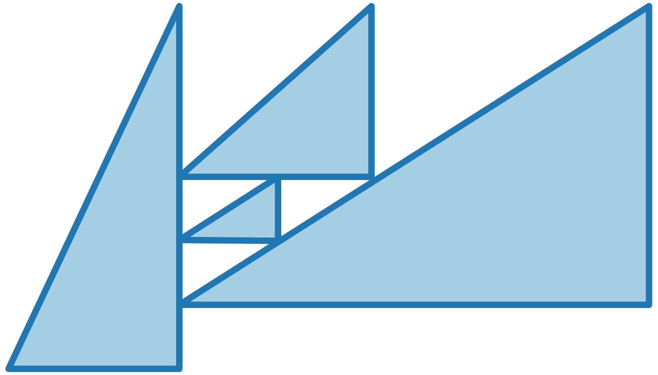
Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

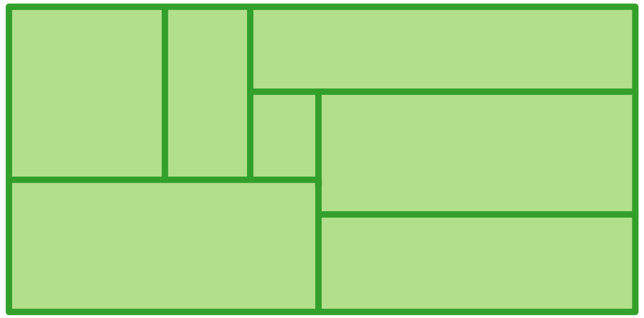
Visualization of Graphs

Lecture 8:
Conact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



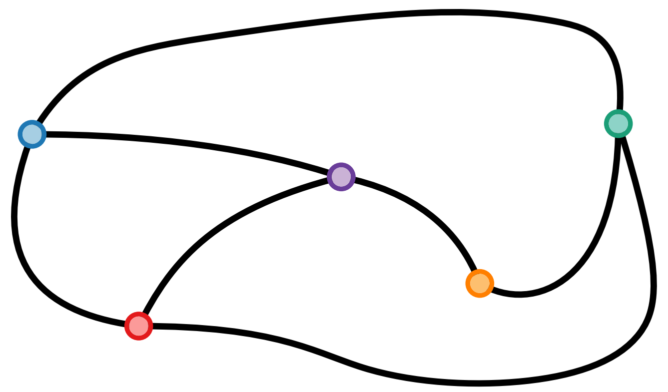
Part I:
Geometric Representations

Jonathan Klawitter



Intersection Representation

In an **intersection representation** of a graph each vertex is represented as a set



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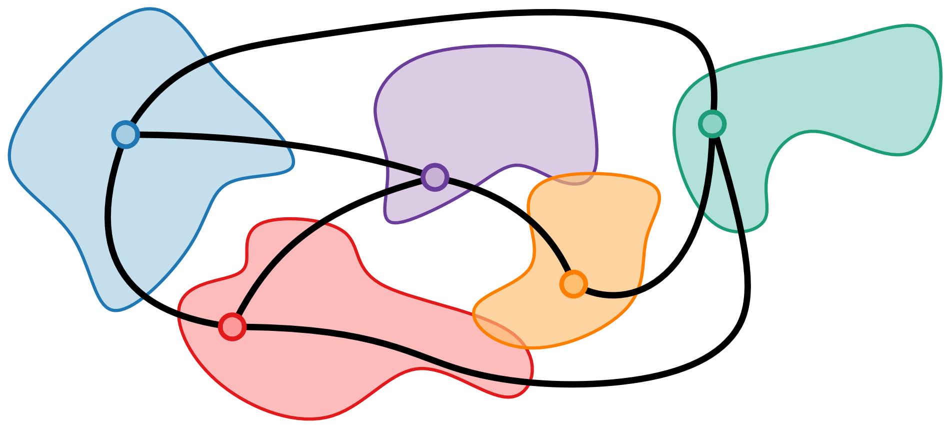

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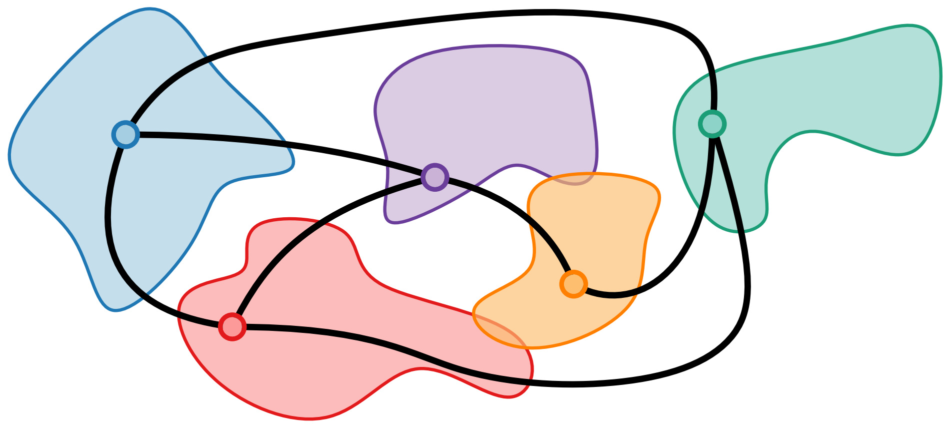
In an **intersection representation** of a graph each vertex is represented as a set such that two sets intersect if and only if the corresponding vertices are adjacent.



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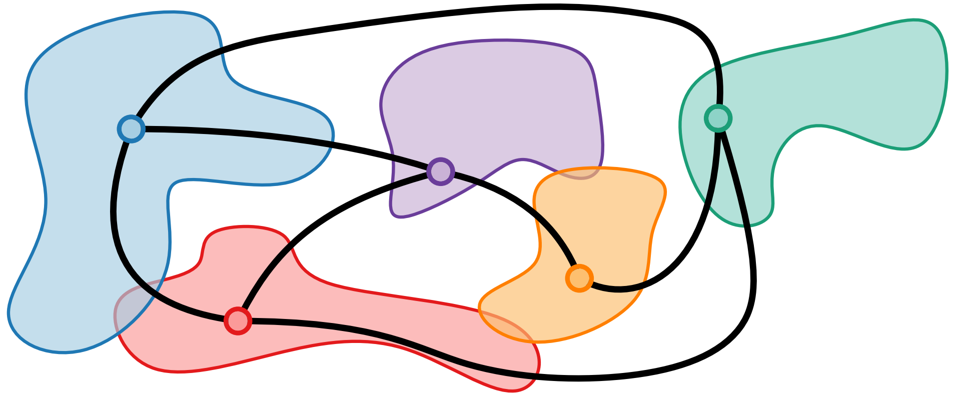

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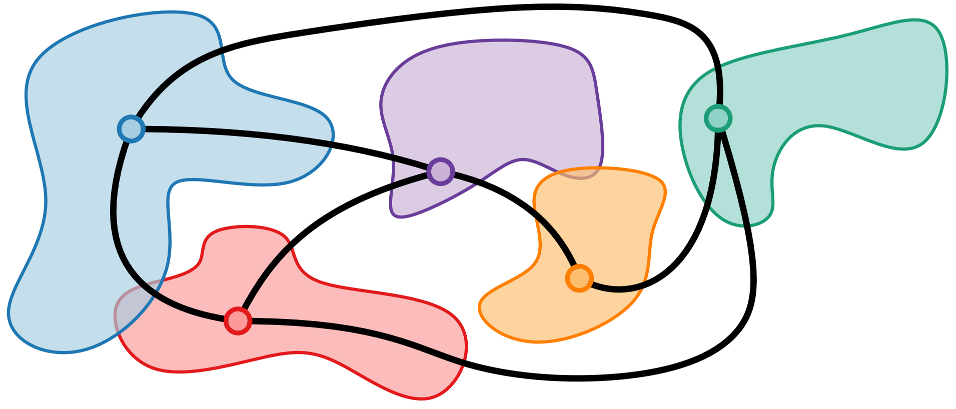

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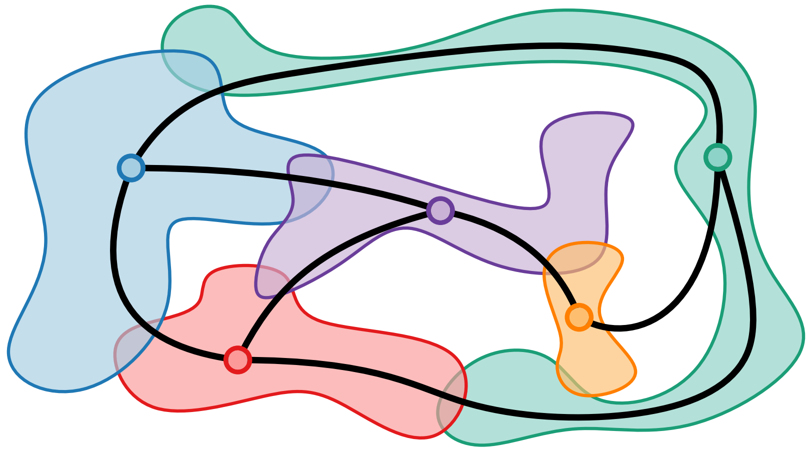

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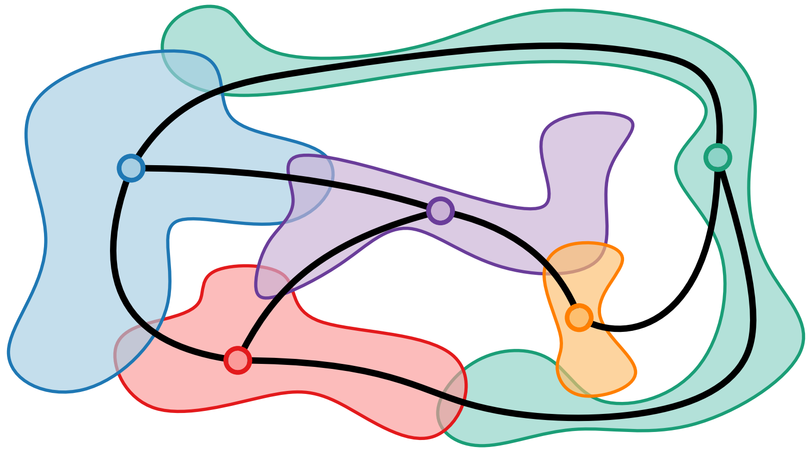
For a collection $\mathcal{S}$ of sets $S_1, \ldots, S_n$, the **intersection graph** $G(\mathcal{S})$ of $\mathcal{S}$ has vertex set $\mathcal{S}$ and edge set $\{S_iS_j : i, j \in \{1, \ldots, n\}, i \neq j,$ and $S_i \cap S_j \neq \emptyset\}$.



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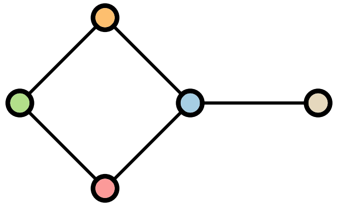

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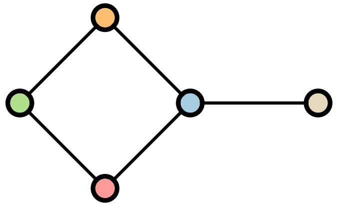

Represent each vertex v by a geometric object $S(v)$

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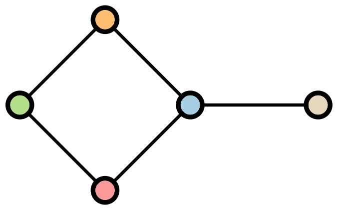

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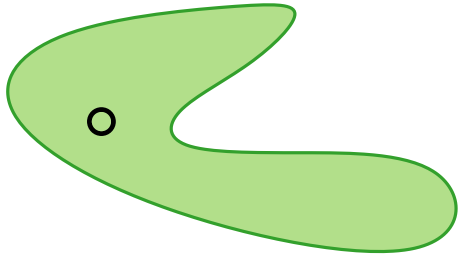

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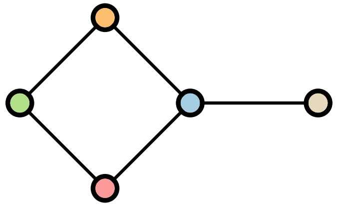

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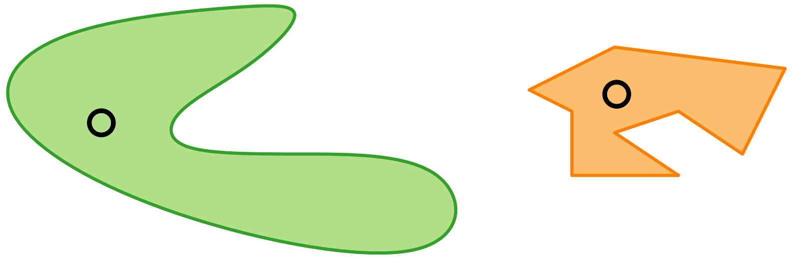

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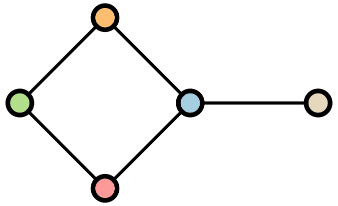

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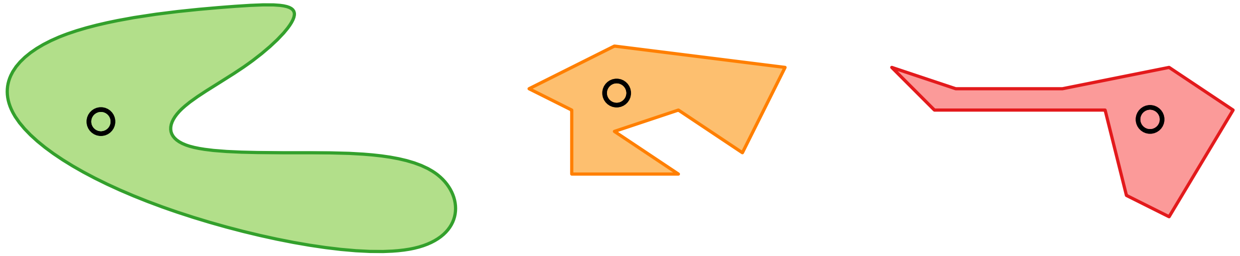

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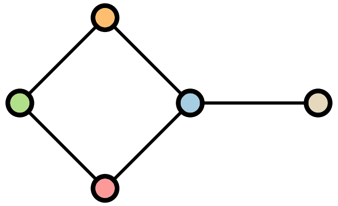

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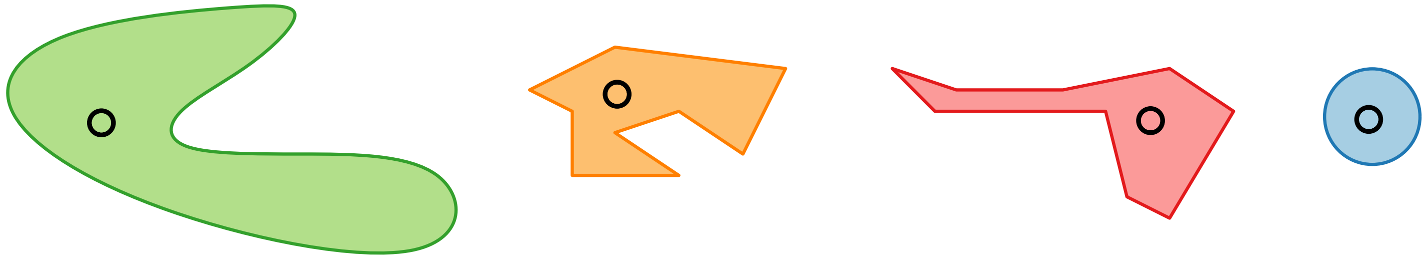

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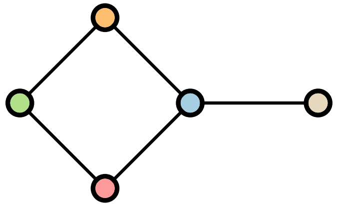

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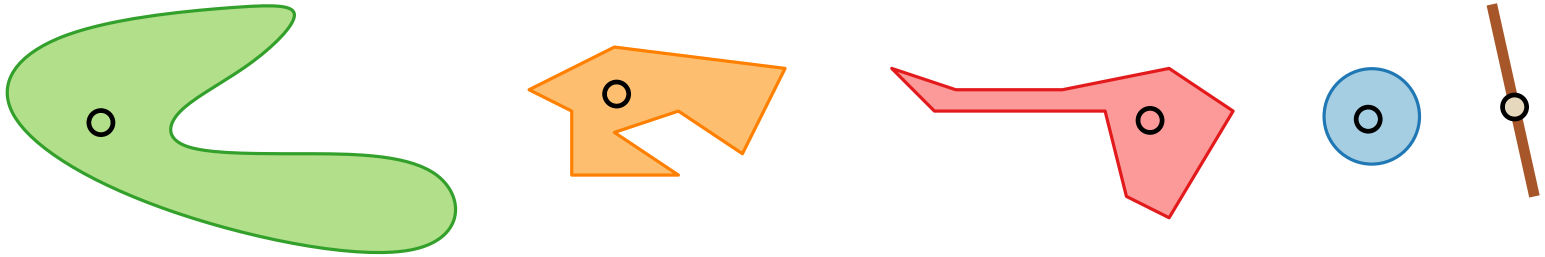

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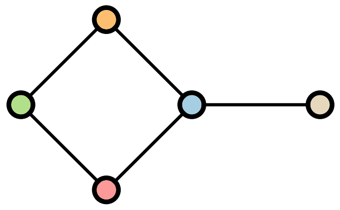

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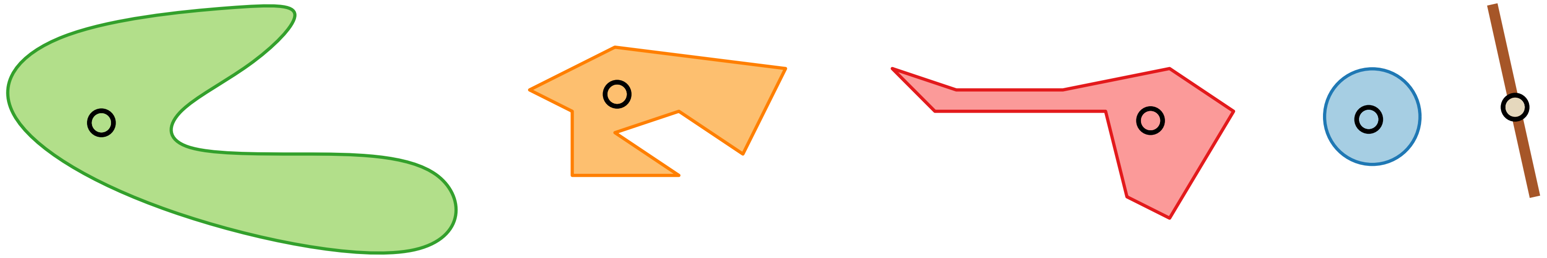

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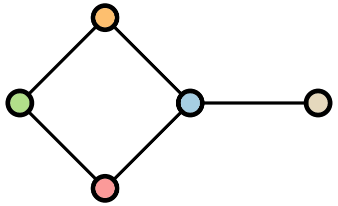

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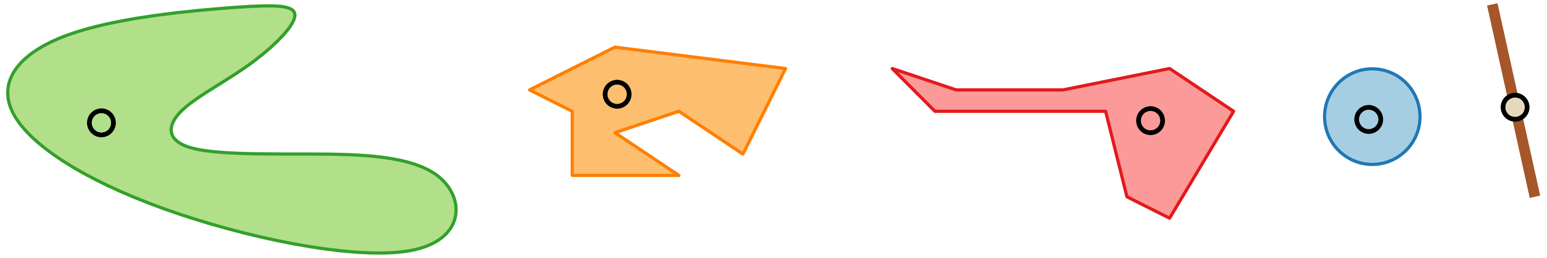

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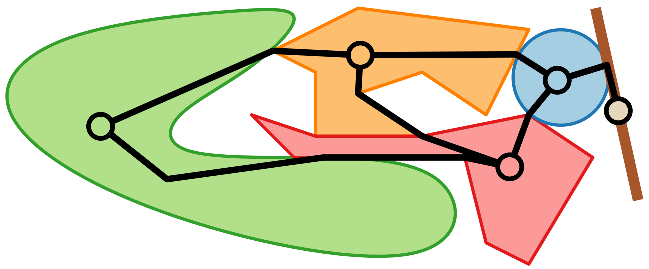

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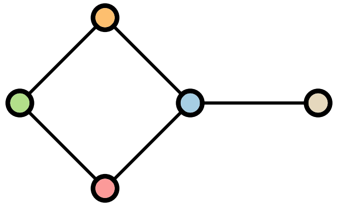

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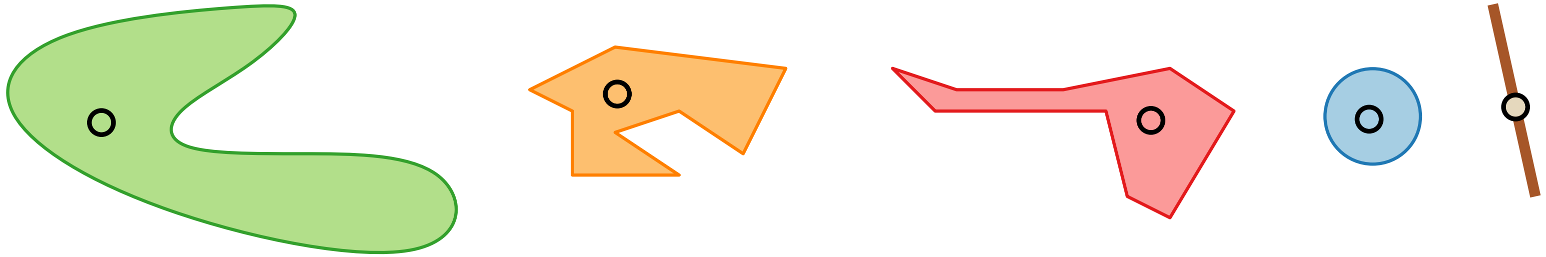

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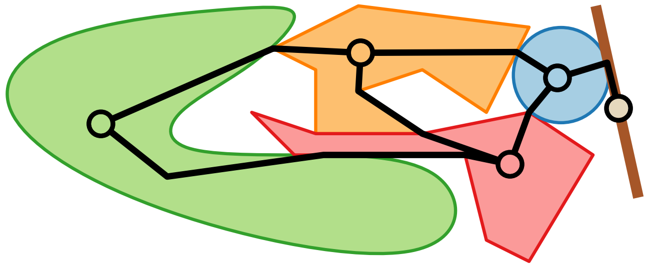

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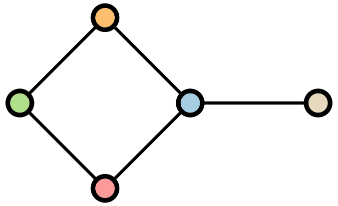

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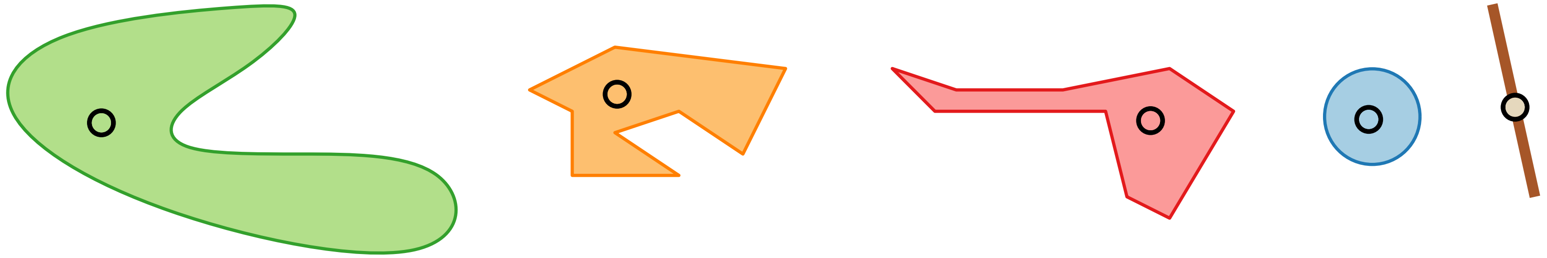

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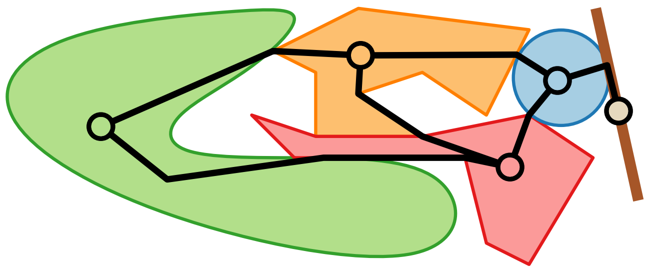

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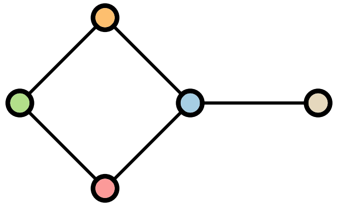

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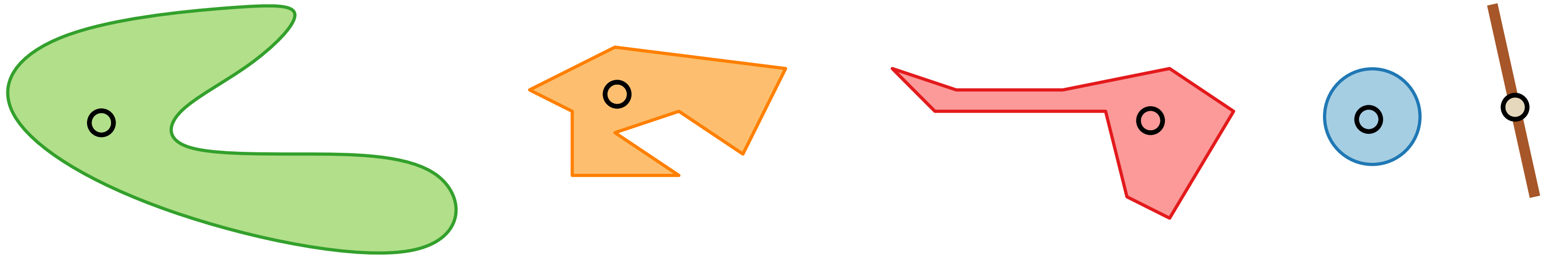

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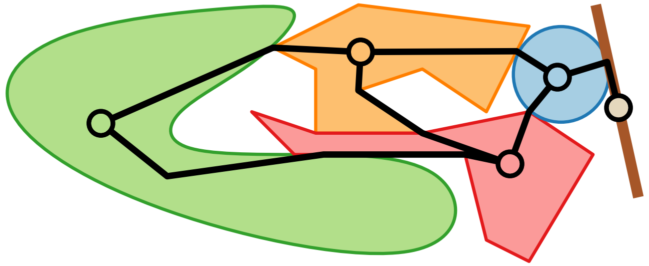

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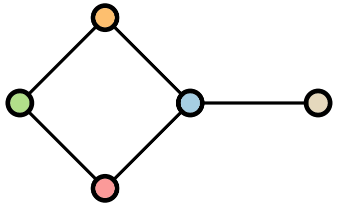

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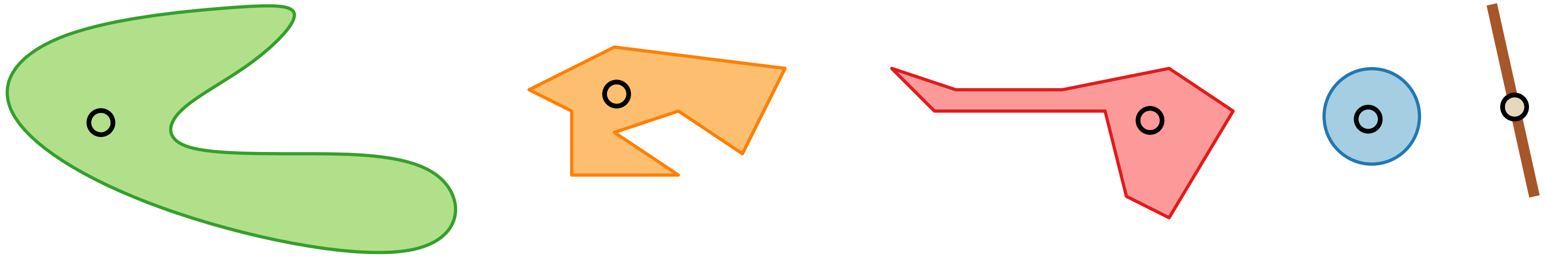

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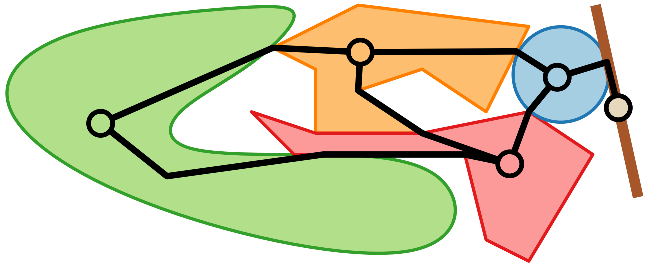

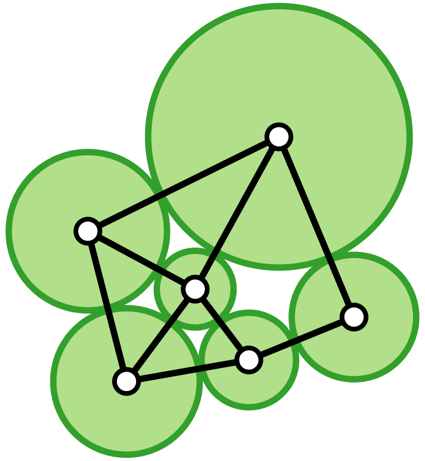

disks

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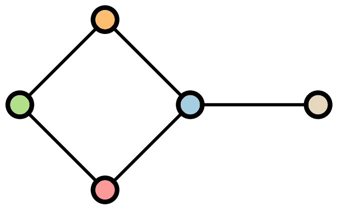
Let G be a graph.  Let $\mathcal{S}$ be a set of geometric objects

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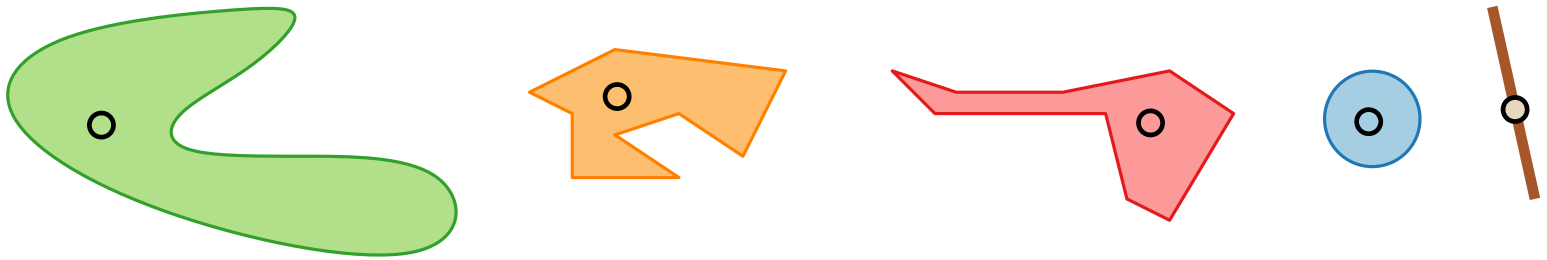

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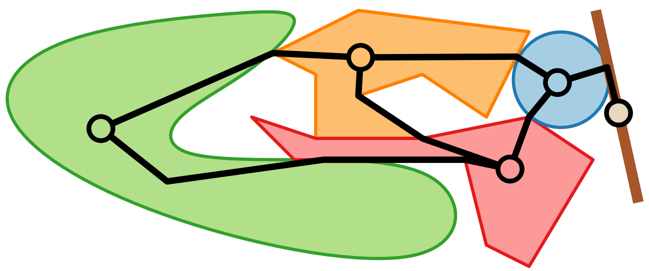

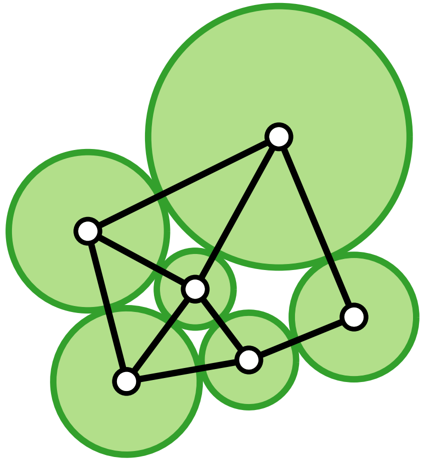

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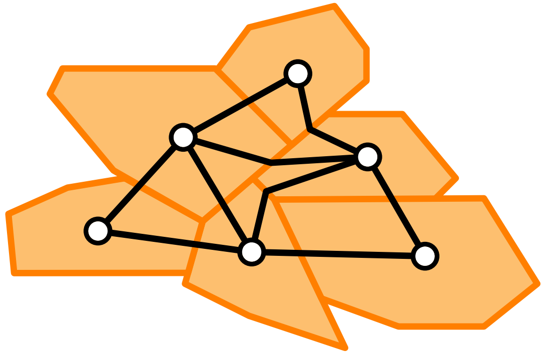

polygons

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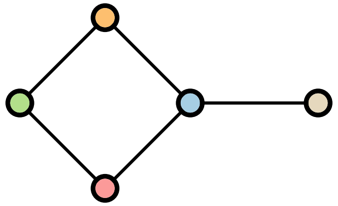

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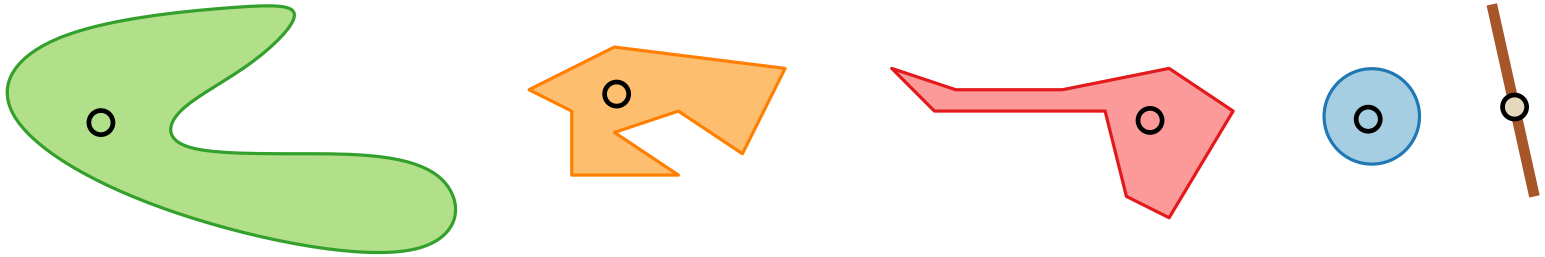
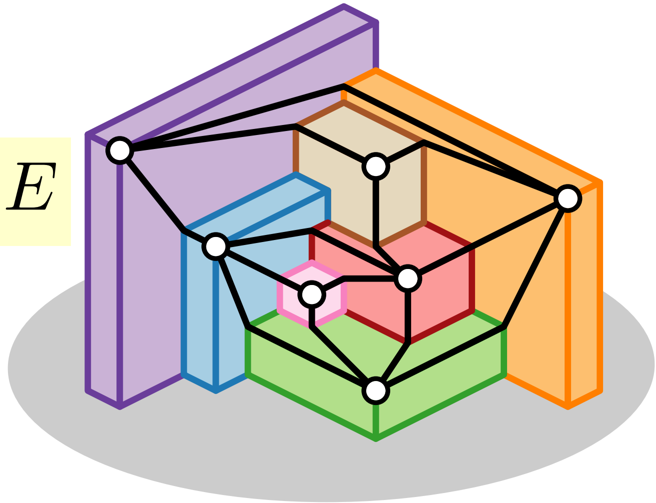


rectangular cuboids



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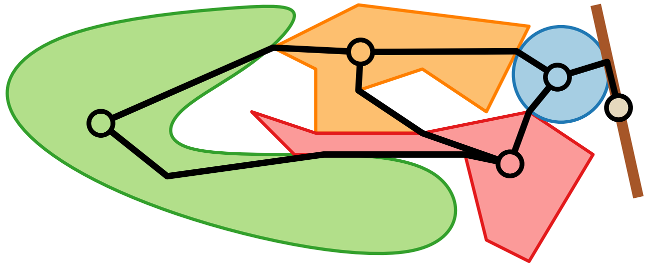

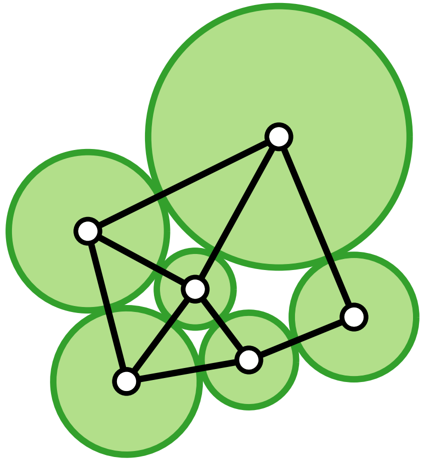

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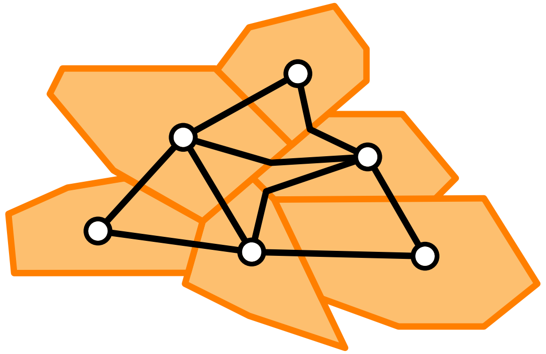

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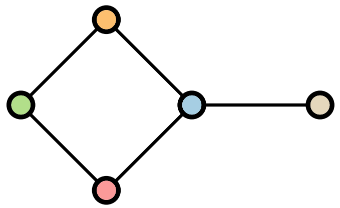

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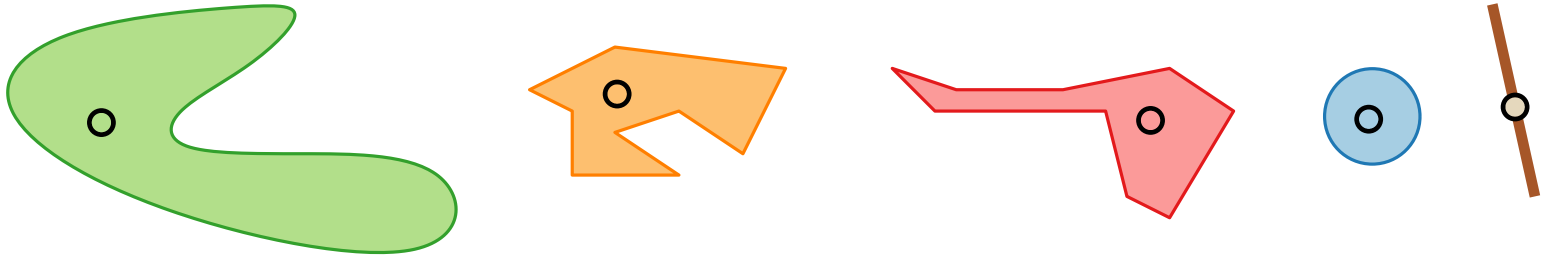
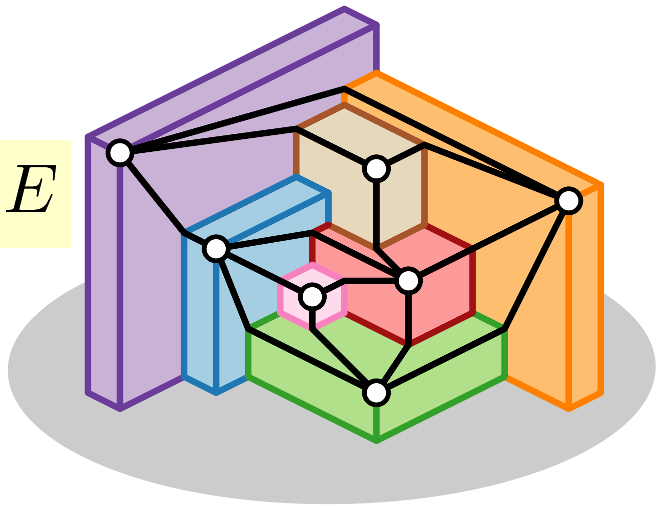


rectangular cuboids



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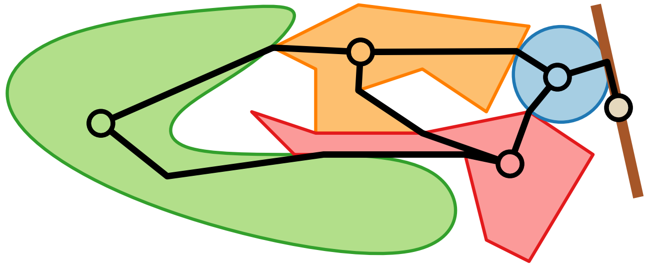
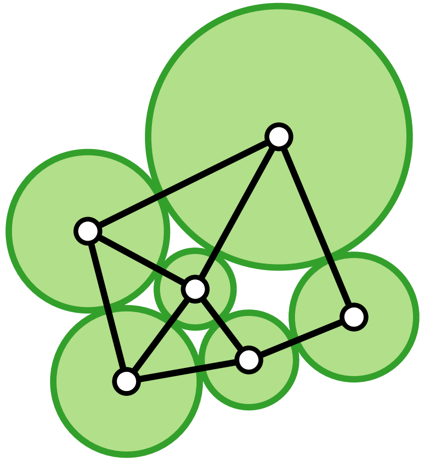


G is planar



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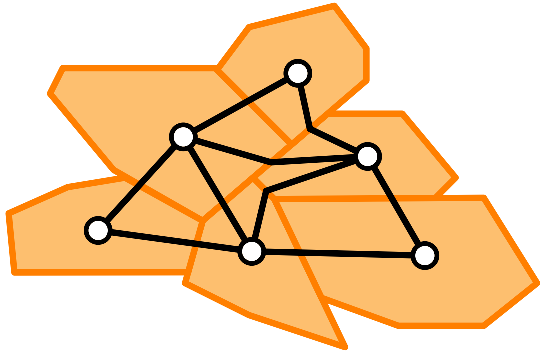

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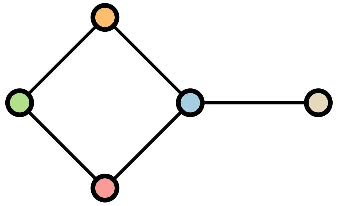

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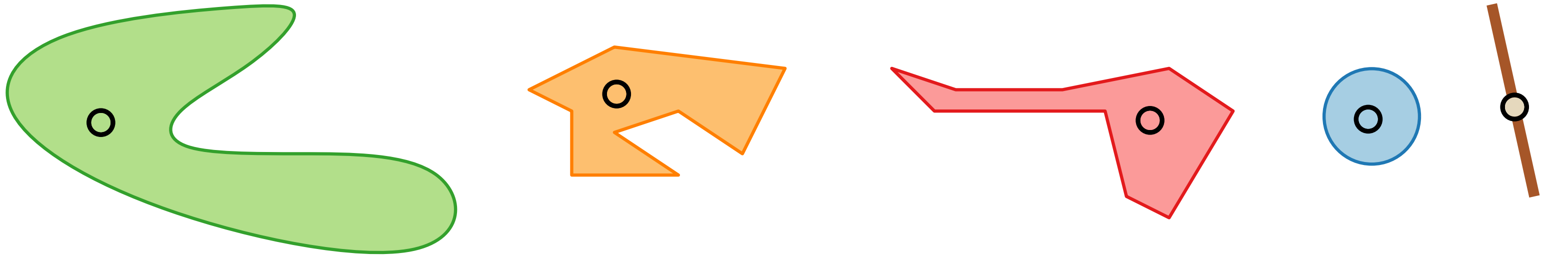

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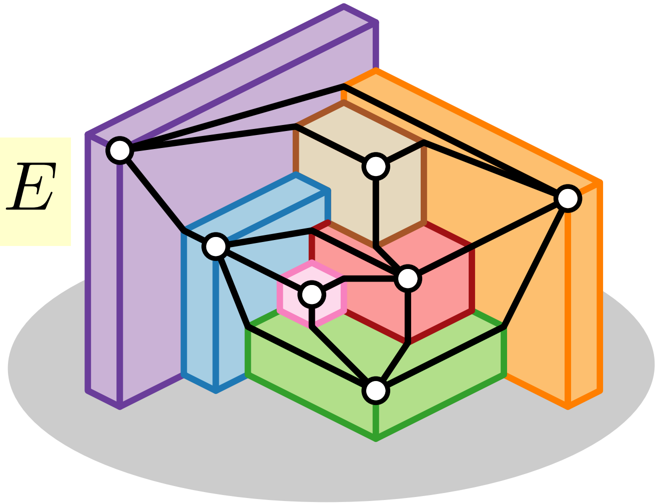

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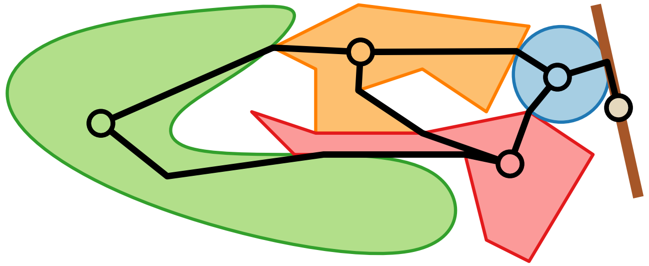

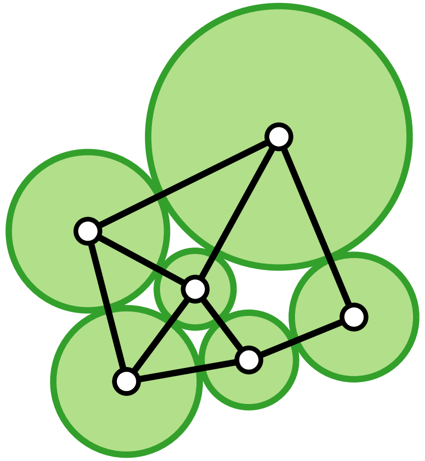

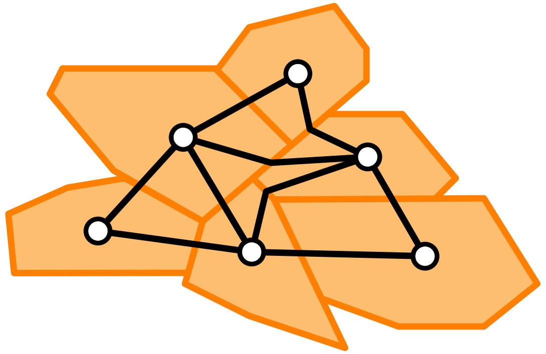

G is planar → disks [Koebe 1936]

polygons

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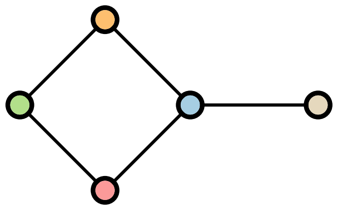

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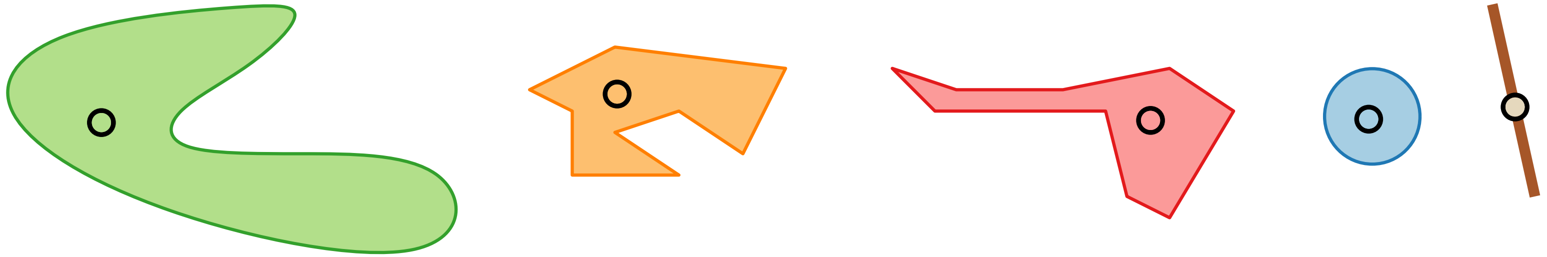

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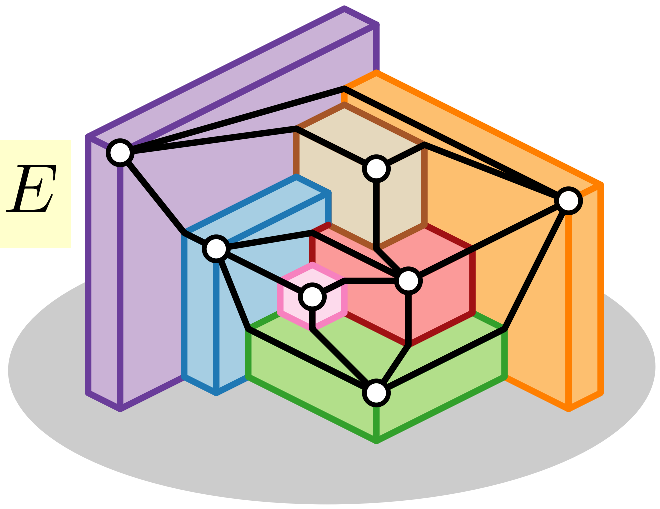

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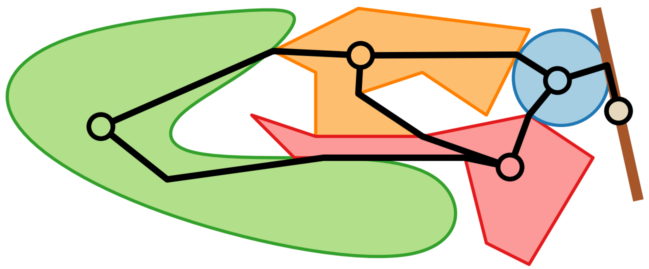
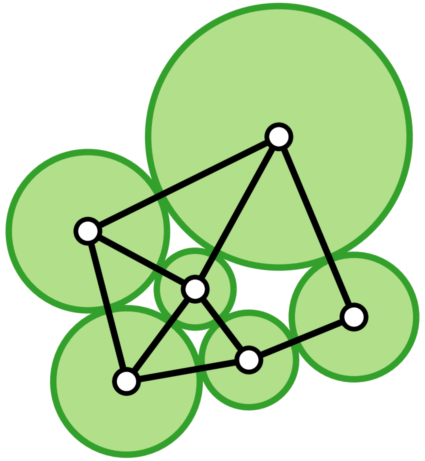
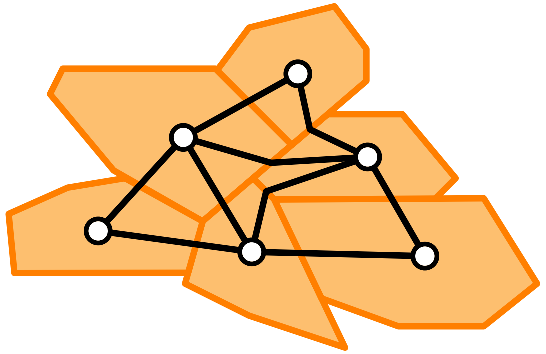






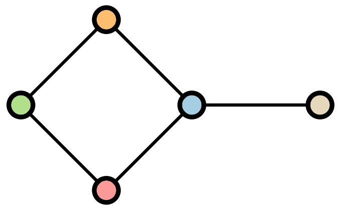
G is planar → disks → polygons

[Koebe 1936]

Contact Representation of Graphs

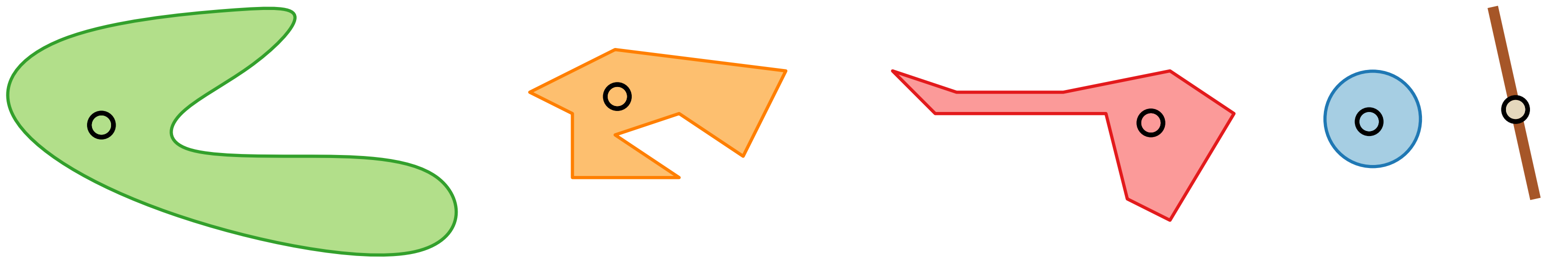
A contact representation is an intersection representation with interior-disjoint sets.

Let G be a graph.



Let $\mathcal{S}$ be a set of geometric objects

Represent each vertex v by a geometric object $S(v) \quad \in \mathcal{S}$



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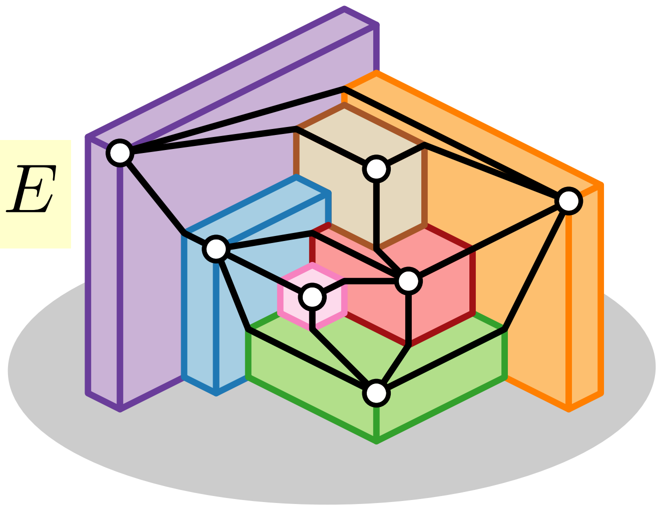

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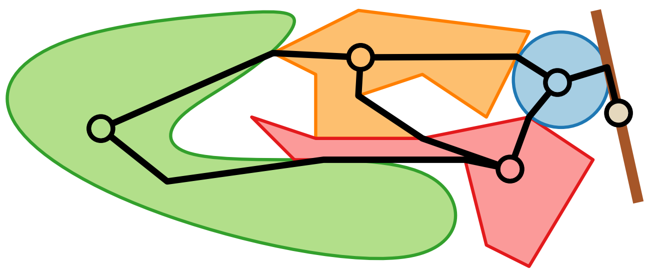

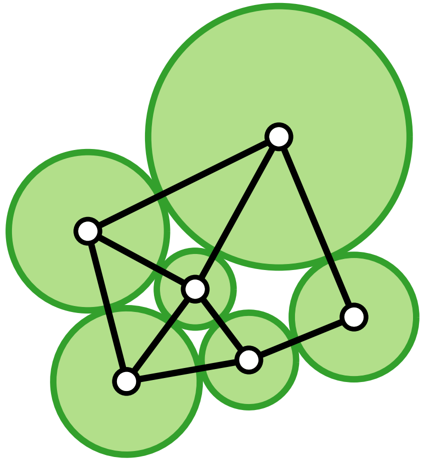

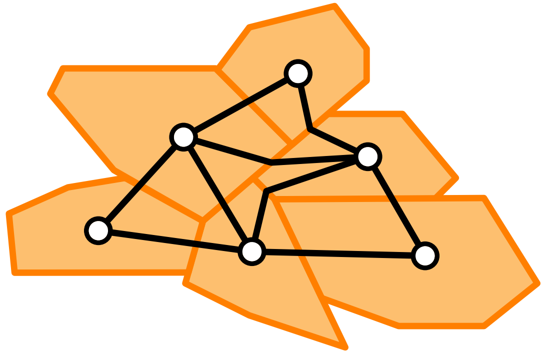

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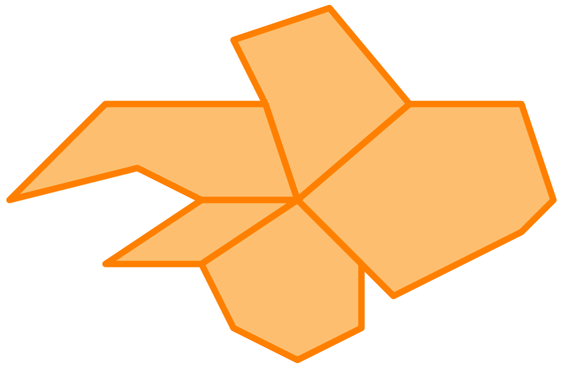

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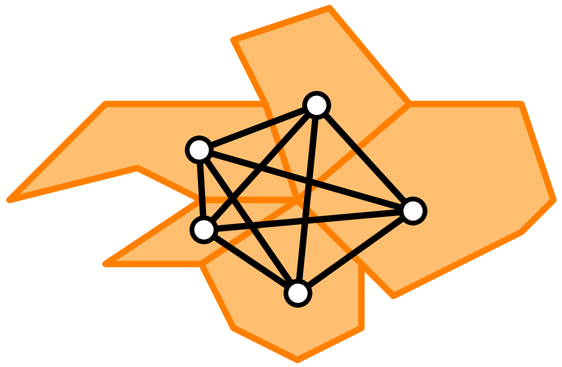

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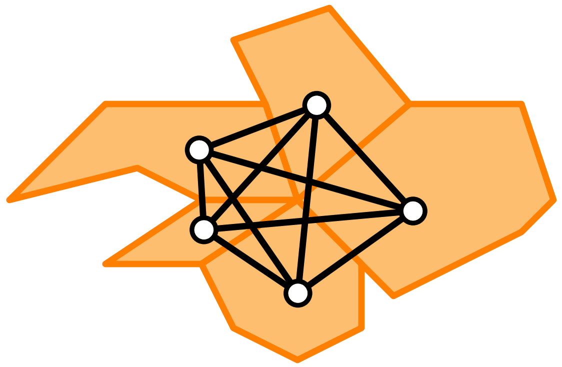

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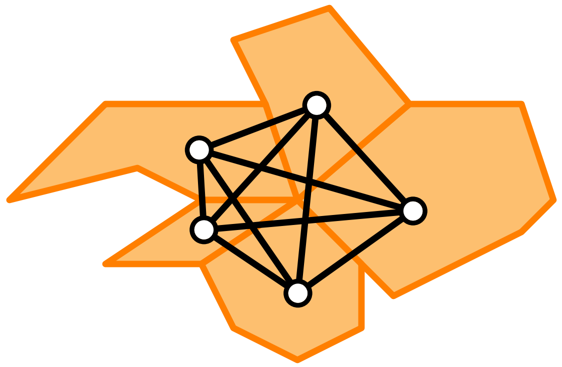

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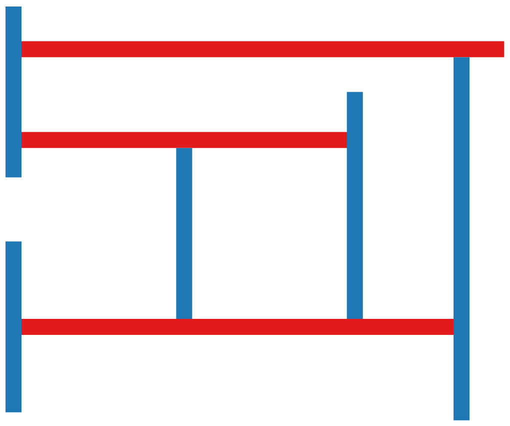

bipartite graphs

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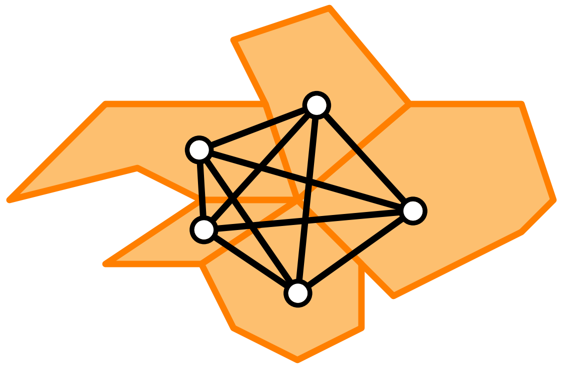

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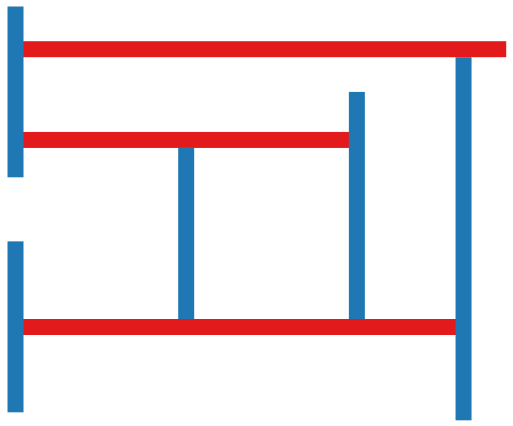

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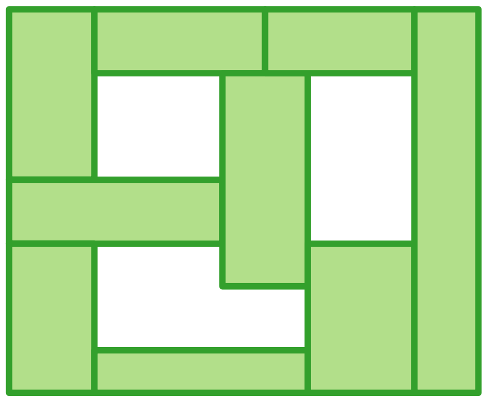

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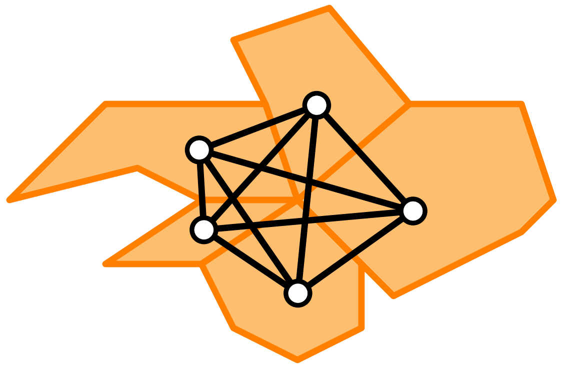

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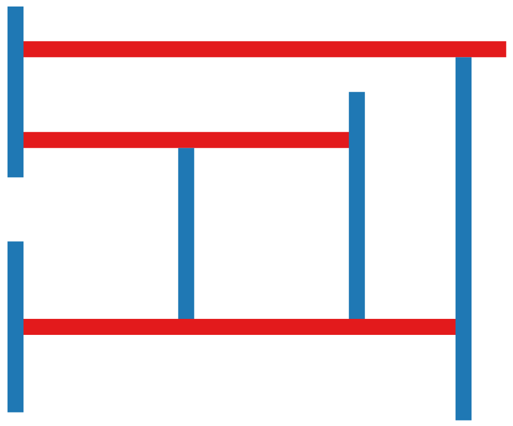

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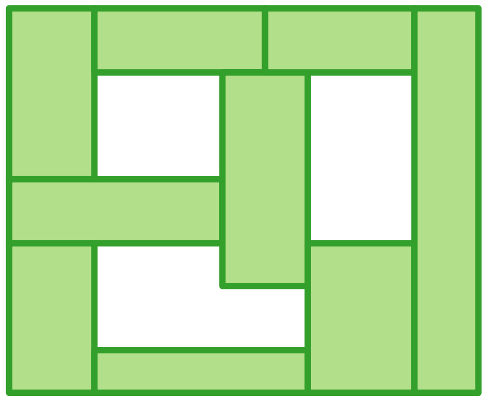

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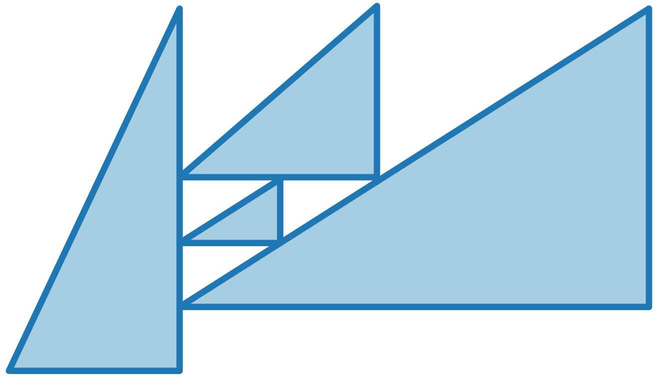

planar triangulations

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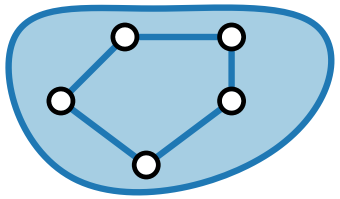

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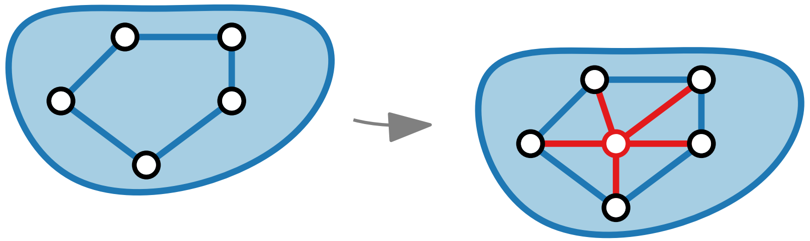

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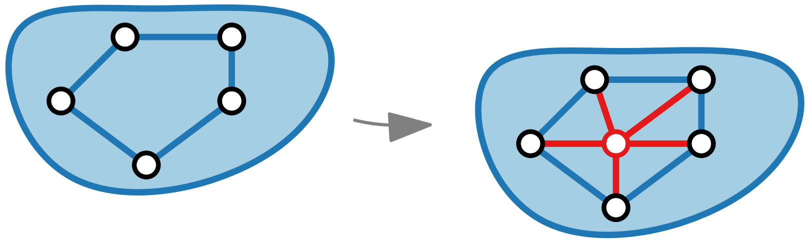

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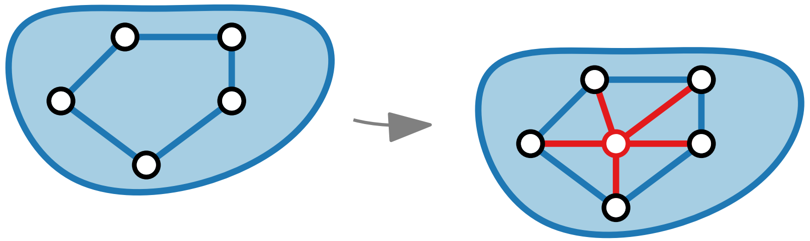

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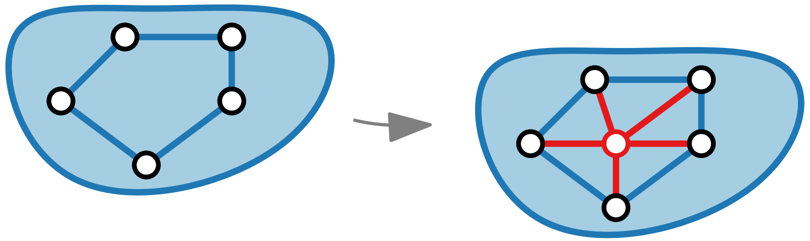

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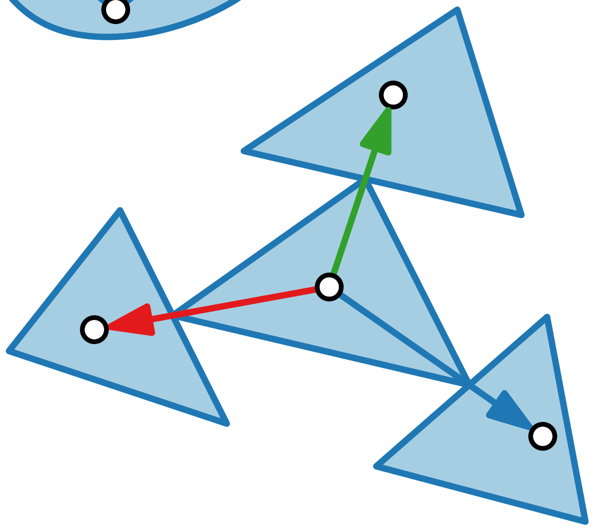

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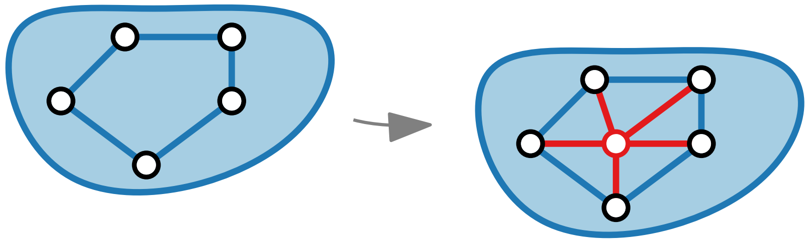

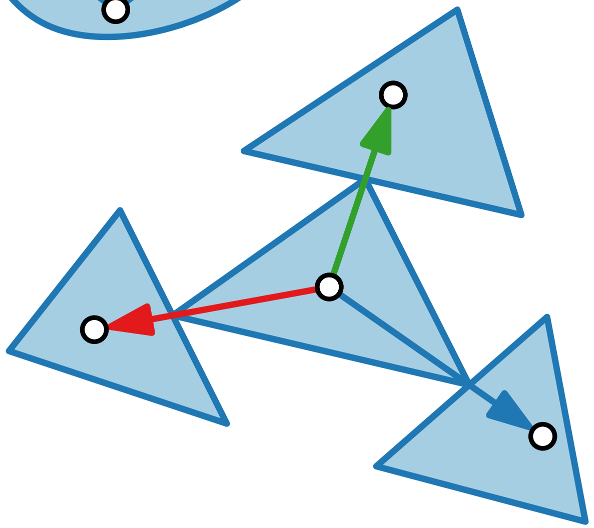

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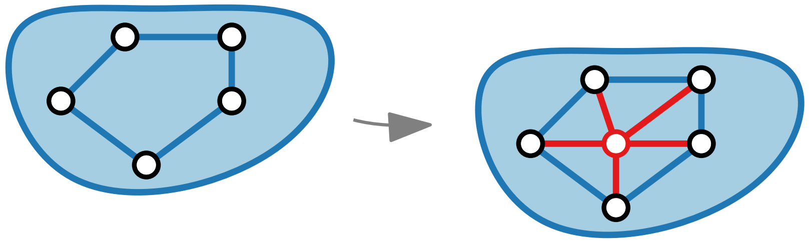

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  - Which objects contact each other in which way?
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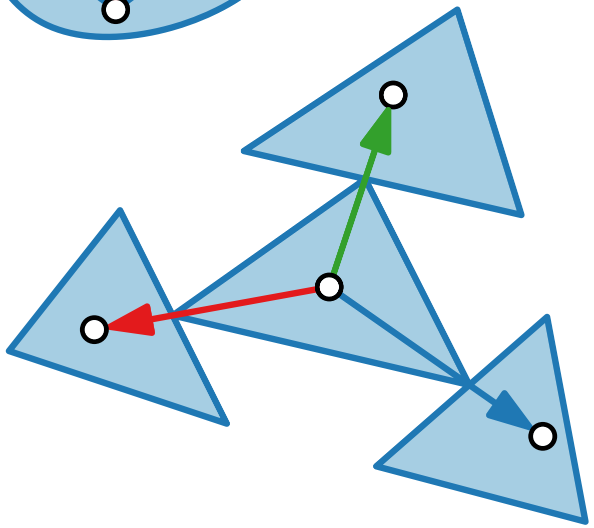

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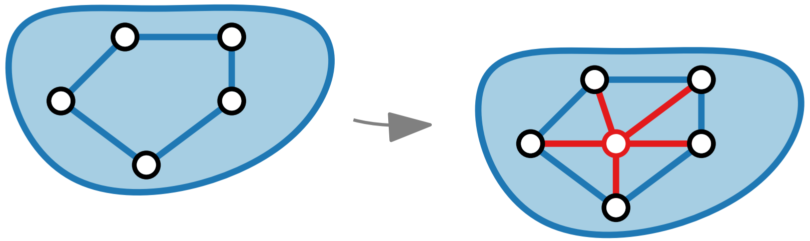
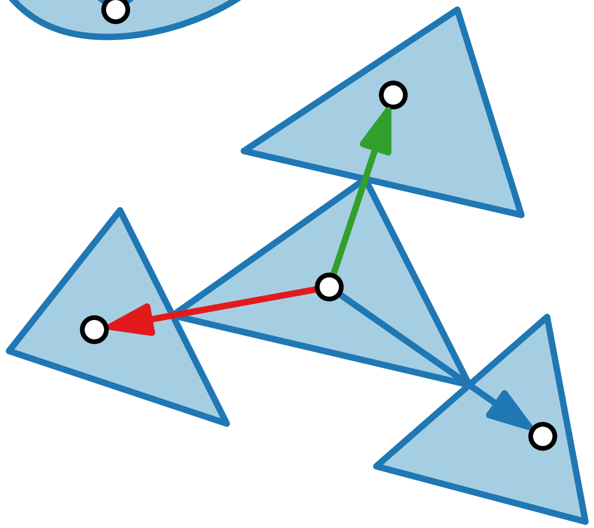
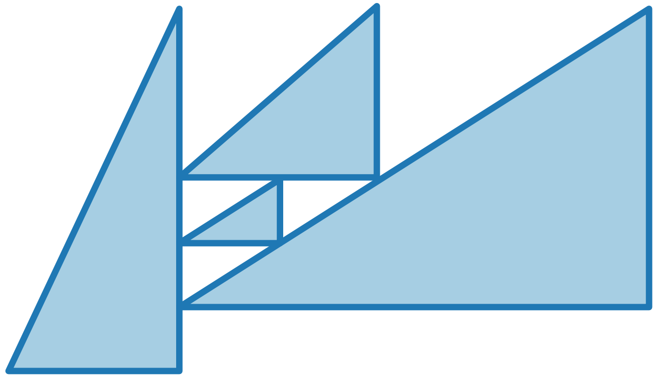
- Compute combinatorical description.
- Show that combinatorical description can be used to construct drawing.





In This Lecture

Representations with right-triangles and corner contact



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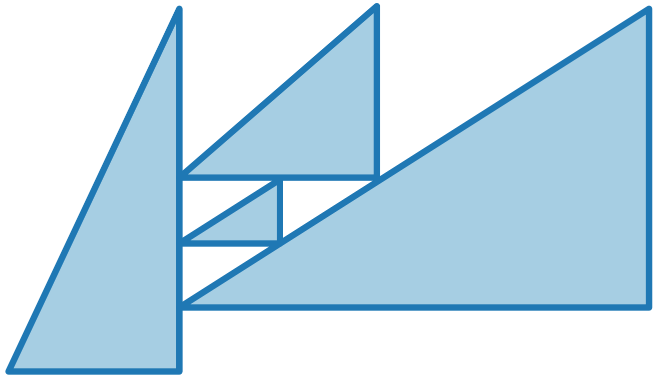
Representations with right-triangles and corner contact

- Use Schnyder realizer to describe contacts between triangles



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- Use Schnyder realizer to describe contacts between triangles
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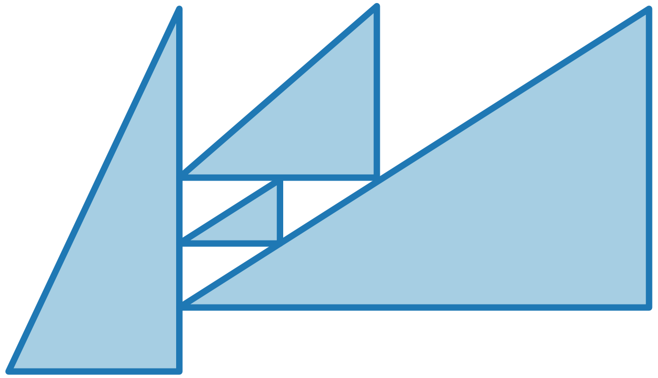

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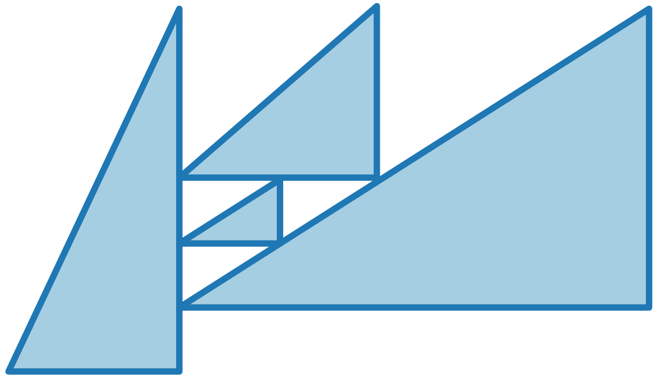
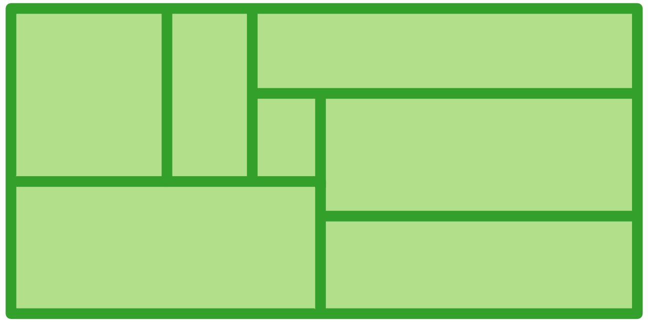


Representation with dissection of a rectangle, called **rectangular dual**



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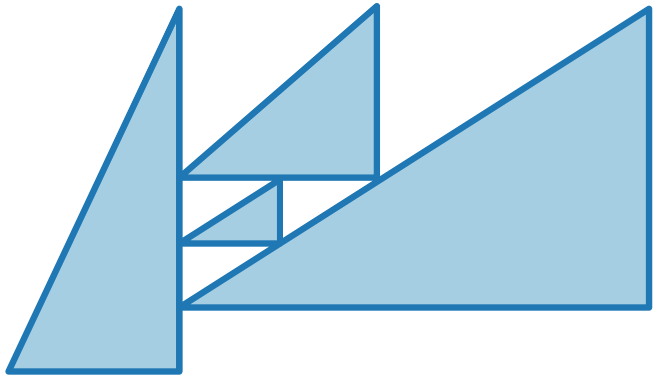

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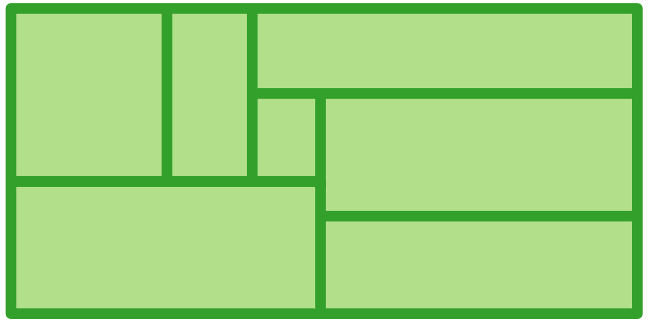

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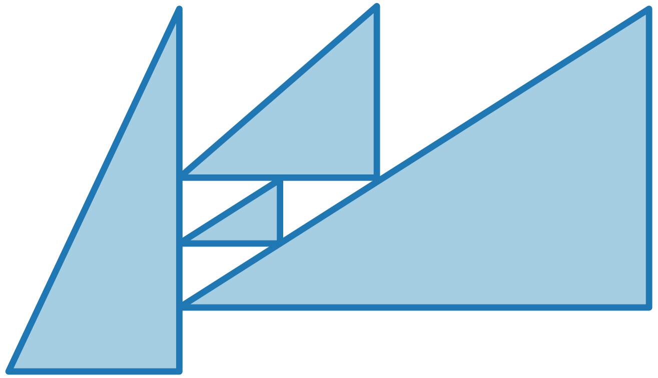
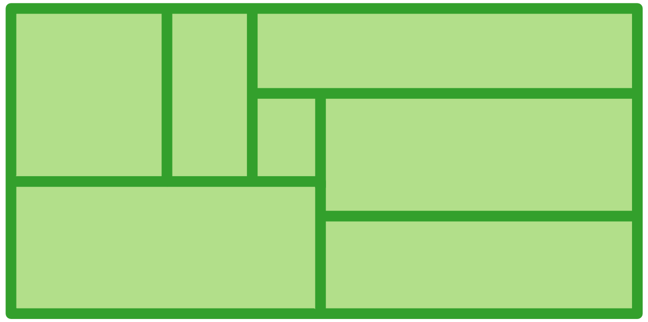


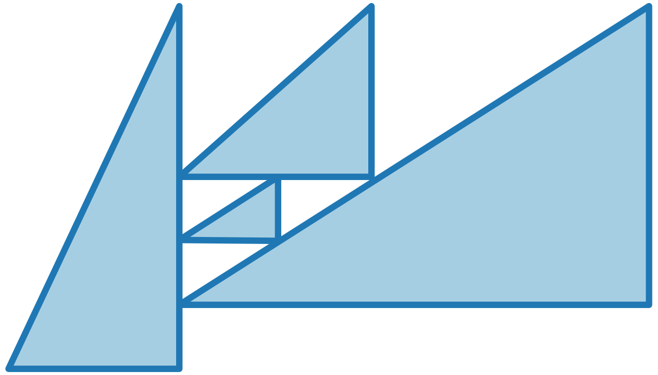
Representation with dissection of a rectangle, called **rectangular dual**

- Find similar description like Schnyder realizer for rectangles
- Construct drawing via st-digraphs, duals, and topological sorting



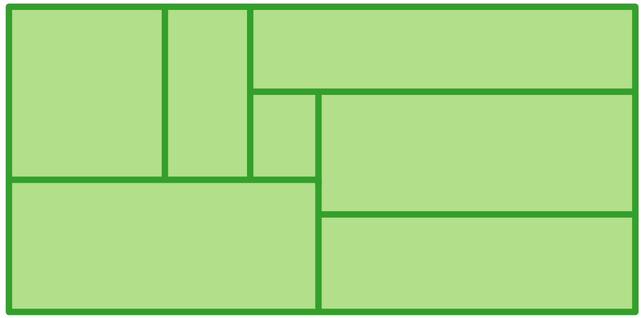
Visualization of Graphs

Lecture 8:
Conact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



Part II:
Triangle Contact Representations

Jonathan Klawitter



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Idea.
Use canonical order and Schnyder realizer to find coordinates for triangles.

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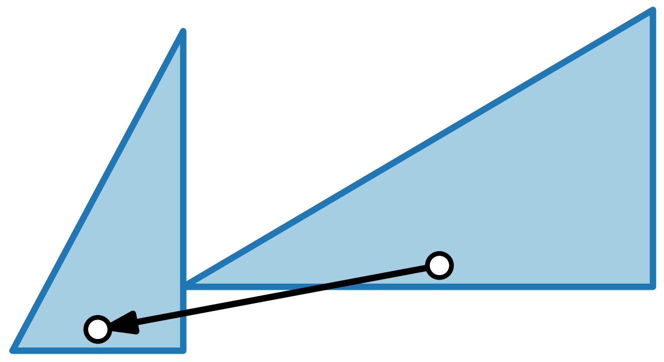

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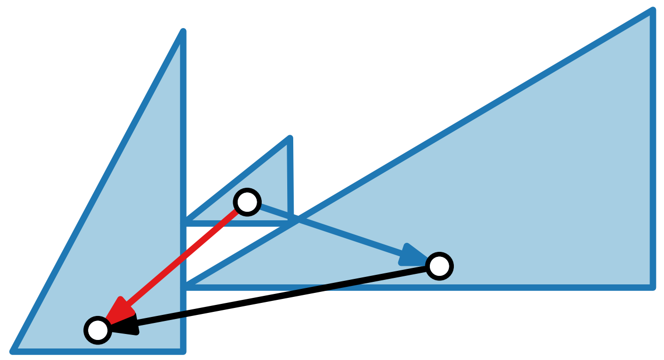

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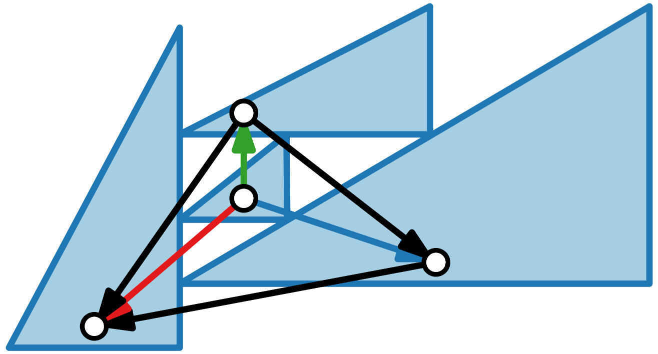

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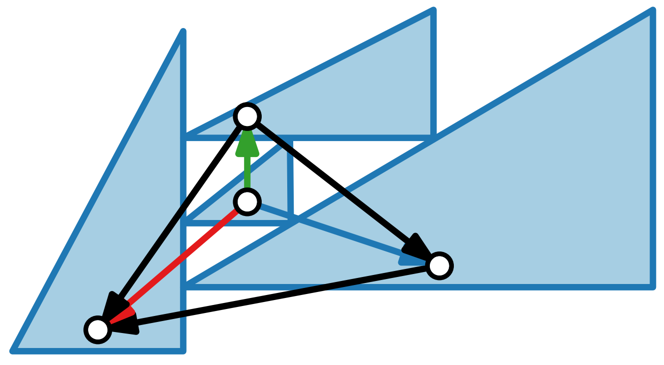

Observation.

- Can set base of triangle at height equal to position in canonical order.

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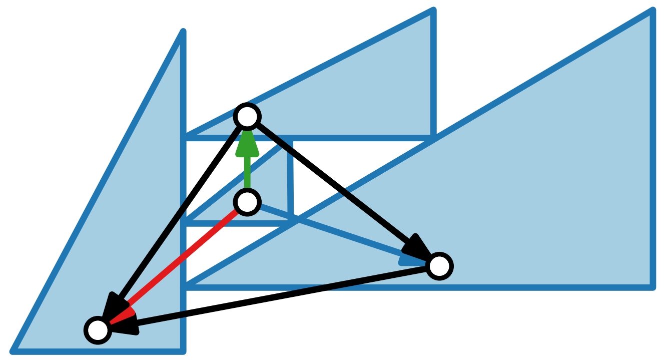

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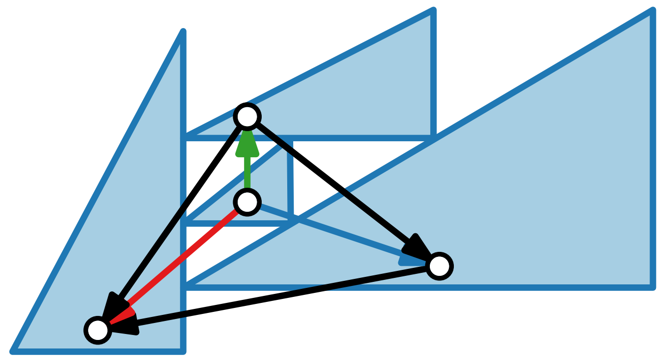

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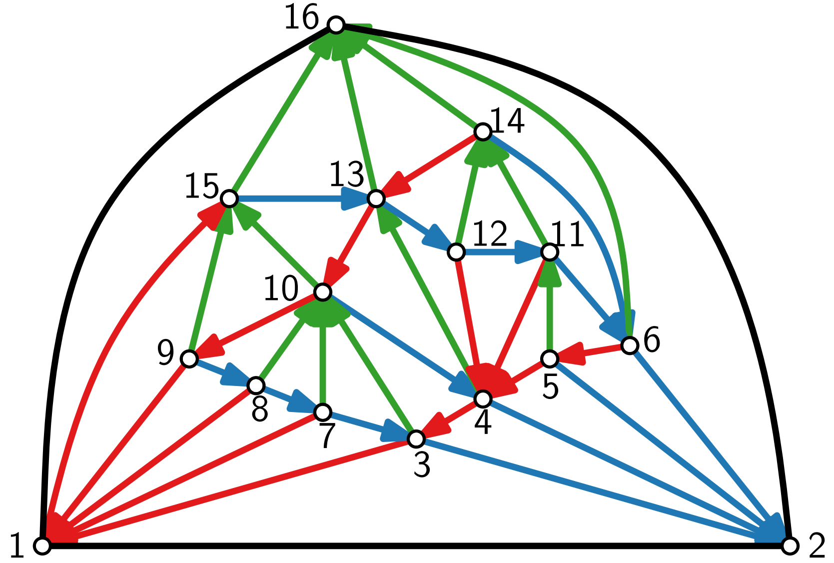
- Outgoing edges in Schnyder forest indicate corner contacts.

Triangle Contact Representation Example



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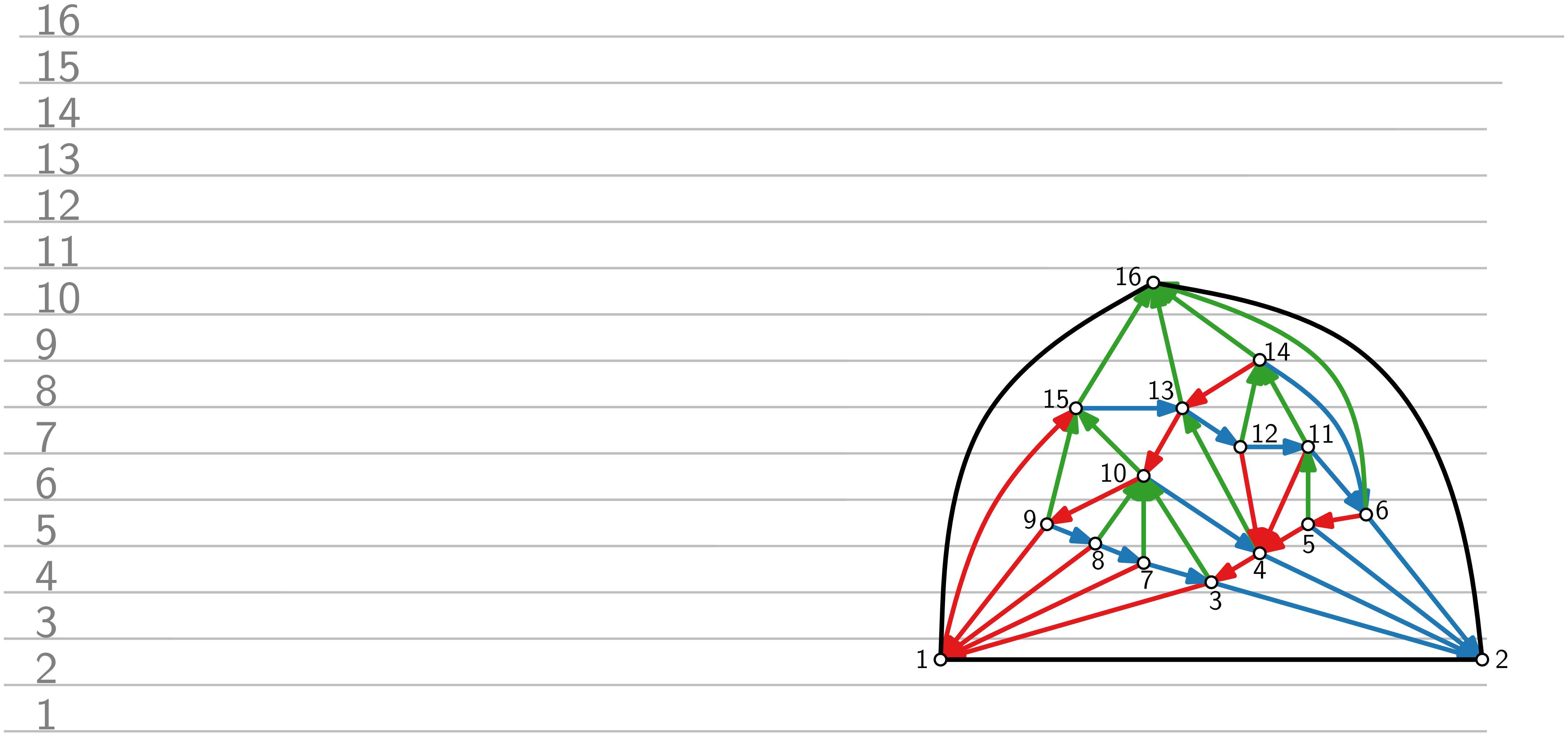

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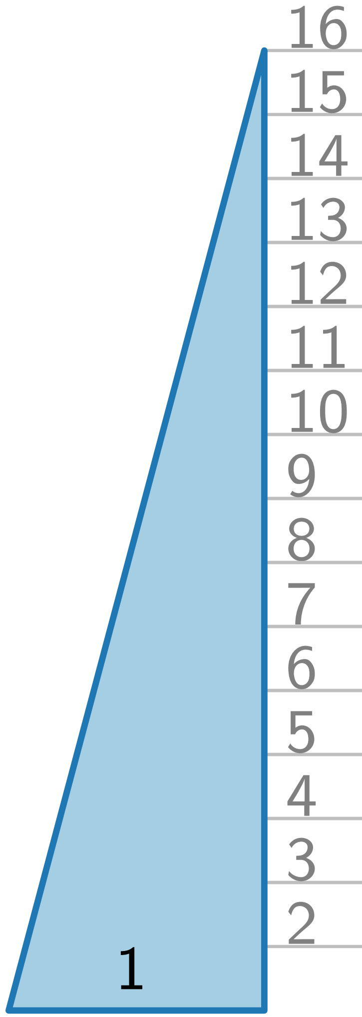

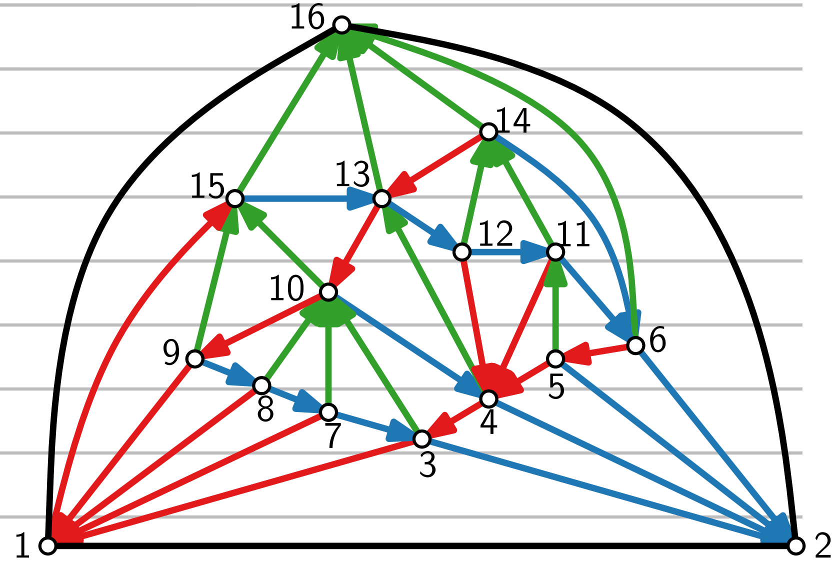

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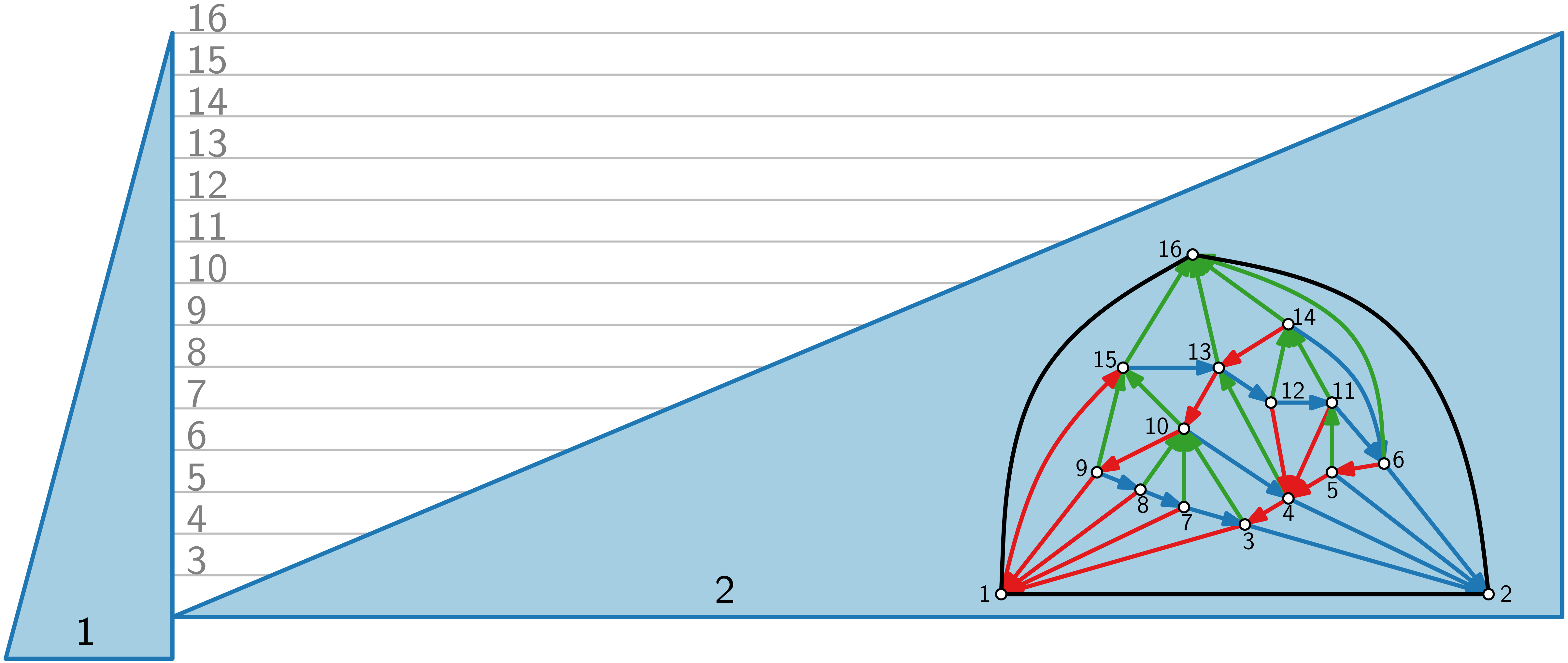

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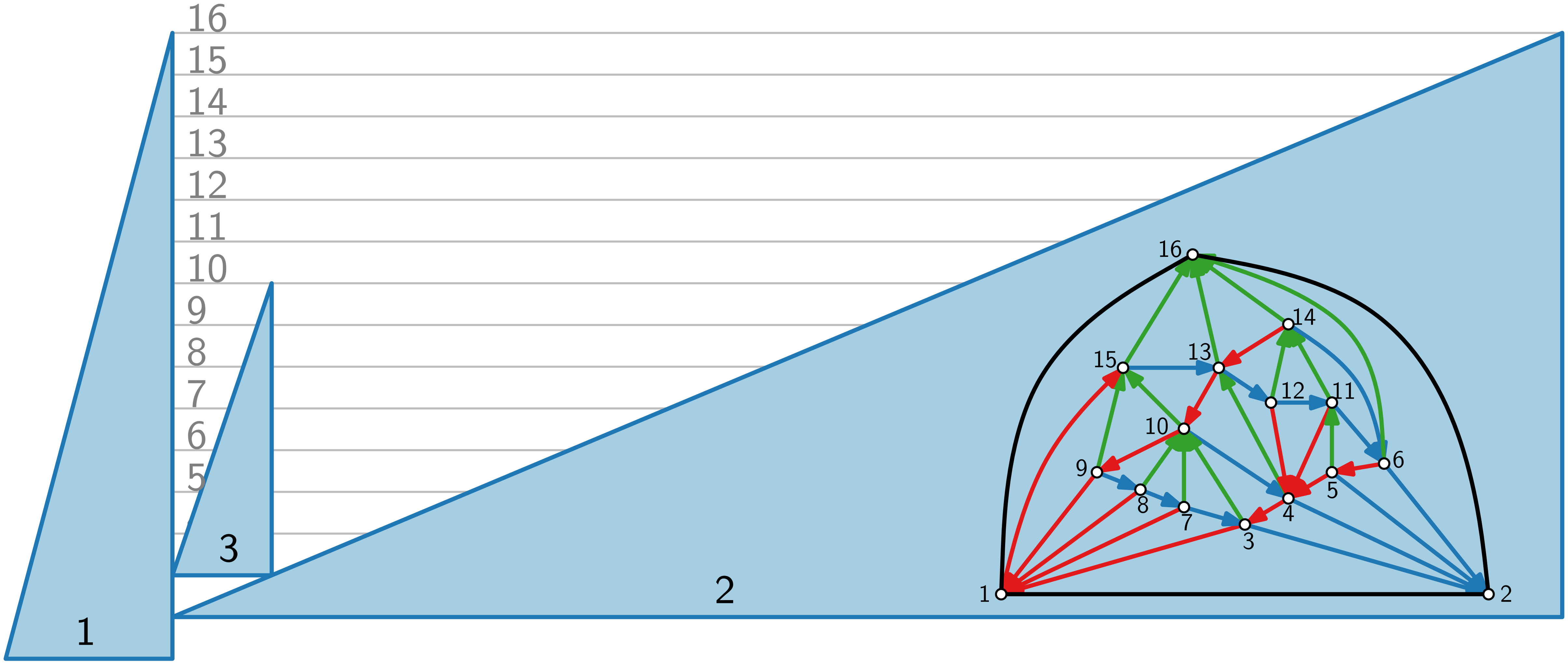

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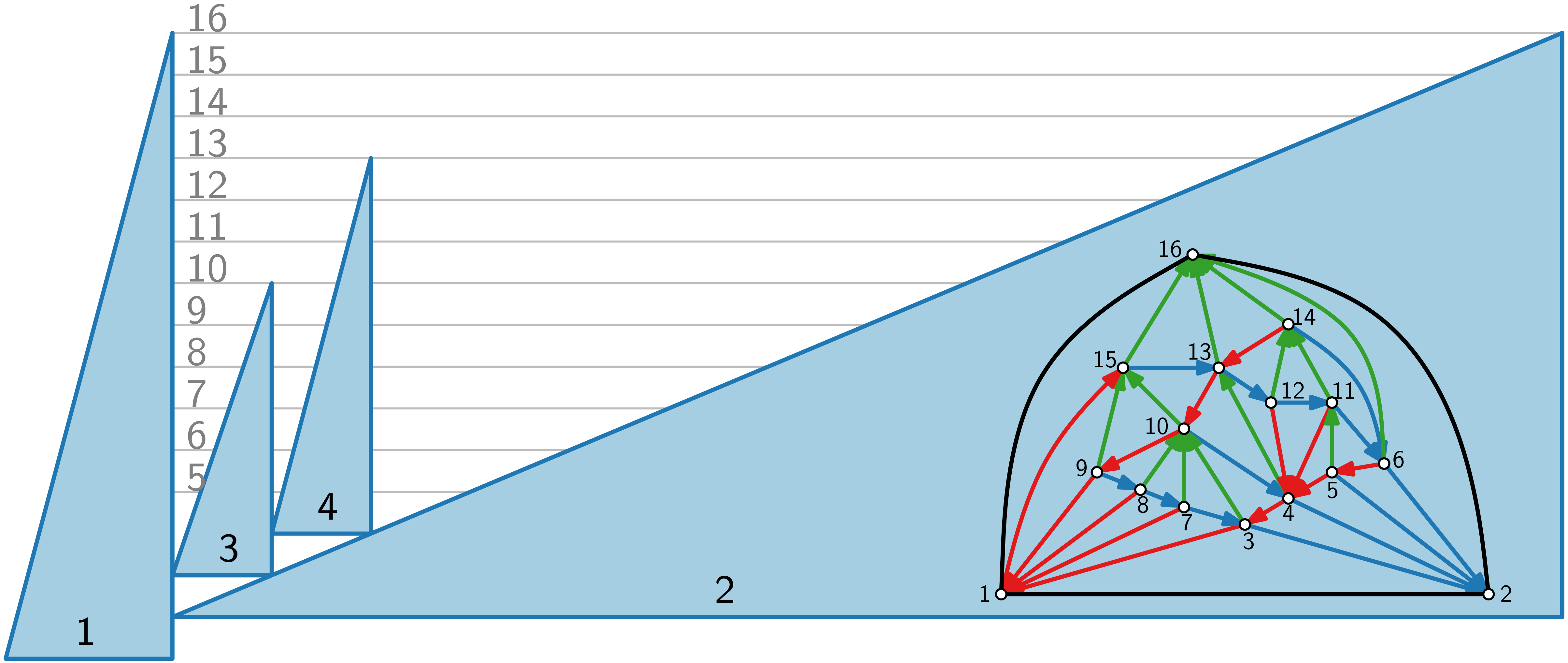
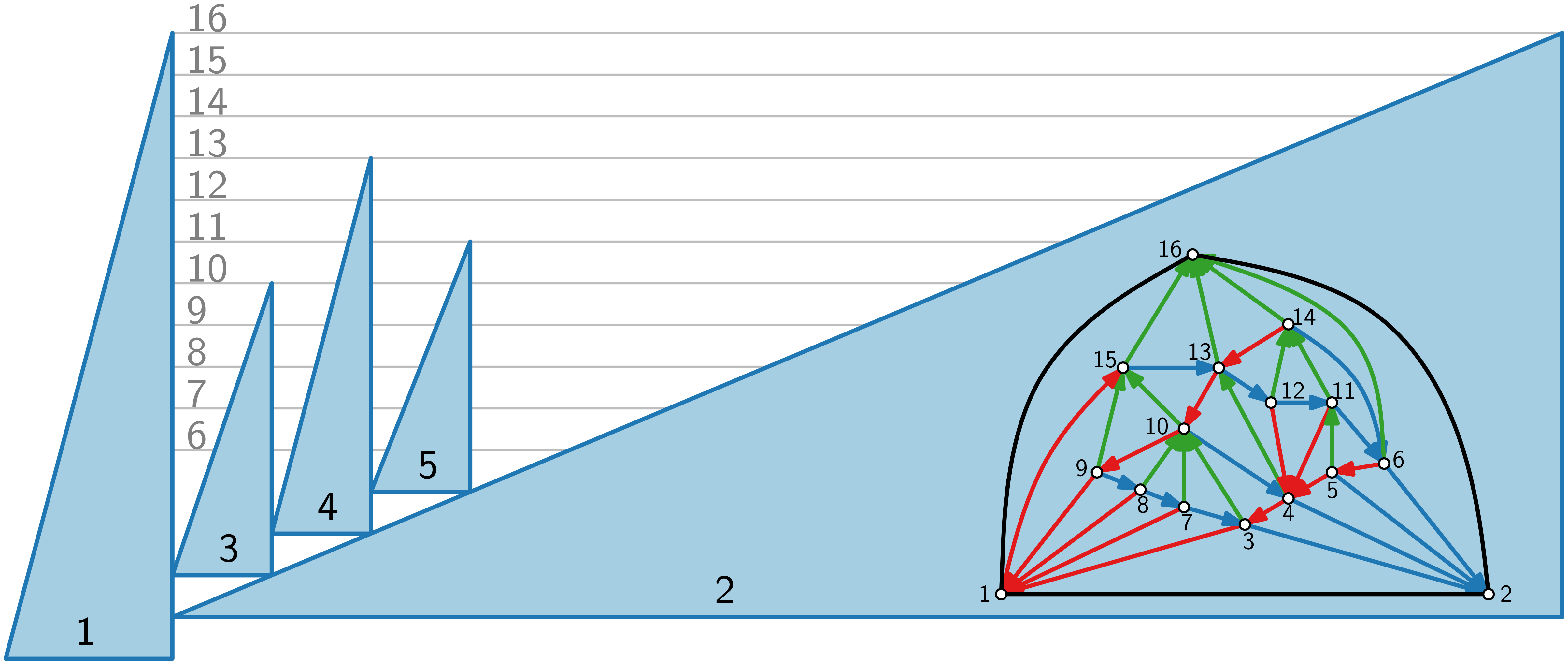


Triangle Contact Representation Example



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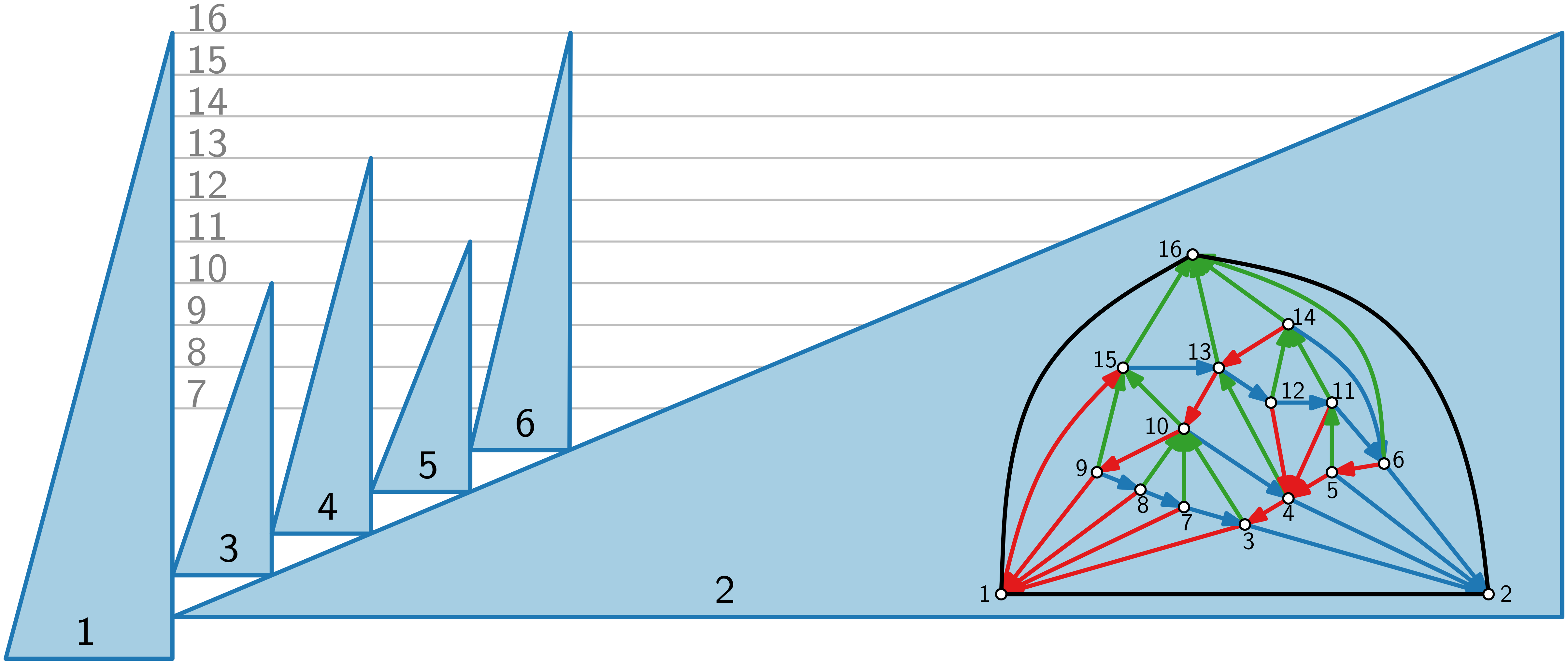

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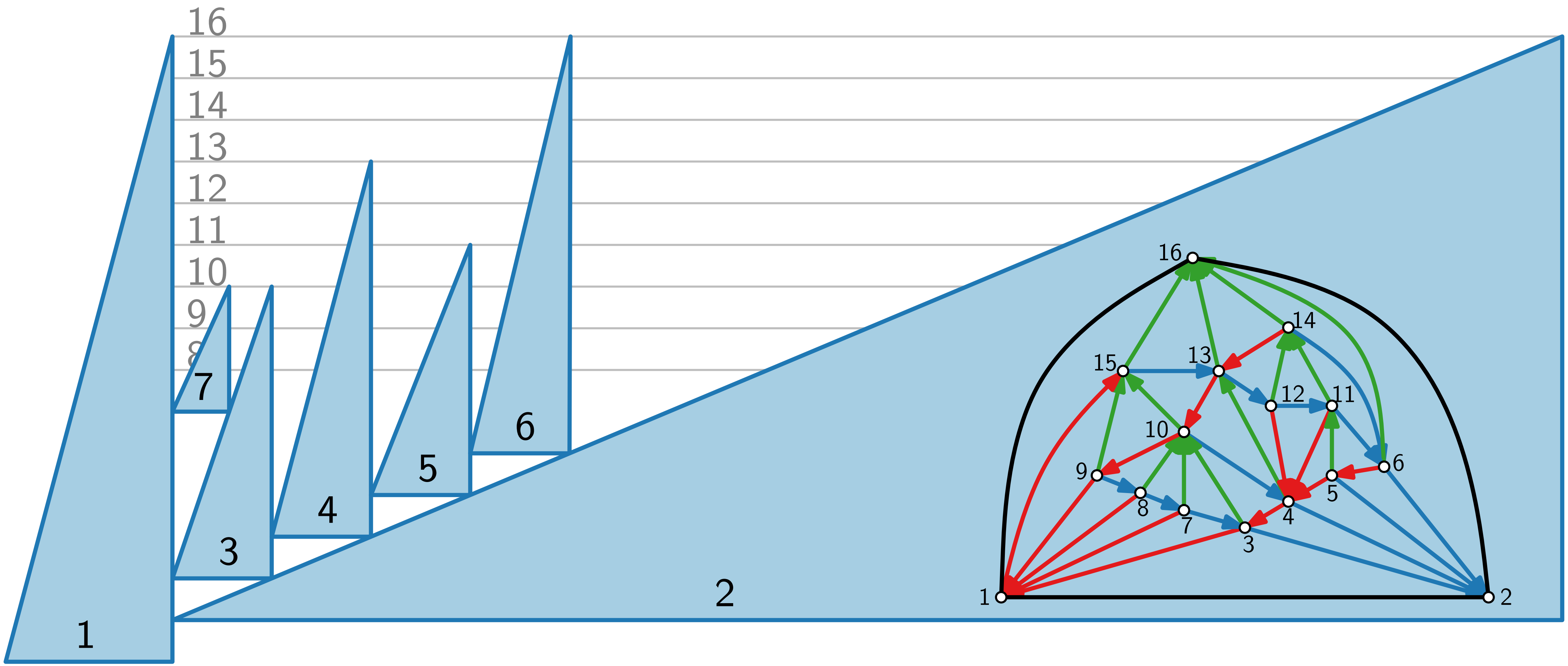

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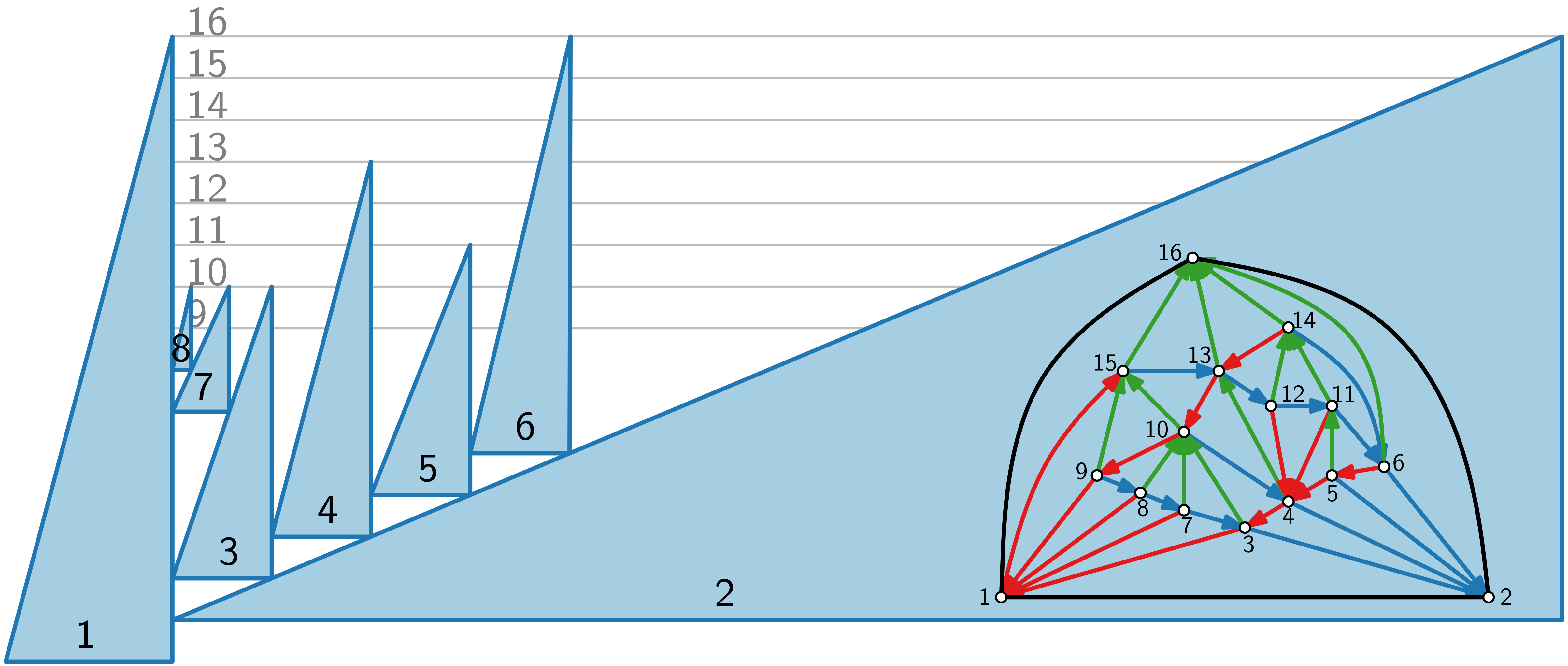

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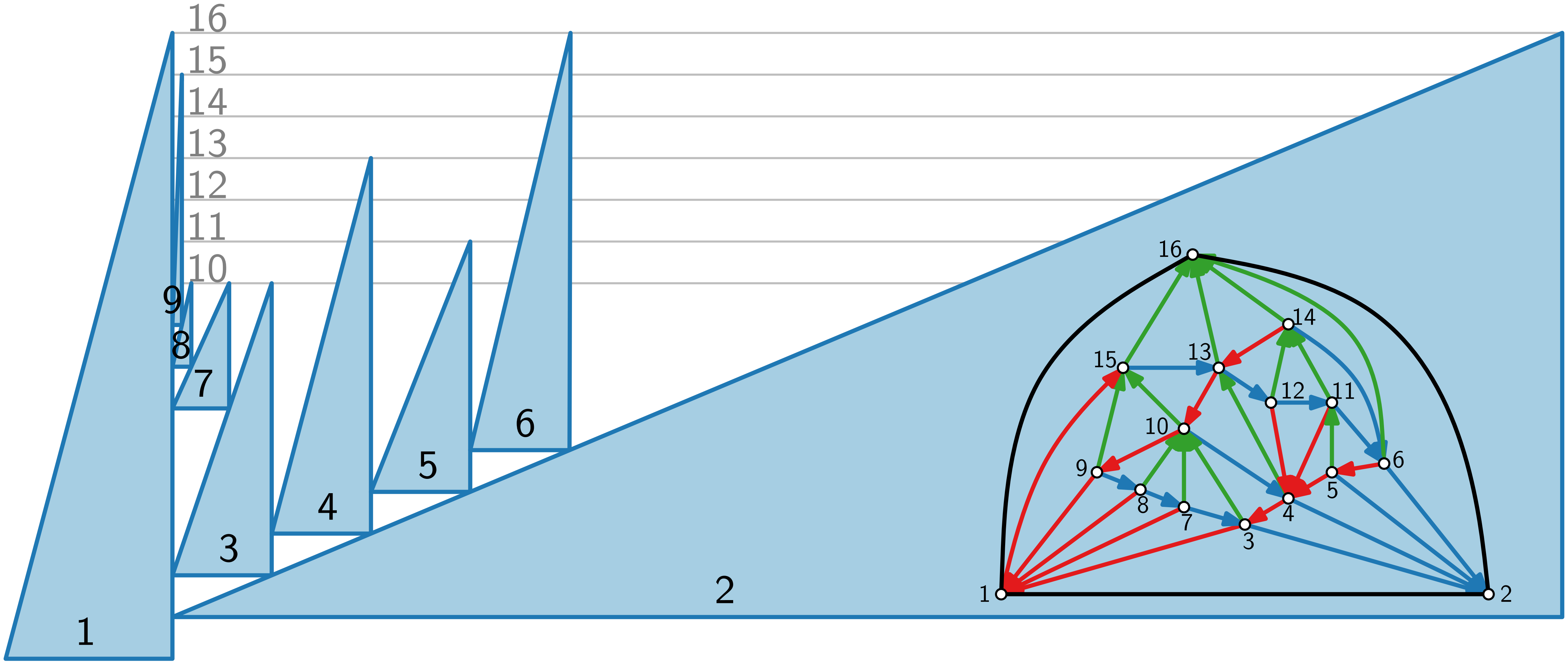

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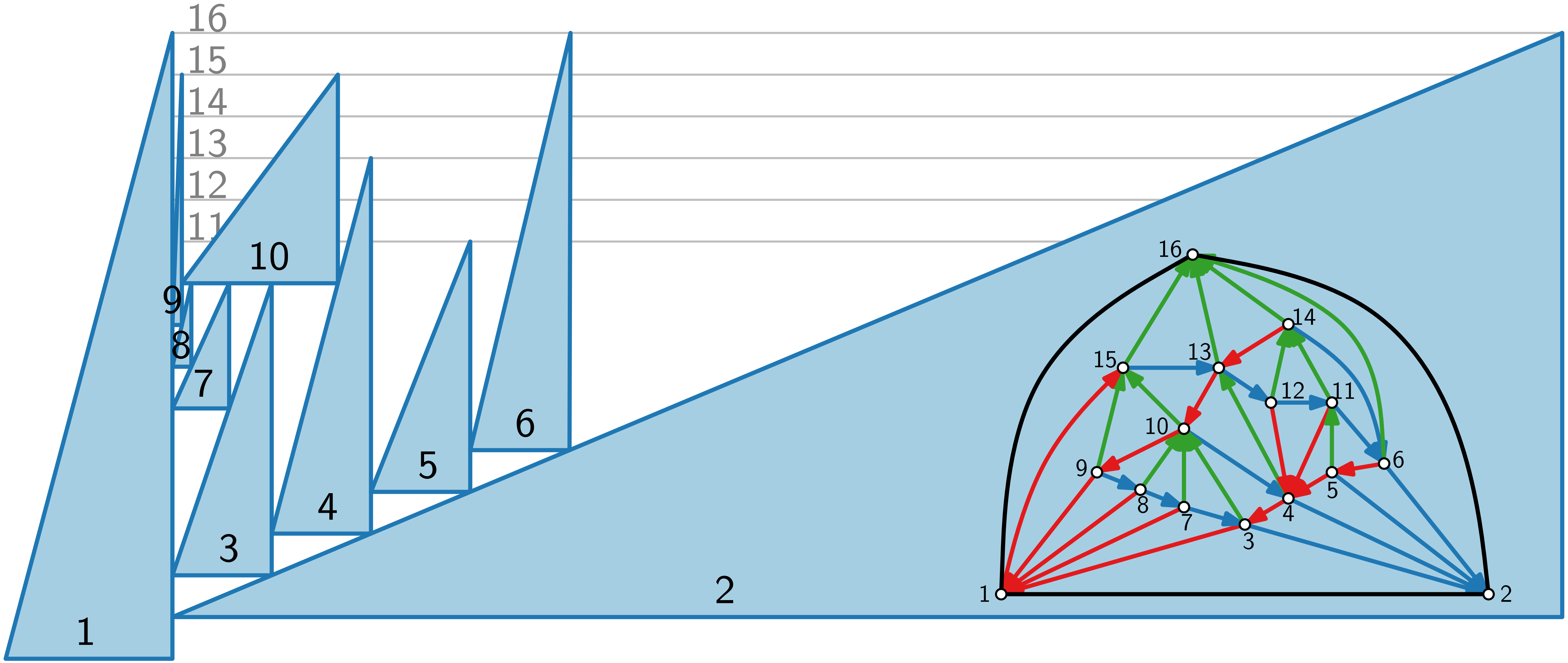

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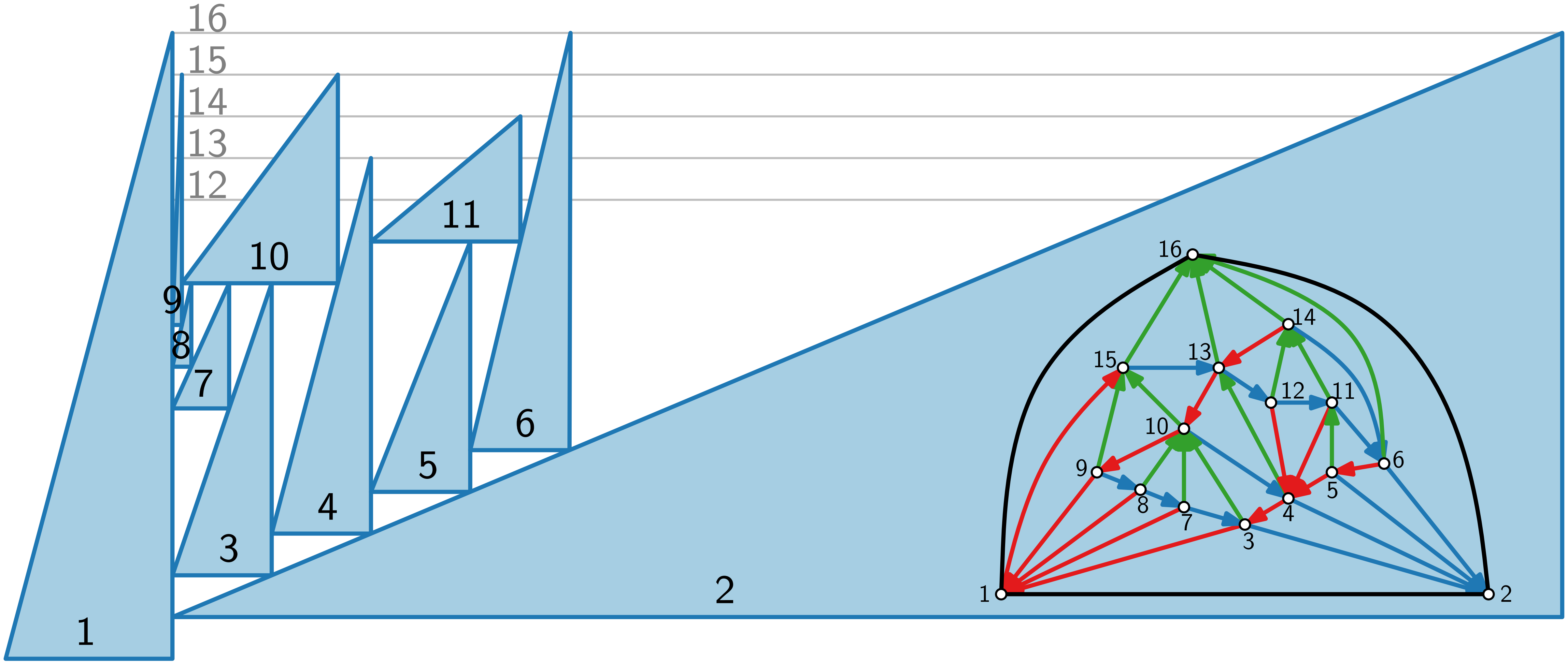

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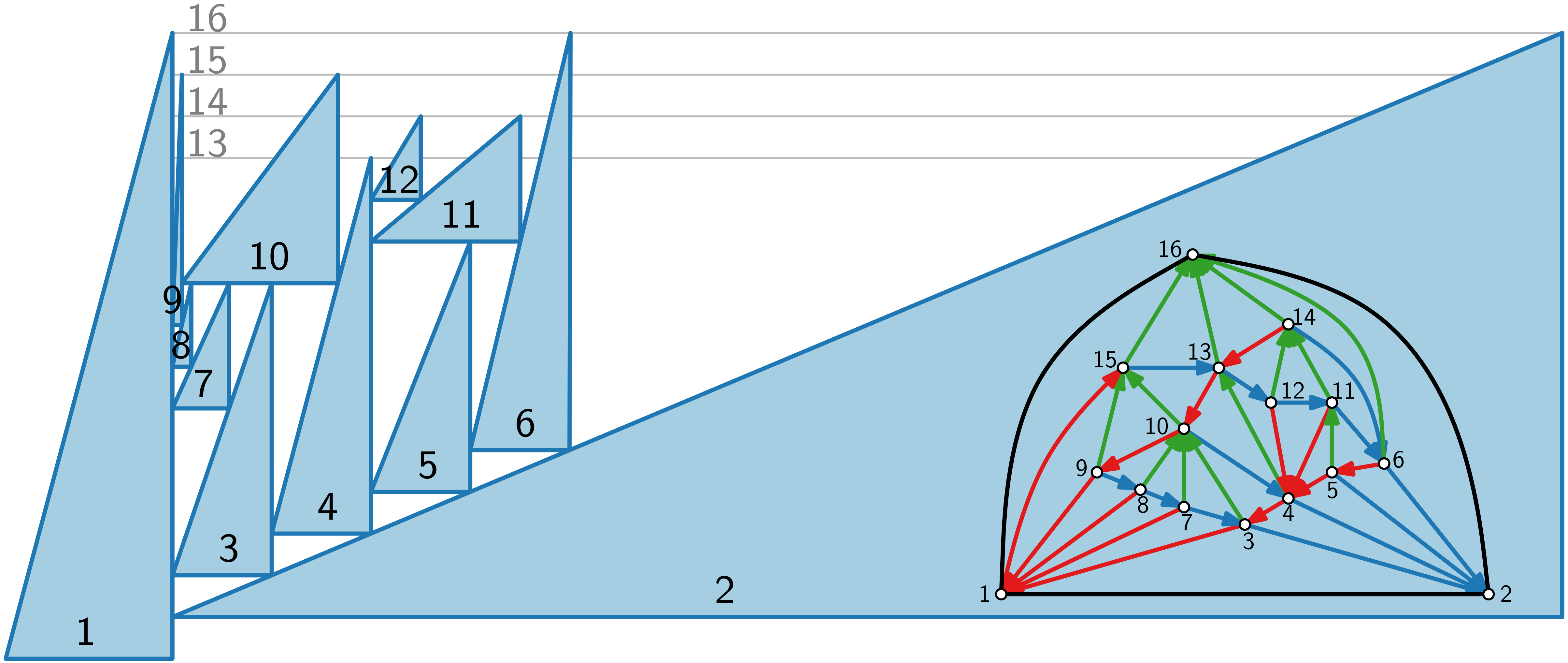

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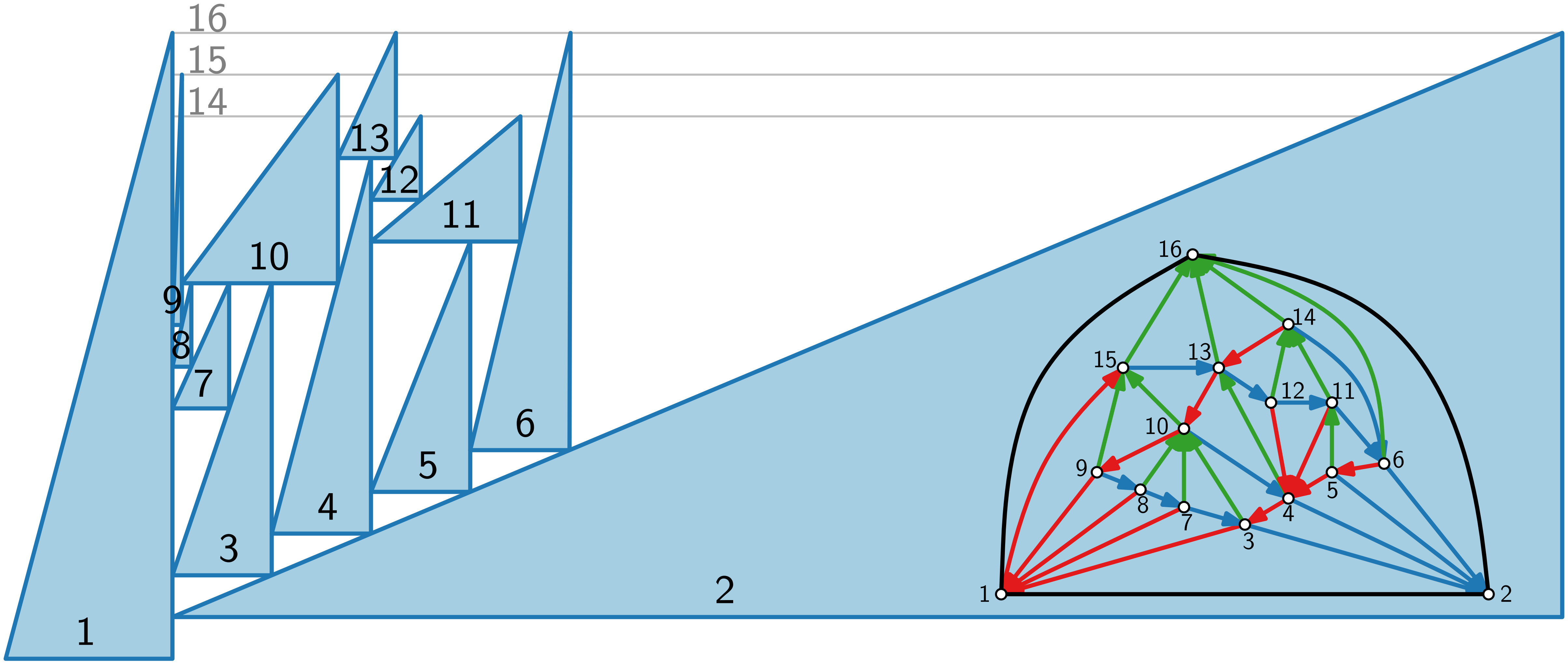

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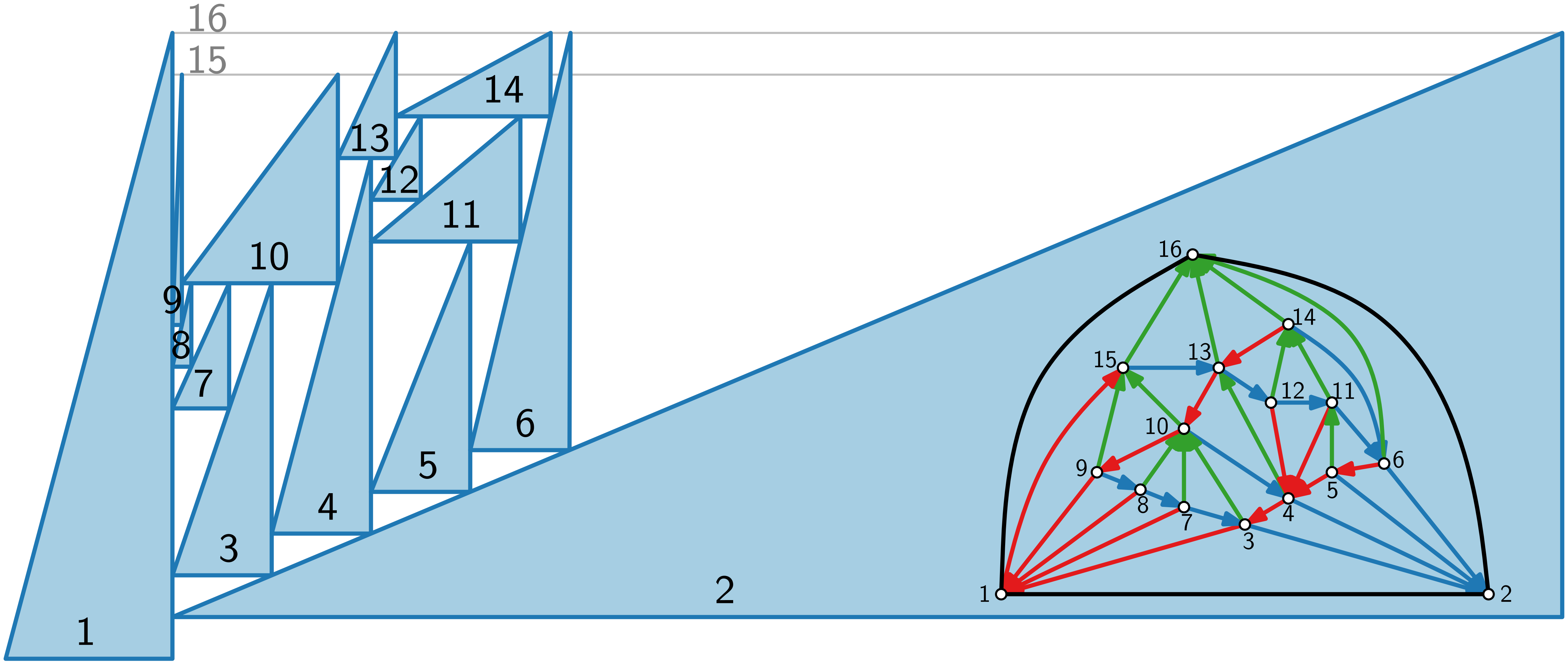

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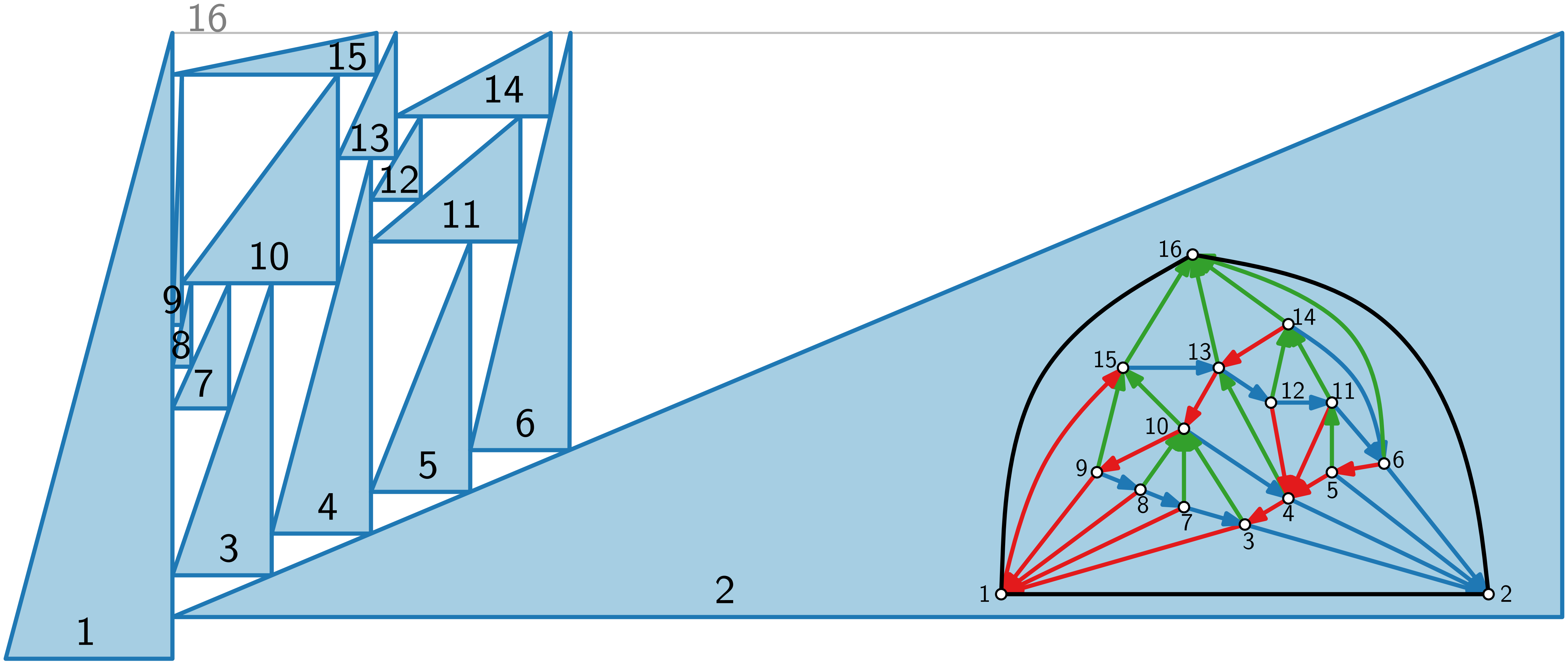

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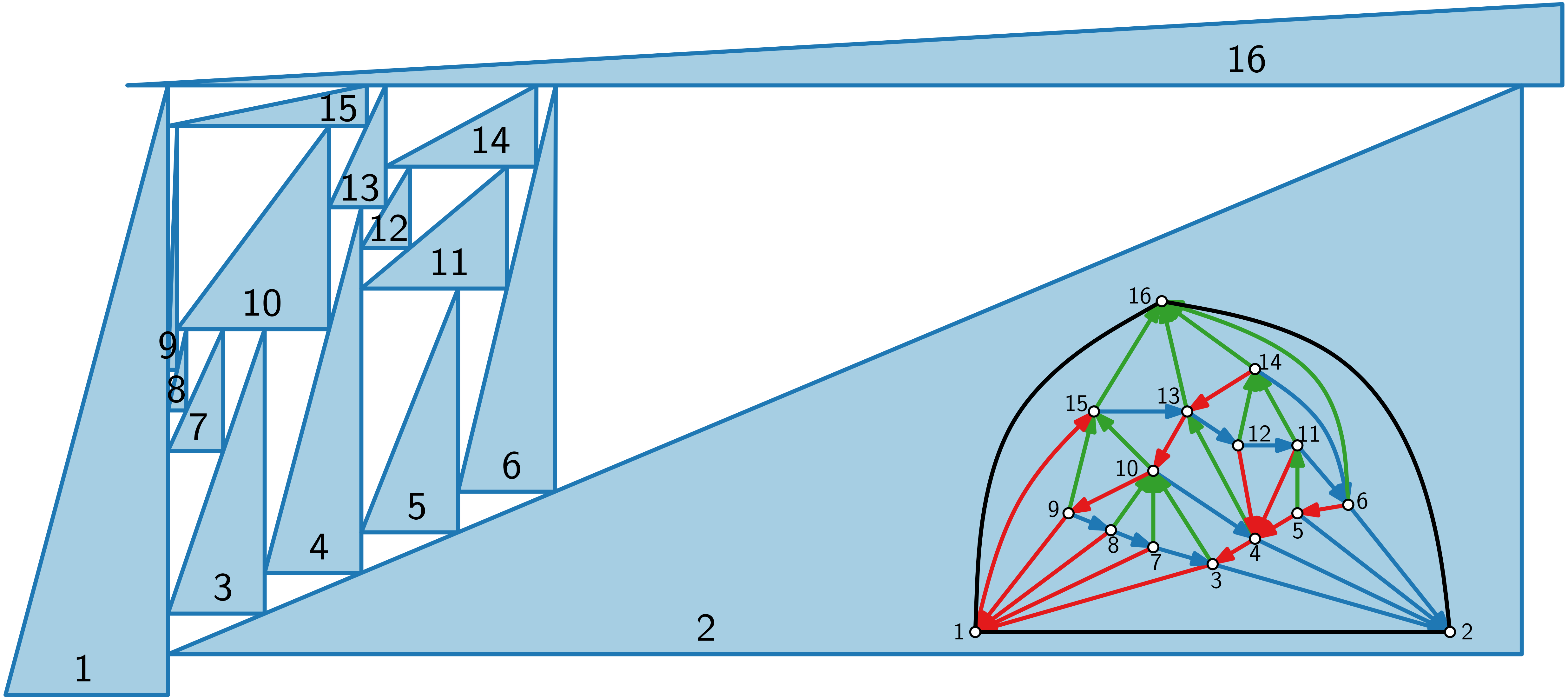
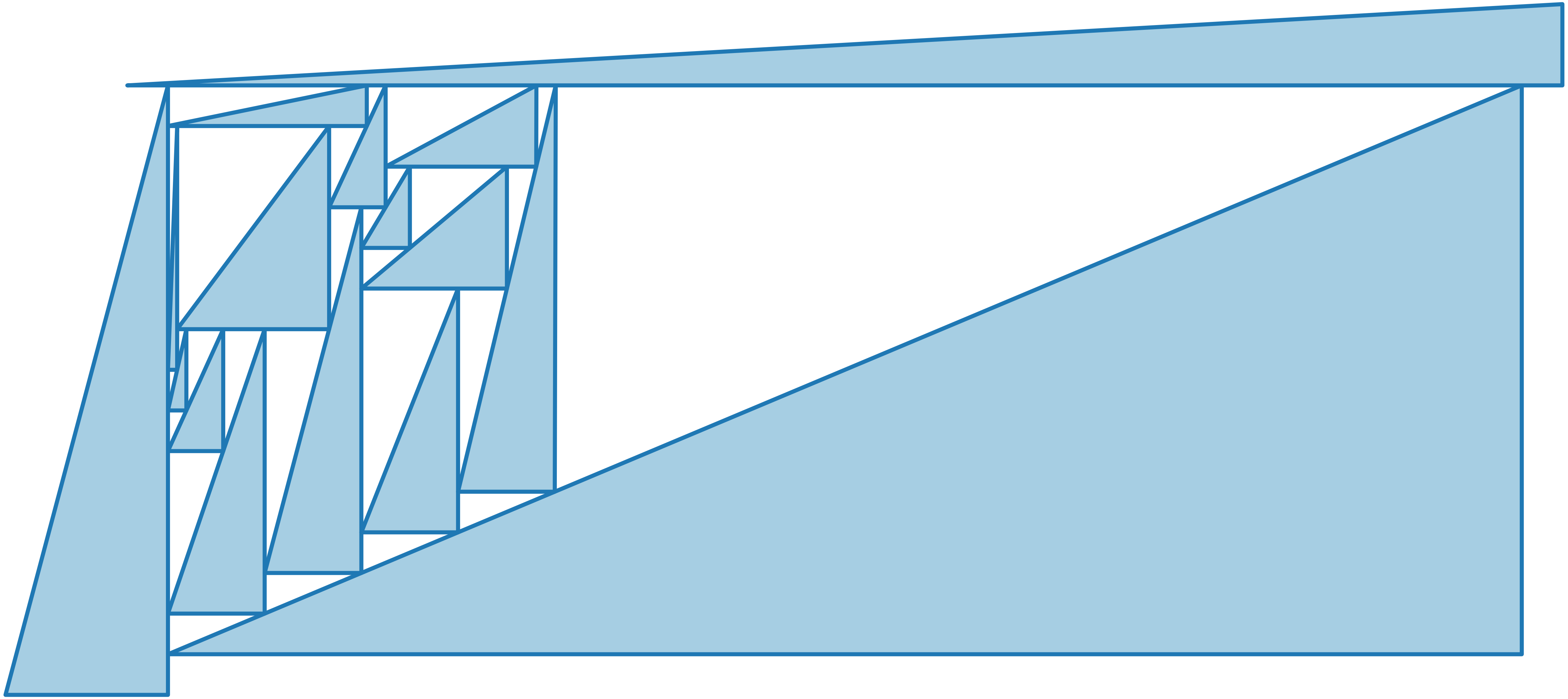
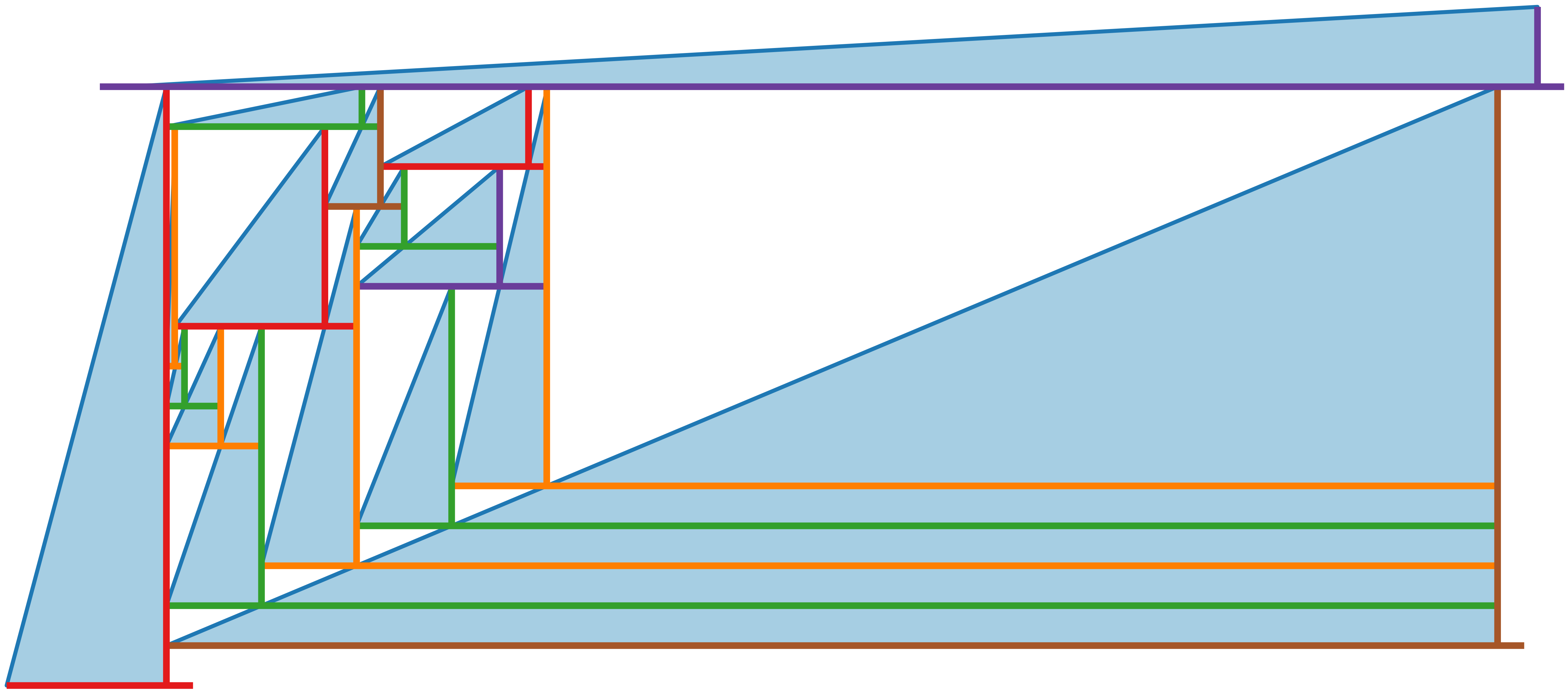


T-shape Contact Representation



T-shape Contact Representation



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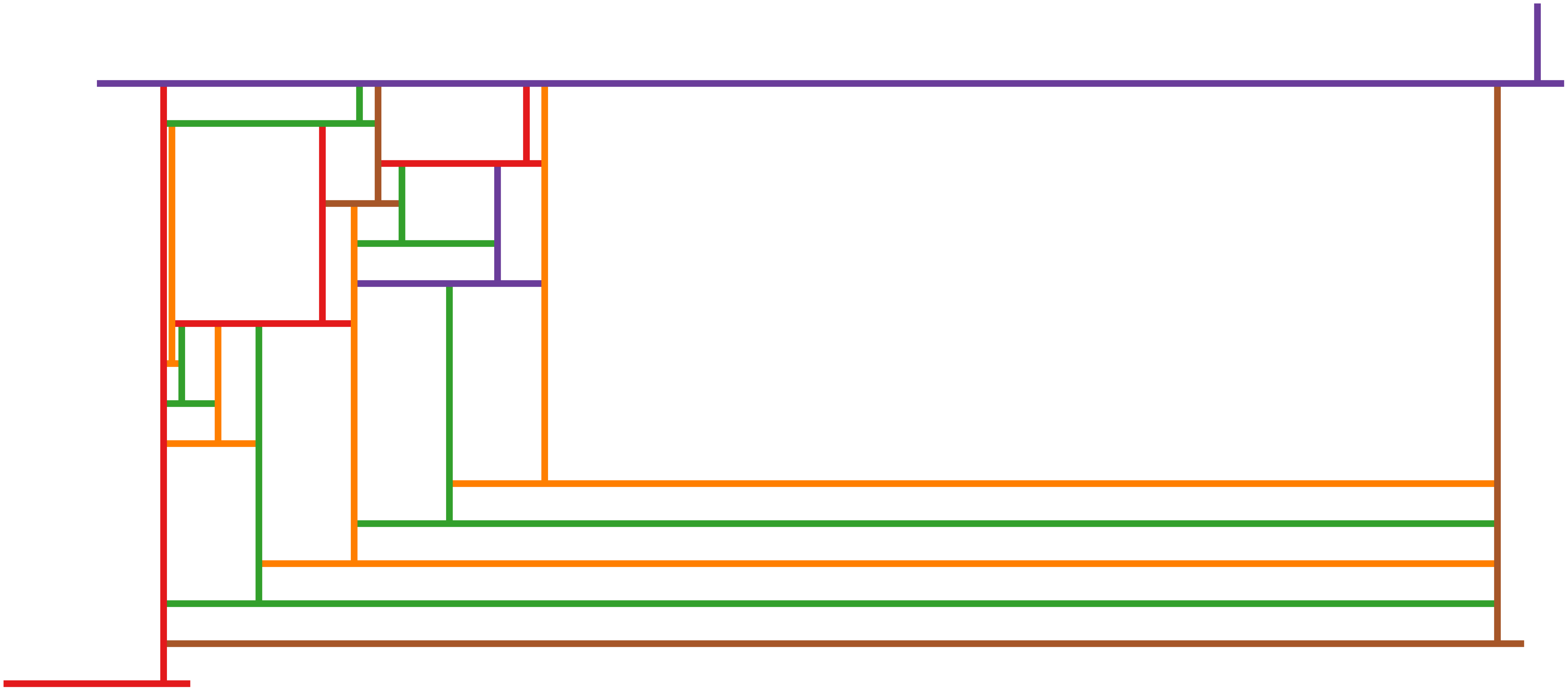

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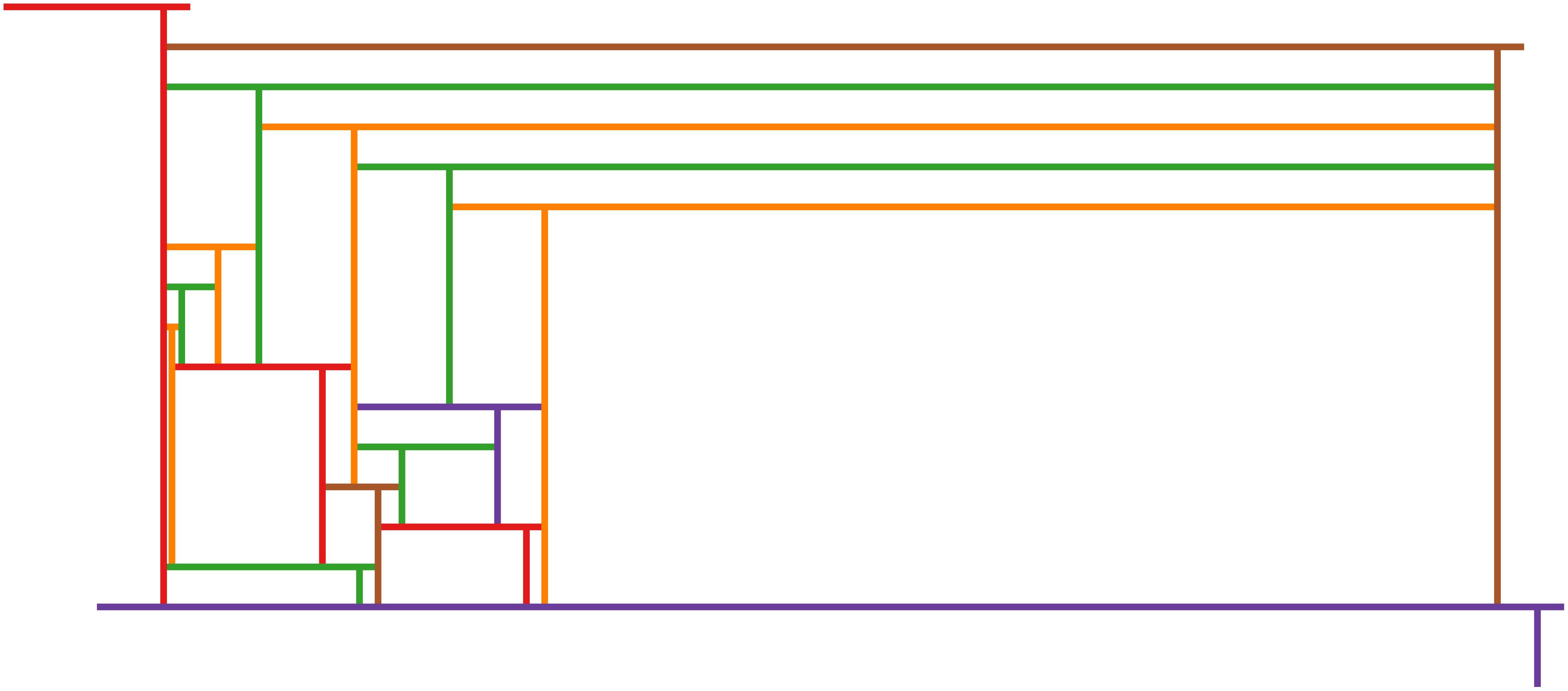

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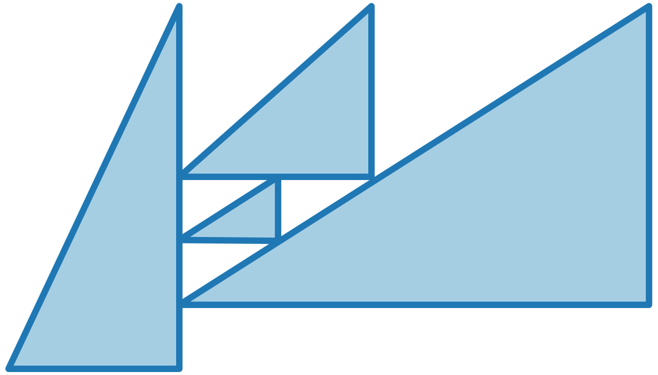
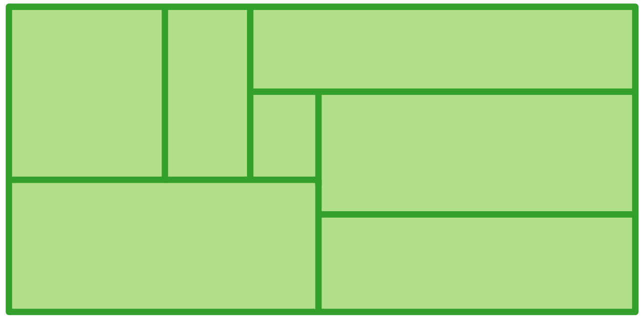
Visualization of Graphs

Lecture 8:
Conact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



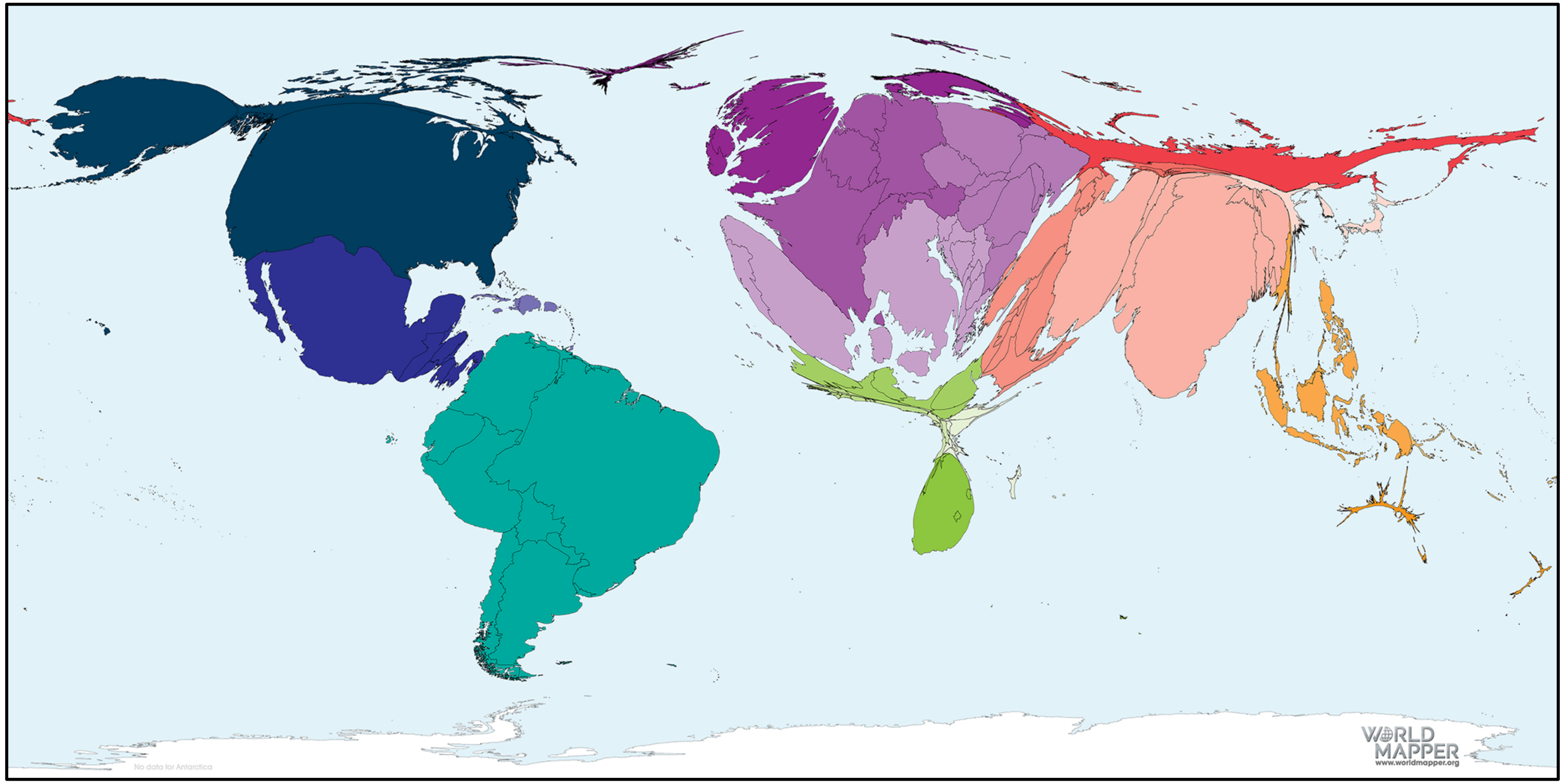
Part III:
Rectangular Duals

Jonathan Klawitter



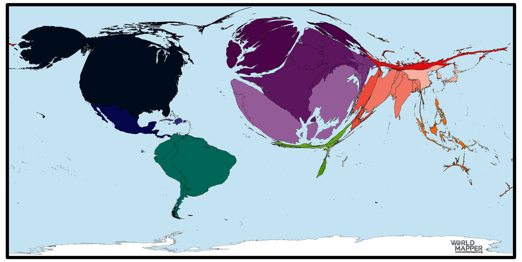
Cartograms

Cartograms



COVID19 reported deaths (January 1, 2021)

Cartograms



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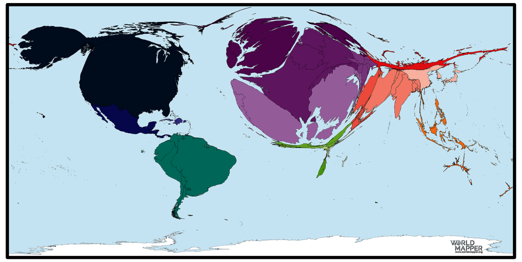

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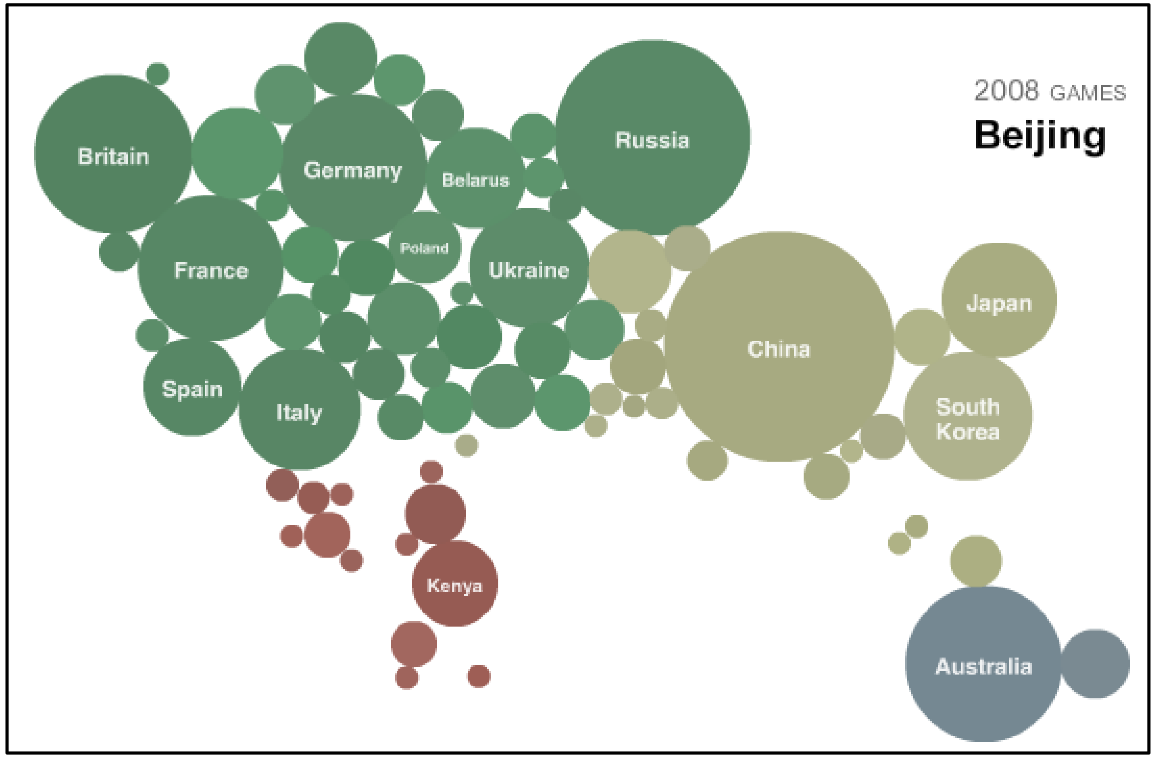

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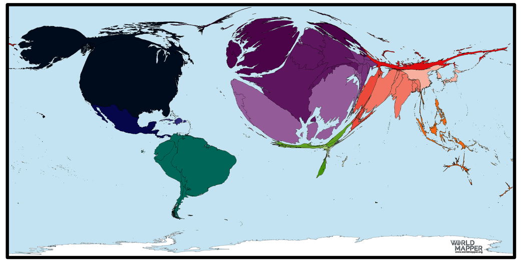

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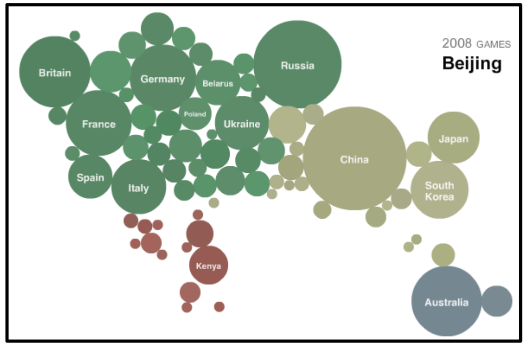

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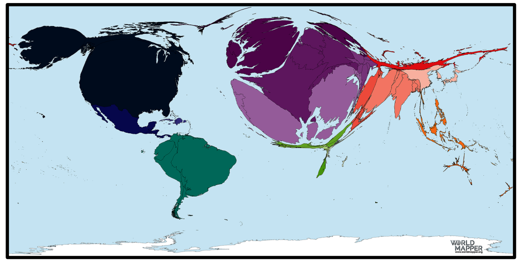

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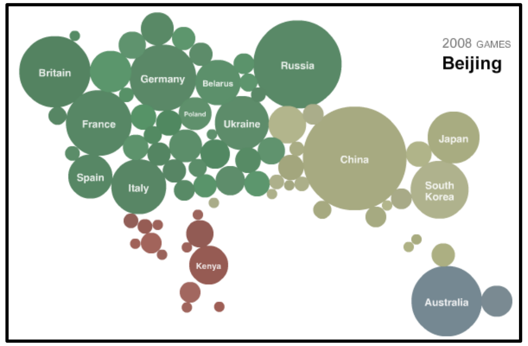

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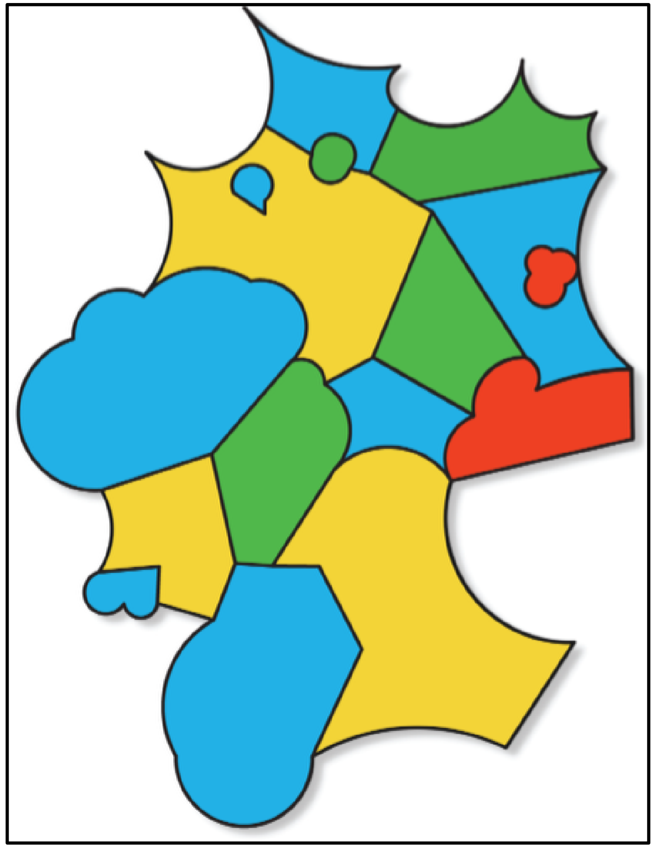

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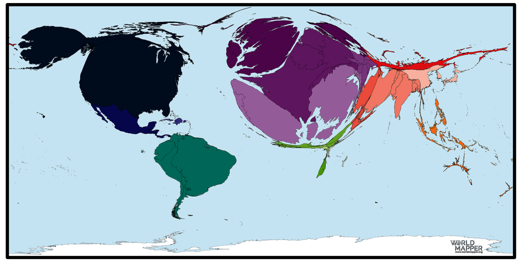

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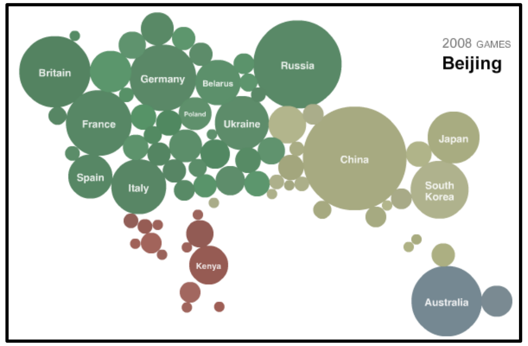

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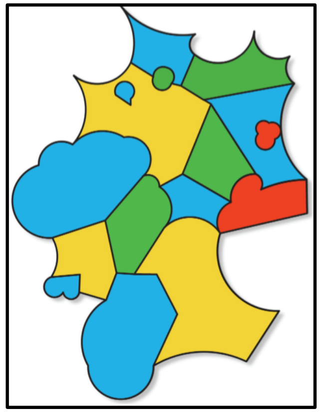

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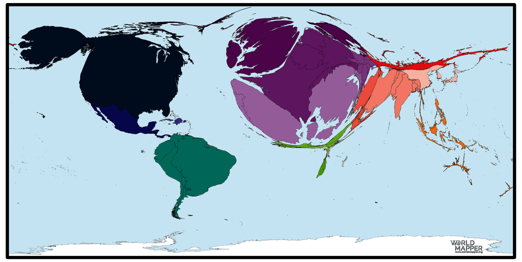

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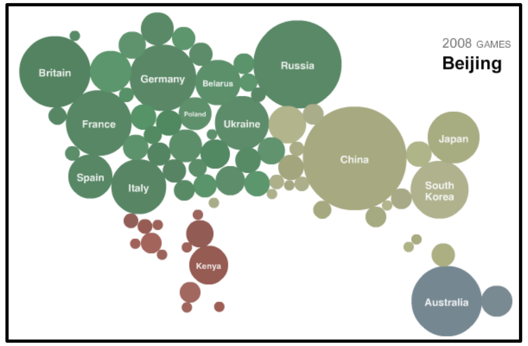

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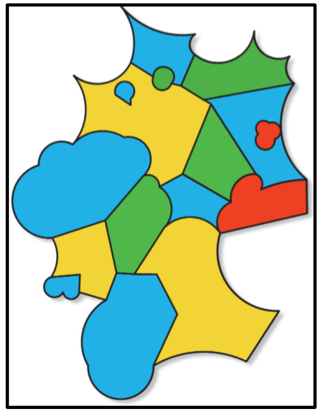

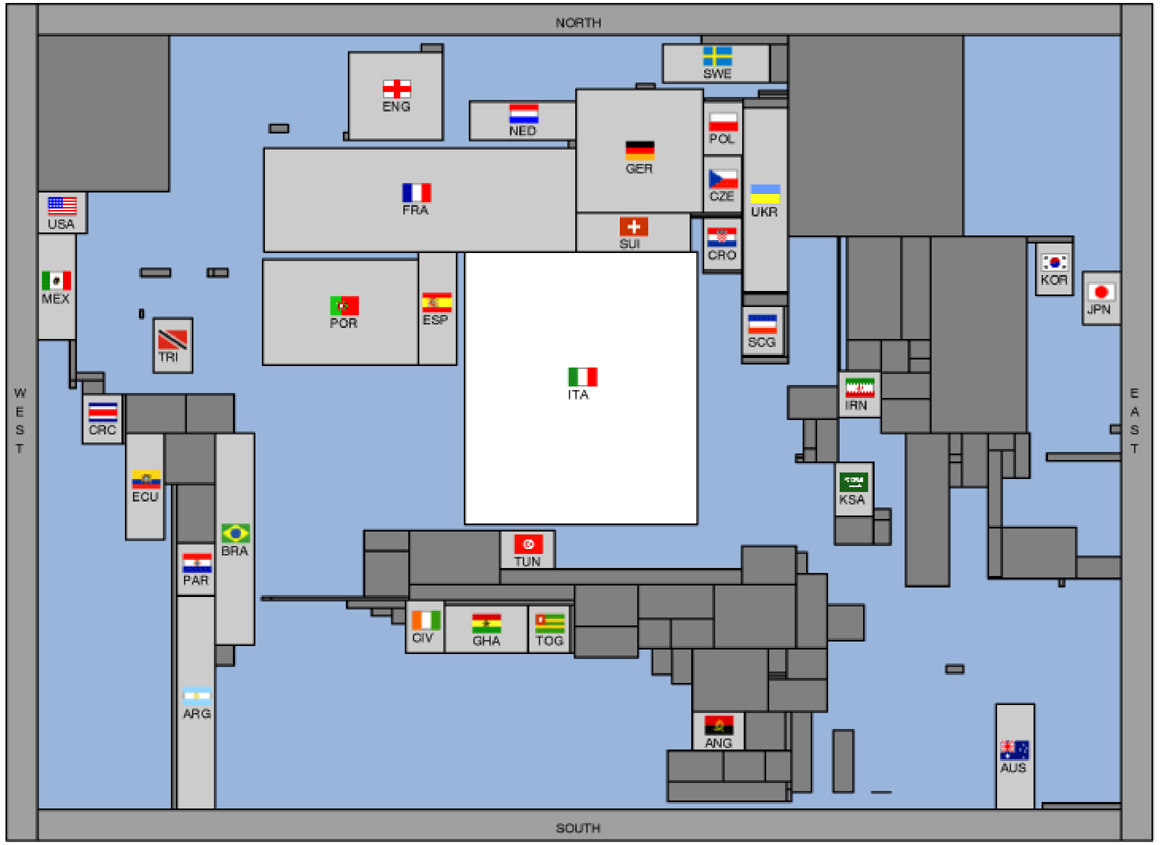

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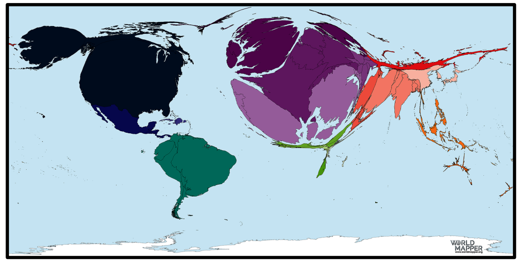

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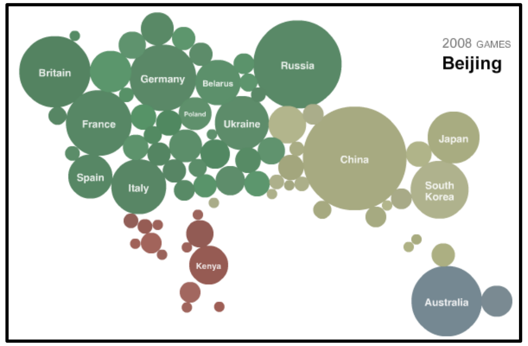

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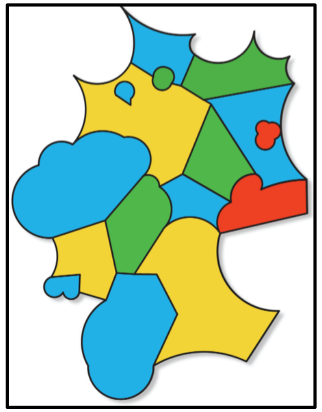

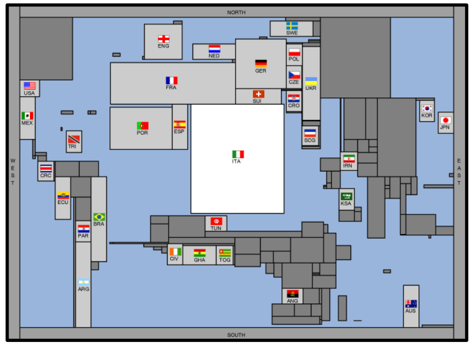

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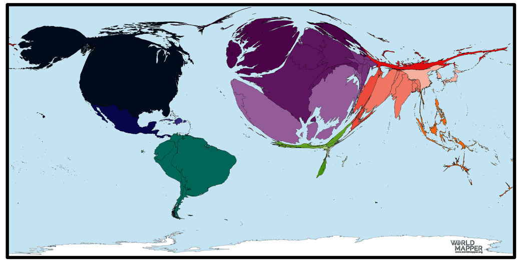

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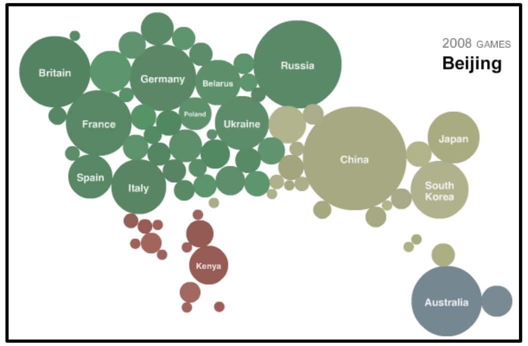

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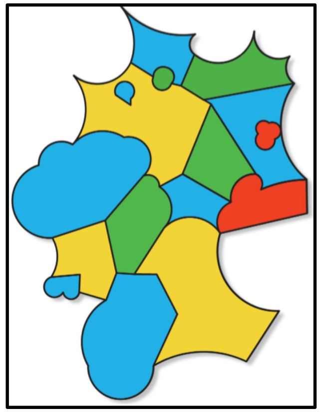

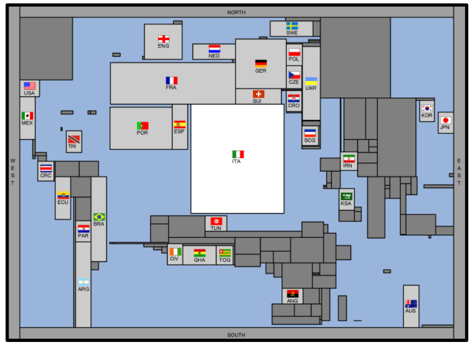

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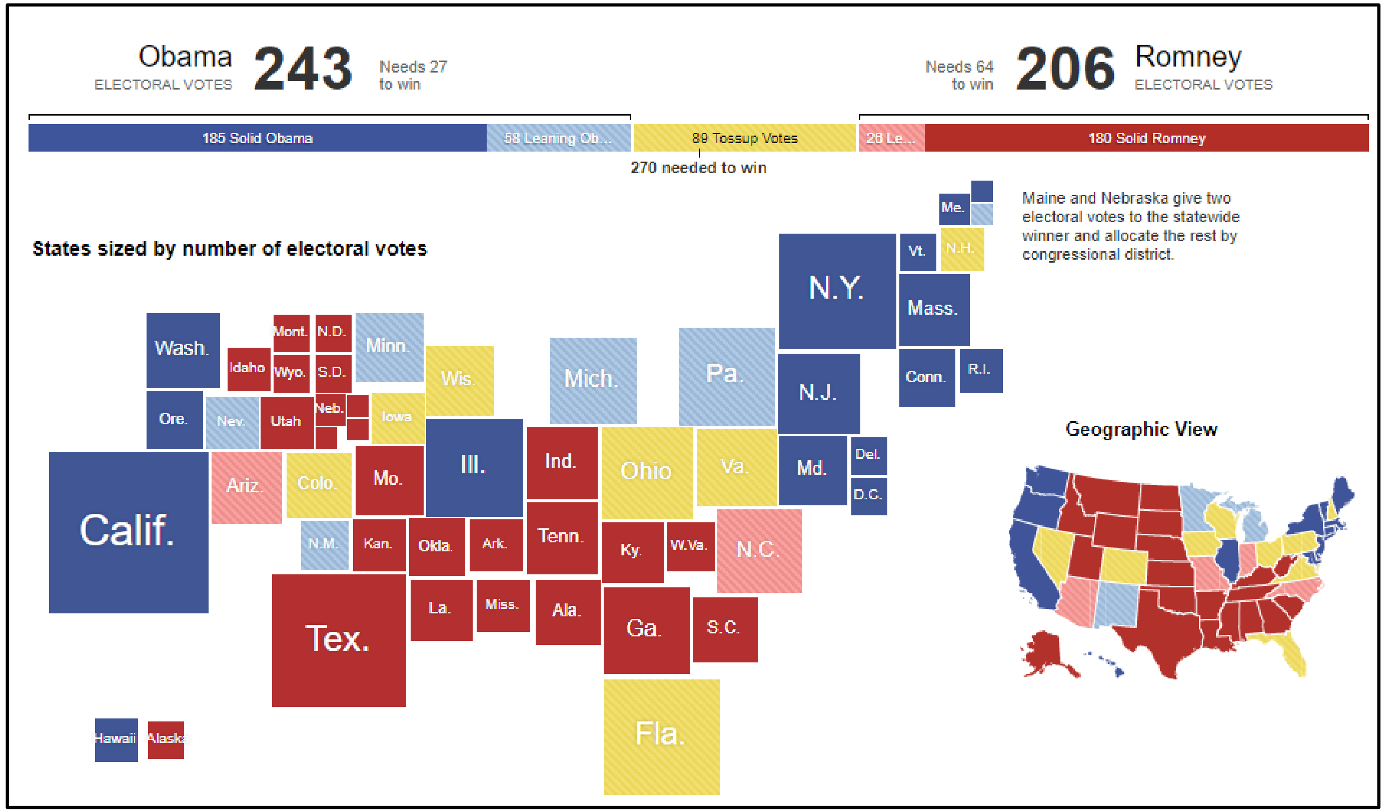

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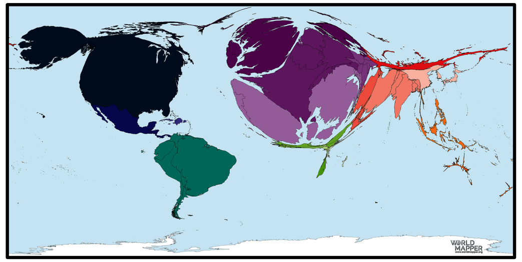

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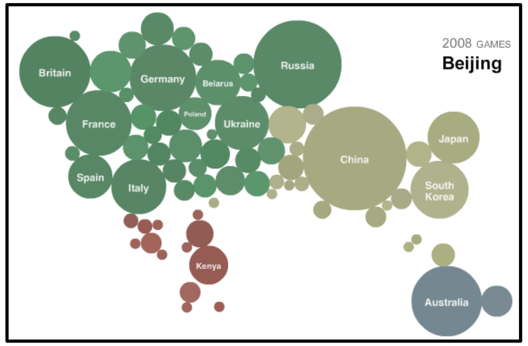

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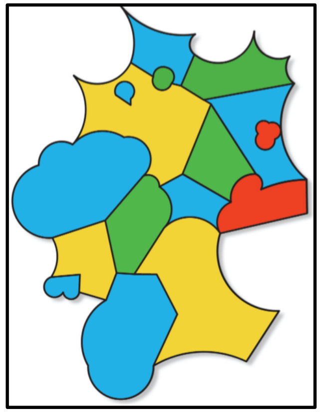

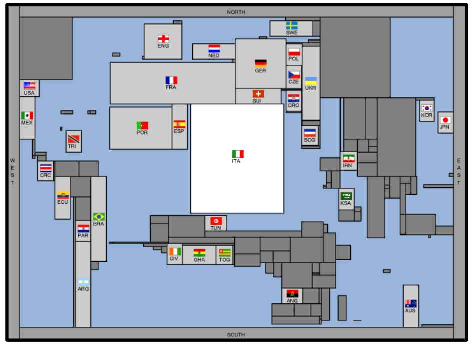

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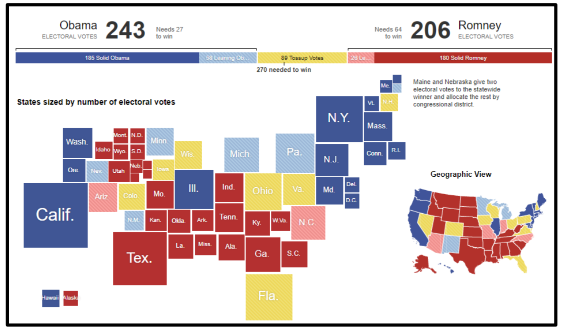

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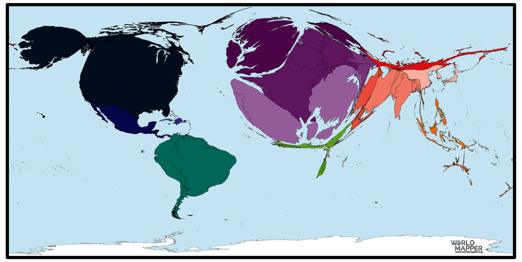

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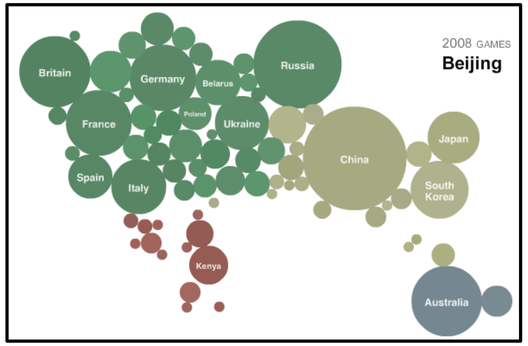

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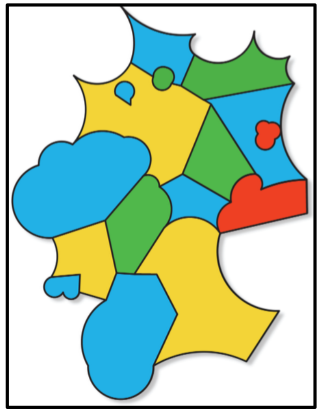

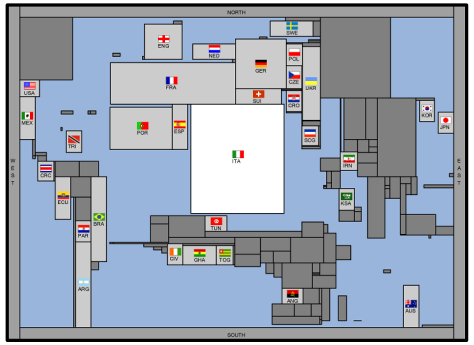

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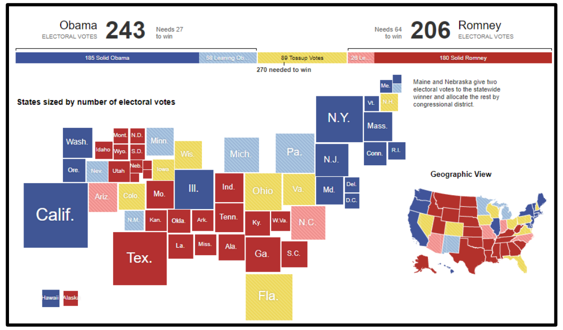

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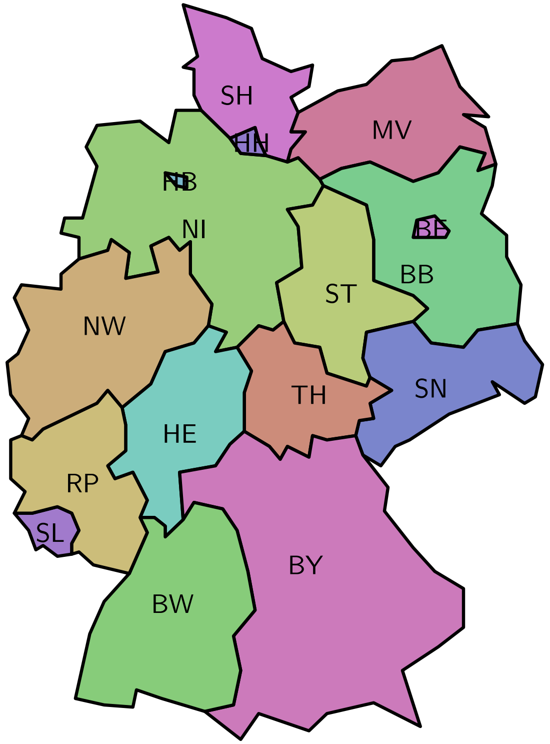

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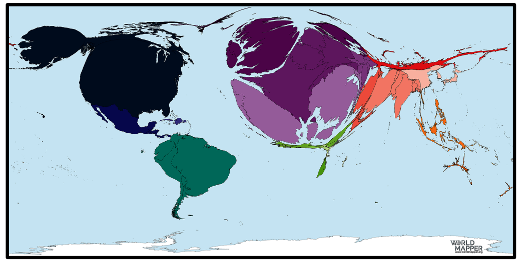

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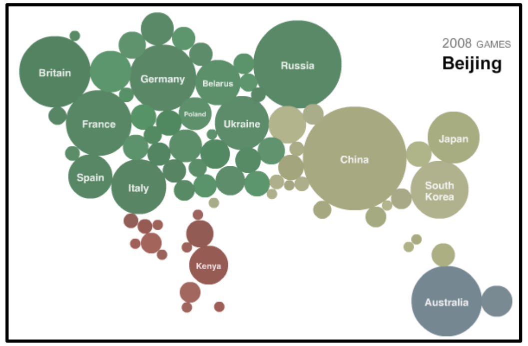

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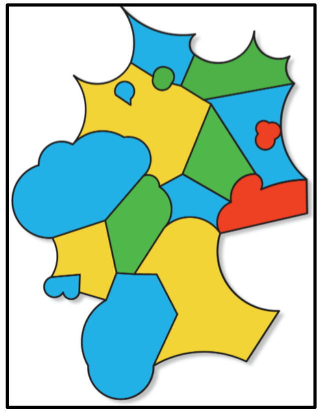

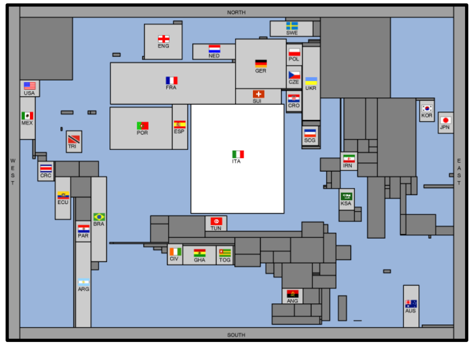

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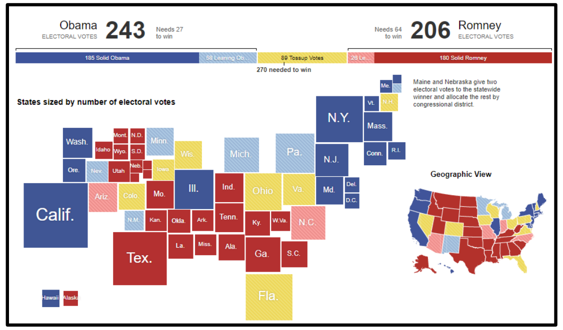

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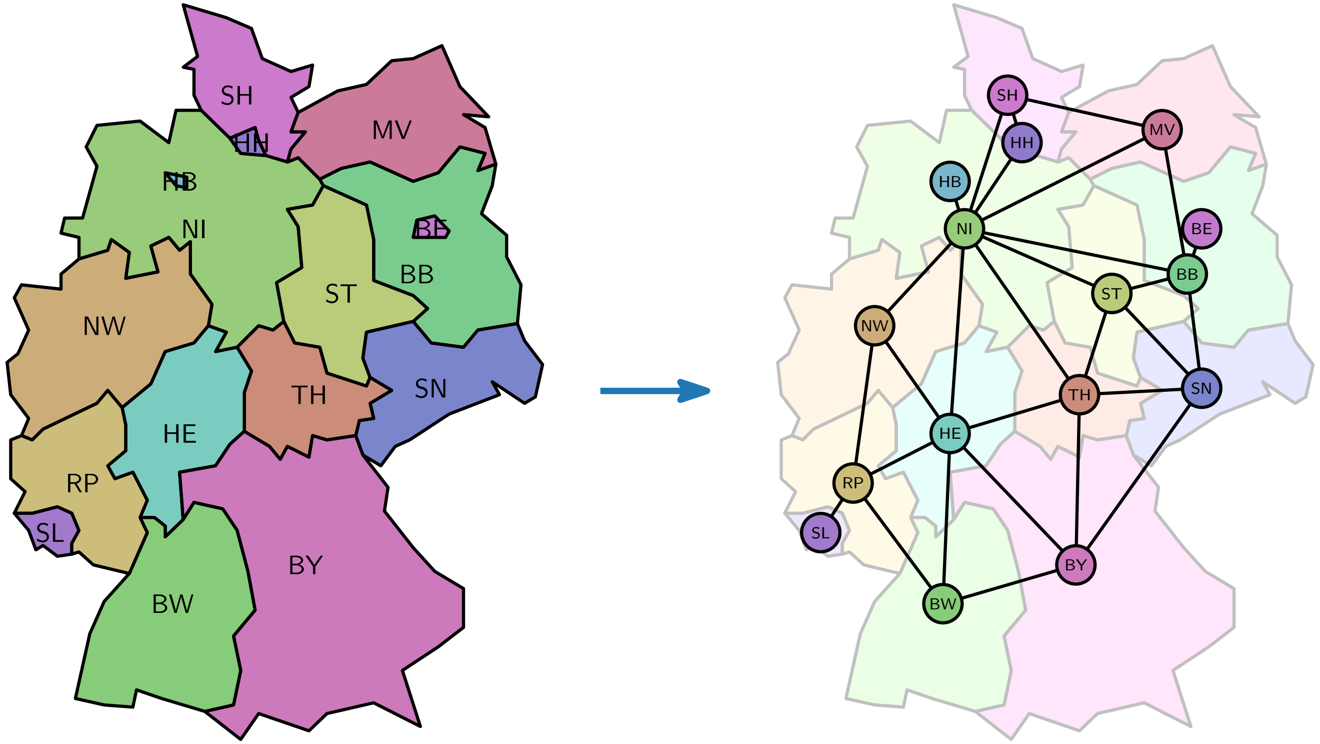

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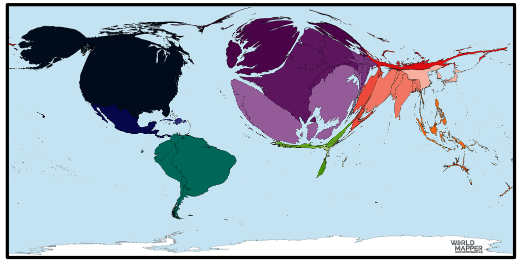

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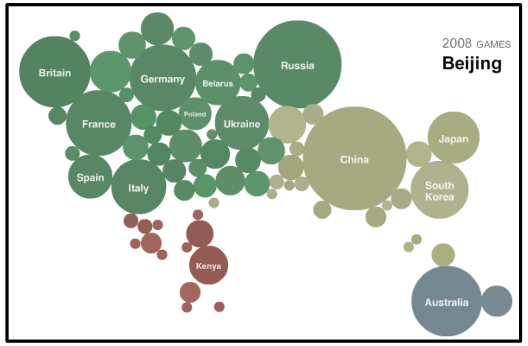

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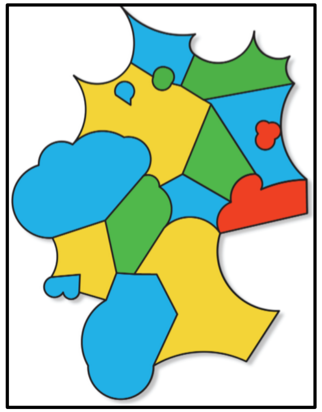

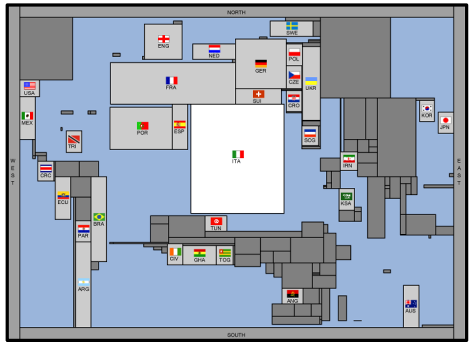

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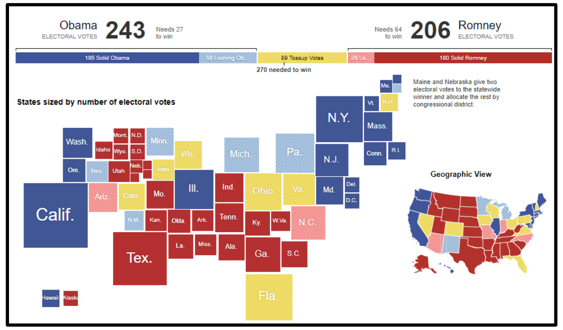

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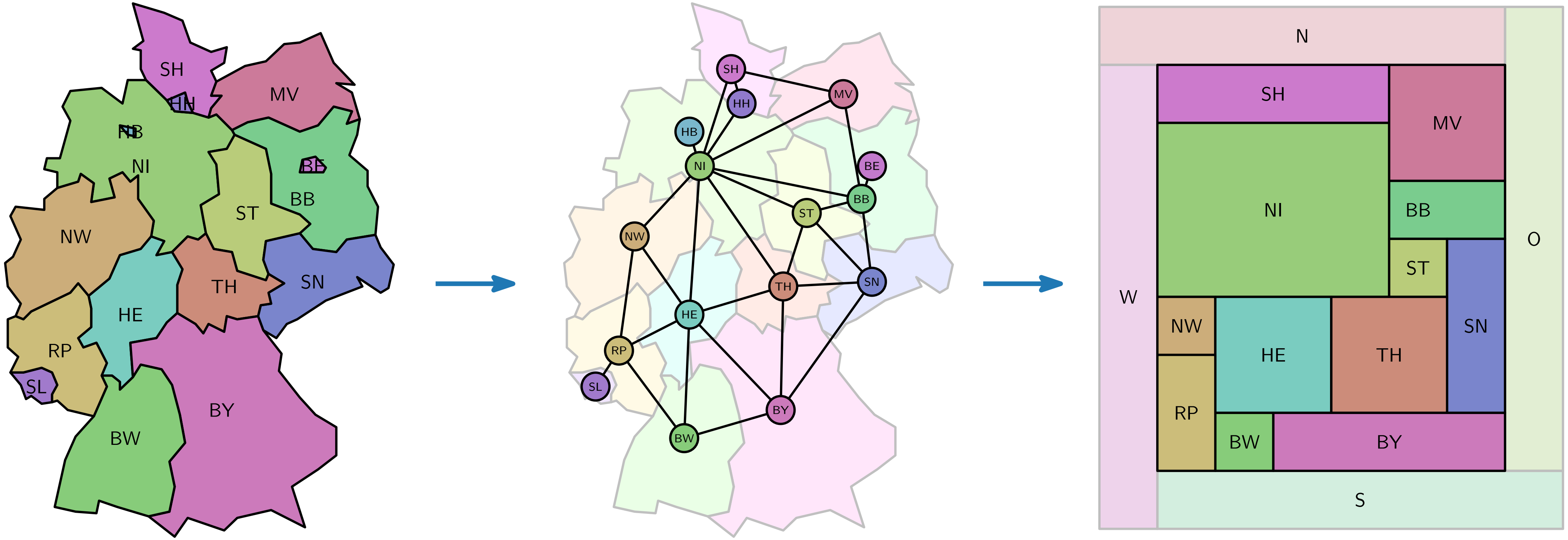
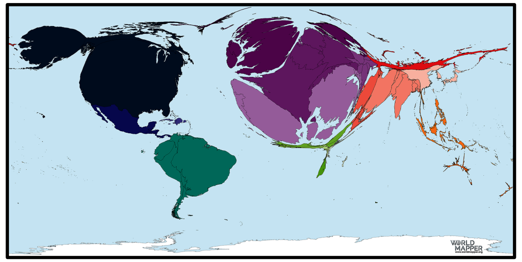


Cartograms



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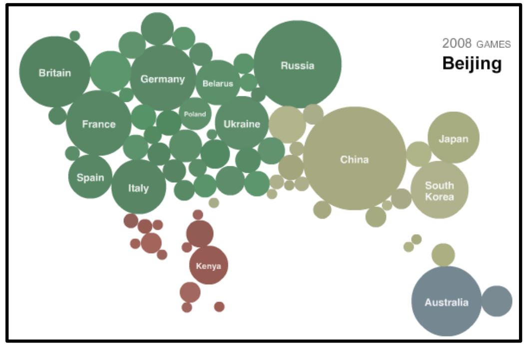

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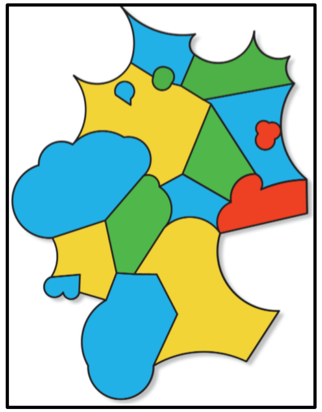

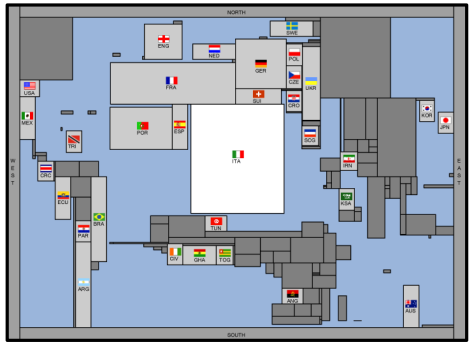

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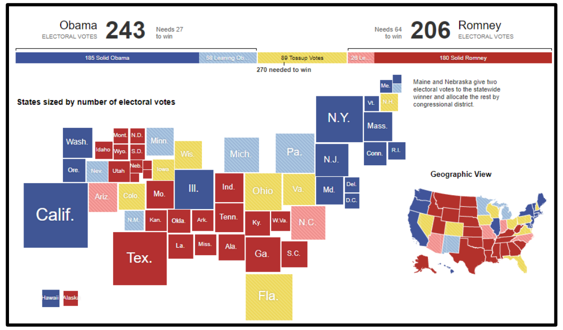

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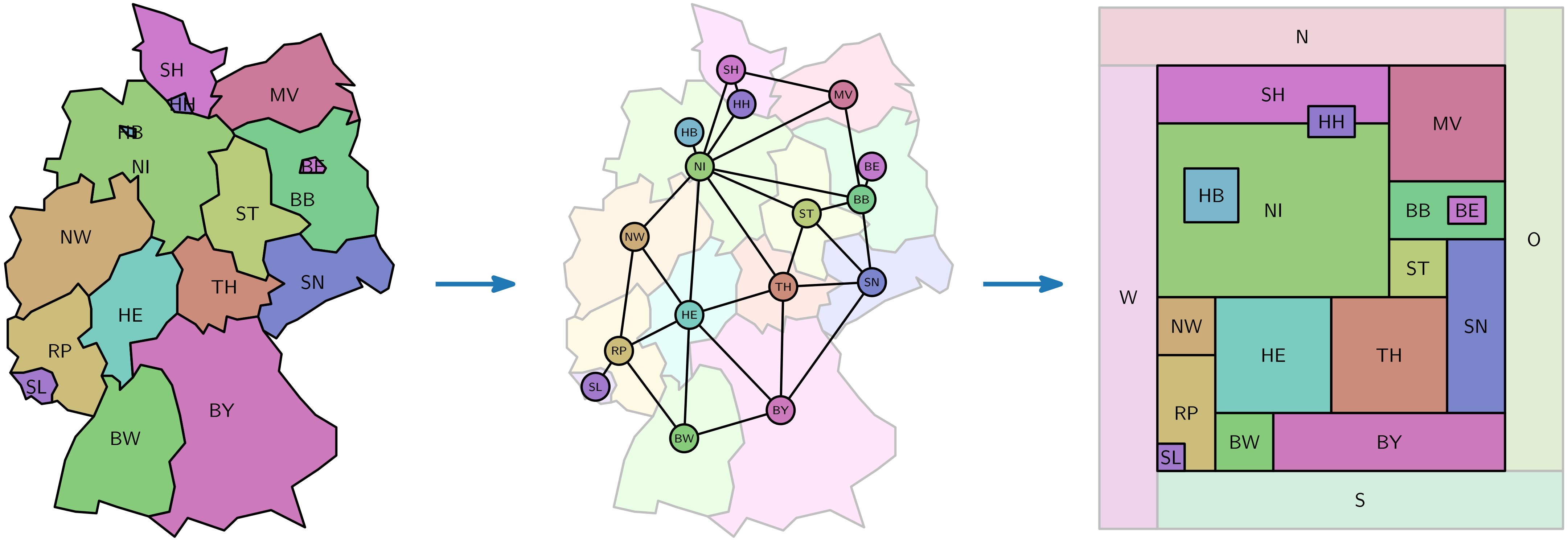
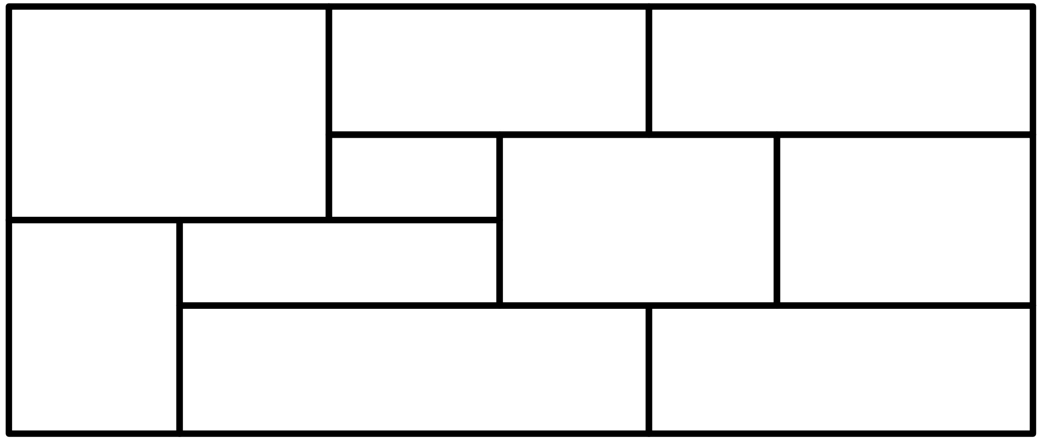


Rectangular Dual

Rectangular Dual



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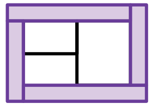
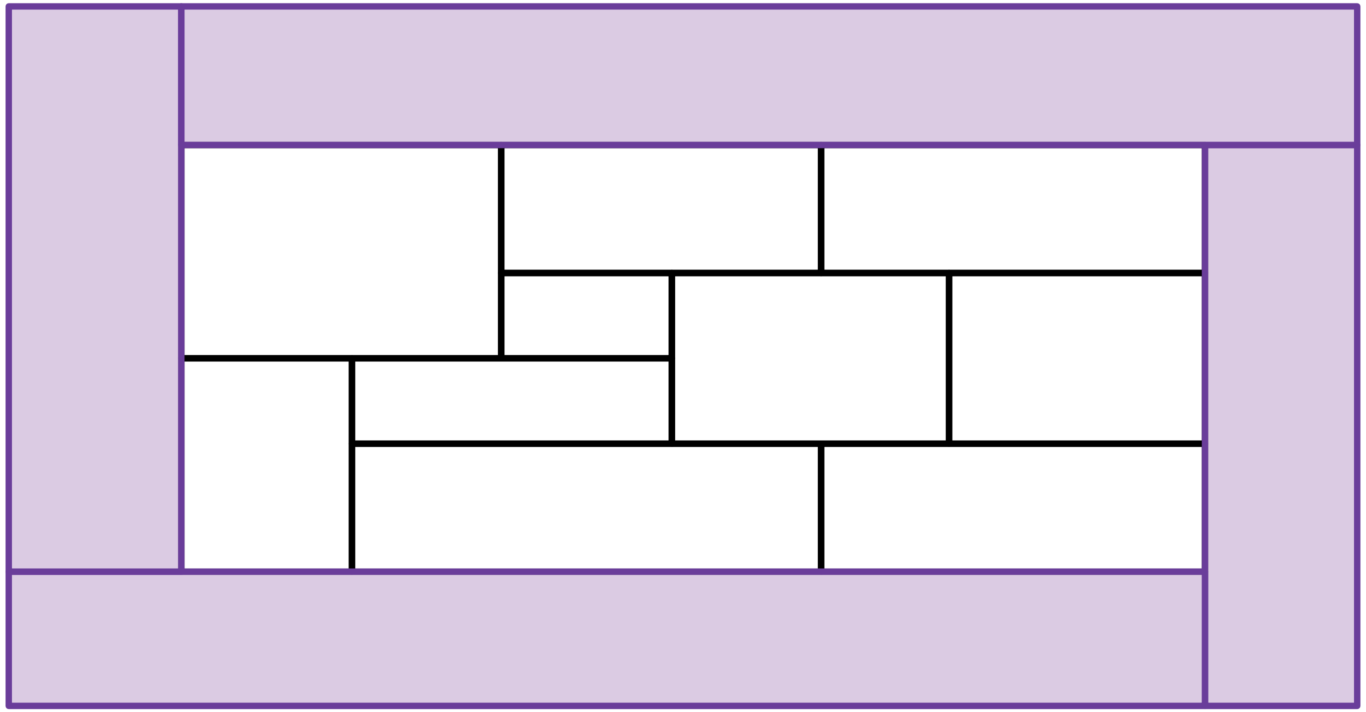


RD

Rectangular Dual $\mathcal{R}$



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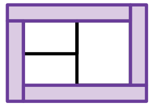

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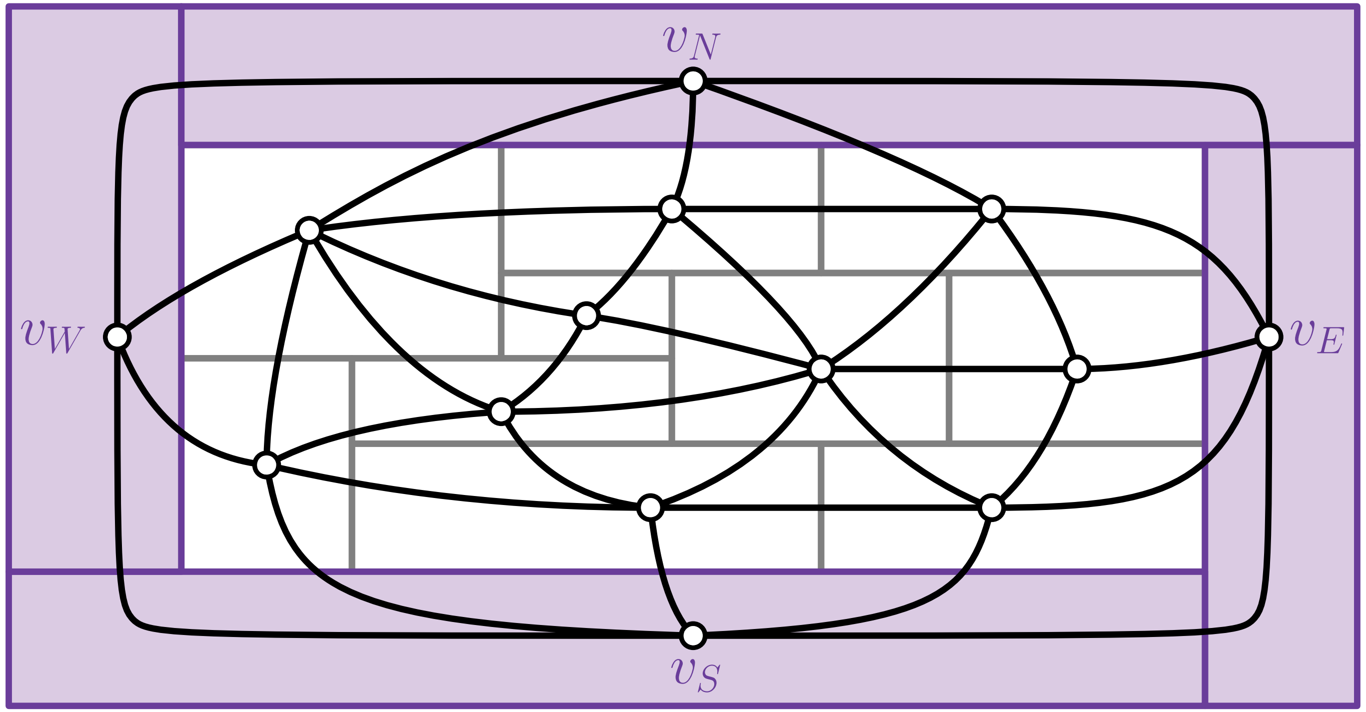

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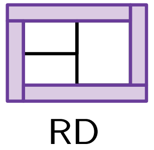

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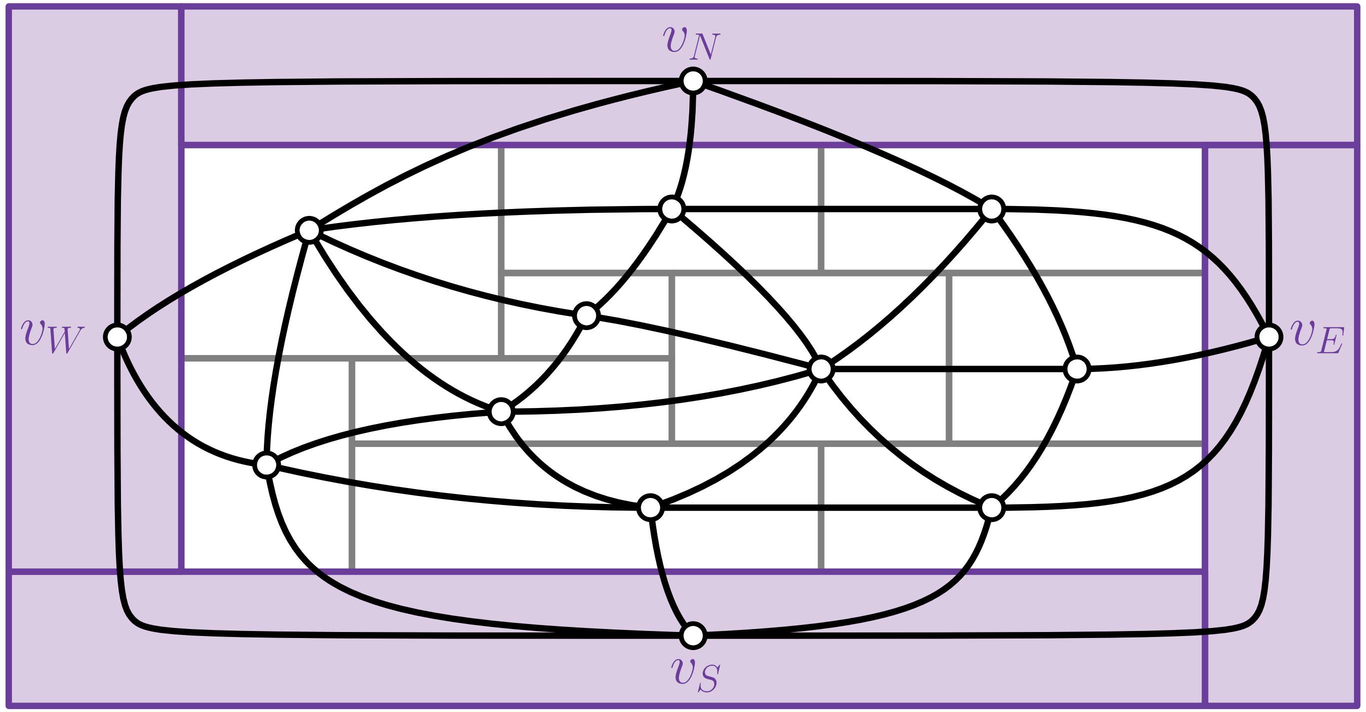

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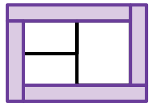

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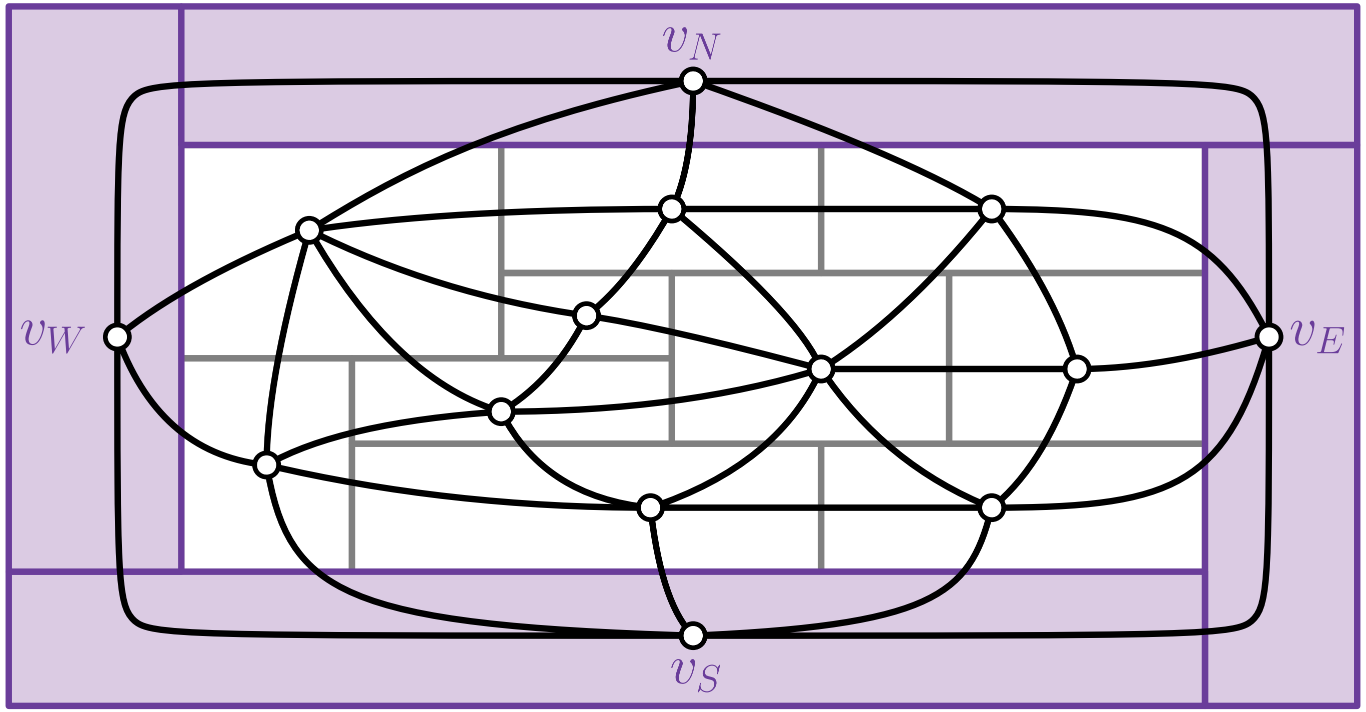

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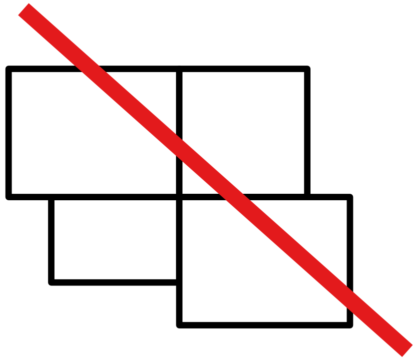

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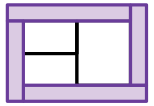

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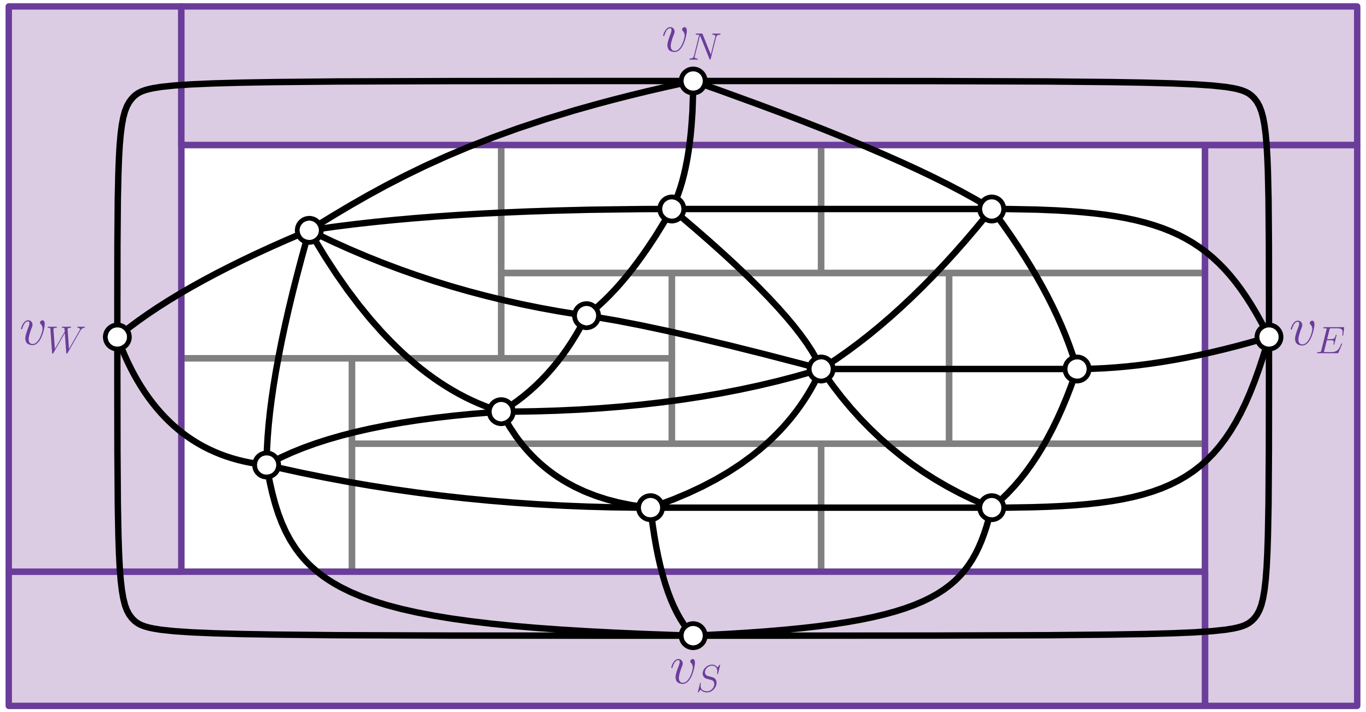

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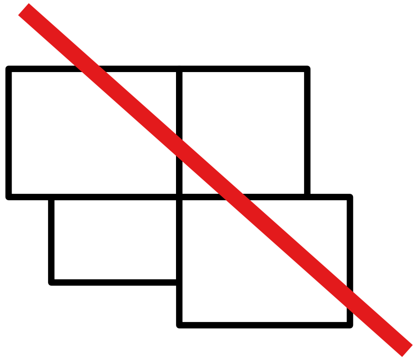

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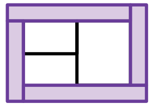

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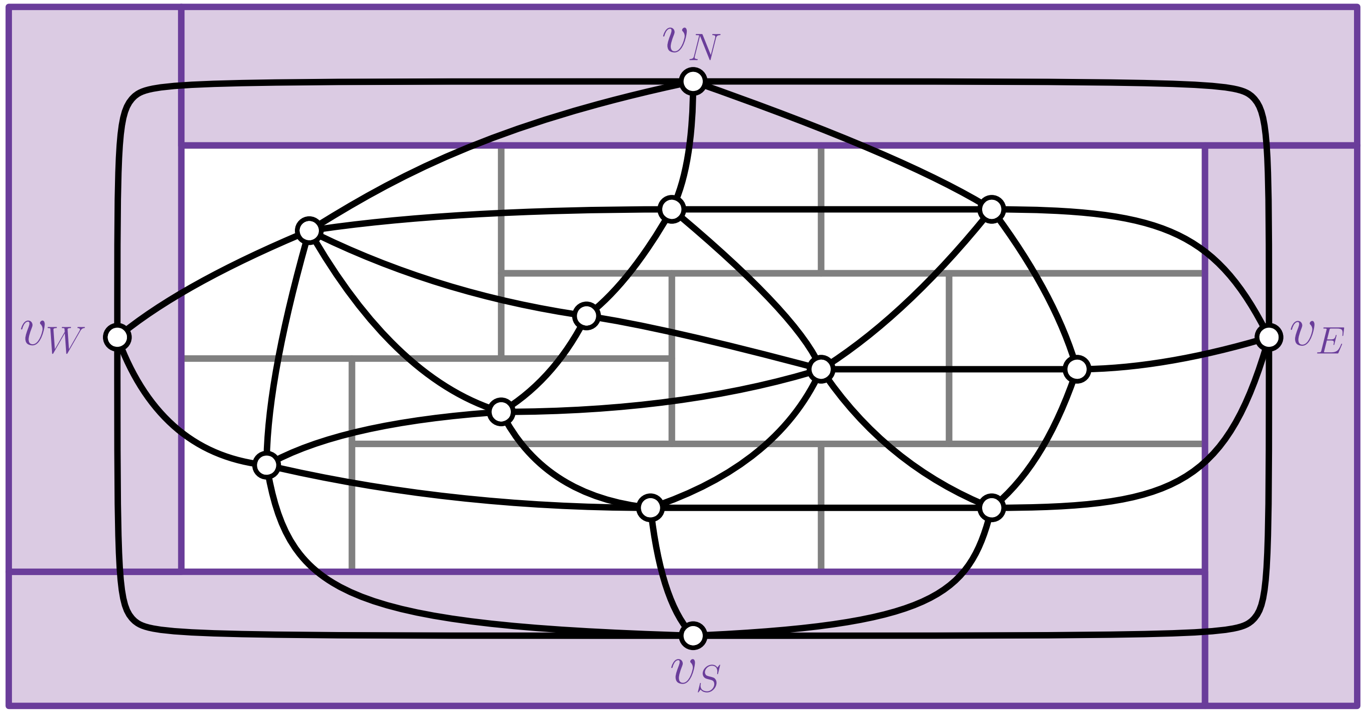

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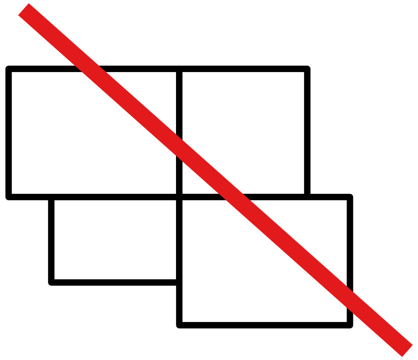

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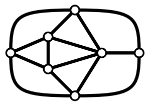
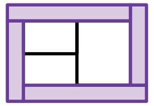
Rectangular Dual


PTP

Properly Triangulated Planar Graph G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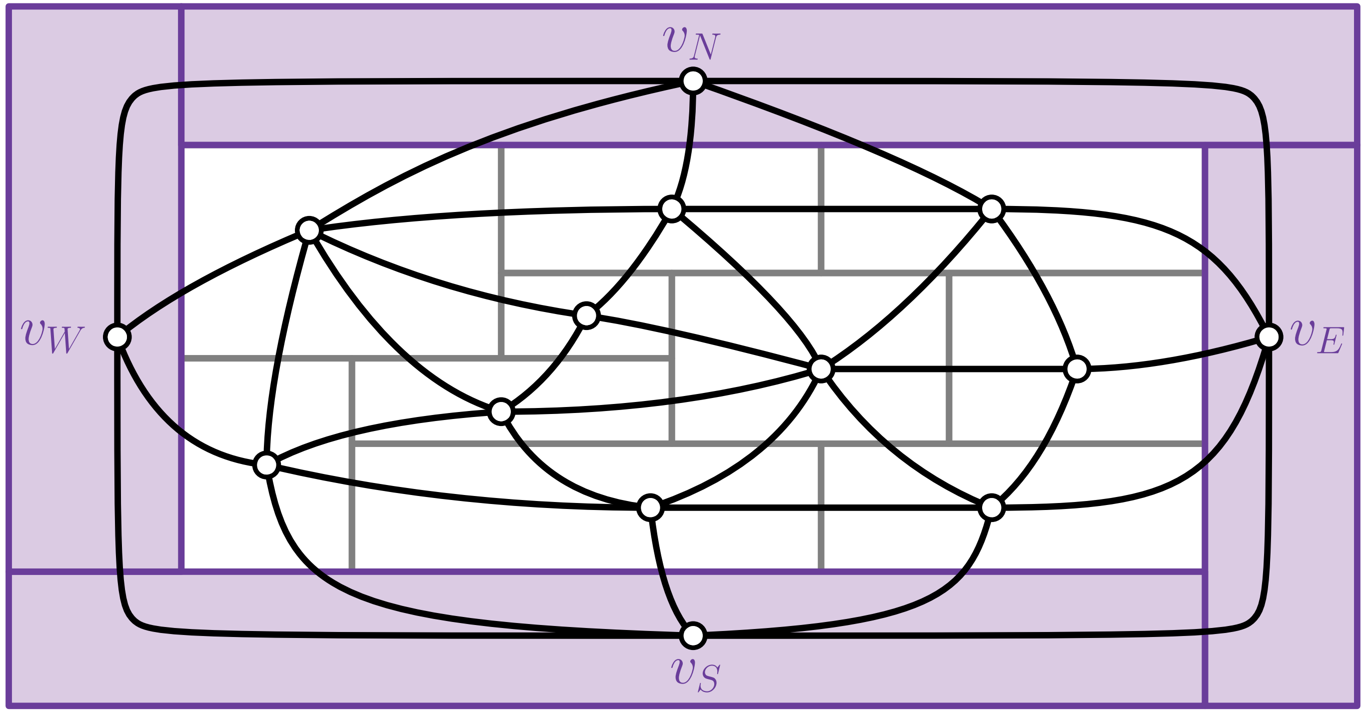

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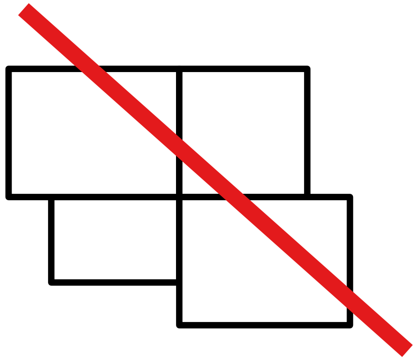

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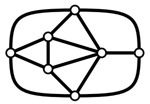

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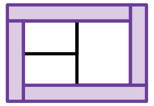

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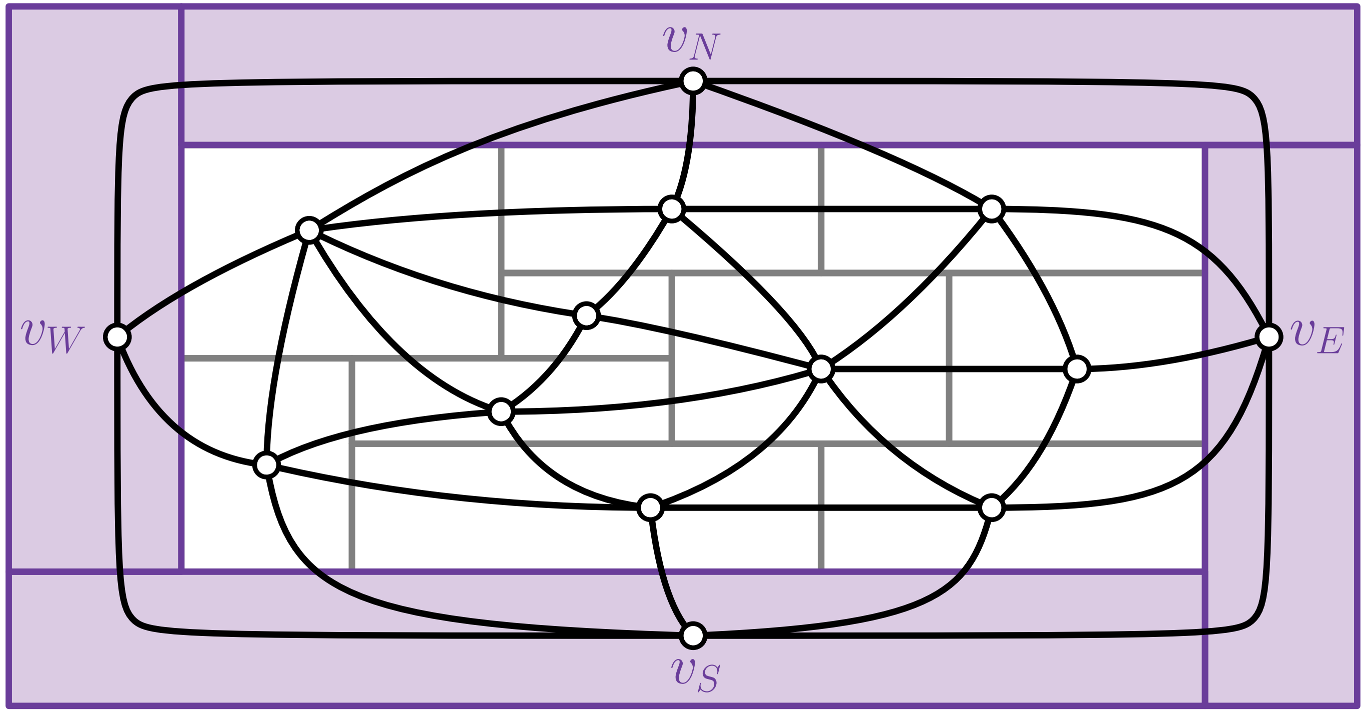

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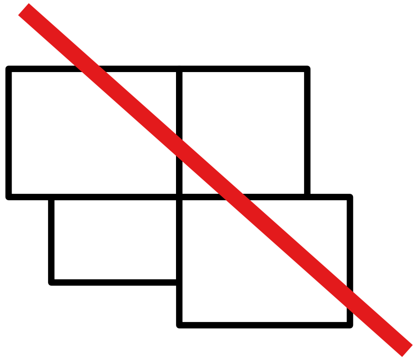

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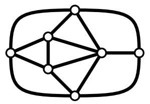

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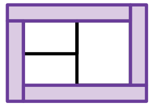

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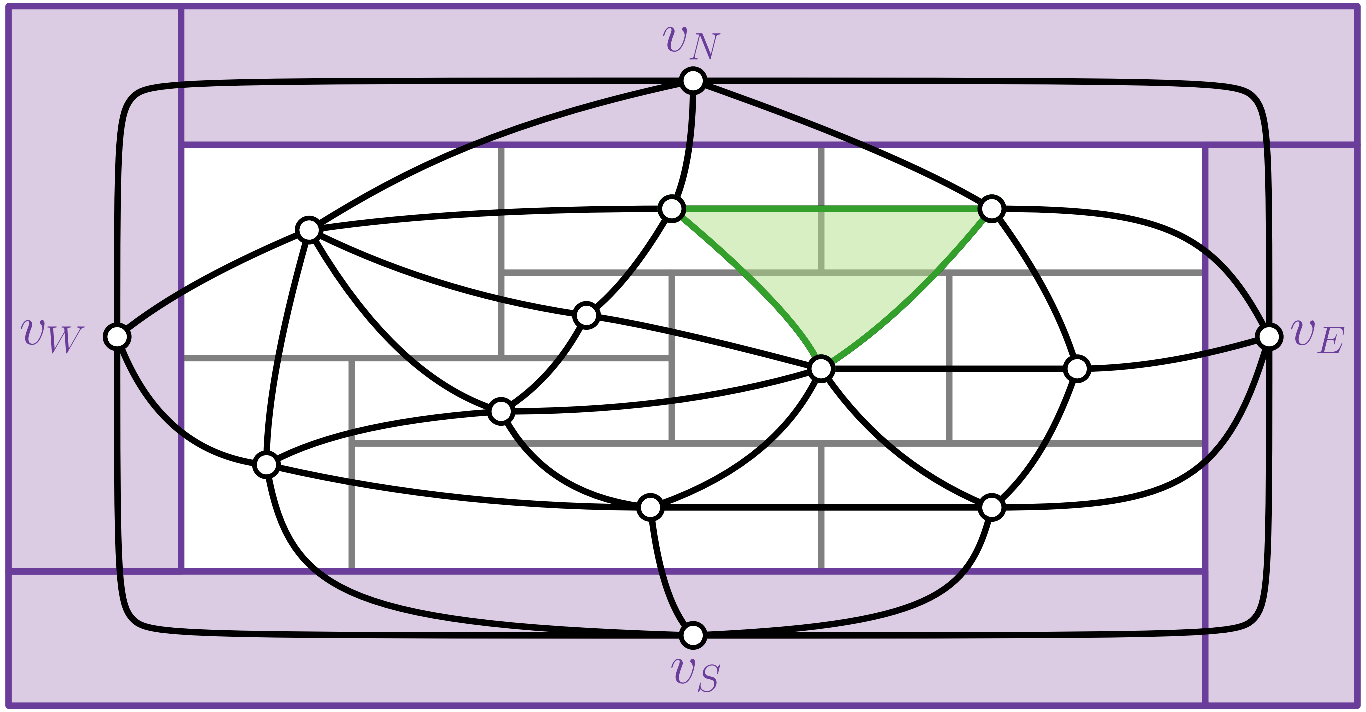

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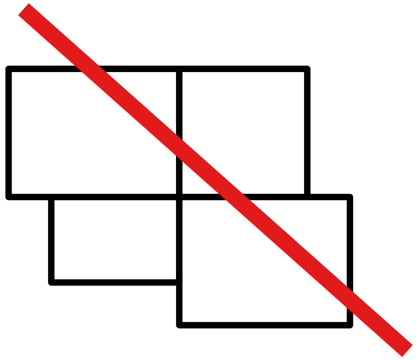

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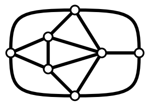

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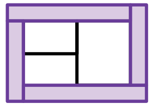

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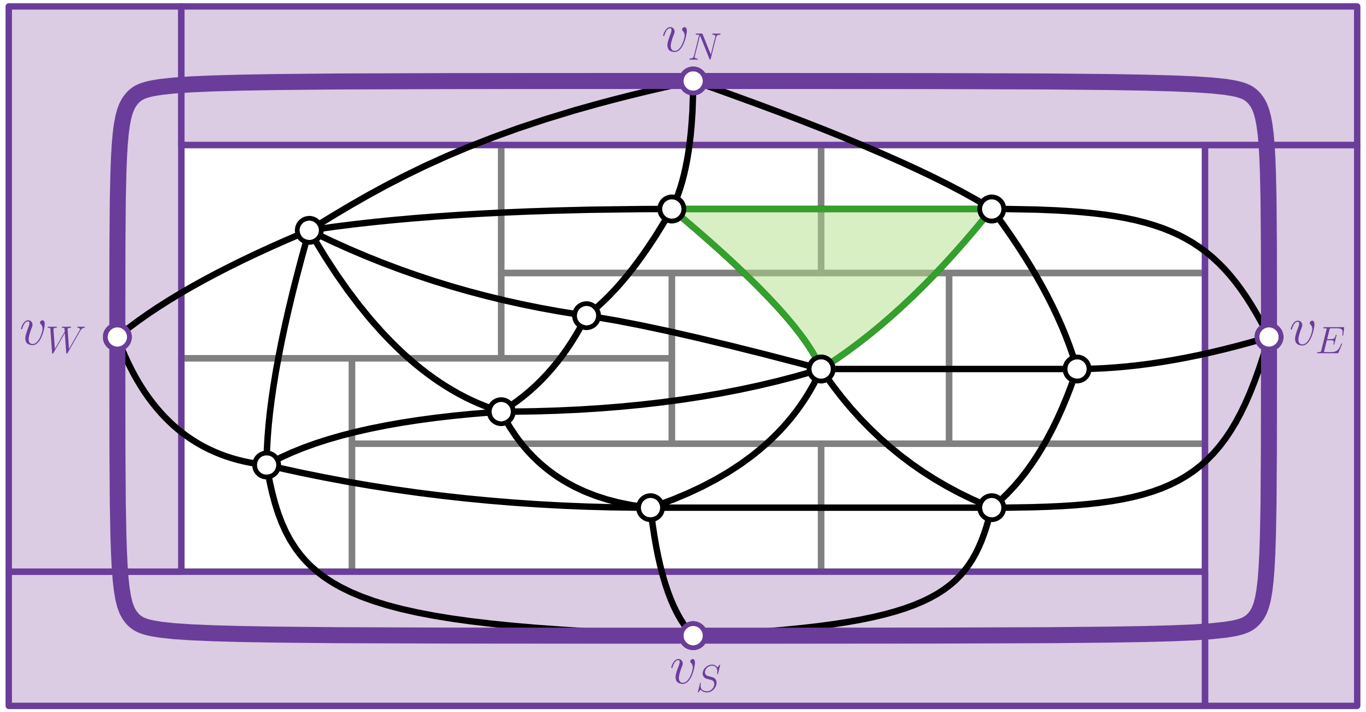

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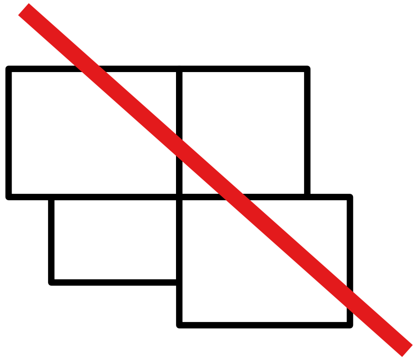

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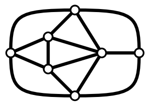

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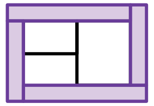

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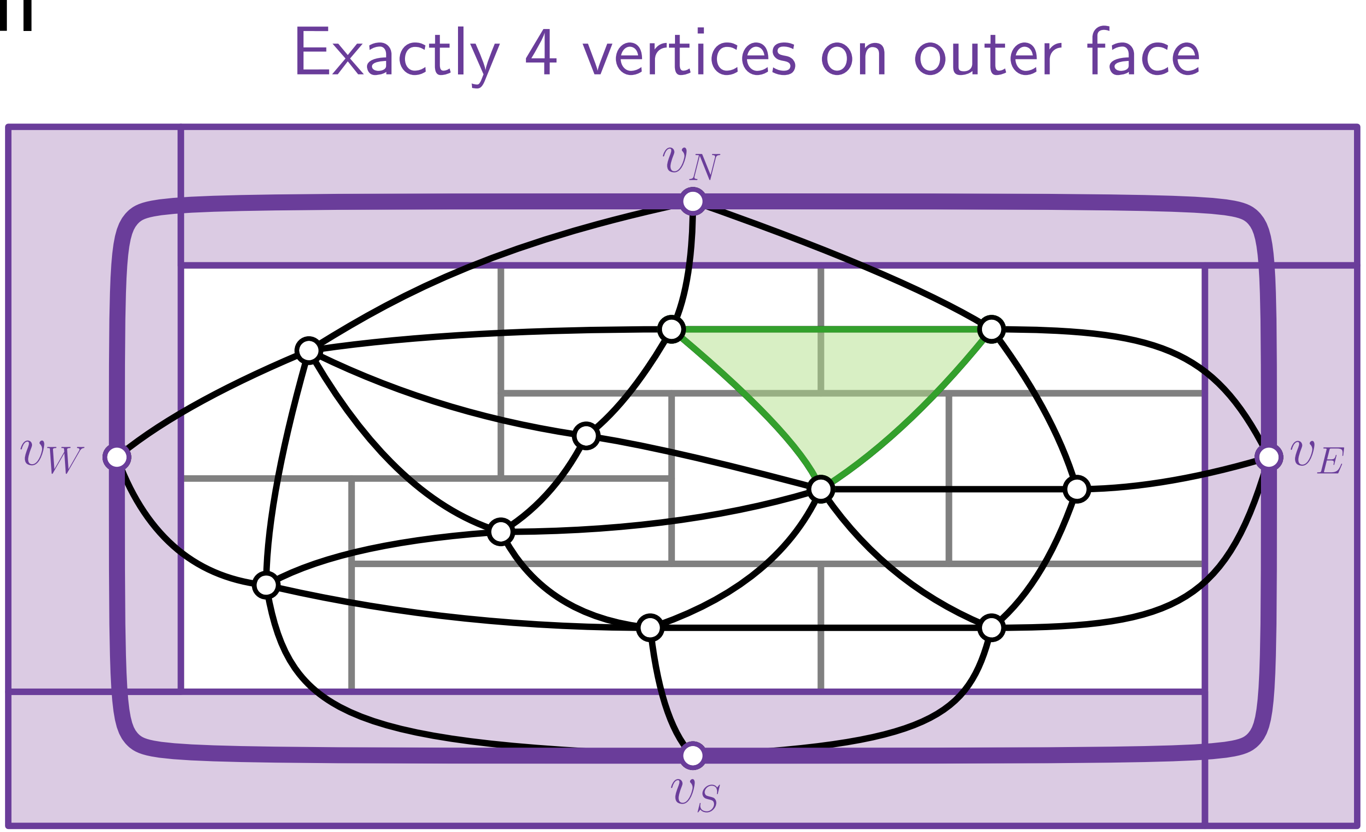

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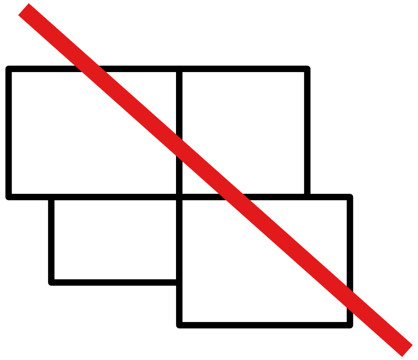

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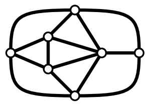

PTP

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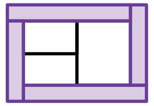
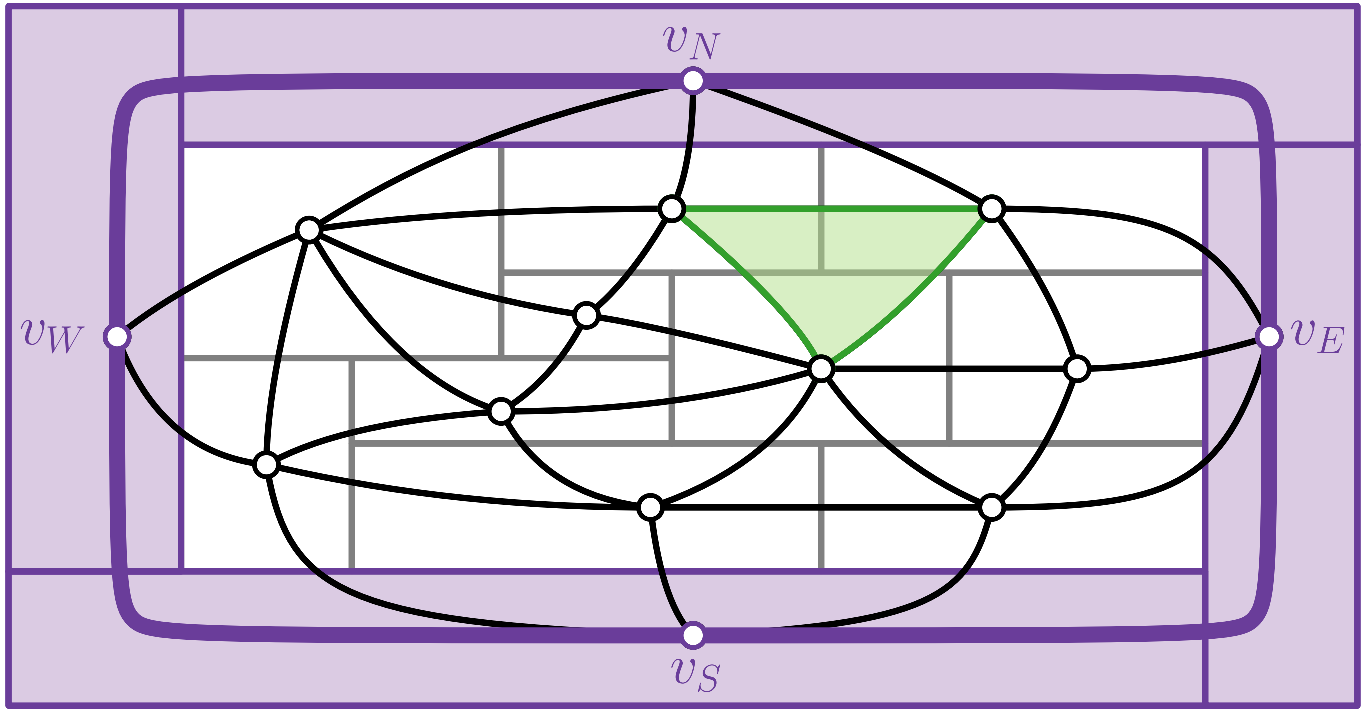


RD

Rectangular Dual $\mathcal{R}$

Exactly 4 vertices on outer face



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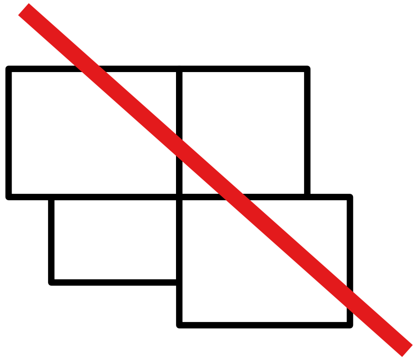

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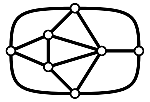

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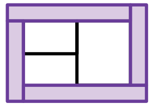

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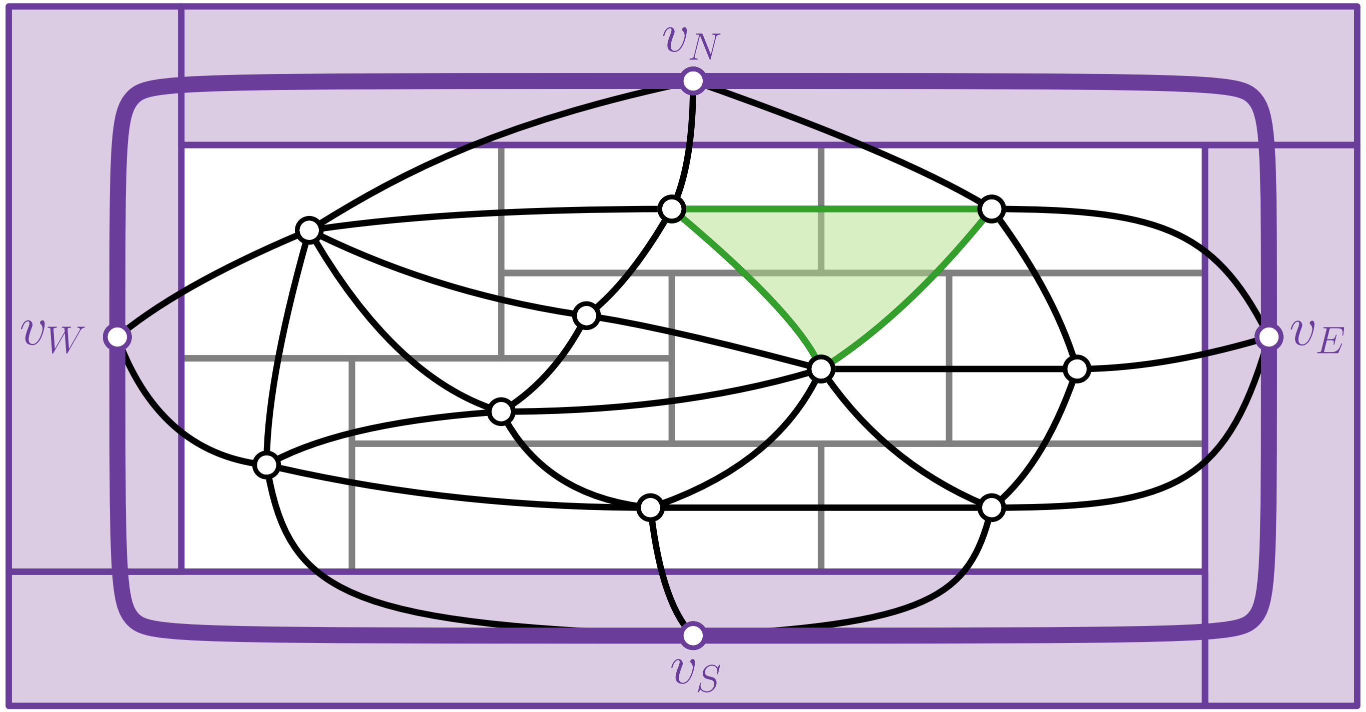

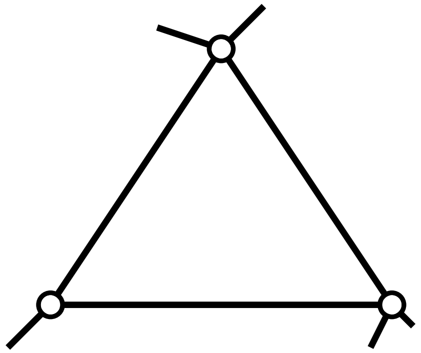

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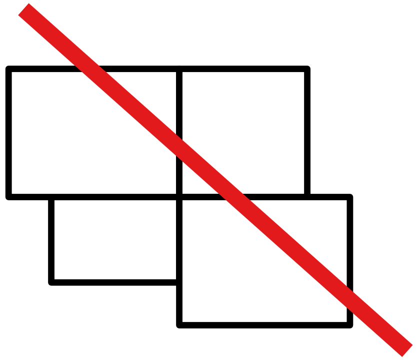

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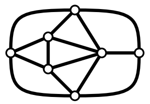

PTP

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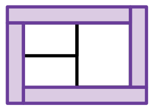

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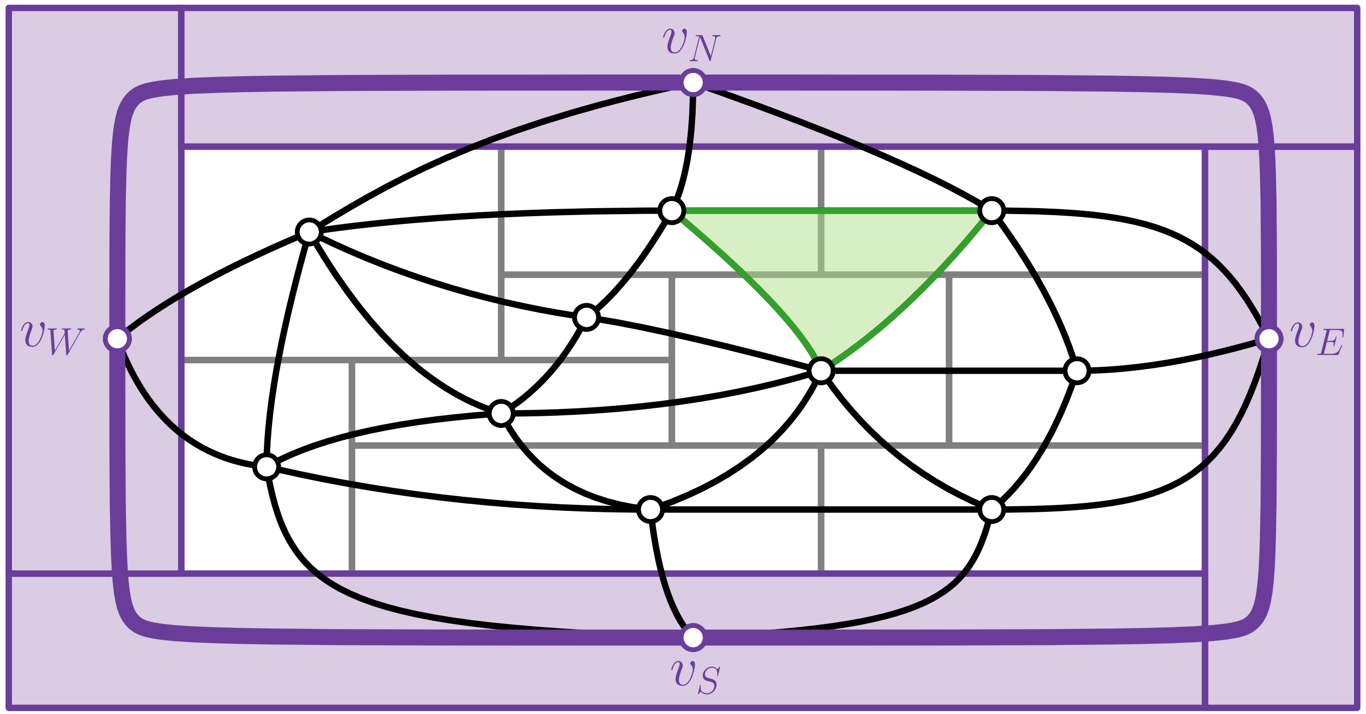

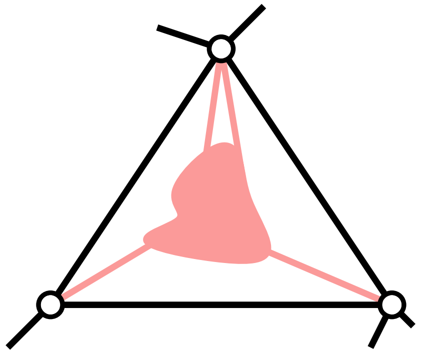

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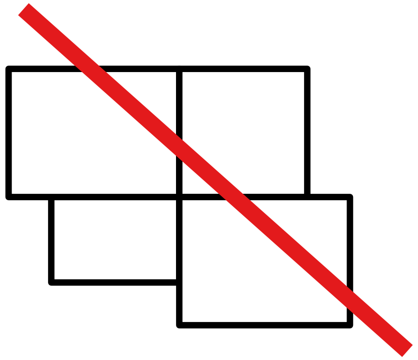

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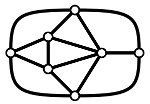
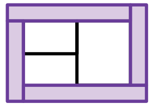


Properly Triangulated Planar Graph G



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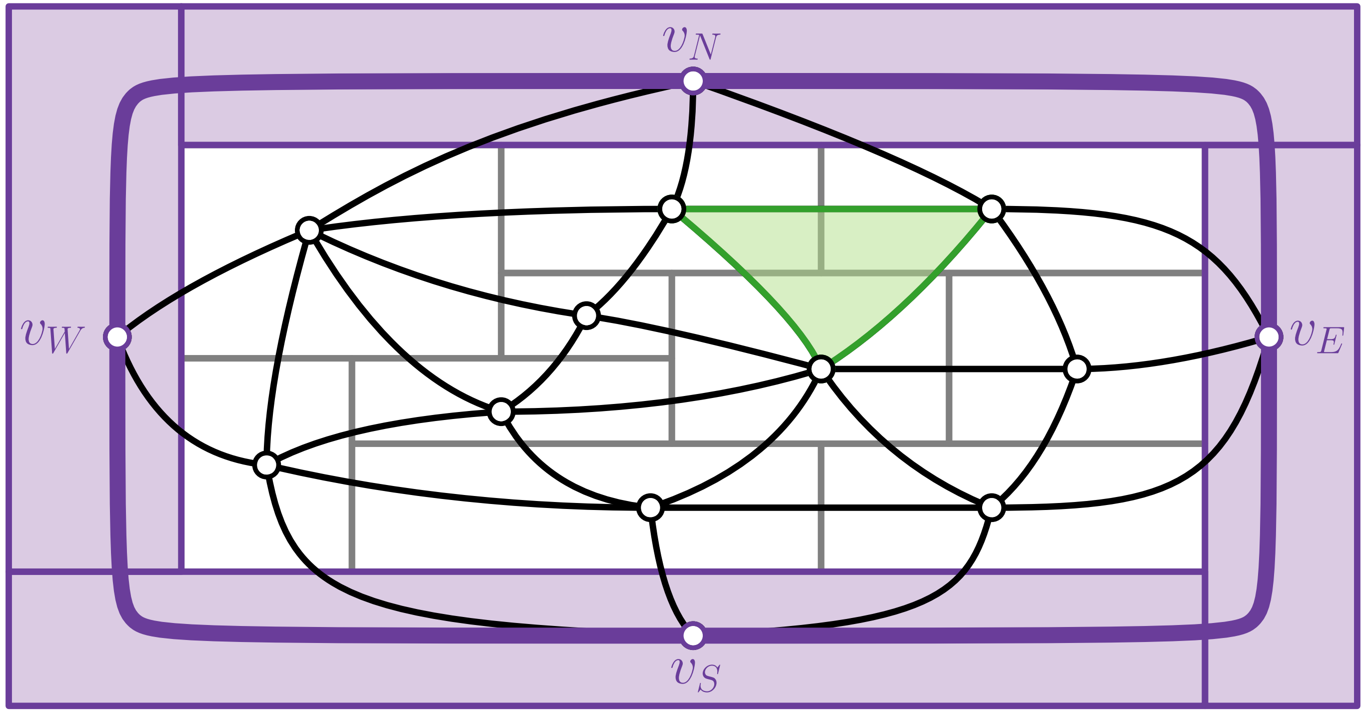

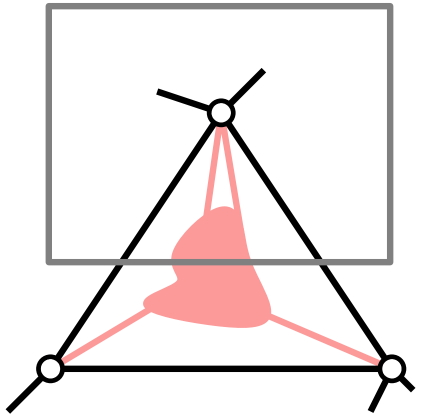

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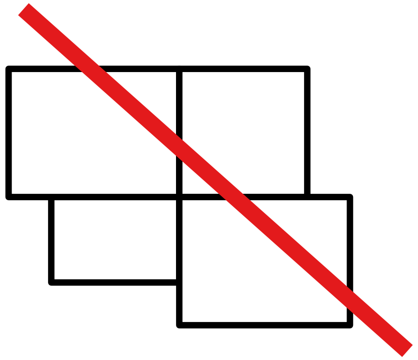

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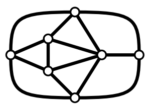

PTP

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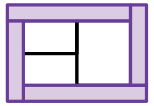

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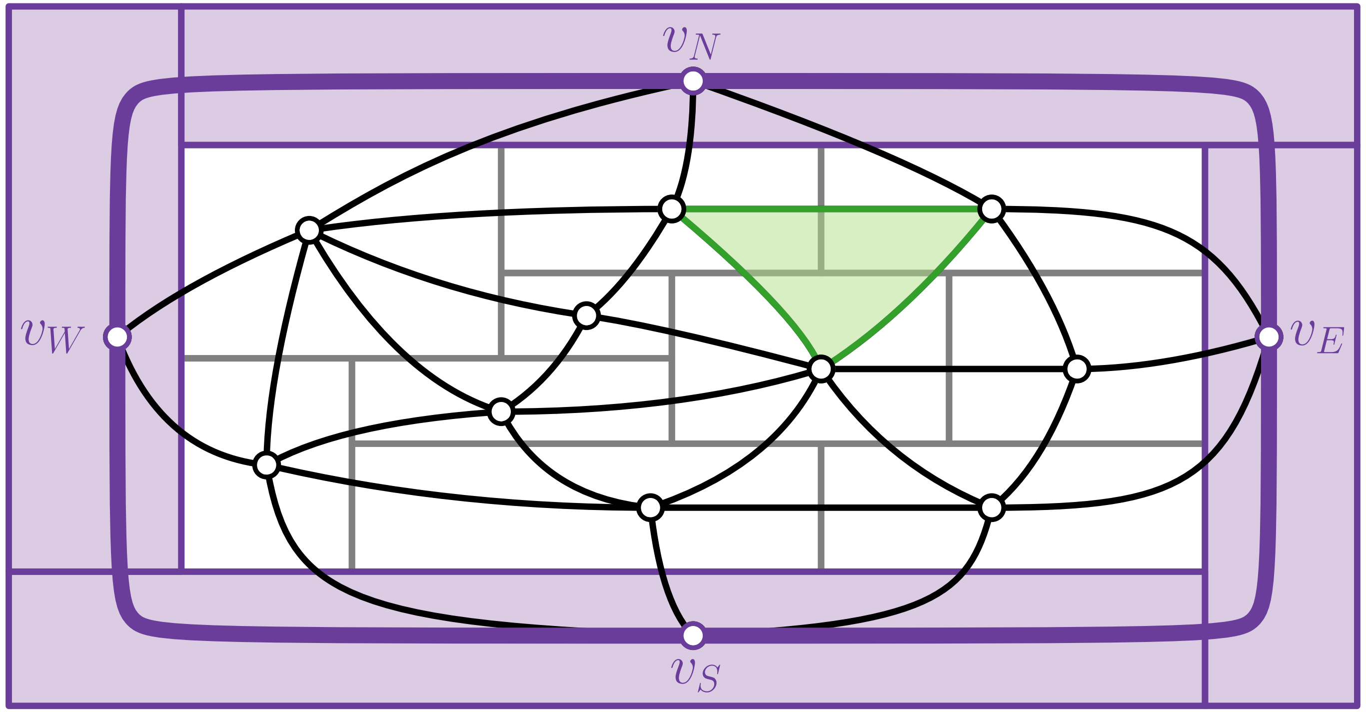

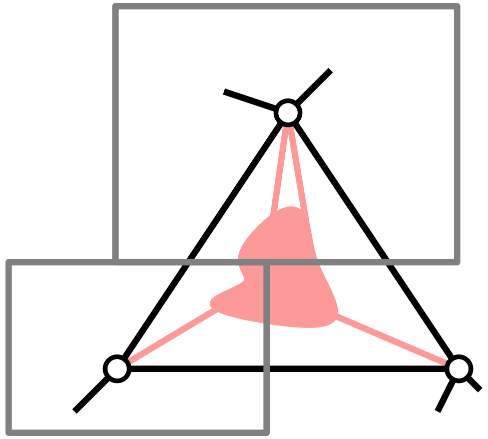

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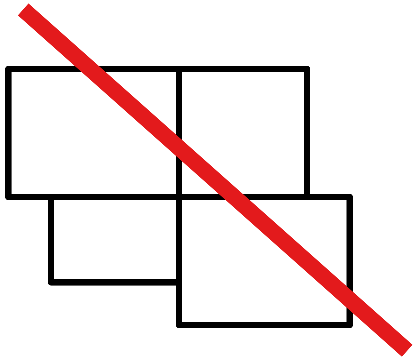

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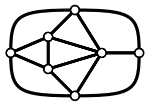

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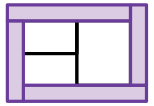

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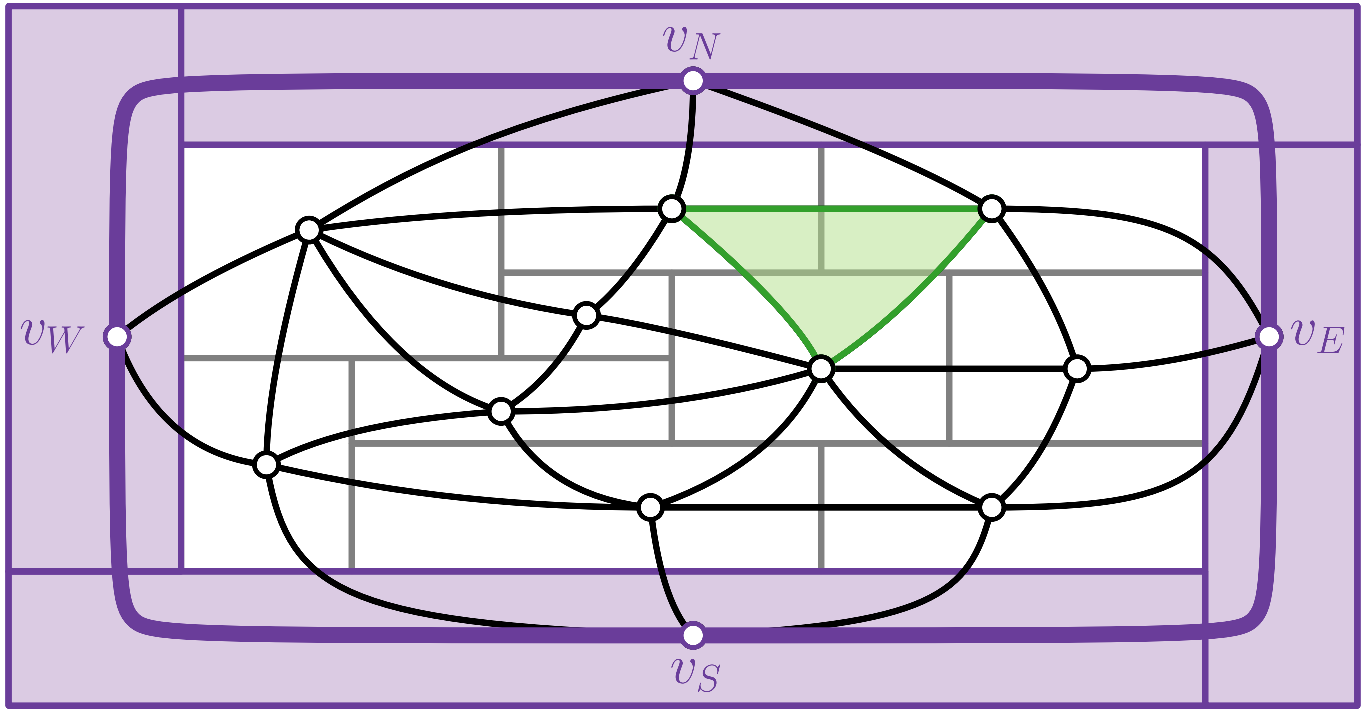
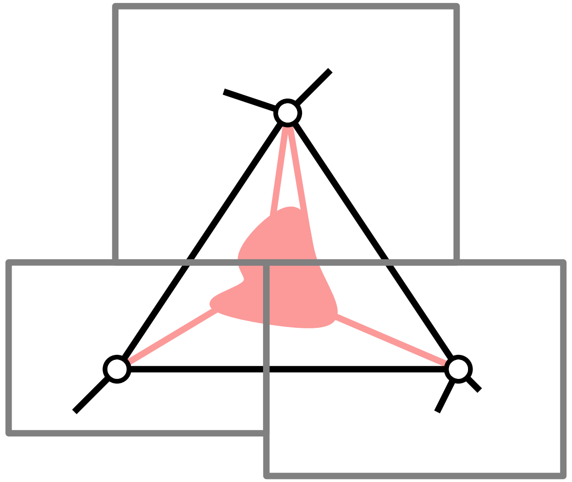
Exactly 4 vertices on outer face

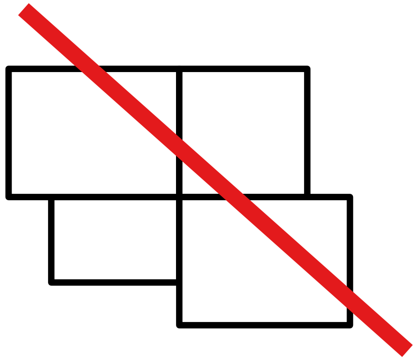




no separating triangle

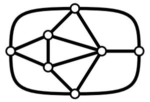
A **rectangular dual** of a graph G is a contact representation with axis aligned rectangles s.t.

- no four rectangles share a point, and
- the union of all rectangles is a rectangle

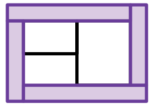


Theorem. [Koźmiński, Kinnen '85]
A graph G has a rectangular dual $\mathcal{R}$ if and only if G is a PTP graph.

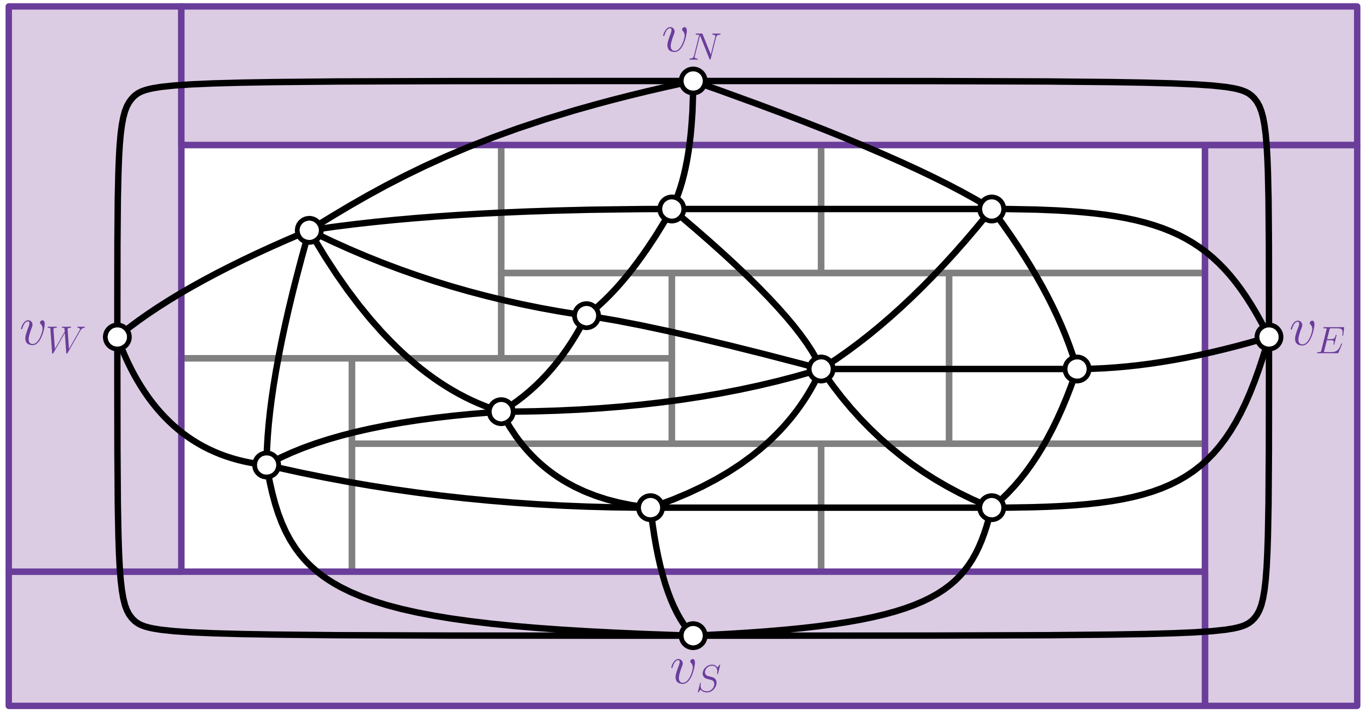
Regular Edge Labeling



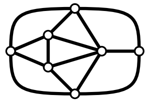
Properly Triangulated Planar Graph G



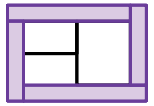
Rectangular Dual $\mathcal{R}$



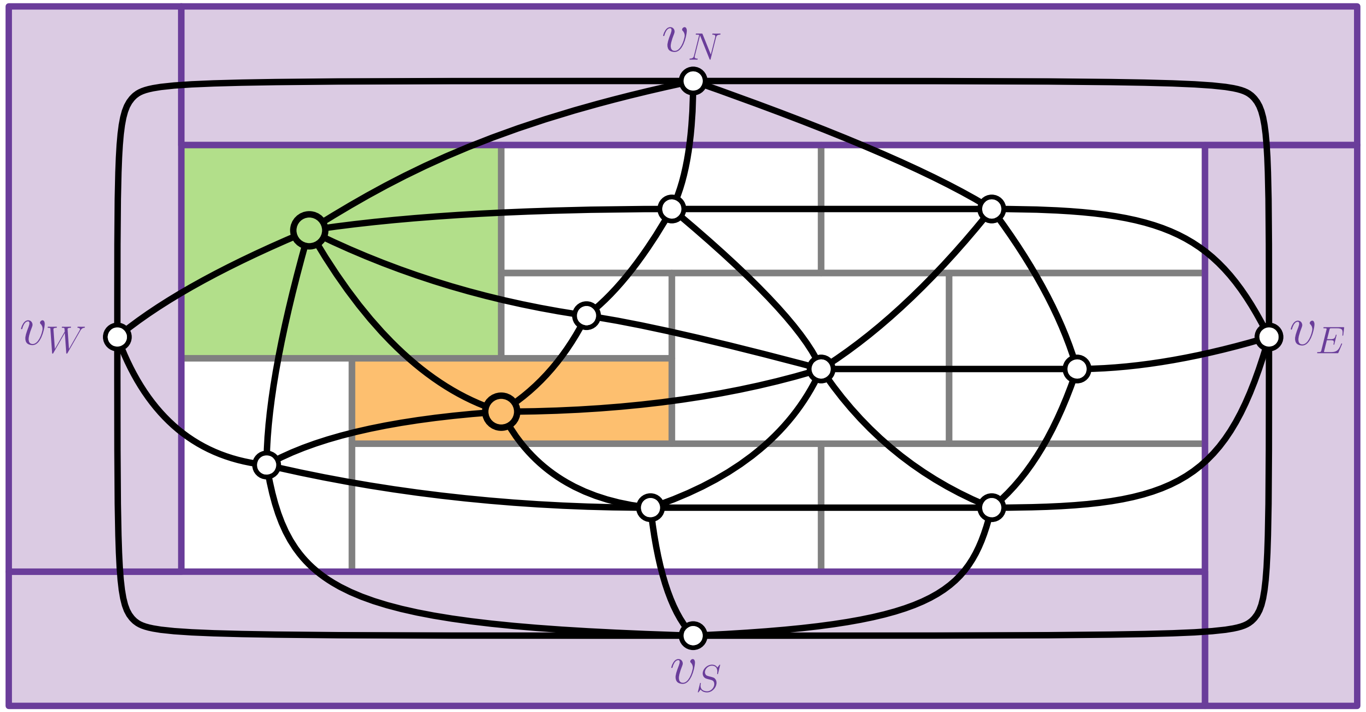
Regular Edge Labeling



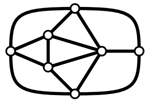
Properly Triangulated Planar Graph G



Rectangular Dual $\mathcal{R}$

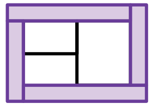


Regular Edge Labeling



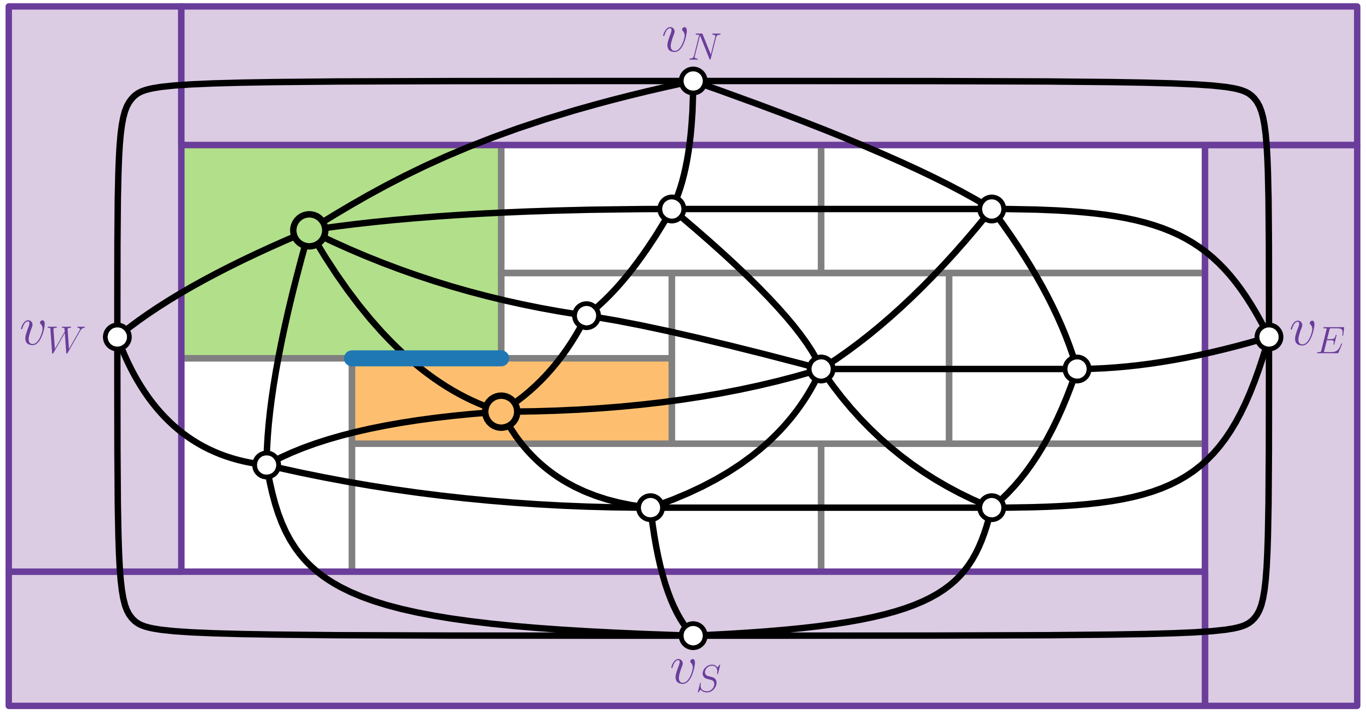
PTP

Properly Triangulated Planar Graph G

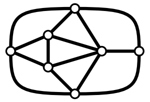


RD

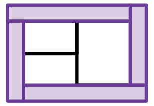
Rectangular Dual $\mathcal{R}$



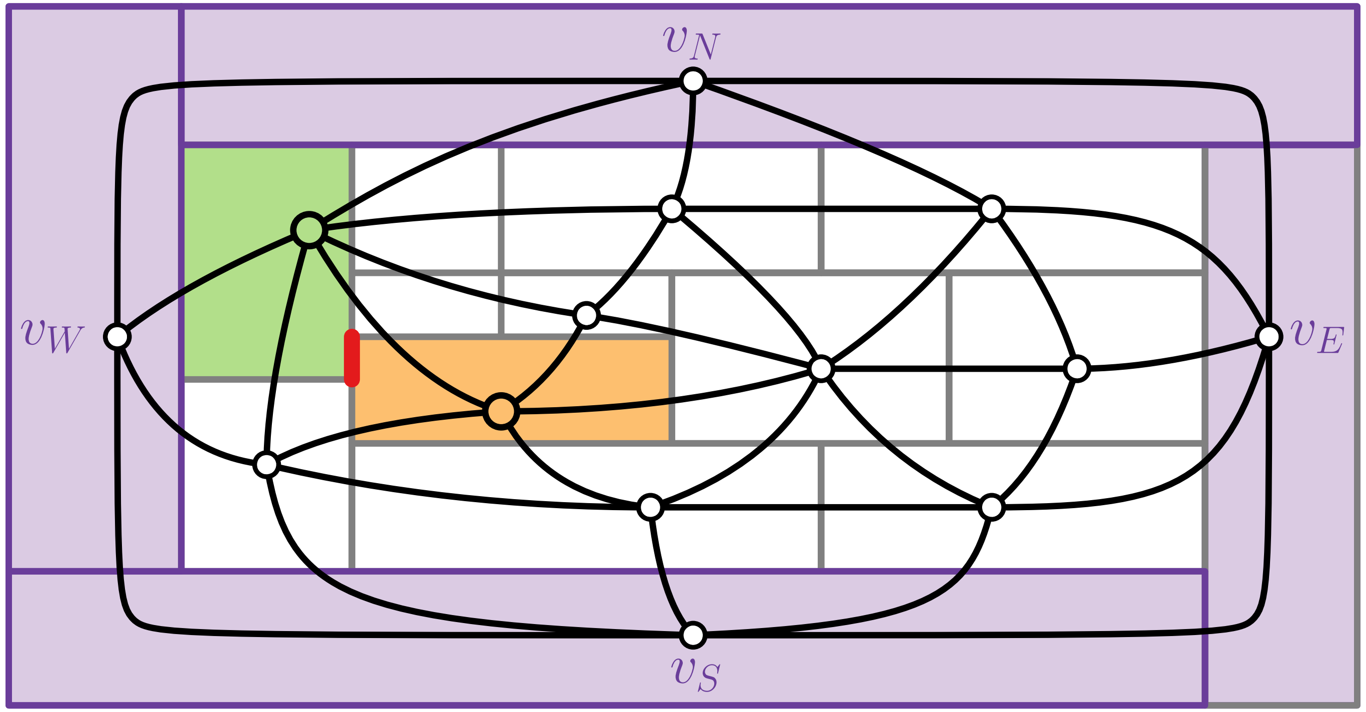
Regular Edge Labeling



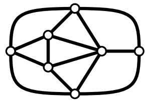
Properly Triangulated Planar Graph G



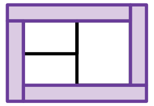
Rectangular Dual $\mathcal{R}$



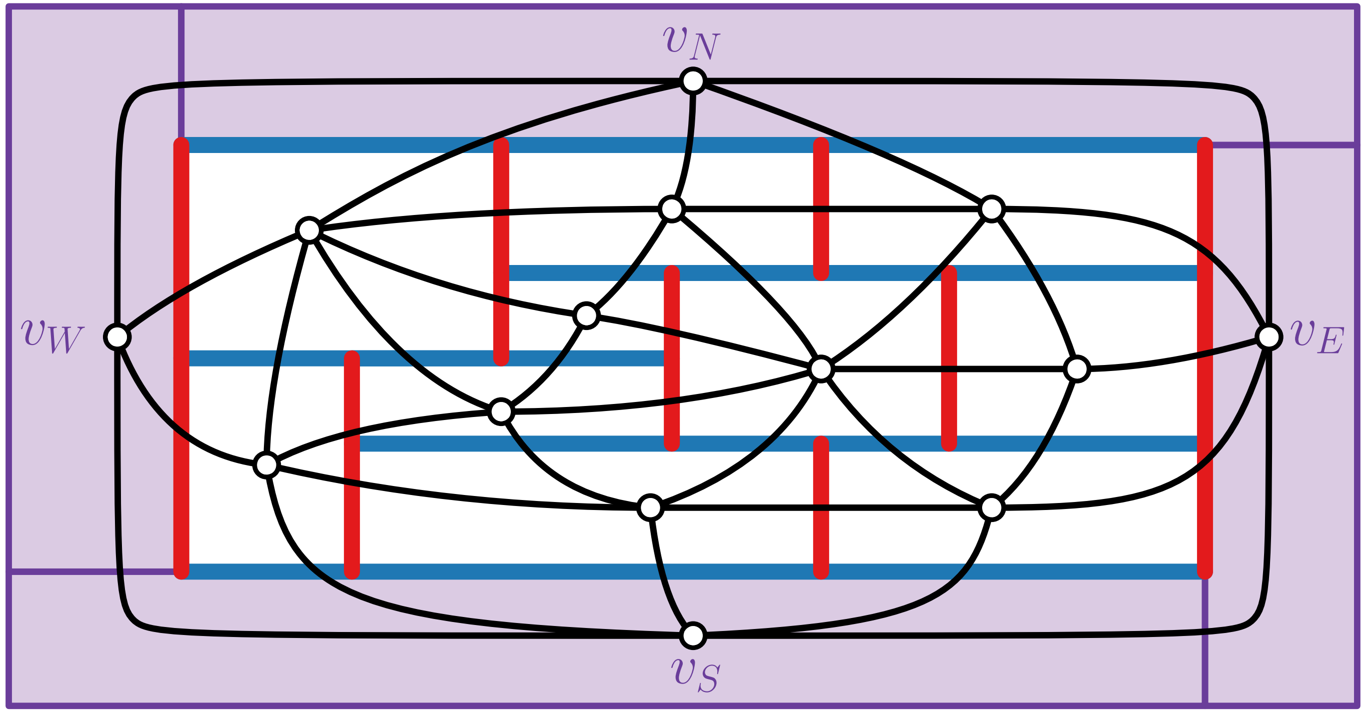
Regular Edge Labeling



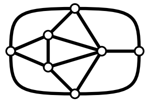
Properly Triangulated Planar Graph G



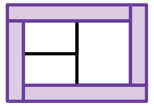
Rectangular Dual $\mathcal{R}$



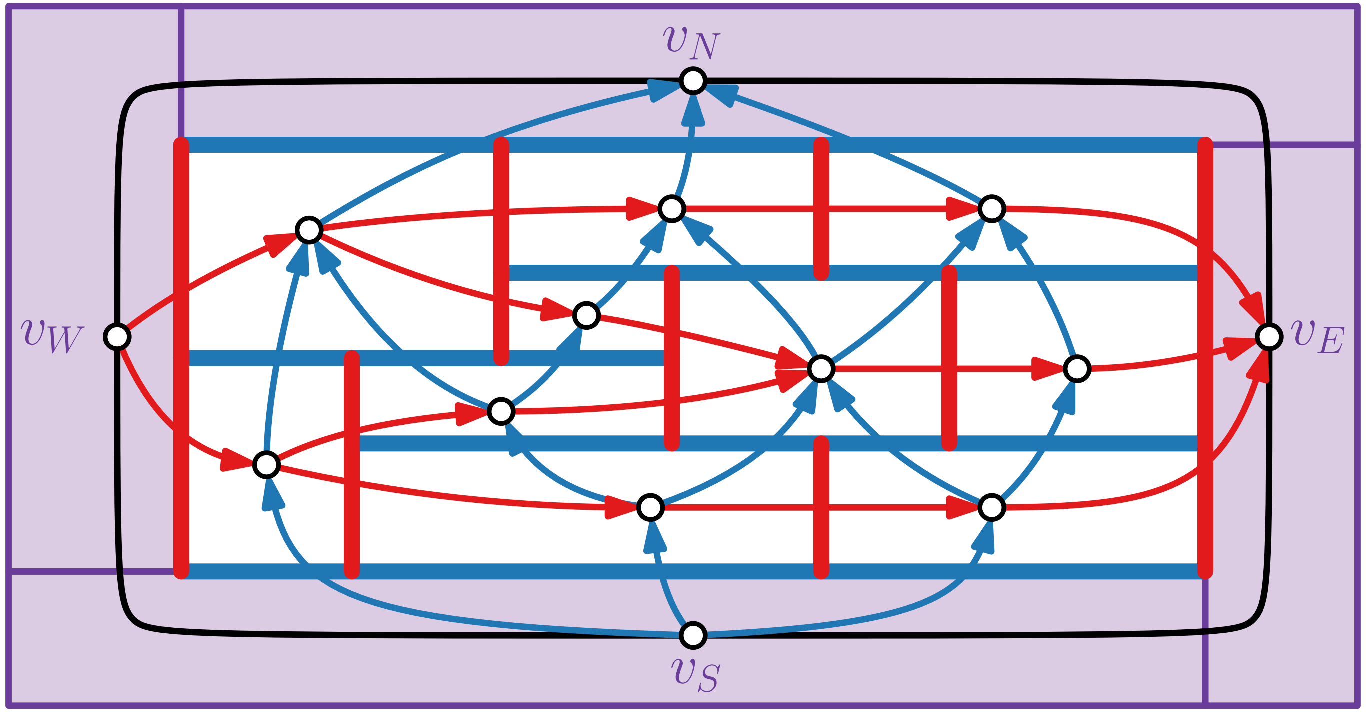
Regular Edge Labeling



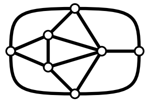
Properly Triangulated Planar Graph G



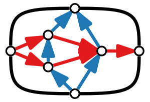
Rectangular Dual $\mathcal{R}$



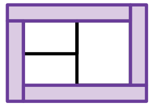
Regular Edge Labeling



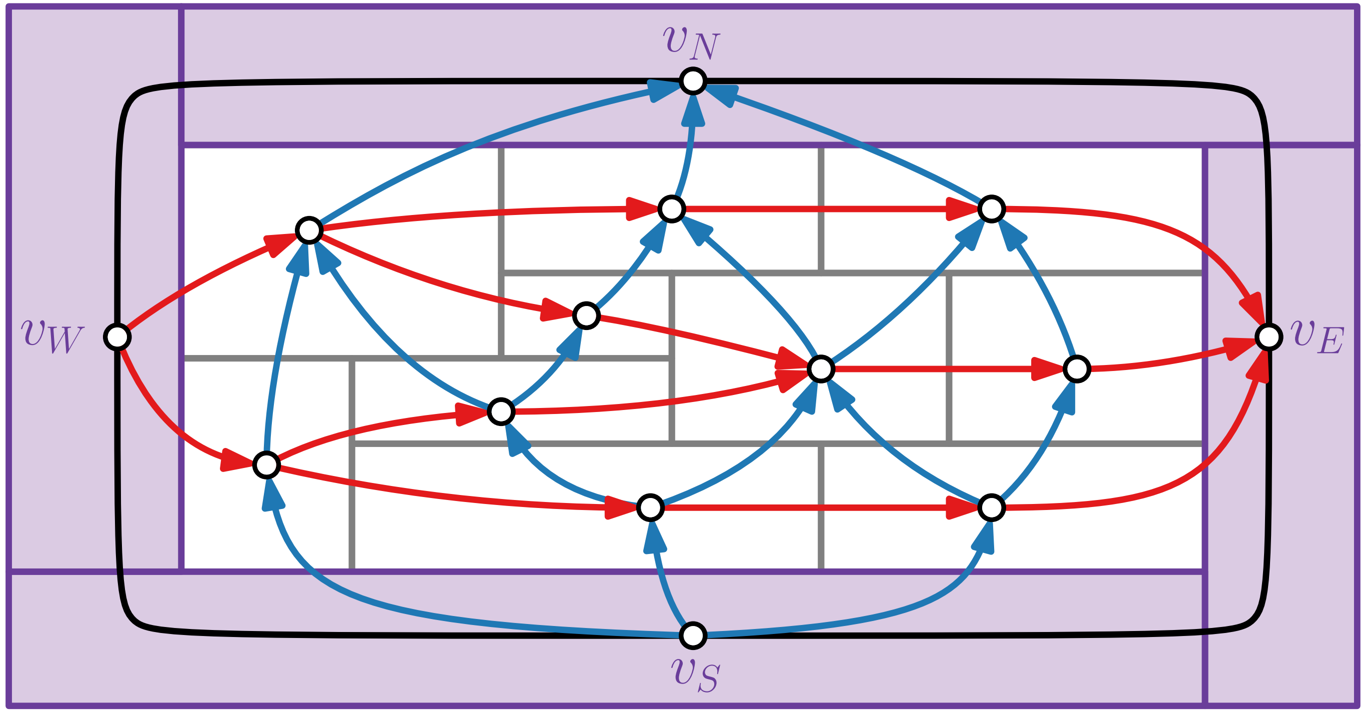
Properly Triangulated Planar Graph G



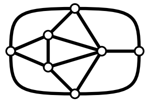
Regular Edge Labeling



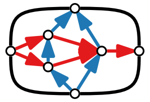
Rectangular Dual $\mathcal{R}$



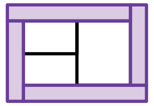
Regular Edge Labeling



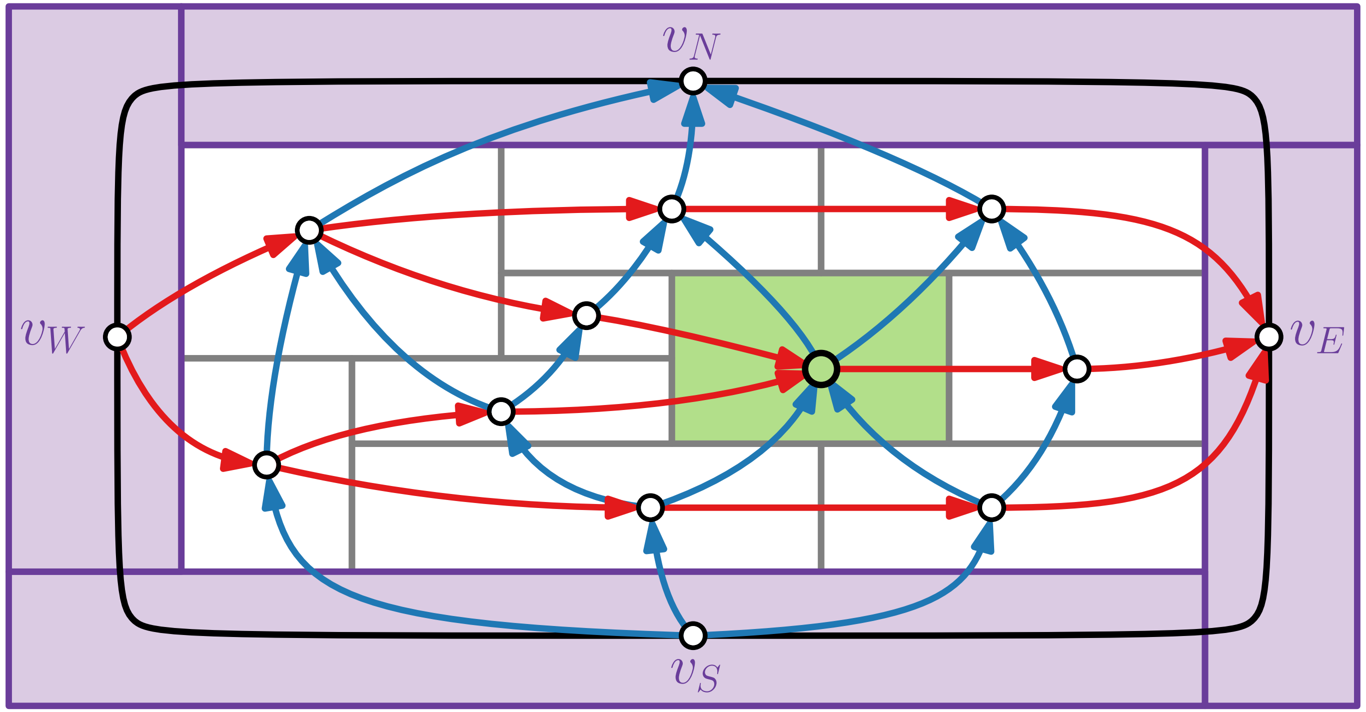
Properly Triangulated Planar Graph G



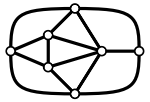
Regular Edge Labeling



Rectangular Dual $\mathcal{R}$

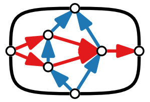


Regular Edge Labeling



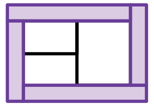
Properly Triangulated Planar Graph G

PTP



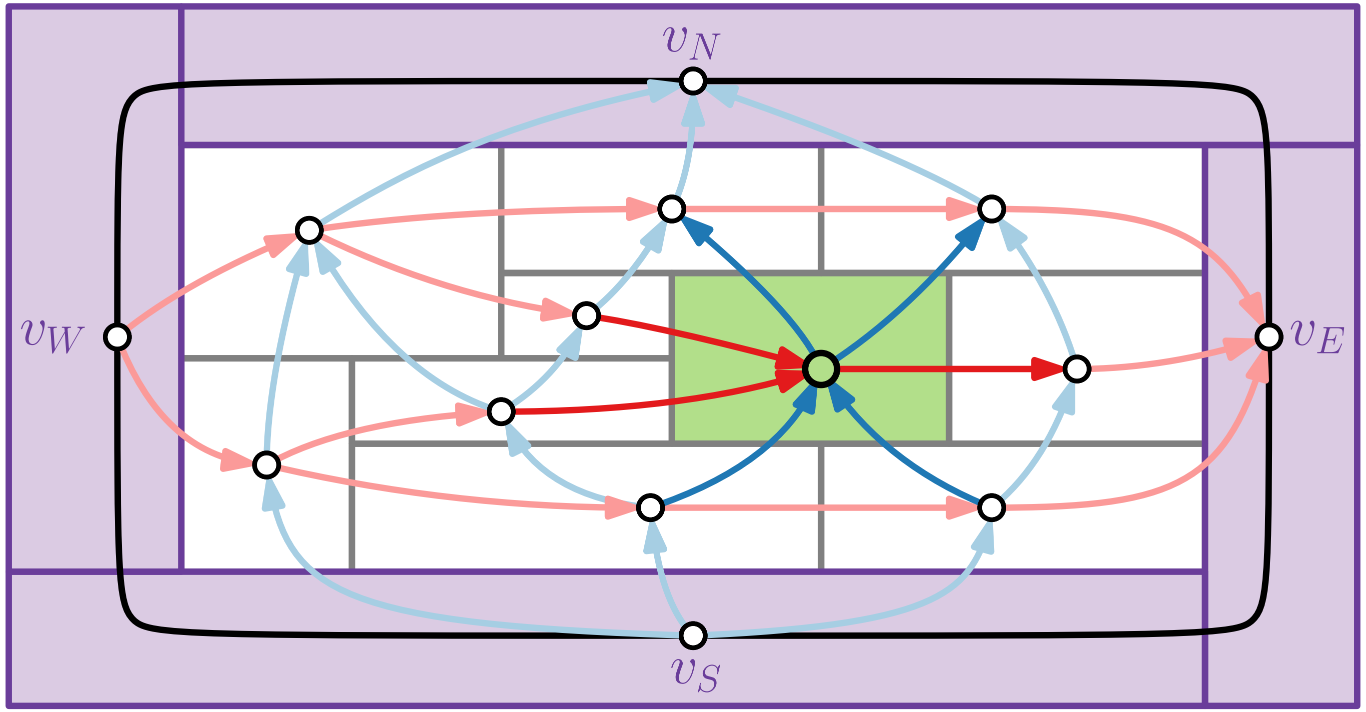
Regular Edge Labeling

REL

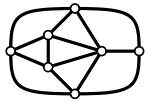


Rectangular Dual $\mathcal{R}$

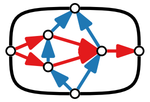
RD



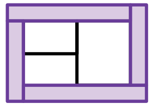
Regular Edge Labeling



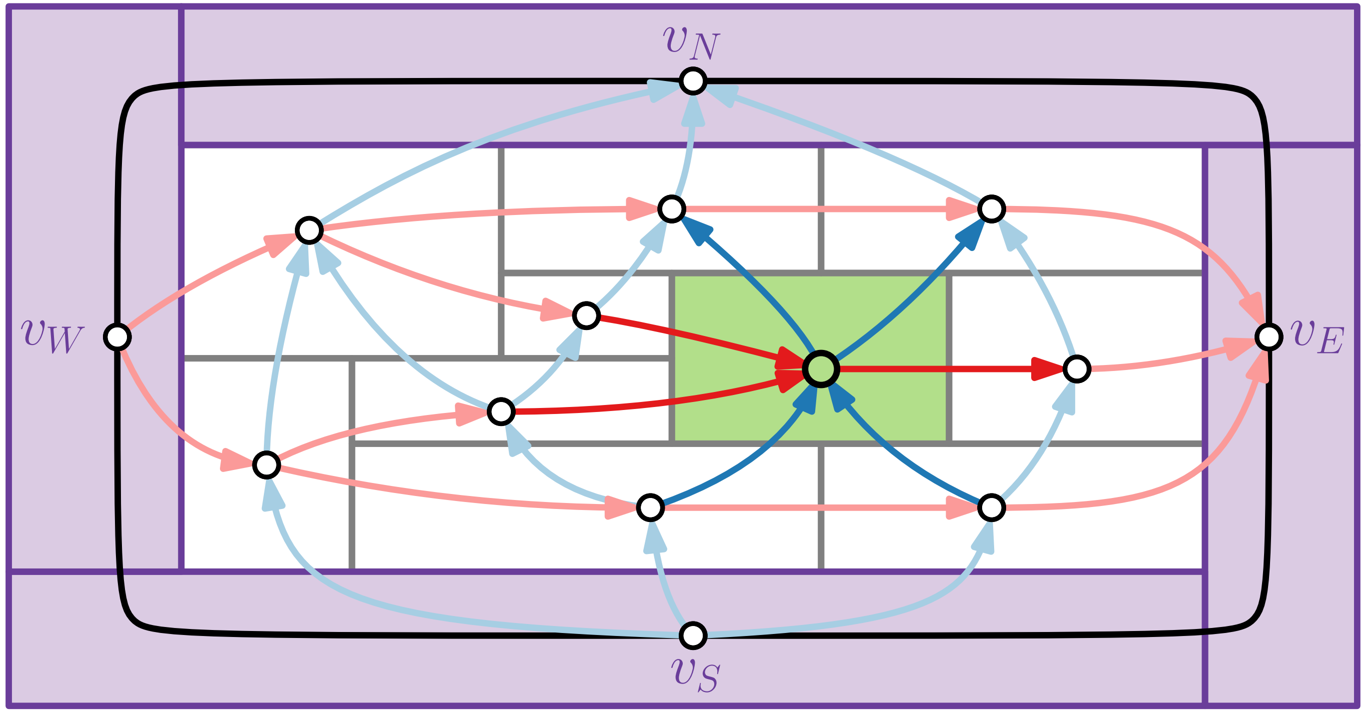
Properly Triangulated Planar Graph G

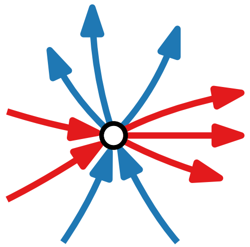


Regular Edge Labeling



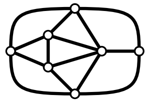
Rectangular Dual $\mathcal{R}$



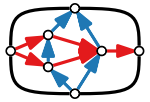


for every inner vertex

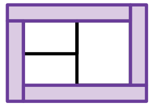
Regular Edge Labeling


PTP

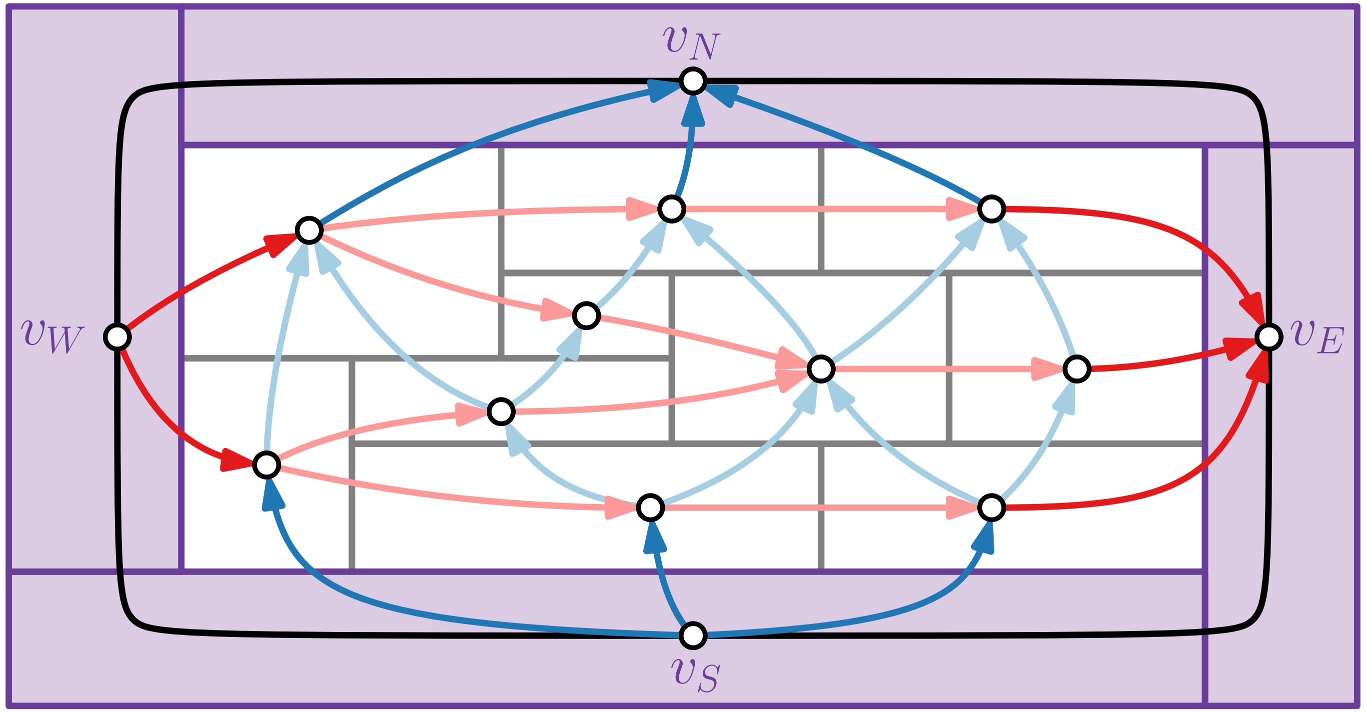
Properly Triangulated Planar Graph G

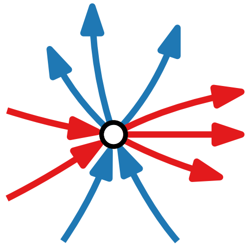

REL

Regular Edge Labeling


RD

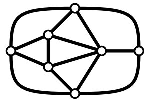
Rectangular Dual $\mathcal{R}$



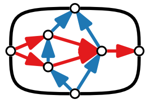


for every inner vertex

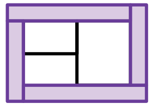
Regular Edge Labeling



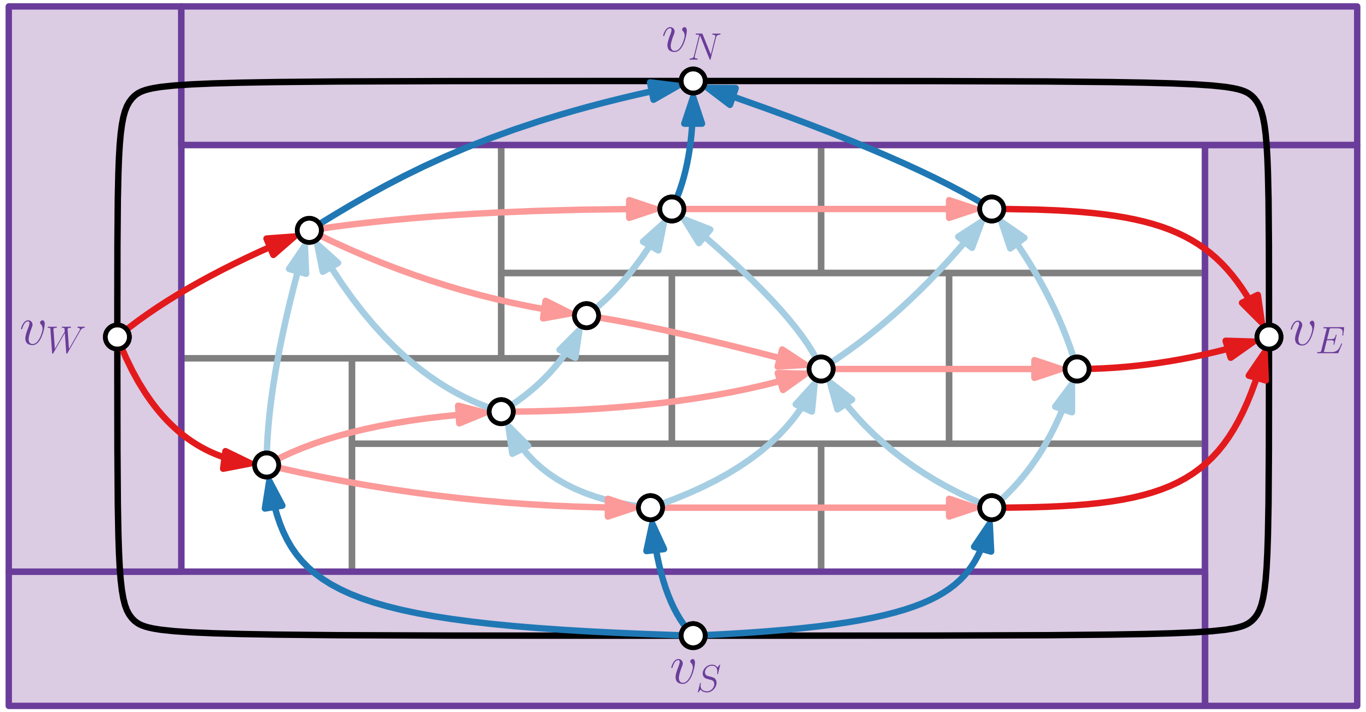
Properly Triangulated Planar Graph G

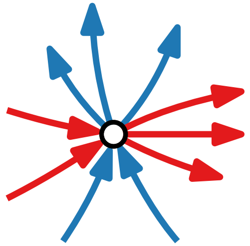


Regular Edge Labeling

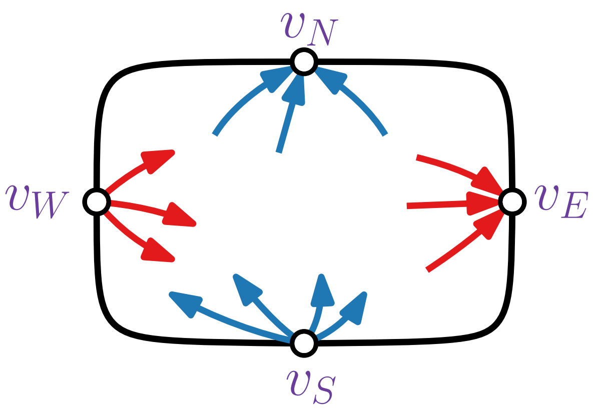


Rectangular Dual $\mathcal{R}$



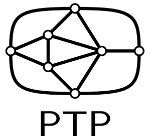


for every inner vertex

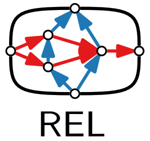


for four outer vertices

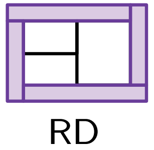
Regular Edge Labeling



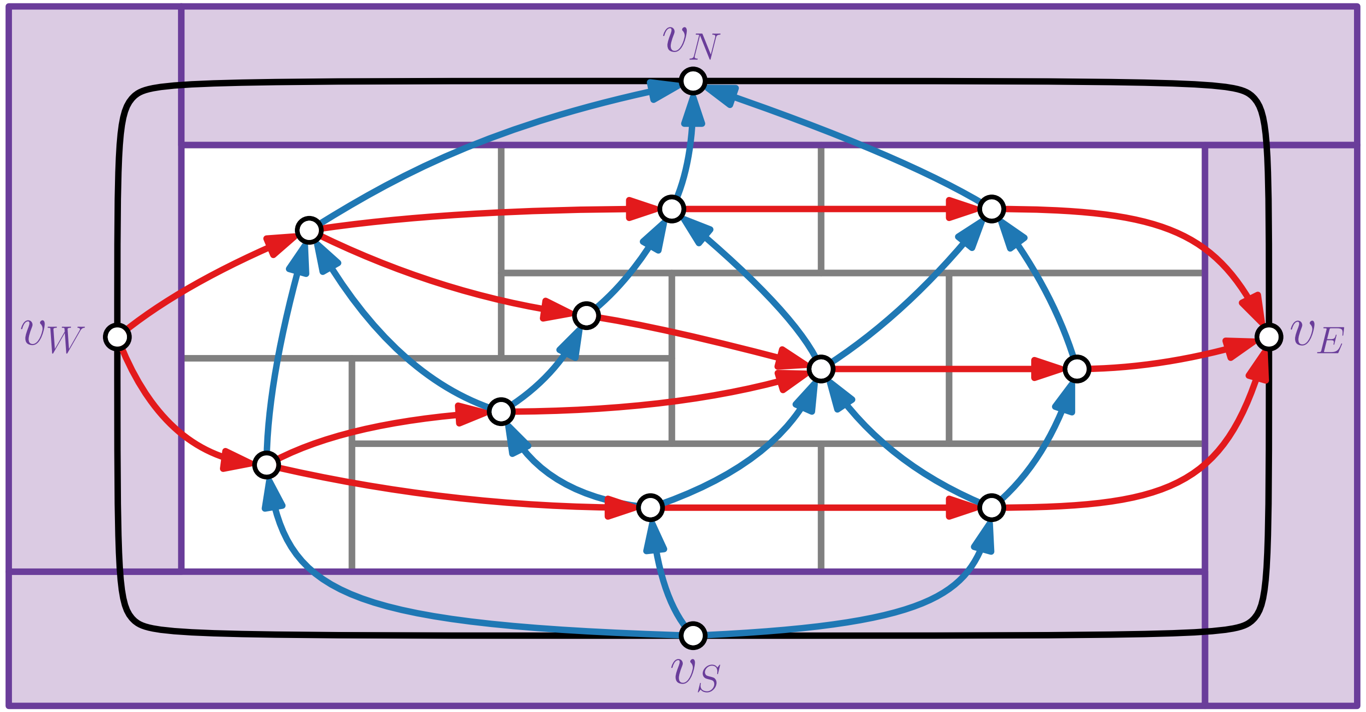
Properly Triangulated Planar Graph G

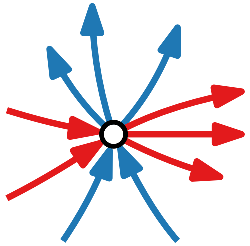


Regular Edge Labeling

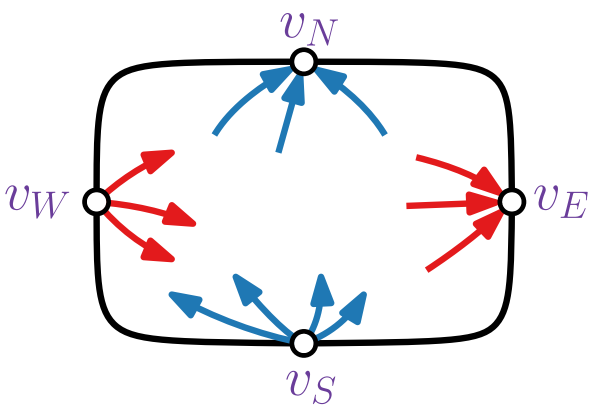


Rectangular Dual $\mathcal{R}$



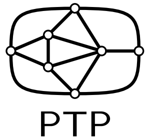


for every inner vertex

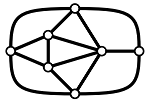


for four outer vertices

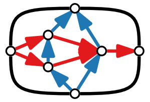
[Kant, He '94]: In linear time



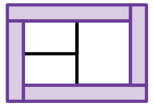
Regular Edge Labeling



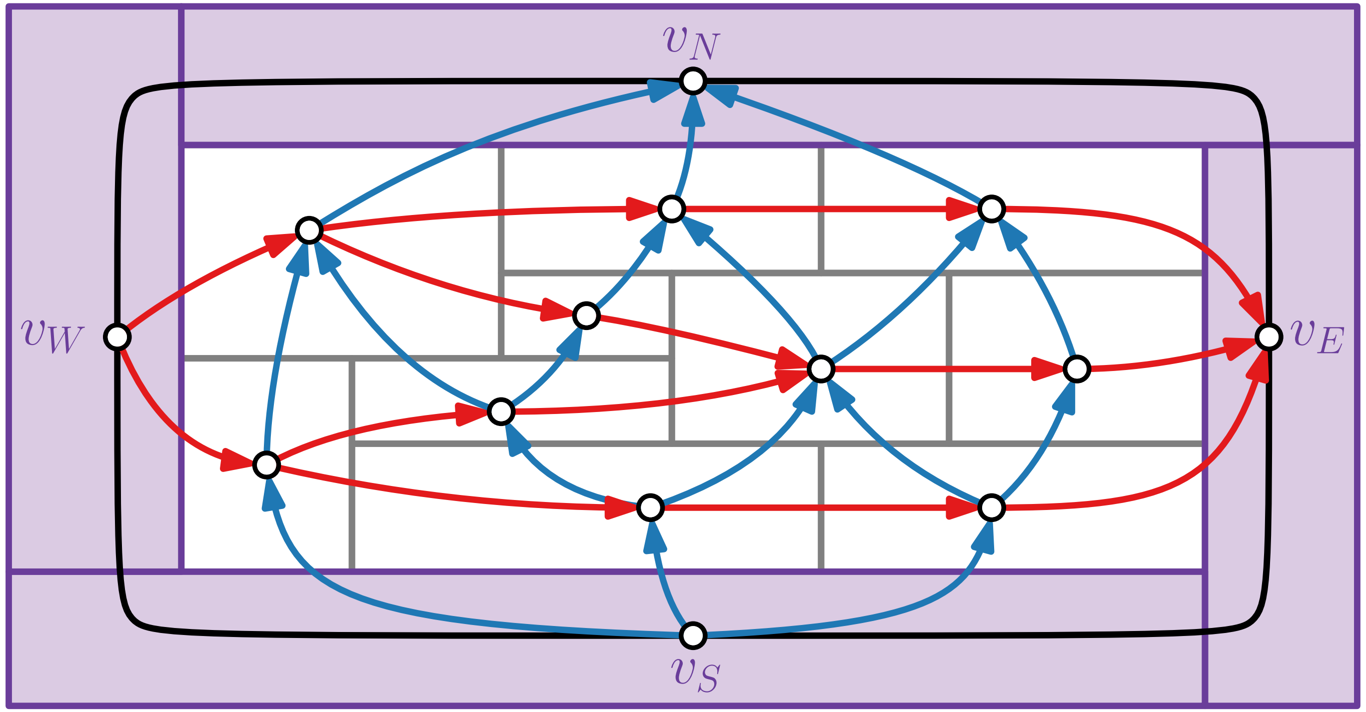
Properly Triangulated Planar Graph G

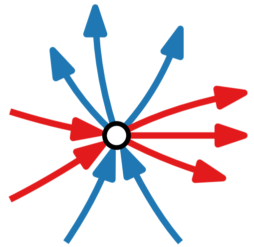


Regular Edge Labeling

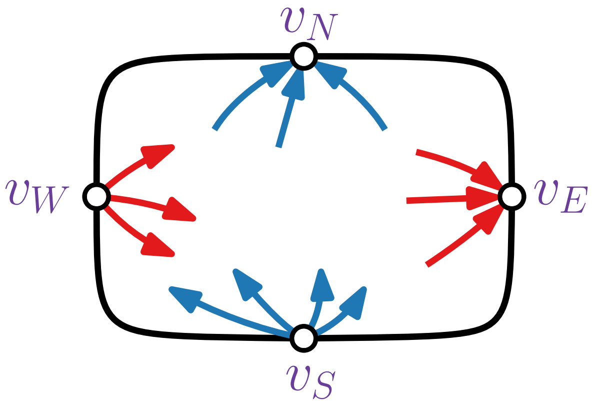


Rectangular Dual $\mathcal{R}$



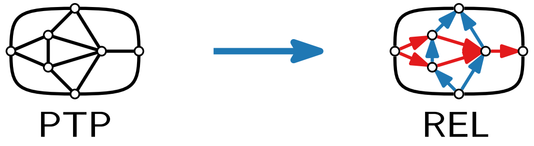


for every inner vertex

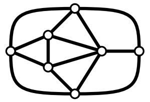


for four outer vertices

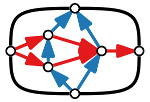
[Kant, He '94]: In linear time



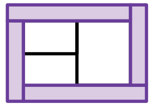
Regular Edge Labeling



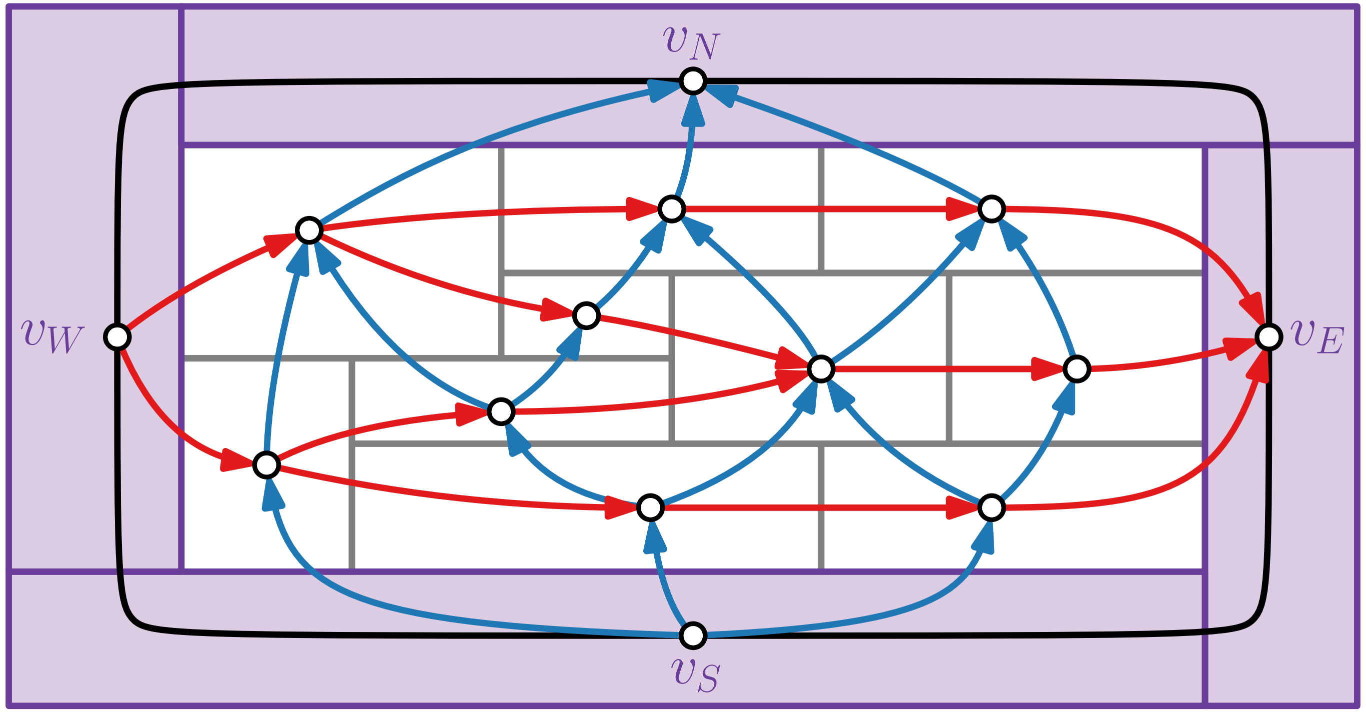
Properly Triangulated Planar Graph G

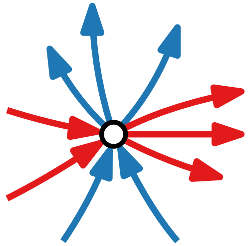


Regular Edge Labeling

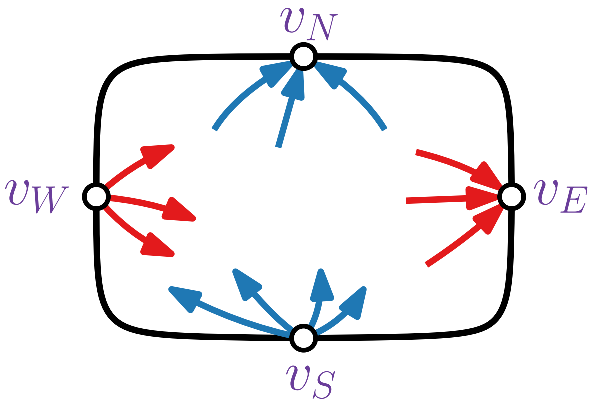


Rectangular Dual $\mathcal{R}$



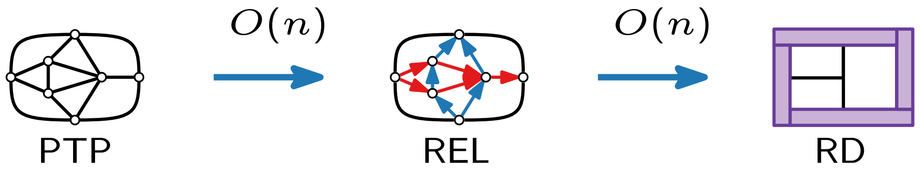


for every inner vertex

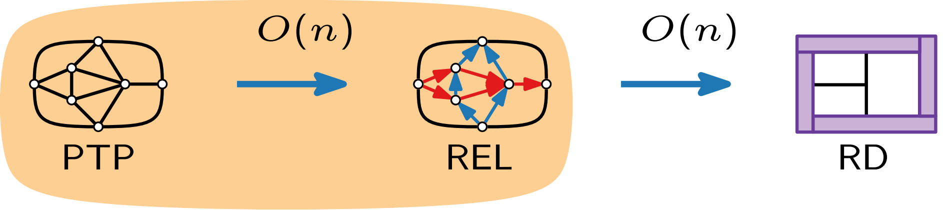


for four outer vertices

[Kant, He '94]: In linear time

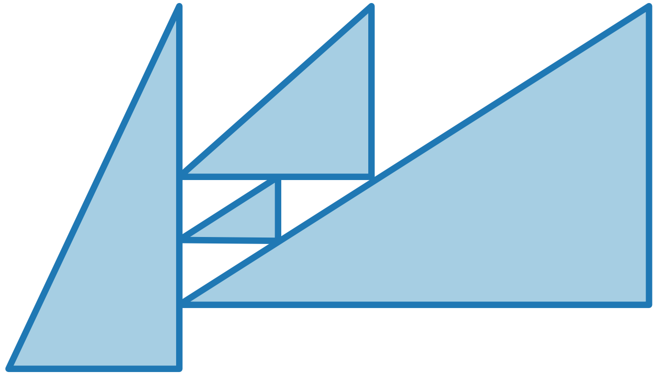


[Kant, He '94]:

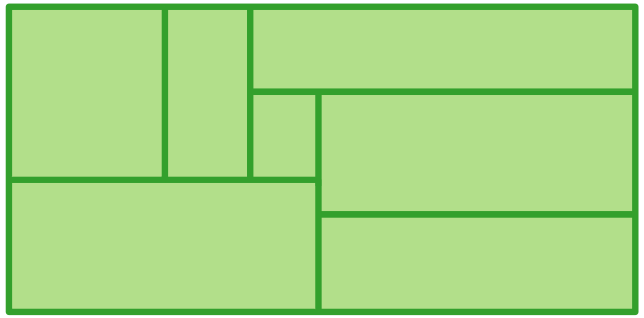


Visualization of Graphs

Lecture 8:
Conact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals



Part IV:
Computing a REL



Jonathan Klawitter

Refined Canonical Order

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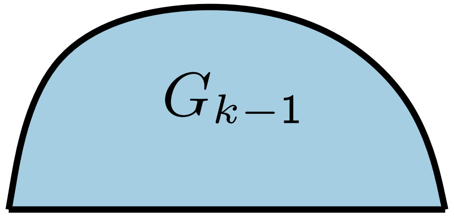

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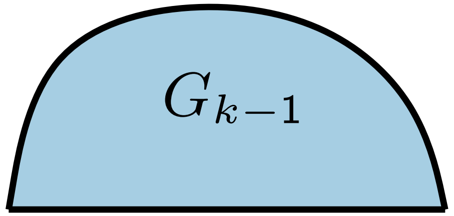

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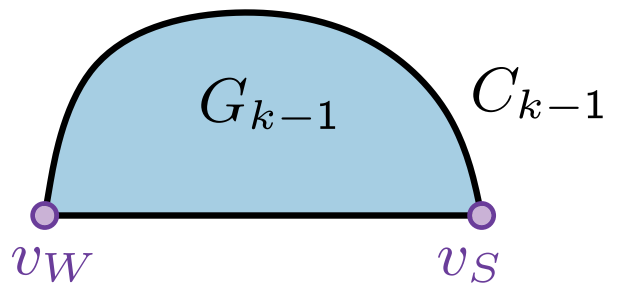

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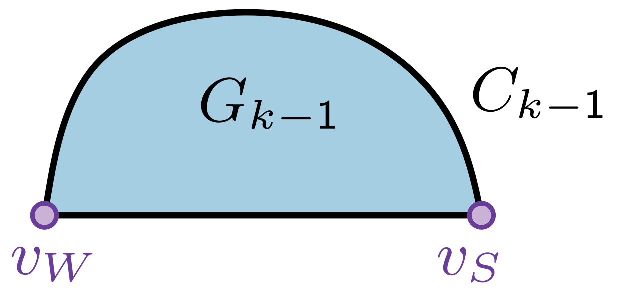

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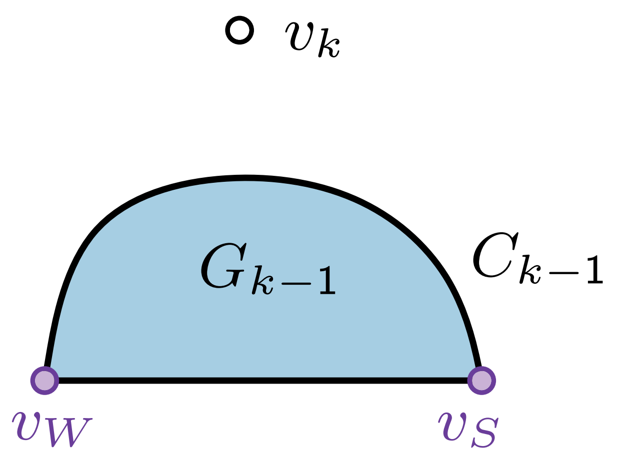

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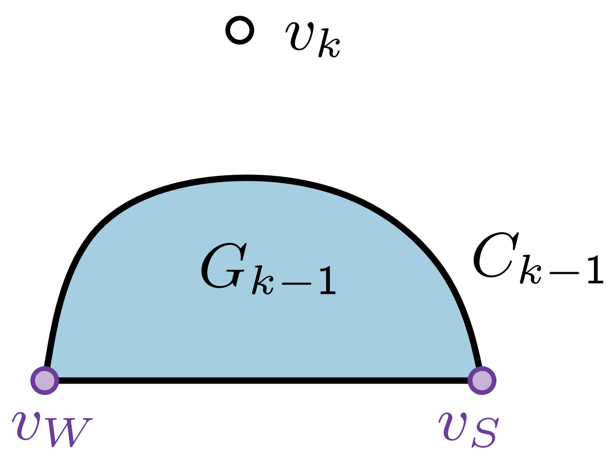

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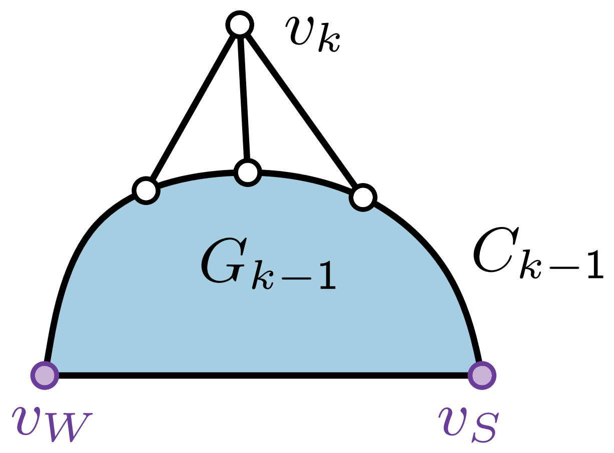

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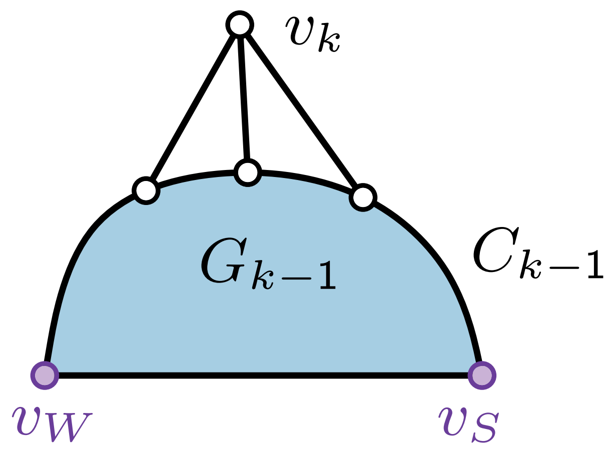

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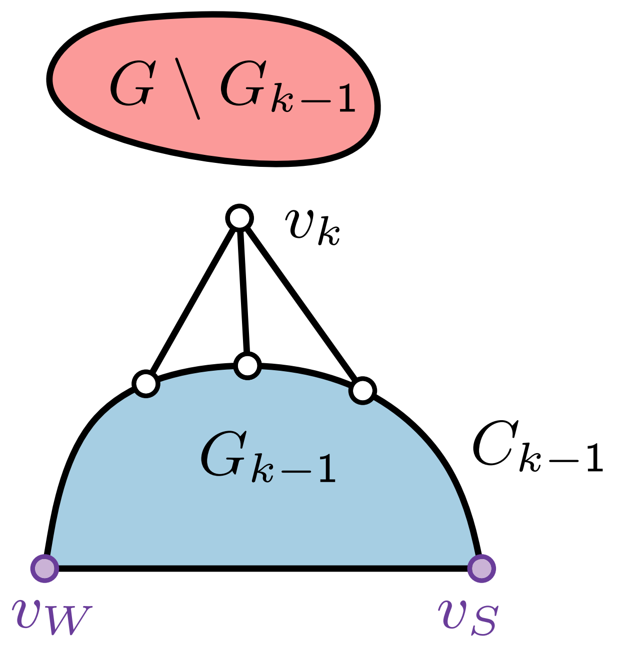

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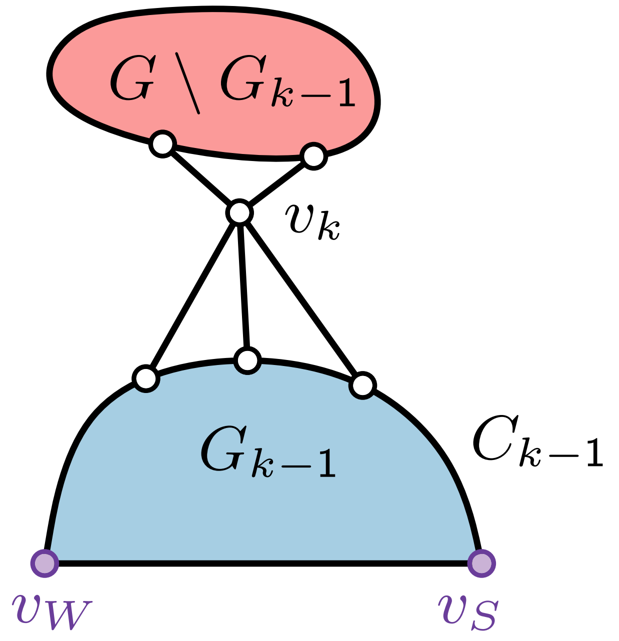

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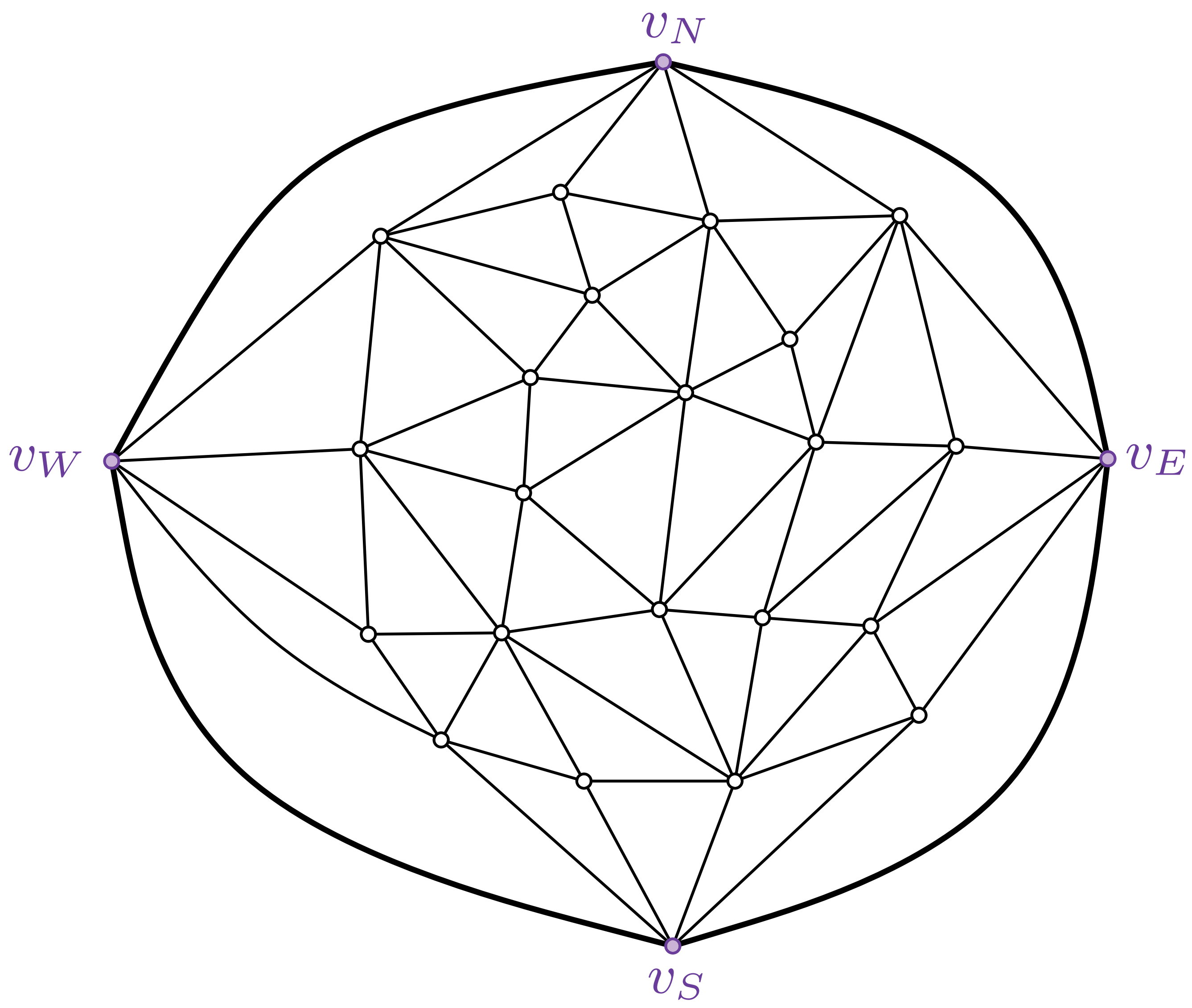

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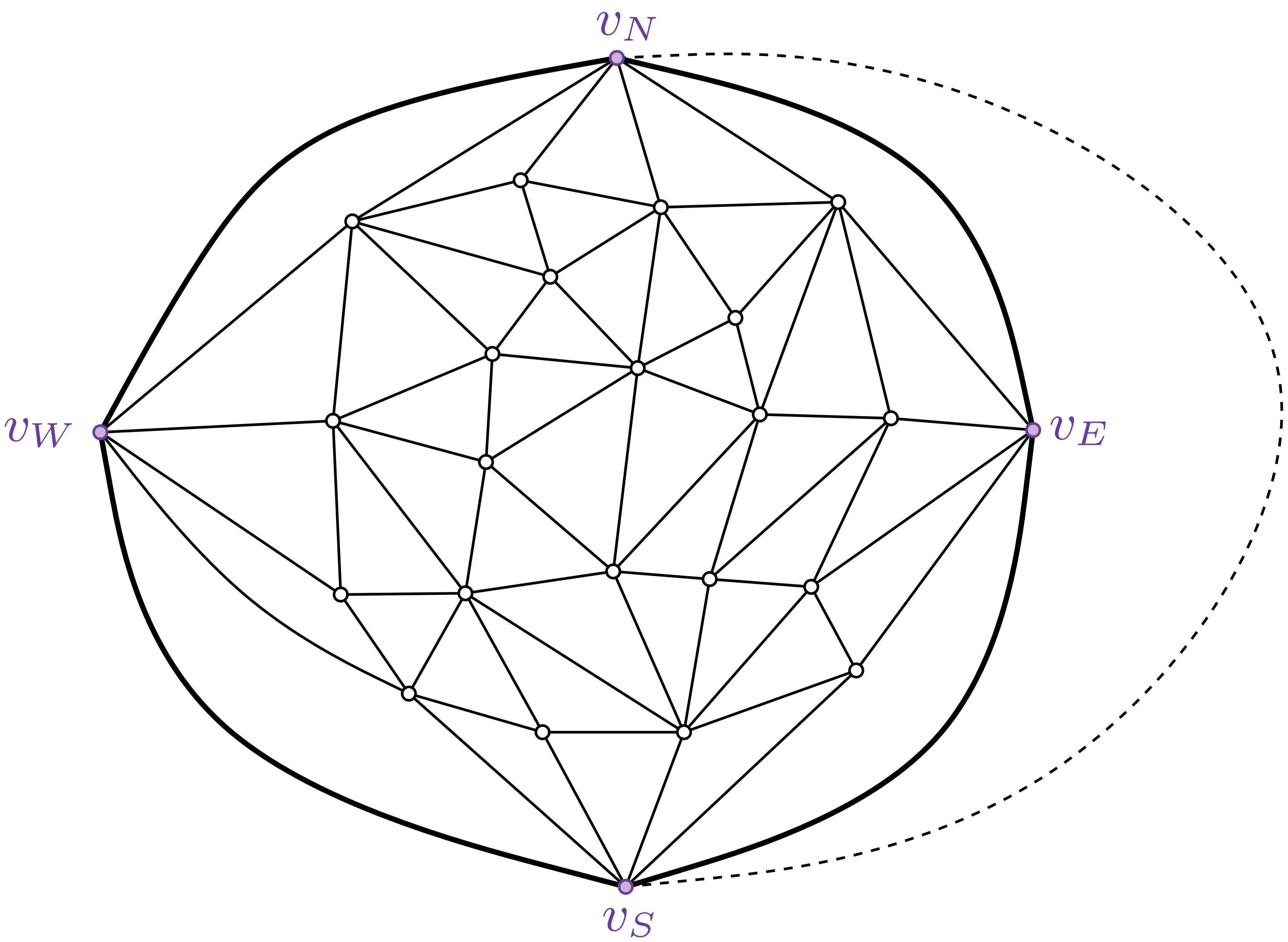

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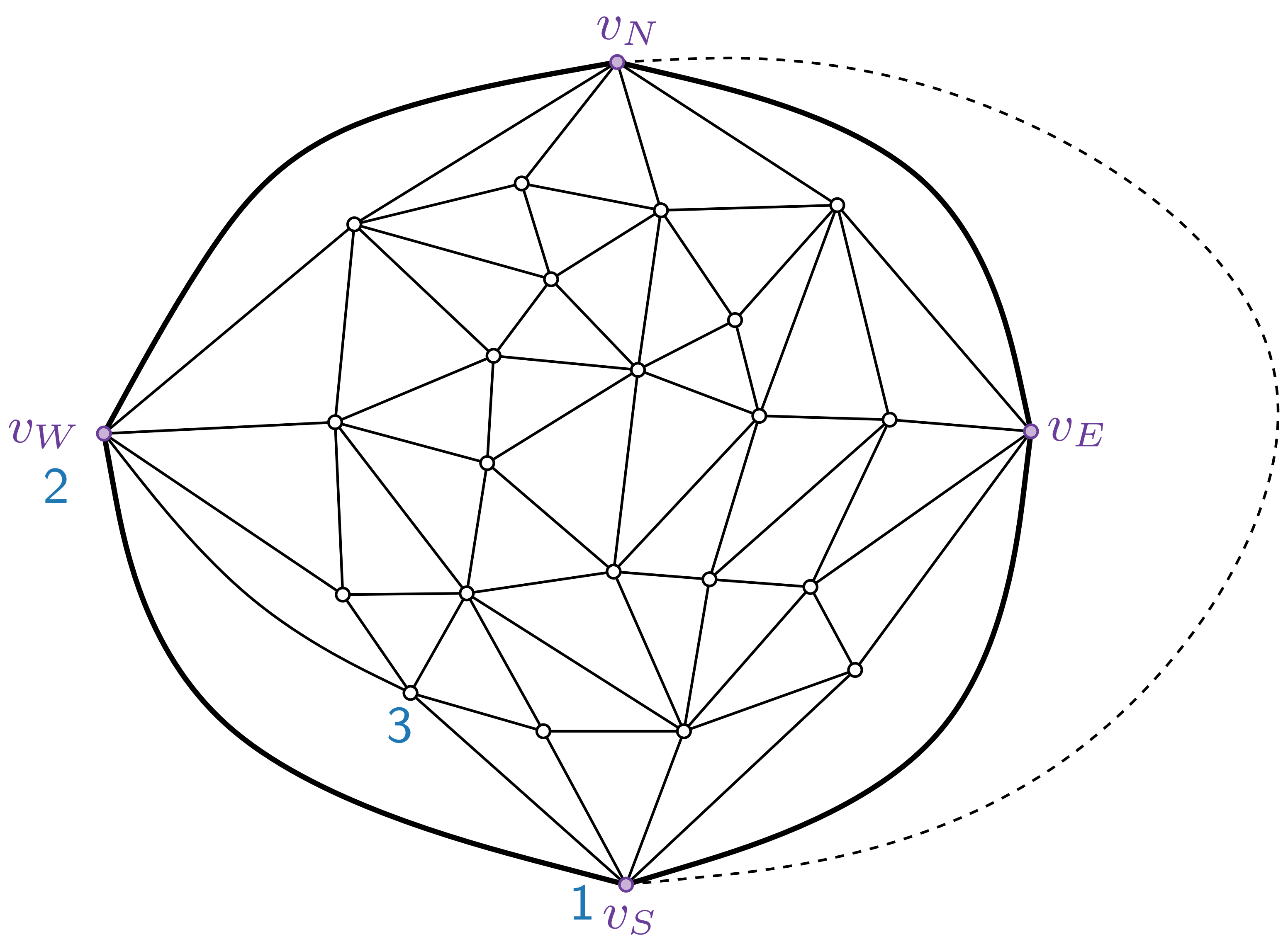

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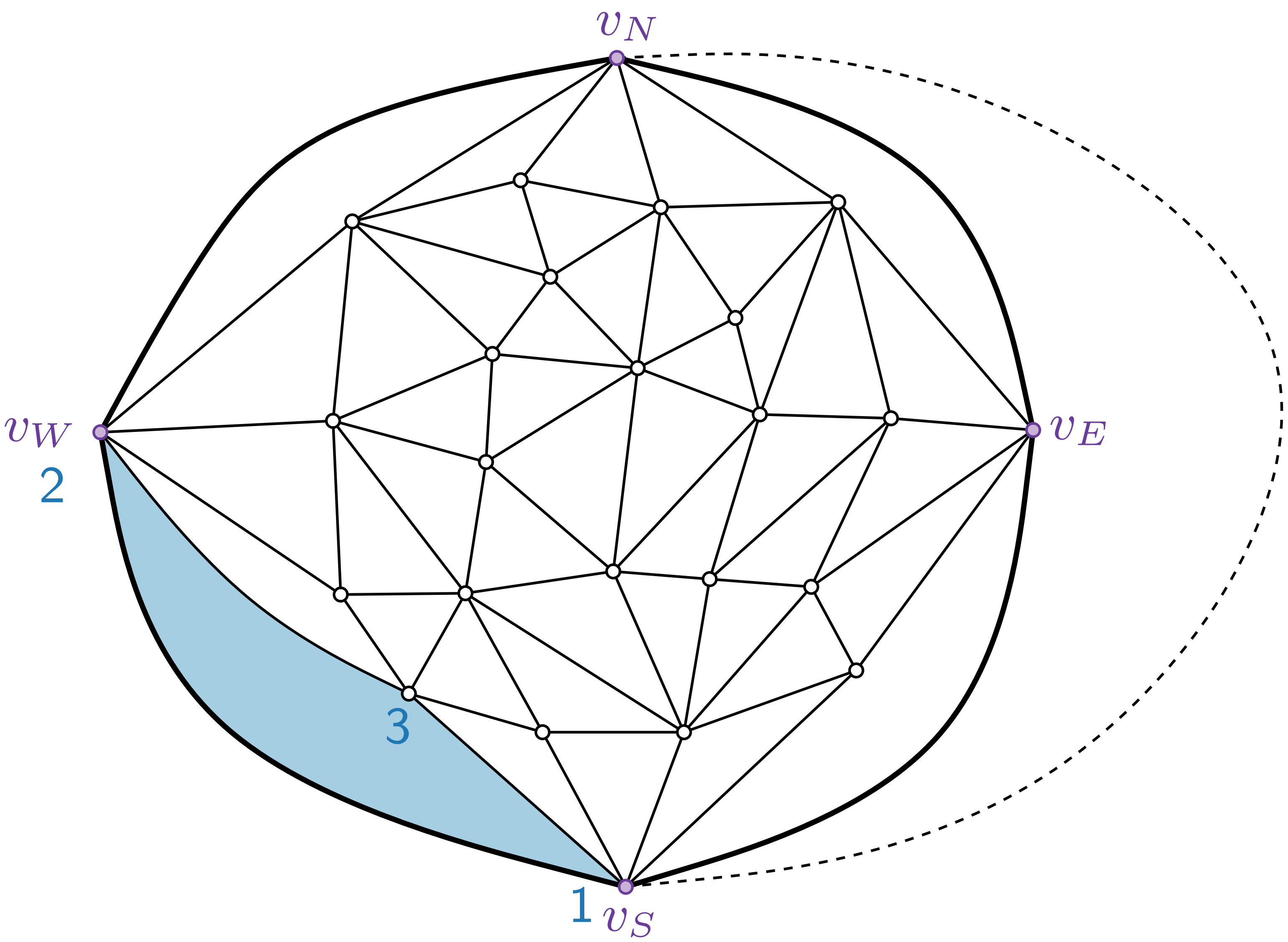

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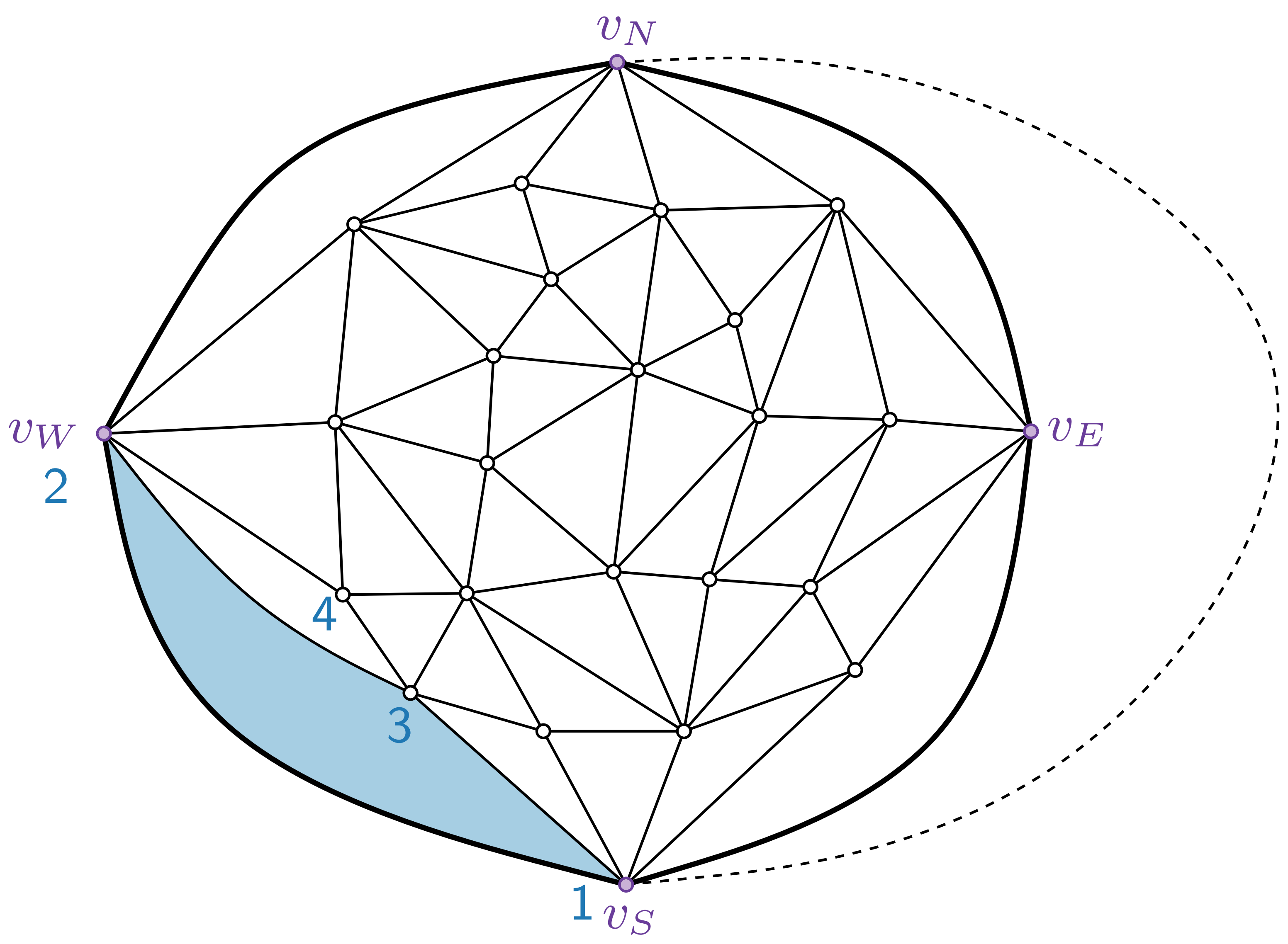

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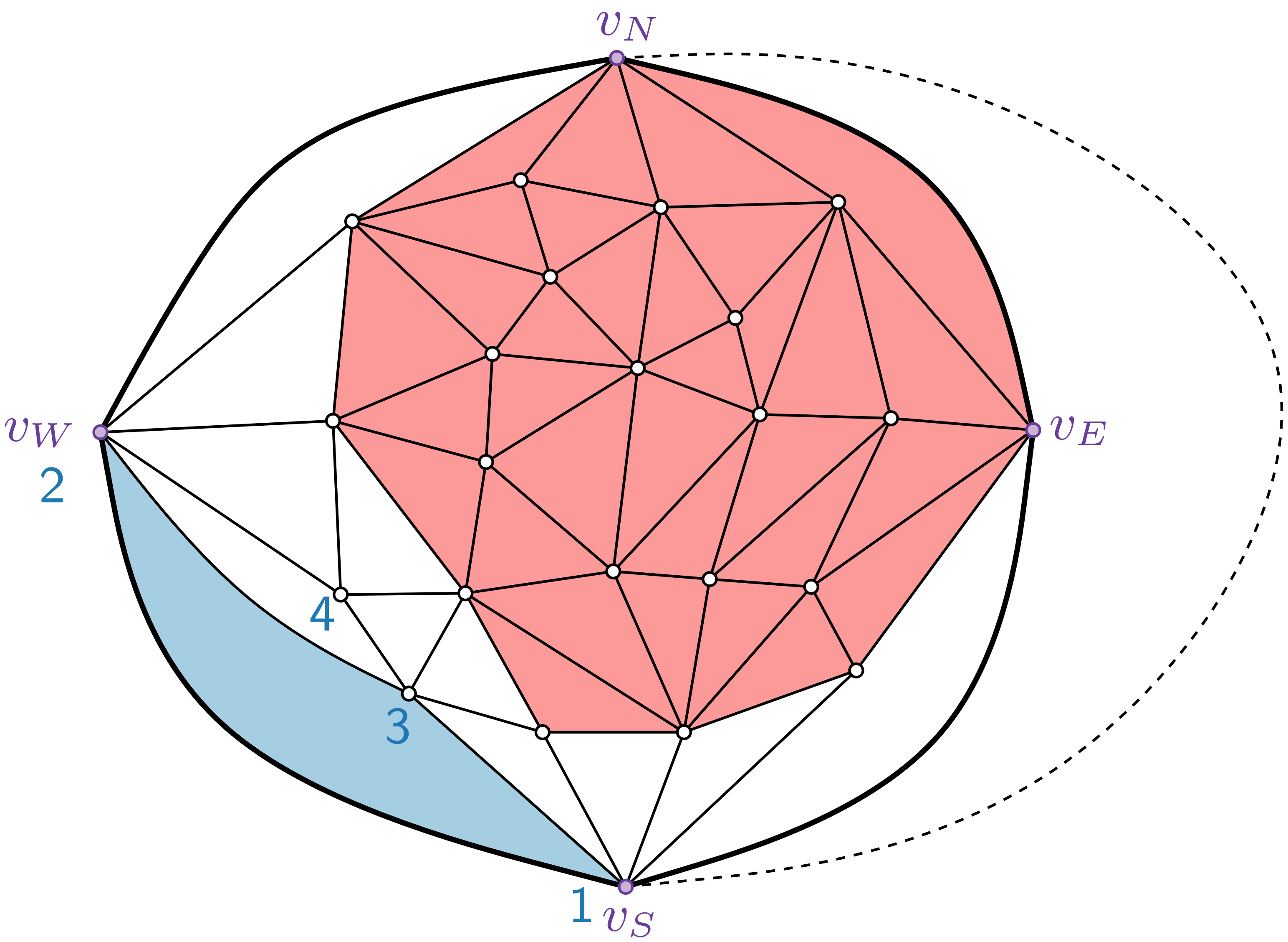

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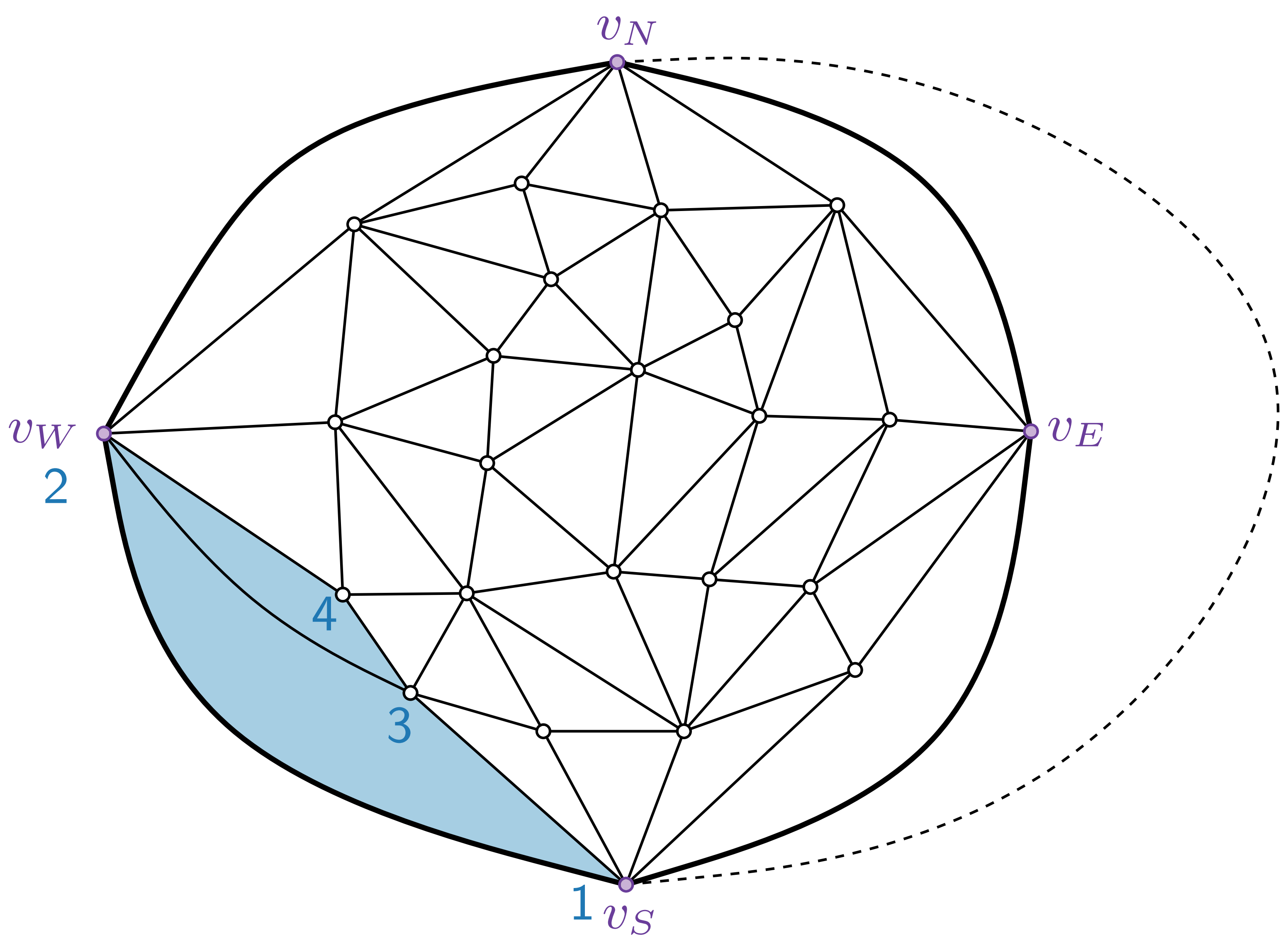

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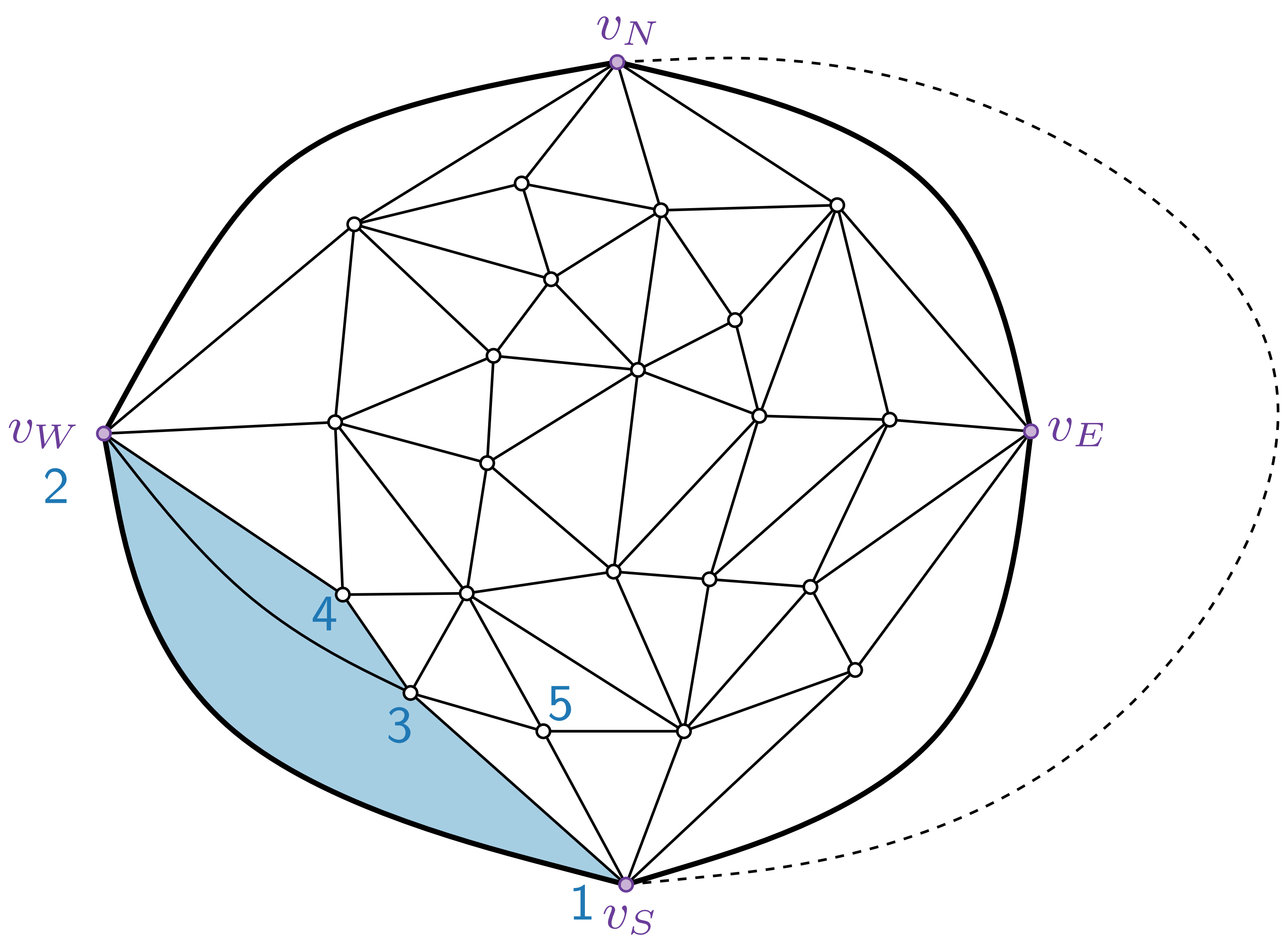

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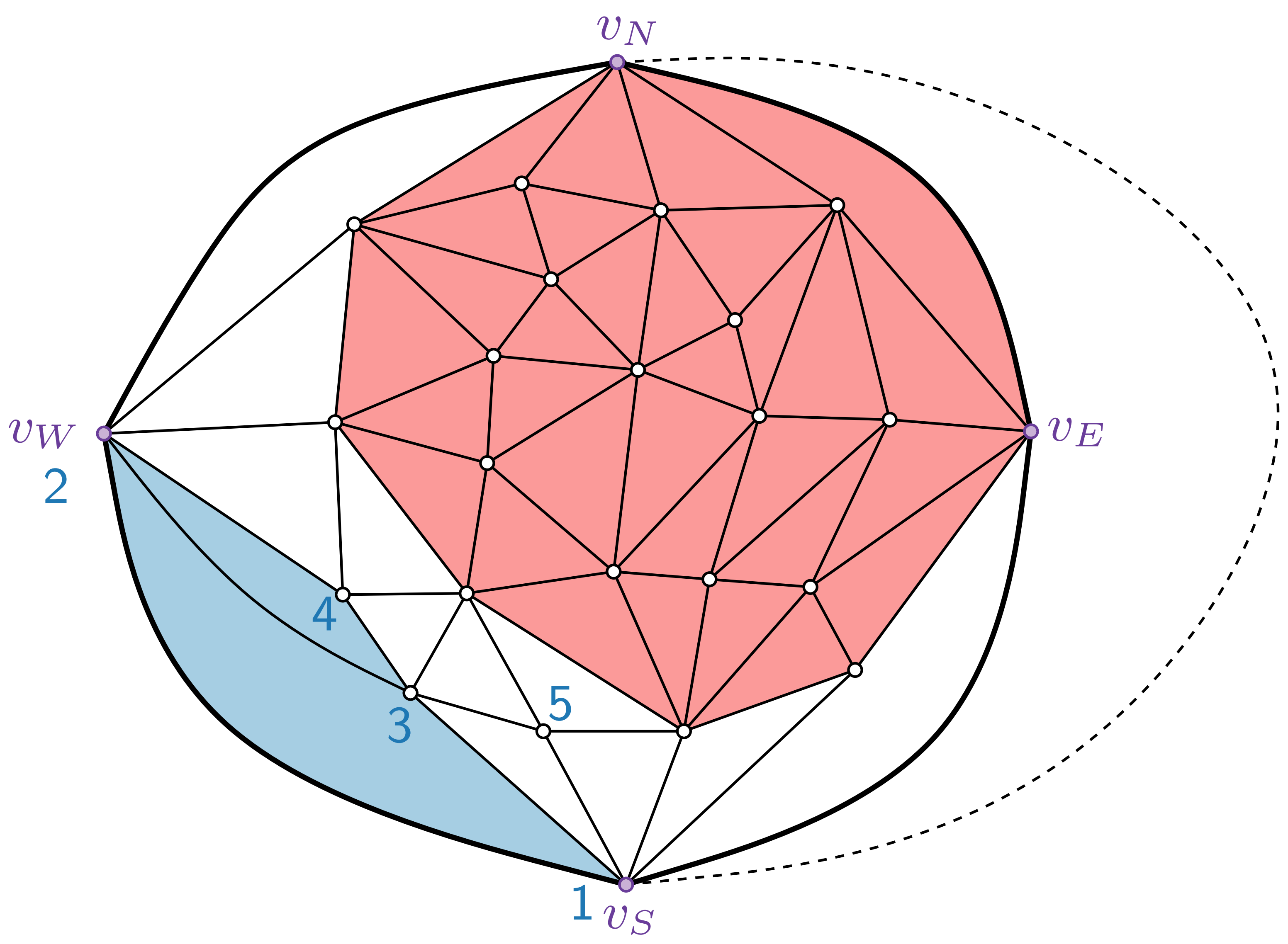

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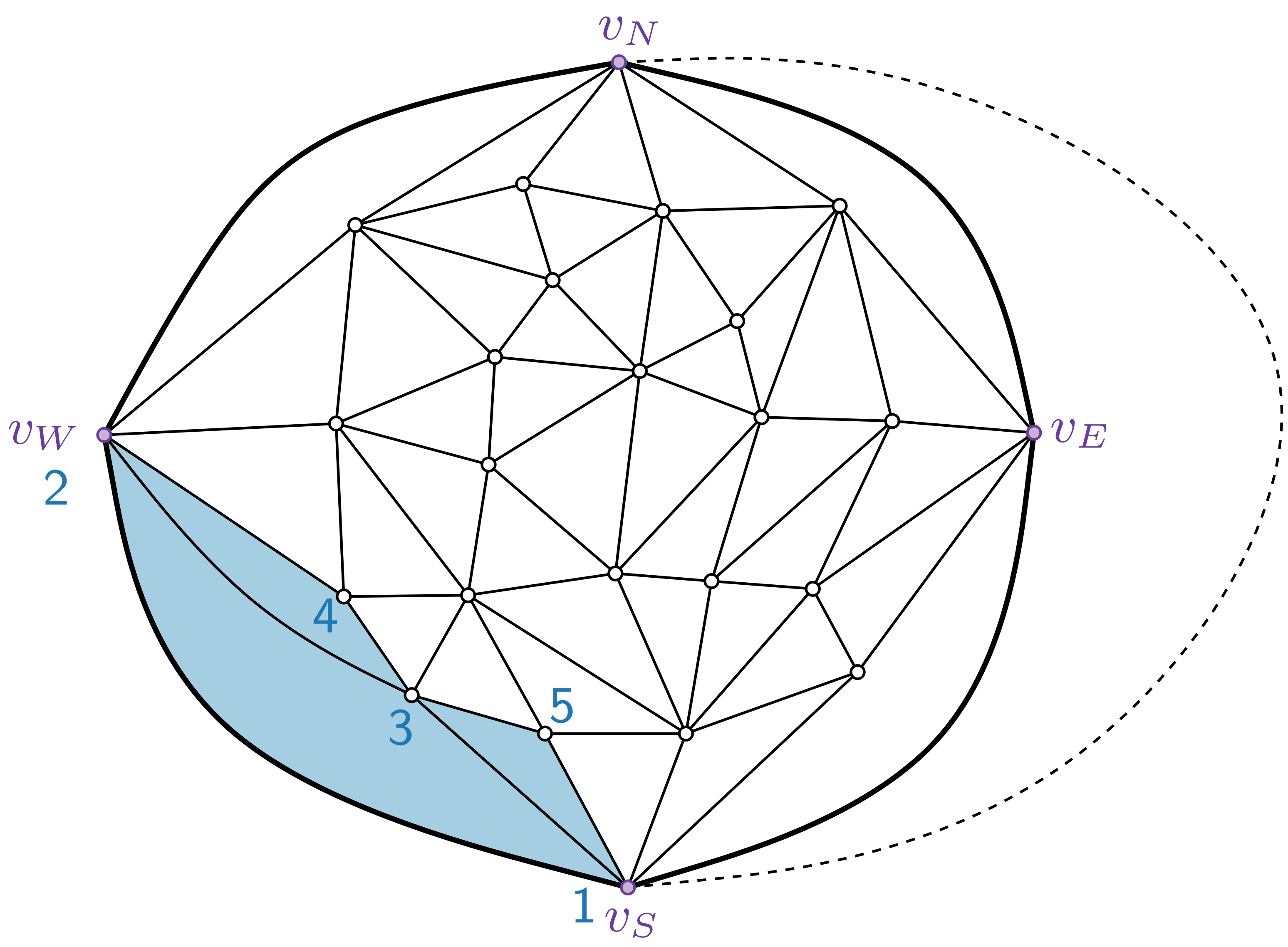

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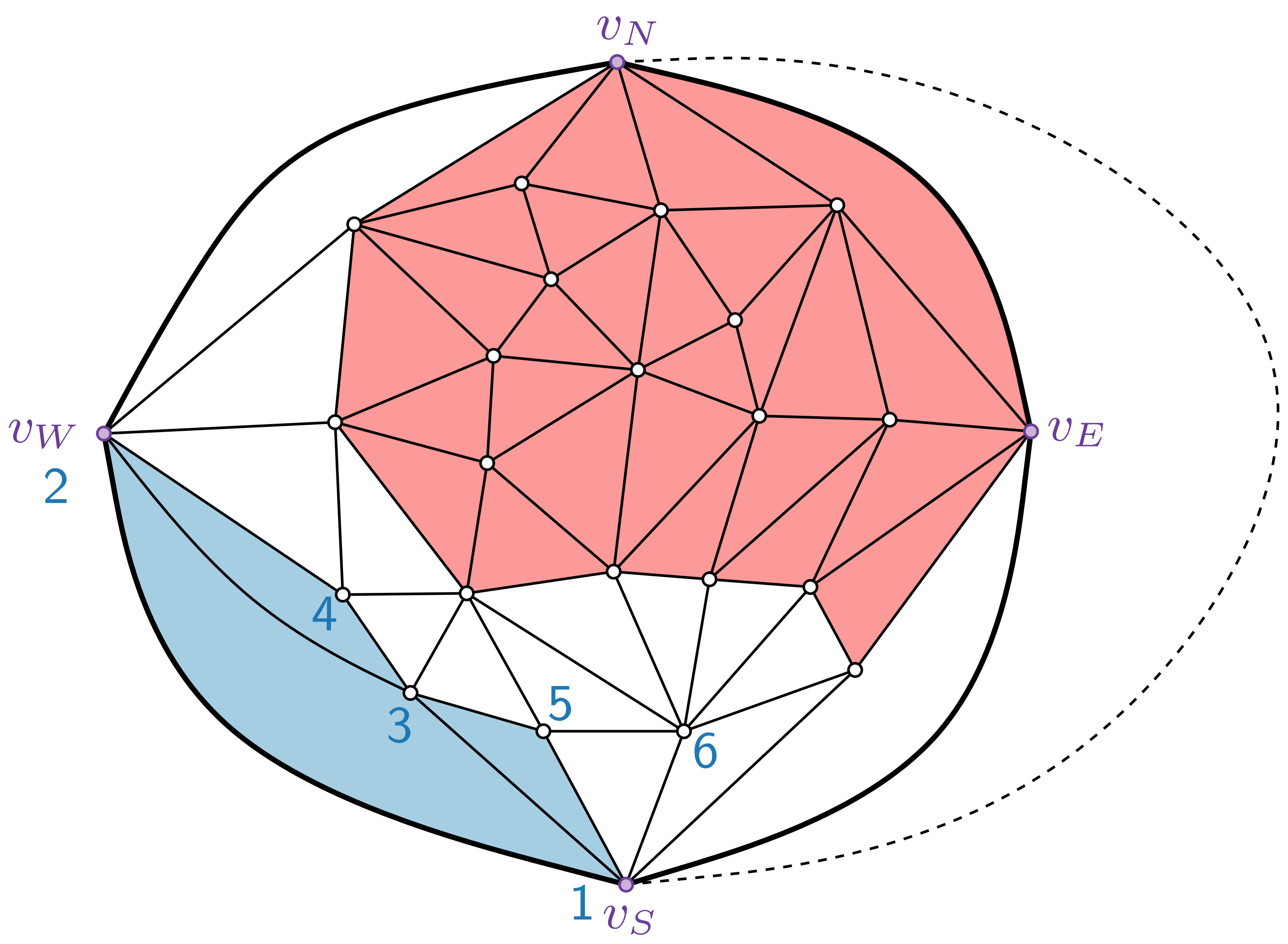

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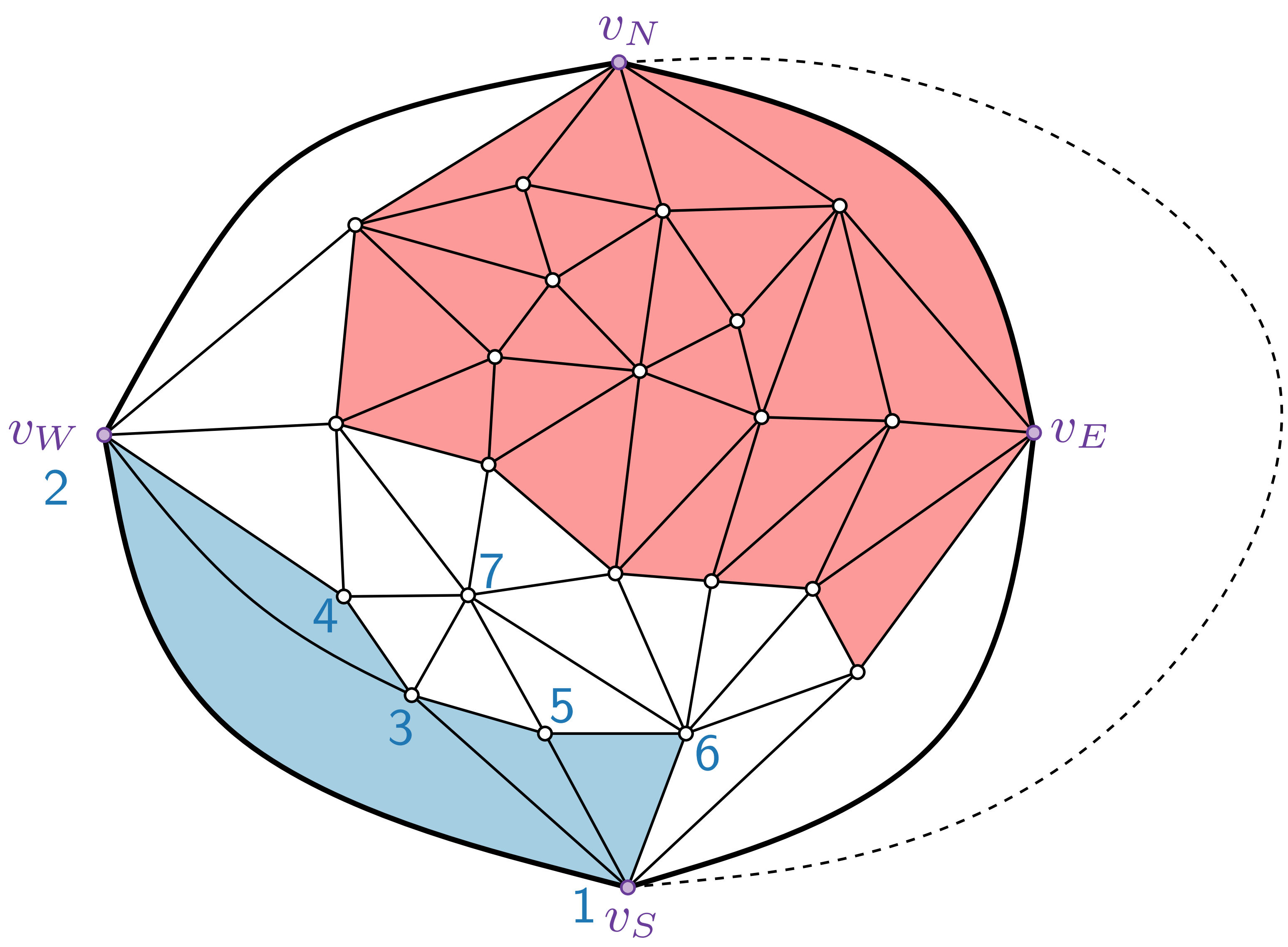

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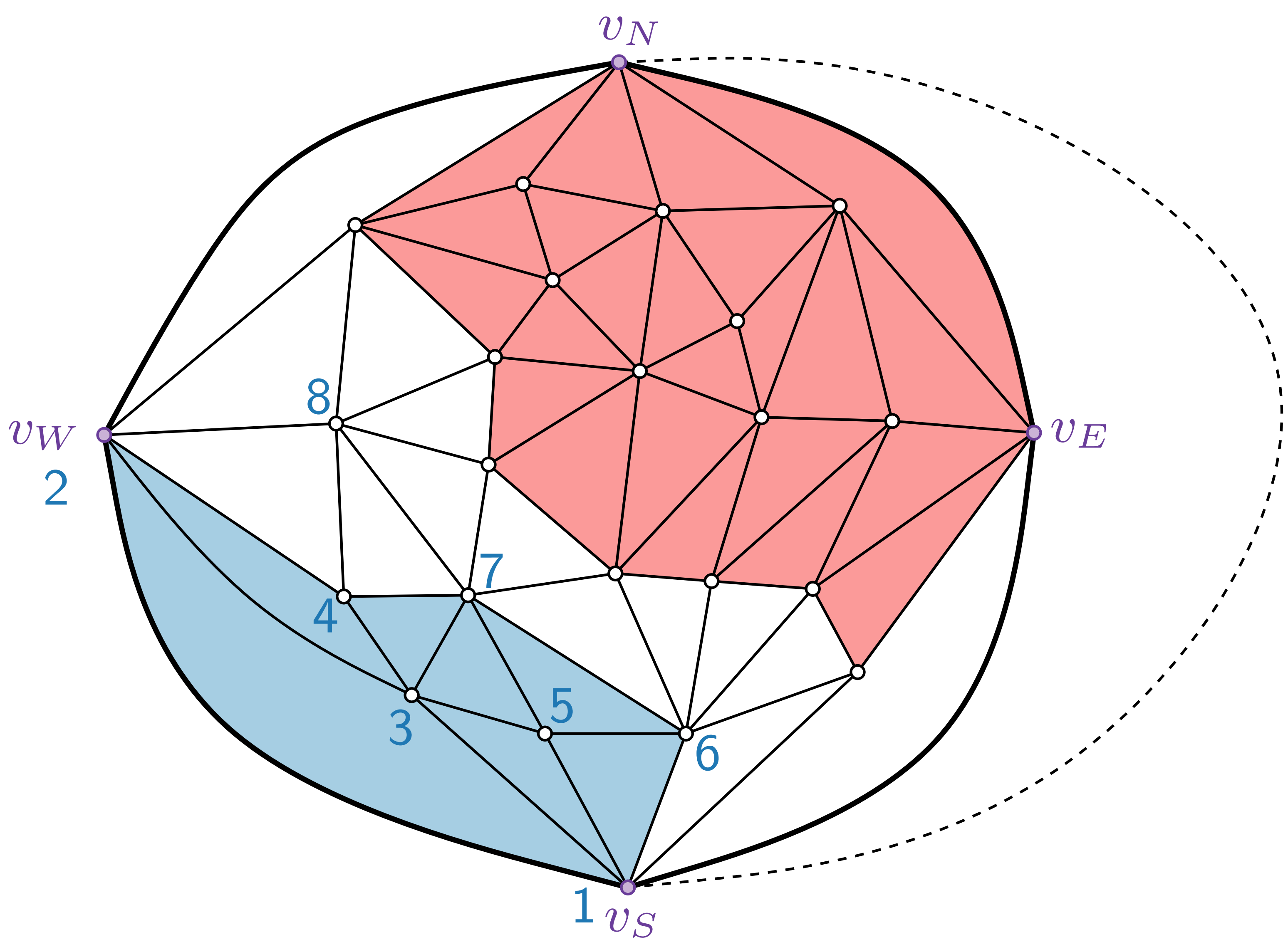

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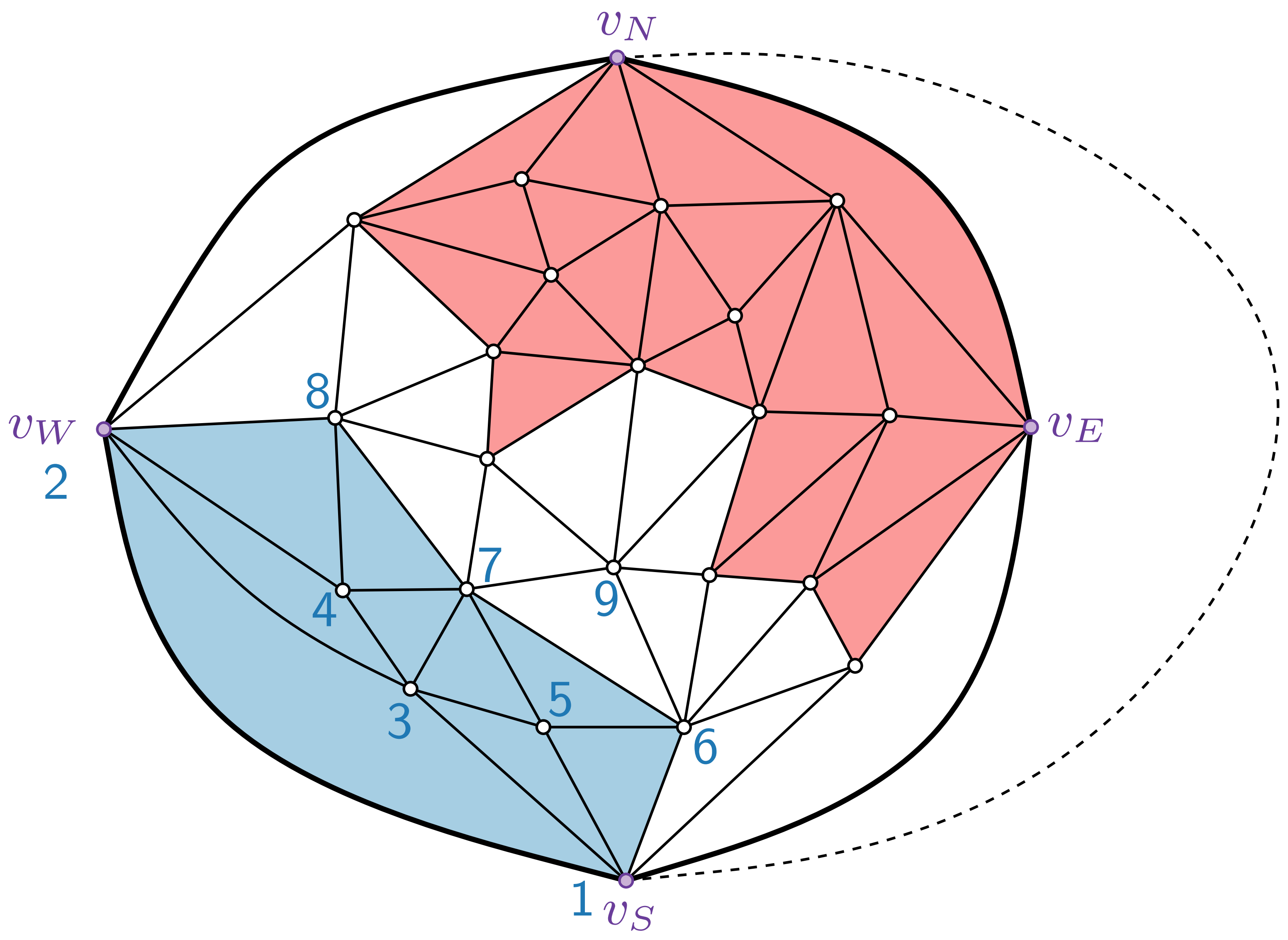

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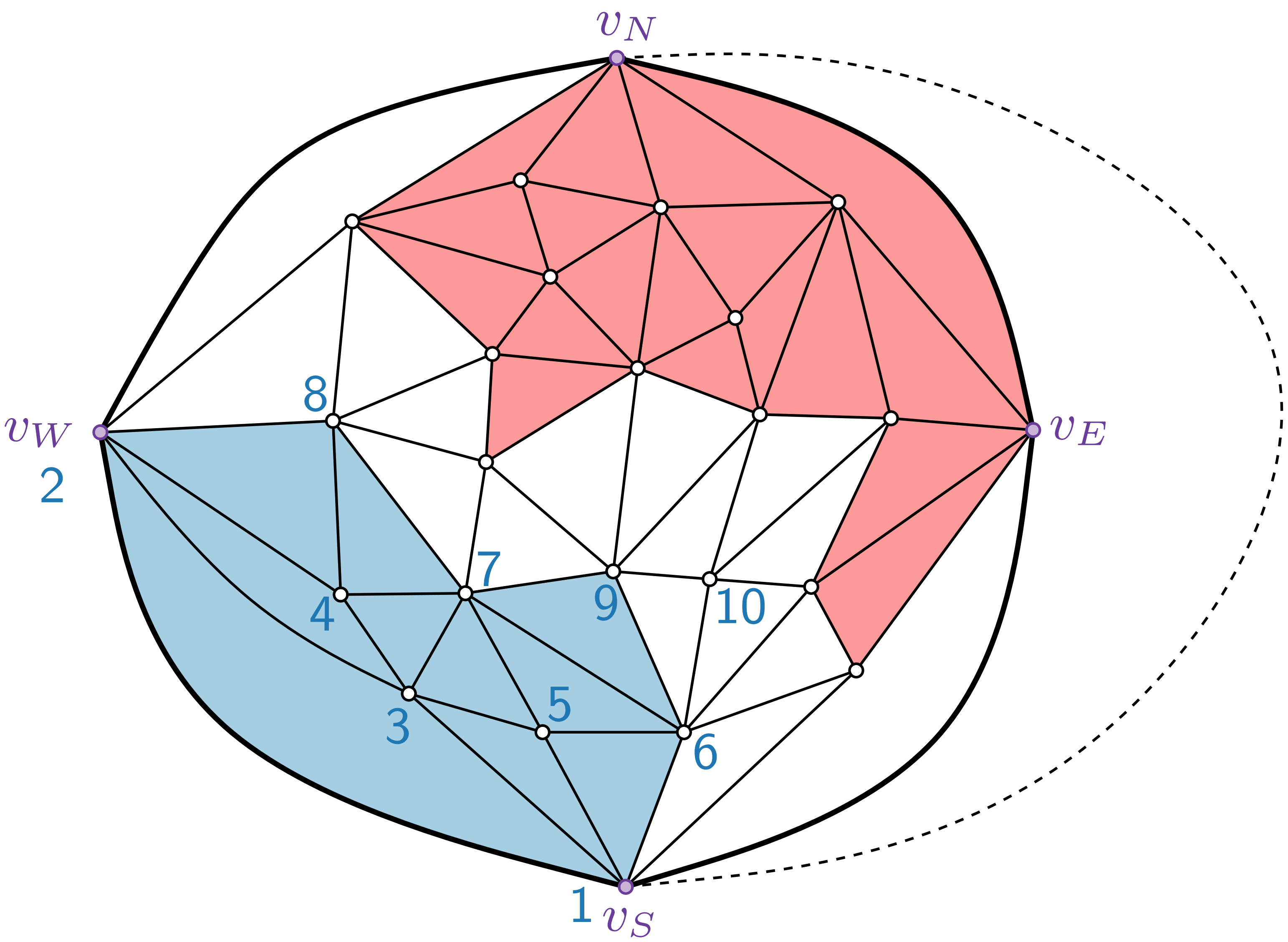

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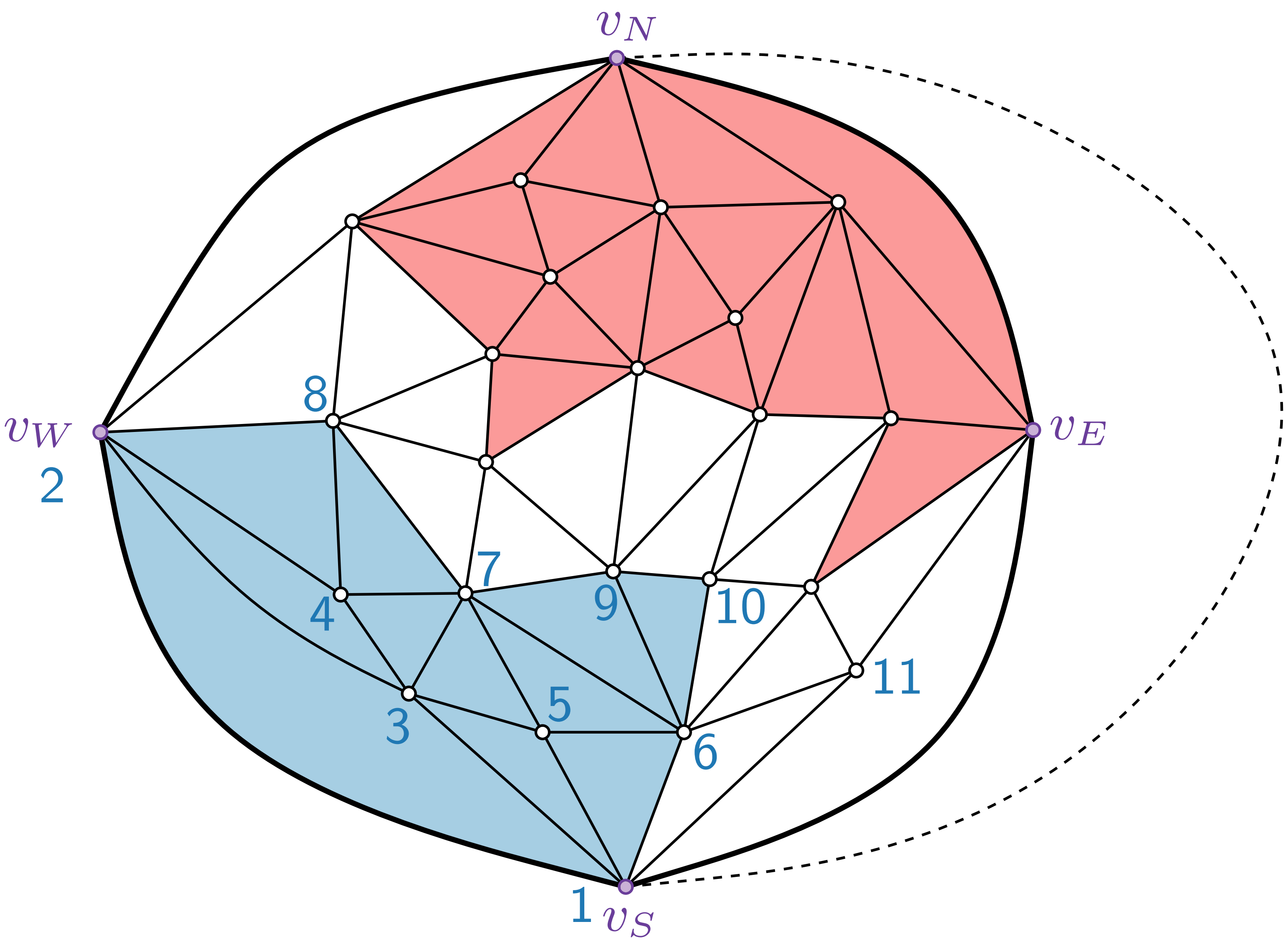

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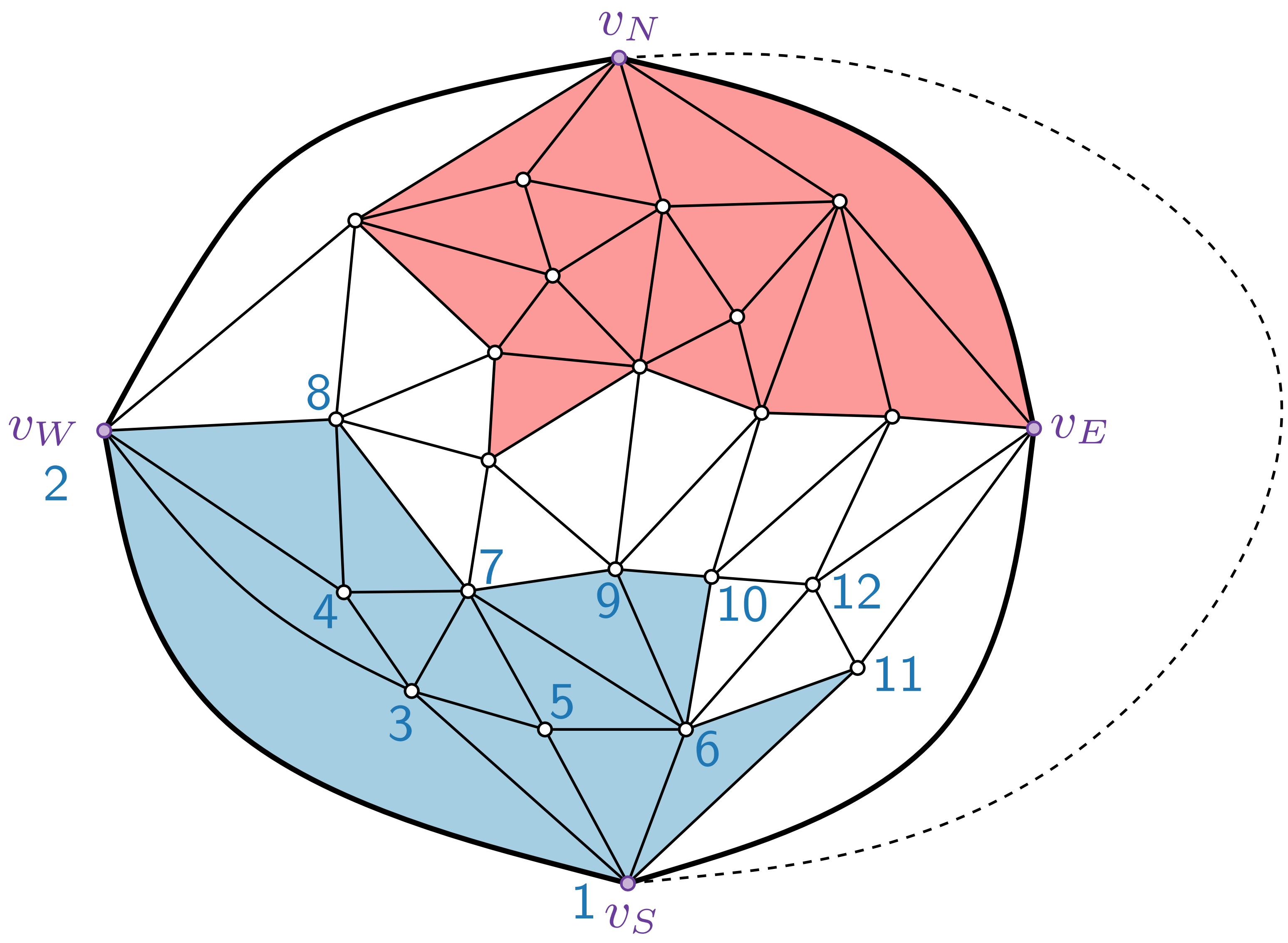

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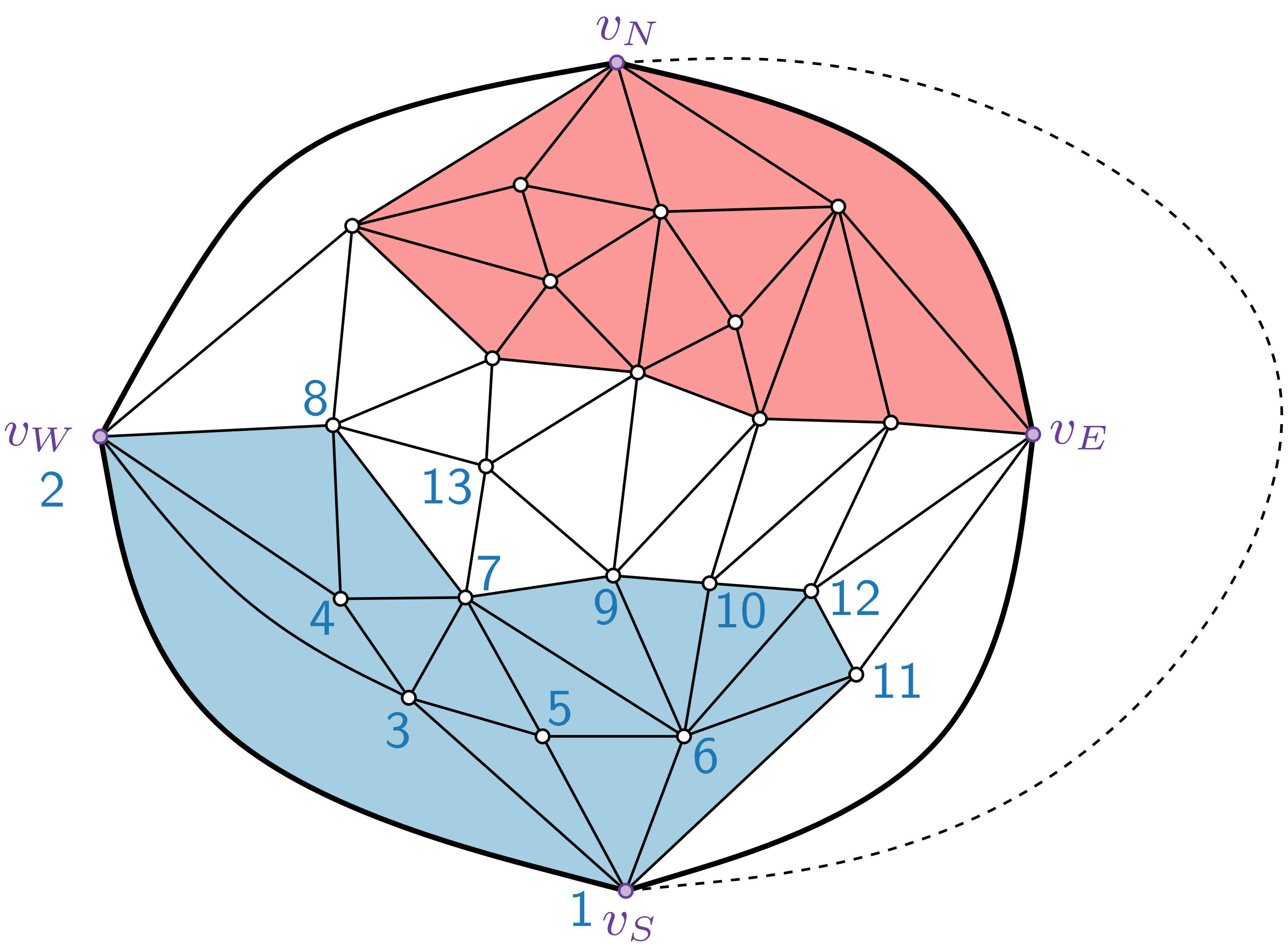

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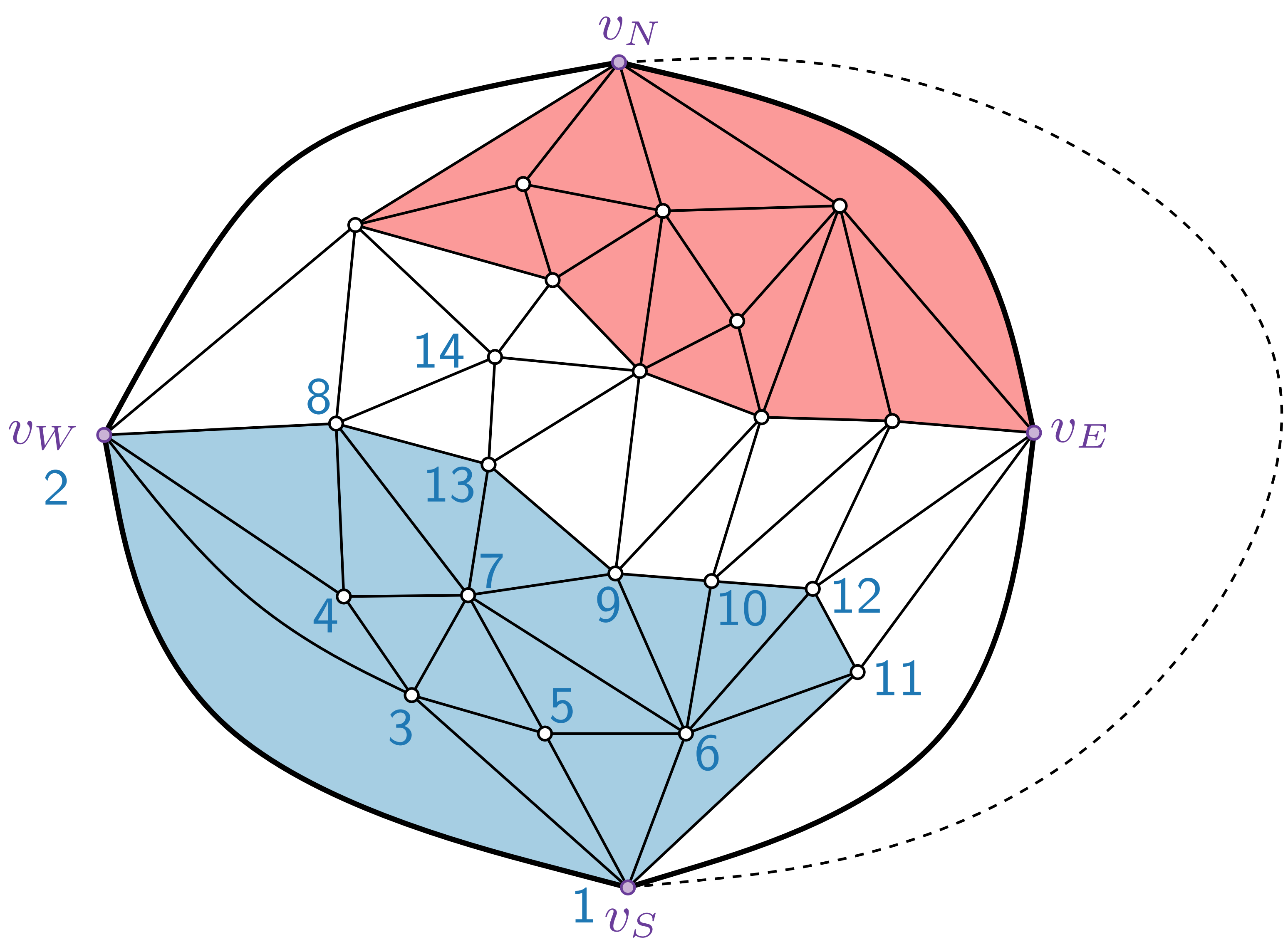

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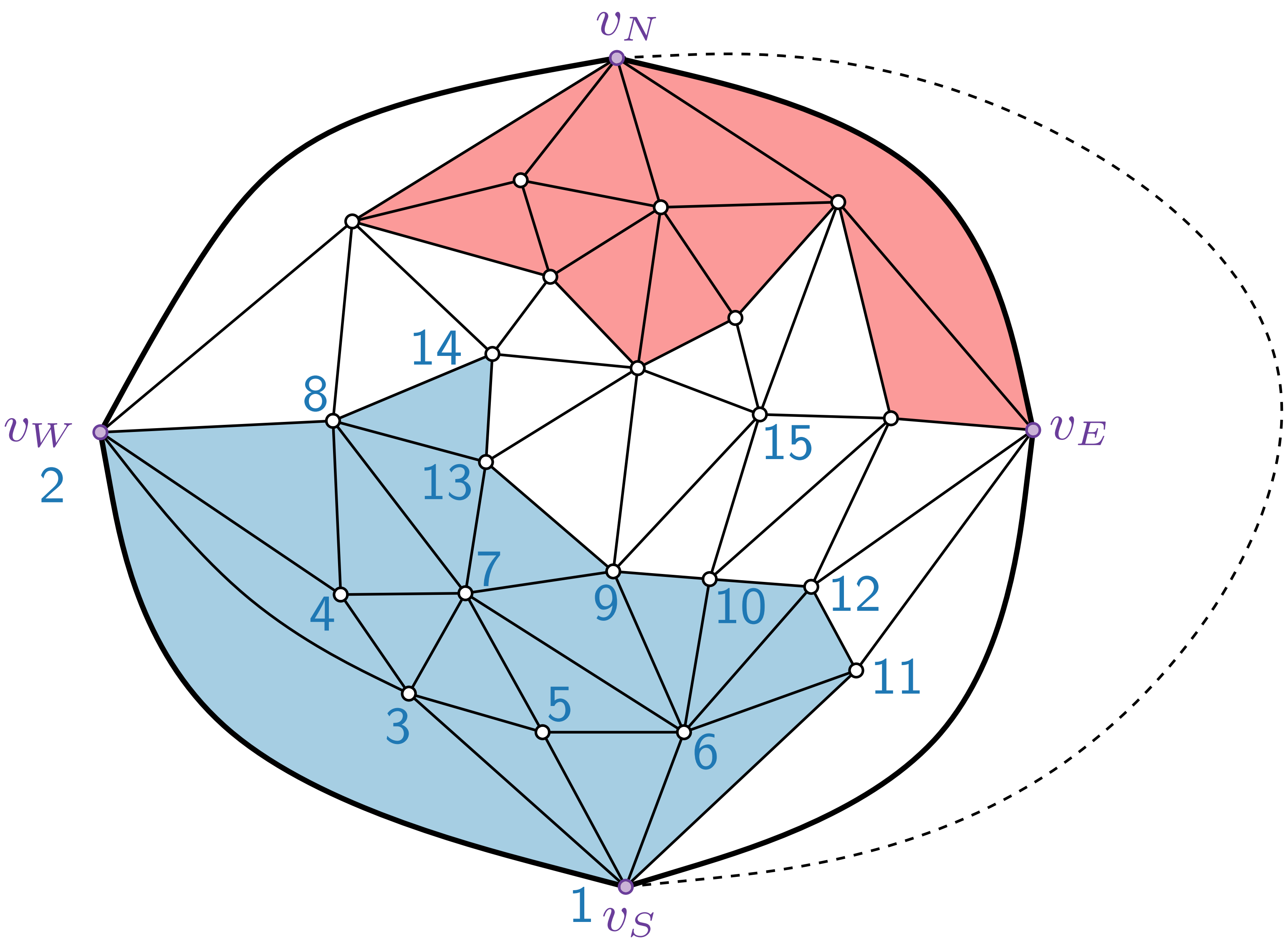

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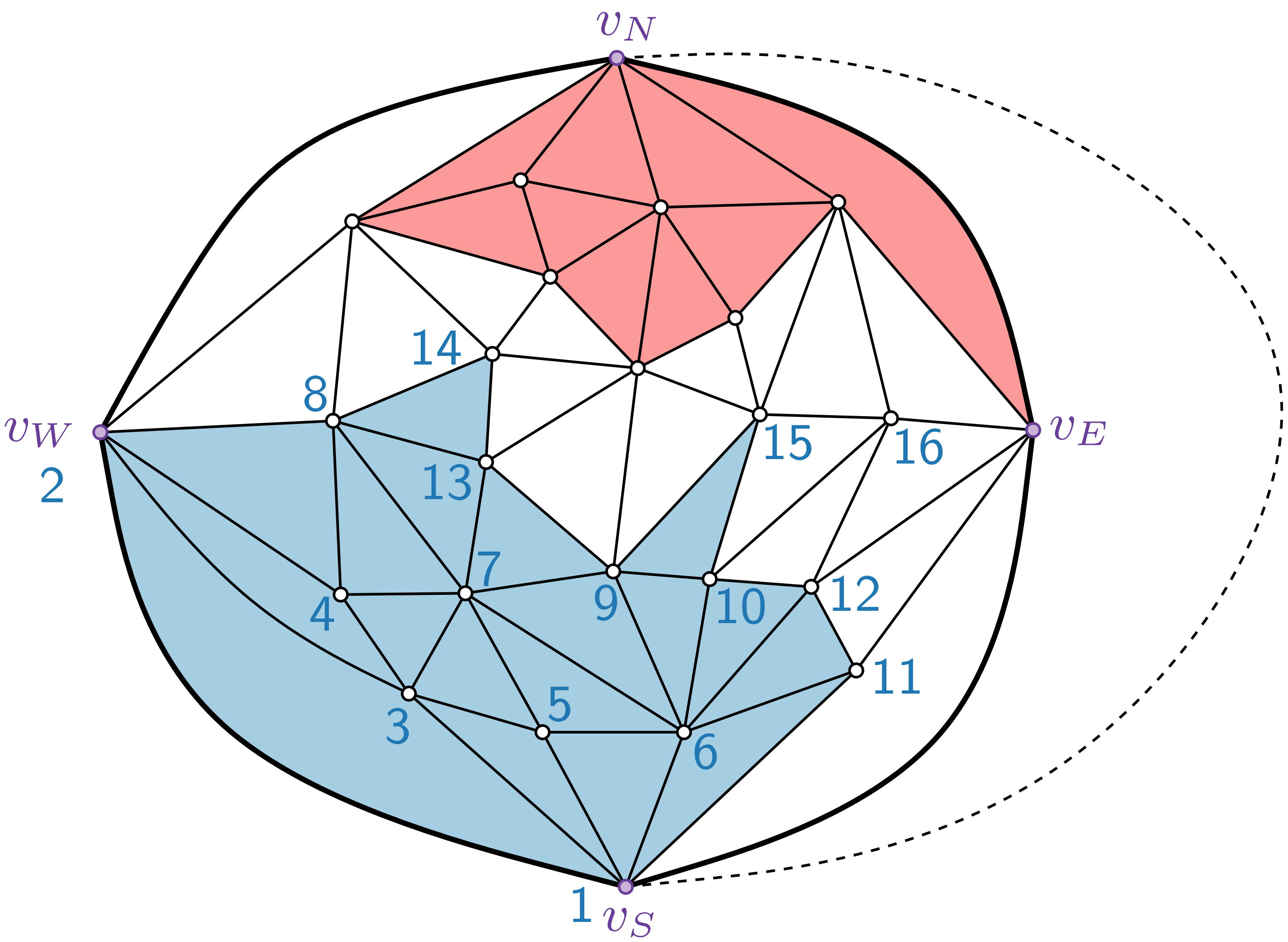

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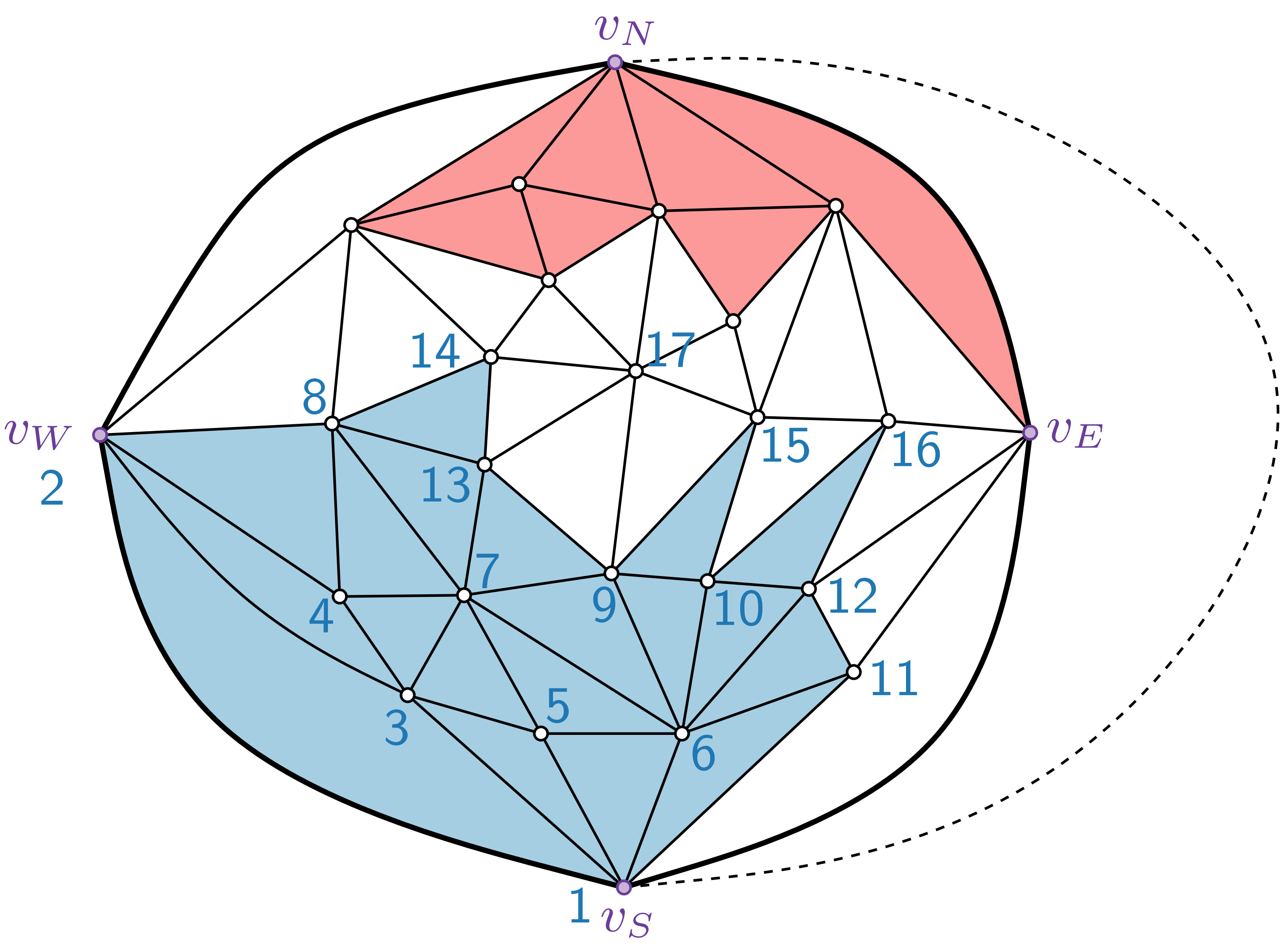

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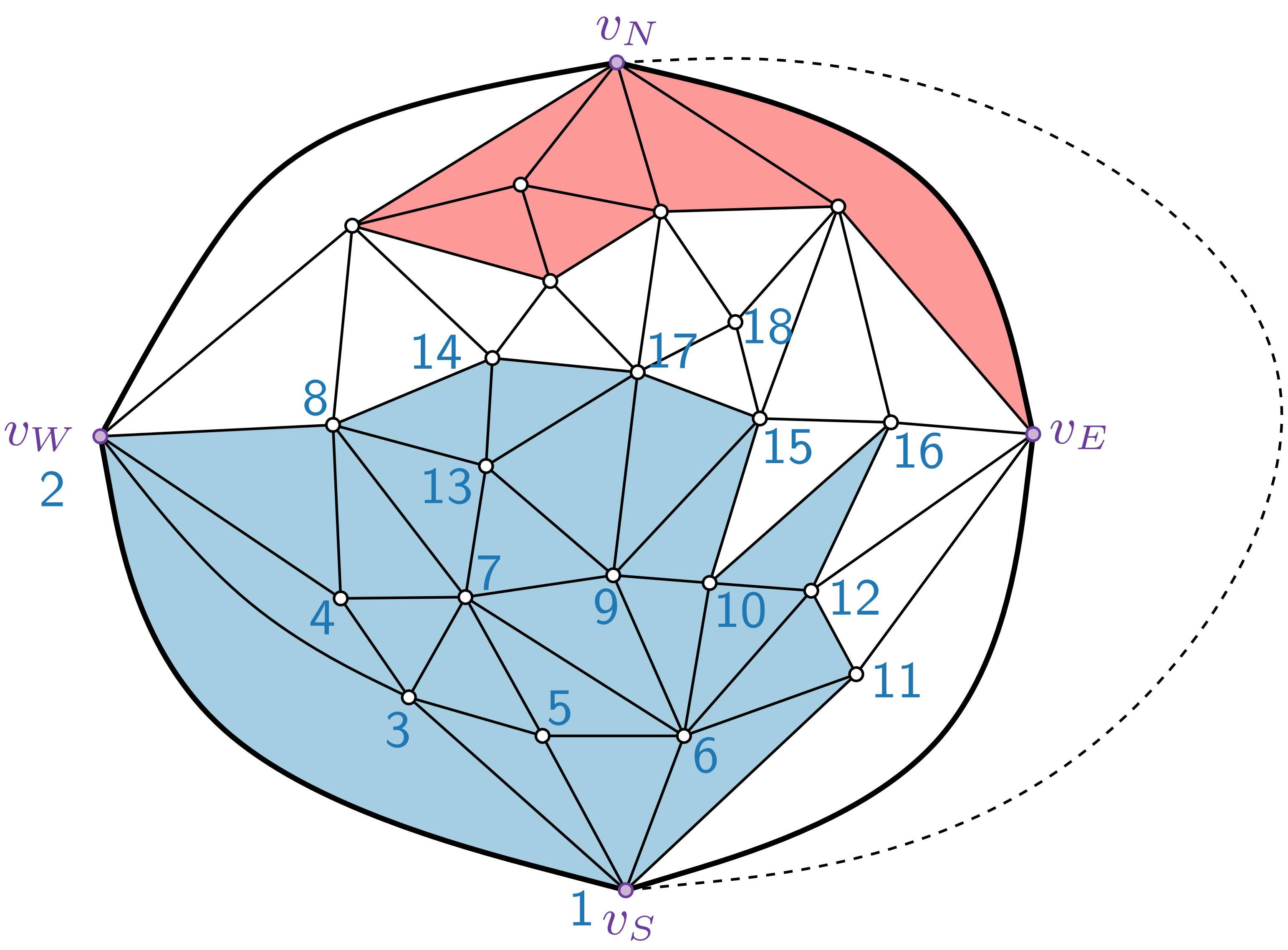

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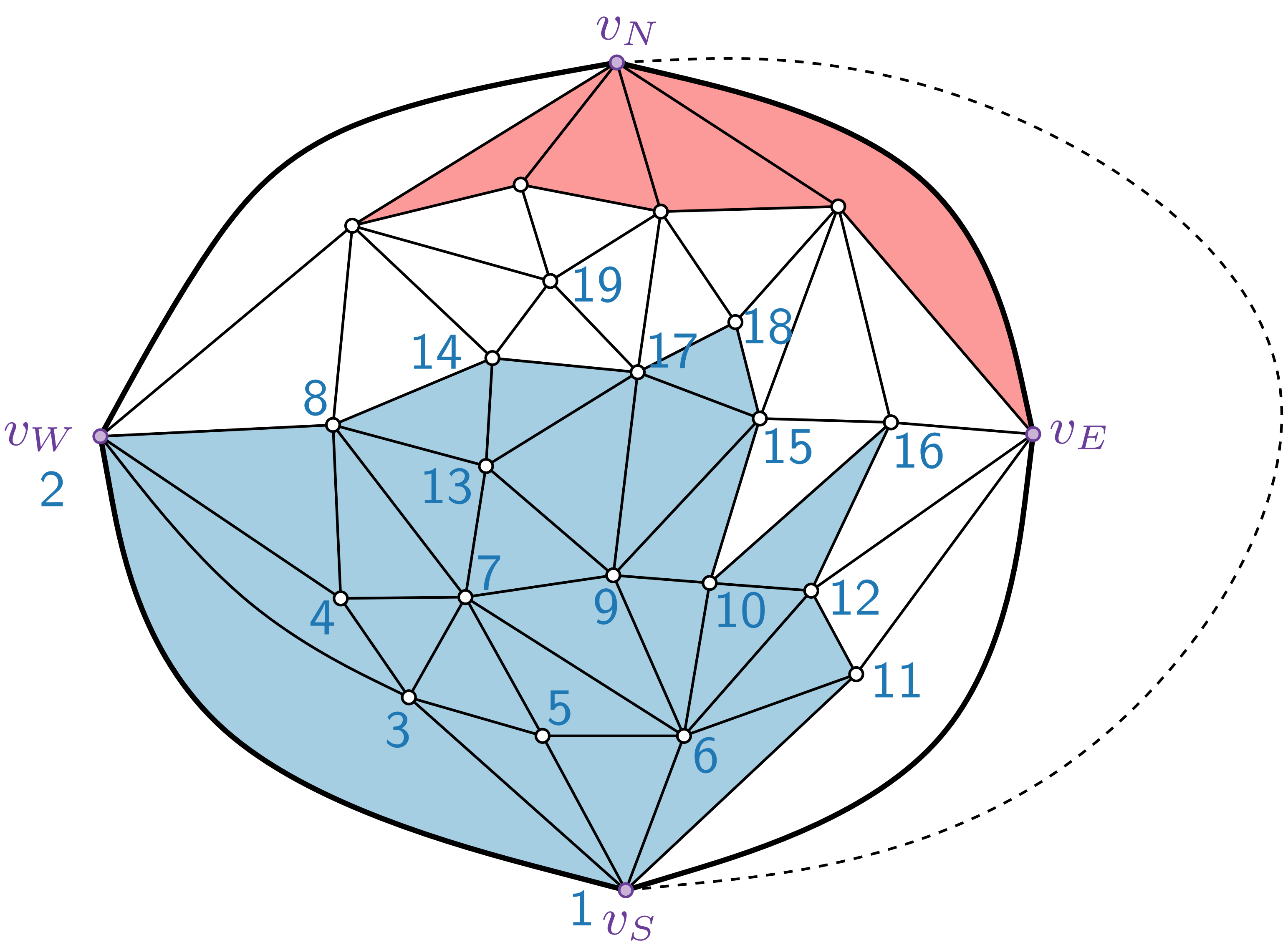

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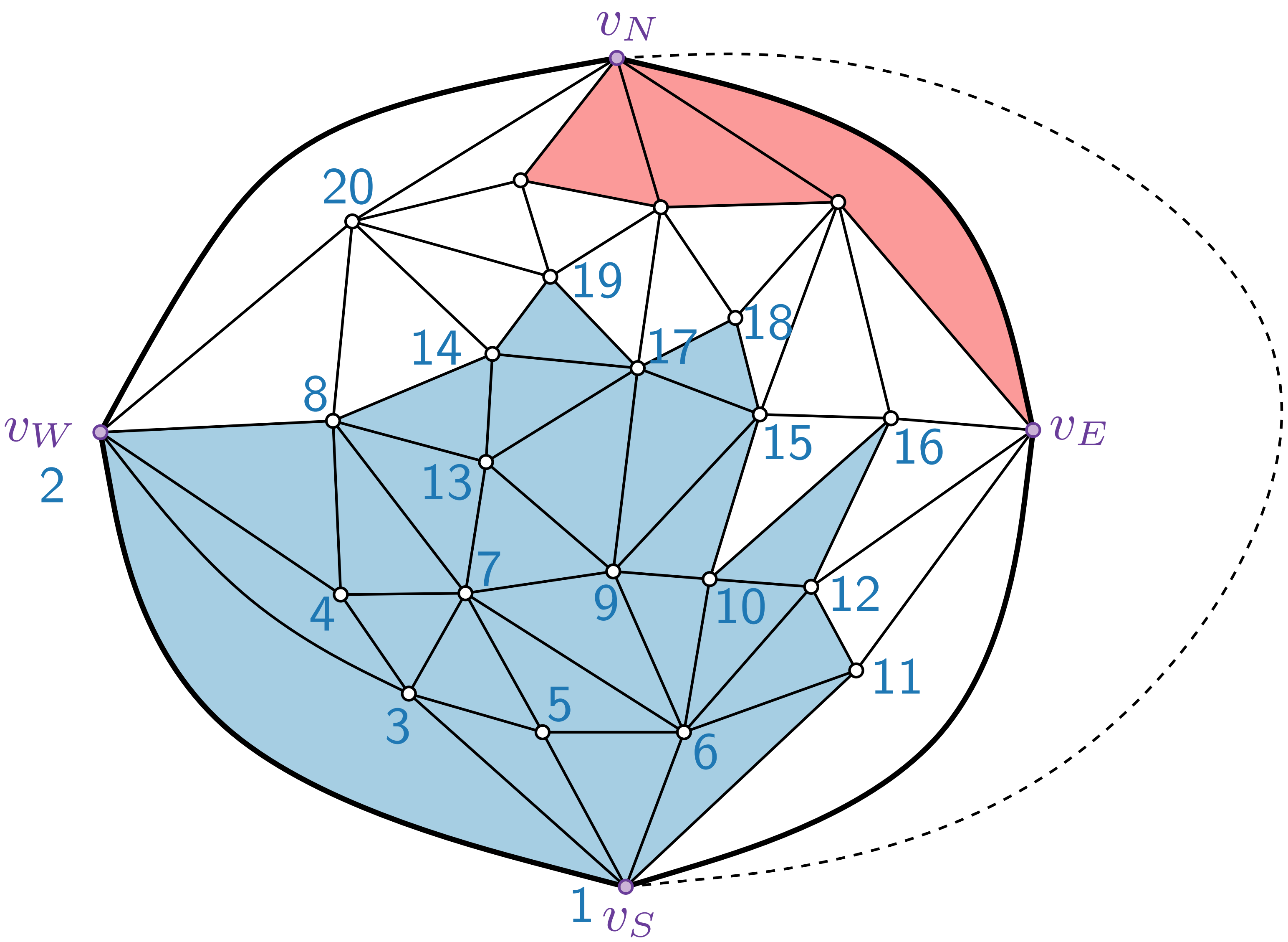

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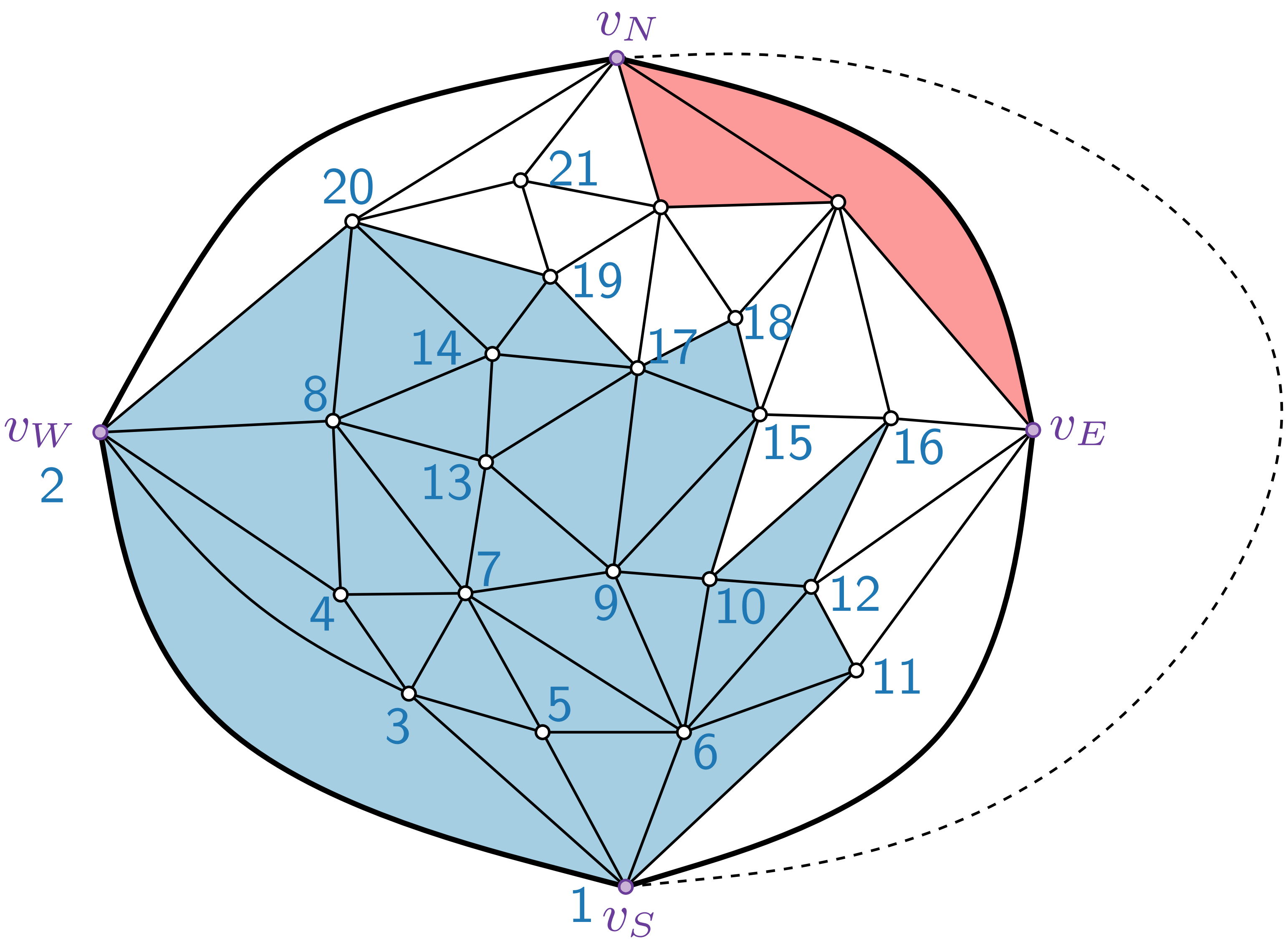

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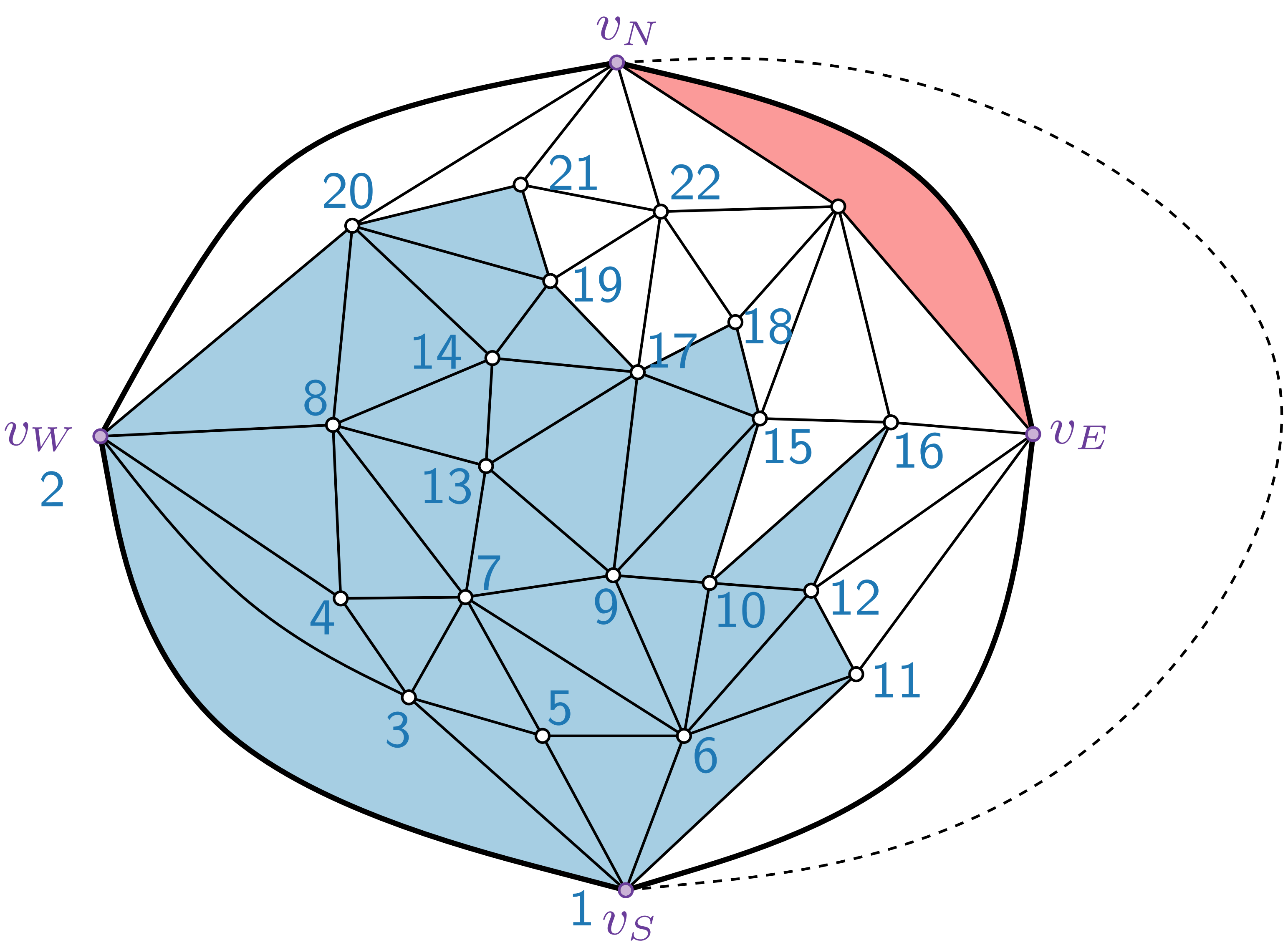

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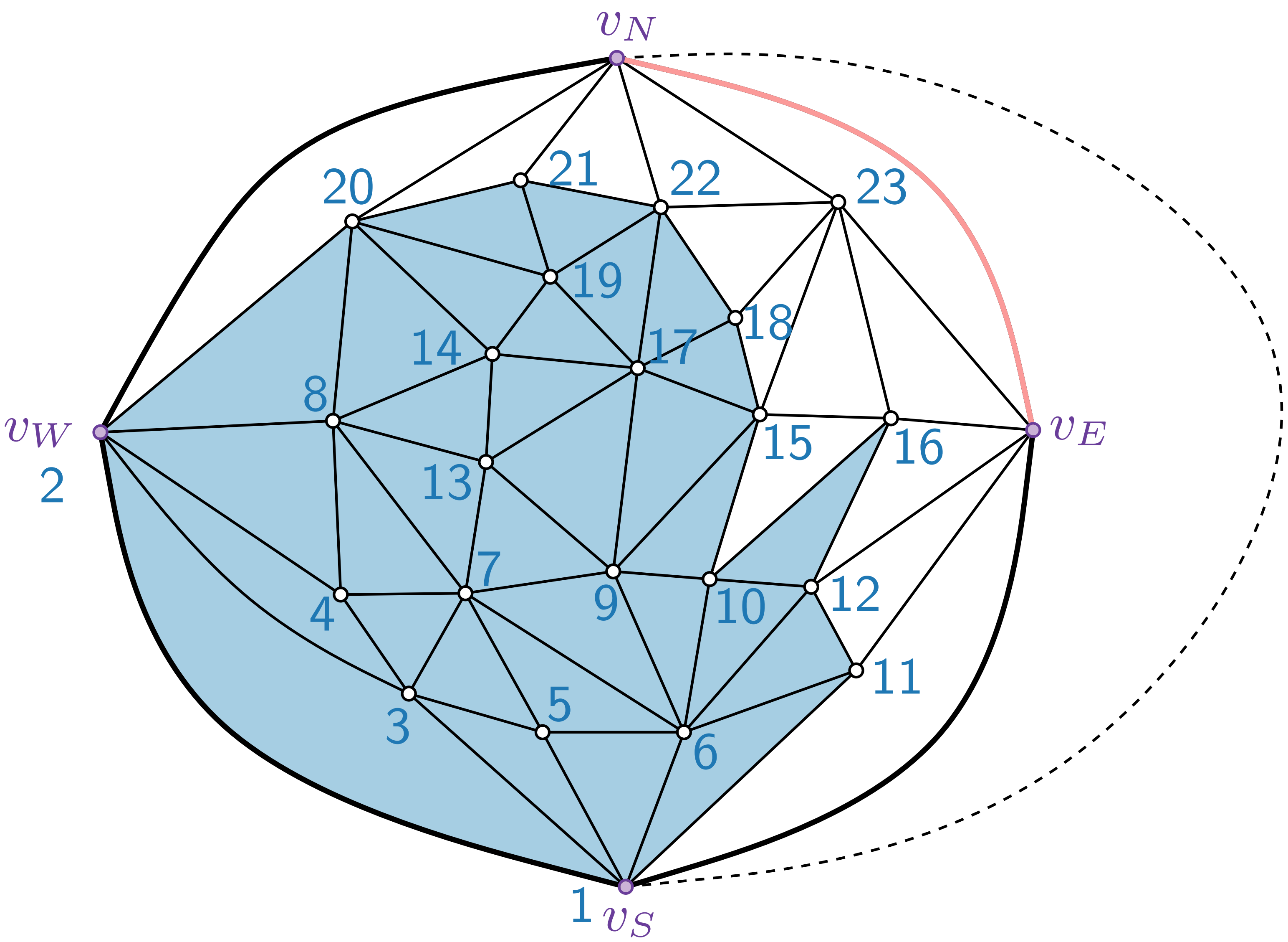

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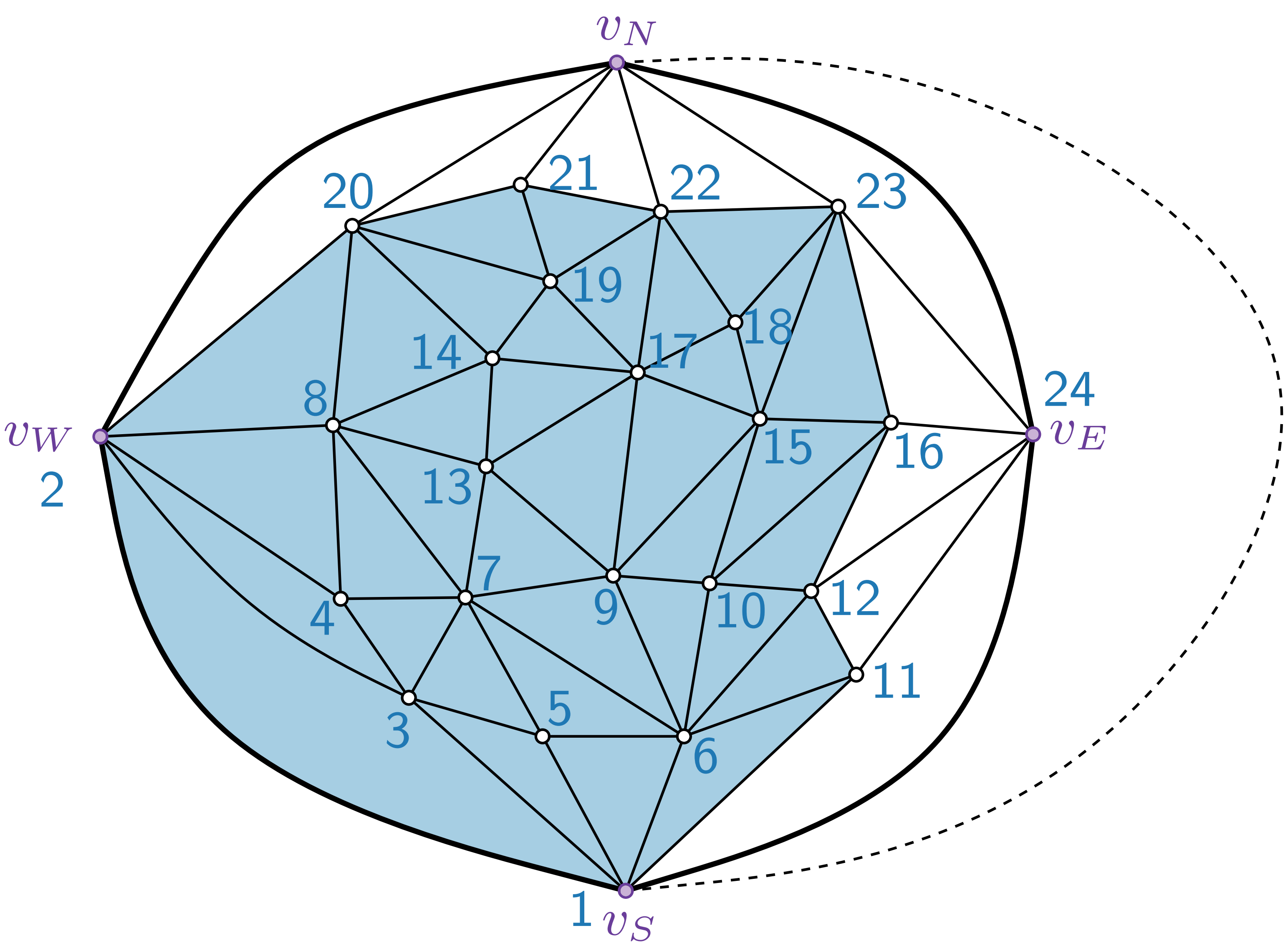

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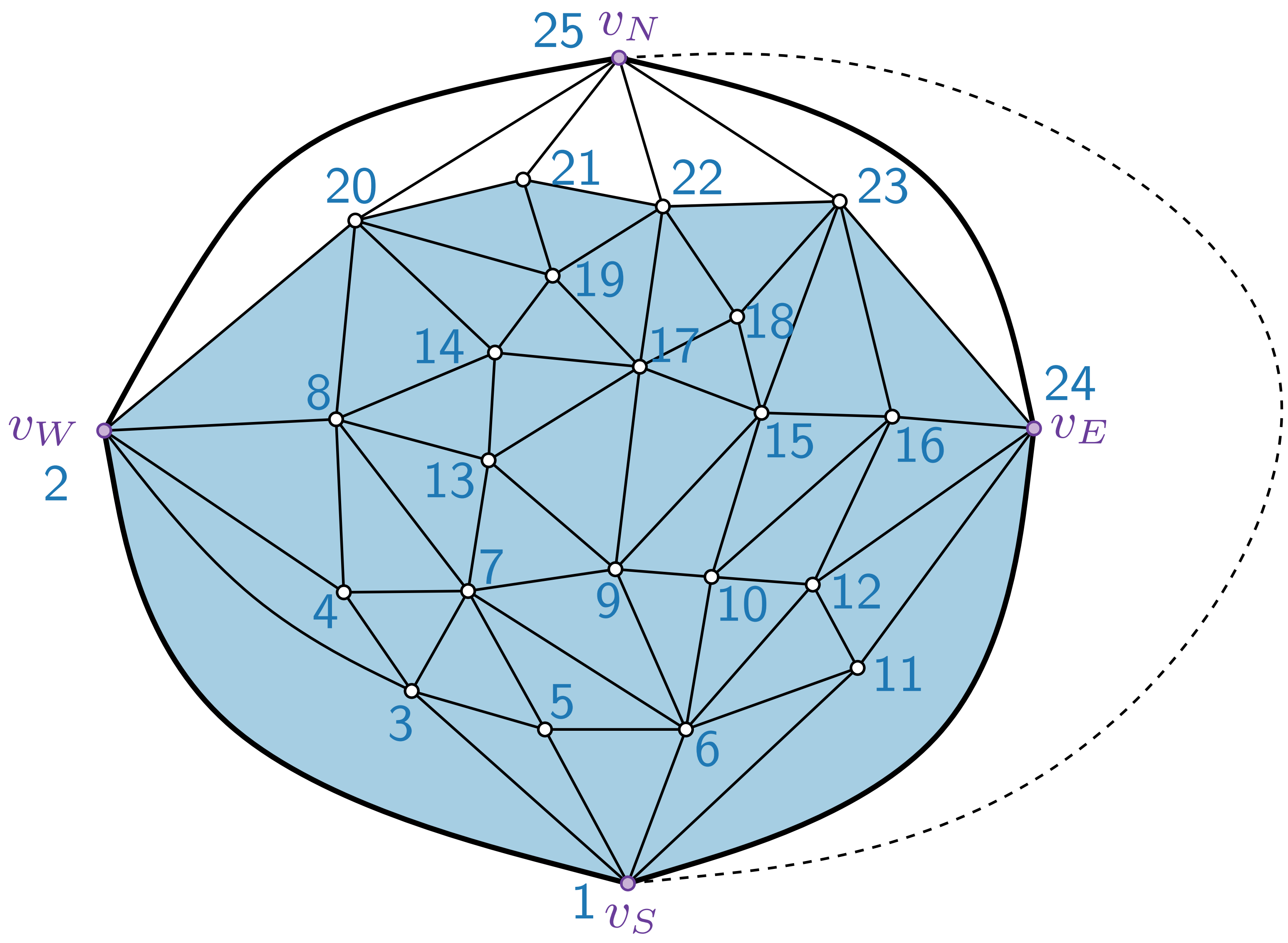

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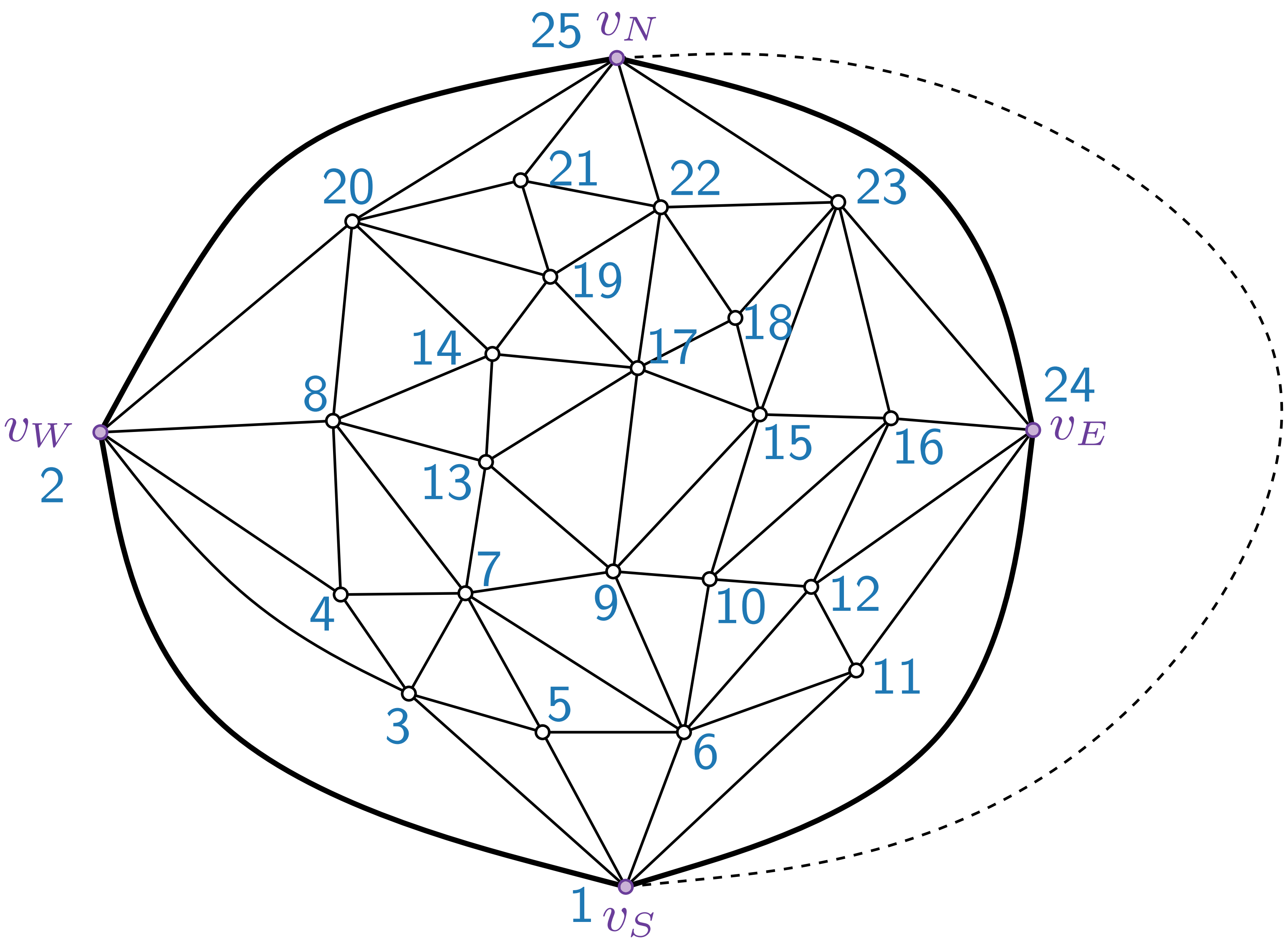

Refined Canonical Order → REL

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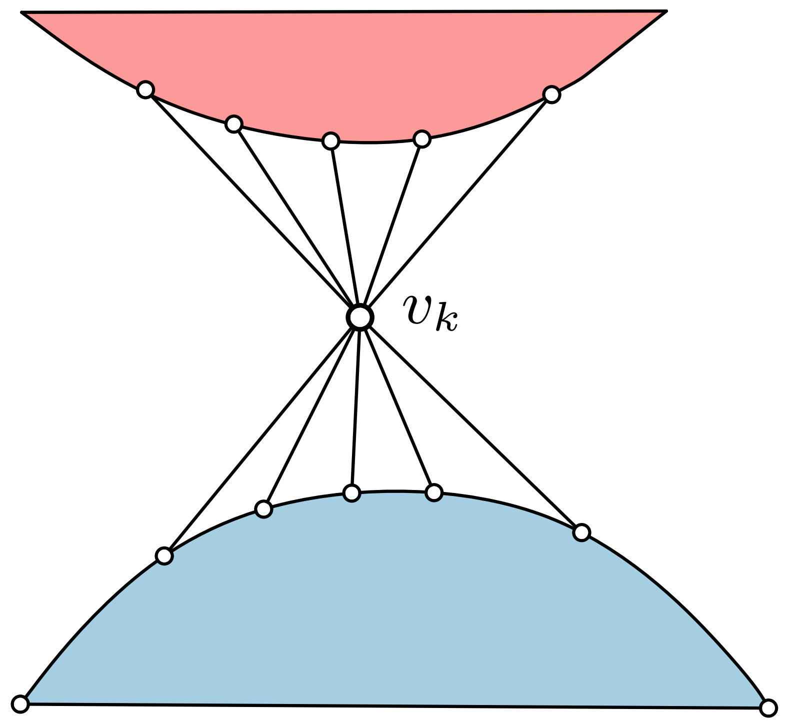

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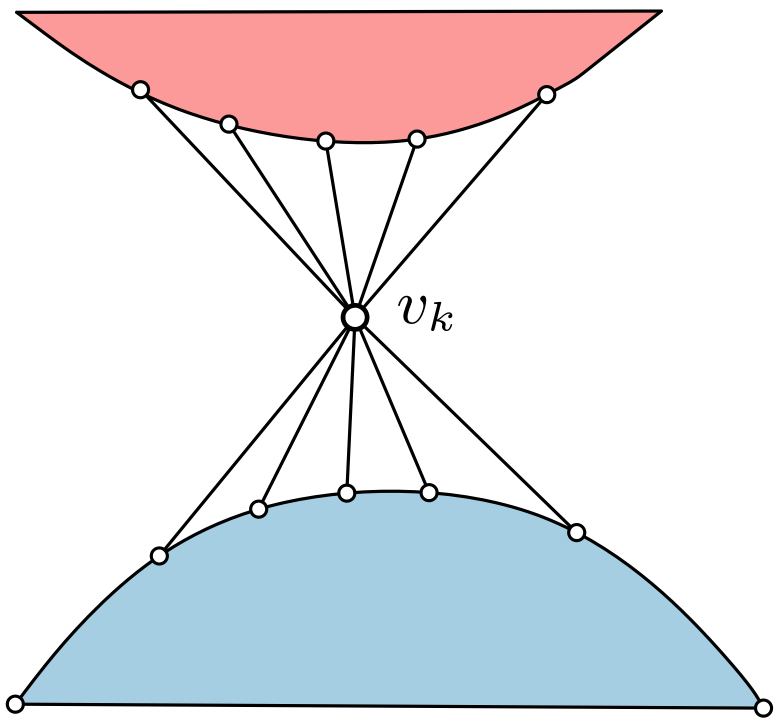

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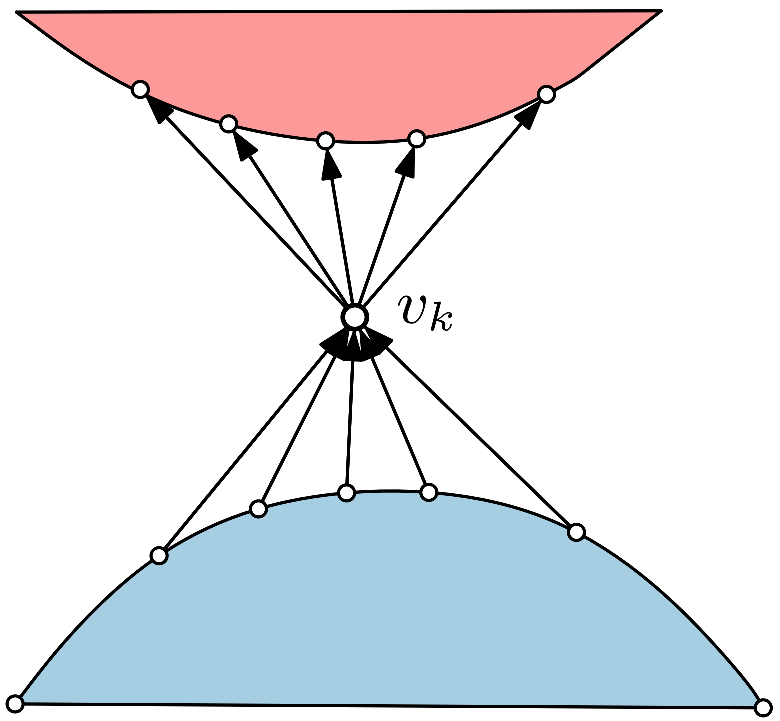

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- For $i < j$, orient (v_i, v_j) from v_i to v_j;
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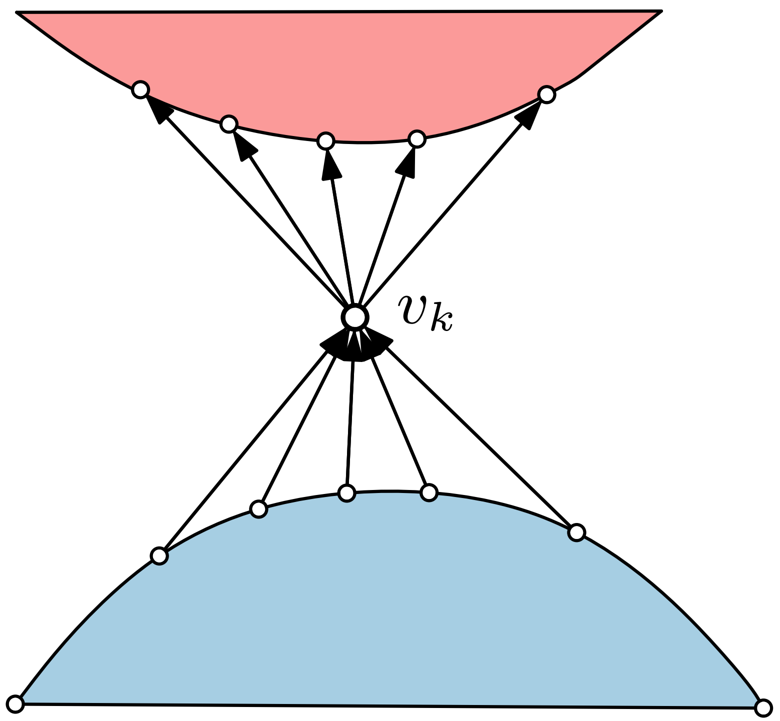

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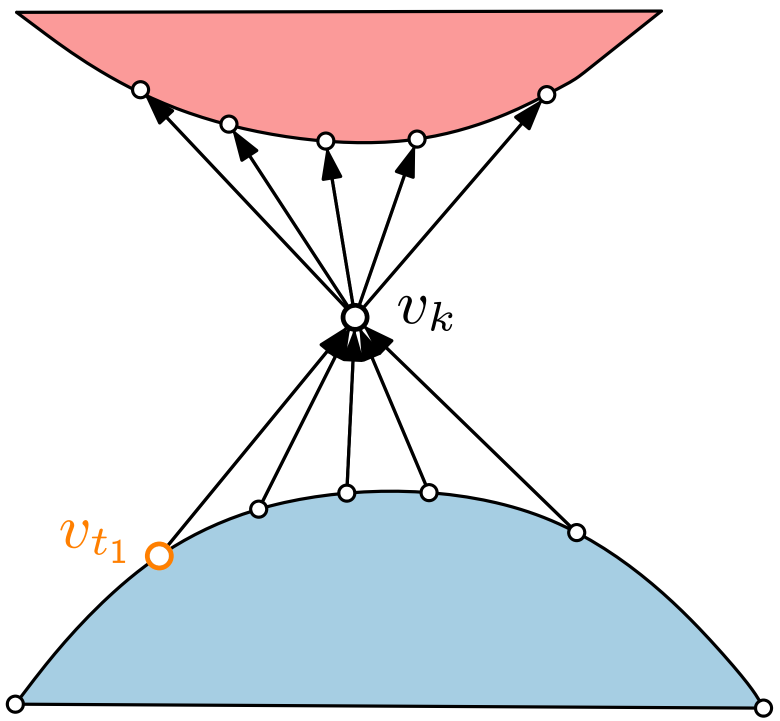

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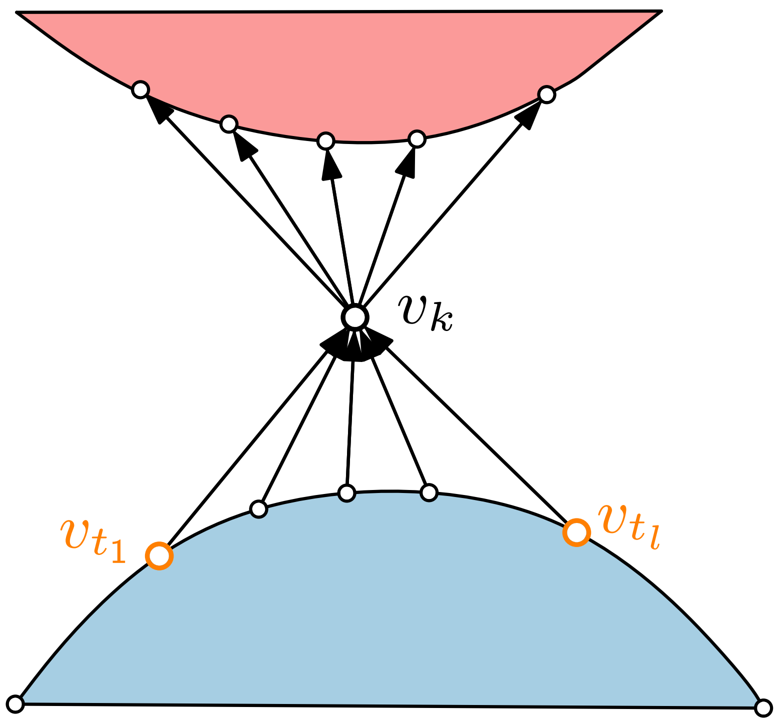

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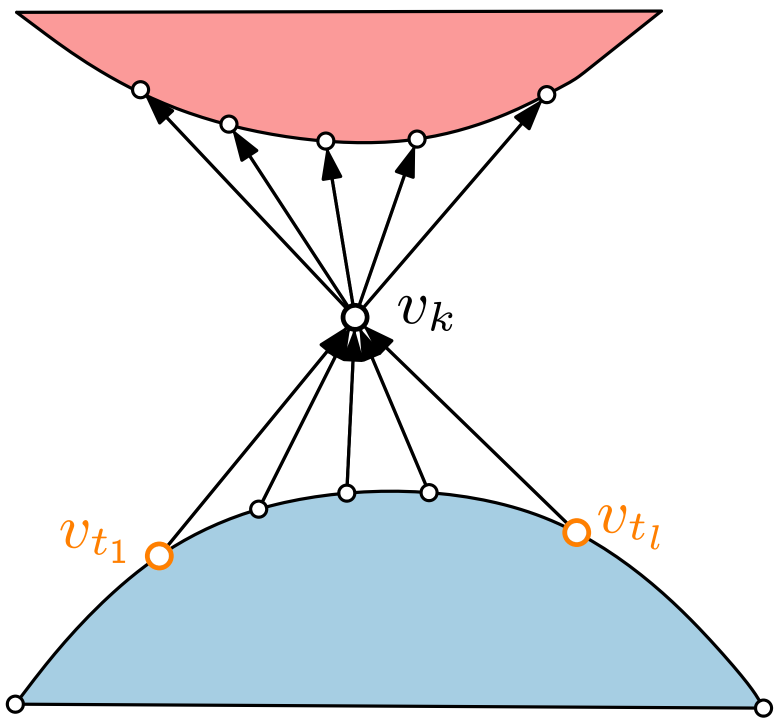
- **Base edge** of v_k is (v_{t_a}, v_k), where $t_a < k$ is minimal.



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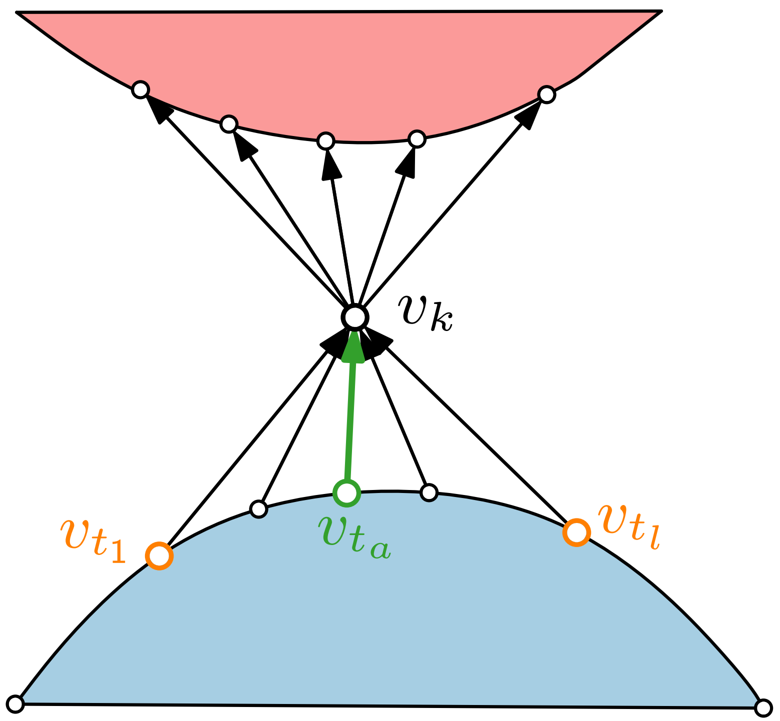

Refined Canonical Order $\rightarrow$ REL

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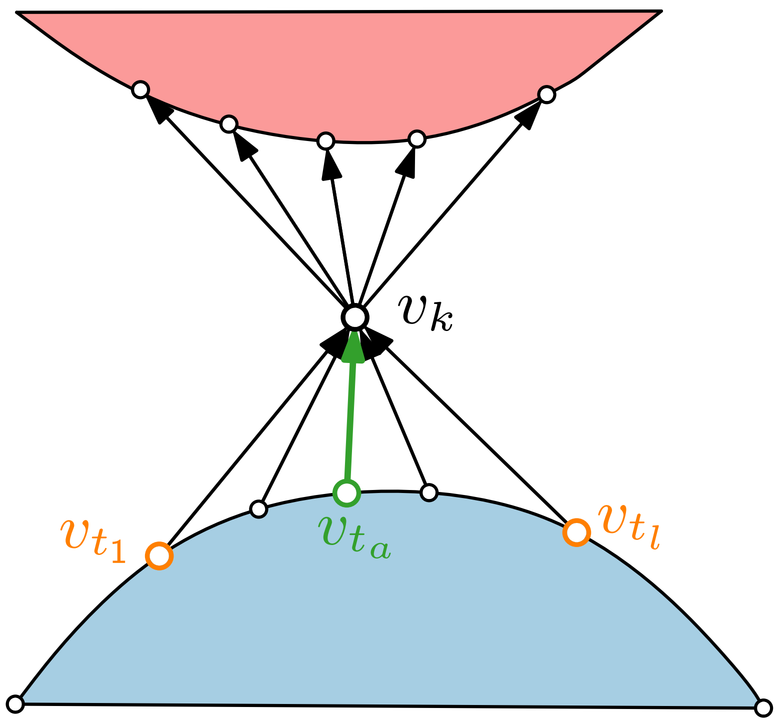

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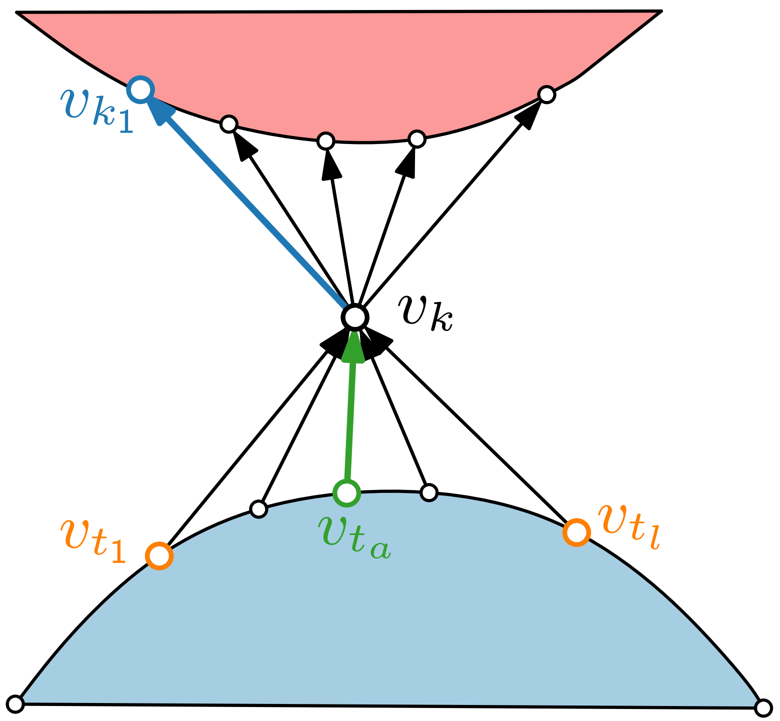

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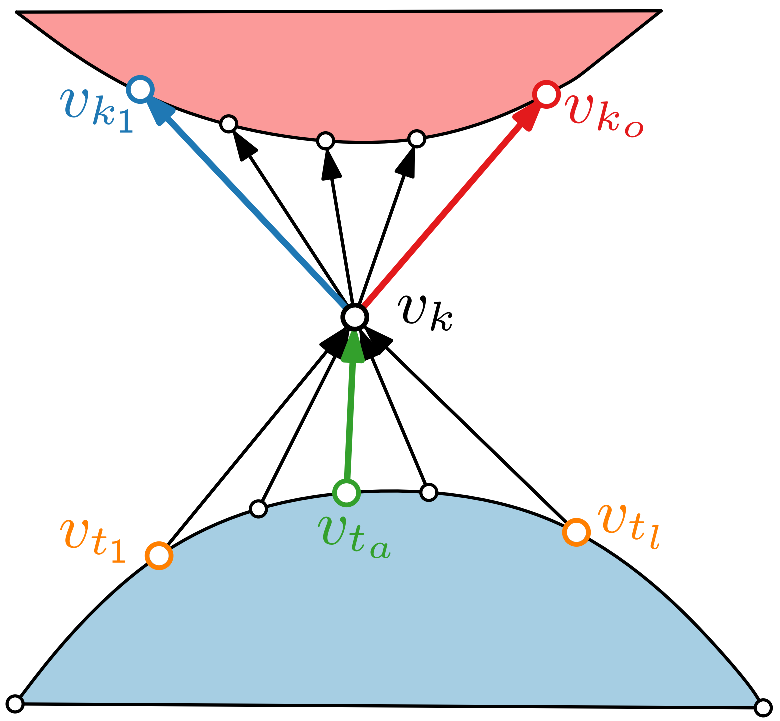

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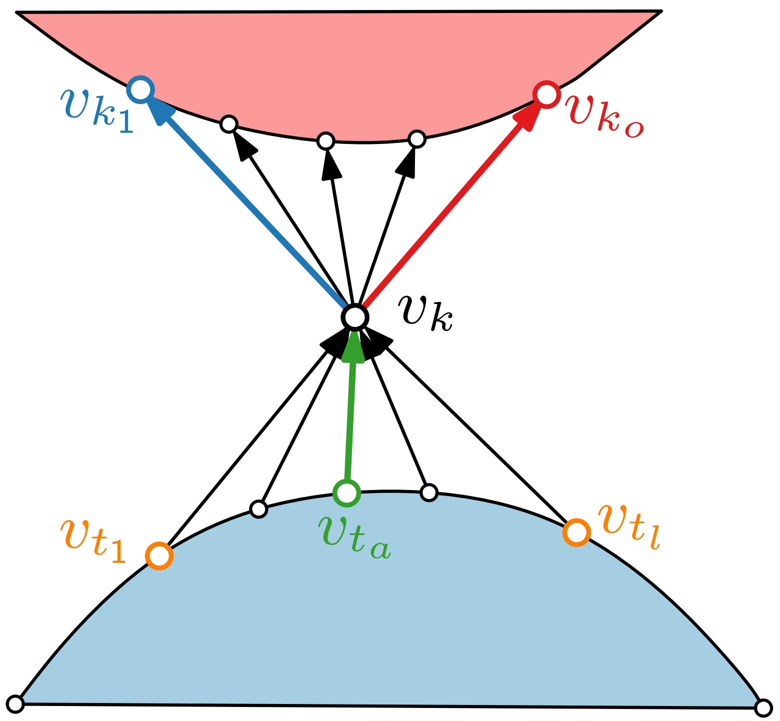

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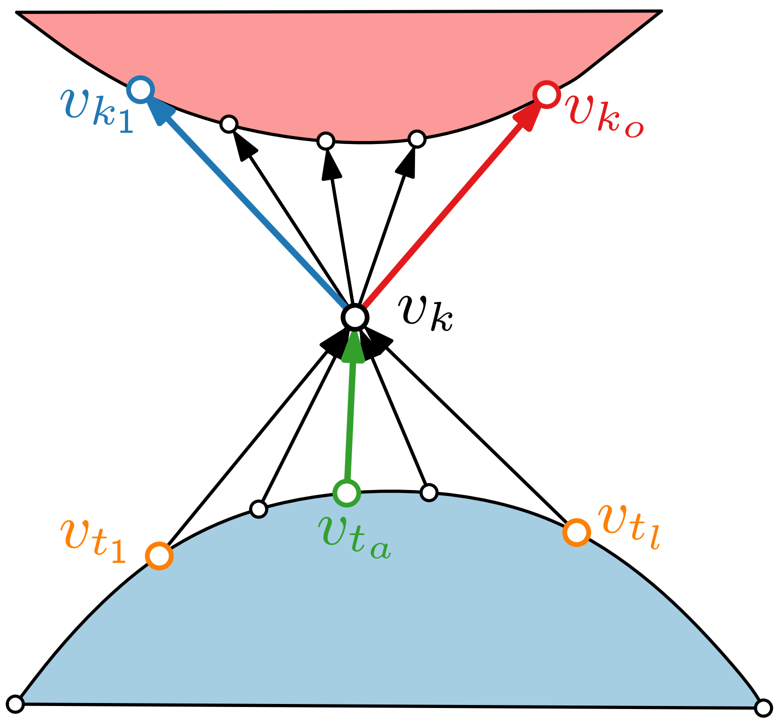

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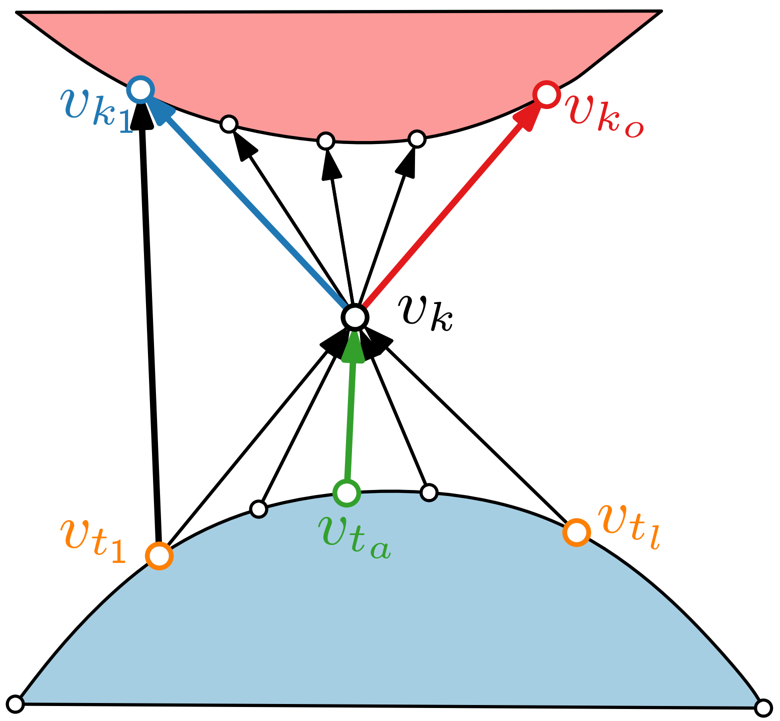

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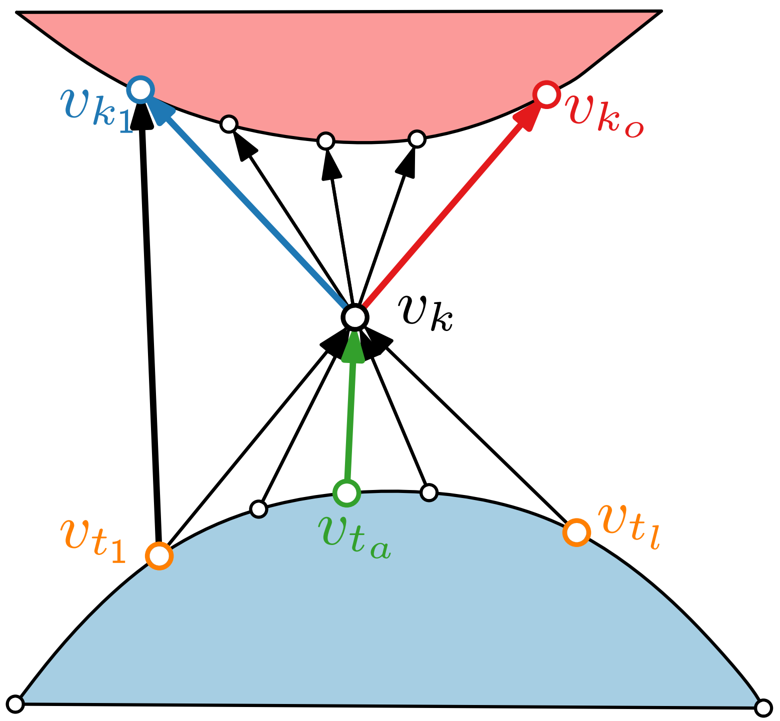

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Contradiction since $v_k > v_{t_1}$.

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Lemma 2.
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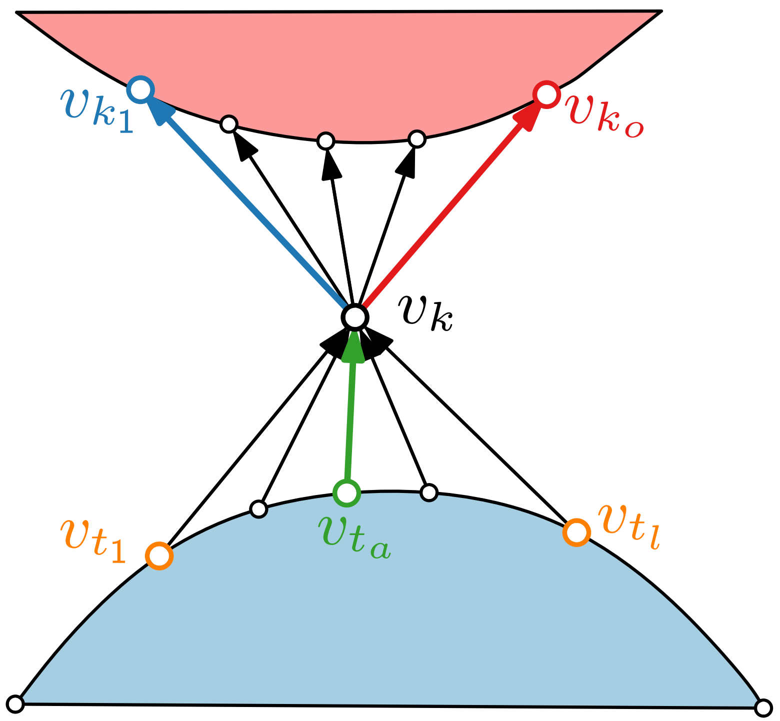

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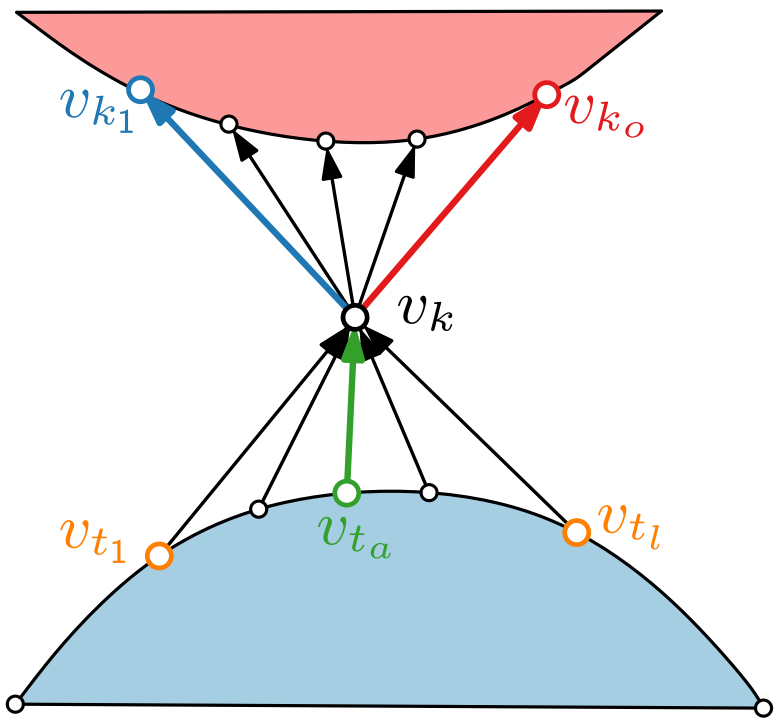

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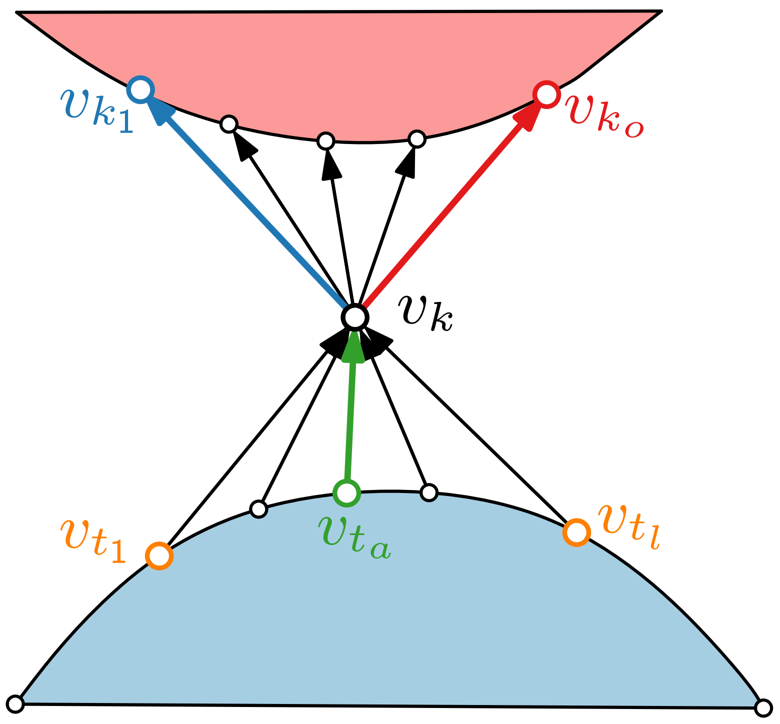

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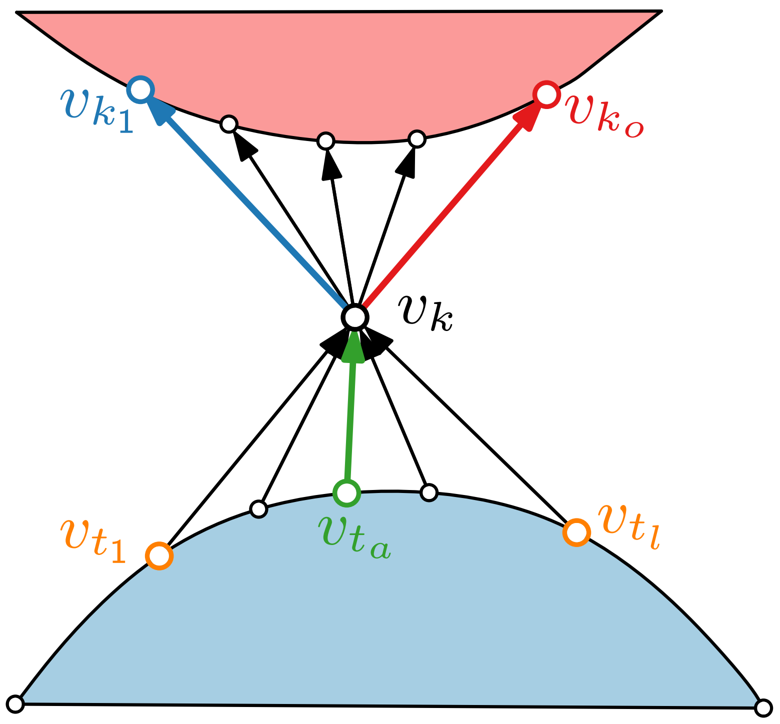

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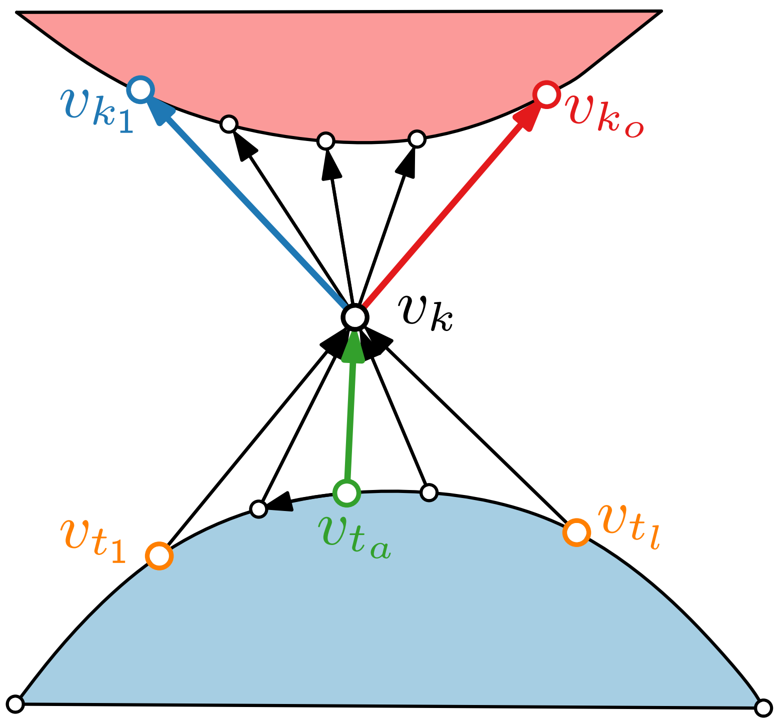

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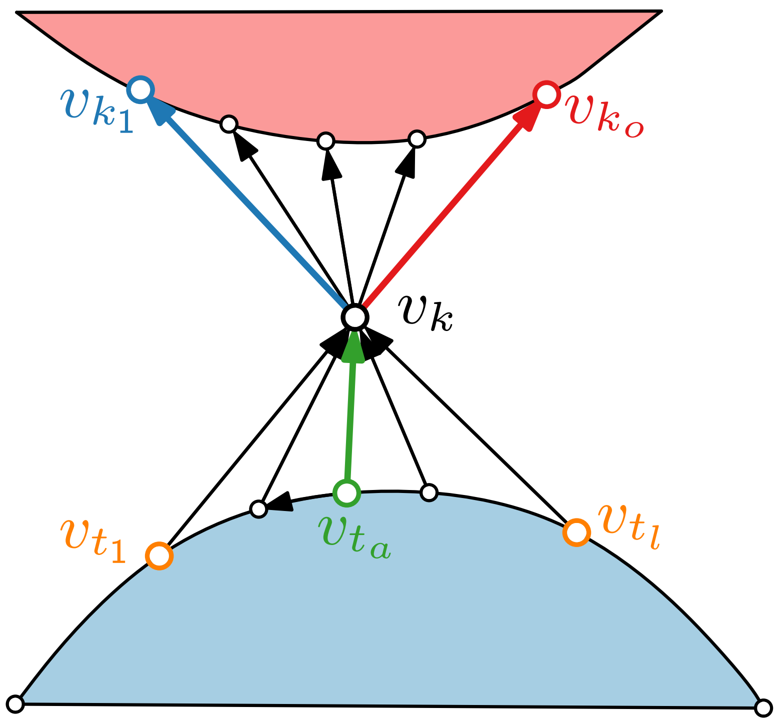

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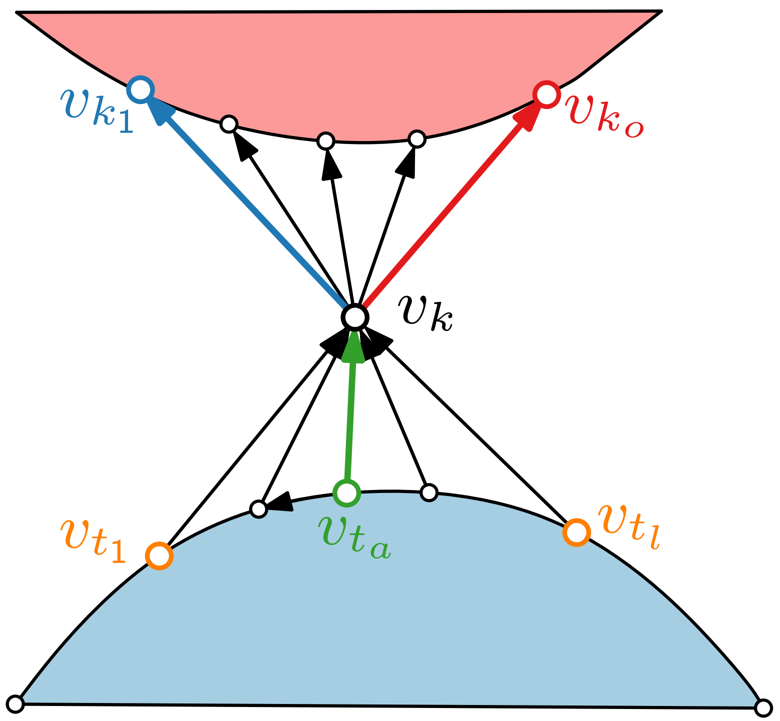

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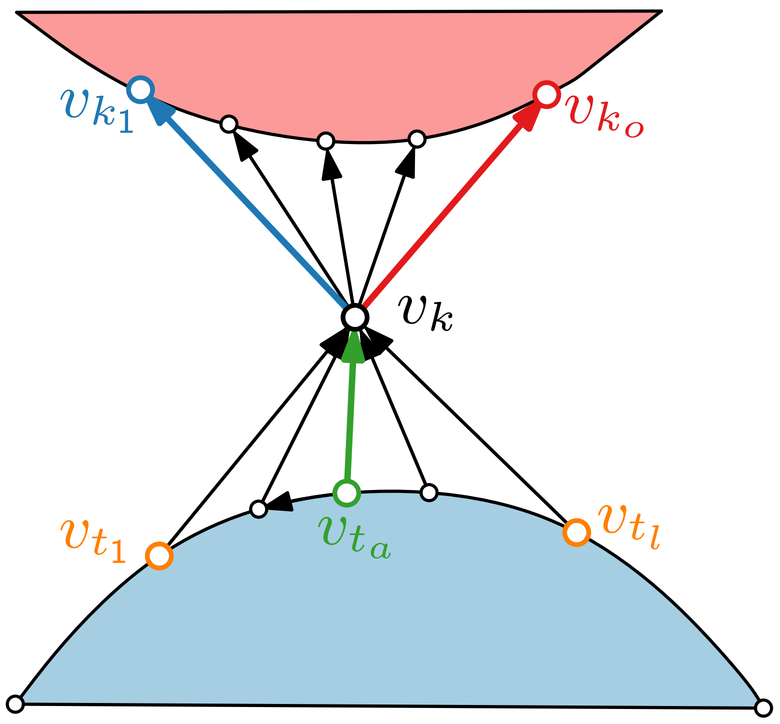

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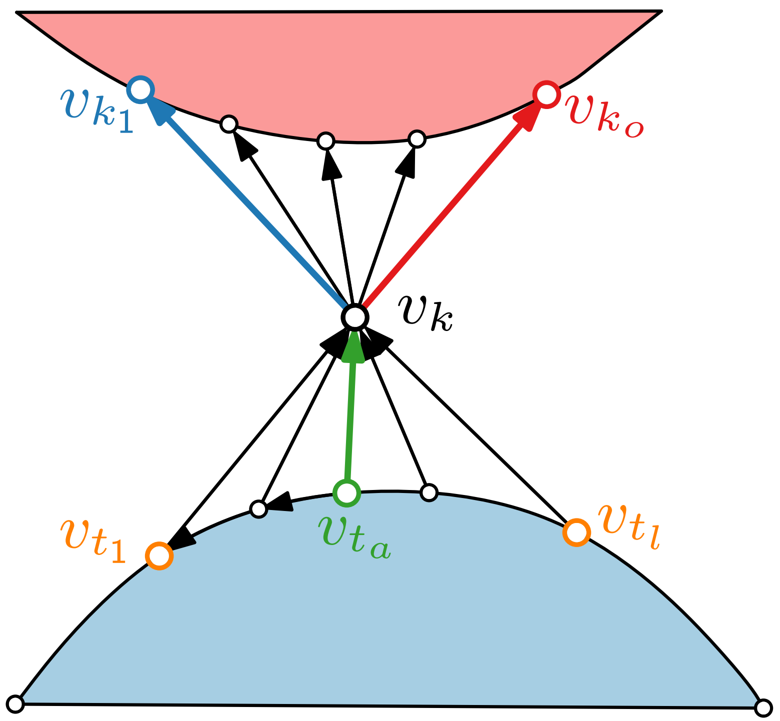

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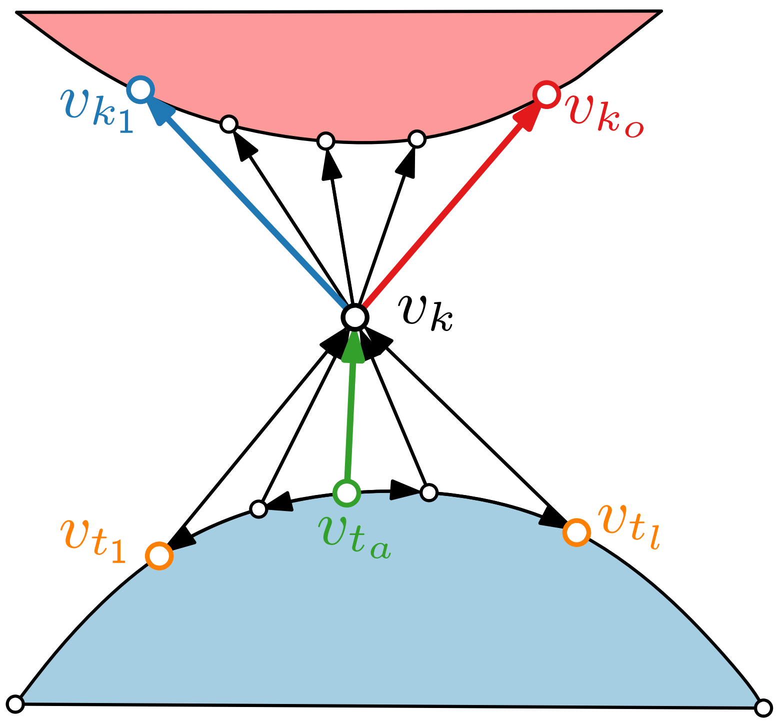

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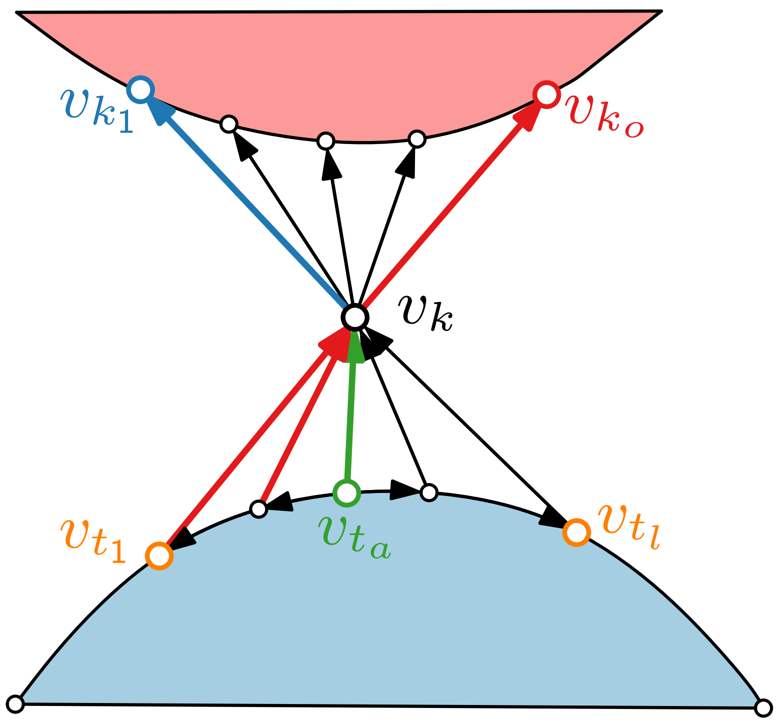

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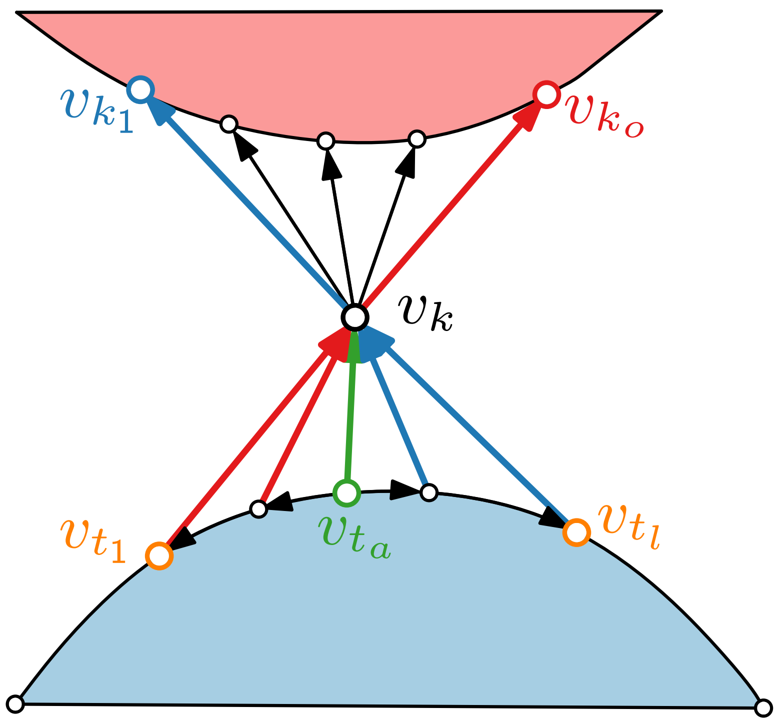

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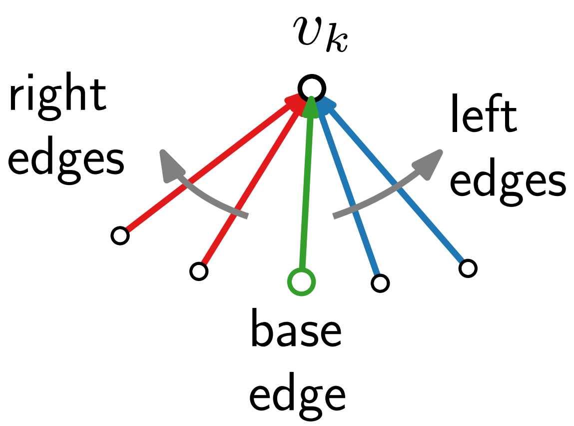

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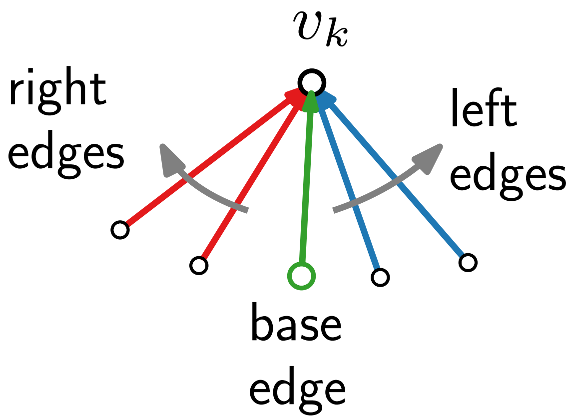

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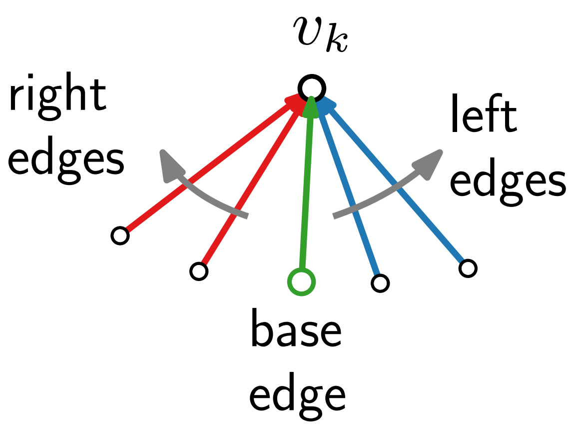

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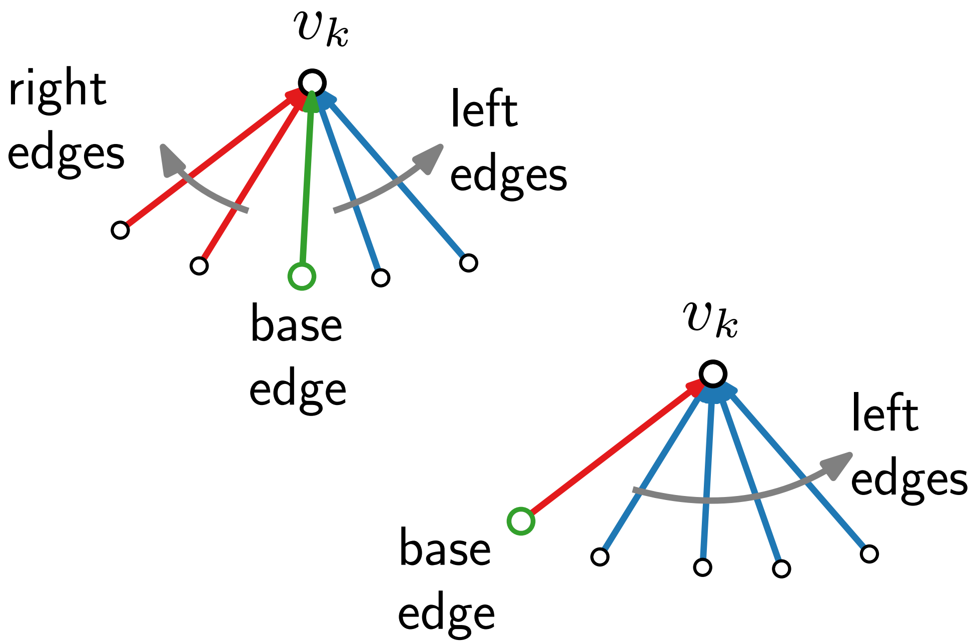

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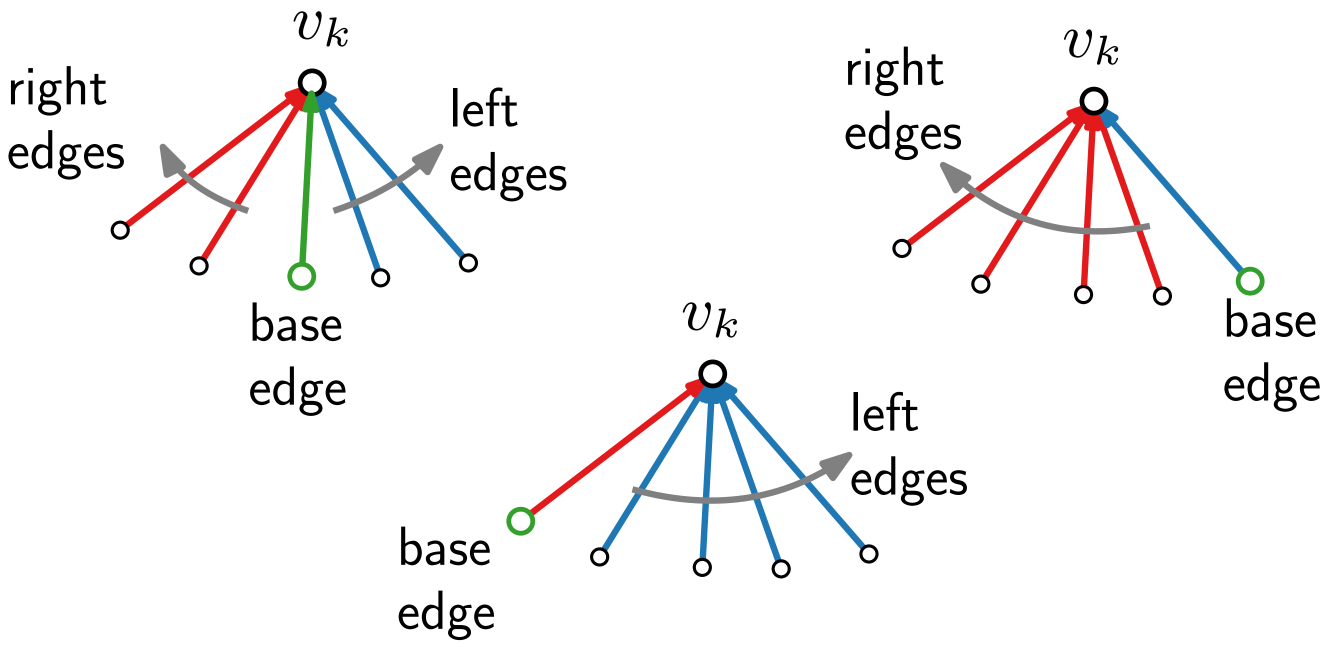

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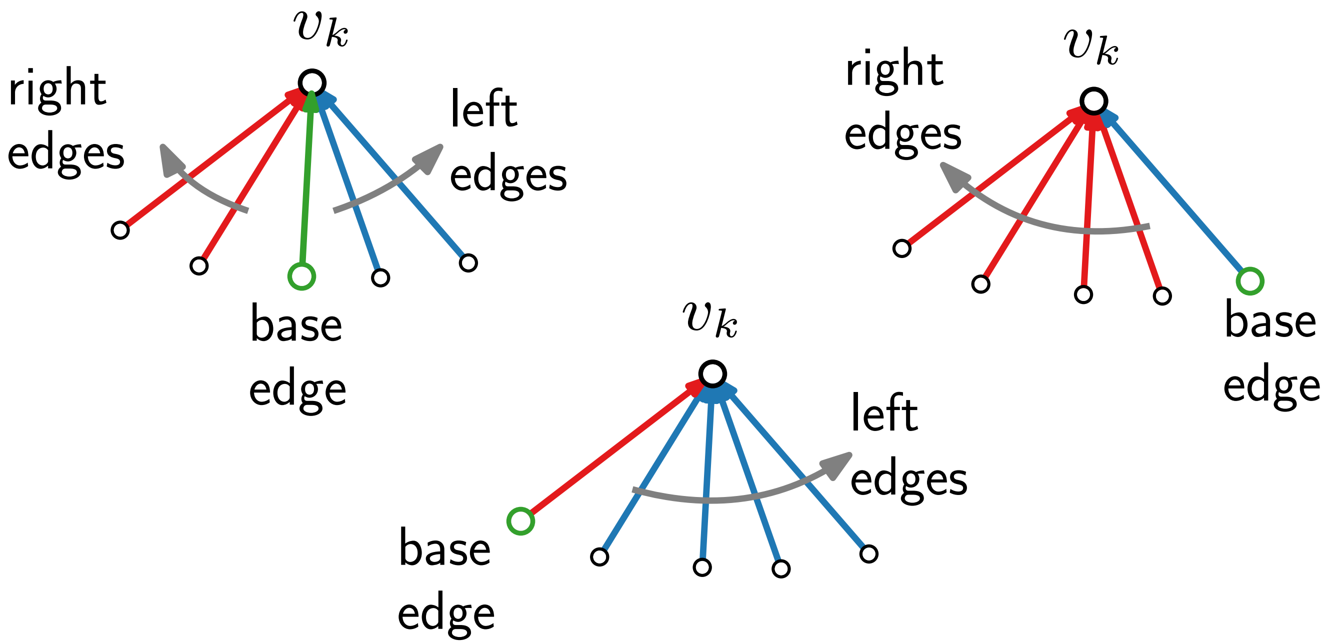

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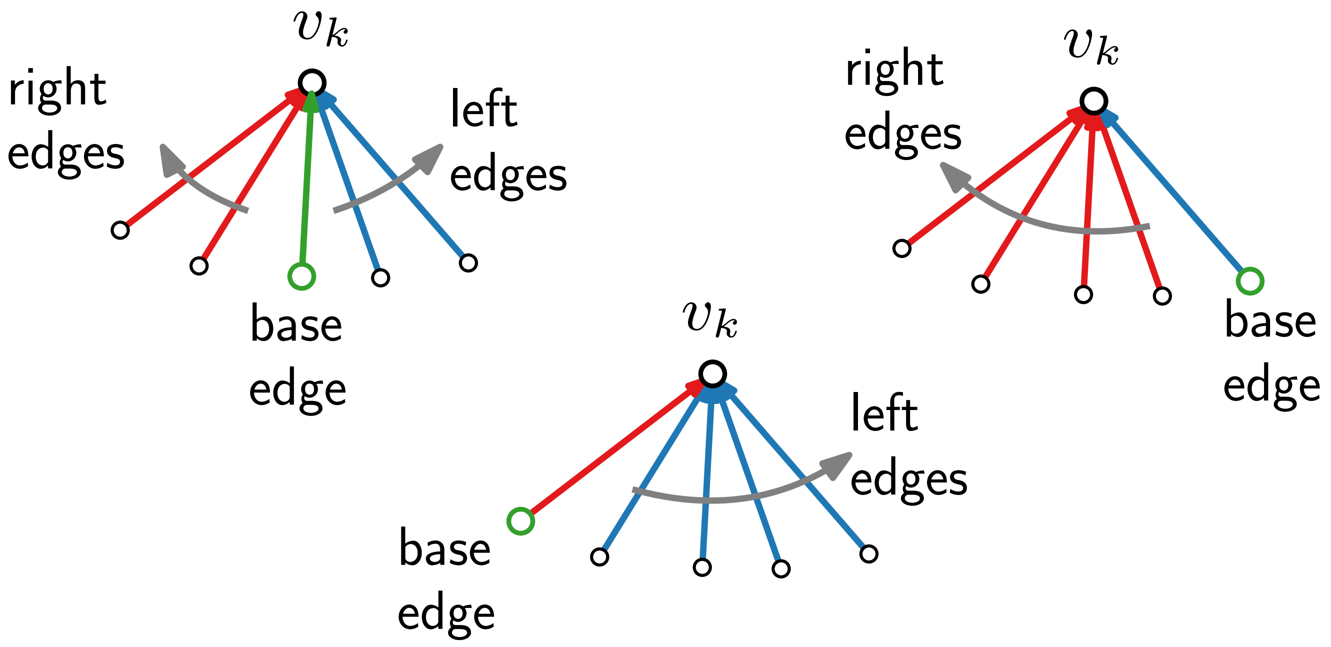

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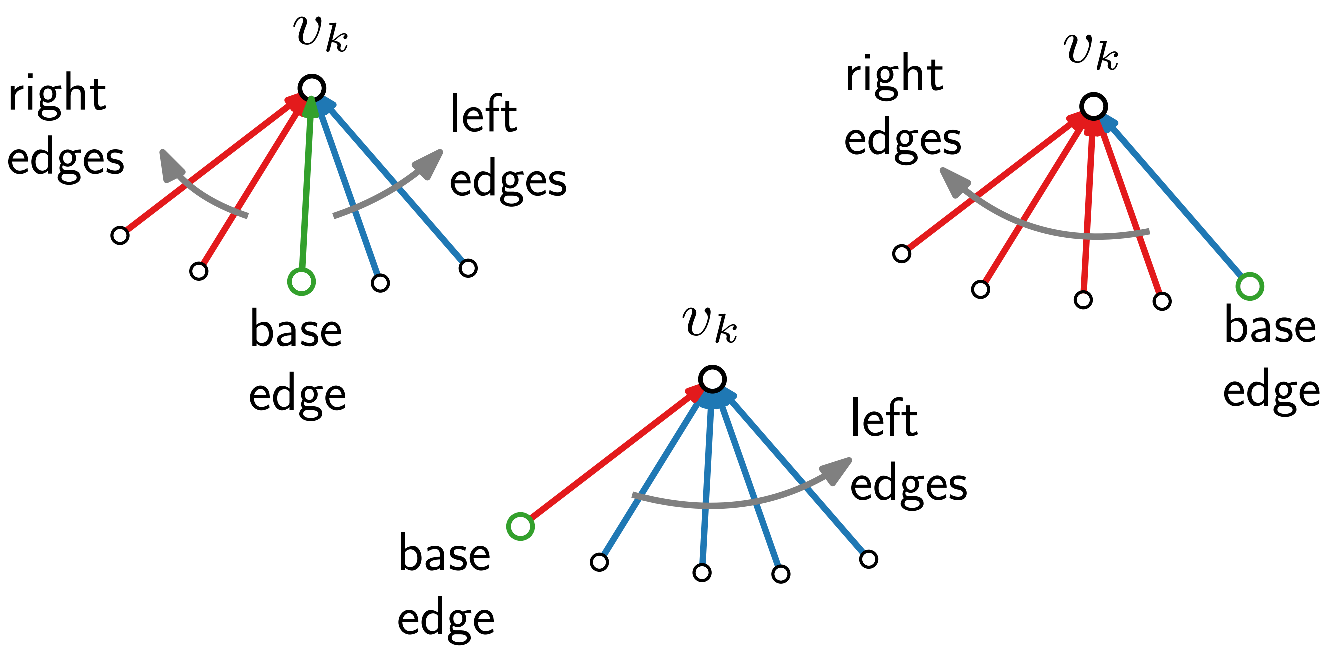

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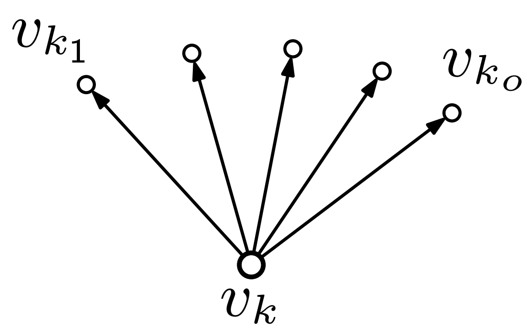

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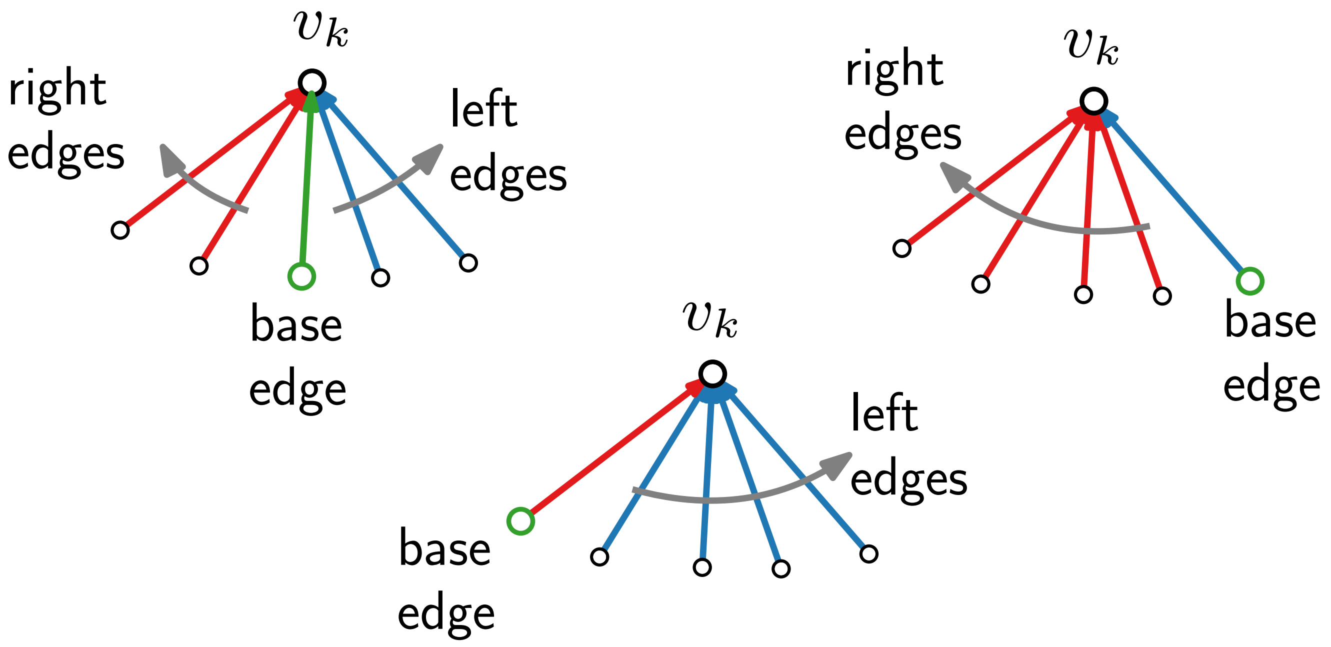

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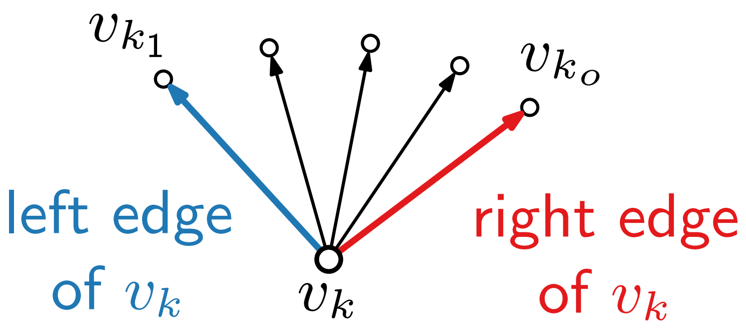

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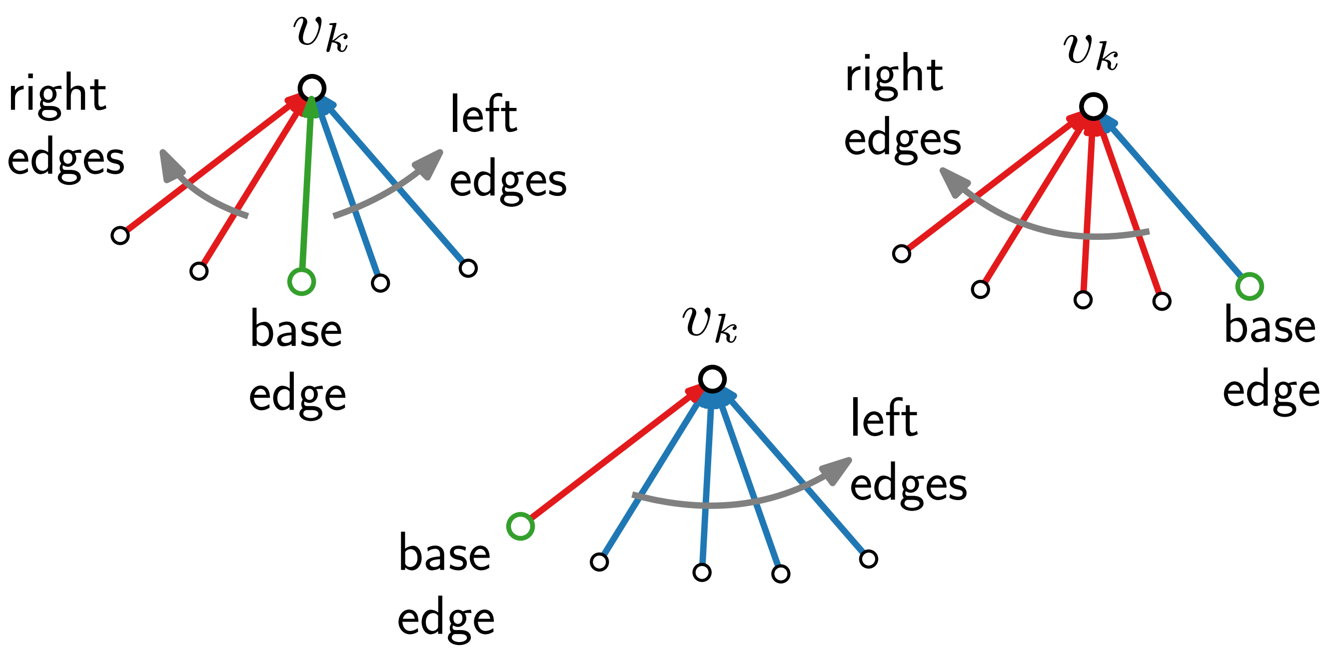

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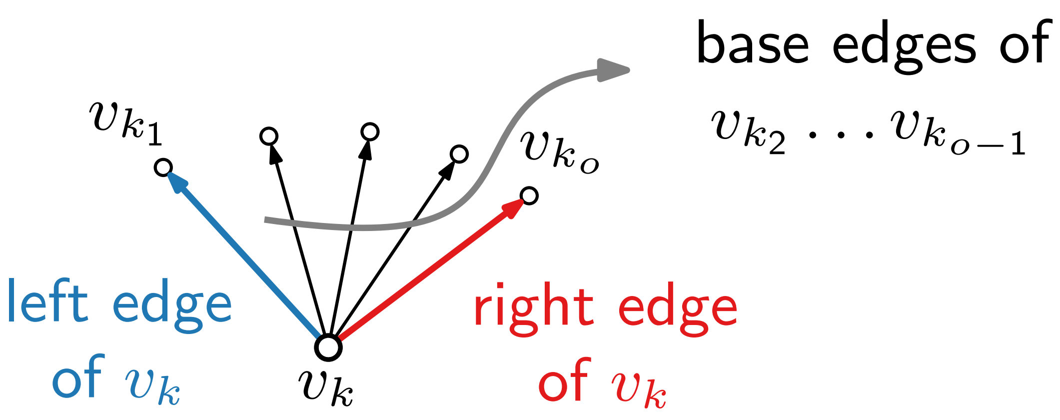

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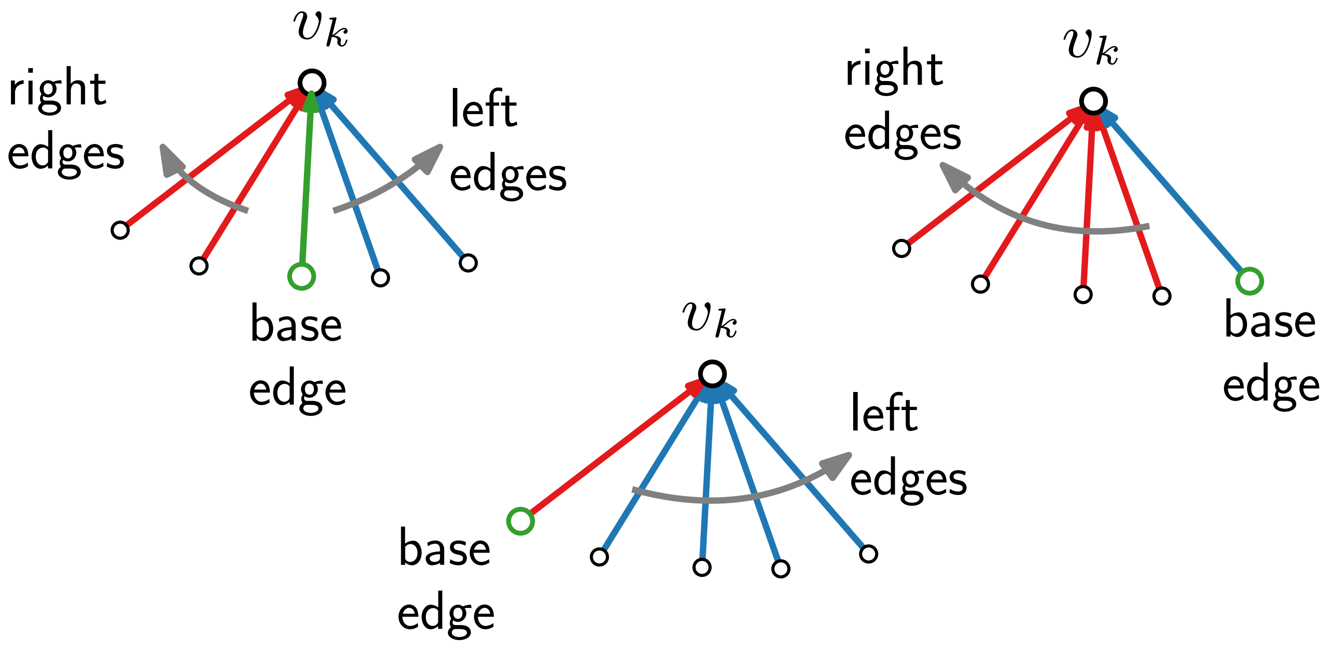

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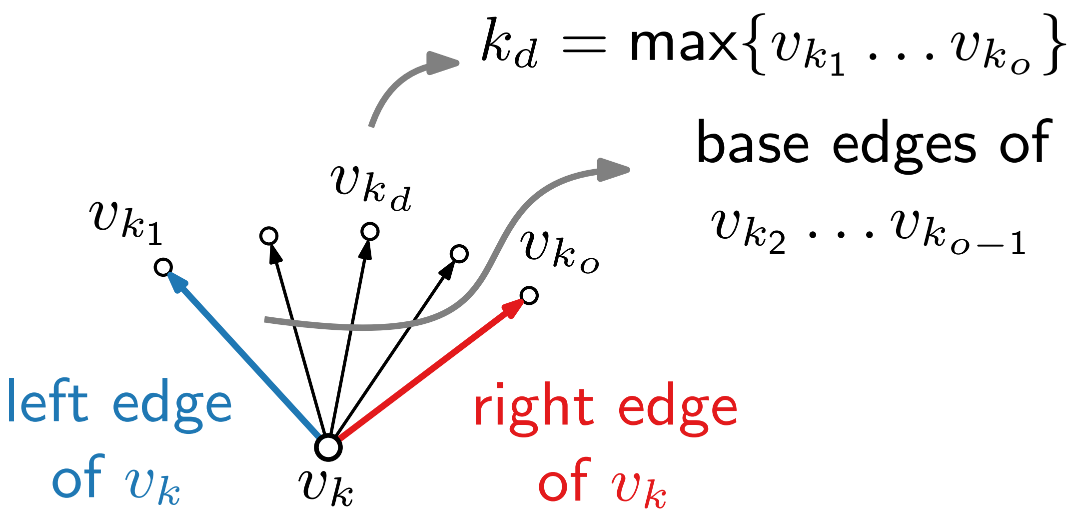

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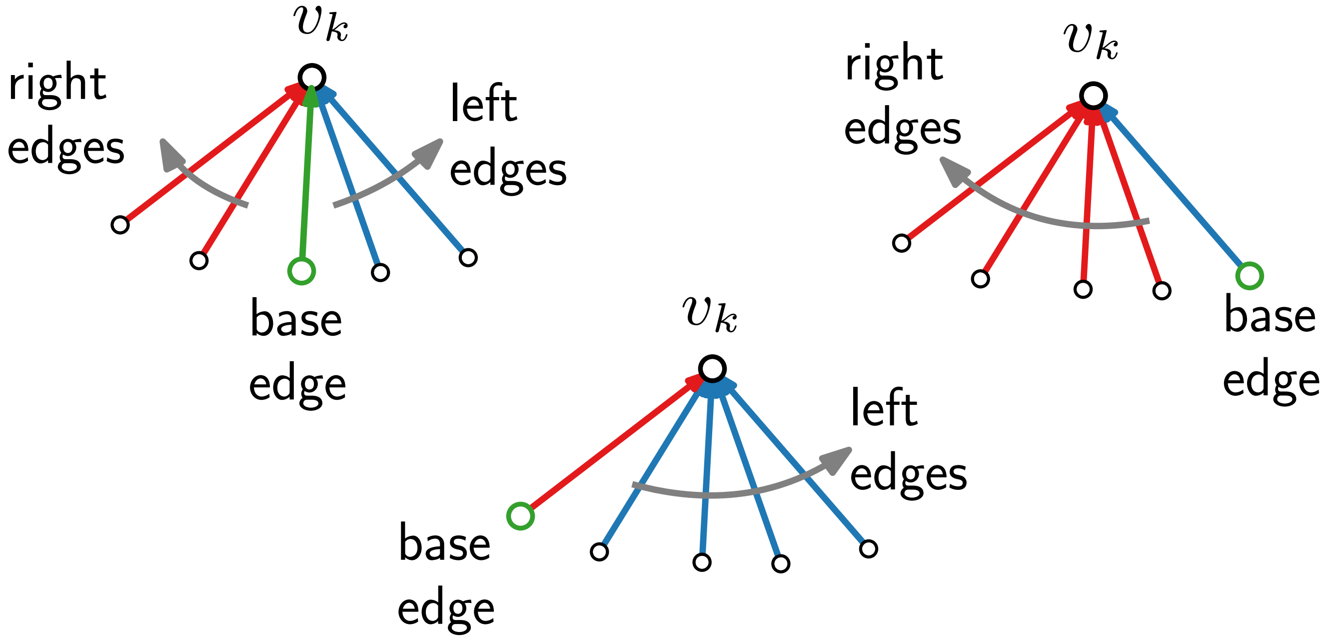
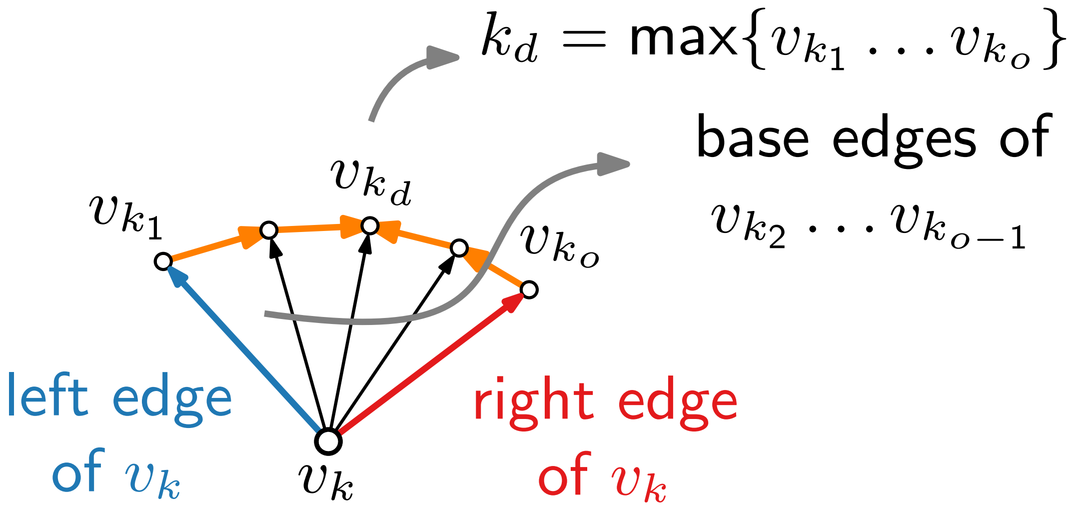


Let T_r be the red edges and T_b the blue edges.

Lemma 3.
$\{T_r, T_b\}$ is a regular edge labeling.

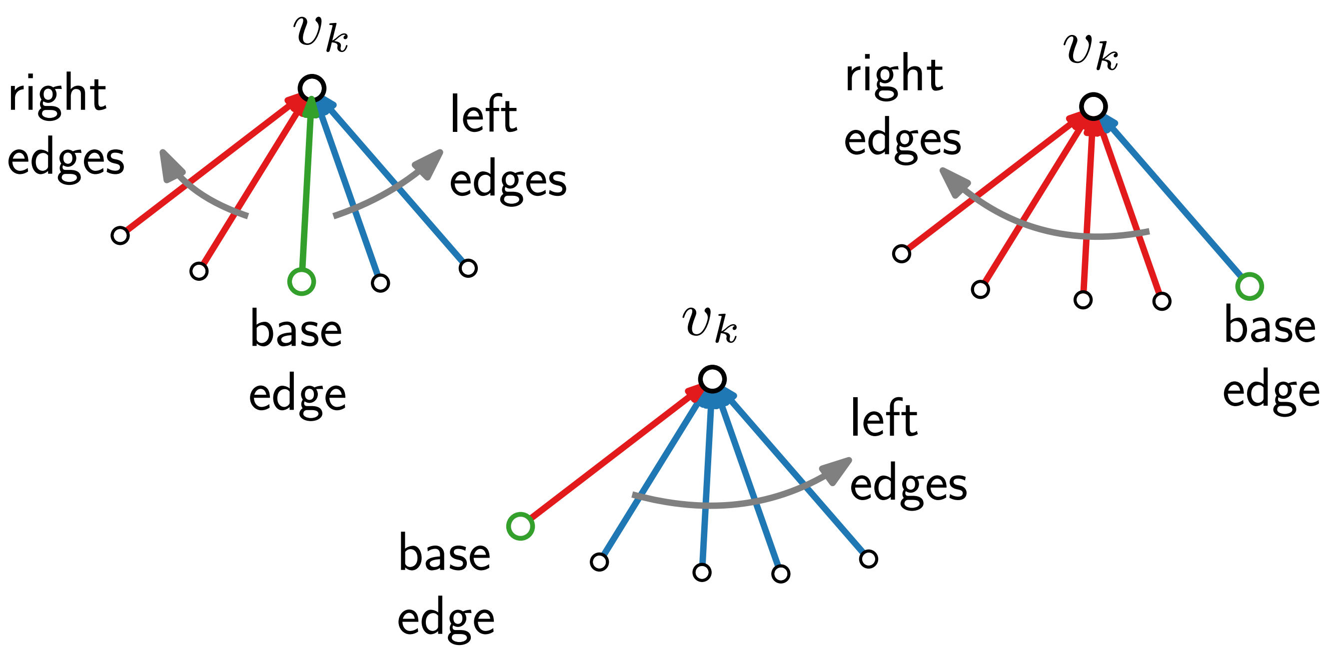
Proof.

$k_o \geq 2$



- $k_1 < k_2 < \dots < k_d$ and $k_d > k_{d+1} > \dots > k_o$

Refined Canonical Order → REL



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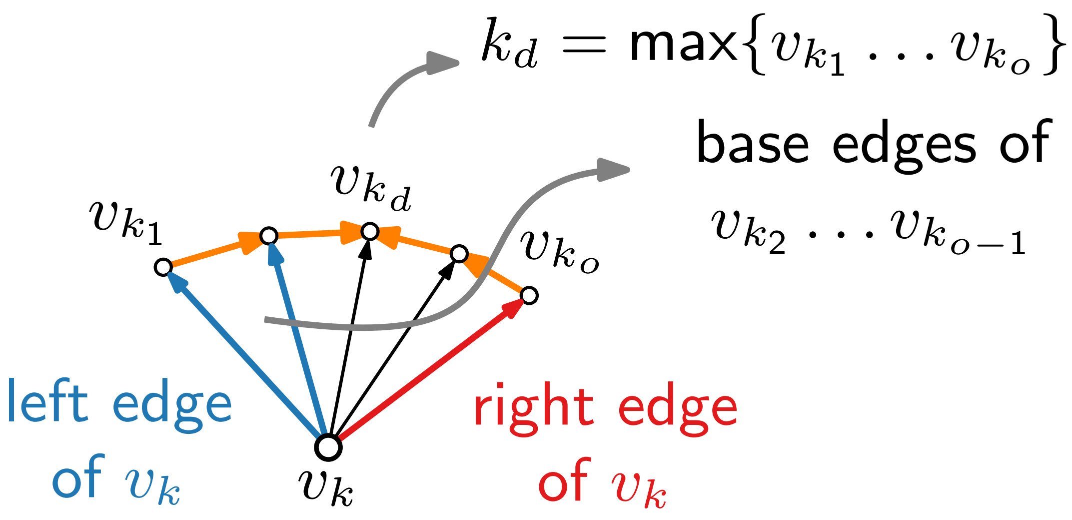

- $k_1 < k_2 < \dots < k_d$ and $k_d > k_{d+1} > \dots > k_o$
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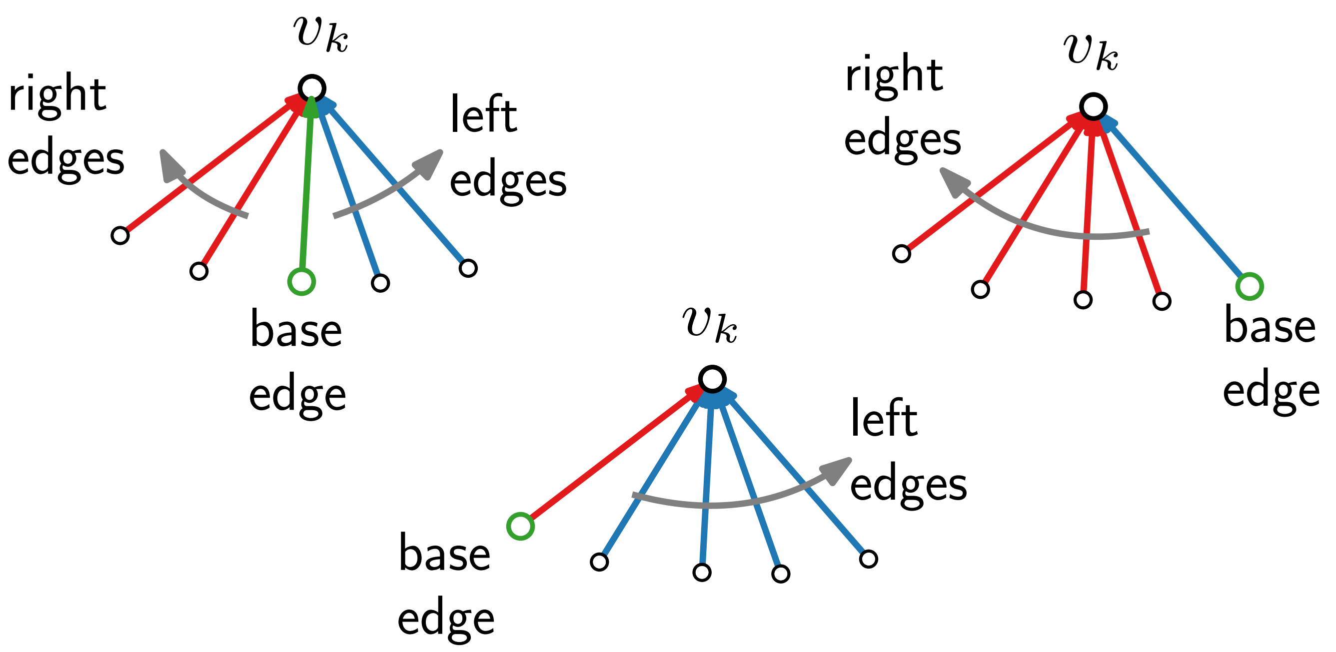
Refined Canonical Order → REL



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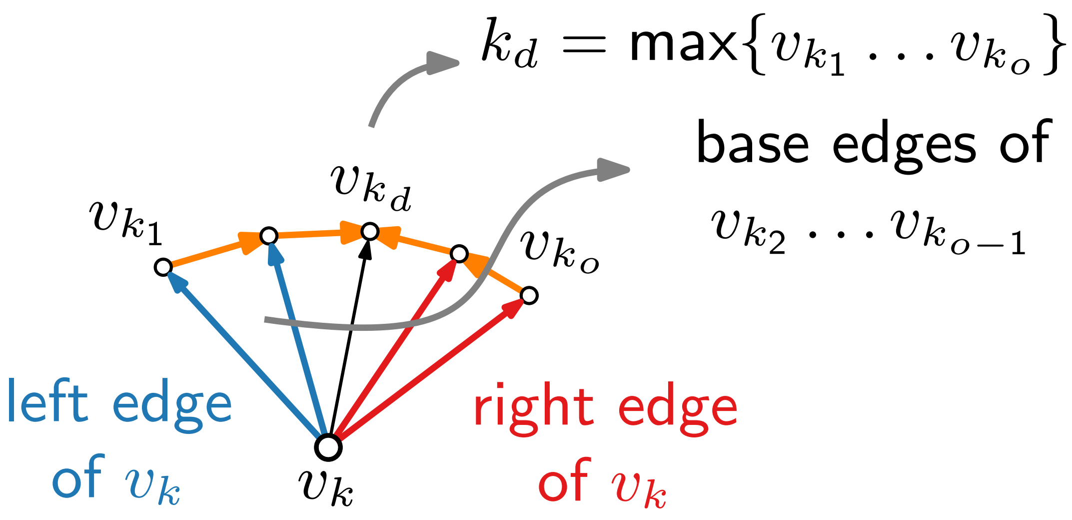

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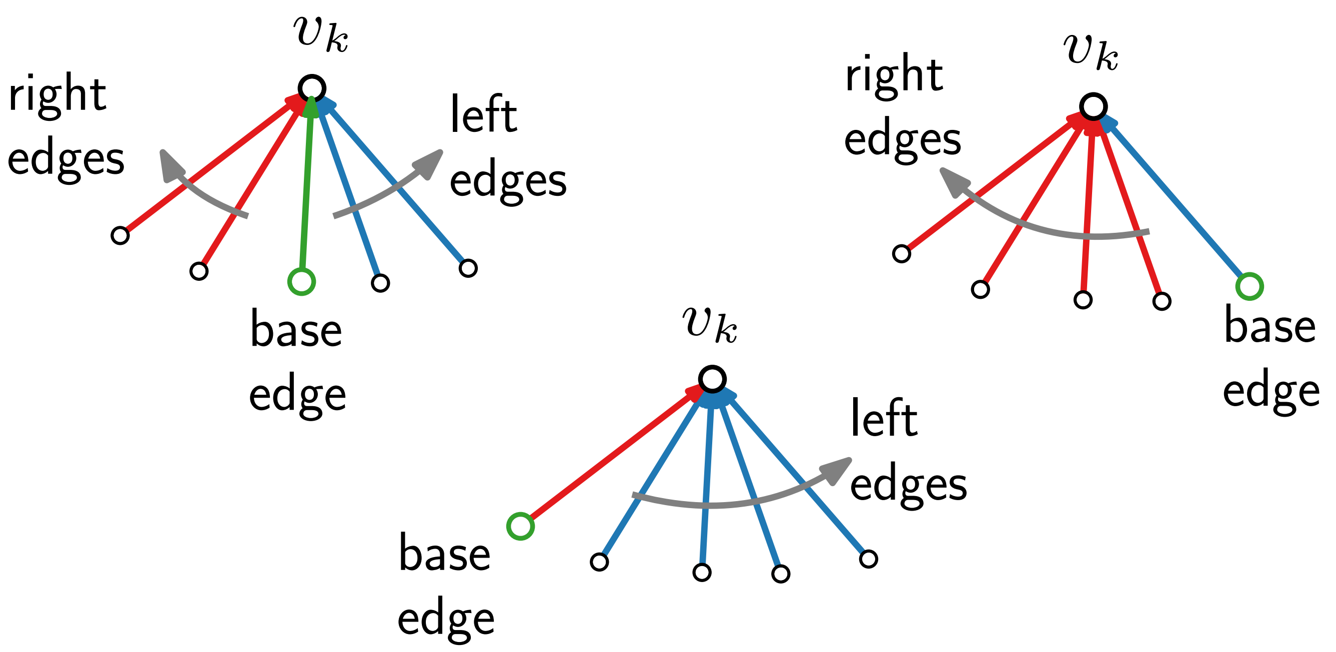

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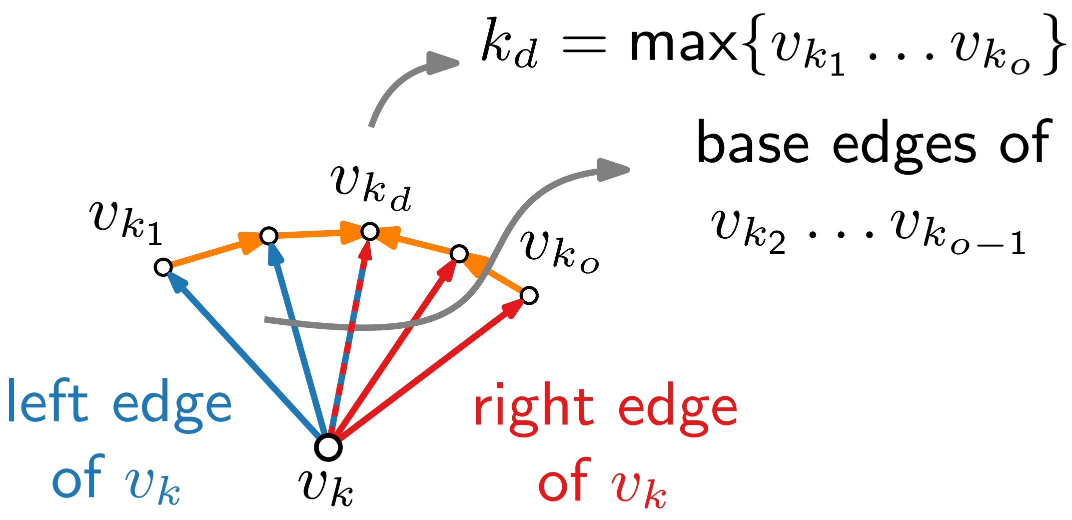

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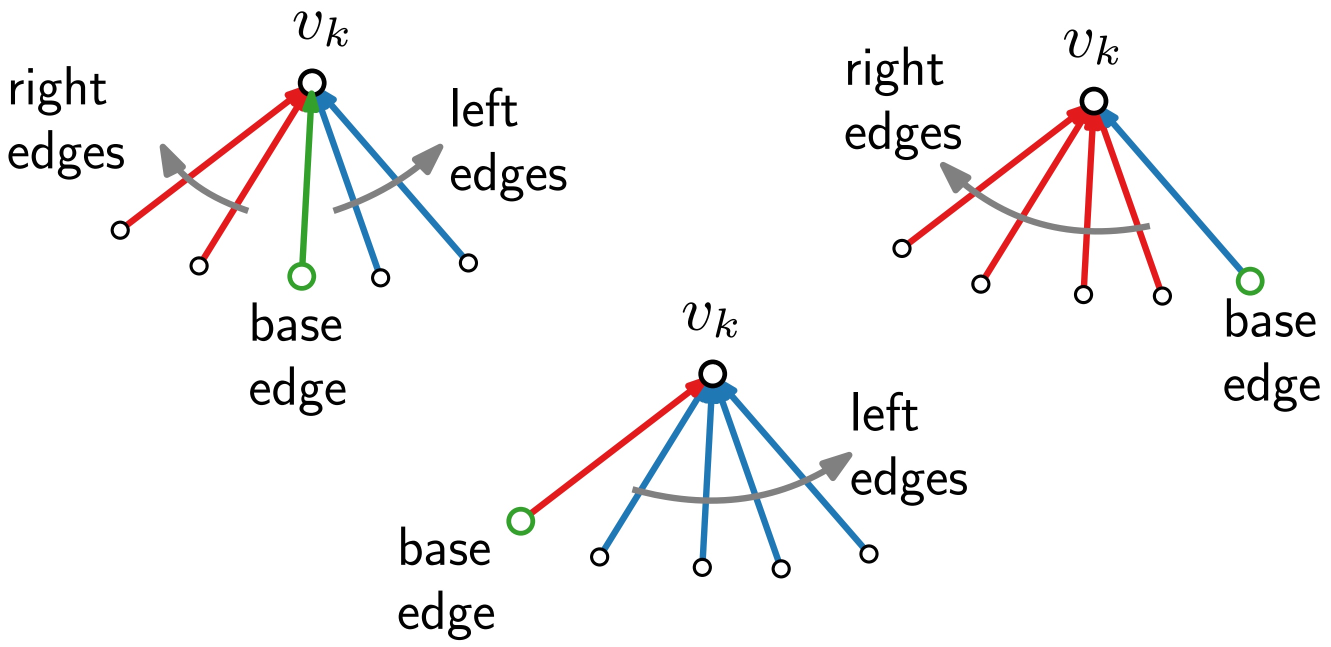
- (v_k, v_{k_d}) is either red or blue

Refined Canonical Order → REL



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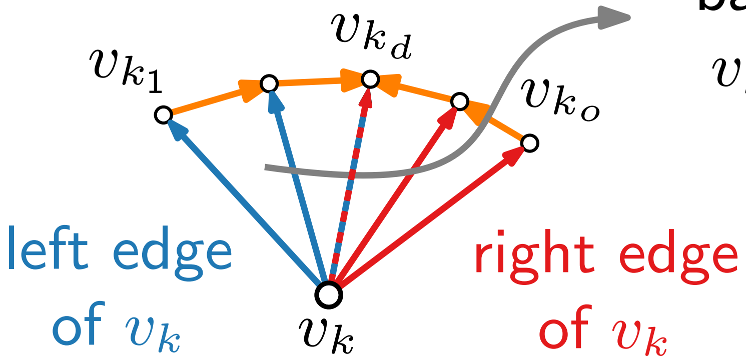

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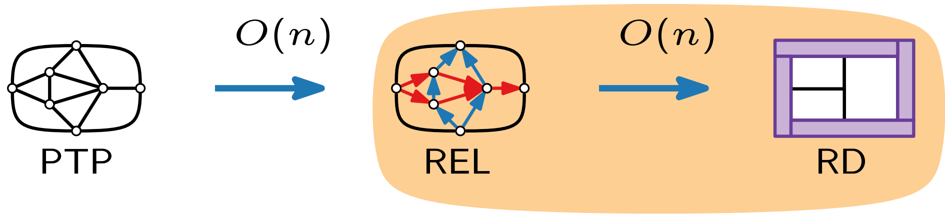
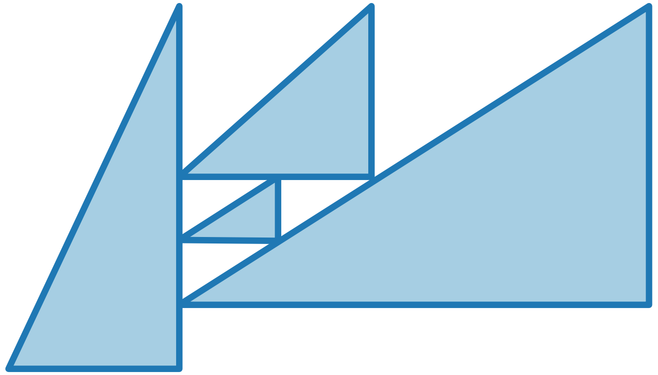
$\Rightarrow$ circular order of outgoing edges at v_k correct

[Kant, He '94]:

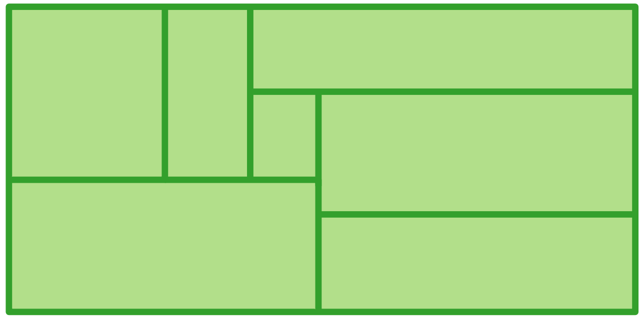


Visualization of Graphs

Lecture 8:
Conact Representations of Planar Graphs:
Triangle Contacts and Rectangular Duals

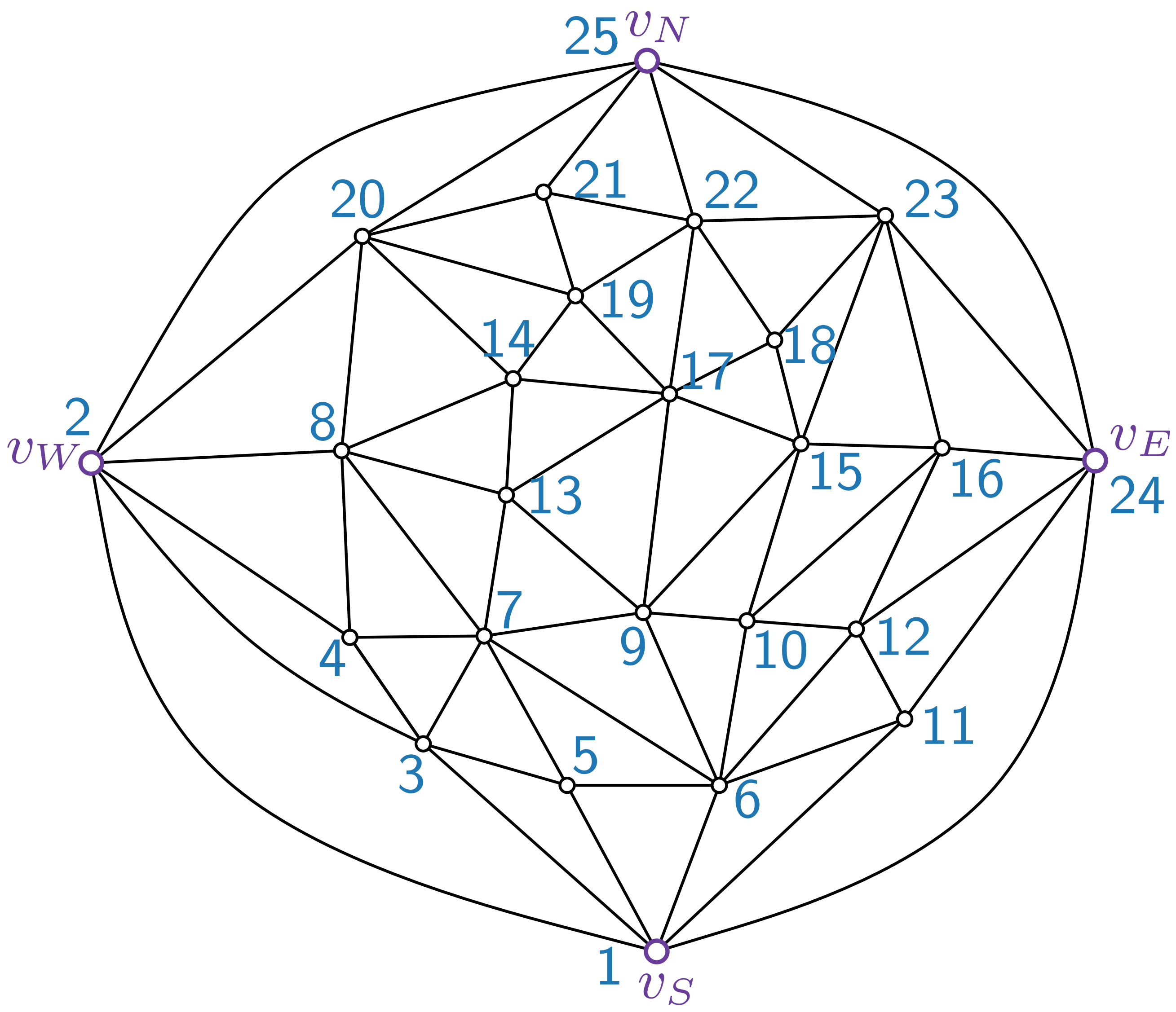


Part V:
Computing the Coordinates



Jonathan Klawitter

From REL to st-digraphs to Coordinates



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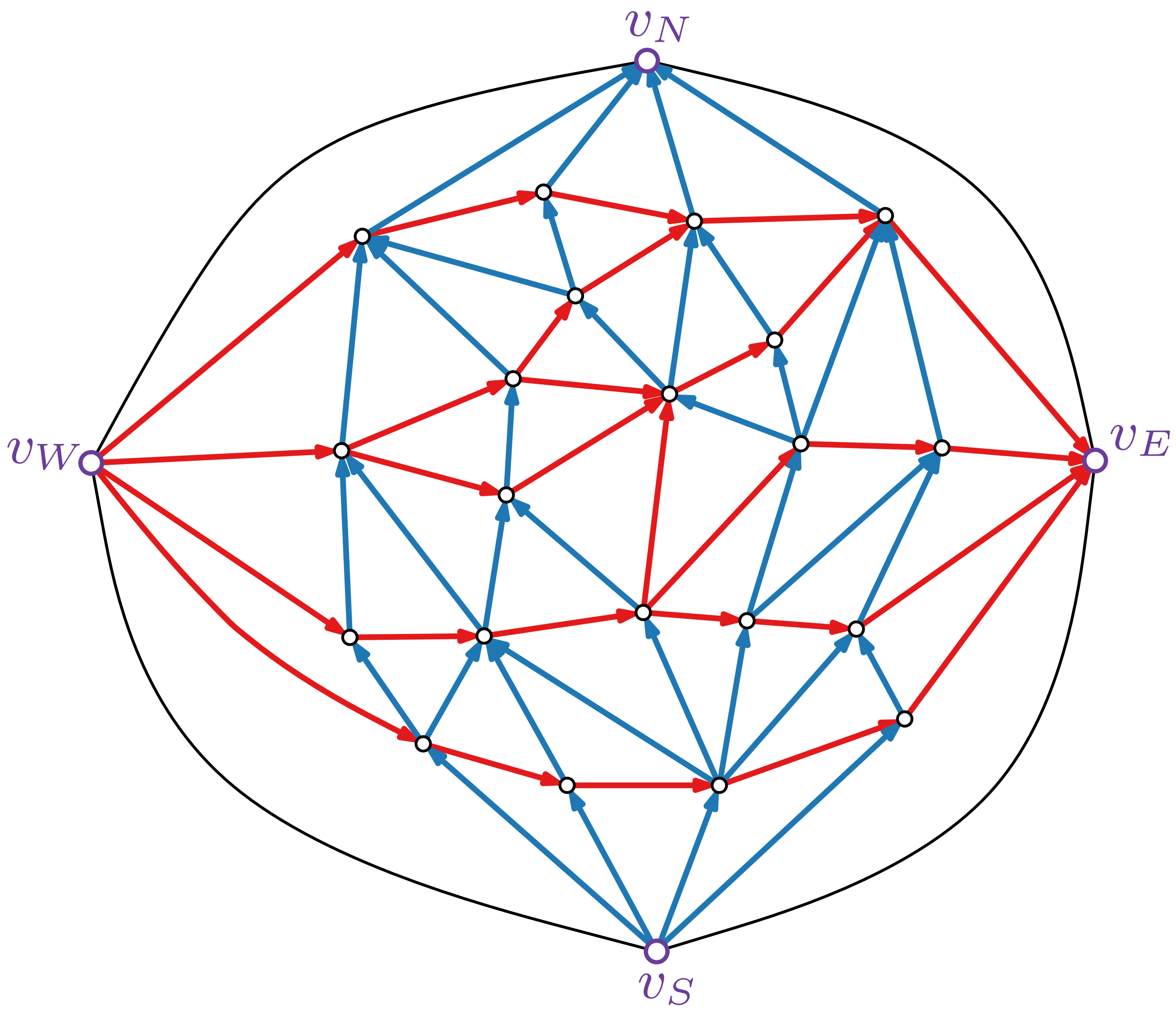

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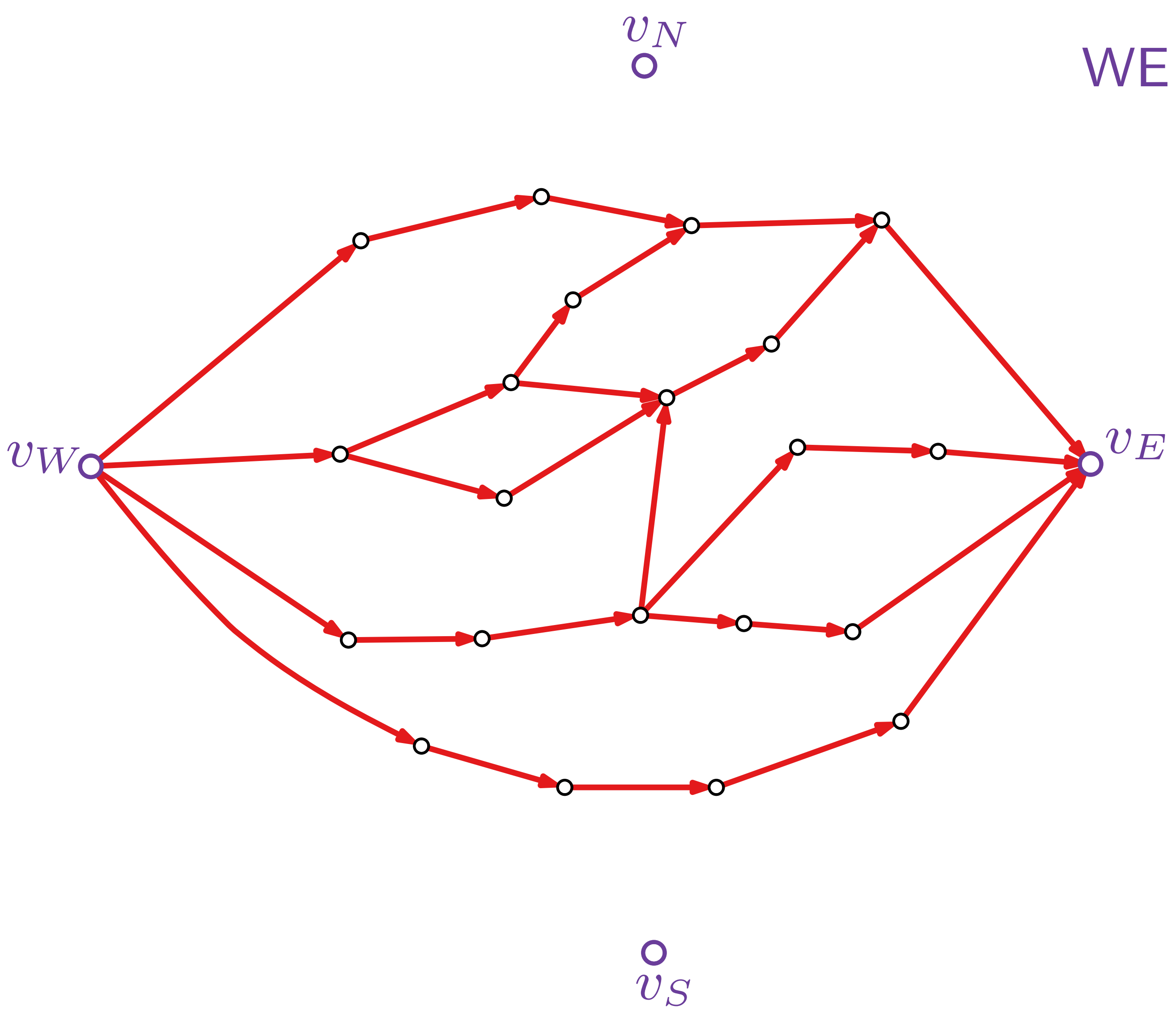
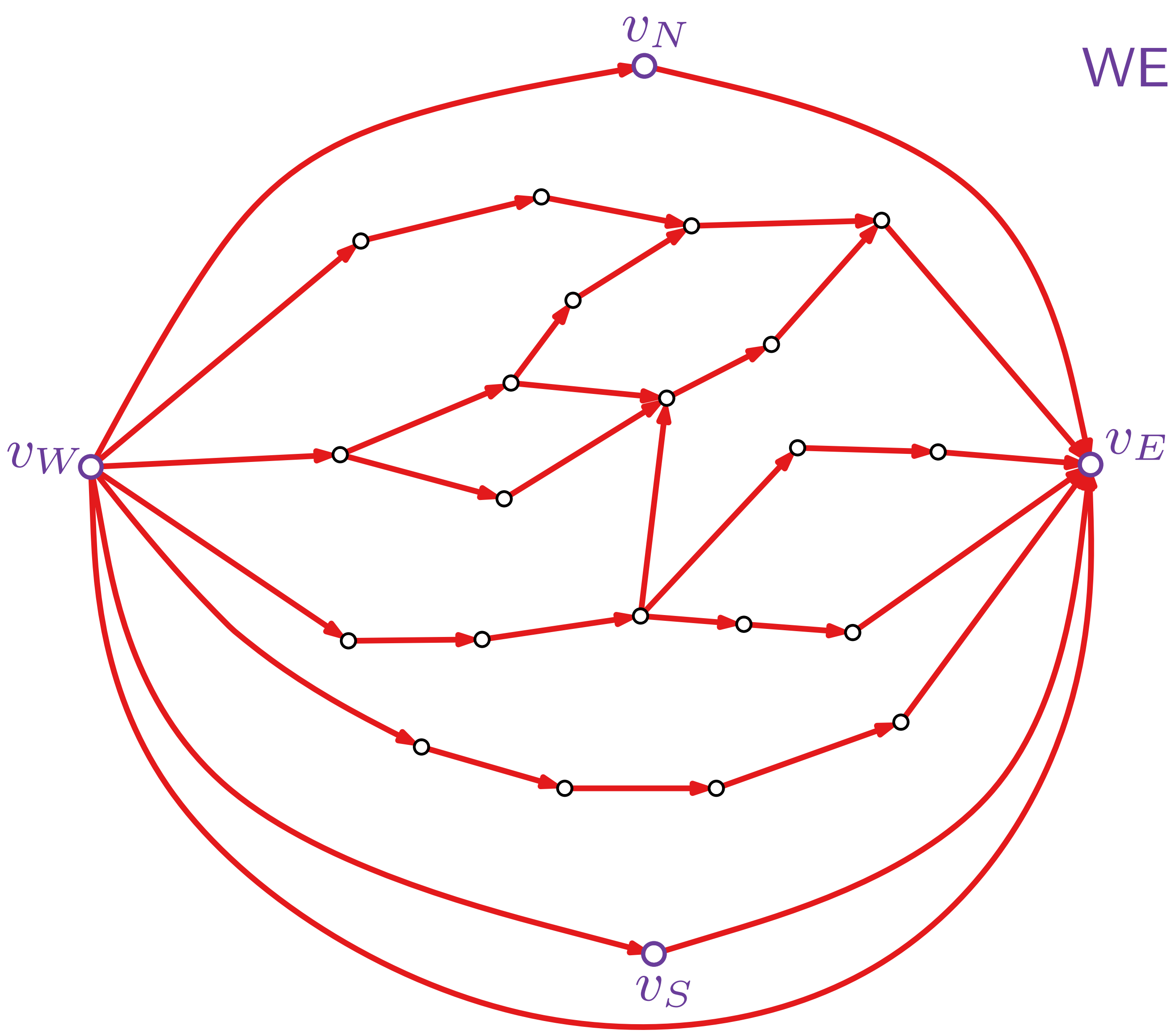


WE network G_{hor}

From REL to st-digraphs to Coordinates



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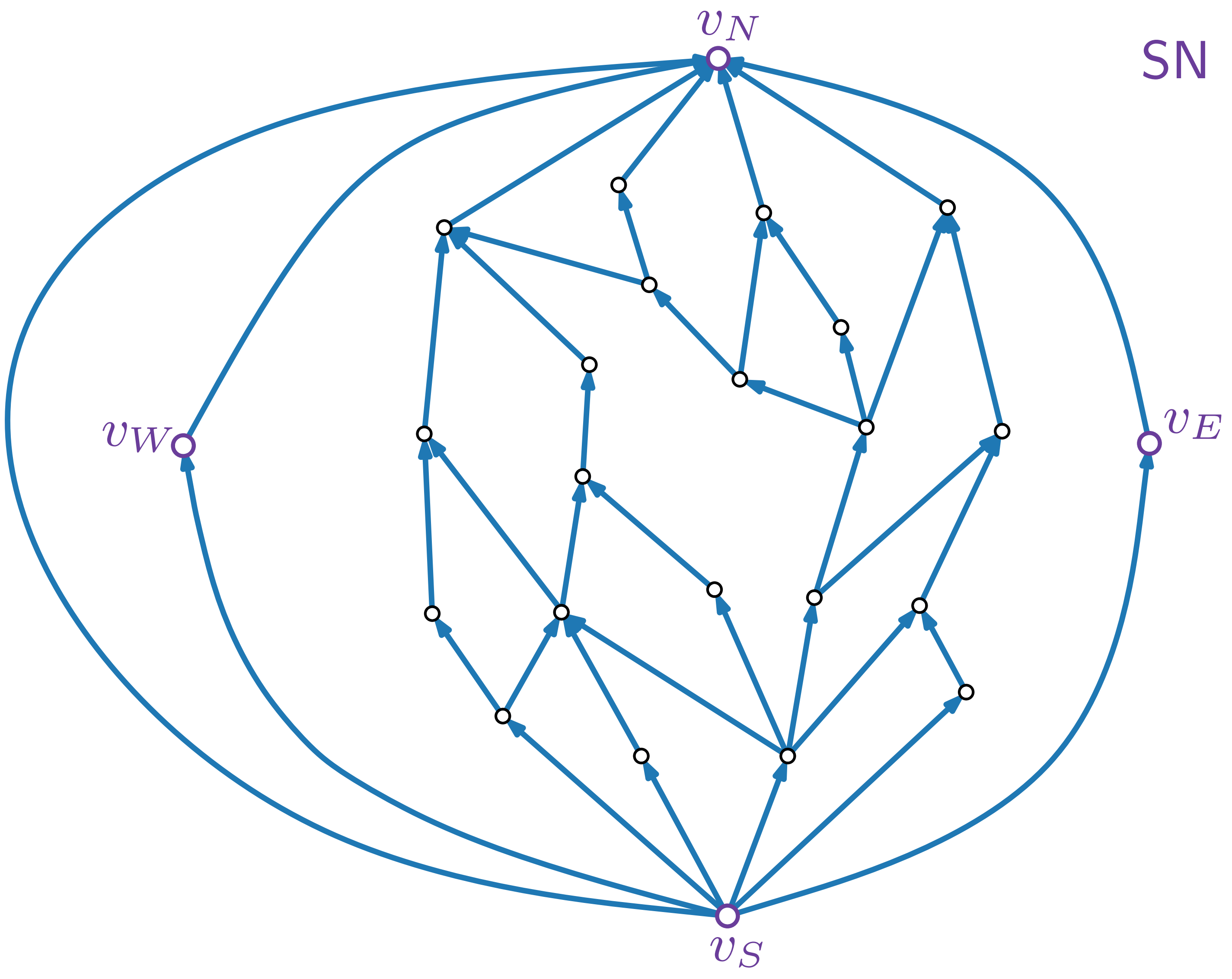
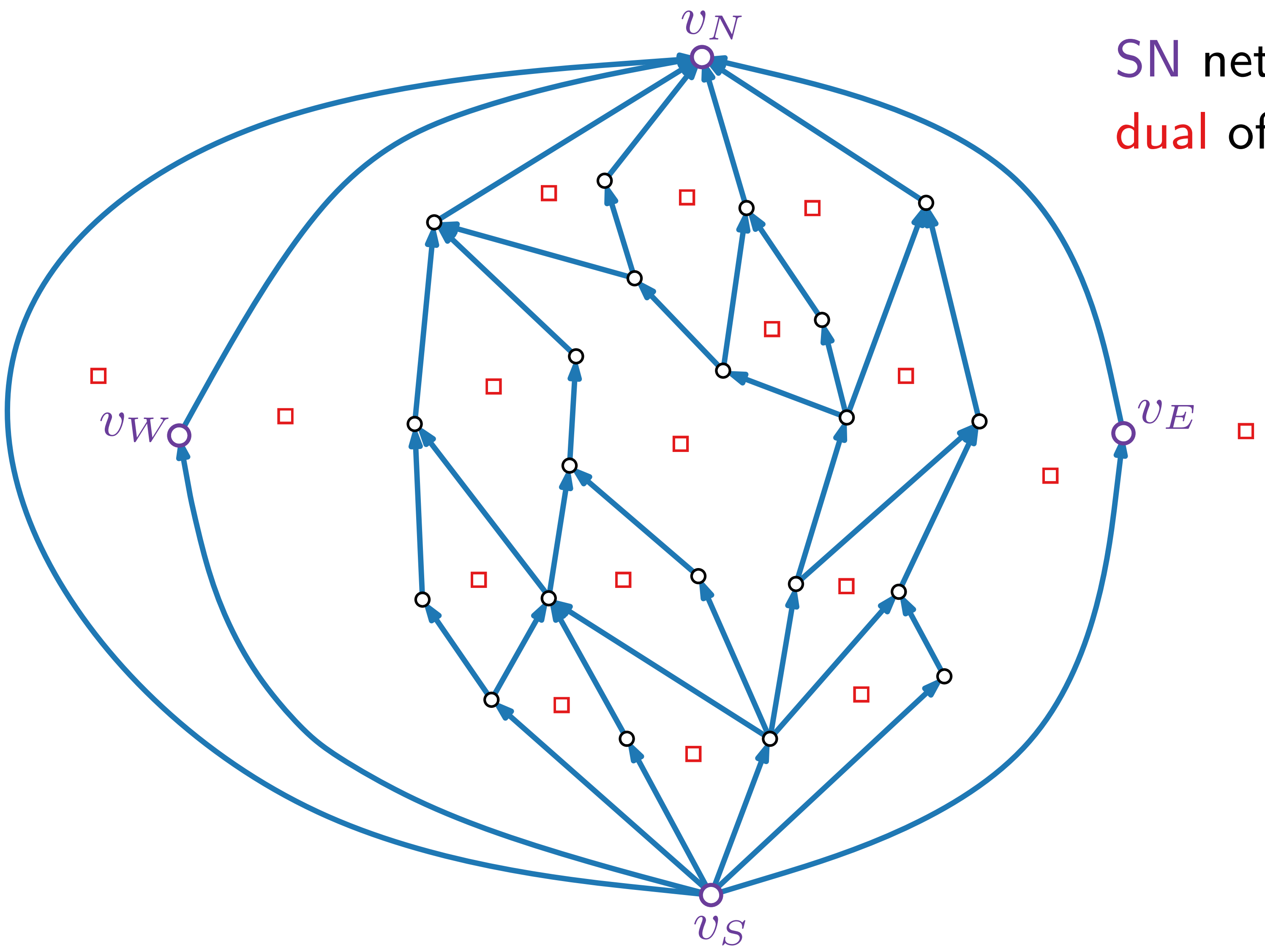


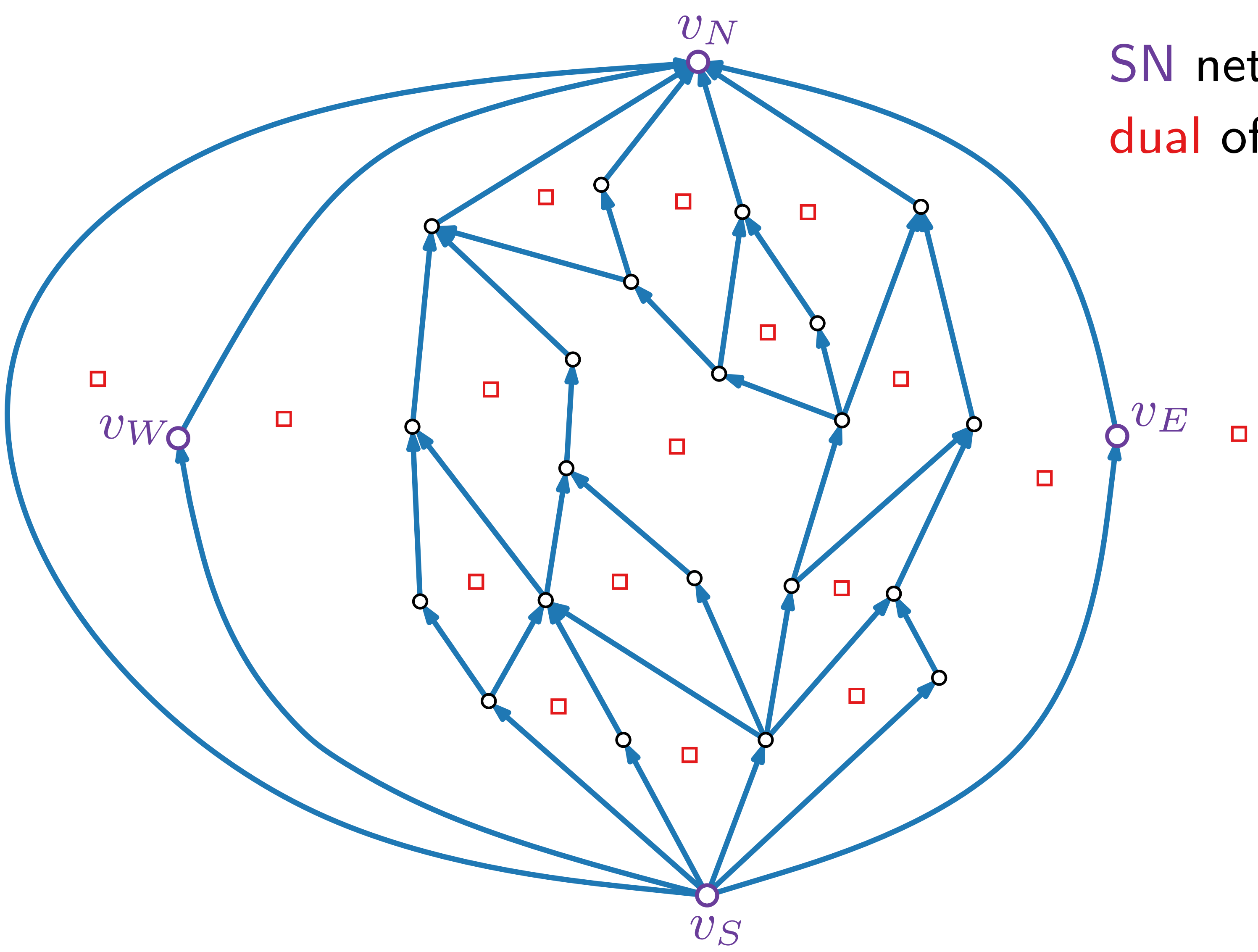
SN network G_{ver}

From REL to st-digraphs to Coordinates



SN network G_{ver}
dual of G_{ver}

From REL to st-digraphs to Coordinates



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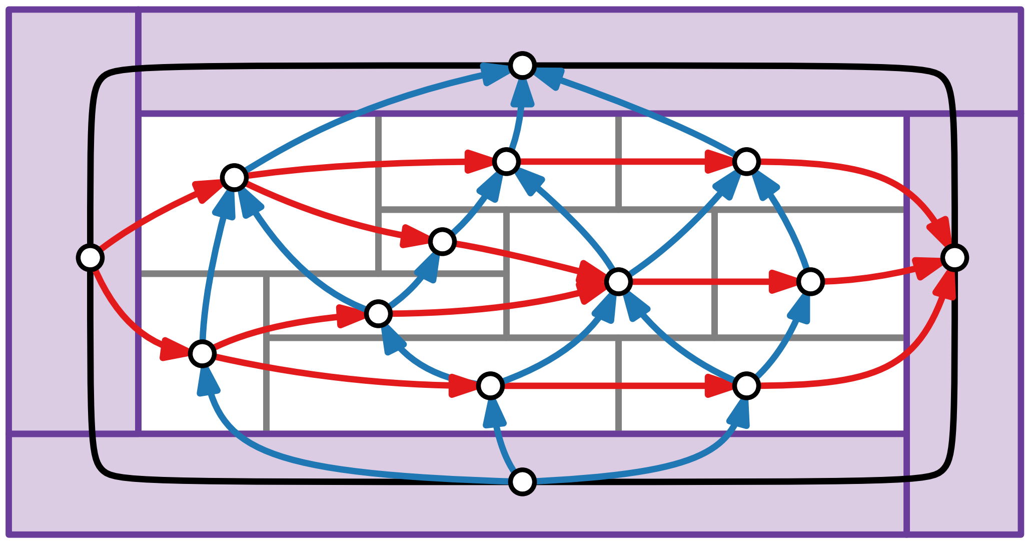

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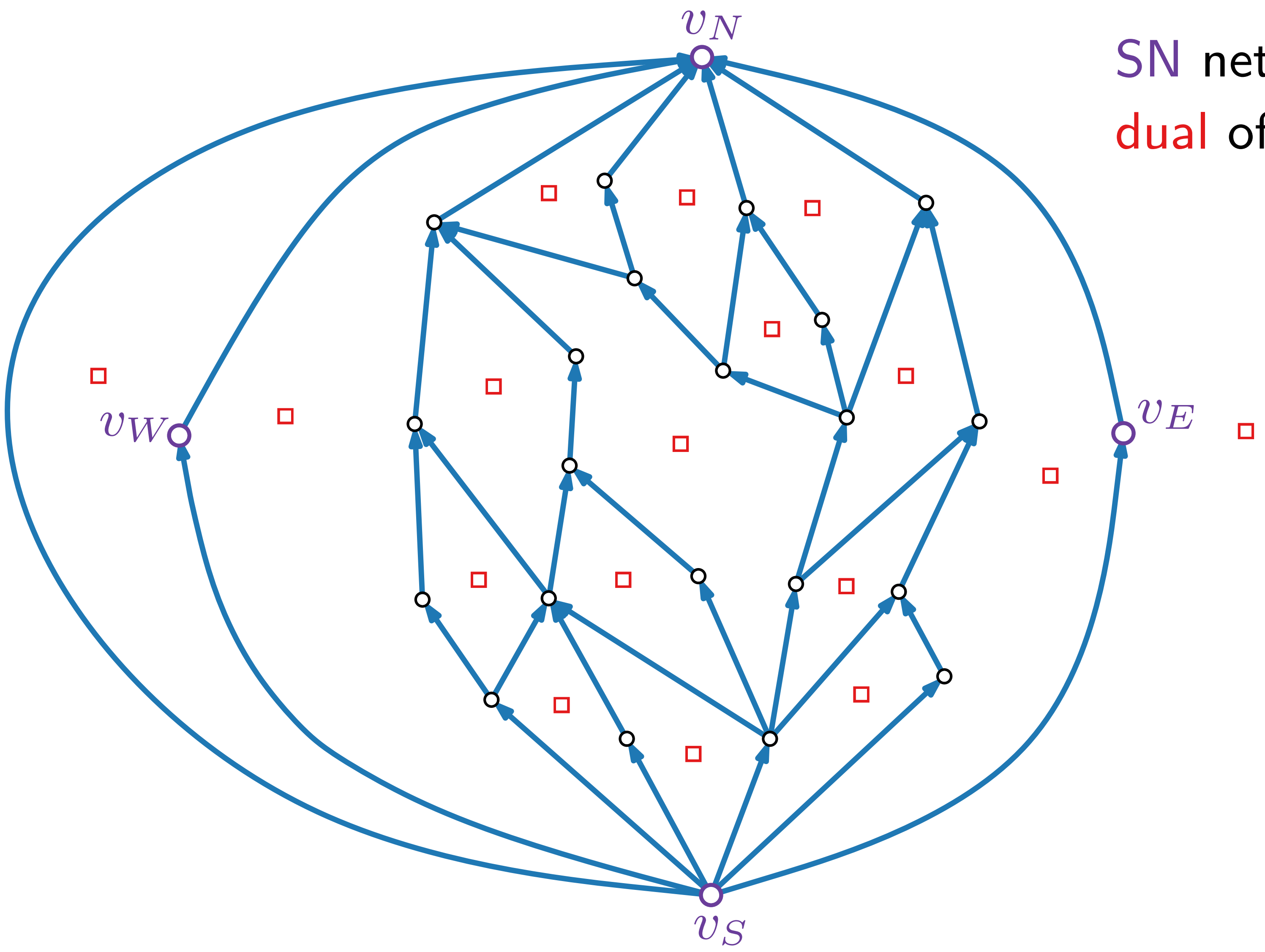

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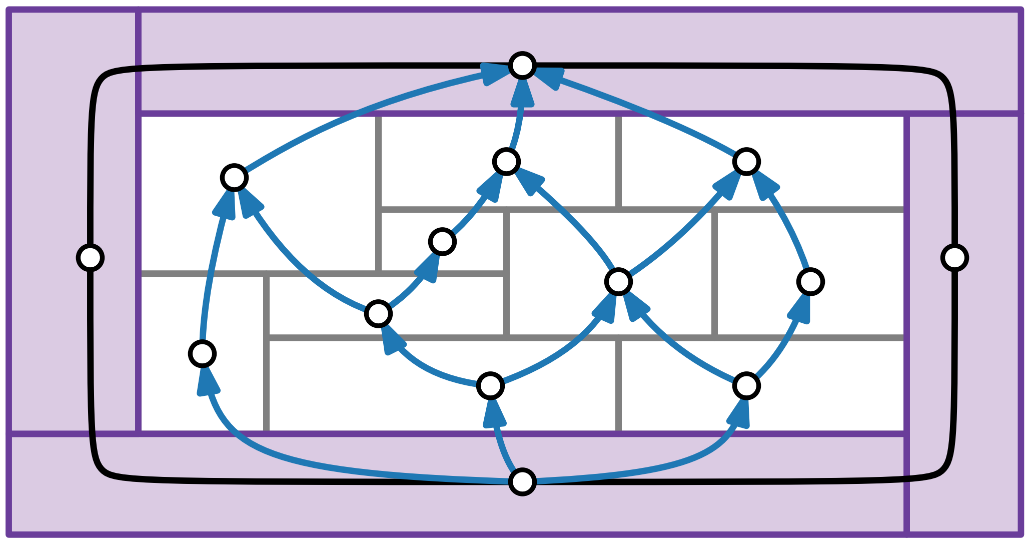

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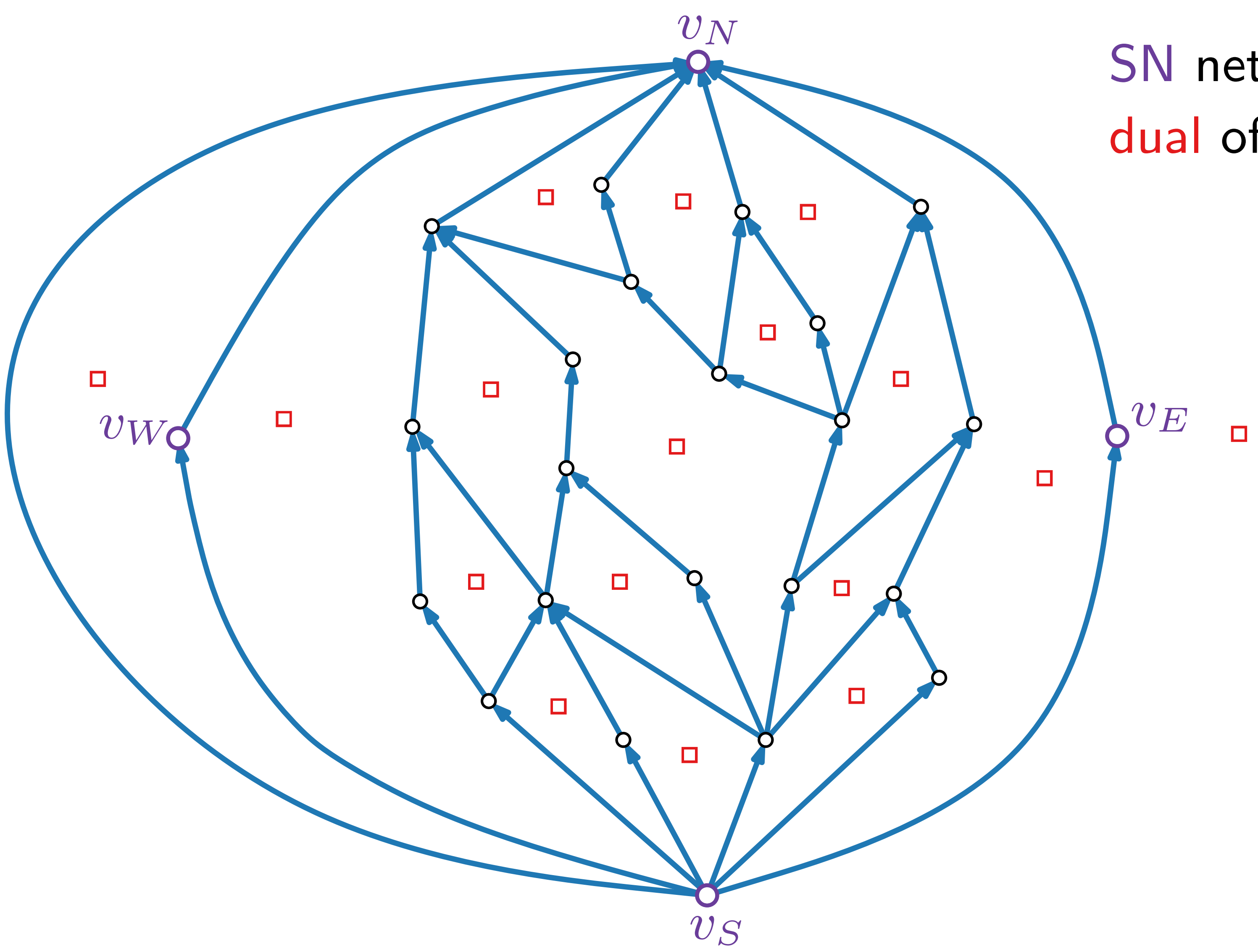

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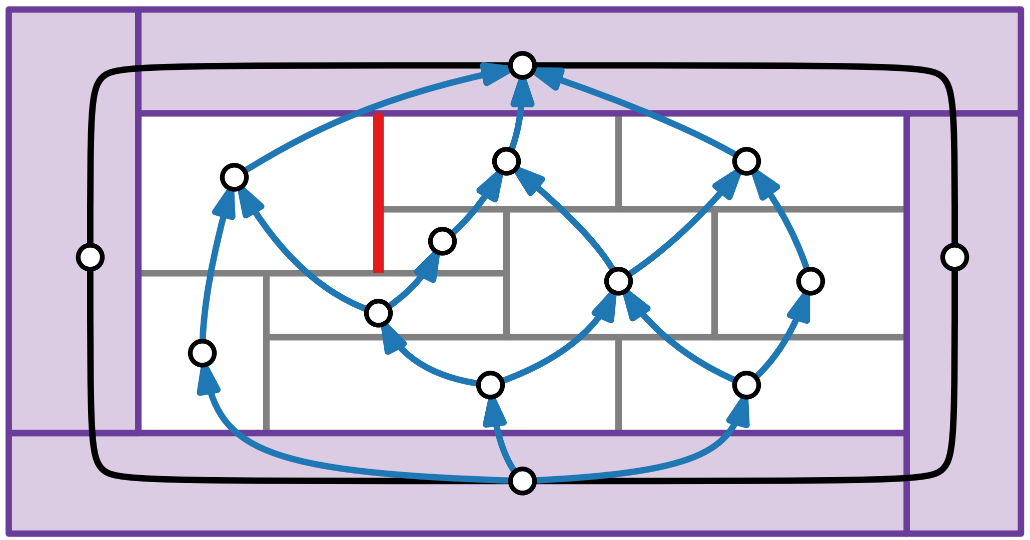

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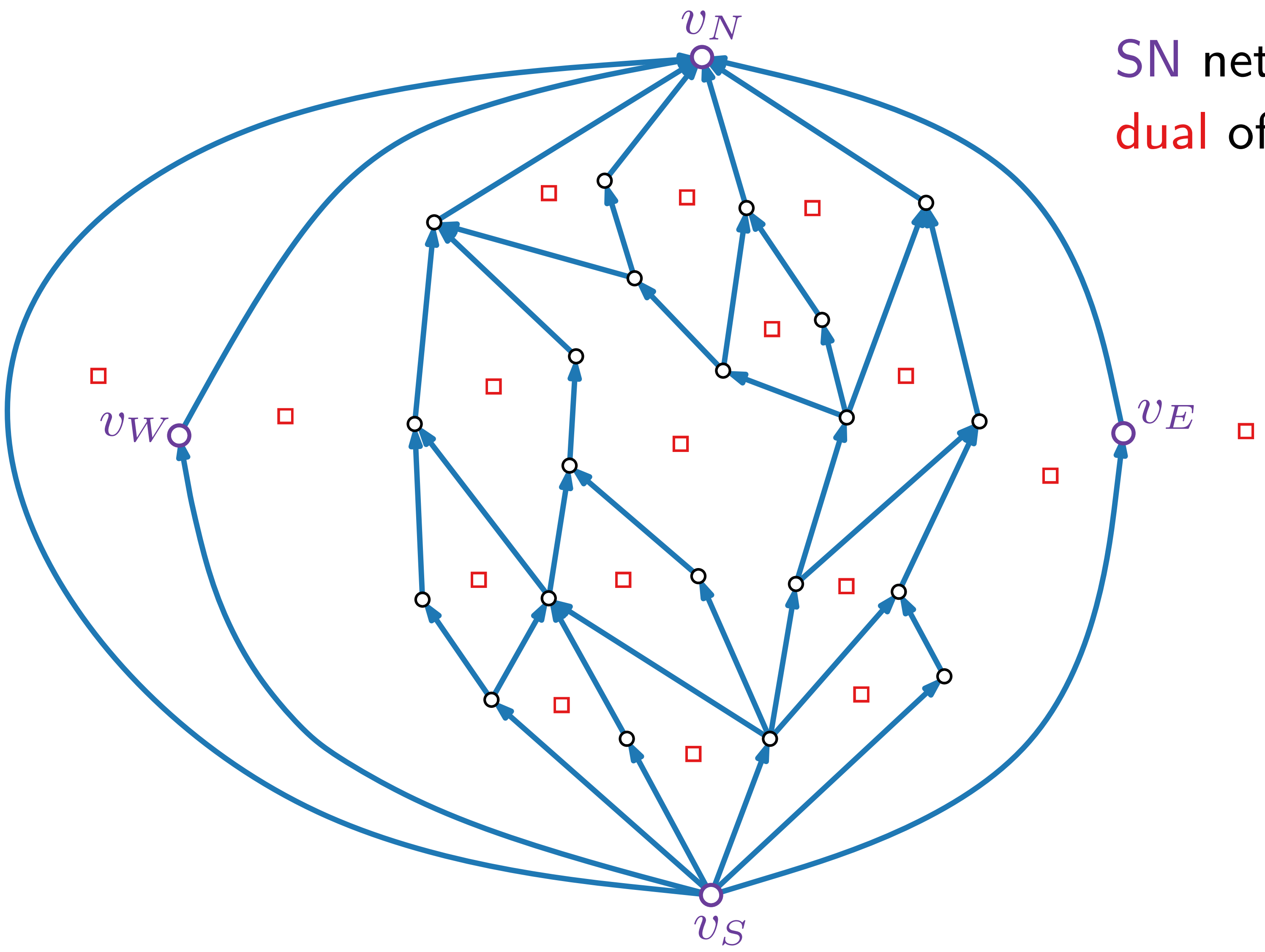

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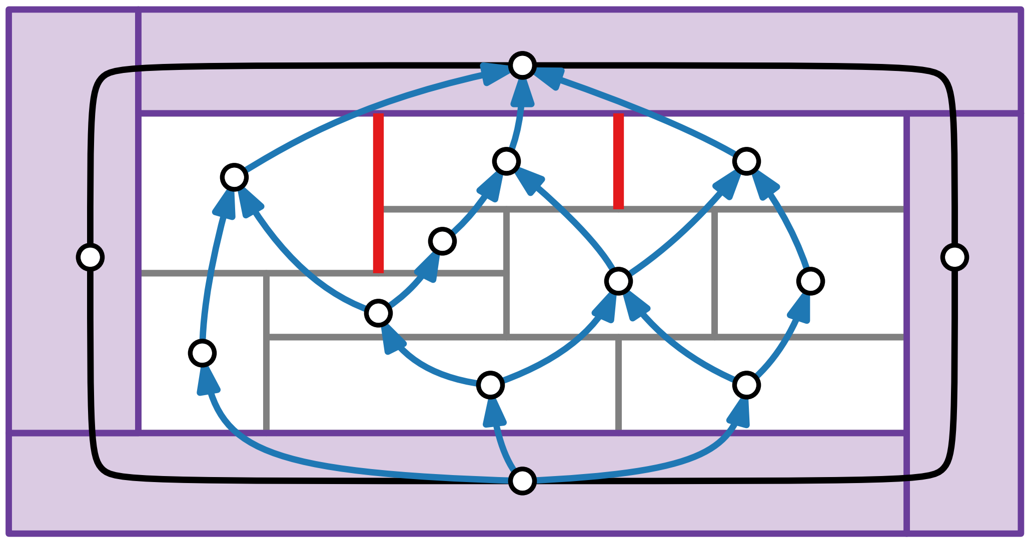

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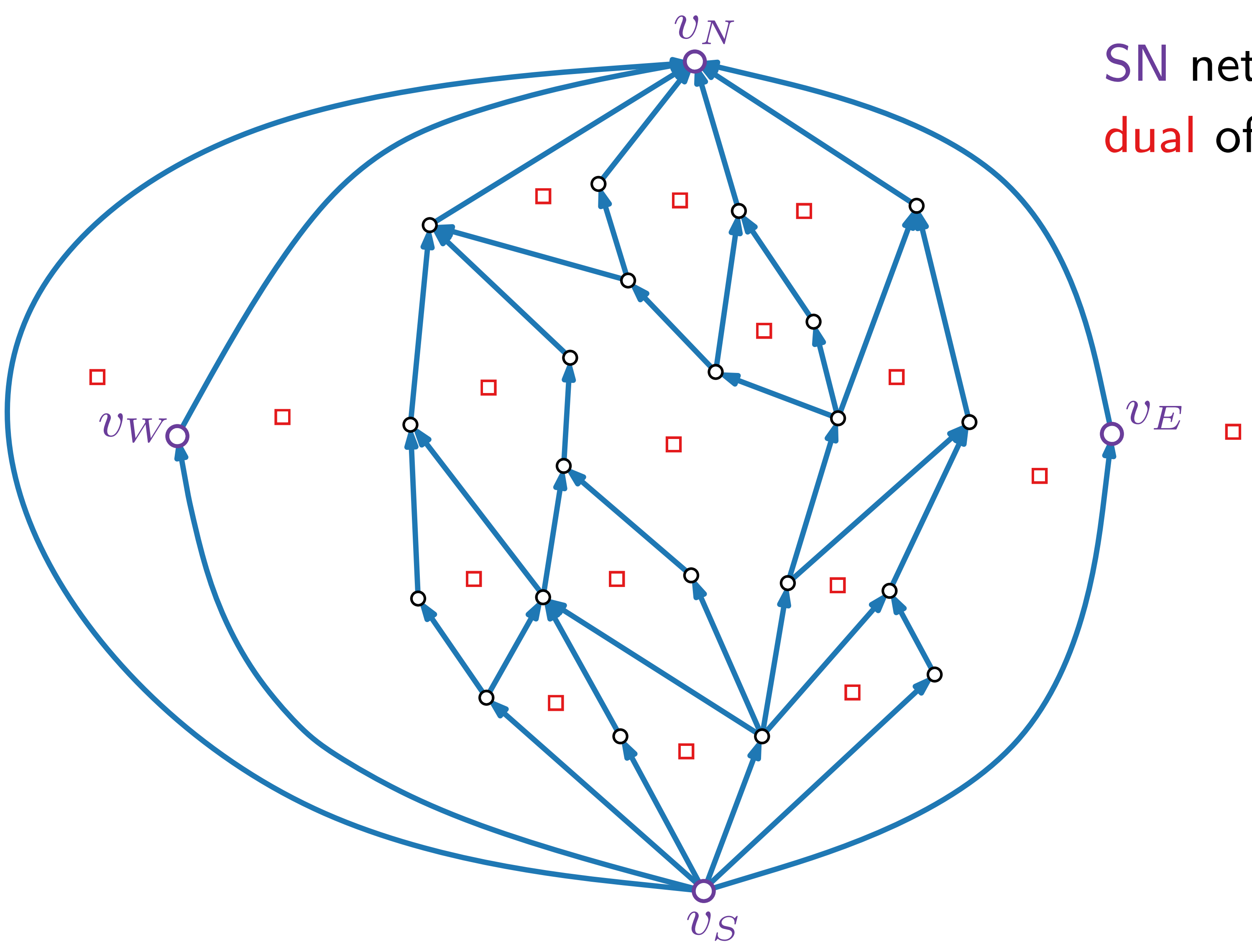

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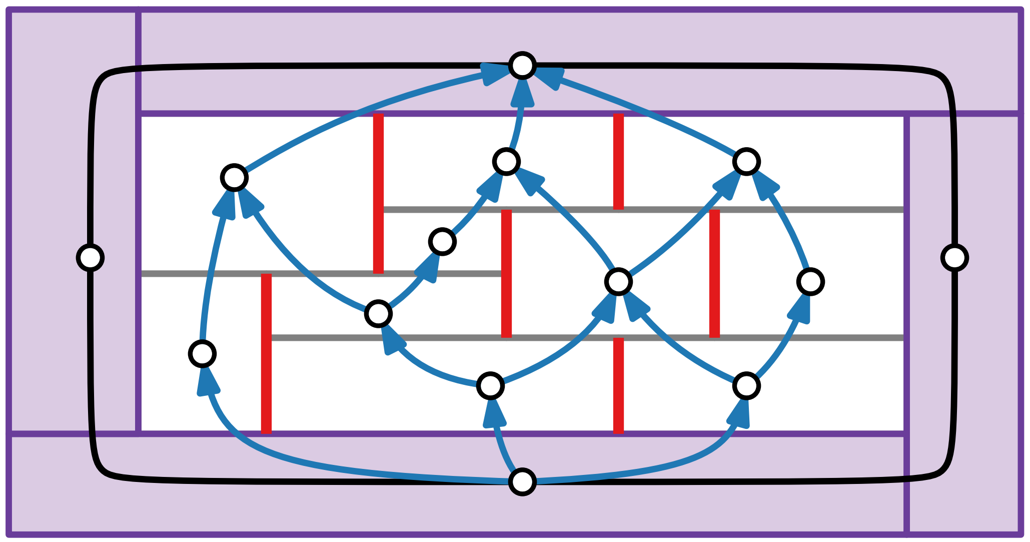

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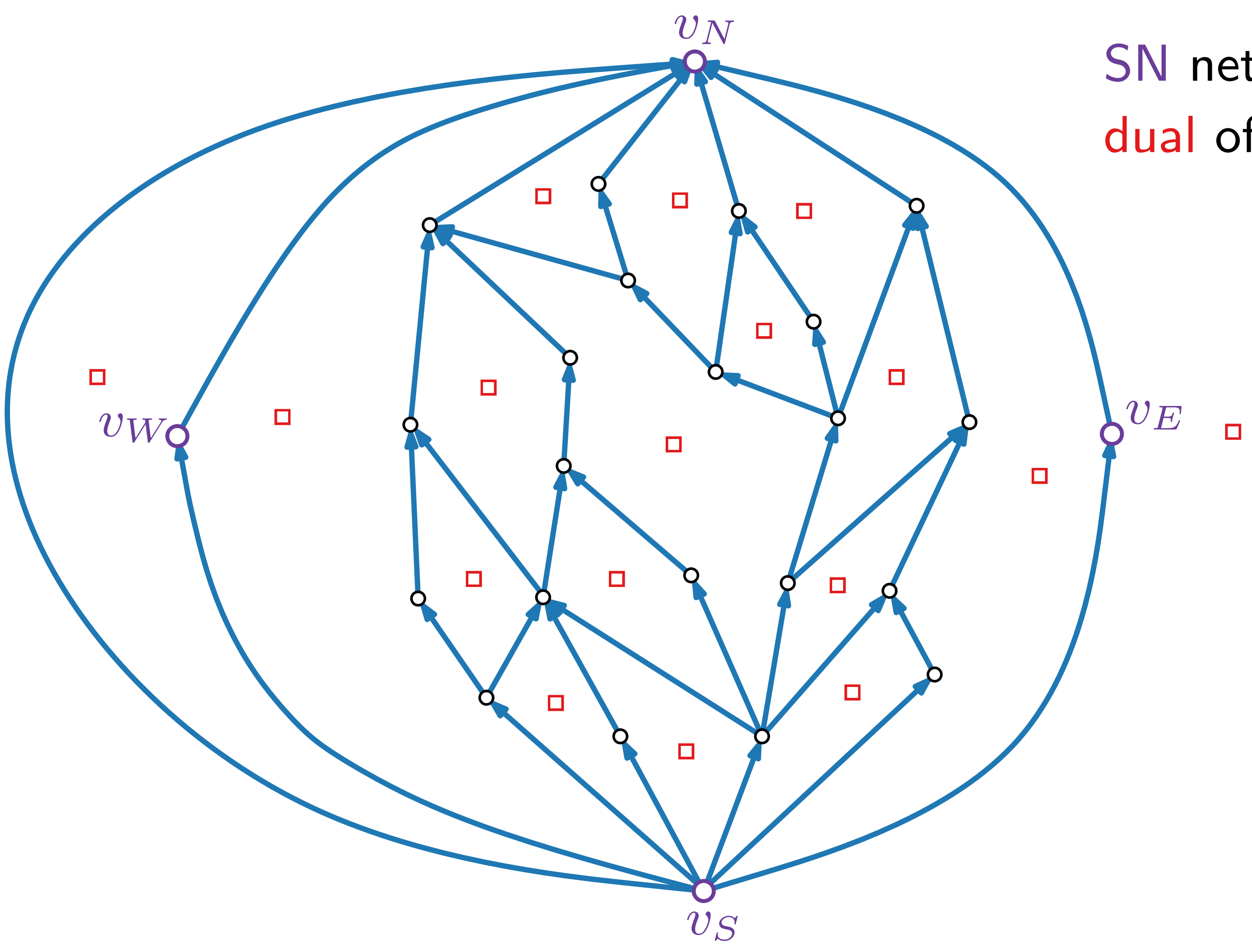

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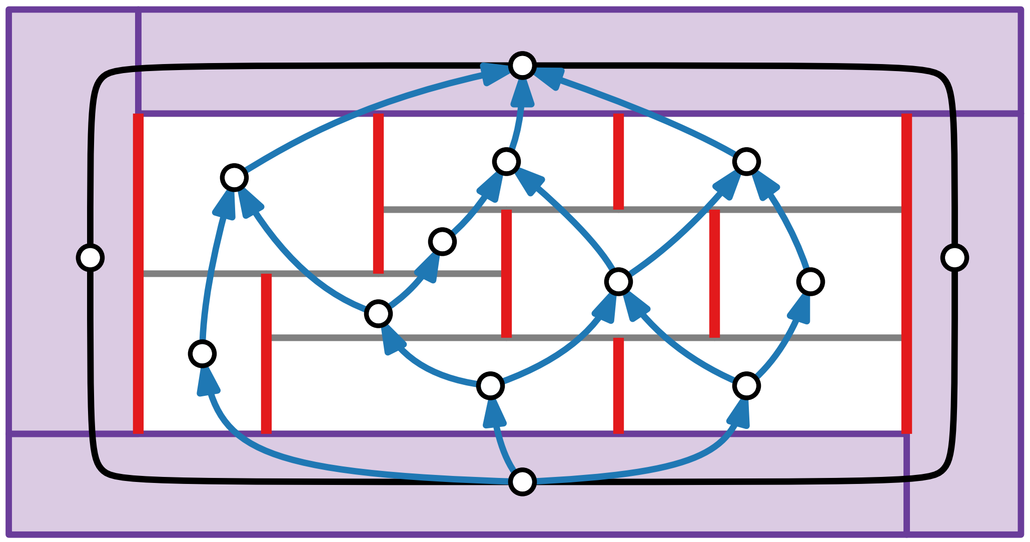

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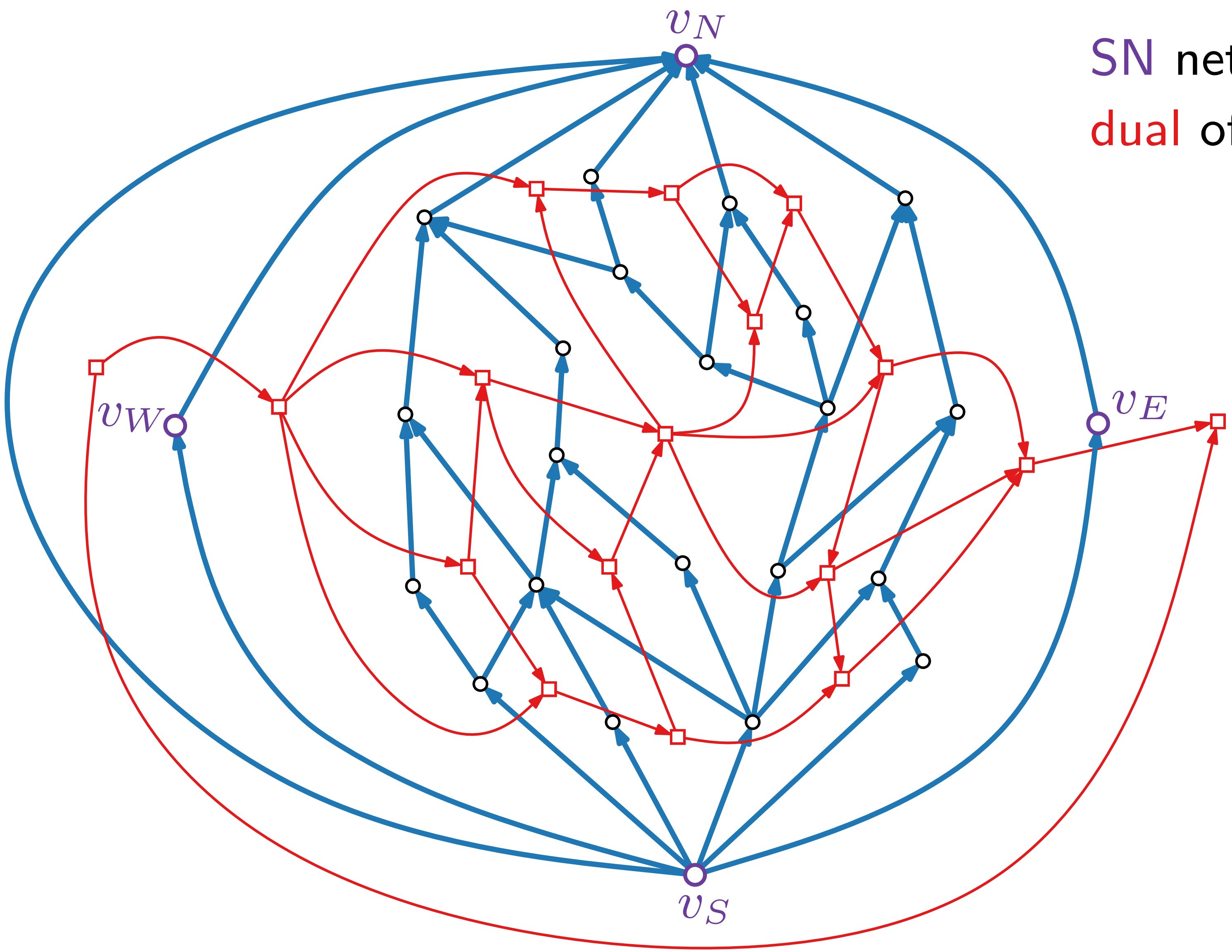

SN network G_{ver}

dual of G_{ver}

From REL to st-digraphs to Coordinates

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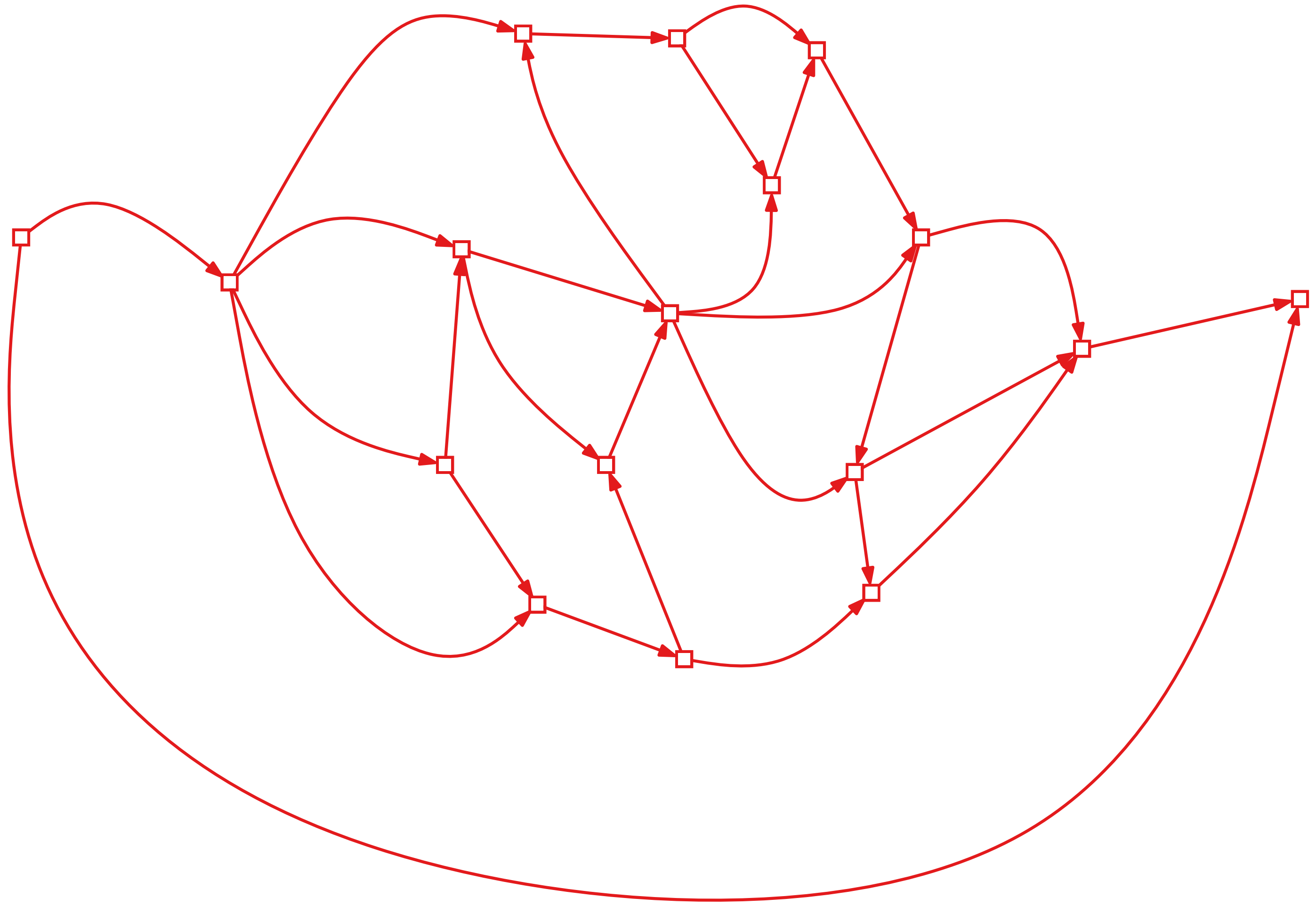

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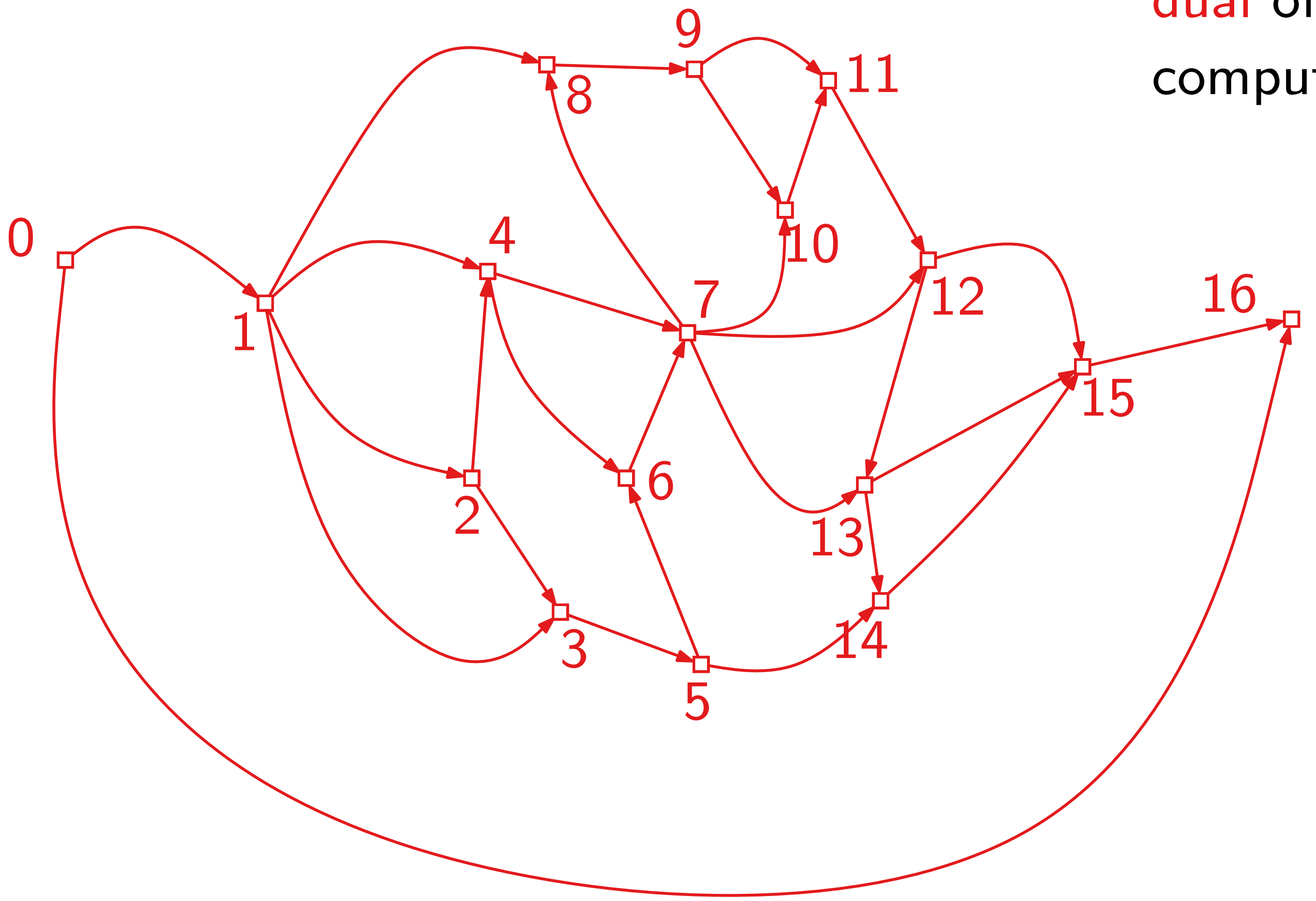
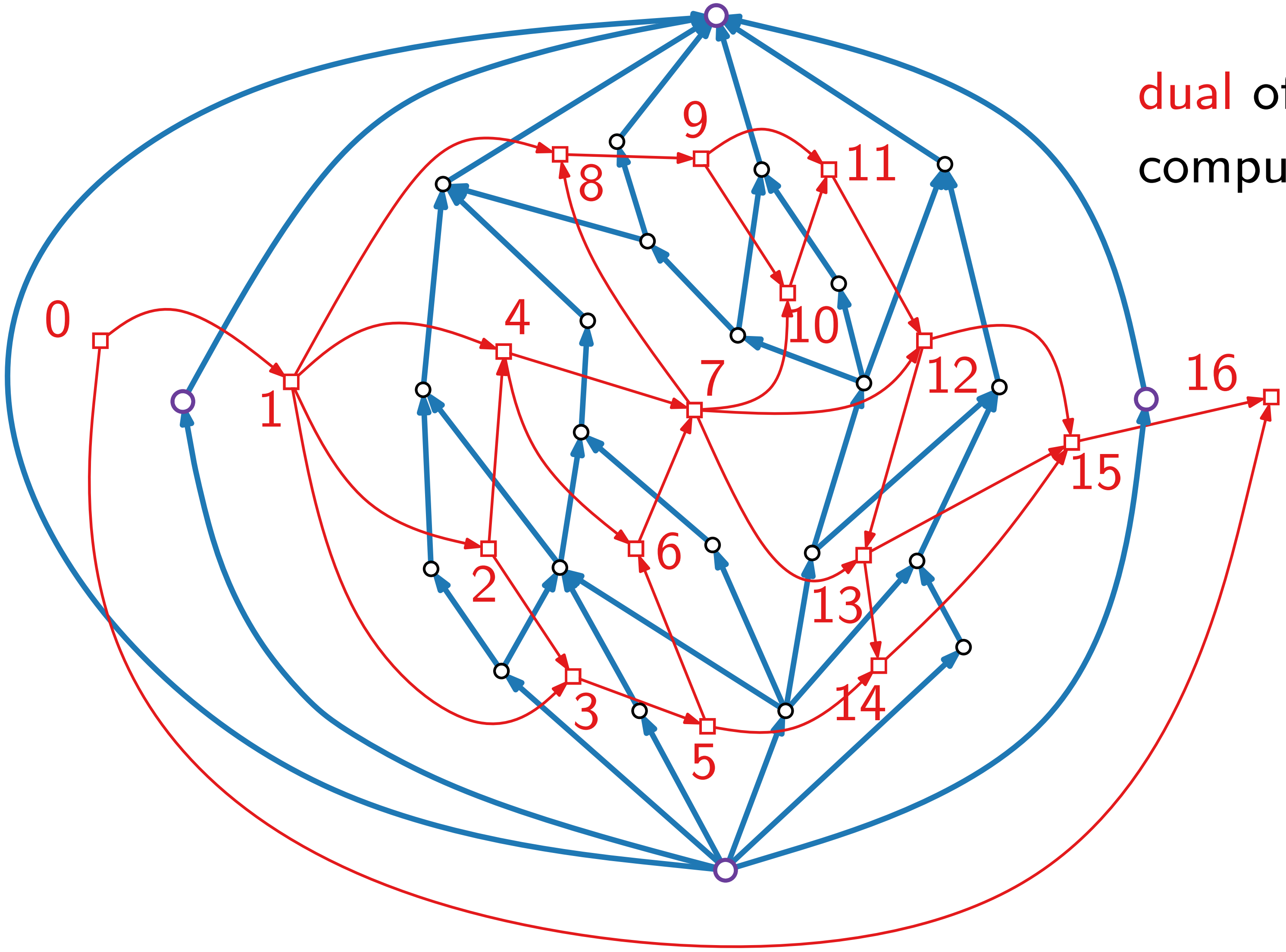


dual of G_{ver}

compute topological order

From REL to st-digraphs to Coordinates



dual of G_{ver}

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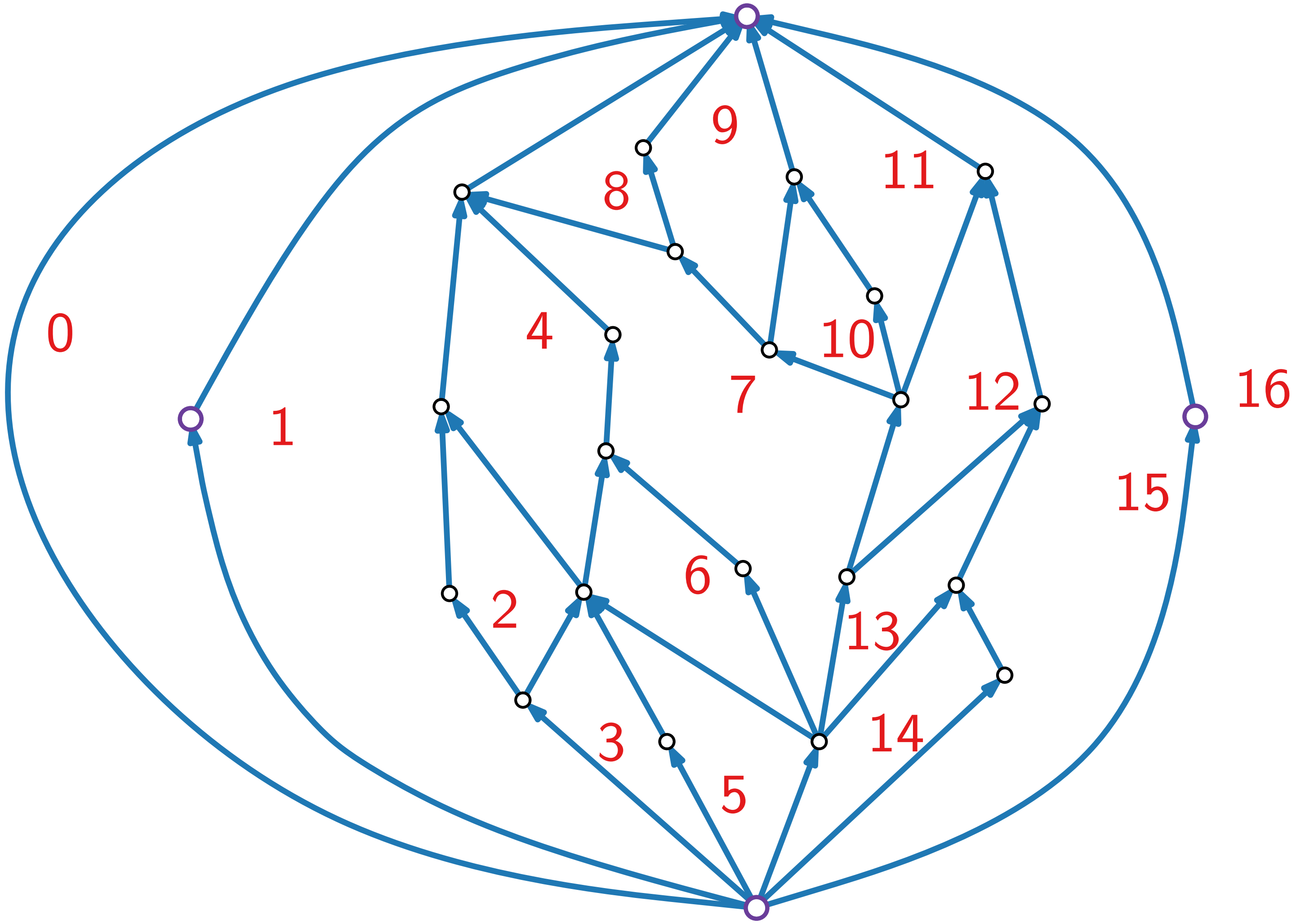
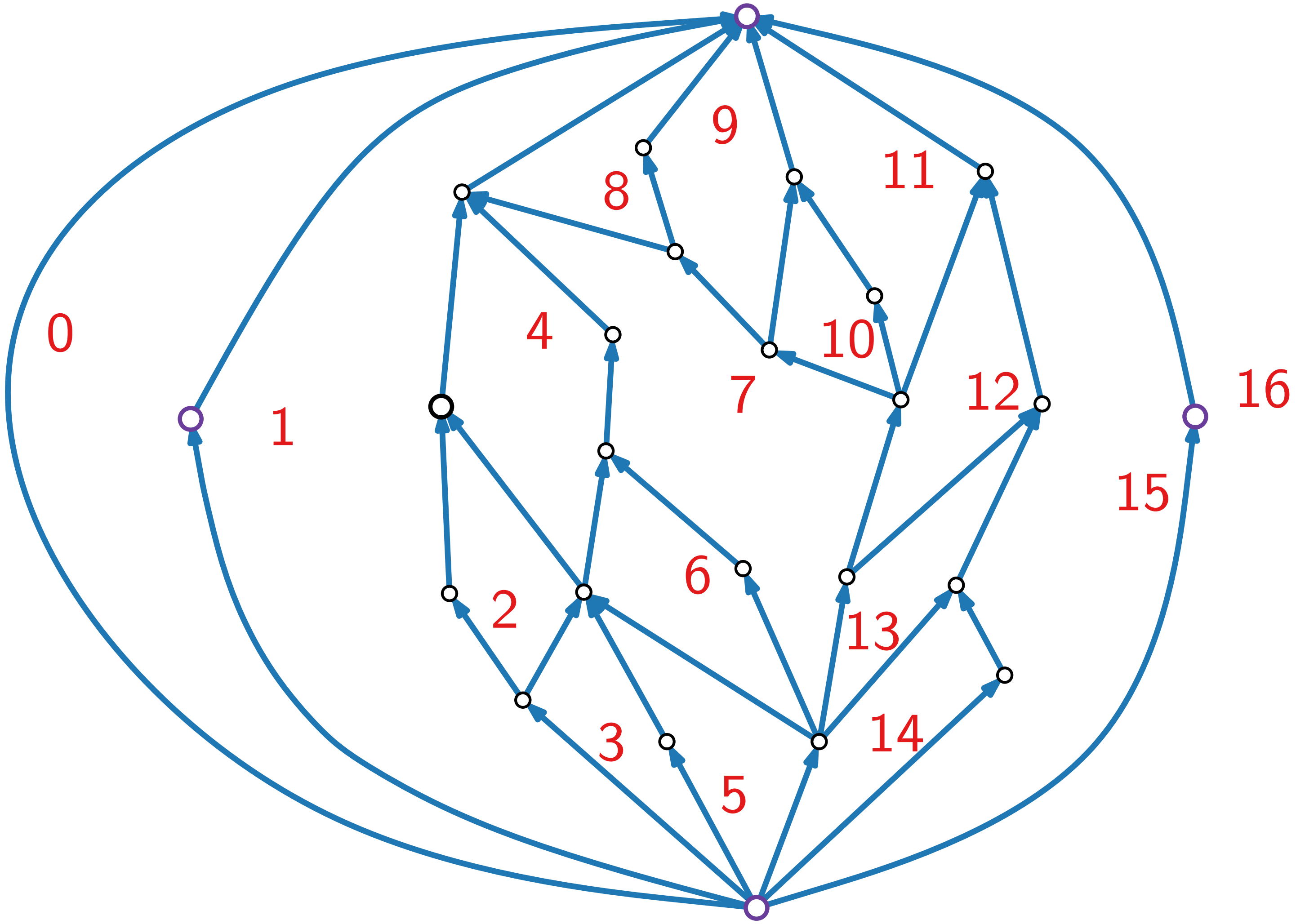


From REL to st-digraphs to Coordinates



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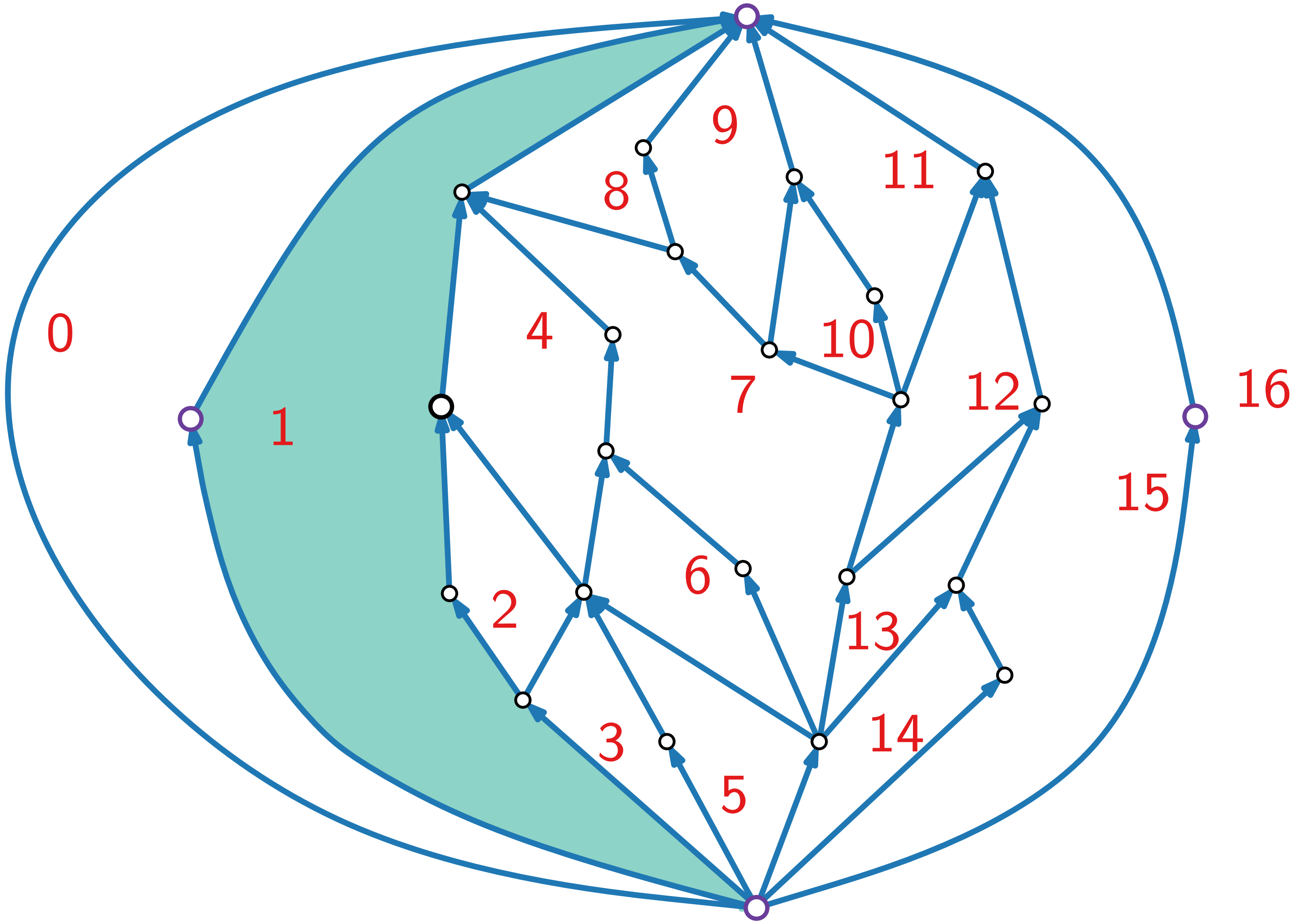

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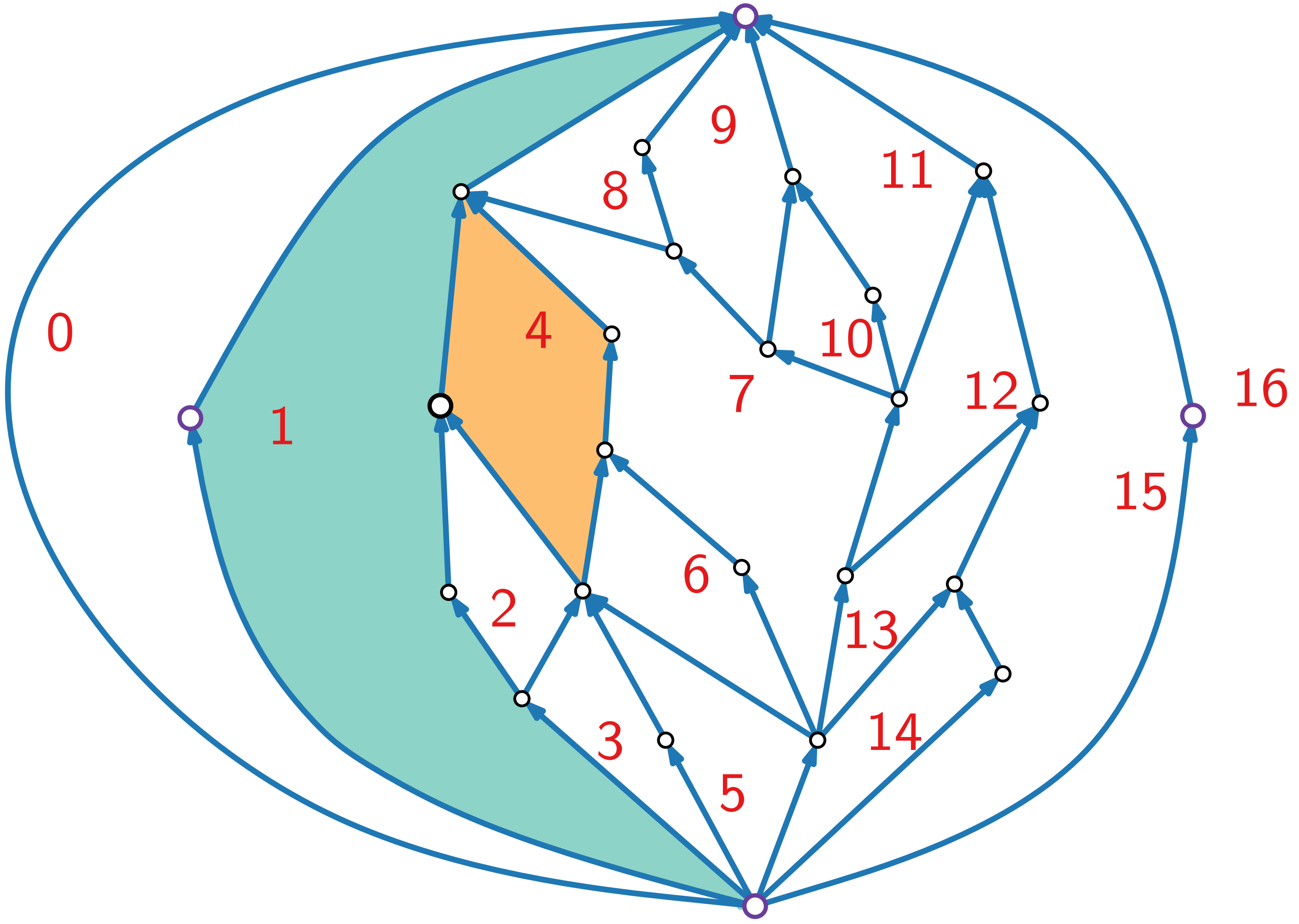

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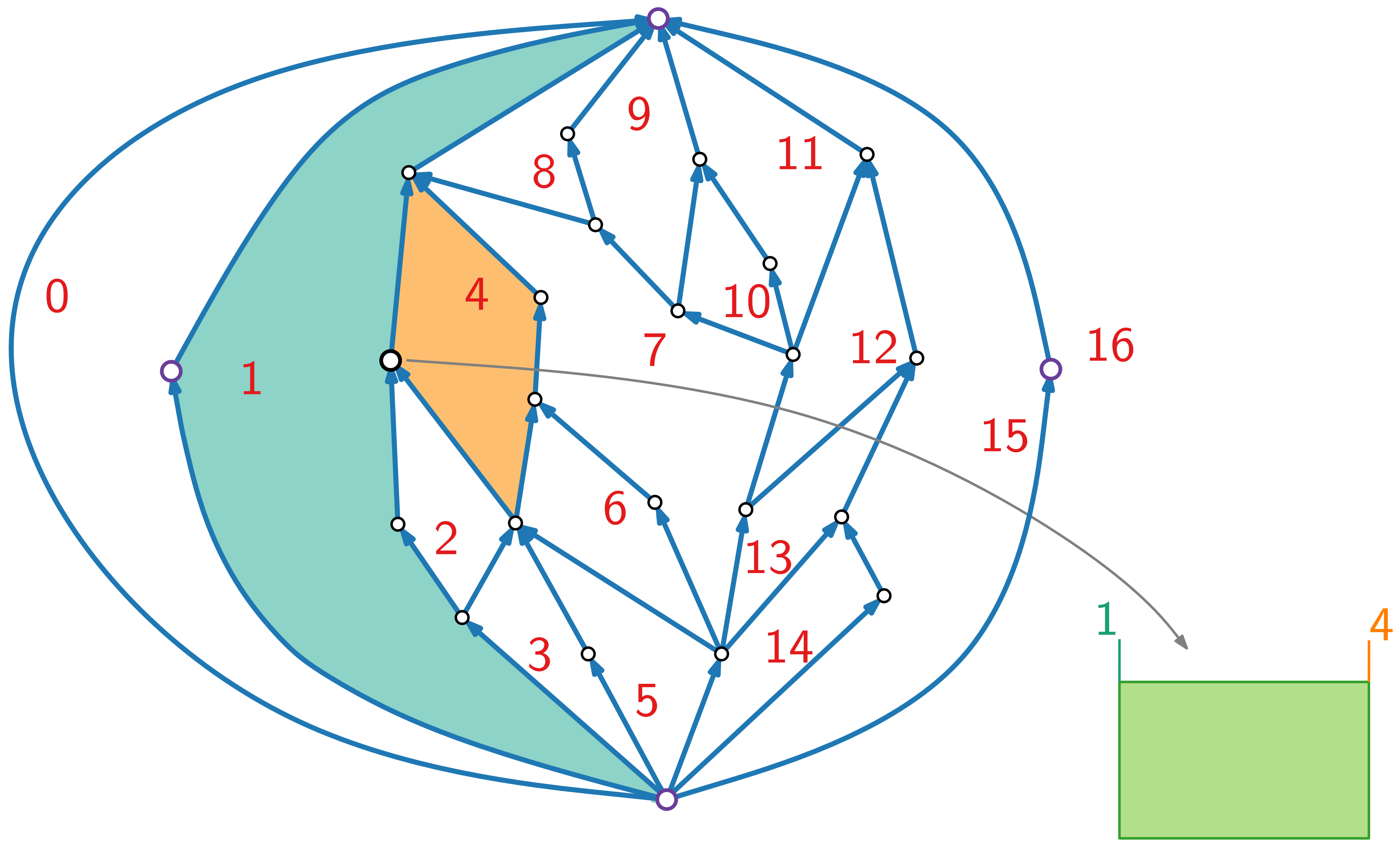

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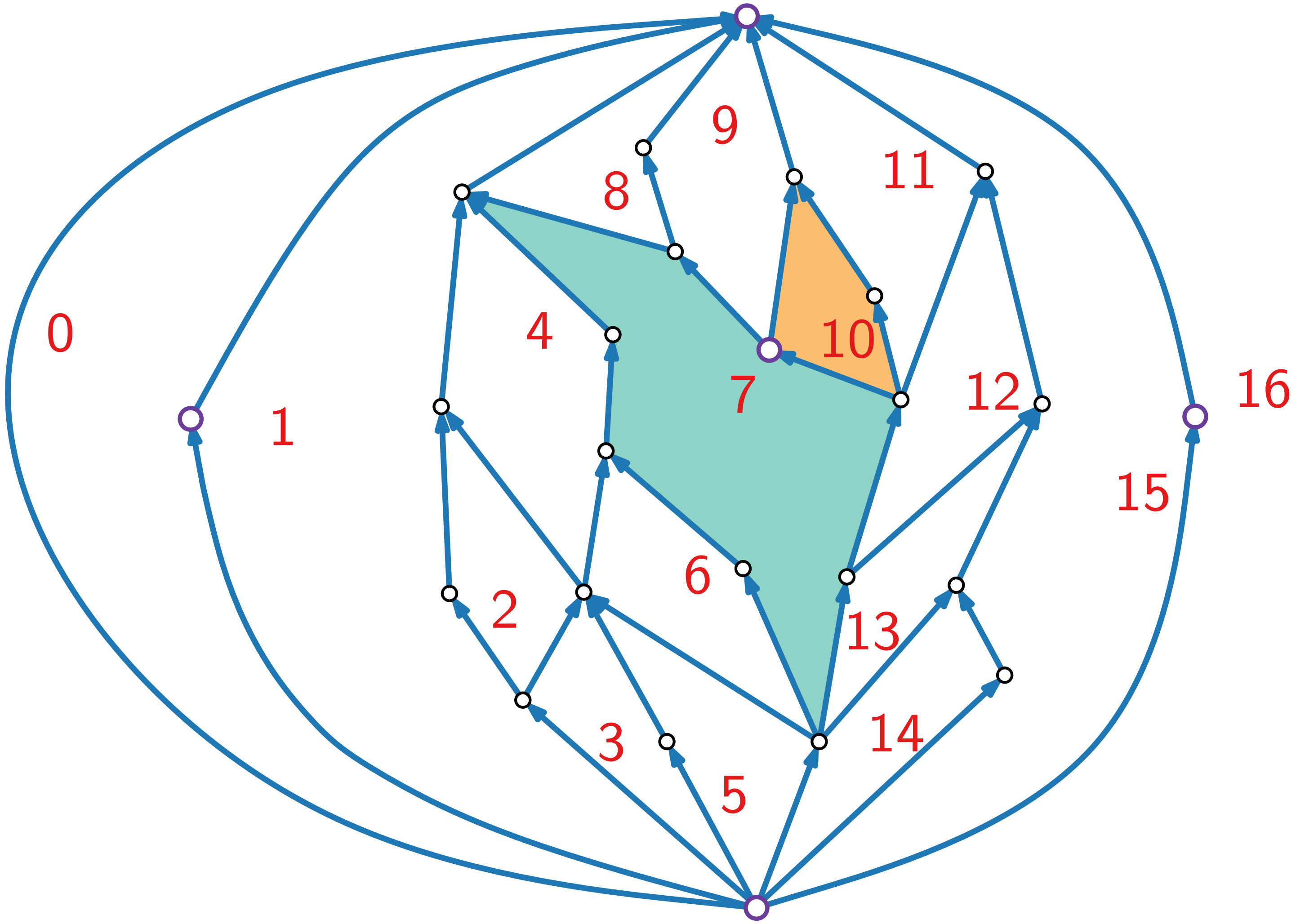

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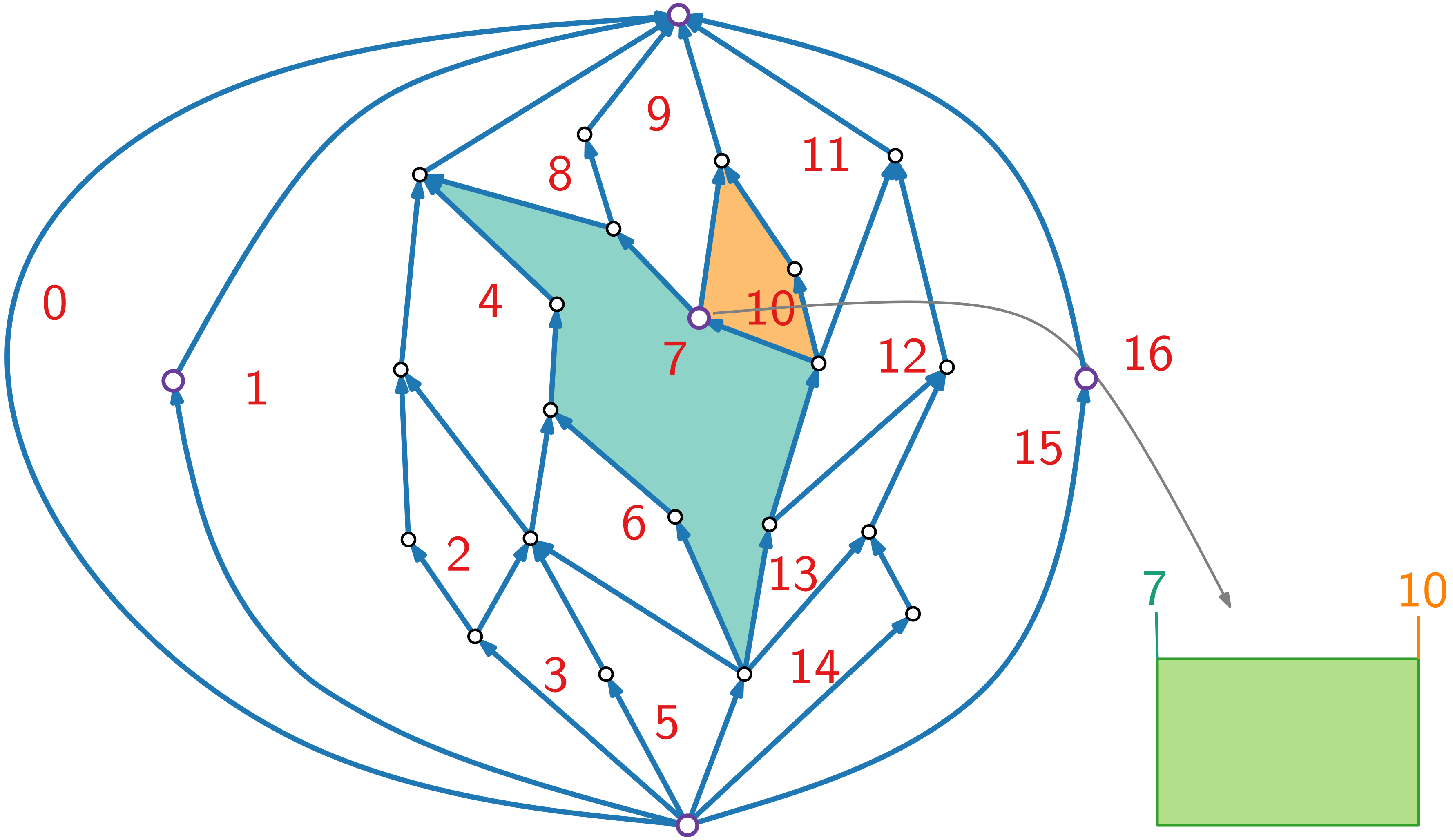

Rectangular Dual Algorithm

For a PTP graph $G = (V, E)$:

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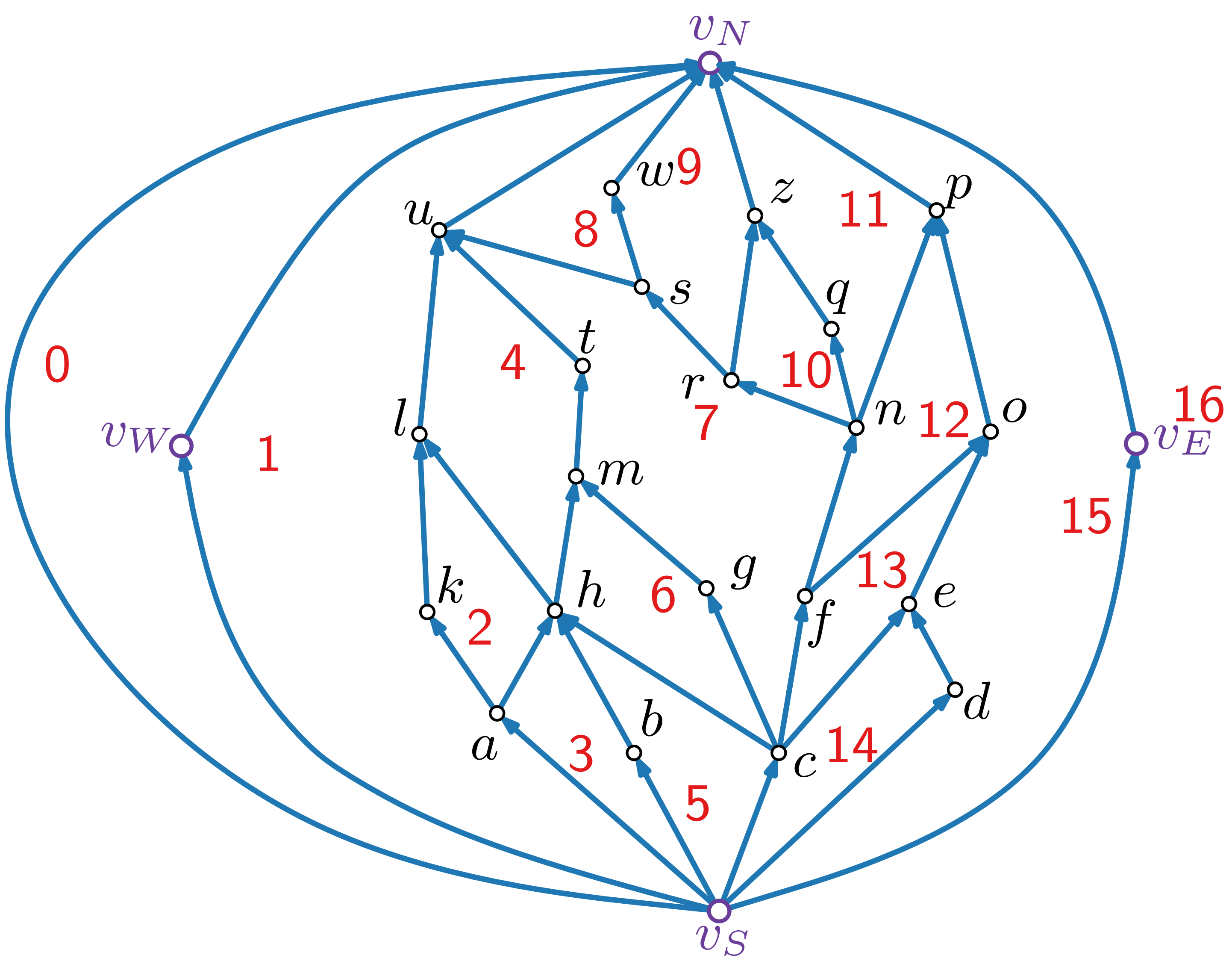
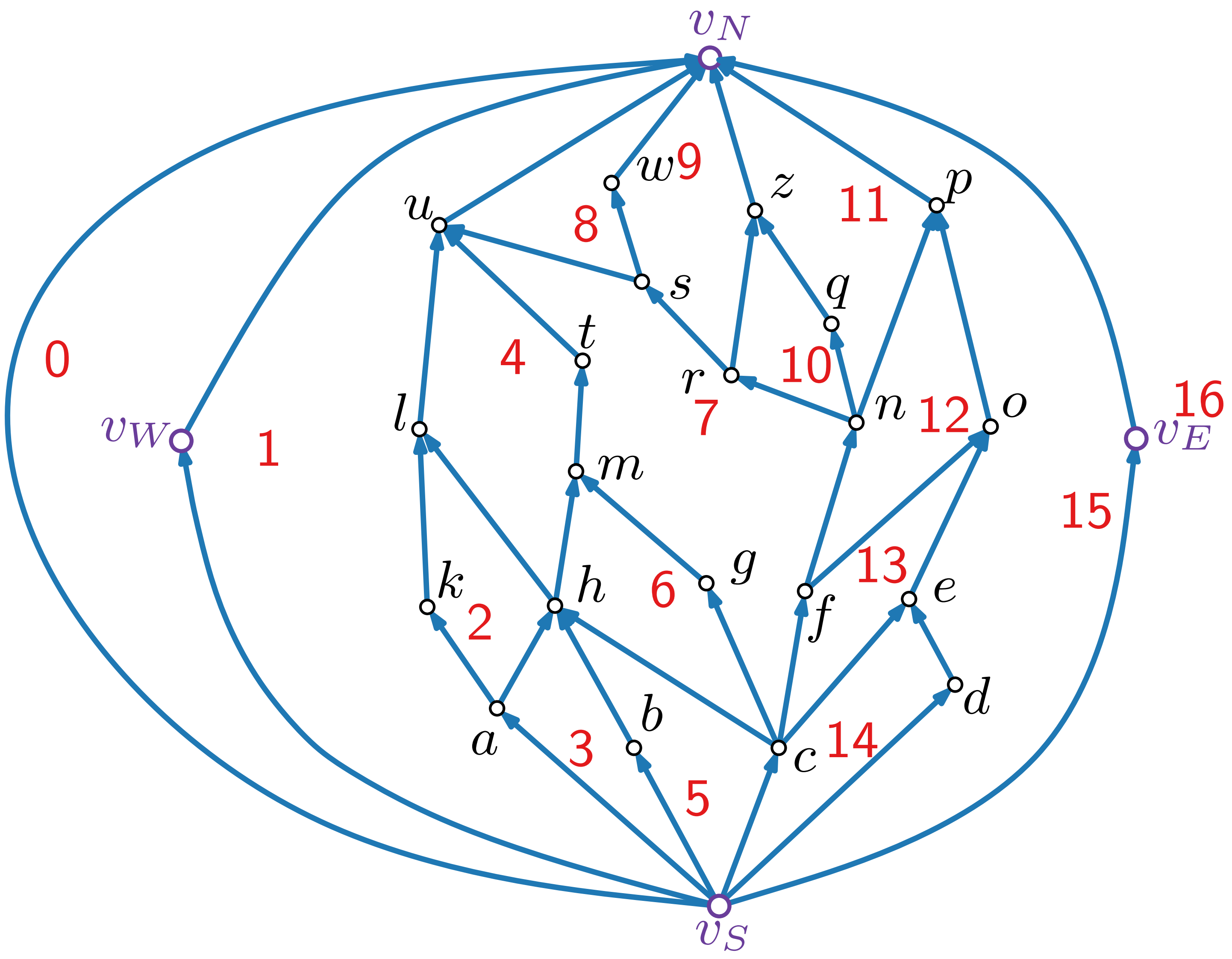
- Analogously compute y_1 and y_2 with G_{hor}.
- For each $v \in V$, assign a rectangle $R(v)$ bounded by x-coordinates $x_1(v),\ x_2(v)$ and y-coordinates $y_1(v),\ y_2(v)$.

Reading off Coordinates to get Rectangular Dual

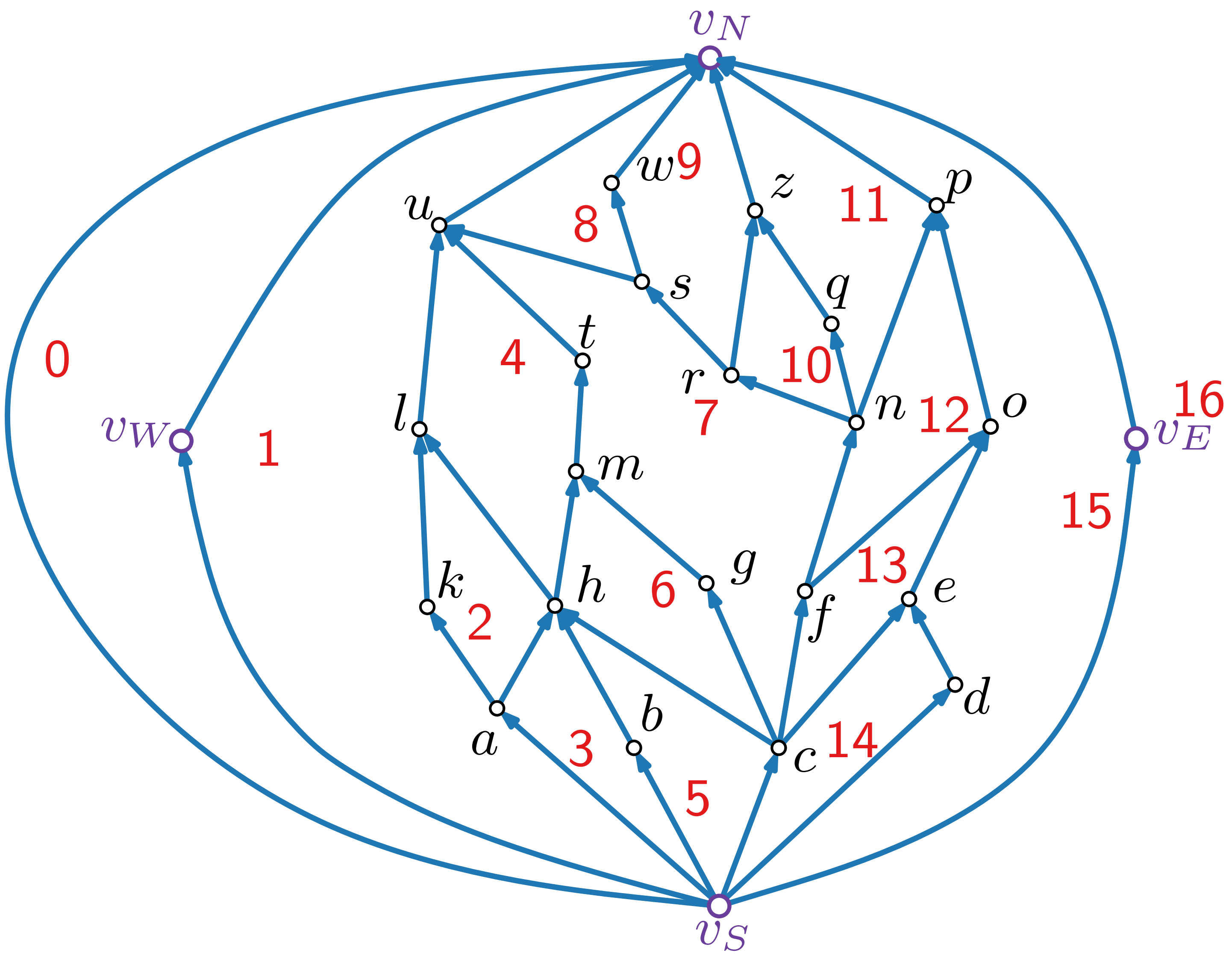


Reading off Coordinates to get Rectangular Dual

$x_1(v_N) = 1,\ x_2(v_N) = 15$

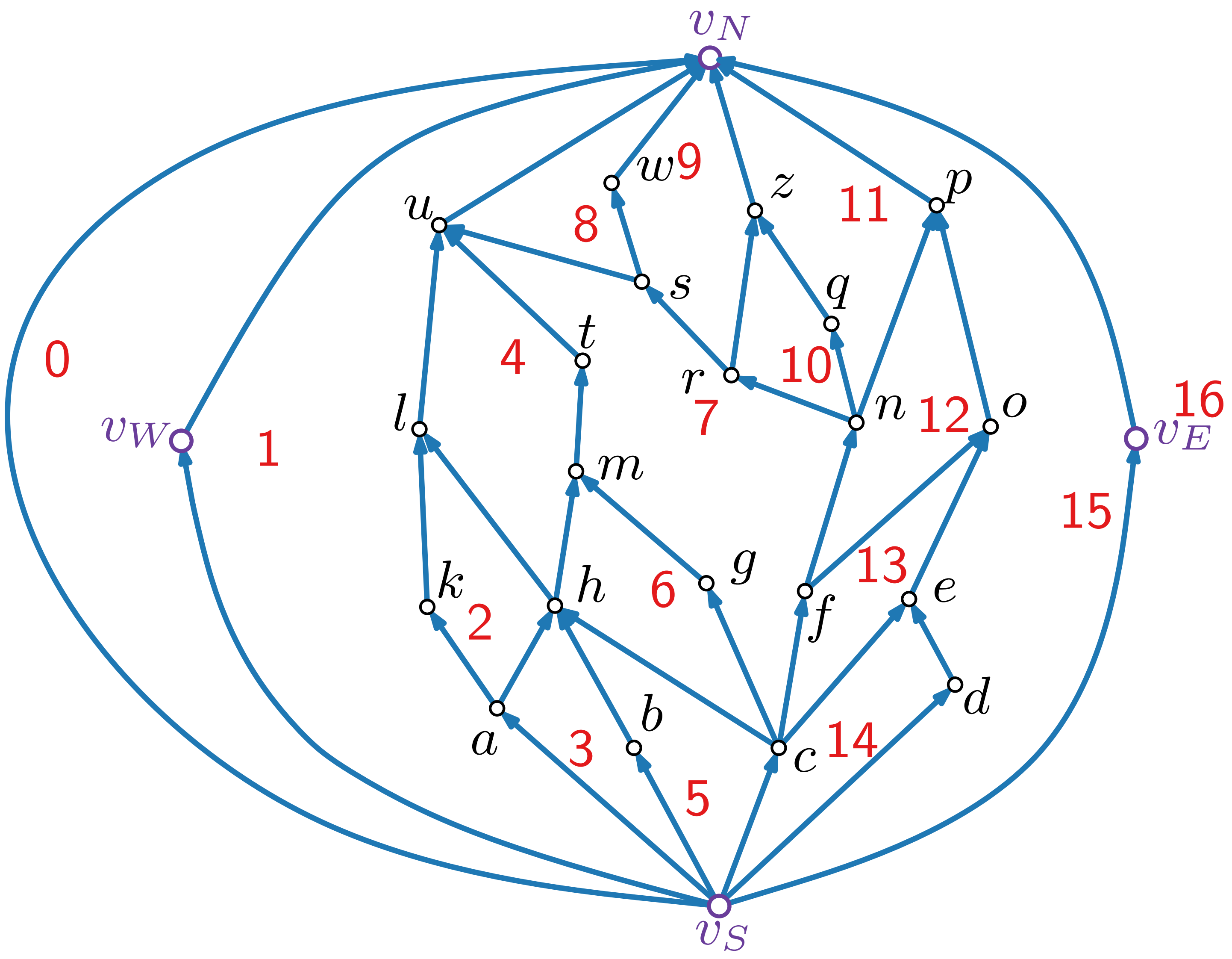


Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$

Reading off Coordinates to get Rectangular Dual

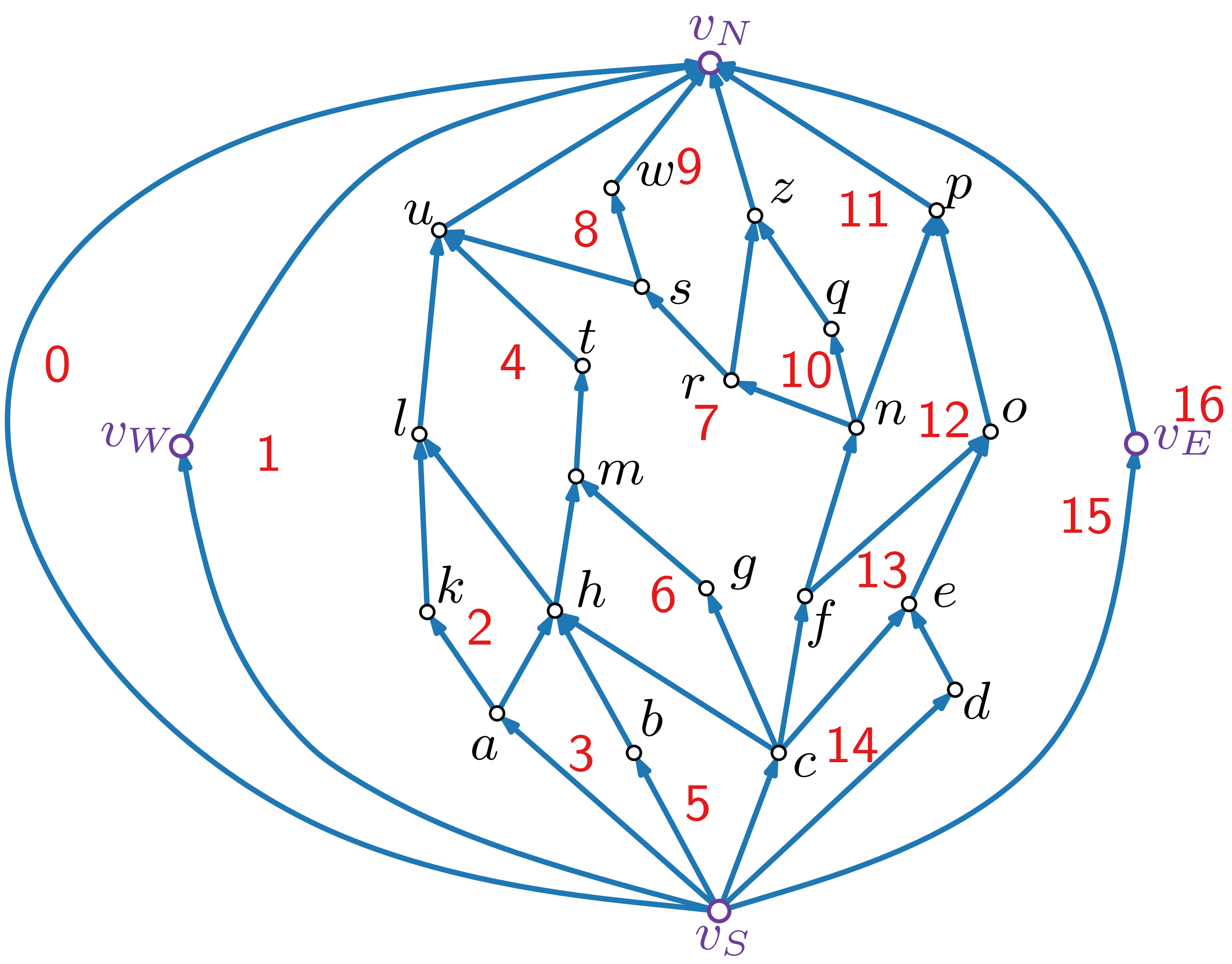


$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

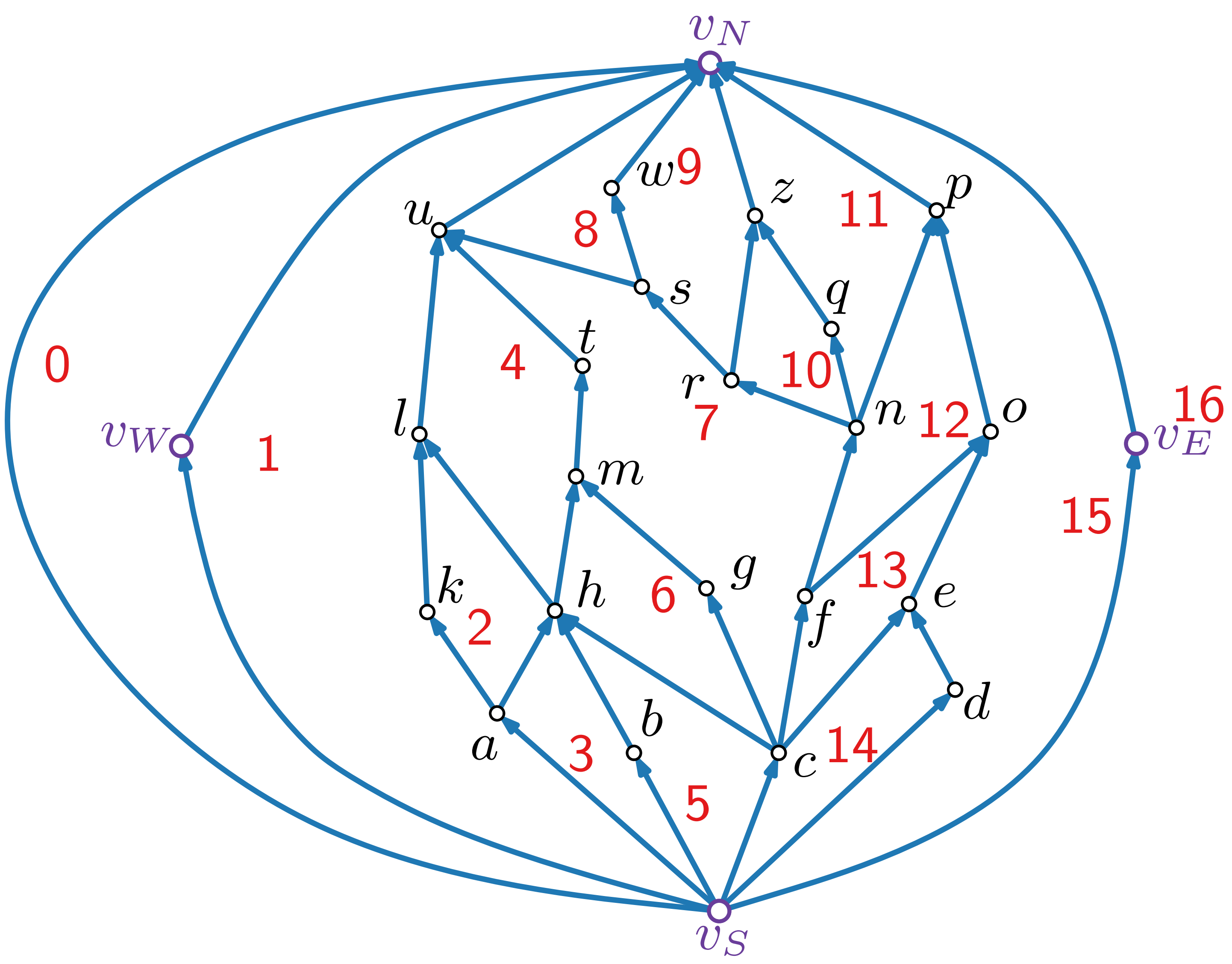
$x_1(v_W) = 0, x_2(v_W) = 1$

Reading off Coordinates to get Rectangular Dual



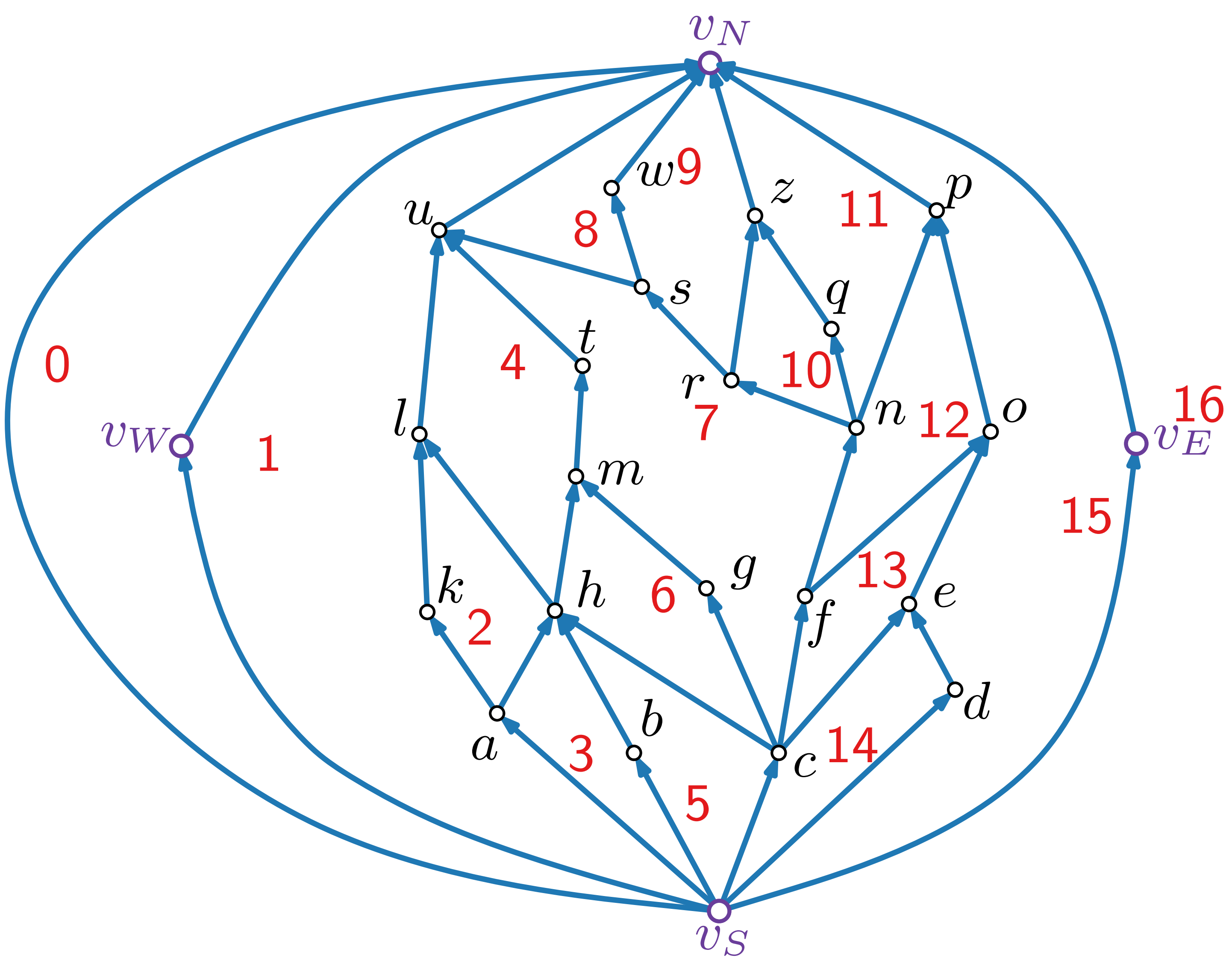
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$

Reading off Coordinates to get Rectangular Dual



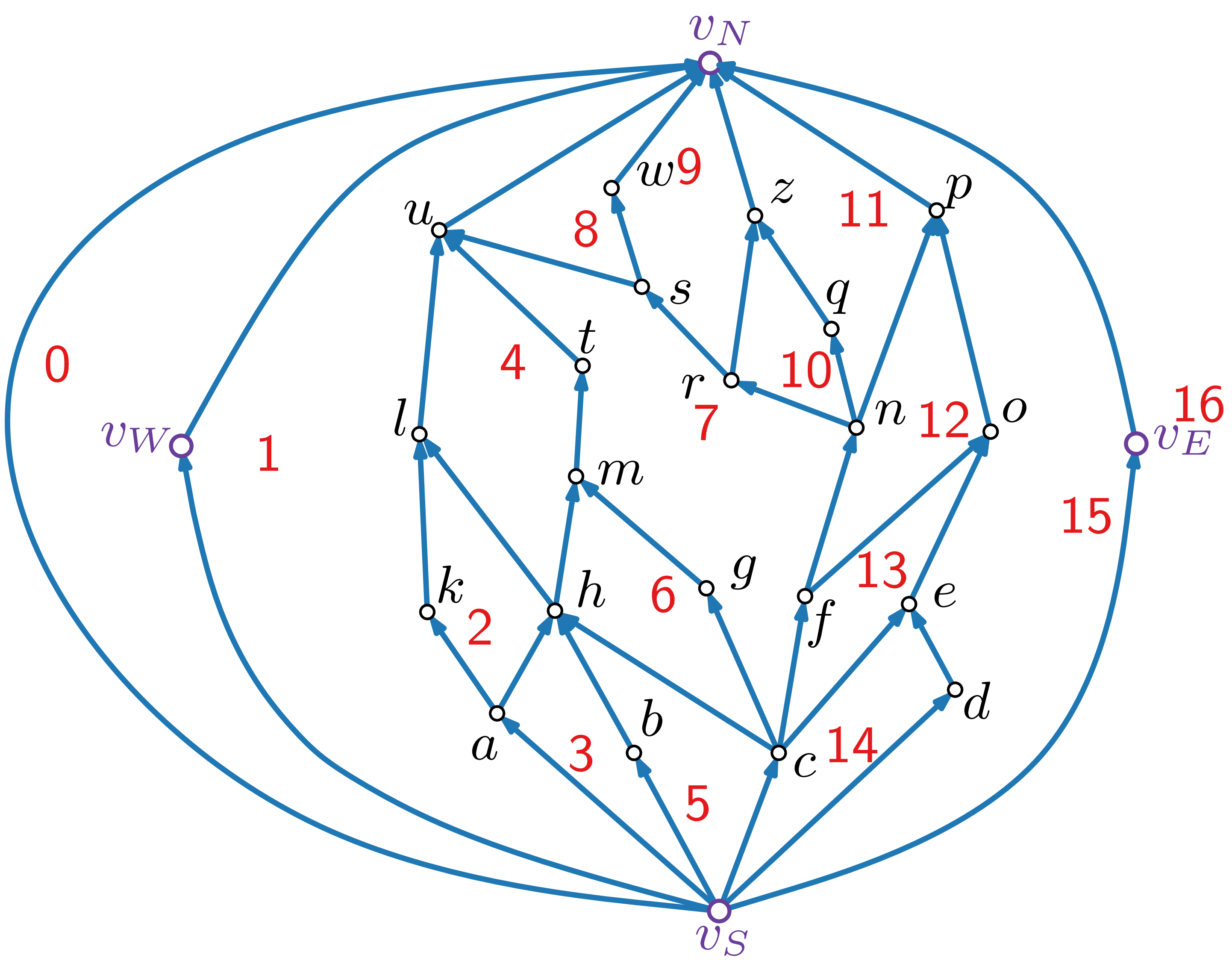
$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

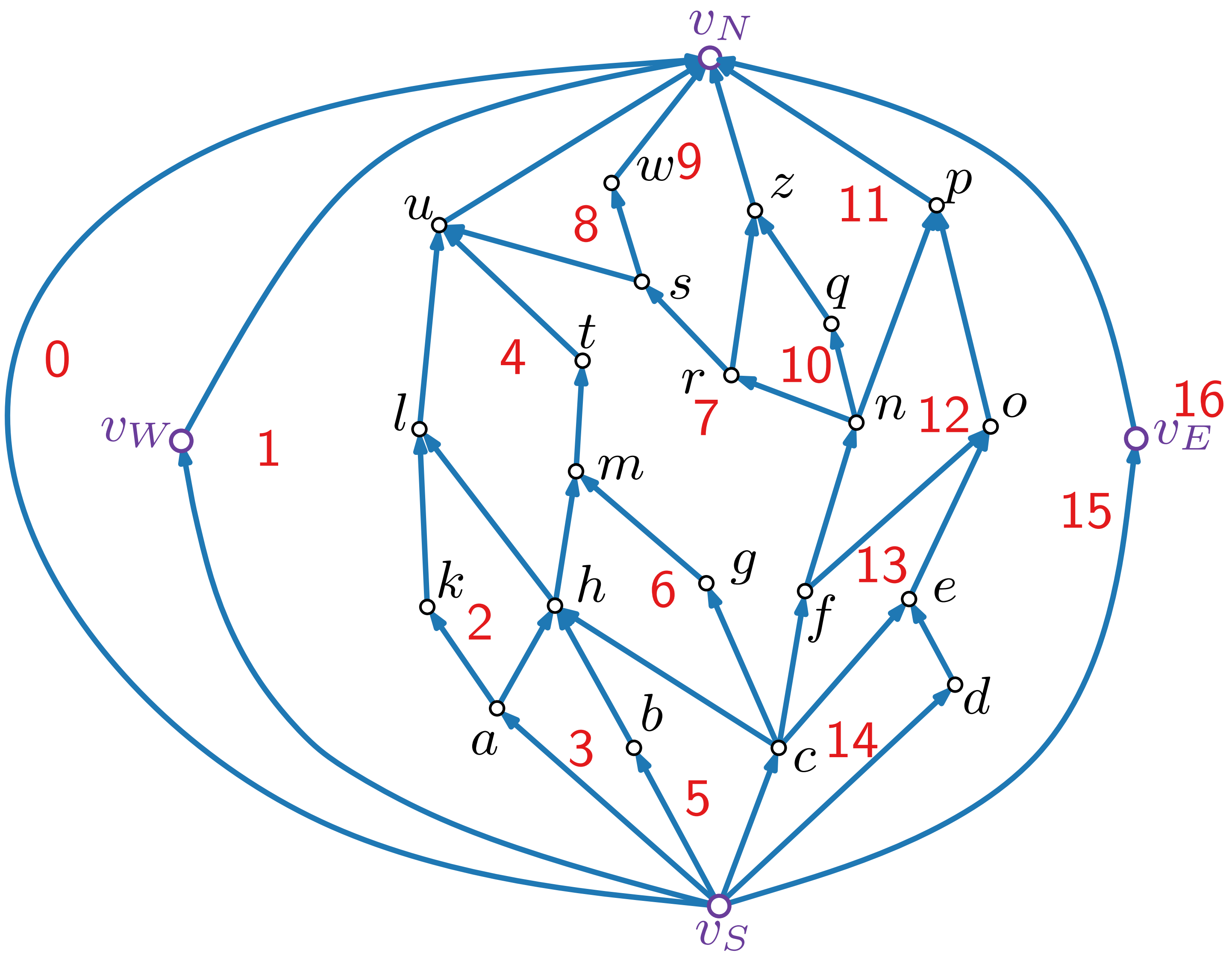
$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

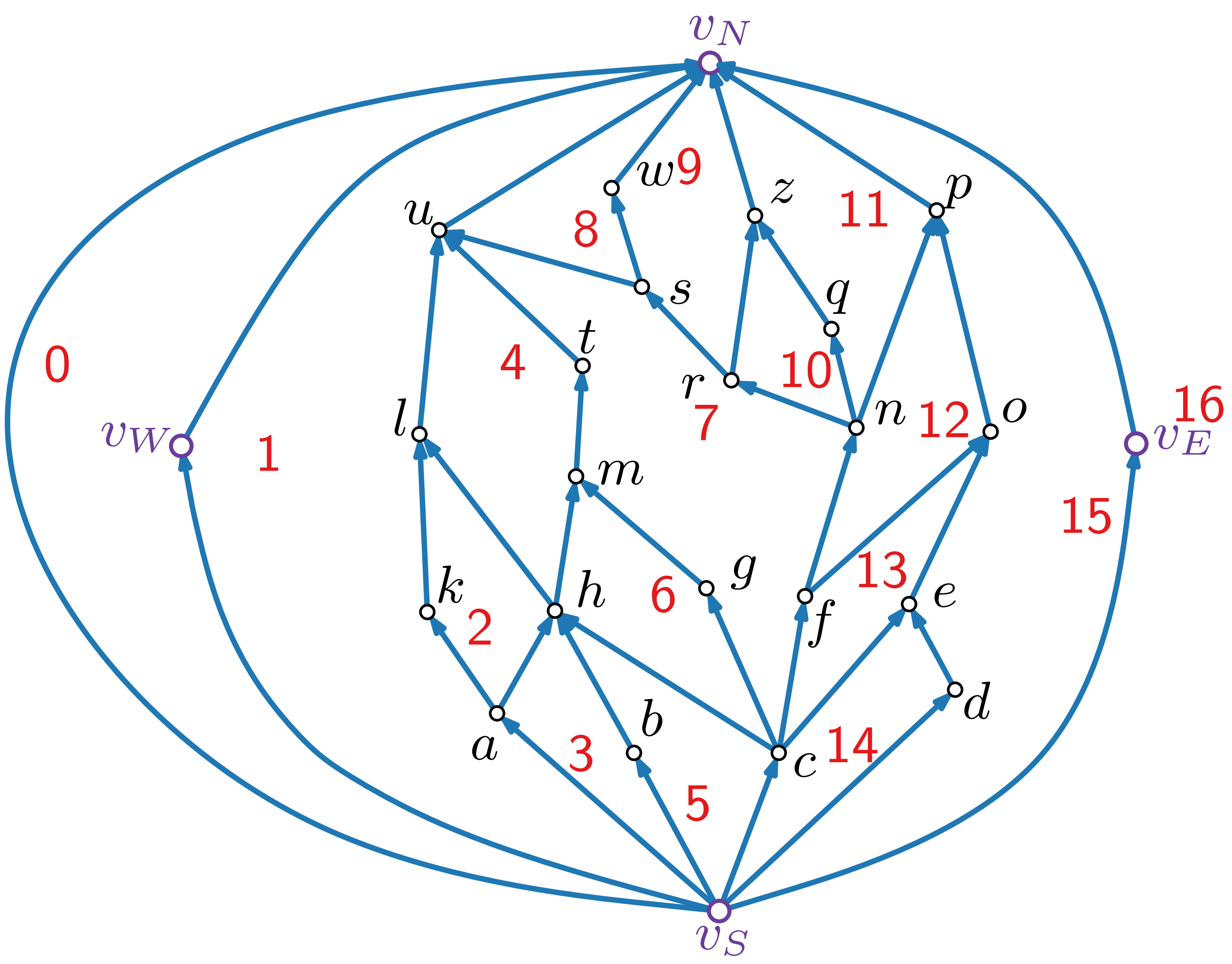
$x_1(c) = 5,\ x_2(c) = 14$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

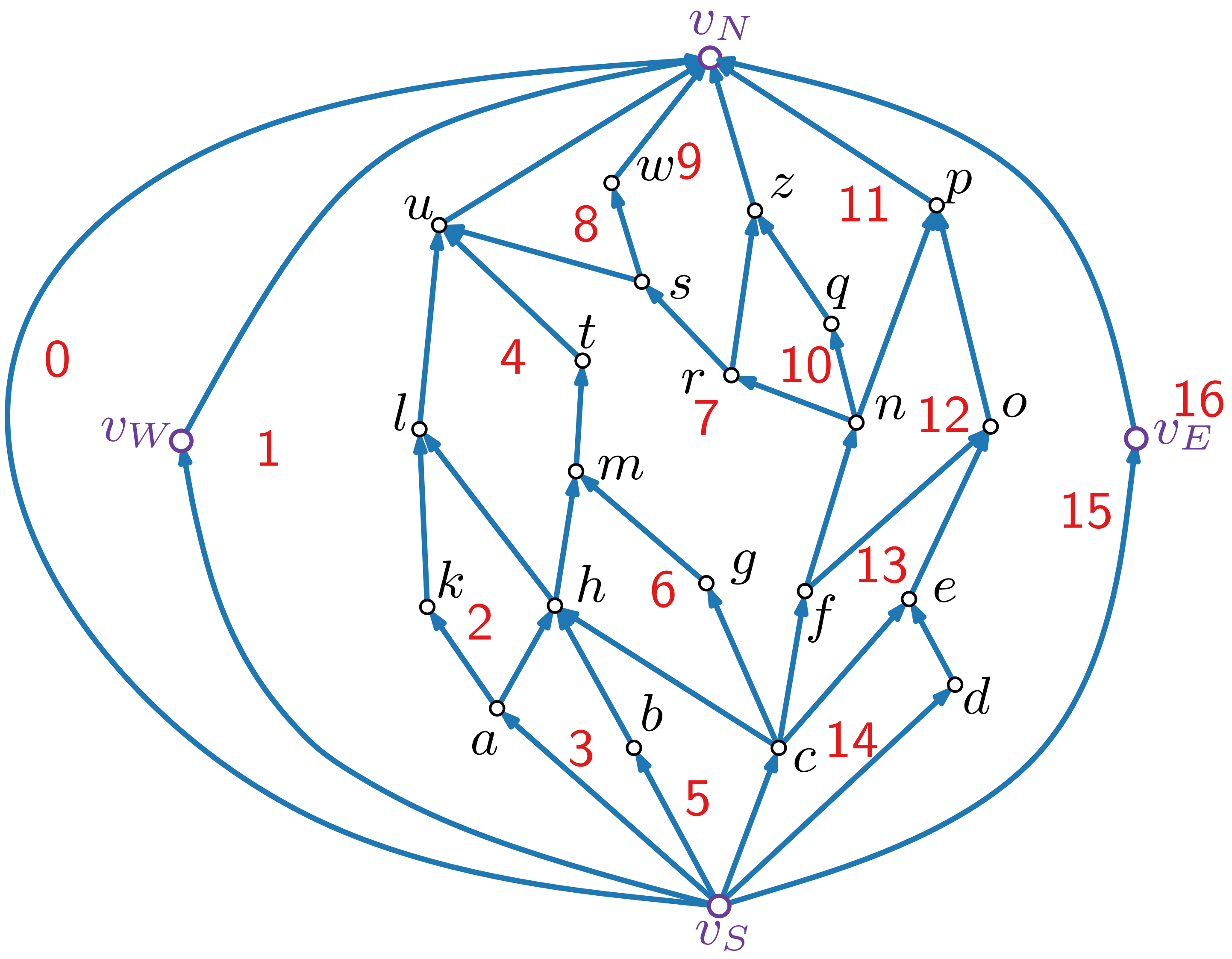
$x_1(b) = 3,\ x_2(b) = 5$

$x_1(c) = 5,\ x_2(c) = 14$

$x_1(d) = 14,\ x_2(d) = 15$

$x_1(e) = 13,\ x_2(e) = 15$

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$

$x_1(v_S) = 2,\ x_2(v_S) = 16$

$x_1(v_W) = 0, x_2(v_W) = 1$

$x_1(v_E) = 15,\ x_2(v_E) = 16$

$x_1(a) = 1,\ x_2(a) = 3$

$x_1(b) = 3,\ x_2(b) = 5$

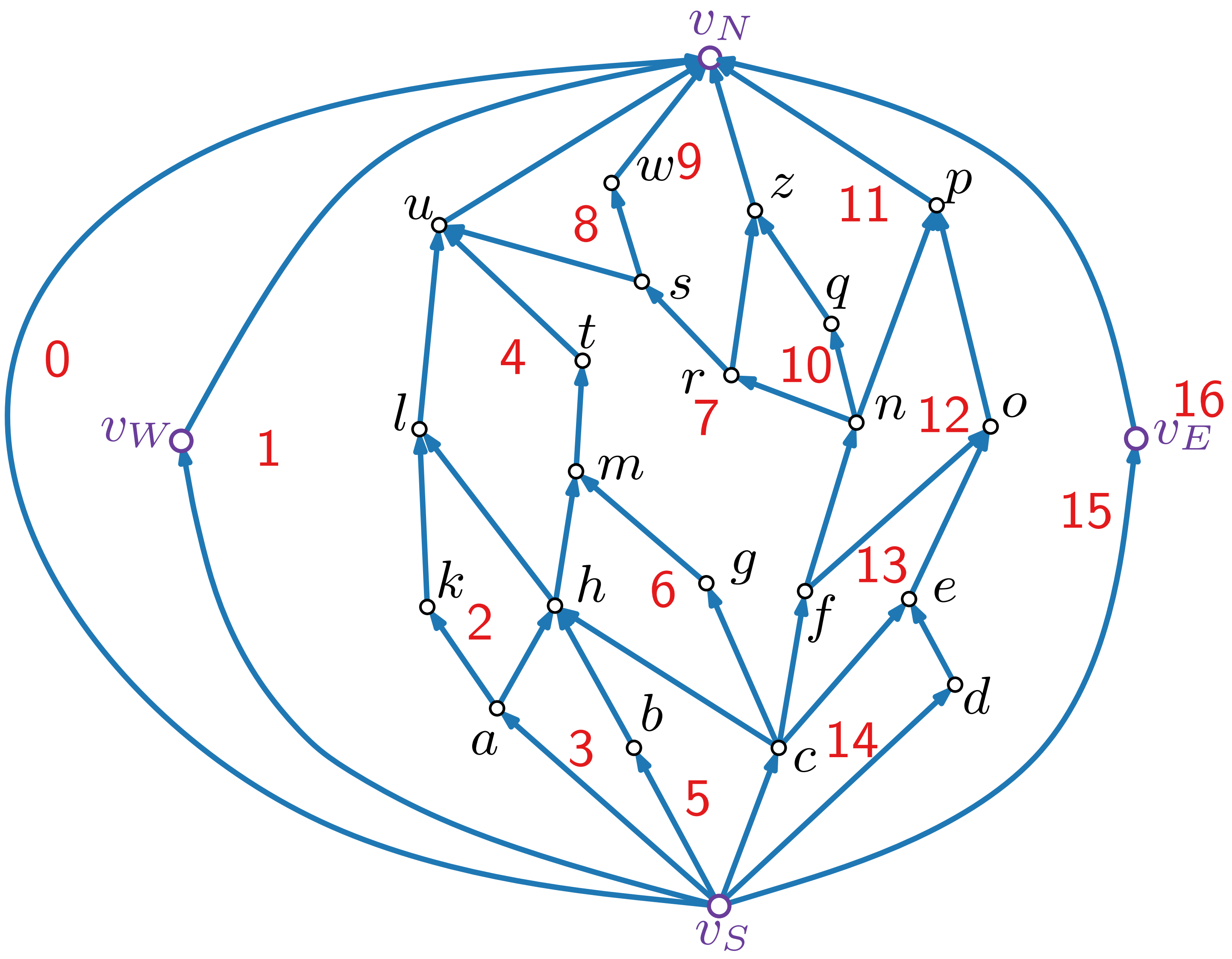
$x_1(c) = 5,\ x_2(c) = 14$

$x_1(d) = 14,\ x_2(d) = 15$

$x_1(e) = 13,\ x_2(e) = 15$

$\ldots$

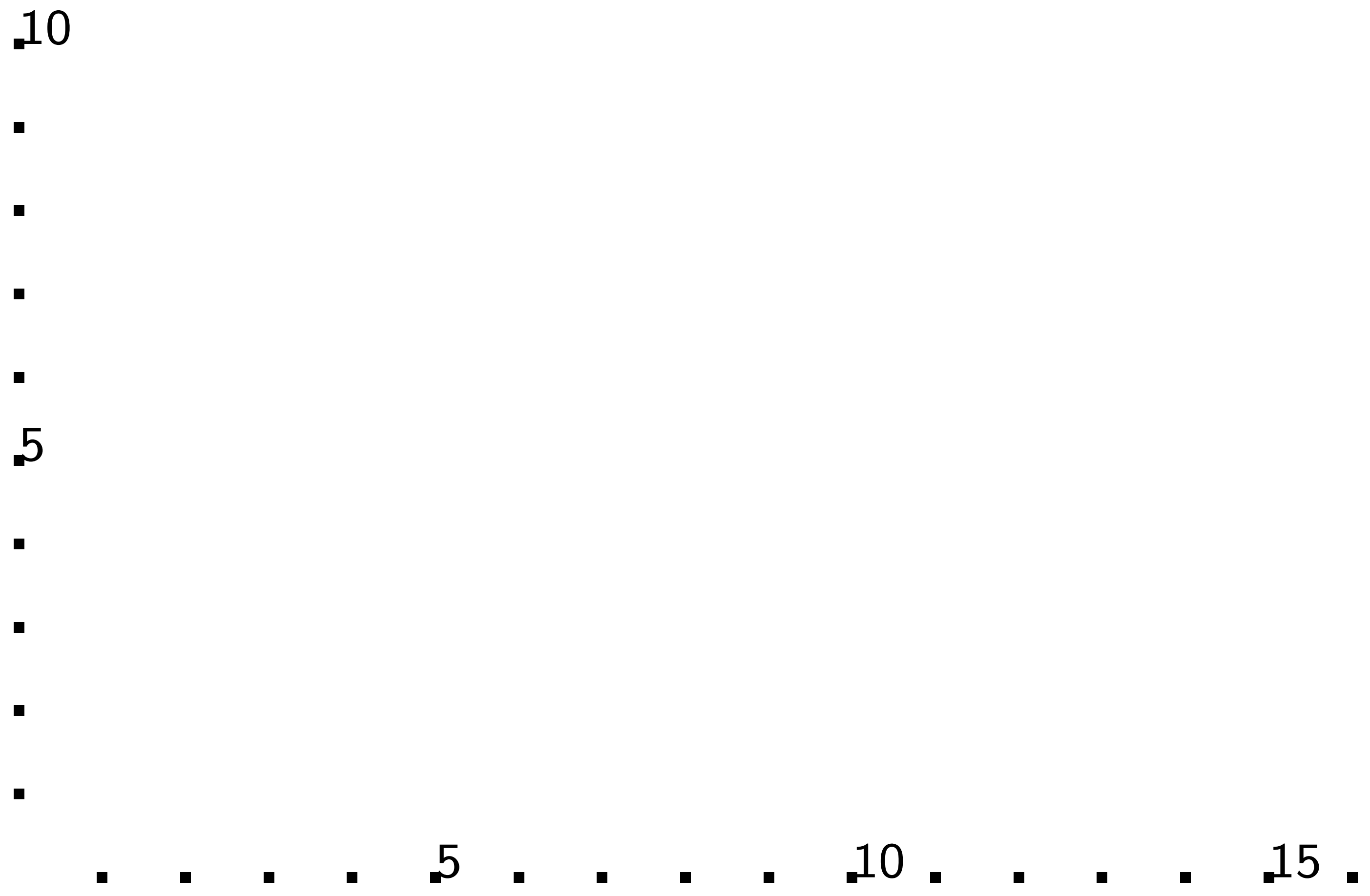
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

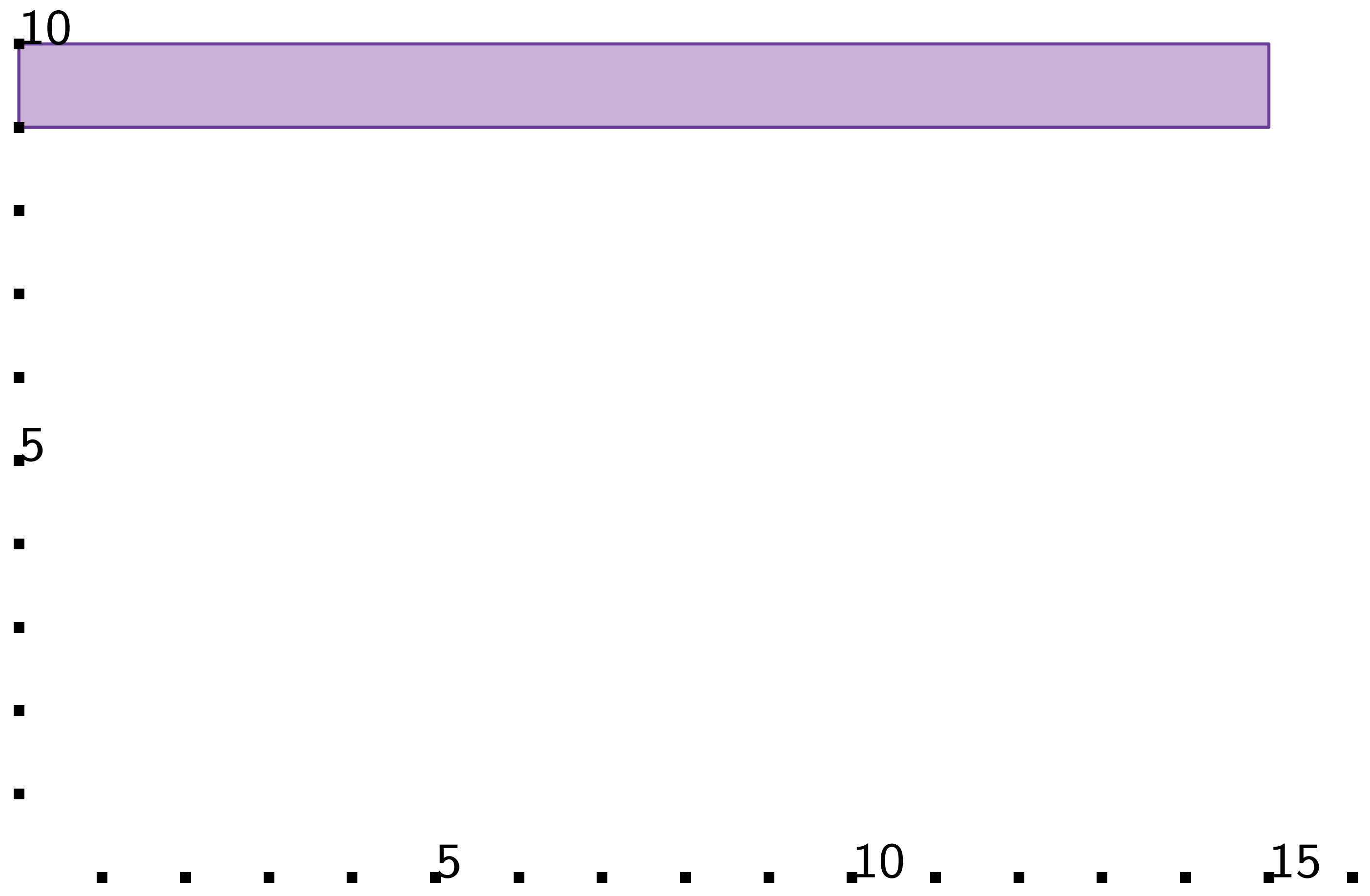
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

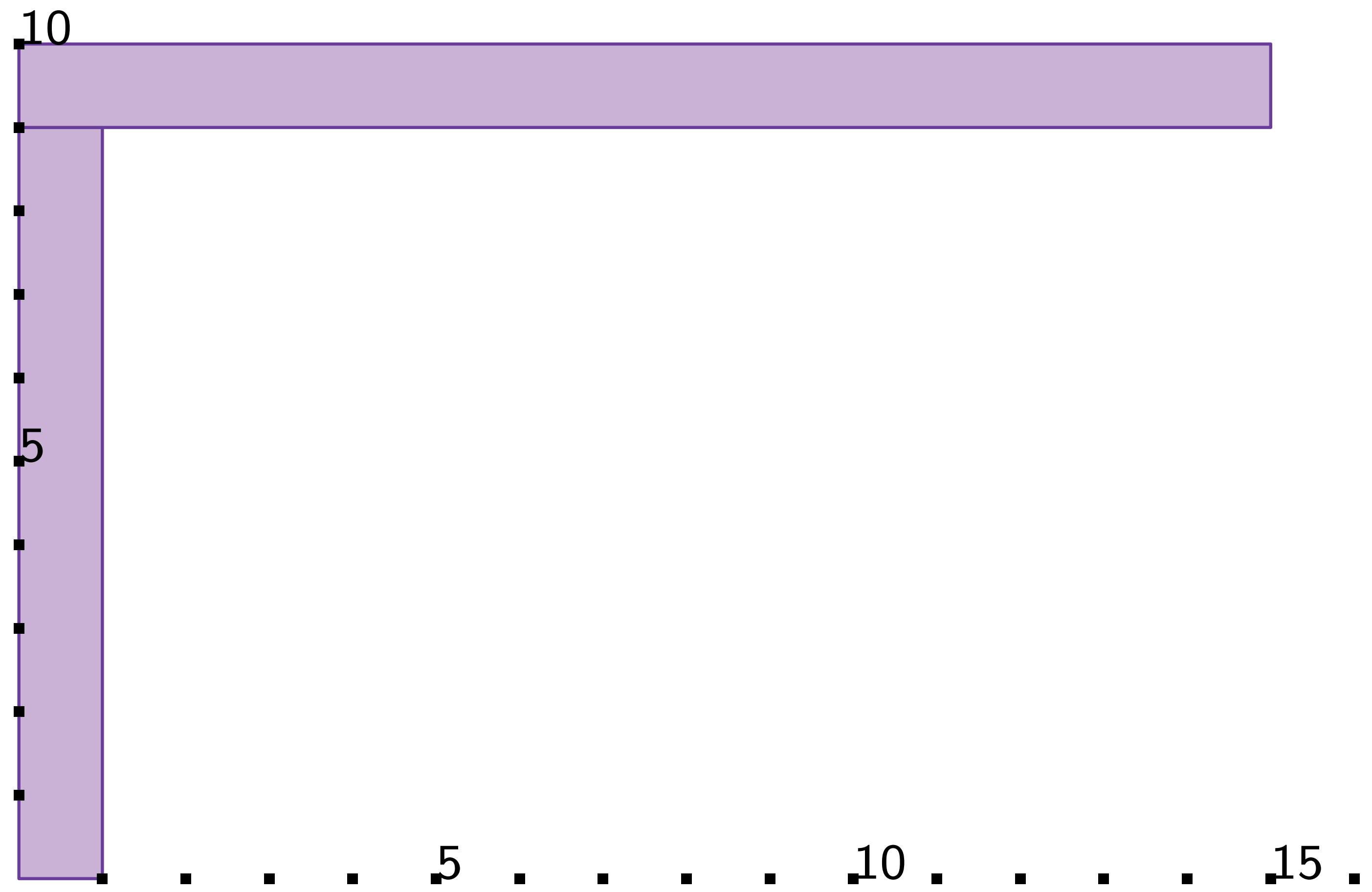
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

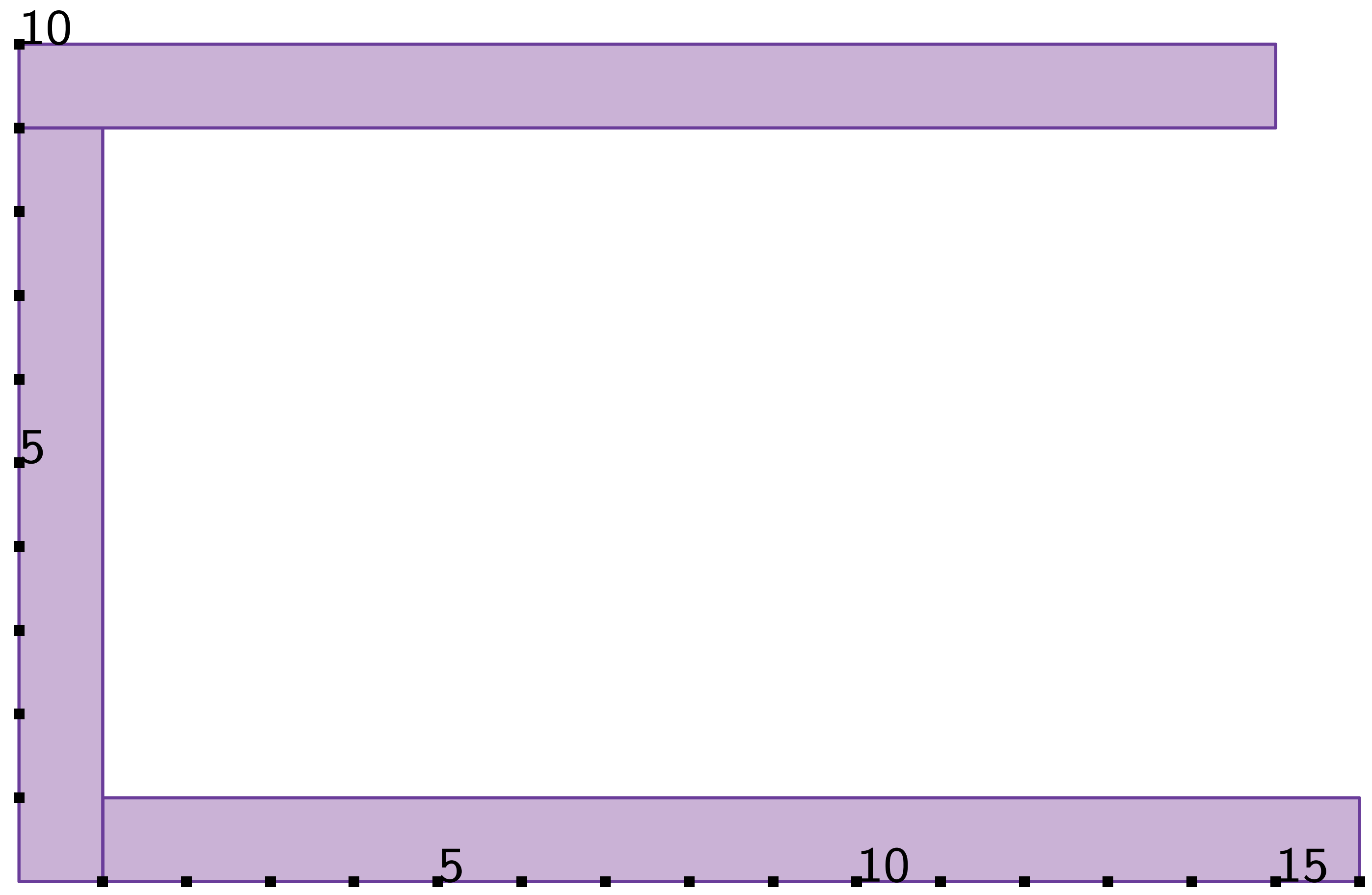
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

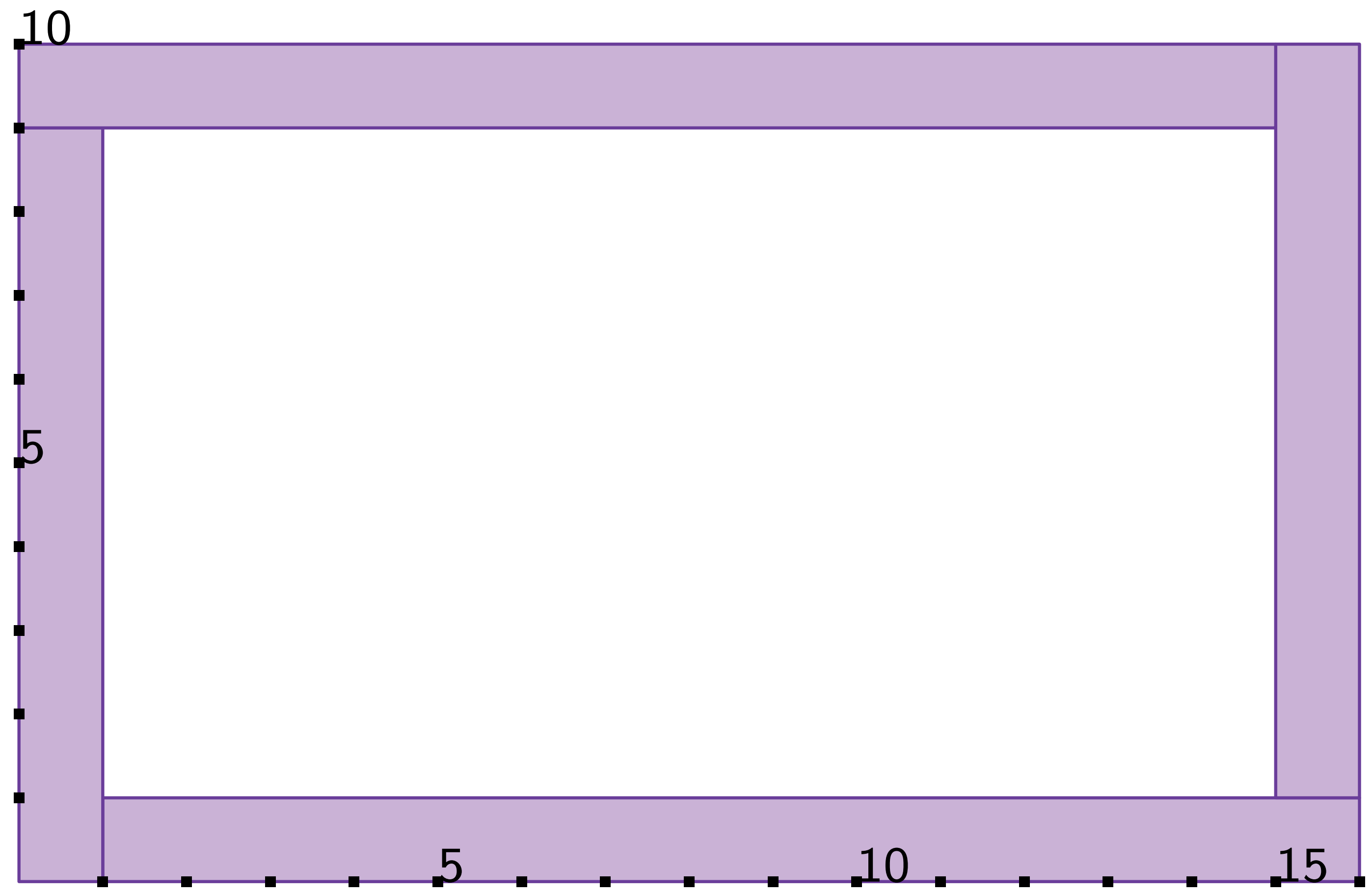
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

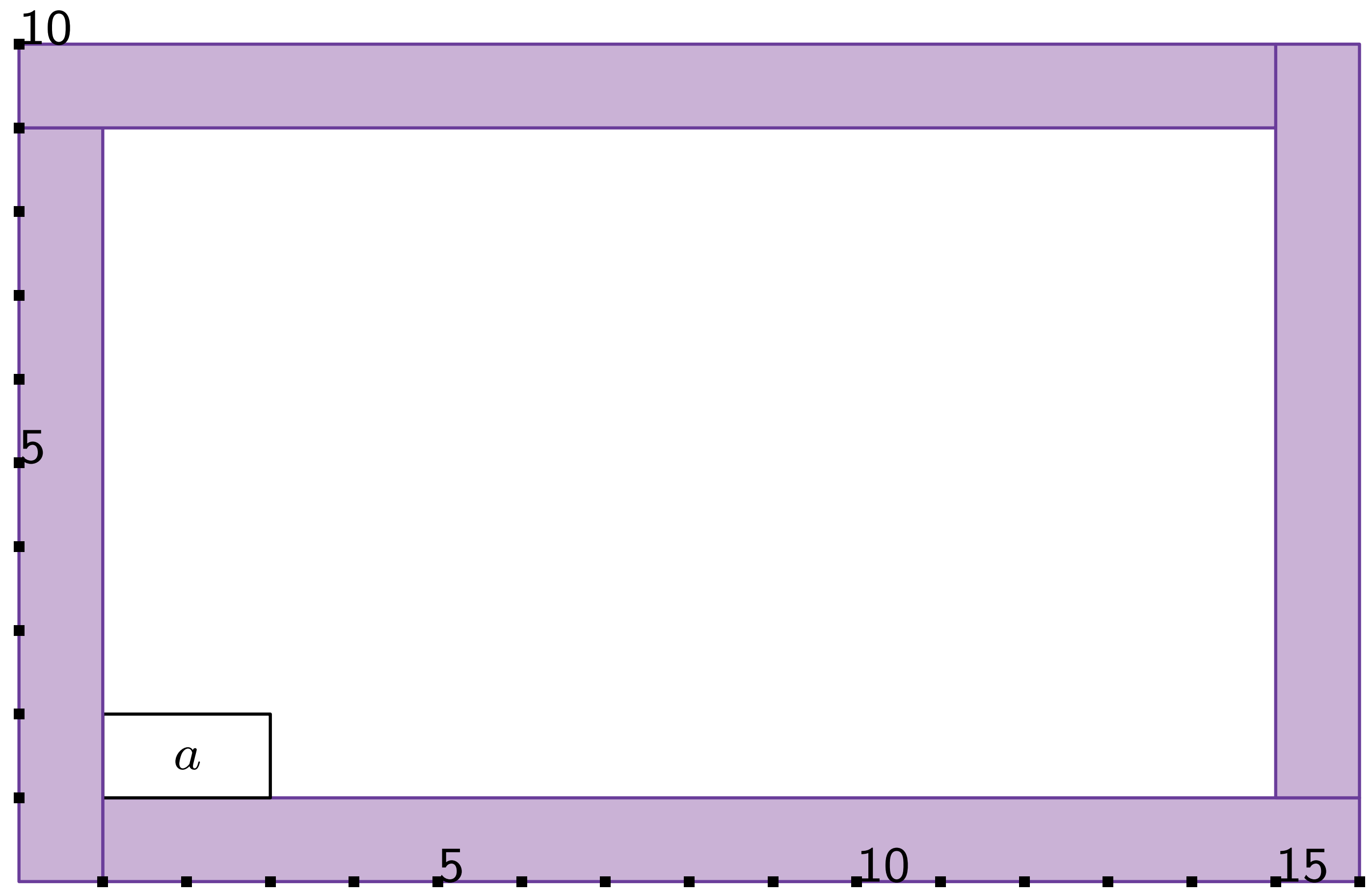
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

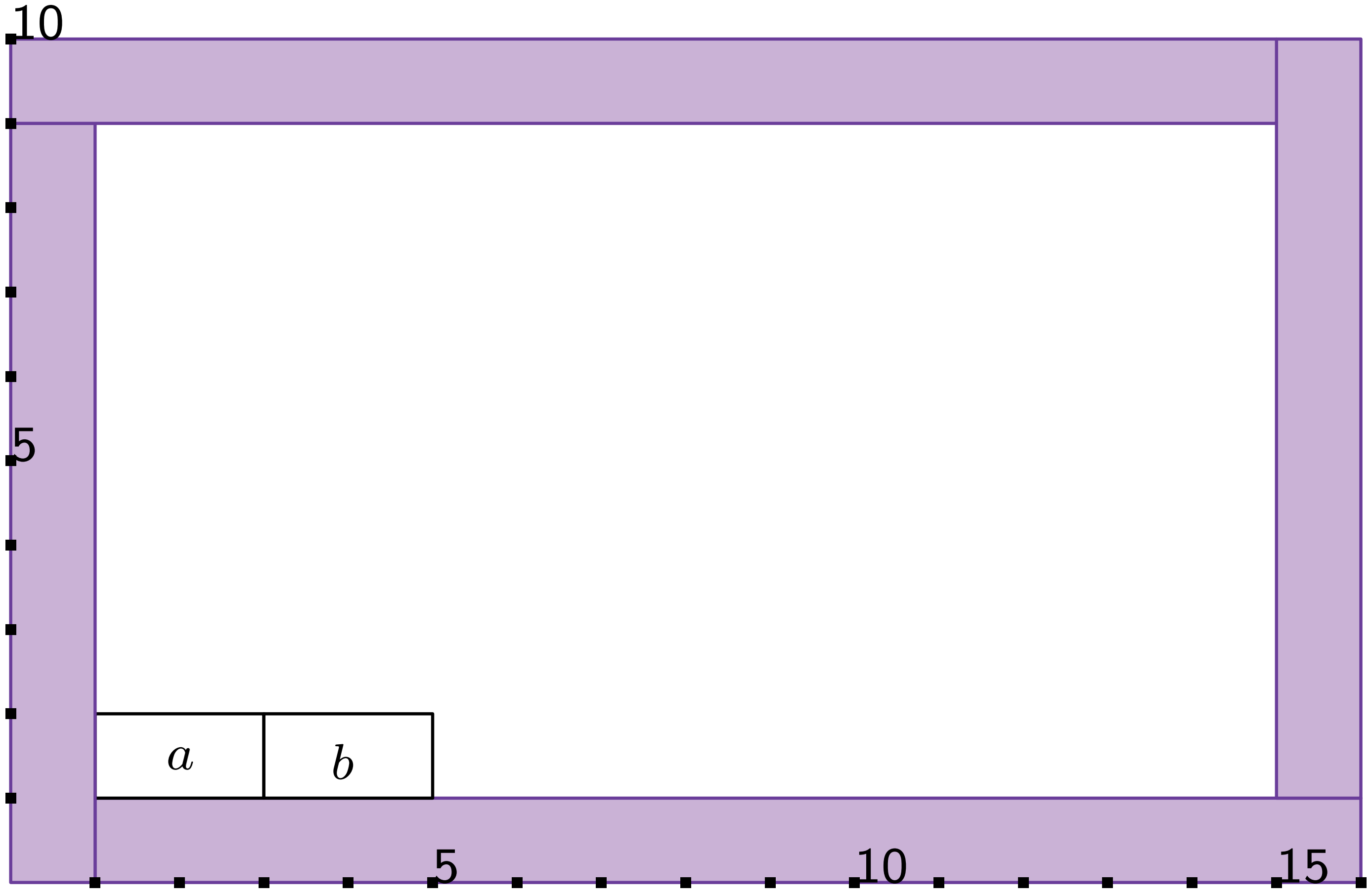
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\ldots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\ldots$

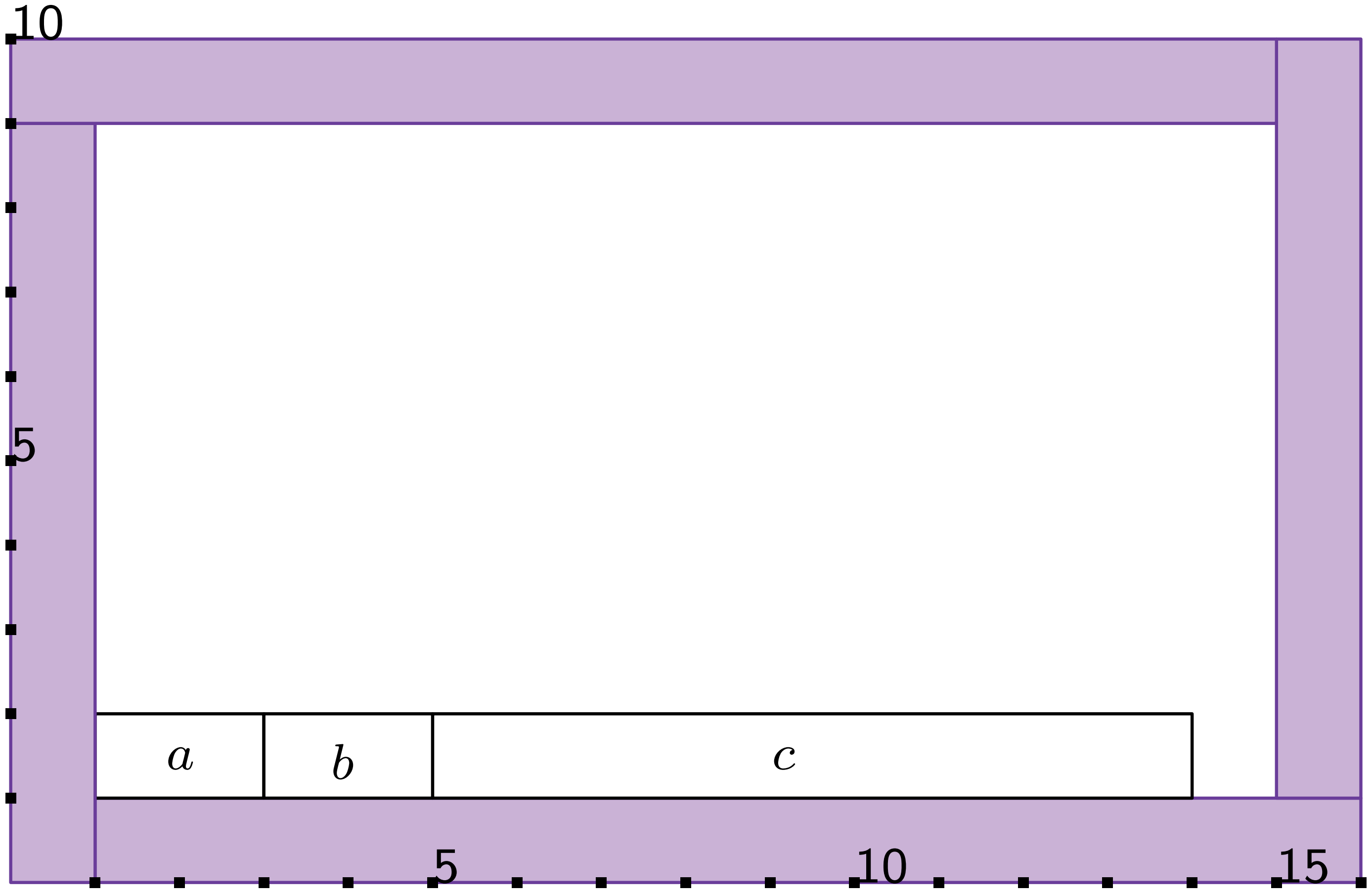
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

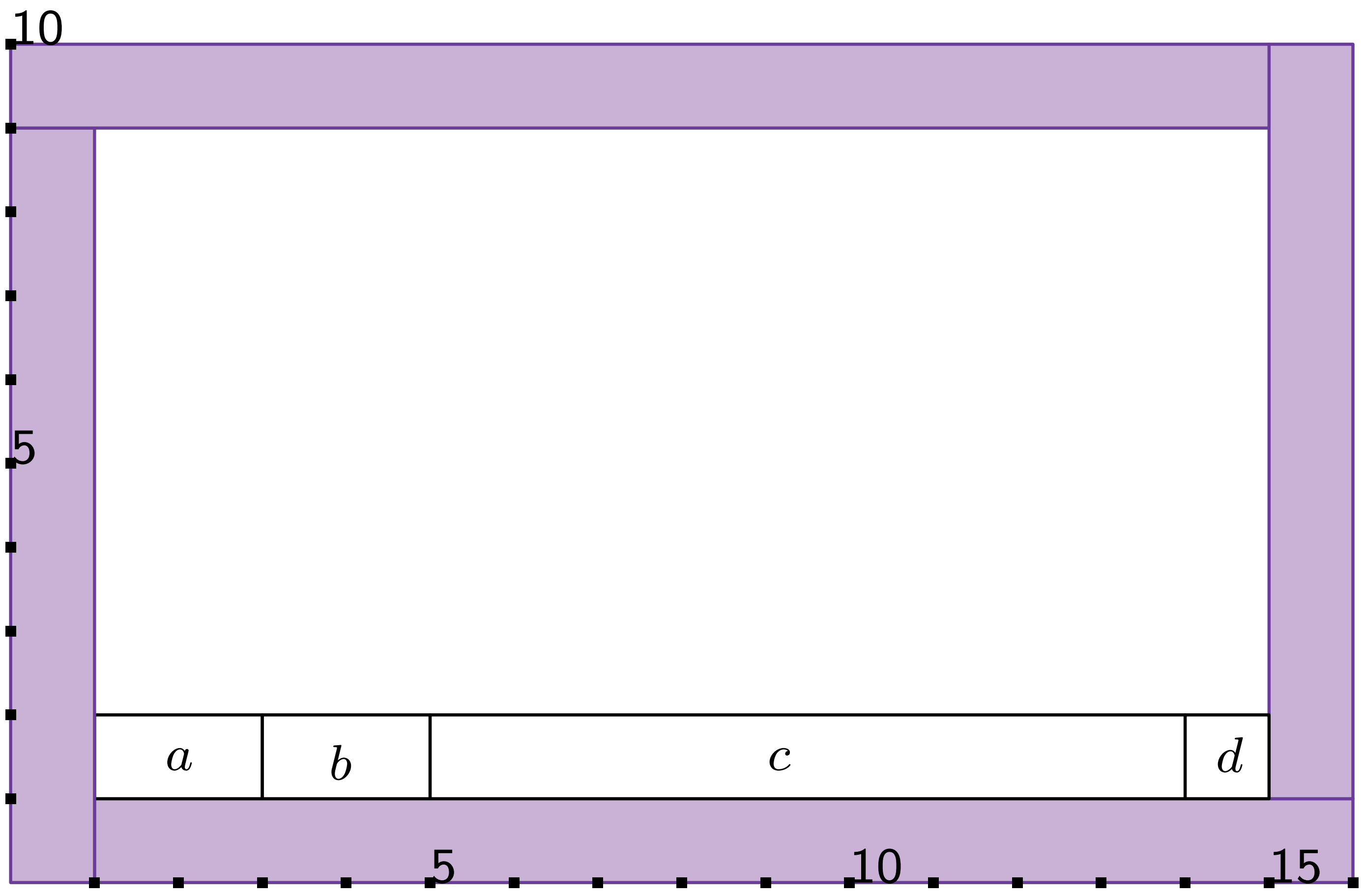
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

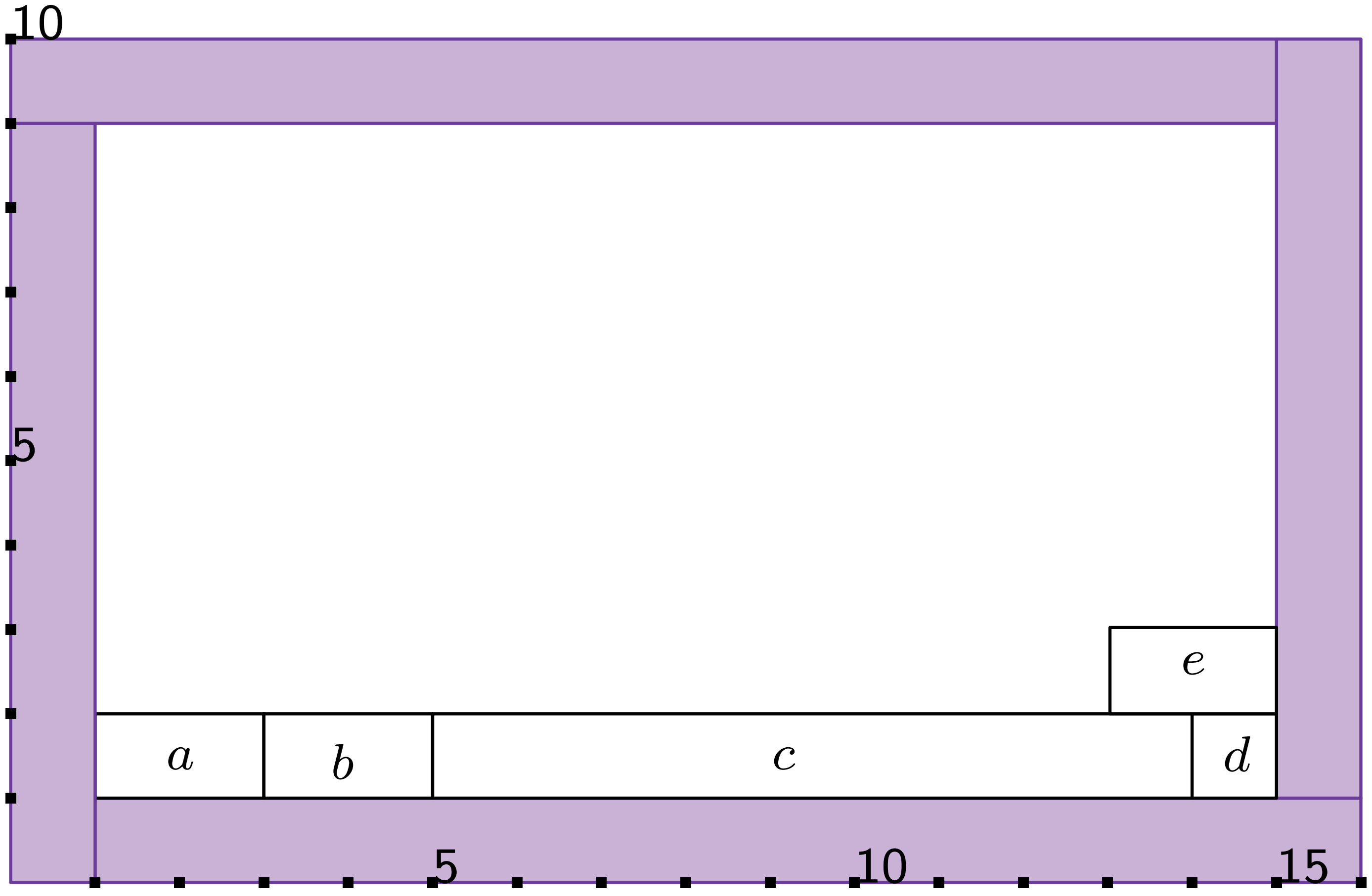
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

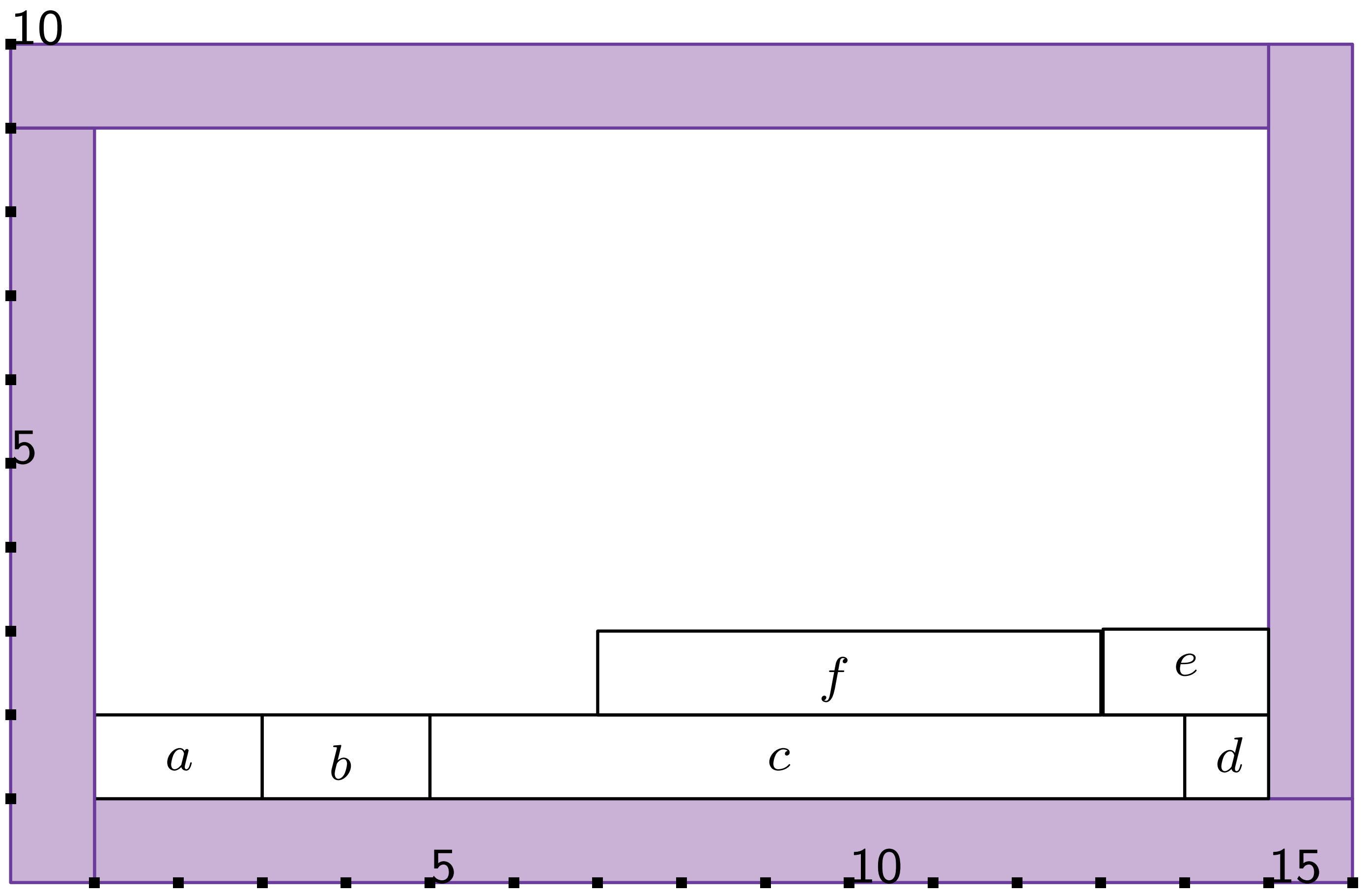
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

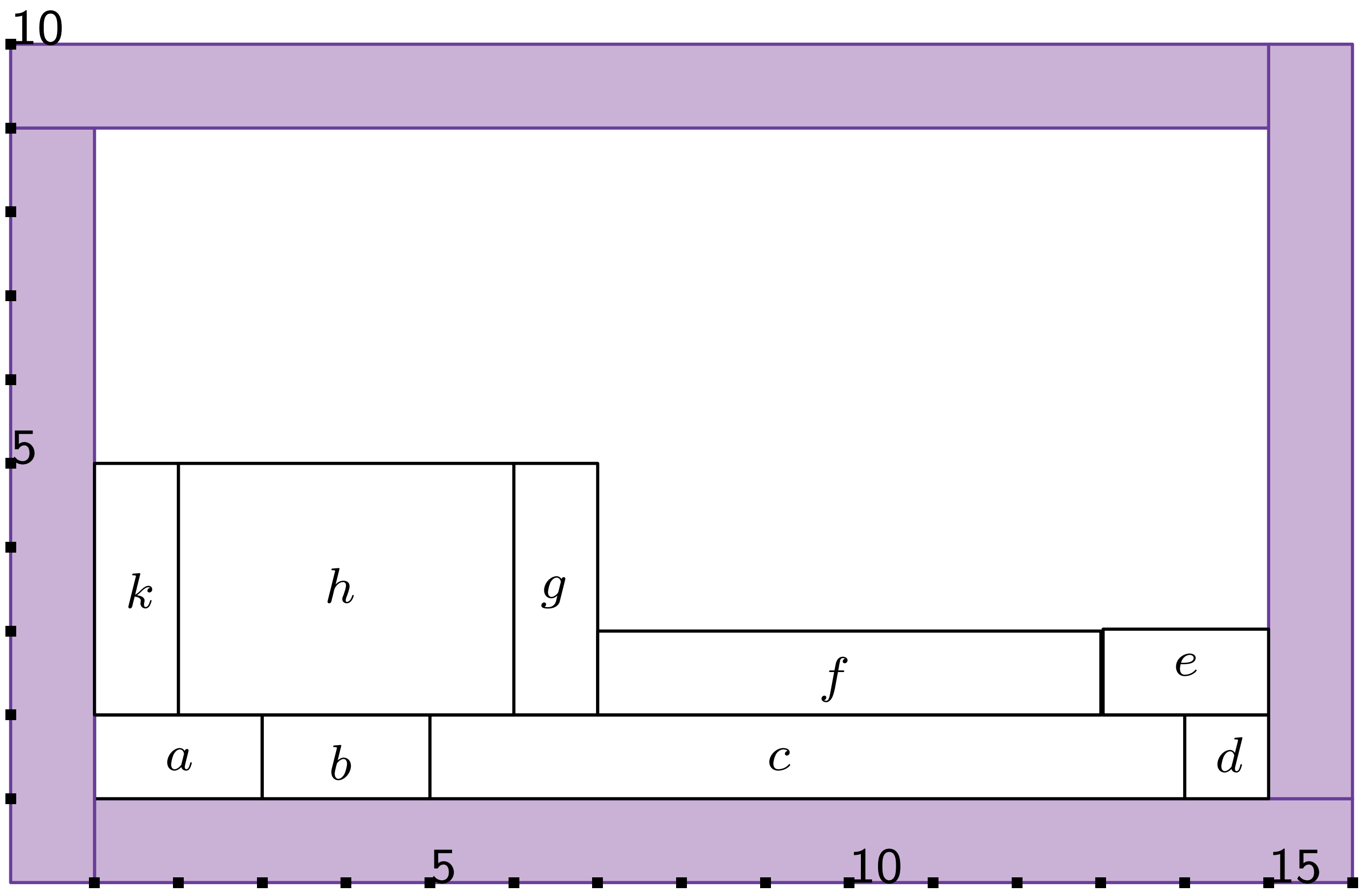
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
$\dots$

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
$\dots$

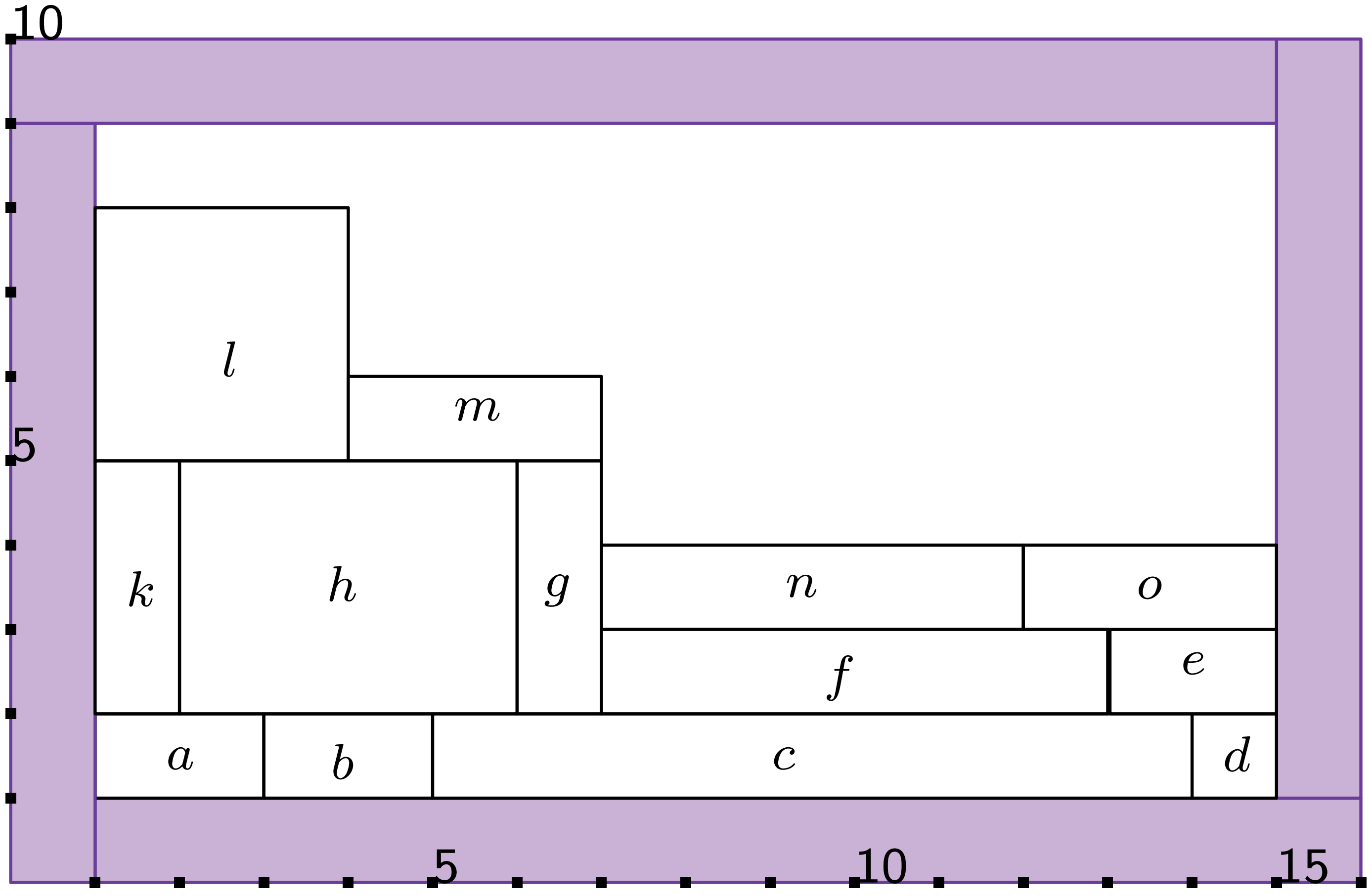
Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
$y_1(b) = 1,\ y_2(b) = 2$
...

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
$x_1(e) = 13,\ x_2(e) = 15$
...

$y_1(v_W) = 0, y_2(v_W) = 9$
$y_1(v_E) = 1,\ y_2(v_E) = 10$
$y_1(v_N) = 9,\ y_2(v_N) = 10$
$y_1(v_S) = 0,\ y_2(v_S) = 1$
$y_1(a) = 1,\ y_2(a) = 2$
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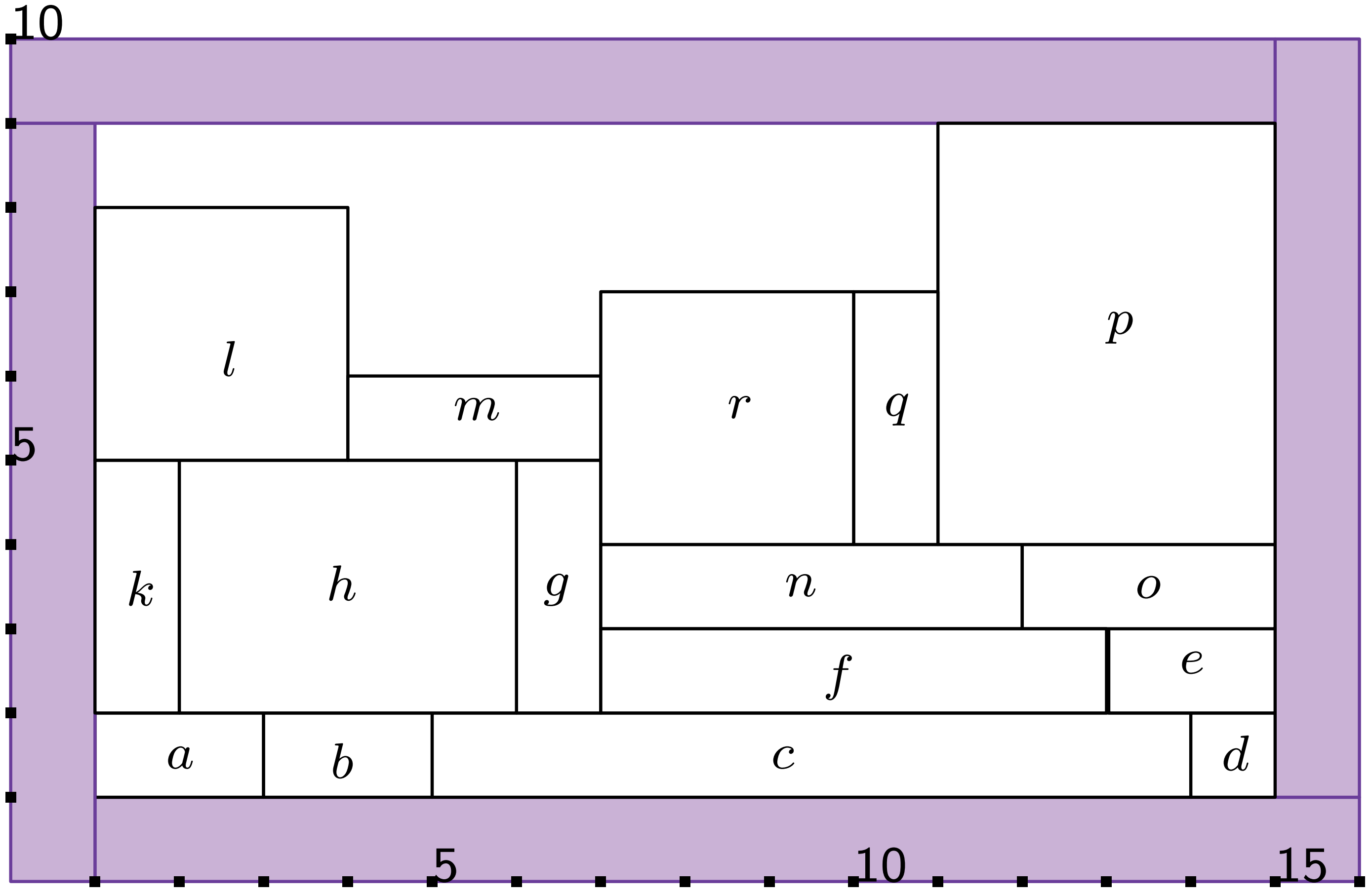
...

Reading off Coordinates to get Rectangular Dual



$x_1(v_N) = 1,\ x_2(v_N) = 15$
$x_1(v_S) = 2,\ x_2(v_S) = 16$
$x_1(v_W) = 0, x_2(v_W) = 1$
$x_1(v_E) = 15,\ x_2(v_E) = 16$
$x_1(a) = 1,\ x_2(a) = 3$
$x_1(b) = 3,\ x_2(b) = 5$
$x_1(c) = 5,\ x_2(c) = 14$
$x_1(d) = 14,\ x_2(d) = 15$
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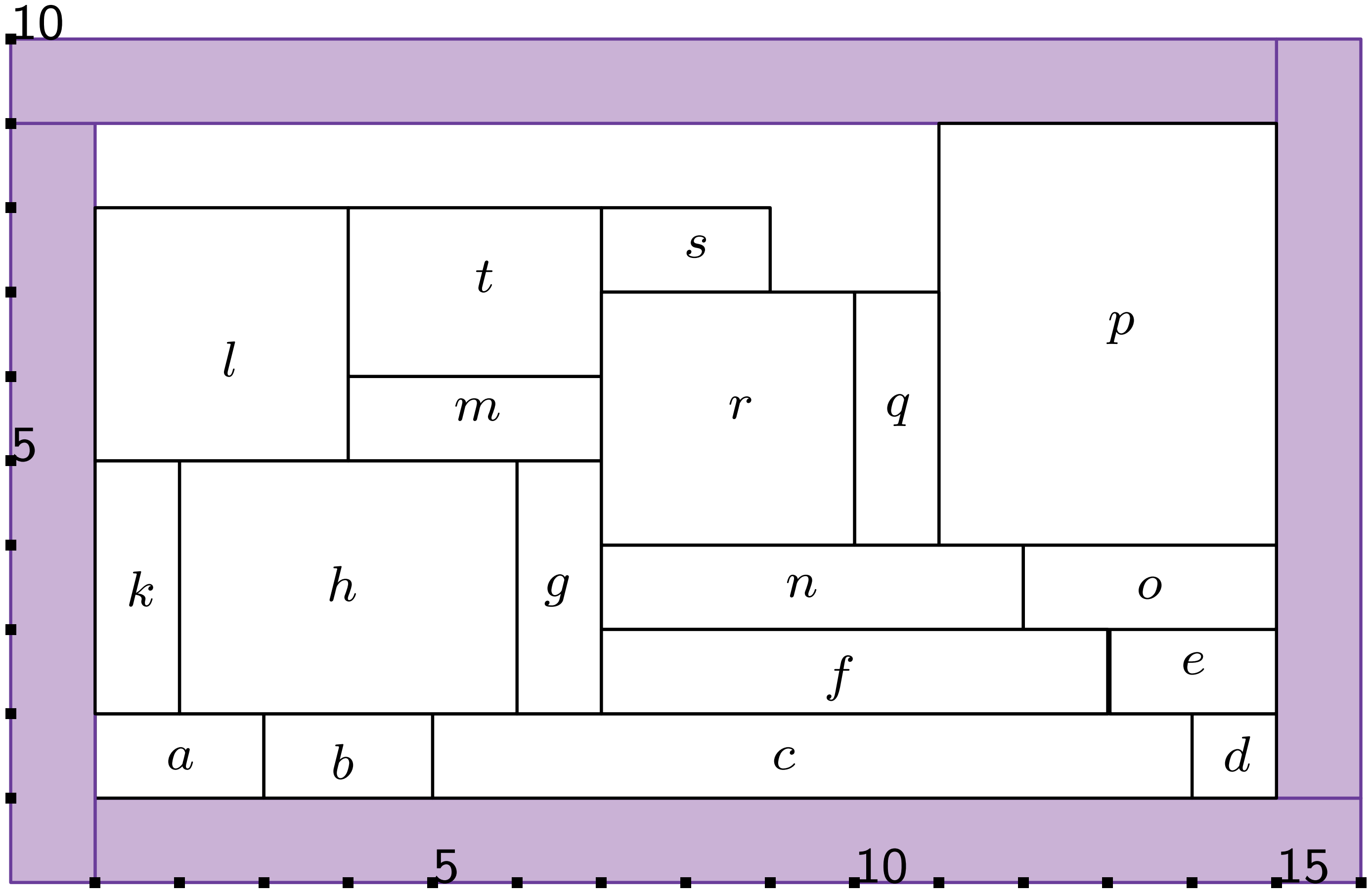

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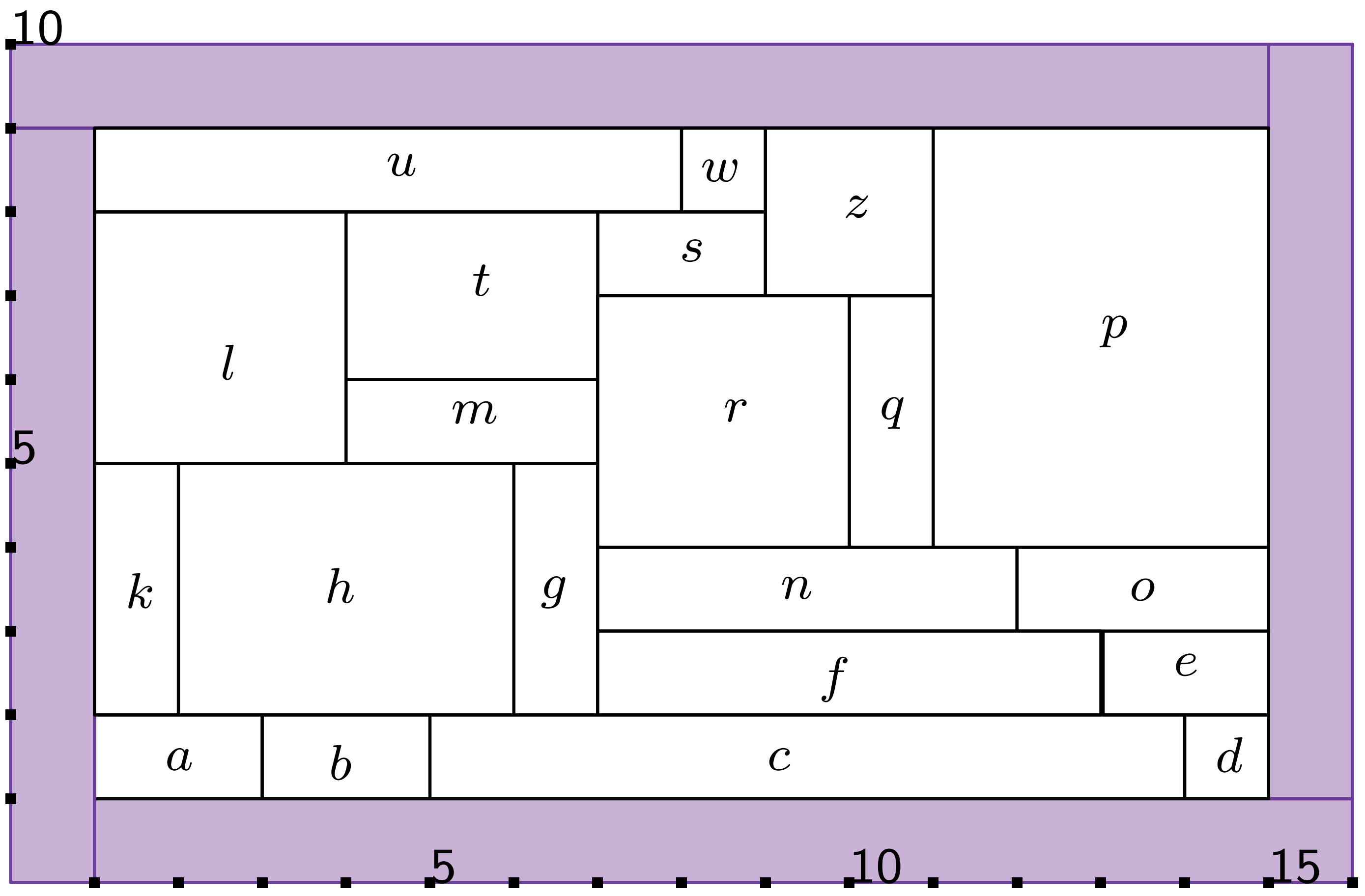

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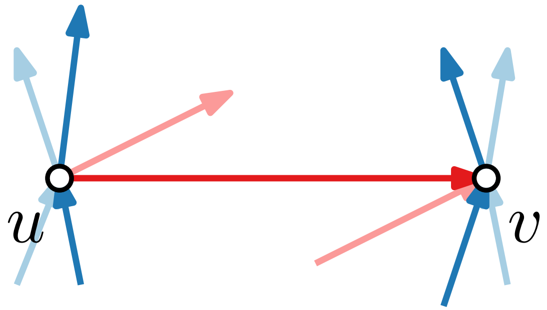

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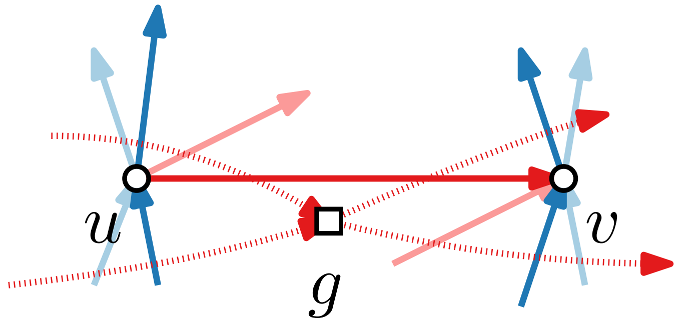

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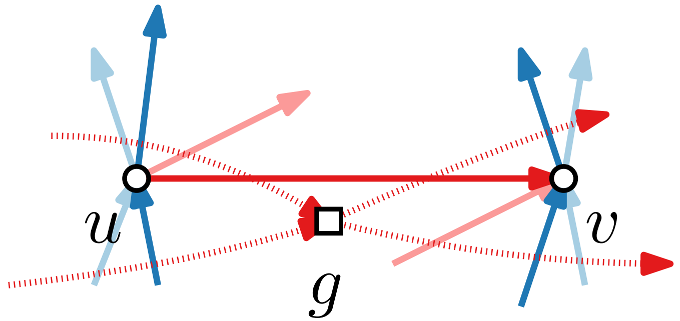

$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

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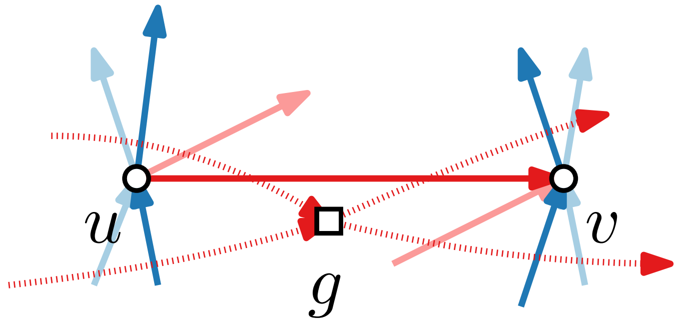

$$x_2(u) = f_{\text{ver}}(g) = x_1(v)$$

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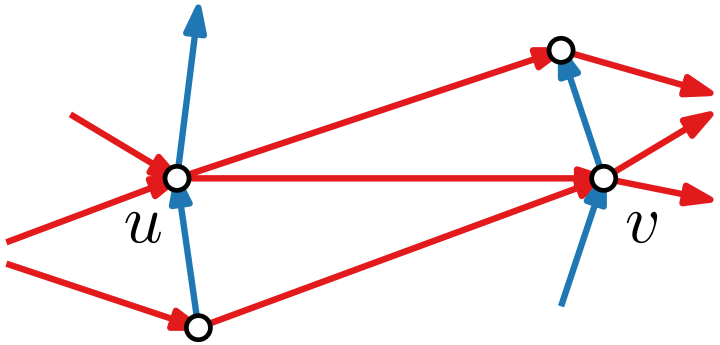

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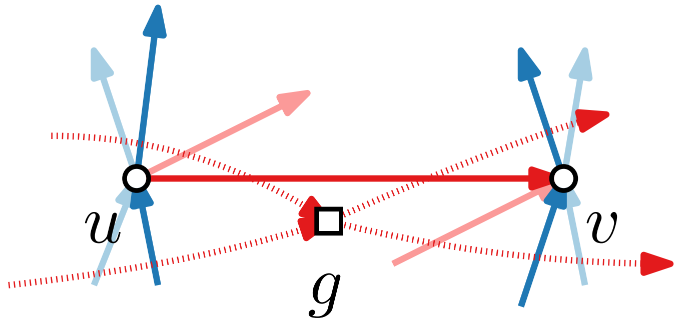

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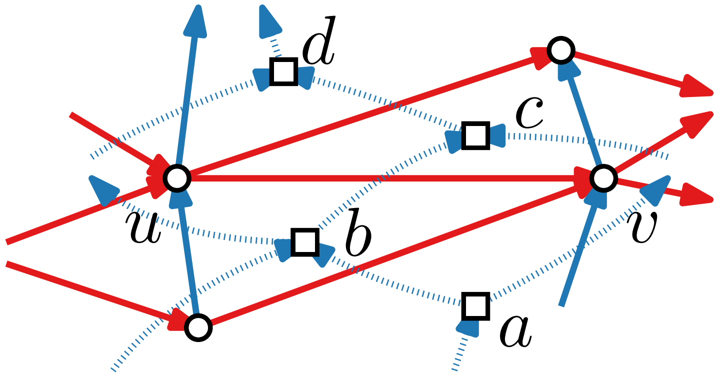

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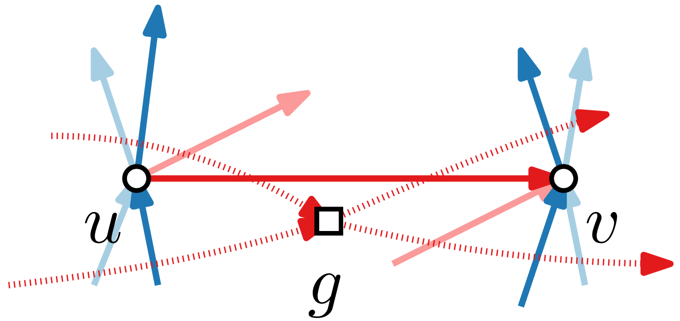

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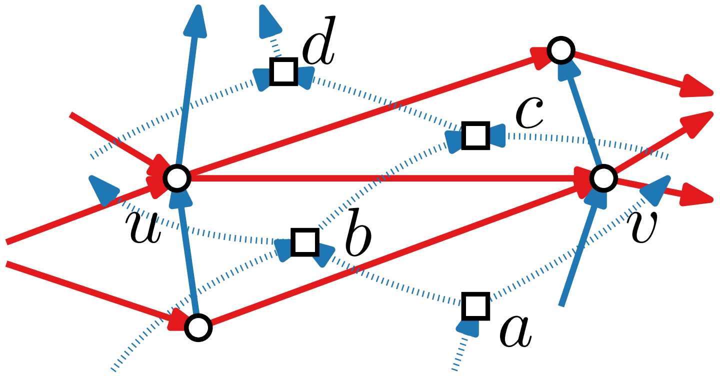

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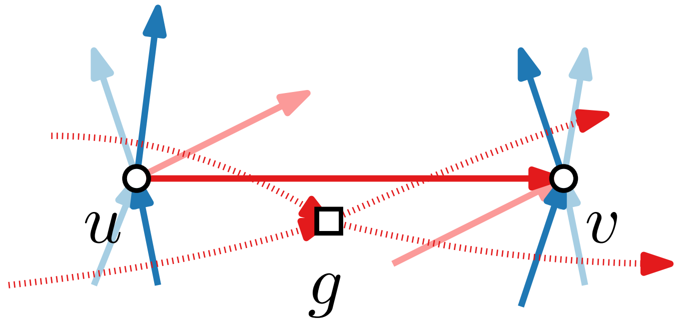

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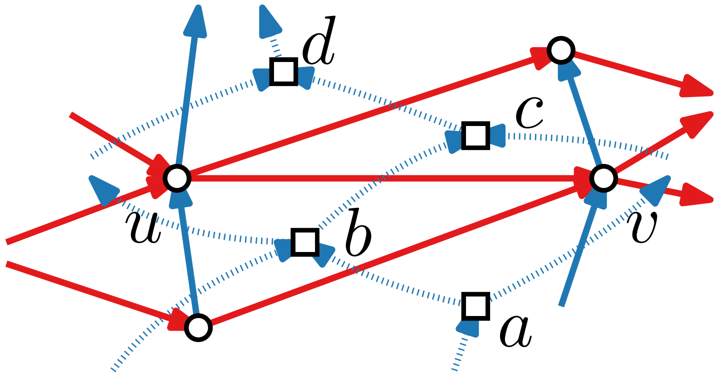

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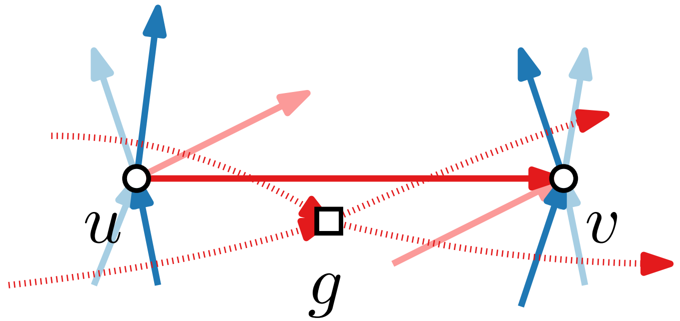

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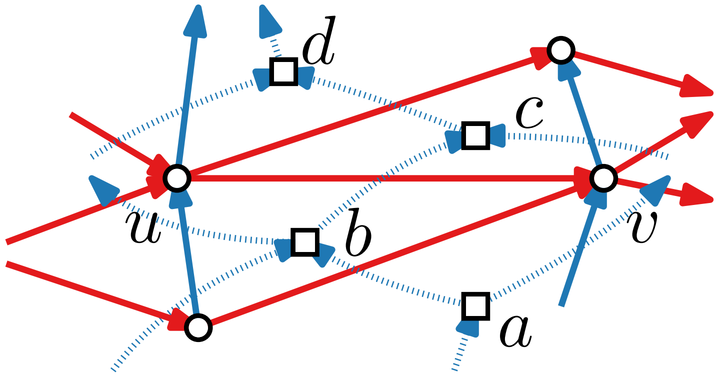

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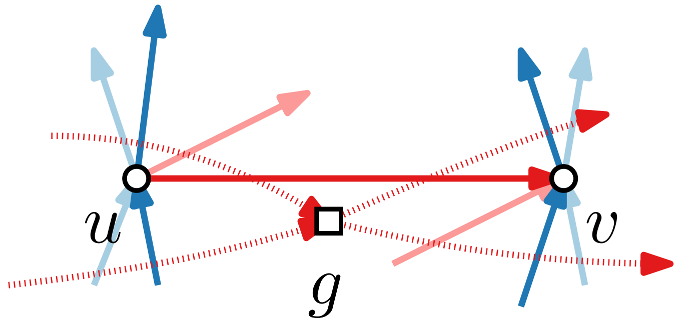

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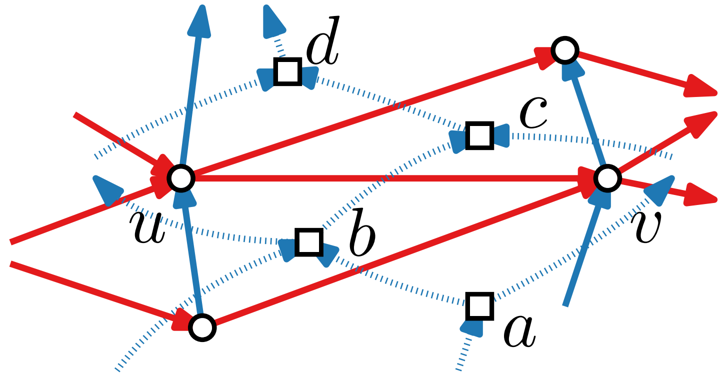

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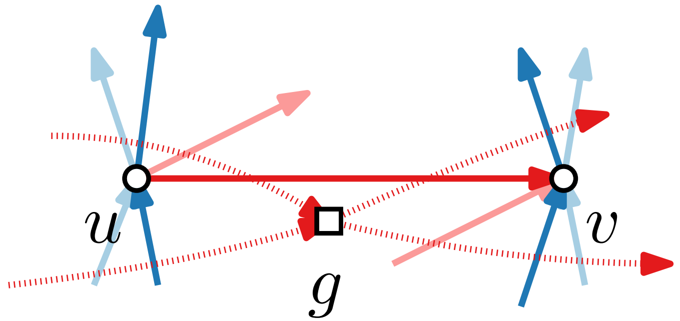

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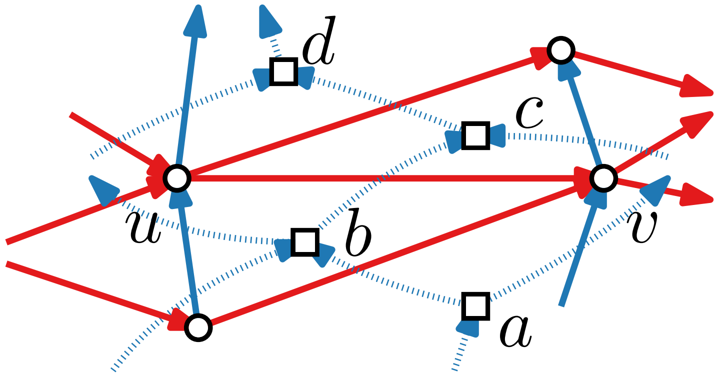

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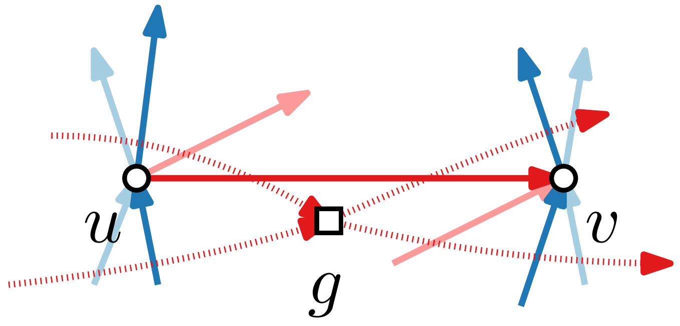

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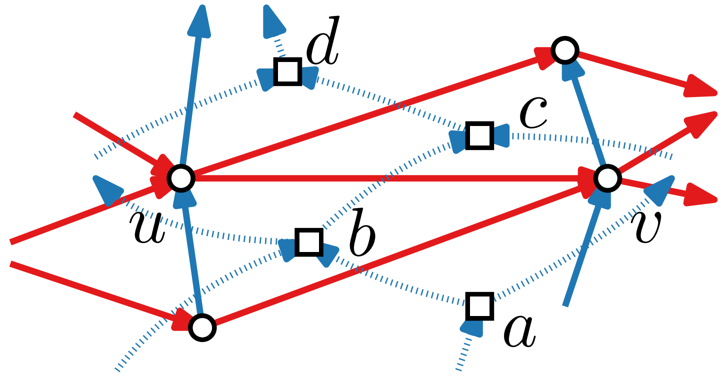

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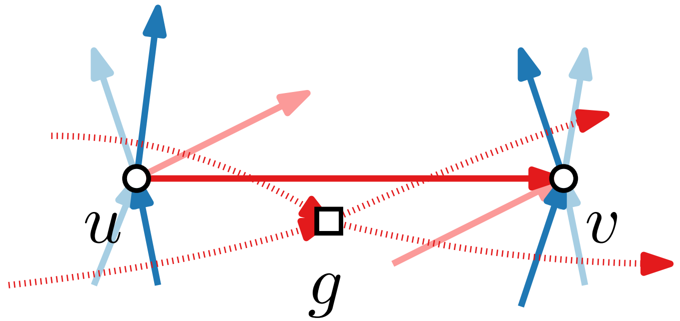

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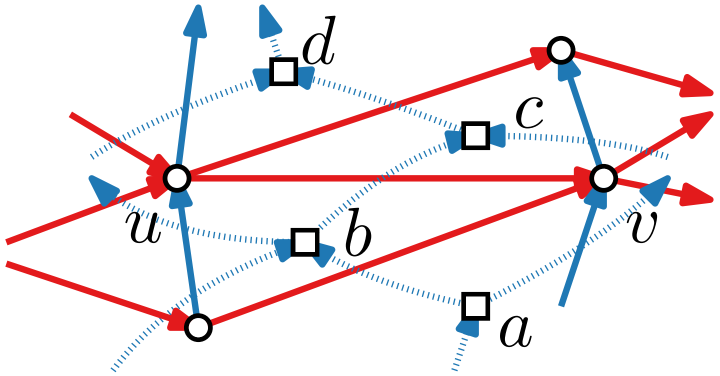

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- for details see He's paper [He '93]

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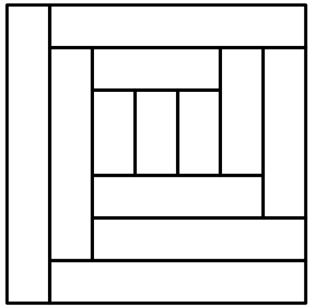

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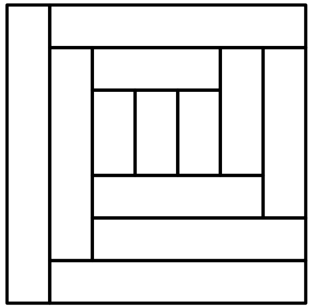

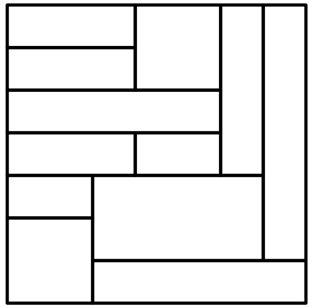

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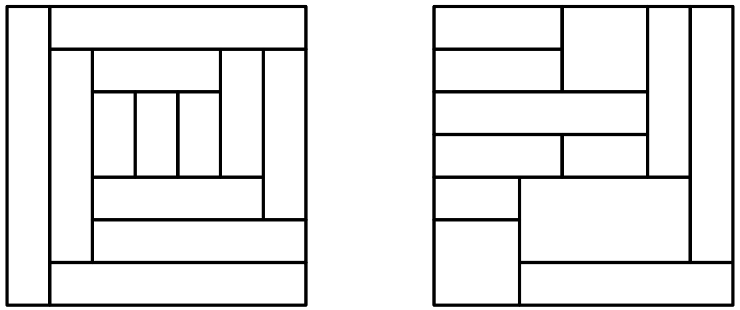

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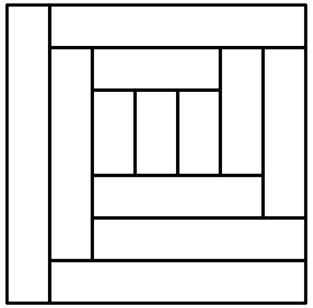

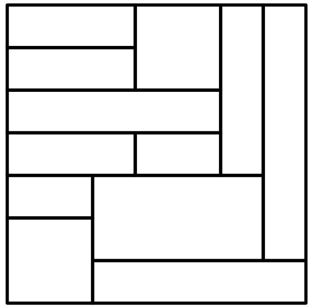

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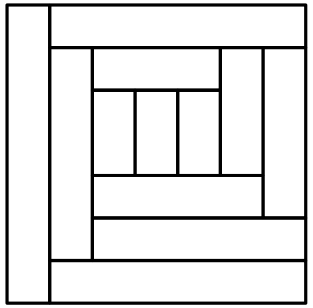
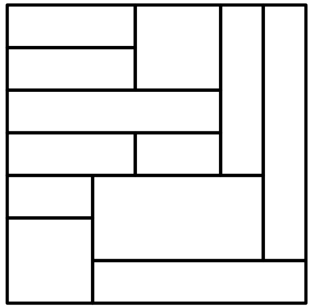
one-sided   not one-sided

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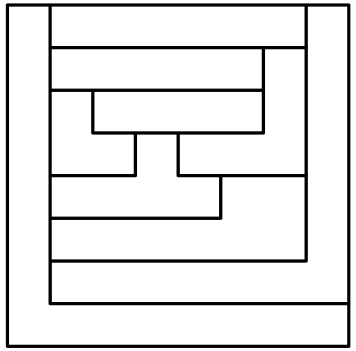

Literature

Construction of triangle contact representations based on

- [de Fraysseix, de Mendez, Rosenstiehl ’94] On Triangle Contact Graphs

Construction of rectangular dual based on

- [He ’93] On Finding the Rectangular Duals of Planar Triangulated Graphs
- [Kant, He ’94] Two algorithms for finding rectangular duals of planar graphs

and originally from

- [Koźmiński, Kinnen ’85] Rectangular Duals of Planar Graphs