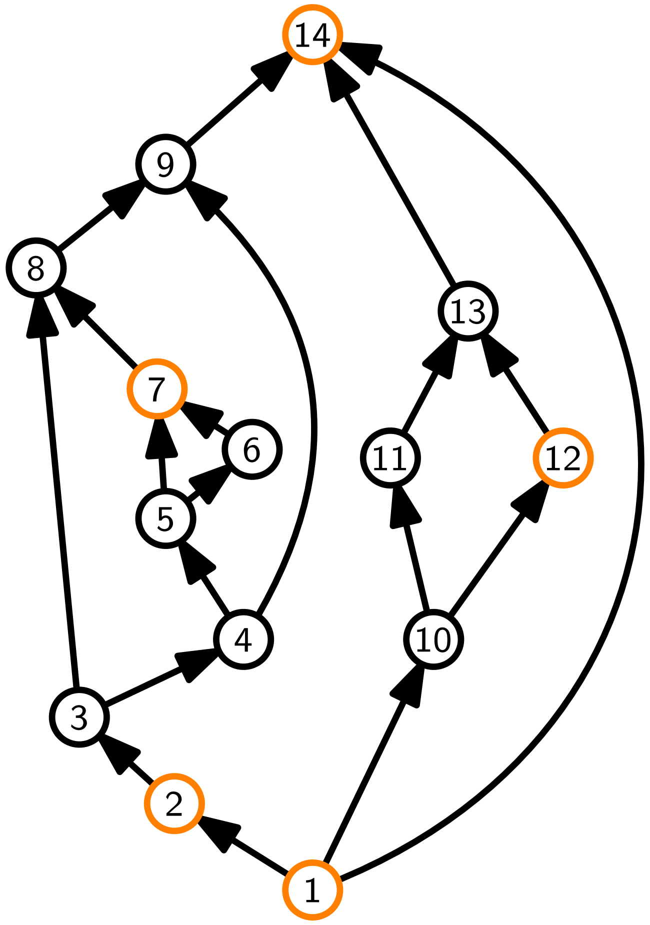

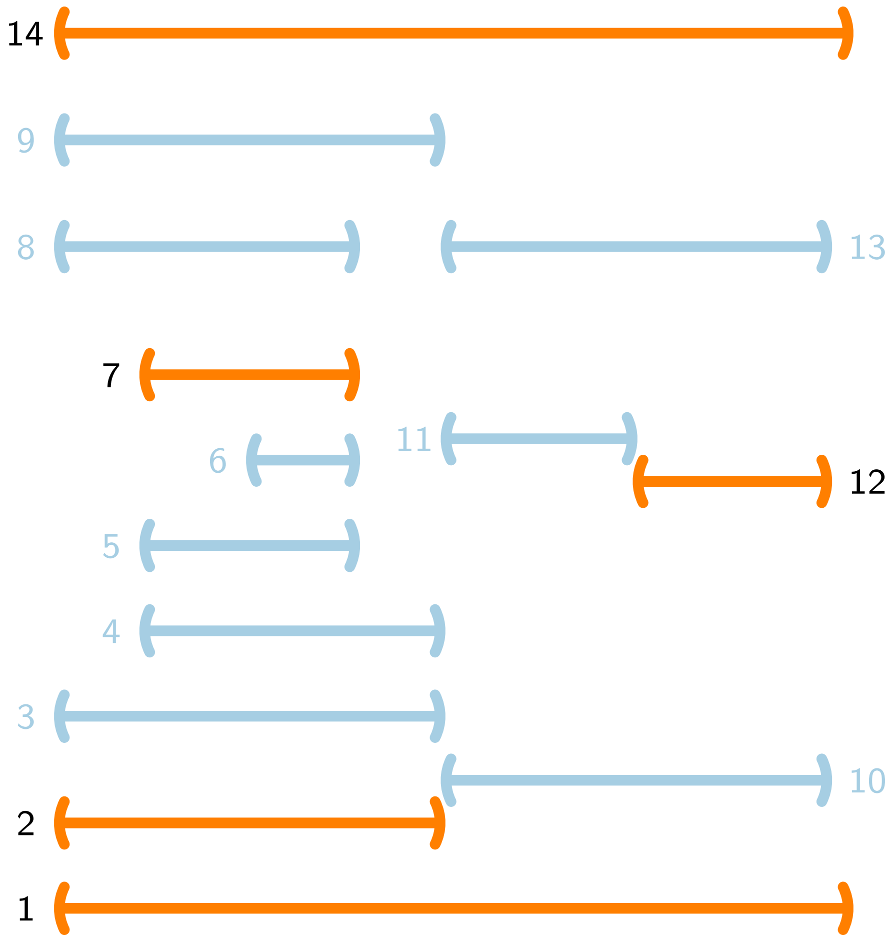

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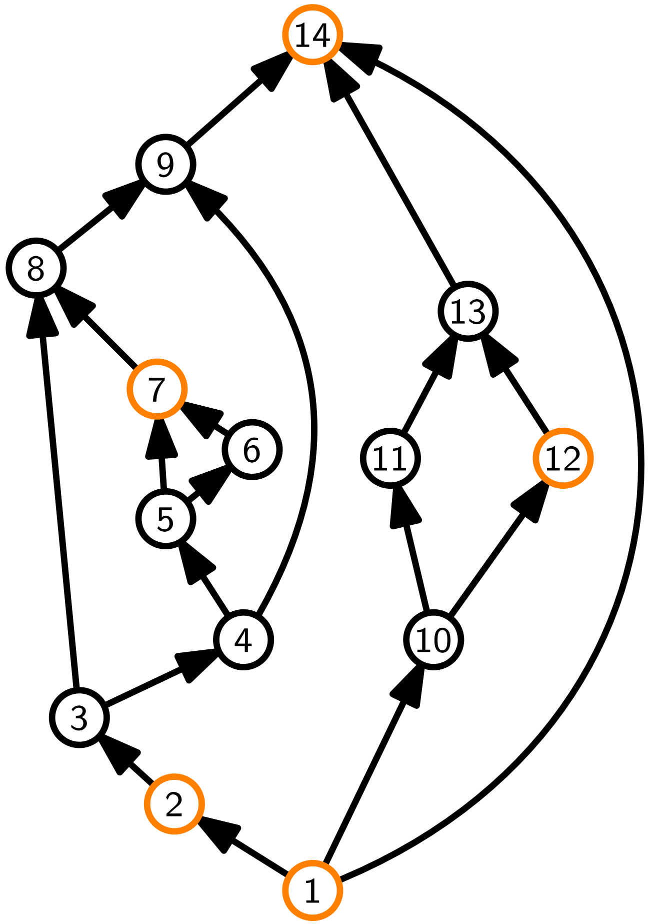

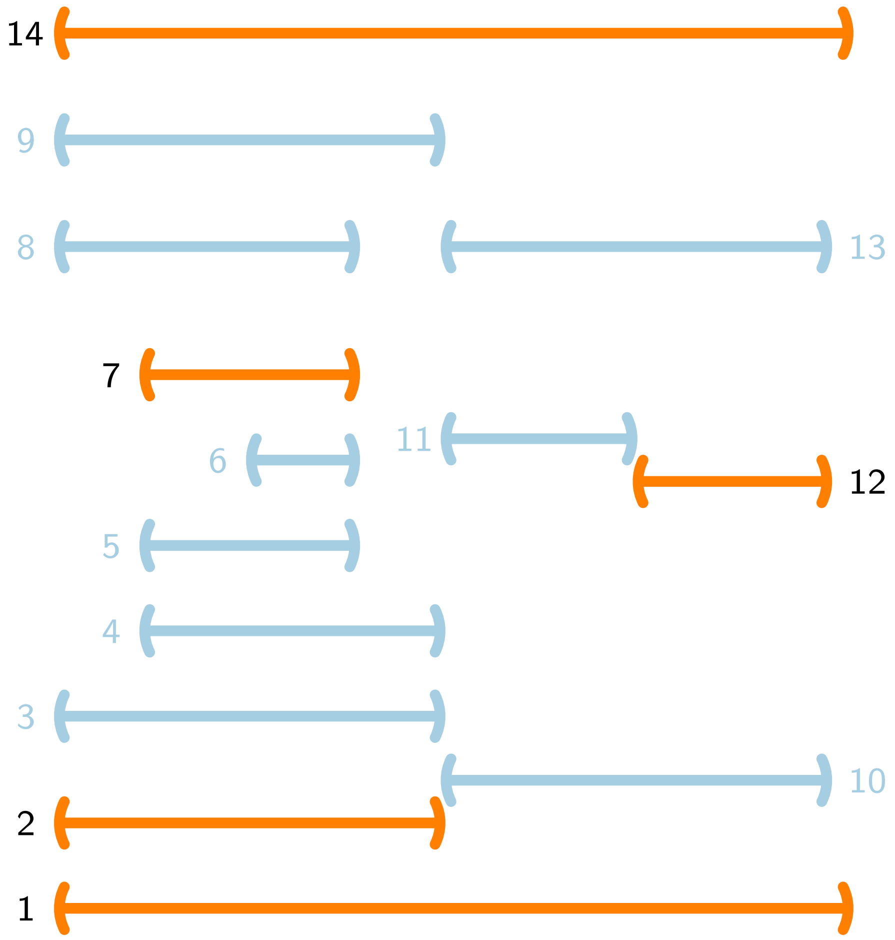

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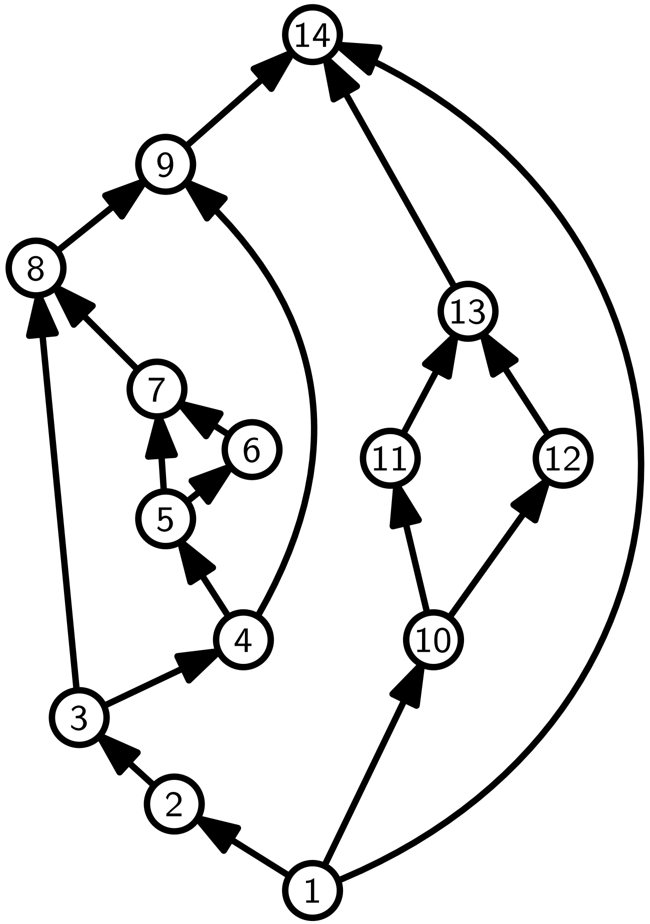

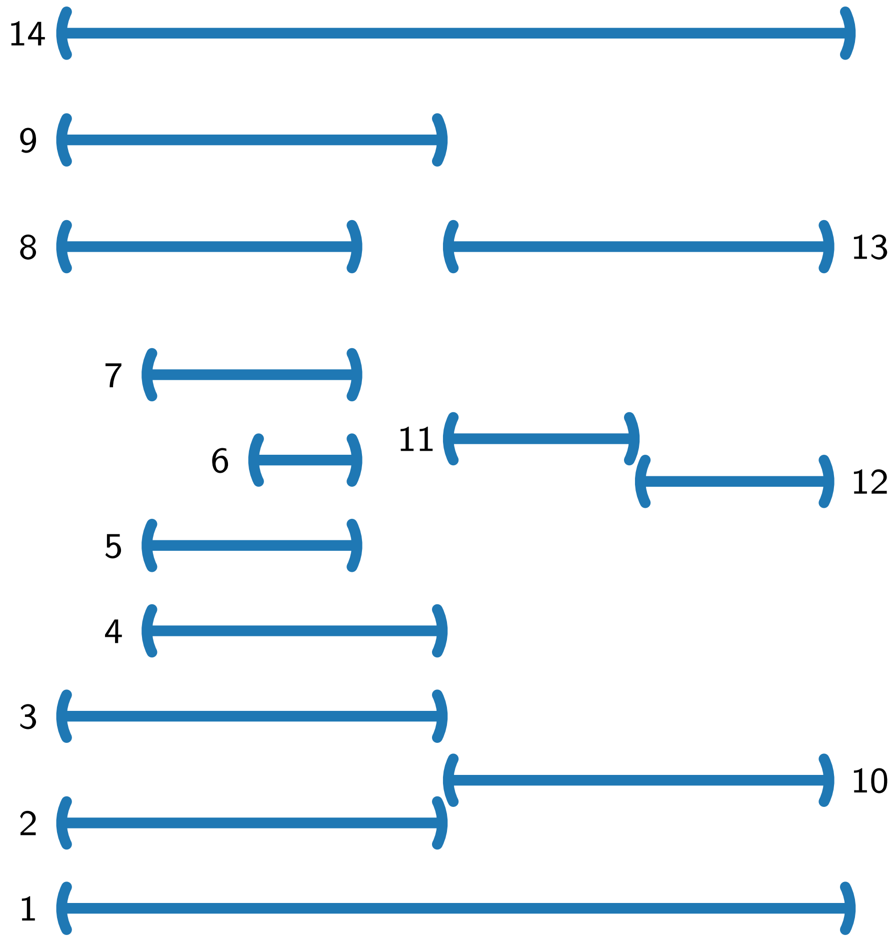

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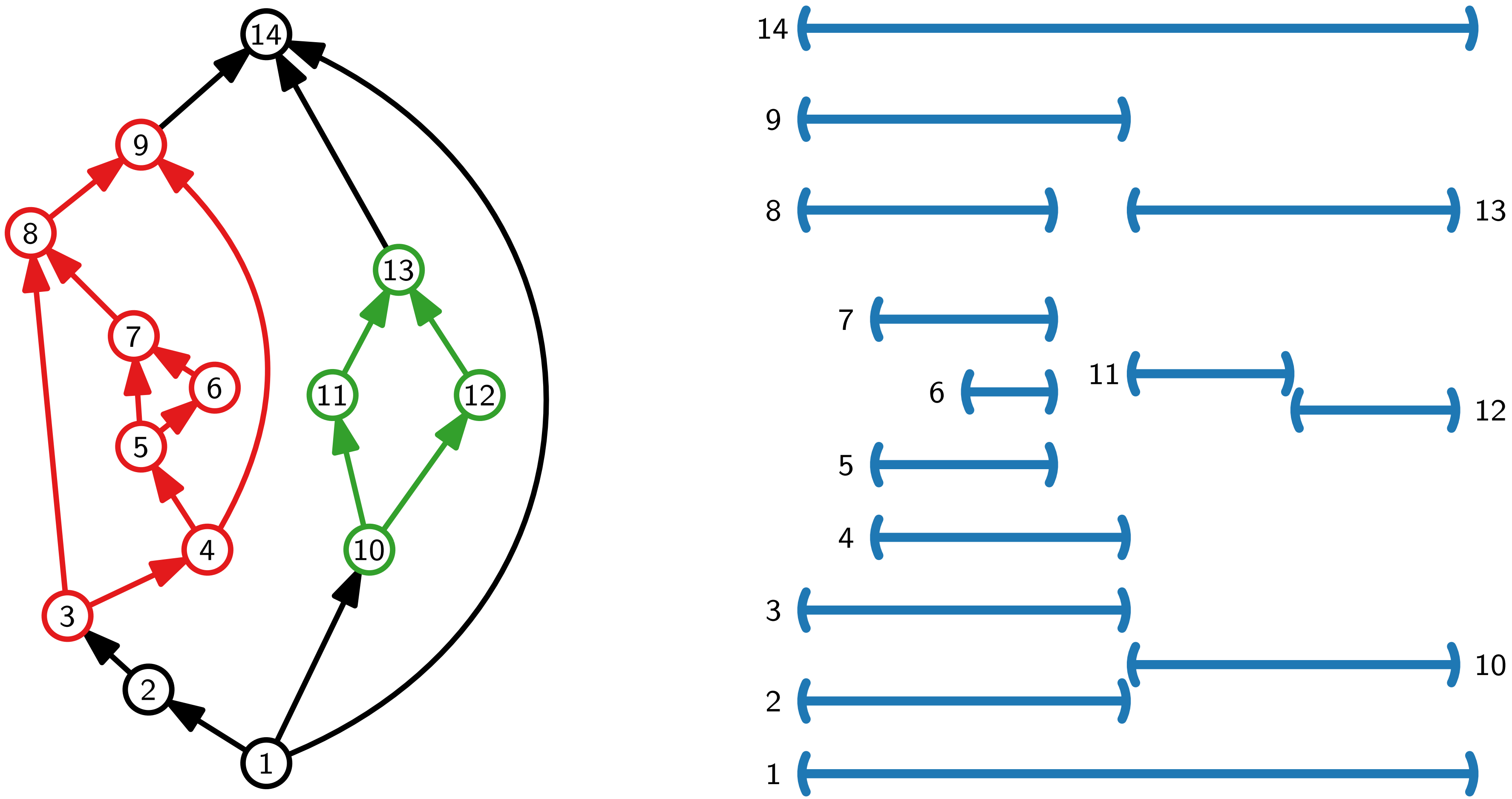

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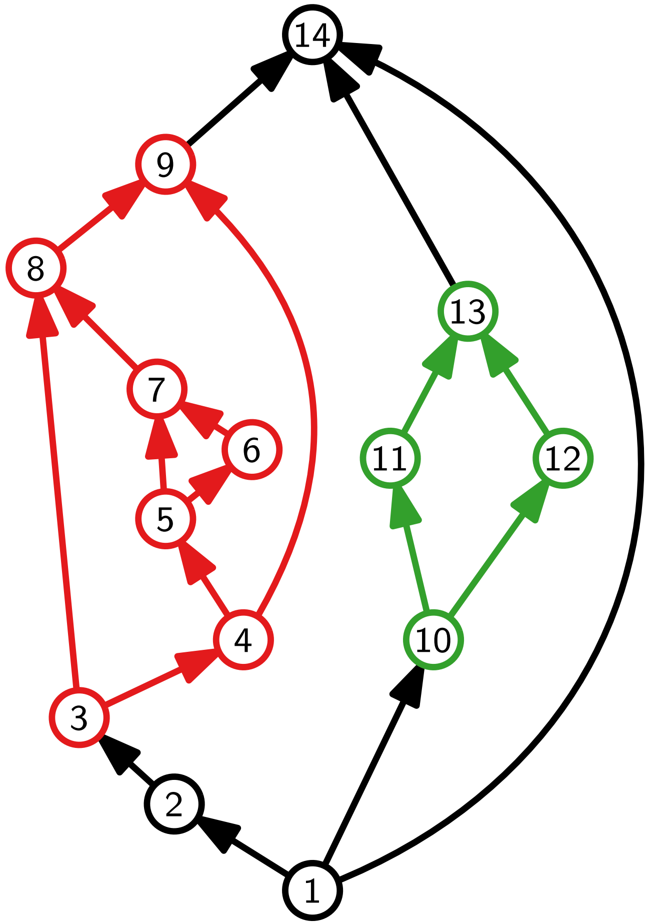

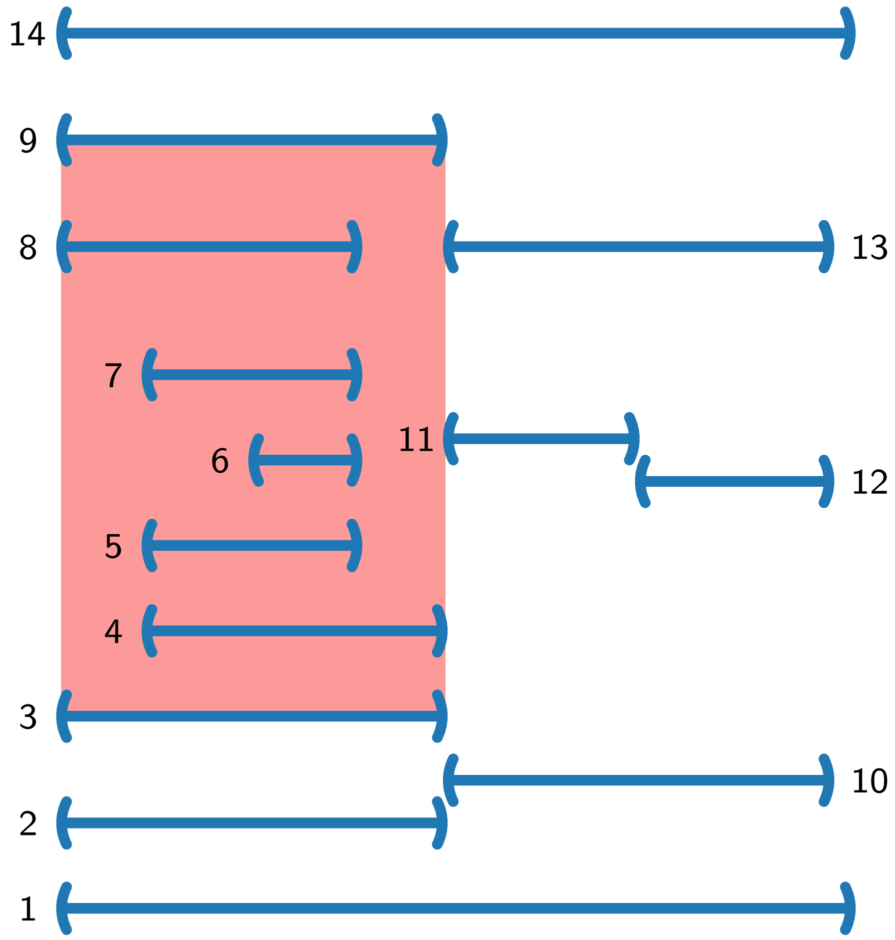

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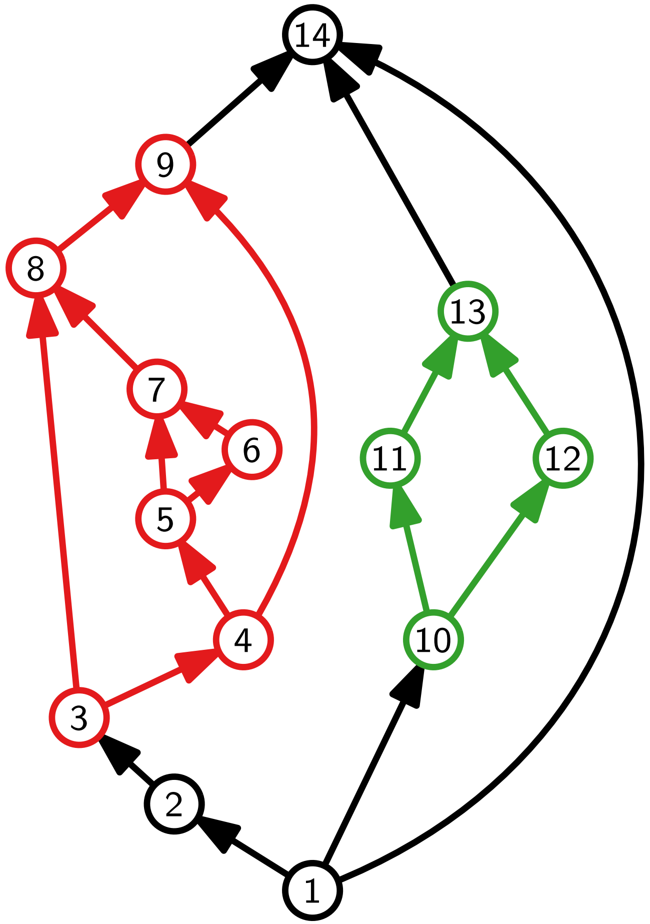
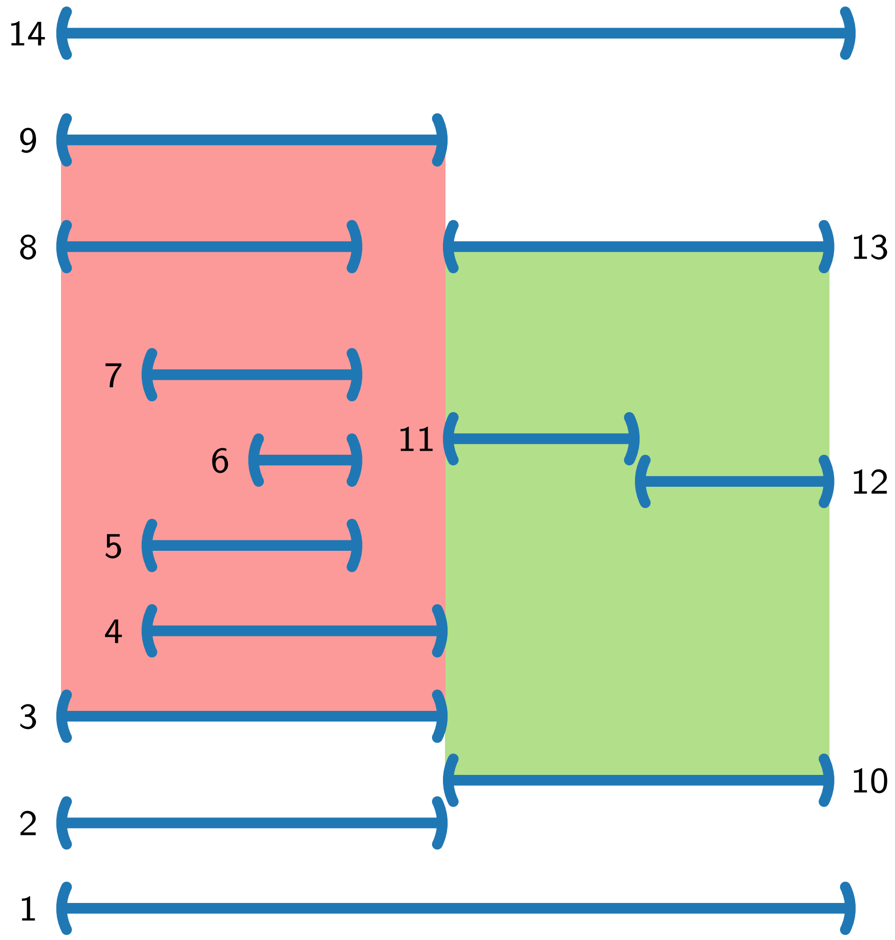
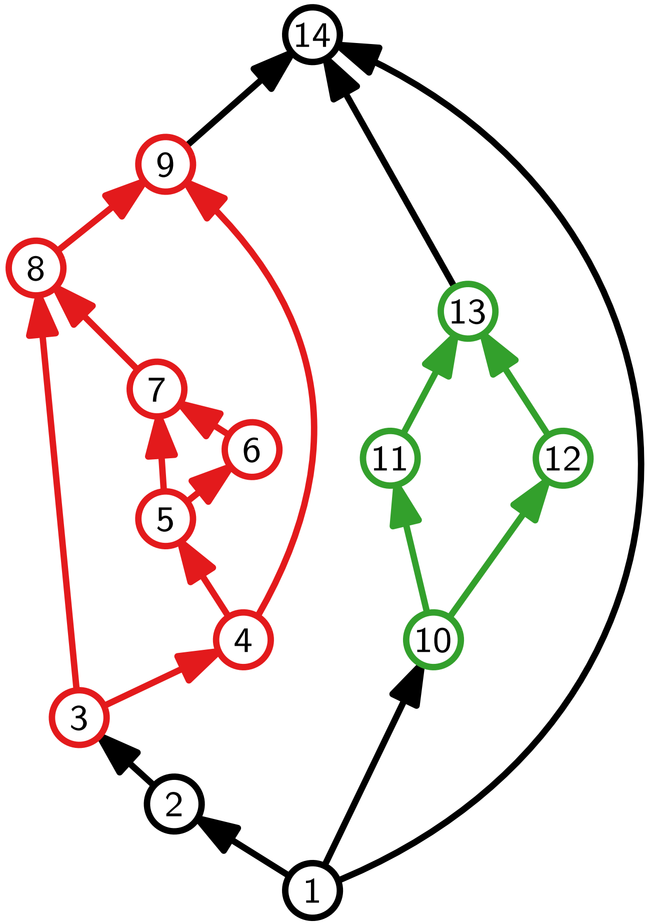
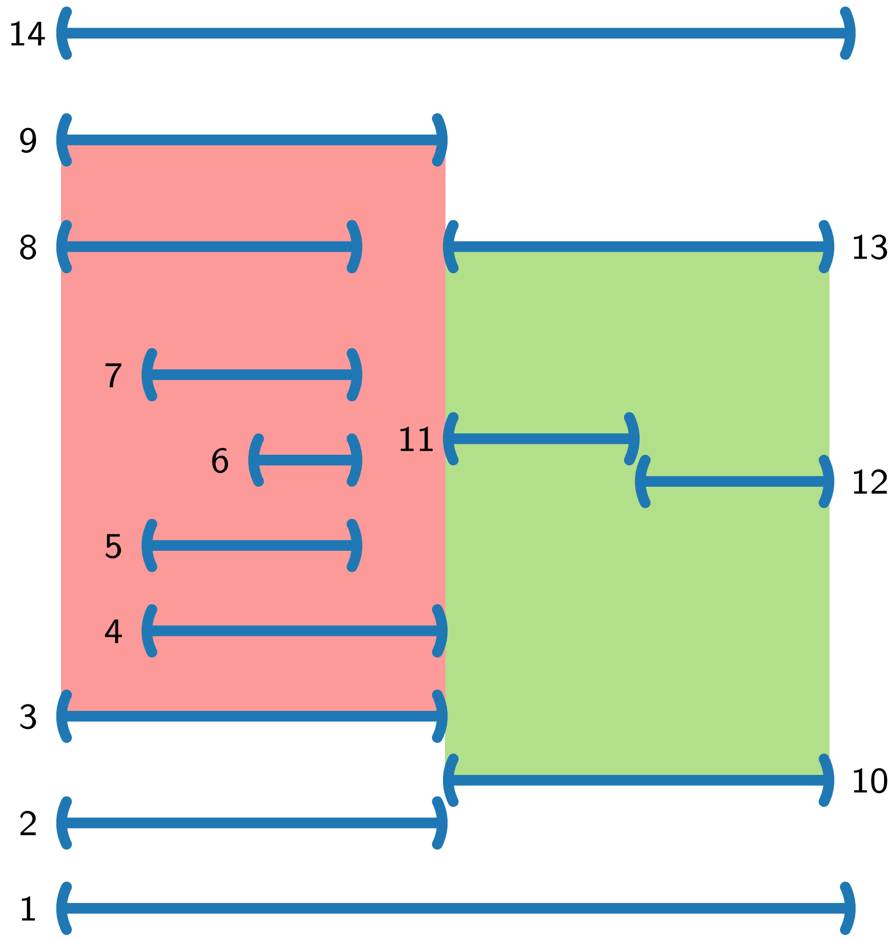




But why do SPQR-Trees help?

Lemma 2.
The SPQR-tree of an st-graph G induces a recursive **tiling** of any ε-bar visibility representation of G.





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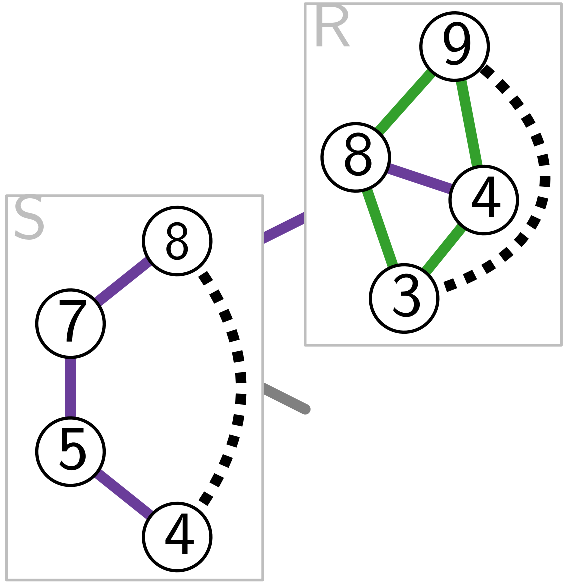
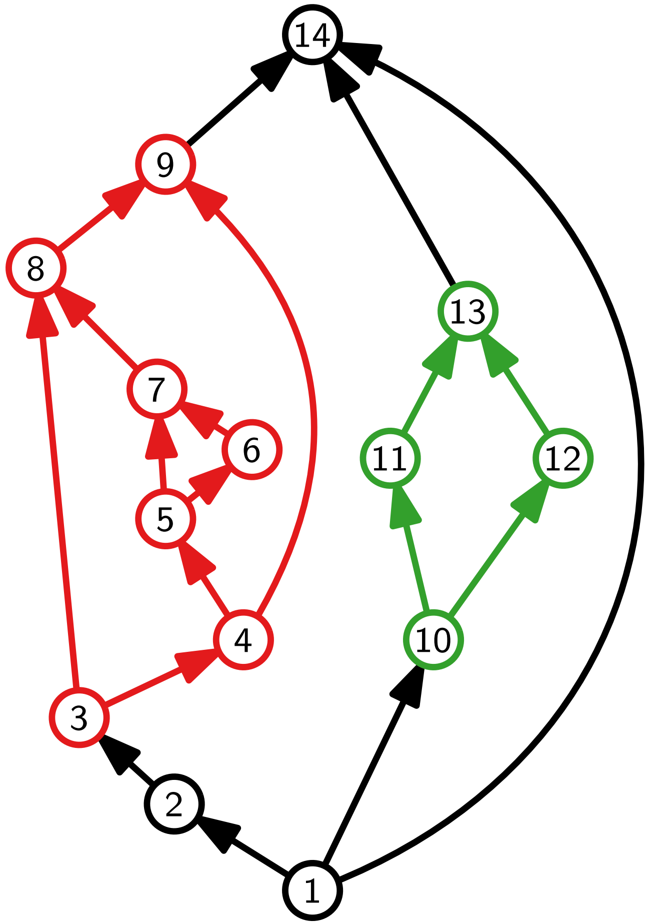
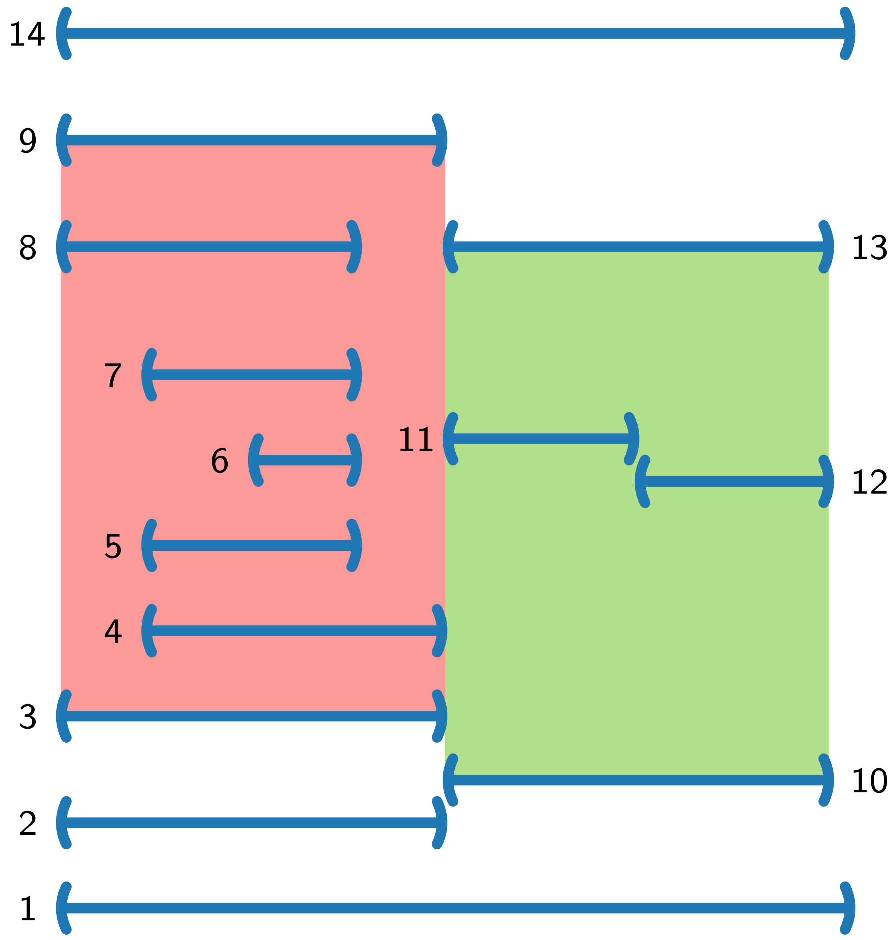


Solve tiles bottom-up





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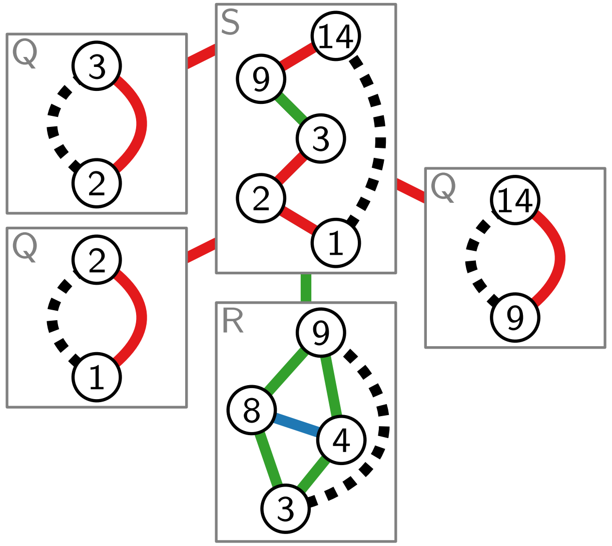
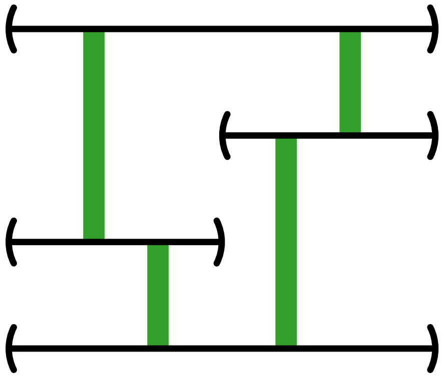
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



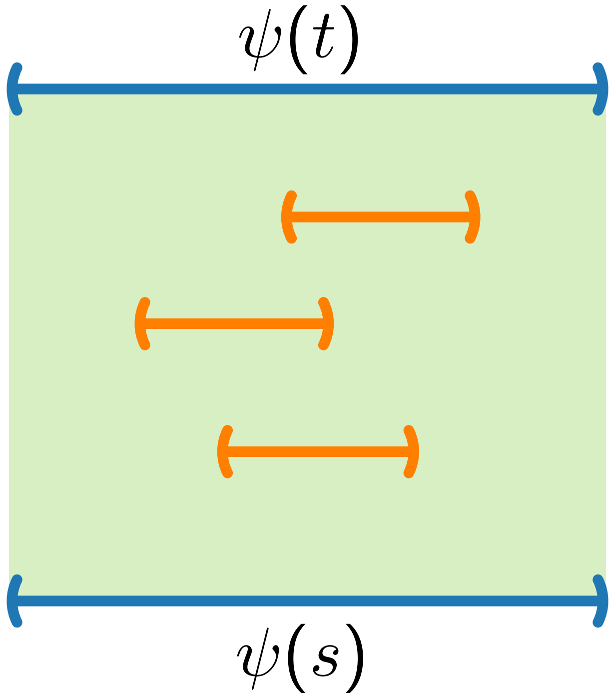
Part V:
Dynamic Program

Jonathan Klawitter



Tiles

Convention. **Orange** bars are from the partial representation



Tiles

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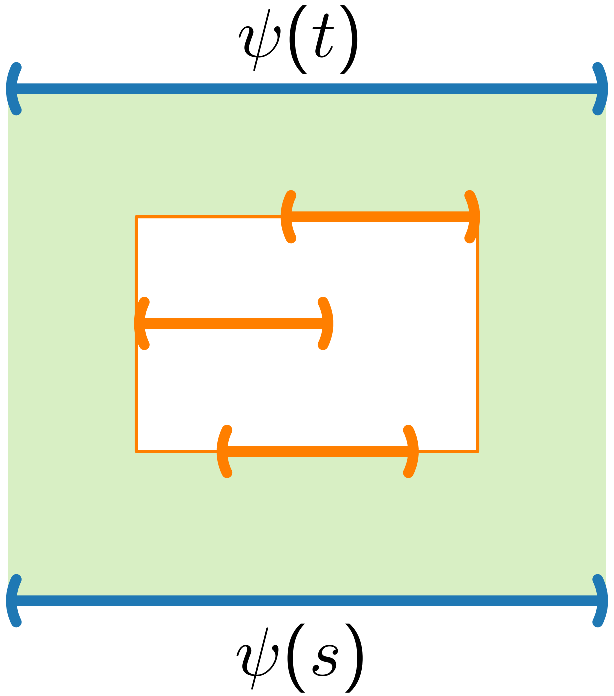

Tiles

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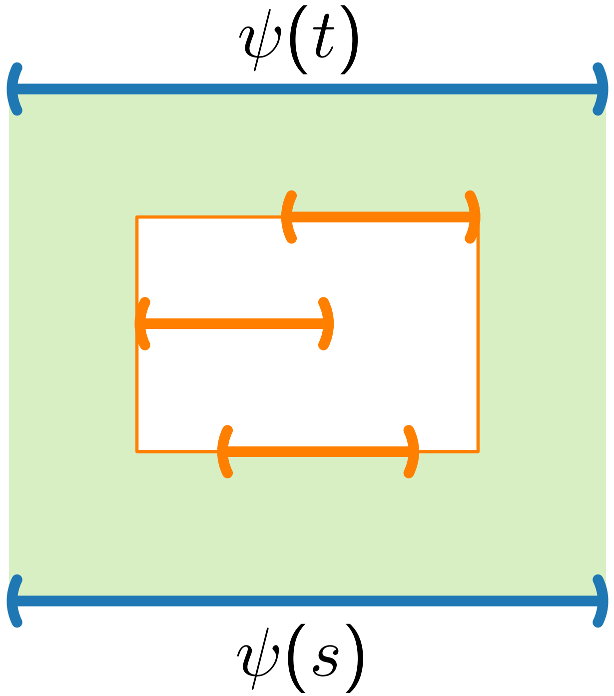
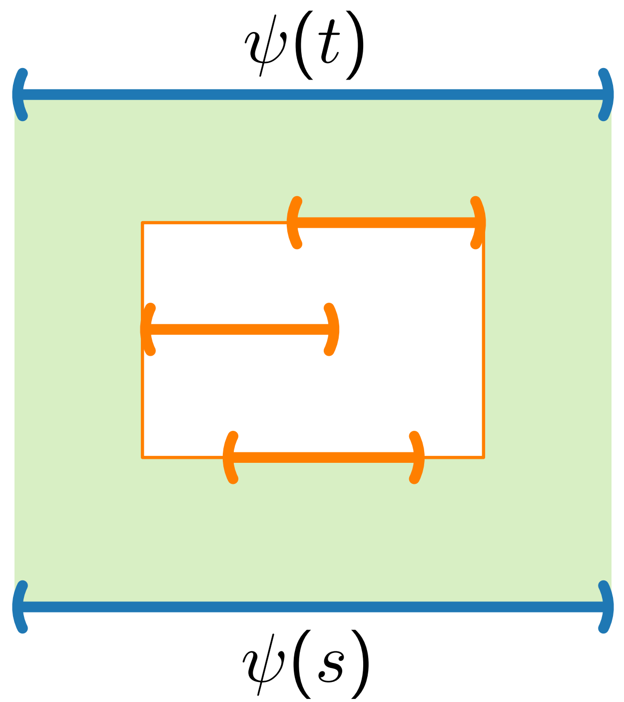


Observation.
The bounding box (tile) of any solution ψ **contains** the bounding box of the partial representation.

Tiles

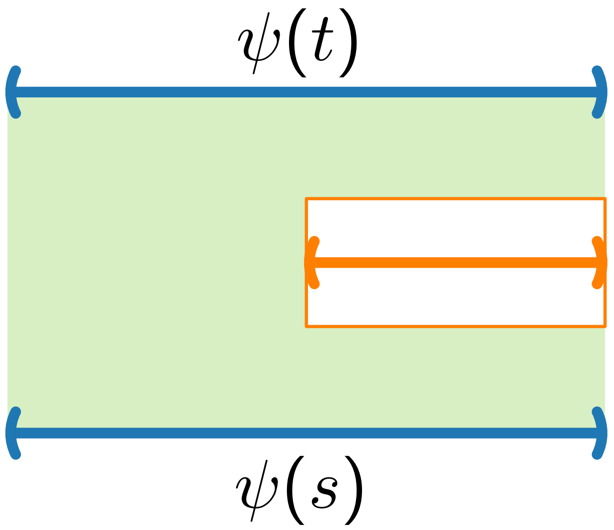
Convention. **Orange** bars are from the partial representation



Observation.
The bounding box (tile) of any solution ψ **contains** the bounding box of the partial representation.

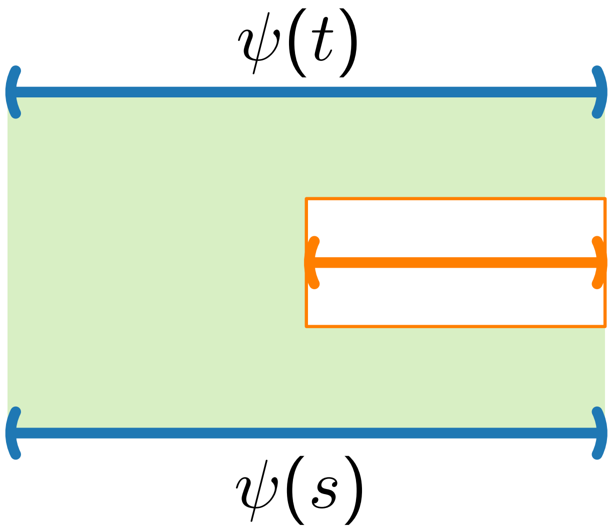
How many **different** tiles can we really have?

Types of Tiles



- Right **F**ixed – due to the orange bar
- Left **L**oose – due to the orange bar

Types of Tiles



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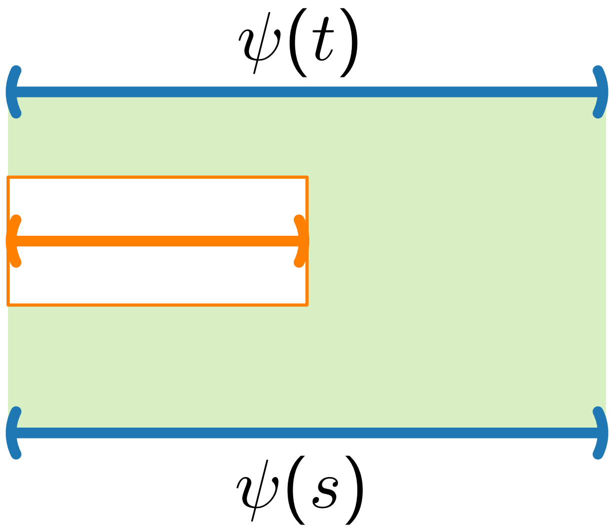

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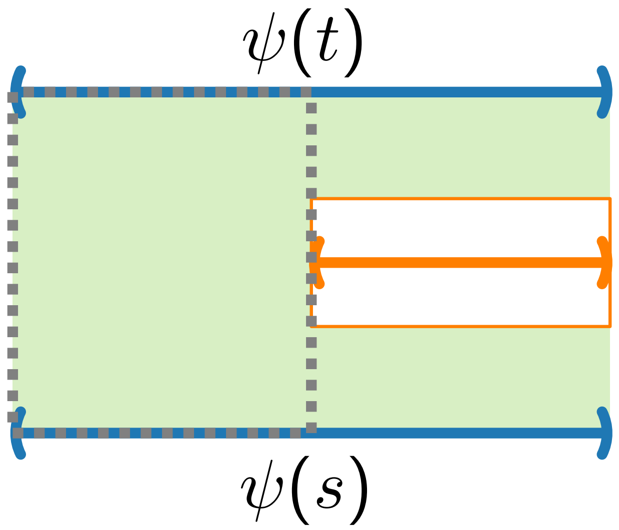
Types of Tiles



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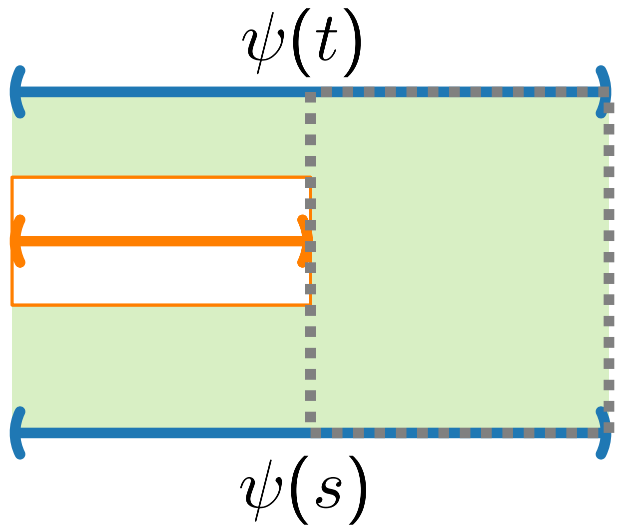

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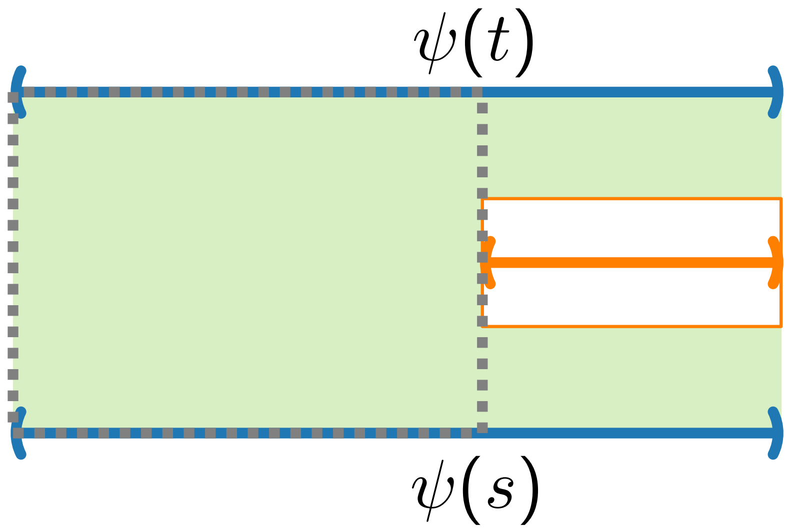
Types of Tiles



- Right **F**ixed – due to the orange bar
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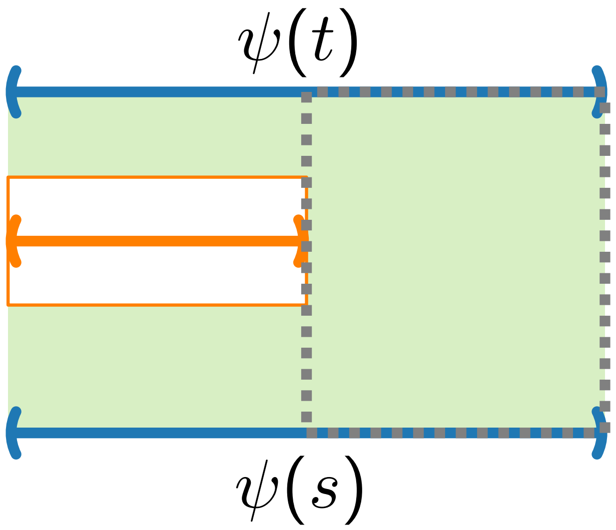

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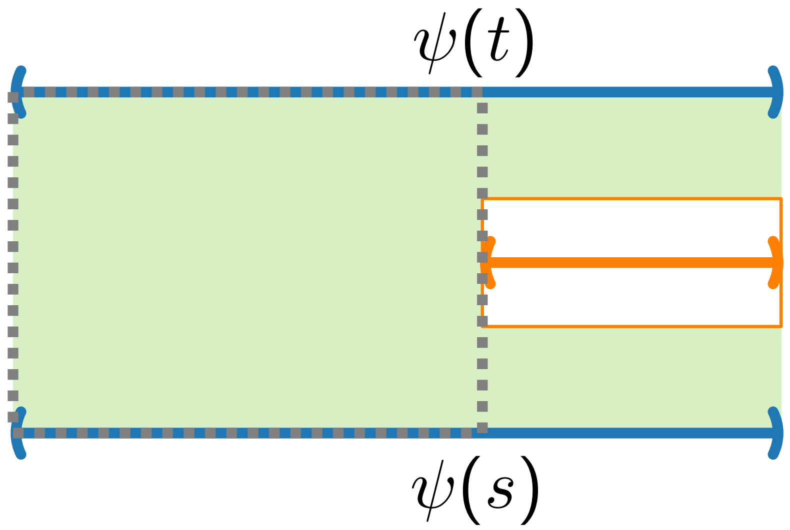
Types of Tiles



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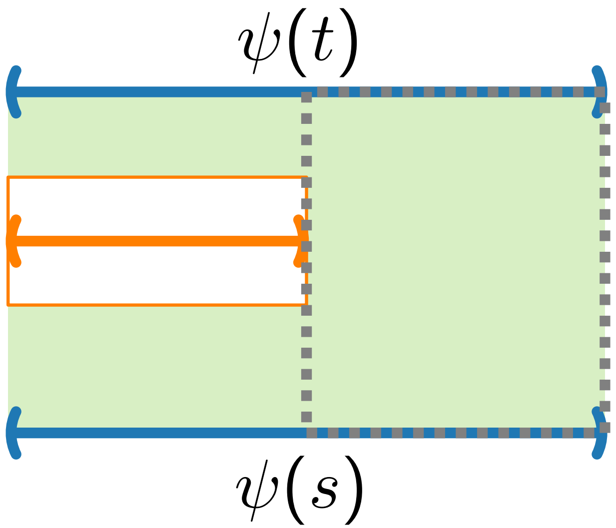

- Left **F**ixed – due to the orange bar
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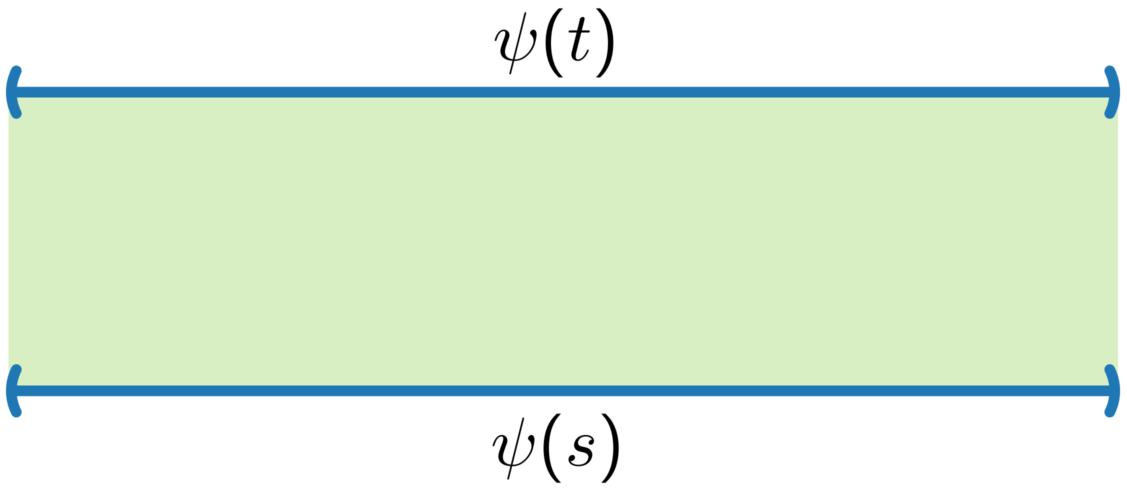
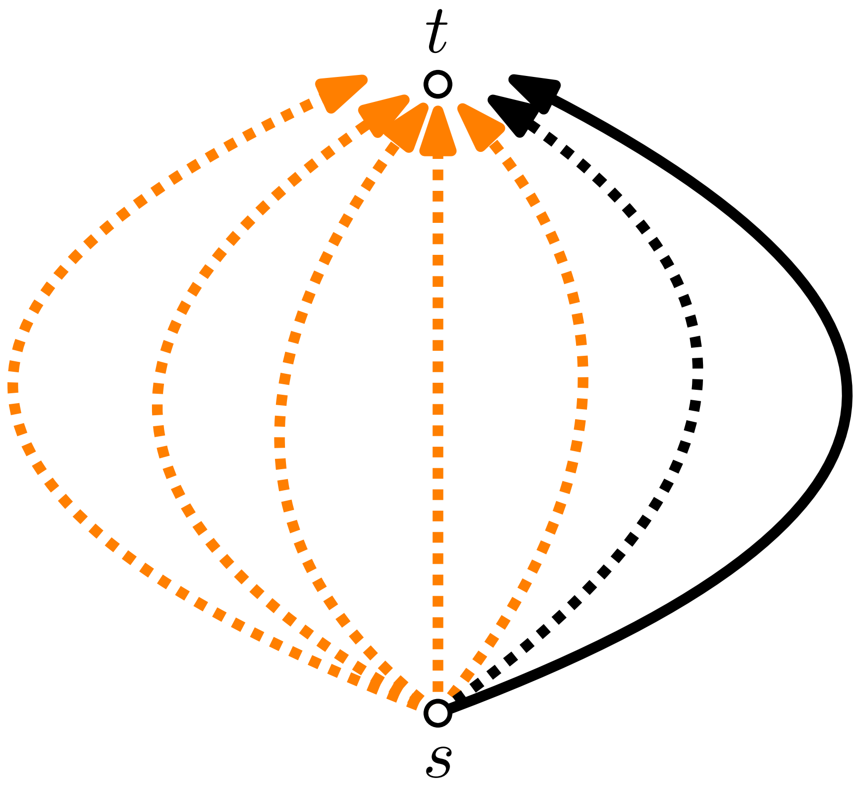
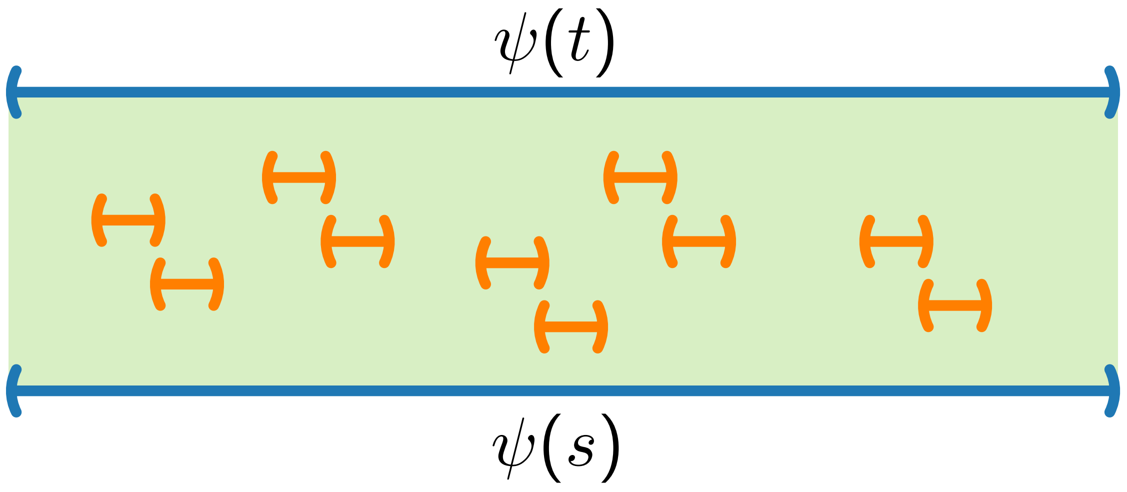
Four different types: **FF, FL, LF, LL**

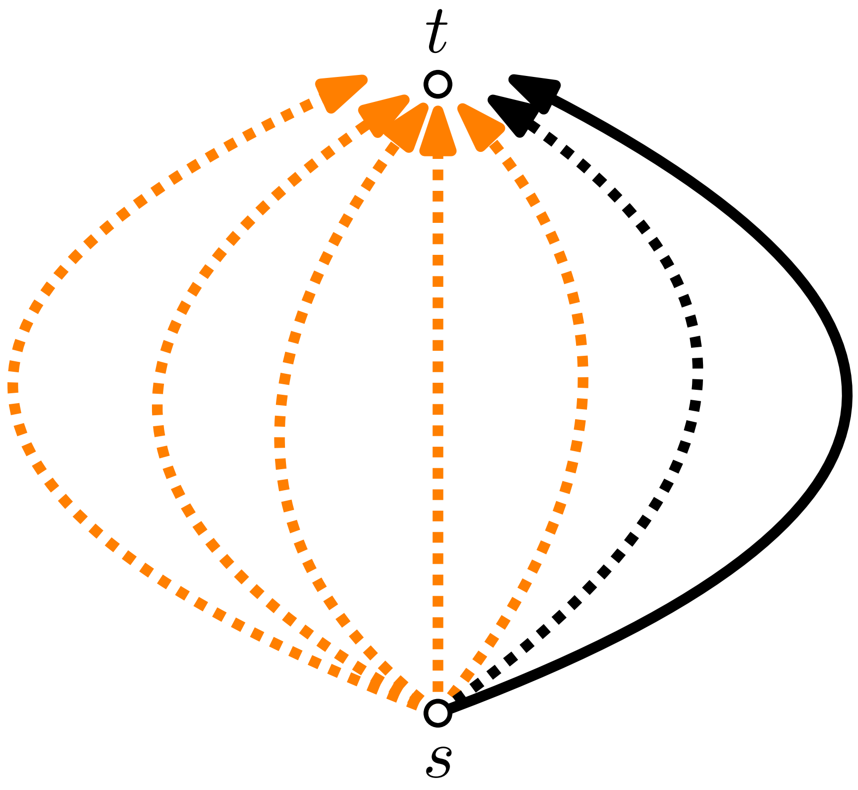
P Nodes





P Nodes





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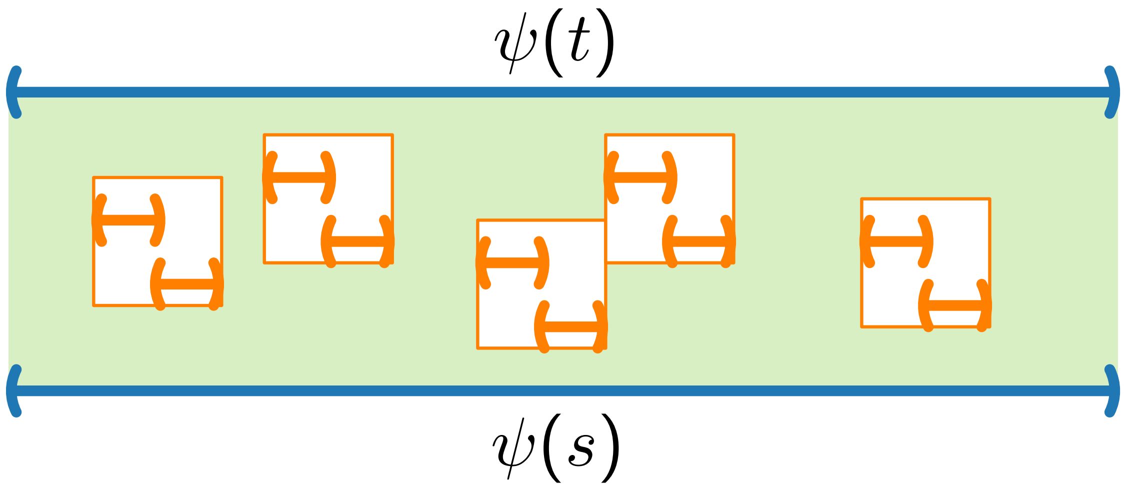

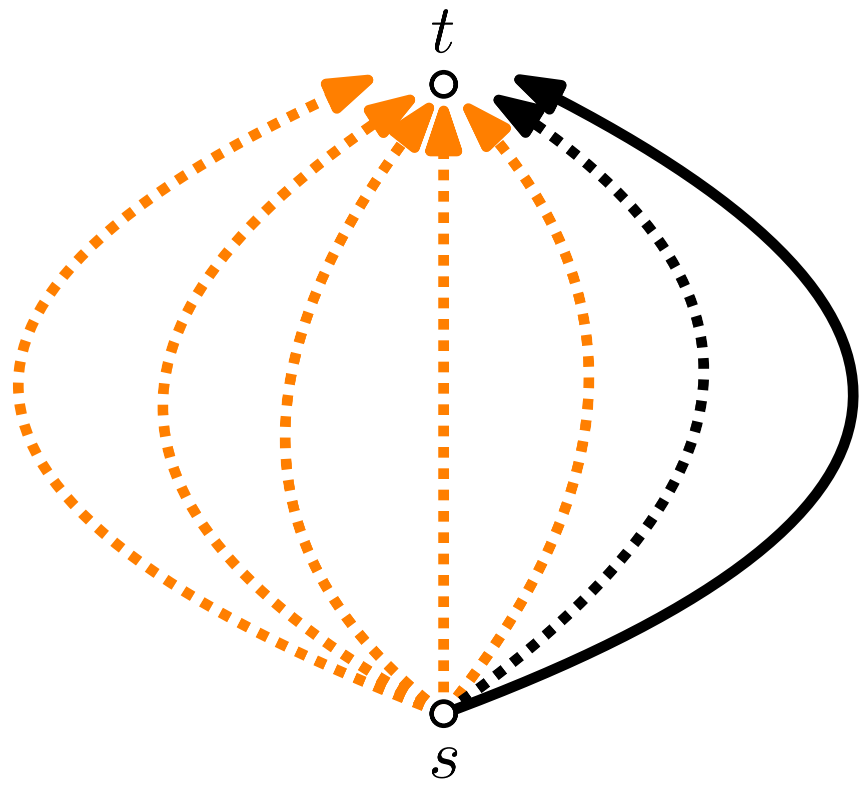

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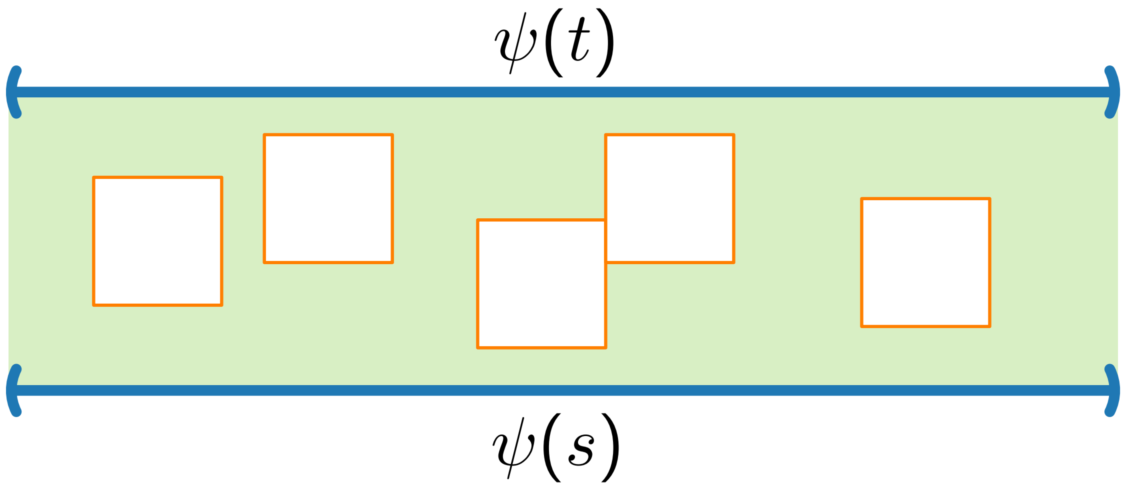

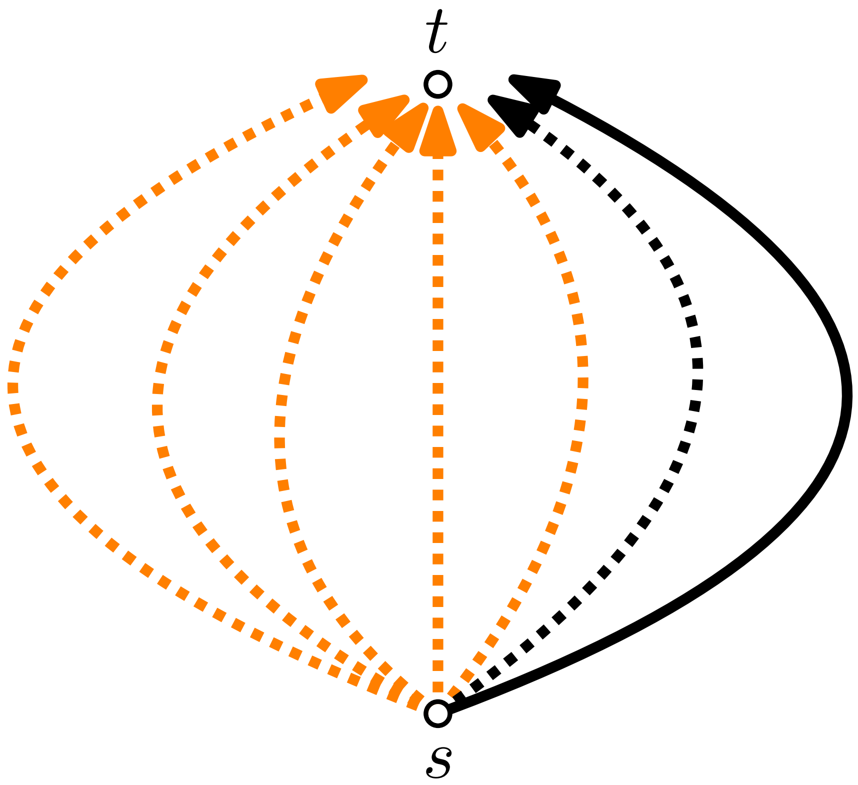

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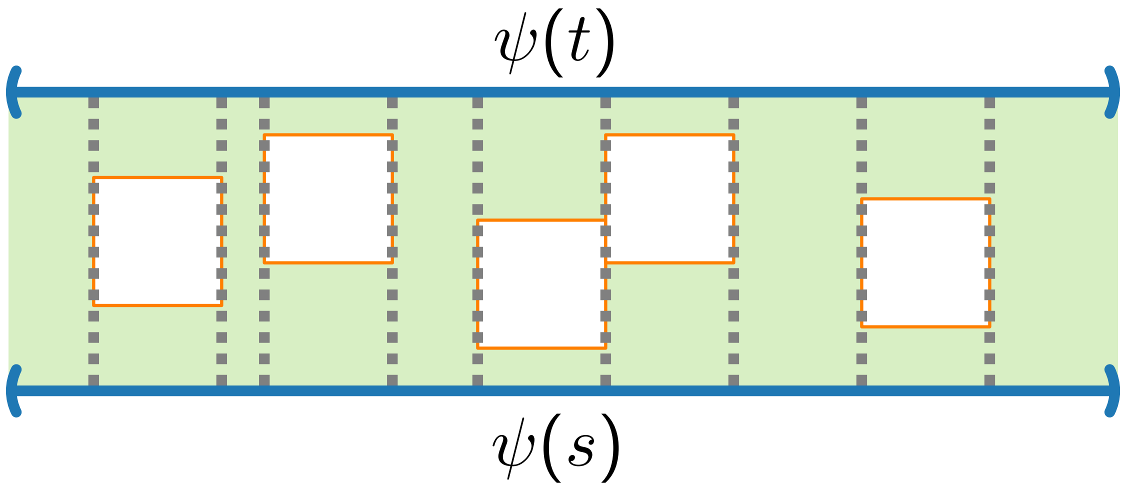

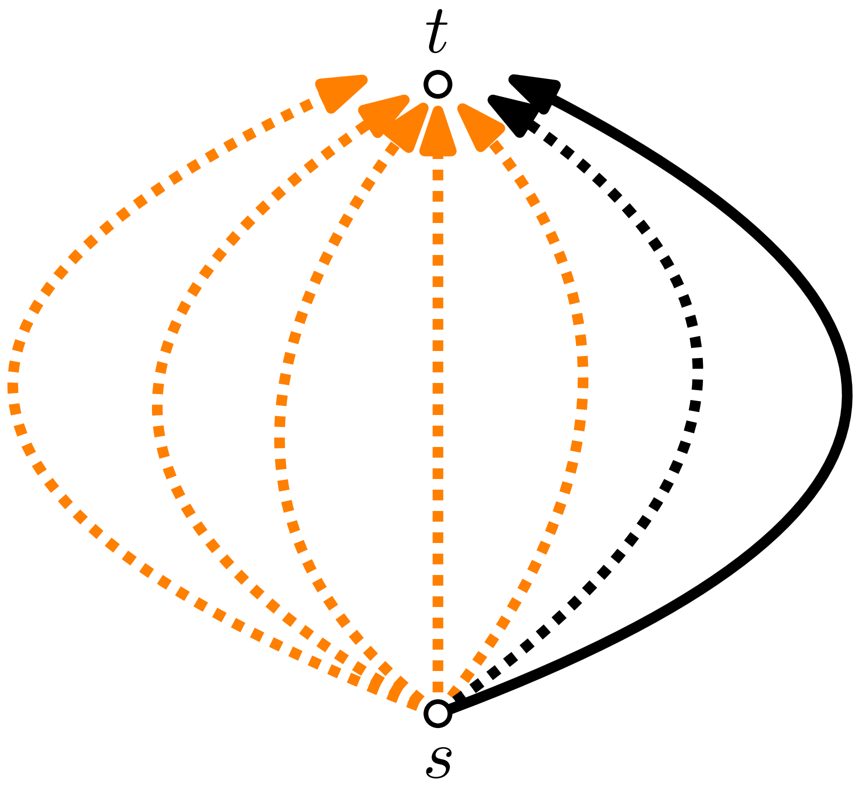

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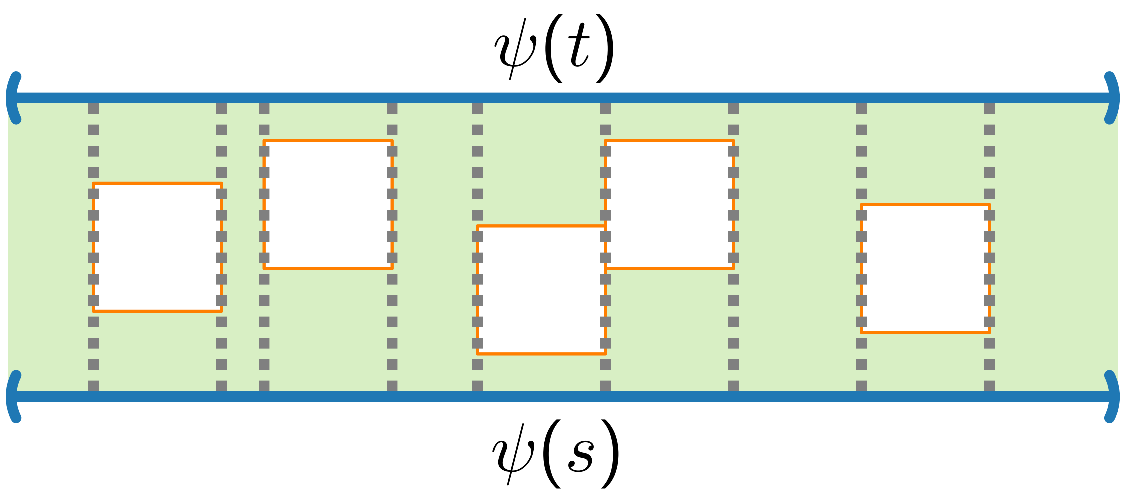
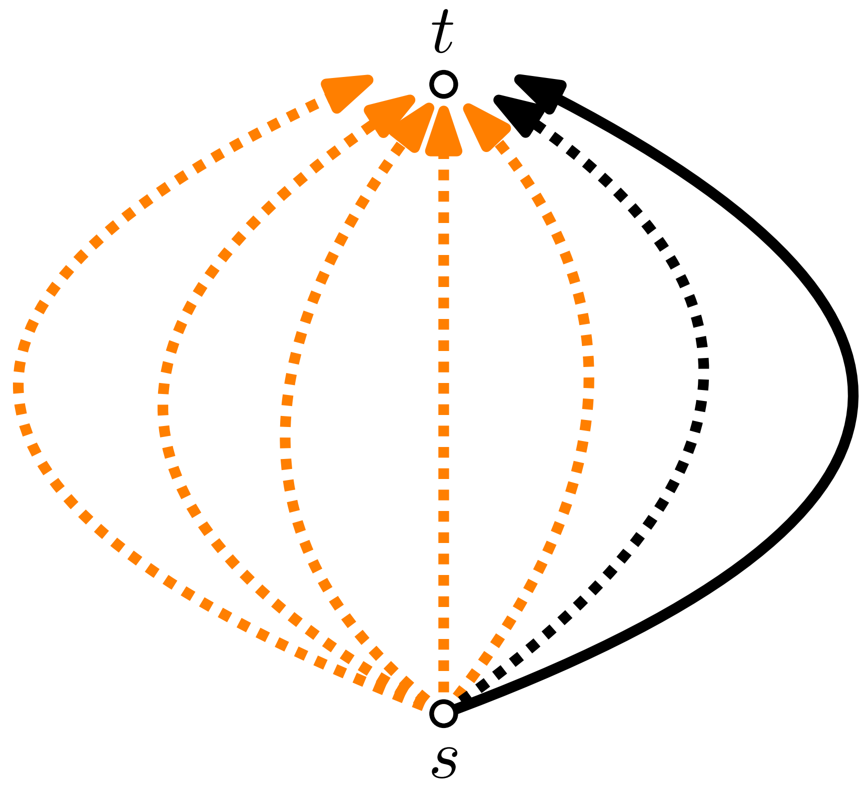
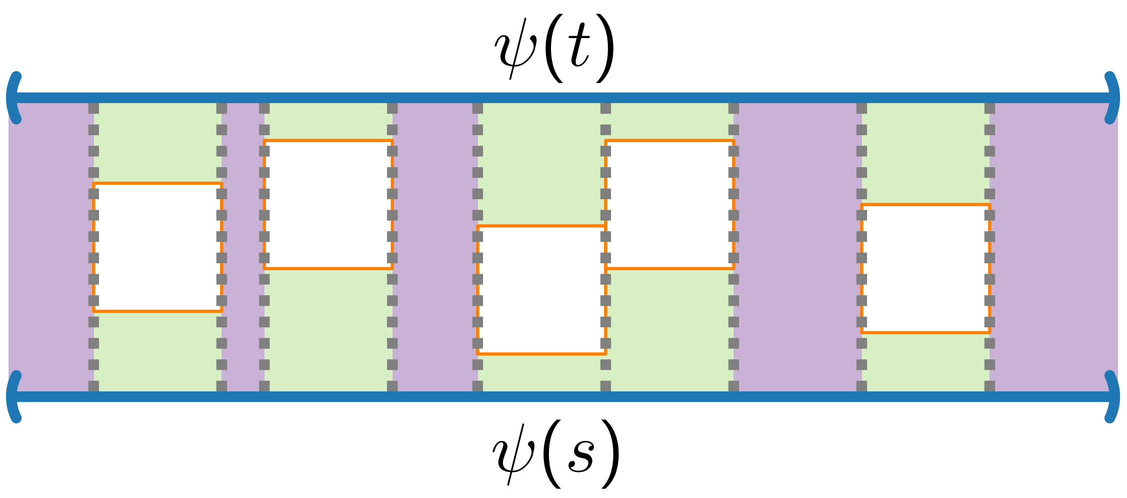


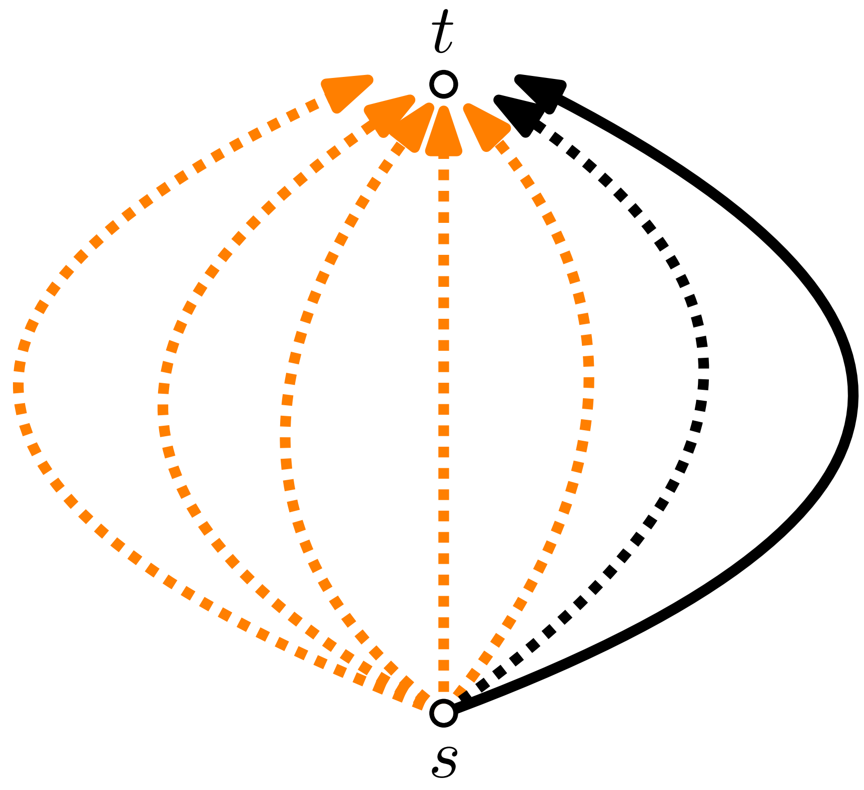


- Children of **P** node with prescribed bars occur in given left-to-right order

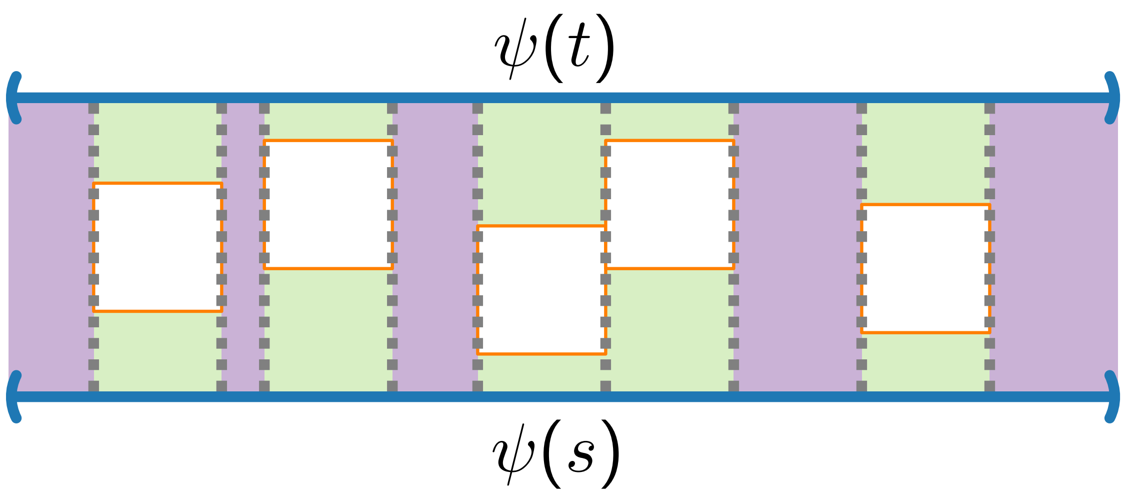
P Nodes

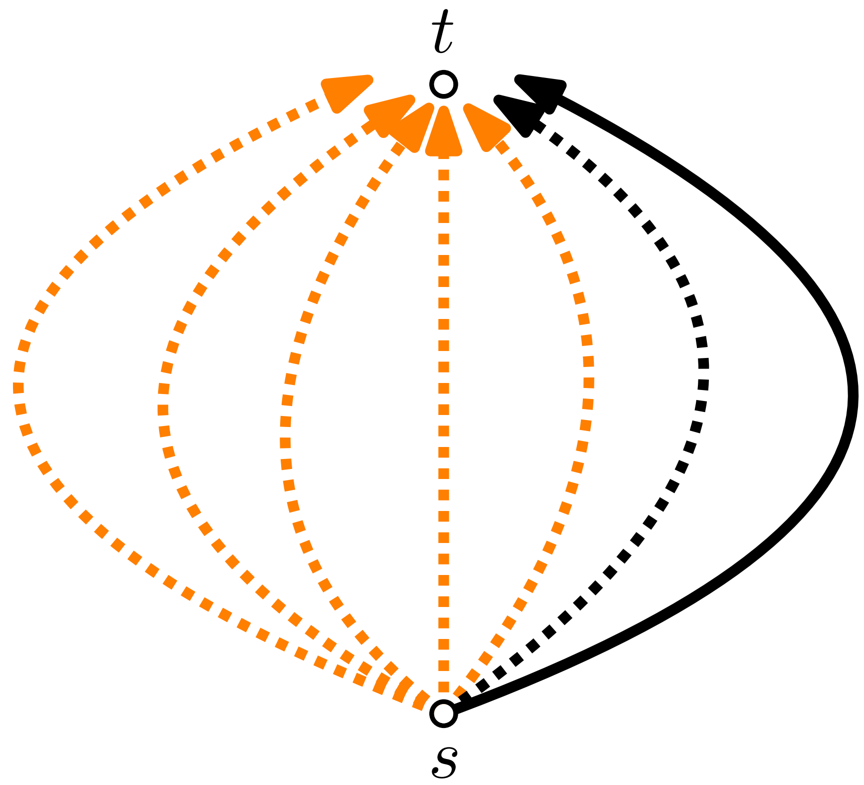


- Children of **P** node with prescribed bars occur in given left-to-right order
- But there might be some gaps...



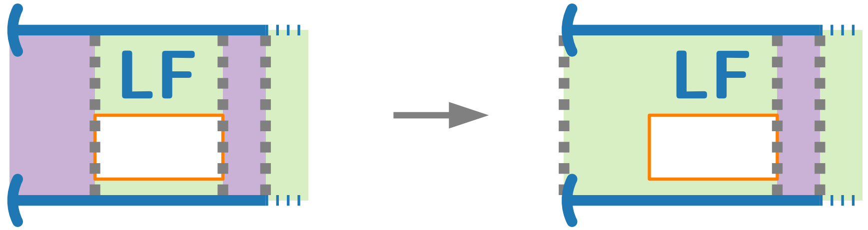
P Nodes





- Children of **P** node with prescribed bars occur in given left-to-right order
- But there might be some gaps. . .

Idea.
Greedily *fill* the gaps by preferring to “stretch” the children with prescribed bars.



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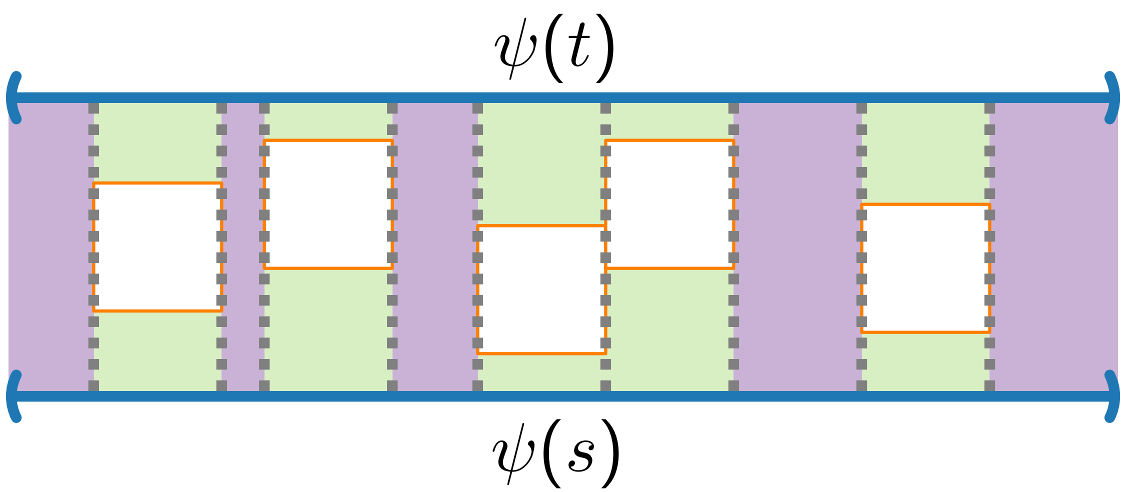

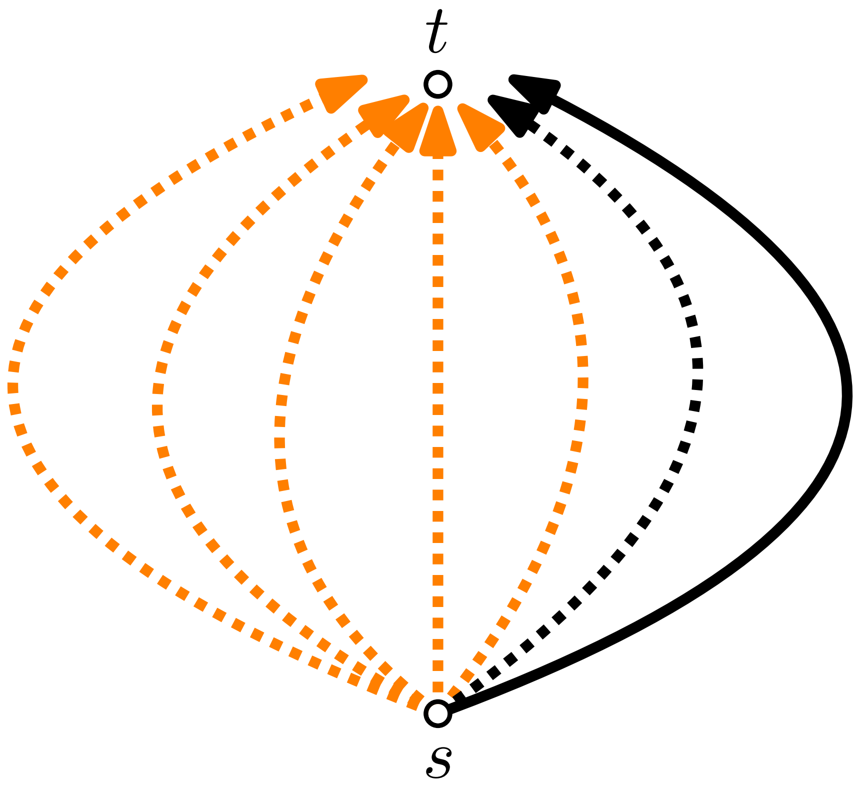

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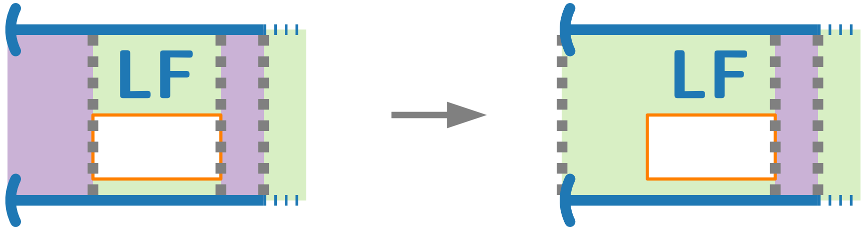
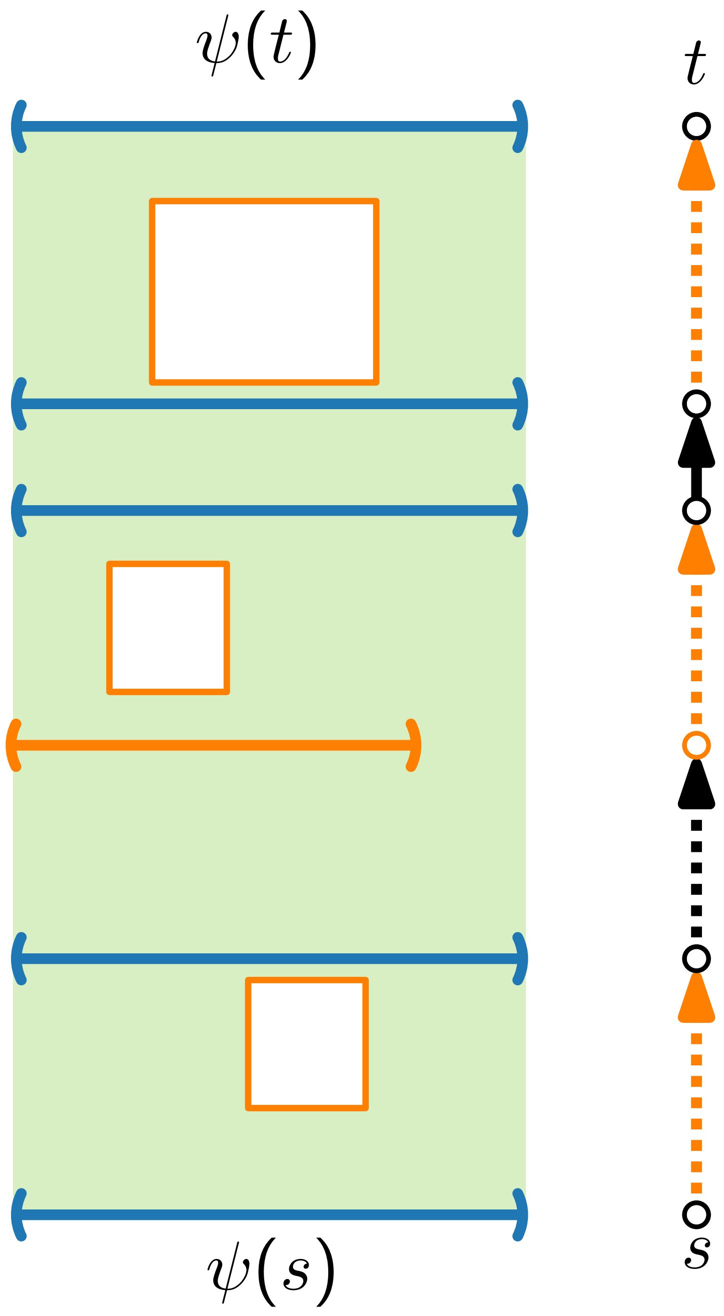
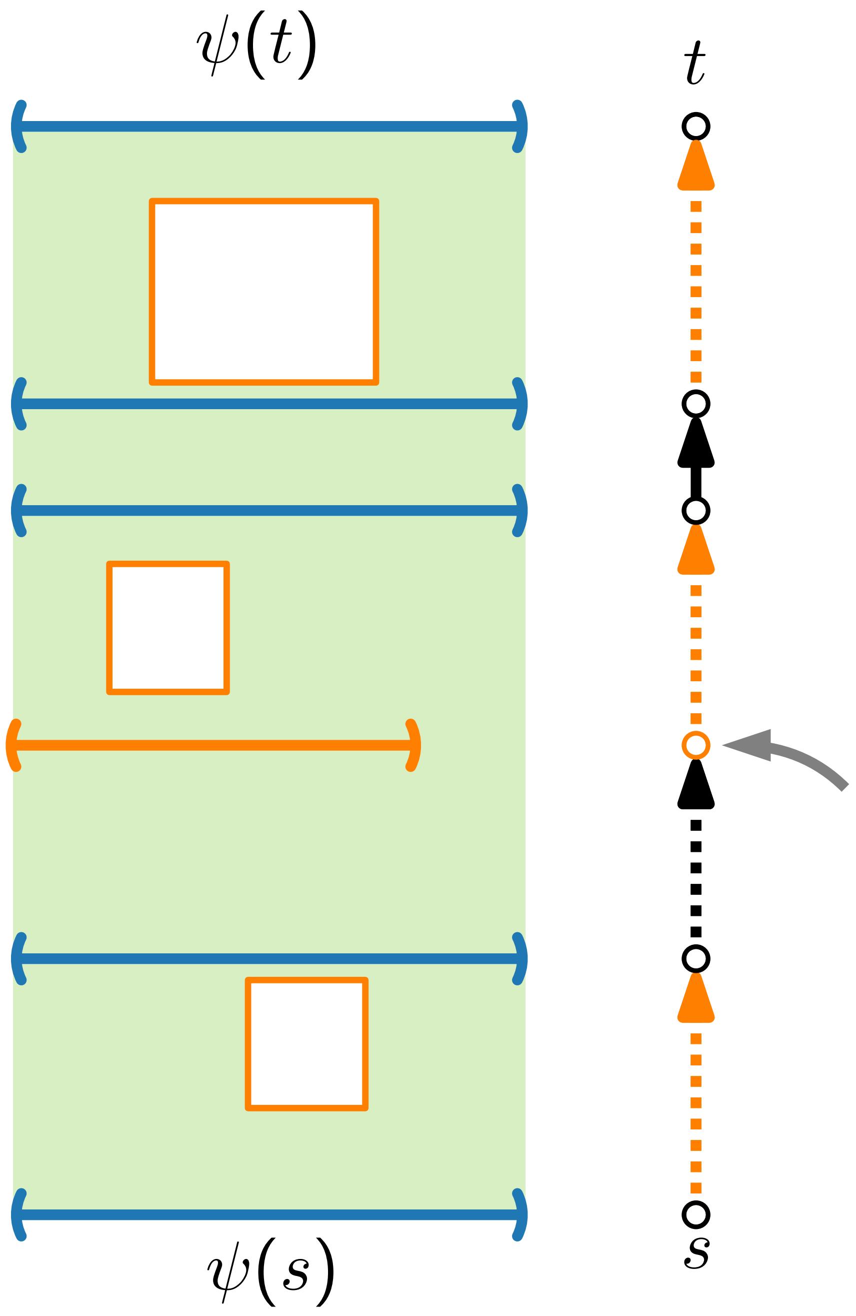


Outcome.
After processing, we must know the valid types for the corresponding subgraphs.

S Nodes

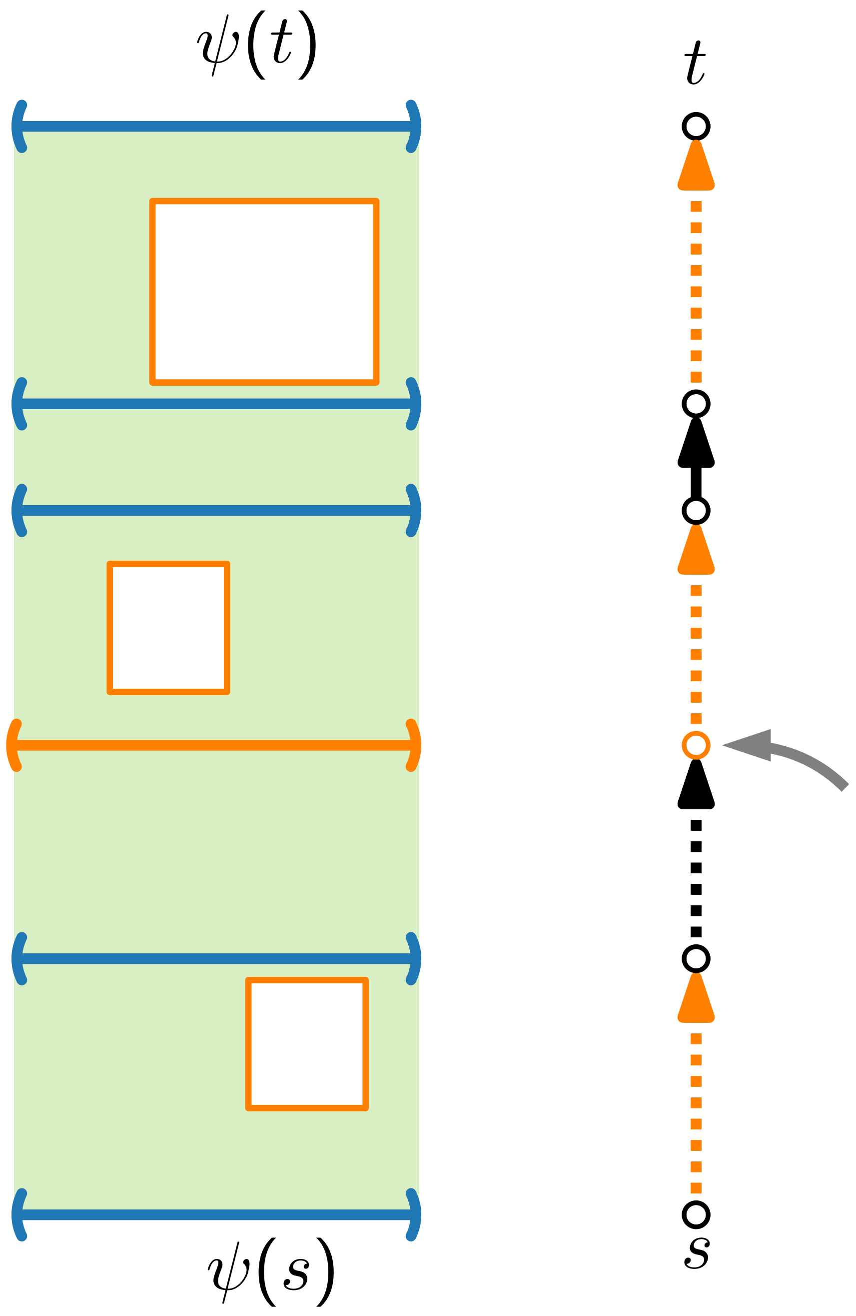


S Nodes



This fixed vertex means we can only make a Fixed-Fixed representation!

S Nodes



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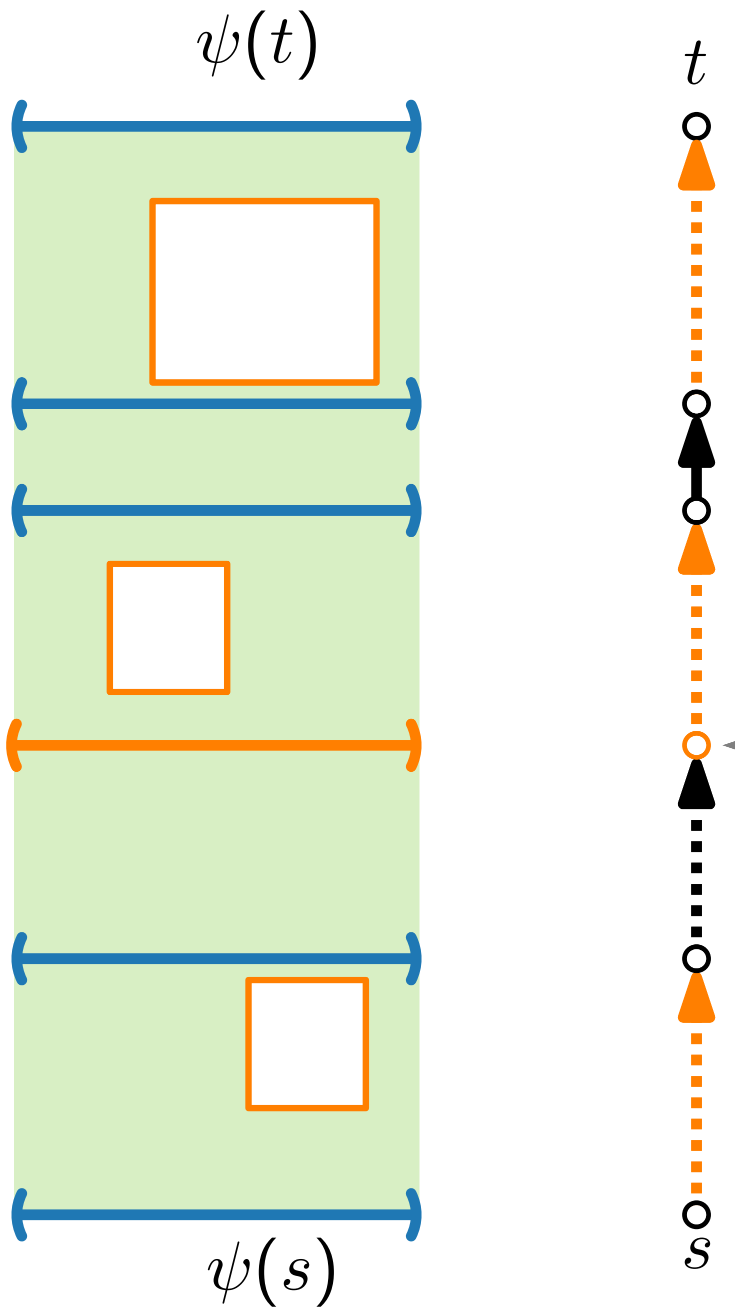

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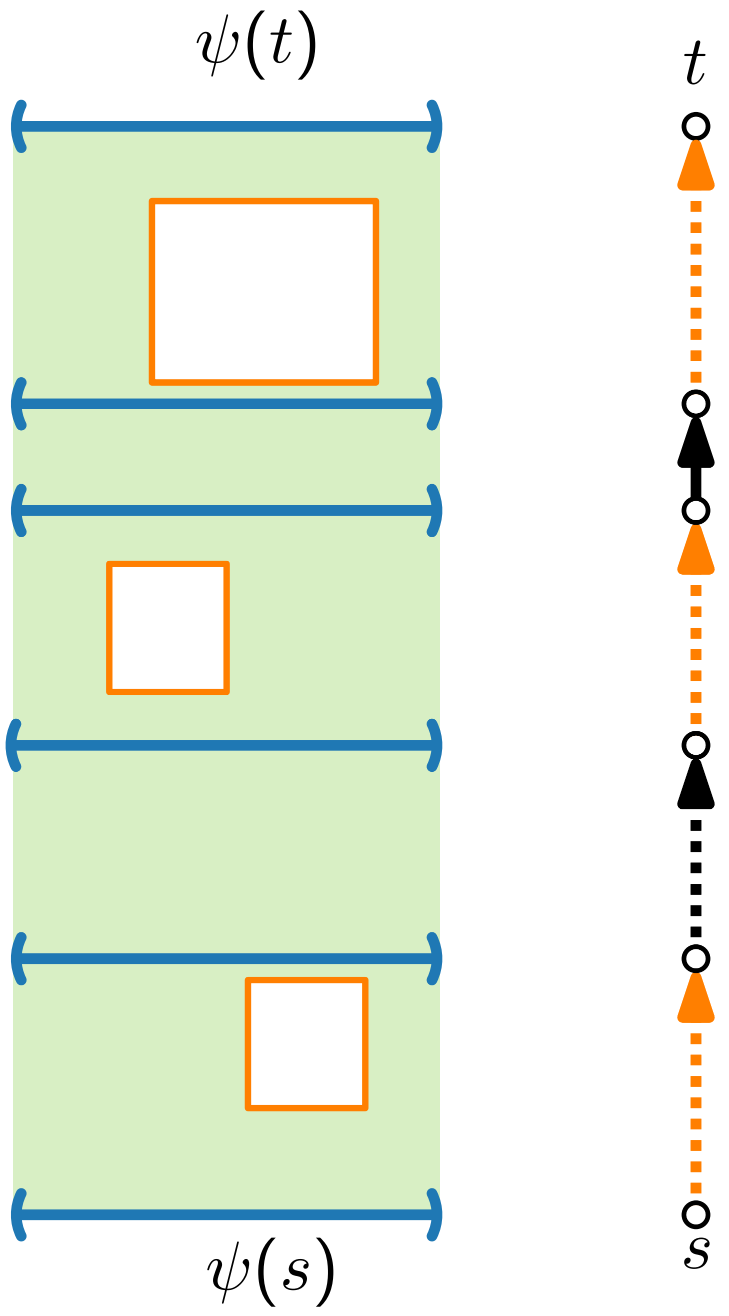
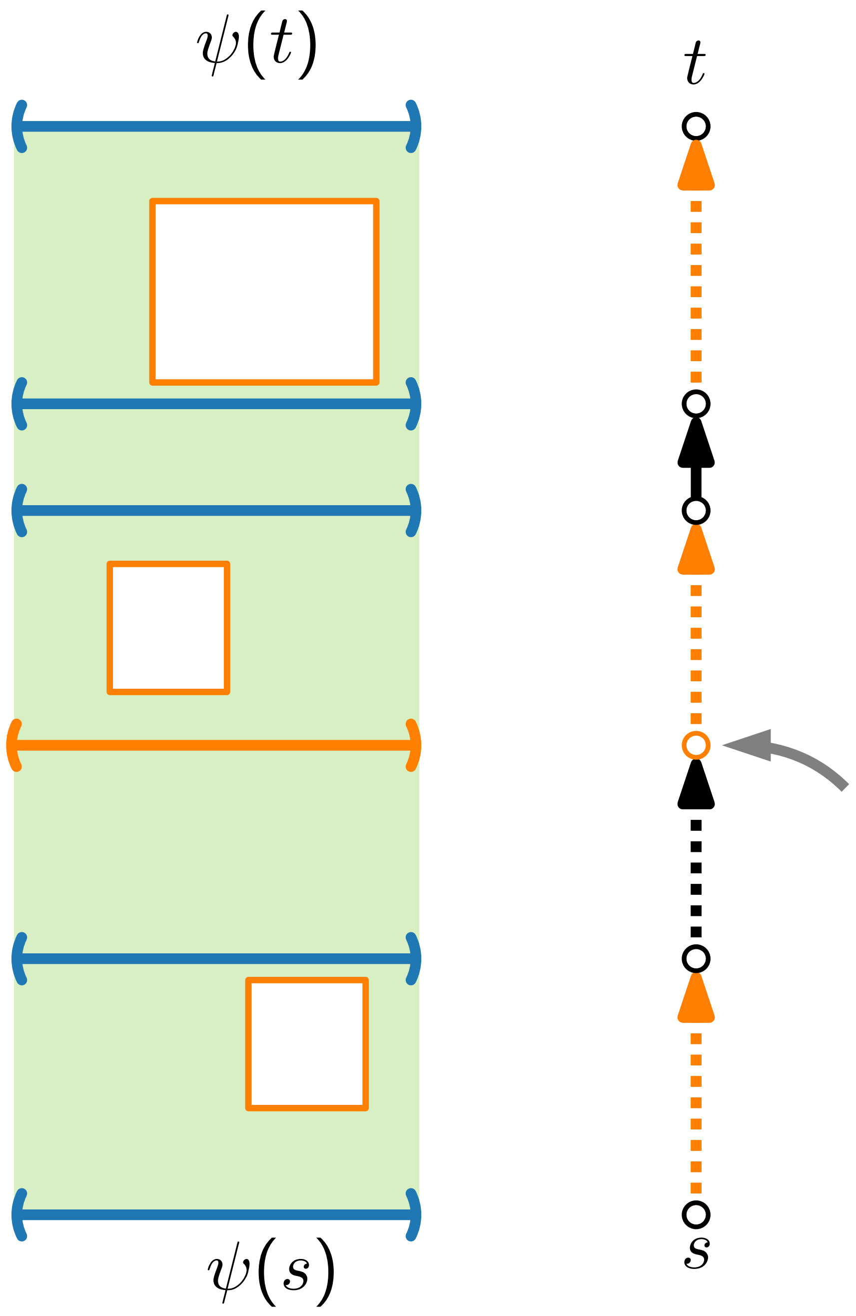
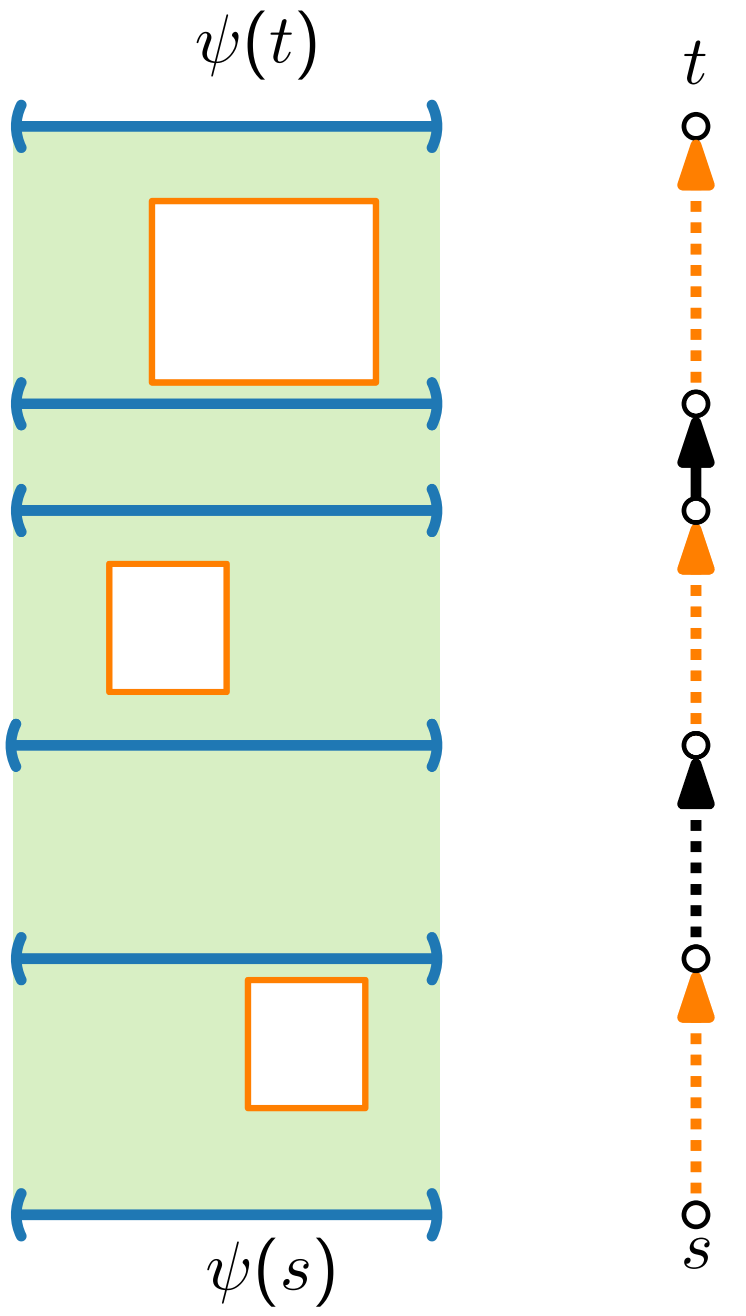


S Nodes

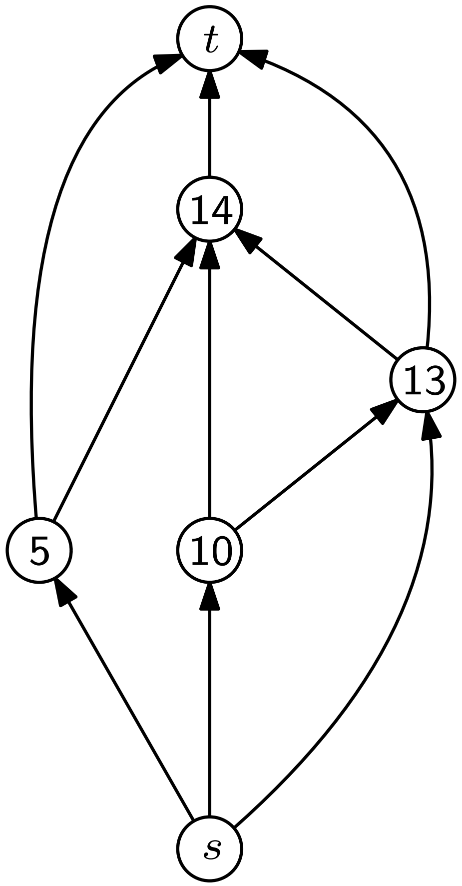


This fixed vertex means we can only make a Fixed-Fixed representation!

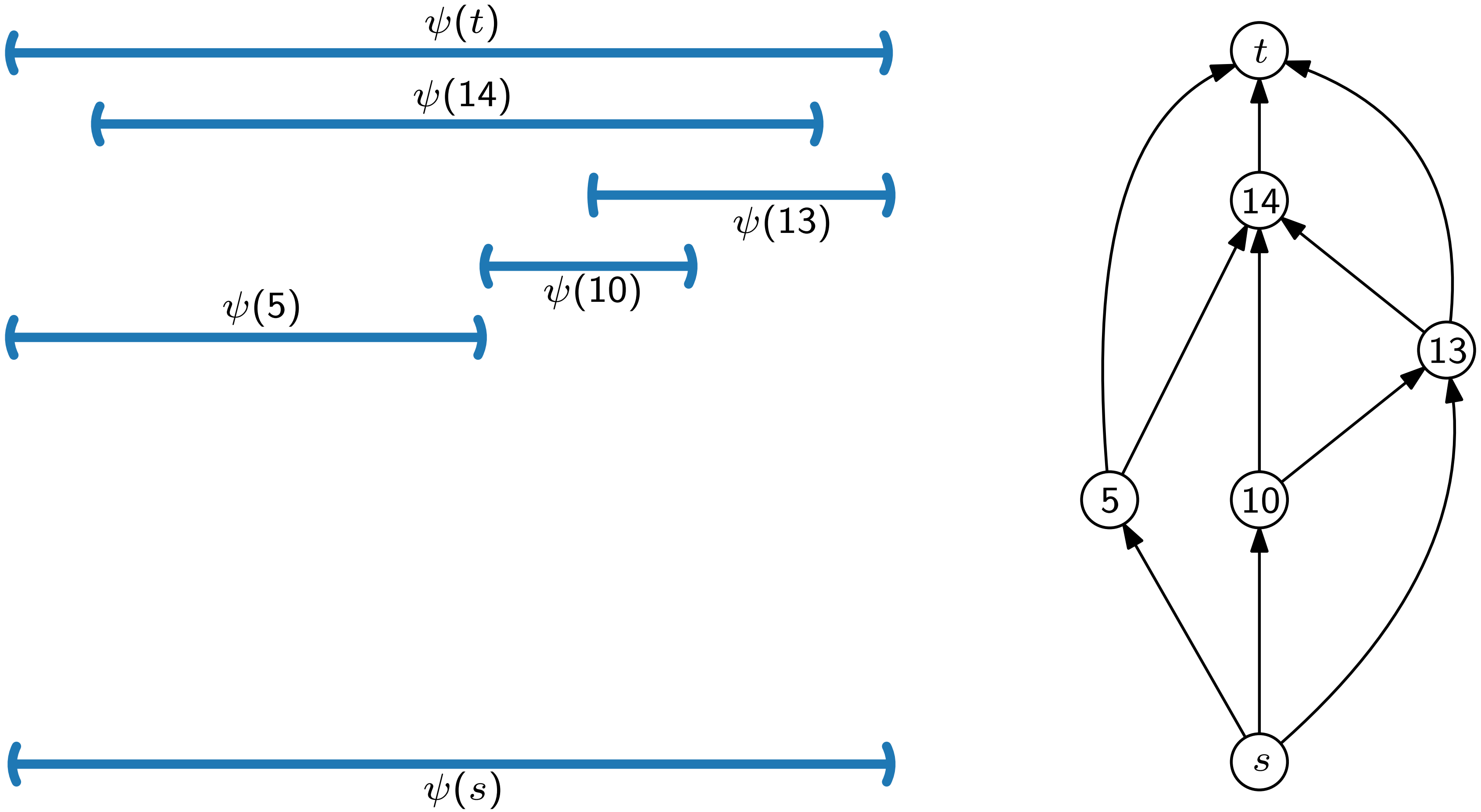
Here we have a chance to make all (**LL, FL, LF, FF**) types.



R Nodes



R Nodes



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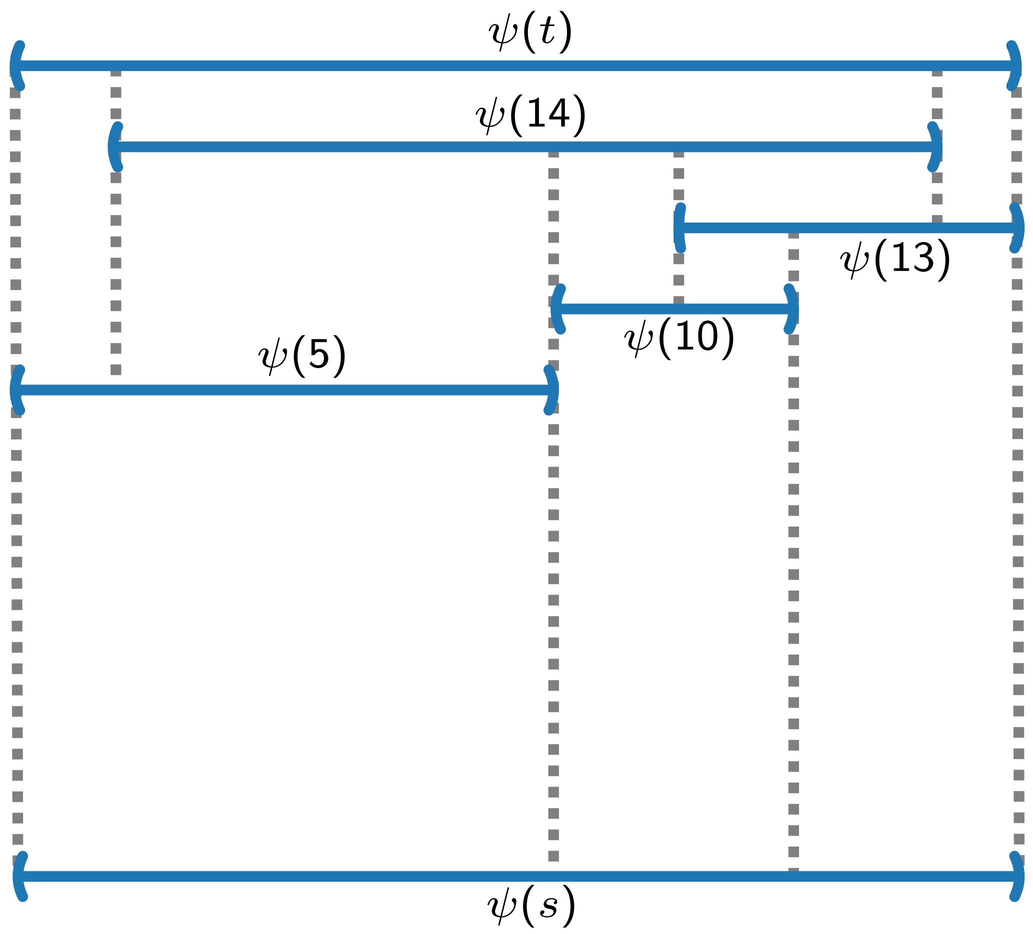

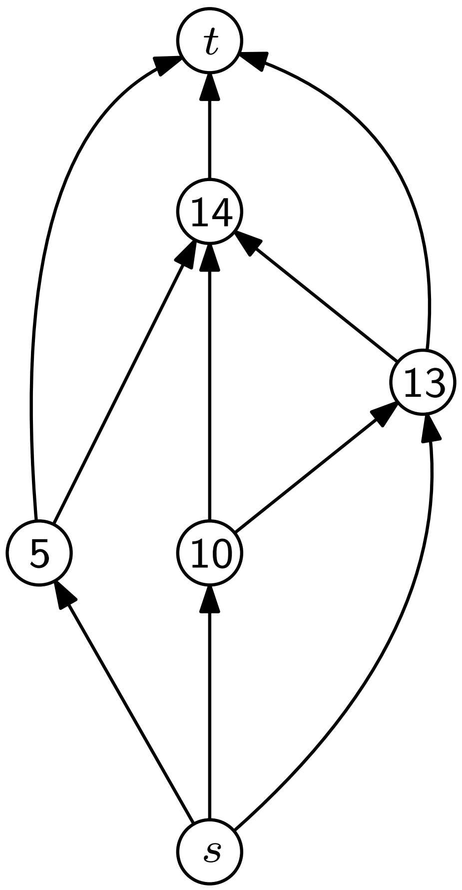

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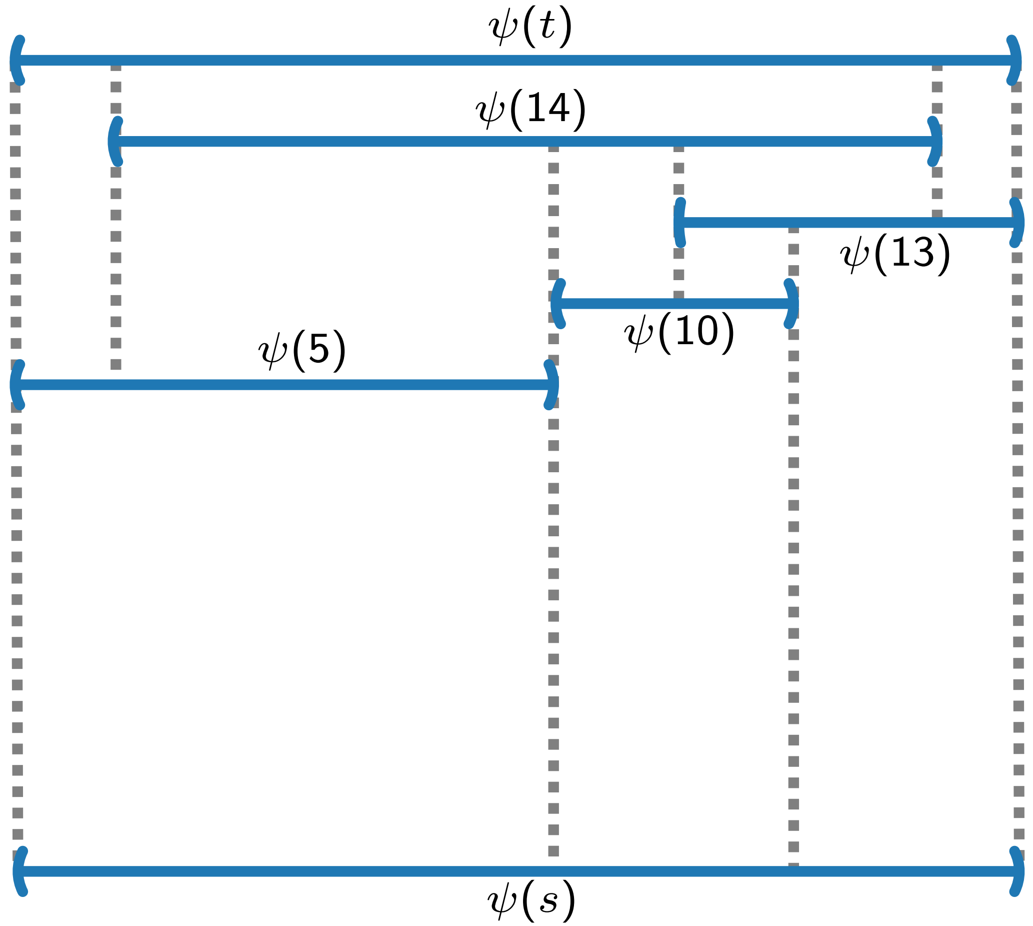

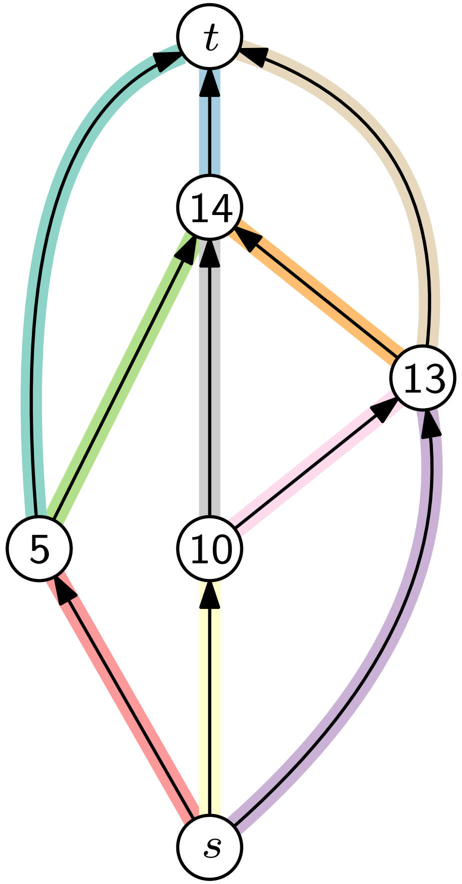

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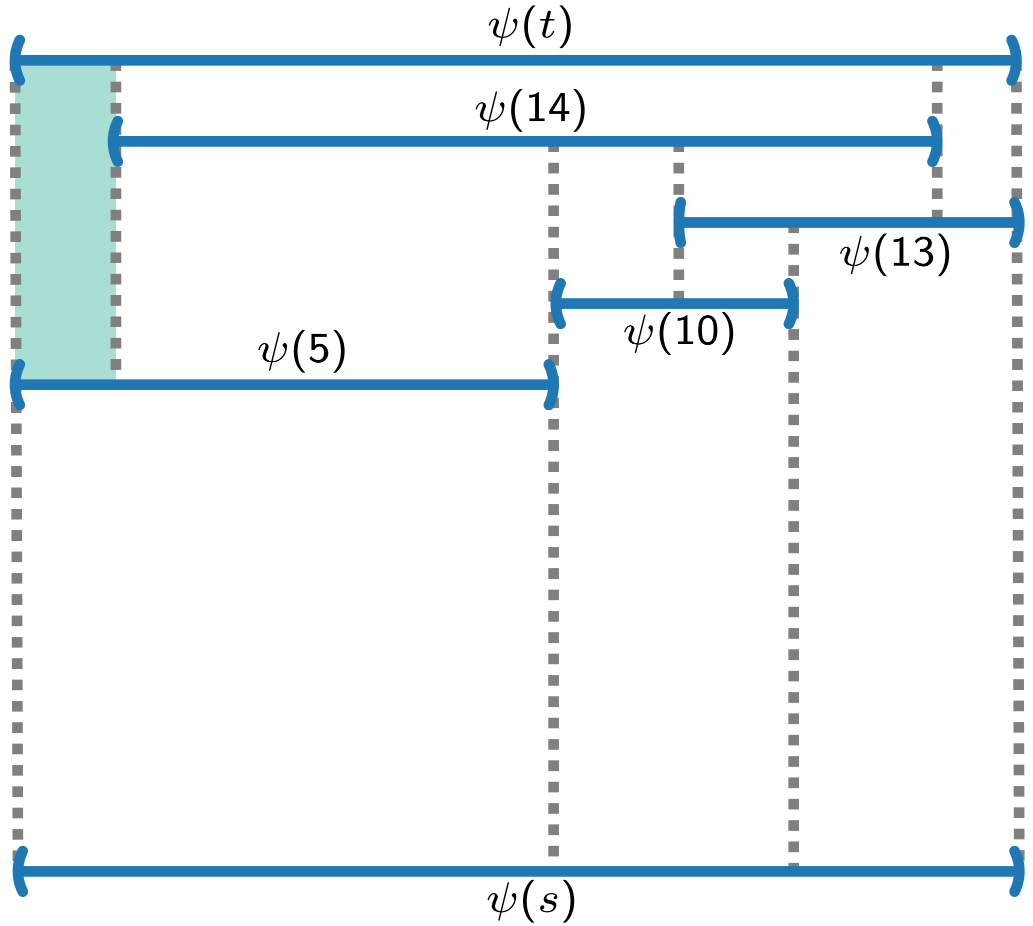

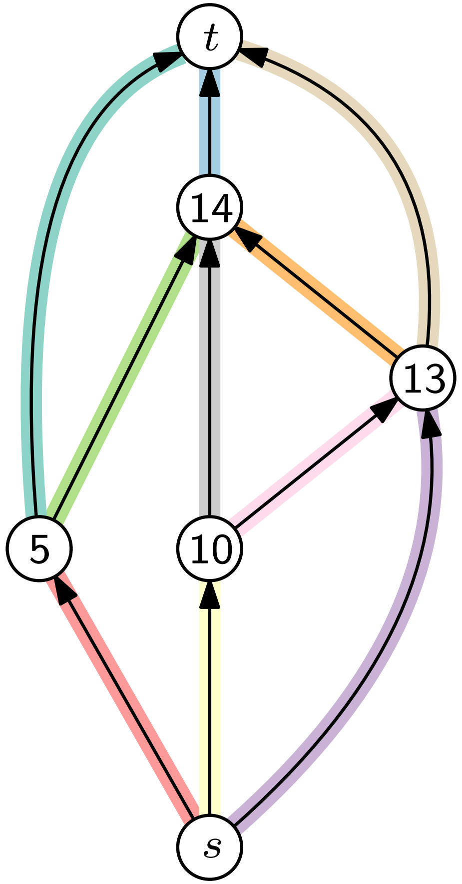

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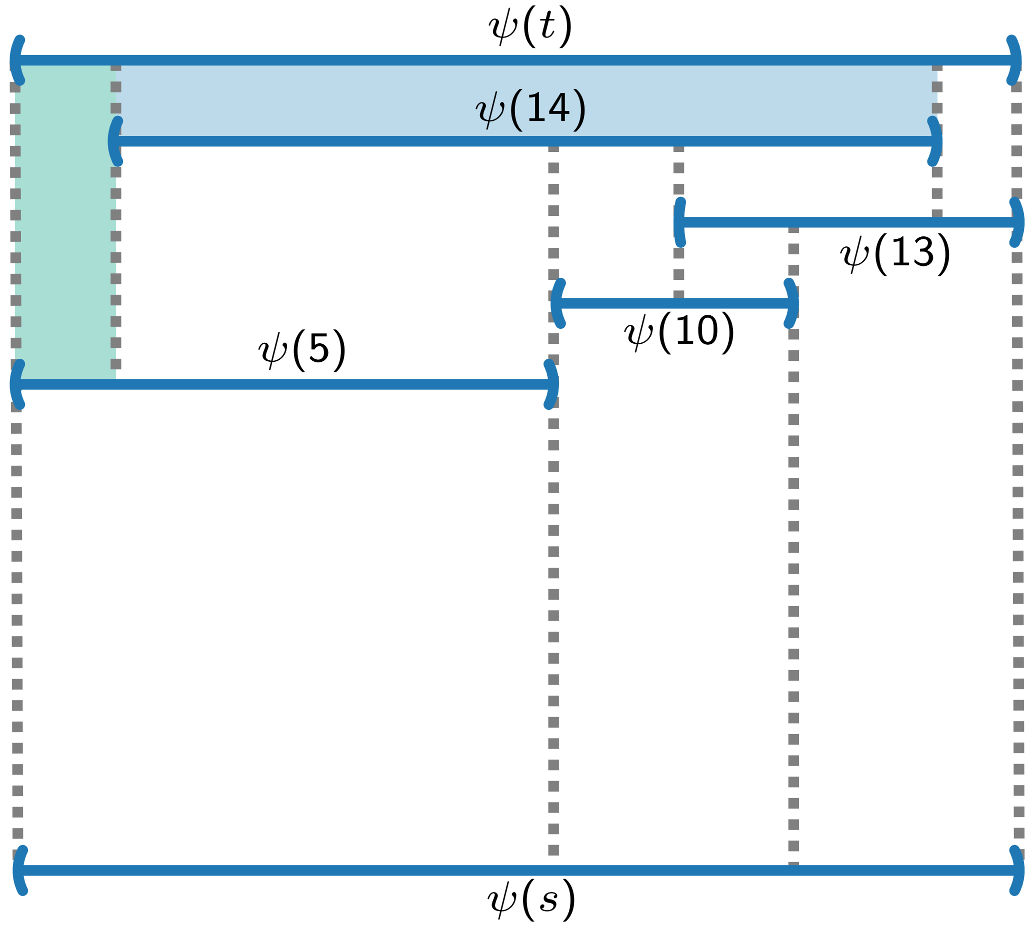

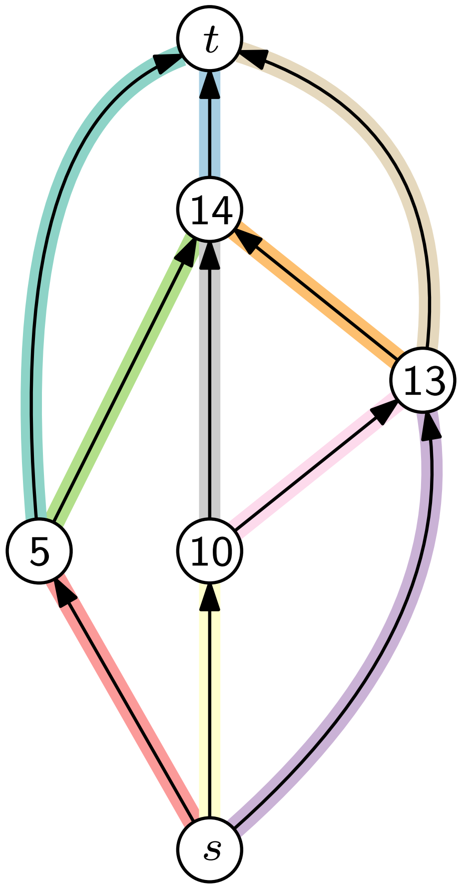

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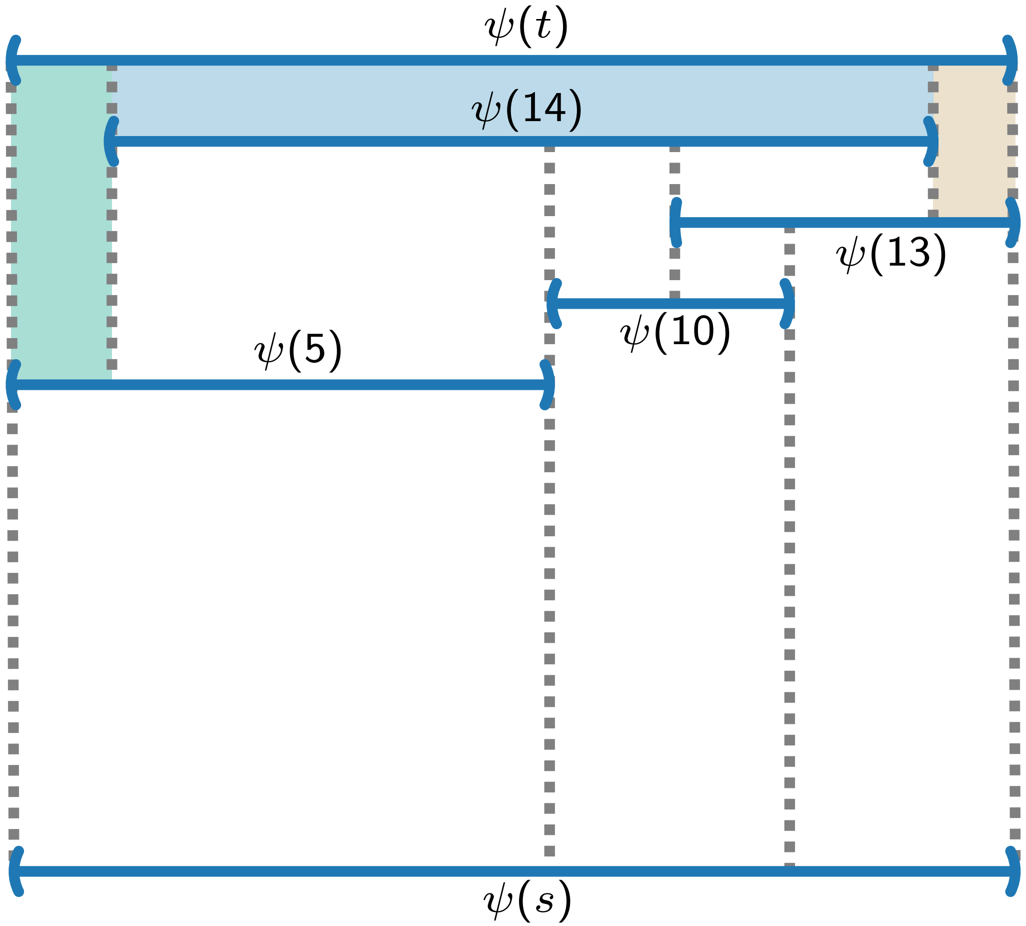

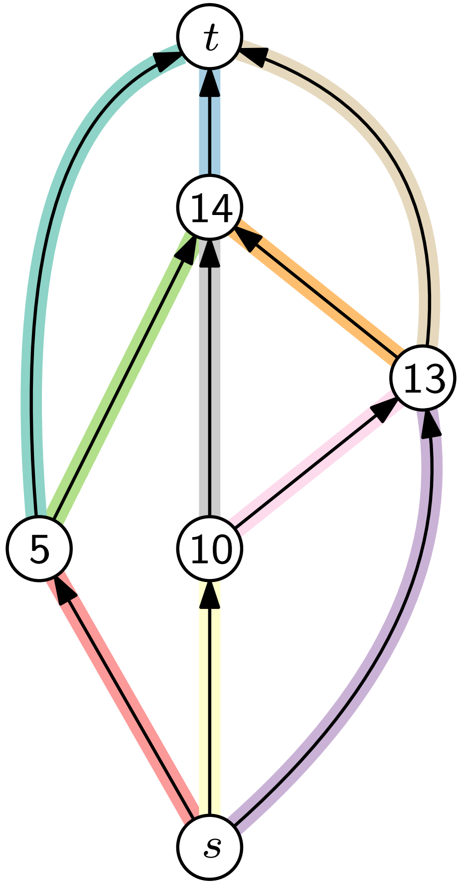

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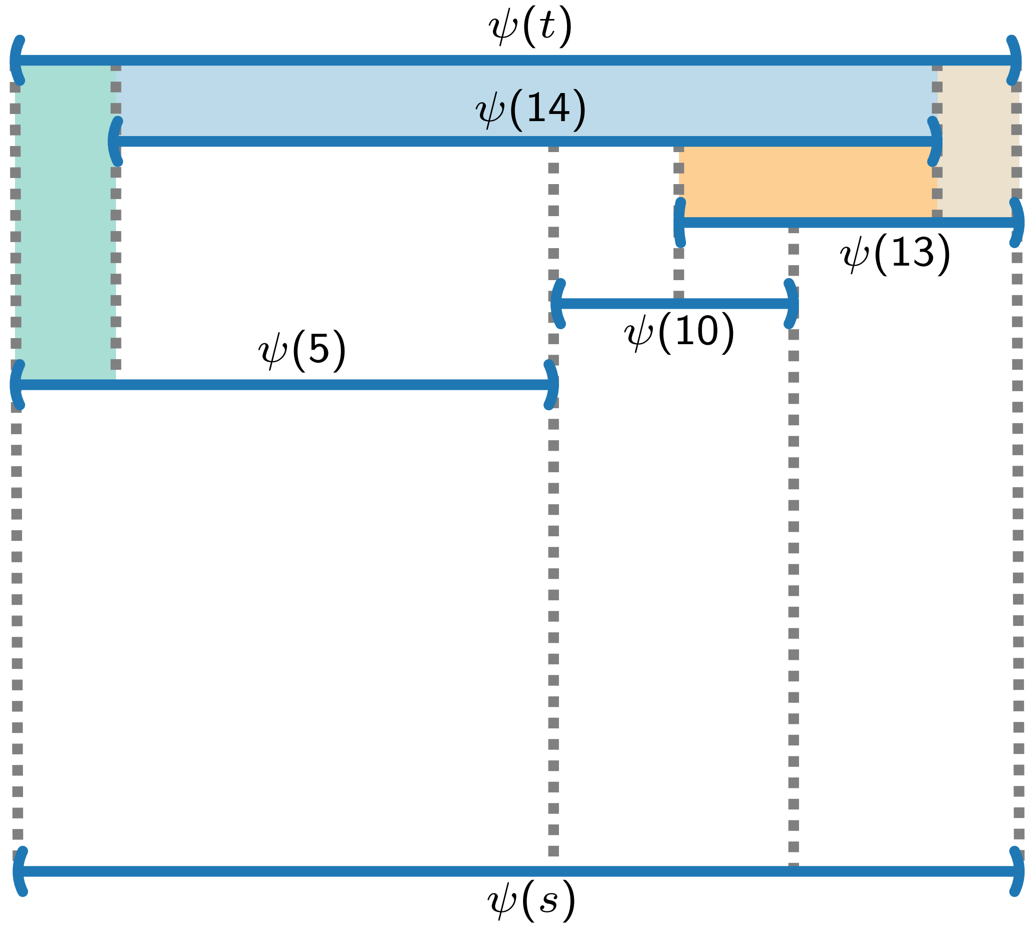

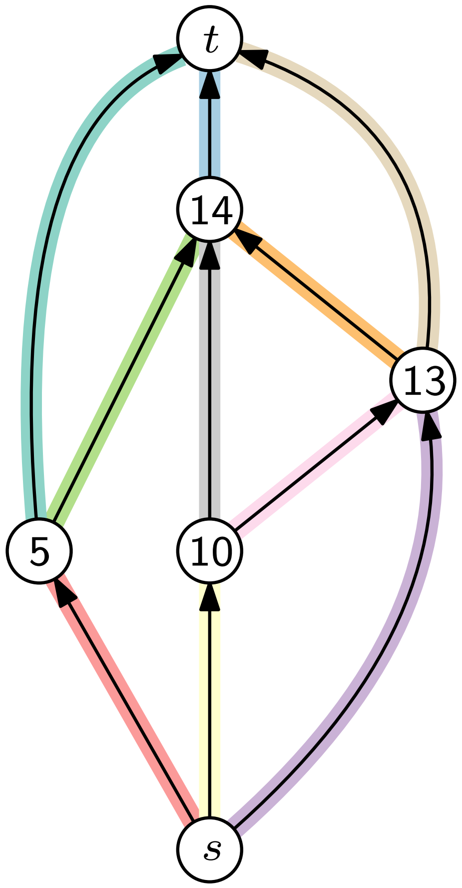

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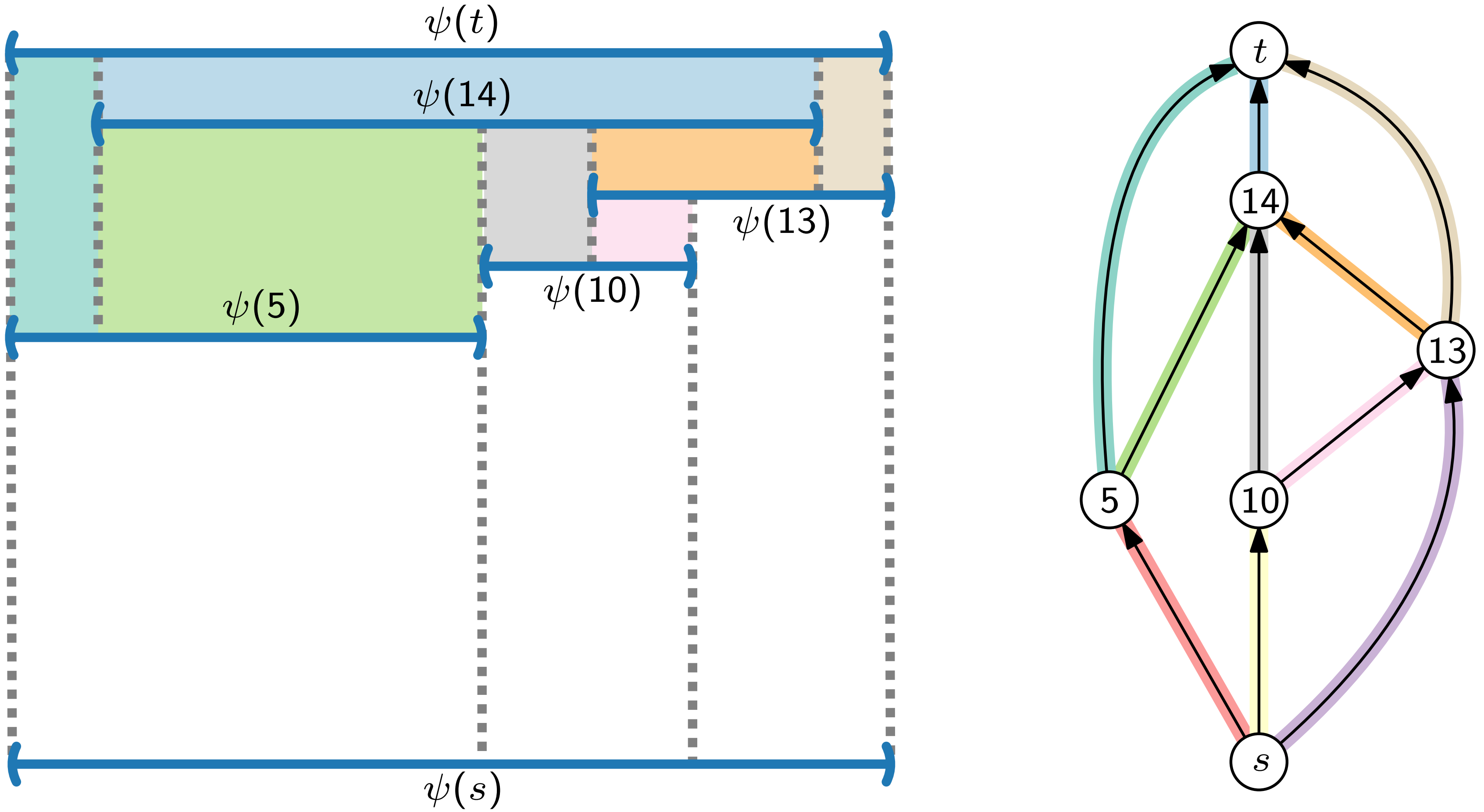

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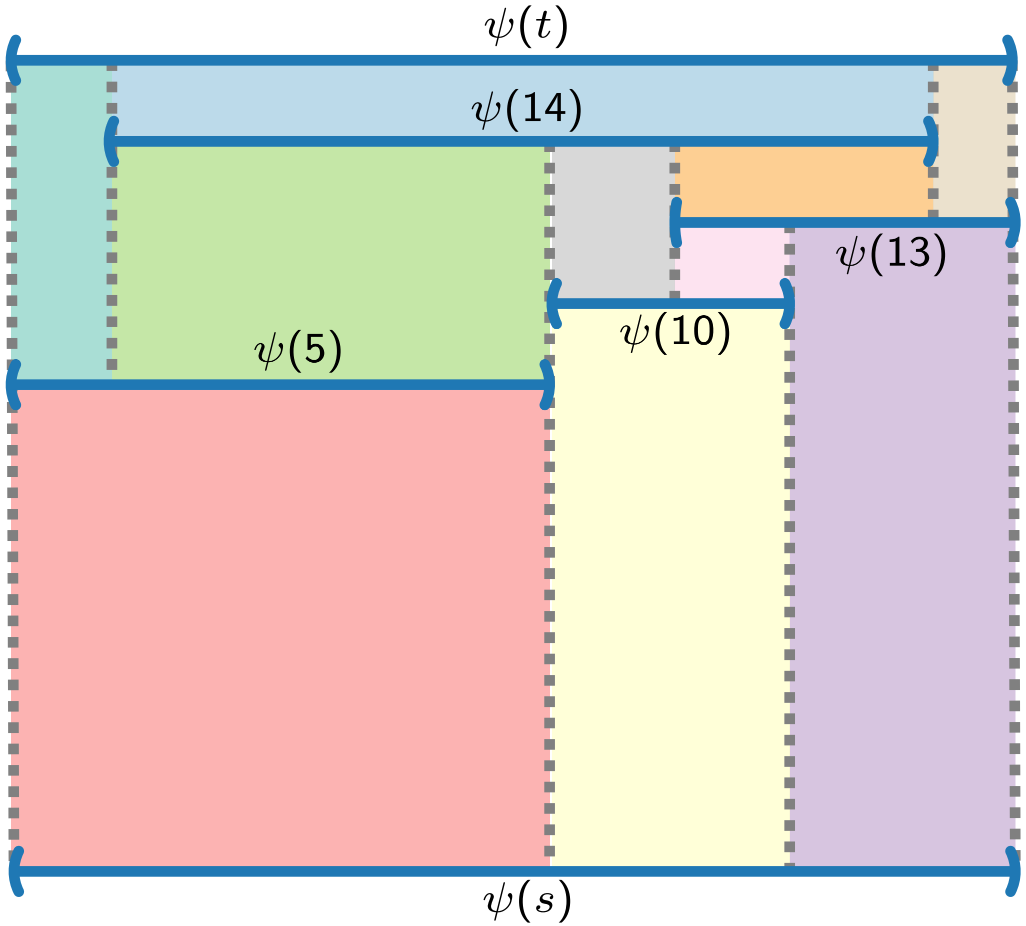

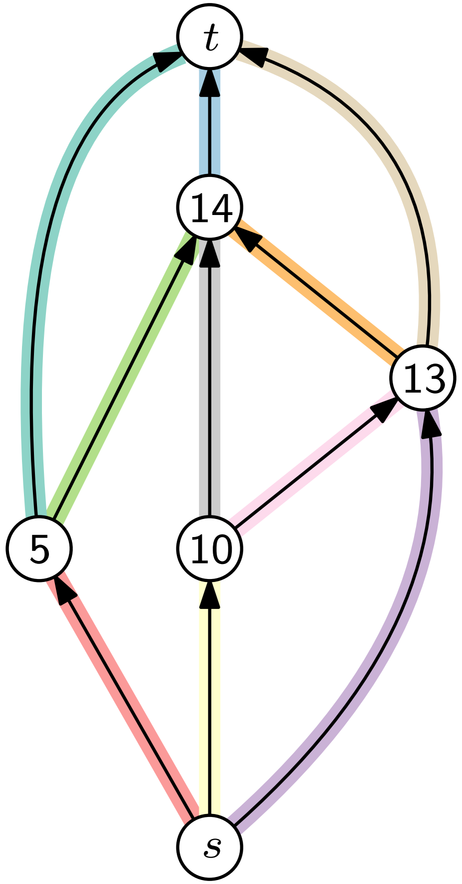

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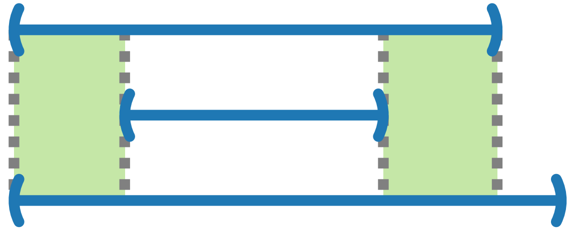
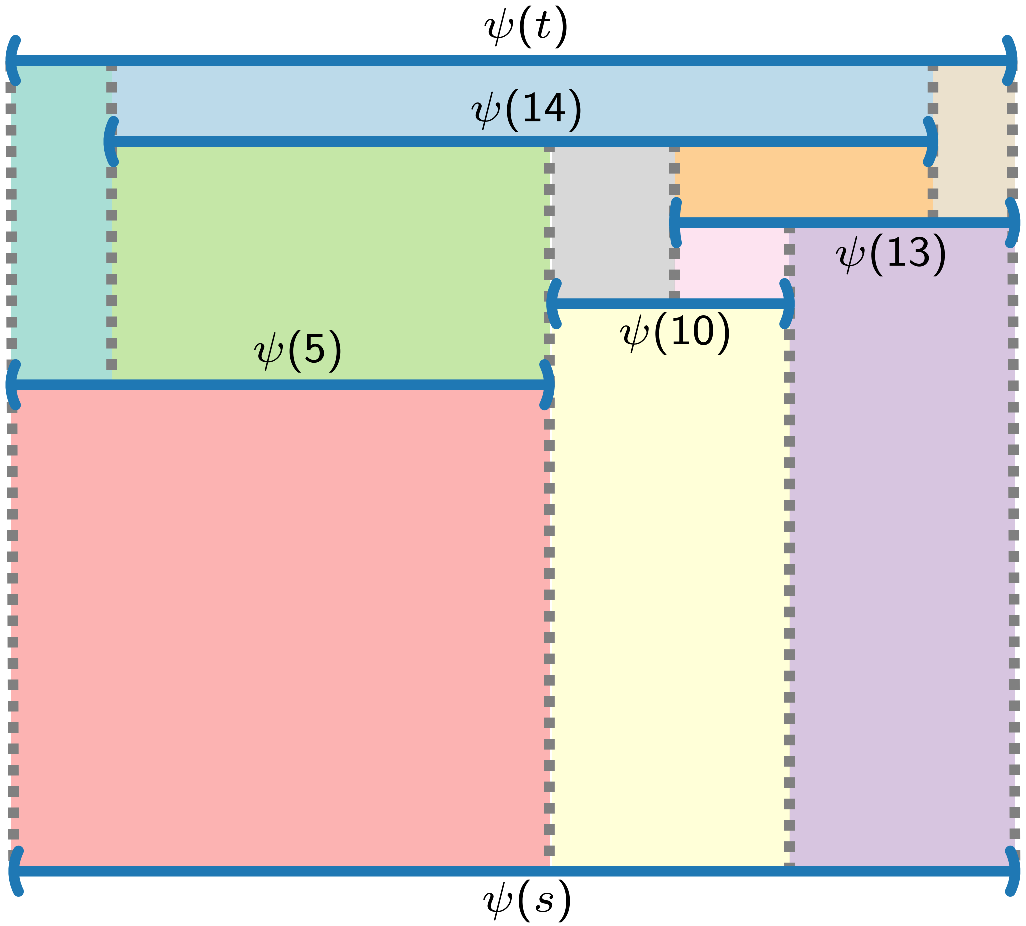
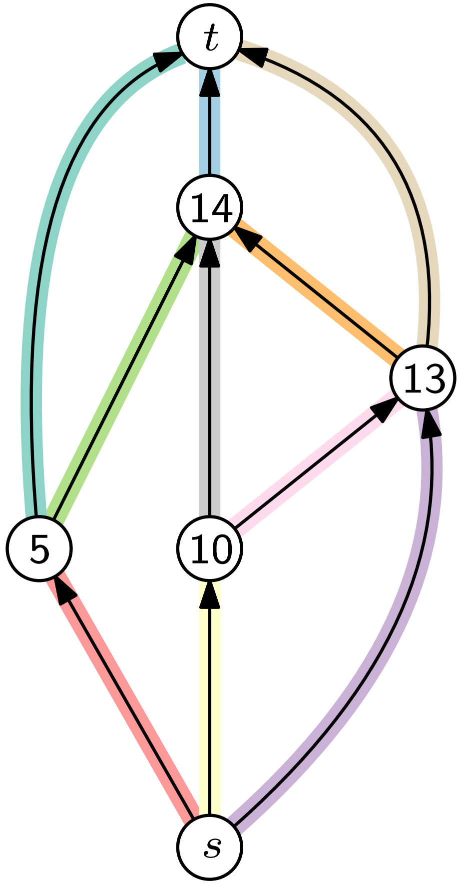
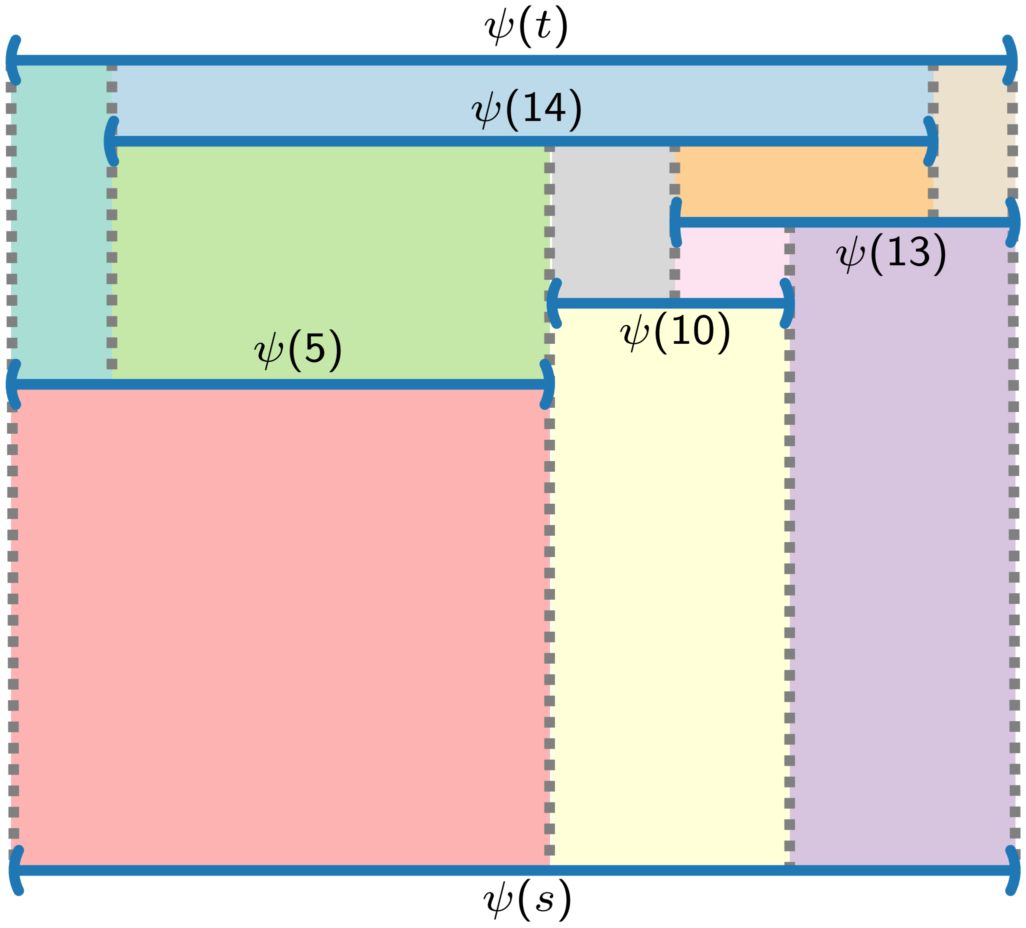
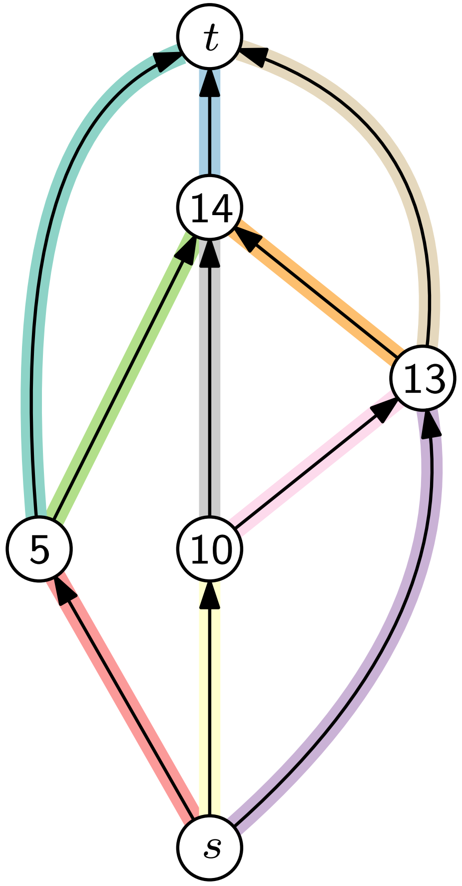


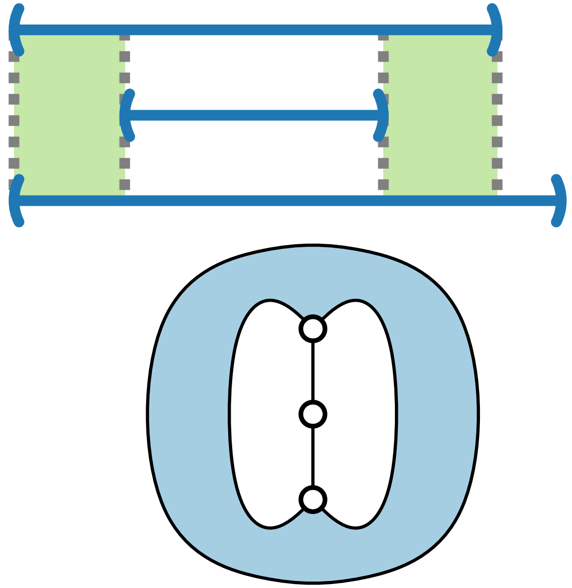




R Nodes







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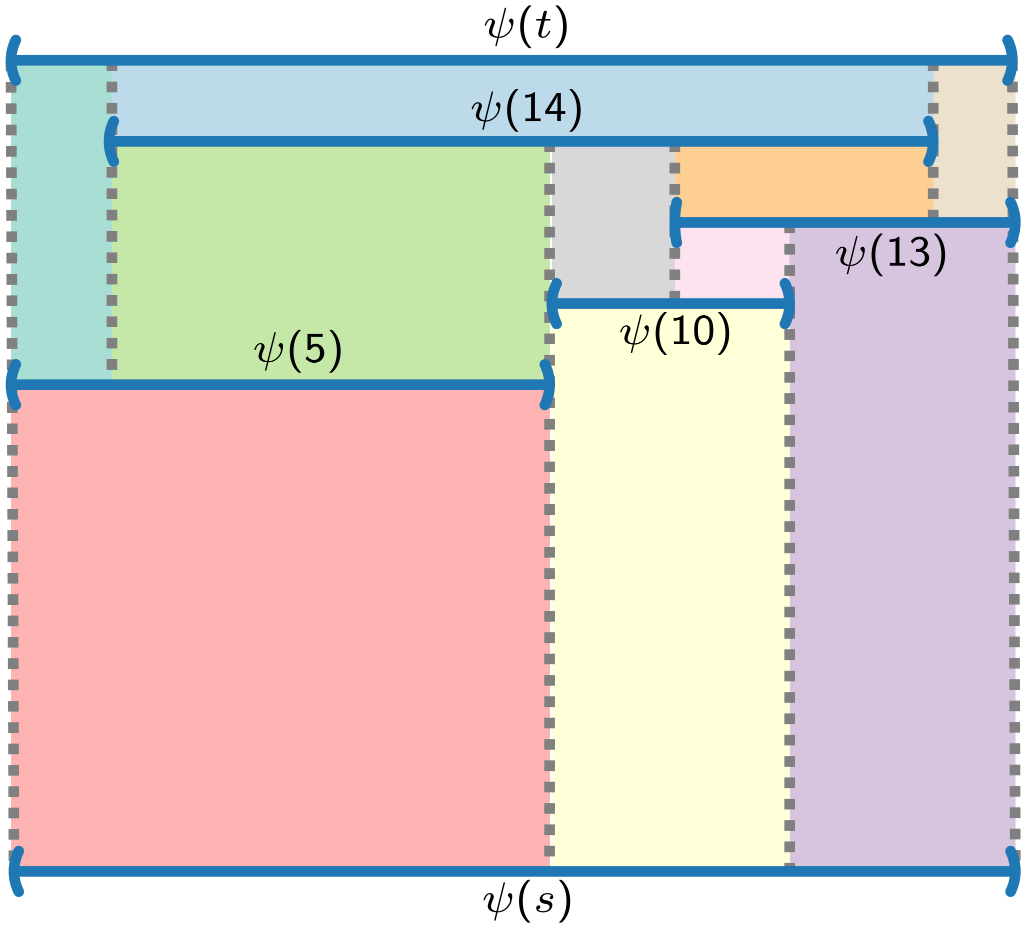

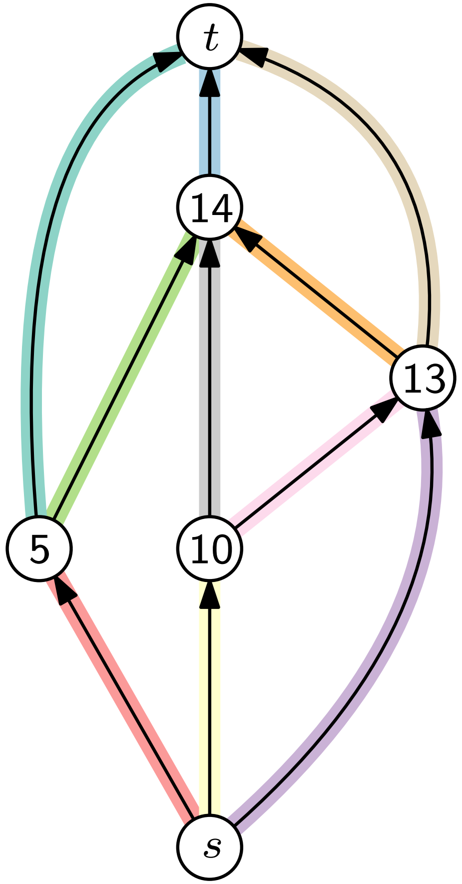

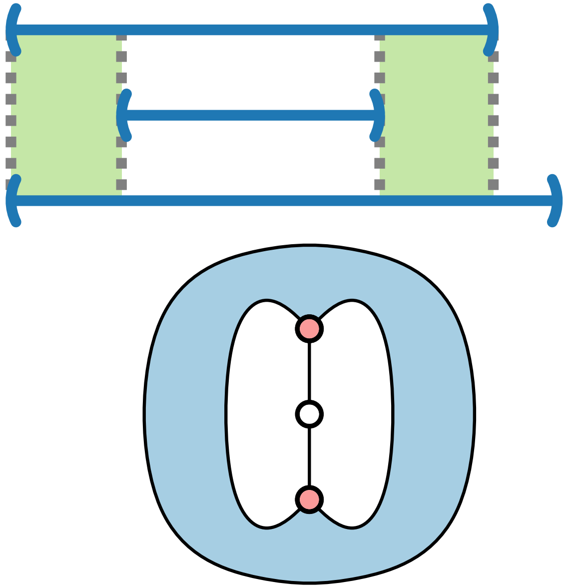

cutting pair!

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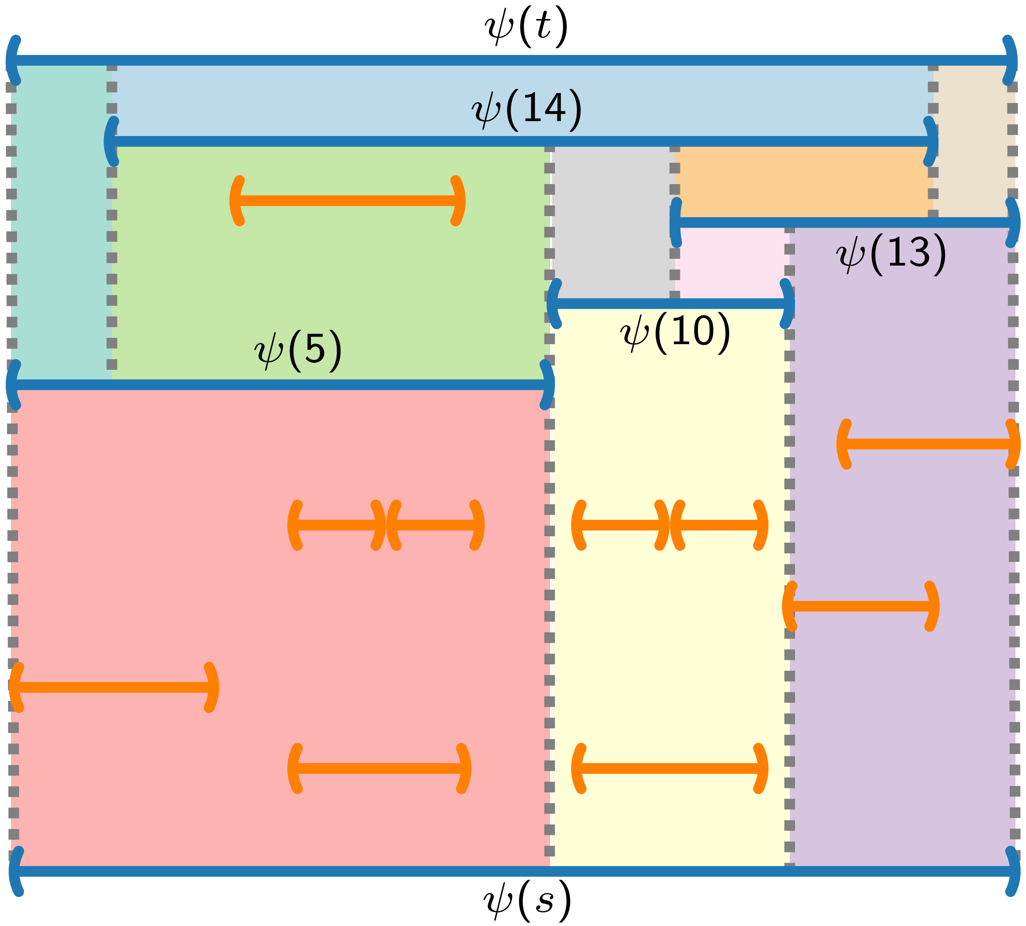

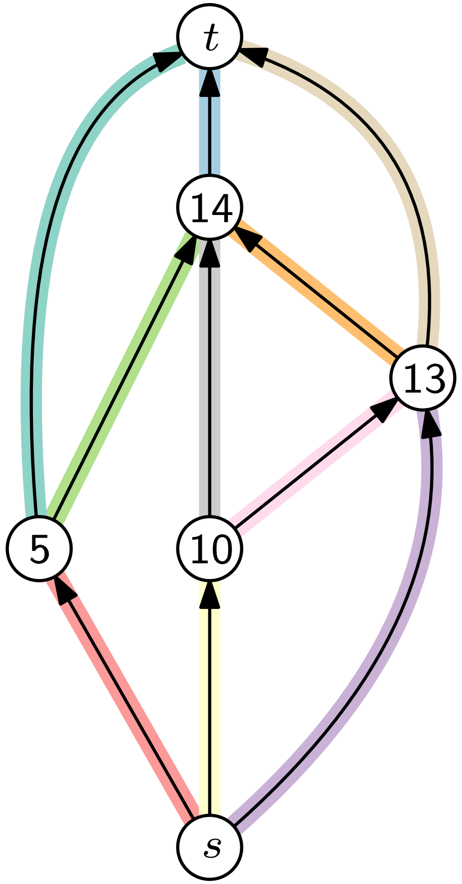

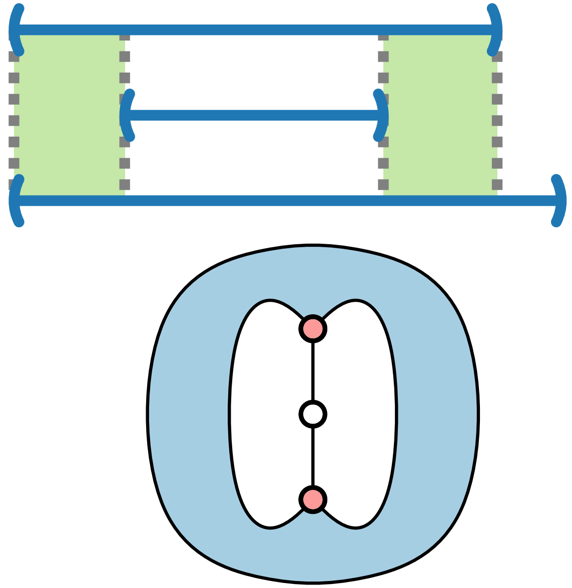

cutting pair!

R Nodes

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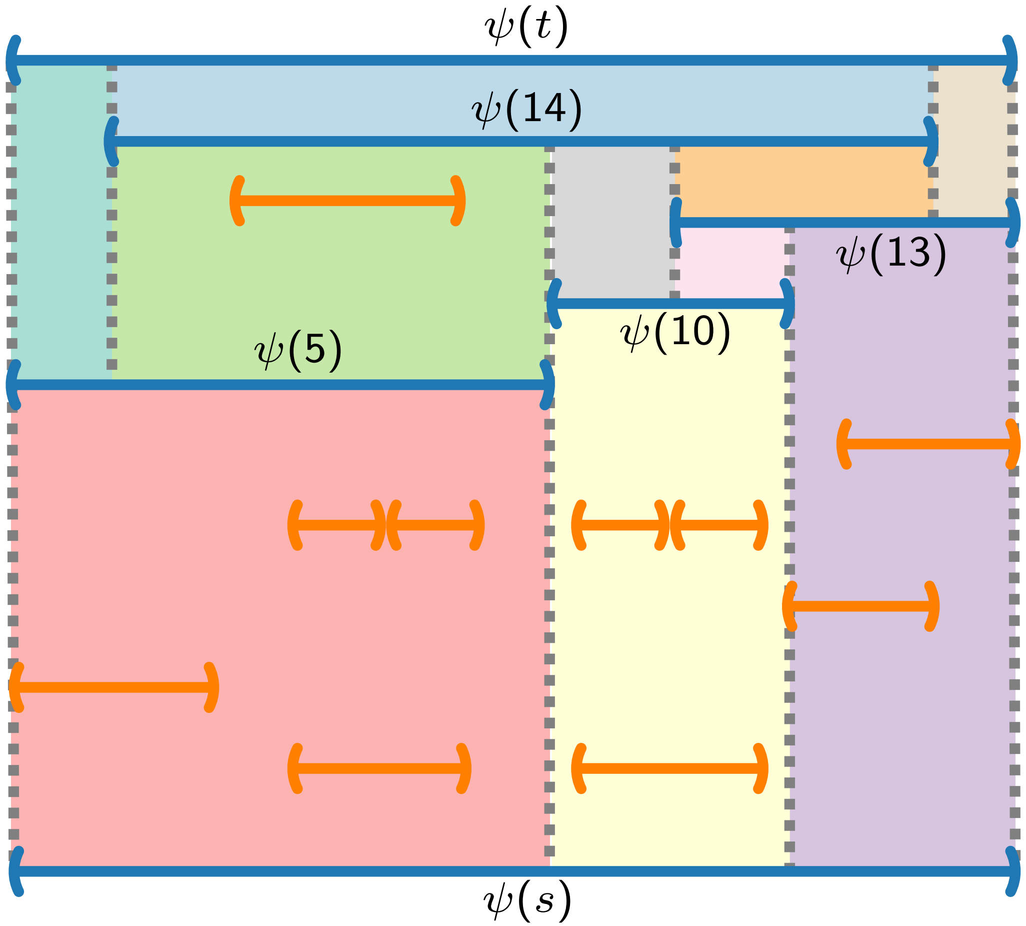

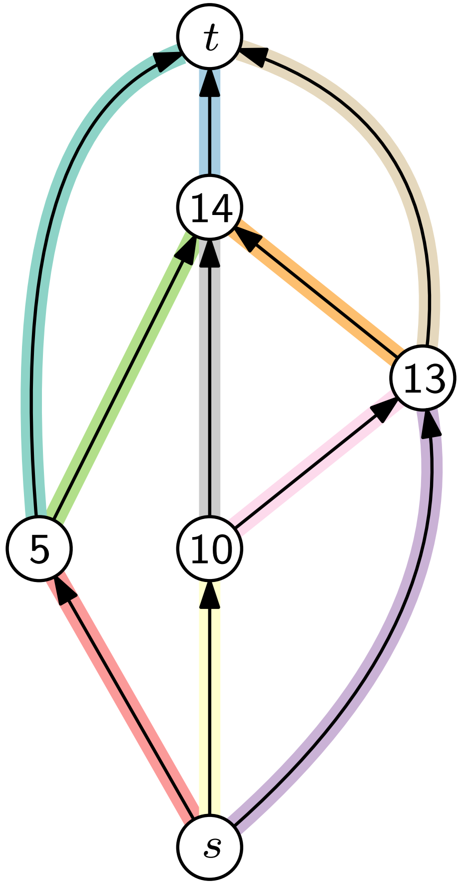

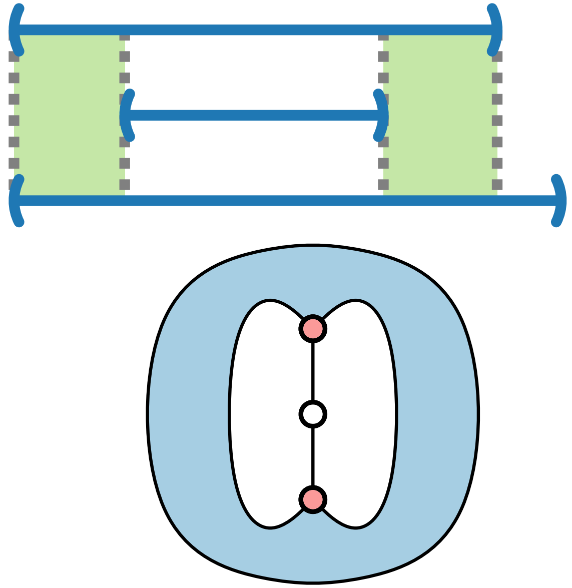

cutting pair!

R Nodes

- for each child (edge) e:
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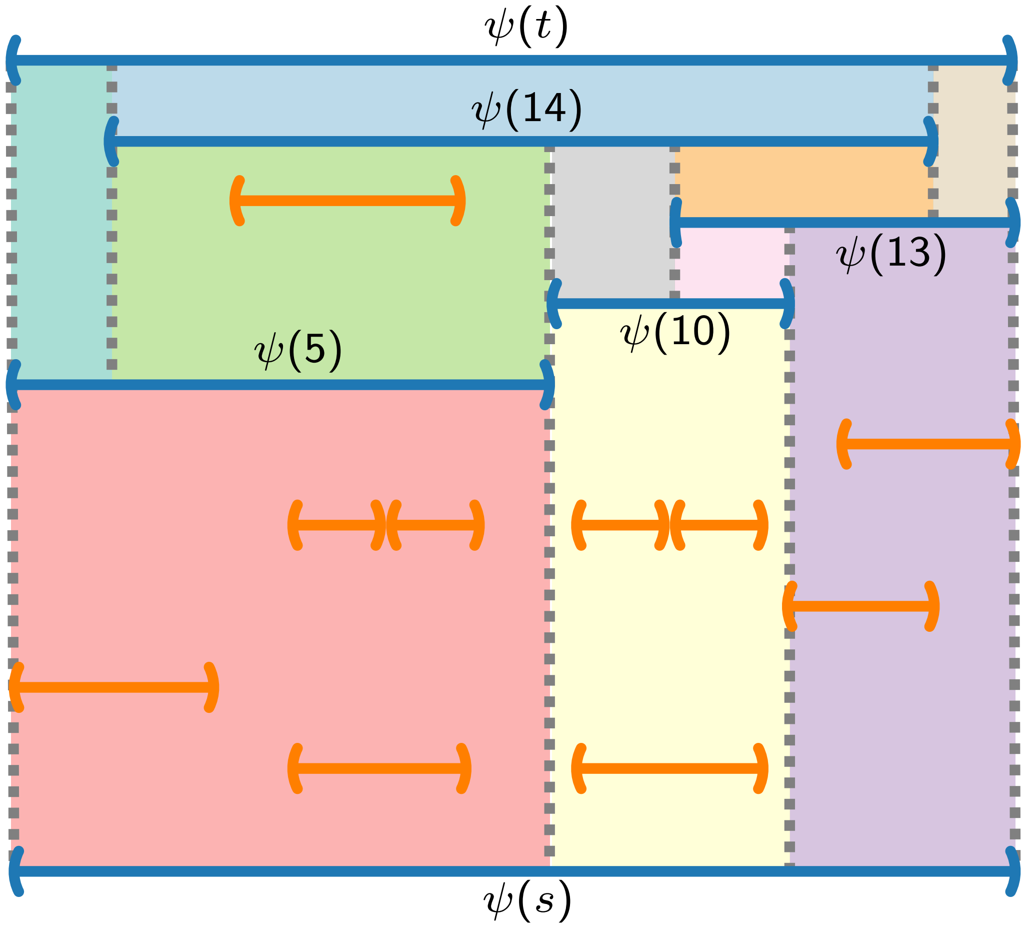

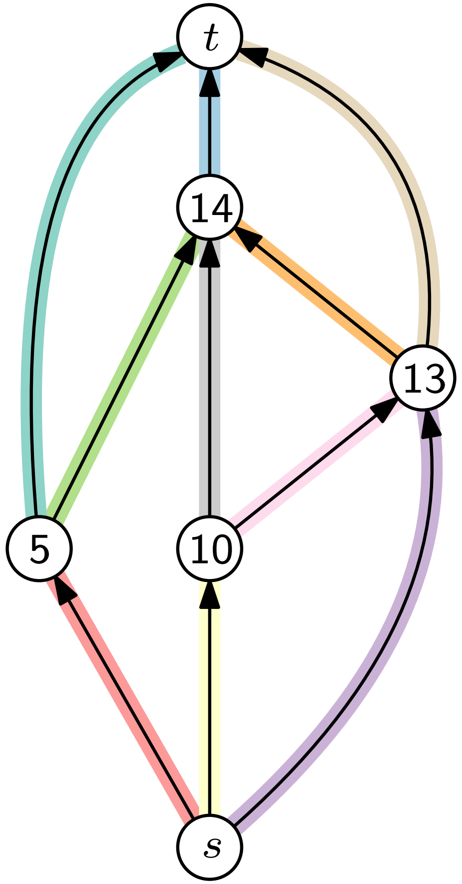

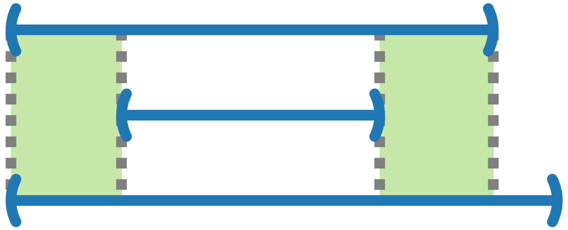

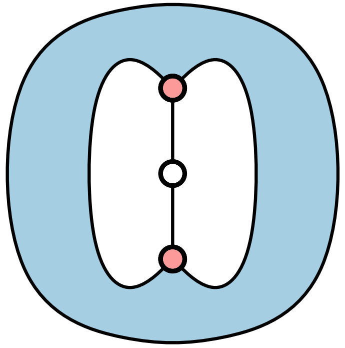

cutting pair!

R Nodes with 2-SAT Formulation

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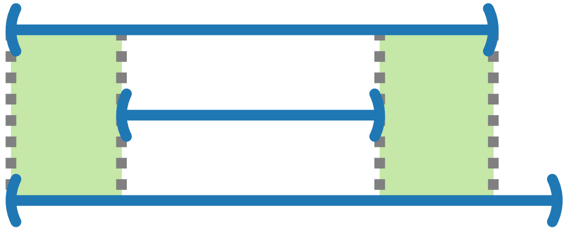

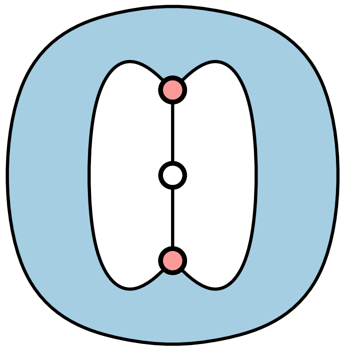

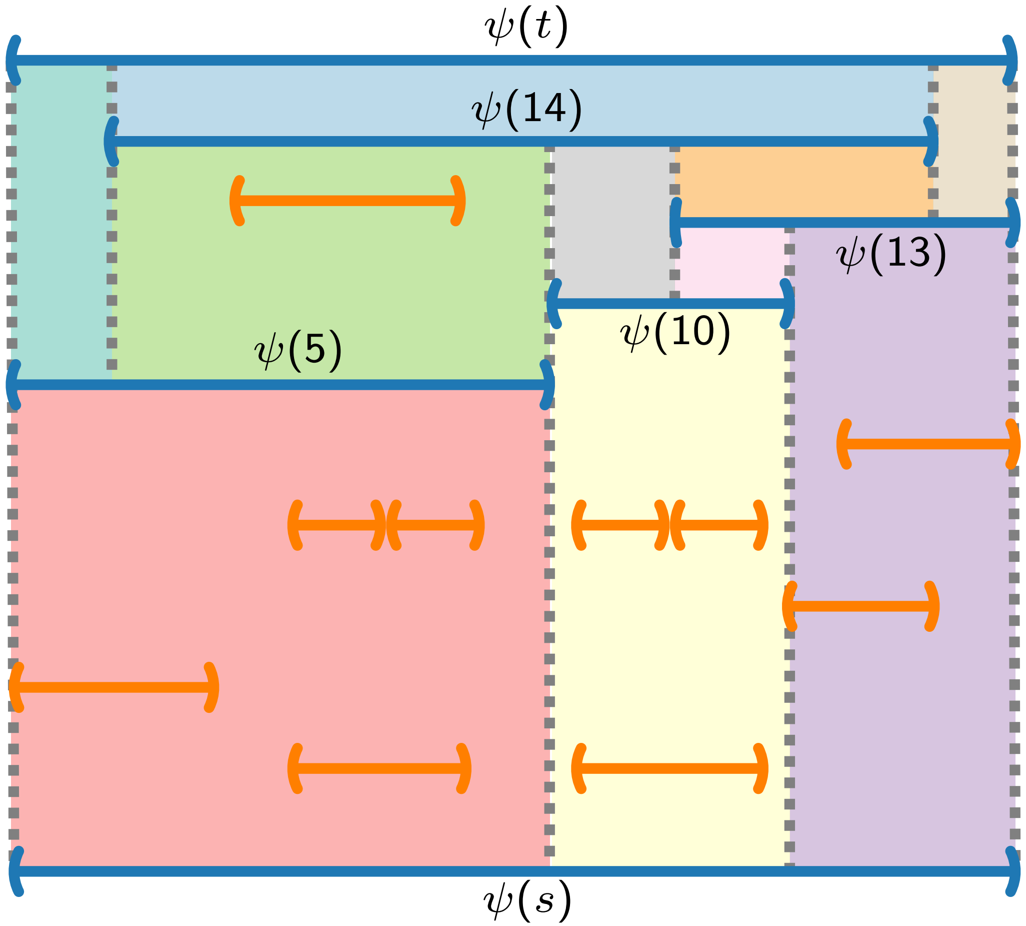

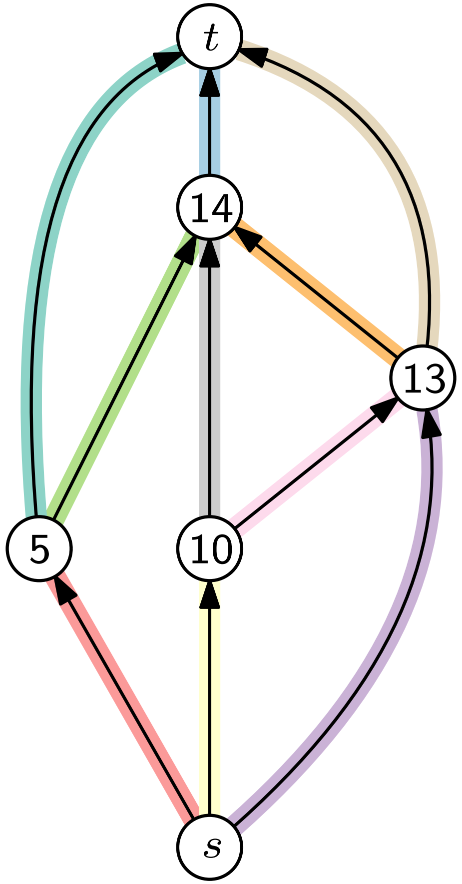

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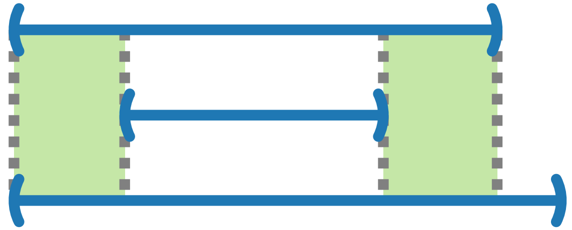

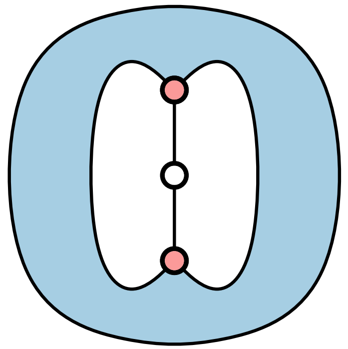

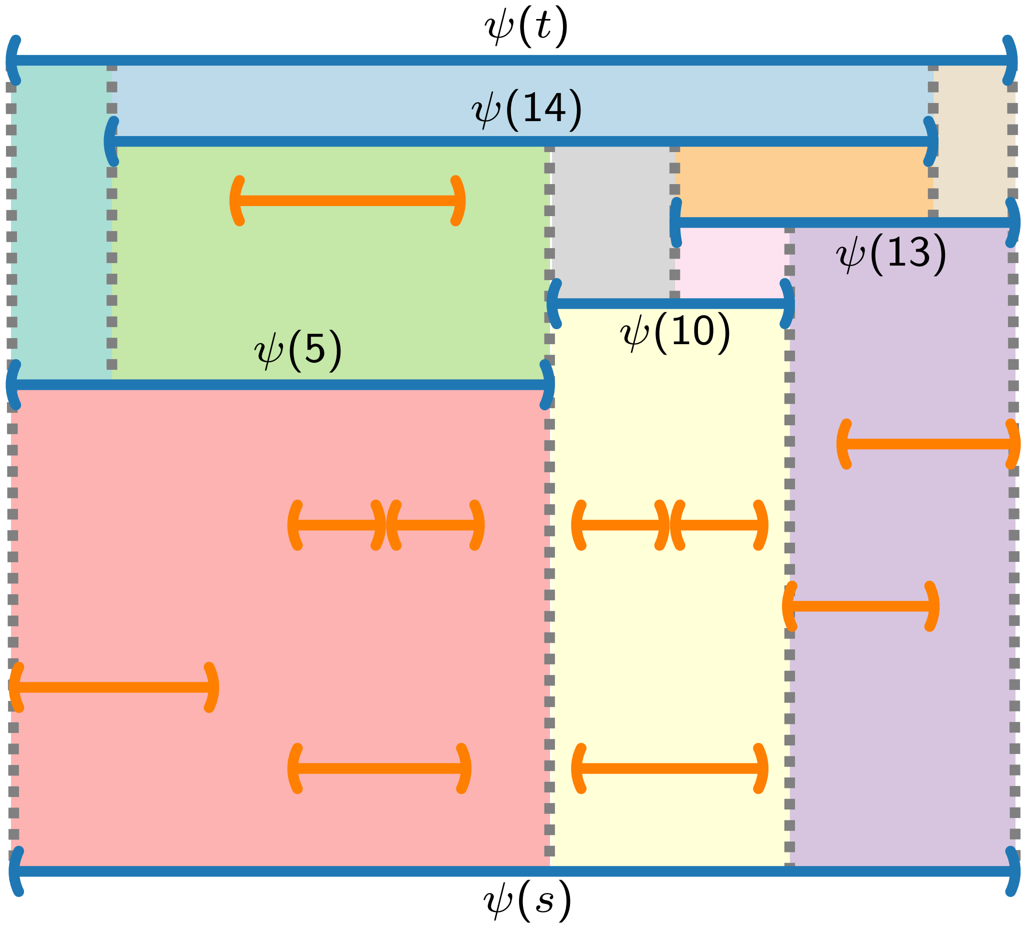

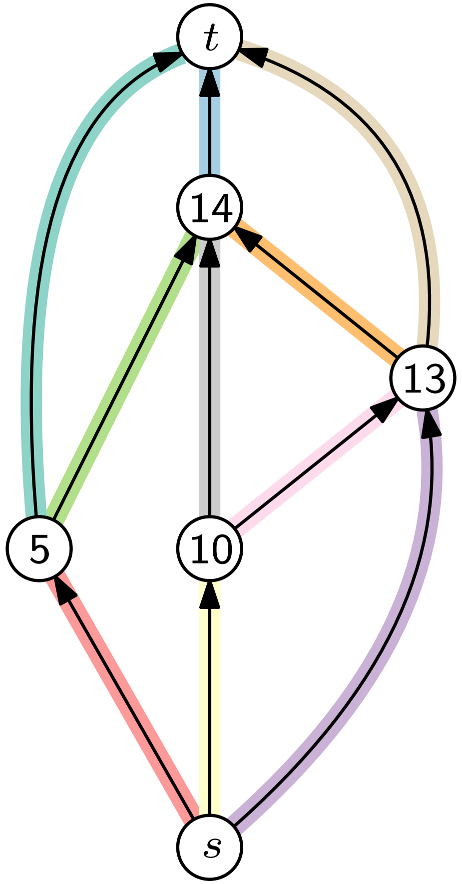

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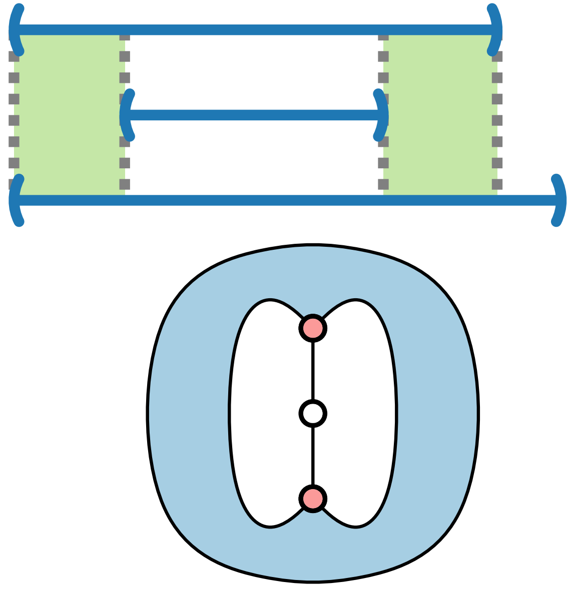

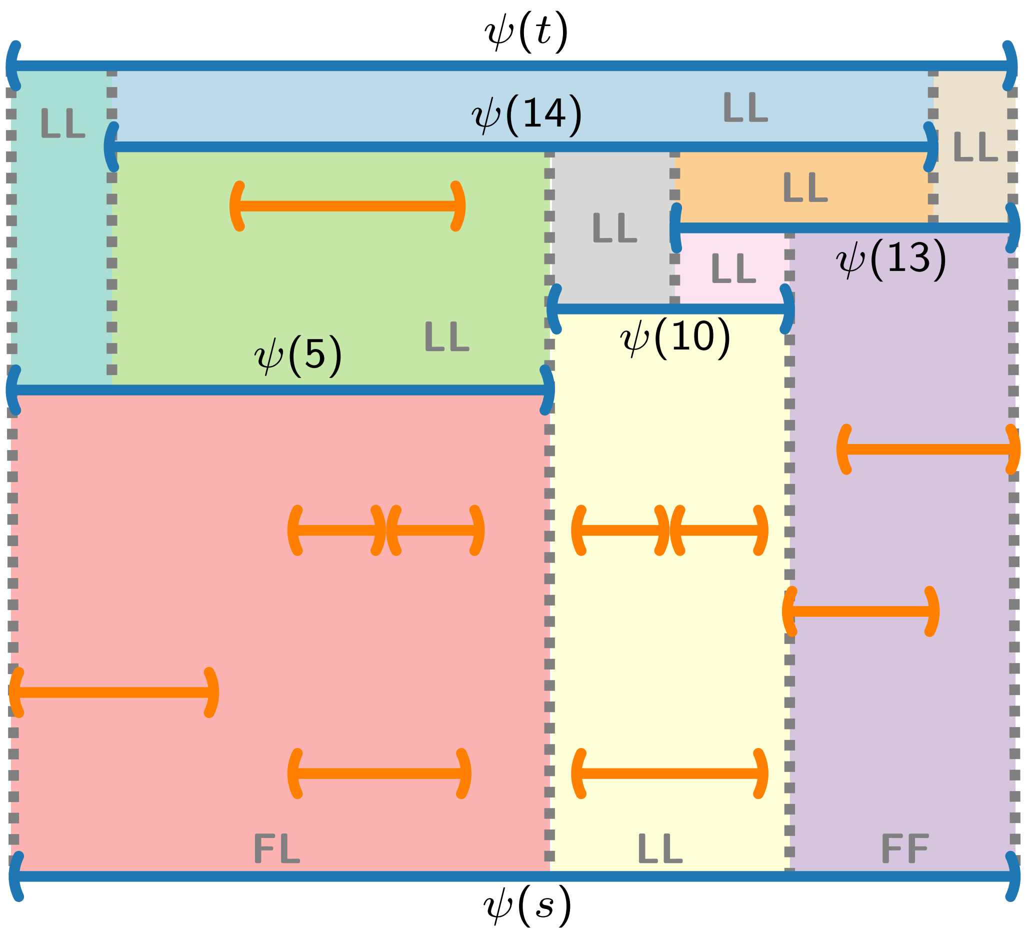

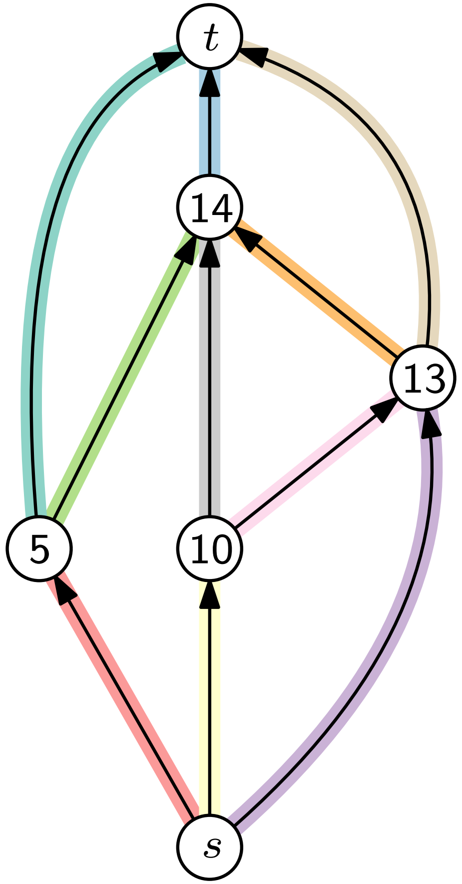

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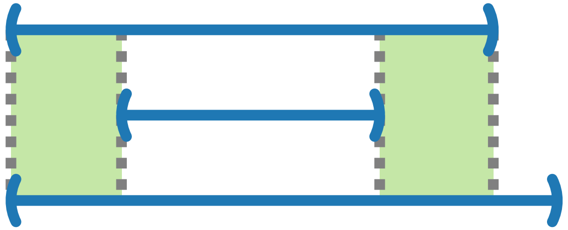

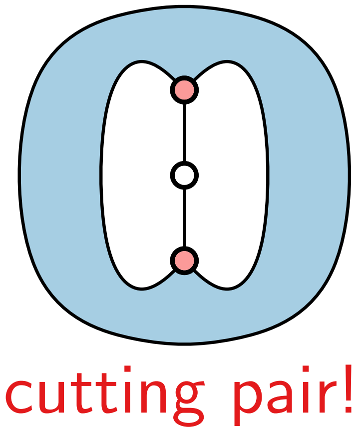

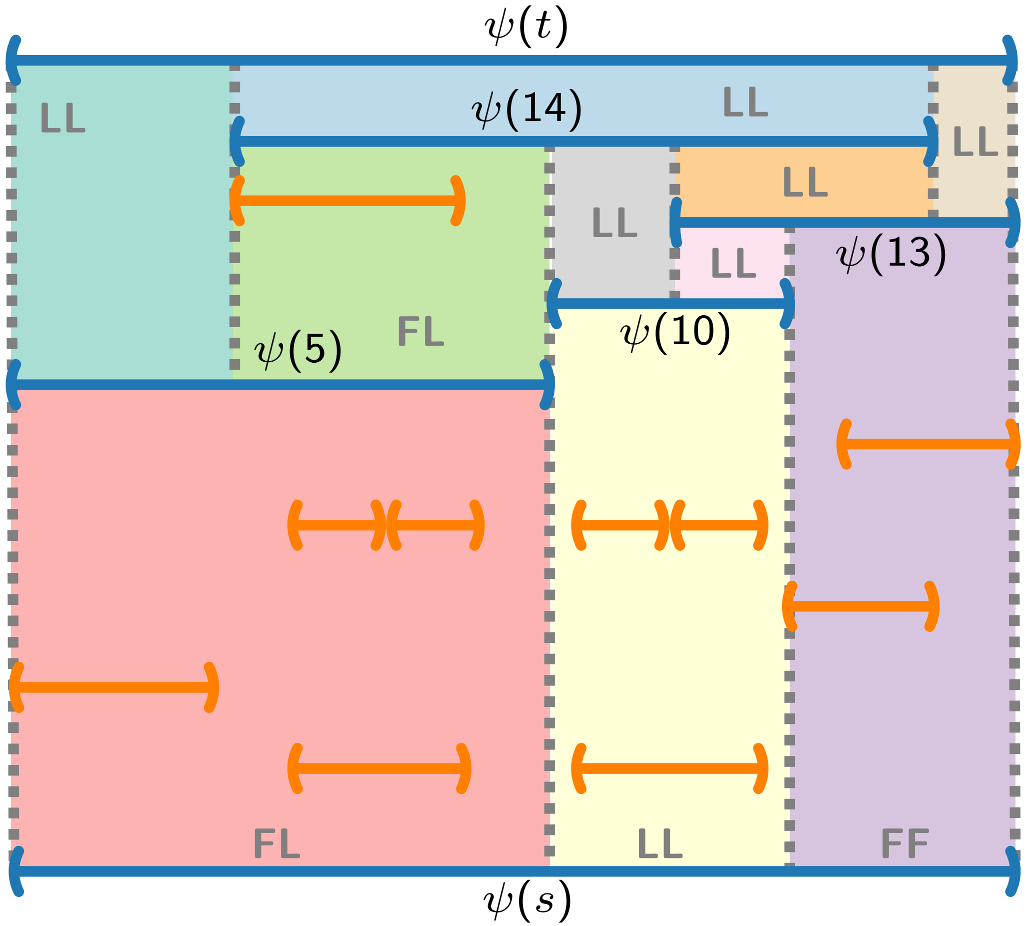

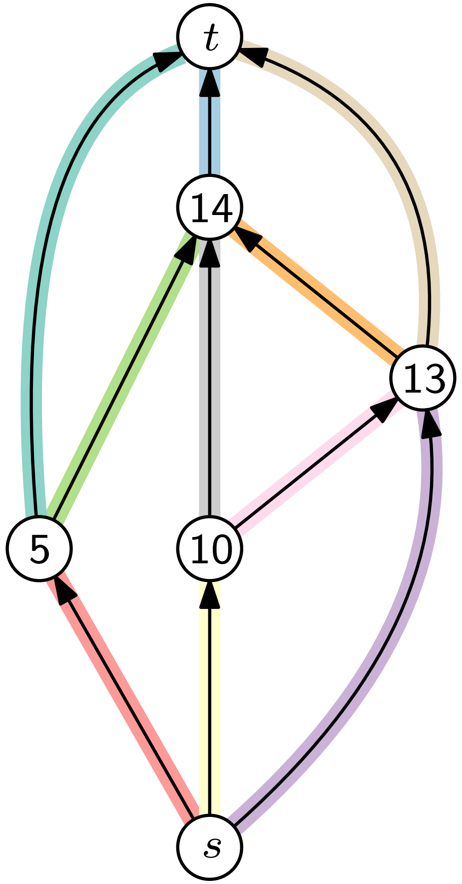

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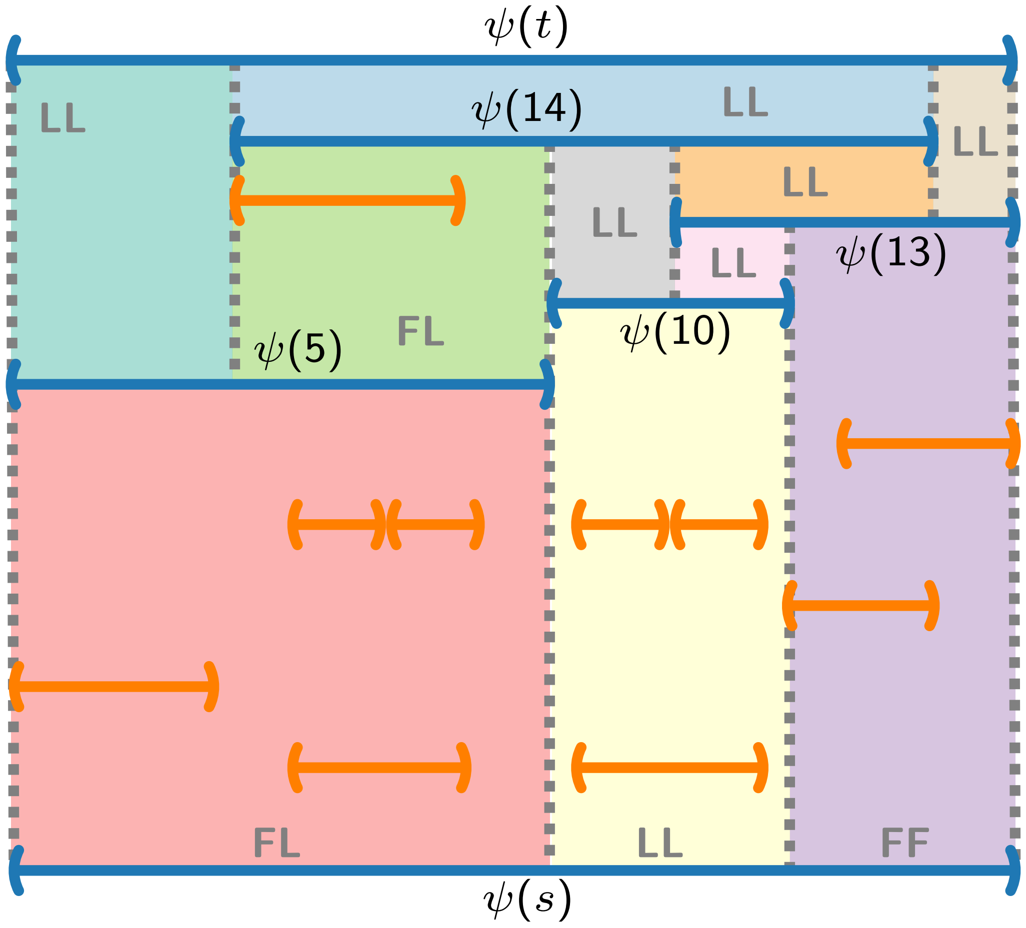

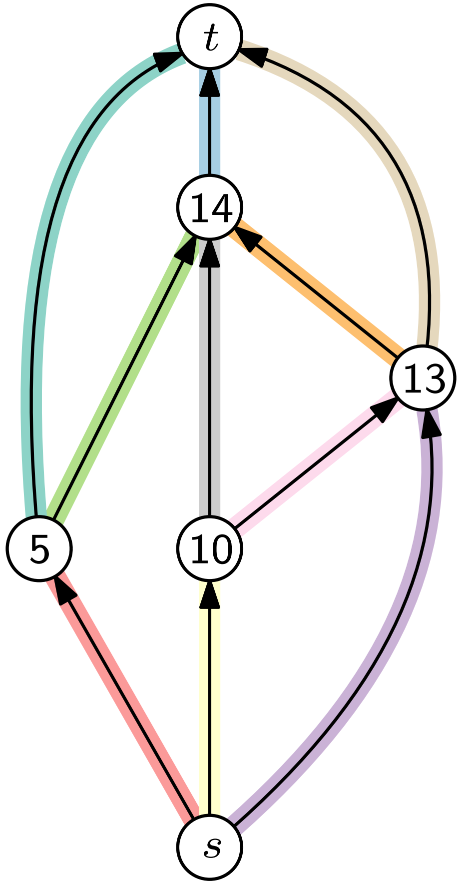

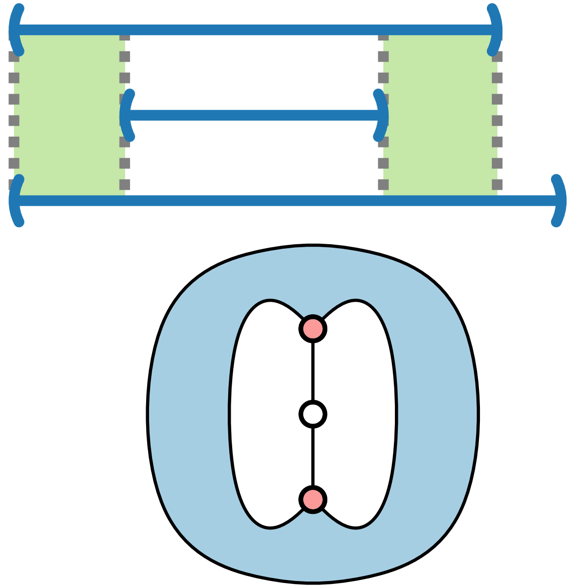

cutting pair!

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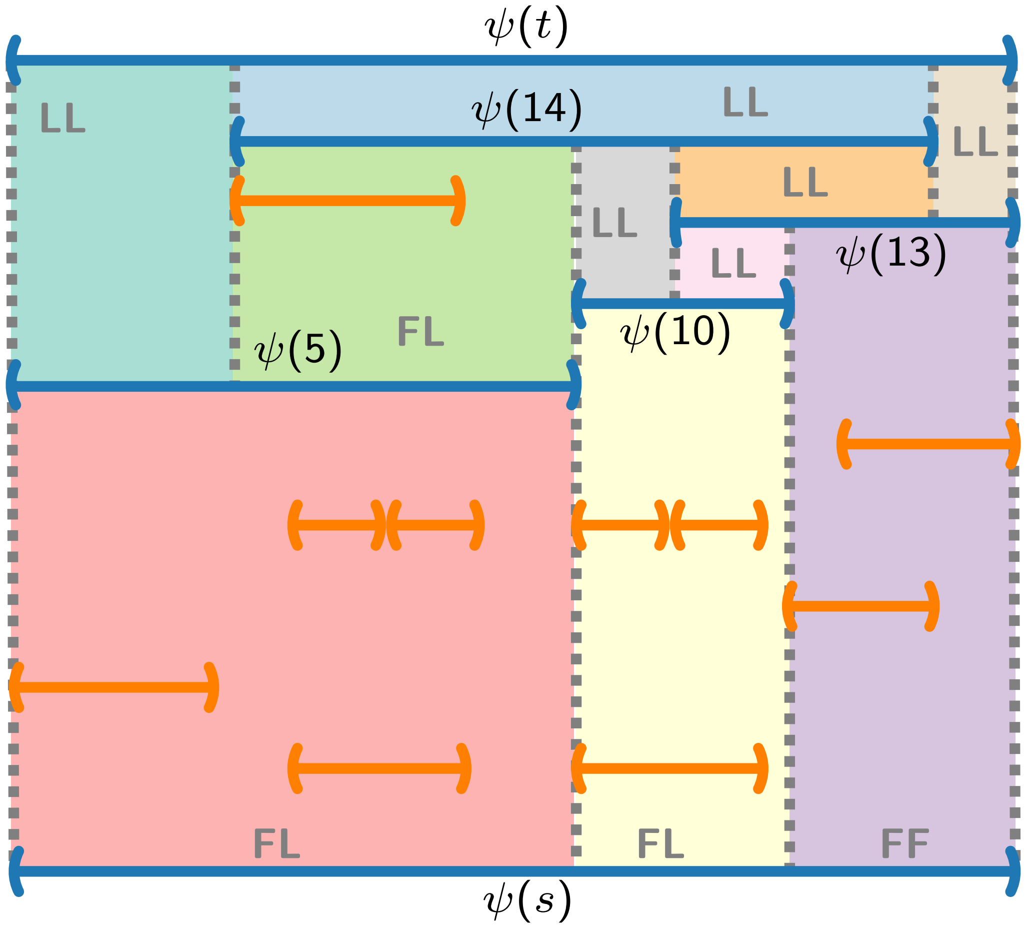

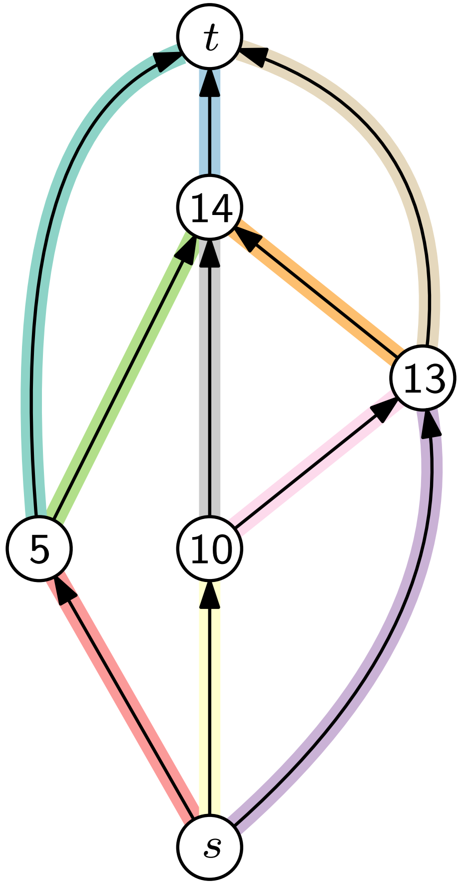

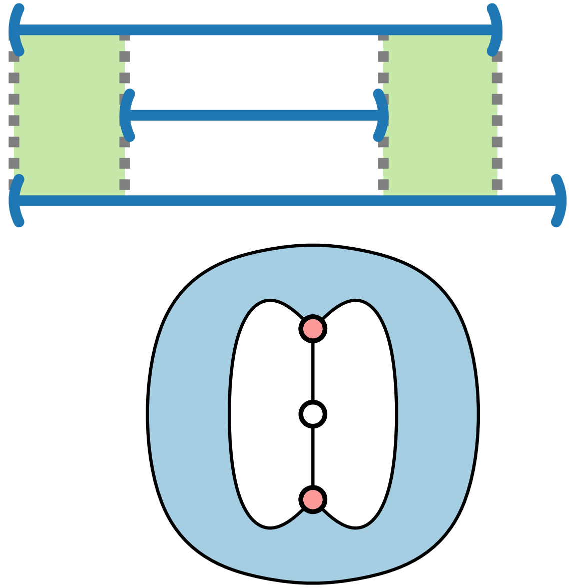

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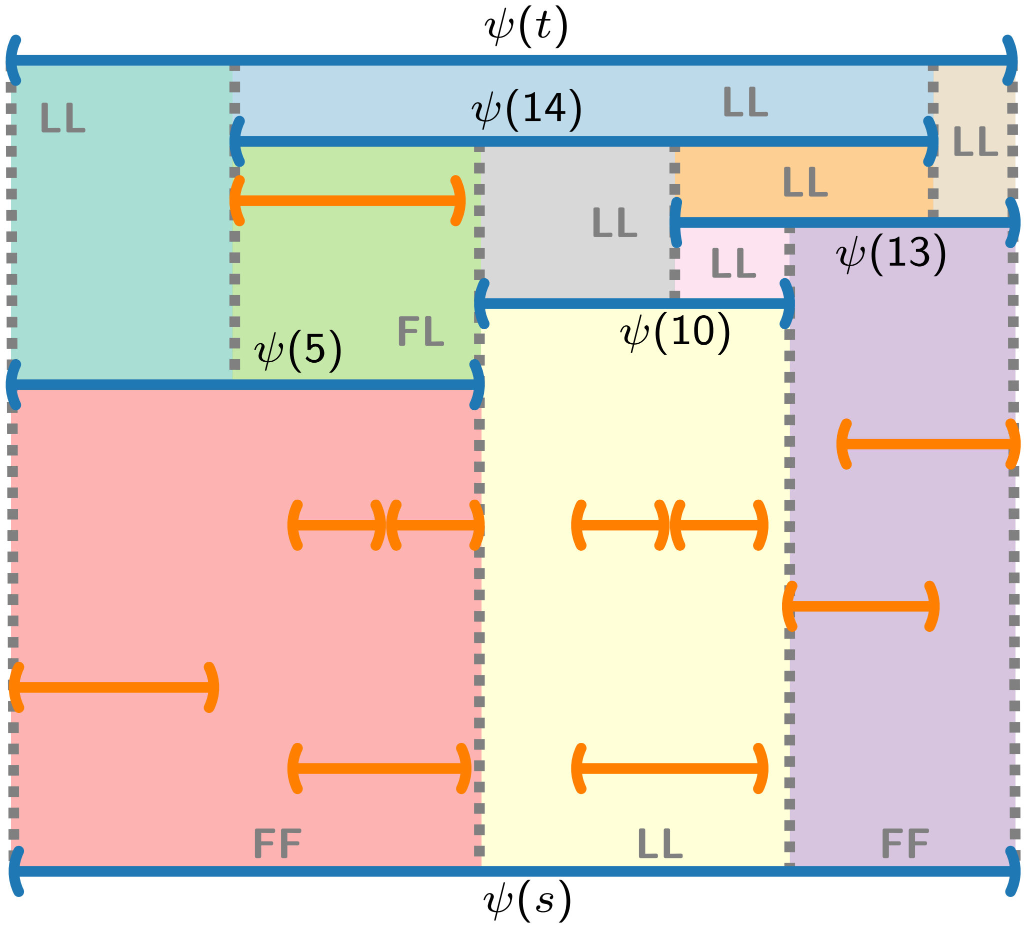

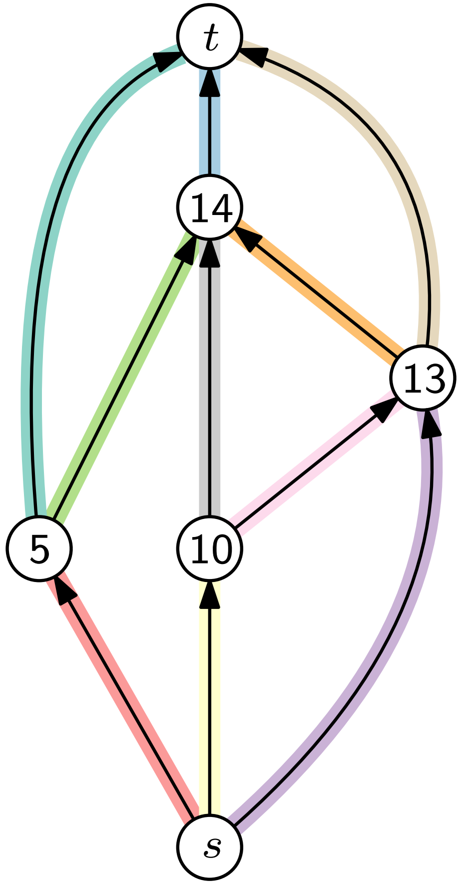

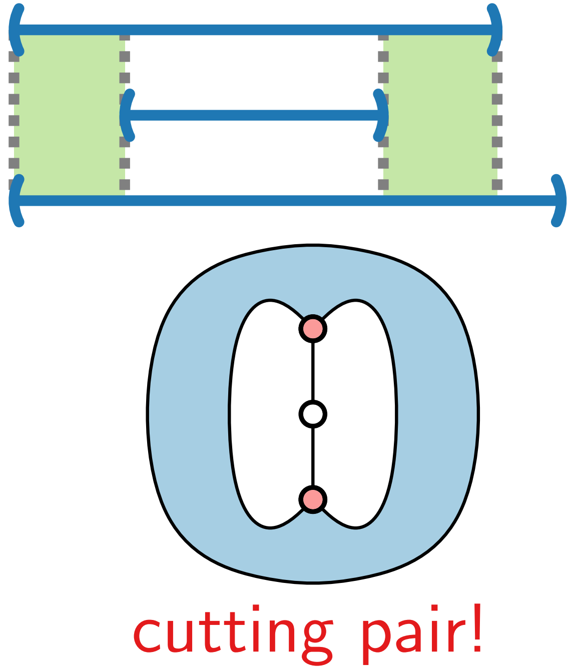

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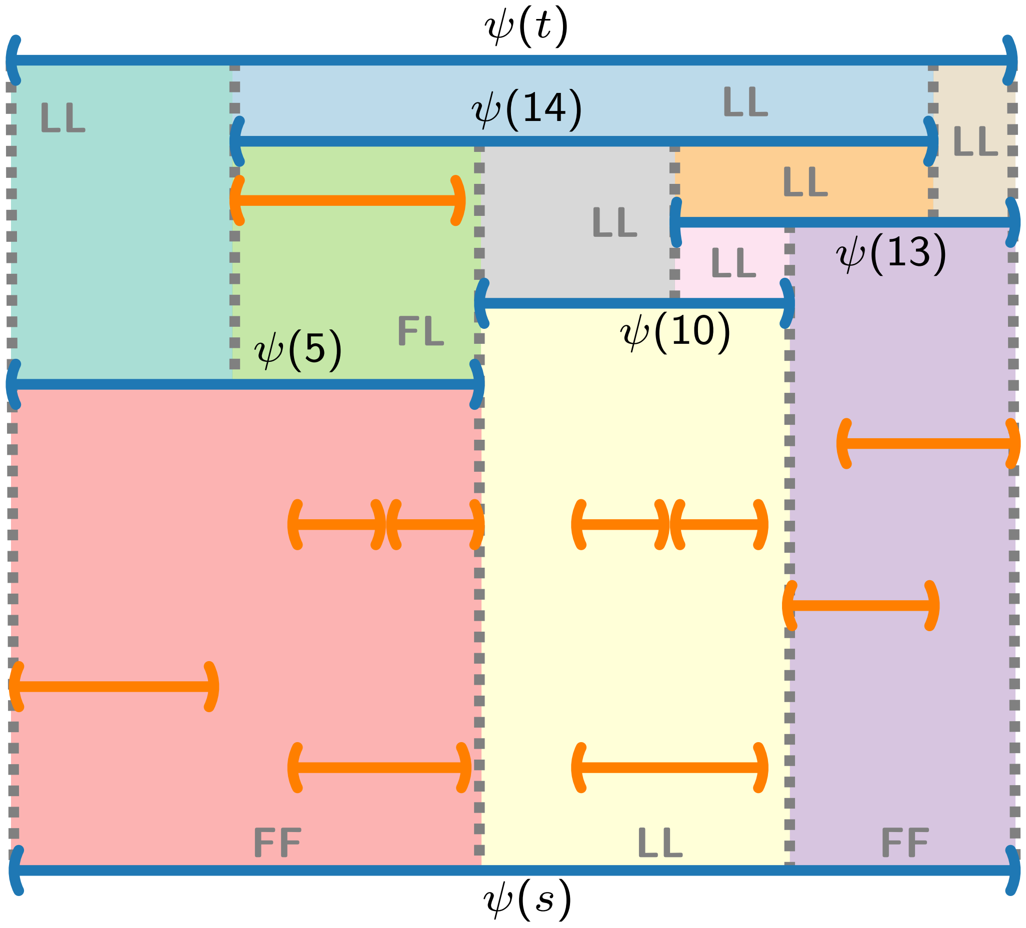

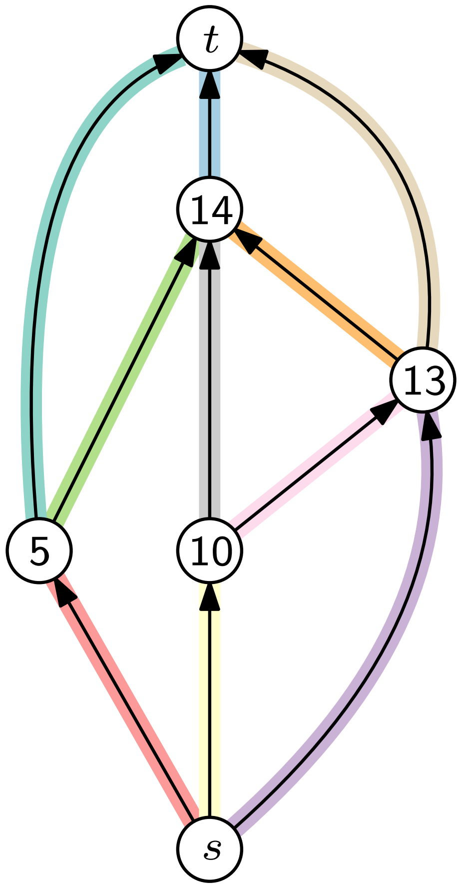

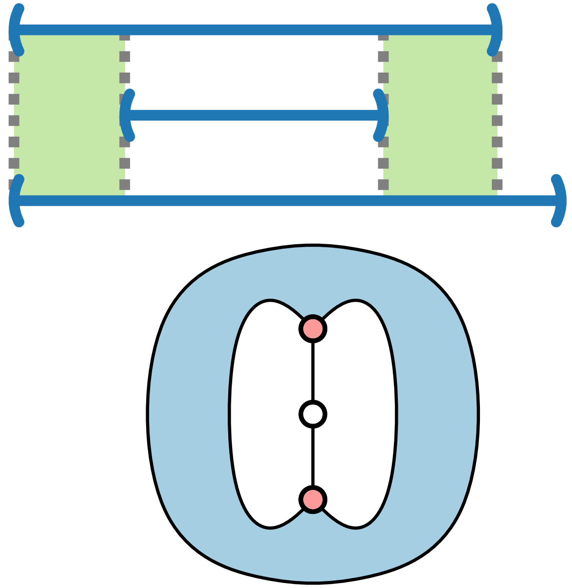

cutting pair!

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- for each child (edge) e:
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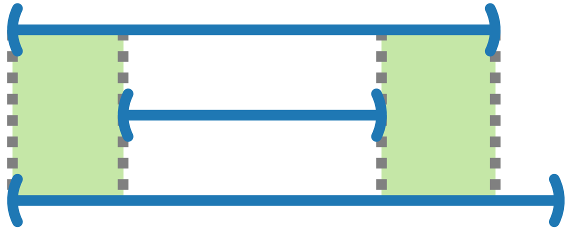
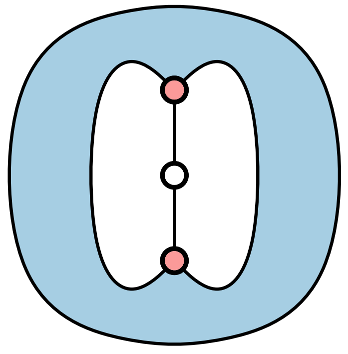
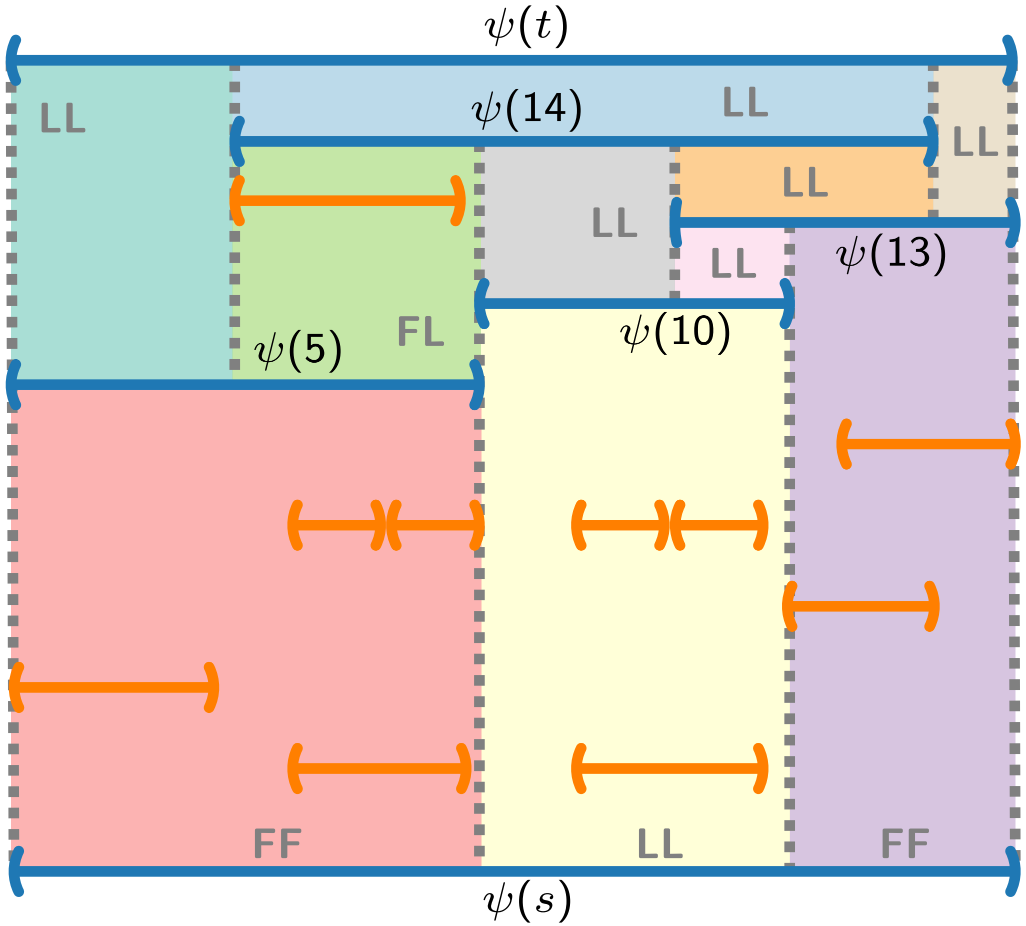
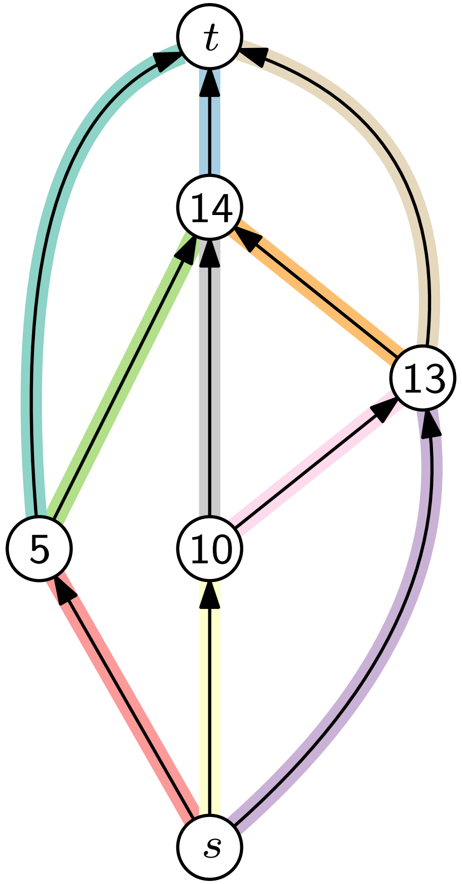
  - 2 variables l_e, r_e encoding fixed/loose type of its tile
  - consistency clauses – $O(n^2)$ many, but can be reduced to $O(n \log^2 n)$









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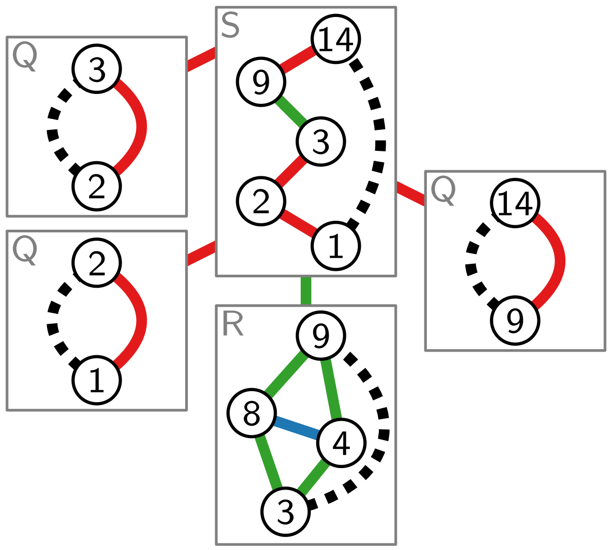
WÜRZBURG

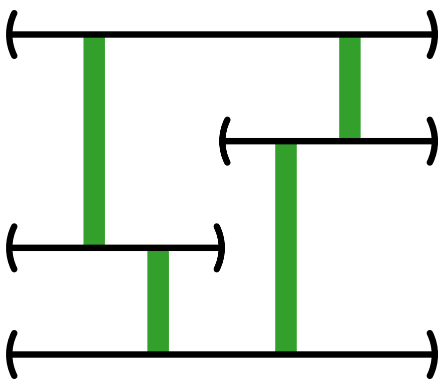
Visualization of Graphs

Lecture 9:
Partial Visibility Representation Extension



Part VI:
NP-Hardness
of General Case

Jonathan Klawitter



NP-Hardness of RepExt in General Case

Theorem 2.
ε-Bar Visibility Representation Ext. is NP-complete.

- Reduction from Planar Monotone 3-SAT

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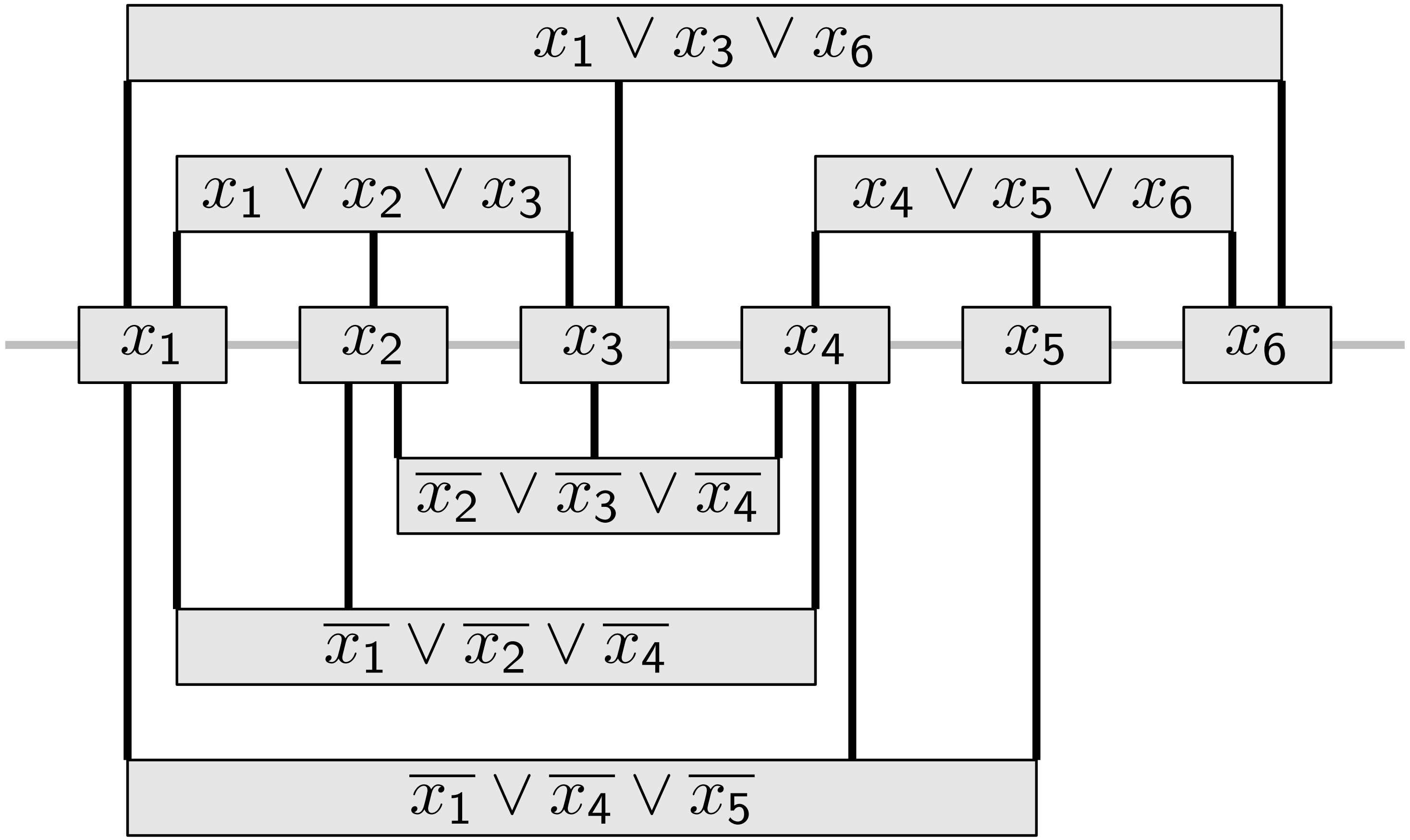

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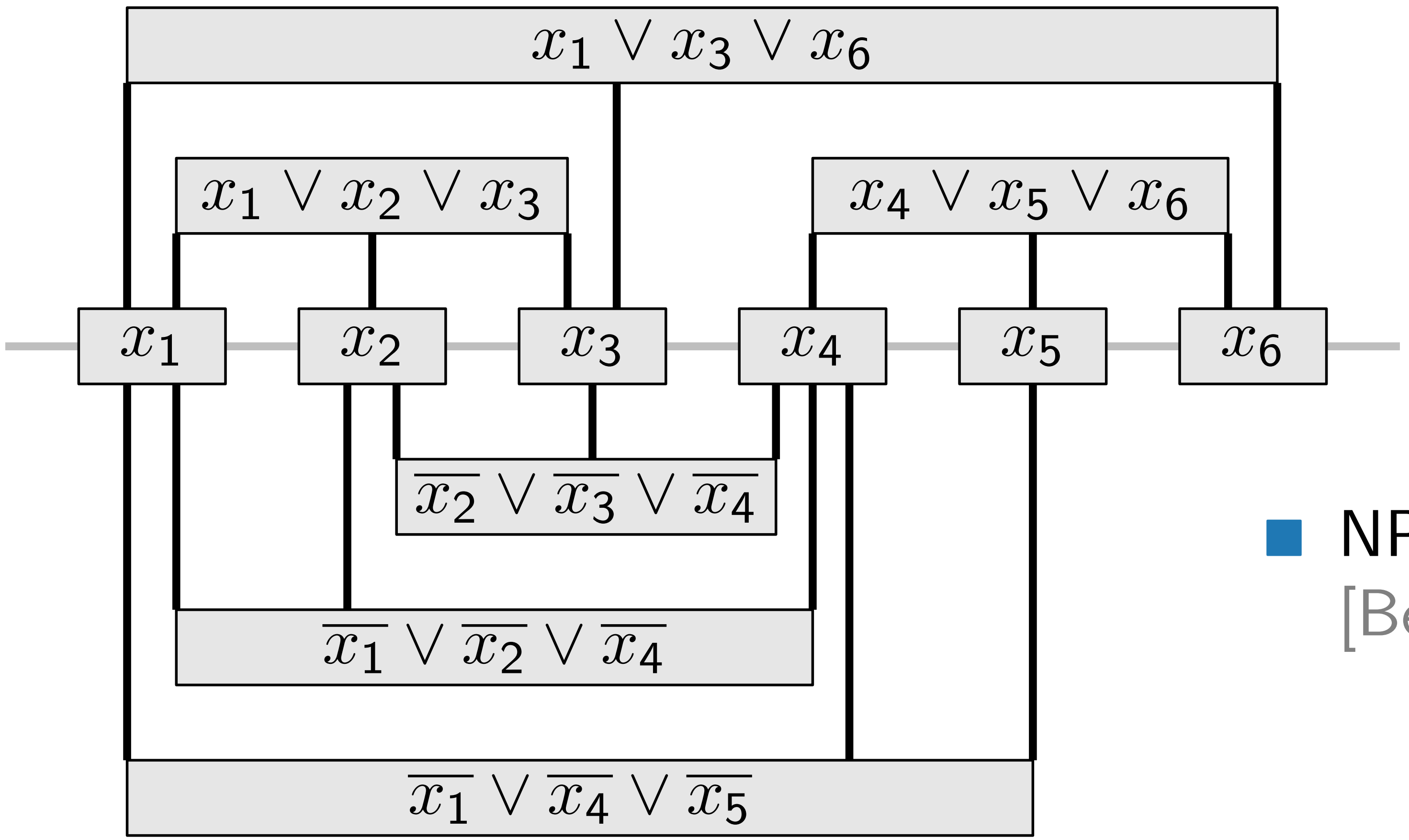

- NP-complete
  [Berg & Khosravi '10]

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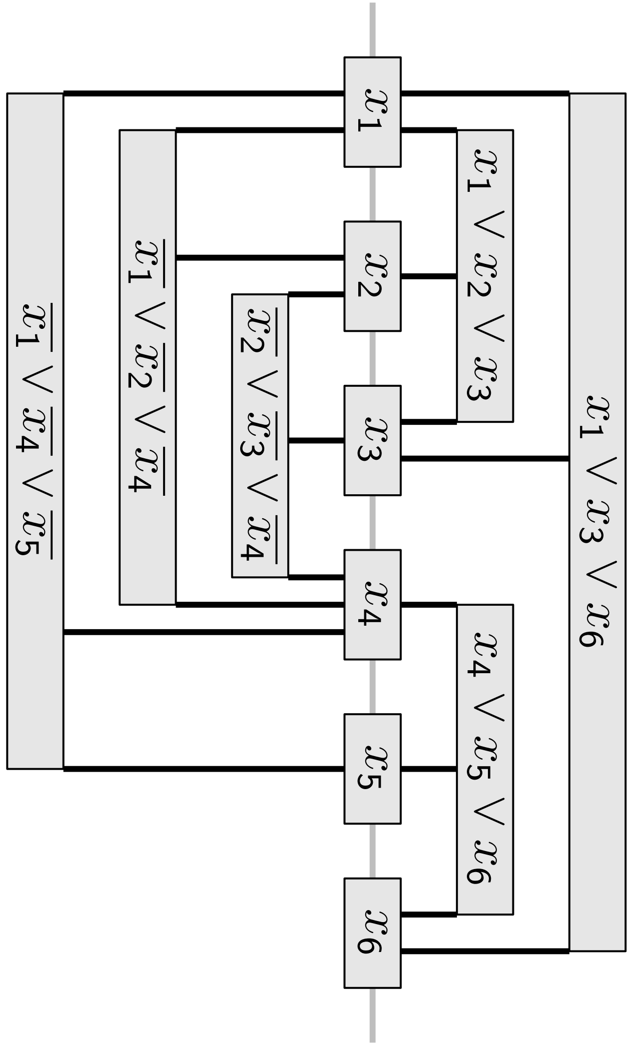

- NP-complete
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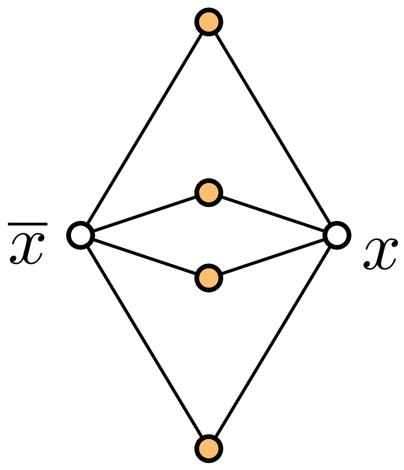
Variable Gadget



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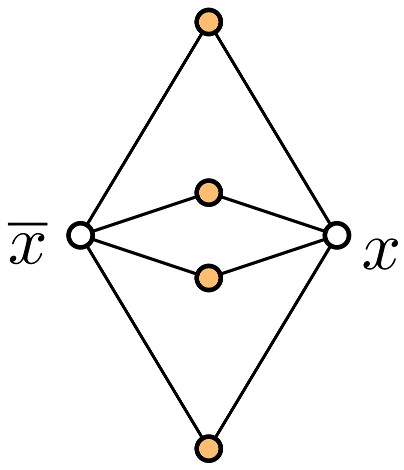

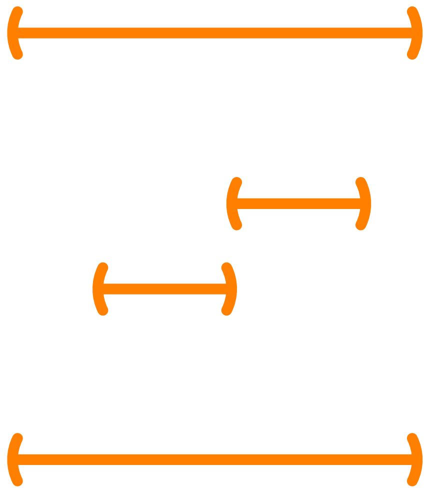

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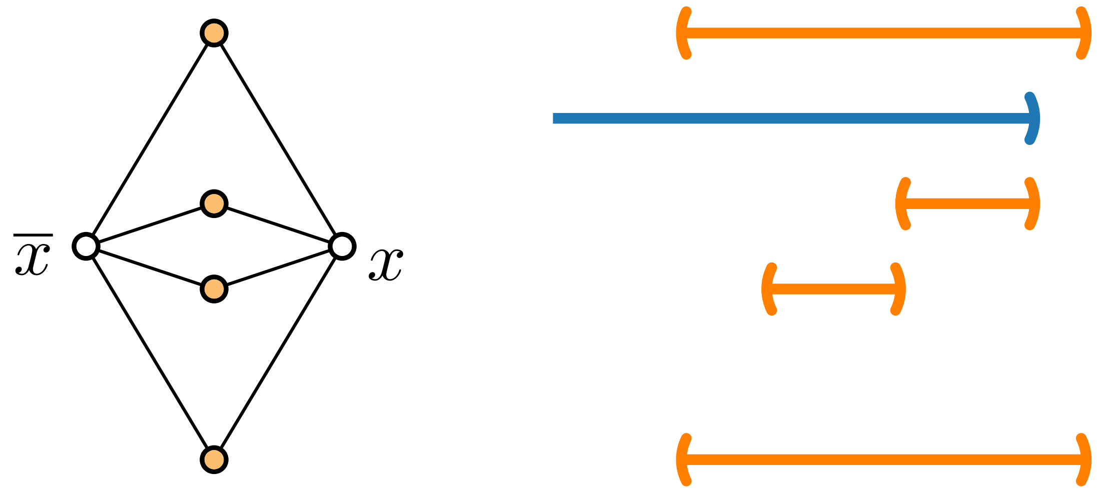

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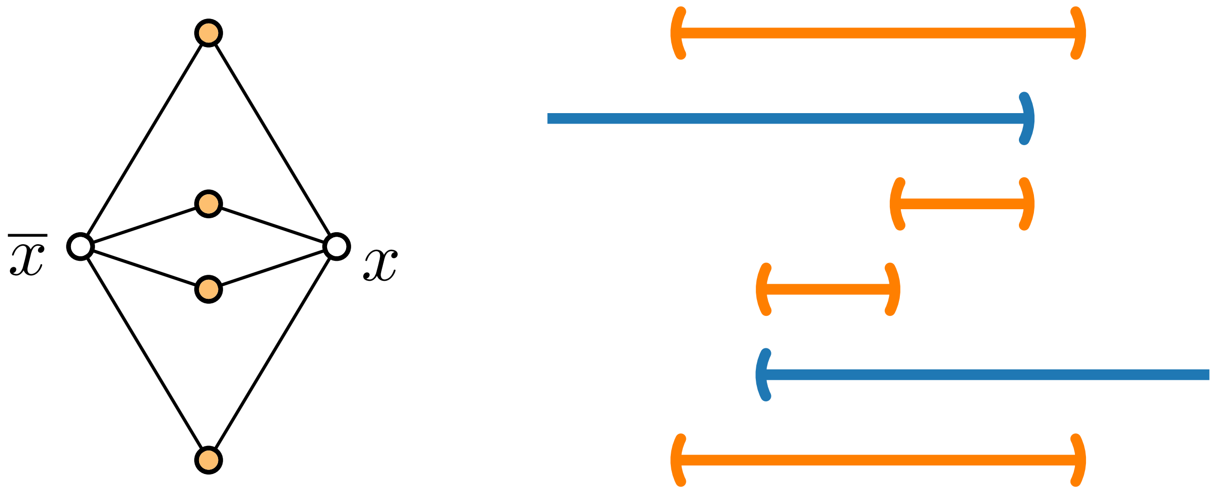

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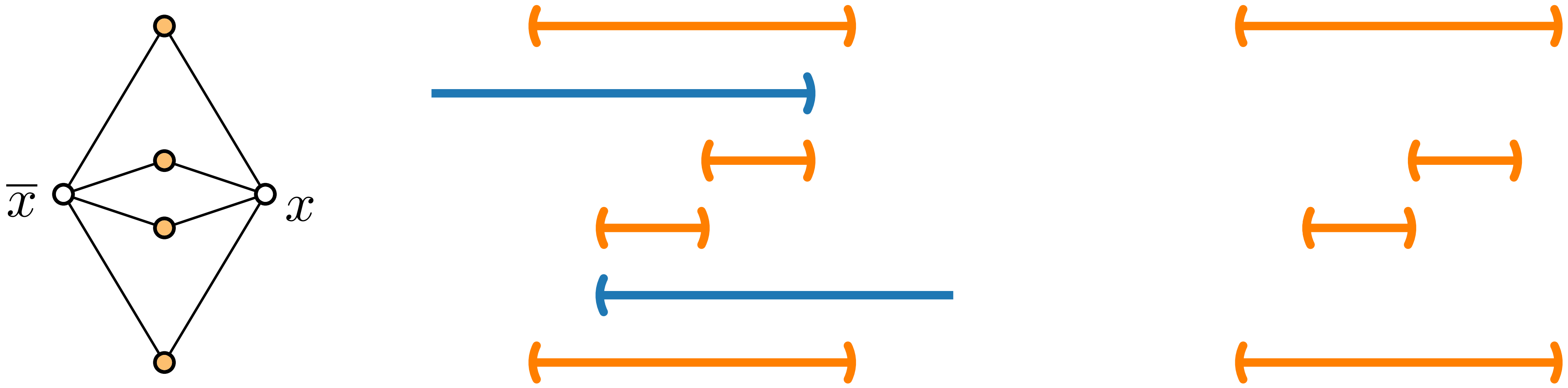

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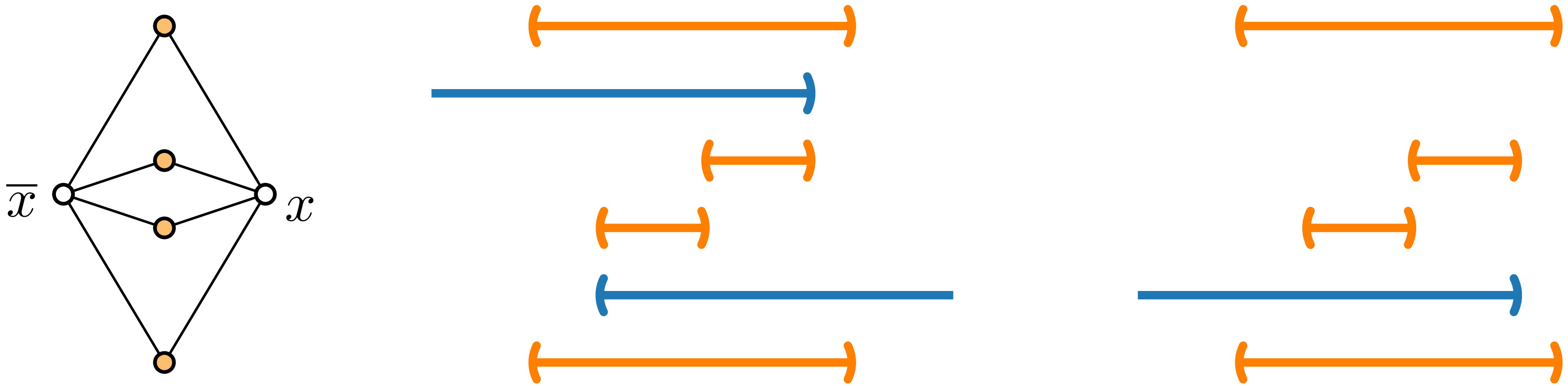

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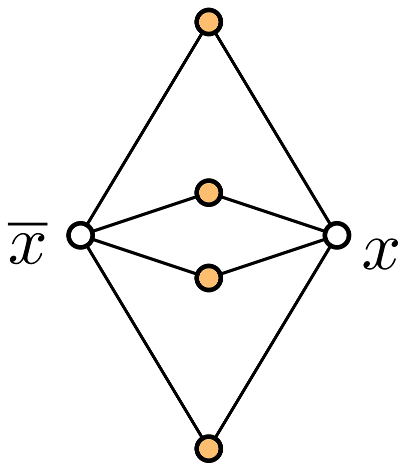

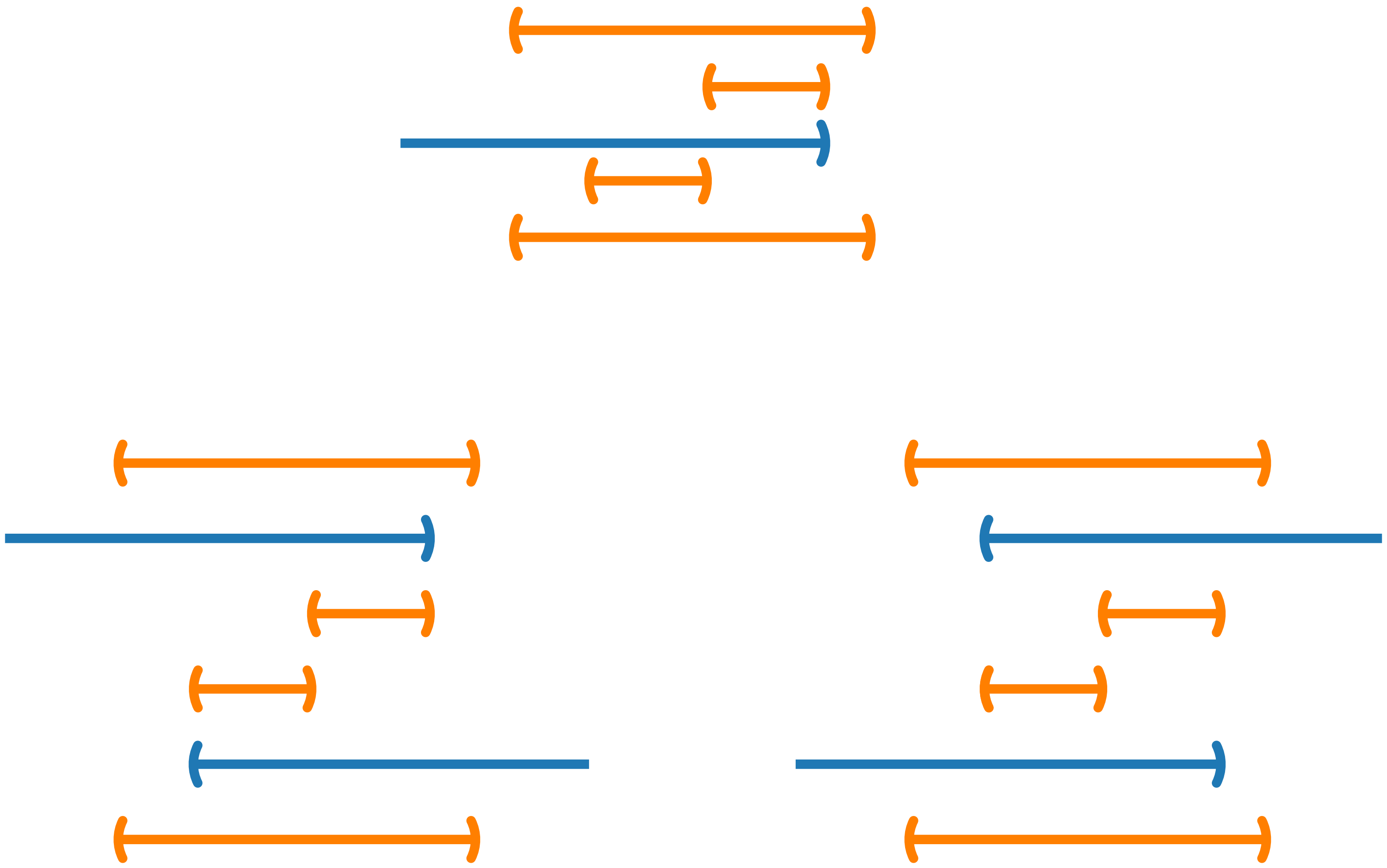

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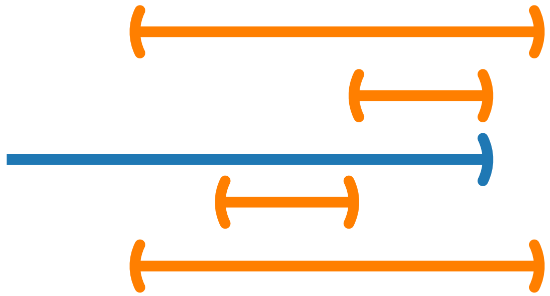

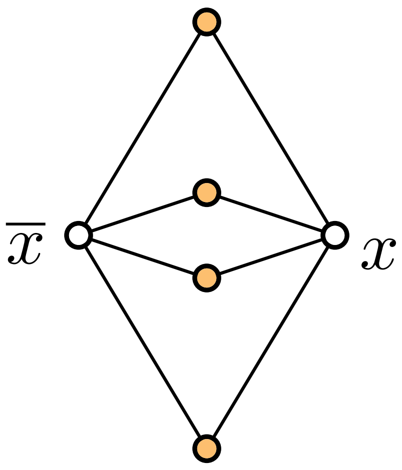

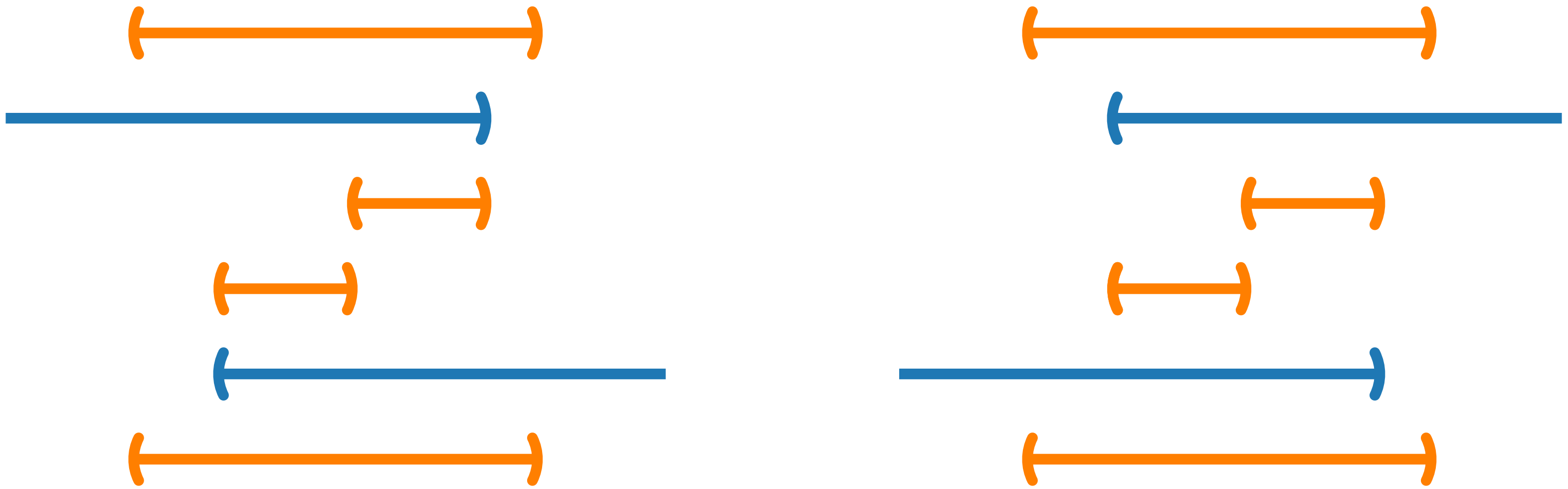

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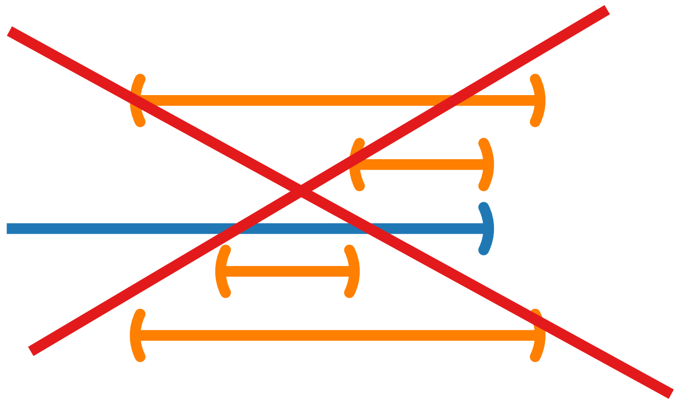

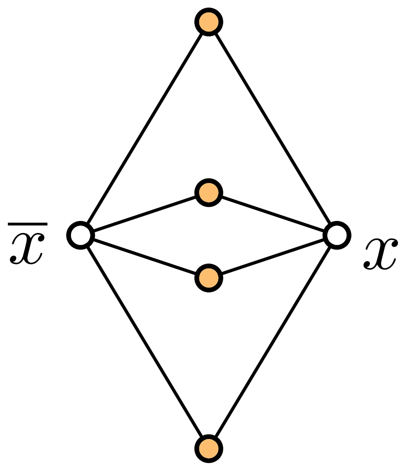

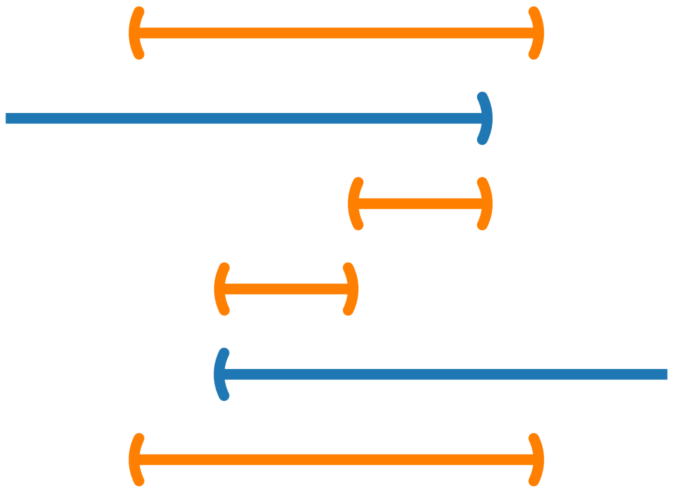

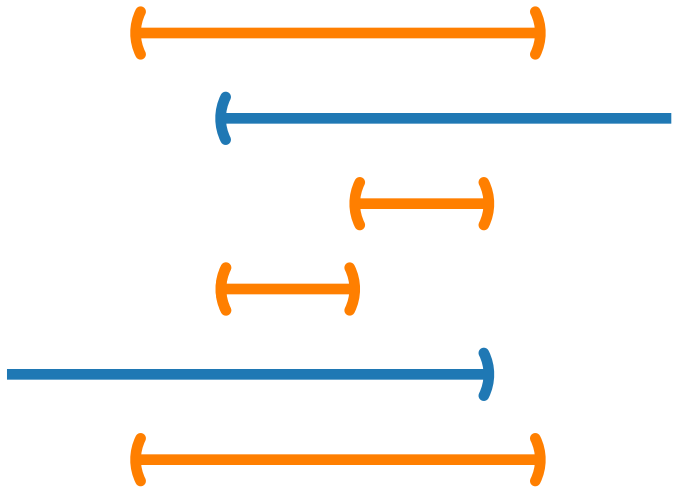

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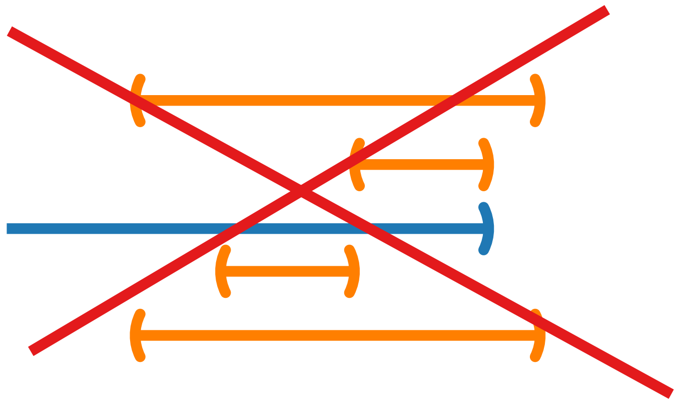

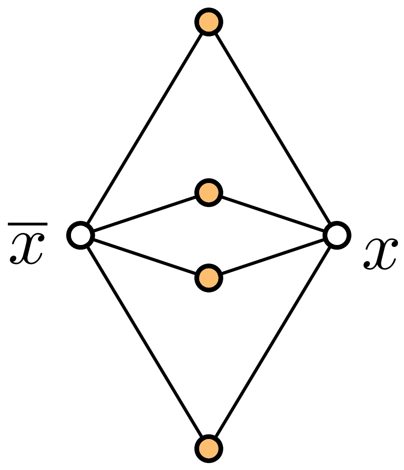

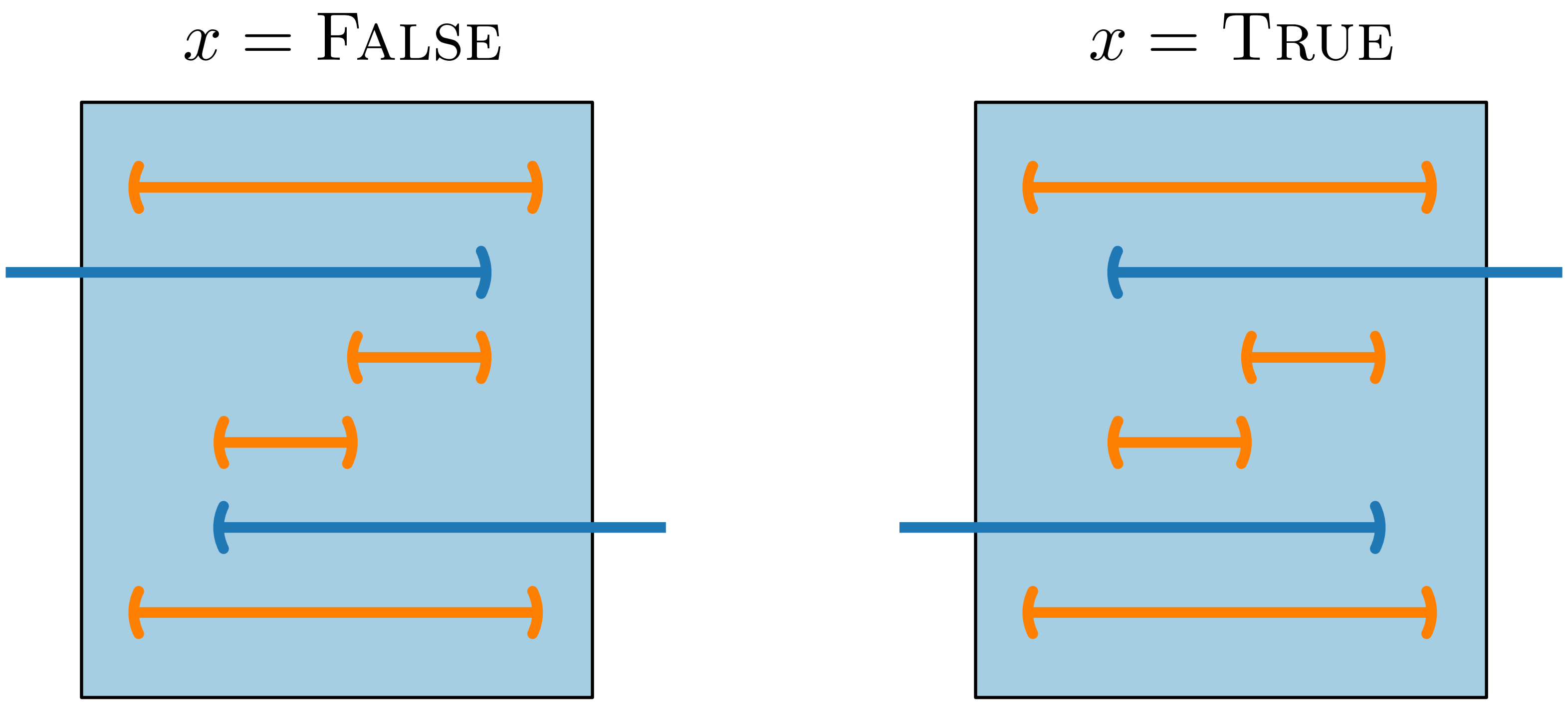

Clause Gadget

$$x \vee y \vee z$$

Clause Gadget

$$x \lor y \lor z$$

Clause Gadget

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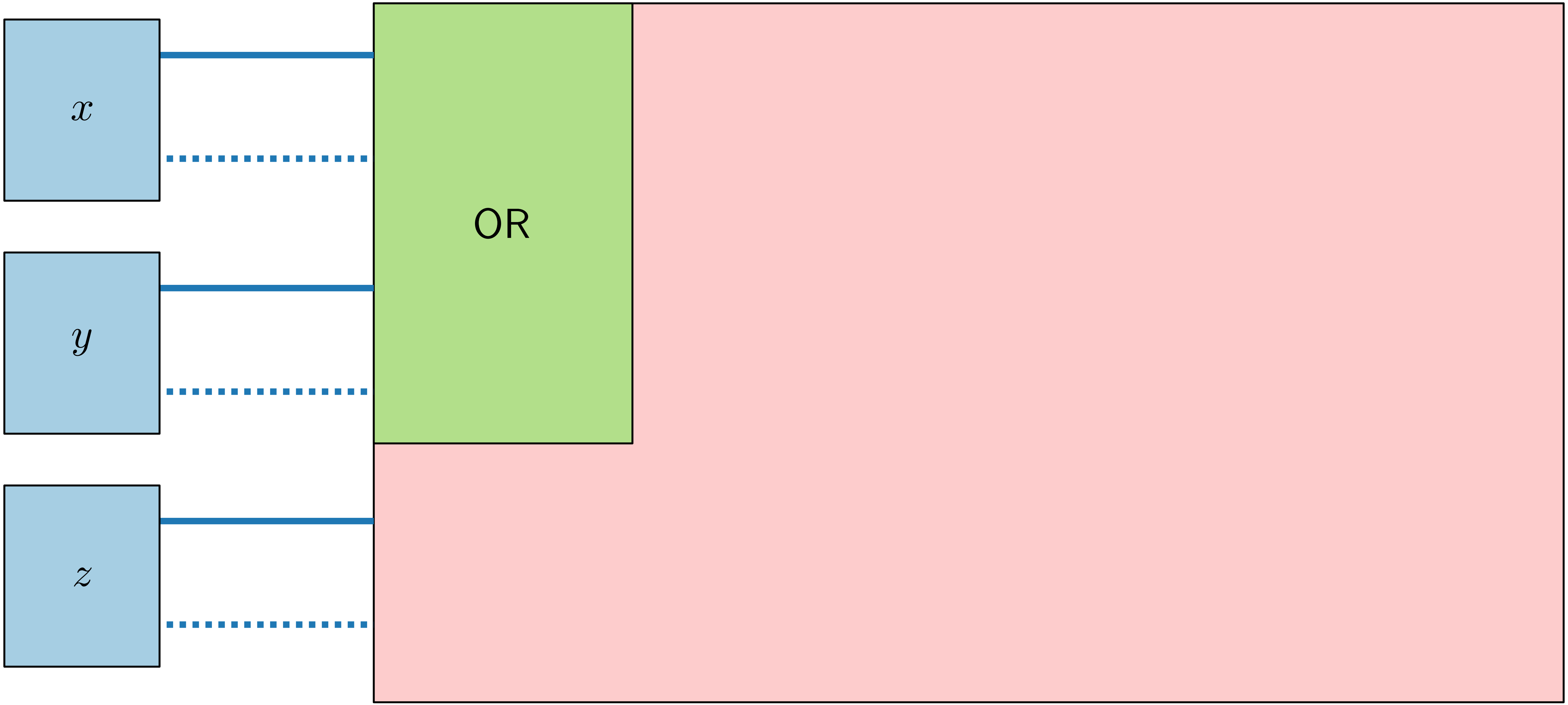

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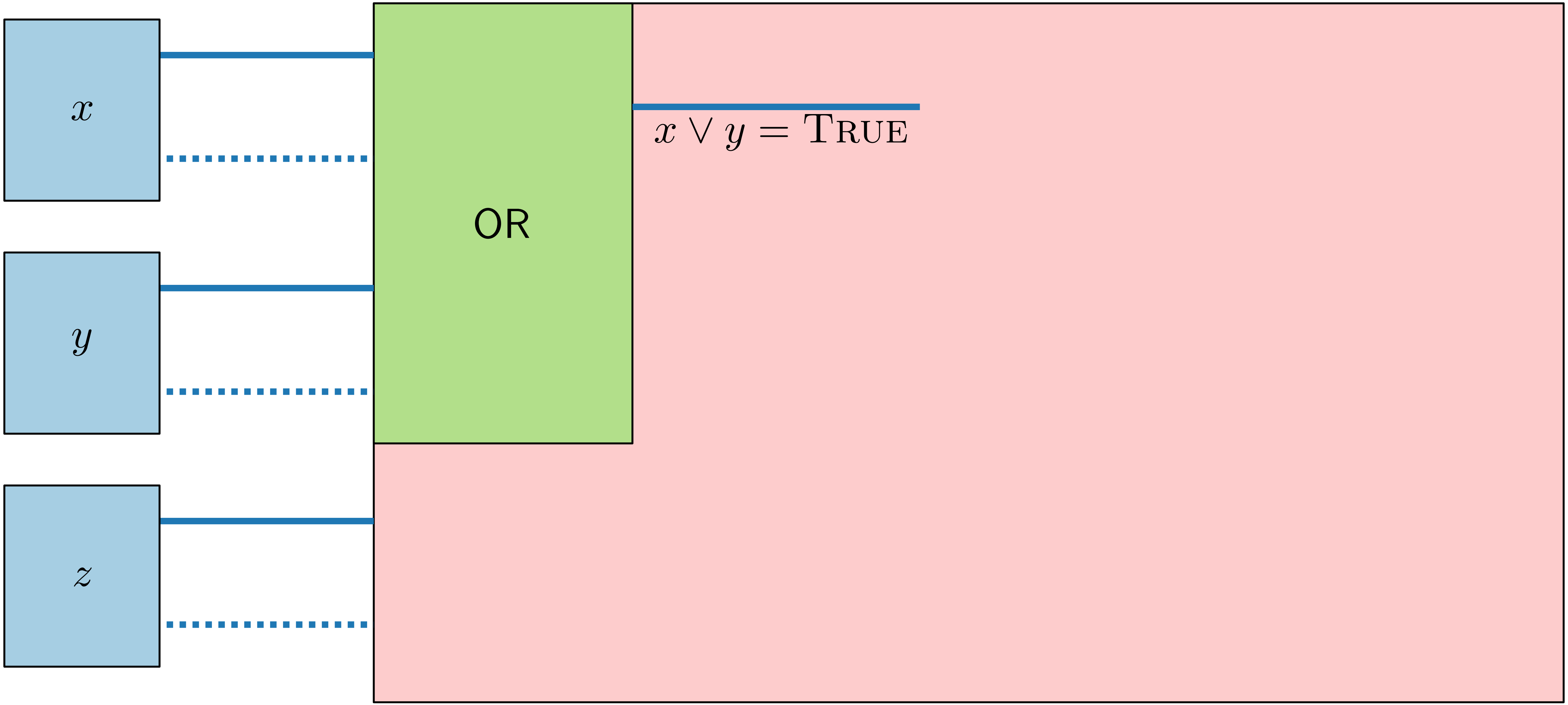

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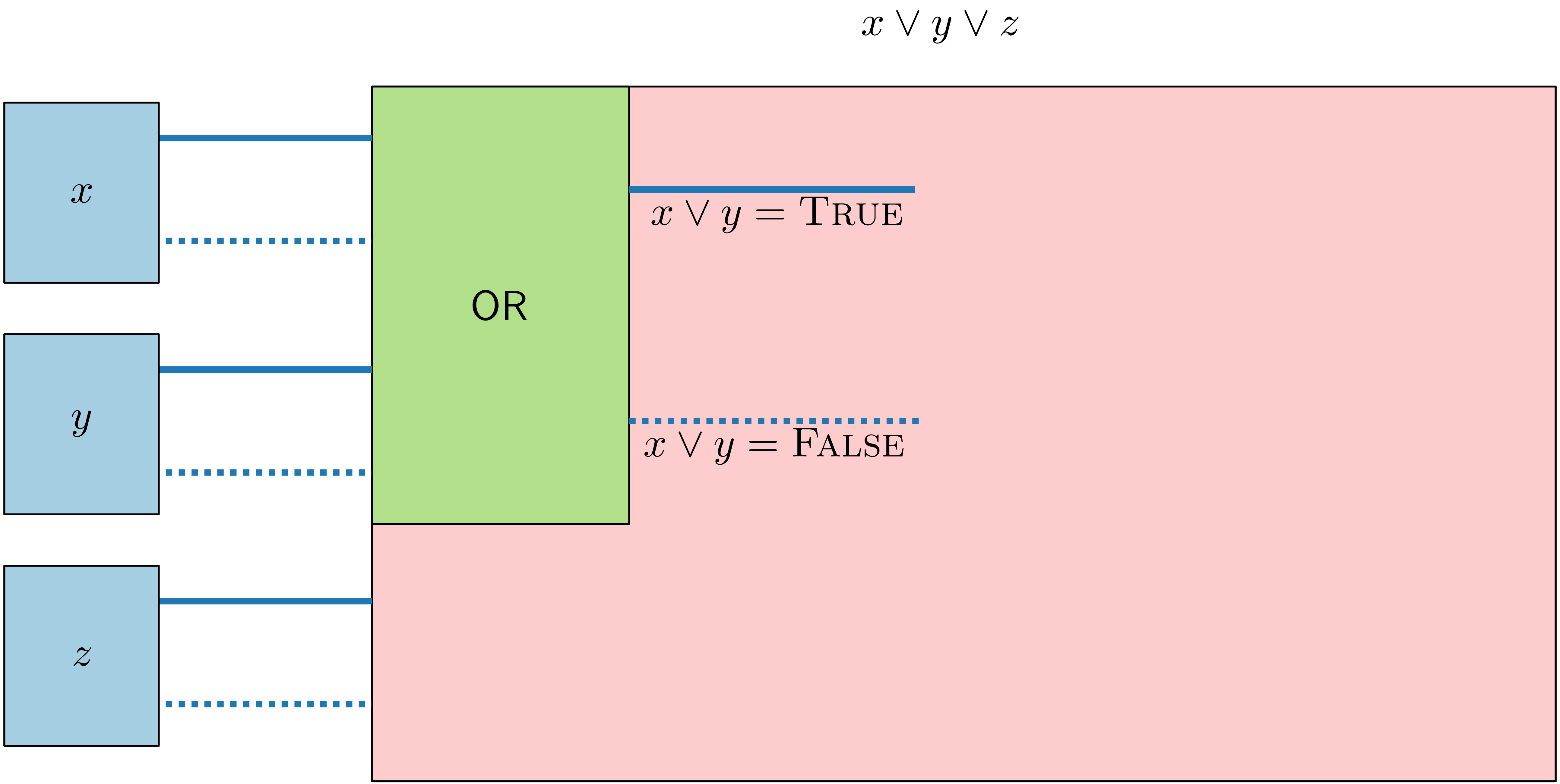

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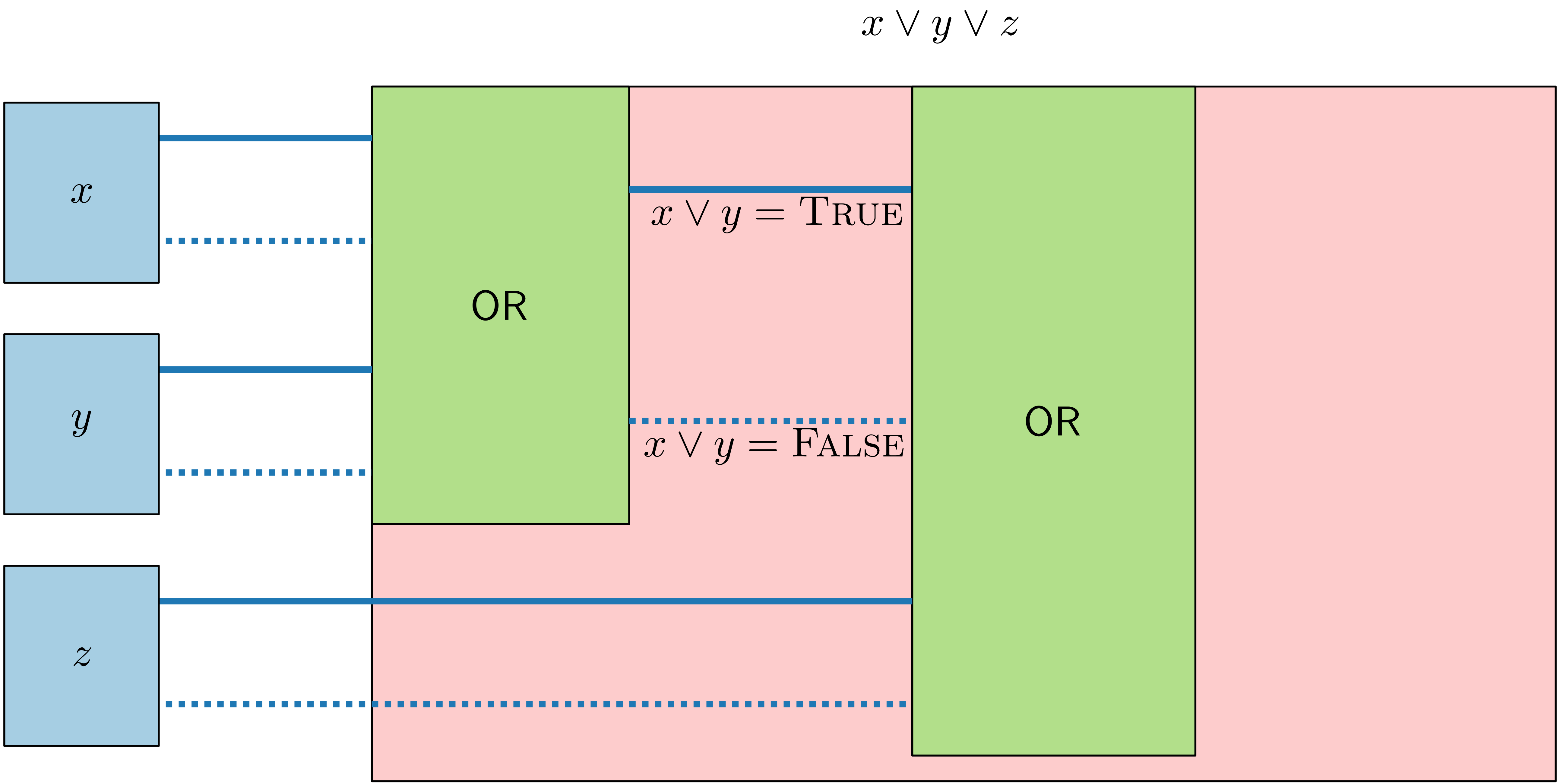
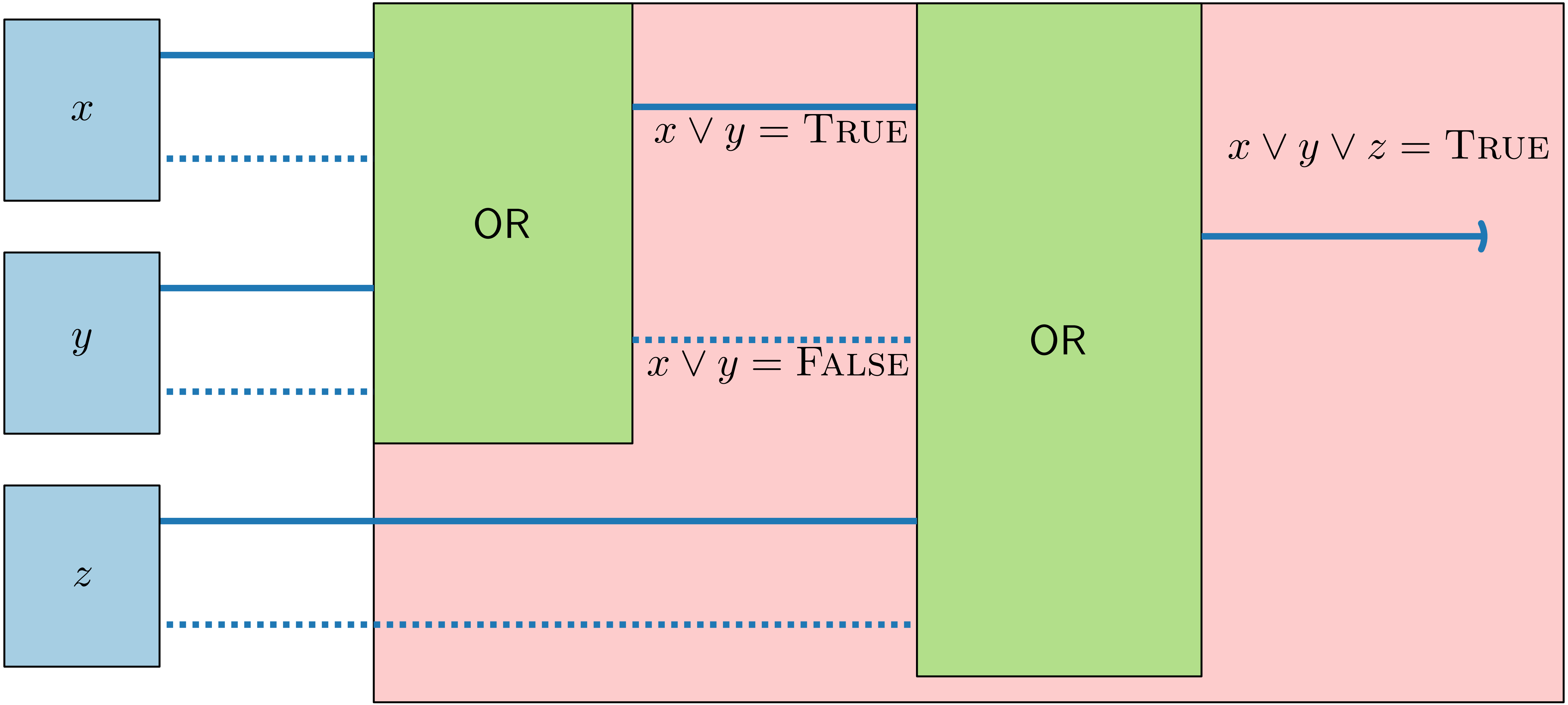


Clause Gadget

$x \lor y \lor z$



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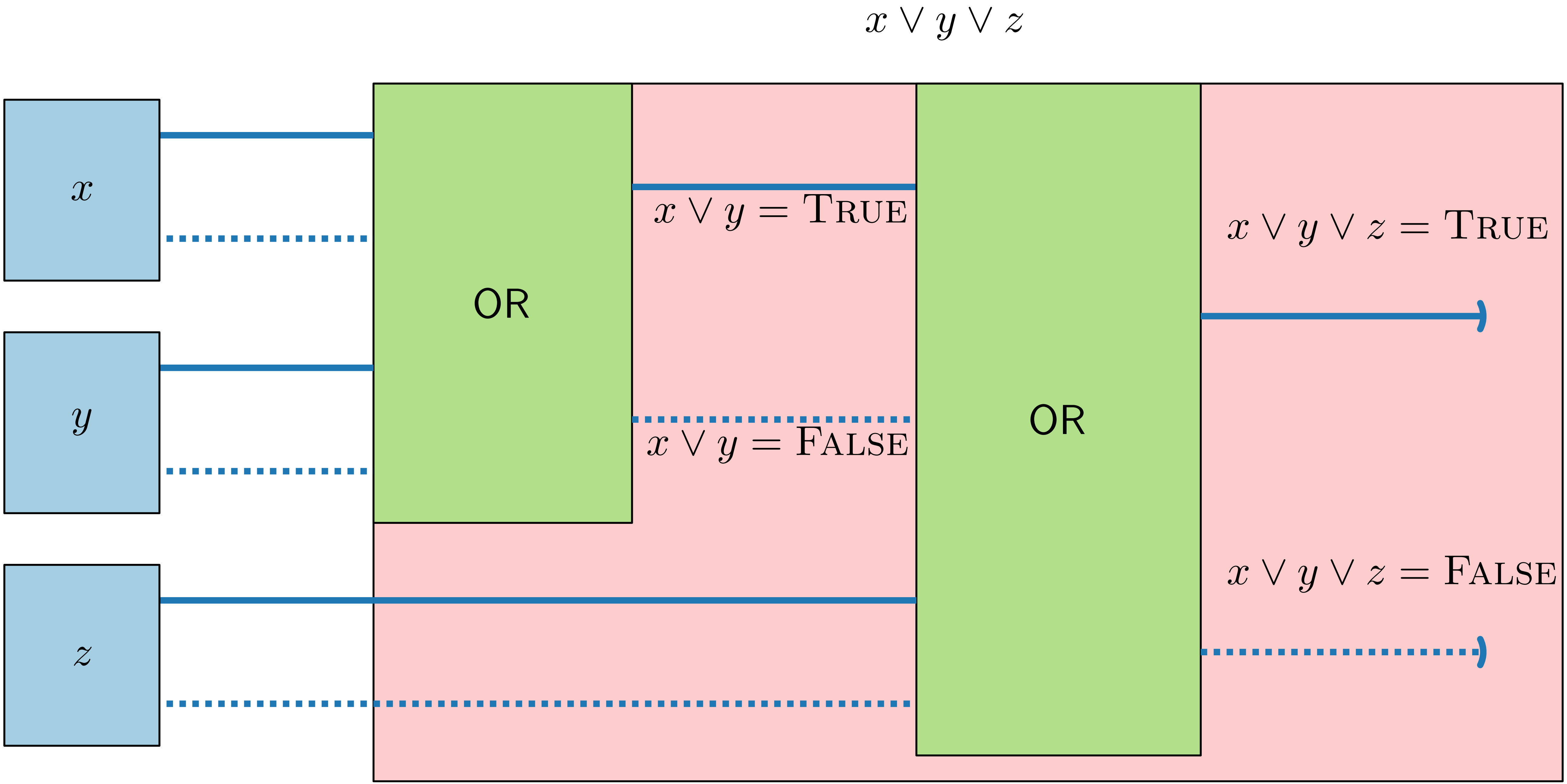

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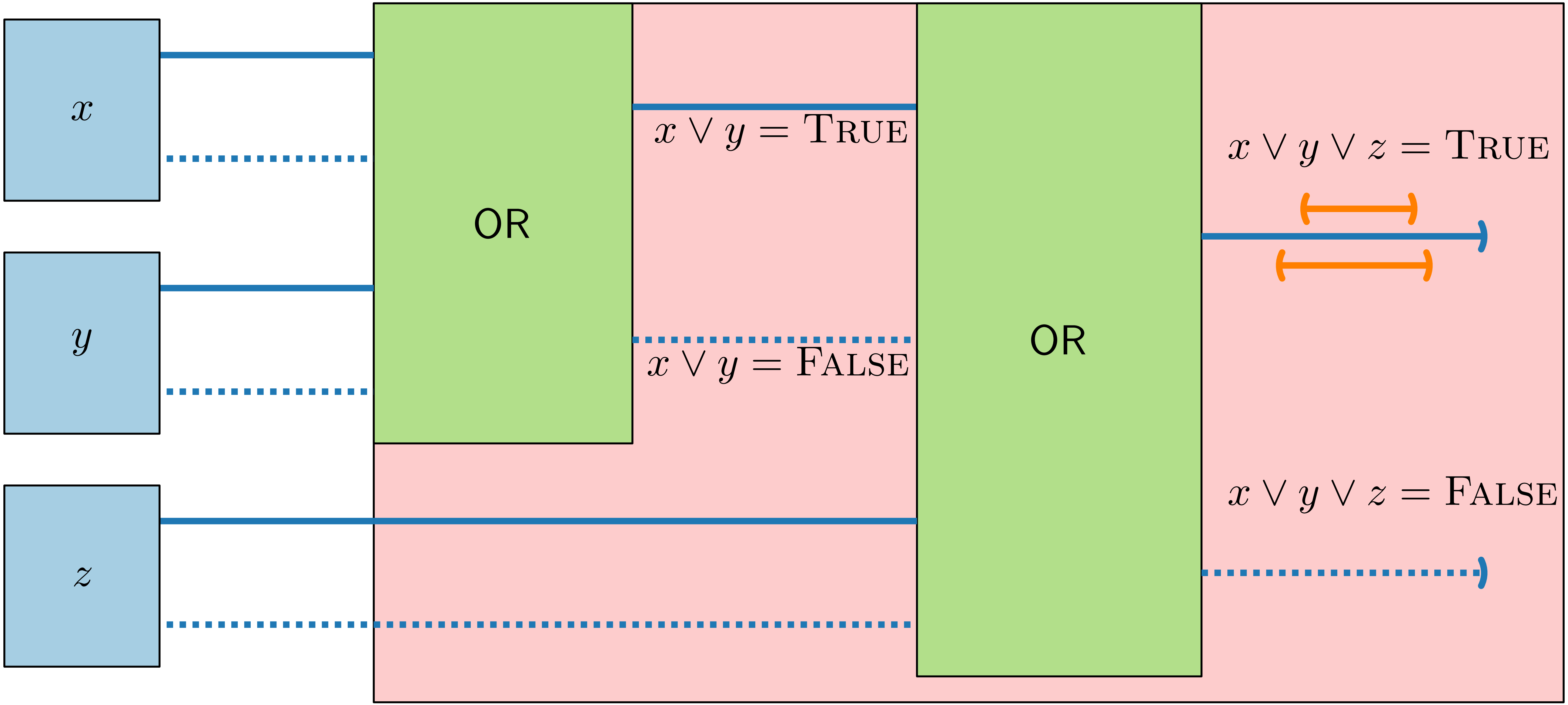

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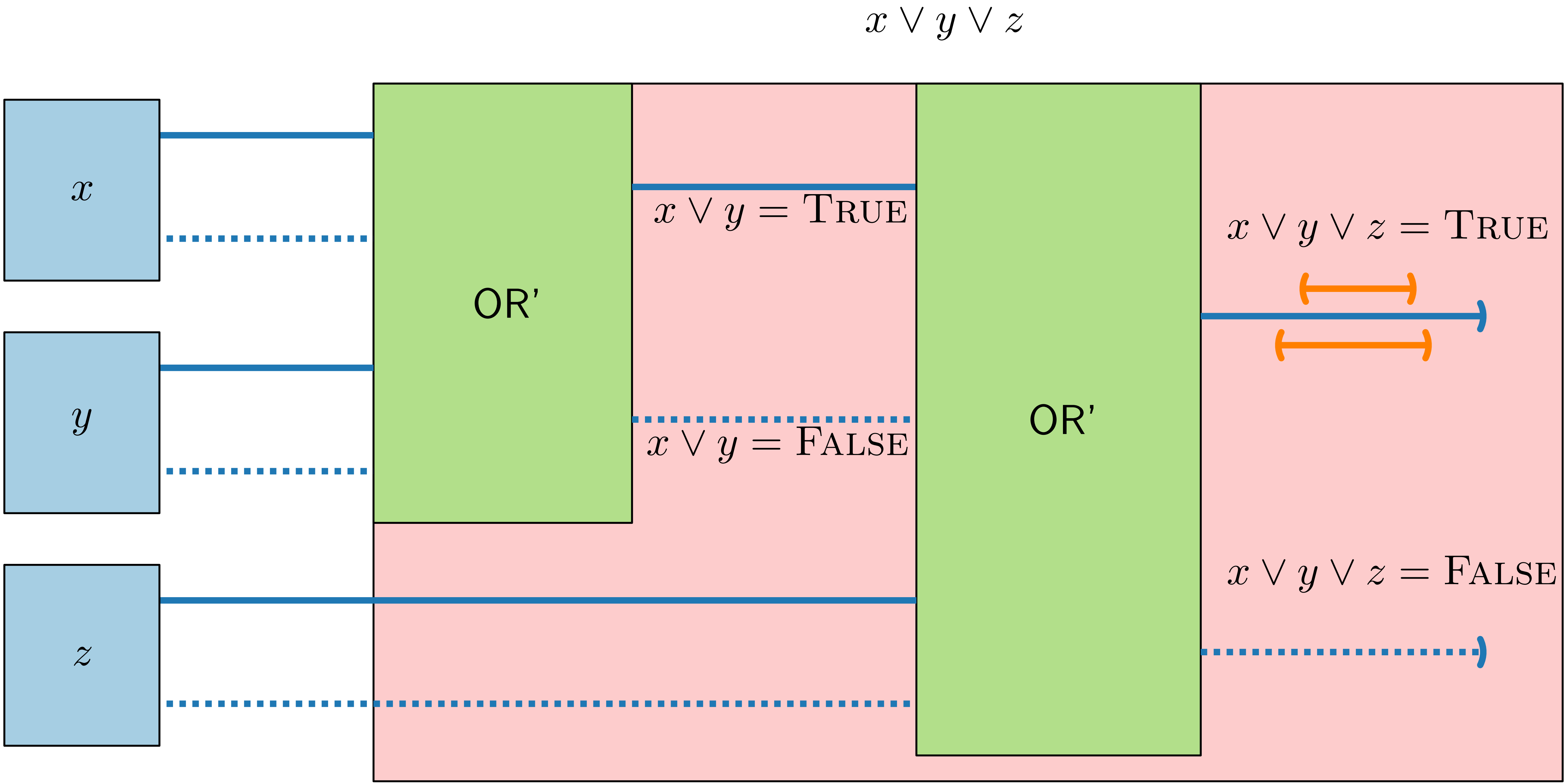

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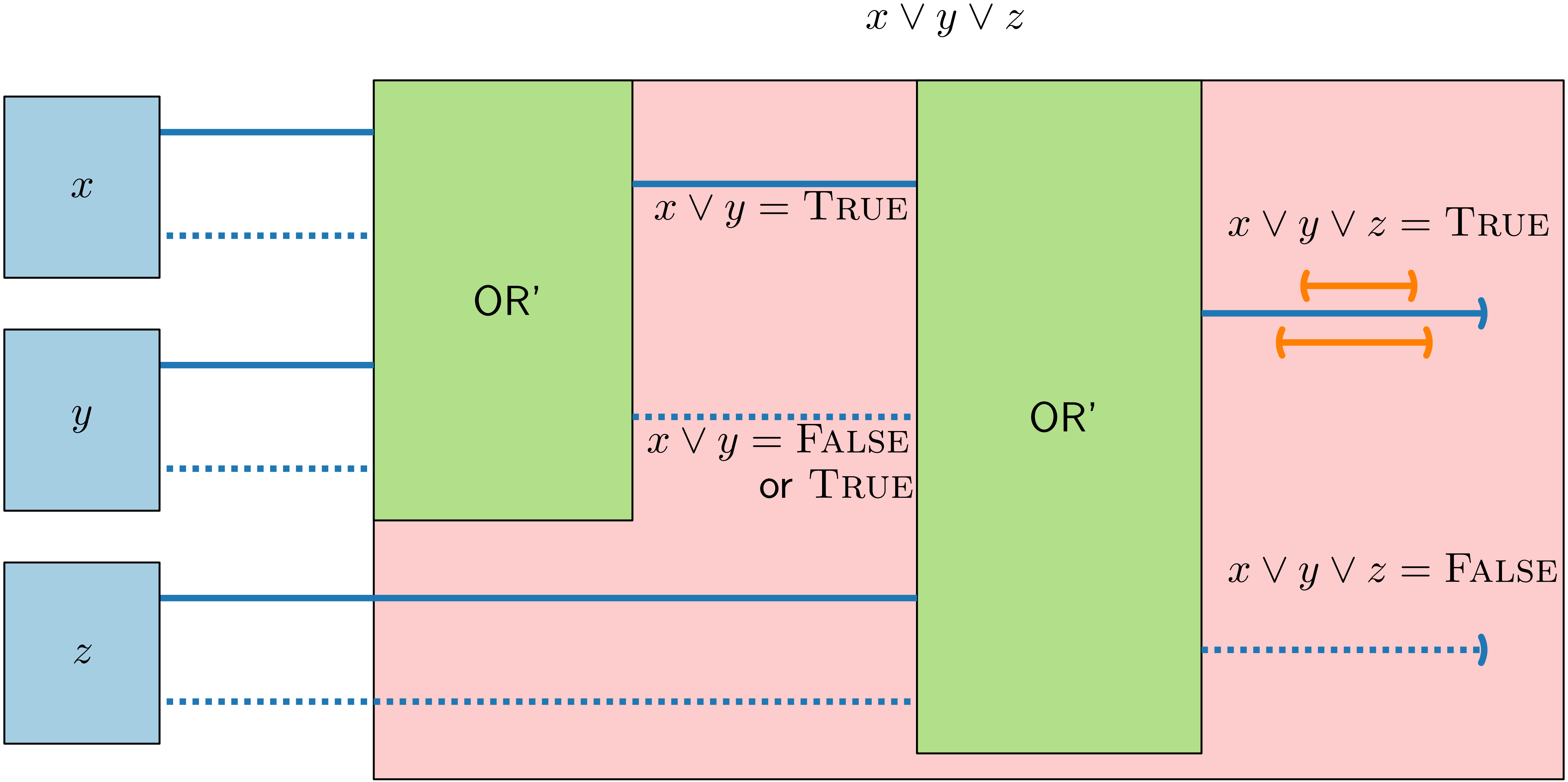

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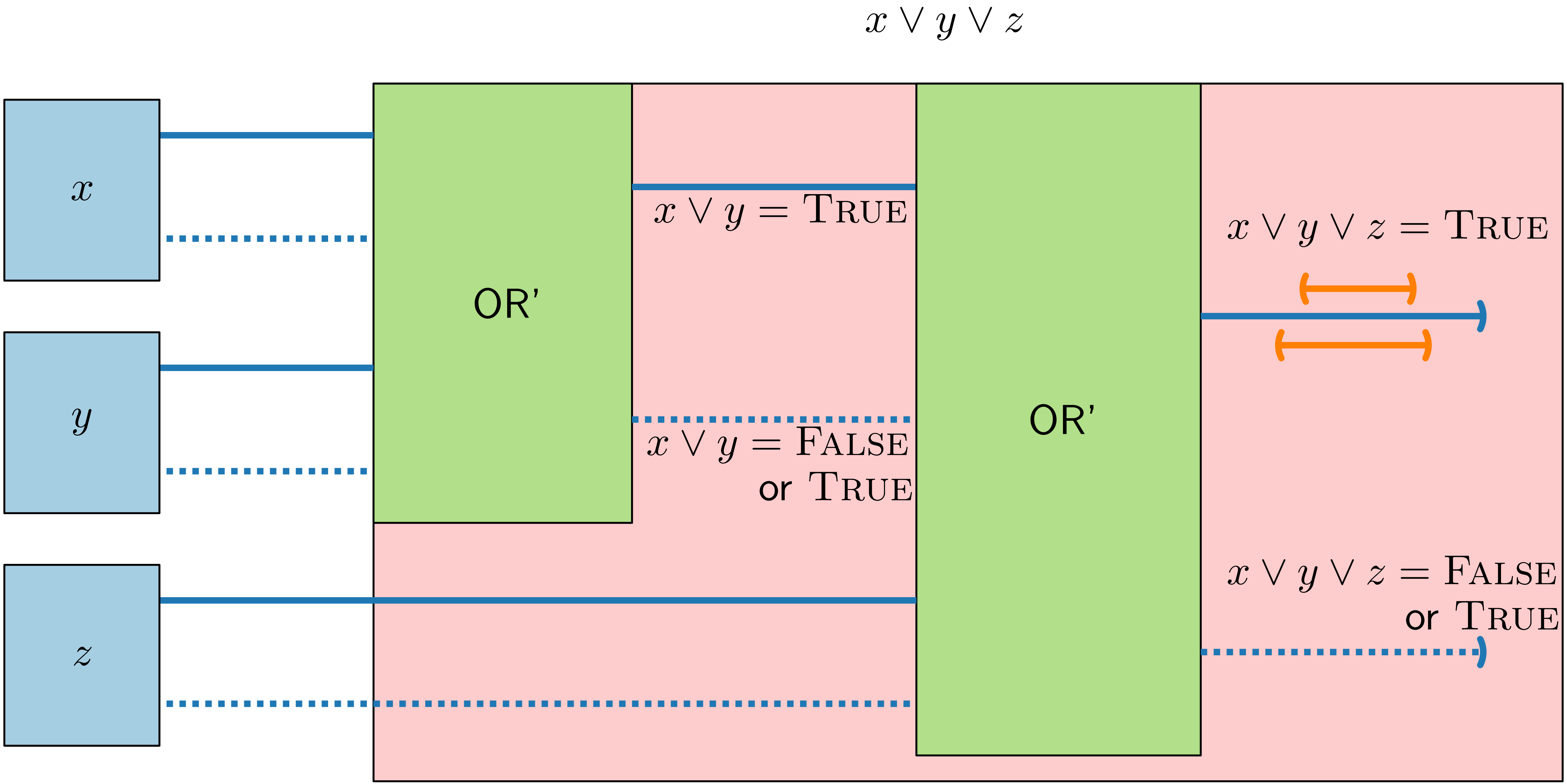

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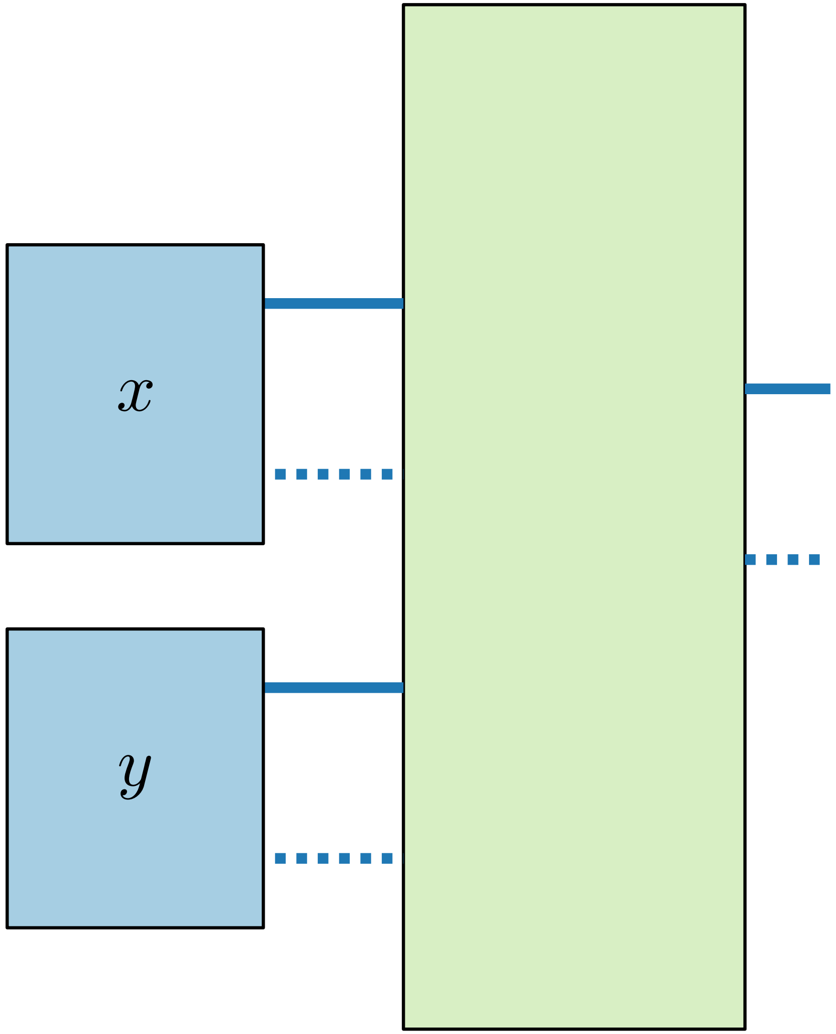

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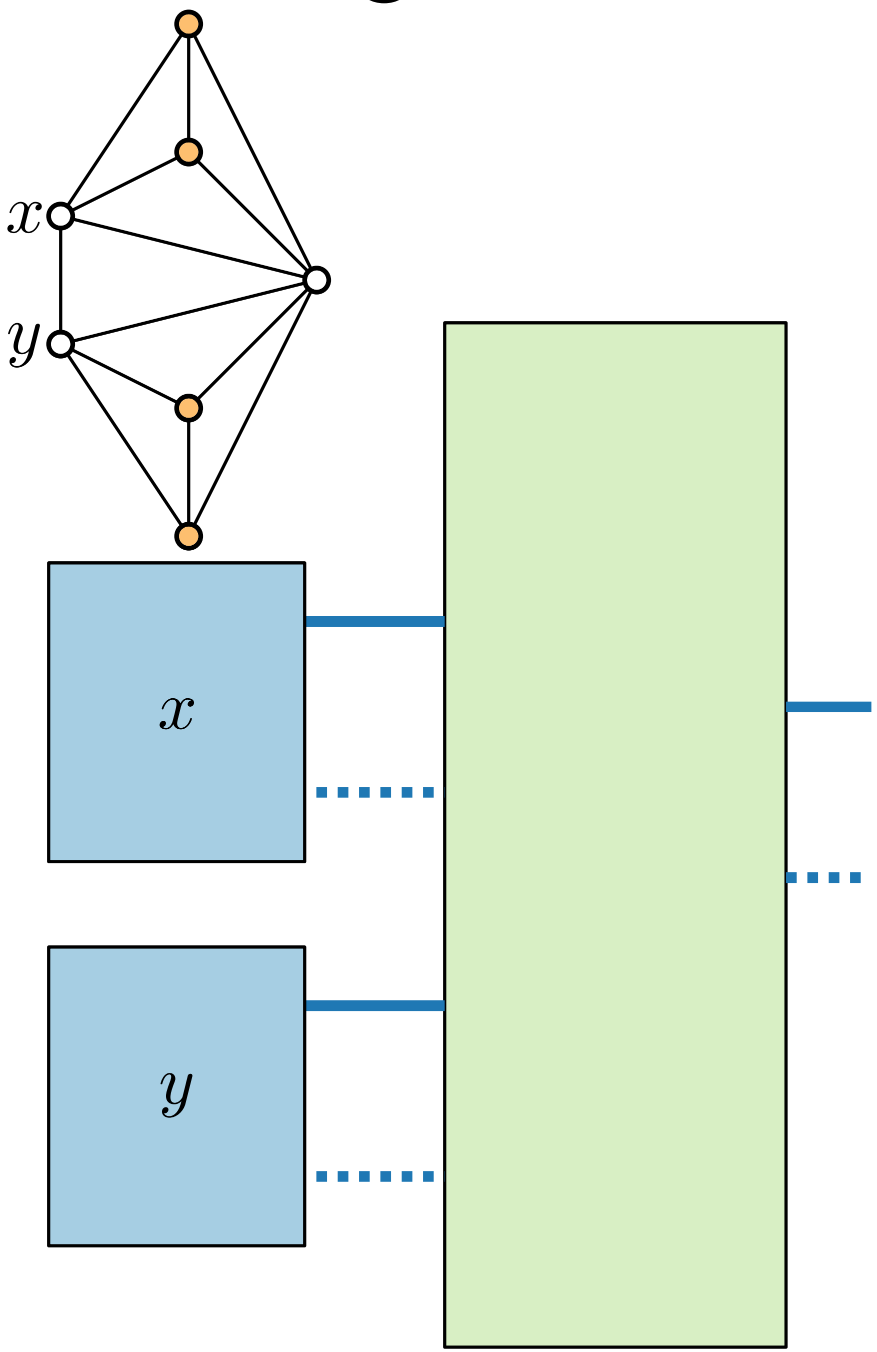

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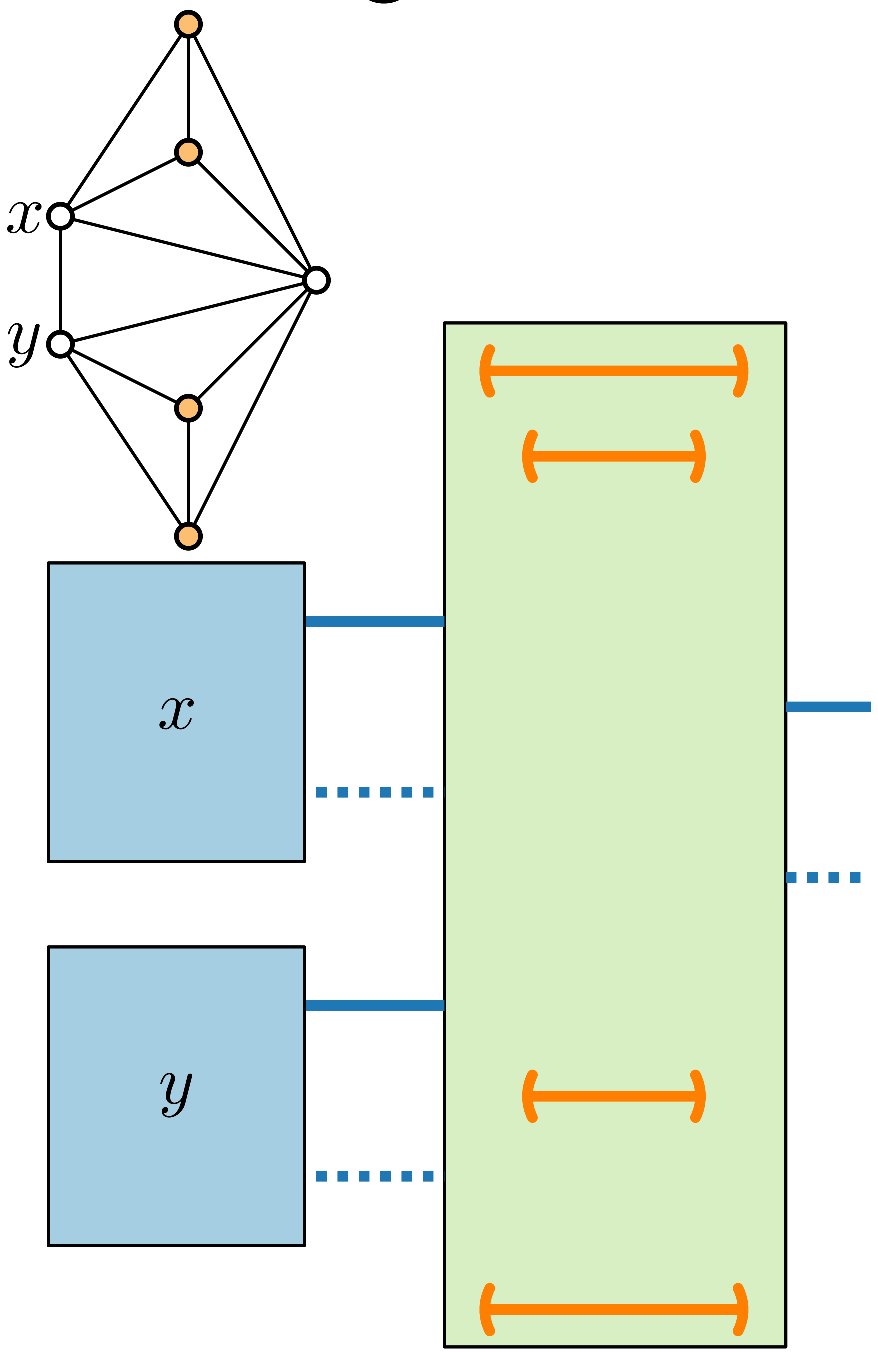

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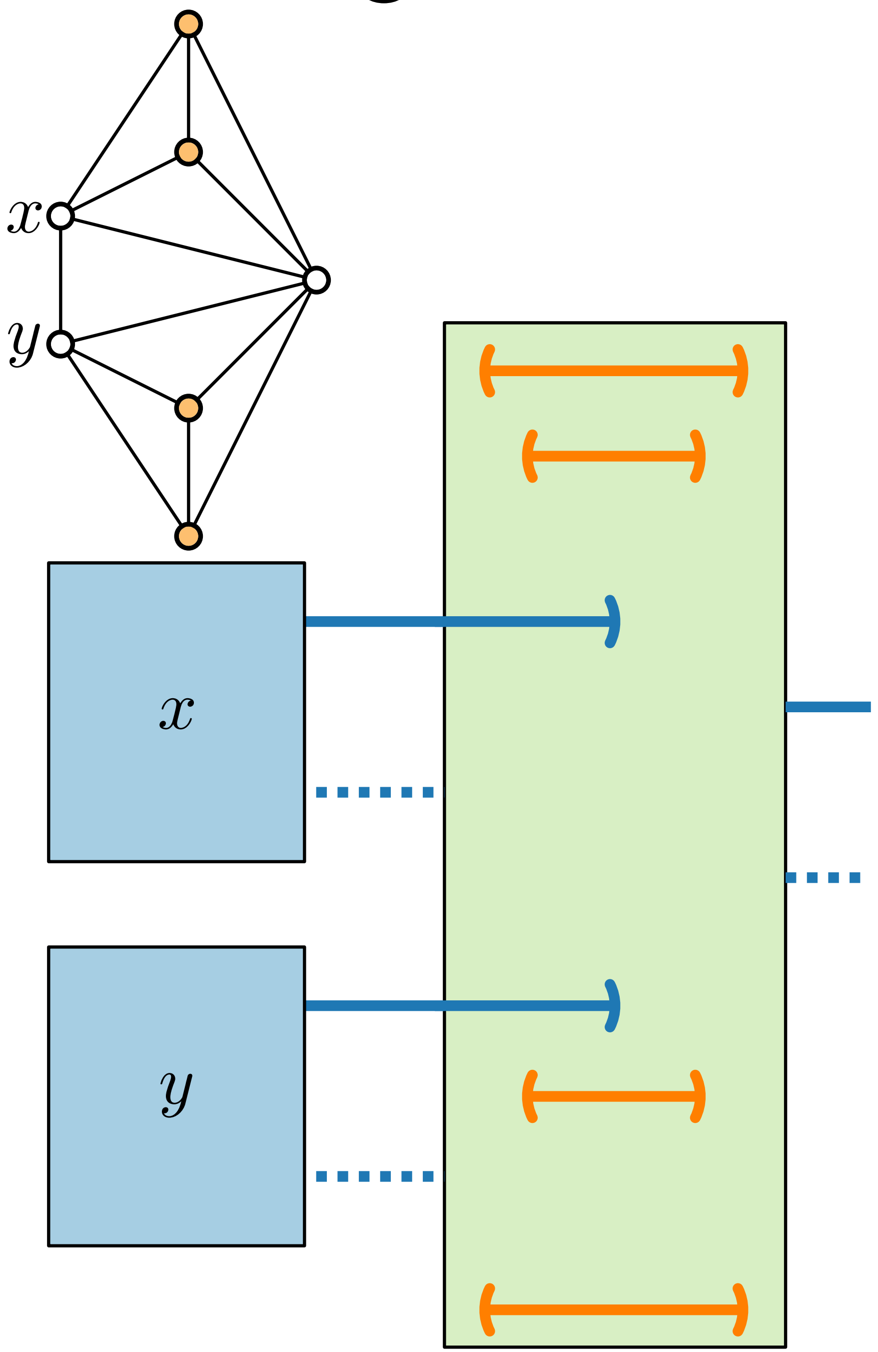

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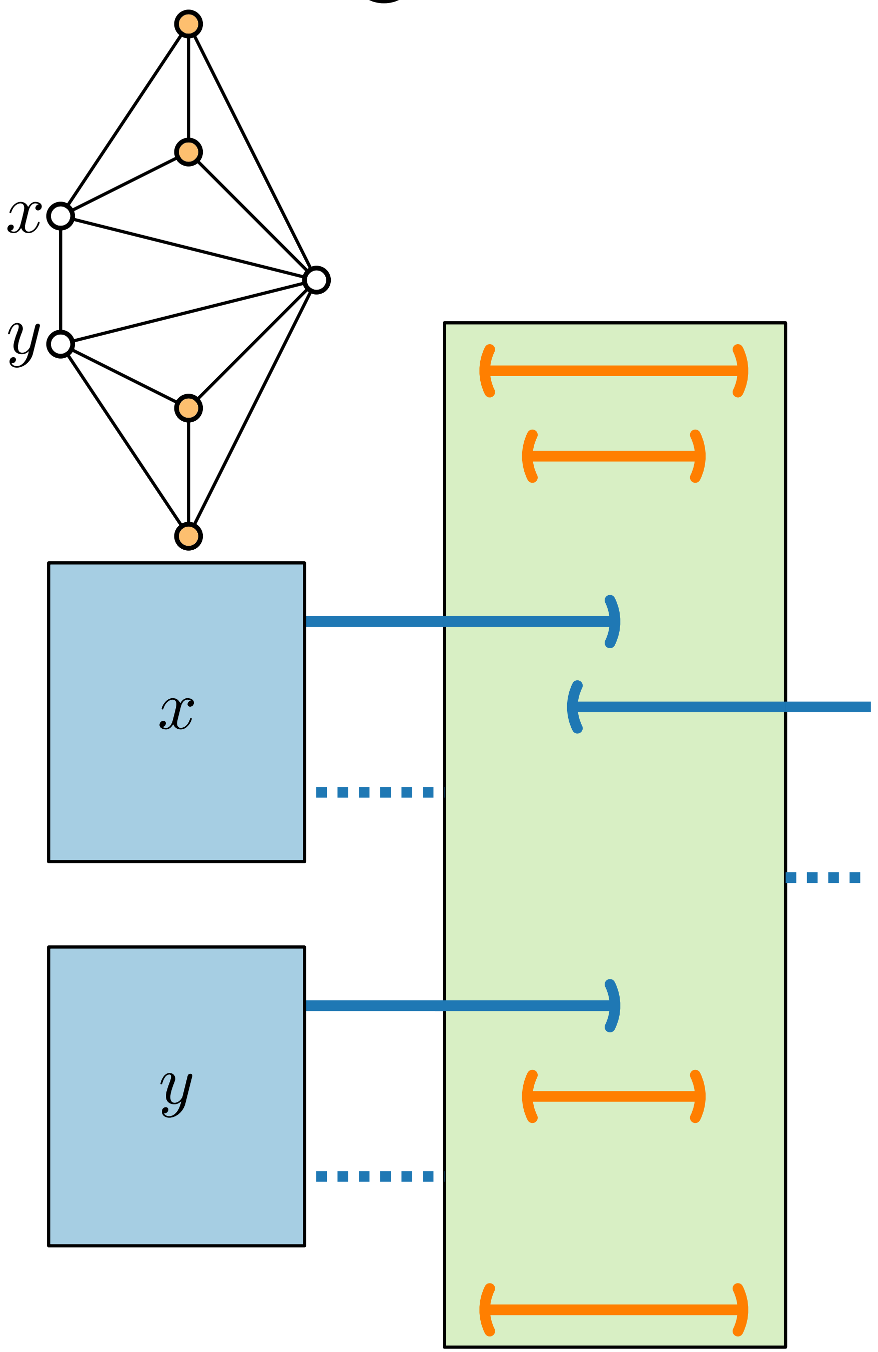

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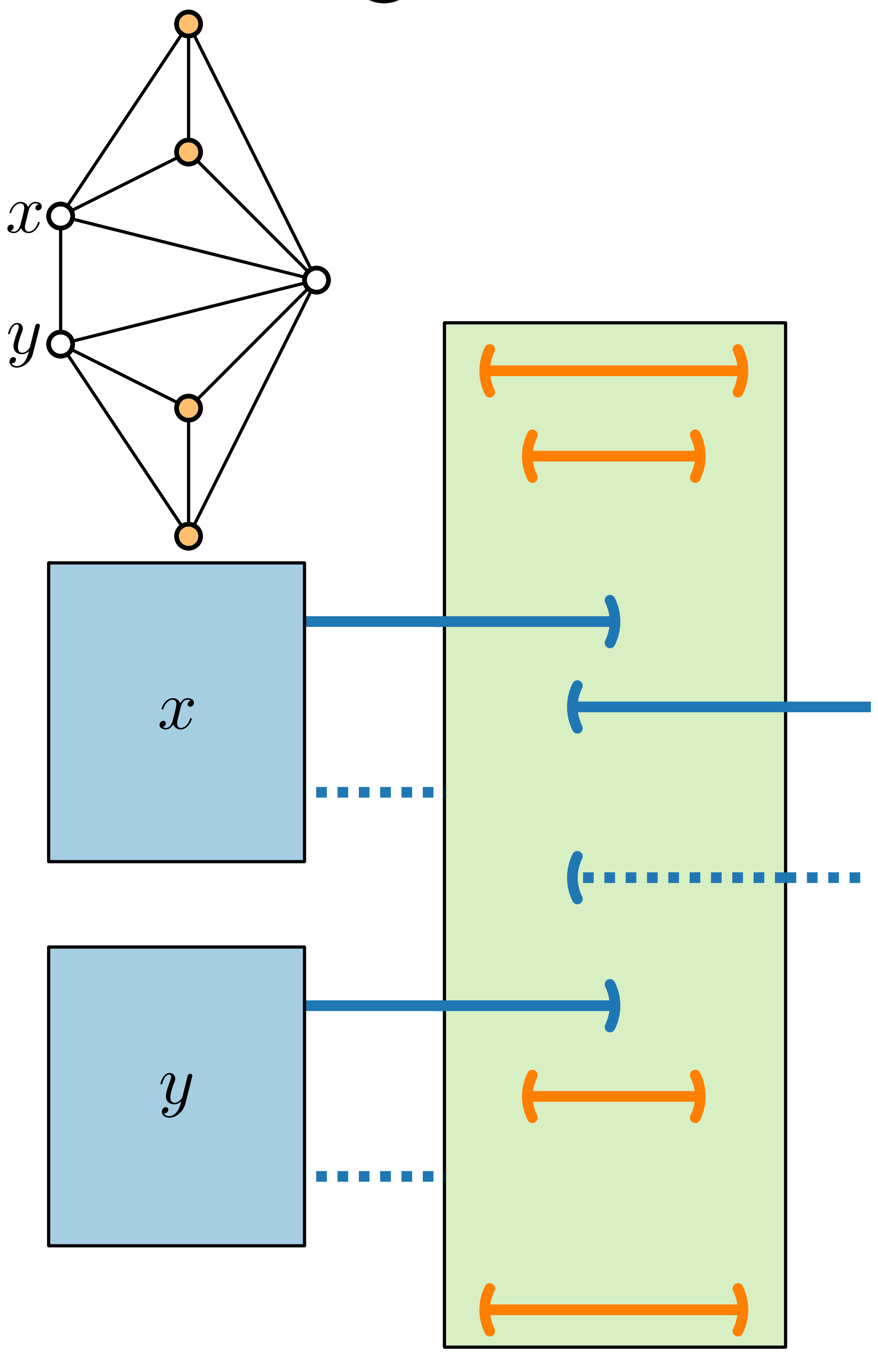

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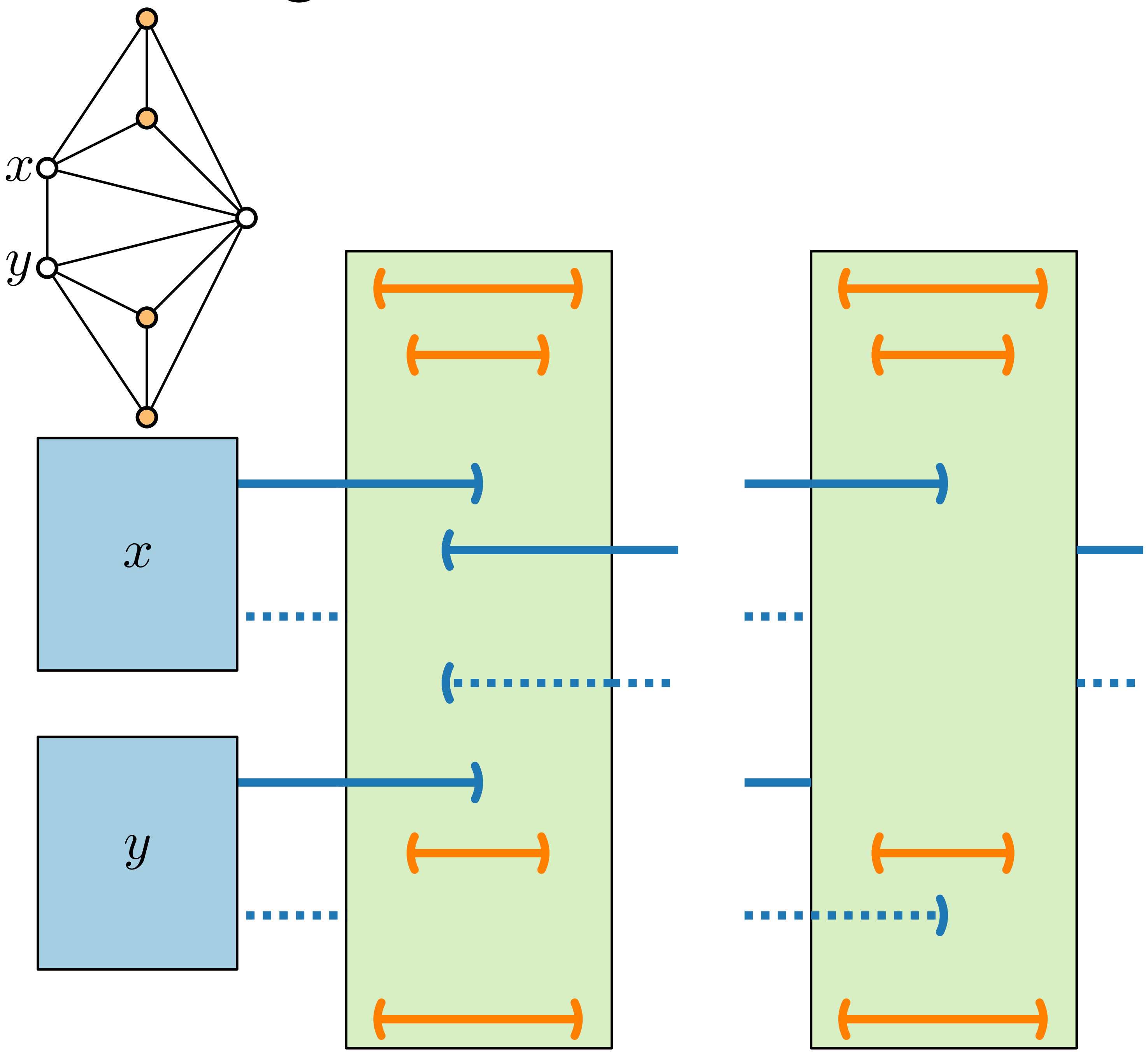

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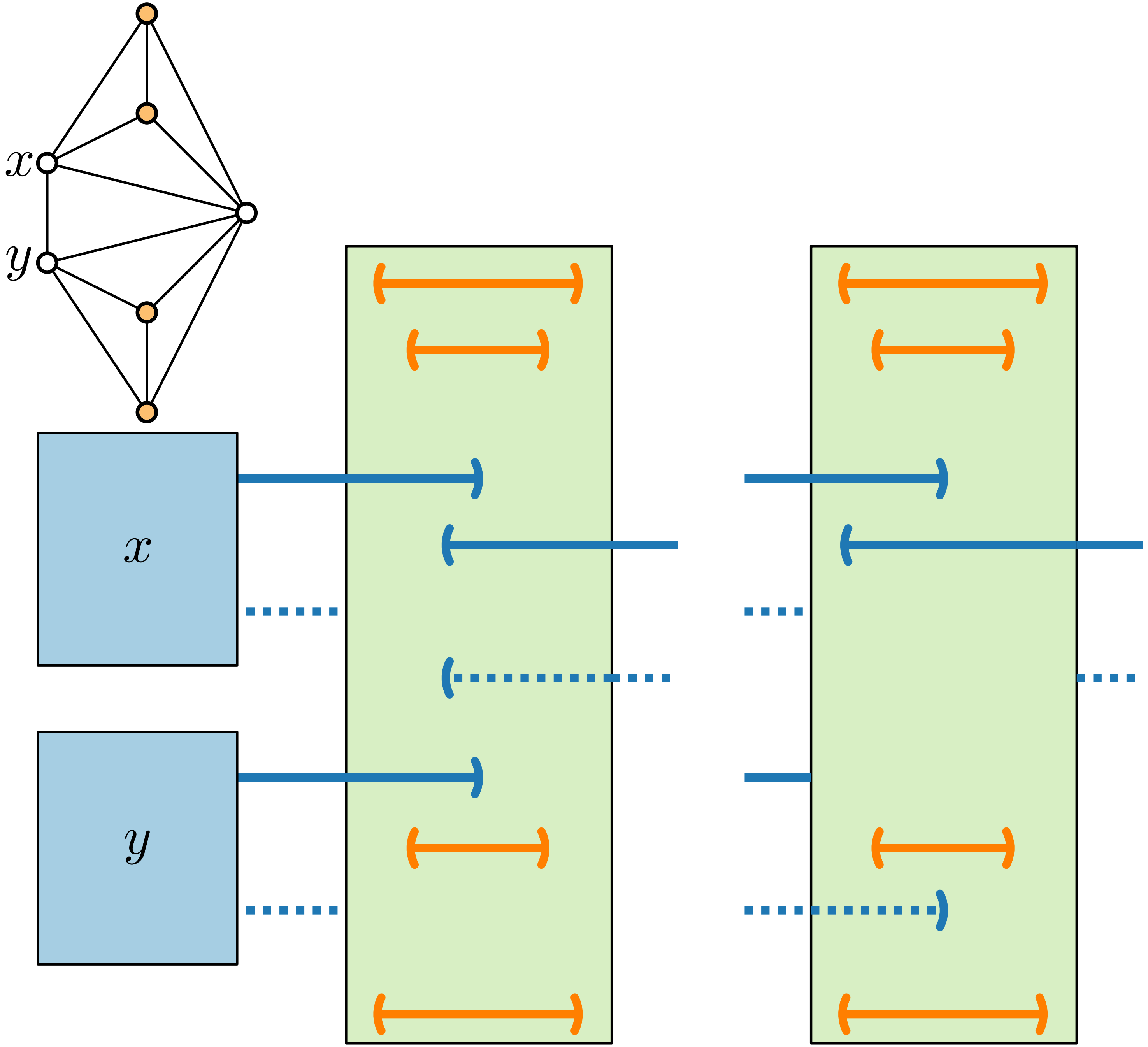

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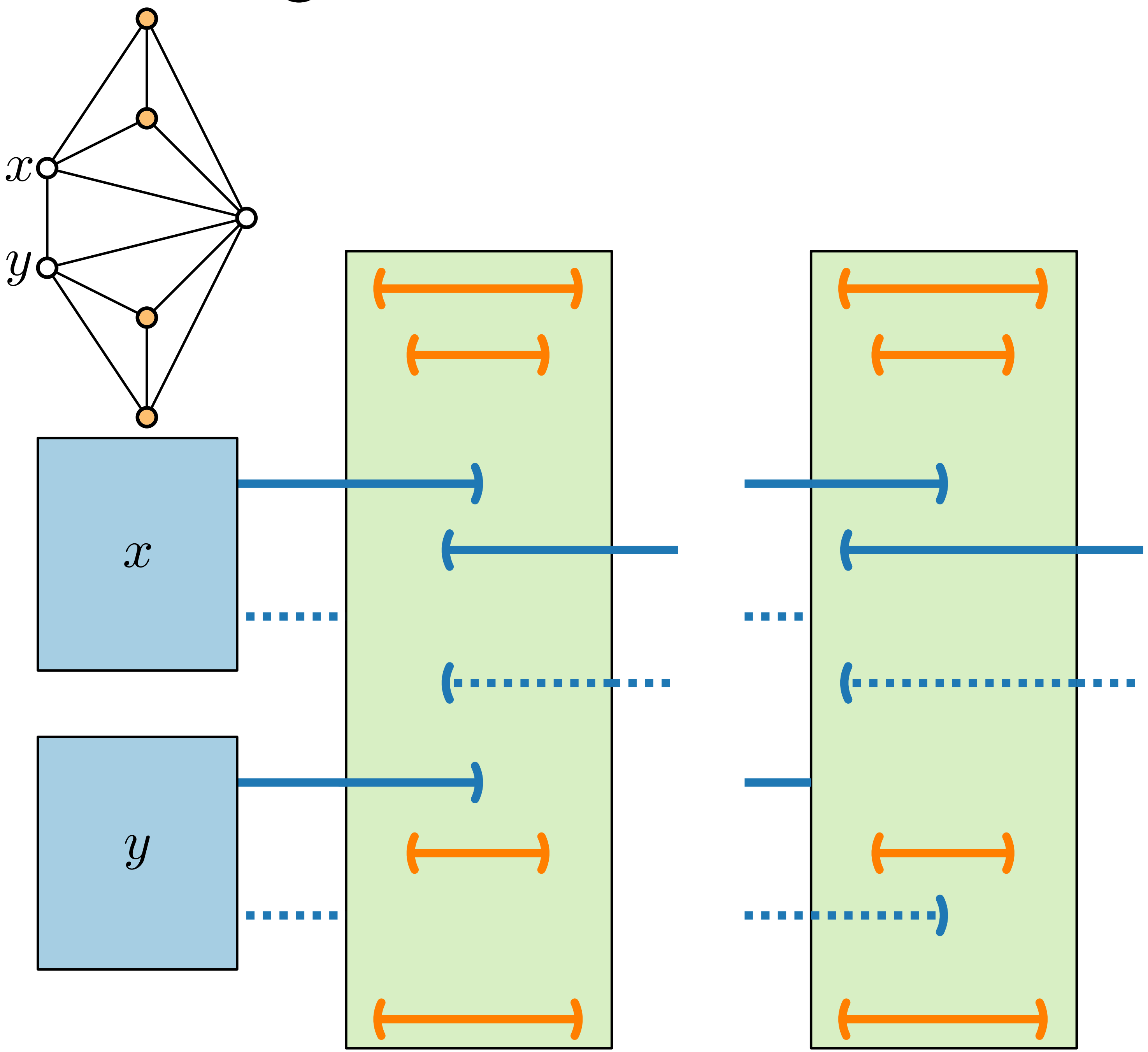

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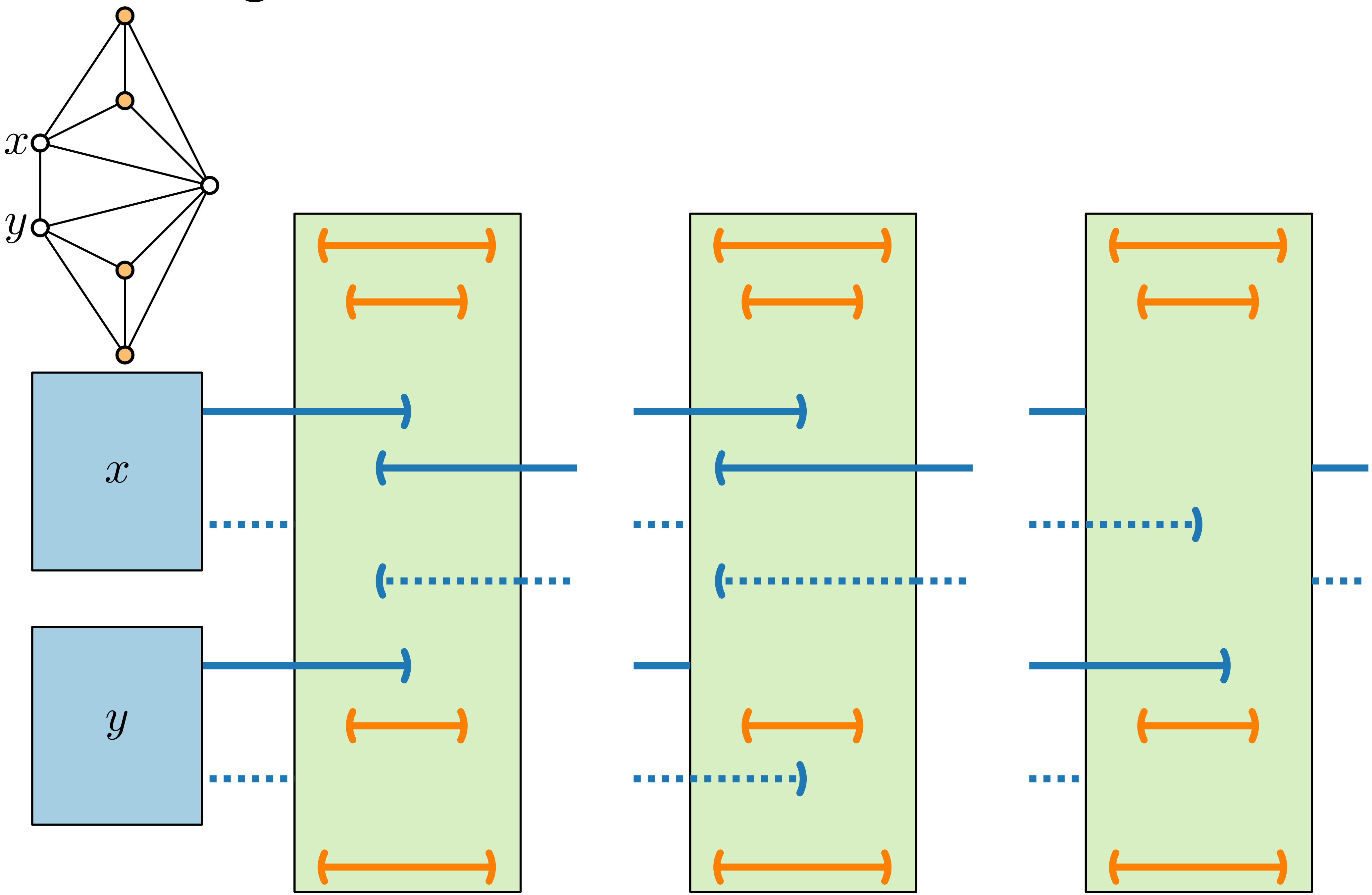

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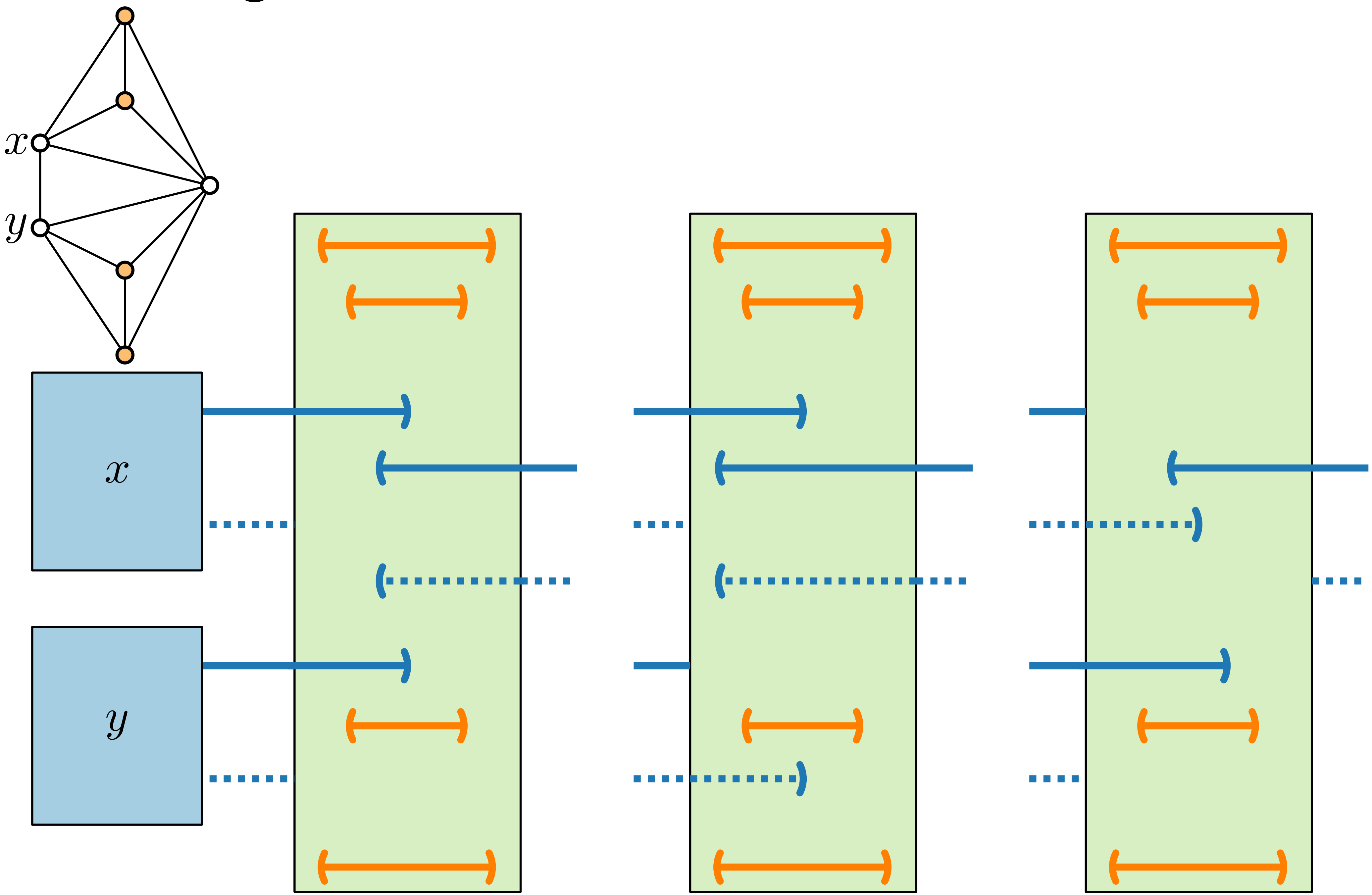

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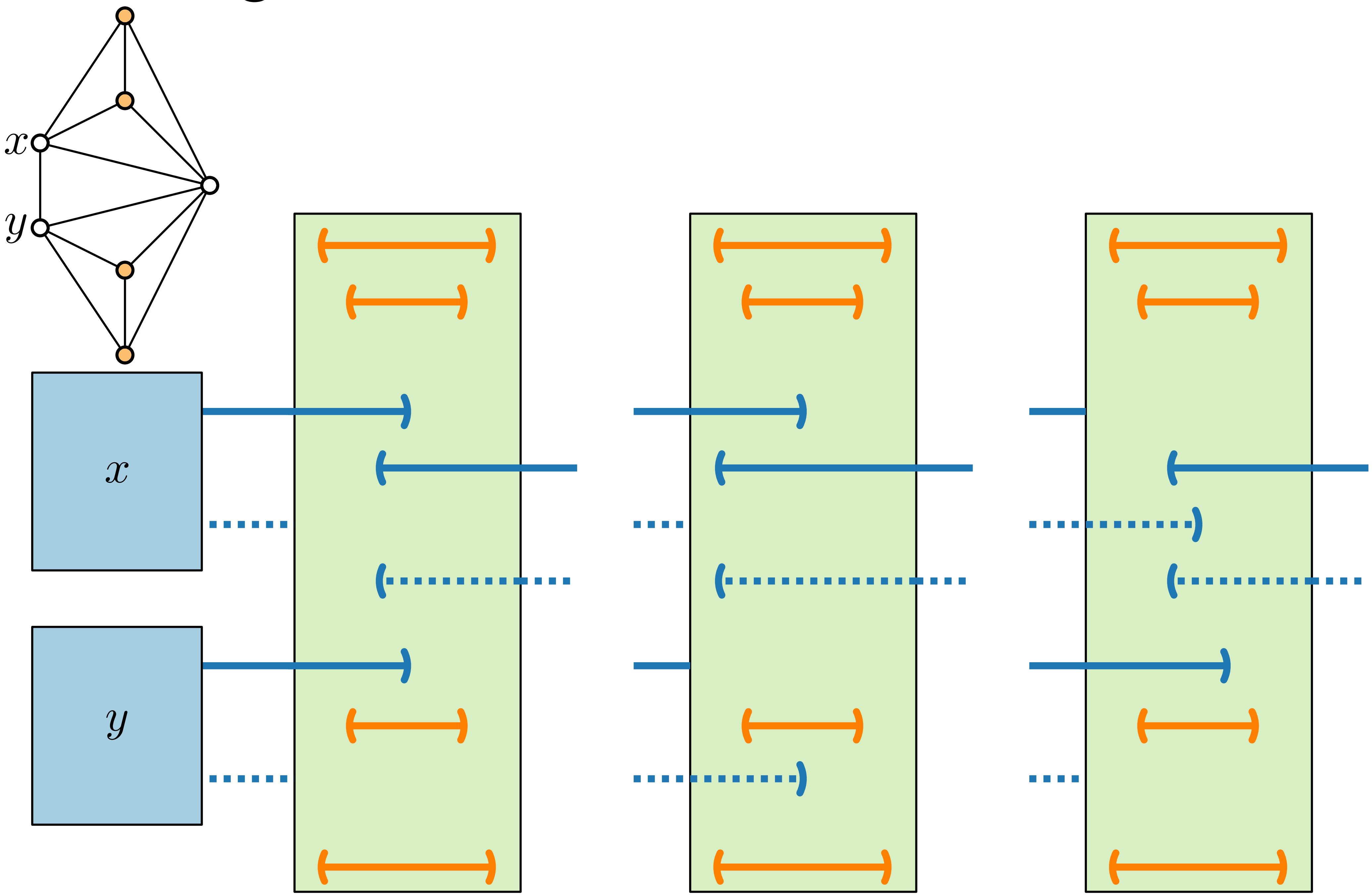

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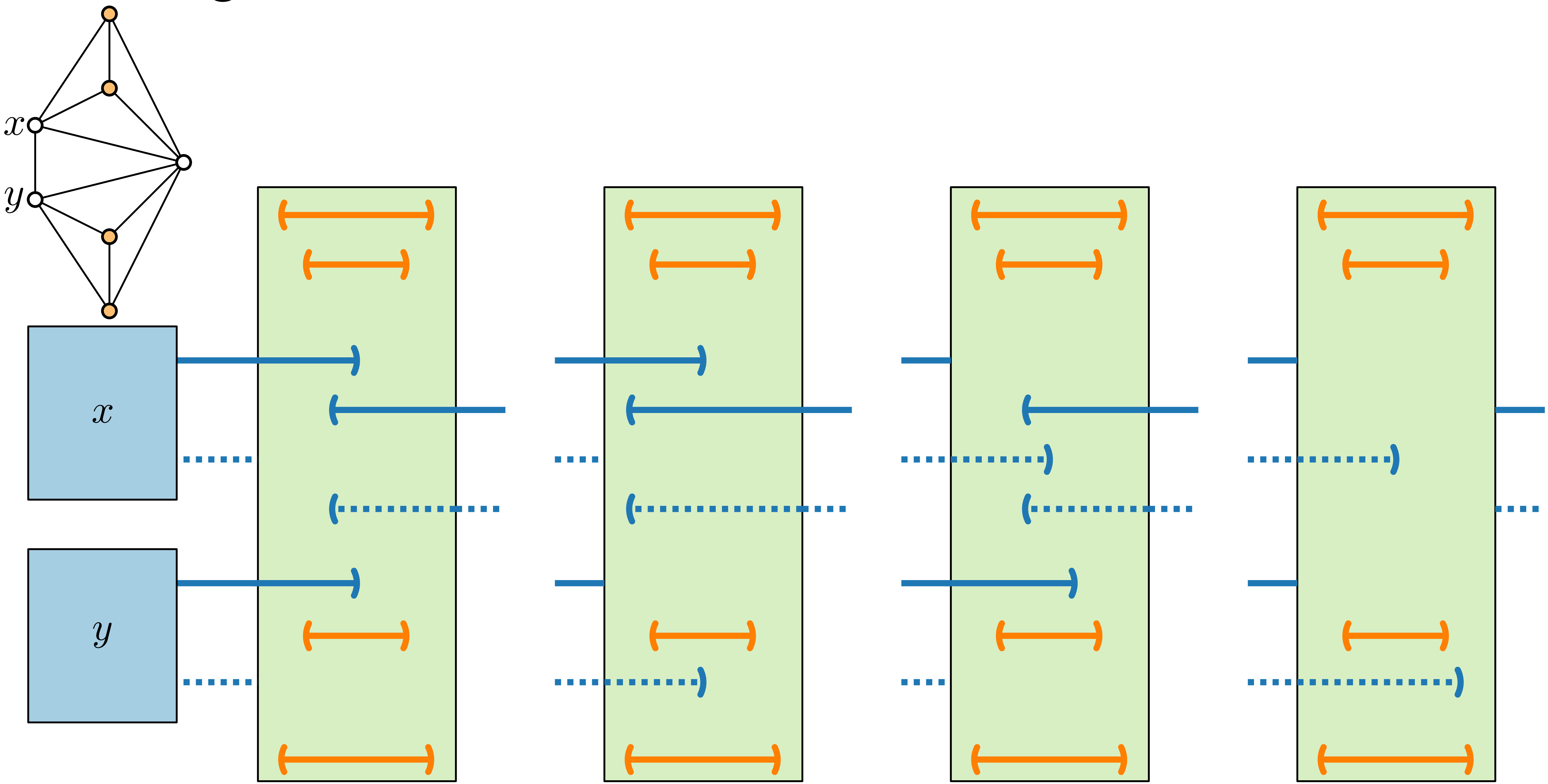

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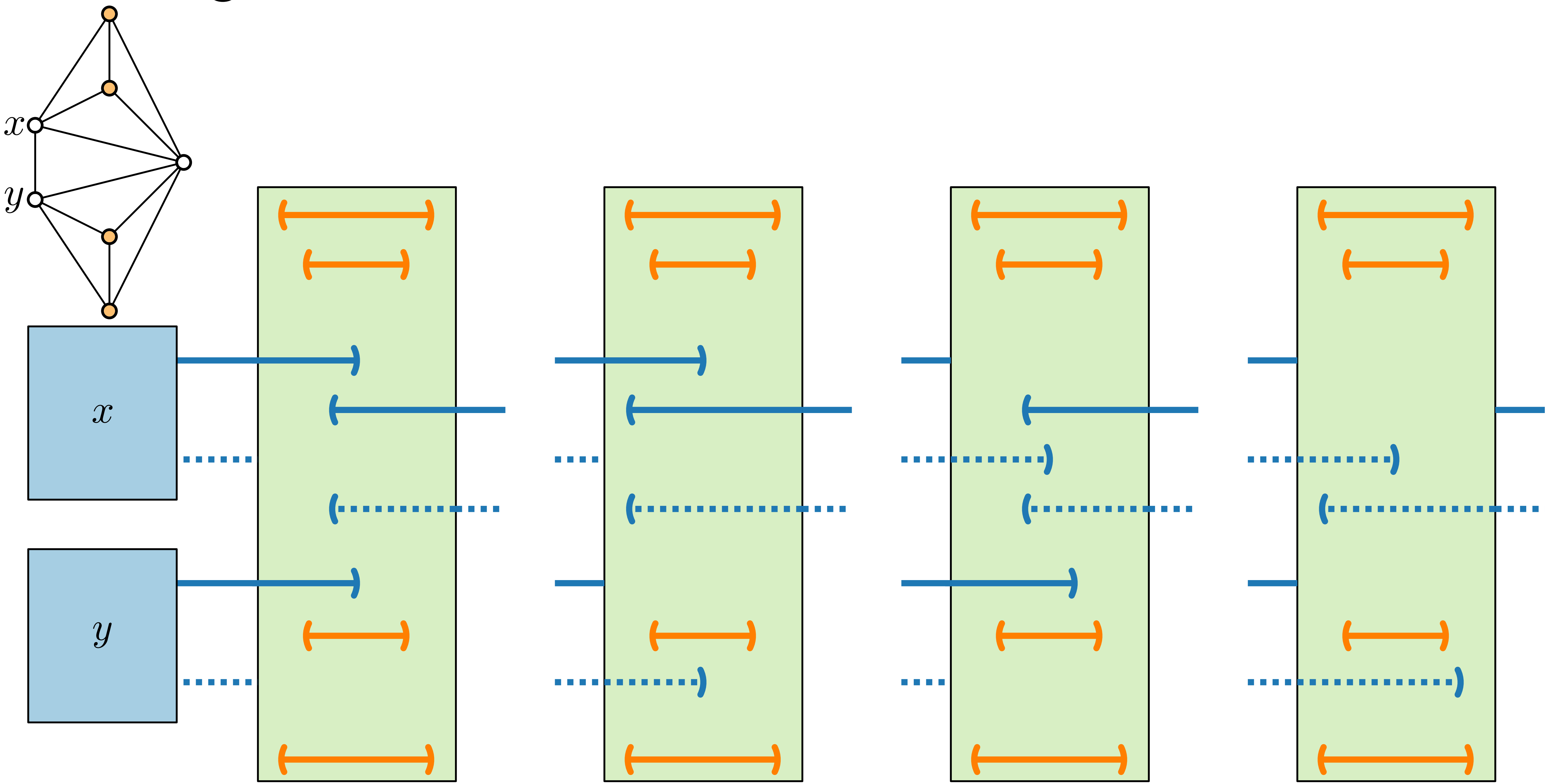

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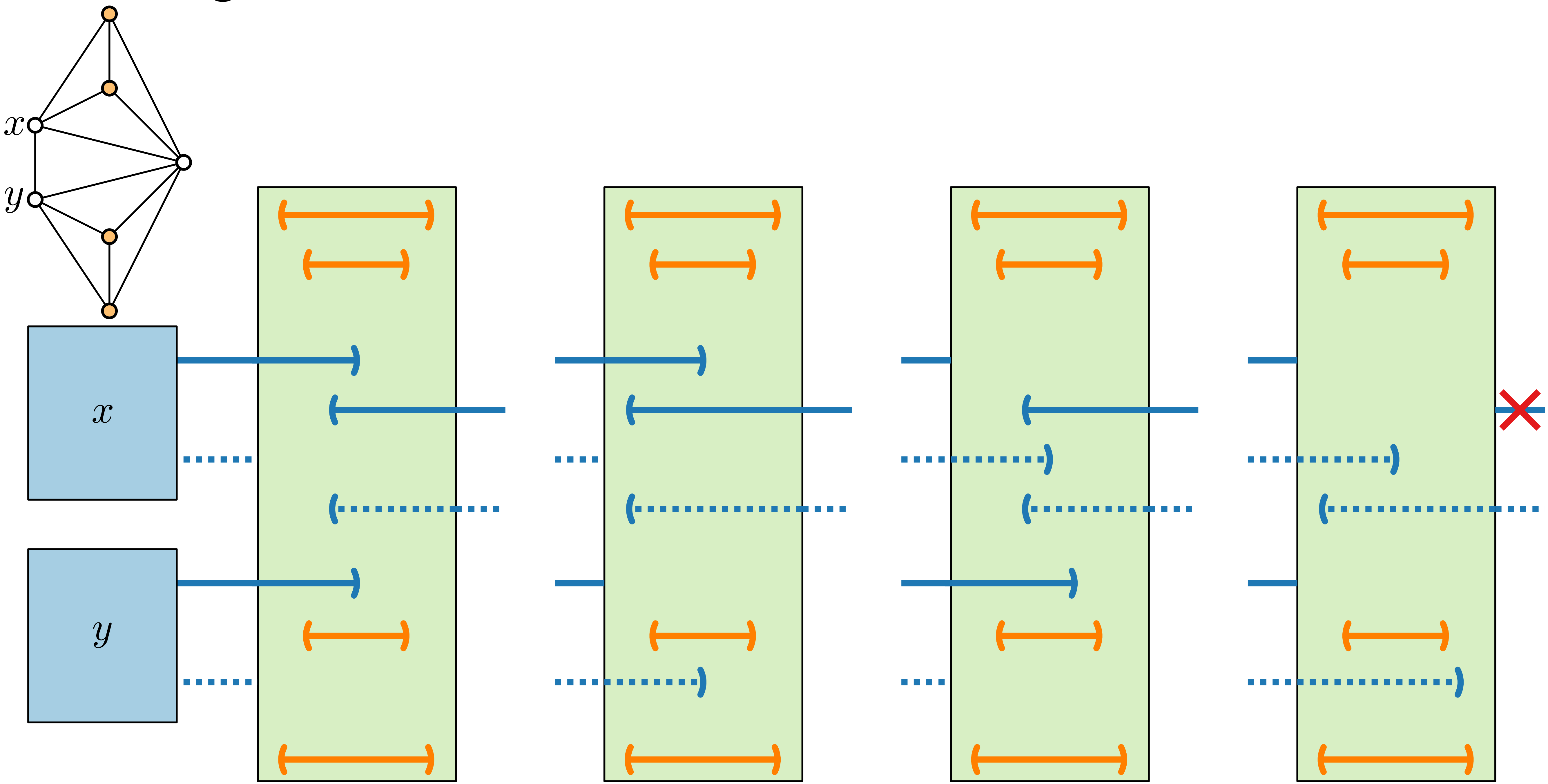

Discussion

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- Can **Strong** Bar Visibility Recognition / Representation Extension can be solved in polynomial time on st-graphs?

Literature

Main source:

- [Chaplick, Guśpiel, Gutowski, Krawczyk, Liotta '18] The Partial Visibility Representation Extension Problem

Referenced papers:

- [Gutwenger, Mutzel '01] A Linear Time Implementation of SPQR-Trees
- [Wismath '85] Characterizing bar line-of-sight graphs
- [Tamassia, Tollis '86] Algorithms for visibility representations of planar graphs
- [Andreae '92] Some results on visibility graphs
- [Chaplick, Dorbec, Kratchovíl, Montassier, Stacho '14] Contact representations of planar graphs: Extending a partial representation is hard