Recoverable Robust Shortest Path Problems

Paper-Autorin: Christina Büsing

Seminar: Robust Optimization/Algorithms for Optimization under Uncertainty (SS24)

Viktor Charin, 22.05.24

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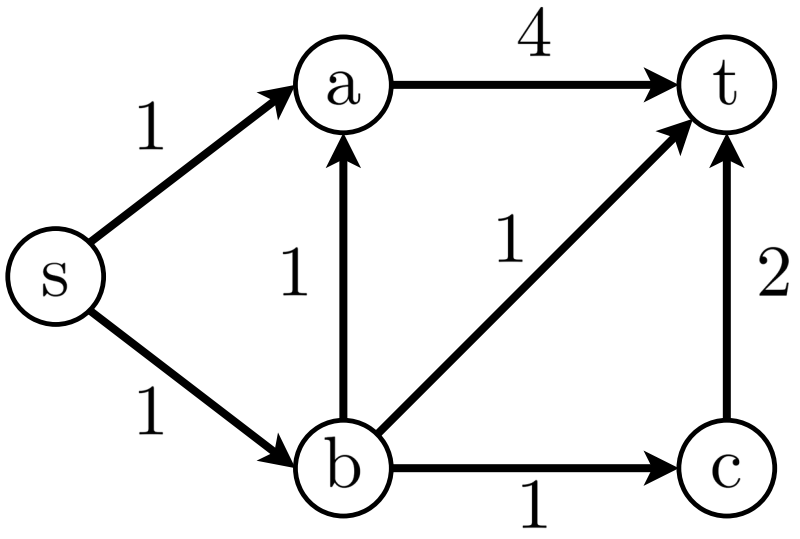
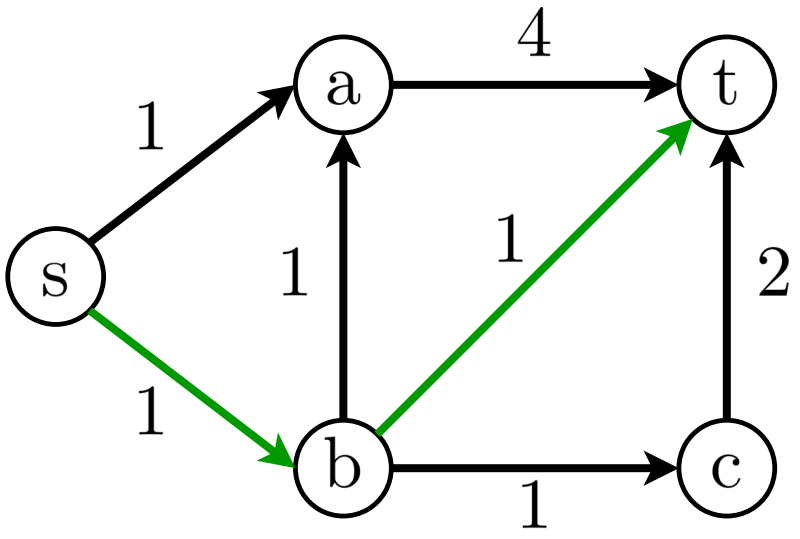
Wiederholung: Robust Shortest Path Problem

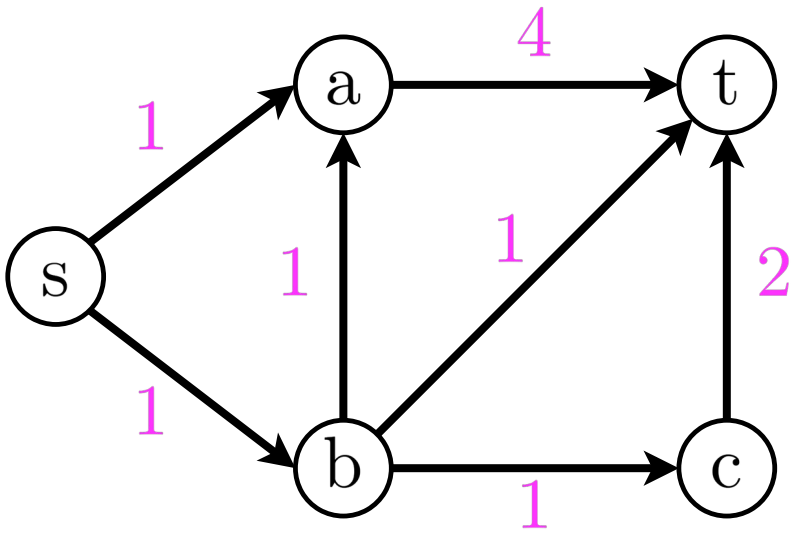


Wiederholung: Robust Shortest Path Problem



Dijkstra

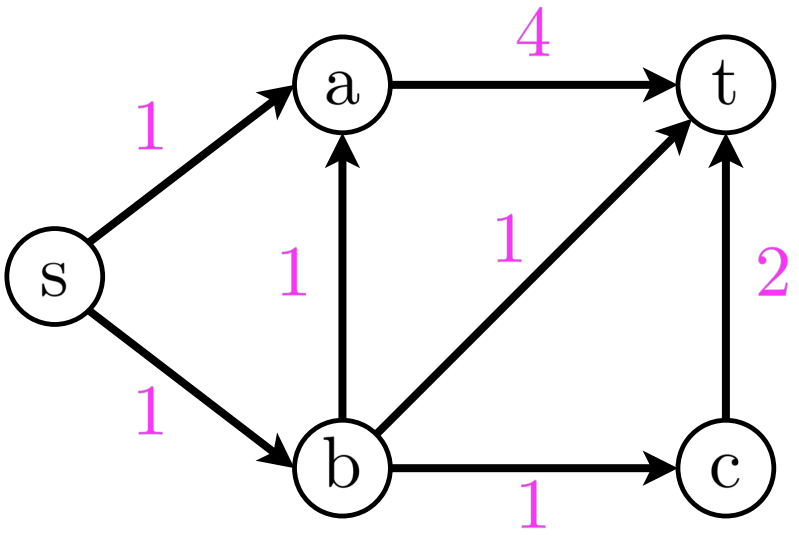
Wiederholung: Robust Shortest Path Problem



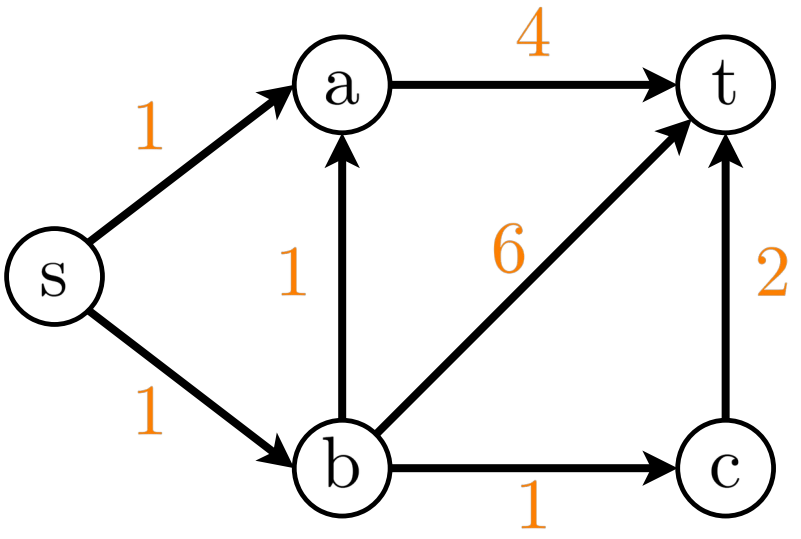
Szenario s_1

$r: E \to \mathbb{N}$

Wiederholung: Robust Shortest Path Problem

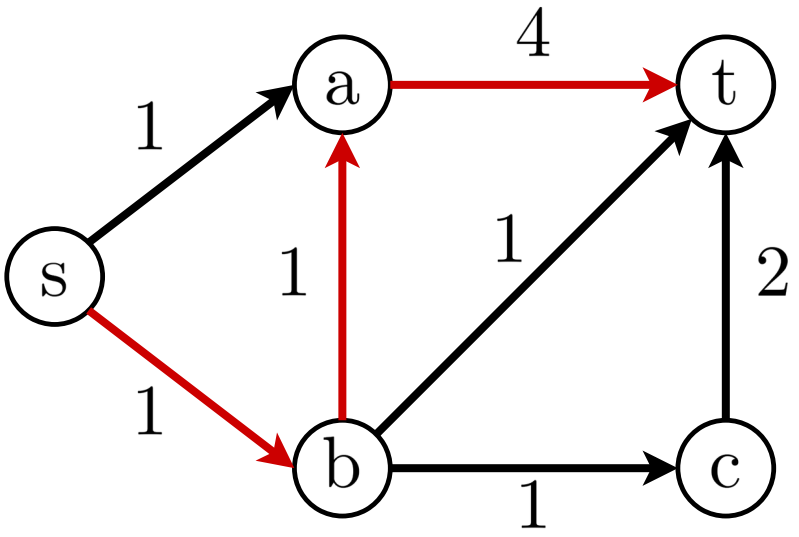


Szenario s_1
$r: E \to \mathbb{N}$

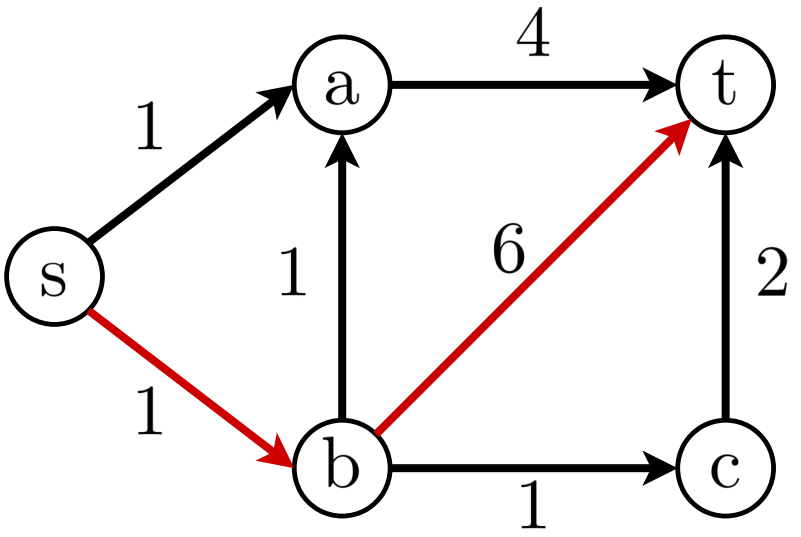


Szenario s_2
$r: E \to \mathbb{N}$

Wiederholung: Robust Shortest Path Problem

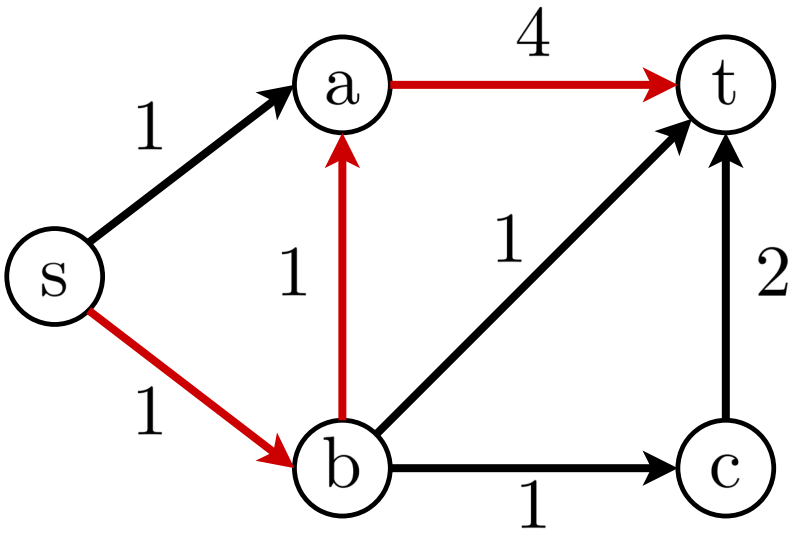


Worst-Case: 6 (s_1)

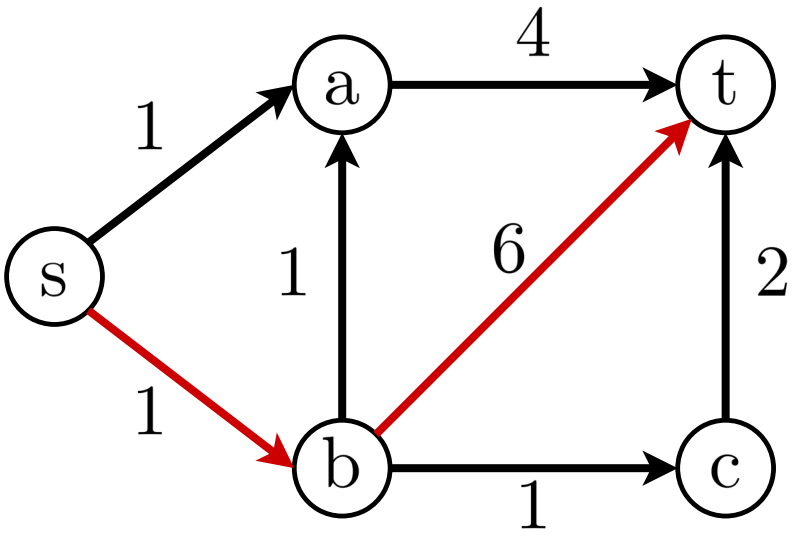


Worst-Case: 7 (s_2)

Wiederholung: Robust Shortest Path Problem



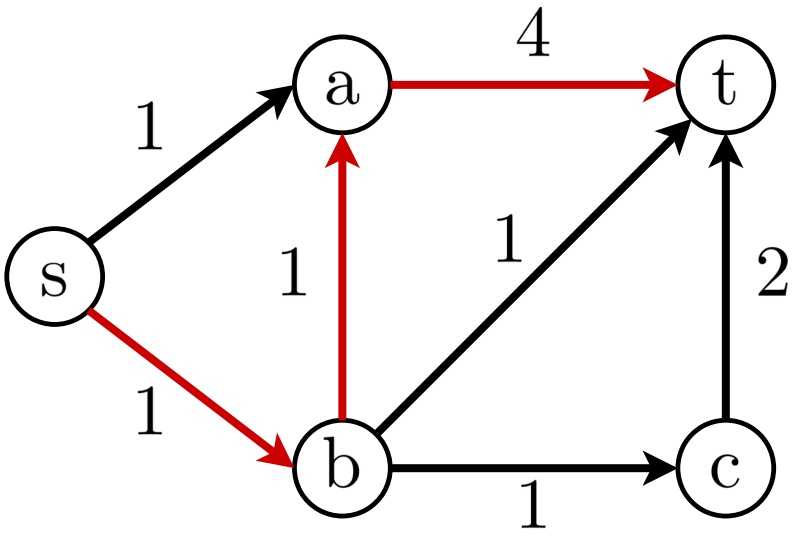
Worst-Case: 6 (s_1)

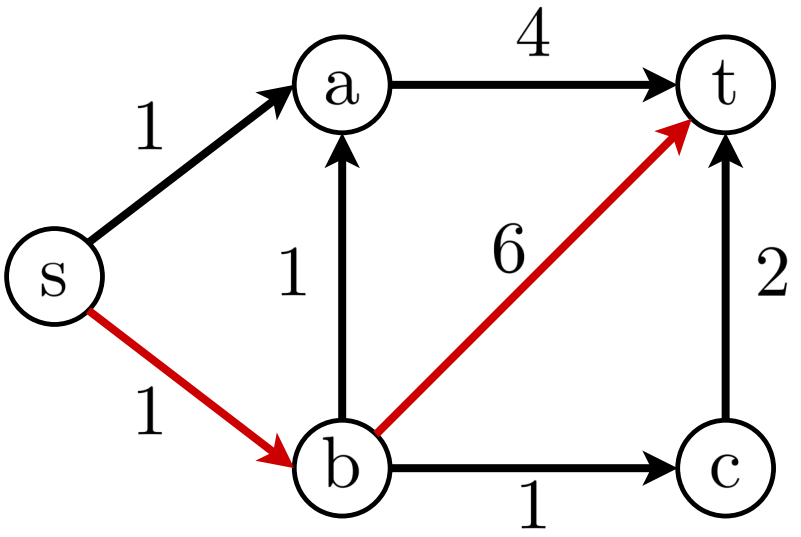


Worst-Case: 7 (s_2)

min (ARSP)

Wiederholung: Robust Shortest Path Problem





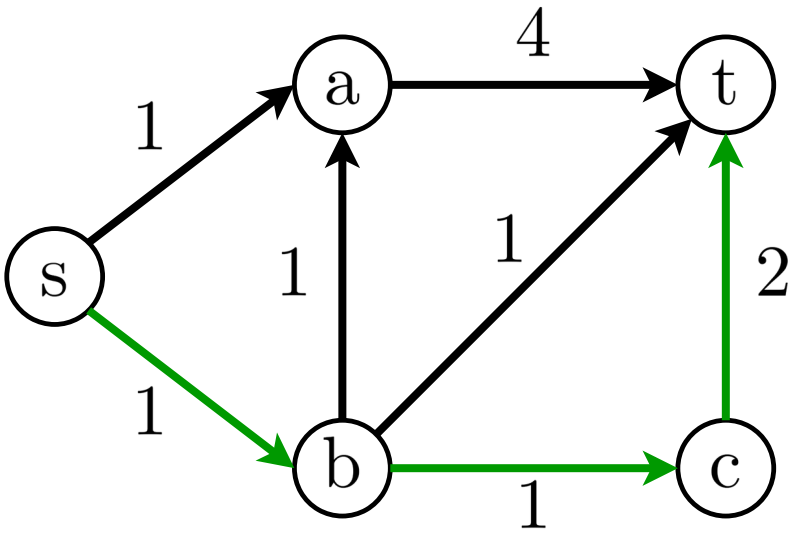
Worst-Case: 6 (s_1)

Worst-Case: 7 (s_2)

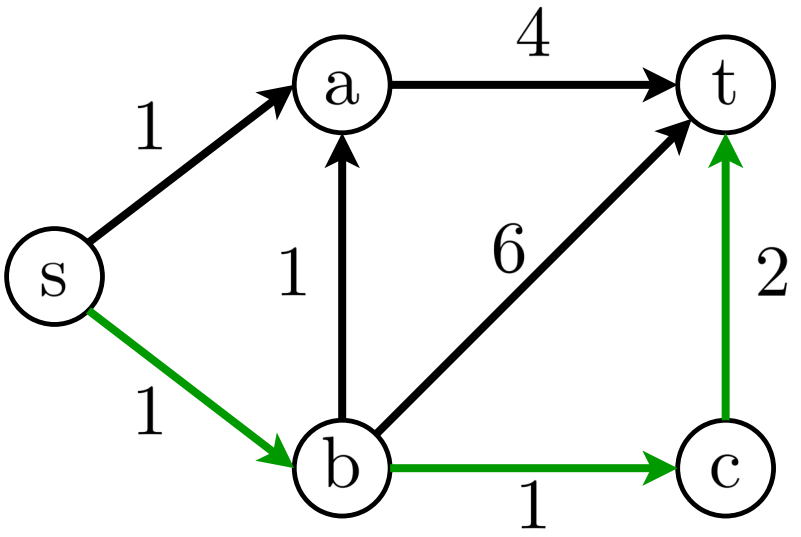
min (ARSP)

PROBLEM?

Wiederholung: Robust Shortest Path Problem



Worst-Case: 6 (s_1)



Worst-Case: 7 (s_2)

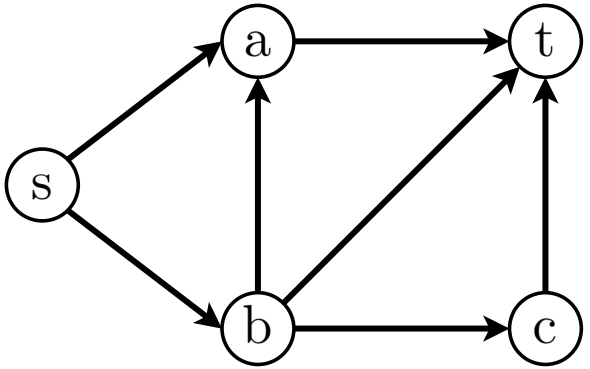
Pfad zu konservativ!

Erweiterung: Recoverable Robust

Wir wollen robuste Lösungen (d. h. Lösungen, die im Vorfeld festgelegt werden und unter allen Szenarien realisierbar sind)!

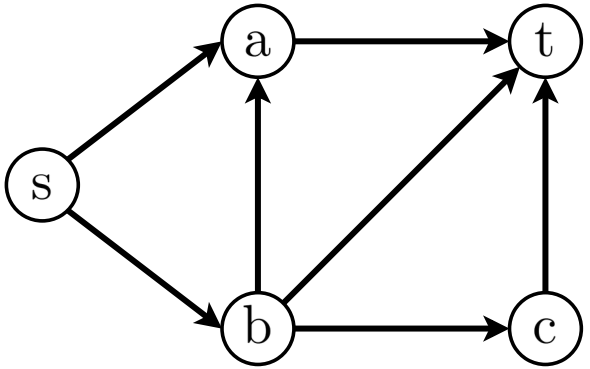
Idee Recoverable: Nachdem ein Szenario realisiert wird (exakte Daten bekannt), erlaube dem Problem sich zu erholen (vorher festgelegter Plan wird minimal an das Szenario angepasst).

k-dist-RR Shortest Path

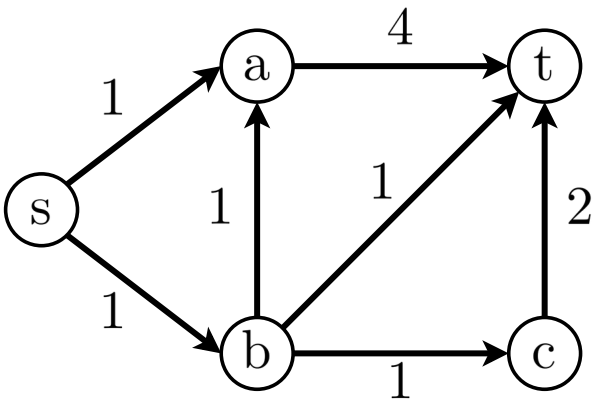


$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases}$$

k-dist-RR Shortest Path

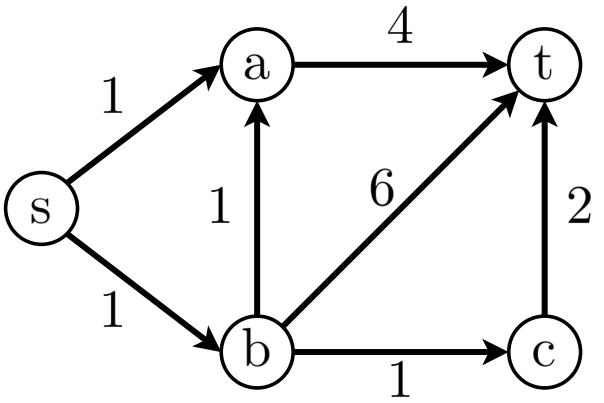


s_1

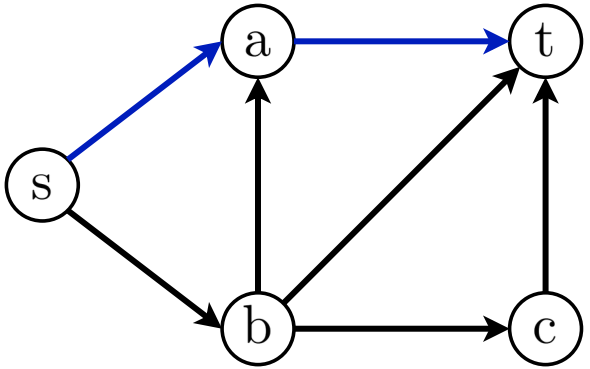


$$\Pi = \begin{array}{c|cc} & s_1 & s_2 \\ \hline \text{sat} & & \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{array}$$

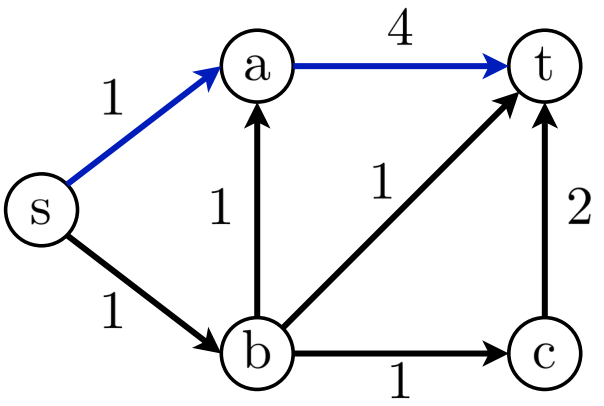
s_2



k-dist-RR Shortest Path

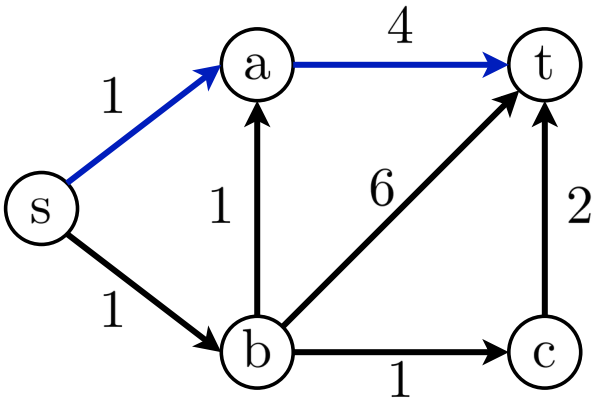


s_1

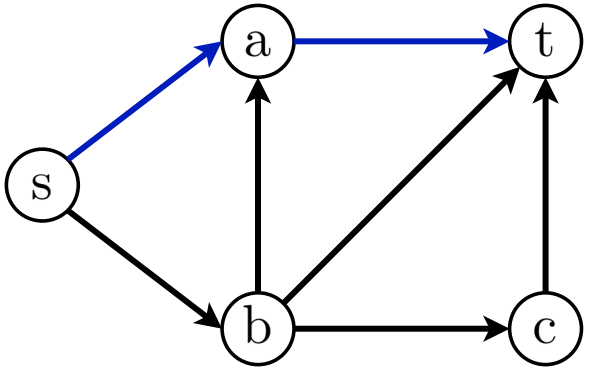


$$\Pi = \begin{array}{c|cc} & s_1 & s_2 \\ \hline \text{sat} & & \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{array}$$

s_2



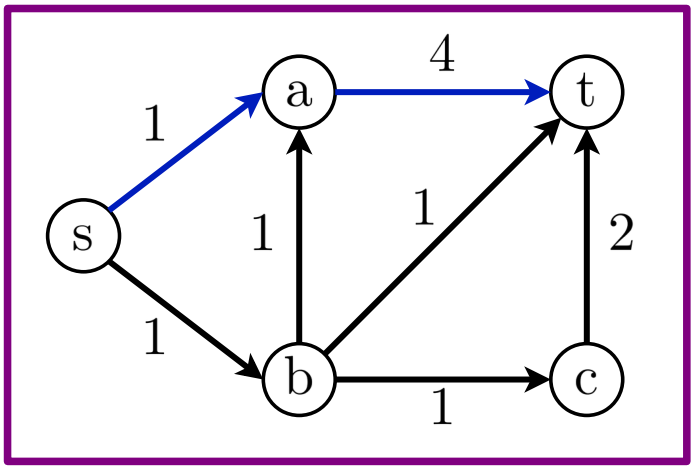
k-dist-RR Shortest Path



$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases}$$

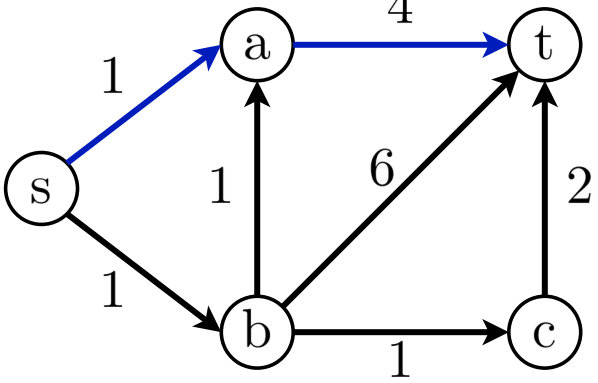
	s_1	s_2
sat		
sbat		
sbt		
sbct		

s_1

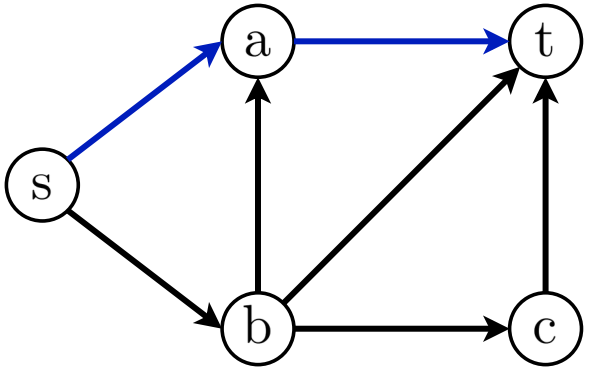


$k \leq 1$

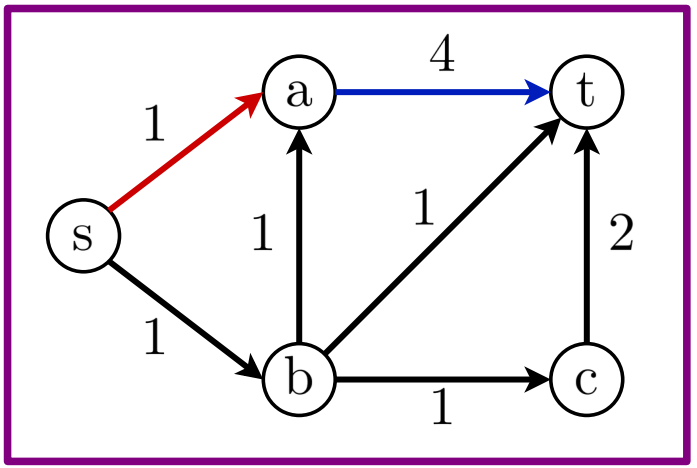
s_2



k-dist-RR Shortest Path

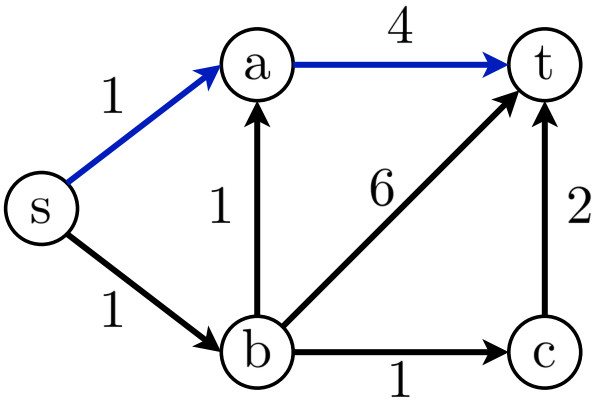


s_1



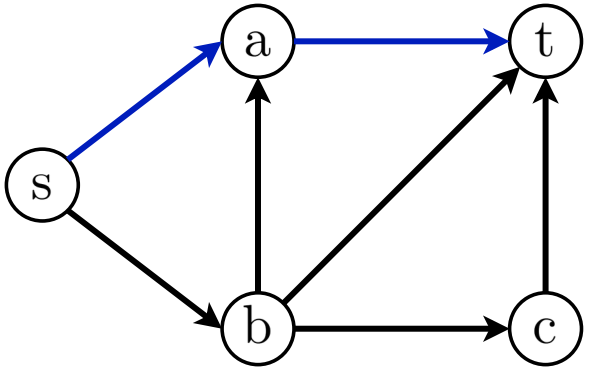
$k \leq 1$

s_2

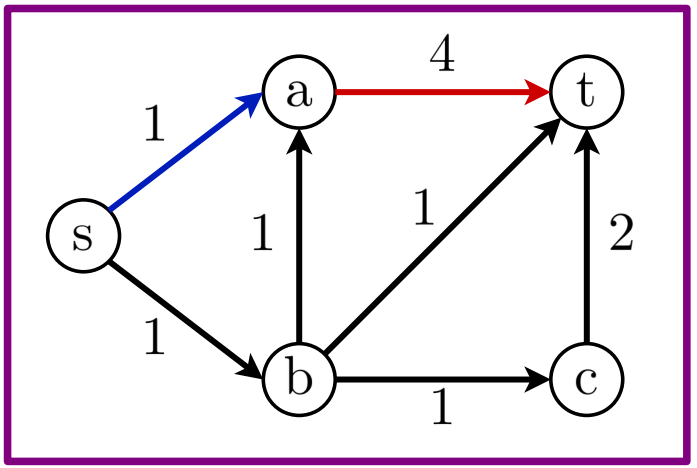


$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases} \quad \begin{array}{c|c} s_1 & s_2 \\ \hline & \\ \hline & \\ \hline & \\ \hline & \end{array}$$

k-dist-RR Shortest Path



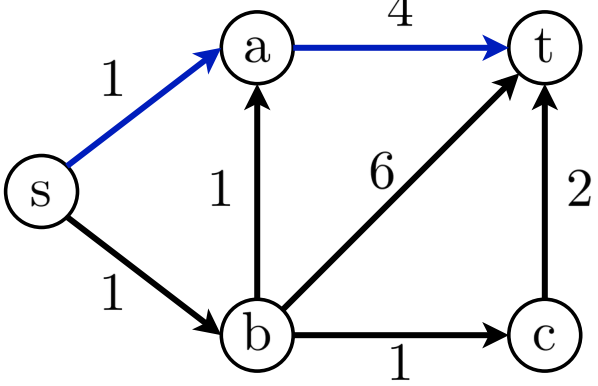
s_1



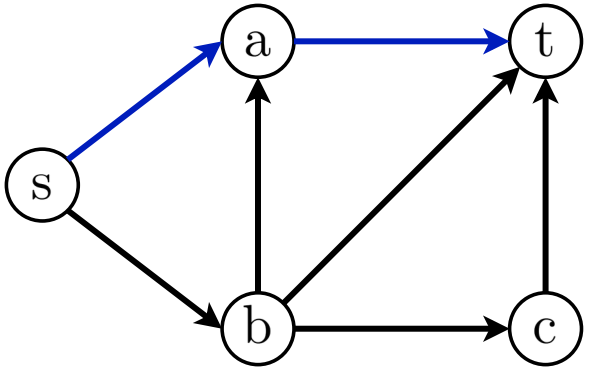
$k \leq 1$

$$\Pi = \begin{array}{c|cc} & s_1 & s_2 \\ \hline \text{sat} & & \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{array}$$

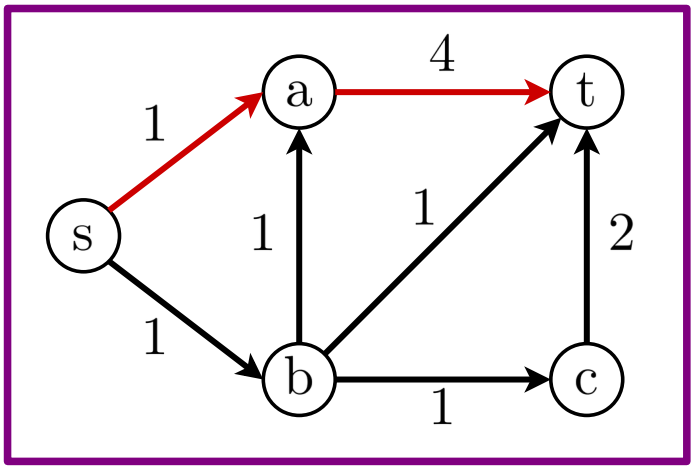
s_2



k-dist-RR Shortest Path

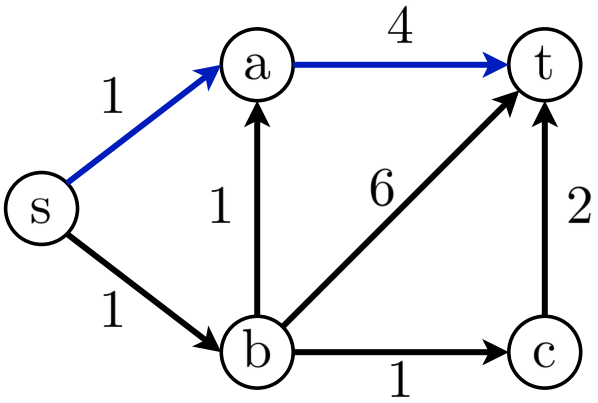


s_1



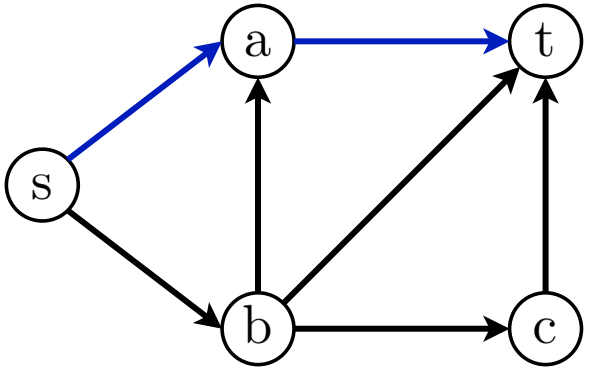
$k \leq 1$

s_2

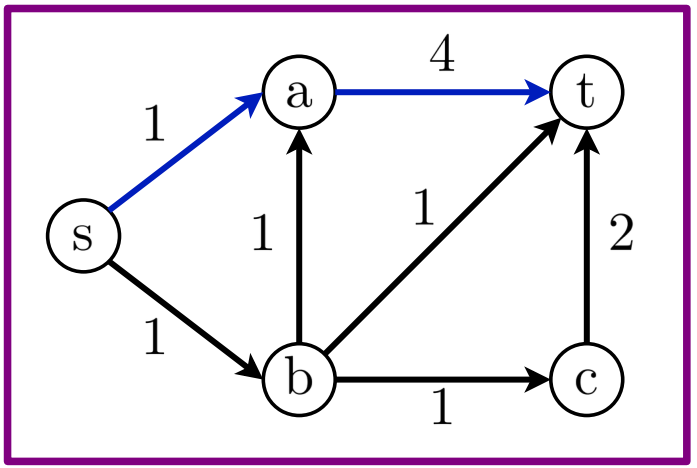


$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases} \quad \begin{array}{c|c} s_1 & s_2 \\ \hline & \\ \hline & \\ \hline & \\ \hline & \end{array}$$

k-dist-RR Shortest Path

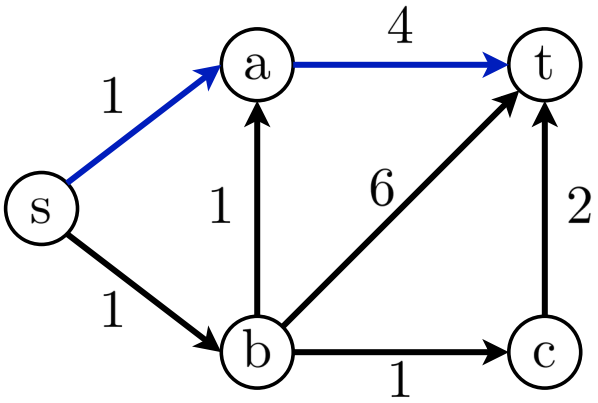


s_1



$k \leq 1$

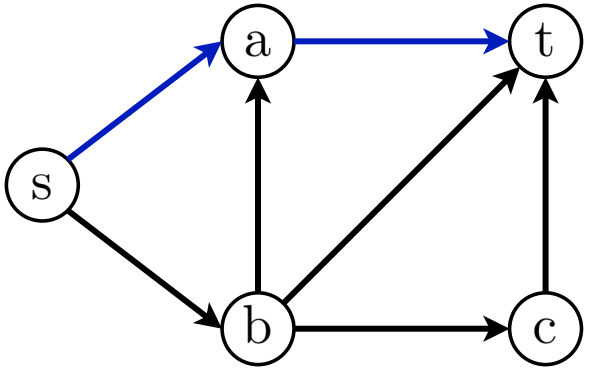
s_2



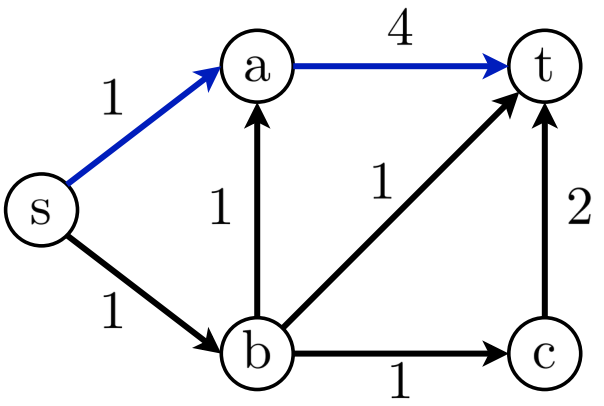
$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

	s_1	s_2
sat	5	
sbat		
sbt		
sbct		

k-dist-RR Shortest Path



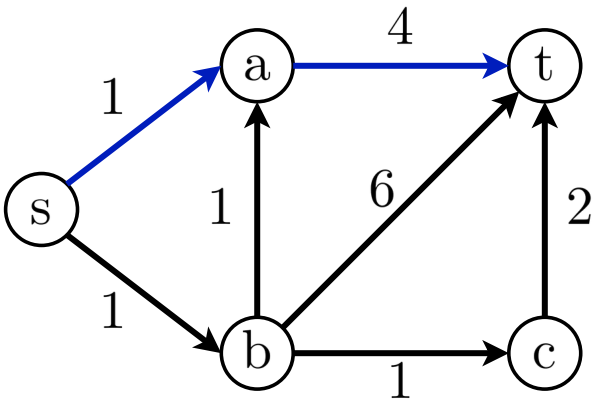
s_1



s_1 s_2

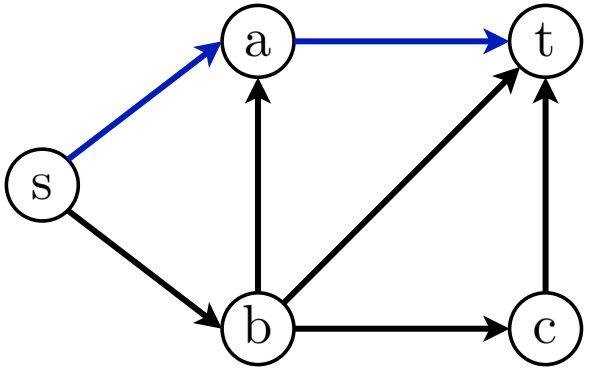
$$\Pi = \begin{cases} \text{sat} & 5 \mid \\ \text{sbat} & \mid \\ \text{sbt} & \mid \\ \text{sbct} & \mid \end{cases}$$

s_2

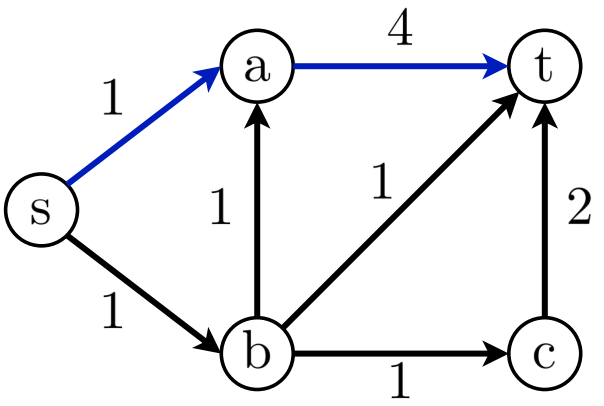


$k \leq 1$

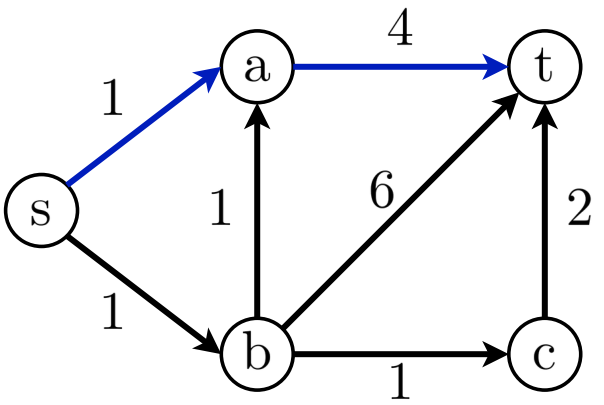
k-dist-RR Shortest Path



s_1



s_2

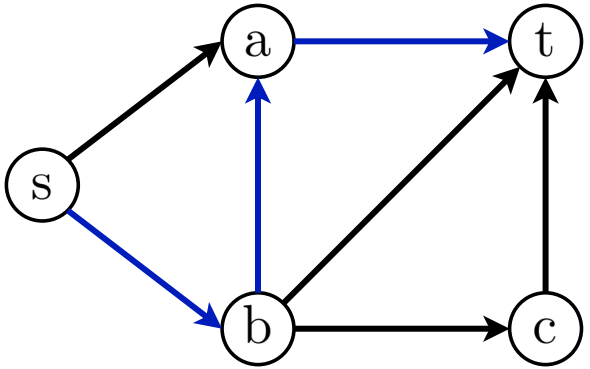


$k \leq 1$

$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

	s_1	s_2
sat	5	5
sbat		
sbt		
sbct		

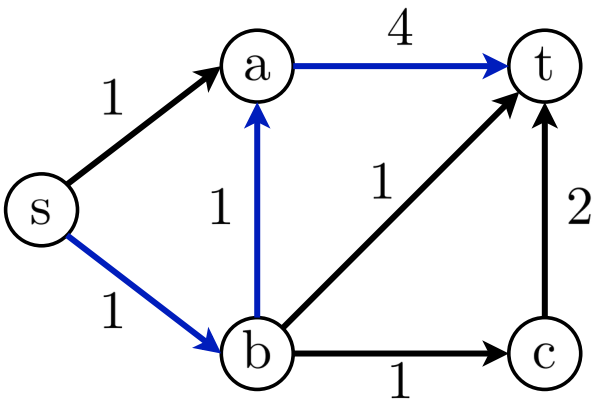
k-dist-RR Shortest Path



$\Pi =$

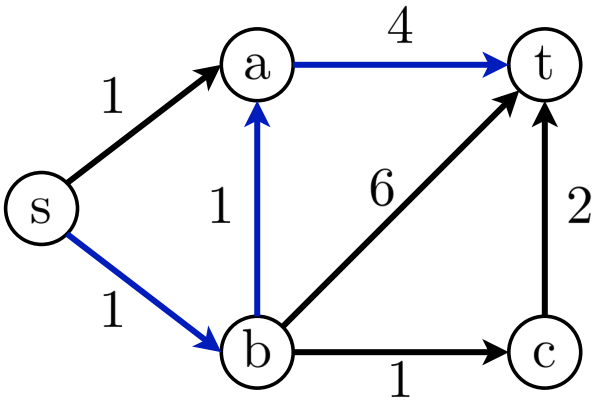
	s_1	s_2
sat	5	5
sbat		
sbt		
sbct		

s_1

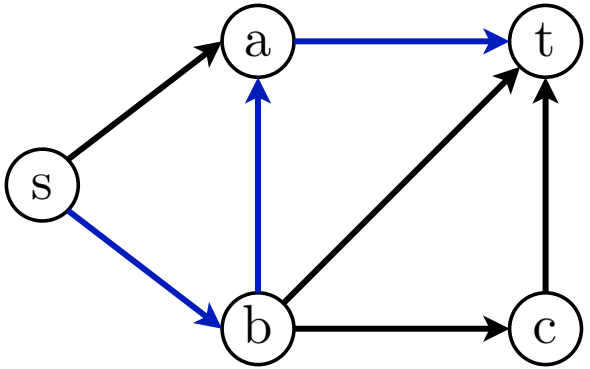


$k \leq 1$

s_2



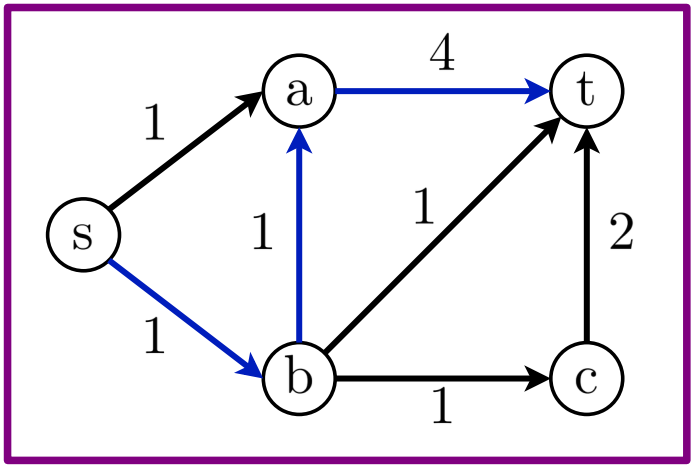
k-dist-RR Shortest Path



$\Pi =$

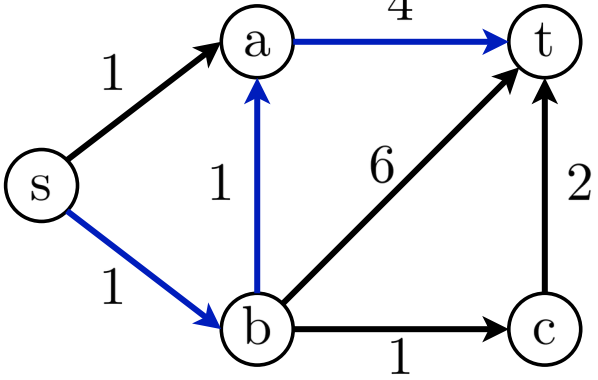
	s_1	s_2
sat	5	5
sbat		
sbt		
sbct		

s_1

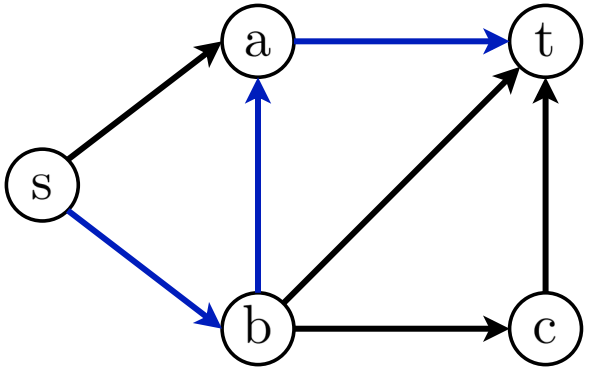


$k \leq 1$

s_2



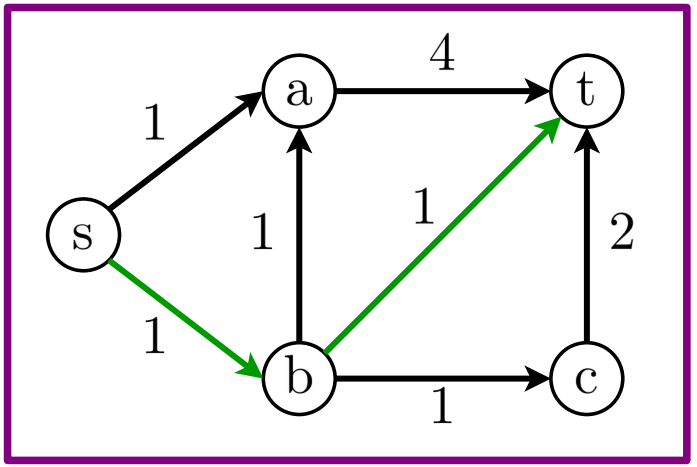
k-dist-RR Shortest Path



$\Pi =$

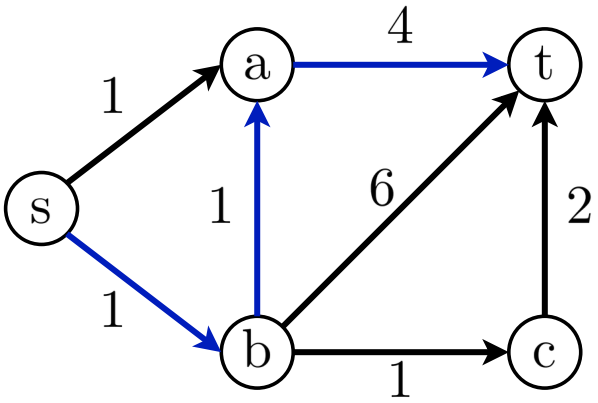
	s_1	s_2
sat	5	5
sbat	2	
sbt		
sbct		

s_1

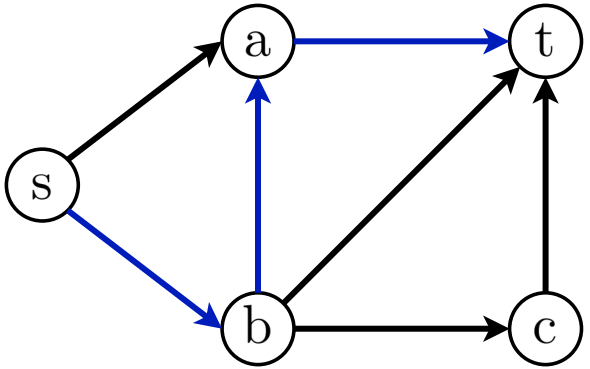


$k \leq 1$

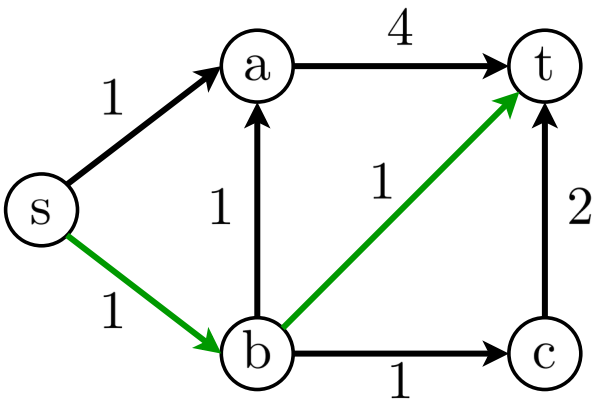
s_2



k-dist-RR Shortest Path



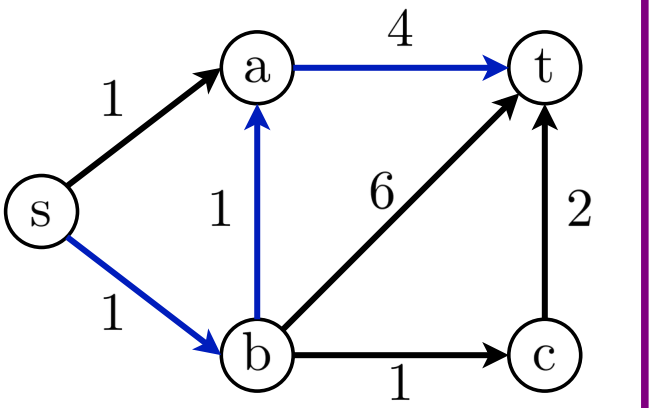
s_1



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

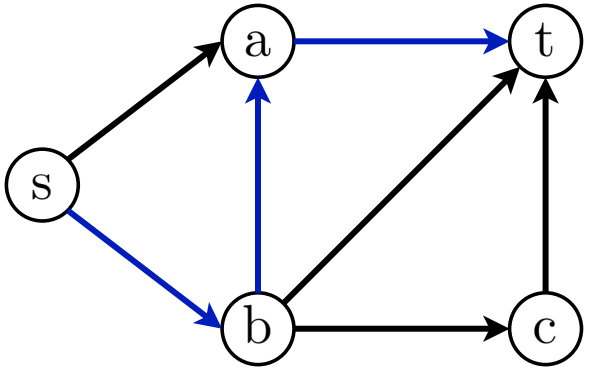
s_1	s_2
5	5
2	

s_2

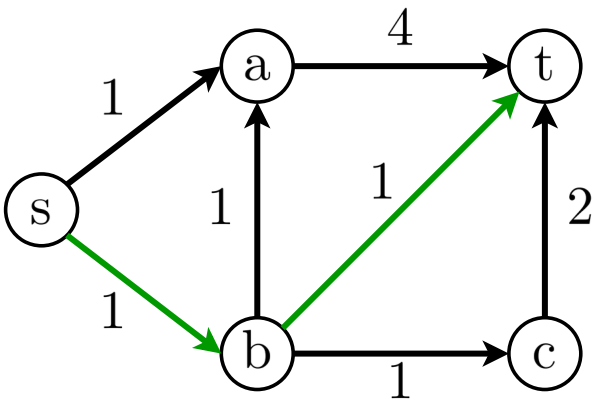


$k \leq 1$

k-dist-RR Shortest Path



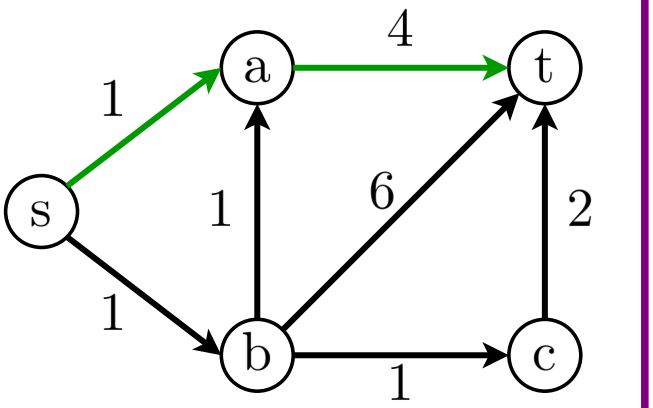
s_1



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

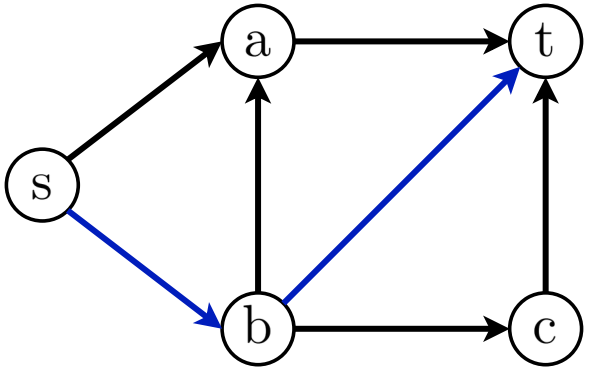
s_1	s_2
5	5
2	5

s_2



$k \leq 1$

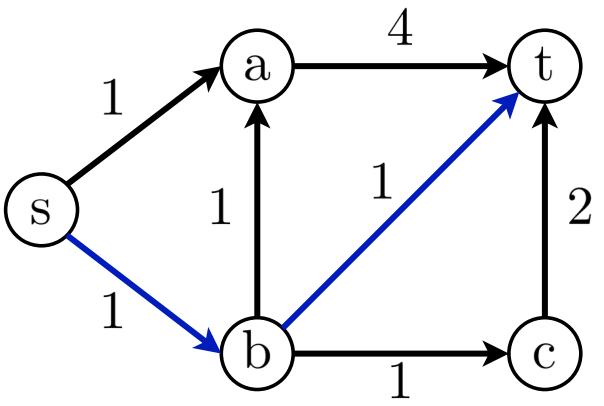
k-dist-RR Shortest Path



$\Pi =$

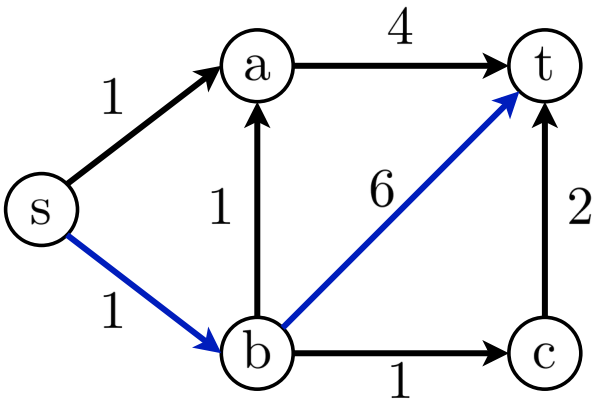
	s_1	s_2
sat	5	5
sbat	2	5
sbt		
sbct		

s_1

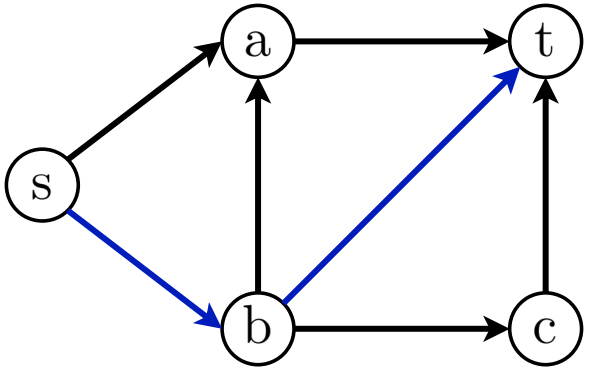


$k \leq 1$

s_2



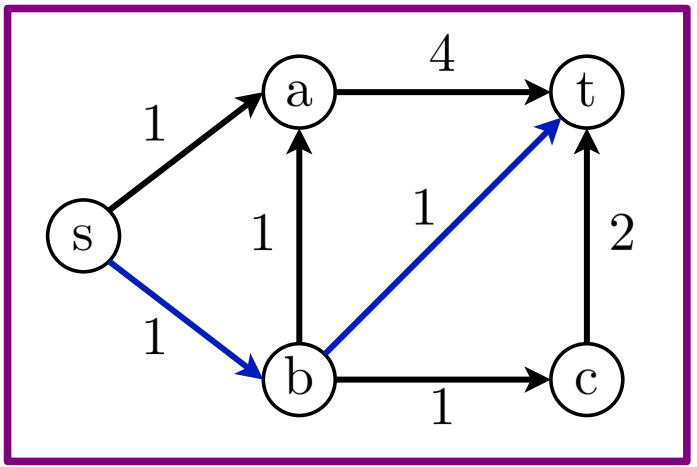
k-dist-RR Shortest Path



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

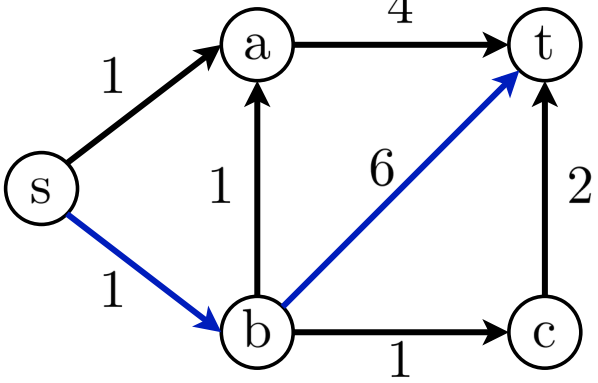
	s_1	s_2
sat	5	5
sbat	2	5
sbt		
sbct		

s_1

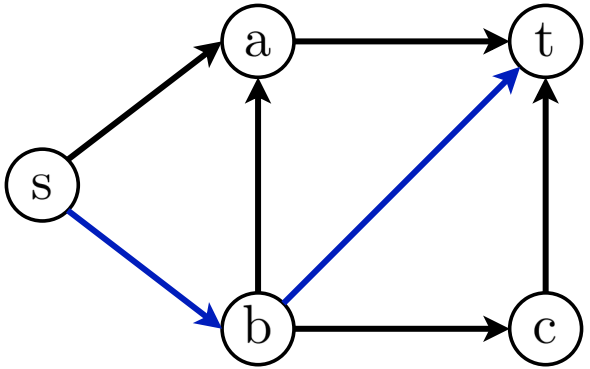


$k \leq 1$

s_2



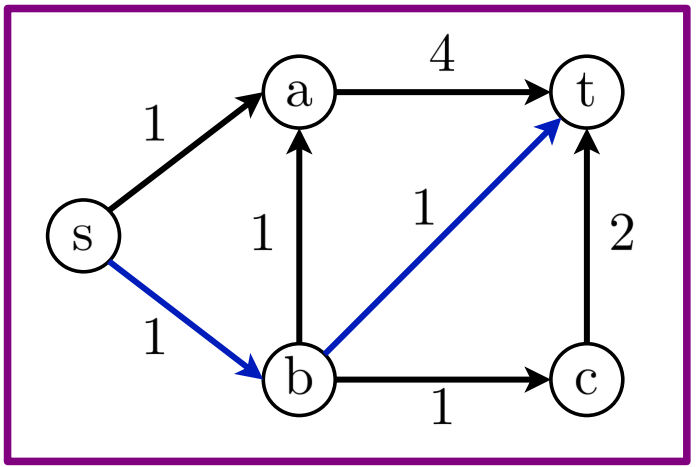
k-dist-RR Shortest Path



$\Pi =$

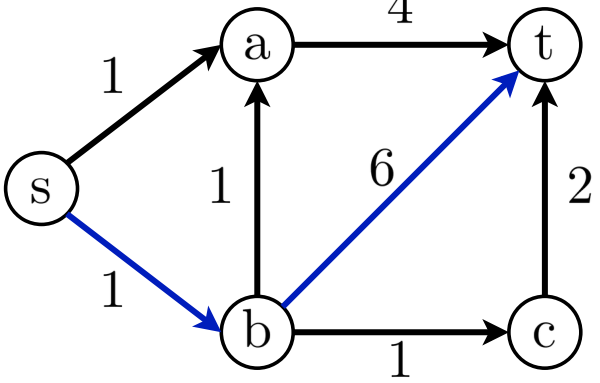
	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	
sbct		

s_1

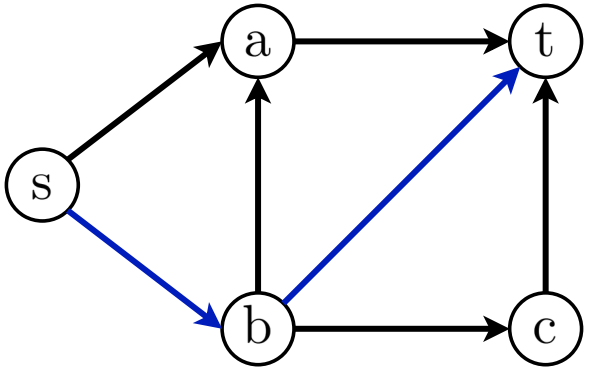


$k \leq 1$

s_2



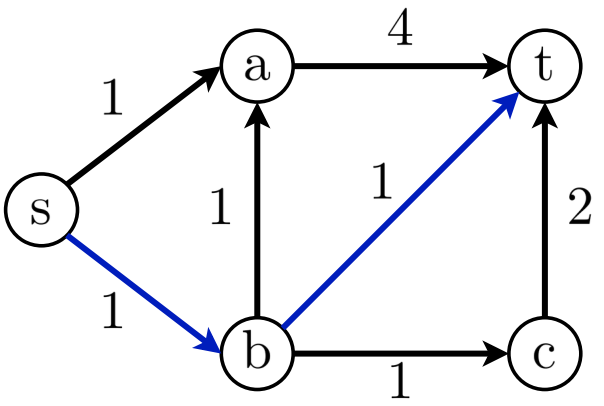
k-dist-RR Shortest Path



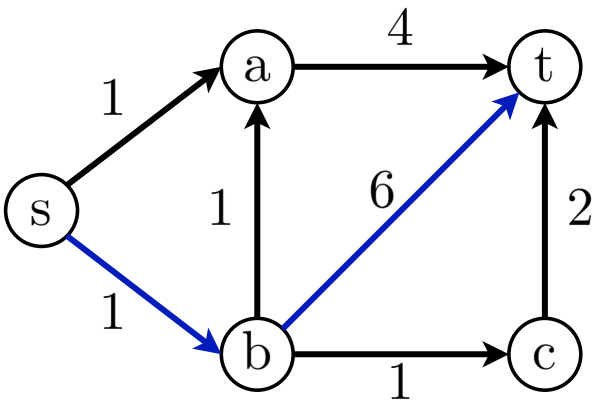
$\Pi =$

	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	
sbct		

s_1

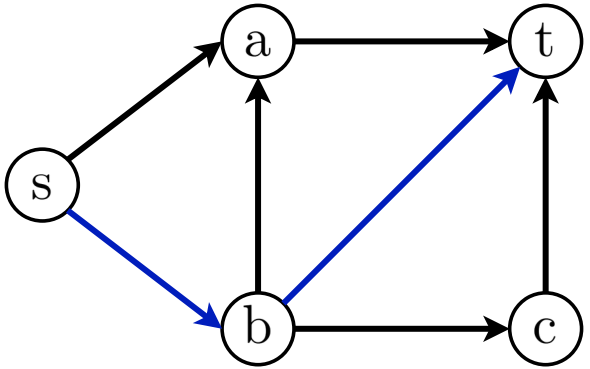


s_2



$k \leq 1$

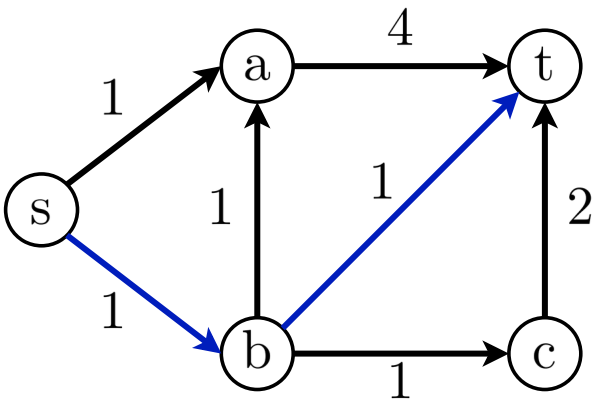
k-dist-RR Shortest Path



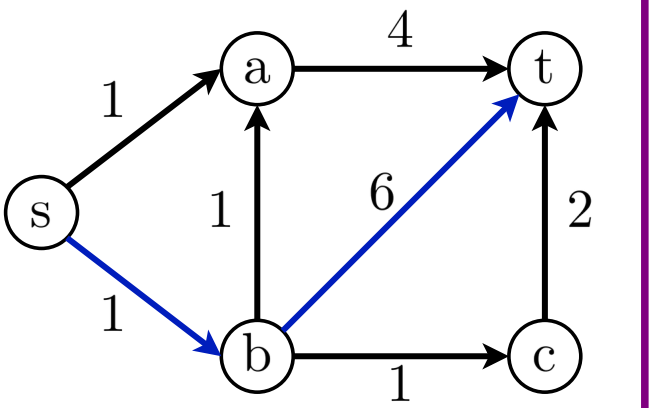
$\Pi =$

	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct		

s_1

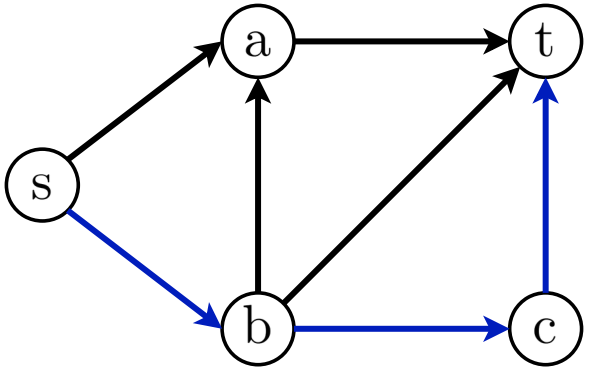


s_2

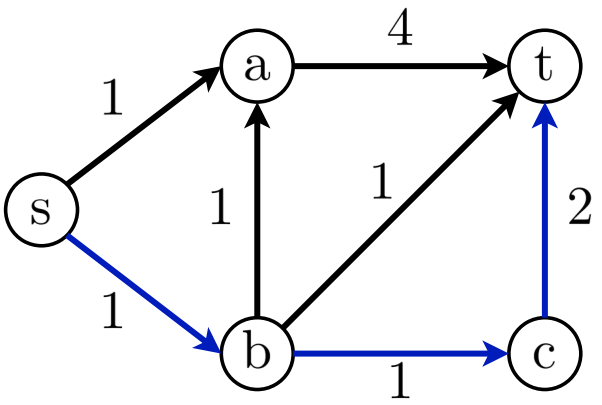


$k \leq 1$

k-dist-RR Shortest Path



s_1

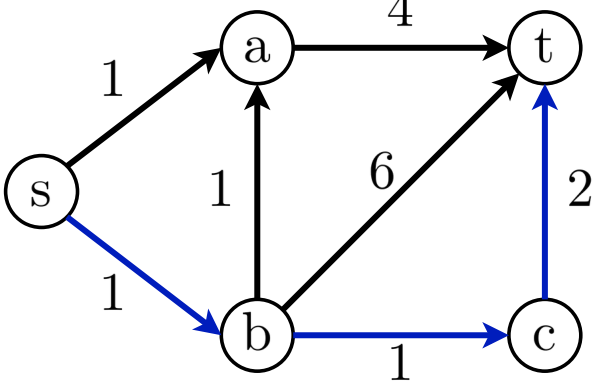


$k \leq 1$

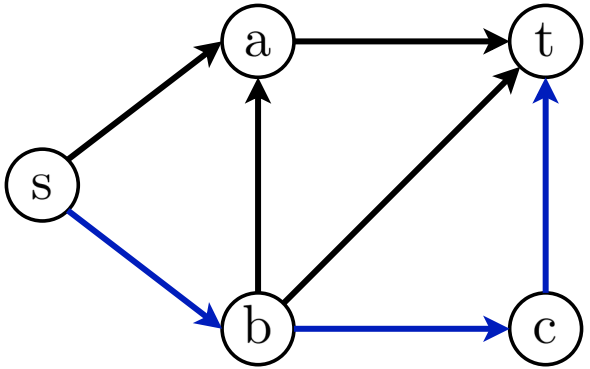
$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct		

s_2



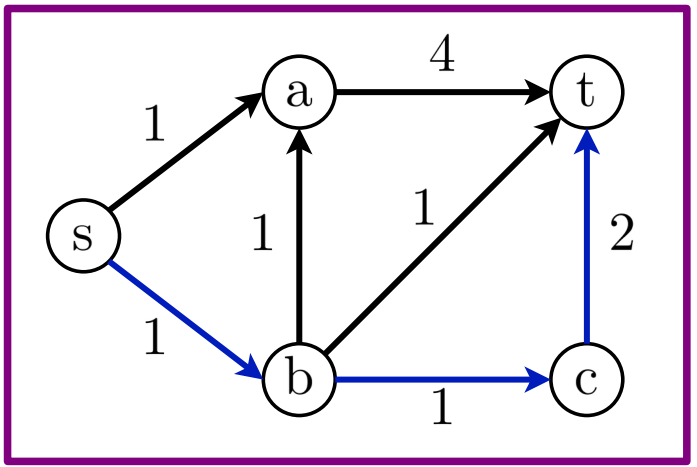
k-dist-RR Shortest Path



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

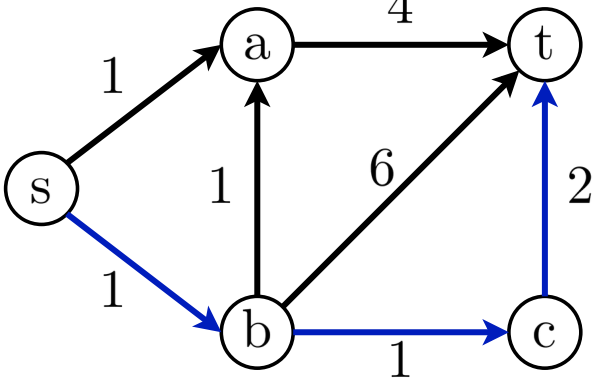
	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct		

s_1

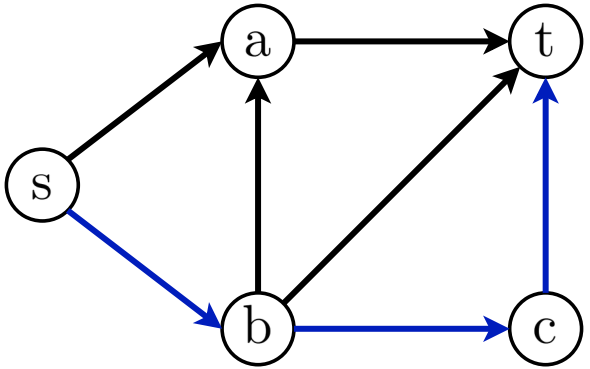


$k \leq 1$

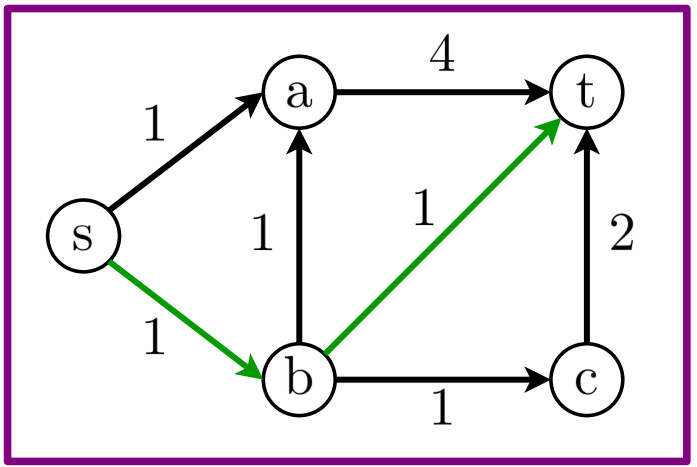
s_2



k-dist-RR Shortest Path



s_1

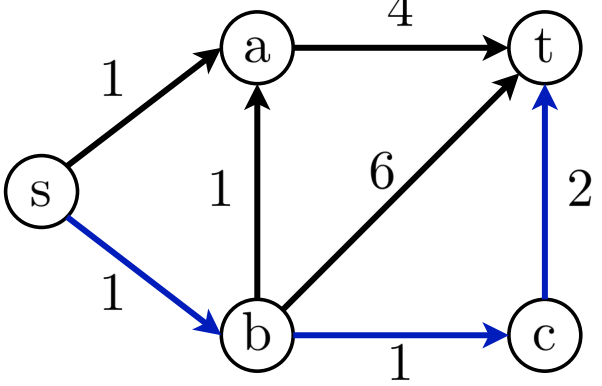


$k \leq 1$

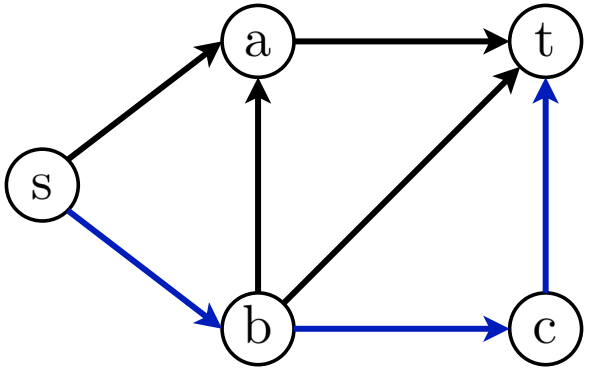
$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct	2	

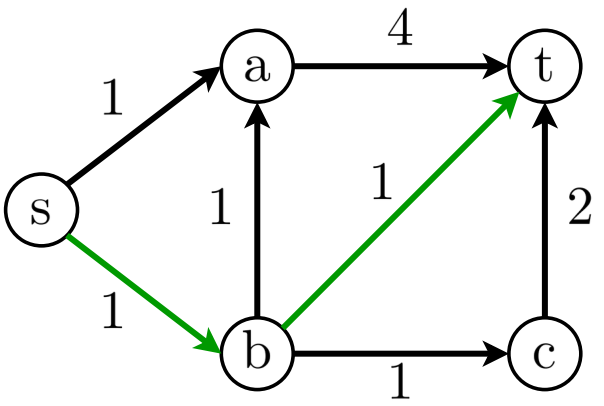
s_2



k-dist-RR Shortest Path



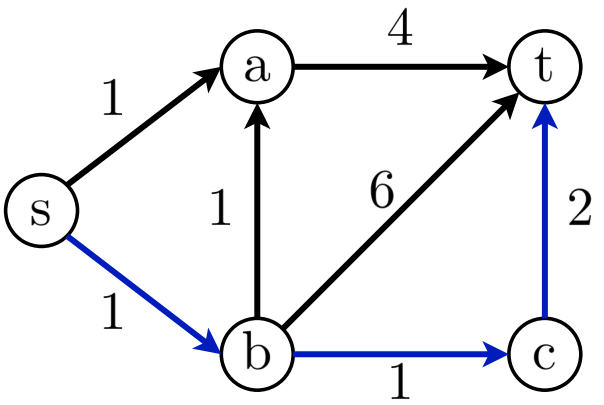
s_1



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

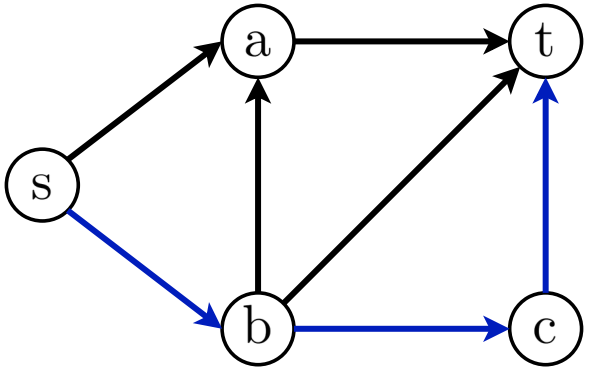
	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct	2	

s_2

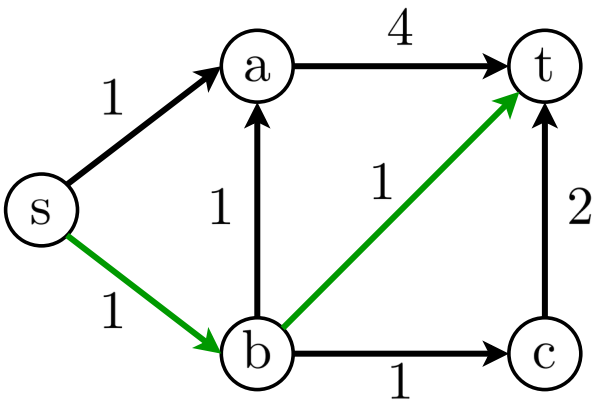


$k \leq 1$

k-dist-RR Shortest Path



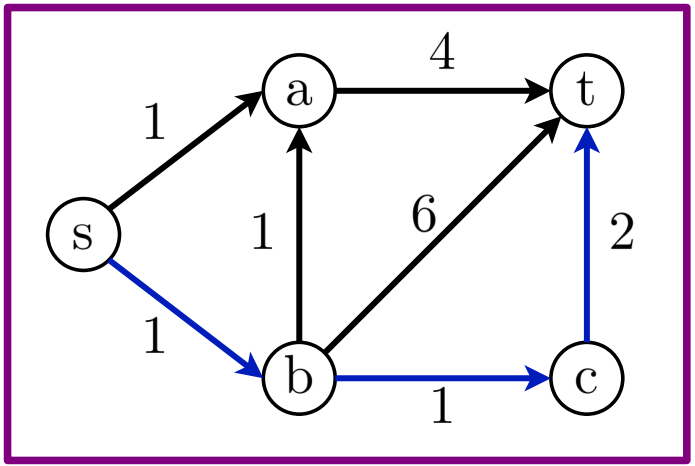
s_1



$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

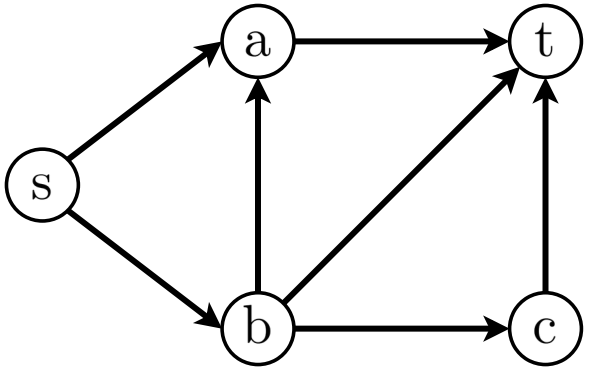
	s_1	s_2
sat	5	5
sbat	2	5
sbt	2	7
sbct	2	4

s_2

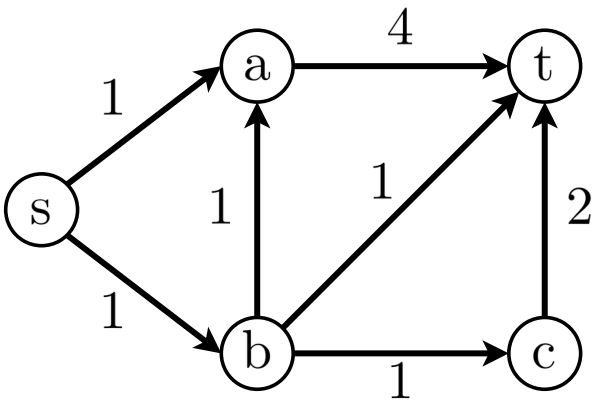


$k \leq 1$

k-dist-RR Shortest Path

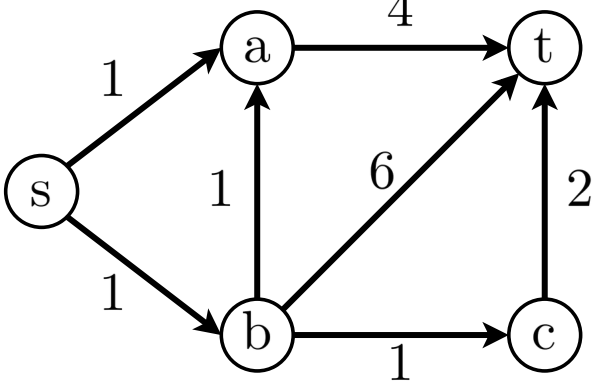


s_1



$k \leq 1$

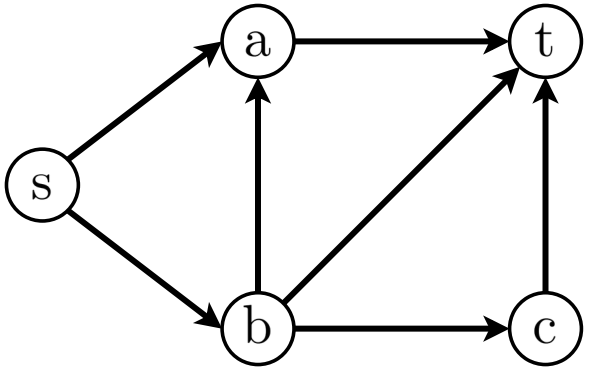
s_2



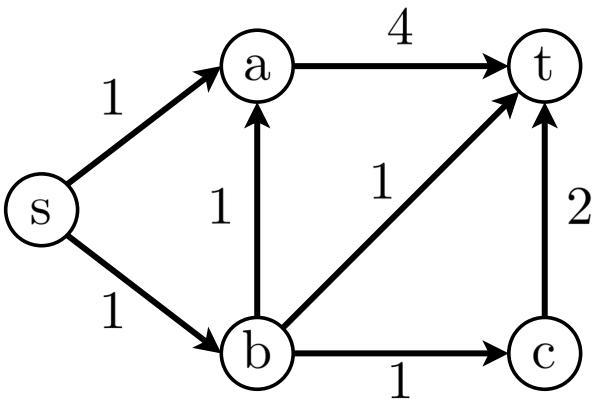
$$\Pi = \left\{ \begin{array}{l|cc} & s_1 & s_2 \\ \hline \text{sat} & 5 & 5 \\ \text{sbat} & 2 & 5 \\ \text{sbt} & 2 & 7 \\ \text{sbct} & 2 & 4 \end{array} \right.$$

Welchen Pfad wählen wir?

k-dist-RR Shortest Path



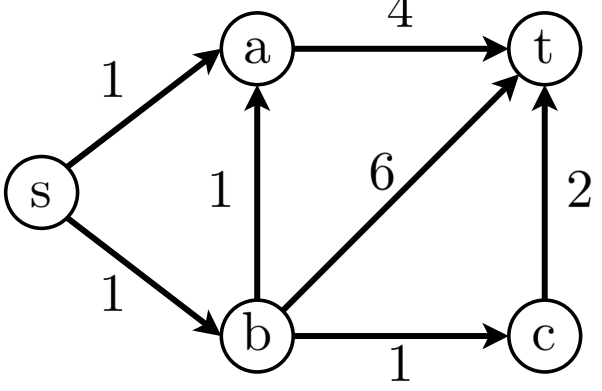
s_1



$k \leq 1$

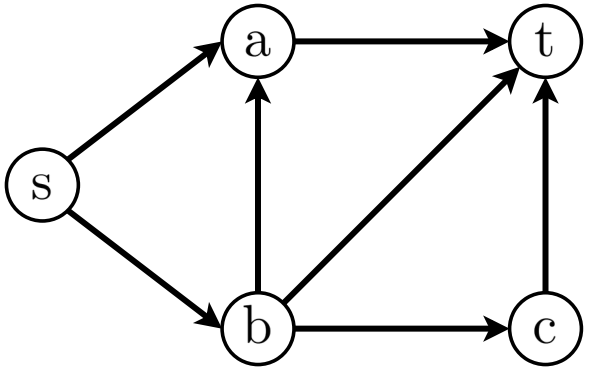
$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right. \begin{array}{c|c} s_1 & s_2 \\ \hline 5 & 5 \\ \hline 2 & 5 \\ \hline 2 & 7 \\ \hline 2 & 4 \end{array} \xrightarrow{\max} \begin{array}{c} 5 \\ 5 \\ 7 \\ 4 \end{array}$$

s_2



Welchen Pfad wählen wir?

k-dist-RR Shortest Path



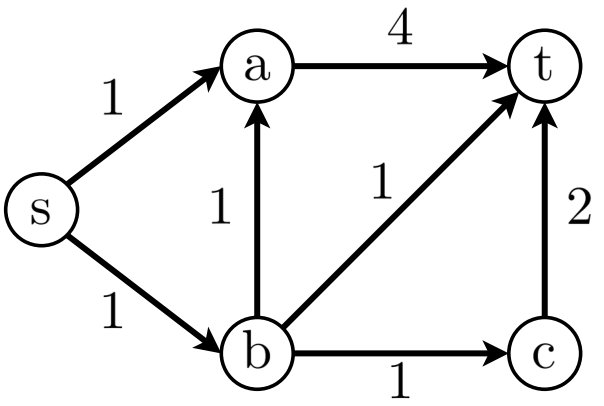
$\Pi =$

	s_1	s_2	
sat	5	5	5
sbat	2	5	5
sbt	2	7	7
sbct	2	4	4

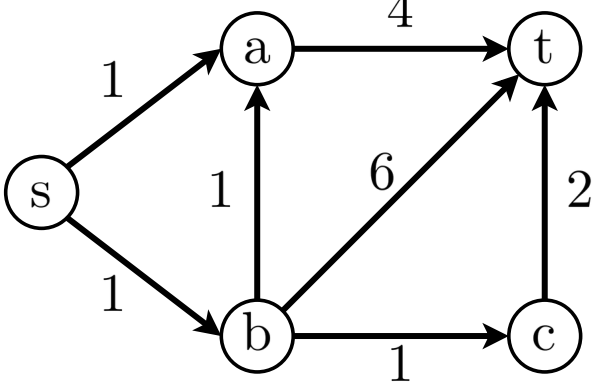
max → 4 ↑ min

Welchen Pfad wählen wir?

s_1

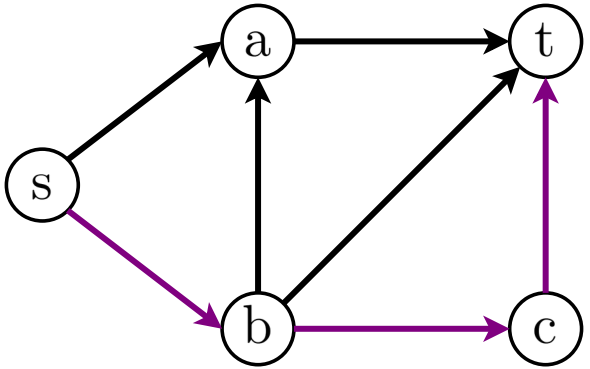


s_2

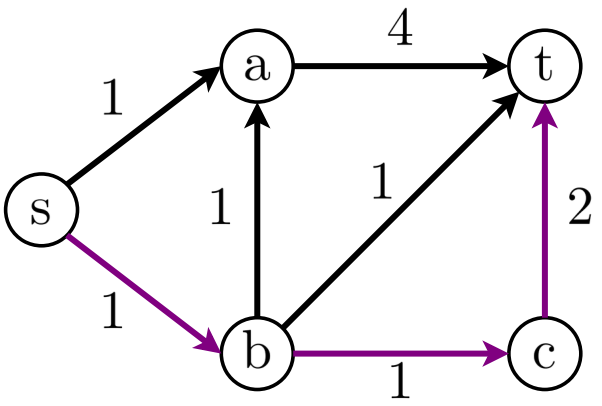


$k \leq 1$

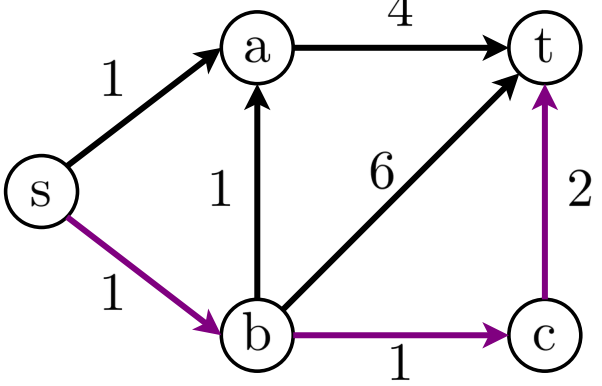
k-dist-RR Shortest Path



s_1



s_2

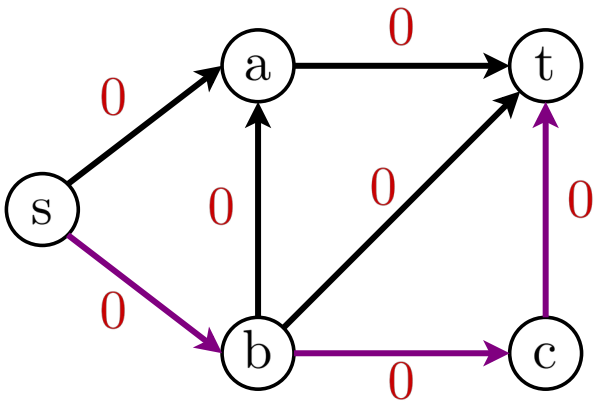


$k \leq 1$

$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right. \begin{array}{|c|c|} \hline 5 & 5 \\ \hline 2 & 5 \\ \hline 2 & 7 \\ \hline 2 & 4 \\ \hline \end{array} \begin{array}{l} \\ \\ \xrightarrow{\max} \\ \\ \end{array} \begin{array}{l} 5 \\ 5 \\ 7 \\ 4 \end{array} \quad \begin{array}{l} 4 \\ \uparrow \min \end{array}$$

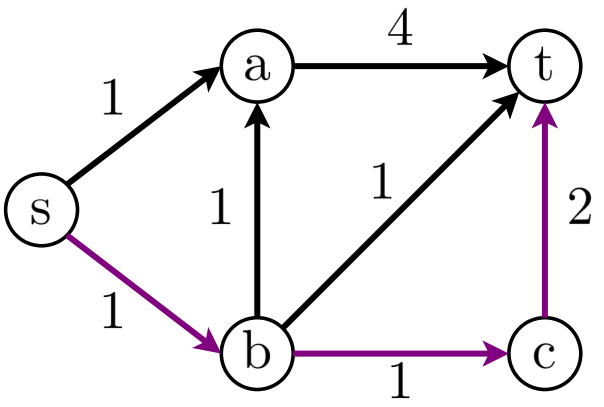
s_1 s_2

k-dist-RR Shortest Path

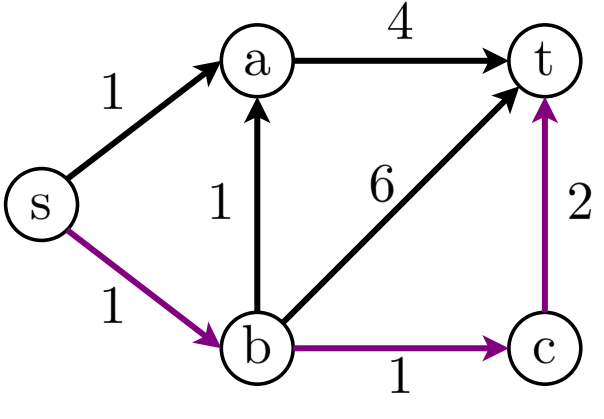


$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right. \quad \begin{array}{c|c} s_1 & s_2 \\ \hline 5 & 5 \\ \hline 2 & 5 \\ \hline 2 & 7 \\ \hline 2 & 4 \end{array} \quad \xrightarrow{\max} \quad \begin{array}{l} 5+0 \\ 5+0 \\ 7+0 \\ 4+0 \end{array} \quad \begin{array}{c} 4 \\ \uparrow \\ \min \end{array}$$

s_1



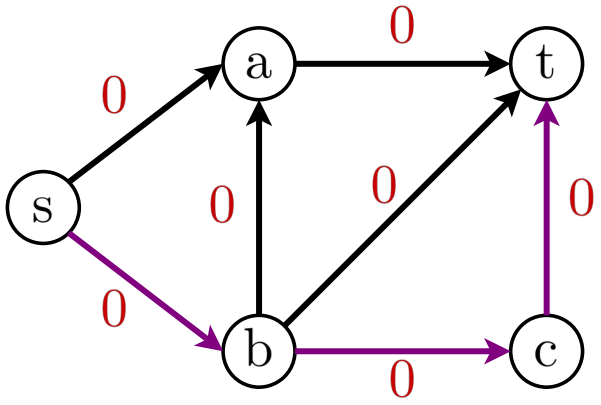
s_2



$k \leq 1$

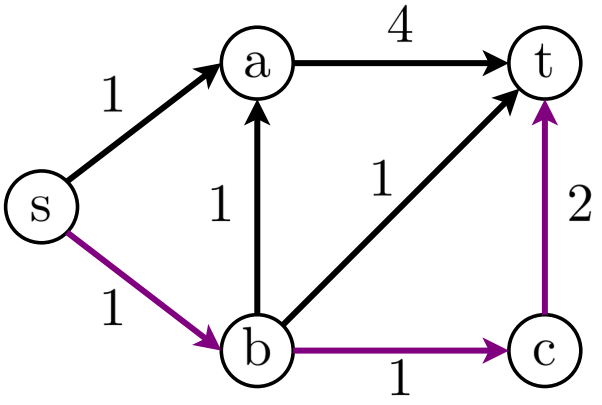
k-dist-RR Shortest Path

first-stage

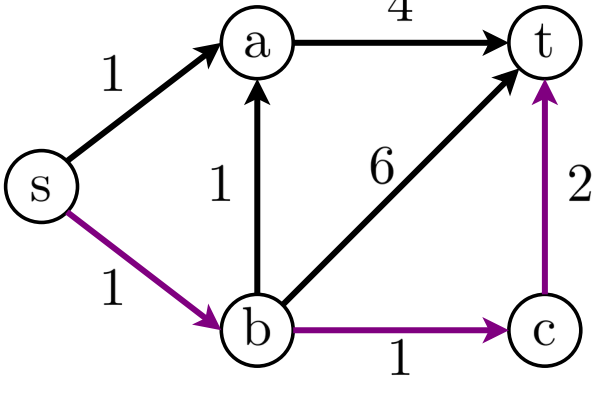


second-stage

s_1



s_2



$k \leq 1$

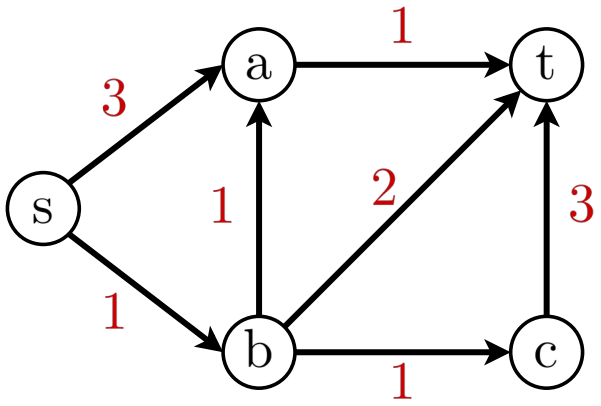
$$\Pi = \left\{ \begin{array}{l} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{array} \right.$$

	s_1	s_2	
sat	5	5	5+0
sbat	2	5	5+0
sbt	2	7	7+0
sbct	2	4	4+0

max → 4 ↑ min

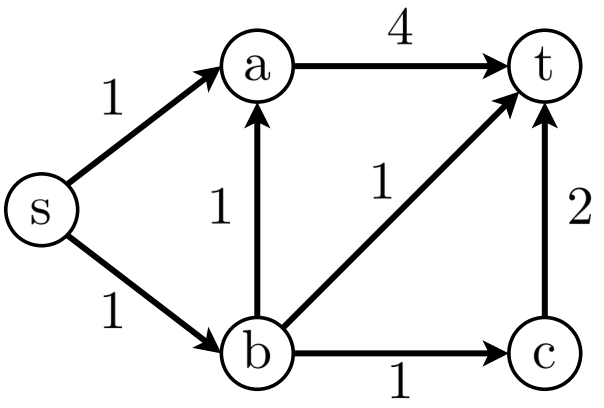
k-dist-RR Shortest Path

first-stage

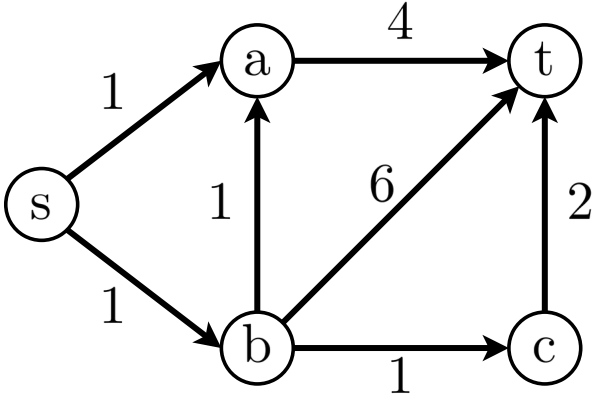


second-stage

s_1



s_2



$k \leq 1$

$$\Pi = \begin{cases} \text{sat} & 5 \mid 5 \\ \text{sbat} & 2 \mid 5 \\ \text{sbt} & 2 \mid 7 \\ \text{sbct} & 2 \mid 4 \end{cases}$$

$s_1 \; s_2$

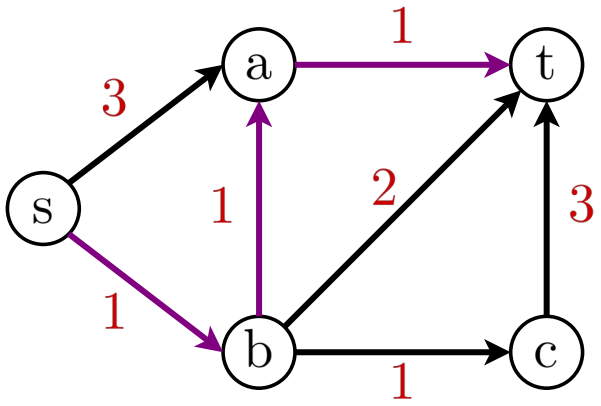
max →

$5+?$
$5+?$
$7+?$
$4+?$

min ↑ ?

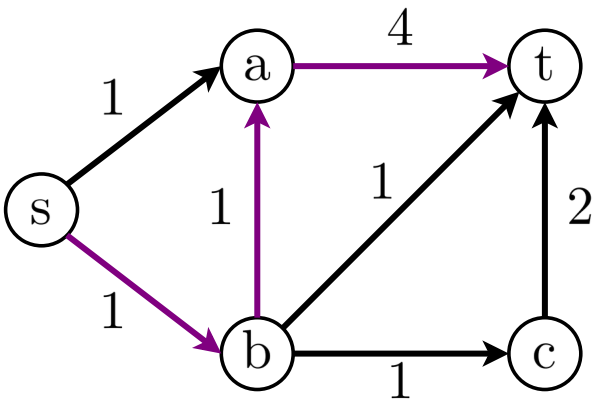
k-dist-RR Shortest Path

first-stage

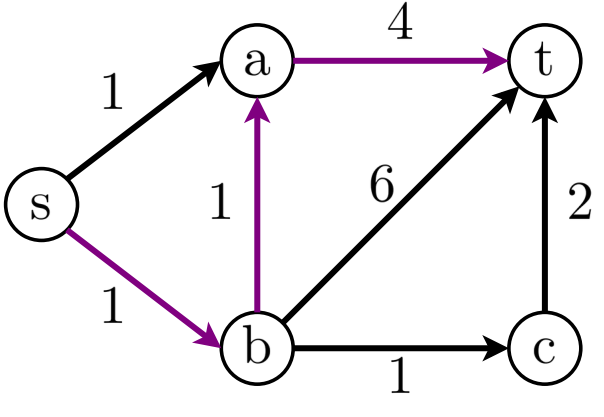


second-stage

s_1



s_2



$k \leq 1$

$$\Pi = \begin{cases} \text{sat} & 5 \mid 5 \\ \text{sbat} & 2 \mid 5 \\ \text{sbt} & 2 \mid 7 \\ \text{sbct} & 2 \mid 4 \end{cases}$$

$s_1 \; s_2$

max →

$5+4$
$5+3$
$7+3$
$4+5$

min ↑ 8

k-dist-RR Shortest Path

Definition:
Sei $G{=}(V, E)$ ein gerichteter Graph mit Startknoten s und Zielknoten t. Sei durch $r^1{:}\, E \to \mathbb{N}$ eine „first-stage“ Kostenfunktion gegeben und S die Menge an möglichen Szenarien, wobei jedes Szenario $s \in S$ eine Kostenfunktion $r^s : E \to \mathbb{N}$ definiert. Π sei die Menge aller s-t-Pfade π in G, $k \in \mathbb{N}$ ein recovery-Parameter und $\Pi(\pi, k)$ die Menge der recovery-Pfade π' von π mit $|\pi' \backslash \pi \,| \leq k$.

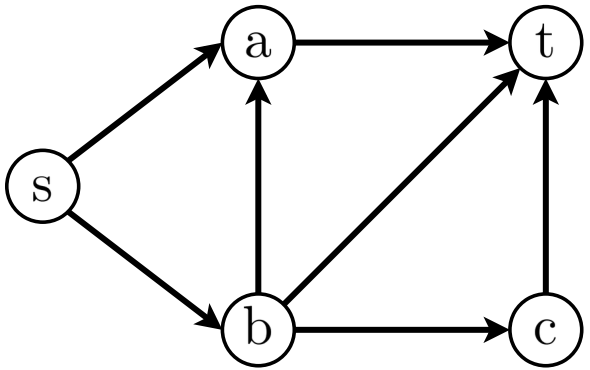
Das k-dist-RR Shortest Path Problem besteht darin, einen Pfad $\pi^* \in \Pi$ zu finden, der die Gesamtkosten minimiert:

$$\min_{\pi \in \Pi} \left(r^1(\pi) + r_{RR}(\pi) \right)$$

$$\text{mit } r^1(\pi) = \sum_{e \in \pi} r^1(e) \text{ und } r_{RR}(\pi) = \max_{s \in S} \min_{\pi' \in \Pi(\pi,k)} \sum_{e \in \pi'} r^s(e)$$

Rent-RR Shortest Path

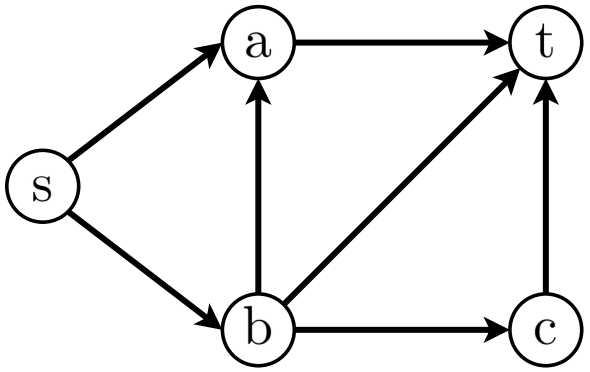
heute morgen übermorgen



Planung

Rent-RR Shortest Path

heute morgen übermorgen

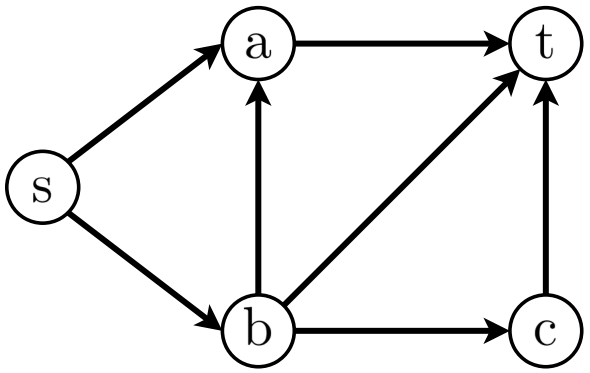


Planung

> Marktpreise unbekannt

Rent-RR Shortest Path

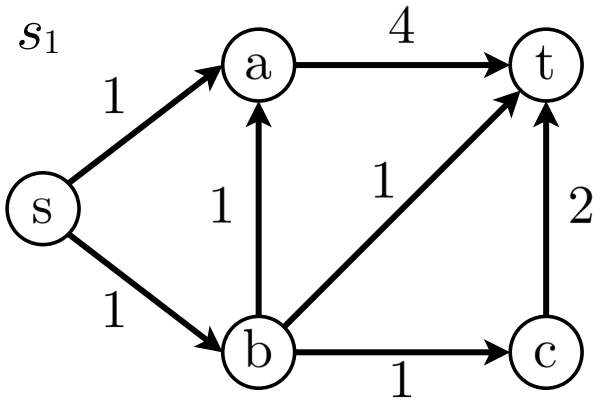
heute morgen übermorgen



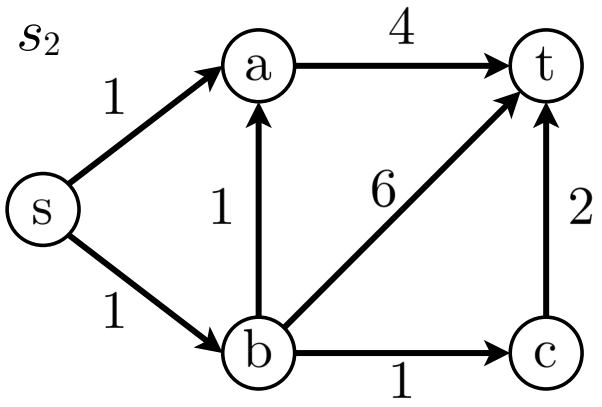
Planung

> Marktpreise unbekannt

s_1

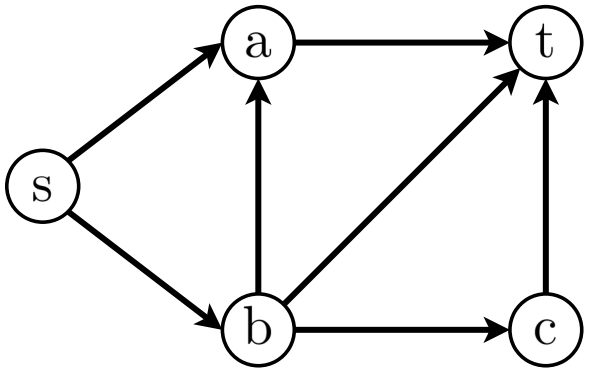


s_2



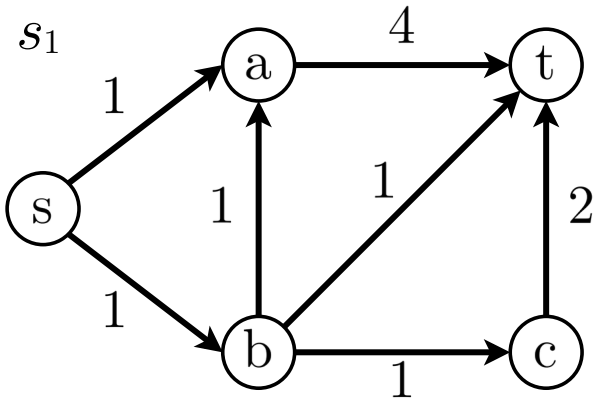
Rent-RR Shortest Path

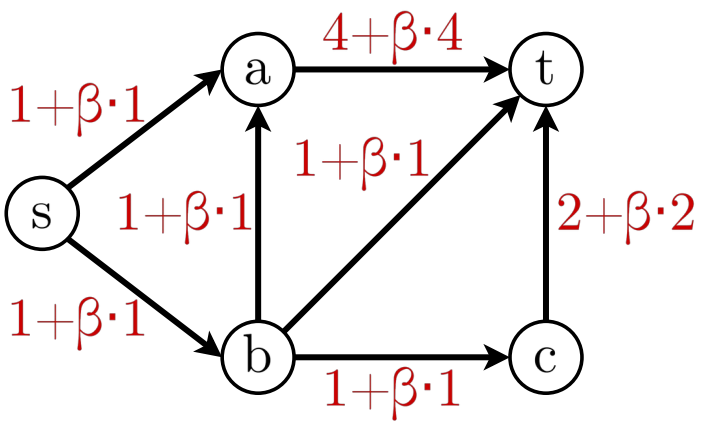
heute — morgen — übermorgen

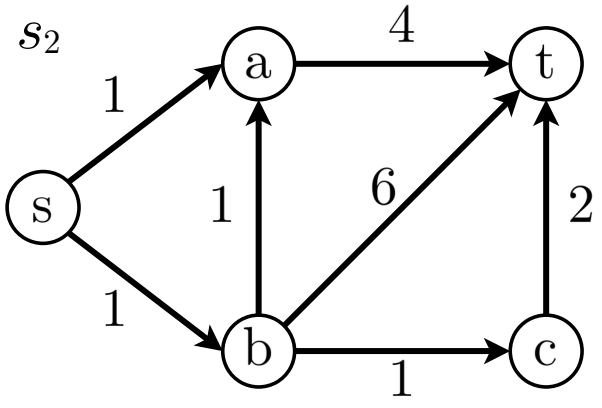


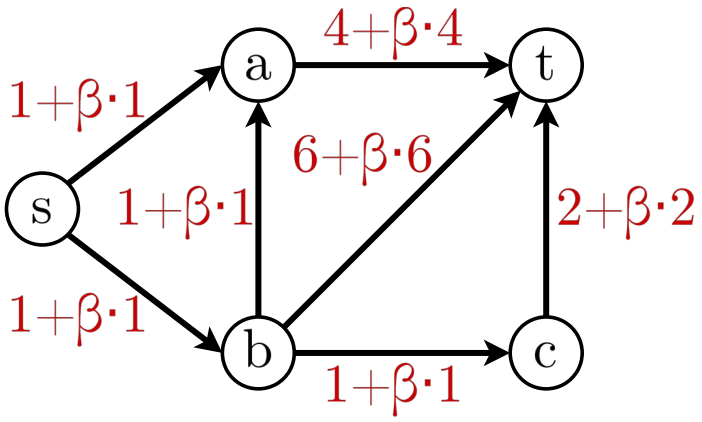
Planung

> Marktpreise unbekannt



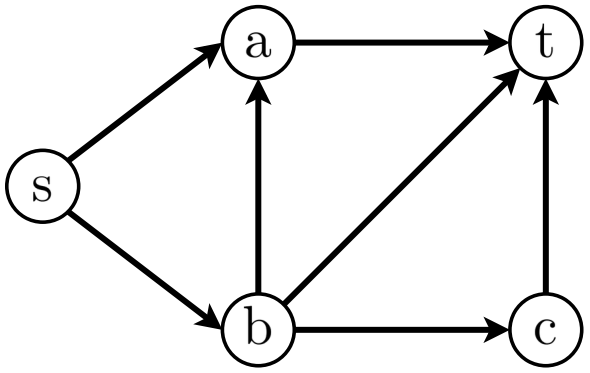






Rent-RR Shortest Path

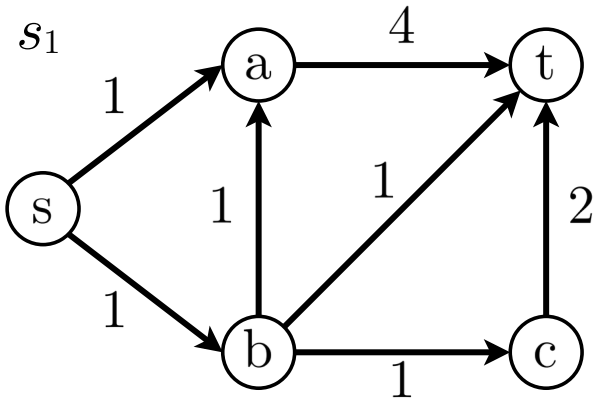
heute — morgen — übermorgen

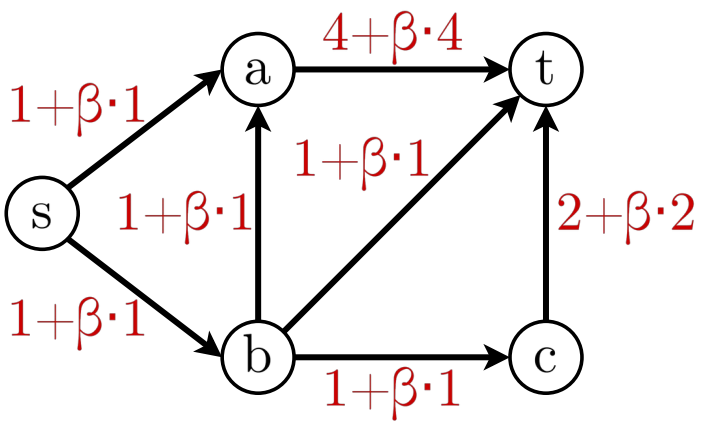


Planung
> Marktpreise unbekannt

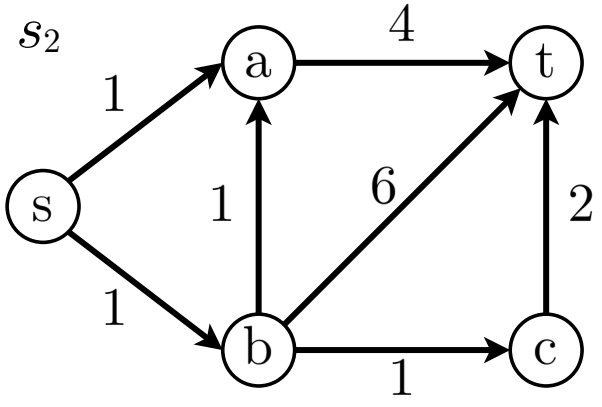
Beispiel: $\beta = 1$

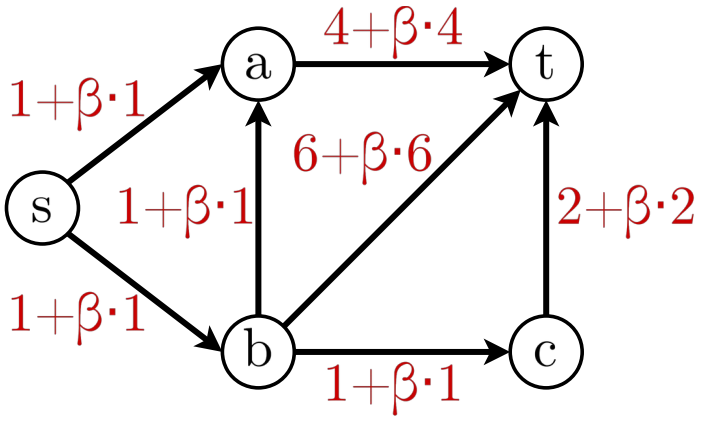
s_1





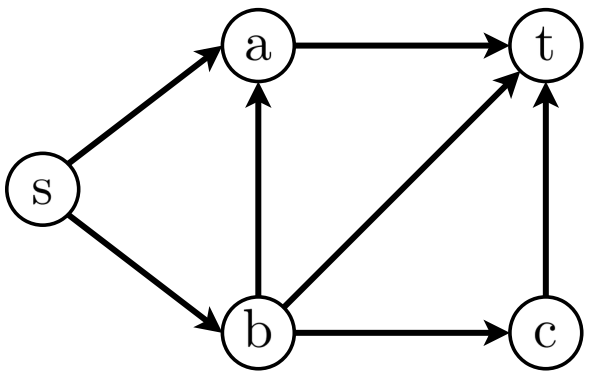
s_2





Rent-RR Shortest Path

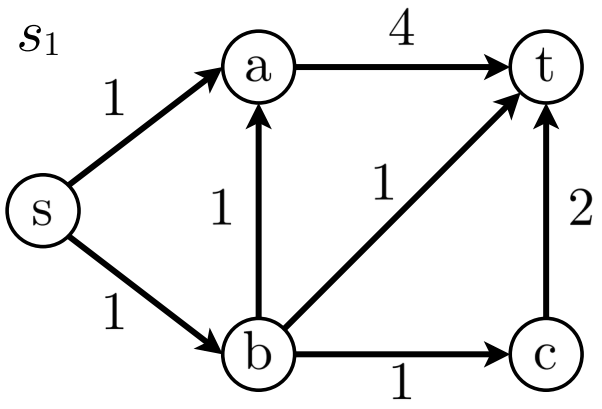
heute morgen übermorgen

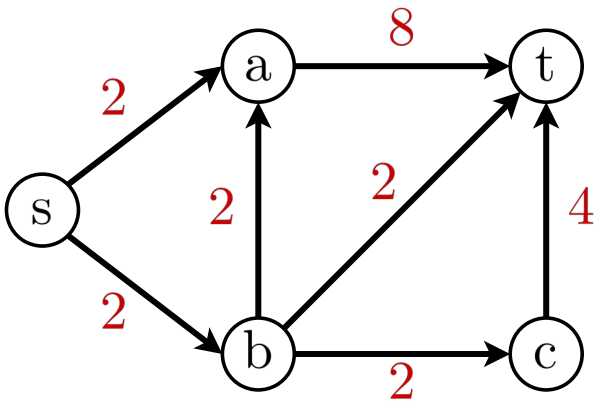


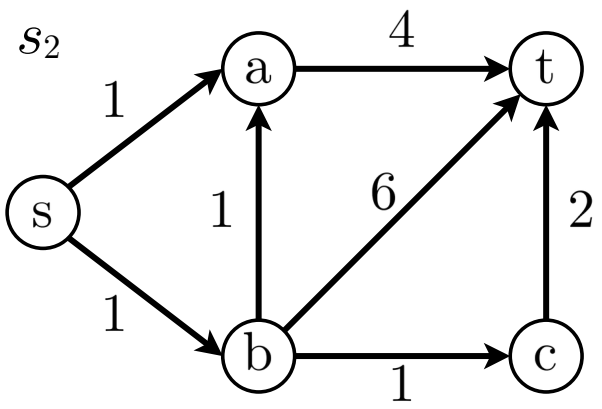
Planung

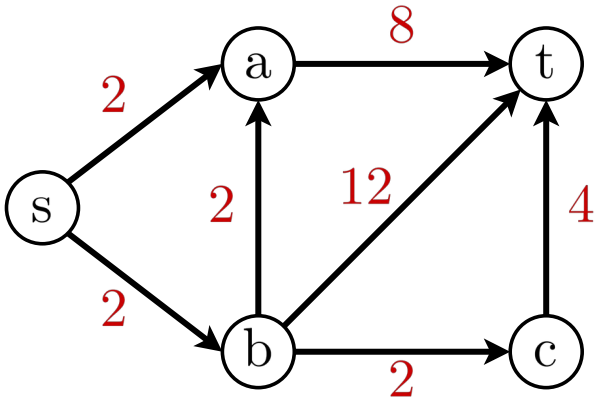
> Marktpreise unbekannt

Beispiel: $\beta = 1$



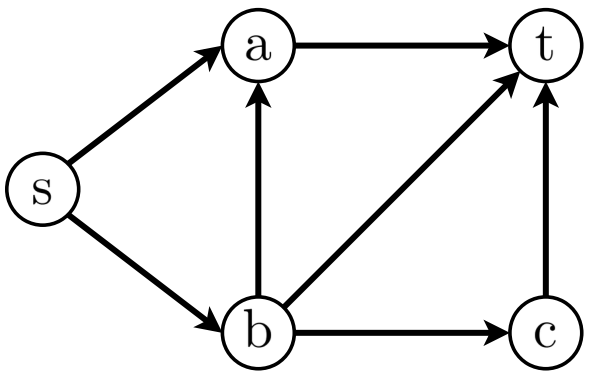






Rent-RR Shortest Path

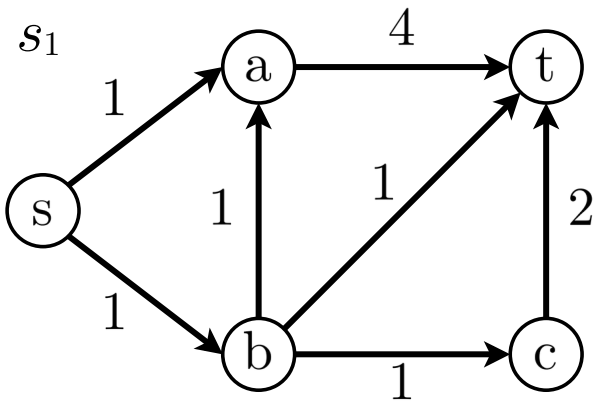
heute | morgen | übermorgen

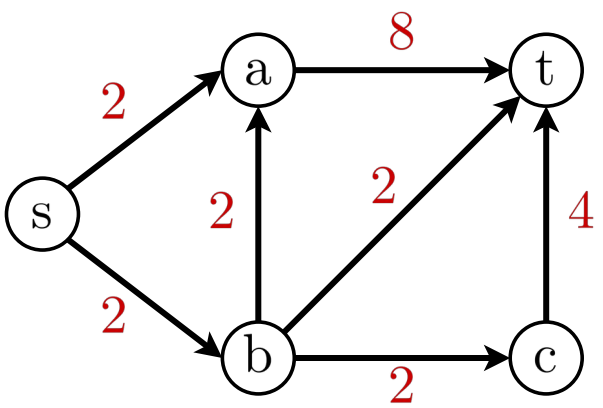


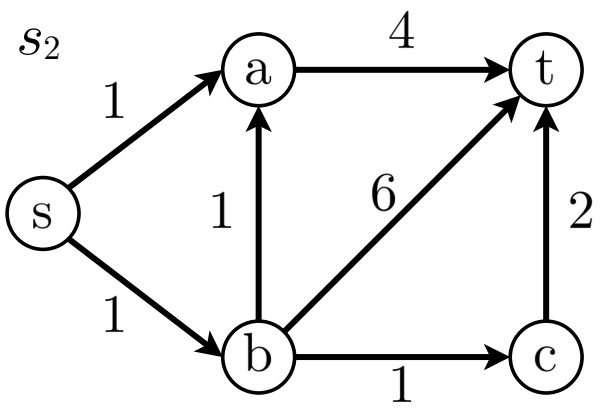
Planung

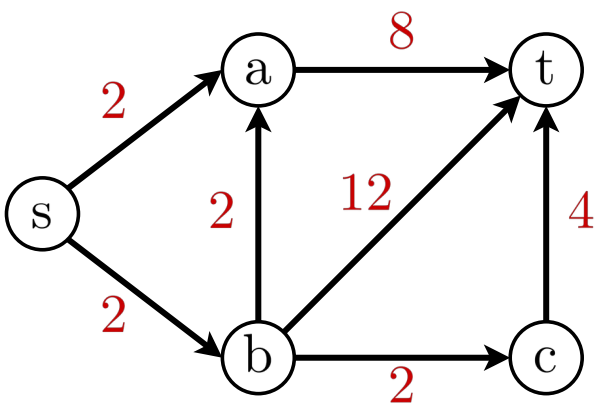
> Marktpreise unbekannt

Beispiel: $\beta = 1, \alpha = 0{,}5$



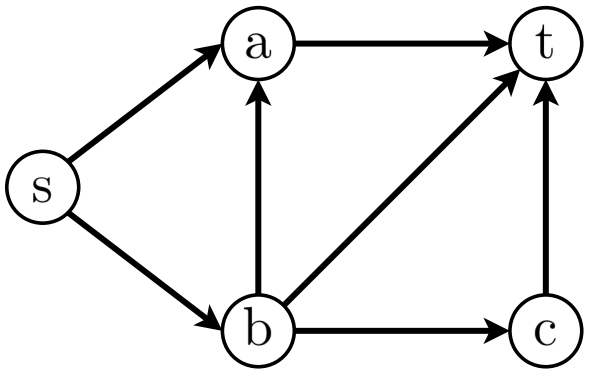


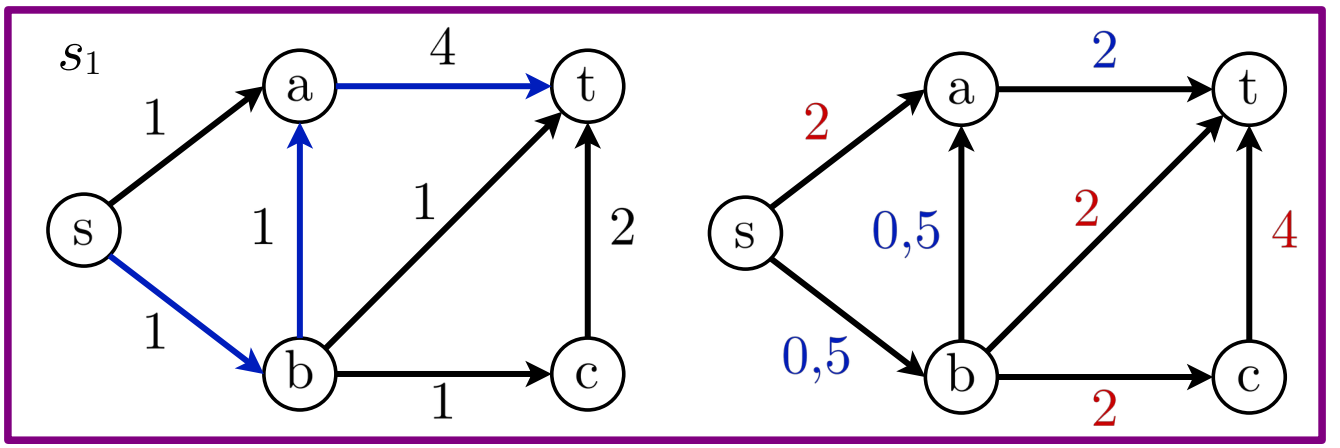




Rent-RR Shortest Path

heute morgen übermorgen



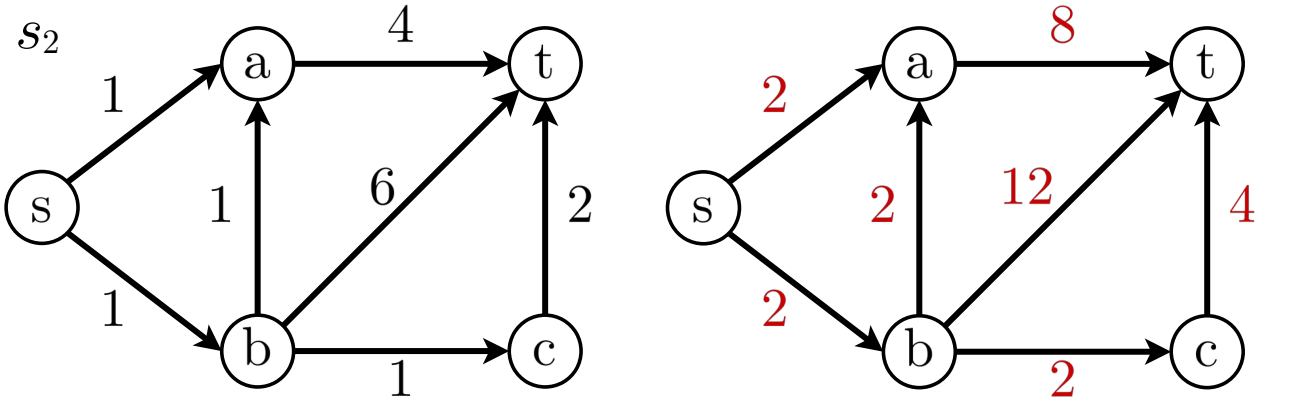


Planung

> Marktpreise unbekannt

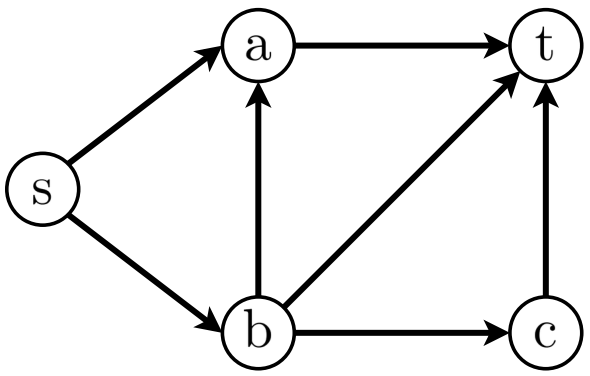
Beispiel: $\beta = 1, \alpha = 0{,}5$

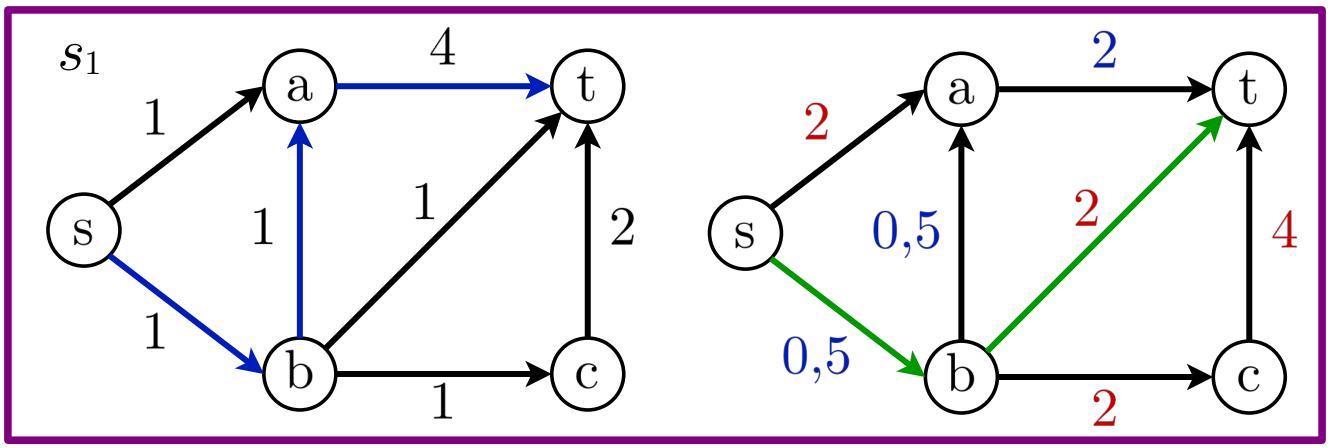
Miete sbat = 0,5+0,5+2



Rent-RR Shortest Path

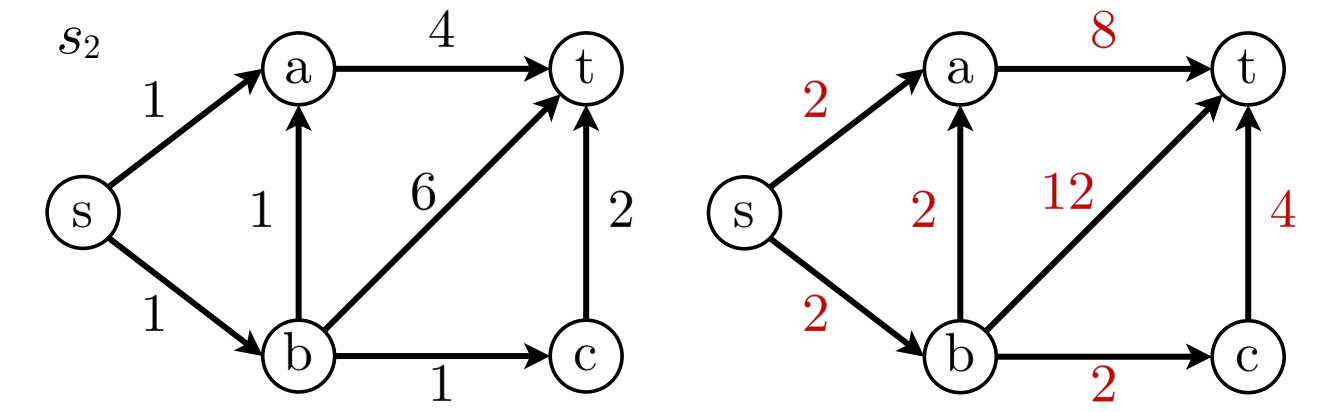
heute morgen übermorgen





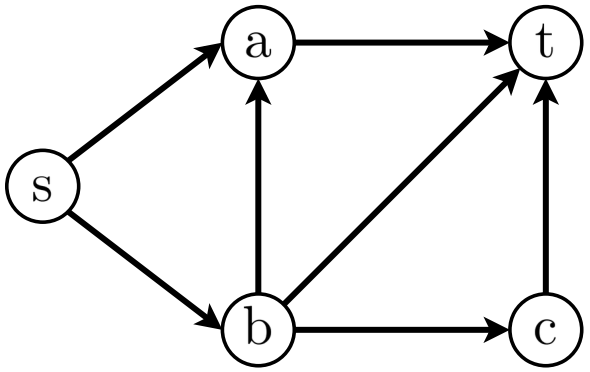
Planung
> Marktpreise unbekannt

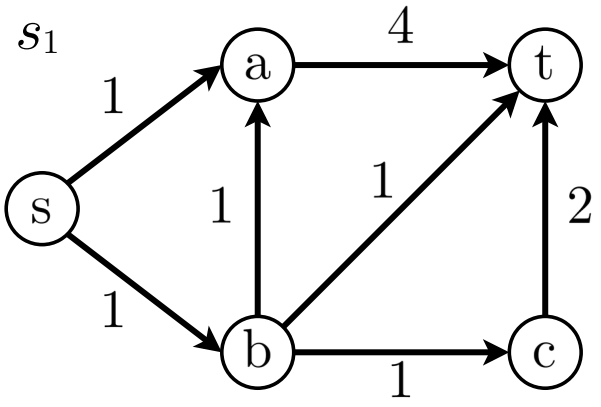
Beispiel: $\beta = 1, \alpha = 0{,}5$
Miete sbat = 0,5+0,5+2
Implementierung = 0,5+2
Gesamtkosten = 3+2,5

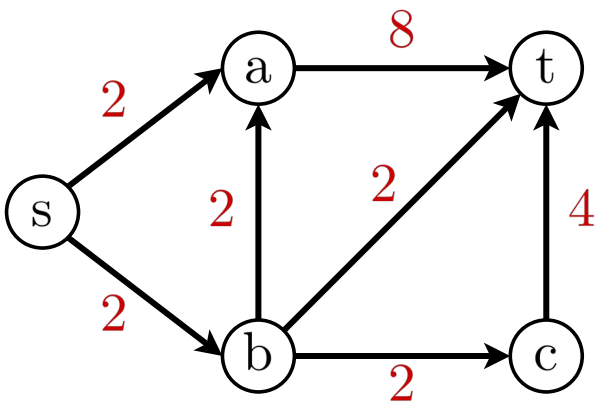


Rent-RR Shortest Path

heute — morgen — übermorgen

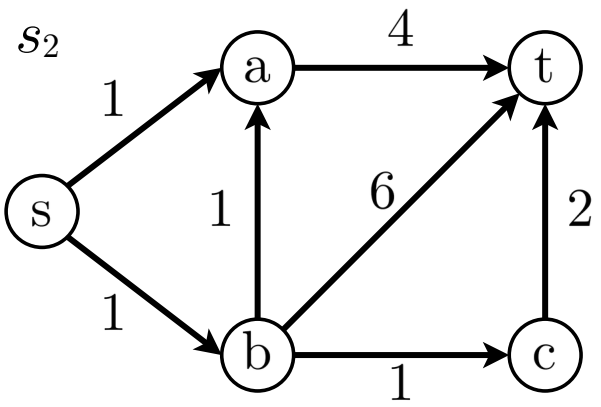


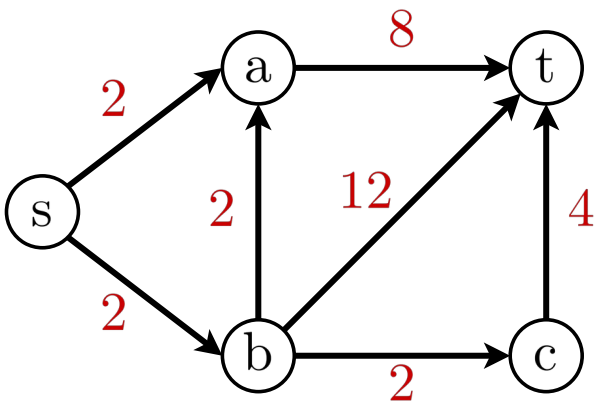




Planung

Welchen s-t-Pfad sollen wir morgen anmieten, um die Gesamtkosten des Einkaufs von übermorgen unter allen Szenarien zu minimieren?





Rent-RR Shortest Path

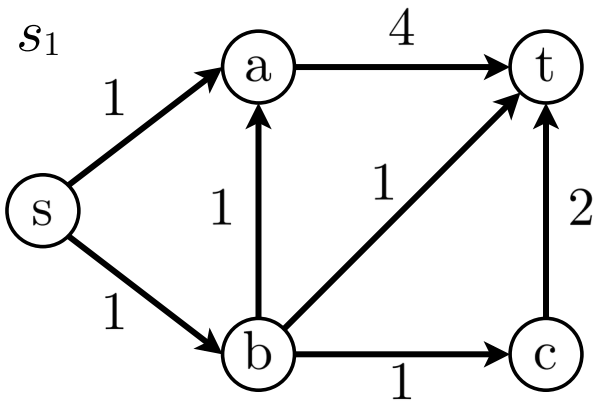
heute — morgen — übermorgen

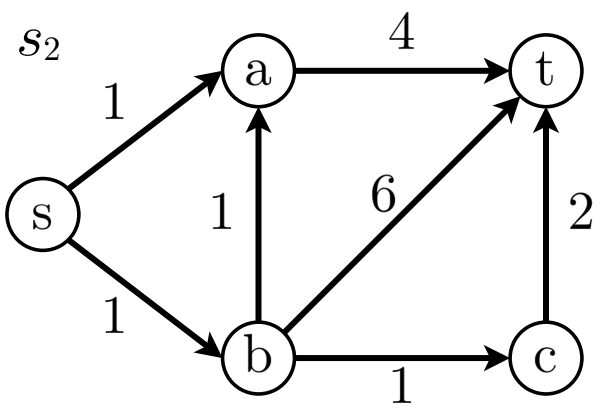
Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases}$$

	s_1	s_2
sat		
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





Rent-RR Shortest Path

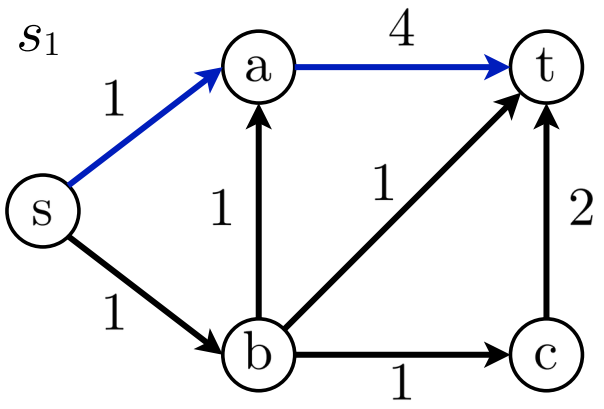
heute morgen übermorgen

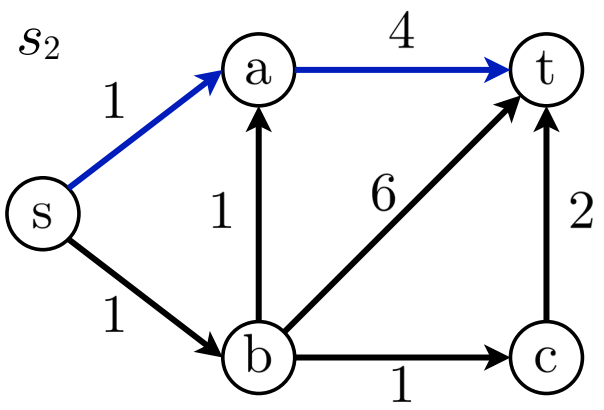
Gesamtkosten
= Miete + Implementierung

$\Pi = \{$ sat, sbat, sbt, sbct

	s_1	s_2
sat		
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





Rent-RR Shortest Path

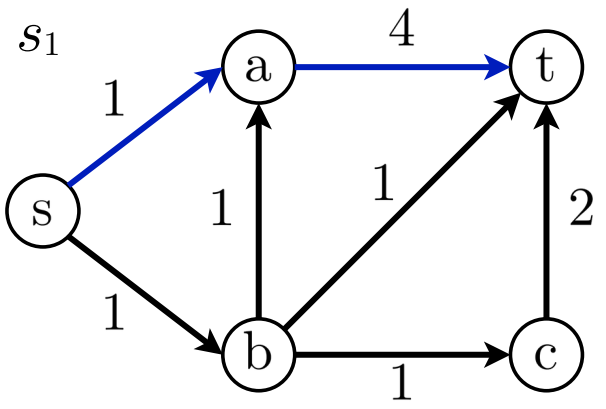
heute — morgen — übermorgen

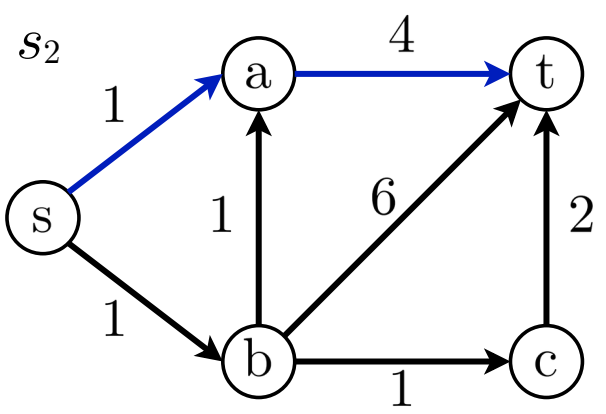
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

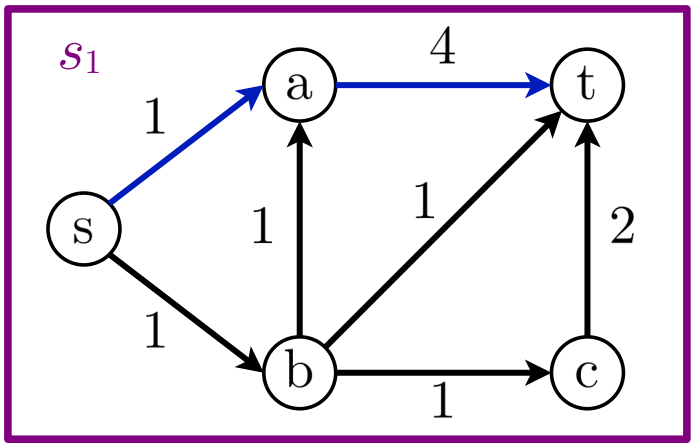




Rent-RR Shortest Path

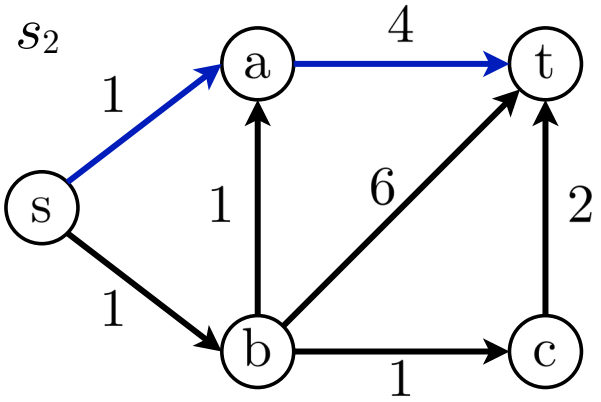
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

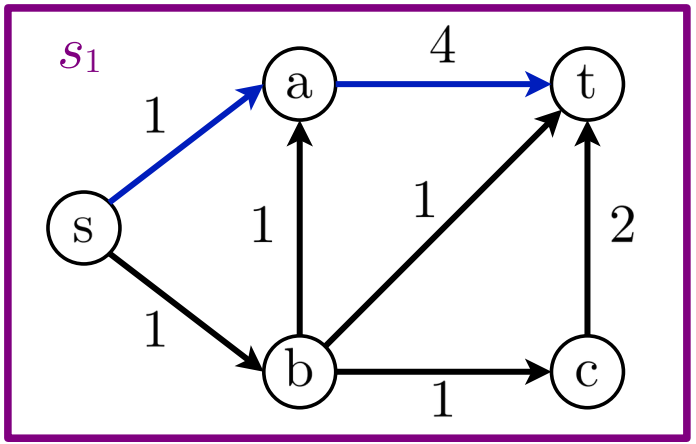


$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

heute — morgen — übermorgen

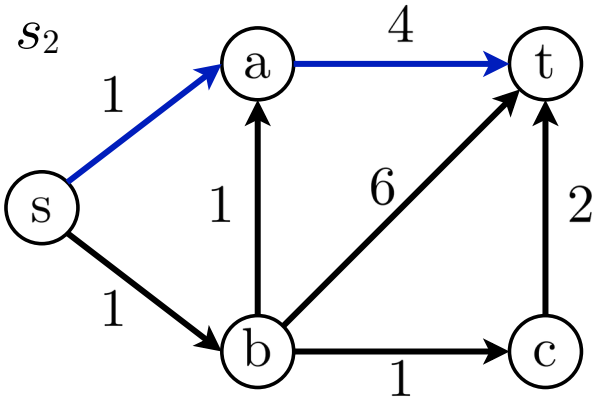
Gesamtkosten
= Miete + Implementierung



π'	Restpreis	Inflation	Σ
sat	0,5+2		
sbat			
sbt			
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

(s_1 | s_2)



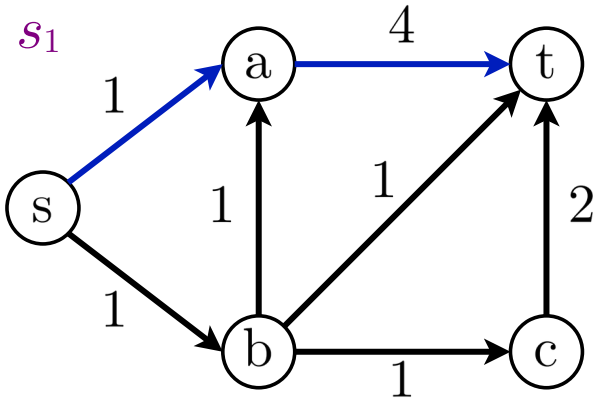
$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

heute — morgen — übermorgen

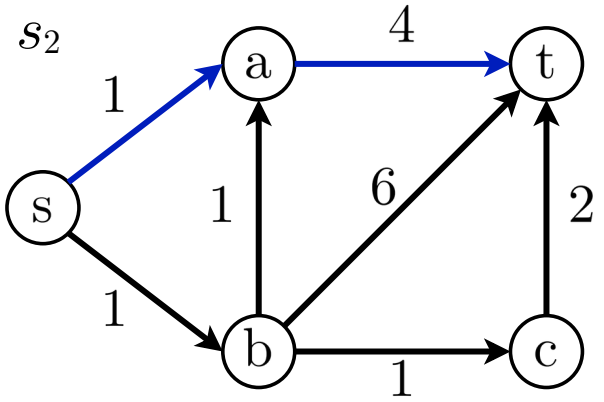
Gesamtkosten
= Miete + Implementierung

s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	
sbat			
sbt			
sbct			

s_2



$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

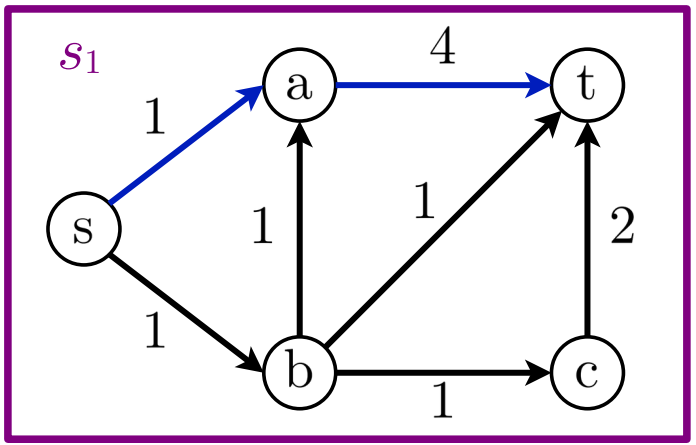
(s_1 | s_2)

$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

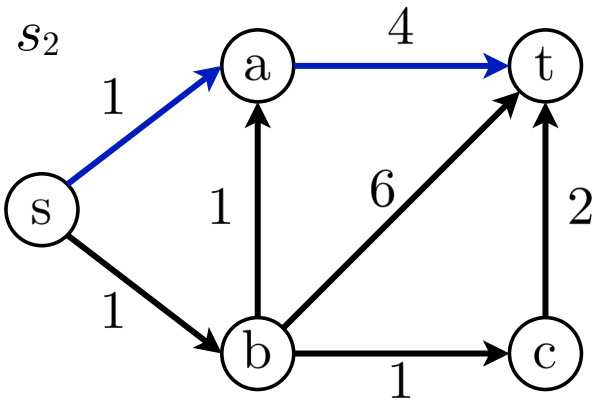


π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat			
sbt			
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

(s_1 | s_2)

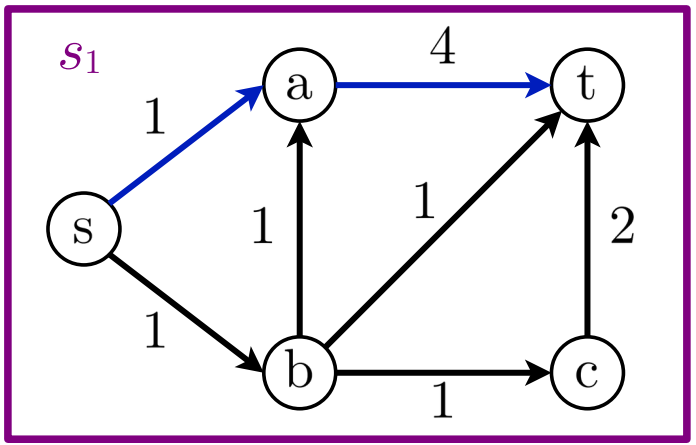
$\alpha = 0{,}5$
$\beta = 1$



Rent-RR Shortest Path

heute — morgen — übermorgen

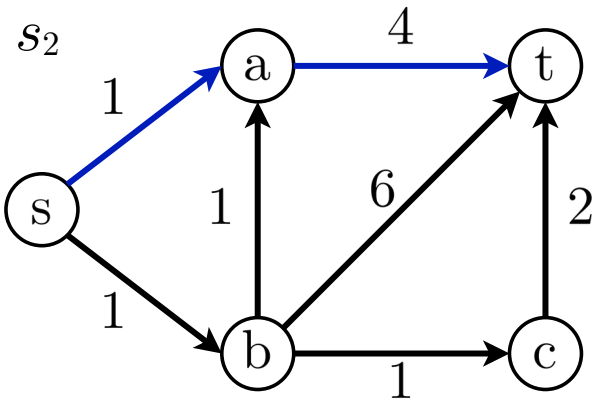
Gesamtkosten
= Miete + Implementierung



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2		
sbt			
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

(s_1 | s_2)

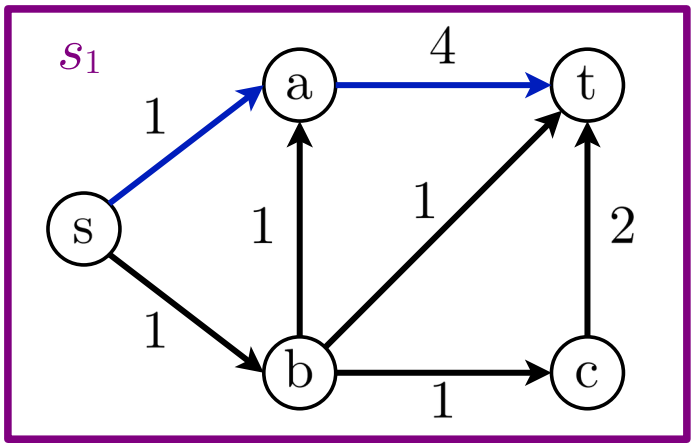


$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

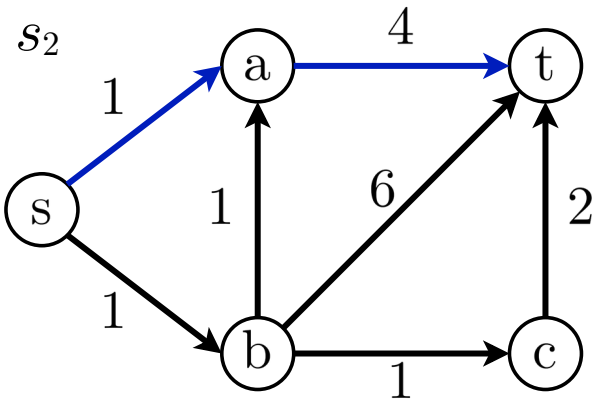
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	
sbt			
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$
(s_1, s_2)



$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

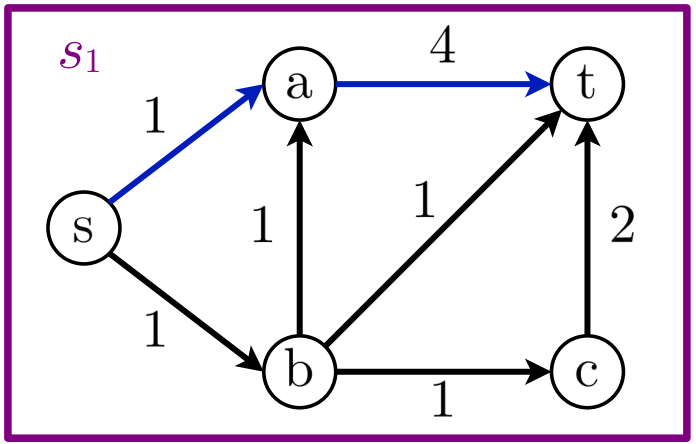
heute — morgen — übermorgen

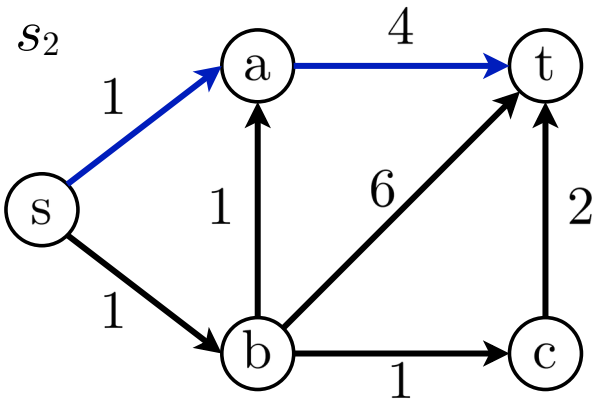
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





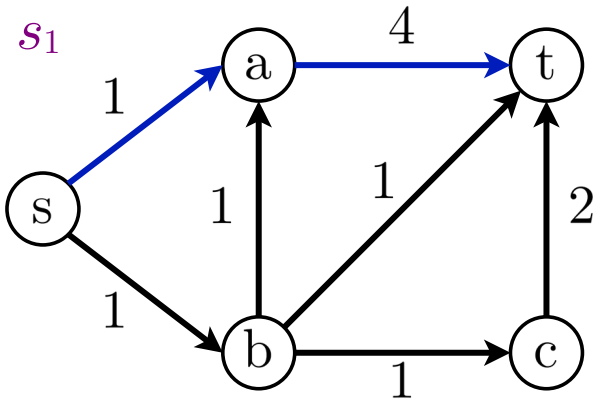
π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt			
sbct			

Rent-RR Shortest Path

heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

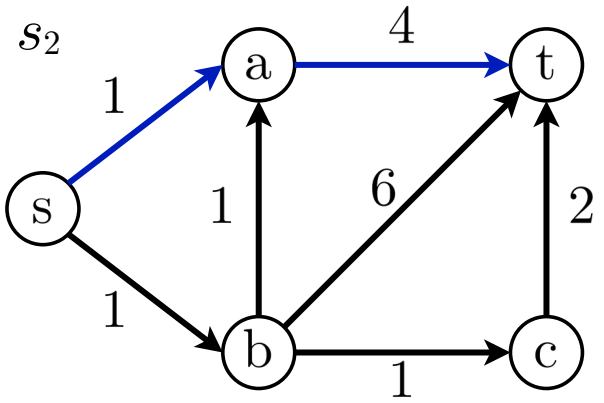
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0		
sbct			

$$\Pi = \begin{cases} \text{sat} & 2{,}5 & 2{,}5 \\ \text{sbat} & & \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$
(s_1, s_2)

s_2



$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

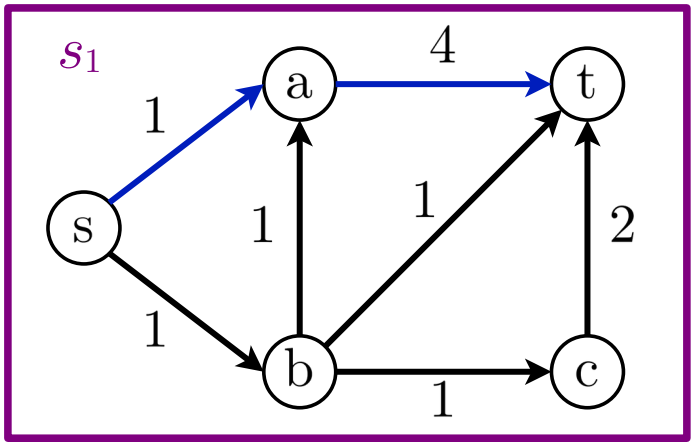
heute — morgen — übermorgen

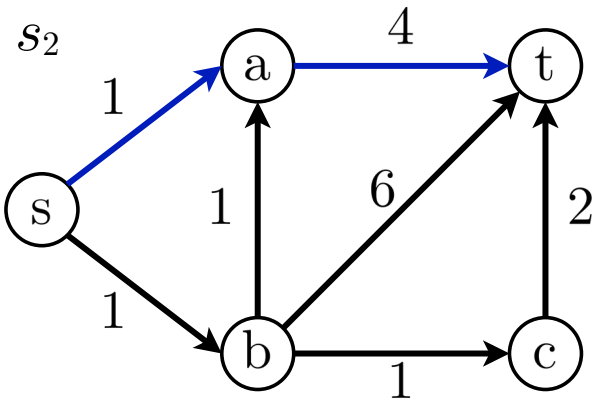
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct			

Rent-RR Shortest Path

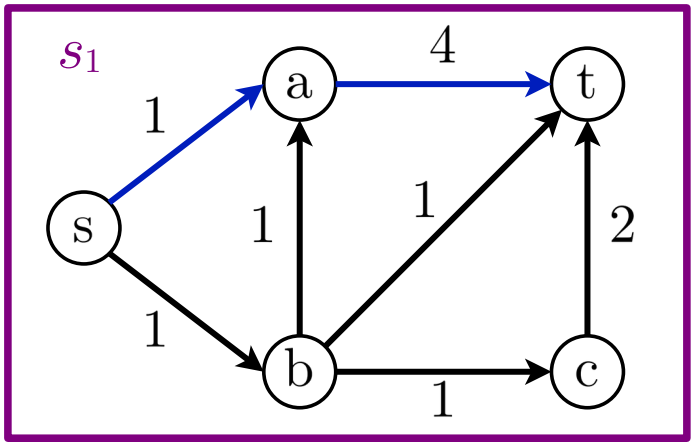
heute — morgen — übermorgen

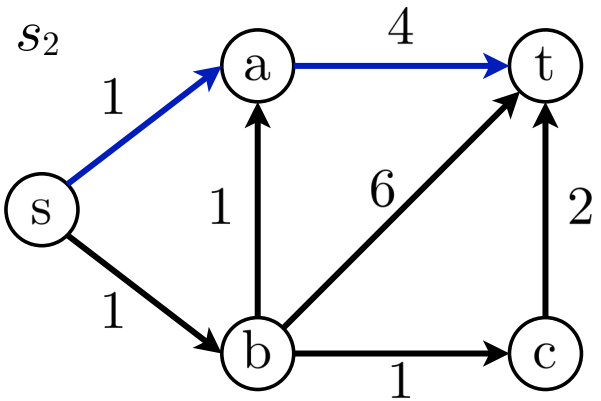
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Rent-RR Shortest Path

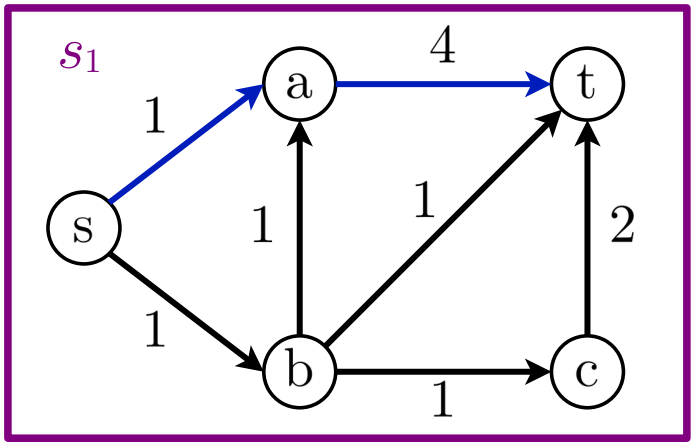
heute — morgen — übermorgen

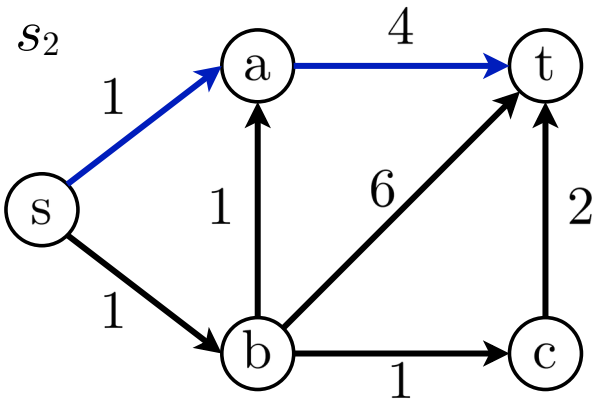
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung?

Rent-RR Shortest Path

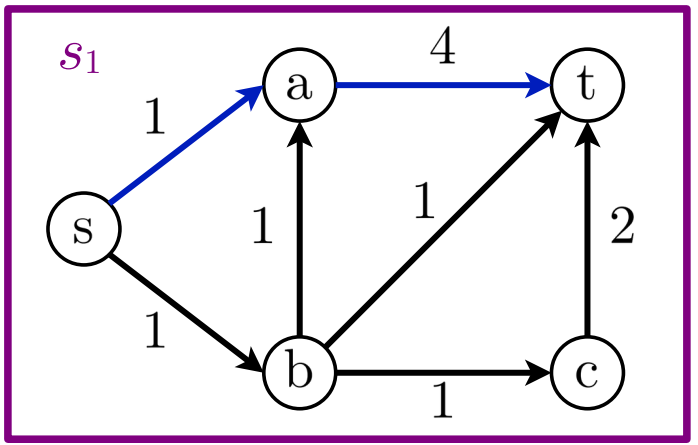
heute — morgen — übermorgen

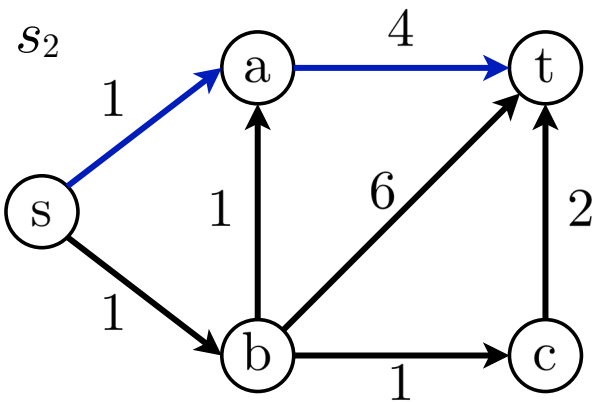
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	2,5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$





π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

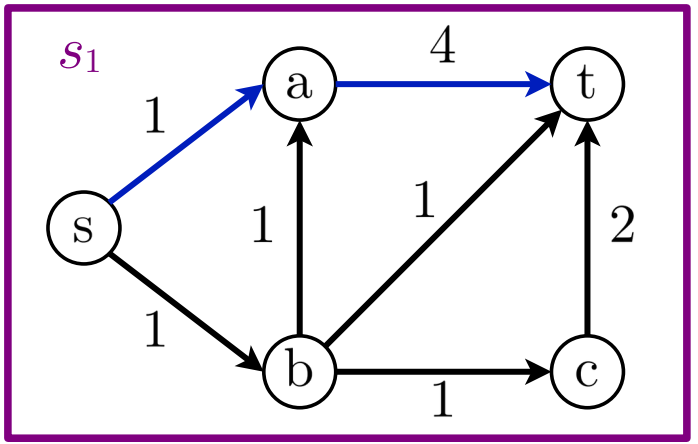
Rent-RR Shortest Path

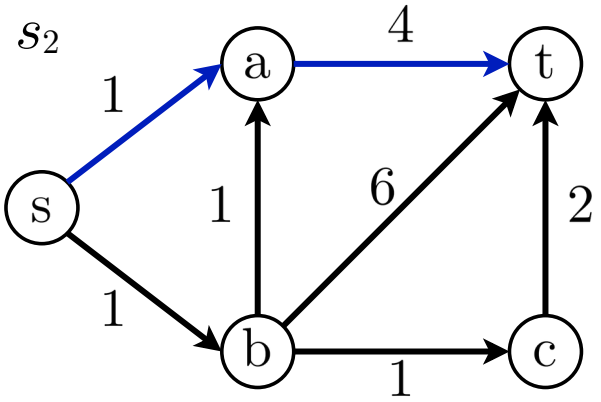
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

$\Pi = \left\{ \begin{array}{l|c|c} & s_1 & s_2 \\ \hline \text{sat} & 5 & 2{,}5 \\ \hline \text{sbat} & & \\ \hline \text{sbt} & & \\ \hline \text{sbct} & & \end{array} \right.$





π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

Rent-RR Shortest Path

heute — morgen — übermorgen

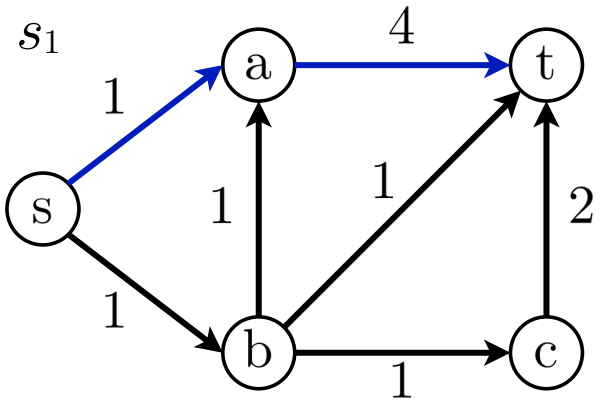
Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

$\Pi =$

	s_1	s_2
sat	5	2,5
sbat		
sbt		
sbct		

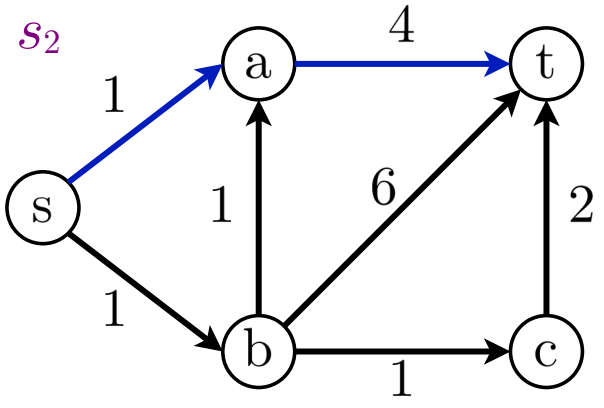
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

heute — morgen — übermorgen

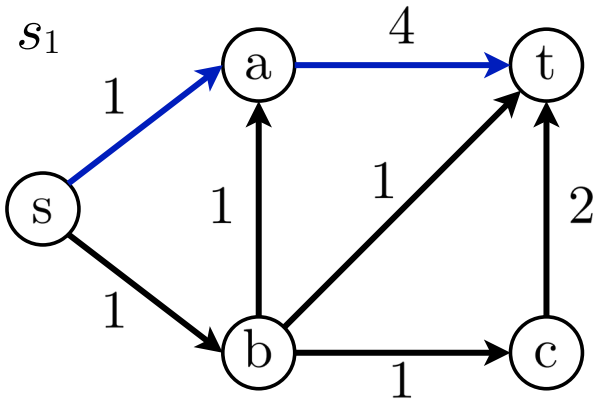
Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

$\Pi =$

	s_1	s_2
sat	5	2,5
sbat		
sbt		
sbct		

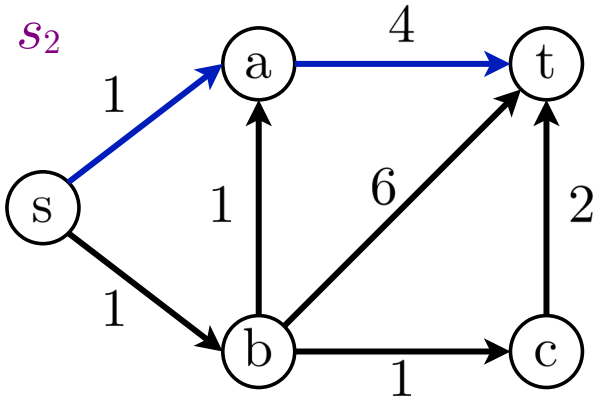
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt			
sbct	0	$4+\beta\cdot 4$	8

Implementierung:

Rent-RR Shortest Path

heute — morgen — übermorgen

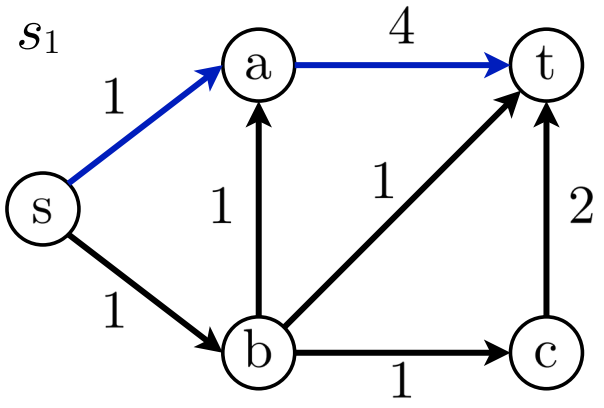
Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

$\Pi =$

	s_1	s_2
sat	5	2,5
sbat		
sbt		
sbct		

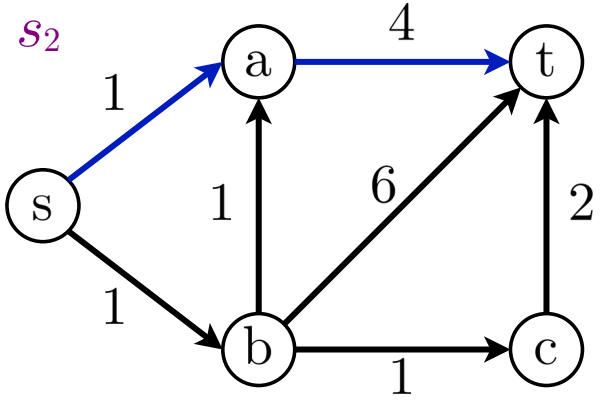
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0		
sbct	0	$4+\beta\cdot 4$	8

Implementierung:

Rent-RR Shortest Path

heute — morgen — übermorgen

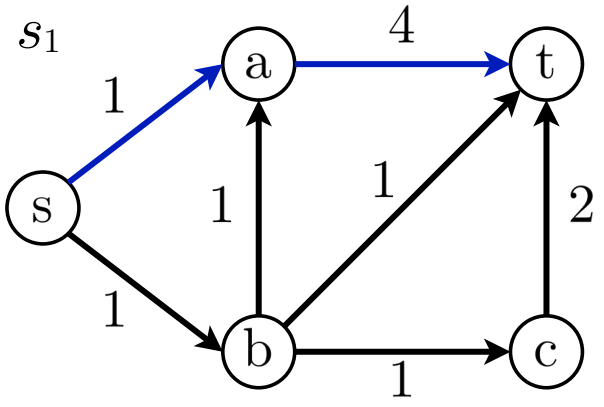
Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} \\ \text{sbat} \\ \text{sbt} \\ \text{sbct} \end{cases}$$

	s_1	s_2
sat	5	2,5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

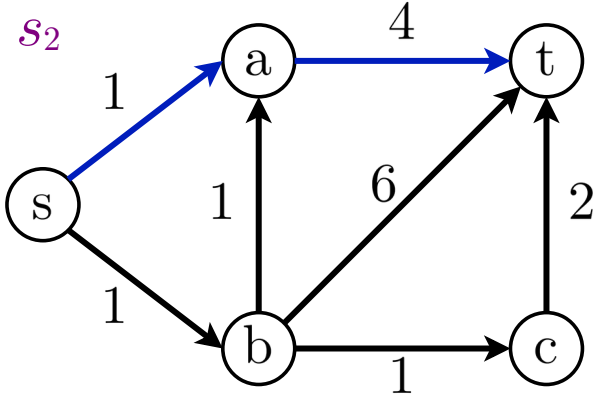
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$7+\beta\cdot 7$	14
sbct	0	$4+\beta\cdot 4$	8

Implementierung:

Rent-RR Shortest Path

heute — morgen — übermorgen

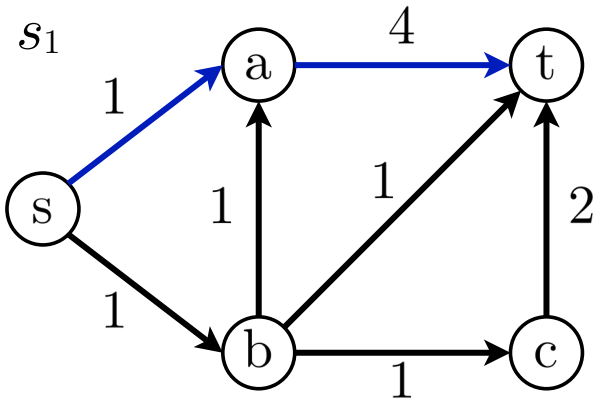
Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

$\Pi =$

	s_1	s_2
sat	5	2,5
sbat		
sbt		
sbct		

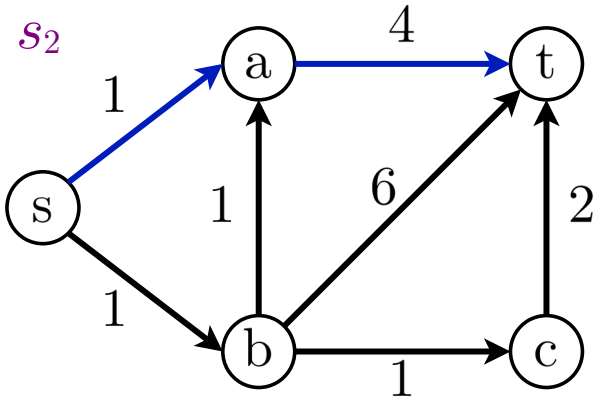
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$7+\beta\cdot 7$	14
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

Rent-RR Shortest Path

heute — morgen — übermorgen

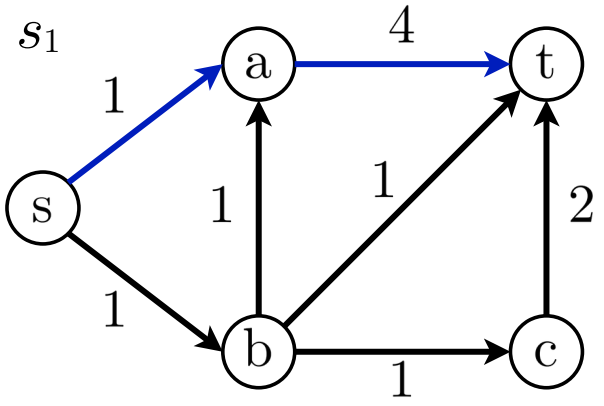
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

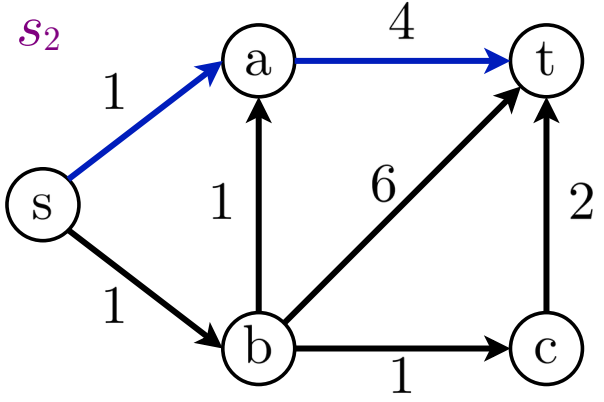
s_1



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$2+\beta\cdot 2$	4
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	0,5+2	0	2,5
sbat	2	$2+\beta\cdot 2$	6
sbt	0	$7+\beta\cdot 7$	14
sbct	0	$4+\beta\cdot 4$	8

Implementierung: 2,5

Rent-RR Shortest Path

heute — morgen — übermorgen

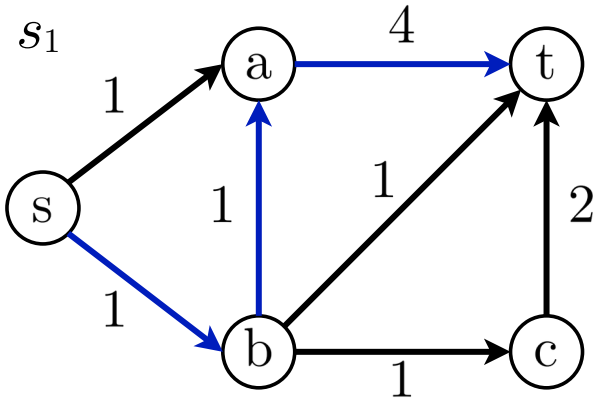
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat		
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

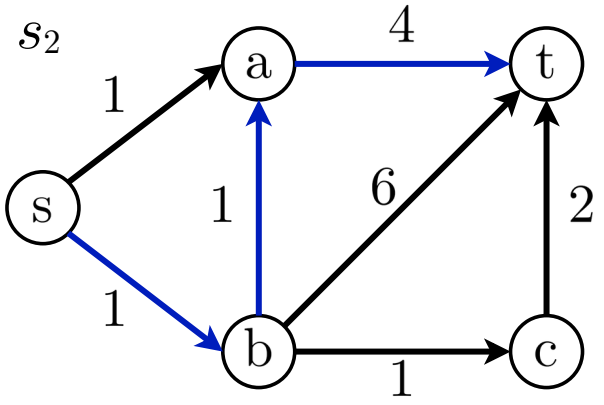
s_1



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

s_2



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

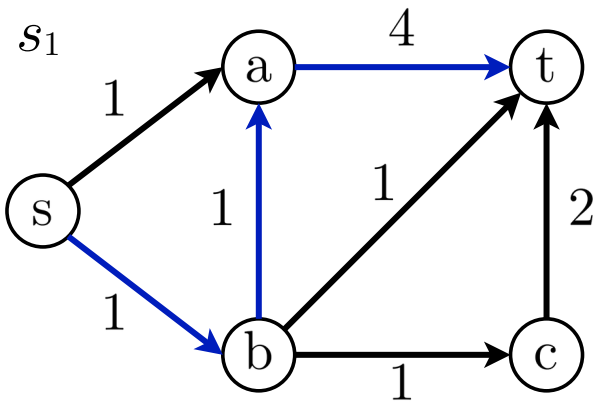
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 3 & 3 \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

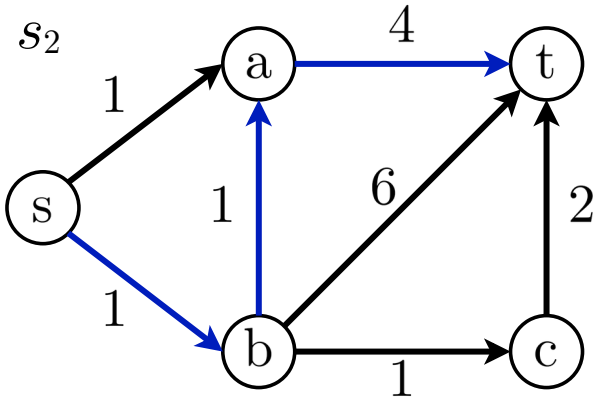
s_1 s_2

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

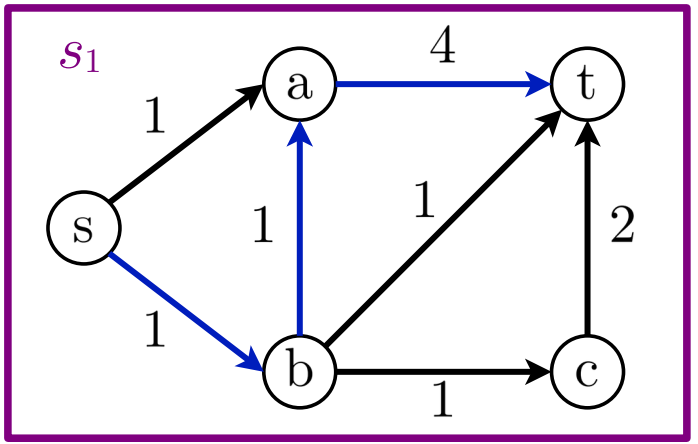
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$\Pi =$

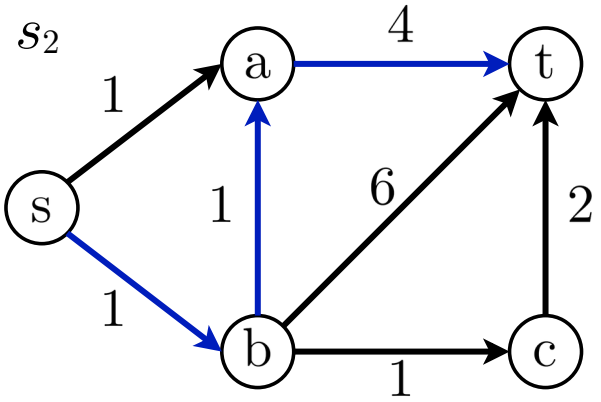
	s_1	s_2
sat	5	5
sbat	3	3
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 2,5



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

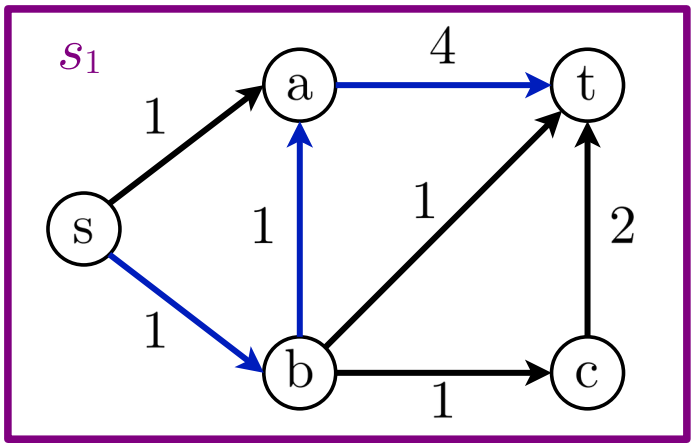
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$\alpha = 0{,}5$
$\beta = 1$

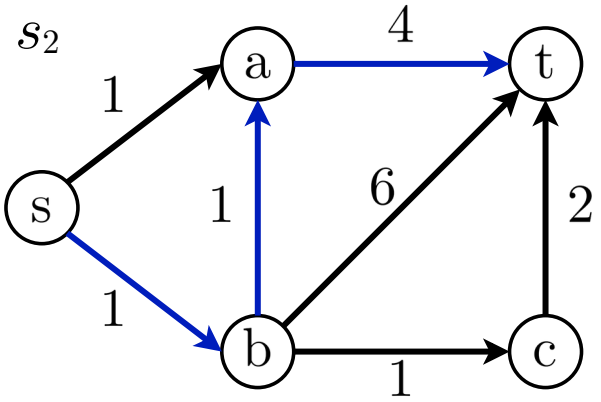
$\Pi =$

	s_1	s_2
sat	5	5
sbat	5,5	3
sbt		
sbct		



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 2,5



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

heute — morgen — übermorgen

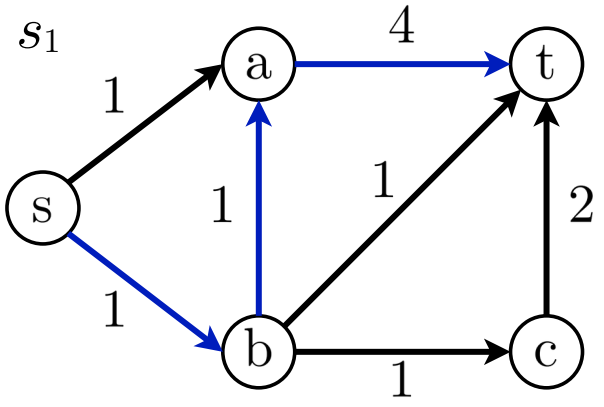
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat	5,5	3
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

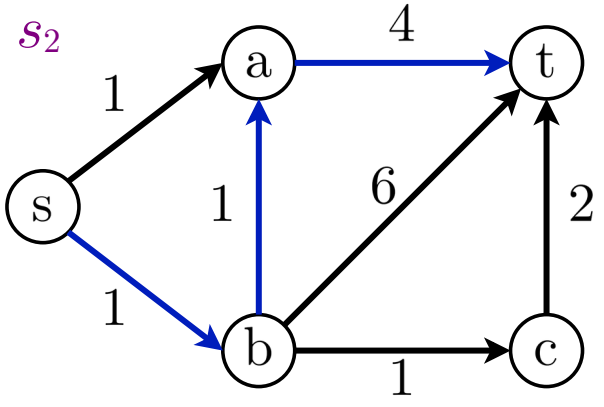
s_1



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$6+\beta\cdot 6$	12,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 3

Rent-RR Shortest Path

heute — morgen — übermorgen

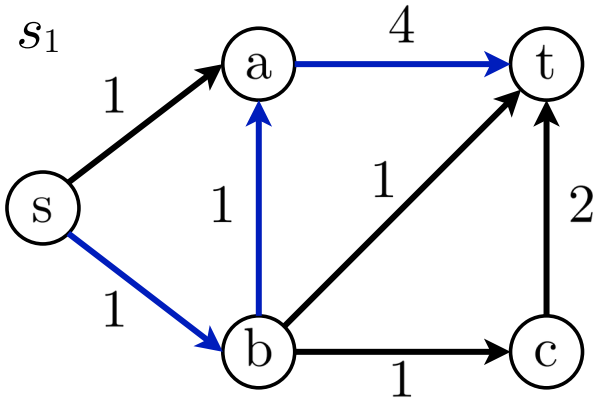
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat	5,5	6
sbt		
sbct		

$\alpha = 0{,}5$
$\beta = 1$

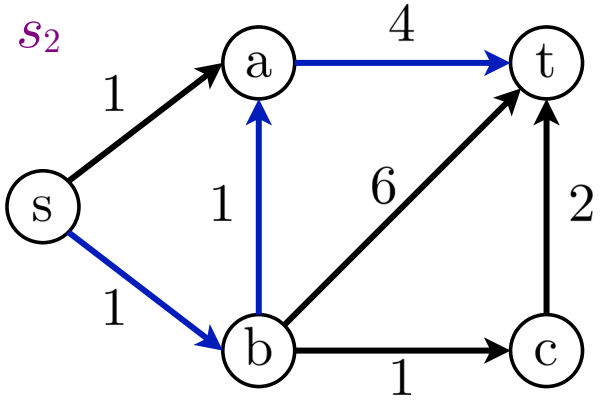
s_1



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 2,5

s_2



π'	Restpreis	Inflation	Σ
sat	2	$1+\beta\cdot 1$	4
sbat	0,5+0,5+2	0	3
sbt	0,5	$6+\beta\cdot 6$	12,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 3

Rent-RR Shortest Path

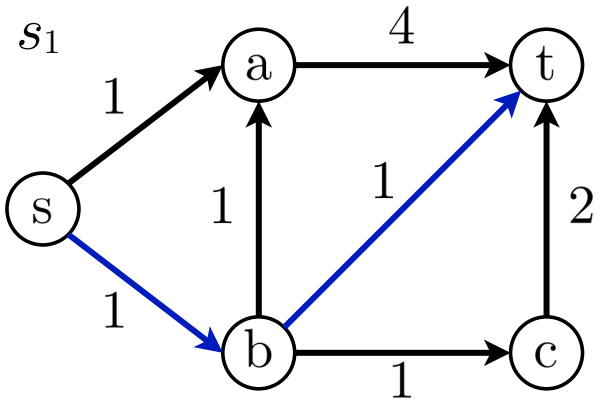
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & & \\ \text{sbct} & & \end{cases}$$

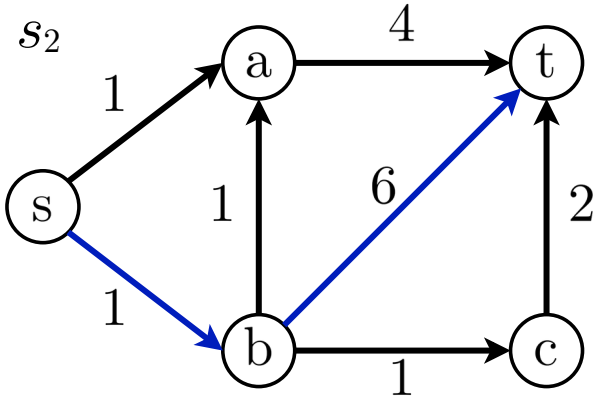
s_1 s_2

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

Rent-RR Shortest Path

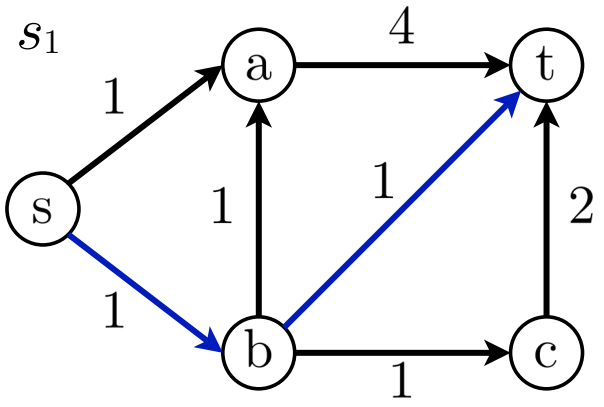
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & 1 & 3, \\ \text{sbct} & & 5 \end{cases}$$

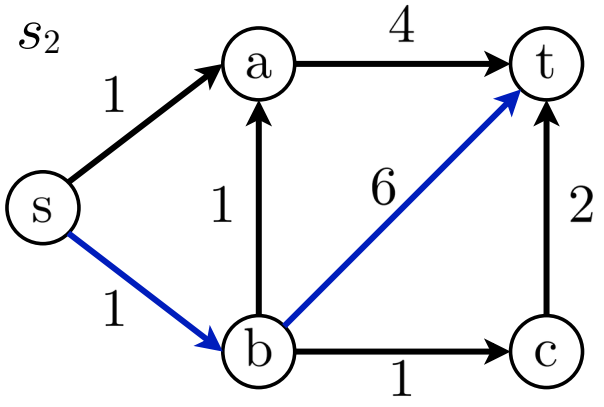
s_1 s_2

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5+0,5	0	1
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 1



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5+3	0	3,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 3,5

Rent-RR Shortest Path

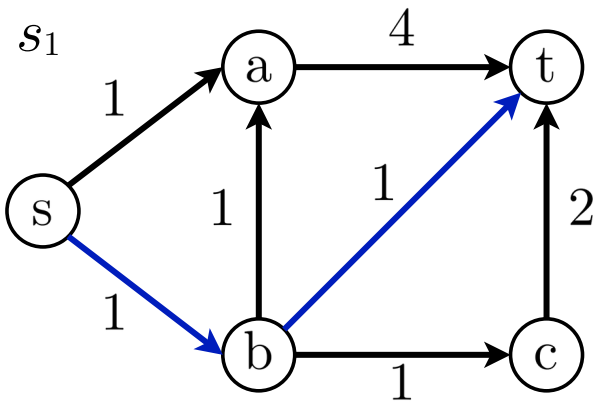
heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & 2 & 7 \\ \text{sbct} & & \end{cases}$$

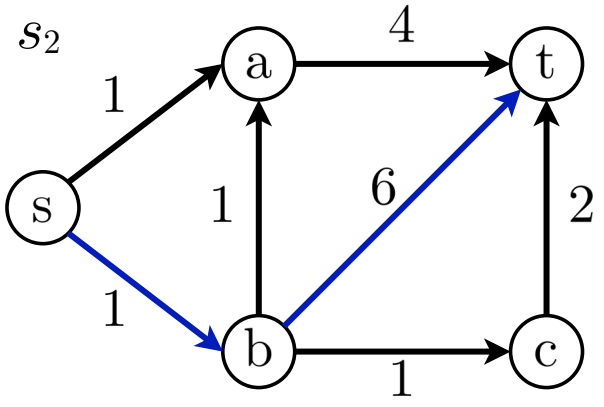
s_1 s_2

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5+0,5	0	1
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 1



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5+3	0	3,5
sbct	0,5	$3+\beta\cdot 3$	6,5

Implementierung: 3,5

Rent-RR Shortest Path

heute — morgen — übermorgen

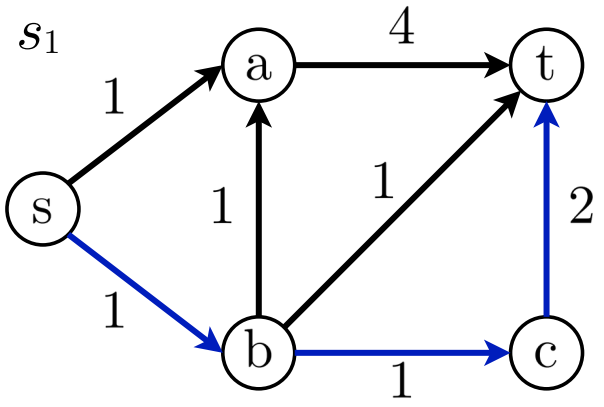
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat	5,5	6
sbt	2	7
sbct		

$\alpha = 0{,}5$
$\beta = 1$

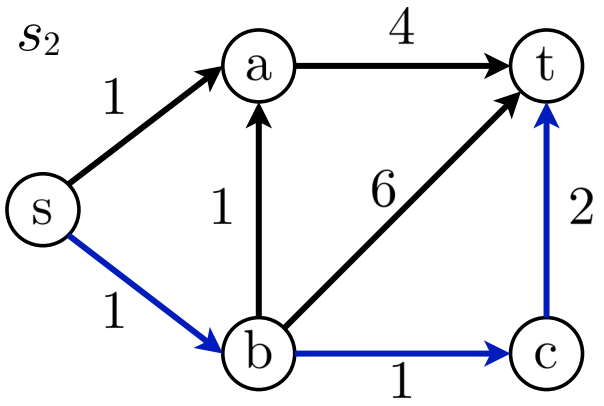
s_1



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

s_2



π'	Restpreis	Inflation	Σ
sat			
sbat			
sbt			
sbct			

Implementierung:

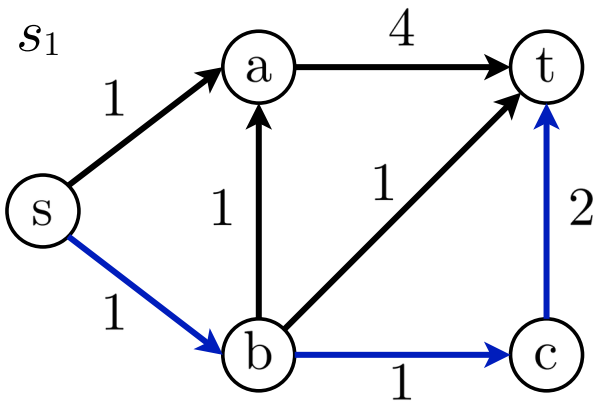
Rent-RR Shortest Path

heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung

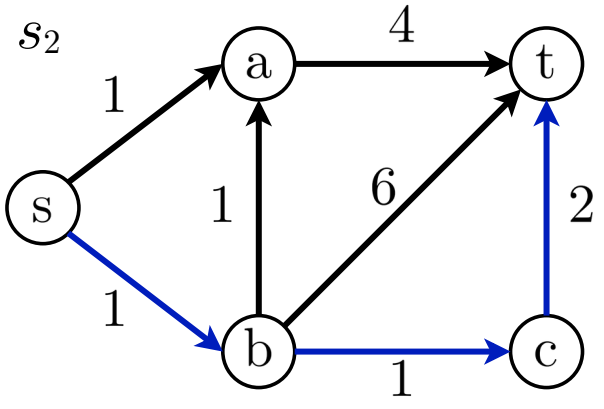
$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & 2 & 7 \\ \text{sbct} & 2 & 2 \end{cases} \qquad \begin{matrix} & s_1 & s_2 \end{matrix}$$

$\alpha = 0{,}5$
$\beta = 1$



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5+0,5+1	0	2

Implementierung: 2



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5	$6+\beta\cdot 6$	12,5
sbct	0,5+0,5+1	0	2

Implementierung: 2

Rent-RR Shortest Path

heute — morgen — übermorgen

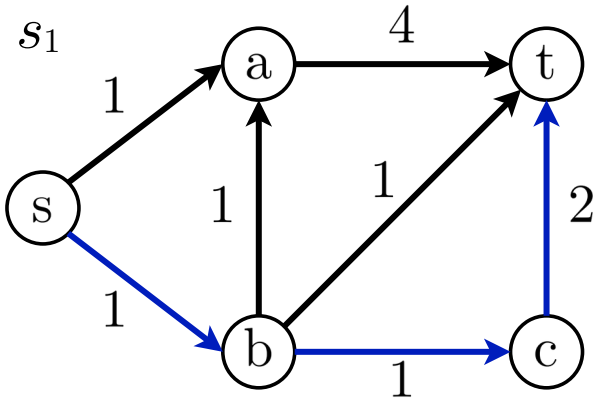
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2
sat	5	5
sbat	5,5	6
sbt	2	7
sbct	4	4

$\alpha = 0{,}5$
$\beta = 1$

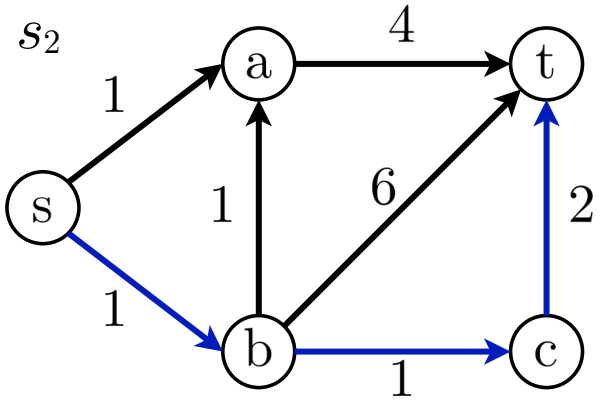
s_1



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5	$1+\beta\cdot 1$	2,5
sbct	0,5+0,5+1	0	2

Implementierung: 2

s_2



π'	Restpreis	Inflation	Σ
sat	0	$5+\beta\cdot 5$	10
sbat	0,5	$5+\beta\cdot 5$	10,5
sbt	0,5	$6+\beta\cdot 6$	12,5
sbct	0,5+0,5+1	0	2

Implementierung: 2

Rent-RR Shortest Path

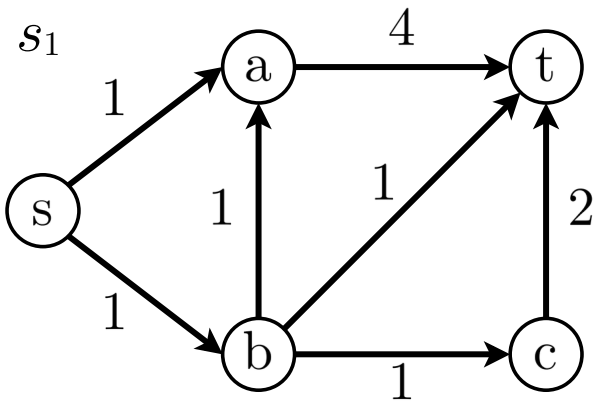
heute — morgen — übermorgen

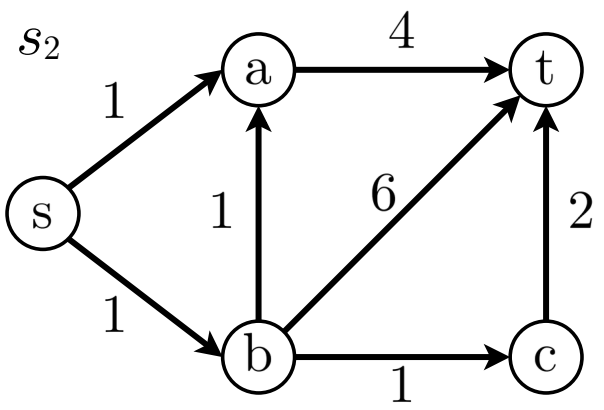
Gesamtkosten
= Miete + Implementierung

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & 2 & 7 \\ \text{sbct} & 4 & 4 \end{cases}$$

s_1 s_2

$\alpha = 0{,}5$
$\beta = 1$

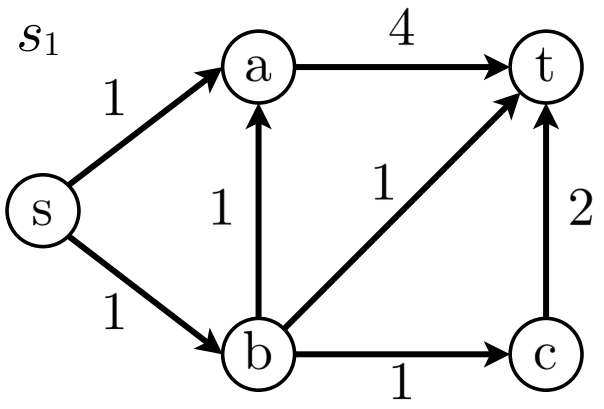


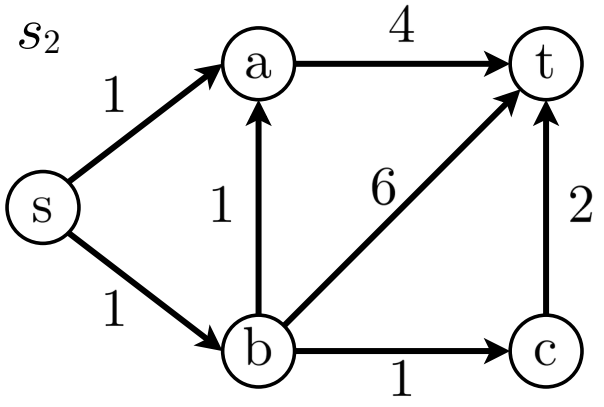


Rent-RR Shortest Path

heute — morgen — übermorgen

Gesamtkosten
= Miete + Implementierung





max →

$$\Pi = \begin{cases} \text{sat} & 5 & 5 \\ \text{sbat} & 5{,}5 & 6 \\ \text{sbt} & 2 & 7 \\ \text{sbct} & 4 & 4 \end{cases}$$

	s_1	s_2
sat	5	5
sbat	5,5	6
sbt	2	7
sbct	4	4

$\alpha = 0{,}5$
$\beta = 1$

Rent-RR Shortest Path

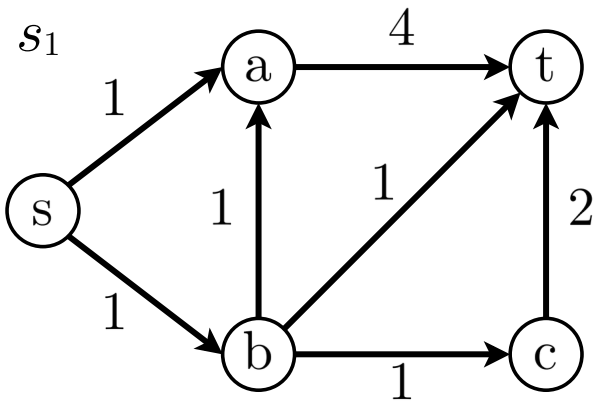
heute — morgen — übermorgen

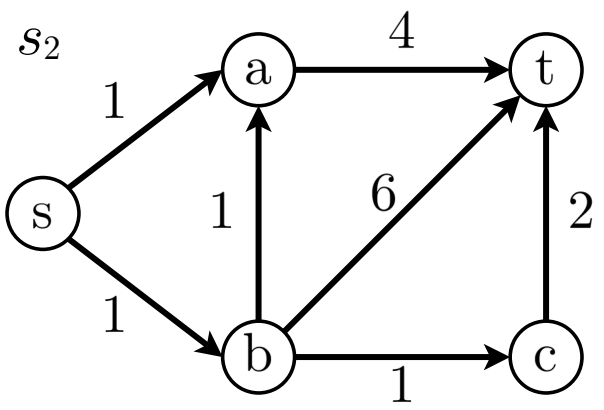
Gesamtkosten
= Miete + Implementierung

$\Pi =$

	s_1	s_2	
		max →	4
sat	5	5	↑ min
sbat	5,5	6	
sbt	2	7	
sbct	4	4	

$\alpha = 0{,}5$
$\beta = 1$





Rent-RR Shortest Path

Definition:
Sei $G{=}(V, E)$ ein gerichteter Graph mit Startknoten s und Zielknoten t, S die Menge an möglichen Szenarien, wobei jedes Szenario $s \in S$ eine Kostenfunktion $r^s : E \to \mathbb{N}$ definiert, Π die Menge aller s-t-Pfade π in G, $\alpha \in]\,0, 1\,[$ ein „rental“ Faktor und $\beta \geq 0$ eine Inflationsrate.

Das Rent-RR Shortest Path Problem besteht darin, einen Pfad $\pi^* \in \Pi$ zu finden, der die Gesamtkosten minimiert:

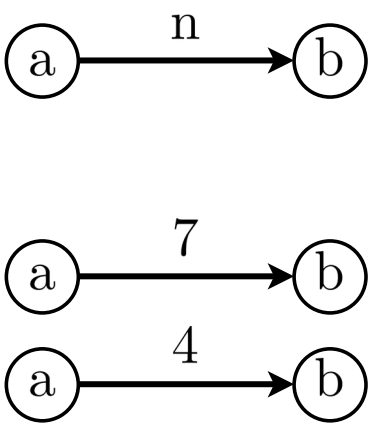
$$\min_{\pi \in \Pi} \max_{s \in S} \left(r_R^s(\pi) + r_I^s(\pi) \right)$$

$$\text{mit } r_R^s(\pi) = \alpha \cdot r^s(\pi) \text{ und } r_I^s(\pi) = \min_{\pi' \in \Pi} \left((1-\alpha) r^s(\pi') + (\alpha + \beta) \sum_{e \in \pi' \setminus \pi} r^s(e) \right)$$

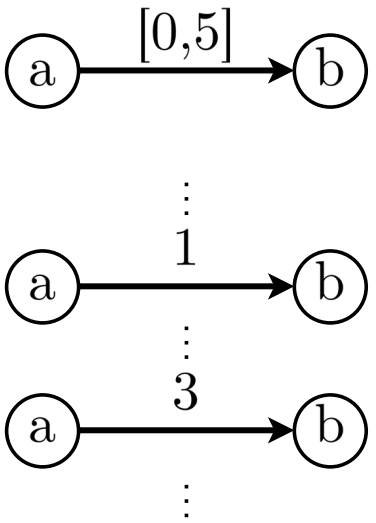
Komplexität

Szenario-Mengen

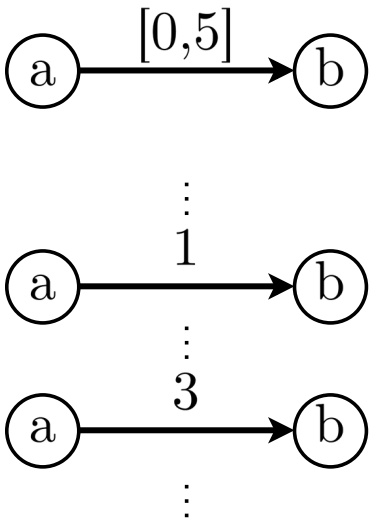
Diskrete Szenarien



Intervall Szenarien



Γ Szenarien



Höchstens Γ-viele Kanten dürfen Kosten von 0 auf 5 erhöhen

Komplexität

	Diskrete Szenarien	Intervall Szenarien	Γ Szenarien
k-dist RR	(sogar für 2 Szenarien) Strongly NP-hard	Strongly NP-hard (aber polynomiell für seriell-parallele Graphen)	Strongly NP-hard
Rent RR	Weakly NP-hard (gleiche Komplexität wie ARSP)	Strongly NP-hard (gleiche Komplexität wie ARSP)	Strongly NP-hard (anders als bei ARSP)

Quellen

Büsing, C. (2011). *Recoverable Robust Shortest Path Problems*. Wiley Periodicals, Inc.

Büsing, C. (2011). *Recoverable Robustness in Combinatorial Optimization*. Cuvillier.