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The Canadian Traveller Problem

Robust Optimization/Algorithms for Optimization under Uncertainty

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Structure of the paper

- “The canadian traveller problem.” by Amotz Bar-Noy and Baruch Schieber
- 4 different Topics regarding the Canadian Traveller Problem
  - The stochastic Recoverable-CTP
  - The deterministic Recoverable-CTP
  - The k-Canadian Traveller Problem
  - The k-Vital Edges Problem

But what is the Canadian Traveller Problem?

Motivation: The Canadian Traveller Problem

- Given a roadmap
- Some roads might be unsuitable for travel at certain times
- Blockage is only revealed upon reaching an adjacent site

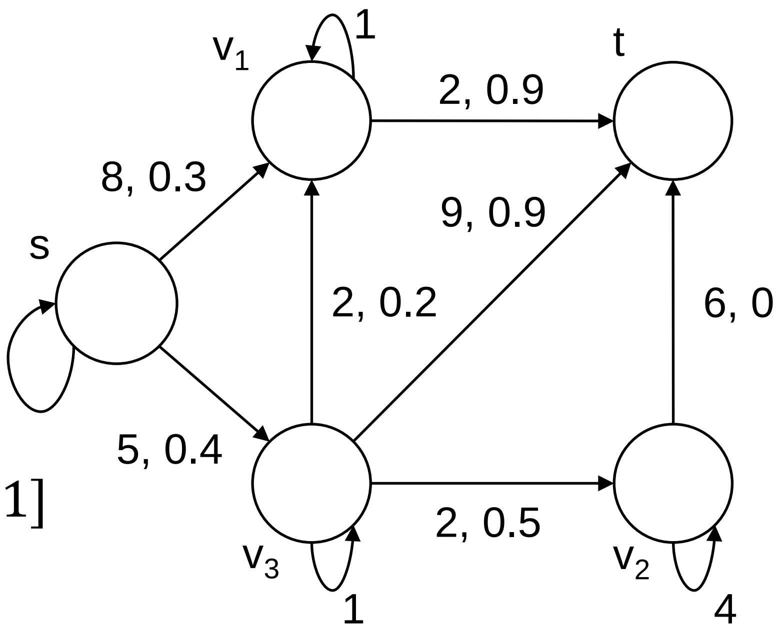
Structure of the paper

- 4 different Topics regarding the Canadian Traveller Problem
  - The stochastic Recoverable-CTP
  - The deterministic Recoverable-CTP
  - The k-Canadian Traveller Problem
  - The k-Vital Edges Problem

THE STOCHASTIC RECOVERABLE-CTP

The stochastic Recoverable-CTP

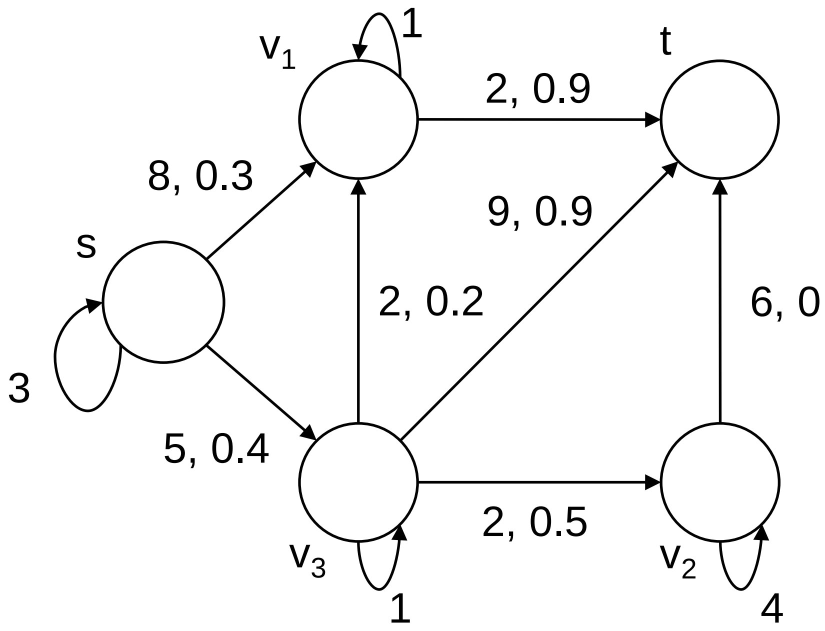
- Acyclic, directed graph G
- Consisting of $v_i \in V$ and $e_i \in E$
- cost function $c: E \rightarrow \mathbb{N}$
- $\forall v \in V, \exists\{v,v\}\ such\ that\ c: c(\{v,v\}) < c(\{v,x\})\ \forall x \in adj(v)$
- Blockage probability $p: E \rightarrow [0;1[$, counter probability q: $E \rightarrow]0;1]$
- Expected travel time from $x \in V$ to t: E(x)



Find: shortest expected travel time from chosen $s \in V$ to t $\in V$

The stochastic Recoverable-CTP

Find: shortest expected travel time from chosen $s \in V$ to $\mathrm{t} \in V$



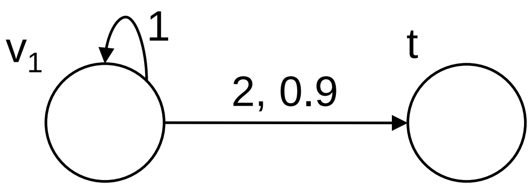
The stochastic Recoverable-CTP

Algorithm:

- Set L of labeled nodes
- For each $x \in L$, its priority list and E(x) are known
- Initially, $E(t) = 0$ and $E(x) = \infty \; \forall x \in V - \{t\}$
- Iterate:
  - Labeling step: find node x with minimum E(x) and add it to L
  - Updating step: $\forall y \in Adj(x) - L$: check if inserting edge {y, x} to y's priority list improves E(y)

The stochastic Recoverable-CTP

But how to calculate E(x)?



The stochastic Recoverable-CTP

	probability	cost to neighbour	waiting cost	weighted cost	total summed up
1	0.1	2	0	0.2	0.2
2	0.09	2	1	0.27	0.47
3	0.081	2	2	0.324	0.794
4	0.0729	2	3	0.3645	1.1585
5	0.06561	2	4	0.39366	1.55216
6	0.059049	2	5	0.413343	1.965503
7	0.0531441	2	6	0.4251528	2.3906558
8	0.04782969	2	7	0.43046721	2.82112301
9	0.043046721	2	8	0.43046721	3.25159022
10	0.038742049	2	9	0.426162538	3.677752758
11	0.034867844	2	10	0.418414128	4.096166886
12	0.03138106	2	11	0.407953775	4.504120661
13	0.028242954	2	12	0.395401351	4.899522012
14	0.025418658	2	13	0.381279874	5.280801886
15	0.022876792	2	14	0.366028679	5.646830566
16	0.020589113	2	15	0.350014925	5.99684549
17	0.018530202	2	16	0.333543634	6.330389124
18	0.016677182	2	17	0.316866452	6.647255576
19	0.015009464	2	18	0.300189271	6.947444847

The stochastic Recoverable-CTP

$$\sum_{i=0}^{\infty} p^i = \frac{1}{1-p}; \sum_{i=0}^{\infty} p^i * i = \frac{p}{(1-p)^2}$$

$$\sum_{i=0}^{\infty} p^i * q(c(v_1, t) + i * c(v_1, v_1))$$

$$\sum_{i=0}^{\infty} p^i (q * c(v_1, t) + q * i * c(v_1, v_1))$$

$$\sum_{i=0}^{\infty} p^i * q * c(v_1, t) + p^i * q * i * c(v_1, v_1)$$

$$q * c(v_1, t) * \sum_{i=0}^{\infty} p^i + q * c(v_1, v_1) * \sum_{i=0}^{\infty} p^i * i$$

$$\frac{q * c(v_1, t)}{1-p} + \frac{q * c(v_1, v_1) * p}{(1-p)^2} = \frac{q * c(v_1, t) + p * c(v_1, v_1)}{1-p}$$

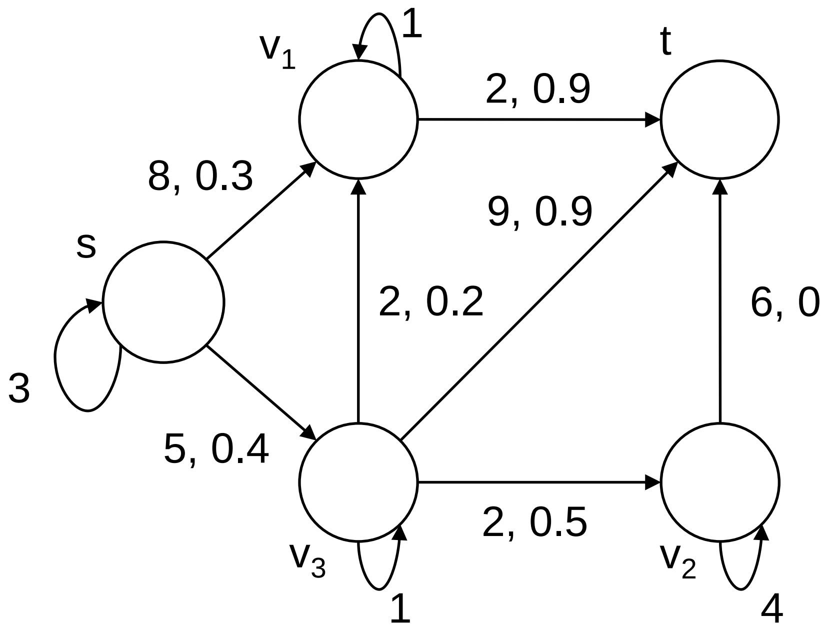
The stochastic Recoverable-CTP

- Iterate:
  - Labeling step: find node x with minimum E(x) and add it to L
  - Updating step: $\forall y \in Adj(x) \ - L$: check if inserting edge {y, x} to y's priority list improves E(y)

Given k neighbors of y with known expected travel time:

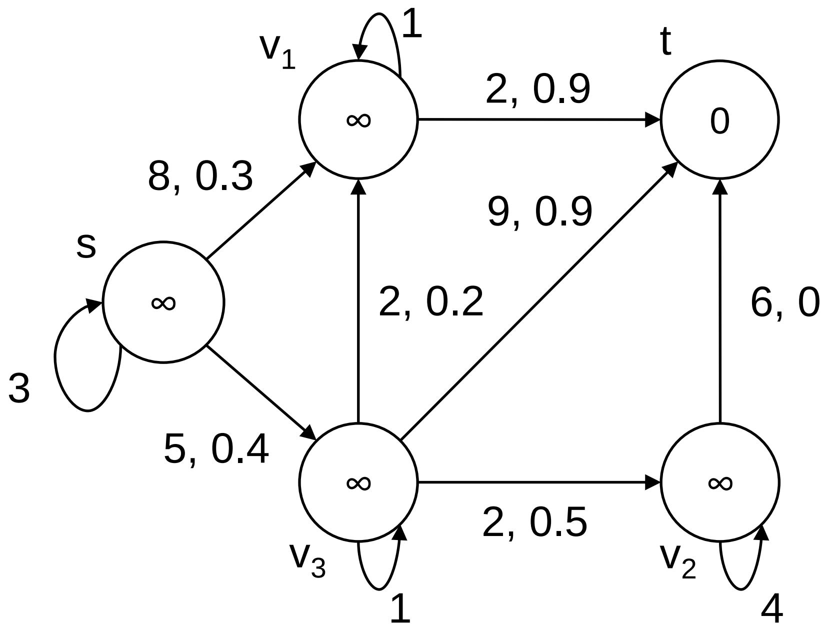
- $E(y) = \frac{\alpha_h + P_h c(y,y)}{1 - P_h}$
- $\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j (c(y, y_j) + E(y_j))$
- $P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$
- Chose $1 \leq h \leq k$ to minimize E(y)
- Minima is reached with h for which $\frac{a_h + c(y,y)}{1 - P_h} < c(y, y_{h+1}) + E(y_{h+1})$

The stochastic Recoverable-CTP



The stochastic Recoverable-CTP

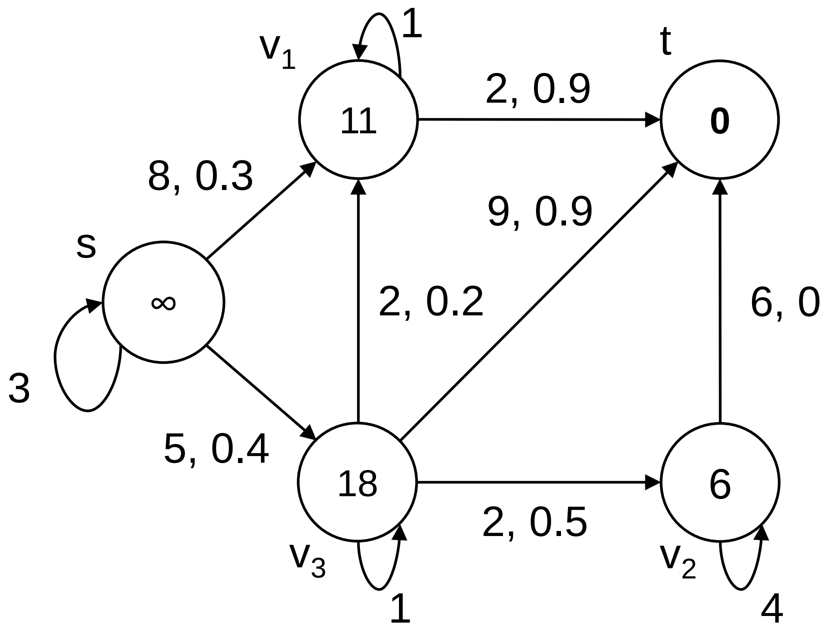
$L = \{\}$



The stochastic Recoverable-CTP

	Priority List
t	**{}**
v_1	{t}
v_2	{t}
v_3	{t}

$$E(v_1) = \frac{0.2 + 0.9 * 1}{1 - 0.9} \quad \alpha_1 = 1 * 0.1(2 + 0) \qquad L = \{t\}$$



$$E(v_2) = \frac{(1 * 1 * (6 + 0)) + 0 * 4}{1 - 0}$$

$$E(x) = \frac{\alpha_h + P_h c(x, x)}{1 - P_h}$$

$$\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j (c(x, x_j) + E(x_j))$$

$P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$

The stochastic Recoverable-CTP

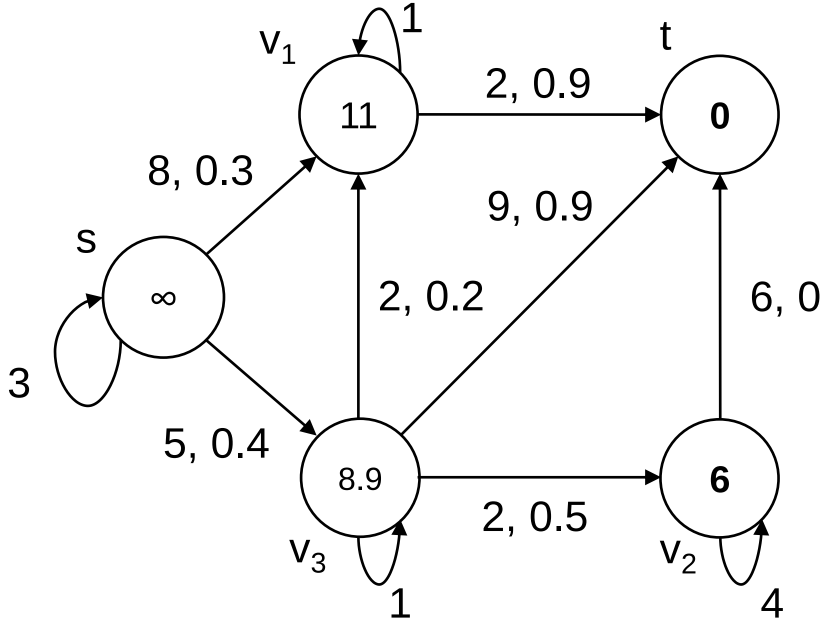
	Priority List
t	**{}**
v_1	{t}
$\mathbf{v_2}$	**{t}**
v_3	$\{v_2$, t}

$L = \{t, v_2\}$

$$\frac{a_h + c(x,x)}{1 - P_h} < c(x, x_{h+1}) + E(x_{h+1})$$

$$\alpha_1 = 1 * 0.5(2 + 6)$$

$$\frac{4+1}{1-0.5} < 9 + 0$$



$$E(x) = \frac{\alpha_h + P_h c(x,x)}{1 - P_h}$$

$$\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j \left(c(x, x_j) + E(x_j)\right)$$

$P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$

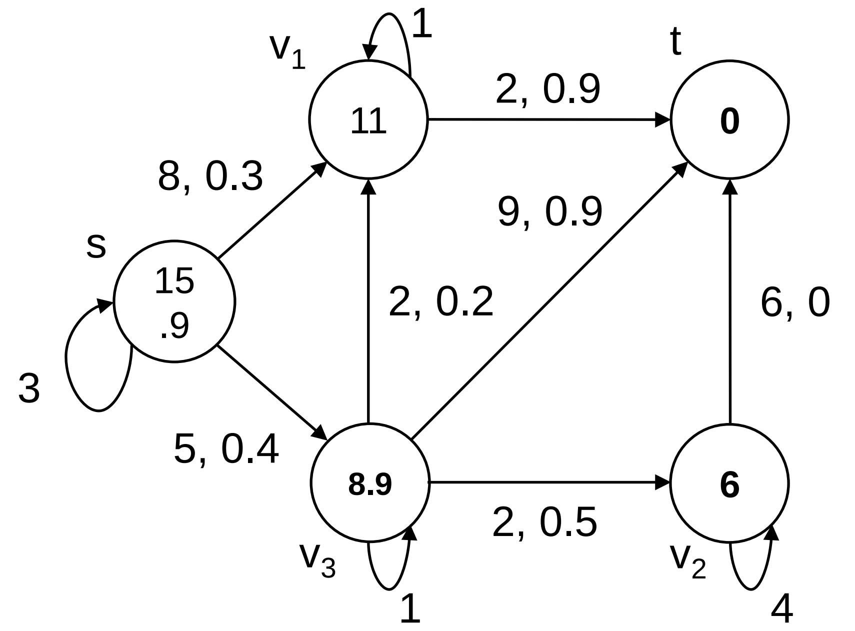
$$E(v_3) = \frac{4.45 + 0.45 * 1}{1 - 0.45}$$

$$\alpha_2 = \alpha_1 + 0.5 * 0.1 * (9 + 0)$$

The stochastic Recoverable-CTP

	Priority List
t	**{}**
v_1	{t}
v_2	**{t}**
v_3	**{v_2, t}**
s	{v_3}

$L = \{t, v_2, v_3\}$



$$E(x) = \frac{\alpha_h + P_h c(x,x)}{1 - P_h}$$

$$\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j \left(c(x, x_j) + E(x_j)\right)$$

$P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$

$$E(s) = \frac{8.34 + 0.4 * 3}{1 - 0.4} \qquad \alpha_1 = 1 * 0.6 * (5 + 8.9)$$

The stochastic Recoverable-CTP

	Priority List
t	{}
v_1	{t}
v_2	{t}
v_3	{v_2, t}
s	{v_3}

$$\frac{\alpha_h + c(x,x)}{1-P_h} < c(x, x_{h+1}) + E(x_{h+1})$$

$$\alpha_1 = 1 * 0.6(5 + 8.9)$$

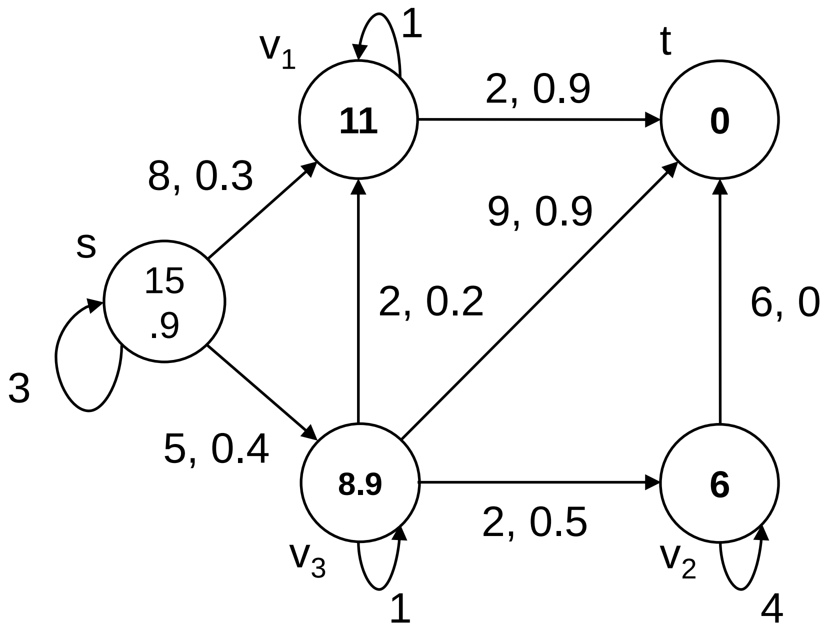
$$\frac{8{,}34 + 3}{1 - 0.4} = 18.9 < 8 + 11$$

$$E(x) = \frac{\alpha_h + P_h c(x,x)}{1 - P_h}$$

$$\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j (c(x, x_j) + E(x_j))$$

$P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$

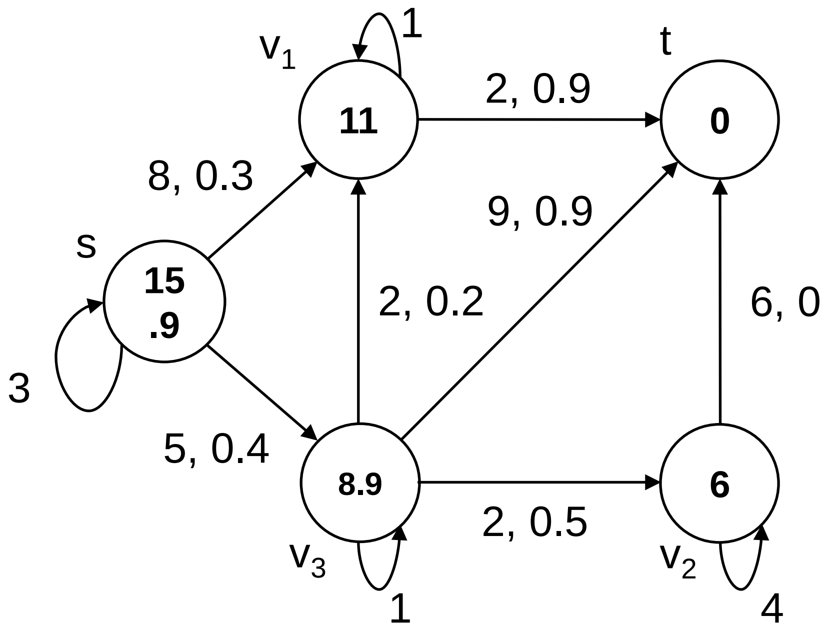
$$L = \{t, v_2, v_3, v_1\}$$



The stochastic Recoverable-CTP

	Priority List
t	{}
v_1	{t}
v_2	{t}
v_3	{v_2, t}
s	{v_3}

$L = \{t, v_2, v_3, v_1, s\}$



$$E(x) = \frac{\alpha_h + P_h c(x,x)}{1 - P_h}$$

$$\alpha_i = \sum_{j=1}^{i} P_{j-1} q_j (c(x, x_j) + E(x_j))$$

$P_i = \prod_{j=1}^{i} p_j$, for I = 1, …, k with $P_0 = 1$

→ **Expected travel time from s to t = 15.9**
→ **Policy does not "contain" the shortest deterministic path**
→ **Complexity: $O(|E| \, log \, |V|)$**

Sources

- Bar-Noy, Amotz, and Baruch Schieber. "The canadian traveller problem." *Proceedings of the second annual ACM-SIAM symposium on Discrete algorithms*. 1991.
- https://www.autentic.com/68/pid/216/Highway-to-the-Arctic-%E2%80%93-Canadas-Ice-Roads.htm
- https://en.wikipedia.org/wiki/Geometric_series
- https://se.informatik.uni-wuerzburg.de/software-engineering-group/teaching/templates/